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CHARGE AND SPIN TRANSPORT IN 2D LIGHT-MATTER COUPLING SYSTEMS

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Abstract

In this thesis, the candidate studies the effects of externally applied fields on spin and charge transport in two dimensional systems strongly coupled to light. On one hand, the influence of a strong monochromatic laser field on charge conductivity and spin relaxation in a two dimensional electron gas and graphene is investigated. On the other hand, the spin dynamics of microcavity exciton-polaritons is simulated under a magnetic field applied normally to the cavity plane and spin-orbit interaction as well. In both cases, the externally applied fields are shown to substantially change the transport properties.

For the work on the two dimensional electron gas and graphene, standard approaches such as the Boltzmann transport equation and Kubo formalism were employed. The polariton spin polarization patterns were calculated using the previously developed pseudospin formalism and a coherent Schrödinger type equation accounting for pump and decay.

*To my parents,
Violeta and Ferhat Morina.*

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1. Introduction

1.1. 2D electron gas systems

Particular carriers of net charge, no matter the medium where they are situated, tend to move from a higher electric potential to a lower one and the itinerant particles constitute an electric current. Electric charge flows through solid-state devices such as transistors which are the building blocks of the modern computer structures. The general study of charge and spin transport in solids is not young and has brought us a wealth of scientific breakthrough and application in the last two centuries. Much of what we understand about the electronic structure of solids is deduced from observing their crystallographic symmetry groups and from analyzing the orbitals of the constituent atoms. A remarkable achievement of solid-state physics is band theory with the fact that electrons in solids form energy bands of quantum states which they occupy. Based on the structure and landscape of these bands, one can classify solids into metals, semiconductors and insulators and is able to quantify the charge carrier dynamics in solids [1,2]. The band structure of semiconductors at low excitation energies, i.e. close to the ground state, can be obtained by $\mathbf{k} \cdot \mathbf{p}$ perturbation theory [3]. To provide an example, for a semiconductor with zincblende structure like galli-

um arsenide (GaAs), the band structure has the prototypical profile shown on Fig. 1.1.

Figure 1.1: The bandstructure of a typical direct-gap semiconductor with a z-icblende crystal lattice. The s -type conduction band lies above the Fermi level while the valence band has a triple subband structure due to the mixing of three p -type orbital states.

In a pure semiconductor, usually called an intrinsic semiconductor, the Fermi level lies in-between the filled valence band and the empty conduction band. The valence band is split into three bands due to spin-orbit interaction; the two hole bands and the split-off band. The two hole bands containing the light holes (lh) and the heavy holes (hh) are degenerate at zero momentum (Γ -point). Since the valence band maximum and the conduction band minimum are both at the same momentum value, GaAs is said to be a direct-gap semiconductor. In such semiconductors, the orbital symmetry of the conduction band inherited from the constituent crystal atoms is s -type while

the three other bands are a superposition of the three p orbitals.

The occupation probability distribution of electrons, as of all other fermions, is the Fermi-Dirac distribution

$$f(E) = \frac{1}{e^{(E-\mu)/k_B T} + 1}, \quad (1.1)$$

where E is the energy of the electron, μ is the chemical potential, k_B is the Boltzmann constant and T is the temperature. In equilibrium at $T = 0$, all states up to the Fermi energy $\mu = E_F$ are filled while those lying above are empty. The distribution 1.1 is symmetric under spatial reflection and therefore no electric current is able to flow as there is an electron going in the opposite orientation for each electron propagating in any specific direction [4, 5]. For finite temperatures $T \neq 0$ the region of finite occupation probability around E_F has a width of $2k_B T$ and in a semiconductor, where E_F lies in the forbidden region between the valence and conduction band, this region can overlap with the two bands. In this case, electrons can get thermally excited from the valence band to the conduction band where an externally applied electric field can accelerate them to create a current. The “holes” the electrons leave behind in the valence band occupancy, have net positive charges and are accelerated in the opposite direction and contribute to the current as well. In insulators, the band-gap is very large, i.e. $2k_B T \ll E_g$ for moderate temperatures and no overlap occurs between this region of finite occupancy and the conduction band. This hinders insulators from carrying a current. The dispersion of electrons and holes near the band edges is to a good approximation a parabolic one and the electrons and holes

Table 1.1: Comparison of electronic systems of different dimensions. See Fig. 1.2 for visualization.

Dimension	Designation	Condition
0	Quantum dot	$L_1, L_2, L_3 \leq \lambda$
1	Quantum wire	$L_1, L_2 \leq \lambda \ll L_3$
2	Quantum well	$L_3 \leq \lambda \ll L_1, L_2$
3	Bulk	$\lambda \ll L_1, L_2, L_3$

in these regions are considered as free albeit with a mass different from the vacuum electron rest mass although in general, this effective mass is a tensor quantity. For this reason, one speaks of a free electron gas or a free hole gas in solids. Before we proceed to the specific properties of two dimensional electronic systems, let us make a geometrical classification, which will be applied throughout this thesis, of systems of different dimensionality based on the de Broglie wavelength of electrons $\lambda = h/p$, where h is the Planck's constant and p is the electron momentum.

A bulk is a metal, insulator or semiconductor where electrons are unconfined and free to move in all the three spatial directions whereas the other extreme is the quantum dot in which the charge carriers are confined in a cell with lateral dimensions comparable or smaller than the electron wavelength. Then, a quantum wire and a quantum well are systems where electrons are only confined along one and two dimensions, respectively (see Table 1.1 and Fig. 1.2).

Figure 1.2: An electronic confinement medium can be classified on the basis of its lateral dimensions compared with the de Broglie wavelength, $\lambda = h/p$, of electrons.

Consider the quantum mechanical nature of the quantum well. Its energy spectrum is of mixed continuous and discrete nature. On one hand, the charge carriers are free to move along the plane of the quantum well with a parabolic energy dispersion. On the other hand, due to confinement in the normal direction, we have the emergence of discrete subbands, which are parabolic due to the aforementioned property. The confinement potential of a quantum well can be of different shapes, e.g. square, harmonic or triangular. For the observation of quantum effects, certain conditions have to be satisfied [6]. First of all, the well has to be narrow in order for the subband edge separation to be large enough. Quantitatively speaking, the thermal energy of the carriers must be much smaller than the energy separation of the subbands and the Fermi level should lie between them to effectively suppress transitions between adjacent levels. The energy broadening due to momentum scattering should also be much smaller than the energy separation between subbands. In this thesis, such quantum well systems are considered where the motion of electrons takes place only in the lowest subband and about which the term two-dimensional electron gas (2DEG) is used.

There are numerous ways in creating 2DEGs and some of them are semiconduc-

tor heterojunctions, thin films, and metal-oxide-semiconductor (MOS) structures to mention a few. These last are extensively applied in electronic components, for example in field effect transistors [7]. They are usually made of a silicon dioxide (SiO_2) layer sandwiched between a metal layer and a bulk semiconductor, e.g. silicon. By applying a bias voltage to the metal, an electron gas can be trapped along the oxide-semiconductor interface in a triangular potential well.

When an external electric DC field is applied along the plane of the quantum well, it provides the charge carriers with acceleration which is balanced out by scattering with impurities, phonons, defects etc.. In the Drude model, the conductivity, which is a proportionality constant between current and applied field, is given by

$$\sigma = \frac{ne^2}{m}\tau, \quad (1.2)$$

where n is the charge carrier concentration, e is the electron charge, m is the charge carrier effective mass and τ is the transport relaxation time [8]. It can be viewed as a mean time between collision with scatterers. Another way to express the conductivity is by using the definition of electron mobility

$$\mu_e = \frac{e\tau}{m} \quad (1.3)$$

for the case of electrons, while the hole mobility has opposite sign, along with the insertion of the appropriate effective mass.

Fermi's golden rule says that the probability $w_{\mathbf{k}\mathbf{k}'}$ that a particle scatters from a

state with a wavevector $\mathbf{k}$ to a state with wavevector $\mathbf{k}'$ per unit time per unit volume is given by

$$w_{\mathbf{k}\mathbf{k}'} = \frac{2\pi}{\hbar} |M(\mathbf{k}, \mathbf{k}')|^2 \delta(E_{\mathbf{k}} - E_{\mathbf{k}'}) \quad (1.4)$$

where $M(\mathbf{k}, \mathbf{k}') = \langle \mathbf{k} | \hat{V} | \mathbf{k}' \rangle$ is the matrix element of the operator $\hat{V}$ which acts as a perturbation on the free electron Hamiltonian, δ is the Dirac delta function, and $E_{\mathbf{k}}$ is the energy of the electron in the state $\mathbf{k}$. The motion of free electrons in quantum wells is perturbed by four important scattering processes whose contribution to the electron mobility can be quantified by calculating the corresponding matrix element $M(\mathbf{k}, \mathbf{k}')$. These processes are, namely, scattering by phonons, alloy scattering, surface and interface scattering, and ionized impurity scattering [9].

Scattering of electrons by longitudinal acoustic and optical phonons is the major scattering process at room temperatures. Momentum transfer by acoustic phonons is nearly elastic where these lattice vibrations interact with electrons by shifting the electron energy through oscillating strain. This is called deformation potential scattering. Interaction of electrons with optical phonons takes place when the unit cell contains more than two atoms with opposite effective electric charges. Out-of-phase movement of these atoms sets up an oscillating electric field which scatters the carriers. However, momentum relaxation of electrons by phonons becomes negligible in temperatures close to zero where lattice vibrations don't occur. In semiconductor alloy quantum wells, electrons may relax their momentum through scattering with a space-dependent random potential arising from sites where the local composition of the material is different than the global average. Such momentum scattering is isotropic, like scattering by acoustic phonons, and short-range, which means that the

scattering matrix element is effectively a constant.

As the confinement walls are not exactly flat for realizable quantum wells, electrons can undergo non-specular reflection when they bounce from surfaces of thin films or the interfaces in heterostructures. Such scattering decreases in intensity as the well width becomes larger. In heterostructures surface disorder is not important as modern growth techniques can provide high-quality quantum wells, i.e. with barriers which can be considered as flat. Thus, for high-quality samples at low temperatures, the scattering of carriers by charged impurities such as donor ions in MOS-structures (Coulomb scattering) is a crucial mechanism. An important parameter describing this is the distance of a localized scatterer from the two-dimensional layer containing the carriers. Scattering by impurities in the presence of an intense non-resonant laser field will be considered Chapter 2.

A simple approach in quantifying the movement of conduction charges in a medium is one based on the Boltzmann semi-classical equation for the electron distribution function $f(\mathbf{r}, \mathbf{p}, t)$ [2],

$$\frac{\partial f(\mathbf{r}, \mathbf{p}, t)}{\partial t} = -\dot{\mathbf{r}} \cdot \nabla_{\mathbf{r}} f(\mathbf{r}, \mathbf{p}, t) - \dot{\mathbf{p}} \cdot \nabla_{\mathbf{p}} f(\mathbf{r}, \mathbf{p}, t) + I[f(\mathbf{r}, \mathbf{p}, t)], \quad (1.5)$$

which describes its rate of change due to three following processes. The first term on the right-hand side of Eq. 1.5 stands for diffusion in and out of a small region around the real space coordinate $\mathbf{r}$ with $\nabla_{\mathbf{r}}$ being the standard spatial gradient. Hence, this term is proportional to the velocity of the electrons $\mathbf{v} = \dot{\mathbf{r}}$. The acceleration due

to external fields is accounted for in the second term of the right-hand side of the equation above. From Newton's second law coupled with the Lorentz force, we know that in the presence of external fields we have [10],

$$\dot{\mathbf{p}} = e \left(\mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{H} \right), \quad (1.6)$$

where $\mathbf{E}$ and $\mathbf{H}$ are externally applied electric and magnetic fields, respectively, and c is the speed of light. The reciprocal space gradient is given by $\nabla_{\mathbf{k}} = \partial/\partial\mathbf{k}$. The last term in the equation describes the scattering of electrons due to the various mechanisms discussed above. The law of microscopic reversibility says that the rate of scattering of charge carriers from a state with momentum $\mathbf{p} = \hbar\mathbf{k}$ to another state with momentum $\mathbf{p}' = \hbar\mathbf{k}'$ is the same as the reverse process, namely

$$w_{\mathbf{k}\mathbf{k}'} = w_{\mathbf{k}'\mathbf{k}}, \quad (1.7)$$

where $w_{\mathbf{k}\mathbf{k}'}$ is given by Eq. 1.4. This enables us to write

$$I[f(\mathbf{r}, \mathbf{k}, t)] = \int_{\text{1stBZ}} \{f(\mathbf{r}, \mathbf{k}', t) [1 - f(\mathbf{r}, \mathbf{k}, t)] - f(\mathbf{r}, \mathbf{k}, t) [1 - f(\mathbf{r}, \mathbf{k}', t)]\} w_{\mathbf{k}\mathbf{k}'} d\mathbf{k}' \quad (1.8)$$

where 1stBZ signifies that the integration is performed on the first Brillouin zone. This term is responsible for taking the system back to the steady-state when external fields are introduced whence the time derivative of the distribution function vanishes, i.e. $\partial f(\mathbf{r}, \mathbf{p}, t)/\partial t = 0$, by definition. The Drude formula, Eq. 1.2, can be obtained straight from the Boltzmann equation using the definition of the current density and that of the conductivity along with the introduction of a phenomenological relaxation

time.

Although using the Boltzmann equation one is able to predict many aspects of charge transport in solid-state systems, it doesn't allow for the explanation of phenomena which rely on quantum interference. One such is the phenomenon of weak localization which is a pure quantum effect [11]. Classically, the probability for an electron to return back to an original point in space along a closed path is a sum of the probability that it propagates, say, clockwise along this path while undergoing multiple scattering with impurities and the probability that it travels in the opposite direction, counter-clockwise, along the path. In quantum mechanics, however, when calculating the probability that an electron returns to the original point, one must sum the probability amplitudes and then take the modulus of the sum as the desired probability result. This gives rise to additional interference terms in the probability for the electrons to stay localized in one place, see Fig. 1.3.

Figure 1.3: An electron, through elastic scattering with impurities, returns to an initial point O after traveling in a closed path along which it can move in two opposite directions, A and B. The quantum probability that the electron takes this path is larger than the classical one due to constructive interference of electronic waves propagating along the two directions.

Since the kinetics of charge carriers are accounted for in a classical fashion in the Boltzmann transport equation, one cannot expect to predict weak localization corrections to the conductivity. To account for such effects, the Kubo formalism is usually applied where the non-equilibrium quantities such as conductivity are calculated via the Green's functions [12, 13].

1.2. Spin Physics in 2D systems

Spintronics is an extensive field within condensed matter physics with aims of harnessing electron and hole spin in solid-state systems for the development of nanodevices. First of all, electrons and holes carry spin which can have discrete values about some quantization axis and therefore can be considered as stored information. Secondly, they are mobile; conduction electrons, once injected at a specific point in space with a specific spin polarization can travel in the solid to another point in space. The distance carriers can travel while “remembering” their spin state is pretty much of fundamental importance for the design and functionality of spintronic devices. The mechanisms by which carriers can relax their spin will be discussed below. Finally, carrier spin, in the similar way by which it arises, can be detected. Information about the spin polarization of a carrier population can, for example, be obtained by measuring the polarization state of emitted light by electron-hole recombination [15–19].

It can be said that research and design of spintronic devices has been focused mainly on ferromagnetic materials where many important phenomena have been experimentally observed, such as giant magnetoresistance (GMR) for whose discovery the Nobel Prize in Physics was awarded in 2007 [20]. As an example, GMR is the effect

which allows the reading and writing of data on magnetic hard disks. Nevertheless, the field of research in non-magnetic semiconductors is extensive as well where the manipulation of spin transport in such media relies heavily on spin-orbit interaction and spin-dependent scattering [21].

The electron spin degree of freedom in 2DEG systems is easily accessed by external magnetic fields and a spin population can be generated by spin injection from a ferromagnet to a nonmagnet where the two are in interfacial contact [22]. Another effective way is to optically excite a semiconductor. The transition from the valence band to the conduction band of a semiconductor is proportional to the dipole moment between two bands which is a vector aligned in different directions with varying amplitudes depending on the three hole subbands and thus allows for selectively populating carrier states with different spin polarization by tuning the polarization of the optical pump [23].

Spin-orbit interaction is responsible for the fine structure in atoms. In an electron's reference frame, when it is orbiting around an atomic nucleus at relativistic speeds, the electric field of the nucleus is projected to a magnetic one through a Lorentz transformation of the electromagnetic field tensor [24]. This results in an added term to the Hamiltonian of the form,

$$H_{\text{SO}} = -\frac{\hbar}{4m_0^2c^2}\boldsymbol{\sigma} \cdot (\nabla V \times \mathbf{p}) \quad (1.9)$$

where m_0 is the electron rest mass, $\boldsymbol{\sigma}$ is a vector of Pauli spin matrices, V is the

nuclear electrostatic potential and $\mathbf{p}$ is the electron's momentum. This result can be obtained by solving the Dirac equation for the electron under the influence of any arbitrary spatially inhomogeneous electric potential [25].

In semiconductors, spin-orbit interaction is provided by the symmetry properties of the host lattice and the heterostructure system [26]. In the absence of any magnetic field, the energy spectrum of electrons $E_{\pm}(\mathbf{k})$ is both space inversion symmetric, i.e.

$$E_{\pm}(\mathbf{k}) = E_{\pm}(-\mathbf{k}), \quad (1.10)$$

and time inversion symmetric, i.e.

$$E_{\pm}(\mathbf{k}) = -E_{\mp}(-\mathbf{k}), \quad (1.11)$$

where $\pm$ stands for the two spin polarizations, resulting in

$$E_{+}(\mathbf{k}) = E_{-}(\mathbf{k}). \quad (1.12)$$

This spin degeneracy is lifted in semiconductor quantum well structures, however, due to the breaking of the inversion symmetry which occurs in non-centrosymmetric crystals such as GaAs. This kind of symmetry breaking is known as bulk inversion asymmetry (BIA). An effect of the static potential confining the electron gas along one dimension in a heterostructure system is to produce an effective field which further splits the spin polarizations in energy and is known as structural inversion asymmetry (SIA).

The energy splitting between the two spin polarizations is taken into account with an added term to the Hamiltonian of the form

$$H_{\text{SO}} = \frac{\hbar}{2} \boldsymbol{\Omega}_{\mathbf{k}} \cdot \boldsymbol{\sigma}, \quad (1.13)$$

where the spin-orbit field $\boldsymbol{\Omega}_{\mathbf{k}}$, which is linear in the wavevector $\mathbf{k}$, depends on the origin of the inversion symmetry breaking and $\boldsymbol{\sigma}$ is the Pauli spin matrix vector. From elementary quantum mechanics we know that in the presence of an externally applied magnetic field B_{ext} , say along axis z normal to a 2DEG, the spin degeneracy of an electron is lifted [24]. In particular, the projection of the electron spin angular momentum along this axis can be quantized and take the values either $-\hbar/2$ or $+\hbar/2$. If the external field is not too strong, the energy difference between these two states Δ_Z , termed Zeeman splitting, is given by

$$\Delta_Z = \mu_B \gamma_e B \quad (1.14)$$

where μ_B is the electron Bohr magneton given by

$$\mu_B = \frac{e\hbar}{2m_e} \quad (1.15)$$

and γ_e is the electron gyromagnetic ratio. The spin vector of the electron undergoes Larmor precession around the magnetic field. Since the added term to the Hamiltonian

due to the magnetic field is given by

$$\hat{H}_B = \gamma_e \mu_B \sigma_z B_z = \frac{\hbar}{2} \boldsymbol{\Omega}_B \cdot \boldsymbol{\sigma} \quad (1.16)$$

where $\boldsymbol{\Omega}_B = 2\gamma_e \mu_B \mathbf{B}/\hbar$. In the same way, the spin-orbit field $\boldsymbol{\Omega}_k$ in Eq. 1.13 causes the spin of carriers in a semiconductor quantum well to rotate.

Dresselhaus spin-orbit interaction takes place in semiconductors which have the zinc-blende crystal structure, e.g. GaAs, where two face-centered cubic sublattices of different species interpenetrate each other [1]. The spin-orbit interaction in such systems comes from the crystallographic inversion asymmetry in the bulk material (BIA). For example, in GaAs quantum wells grown along the [001] direction, the effective (linear-in-momentum) split-orbit coupling term,

$$\hat{H}_{SO}^{(D)} = \beta(\sigma_x k_x - \sigma_y k_y) \quad (1.17)$$

can be added to the Hamiltonian where β is constant depending on the average value of the squared momentum along the z-direction, $\langle p_z^2 \rangle$, and gives the strength of the spin-orbit interaction [27, 28].

Rashba spin-orbit interaction is more controllable, as it depends on the confining potential profile along the growth direction of heterostructures where asymmetry creates an average electric field [29]. The Rashba (or Rashba-Bychkov) term is given by

$$\hat{H}_{SO}^{(R)} = \alpha(\sigma_x k_y + \sigma_y k_x), \quad (1.18)$$

where the constant α depends on the material properties of the heterostructure. In semiconductor quantum wells, however, the Rashba constant is usually much larger than the Dresselhaus constant, i.e. $\alpha \gg \beta$, with values around $\alpha \approx 100 \text{ meV \AA}$ [30].

In a planar system like 2DEG, one can thus introduce in the same manner effective magnetic fields arising from the Dresselhaus or Rashba spin-orbit interactions. One sees that according to Eqs. 1.17 and 1.18 such effective magnetic fields lie on the plane of the 2DEG and are given by

$$\vec{B}_D(\mathbf{k}) = \frac{2\alpha}{g\mu_B}(\mathbf{k} \cdot \mathbf{z}) \quad (1.19)$$

for the Dresselhaus spin-orbit interaction and

$$\vec{B}_R(\mathbf{k}) = \frac{2\alpha}{g\mu_B}(\mathbf{k} \times \mathbf{z}), \quad (1.20)$$

for corresponding Rashba interaction where $\mathbf{z}$ is a unit vector pointing along the growth direction, normal to the 2DEG plane [26, 31]. Note that the Dresselhaus and Rashba fields point out of and along the QW plane, respectively. In the presence of a combination of discussed magnetic fields, the carrier spin precesses around the vectorial sum [32], given by

$$\vec{B}_{\text{tot}} = \vec{B}_{\text{ext}} + \vec{B}_D + \vec{B}_R. \quad (1.21)$$

Given that carriers are introduced to the quantum well by either photoexcitation or injection with definite spin, they can't retain their spin indefinitely; their spin is bound to change orientation by the influence of various electromagnetic fields and interactions. Neglecting nuclear hyperfine interaction between electrons and nuclear spins, the spin polarization of a induced carrier population in semiconductor heterostructures can relax to an equilibrium value through three important mechanisms which will be presented here [33]. A spin population can also decay through the loss of spin carrying particles, as in the recombination of electrons and holes.

The spin relaxation time τ_s can be roughly defined as a characteristic time during which a carrier population can keep its spin polarization. As a result of spin orbit coupling, the pure carrier spin states are not the eigenstates of the Hamiltonian. The pseudospin of the eigenstate is rather a mixture of the two spin states in vacuum with the superposition coefficients depending on the wavevector $\mathbf{k}$. Upon scattering with impurities or phonons, as $\mathbf{k}$ changes, so does the orientation of the pseudospin along with the probability that a carrier be either in the Pauli spin-up or spin-down state. This channel of spin relaxation is known as the Elliott-Yafet mechanism in whose regime the spin relaxation time is directly proportional to the momentum scattering time. The proportionality constant generally depends on the semiconductor parameters, scattering mechanism, SOI strength and the carrier energy [34].

In the D'yakonov-Perel' (DP) mechanism, the Rashba and Dresselhaus spin-orbit interactions come into play. This mechanism is the dominating one in non-centrosymmetric semiconductors such as GaAs. Due to the aforementioned effective magnetic fields,

$\vec{B}_R$ and $\vec{B}_D$, the spin vector of the carriers precesses around some axis which depends on the momentum of the carriers. If the period of a single precession is much larger than the mean time between scattering events, then the spin vector doesn't manage to make any significant rotation during the brief time window between successive collisions. Since the axis of precession fluctuates very rapidly and in a random manner, spin orientation is conserved leading to motional narrowing, where spin relaxation is counter-intuitively weaker as momentum scattering increases. The spin relaxation time in this scenario is inversely proportional to the momentum scattering time where the proportionality constant depends on the effective magnetic fields in the sample which in turn depend on the heterostructure geometry and sample composition.

Finally, an important mechanism where a spin population relaxes through the exchange interaction, is known as the Bir-Aronov-Pikus (BAP) spin relaxation [35]. This mechanism is particularly important in a p-type semiconductor where holes are the majority carriers. There, an electron coupled with a hole via the exchange interaction flips its spin when the hole flips its own resulting in spin relaxation. The spin relaxation time is inversely proportional to the disorder strength in this case as well which is provided by the hole population. To suppress the BAP mechanism, electrons and holes can be spatially separated [36]. Detailed derivation of the relaxation times for the mechanisms discussed above can be found in Ref. [37].

The ability to generate and detect spin currents is crucial for any spintronic application. To this effect, the spin-orbit interaction in semiconductors discussed above is often the underlying mechanism. A spin-unpolarized electric current flowing through

a slab of metal leads to the asymmetric accumulation of antiparallel spin along the edges, in effect generating a transverse spin current. This phenomenon, which doesn't require external magnetic fields, is called the spin Hall effect (SHE) and is divided into the extrinsic and intrinsic SHE depending on its microscopic origin. In the extrinsic SHE, which is the one originally predicted by D'yakonov and Perel' in their paper in 1971 and more recently by Hirsch in 1999 where he proposed the subsequent experimental work confirming its existence, the scattering of carriers by charged impurities or any external stray electric fields is spin-dependent [38,39]. The intrinsic SHE, however, doesn't rely on spin-dependent scattering but rather on the effective magnetic fields discussed above arising from Rashba or Dresselhaus spin-orbit interactions. The SHE has been observed in bulk semiconductors and two dimensional hole or electron systems by optically (Kerr rotation) measuring the spin density along the edges [40–43].

Figure 1.4: A current with no net spin-polarization perpendicular to the plane generates a transverse spin current by skew scattering (extrinsic) with impurities or via Rashba and Dresselhaus spin-orbit interaction (intrinsic).

By the same mechanisms, as Hirsch pointed out, a spin density current will create a transverse charge imbalance across a contact strip connecting the edges of an insulator containing electrons with antiparallel spin. This inverse SHE is then easily

detected by a voltmeter. It is remarkable that it has been detected up to room temperatures where quantum effects tend to be smeared out [44, 45].

1.3. Graphene and Dirac dispersion

Without a doubt, we can state that research in graphene is one of the most active fields in contemporary material science and nanotechnology. The search for improved technological efficiency has put this newly obtainable two-dimensional allotrope of carbon in the front seat. The production of graphene is a feat that has only been accomplished in 2003, with a scientific paper devoted to its discovery being published in the following year [46]. A layer of graphene is a one-atom thick sheet of carbon atoms arranged in a honeycomb lattice as shown in Fig. 1.5(a). This structure originates in the sp^2 hybridization of three orbitals of a C atom which, when bonding covalently with other C atoms via σ -bonds, lie on a plane, say the xy -plane. The fourth valence electron occupies the p_z orbital which is perpendicular to the plane and can form π -bonds with other p_z orbitals belonging to parallel graphene sheets via van der Waals interaction [47].

Figure 1.5: (a). The graphene grid is composed of two equivalent sublattices, whose carbon atoms are designated by A and B, respectively. (b). The Brillouin zone (BZ) of graphene (grey region) with its two inequivalent corners, K and K' , constructed in the usual way. Note that the only thick part of the border belongs to the BZ to avoid double counting of the wavevectors.

In a single layer of graphene the hexagon does not form a Bravais lattice. Let a be the distance between nearest neighbors in the lattice and imagine that we shift a single hexagon by a towards a nearest neighbor. Then, the shifted hexagon doesn't fall into the original lattice structure which means that translational invariance, which is what defines a Bravais lattice, is absent. Instead, one can look at the hexagon as two triangular cells interpenetrating each other and hence at the whole graphene layer as a superposition of two triangular Bravais sublattices. The vectors which span sublattice A in this case are given by

$$\mathbf{a}_1 = \sqrt{3}a\hat{\mathbf{x}}, \quad \mathbf{a}_2 = \frac{\sqrt{3}a}{2}(\hat{\mathbf{x}} + \sqrt{3}\hat{\mathbf{y}}), \quad (1.22)$$

where $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ are unit vectors in the x and y directions, respectively. Constructing

the reciprocal lattice in the usual manner, where the reciprocal lattice vectors are

$$\mathbf{b}_1 = \frac{2\pi}{\sqrt{3}a} \left(\hat{\mathbf{x}} - \frac{\hat{\mathbf{y}}}{\sqrt{3}} \right), \quad \mathbf{b}_2 = \frac{4\pi}{3a} \hat{\mathbf{y}}, \quad (1.23)$$

we arrive at the first Brillouin zone shown in Fig. 1.5(b). The points K and K' , called Dirac points, at the corners of the Brillouin zone are inequivalent in the sense that one is not a reciprocal lattice vector shift of the other while all the other corners are given by adding one of the vectors in Eq. 1.23 to either K or K' . All of the electron transport properties can be explained in a region close to these points and so they play a role similar to the Γ points in direct band-gap semiconductors.

To describe the transport properties of graphene, we need to find its band structure first and then the low-excitation spectrum. Since there are two atoms in a unit cell, the electron wavefunction is expressed as

$$\psi_{\mathbf{k}} = a_{\mathbf{k}}\psi_{\mathbf{k}}^{(A)} + b_{\mathbf{k}}\psi_{\mathbf{k}}^{(B)} \quad (1.24)$$

i.e. expanded into two parts belonging to the two sublattices, A and B, respectfully. The component of the wavefunction belonging to the A (B) sublattice can be viewed as a “spin-up” (“spin-down”) component and one thus speaks of sublattice pseudospin [48]. The time-independent Schrödinger equation,

$$H\psi_{\mathbf{k}} = \epsilon_{\mathbf{k}}\psi_{\mathbf{k}}, \quad (1.25)$$

is solved to find the electron energy $\epsilon_{\mathbf{k}}$ where the Hamiltonian H acts on the wave-

functions $\psi_{\mathbf{k}}^{(A)}$ and $\psi_{\mathbf{k}}^{(B)}$ independently.

As mentioned, there are three electrons for each atom occupying the s , p_x and the p_y states which form the covalent σ bonds with other electrons in these orbitals belonging to other atoms. It is also possible for these three electrons to form the anti-bonding σ^* orbitals, but this band has much higher energies (≈ 12 eV) than the σ band and so these three atomic states are not relevant in the discussion of electron transport properties [49]. It is the normal-to-the-plane atomic state p_z which upon forming the bonding π and the higher-lying antibonding π^* orbitals with neighboring p_z states contributes to electrons transport. For an electrically neutral graphene sheet, all the p_z (as well as the s , p_x and p_y) states are singly occupied so the π band is full while the π^* band is empty.

Writing out the Schrödinger equation in its 2×2 matrix form using Eq. 1.24, we obtain the valence (with index $\lambda = -1$) and the conduction ($\lambda = 1$) band of graphene in the tight-binding approximation. The energy dispersion, plotted in Fig. 1.6, is given by

$$\epsilon_{\mathbf{k}}^{\lambda} = t'_{NNN} \left[3 + 2 \sum_{i=1}^3 \cos(\mathbf{k} \cdot \mathbf{a}_i) \right] + \lambda t \left[3 + 2 \sum_{i=1}^3 \cos(\mathbf{k} \cdot \mathbf{a}_i) \right]^{\frac{1}{2}} \quad (1.26)$$

where t is the nearest neighbor hopping parameter

$$t = \int d^2r \phi^{(A)*}(\mathbf{r}) \Delta V \phi^{(B)}(\mathbf{r} - a\hat{\mathbf{y}}) \quad (1.27)$$

with $\phi^{(j)}$ being the atomic wave functions of the two atoms. More specifically, these

are the p_z orbitals discussed above. This replacement is the essence of the π - π^* band approximation [49]. In the above equation, t'_{NNN} gives the effective amplitude of next nearest neighbor hopping. It is an “effective” amplitude because it’s not a parameter independent of t . It is experimentally established that $t \approx -3$ eV and that the relation $t'_{NNN} \approx 0.1t$ holds [48].

Figure 1.6: The energy dispersion of graphene calculated by the tight-binding approach. The conduction (π^*) and valence (π) bands touch at the two inequivalent K points positioned at the corners of the Brillouin zone, about which the energy is linearly dependent on the wavevector. Figure adapted from Ref. [49].

Eq. 1.26 gives the energy dispersion of two bands which touch at the values of $\mathbf{k}$ given by

$$\epsilon_{\mathbf{k}}^{\lambda} = \epsilon_{\mathbf{k}}^{-\lambda} = 0 \quad (1.28)$$

which gives the two inequivalent Dirac points

$$\mathbf{K} = -\mathbf{K}' = \frac{4\pi}{3\sqrt{3}a} \hat{\mathbf{x}}. \quad (1.29)$$

These points coincide with the corners of the Brillouin zone in Fig. 1.5 shown above.

Expansion of the energy dispersion around these point to first order in the wavevector gives

$$\epsilon_{\mathbf{q}}^{\lambda} = \lambda \hbar v_F |\mathbf{q}| \quad (1.30)$$

for both the K and K' point. Here, $\mathbf{q}$ is a wavevector in a coordinate system centered along these two points, effectively a low-excitation vector. The “Fermi velocity”

$$v_F \equiv -\frac{3ta}{2\hbar} \quad (1.31)$$

is the value of the electron group velocity in graphene. The fact that it is a constant for any given value of the wavevector for low energy excitations is an important peculiarity of graphene. The linear energy dispersion of graphene, Eq 1.30, is the same as that of a relativistic particle with zero rest mass which is the photon. The “light speed” in graphene is, however, $v_F \approx 10^6$ m/s. The twofold (K and K') degeneracy of the energy spectrum at the neutrality point gives the a so-called valley pseudospin similar to the sublattice pseudospin. An effective Hamiltonian which gives this dispersion around the two Dirac points and incorporates both the sublattice and valley pseudospins is written as

$$H = \hbar v_F \begin{pmatrix} k_x \sigma_x + k_y \sigma_y & \mathbf{0} \\ \mathbf{0} & k_x \sigma_x - k_y \sigma_y \end{pmatrix} \quad (1.32)$$

where $\mathbf{q}$ has been replaced with $\mathbf{k} = -i\nabla$ to simplify notation, σ_x and σ_y are the Pauli spin matrices and $\mathbf{0}$ is a 2×2 matrix with all components set to zero.

An important property of the valley degenerate energy dispersion is it’s chiral nature. The effective Hamiltonian at the Dirac points at K and K' is proportional

to $\mathbf{k} \cdot \boldsymbol{\sigma}$ and $\mathbf{k} \cdot \boldsymbol{\sigma}^T$, respectively. Now, for a relativistic particle, its helicity is the projection of its angular momentum on the propagation direction, given by $\mathbf{k}$. An operator corresponding to this observable is given by

$$\mathcal{H} = \frac{\mathbf{k} \cdot \boldsymbol{\sigma}}{|\mathbf{k}|}. \quad (1.33)$$

For zero-mass particles, helicity and chirality are the same. It's easy to show that $\mathcal{H}$ commutes with the Hamiltonian H so they share common wavefunctions and chirality is conserved. But since the Hamiltonian is proportional to $\boldsymbol{\sigma}^T$ at the K' point, electrons' chirality there is anti-parallel to their chirality at the K point. Since chirality is conserved, electrons cannot scatter via smooth scattering potentials between these two points. This means that electron backscattering is inherently suppressed in graphene, an effect which profoundly increases carrier mobility in graphene. For any scattering potential which is smooth over the unit cell, however, the carrier states close to the two Dirac points are uncoupled and we can focus our attention to the K point when solving the Schrödinger equation.

If there is a difference in electron potential on the A and B sublattices, a band gap is opened at the Dirac points [50]. The gap can be achieved, for example, in a graphene sheet epitaxially grown on a SiC substrate. The energy splitting between the sublattices results in an off-diagonal term in the Hamiltonian close to the Dirac points. For the K point, the Hamiltonian becomes

$$H_K = \begin{pmatrix} \Delta/2 & \hbar v_F(k_x - ik_y) \\ \hbar v_F(k_x + ik_y) & -\Delta/2 \end{pmatrix} \quad (1.34)$$

where $\Delta \geq 0$ is the energy splitting between the two sublattices and has been taken to be positive without a loss of generality. The eigenvalues of the above matrix gives the energy spectrum

$$\epsilon_{\lambda}^{(g)} = \lambda \sqrt{(\hbar v_F k)^2 + \Delta^2} \quad (1.35)$$

so the opening of a gap can be considered as an acquisition of rest mass for the Dirac-like charge carriers in graphene. The band-gap opening of graphene can also be achieved by applying a strong monochromatic laser field in the mid-infrared range to the sheet [51, 52]. In Chapter 3, DC transport in the presence of such a gap will be discussed.

1.4. Normally applied magnetic fields

Fig. 1.7 shows the longitudinal resistance of a copper thin film for different temperatures as a function of an externally applied magnetic field normal to the plane. If one calculates the current density by using the classical Eq. 1.6, one gets that the longitudinal conductivity is given by

$$\sigma_{xx} = \frac{ne^2\tau}{m} \frac{1}{1 + (\omega_c\tau)^2} \quad (1.36)$$

where $\omega_c = He/m$ is the cyclotron frequency which alone in this expression takes into account the magnetic field effect. For the fields shown, the quantity $(\omega_c\tau)^2 \leq 10^{-8}$ so that the resistance should appear to be a constant on the plot.

Figure 1.7: The resistance of a copper thin-film decays with increasing normal-to-the-plane magnetic field. This dependence is a confirmation of weak localization in dirty metals. Figure adapted from Ref. [14].

The magnetic field dependence of the transport coefficients can be explained by studying the influence of the magnetic field on weak localization discussed earlier. The application of a weak magnetic field B normal to the plane breaks the time-reversal symmetry in the 2DEG plane. Two electrons moving in opposite orientations are deflected along two opposite transverse directions. For heavily disordered systems, e.g. dirty metals, the amount of deflection is negligible between two collisions with impurities.

Nevertheless, an electron moving along a closed loop $\mathcal{C}$, which encircles an area

A_C , in addition to the kinetic phase, acquires the phase

$$\theta_B^\pm = i \frac{e}{\hbar c} \oint_C \mathbf{A} d\mathbf{l} = \pm i \frac{\Phi}{\Phi_0} \pi \quad (1.37)$$

with the “+” (“-”) standing for (anti-) clockwise motion around the magnetic flux, $\Phi = BA_C$ is the magnetic flux penetrating the area A_C and $\Phi_0 = \pi \hbar c / e$ is the quantum flux. Since the area A_C is at most of the order $2\pi r^2(t)$, where $r(t) \approx \sqrt{Dt}$ is the mean distance which electrons diffuse at time t and D is a diffusion constant, the difference in phase for the two electrons moving in the two opposite orientations is given by

$$\Theta = \theta_B^+ - \theta_B^- = 2\pi \frac{\Phi}{\Phi_0} \approx \frac{Br^2(t)}{\Phi_0} = \frac{BDt}{\Phi_0}. \quad (1.38)$$

The probability of finding the electron at the original position for motion along a single closed path is given by

$$|\mathcal{A}_1 + \mathcal{A}_2|^2 = 2\mathcal{A}^2(1 + \cos \Theta) \quad (1.39)$$

where $\mathcal{A}$ is the probability amplitude in the absence of a magnetic field. As the time t in Eq. 1.38 increases, the number of possible closed loops accommodated inside the area $2\pi r^2(t)$ grows larger and so the phase Φ for each such loop is a random number with a mean value quite larger than zero. Therefore, in the summing over the probabilities of the above form, the mean of the factor $\cos \Theta$ amounts to zero and weak localization is destroyed.

A criterion for when the phase-breaking due to the magnetic field is of any impor-

tance can be formulated as

$$\Theta \approx \frac{BDt}{\Phi_0} \geq 1, \quad (1.40)$$

which occurs for the characteristic magnetic phase-breaking time

$$\tau_{\text{ph,B}} = \frac{\Phi_0}{BD}, \quad (1.41)$$

which for appropriate strengths of the magnetic field can be smaller than the inelastic phase-breaking time $\tau_{\text{ph,ie}}$ and therefore introduces a new upper limit in the summation of allowed loops contributing to weak localization. The fact that the resistance shown in Fig. 1.7 is smaller for higher temperatures arises because the inelastic collisions become more effective and lead to smaller values of $\tau_{\text{ph,ie}}$.

For stronger magnetic fields, when the magnetic length $l_B = \sqrt{\hbar c/eB}$ is smaller than the mean free path $v_F\tau$, the energy spectrum of the 2DEG within a single subband is given by

$$E_n = \hbar\omega_c \left(n + \frac{1}{2} \right), \quad (1.42)$$

which is obtained from the stationary Schrödinger equation. The discrete energy levels E_n are known as Landau levels and their existence leads to such remarkable effects as the quantum Hall effect (QHE), De Haas-van Alphen and Shubnikov-de Haas oscillations to name a few.

The QHE, observed by v. Klitzing in 1980 and earning him a Nobel prize, is the exactly quantized behaviour of the resistance of a 2DEG in an increasing magnetic

field or carrier concentration [53, 54]. The Hall resistivity is related to the number N of filled Landau levels as

$$\rho_{xy} = \frac{h}{Ne^2} \quad (1.43)$$

where h is the Planck's constant and N is the number of completely filled Landau levels and so it doesn't depend on any geometrical aspects of the system. By fixing the carrier concentration, an increasing normal magnetic field increases the cyclotron frequency ω_c and the degeneracy of each Landau level while the Fermi level remains fixed. On the contrary, since the Landau level separation is proportional to ω_c , the number N decreases as the discrete levels deplete after crossing the Fermi level as a result of increasing magnetic field. This leads to the noticeable jumps in the resistivity shown in the upper part of Fig. 1.8.

In the lower plot in Fig. 1.8, the longitudinal resistivity ρ_{xx} is shown. For the magnetic field strengths where ρ_{xy} has plateaus, ρ_{xx} takes vanishing values. These regions correspond to completely filled Landau levels where electrons don't contribute to a net electric current. For small B , where the broadened Landau levels overlap, the resistivity shows the Shubnikov-de Haas oscillations which depend on disorder in contrast to the Hall resistivity which is not affected by it.

Since the low-excitation dispersion of graphene is fundamentally different from the one of the free parabolic 2DEG, a normal magnetic field has a different effect on the Dirac-like energy spectrum. If the magnetic length is much larger than the lattice constant of a perfect graphene lattice, one does the usual substitution of the canonical momentum with the dynamical one Π which takes into account the vector potential of the magnetic field, usually in the Landau gauge. Defining the creation and

Figure 1.8: (a). The transverse resistivity ρ_{xy} as a function of an applied magnetic field normal to the 2DEG plane takes jumps of value h/e^2 confirming the quantum Hall effect. (b). The longitudinal resistivity ρ_{xx} shows the Shubnikov-de Haas oscillations at low magnetic fields while vanishing at the regions where ρ_{xy} has plateaus, which correspond to completely full Landau levels. Figure adapted from [55].

annihilation operators as

$$\begin{aligned}\hat{a} &= \frac{l_B}{\sqrt{2}\hbar}(\Pi_x - i\Pi_y), \\ \hat{a}^\dagger &= \frac{l_B}{\sqrt{2}\hbar}(\Pi_x + i\Pi_y),\end{aligned}\tag{1.44}$$

respectively, which satisfy the commutation relation $[\hat{a}, \hat{a}^\dagger] = 1$, one is able to rewrite the Hamiltonian around the K -point as

$$\hat{H}_B = \frac{\sqrt{2}\hbar v_F}{l_B} \begin{pmatrix} 0 & \hat{a} \\ \hat{a}^\dagger & 0 \end{pmatrix}. \quad (1.45)$$

Its eigenvalues, E_n , are known as relativistic Landau levels. Writing the eigen-spinors in the form

$$\psi_n = \begin{pmatrix} \phi_{1,n} \\ \phi_{2,n} \end{pmatrix}, \quad (1.46)$$

the stationary Schrödinger equation yields the system of equations

$$\begin{aligned} \frac{\sqrt{2}\hbar v_F}{l_B} \hat{a} \phi_{1,n} &= E_n \phi_{2,n} \\ \frac{\sqrt{2}\hbar v_F}{l_B} \hat{a}^\dagger \phi_{2,n} &= E_n \phi_{1,n} \end{aligned} \quad (1.47)$$

from which one gets

$$\hat{a}^\dagger \hat{a} \phi_{2,n} = E_n^2 \left(\frac{l_B}{\sqrt{2}\hbar v_F} \right)^2 \phi_{2,n} \quad (1.48)$$

which equation resembles the eigenvalue problem for the harmonic oscillator $\hat{a}^\dagger \hat{a} |n\rangle = n |n\rangle$ and is solved in the same way. Eq. 1.47 is then used to find $\phi_{1,n}$. The relativistic Landau levels,

$$E_n^\pm = \pm \frac{\hbar v_F}{l_B} \sqrt{2n}, \quad (1.49)$$

where the “+” (“-”) stands for the conduction (valence) band, are also proportional to the the magnetic field strength B just as the semiconductor Landau levels, while the separation between adjacent levels is not a constant but varies with the level index n .

1.5. Light-Matter Interaction

It is well known that it's impossible for a free electron to absorb a photon of an incident optical field $\vec{\mathbf{E}}(t)$. This is so because such a process cannot simultaneously abide by the laws of conservation of both energy and momentum. Classically, a particle with charge q interacts with radiation or any external electromagnetic field as described by the Newton's second law Eq. 1.6 produced above. By introducing a phenomenological relaxation time τ and neglecting influences from the magnetic component of a radiation field illuminating a 2DEG plane, we can express the time derivative of the in-plane momentum vector $\vec{\mathbf{p}}$ as

$$\dot{\vec{\mathbf{p}}} = -\frac{\vec{\mathbf{p}}}{\tau} + e\vec{\mathbf{E}}_{\parallel}(t). \quad (1.50)$$

where $\vec{\mathbf{E}}_{\parallel}(t)$ is the electric field of the incoming wave which lies along the quantum well since an out-of-the-plane electric field doesn't cause electrons to accelerate in the limit of very thin quantum well. For a continuous harmonic wave,

$$\vec{\mathbf{E}}_{\parallel}(t) = \Re\{\vec{\mathbf{E}}_{\parallel}(\omega)e^{-i\omega t}\} \quad (1.51)$$

and we expect the momentum to have the same oscillation frequency, i.e. $\vec{\mathbf{p}} = \Re\{\vec{\mathbf{E}}_{\parallel}(\omega)e^{-i\omega t}\}$, and using Eq. 1.50, the momentum and electric field vectors are related by the equation

$$\vec{\mathbf{p}}(\omega) = \frac{e\vec{\mathbf{E}}_{\parallel}(\omega)}{-\tau^{-1} + i\omega}. \quad (1.52)$$

Using the equation $\vec{\mathbf{j}}(t) = ne\vec{\mathbf{v}}(t) = ne\vec{\mathbf{p}}(t)/m$ for the electric current density $\vec{\mathbf{j}}(t)$ in the definition of the conductivity (for an isotropic medium), $\vec{\mathbf{j}}(t) = \sigma\vec{\mathbf{E}}(t)$, one obtains that

$$\sigma(\omega) = \sigma_0 \frac{1 + i\omega\tau}{1 + \omega^2\tau^2}, \quad (1.53)$$

where σ_0 is the Drude DC conductivity. In the limit $\omega\tau \gg 1$ the conductivity therefore vanishes. The absorption coefficient contribution from intrasubband scattering of free carriers is calculated to be

$$\alpha(\omega) = \frac{\sigma_0}{\epsilon_0 c \eta_r} \frac{1}{1 + \omega^2\tau^2}, \quad (1.54)$$

where η_r is the refractive index of the medium [57]. For low frequencies, $\omega\tau \ll 1$, the absorption coefficient is a constant,

$$\alpha(\omega) = \frac{\sigma_0}{\epsilon_0 c \eta_r}, \quad (1.55)$$

while in the high-frequency limit, $\omega\tau \gg 1$, the absorption coefficient

$$\alpha(\omega \gg \tau^{-1}) = \sigma_0 \frac{1}{\epsilon_0 c \eta_r (\omega\tau)^2} \quad (1.56)$$

evidently behaves as ω^{-2} so that free carrier contribution to the optical response becomes negligible.

In order to calculate the interband transitions and those between subbands which arise due to the confinement of finite width along the growth axis, the absorption coefficient needs to be calculated in a quantum fashion. The electric component $\mathbf{E}$

and magnetic component $\mathbf{B}$ of an EM field are given by the equations

$$\mathbf{E} = -\nabla\varphi - \frac{\partial\mathbf{A}}{\partial t} \quad (1.57)$$

$$\mathbf{B} = \nabla \times \mathbf{A}, \quad (1.58)$$

respectively, in terms of a vector potential $\mathbf{A}$ and a scalar potential φ [10]. In quantum mechanics the interaction is introduced by using the canonical momentum operator

$$\hat{\mathbf{p}} \mapsto \hat{\mathbf{p}}' = \hat{\mathbf{p}} - \frac{e}{c}\mathbf{A}. \quad (1.59)$$

in the Hamiltonian. In a simple particle-in-a-box model, the photons of the optical field can be absorbed when an electron jumps from the valence band to the conduction band, leaving a hole behind. If the optical field is weak, the interaction is treated perturbatively, where the Hamiltonian is written as

$$\hat{H} = \hat{H}_0 + \hat{H}'(t) \quad (1.60)$$

where $\hat{H}_0$ is the unperturbed part and $\hat{H}'(t)$ is a time-dependent term much smaller than $\hat{H}_0$. In the case of a spatially homogeneous continuous wave, the interaction term is expressed as

$$\hat{H}'(t) = \frac{e}{2m_0c}\mathbf{A} \cdot \hat{\mathbf{p}}. \quad (1.61)$$

which is obtained by making the above momentum replacement and keeping only linear terms in the expansion of the original Hamiltonian. The Fermi Golden Rule gives the probability $P(\omega)$ of an electron jumping between the valence band, $|v\rangle$, and

conduction band, $|c\rangle$, by absorbing a single photon with frequency ω , i.e.

$$P(\omega) = \frac{2\pi}{\hbar} |\langle c | \hat{H}' | v \rangle|^2 \delta(E_c - E_v - \hbar\omega). \quad (1.62)$$

where E_v and E_c are the energies of levels $|v\rangle$ and $|c\rangle$, respectively. This gives the absorption coefficient, in the case of a degenerate electron gas where the conduction band is empty and the valence band is full, to be

$$\alpha(\omega) = \frac{4\pi^2 e^2}{\eta_r c m_0^2 \omega V_{QW}} |\mathbf{e} \cdot \mathbf{p}_{vc}|^2 \delta(E_c - E_v - \hbar\omega) \quad (1.63)$$

where $\mathbf{p}_{vc} = \langle c | \hat{\mathbf{p}} | v \rangle$ [56, 57]. In a quantum well, the absorption edge doesn't coincide with the energy difference between the bulk conduction and valence bands at zero in-plane momentum. This is so because the confinement shifts the valence band maximum and conduction band minimum. In addition to this, due to the long range nature of the Coulomb interaction, carrier correlations cannot be neglected even in the limit of a weak optical pump.

Excitons

Taking into account the attractive interaction between the positively charged electron and negatively charged hole after their creation by the interband absorption of a photon, one has set up the two-body problem of the hydrogen atom. A semiconductor Wannier-Mott exciton is a bound state of an electron and hole with binding energies much smaller than the rate of inter-cell hopping of each constituent quasiparticle and large Bohr radii, usually extending to around 10 times the lattice constant. Beside

the two-particle discrete energy structure, the excitons have a continuous part which corresponds to translational motion of the center of mass. Excitons manifest themselves in the appearance of sharp peaks in the absorption spectrum of semiconductors below the fundamental band edge, see Fig. 1.9. Frenkel excitons, for comparison, are of a size comparable to a crystal cell of a material with a small dielectric constant which allows for stronger electron-hole Coulomb coupling.

The energy spectrum of the bound system is found by the Wannier equation for the wavefunction $\Psi(\mathbf{r})$ which describes the motion of the electron and the hole relative to each other

$$-\left[\frac{\hbar^2 \nabla^2}{2m_r} + V(r)\right] \Psi(\mathbf{r}) = E\Psi(\mathbf{r}) \quad (1.64)$$

where $\mathbf{r}$ is the electron-hole spatial separation, m_r is the electron-hole reduced mass defined above, E is the energy and the Coulomb potential is given by

$$V(r) = -\frac{e^2}{4\pi\epsilon\epsilon_0 r} \quad (1.65)$$

In a purely 2D quantum well, the bound states energies have the form

$$E_n = -\frac{e^4 m_r}{2\epsilon_0^2 \hbar^2} \frac{1}{n^2} \quad (1.66)$$

with n being a positive integer. The ground state (1s) radial wavefunction is given by

$$\varphi_0(r) = \frac{4}{a_0} \exp\left(-\frac{2r}{a_0}\right) \quad (1.67)$$

where the exciton the Bohr radius

$$a_0 = \frac{\hbar^2 \varepsilon_0}{e^2 m_r} \approx 150 \text{ \AA}, \quad (1.68)$$

in a GaAs based quantum well is two times smaller than in the bulk. The absorption coefficient of a QW accounting for the excitonic effects is obtained by the 2D Elliott formula

$$\alpha(\omega) = \alpha_0 \frac{\hbar\omega}{E_0} \left[\sum_{n=1}^{\infty} 4 \left(n - \frac{1}{2} \right)^{-3} \delta \left(\Delta + \left[n - \frac{1}{2} \right]^{-2} \right) + \frac{\Theta(\Delta) \exp(\pi/\sqrt{\Delta})}{\cosh(\pi/\sqrt{\Delta})} \right] \quad (1.69)$$

where E_0 is the exciton ground state and

$$\Delta = \frac{E_g - \hbar\omega}{E_0} \quad (1.70)$$

is the detuning of the pump frequency with respect to the bandgap and α_0 is a constant proportional to the interband dipole moment [58]. Scattering by impurities and phonons leads to the broadening of the exciton lines while fluctuations in the quantum well thickness contribute to the production of disorder as well.

Figure 1.9: The absorption of a quantum well as a function of detuning of the pump relative to the fundamental band edge. The sharp lines at negative Δ correspond to excitons. Figure adapted from Ref. [58].

The spin of excitons is directly related to the free carrier spin. Since the conduction band is s -type (zero angular momentum), the projection of their total angular momentum onto an axis z defined to be along the growth direction takes values $J_z^e = \pm 1/2$. For the p -type valence band, the light hole (lh) states are established, from the standard addition of angular momenta [59], when the hole spin and angular momentum lie in anti-parallel giving the projection $J_z^{\text{lh}} = \pm 1/2$. The heavy hole states, having the projection values $J_z^{\text{hh}} = \pm 3/2$, are formed with the parallel alignment of spin and angular momentum. While in bulk semiconductors the hh and lh subbands are degenerate at zero momentum, the confinement along the z -axis in a QW lifts this degeneracy.

Since an exciton is formed by bound electron-hole states, its total angular momentum in the ground state has the projections $J_z^{\text{exc}} = \pm 1, \pm 2$ onto the z -axis. Adhering

to the law of conservation of angular momentum, only the bright exciton state with $J_z = \pm 1$ can be populated by an optical field, see Fig. 1.10. The optically inactive $J_z^{\text{exc}} = \pm 2$ states are known as dark excitons. In the presence of disorder, the exciton spin relaxes via the mechanisms discussed earlier.

Figure 1.10: Light having certain polarization can induce optical transitions between the energy-split hole subbands and the electron band, depending on the indicated projection of the total angular momentum along the growth axis.

To see how a strong field affects interlevel optical transitions, we consider a general two-level system whose interaction with the field is taken into account via the dipole approximation. Consider levels $|a\rangle$ and $|b\rangle$ where $\hat{H}_0|a\rangle = E_a|a\rangle$ and $\hat{H}_0|b\rangle = E_b|b\rangle$ with $\hat{H}_0$ being the free Hamiltonian of the system and take $E_b > E_a$. The absorption spectrum in this case has a single line with frequency $\omega_{ba} = (E_b - E_a)/\hbar$. The total Hamiltonian of the system reads

$$\hat{H} = \hat{H}_0 + \hat{H}_I \quad (1.71)$$

where

$$H_I = -ex\mathcal{E}(t) \quad (1.72)$$

is the interaction term and the optical field is assumed to be aligned along the x -direction. Writing the general time-dependent wavefunction as

$$|\psi(t)\rangle = c_a(t)|a\rangle + c_b(t)|b\rangle \quad (1.73)$$

the Schrödinger equation gives two coupled equations

$$\begin{aligned} i\hbar\dot{c}_a &= -\mathcal{E}(t)\exp(-i\omega_{ba}t)dc_b \\ i\hbar\dot{c}_b &= -\mathcal{E}(t)\exp(-i\hbar\omega_{ba}t)d^*c_a \end{aligned} \quad (1.74)$$

where $d = \langle b|ex|a\rangle$ matrix element of the dipole transition which for the same levels is zero.

Around the resonance condition $\omega_{ba} - \omega \approx 0$, that is to say for small detuning $\delta \equiv \omega_{ba} - \omega$ of the pump, one gets the solutions

$$\Psi(\mathbf{r}, t) = c_a(0)\exp(-i[\Omega_{\pm} + E_a/\hbar]t)\varphi_a(\mathbf{r}) + c_b(0)\exp(-i[\Omega_{\pm} + E_b/\hbar]t)\varphi_b(\mathbf{r}) \quad (1.75)$$

with

$$\Omega_{\pm} = -\frac{\delta}{2} \pm \frac{1}{2}\sqrt{\delta^2 + \omega_{Rabi}^2} \quad (1.76)$$

where

$$\omega_{Rabi} = \frac{|\mathcal{E}(\omega)d|^2}{4\hbar^2} \quad (1.77)$$

is called the Rabi frequency. In the case of zero detuning, $\delta = 0$, each energy level E_i is

split into two with energies $E_i \pm \hbar\omega_{\text{Rabi}}$ as shown in Fig. 1.11. The optical field, besides altering the energy levels, mixes the two states, $|a\rangle$ and $|b\rangle$, so that the probability of the system being in either state oscillates with the frequency ω_{Rabi} . Applying a weak probing field to measure the absorption spectrum, one thus sees a (Mollow) triplet instead of the original singlet. This effect is reminiscent of the Stark effect in which a constant electric field induces a shift in the energy spectrum of an atom, for example, and is called AC, dynamic or optical Stark effect.

Figure 1.11: AC Stark Effect. The single absorption line of a two-level system splits into a triplet in the presence of a strong monochromatic optical field in resonance with the bare transition. One speaks of a system dressed by light when the energy states are altered by the electromagnetic field.

To observe energy shifts away from resonance, strong laser fields are required and are available only in the form of short pulses. This effect has been observed, for example in [60], by illuminating a sodium atomic beam with a continuous wave in resonance with a hyperfine transition and it was also observed in the mixing of 1s and 2p excitonic states in a bulk Cu_2O semiconductor [61]. Pumping a quantum well in the transparency region results in a shift of the first heavy hole peak in the transmis-

sion spectrum which lasts as long as the nonresonant pulse, implying the absence of interband transitions [62,63], see Fig. 1.12. This means that the shift is a pure electric field effect and so one speaks of dressed excitons [64–66].

Figure 1.12: Pump-probe absorption spectrum of a GaAs-AlGaAs multiple quantum well system. The different lines correspond to different delays between pump and probe. It is clear that for zero delay, the absorption peak is shifted in the presence of the pulse, whose spectral position is shown on top of the wavelength axis. The lengths of the pump and probe pulses are under a picosecond. Figure adapted from Ref. [62].

While coherent modification of transport phenomena by a strong ac field has been extensively investigated in the near past in all D dimensions, most of the research activity has been concentrated in driving systems such as superlattices, quantum dots

and double quantum dots with the aim of improving or suppressing tunneling [67–70]. When the ac field is the electric component of light, the processes are called photon-assisted tunneling. Beside controlling the charge carrier density by photogeneration little has been done in controlling conductivity by light in quantum wells within a single quantum well, even though investigation of magnetoresistance in the presence of strong radiation has taken place [71,72].

1.6. Microcavity polaritons

The most basic tool for confining light in a small region of space is the Fabry-Perot cavity which consists of two planar mirrors aligned parallel to each other and displaced by the distance L_0 as shown in Fig. 1.13(a). From classical optics we find that the

Figure 1.13: (a). The Fabry-Perot resonator consists of two mirrors of reflectance R_1 and R_2 separated by a distance L_0 . (b). The transmission spectrum of a Fabry-Perot resonator.

resonant frequencies of such a structure, where the transmittance takes maxima, are

given by

$$\omega_r = j \frac{\pi c}{L_0}, \quad \text{for integer } j, \quad (1.78)$$

from which it is understood that as the distance between the mirrors is reduced, the separation between resonant frequencies is increased. The resonant levels do not form sharp lines, but have a finite width, as seen in Fig. 1.13(b). A measure of how a cavity performs in resolving the individual modes is called finesse $\mathcal{F}$ which is the ratio of the resonance mode separation in frequency $\Delta\omega$ to the linewidth of a cavity mode $\delta\omega$, i.e.

$$\mathcal{F} = \frac{\Delta\omega}{\delta\omega}, \quad (1.79)$$

and so, the larger the finesse $\mathcal{F}$, the sharper the individual lines. If the mirror reflectance is close to 1, then

$$\mathcal{F} \approx \frac{\pi}{T} \quad (1.80)$$

where $T = 1 - R$ is their transmittance.

If W is the energy of the optical field trapped between the mirrors, the quality factor Q , which is inversely proportional to the rate of energy loss in the cavity, is defined by the equation

$$\frac{dW}{dt} = -\frac{\omega}{Q}W \quad (1.81)$$

which for the case of high reflectance is given by

$$Q \approx \mathcal{F} \frac{2L_0}{\lambda}. \quad (1.82)$$

Figure 1.14: (a). A planar microcavity consists of a quantum well sandwiched between distributed Bragg resonators (DBRs) which are made by growing alternating layers of differing refractive indices. (b). DBR mirror reflectance and transmittance of an organic semiconductor microcavity at normal incidence. The microcavity has a single peak in the transmittance spectrum corresponding to the cavity mode. Experimental data from Ref. [73].

Alternatively, one may look at the quantity $1/Q$ as the fraction of energy lost in one round-trip around the cavity. Therefore, a confined photon of energy $\hbar\omega_C$ has a lifetime of $\tau_C = Q/\omega_C$. To get high values of $\mathcal{F}$ and Q , that is to say to get a sharp and lossless spectral resolvent, one thus needs high mirror reflectivity. Metallic mirrors provide an easy option but another way of confining light along one dimension is by using distributed Bragg reflectors (DBR) to engineer so-called planar microcavities, see Fig. 1.14.

A DBR is a stack of dielectric layers of thickness $\lambda/4$ with alternating refractive indices, where λ is the wavelength of the single cavity mode, usually in the micrometer range. For a large number of layers, a spacer, which is a multiple of $\lambda/2$, between the DBRs can be considered as a defect in an otherwise periodic lattice along one

direction [74, 75]. The spacer is an interruption of refractive index modulation where the electric field of the cavity mode takes a maximum. These kind of heterostructures can be grown with such methods as molecular beam epitaxy and reactive ion beam sputtering. The finesse and quality factor of microcavities are determined by the number of dielectric layers and the difference in their refractive indexes while layer imperfections tend to reduce the confinement ability.

For a cavity mode making a finite angle with the growth axis, the in-plane dispersion of a Fabry-Perot resonator is given by

$$E_C(\mathbf{k}) = \sqrt{E_C(0)^2 + \frac{\hbar^2 c^2 k^2}{n_{\text{eff}}^2}}, \quad (1.83)$$

where $\mathbf{k}$ is the in-plane wavevector and

$$E_C(0) = \frac{\hbar c \pi}{n_{\text{eff}} L_c}, \quad (1.84)$$

is the resonant mode energy at zero angle (normal incidence) obtained from Eq. 1.78 with the insertion of the appropriate refractive index. The effective refractive index of the cavity, n_{eff} , is an average over the DBR layers and the central defect,

$$n_{\text{eff}} = \frac{\int_0^L n(z) \mathcal{E}^2(z) dz}{\int_0^L \mathcal{E}^2(z) dz}, \quad (1.85)$$

where z is the coordinate along the growth axis and $\mathcal{E}$ is the in-plane electric field

amplitude of the cavity mode. Expanding Eq. 1.83 about $k = 0$ gives

$$E_C(\mathbf{k}) \approx E_C(0) + \frac{\hbar^2 k^2}{2m_C}, \quad (1.86)$$

where the in-plane photon effective mass is defined as

$$m_C = \frac{n_{\text{eff}}\pi}{cL_c}, \quad (1.87)$$

Inside a cavity therefore, light shows a parabolic dispersion characteristic of massive particle, a fascinating behavior since we know that photons are massless in the outside world.

Because the two orthogonal polarization modes of the cavity have different penetration depths into the DBR mirrors, a splitting in photon energies for the transverse-electric (TE) and transverse-magnetic (TM) mode occurs. The dependence of the TE-TM splitting, also known as longitudinal-transverse (LT) splitting, on the in-plane momentum is expressed as [76]

$$\begin{aligned} \Delta_{LT} &= \hbar\omega^{\text{TM}}(\theta) - \hbar\omega^{\text{TE}}(\theta) \\ &\approx \frac{\delta_s L_c L_{\text{DBR}}}{(L_c + L_{\text{DBR}})^2} \frac{2 \cos \theta_{\text{eff}} \sin^2 \theta_{\text{eff}}}{1 - 2 \sin^2 \theta_{\text{eff}}}, \end{aligned} \quad (1.88)$$

where δ_s is the mismatch between the resonant frequency of the Fabry-Perôt resonator and the center of the stopband. For perfect layers, that is to say homogeneous layer thickness across the DBRs, this quantity vanishes but assumes finite values in realistic samples. Here L_{DBR} is an effective length of the mirrors and the effective angle

is related to the effective refractive index by the equation $\arcsin(n_0/n_{\text{eff}})\sin(\theta)$. At normal incidence, when $\theta = 0$, the TE and TM modes are of course equivalent so that the TE-TM splitting is zero.

Placing a number of quantum wells inside the center defect of a DBR resonator, with the excitonic transition close to resonance with the cavity mode allows for strong coupling between light and matter. In high-Q microcavities, where a photon is able to be absorbed and emitted via interband transitions a large number of times before eventually escaping the resonator, a new mode, neither a photon nor an exciton, appears which combines the properties of both particles.

In second quantization, the total Hamiltonian is written as

$$\hat{H} = \sum_{\mathbf{k}} E_X(\mathbf{k}) \hat{X}_{\mathbf{k}}^\dagger \hat{X}_{\mathbf{k}} + \sum_{\mathbf{k}} E_C(\mathbf{k}) \hat{C}_{\mathbf{k}}^\dagger \hat{C}_{\mathbf{k}} + \frac{1}{2} \sum_{\mathbf{k}} V_R (\hat{X}_{\mathbf{k}}^\dagger \hat{C}_{\mathbf{k}} + \hat{X}_{\mathbf{k}} \hat{C}_{\mathbf{k}}^\dagger) \quad (1.89)$$

with $\hat{X}_{\mathbf{k}}$ ($\hat{X}_{\mathbf{k}}^\dagger$) and $\hat{C}_{\mathbf{k}}$ ($\hat{C}_{\mathbf{k}}^\dagger$) being the exciton and cavity photon annihilation (creation) operators, respectively. Plotting the parabolic exciton dispersion $E_X(\mathbf{k})$ on the same grid as the photonic one $E_C(\mathbf{k})$, it appears to be flat since the excitonic effective mass is orders of magnitude larger than the photonic one which is of the order of $10^{-4}m_0$. The last term of the above Hamiltonian stems from the interaction between photons and excitons where the Rabi splitting V_R is proportional to the interband dipole matrix element and the in-plane electric field of the cavity mode and depends on the number of embedded quantum wells between the DBRs [77].

The Hamiltonian of Eq. 1.89 can be diagonalized so that it is written as

$$\hat{H} = \sum_{\mathbf{k}} E_P(\mathbf{k}) \hat{P}_{\mathbf{k}}^\dagger \hat{P}_{\mathbf{k}} \quad (1.90)$$

by combining the exciton and photon operators,

$$\hat{P}_{\mathbf{k}} = \mathcal{U}_{\mathbf{k}} \hat{X}_{\mathbf{k}} + \mathcal{V}_{\mathbf{k}} \hat{C}_{\mathbf{k}}, \quad (1.91)$$

and demanding that they obey the bosonic algebra

$$[\hat{P}_{\mathbf{k}}, \hat{P}_{\mathbf{k}}^\dagger] = |\mathcal{U}_{\mathbf{k}}|^2 + |\mathcal{V}_{\mathbf{k}}|^2 = 1 \quad (1.92)$$

where the squared moduli of the Hopfield coefficients $\mathcal{U}_{\mathbf{k}}$ and $\mathcal{V}_{\mathbf{k}}$ are the excitonic and photonic fraction, respectively, of the hybrid new polariton excitations [78]. The fractions give the probability that the polariton is to be found in either constituent state.

The polariton dispersion has two distinct branches,

$$E_{U,L}(\mathbf{k}) = \frac{1}{2}(E_X(\mathbf{k}) + E_C(\mathbf{k})) \pm \frac{1}{2}\sqrt{\delta E_{\mathbf{k}}^2 + V_R^2}, \quad (1.93)$$

where the “+” (“-”) stands for the upper (lower) polariton branch $E_U(\mathbf{k})$ ($E_L(\mathbf{k})$) and the energy difference between the photonic and excitonic energies is defined as $\delta E_{\mathbf{k}} \equiv E_C(\mathbf{k}) - E_X(\mathbf{k})$, see Fig. 1.15.

Figure 1.15: Calculations of the two polariton branches (solid lines) for photon effective mass $m_C = 0.5 \times 10^{-4}m_0$, Rabi-splitting $V_R = 50$ meV and detuning (a) $\delta E_0 < 0$, (b) $\delta E_0 = 0$, (c) $\delta E_0 > 0$. The dashed curves are the free exciton and cavity photon modes. The exciton parabola seems to be flat because of the high effective mass of excitons. Plots adapted from Ref. [79].

It should be noted that polaritonic Rabi doublet is only observed in the strong coupling regime, where the Rabi splitting V_R is larger than broadening of the exciton and cavity photon modes, the former being caused by interparticle and phonon interactions while the second by leakage of optical energy from the cavity characterized by the Q factor [74].

The Hopfield coefficient depend on the polariton branch and the exciton and photon energies as well,

$$\mathcal{U}_L = \frac{V_R}{\sqrt{V_R^2 + (E_U - E_X)^2}}, \quad (1.94)$$

$$\mathcal{V}_L = \frac{-(E_U - E_X)}{\sqrt{V_R^2 + (E_U - E_X)^2}}, \quad (1.95)$$

for the lower branch and while $\mathcal{U}_U = -\mathcal{V}_L$ and $\mathcal{V}_U = \mathcal{U}_L$. We see that for no exciton-photon interaction ($V_R = 0$), we retain the uncoupled exciton and photon modes.

Polariton pseudospin

It turns out that the conversion of optically active bright exciton states into dark excitons is very inefficient in a QW and as a result, polaritons can be considered as a two-level system in regards to their pseudospin having projections $J_z = \pm 1$ on the growth axis [80]. In the study and description of (pseudo) spin relaxation in a polariton ensemble, it is convenient to decompose the density matrix in spin space using the identity matrix $\mathbf{I}$ and the three Pauli spin matrices vector $\boldsymbol{\sigma}$ as is usually done for two-level systems [81],

$$\rho_{\mathbf{k}} = \frac{N_{\mathbf{k}}}{2}\mathbf{I} + \mathbf{S}_{\mathbf{k}} \cdot \boldsymbol{\sigma}, \quad (1.96)$$

where $\mathbf{S}_{\mathbf{k}}$ is the mean value of the pseudospin in the state $\mathbf{k}$, $N_{\mathbf{k}}$ is the population of polaritons having wavevector $\mathbf{k}$. In relation to the Stokes parameters of the emitted light from a strong coupling microcavity, the components $S_{\mathbf{k},x}, S_{\mathbf{k},y}, S_{\mathbf{k},z}$ of the spin vector $\mathbf{S}_{\mathbf{k}}$ correlate with the circular, linear and diagonal polarization degrees, respectively [74, 80].

To see the influence of a magnetic field, either effective or real, on the in-plane spin pattern, $\mathbf{S}_{\mathbf{k}}$ is mapped onto the Poincaré sphere, Fig. 1.16. The evolution of the

Figure 1.16: The Poincaré sphere. The pseudospin vector $\mathbf{S}$ (light blue arrow) describes the polarization state of a polariton. Its projection onto the horizontal plane gives the linear polarization degrees, defined by two sets of orthogonal axes. The circular polarization degree is given by the projection of $\mathbf{S}$ onto the vertical axis.

density matrix 1.96 is deduced using the Liouville-Neumann equation

$$i\hbar\dot{\rho}_{\mathbf{k}} = [H_{\mathbf{k}}^{(j)}, \rho_{\mathbf{k}}] \quad (1.97)$$

where $j = L, U$ stands for the lower and upper polariton branch, respectively. For the lower polariton branch, the Hamiltonian, including magnetic fields, is given by

$$H = E_L(\mathbf{k}) + \frac{\hbar}{2} \boldsymbol{\Omega}_{\mathbf{k}} \cdot \boldsymbol{\sigma} \quad (1.98)$$

where the spectrum $E_L(\mathbf{k})$ is given by Eq. 1.93. The effective magnetic field $\Omega_{\mathbf{k}}$ is a consequence of the photonic and excitonic TE-TM splitting. The former was discussed earlier while the LT splitting of QW excitons is a consequence of the long-range exchange interaction which leads to polariton spin flips via the Bir-Pikus mechanism [35, 82].

Figure 1.17: The orientation of the effective magnetic field induced by the TE-TM splitting along the elastic circle. Polaritons fill this circle by scattering with impurities with the effective magnetic field reorientating in the process.

Different from the effective magnetic fields induced by Rashba and Dresselhaus spin-orbit interaction, the effective magnetic field has a 2ϕ angle dependence on the wavevector $\mathbf{k}$, see Fig.1.17, and the interaction gives off-diagonal terms to the polariton reciprocal space Hamiltonian in the form

$$H_{\text{LT}} = \frac{\Delta_{\text{LT}}(k)}{k^2} \begin{pmatrix} 0 & (k_x - ik_y)^2 \\ (k_x + ik_y)^2 & 0 \end{pmatrix}. \quad (1.99)$$

Finally, Eq. 1.97 can be rewritten as

$$\frac{\partial \mathbf{S}_{\mathbf{k}}(t)}{\partial t} = \mathbf{S}_{\mathbf{k}}(t) \times \boldsymbol{\Omega}. \quad (1.100)$$

To account for the finite lifetime of polaritons and disorder in the spin relaxation, phenomenological loss and scattering terms are added to the equation.

When a polariton is excited with a definite in-plane wavevector its spin precesses according to Eq. 1.100 around the effective magnetic field $\boldsymbol{\Omega}$. Even though the initial polariton state and the effective magnetic field may initially point along the same direction, elastic scattering with disorder changes the orientation of $\boldsymbol{\Omega}$ which in turn leads to spin precession. This way of polariton spin evolution is the underlying mechanism of the optical spin Hall effect (OSHE) predicted in [83] and experimentally observed in [84], see Fig. 1.18, by measuring the far-field Stokes parameters of emitted light. In the OSHE, the anisotropy of the effective magnetic field leads to the anisotropic redistribution of polaritons along the four quadrants of the reciprocal space.

Finally, a real magnetic field applied normally to the microcavity plane induces an energy split between exciton states having spin projections $+1$ and -1 on the growth axis, respectively. The Zeeman splitting can be estimated as

$$\delta E_B = \mu_B g_X B, \quad (1.101)$$

where $\mu_B \approx 0.062$ meV/T is the Bohr magneton and the g_X is the excitonic g -factor which depends on the QW properties and magnetic field orientation [74]. The ex-

Figure 1.18: The optical spin Hall effect. Elastic Rayleigh scattering redistributes polaritons excited at a finite in-plane wavevector on the k_x axis. Longitudinal-transverse splitting reorientates the polariton spin and leads to change in the polarization of the emitted light in comparison with the pump polarization. (a) Far-field measurement of circular polarization of light showing the circular polarization degree. (b). Theoretical predictions using a Schrödinger-type equation for the polariton spin. Figures adapted from Ref. [84]

citonic Zeeman splitting rotates the polarization plane of a linearly polarized light passing through a QW, an effect known as resonant Faraday rotation. In Chapter 4, the influence of an external magnetic field and one arisen from free carrier spin-orbit coupling on the OSHE is studied using the discussed techniques. The effects of a real magnetic field on the polariton spin patterns have up to now been largely neglected.

Finally, although in the present thesis the spin patterns of polaritons are discussed only in the linear regime where polariton-polariton interactions are not important,

there is a rich variety of spin related phenomena which arise due to nonlinearities, such as the nonlinear optical spin Hall effect and spin switching to name a few [85–87].

2. Transport and spin properties of 2DEG in the regime of strong coupling with light

2.1. Introduction

In this chapter, I consider the effect of an external electromagnetic (EM) wave on the electron transport properties of two dimensional electron systems. Corresponding papers can be found in references [88] and [89]. Studies in the 70's showed that a continuous EM wave can alter the electron-atom scattering amplitude [90, 91]. In particular, terms appear in the scattering rate corresponding to the emission and absorption of radiation which in turn lead to inelastic collisions. Few years ago, a theoretical work by Kibis [92] suggested that in a condensed matter system where there is a disorder broadening of energy levels, the inelastic terms can be neglected for photon energies well above the collision-induced broadening.

In order to provide a testing ground for this hypothesis, a physical quantity needed

to be related to the modified scattering rate and examples of such a quantity are given by the DC electrical conductivity of a two dimensional electron gas (2DEG) and the spin relaxation rate. This chapter is therefore devoted to work done on calculating these quantities taking into account the modified impurity scattering. The criterion for the electron gas system to be strongly coupled to the EM wave is that the interaction strength between a free electron and the wave, characterized by the coefficient

$$g = \frac{eE\hbar k_F}{m\omega}, \quad (2.1)$$

should be far larger than the electron dephasing rate τ_0 which depends in turn on its interaction with disorder. In the single band model system studied in this thesis, this dephasing rate is approximately the momentum scattering rate. In other words, $g \gg \hbar/\tau_0$.

The chapter is divided as follows. In Section 2.2. I derive the electron scattering rate in a 2DEG system where the energy spectrum is parabolic. In Section 2.3. the conductivity of a 2DEG is calculated using the Boltzmann semi-classical equation and the results discussed. Section 2.3. deals with the weak localization in the presence of an EM wave while in the last section the spin relaxation is considered using a Green's function formalism.

2.2. Dressed parabolic systems

A free 2DEG gas with electron effective mass m in a semiconductor quantum well is described by the single particle Hamiltonian

$$\hat{H}_0 = \frac{\hat{\mathbf{p}}^2}{2m} \quad (2.2)$$

where $\hat{\mathbf{p}} = i\hbar\nabla$. Let the gas lie along the xy -plane and let an intense homogeneous continuous wave with frequency ω , say, a laser field, illuminate the plane perpendicularly. For arbitrary strengths of the irradiation, the interaction of the electron gas with the field has to be considered unperturbatively. The bare 2DEG dispersion, without irradiation, is given by $E(\mathbf{k}) = \hbar^2 k^2 / 2m$. If $\mathbf{A}_\omega(t)$ is the time-periodic vector potential of the laser field then the substitution

$$\hat{\mathbf{p}} = i\hbar\nabla \mapsto \hat{\mathbf{p}} = i\hbar\nabla - e\mathbf{A}_\omega(t) \quad (2.3)$$

has to be made in Eq. 2.2, so that the time-dependent Hamiltonian of the electrons interacting with the field is given by

$$\hat{H}(t) = \frac{[i\hbar\nabla - e\mathbf{A}_\omega(t)]^2}{2m}. \quad (2.4)$$

Assuming that the laser field doesn't induce any inter-subband transitions, the momentum operator acts only on the in-plane coordinates. For this purpose, it is therefore necessary that the photon energy of the field, $\hbar\omega$, be far away from any such transition resonances. To find the dressed electron energy spectrum, the time-dependent

Schrödinger equation

$$i\hbar \frac{\partial}{\partial t} \Psi(\mathbf{r}, t) = \hat{H}(t) \Psi(\mathbf{r}, t) \quad (2.5)$$

is solved. The solutions to this equation with a Hamiltonian periodic in time, from a mathematical viewpoint, are derived through Floquet theory. The Floquet theorem states that for a differential operator periodic in time, the solutions have the form

$$\psi_\alpha(\mathbf{r}, t) = \exp(-iE_\alpha t/\hbar) \varphi_\alpha(\mathbf{r}, t) \quad (2.6)$$

where E_α are the quasi-energies and the functions φ_α have the same periodicity as the laser field, i.e. $\varphi_\alpha(t + T) = \varphi_\alpha(t)$ where $T = 2\pi/\omega$ is the period of the wave. The quasi-energies satisfy the equation

$$\left[i\hbar \frac{\partial}{\partial t} - \hat{H}(t) \right] \varphi_\alpha(\mathbf{r}, t) = E_\alpha \varphi_\alpha(\mathbf{r}, t) \quad (2.7)$$

and are only relevant in the region

$$-\frac{\hbar\omega}{2} \leq E_\alpha < \frac{\hbar\omega}{2} \quad (2.8)$$

in the sense that any solution $\varphi_{\alpha'}$ obtained with $\varepsilon_{\alpha'} = \varepsilon_\alpha + n\hbar\omega$ outside the range is equivalent to φ_α . One can look at the Floquet theorem as the time-domain analogue of the Bloch theorem and the time periodic functions φ_α as the space-periodic Bloch functions in a periodic crystal lattice.

We consider the laser field to be perfectly polarized. Let the first case be that of circular polarization, since it preserves the isotropy of the 2DEG. The geometry is

shown in Fig. 2.1(a). The vector potential can be written as

$$\mathbf{A}_\omega(t) = \mathbf{e}_x(\mathcal{E}_\omega/\omega) \sin(\omega t) + \mathbf{e}_y(\mathcal{E}_\omega/\omega) \cos(\omega t) \quad (2.9)$$

with $\mathcal{E}_\omega$ being the electric field amplitude of the wave and $\mathbf{e}_x$ and $\mathbf{e}_y$ being the unit vectors along the x and y directions, respectively.

Figure 2.1: (a). An isotropic 2DEG system is subjected to a circularly polarized electromagnetic field. (b). A linearly polarized electromagnetic waves illuminates the plane of the 2DEG, introducing anisotropy to the impurity scattering of electrons.

In this case, the solutions are given by

$$\psi_{\mathbf{k}}(\mathbf{r}, t) = \exp \left[-i \left(\frac{E_{\mathbf{k}} t}{\hbar} + \frac{\mathcal{E}_\omega^2 e^2 t}{2m\omega^2 \hbar} - \frac{\mathcal{E}_\omega e k_y}{m\omega^2} \sin \omega t + \frac{\mathcal{E}_\omega e k_x}{m\omega^2} \cos \omega t \right) \right] \varphi_{\mathbf{k}}(\mathbf{r}) \quad (2.10)$$

where $\varphi = \exp(i\mathbf{k}\mathbf{r})/\sqrt{S}$ are electron plane waves and S is the surface area of the quantum well. The solution in Eq. 2.10 is better represented if we make a change of coordinates and go to the polar system (k, θ) so that $k_x = k \cos \theta$ and $k_y = k \sin \theta$ where $k = |\mathbf{k}|$, namely

$$\psi_{\mathbf{k}}(\mathbf{r}, t) = \exp \left[-i \left(\frac{E_{\mathbf{k}} t}{\hbar} + \frac{\mathcal{E}_\omega^2 e^2 t}{2m\omega^2 \hbar} + \frac{\mathcal{E}_\omega e k}{m\omega^2} \cos(\omega t + \theta) \right) \right] \varphi_{\mathbf{k}}(\mathbf{r}) \quad (2.11)$$

from which we can recognize

$$E_\alpha = E_{\mathbf{k}} + \frac{\mathcal{E}_\omega^2 e^2}{2m\omega^2}, \quad (2.12)$$

and

$$\varphi_\alpha(\mathbf{r}, t) = \exp\left(-i\frac{\mathcal{E}_\omega e k}{m\omega^2} \cos(\omega t + \theta)\right) \varphi_{\mathbf{k}}(\mathbf{r}), \quad (2.13)$$

In the absence of irradiation $E_\alpha = E(\mathbf{k})$ and the wavefunctions are the bare Bloch states.

On the other hand, if the polarization of the laser field is linear, see Fig.2.1(b), the vector potential can be chosen as

$$\mathbf{A}_\omega(t) = \frac{\mathbf{E}_\omega}{\omega} \cos \omega t \quad (2.14)$$

and the dressed electron states are found to be

$$\psi_{\mathbf{k}}(\mathbf{r}, t) = \exp\left[-i\left(\frac{E_{\mathbf{k}} t}{\hbar} + \frac{\mathcal{E}_\omega^2 e^2 t}{4m\omega^2 \hbar} + \frac{\mathcal{E}_\omega^2 e^2}{8m\omega^3 \hbar} \sin 2\omega t - \frac{e\mathbf{E}_\omega \mathbf{k}}{m\omega^2} \sin \omega t\right)\right] \varphi_{\mathbf{k}}(\mathbf{r}) \quad (2.15)$$

Any single-particle state can be expressed as a linear superposition of the wavefunctions appearing in Eqs. 2.11 and 2.15,

$$\Psi(\mathbf{r}, t) = \sum_{\mathbf{k}'} a_{\mathbf{k}'}(t) \psi_{\mathbf{k}'}(\mathbf{r}, t). \quad (2.16)$$

with time-dependent coefficients $a_{\mathbf{k}'}$. The field dressing of the free electron states

doesn't change the kinetic observable expectation values. Especially, the expectation value of the canonical momentum $\hat{\mathbf{p}} - e\mathbf{A}(t)$ averaged over the period of the wave T

$$\mathbf{p}(\mathbf{k}) = \frac{1}{T} \int_0^T \langle \hat{\mathbf{p}} - e\mathbf{A}(t) \rangle dt = \hbar\mathbf{k} \quad (2.17)$$

so that it is not affected by the field. This allows for the treatment of electrons within the Fermi-Dirac statistics.

2.3. Impurity scattering

The calculation of the DC conductivity of a 2DEG in the presence of such a dressing field requires first of all knowledge of how free electrons are scattered by disorder. At zero temperature, where all phonon modes are frozen and don't provide disorder, the main scattering occurs due to charged impurities and defects. A single scatterer is attributed the potential $U_i(\mathbf{r} - \mathbf{r}_i)$ where $\mathbf{r}_i$ is the scatterer's position. The total Hamiltonian can then be written as

$$\hat{H}_t(\mathbf{r}, t) = \hat{H}(t) + \hat{H}'(\mathbf{r}) \quad (2.18)$$

where $\hat{H}(t)$ is the time-periodic Hamiltonian containing the dressing field and the disorder term $\hat{H}'$ is a summation of all the single-scatterer potentials at position $\mathbf{r}_i$,

$$\hat{H}' = \sum_{i=1}^N U_i(\mathbf{r} - \mathbf{r}_i), \quad (2.19)$$

The disorder term satisfies the inequality

$$\hat{H}' \ll \hat{H}(t), \quad (2.20)$$

which allows us to treat the scattering perturbatively using the solutions above. In the scope of this thesis, the simplifying assumption is made that all the scatterers on the 2DEG plane can be represented by the same potential so that the subscript i can be omitted in the disorder term in the total Hamiltonian.

Given that at time $t = 0$, an electron is in the state $\psi_{\mathbf{k}}(r, t)$, we can find the probability that it scatters to the state $\psi_{\mathbf{k}'}(\mathbf{r}, t)$ by solving the full Schrödinger equation with the insertion of the wavefunction from Eq. 2.16. Namely,

$$i\hbar \frac{\partial}{\partial t} \sum_{\mathbf{k}'} a_{\mathbf{k}'}(t) \psi_{\mathbf{k}'}(\mathbf{r}, t) = \hat{H}_t \sum_{\mathbf{k}'} a_{\mathbf{k}'}(t) \psi_{\mathbf{k}'}(\mathbf{r}, t) \quad (2.21)$$

with the initial conditions

$$a_{\mathbf{k}'} = \delta_{\mathbf{k}', \mathbf{k}} \quad (2.22)$$

where $\delta_{\mathbf{k}', \mathbf{k}}$ is the Kronecker delta symbol. Applying perturbation theory to first order, the scattered state coefficient $a_{\mathbf{k}'}$ can be reduced to the form

$$a_{\mathbf{k}'} = -i \frac{U_{\mathbf{k}', \mathbf{k}} Z_{\mathbf{k}', \mathbf{k}}(t)}{\hbar} \quad (2.23)$$

where

$$U_{\mathbf{k}', \mathbf{k}} = \langle \varphi_{\mathbf{k}'} | \hat{H}' | \varphi_{\mathbf{k}} \rangle \quad (2.24)$$

is the matrix element of the disorder potential and

$$Z_{\mathbf{k}',\mathbf{k}}(t) = \int_0^t \exp(i[E_{\mathbf{k}'} - E_{\mathbf{k}}]t'/\hbar) \exp(ig_{\mathbf{k}',\mathbf{k}}(t')) dt' \quad (2.25)$$

contains the scattering adjustment induced by the dressing field. Here, the function $g_{\mathbf{k}',\mathbf{k}}(t)$ depends on whether the dressing field is linearly or circularly polarized. In the former case, it is given by

$$g_{\mathbf{k}',\mathbf{k}}^L(t) = \frac{e\mathbf{E}_\omega(\mathbf{k}' - \mathbf{k})}{m\omega^2} \sin \omega t \quad (2.26)$$

while in the latter it is

$$g_{\mathbf{k}',\mathbf{k}}^R(t) = \frac{2e\mathcal{E}_\omega k}{m\omega^2} \sin\left(\frac{\zeta}{2}\right) \sin\left(\omega t + \frac{\theta + \theta'}{2}\right) \quad (2.27)$$

where $k = |\mathbf{k}'| = |\mathbf{k}|$ from the condition of elastic scattering and $\zeta = \theta - \theta'$ is the angle from $\mathbf{k}'$ to $\mathbf{k}$.

The probability that the electron be found in the scattered state $\psi_{\mathbf{k}'}(\mathbf{r}, t)$ is then the squared modulus of $a_{\mathbf{k}'}(t)$, namely

$$|a_{\mathbf{k}'}(t)|^2 = \frac{|U_{\mathbf{k}',\mathbf{k}}|^2 |Z_{\mathbf{k}',\mathbf{k}}(t)|^2}{\hbar^2} \quad (2.28)$$

Calculating $|U_{\mathbf{k}',\mathbf{k}}|^2$ from Eq. 2.24 using Eq. 2.19 we write out

$$\begin{aligned} |U_{\mathbf{k}',\mathbf{k}}|^2 &= \frac{1}{S^2} \sum_{i=1}^N \left| \int \exp(-i[\mathbf{k}' - \mathbf{k}] \cdot \mathbf{r}) U(\mathbf{r} - \mathbf{r}_i) d\mathbf{r} \right|^2 \\ &+ \frac{1}{S^2} \sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N \int \exp(-i[\mathbf{k}' - \mathbf{k}] \cdot \mathbf{r}_1) U(\mathbf{r}_1 - \mathbf{r}_i) d\mathbf{r}_1 \\ &\times \int \exp(-i[\mathbf{k}' - \mathbf{k}] \cdot \mathbf{r}_2) U(\mathbf{r}_2 - \mathbf{r}_j) d\mathbf{r}_2. \end{aligned} \quad (2.29)$$

Considering we have a macroscopically large number of randomly distributed identical scatterers, the long second term in the above equation adds up to zero, so effectively

$$|U_{\mathbf{k}',\mathbf{k}}|^2 = N|U(\mathbf{k}' - \mathbf{k})|^2, \quad (2.30)$$

where

$$U(\mathbf{k}' - \mathbf{k}) = \frac{1}{S} \int \exp(-i[\mathbf{k}' - \mathbf{k}] \cdot \mathbf{r}) U(\mathbf{r}) d\mathbf{r} \quad (2.31)$$

is the Fourier transform of the single scatterer potential. If we approximate it by a Dirac delta potential, $U(\mathbf{r}) = U_0\delta(\mathbf{r})$, which is valid for long wavelength s-wave scattering in the low-excitation energy regime, then $U(\mathbf{k}' - \mathbf{k})$ is a constant which doesn't depend on any wavevector characteristics.

Now, for the calculation of $|Z_{\mathbf{k}',\mathbf{k}}(t)|^2$ in Eq.2.23 we treat the two cases of dressing field polarization individually. By inserting Eq. 2.26 in Eq. 2.25 and using the Jacobi-

Anger identity for exponential functions with periodic arguments,

$$\exp(iz \sin \phi) = \sum_{n=-\infty}^{\infty} J_n(z) \exp(in\phi) \quad (2.32)$$

for a real number z where J_n is an n -th order Bessel function of the first kind, we get

$$\begin{aligned} |Z_{\mathbf{k}',\mathbf{k}}(t)|^2 &= \left| \exp(i[E_{k'} - E_k]t/\hbar) \exp(ig_{\mathbf{k}',\mathbf{k}}^L(t')dt') \right|^2 \\ &= \left| \sum_{n=-\infty}^{\infty} J_n \left(\frac{e\mathbf{E}_\omega(\mathbf{k}' - \mathbf{k})}{m\omega^2} \right) \exp(i[E_{k'} - E_k + n\hbar\omega]t/2\hbar) \right. \\ &\quad \times \left. \int_{-t/2}^{t/2} \exp(i[E_{k'} - E_k + n\hbar\omega]t'/\hbar) dt' \right|^2. \end{aligned} \quad (2.33)$$

For a sufficiently long time, the integrals above, numbered by n , become Dirac delta functions, i.e.

$$\frac{1}{2\pi\hbar} \lim_{t \rightarrow \infty} \int_{-t/2}^{t/2} \exp(iEt'/\hbar) dt' = \delta(E) \quad (2.34)$$

and so

$$|Z_{\mathbf{k}',\mathbf{k}}|_{t=\infty}^2 = (2\pi\hbar)^2 \sum_{n=-\infty}^{\infty} J_n^2 \left(\frac{e\mathbf{E}_\omega(\mathbf{k}' - \mathbf{k})}{m\omega^2} \right) \delta^2(E_{k'} - E_k + n\hbar\omega). \quad (2.35)$$

A little mathematical trick using Eq. 2.34 allows us to write

$$\begin{aligned}
 \delta^2(E_{k'} - E_k + n\hbar\omega) &= \delta(E_{k'} - E_k + n\hbar\omega)\delta(0) \\
 &= \frac{\delta(E_{k'} - E_k + n\hbar\omega)}{2\pi\hbar} \lim_{t \rightarrow \infty} \int_{-t/2}^{t/2} \exp(i0t'/\hbar) dt' \\
 &= \frac{\delta(E_{k'} - E_k + n\hbar\omega)t}{2\pi\hbar}
 \end{aligned} \tag{2.36}$$

and so, finally, using Eqs. 2.28, 2.30, 2.35 and 2.36 we arrive at

$$|a_{\mathbf{k}'}(t)|^2 = \frac{2\pi N |U(\mathbf{k}' - \mathbf{k})|^2 t}{\hbar} \sum_{n=-\infty}^{\infty} J_n^2 \left(\frac{e\mathbf{E}_\omega(\mathbf{k}' - \mathbf{k})}{m\omega^2} \right) \delta(E_{k'} - E_k + n\hbar\omega). \tag{2.37}$$

The transition rate per unit time from the initial state with wavevector $\mathbf{k}$ to a state with wavevector $\mathbf{k}'$ is defined by

$$w_{\mathbf{k}', \mathbf{k}} = \frac{d|a_{\mathbf{k}'}(t)|^2}{dt}, \tag{2.38}$$

$$w_{\mathbf{k}', \mathbf{k}}^{(l)} = \frac{2\pi N |U(\mathbf{k}' - \mathbf{k})|^2}{\hbar} \sum_{n=-\infty}^{\infty} J_n^2 \left(\frac{e\mathbf{E}_\omega(\mathbf{k}' - \mathbf{k})}{m\omega^2} \right) \delta(E_{k'} - E_k + n\hbar\omega) \tag{2.39}$$

Doing the same lengthy calculations for the case of circularly polarized EM field, we arrive at the similar expression as in Eq. 2.39 but with the Bessel function having a different argument,

$$w_{\mathbf{k}', \mathbf{k}}^{(c)} = \frac{2\pi N |U(\mathbf{k}' - \mathbf{k})|^2}{\hbar} \sum_{n=-\infty}^{\infty} J_n^2 \left(\frac{2e\mathcal{E}_\omega k}{m\omega^2} \sin \left(\frac{\zeta}{2} \right) \right) \delta(E_{k'} - E_k + n\hbar\omega), \tag{2.40}$$

where the crucial difference is precisely that argument. In Eq. 2.39, there appears the electric field vector which renders the scattering rate dependent on the absolute

value of the angles of the wavevectors belonging to the initial and final states. In Eq. 2.40, the scattering rate depends solely on the electric field amplitude and only the angle ζ between the scattered and initial electron state takes place. For the linear polarization case, this is bound to have an effect on the geometry of the conductivity tensor, as we will see later on.

In the above expressions for the scattering rates, the terms with $n \neq 0$ describe intraband absorption phenomena, which can be neglected in the regime of the condition

$$\omega\tau_0 \gg 1 \quad (2.41)$$

where τ_0 is the relaxation time representing collisional broadening. One can look at their exclusion as a fact coming from the forbidden absorption of a photon by a free electron since the process doesn't conserve momentum and energy at the same time. We thus end up with the formulas

$$w_{\mathbf{k}',\mathbf{k}}^{(l)} = \frac{2\pi N |U(\mathbf{k}' - \mathbf{k})|^2}{\hbar} J_0^2 \left(\frac{e\mathbf{E}_\omega(\mathbf{k}' - \mathbf{k})}{m\omega^2} \right) \delta(E_{k'} - E_k) \quad (2.42)$$

and

$$w_{\mathbf{k}',\mathbf{k}}^{(c)} = \frac{2\pi N |U(\mathbf{k}' - \mathbf{k})|^2}{\hbar} J_0^2 \left(\frac{2e\mathcal{E}_\omega k}{m\omega^2} \sin \left(\frac{\zeta}{2} \right) \right) \delta(E_{k'} - E_k) \quad (2.43)$$

in their final form which will be used to calculate the conductivity of a 2DEG. The effect coming from the dressing field is therefore to add the factor of a squared Bessel function to the well-known result for the scattering rate in the absence of such a field,

$$w_{\mathbf{k}',\mathbf{k}}^{(0)} = \frac{2\pi N |U(\mathbf{k}' - \mathbf{k})|^2}{\hbar} \delta(E_{k'} - E_k), \quad (2.44)$$

and since

$$0 \leq J_0^2(u) \leq 1 \quad (2.45)$$

for any real number u , the "bare" scattering rate will be reduced for all wavevectors and field parameters, since the electronic energy dispersion is unaffected [92].

2.4. Conductivity of a dressed 2DEG

In the attempt of calculating the conductivity of a 2DEG, the logical step after finding the scattering rate due to collisions with impurities, is to employ the Boltzmann semiclassical transport equation, presented in Eq. 1.5, in the presence of a probe DC electric field $\mathcal{E}_{\text{dc}} = -\nabla V_{\text{dc}}(\mathbf{r})$,

$$\frac{\partial f(\mathbf{r}, \mathbf{k}, t)}{\partial t} = -\frac{e}{\hbar} \mathcal{E}_{\text{dc}} \cdot \nabla_{\mathbf{k}} f(\mathbf{r}, \mathbf{k}, t) + I[f(\mathbf{r}, \mathbf{k}, t)], \quad (2.46)$$

for the distribution function f where spatial homogeneity is presumed, a point valid in the absence of any temperature gradient. The role of the dressing field enters only in the collision integral $I[f]$

$$I[f(\mathbf{r}, \mathbf{k}, t)] = \int_{\text{1stBZ}} [f(\mathbf{r}, \mathbf{k}', t) - f(\mathbf{r}, \mathbf{k}, t)] w_{\mathbf{k}\mathbf{k}'} d\mathbf{k}'. \quad (2.47)$$

Since a current measurement probes the electron flow a long time after the voltage V_{dc} is applied, we seek the steady state distribution function $f(\mathbf{k})$ obtained by setting

the time derivative in Eq. 2.46 to zero by definition, such that

$$\frac{e}{\hbar} \mathcal{E}_{\text{dc}} \cdot \nabla_{\mathbf{k}} f(\mathbf{k}) = \int_{\text{1stBZ}} [f(\mathbf{k}') - f(\mathbf{k})] w_{\mathbf{k}\mathbf{k}'} d\mathbf{k}'. \quad (2.48)$$

If $\mathcal{E}_{\text{dc}}$ is weak, we can write

$$f(\mathbf{k}) = f_0(\mathbf{k}) + \Delta f(\mathbf{k}) \quad (2.49)$$

where $f_0(k)$ is the equilibrium Fermi-Dirac distribution which is taken to be isotropic and thus contributing to no net current and at temperatures close to zero

$$\frac{1}{\hbar} \nabla_{\mathbf{k}} f_0(\mathbf{k}) = \delta(E - E_F) \mathbf{v}(\mathbf{k}) \quad (2.50)$$

while the function $\Delta f(k)$ represents a small change in the distribution introduced by the applied probe field [4]. A general relaxation time $\tau(\mathbf{k})$ which depends not only on the electron energy, but on both the wavevector modulus and its direction can be defined by [93]

$$I[f] = -\frac{\Delta f(\mathbf{k})}{\tau(\mathbf{k})} \quad (2.51)$$

Replacing f by f_0 in the left hand side of Eq. 2.48 where we insert Eqs. 2.50 and 2.51, we get

$$\frac{1}{\tau(\mathbf{k})} = \int \left[1 - \frac{\tau(\mathbf{k}') \mathcal{E}_{\text{dc}} \cdot \mathbf{v}(\mathbf{k}')}{\tau(\mathbf{k}) \mathcal{E}_{\text{dc}} \cdot \mathbf{v}(\mathbf{k})} \right] w_{\mathbf{k}'\mathbf{k}} d\mathbf{k}' \quad (2.52)$$

Inserting the obtained scattering rates for the two cases of laser field polarization, namely Eqs. 2.42 and 2.43, the relaxation time is obtained by a numerical solution of the integral equation using any of the standard quadrature algorithms available.

Upon using the relaxation time method to find the $\Delta f(\mathbf{k})$ from the linearized Boltzmann equation, the current density $\mathbf{J}$ becomes

$$\mathbf{J} = \frac{e^2}{2\pi^2} \int_{\mathbf{k}} [\mathcal{E}_{\text{dc}} \cdot \mathbf{v}(\mathbf{k})] \tau(\mathbf{k}) \mathbf{v}(\mathbf{k}) \delta(E_k - E_F) d\mathbf{k}. \quad (2.53)$$

The components of the conductivity tensor are defined by

$$\sigma_{ij} = \frac{J_i}{\mathcal{E}_j}. \quad (2.54)$$

with i and j being one of the in-plane directions x or y . For the circularly polarized dressing field, the direction of the probe field is of no importance and so $\sigma_{xx} = \sigma_{yy} \equiv \sigma_c$ and its ratio to the bare conductivity σ_0 is given by

$$\frac{\sigma_c}{\sigma_0} = 2\pi \int_0^{2\pi} [1 - \cos(\theta)] J_0^2 \left(\frac{2e\mathcal{E}_\omega k}{m\omega^2} \sin\left(\frac{\theta}{2}\right) \right) d\theta. \quad (2.55)$$

For a 2DEG dressed with a linearly polarized light with polarization axis along the x -direction, the conductivity parallel to it $\sigma_{xx} \equiv \sigma_{\parallel}$ is

$$\frac{\sigma_{\parallel}}{\sigma_0} = \frac{1}{\pi} \int_0^{2\pi} \frac{\tau_{\parallel}(\theta)}{\tau_0} \cos^2(\theta) d\theta \quad (2.56)$$

while the conductivity in the perpendicular direction, $\sigma_{yy} \equiv \sigma_{\perp}$, is given by

$$\frac{\sigma_{\perp}}{\sigma_0} = \frac{1}{\pi} \int_0^{2\pi} \frac{\tau_{\perp}(\theta)}{\tau_0} \sin^2(\theta) d\theta. \quad (2.57)$$

By symmetry considerations it is clear that no current flows in the direction normal to

Figure 2.2: The conductivity of 2DEG in a GaAs quantum well illuminated with a dressing electromagnetic field of frequency $\omega = 10^{11}$ rad/s at zero temperature for Fermi energy $E_F = 10$ meV and electron effective mass $m = 0.067m_0$.

the applied probe field $\mathcal{E}_{dc}$, so the transverse conductivities are zero, i.e. $\sigma_{xy} = \sigma_{yx} = 0$.

The desired conductivities $\sigma_{\parallel}$, $\sigma_{\perp}$ and σ_c depending on the dressing field intensity have been plotted in Fig. 2.2. We see that the application of a high-frequency dressing field leads to the enormous increase of the conductivity which is attributed directly to the suppression of scattering by static disorder. Mathematically speaking, the oscillatory behavior is attributed to the Bessel function appearing in the renormalized scattering probabilities in Eqs. 2.42 and 2.43, which depend on the dressing field amplitude $\mathcal{E}_{\omega}$.

Qualitatively, the scattering probability is described by the wavefunction overlap of waves with different wavevectors in the area of the potential $U(\mathbf{r})$. This overlap is related to the phases of the electron wavefunction in the dressing field which are proportional to $\mathcal{E}_\omega \cos \omega t$ and $\mathcal{E}_\omega \sin \omega t$ and so it can be made to vanish which gives a vanishing scattering rate. This, in turn, gives an increased conductivity.

The anisotropy in the conductivity in the presence of a linearly polarized light, the fact that $\sigma_\perp \neq \sigma_\parallel$, is a courtesy of the scattering anisotropy, as said before. The scattering anisotropy is a consequence of the argument of the Bessel function being dependent on the polarization vector of the dressing field. When calculating the conductivity via the Boltzmann equation, another vectorial quantity is taken into account, namely the probe field $\mathcal{E}_{dc}$, and therefore the current density depends on the relative direction of two vectors instead of one, which is the case for an isotropic medium. These specific values of frequency were chosen in producing the plots of conductivity because they belong to the lowest range which satisfies the above condition on the relative magnitude of EM wave and the relaxation time characteristic of a 2DEG in a GaAs quantum well. According to this model, higher frequencies should only change the conductivity as long as the relation above holds. Note however that this doesn't mean it's likely the results would be able to be reproduced experimentally at higher frequencies, for example at optical ones, because the light intensity required becomes enormous. To see this, we need to look again at the Bessel function appearing in the modified scattering rates. Their arguments are proportional to the dressing field amplitude $\mathcal{E}_\omega$ and inversely proportional to the frequency squared, ω^2 . This means that if ω is increased by a factor of only 10, the dressing field intensity, being proportional

to the field amplitude squared, must be increased by a factor of 10^4 in order to see the same conductivity behavior! This puts a practical upper bound on the frequencies available for experimentation.

Now, why does the conductivity show oscillations and what can be said about the position of the local maxima and minima? To understand roughly what is happening as the dressing field intensity is increased, let's look back to the expression of the conductivity in the circular polarization case, namely Eq. 2.55. The conductivity σ_c takes a local minimum (maximum) when the integral takes a local maximum (minimum). Because the integrand is always larger than zero regardless of the angle θ or system parameters, the integral is always larger than zero. If the field intensity is zero, $\mathcal{E}_\omega = 0$, the largest contribution to the integral is the backscattering process, $\theta = \pi$. As $\mathcal{E}_\omega$ is increased, the integral starts to decrease because for substantial intervals of angles of the elastic scattering circle, the Bessel function is less than unity. Locally, on the horizontal axis of Fig. 2.2, the biggest reduction of the integral, resulting in its minima, occurs when the parameter $\alpha = 2e\mathcal{E}_\omega k_F / m\omega^2$ is a root of the zeroth order Bessel function because its argument becomes exactly this root at the backscattering angle since $\sin \frac{\pi}{2} = 1$. Therefore, the positions of the peaks in the conductivity σ_c are determined by the roots of the zeroth order Bessel function. Unfortunately, there are no closed-form expressions of the Bessel function zeros from which we can predict the conductivity maxima. What is important to note is that the conductivity shows maxima everytime backscattering is suppressed. The analysis of the oscillations of the conductivity in the linear polarization case is little bit more complicated due to the anisotropy but its extrema are related to the Bessel function zeros as well.

The conductivity, calculated coherently in the strong coupling regime between light and free electrons, depends on the electric field intensity of the incoming radiation. This means that we can look at the effect deduced here as belonging to the more general ac Stark effect. However, since only the free-free scattering matrix elements $w_{\mathbf{k}'\mathbf{k}}$ between delocalized states with wavevectors $\mathbf{k}$ and $\mathbf{k}'$ are needed to calculate the conductivity in the presence of radiation, this effect is distinct from previous work done around the ac Stark effect in semiconductors, where the energy shifts of quantum well subbands or of bound electron-hole (excitons) states were considered.

To reiterate, the scatterers were assumed to be uniform, that is, all scattering centers were assumed to possess the same potential. In the presence of an EM wave, the scattering matrix element corresponding to the collision of an electron with a single scatterer is modified by the appearance of the squared Bessel function with arguments which don't depend on a particular scattering potential profile. Therefore, the theory is expected to be applicable also to the case of non-uniform scatterers. Moreover, the theory doesn't rely on any resonance effects, that is, the single impurity potential is assumed to be structureless. The only consideration in terms of electron energy is that the Fermi wavelength should be much larger than the range of the single potentials such that s-wave scattering dominates.

2.5. Weak localization corrections in dressed 2DEG

The weak localization correction to the conductivity, $\Delta\sigma$, stems in the multiple scattering of electrons with disorder. As the scattering probability is affected by the illumination of the 2DEG by a high-frequency dressing field, weak localization is ex-

pected to change as well.

To find $\Delta\sigma$, all the possible closed paths which an electron can traverse need to be taken into consideration. The ratio of $\Delta\sigma$ to the bare conductivity σ_0 is obtained from the well-known formula

$$\frac{\Delta\sigma}{\sigma_0} = -\frac{\hbar}{2\pi m} \int_{\tau_F}^{\tau_{ph}} C(t) dt, \quad (2.58)$$

where $C(t)$ is the probability that the electron is to be found at the starting point of its closed path at time t , averaged over the whole 2DEG plane [94]. The quantum lifetime or the mean free time of the electron with Fermi energy is defined as

$$\frac{1}{\tau_F} = \int_F w_{\mathbf{k}'\mathbf{k}} d\mathbf{k}' \quad (2.59)$$

which is calculated by applying the obtained scattering rates from the last section. In Eq. 2.58, τ_{ph} is the electron phase-breaking time which depends on inelastic scattering and external magnetic fields which diminish weak localization;

$$\frac{1}{\tau_{ph}} = \frac{1}{\tau_{ph,ie}} + \frac{1}{\tau_{ph,B}} \quad (2.60)$$

where the first term on the right hand side is due to inelastic scattering and we take it to be constant which doesn't depend on the dressing field. The magnetic phase breaking time is given by

$$\frac{1}{\tau_{ph,B}} = \frac{4eDB}{\hbar} \quad (2.61)$$

where D is the diffusion constant and B is the strength of a magnetic field applied

normal to the 2DEG plane, as discussed in the introduction of this thesis.

The quantum lifetime at the Fermi level, τ_F , just like the relaxation time of the last section, depends on the initial state of the electron represented by the wavevector $\mathbf{k}$ due to the anisotropic scattering rate $w_{\mathbf{k}'\mathbf{k}}$ in the case when the 2DEG is subjected to a linearly polarized dressing field. The weak localization correction, therefore, is anisotropic.

To address this anisotropy, let $\tau_{F,\parallel}$ and $\tau_{F,\perp}$ be the mean free times when the initial wavevector $\mathbf{k}$ is parallel and perpendicular to the polarization vector of the linearly polarized field $\mathbf{E}_\omega(t)$, respectively. By inserting these relaxation times in the lower limit of the integral in Eq. 2.58, we are able to get the corrections to the conductivity in the directions parallel and perpendicular to $\mathbf{E}_\omega(t)$, $\Delta\sigma_\parallel$ and $\Delta\sigma_\perp$, respectively. In the separate case of a circularly polarized irradiation, the weak localization correction $\Delta\sigma_c$ is isotropic.

The probability $C(t)$ in Eq. 2.58 is calculated by using the Feynman path integral [95, 96]

$$C(t) = \int \mathcal{D}_x \mathcal{D}_y \exp \left[- \int_{-t/2}^{t/2} \frac{\dot{x}^2}{4D_x} + \frac{\dot{y}^2}{4D_y} - \frac{i}{\hbar} e\mathbf{r}[\mathcal{E}_\omega(t') - \mathcal{E}_\omega(-t')]dt' \right], \quad (2.62)$$

and the integration is done along closed curves per unit area. The two terms in the integrand depending on $\dot{x}$ and $\dot{y}$ give the diffusive electron kinetic energy, along the respective directions, and the diffusion constants are given by the usual expressions

Figure 2.3: The dependence of weak localization corrections $\Delta\sigma$ to the conductivity of 2DEG on the intensity of irradiation by a dressing field of frequency $\omega = 10^{11}$ rad/s. Calculations are performed for 2DEG in a GaAs quantum well at absolute zero temperature and Fermi energy $E_F = 10$ meV, inelastic phase breaking time $\tau_{\text{ph.ie.}} = 80$ ps and zero magnetic field. The dash-dotted (full) line represents $\Delta\sigma$ in the case of a circularly (linearly) polarized dressing field.

$D_x = v_F^2 \tau_{F,\parallel} / 2$ and $D_y = v_F^2 \tau_{F,\perp} / 2$, where as before, E_ω is assumed to lie along the x -direction. The Fermi velocity is defined as $v_F = \hbar^{-1} |\nabla_{\mathbf{k}} E(\mathbf{k})|_{|\mathbf{k}|=k_F}$. For a circularly polarized dressing field, all the subscripts pertaining to different directions of the diffusion constants are omitted. The last term in the integrand arises from the electron potential energy in the dressing field and represents the electron self-interference between two oppositely oriented trajectories.

The calculations of the weak localization corrections $\Delta\sigma_c$ and $\Delta\sigma_\parallel$ are plotted against the intensity of the dressing field in Fig. 2.3. Note that the ratio $\Delta\sigma/\Delta\sigma_0$ is

plotted where $\Delta\sigma_0$ is the correction in the absence of the radiation. In the plot, using the typical GaAs electron parameters, the strength of the magnetic field is zero, Fermi energy $E_F = 10$ meV and temperature $T = 0$ K. The phase-breaking time related to inelastic scattering is taken to be $\tau_{\text{ph.ie.}} = 80$ ps.

The main observation is that the corrections are washed out with increasing intensity which is traced back to the suppression of impurity scattering. Since the chances for an electron undergoing multiple scattering to travel a fully closed path inside a given time window depend on its duration, the upper limit being set by the phase-breaking time $\tau_{\text{ph.ie.}}$, while the lower limit is set by the mean free time τ_F which in turn depends on scattering by disorder. Since this last is increased, the time window is shortened and the weak localization corrections are canceled out with increasing dressing field intensity. Due to the non-existence of Joule heating in the electromagnetically dressed 2DEG, following from condition Eq. 2.41, the washing-out of weak localization corrections can't be attributed to the intraband absorption of photons via inelastic scattering.

In Fig. 2.4, the weak localization corrections to the conductivity have been plotted for a fixed value of dressing field intensity $I = 3$ mW/cm², along with the correction in the case of no illumination, versus the applied magnetic field of strength B . By shortening the phase-breaking time, and thus lowering the upper limit of the integral in Eq. 2.58, the external magnetic field destroys the correction effects. In the figure, the effect of the external magnetic field is less for the two curves denoted by $\Delta\sigma_{\parallel}$ and $\Delta\sigma_c$ corresponding to a linearly and circularly polarized dressing field, respectively,

Figure 2.4: The dependence of the weak localization corrections to the conductivity on a magnetic field directed perpendicularly to the 2DEG in the presence of the irradiation with the intensity $I = 3 \text{ mW/cm}^2$. Host parameters the same as in Fig. 2.3.

than the curve for the case of bare 2DEG $\Delta\sigma_0$. Note that in the linear polarization case, only the corrections along the polarization vector have been plotted since $\Delta\sigma_{||} \approx \Delta\sigma_{\perp}$ and so the difference is very difficult to be measured in experiment.

2.6. Spin relaxation regimes in the presence of a dressing field

As spin relaxation is related to both the mechanism of spin-orbit coupling and scattering by disorder, it is easy to comprehend that the spin relaxation time is affected by the dressing laser field. A number of mechanisms of spin evolution in 2D systems, especially semiconductor quantum wells, have been studied to minute details, both in

theory and in experiment, but the effect of strong illumination hasn't captured the attention.

In order to get a qualitative view of the effect of illumination by a strong high-frequency dressing field $\mathcal{E}_\omega$ on the spin-relaxation time τ_s , we consider a 2DEG in the presence of a linearly polarized dressing field and spin-orbit interaction of Rashba type. The frequency ω of the continuous laser wave is not in resonance with any inter-band transitions and satisfies Eq. 2.41 so as to inhibit intraband absorption and heating and therefore it is permissible to treat the electromagnetic field as a pure dressing field.

The Hamiltonian of this system,

$$\hat{H}(t) = \frac{[i\hbar\nabla - e\mathbf{A}_\omega(t)]^2}{2m} + \alpha[\boldsymbol{\sigma} \times (\hbar\mathbf{k} - e\mathbf{A}_\omega)]_z \quad (2.63)$$

is obtained from the Hamiltonian in Eq. 2.4 with the addition of the Rashba term where α is the spin-orbit coupling strength constant, $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ is the Pauli spin matrix vector and the subscript z indicates the component along z direction, i.e. normal to the 2DEG plane. Note that the vector potential of the dressing field has been included in the Rashba term as well.

Applying the unitary transformation

$$\begin{aligned} \hat{U} = & \exp \left(i \frac{k_y e \mathcal{E}_\omega}{m \omega^2} \sin \omega t - i \frac{e^2 \mathcal{E}_\omega t}{4 m \omega^2 \hbar} - i \frac{e^2 \mathcal{E}_\omega}{8 m \omega^3 \hbar} \sin 2 \omega t \right) \\ & \times \begin{pmatrix} \exp \left(i \frac{\alpha e \mathcal{E}_\omega}{\hbar \omega^2} \sin \omega t \right) & \exp \left(-i \frac{\alpha e \mathcal{E}_\omega}{\hbar \omega^2} \sin \omega t \right) \\ \exp \left(i \frac{\alpha e \mathcal{E}_\omega}{\hbar \omega^2} \sin \omega t \right) & - \exp \left(-i \frac{\alpha e \mathcal{E}_\omega}{\hbar \omega^2} \sin \omega t \right) \end{pmatrix} \end{aligned} \quad (2.64)$$

to $\hat{H}(t)$ above, we end up with the transformed Hamiltonian

$$\begin{aligned} \hat{H}'(t) = & \hat{U}^\dagger \hat{H}(t) \hat{U} - i \hbar \hat{U}^\dagger \frac{\partial \hat{U}}{\partial t} \\ = & \begin{pmatrix} \hbar^2 k^2 / 2m + \alpha \hbar k_y & -i \alpha \hbar k_x \exp \left(-i \frac{2 \alpha e \mathcal{E}_\omega}{\hbar \omega^2} \sin \omega t \right) \\ i \alpha \hbar k_x \exp \left(i \frac{2 \alpha e \mathcal{E}_\omega}{\hbar \omega^2} \sin \omega t \right) & \hbar^2 k^2 / 2m - \alpha \hbar k_y \end{pmatrix}. \end{aligned} \quad (2.65)$$

The spinor solutions to the time-dependent Schrödinger equation $i \hbar \psi(\mathbf{r}, t) = \hat{H}' \psi(\mathbf{r}, t)$ are of the form

$$\psi(\mathbf{r}, t) = \begin{pmatrix} a_+(\mathbf{r}, t) \\ a_-(\mathbf{r}, t) \end{pmatrix}. \quad (2.66)$$

Making the substitution, we get

$$i \dot{a}_\pm = \left(\frac{\hbar k^2}{2m} \pm \alpha k_y \right) a_\pm \mp i \alpha k_x a_\mp \exp \left(\mp i \frac{2 \alpha e \mathcal{E}_\omega}{\hbar \omega^2} \sin \omega t \right) \quad (2.67)$$

where we expand the last term of the equation by using the Jacobi-Anger identity of Eq. 2.32. If the photon energy of the dressing field is much larger than the Fermi energy E_F and the spin-orbit interaction energy $\alpha \hbar k_F$, where k_F is the Fermi wavevector, then the terms in the Jacobi-Anger expansion with factors $\exp(i n \omega t)$ oscillate very fast and can be omitted, their negligence being tantamount to the rotating wave ap-

proximation.

The resulting equations

$$i\dot{a}_{\pm} = \left(\frac{\hbar k^2}{2m} \pm \alpha k_y \right) a_{\pm} \mp i\alpha k_x a_{\mp} J_0 \left(\frac{2\alpha e \mathcal{E}_{\omega}}{\hbar \omega^2} \right) \quad (2.68)$$

yield the solutions

$$\psi_{\pm}(\mathbf{k}) = \begin{pmatrix} \left[\frac{\sqrt{k_y^2 + J_0^2 \left(\frac{2\alpha e \mathcal{E}_{\omega}}{\hbar \omega^2} \right) k_x^2} \pm k_y}{2\sqrt{k_y^2 + J_0^2 \left(\frac{2\alpha e \mathcal{E}_{\omega}}{\hbar \omega^2} \right) k_x^2}} \right]^{1/2} \\ \pm i \left[\frac{\sqrt{k_y^2 + J_0^2 \left(\frac{2\alpha e \mathcal{E}_{\omega}}{\hbar \omega^2} \right) k_x^2} \mp k_y}{2\sqrt{k_y^2 + J_0^2 \left(\frac{2\alpha e \mathcal{E}_{\omega}}{\hbar \omega^2} \right) k_x^2}} \right]^{1/2} \end{pmatrix} \exp \left(-i \frac{E_{\pm}(\mathbf{k})t}{\hbar} \right) \quad (2.69)$$

with the spin-split spectrum

$$E_{\pm}(\mathbf{k}) = \frac{\hbar^2 k^2}{2m} \pm \alpha \hbar \sqrt{k_y^2 + J_0^2 \left(\frac{2\alpha e \mathcal{E}_{\omega}}{\hbar \omega^2} \right) k_x^2} \quad (2.70)$$

which for the case $\mathcal{E}_{\omega} = 0$ yields the well-known result.

The spin relaxation in the dressed system is calculated using a standard diagram technique, see Refs. [97–99]. In the general case, the single-electron spin vector $\mathbf{S} = (S_x, S_y, S_z)^T$ evolves in accord with the diffusion equation

$$D^{-1} \mathbf{S} = 0 \quad (2.71)$$

where D is a diffuson, the inverse propagator of the spin density fluctuations, calculated using a conventional diagram technique when the impurity disorder is considered

a white noise [100, 101]. The diffuson can be represented as

$$D = (1 - I_{ij}), \quad i, j = x, y, z, \quad (2.72)$$

where the single joint scattering event diagrams are expressed using the advanced (retarded) Green's functions $G^{A(B)}(\mathbf{k}, E_F)$

$$I_{ij} = \frac{\hbar}{2\pi\nu_F\tau_F} \int_F \text{Tr}[G^A(\mathbf{k}, E_F)\sigma_i G^R(\mathbf{k}, E_F + \hbar\omega')\sigma_j] d\mathbf{k}, \quad (2.73)$$

τ_F being the mean free time on the Fermi level given by Eq. 2.59 and ν_F is the Fermi level density of states. The advanced (retarded) Green's functions in fully inverted space $(\mathbf{k}, \omega')$ are defined by

$$G^{A(R)} = \sum_{n=\pm} \frac{\psi_n(\mathbf{k})\psi_n^\dagger(\mathbf{k})}{E_F - E_n(\mathbf{k}) \mp i\hbar/2\tau_F} \quad (2.74)$$

which is averaged in disorder where the two orthogonal spinors and their corresponding energies are given in Eq. 2.69 and 2.70, respectively.

The influence of the dressing field enters into the spin dynamics adjusting the spin relaxation time through the product $\psi_n(\mathbf{k})\psi_n^\dagger(\mathbf{k})$ as well as by increasing the mean free time τ_F , as discussed in the last section. In the case where the disorder is weak ($\hbar/\tau_F E_F \ll 1$), that is, the mean free time is large with respect to the Fermi energy, and the spin-orbit coupling strength is reasonably weak ($\alpha\hbar k_F/E_F \ll 1$), then the integration in Eq. 2.73 becomes relatively simple and we obtain the spin diffusion

equation

$$\dot{S}_{x,y,z} = -(1/\tau_{x,y,z})S_{x,y,z} \quad (2.75)$$

with $\tau_{x,y,z}$ being the relaxation time of spin projections onto the x, y, z -axes, respectively. The anisotropy of the in-plane spin projection relaxation times τ_x and τ_y , in the general case, is a fact that originates in the spectrum $E_{\pm}(\mathbf{k})$ being anisotropic itself. Nevertheless, for the parameter of the spin-orbit coupling strength α and the dressing field photon energy, the spin is very weakly coupled to the field since $2\alpha e\mathcal{E}_\omega/\hbar\omega^2 \ll 1$. In this regime, the anisotropy of spin relaxation depending on an in-plane quantizing axis is negligibly small and we can approximate $\tau_x = \tau_y = \tau_z/2 \equiv \tau_s$ with τ_s being the spin relaxation time in the electromagnetically dressed 2DEG. We obtain that it is given by

$$\tau_s = \frac{1 + 4\xi^2}{2\xi^2} \tau_F \quad (2.76)$$

where $\xi = \tau_F/\tau_{so}$ with $\tau_{so} = 1/\alpha k_F$ being the period of a single rotation of the spin vector around the effective magnetic field.

The calculated spin relaxation time as a function irradiation intensity is plotted on Fig.2.5 for three different values of the bare mean free time τ_0 , the mean free time in the absence of dressing field. The Fermi energy is taken to be $E_F = 10$ meV and the spin-orbit coupling constant $\alpha = 3.3 \times 10^3$ m/s which set the spin precession time. The frequency of the dressing field is $\omega = 100$ GHz. The dressing field is able to change the behavior of the spin relaxation time τ_s since the parameter ξ is strongly dependent on it. If in the absence of illumination, an electron scatters once in average while its spin vector $\mathbf{S}$ undergoes many revolutions around the effective magnetic field, i.e. $\tau_0 \gg \tau_{so}$,

Figure 2.5: The spin relaxation time of a 2DEG plotted against the intensity of a dressing laser field of frequency $\omega = 100$ Grad/s for different bare scattering times τ_0 in the absence of dressing. The parameters used are the following; electron effective mass $m = 0.067m_0$, Fermi energy $E_F = 10$ meV and spin-orbit coupling constant $\alpha = 3.3 \times 10^3$ m/s. The insert shows the set-up of the considered system.

we have $\xi \gg 1$ and therefore $\tau_s \approx \tau_0$. This is the case for the Elliott-Yafet mechanism. The spin relaxation time in this case increases, showing the same oscillatory behavior as τ_F with dressing field intensity. This is seen in the dashed curve in Fig. 2.5

If we consider the opposite case when in bare 2DEG we have $\xi \ll 1$, the spin relaxation time is governed by spin-orbit coupling as in the case for the D'yakonov-Perel' mechanism. Here, the electron scatters fast and as a consequence, the effective magnetic field changes direction rapidly and randomly so the spin vector doesn't manage to reorient itself. This means that the spin relaxation time is set by the spin

precession time τ_{so} . By turning up the field irradiation intensity, ξ increases and we are no longer in the $\xi \ll 1$ regime and therefore the spin relaxation time shows a non-monotonic behaviour on τ_F , see Fig. 2.5. Fig. 2.6a shows more clearly the dependence of the spin relaxation time on the parameter ξ .

Figure 2.6: **(a)**. The spin relaxation time as a function of the ratio of the mean free time τ (called τ_F in the main text) to the single spin rotation time τ_{so} . **(b)**. A 3D plot considering the combined dependence of the spin relaxation time on the dressing field intensity I and bare mean free time τ_0 .

The switching from the Elliott-Yafet mechanism for low values of τ_0 to the D'yakonov-Perel' mechanism with increasing irradiation intensity and the inverse process for high values of τ_0 are shown in the 3D plot in Fig. 2.6(b). The oscillating behaviour is a relic of the Bessel function appearing in the dressed scattering rates. Other parameters used in these plots are the same as in Fig. 2.5.

3. Transport and spin properties of graphene in the regime of strong coupling with light

This chapter I have devoted to the laser-assisted transport properties of graphene, more specifically to the calculation of the conductivity and the magnetoelectronic properties. We saw in the last chapter how a non-resonant dressing field drastically changes the conductivity and alters the spin relaxation mechanism in a parabolic 2DEG system. The root of the obtained results lied in the acquisition of non-global wavefunction phase which depended on the electron wavevector. Since the energy dispersion close to the Dirac points in graphene is crucially different from a conventional semiconductor free gas dispersion, one can't expect beforehand to obtain the same results in the two systems. In the case of graphene, the non-resonant condition is set by letting the photon energy of the laser field be much larger than the energy difference between the graphene bands at the Fermi level. This of course allows for the photoexcitation of electrons to the states above the Fermi level, but electrons in these states thermalize very quickly by the emission of optical phonons to the lowest

empty states available in the conduction band and raise the Fermi level. Therefore, the contribution of those states to the conductivity should be negligible and has therefore been neglected in the simple approximation in the scope of the thesis.

The chapter is organized as follows. Section 3.2. consists of analytical calculations of the energy spectrum of a graphene sheet in the presence of the EM wave, which is nothing more than ac Stark effect. This is done for two extreme cases of field polarization, namely linear and circular. In section 3.3, the scattering amplitudes for large and smooth scattering centers are found. Using these and the dressed spectrum, the conductivity is calculated using the semi-classical Boltzmann equation. Optical transitions between dressed Landau levels in the presence of a perpendicular magnetic field are considered in the last section of the chapter. The frequency of the weak optical field lies in this case far from the dressing field frequency. The results have been published in Refs. [102] and [103].

3.1. Electromagnetically dressed graphene

In Fig. 3.1, a hypothetical set-up is shown where a graphene sheet is illuminated by a high-frequency electromagnetic field.

The time-dependent Schrödinger equation with the Hamiltonian

$$\hat{H}(t) = v_F \boldsymbol{\sigma} \cdot (\hbar \mathbf{k} - e \mathbf{A}(t)) \quad (3.1)$$

yields the energy spectrum $E_{\pm}(\mathbf{k})$ along with the wavefunctions $\psi_{\pm}(\mathbf{k}, t)$ where the “plus” and “minus” sign denote the conduction and the valence bands, respectively.

Figure 3.1: A graphene monolayer is subjected to a strong high-frequency electric field $\mathcal{E}_\omega$ of (a) circular polarization and (b) linear polarization. The frequency of the field, ω , is high enough such that no absorption takes place, neither interband nor through carrier heating.

If the vector potential is that of a circularly polarized light, see Eq. 2.9, the resulting energy spectrum,

$$E(\mathbf{k}) = \pm \sqrt{(\Delta_g/2)^2 + (\hbar v_F k)^2} \quad (3.2)$$

is gapped and remains isotropic as is to be expected. It should be stressed that v_F is just the original band parameter and the field induced band opening is given by

$$\Delta_g = \sqrt{(\hbar\omega)^2 + (2v_F e \mathcal{E}_\omega / \omega)^2} - \hbar\omega \quad (3.3)$$

which for vanishing amplitude, $\mathcal{E}_\omega = 0$, is zero. Here, the photon energy is assumed to satisfy the inequality

$$\hbar\omega \gg \sqrt{2\hbar v_F e \mathcal{E}_\omega} \quad (3.4)$$

as well as the dressing condition

$$\hbar\omega \gg \hbar/\tau_0 \quad (3.5)$$

discussed in the last section. The same result, considering the quantized electro-

magnetic field in the limit of large occupation numbers, was obtained in Ref. [104], confirming its accuracy.

The wavefunctions read

$$\begin{aligned} \psi_{\mathbf{k},\pm}(\mathbf{r}) = & \left[\sqrt{\frac{|E_{\pm}(\mathbf{k})| \mp \Delta/2}{2|E_{\pm}(\mathbf{k})|}} \exp\left(-i\frac{\theta}{2}\right) \Phi_1(\mathbf{r}) + \sqrt{\frac{|E_{\pm}(\mathbf{k})| \pm \Delta/2}{2|E_{\pm}(\mathbf{k})|}} \exp\left(i\frac{\theta}{2}\right) \Phi_1(\mathbf{r}) \right] \\ & \times \phi_{\mathbf{k}}(\mathbf{r}) \exp\left(\frac{-iE_{\pm}(\mathbf{k})t}{\hbar}\right) \end{aligned} \quad (3.6)$$

where $\mathbf{k} = k(\cos\theta, \sin\theta)$, $\Phi_{1,2}(\mathbf{r})$ are two orthogonal π -orbital wavefunctions representing the two atoms in a unit cell, and $\phi_{\mathbf{k}}(\mathbf{r})$ are normalized planewaves.

For a linearly polarized continuous wave with a polarization vector along the x -direction, spectrum remains gapless but anisotropy is introduced. Namely,

$$E(\mathbf{k}) = \pm \hbar v_F k f(\theta) \quad (3.7)$$

where the form function

$$f(\theta) = \sqrt{\cos^2 \theta + J_0^2 \left(\frac{2v_F e \mathcal{E}_\omega}{\hbar \omega^2} \right) \sin^2 \theta} \quad (3.8)$$

turns the Dirac circular cone into an elliptic one. Effectively, the axis of the ellipse along the y -direction is mapped from $\hbar v_F k$ to $\hbar v_F k J_0^2 (2v_F e \mathcal{E}_\omega / \hbar \omega^2)$ where J_0 is the

Figure 3.2: (a). An electromagnetic field opens a bandgap Δ_g at the $k = 0$ point of graphene which can be tuned by the frequency and the intensity of the continuous wave. (b). A linearly polarized electromagnetic wave introduces anisotropy to the energy dispersion of the otherwise isotropic graphene sheet. The Fermi velocities along and perpendicular to the direction of the polarization vector are therefore different.

zeroth order Bessel function of the first kind. Here we assumed

$$\hbar\omega \gg \hbar v_F k, \quad (3.9)$$

another inequality for the dressing field photon energy needed for this solution to be valid. This is automatically fulfilled since from the onset it was claimed that frequency of the field lied far away from any resonances. For the direction along the polarization vector the Fermi velocity is still a constant.

The eigenfunctions assume a rather complicated form,

$$\begin{aligned} \psi_{\mathbf{k},\pm}(\mathbf{r}) = & [\Phi_{\pm}(\mathbf{r}) \exp\left(\pm i \frac{v_F e \mathcal{E}_\omega}{\hbar \omega^2} \sin \omega t\right) - i \frac{\sin \theta}{\cos \theta + f(\theta)} J_0\left(\frac{2ve\mathcal{E}_\omega}{\hbar \omega^2}\right) \\ & \times \Phi_{\mp}(\mathbf{r}) \exp\left(\mp i \frac{ve\mathcal{E}_\omega}{\hbar \omega^2} \sin \omega t\right)] \\ & \times \phi_{\mathbf{k}}(\mathbf{r}) \sqrt{\frac{\cos \theta + f(\theta)}{4f(\theta)}} \exp\left(-i \frac{E(\mathbf{k})t}{\hbar}\right) \end{aligned} \quad (3.10)$$

using, for convenience, the definition

$$\Phi_{\pm}(\mathbf{r}) \equiv \Phi_1(\mathbf{r}) \pm \Phi_2(\mathbf{r}). \quad (3.11)$$

3.2. Conductivity calculations

For the discussion of the transport properties, the simplest disorder model is chosen. The steps to the final results are similar to the case of the parabolic 2DEG of the last chapter. The impurity scatterers are assumed to be macroscopically large in number and that the scattering between non-stationary states to be affected only by the single impurity potential $U(\mathbf{r})$. It is taken to be smooth over the elementary cell of the graphene sheet.

The probability that an electrons scatters from from the state $|\Phi_j(\mathbf{r})\phi_{\mathbf{k}}(\mathbf{r})\rangle$ to the state $|\Phi_i(\mathbf{r})\phi_{\mathbf{k}'}(\mathbf{r})\rangle$ is proportional to the matrix element

$$\langle \Phi_i(\mathbf{r})\phi_{\mathbf{k}'}(\mathbf{r}) | U(\mathbf{r}) | \Phi_j(\mathbf{r})\phi_{\mathbf{k}}(\mathbf{r}) \rangle \approx U_{\mathbf{k}'\mathbf{k}} \delta_{ij} \quad (3.12)$$

using $U_{\mathbf{k}'\mathbf{k}} = \langle \phi_{\mathbf{k}'}(\mathbf{r}) | U(\mathbf{r}) | \phi_{\mathbf{k}}(\mathbf{r}) \rangle$ and the Kronecker delta δ_{ij} . Using this matrix element, the Born scattering is given by

$$w_{\mathbf{k}'\mathbf{k}} = \frac{2\pi}{\hbar} |\chi_{\mathbf{k}'\mathbf{k}}|^2 |U_{\mathbf{k}'\mathbf{k}}|^2 \delta(E(\mathbf{k}') - E(\mathbf{k})) \quad (3.13)$$

For the case of the circularly polarized dressing field

$$\begin{aligned} \chi_{\mathbf{k}'\mathbf{k}} = & \sqrt{\frac{|E(\mathbf{k}')| - \Delta/2}{2|E(\mathbf{k}')|}} \sqrt{\frac{|E(\mathbf{k})| - \Delta/2}{2|E(\mathbf{k})|}} \exp\left(i\frac{\theta' - \theta}{2}\right) \\ & + \sqrt{\frac{|E(\mathbf{k}')| + \Delta/2}{2|E(\mathbf{k}')|}} \sqrt{\frac{|E(\mathbf{k})| + \Delta/2}{2|E(\mathbf{k})|}} \exp\left(-i\frac{\theta' - \theta}{2}\right) \end{aligned} \quad (3.14)$$

and

$$\begin{aligned} \chi_{\mathbf{k}'\mathbf{k}} = & \sqrt{\frac{\cos(\theta') + f(\theta')}{2f(\theta')}} \sqrt{\frac{\cos(\theta) + f(\theta)}{2f(\theta)}} \exp\left(i\frac{\theta' - \theta}{2}\right) \\ & \times \left[1 + \frac{\sin \theta'}{\cos \theta' + f(\theta')} \frac{\sin \theta}{\cos \theta + f(\theta)} J_0^2\left(\frac{2v_F e \mathcal{E}_\omega}{\hbar \omega^2}\right) \right] \end{aligned} \quad (3.15)$$

for the linear polarization.

Since there is no heating due to the high-frequency nature of the field, the electron can be considered to be in equilibrium with the corresponding fermionic distribution function, although with a new energy spectrum. In the presence of an external in-plane DC electric field $\mathcal{E}_\omega$, the new energy spectrum and the new renormalized scattering probability are inserted into the Boltzmann transport equation, Eq. 2.46, from which

we get the new transport relaxation time $\tau(\mathbf{k})$.

To find the current density

$$\mathbf{J} = \frac{e^2}{\pi^2} \int_{\mathbf{k}} [\mathcal{E}_{\text{dc}} \cdot \mathbf{v}(\mathbf{k})] \tau(\mathbf{k}) \delta(E(\mathbf{k}) - E_F) d^2\mathbf{k} \quad (3.16)$$

the integration path for a circularly polarized dressing field is not different from the case of bare graphene while in the case of a linearly polarized light, the Fermi line is not a circle but an ellipse.

The conductivity tensor components, $\sigma_{ij} = J_i/\mathcal{E}_j$, are then calculated using the model of $U_{\mathbf{k}'\mathbf{k}}$ being independent of the angle of the wavevectors $\mathbf{k}$ and $\mathbf{k}'$. This is the essence of the s -wave approximation for electrons with long wavelengths which was applied in the last chapter and is valid for wavevectors close to the K and K' points.

For the isotropic case of circularly polarized dressing field, the conductivity tensor is a scalar, $\sigma_c \equiv \sigma_{xx} = \sigma_{yy}$. Its ratio to the conductivity of bare graphene becomes

$$\frac{\sigma_c}{\sigma_0} = \frac{1 - (\Delta/2E_F)^2}{1 + 3(\Delta/2E_F)^2} \quad (3.17)$$

with the Fermi energy E_F being assumed to be bigger than the field induced gap Δ_g .

In Fig. 3.3, the conductivity σ_c is plotted against the field irradiance $I_0 = \epsilon_0 \mathcal{E}_\omega^2 c/2$. The result, that the conductivity is a diminishing monotonic function of I_0 , can be explained by noticing that the Fermi velocity, $\mathbf{v}_F(\mathbf{k}) = (1/\hbar) \nabla_{\mathbf{k}} E(\mathbf{k})|_{E=E_F}$, decreases

Figure 3.3: The conductivity of dressed electrons in graphene for a dressing field with a circular polarization. Parameters used in the models are shown inside the figure. The electromagnetic radiation monotonically reduces the conductivity by decreasing the Fermi velocity.

with increasing irradiance. In particular, for points just around the band edge, the Fermi velocity is not a constant, i.e. $\mathbf{v}_F(\mathbf{k}) \propto k$, which property gives graphene its remarkable transport qualities.

The influence of a dressing field on the transport properties of a two dimensional system is most brightly seen in the situation when a linearly polarized field is applied to a graphene sheet. The conductivity, σ_{xx} , parallel to the probe field $\mathcal{E}_{dc}$, and the conductivity, σ_{yy} perpendicular to it are plotted in Fig. 3.4.

On one hand, both σ_{xx} and σ_{yy} oscillate with the intensity I_0 . This behaviour is inherited from the property of the elongation of the Dirac cone perpendicular to the field polarization vector, which is proportional to $|J_0(2ev_F\mathcal{E}_\omega/\hbar\omega^2)|$. On the other hand, σ_{xx} is around 10 times larger than σ_{yy} for some values of the dressing field irradiance. This comes about because the deformation of the Dirac cones leads to

Figure 3.4: The conductivity of dressed electrons in graphene for the dressing field of linear polarization where the polarization plane lies along the x -axis. The dashed regions in the upper subfigure indicate where the theory breaks down.

different Fermi velocities along the x and y -axes. In the plot, the intervals where the developed theory breaks down on the intensity scale are shown by interrupted dashed segments. These correspond to the roots of the Bessel function where the condition of Eq. 3.9 is not satisfied.

3.3. Effect of dressing under a strong magnetic field

In this section, a tool for the observation of the modified energy spectrum in a field-dressed graphene is established by calculating the photon absorption coefficient under a strong magnetic field applied normally to the sheet, see Fig. 3.6. Since the energy dispersion is strongly modified, one is obliged to believe that the magneto-optic properties be changed as well.

Figure 3.5: A strong high-frequency electric field $\mathcal{E}_\omega$ changes the energy separation between quantized orbits (Landau levels) under a strong magnetic field $\mathbf{B}$ normal to the sheet.

The time-dependent Hamiltonian in this geometry is

$$\hat{H}(t) = v_F \boldsymbol{\sigma} \cdot [\hat{\mathbf{p}} + e(\mathbf{A}_B + \mathbf{A}_\omega(t))] \quad (3.18)$$

where $\mathbf{A}_B = -(By/c)\hat{\mathbf{x}}$ is the vector potential of the static magnetic field in the Landau gauge and $\mathbf{A}_\omega(t)$ is the periodic vector potential of the dressing field, given by

2.14, aligned along the x -axis.

To solve the quantum mechanical problem, the two-atom basis functions $\{|+\rangle, |-\rangle\}$ are superposed to form a new orthonormal set

$$\psi_0^\pm(t) = \frac{|+\rangle + |-\rangle}{\sqrt{2}} \exp\left(\mp \frac{\alpha \sin \omega t}{2}\right). \quad (3.19)$$

where, for convenience, the definition $\alpha = 2ev_F\mathcal{E}_\omega/\hbar\omega^2$ is made. Bear in mind that the instance $\alpha = 0$ corresponds to the absence of a dressing field. The general wavefunction is therefore written out as

$$\Psi(\mathbf{r}, t) = a^+(\mathbf{r}, t)\psi_0^+(t) + a^-(\mathbf{r}, t)\psi_0^-(t) \quad (3.20)$$

where $\mathbf{r}$ is an in-plane coordinate. The time-dependent Schrödinger equation gives

$$i\hbar\dot{a}^\pm = \pm v_F \left[\hat{p}_x - \frac{eBy}{c} \right] a^\pm(\mathbf{r}, t) \pm iv_F \exp(\pm i\alpha \sin \omega t) \hat{p}_y a^\mp(\mathbf{r}, t) \quad (3.21)$$

Courtesy of the Floquet theorem, we know that

$$\Psi(\mathbf{r}, t) = \exp(-i\tilde{E}t/\hbar)\phi(\mathbf{r}, t), \quad (3.22)$$

$\phi(\mathbf{r}, t)$ having the same period as $\mathbf{A}_\omega(t)$ and $\tilde{E}$ being the quasi-energy spectrum.

Since the general wavefunction is periodic in time, the components $a^\pm$ can be

expanded in Fourier series

$$a^\pm(\mathbf{r}, t) = \exp(-i\tilde{E}t/\hbar) \sum_{n=-\infty}^{\infty} a_n^\pm(\mathbf{r}) \exp(in\omega t). \quad (3.23)$$

where $a_n^\pm(\mathbf{r})$ are the Fourier coefficients. Inserting this expansion into Eq. 3.21, and using the Jacobi-Anger expansion, a different and equivalent form for the time-dependent Schrödinger equation with a periodic Hamiltonian is obtained, namely

$$[\tilde{E} - n\hbar\omega]a_n^\pm = \pm v_F \left[\hat{p}_x - \frac{eBy}{c} \right] a_n^\pm \pm iv_F \sum_{n'=-\infty}^{\infty} J_{n-n'}(\pm\alpha) \hat{p}_y a_{n'}^\mp(\mathbf{r}) \quad (3.24)$$

for any integers n and n' .

Assuming now that the photon energy, $\hbar\omega$, of the dressing field is far away from any Landau level separation, the interband photon absorption doesn't take place, such that $|a_{n \neq 0}^\pm(\mathbf{r})| \ll 1$. Furthermore, if the coupling parameter α is not close to the roots of the zeroth order Bessel function, then

$$\left| \frac{J_n(\alpha) \hat{p}_y a_n^\pm(\mathbf{r})}{J_0(\alpha) \hat{p}_y a_0^\pm(\mathbf{r})} \right| \ll 1 \quad (3.25)$$

for integer n different from zero. If we choose to neglect such terms, in effect making the rotating wave approximation, instead of infinitely many coupled equations, one ends up with

$$\tilde{E}a_0^\pm(\mathbf{r}, t) = \pm v_F \left[\hat{p}_x - \frac{eBy}{c} \right] a_0^\pm(\mathbf{r}, t) \pm iv_F J_0(\alpha) \hat{p}_y a_0^\mp(\mathbf{r}, t). \quad (3.26)$$

This single equation is the time-independent Schrödinger equation with the Hamiltonian

$$\hat{H}_0 = v_F \sigma_z \left[\hat{p}_x - \frac{eBy}{c} \right] - v_F J_0(\alpha) \sigma_y \hat{p}_y \quad (3.27)$$

acting on spinors $a_0(\mathbf{r}) = [a_0^+(\mathbf{r}), a_0^-(\mathbf{r})]^T$ such that

$$\hat{H}_0 a_0(\mathbf{r}) = \tilde{E} a_0(\mathbf{r}). \quad (3.28)$$

Upon making a unitary transformation with $\hat{U} = (\sigma_x + \sigma_y)/2$ on $\hat{H}_0$, we get

$$\hat{H}'_0 = \hat{U}^\dagger \hat{H}_0 \hat{U} = \sigma_x v'_x \left[\hat{p}_x - \frac{eBy}{c} \right] + \sigma_y v'_y \hat{p}_y \quad (3.29)$$

using the definitions

$$\begin{aligned} v'_x &\equiv v_F \\ v'_y &\equiv v_F J_0(\alpha) \end{aligned} \quad (3.30)$$

which are the new dressed Fermi velocities along the x and y directions, respectively.

In essence, the linearly polarized field changes the carrier Fermi velocity along the direction perpendicular to its polarization vector, just as we saw in the last section in the absence of a magnetic field, while the Fermi velocity along the time-varying electric field stays the same. If the static magnetic field is switched off, i.e. $B = 0$, Eq. 3.29 has the solutions of the last section. For a non-vanishing magnetic field, however, the solutions are found easily as follows.

Define

$$\begin{aligned}\hat{\Pi}_x &= \hat{p}_x - \frac{eBy}{c}, \\ \hat{\Pi}_y &= \hat{p}_y\end{aligned}\tag{3.31}$$

Using Eq.3.31, we compute

$$\begin{aligned}[\hat{\Pi}_x, \hat{\Pi}_x] &= [\hat{p}_x, \hat{p}_x] = 0, \\ [\hat{\Pi}_x, \hat{\Pi}_y] &= [\hat{p}_x, \hat{p}_y] + \frac{eB}{c}[\hat{p}_y, y]\end{aligned}\tag{3.32}$$

$$= -i\frac{\hbar eB}{c} = -i\frac{\hbar^2}{l_B^2},\tag{3.33}$$

where the magnetic length is defined as

$$l_B = \sqrt{\frac{\hbar c}{eB}}.$$

Put

$$\begin{aligned}\hat{a} &= \frac{l_B}{\sqrt{2\hbar v'}} \left(v'_x \hat{\Pi}_x - i v'_y \hat{\Pi}_y \right), \\ \hat{a}^\dagger &= \frac{l_B}{\sqrt{2\hbar v'}} \left(v'_x \hat{\Pi}_x + i v'_y \hat{\Pi}_y \right)\end{aligned}\tag{3.34}$$

with

$$v' \equiv \sqrt{v'_x v'_y} = v_F \sqrt{|J_0(\alpha)|}\tag{3.35}$$

and we get the relations

$$[\hat{a}^\dagger, \hat{a}] = 1. \quad (3.36)$$

The Hamiltonian, Eq. 3.29, then becomes

$$\hat{H}'_0 = \sqrt{2} \frac{\hbar v'}{l_B} \begin{pmatrix} 0 & \hat{a} \\ \hat{a}^\dagger & 0 \end{pmatrix}, \quad (3.37)$$

which formally is the same as the Hamiltonian of graphene electrons in a magnetic field in the absence of the dressing field but with v_F substituted by v' . Using Eq. 3.36, the eigenenergies $\tilde{E}_n$ are found and the eigenspinors ψ_n are constructed in the same way as has been done before [48]. In particular, one obtains the dressed energy spectrum

$$\tilde{E}_{\pm, n} = \pm \frac{\hbar v'}{l_B} \sqrt{2n} = \pm \frac{\hbar v_F}{l_B} \sqrt{2n |J_0(\alpha)|}, \quad (3.38)$$

which for vanishing dressing field intensity, $\alpha = 0$, is the same as the conventional graphene Landau level spectrum.

The linearly polarized dressing field therefore rescales the energy spectrum and consequently the separation between different Landau levels. This effect has implications for photon absorptions through electron inter-level transitions. Namely, since $|J_0(\alpha)| \leq 1$ for all values of α , all the inter-level transitions are redshifted by a different amount depending on the Landau level index n . Since the energy spectrum of graphene under circularly polarized irradiation doesn't depend on the Bessel function, the optical properties of relativistic Landau level are expected to be less pronounced and therefore are not considered here.

To see how the magneto-optical absorption profile is changed by the the modification of the Fermi velocity by a dressing field, the absorption coefficient β is calculated using linear optical response theory based on first order perturbation calculations with the density matrix approach [105]. Besides changing the Landau level separation, the modified Fermi velocity perpendicular to the polarization axis has an effect on the polarization dependence and oscillator strength of the inter-level transitions.

Let a weak probe field of frequency Ω shine on the dressed graphene system under consideration. In order to calculate the absorption coefficient, the dependence of the matrix elements of the electric dipole moment operator, $\hat{\mu} = e\hat{r}$, on the dressing field needs to be calculated. Using the commutation relation

$$[\hat{r}, \hat{H}'] = i\hbar (v'_x \sigma_x \hat{\mathbf{x}} + v'_y \sigma_y \hat{\mathbf{y}}), \quad (3.39)$$

where $\hat{\mathbf{x}}, \hat{\mathbf{y}}$ are unit vectors along the x and y direction respectively and

$$e\langle m | [\hat{r}, \hat{H}'] | n \rangle = e(\tilde{E}_n - \tilde{E}_m) \langle m | \hat{r} | n \rangle \quad (3.40)$$

we get

$$\begin{aligned}
 \vec{\mu}_{nm} &= \frac{ie\hbar v_F}{(\tilde{E}_n - \tilde{E}_m)} \langle m | \sigma_x \hat{\mathbf{x}} + |J_0(\alpha)| \sigma_y \hat{\mathbf{y}} | n \rangle \\
 &= \frac{i\hbar v_F C_m C_n (-1)^{|m|-1} i^{|m|+|n|-1}}{\epsilon_n - \epsilon_m} \times \\
 &\quad (\text{sgn}(n) \delta_{|m|-1, |n|} [\hat{\mathbf{x}} - i |J_0(\alpha)| \hat{\mathbf{y}}] \\
 &\quad - \text{sgn}(m) \delta_{|m|, |n|-1} [\hat{\mathbf{x}} + i |J_0(\alpha)| \hat{\mathbf{y}}])
 \end{aligned} \tag{3.41}$$

where v'_x and v'_y , given by Eq. 3.30, have been used and $|n\rangle$ are the Landau level wavefunctions.

The absolute value of the dipole matrix elements in the presence of a dressing field is therefore renormalized, namely

$$|\vec{\mu}_{mn}| = \frac{\sqrt{1 + J_0^2(\alpha)}}{\sqrt{|J_0(\alpha)|}} |\vec{\mu}_{mn}^0| \tag{3.42}$$

where $\vec{\mu}_{mn}^0$ are the moments of dipole transition between bare graphene Landau levels.

Applying the modified spectrum and dipole moments derived above and the established theory of magneto-optical absorption in “bare” graphene, we end up with the expression for the absorption coefficient in dressed graphene,

$$\beta_i = \sum_{n,m} \frac{2e^2 \tilde{v}_i^2 \gamma (\rho_m - \rho_n) (1 + \delta_{n,0} + \delta_{m,0}) \delta_{|n|-|m|, \pm 1}}{l_B^2 \hbar c \tilde{\omega}_{nm} [(\tilde{\omega}_{nm} - \Omega)^2 + \gamma^2]}, \tag{3.43}$$

where ρ_n is the equilibrium filling factor for the Landau levels, $\tilde{\omega}_{nm} = (\tilde{E}_n - \tilde{E}_m)/\hbar$ are the resonance frequencies of dressed graphene corresponding to electron transitions

Figure 3.6: The absorption coefficient of intrinsic graphene, β_x , at the temperature $T = 0$ as a function of the probing field frequency, Ω , for the decay rate $\Gamma = 1$ meV. The photon energy of the dressing field is $\hbar\omega = 3$ meV and the different curves correspond to the different field intensities: (1) $I_0 = 0$; (2) $I_0 = 3.51$ W/cm²; (3) $I_0 = 6.24$ W/cm². The insert shows the dependence of the absorption coefficient, β_x , on the photon energy, $\hbar\Omega$, and the electron-field interaction parameter, $\alpha = 2evE_0/\hbar\omega^2$.

between different Landau levels given in 3.38, $\gamma = \Gamma/\hbar$ is the decay rate at Landau levels which is assumed to be independent on the irradiation, Γ is the scatterer-induced broadening of Landau levels, δ_{nm} is the Kronecker symbol, and the index $i = x, y$ corresponds to the two polarizations of the probing field along the x, y axes, respectively. The absorption coefficient (3.43) is plotted in Fig. 3.6 for various intensities of the dressing field, $I_0 = c\mathcal{E}_\omega^2/8\pi$. Since the dressing field induces anisotropy between the x and y components of the matrix dipole moment in Eq. 3.41, the light emitted from

inter-level transitions becomes elliptic. The adjusted separation between the discrete levels in the presence of the field leads to the observed displacement of the absorption peaks.

4. Spin transport of polaritons

Upon the excitation of a polariton, its spin undergoes precession around the effective magnetic field, provided by the TE-TM splitting,

$$\mathbf{H}_{\text{LT}} = \Delta_{\text{LT}}(\mathbf{k}) \begin{pmatrix} \cos 2\phi \\ \sin 2\phi \end{pmatrix}, \quad (4.1)$$

where the splitting in energy between the longitudinally and transverse polarized polaritons with energy $E_L(\mathbf{k})$ and $E_T(\mathbf{k})$, respectively, is given by

$$\Delta_{\text{LT}}(\mathbf{k}) = E_T(\mathbf{k}) - E_L(\mathbf{k}) \quad (4.2)$$

Expressed in this way, the effective magnetic field has units of energy. The angle which the field makes with the x -axis in $\mathbf{k}$ -space is two times larger than the one which the spin-orbit coupling effective fields make, and this results in the four leaves pattern of the optical spin Hall effect (OSHE) discussed in the introduction. In this chapter, the term spin-orbit coupling effective magnetic field will be used in relation with the spin-orbit interaction of free carriers, since technically, the effective field in Eq. 4.1 couples spin and momentum.

In this chapter, the spin polarization of polaritons is studied in two excitation scenarios where either a real magnetic field is externally applied normal to the plane of the microcavity or in the presence of an effective magnetic field linear-in-momentum arisen from free carrier spin-orbit coupling [106,107]. Firstly, the pumping of a disordered semiconductor microcavity with a spatially homogeneous continuous wave in resonance with the lower polariton mode with a specific wavevector $\mathbf{k}$, see Fig. 4.1. The elastic scattering of polaritons with the disorder offers a mechanism for distributing them around the circle defined by $|\mathbf{k}|$. Secondly, a pump spot focused in space excites a clean sample with the polaritons moving radially outwards with their pseudospins precessing around the total effective magnetic field.

Figure 4.1: Polaritons with zero in-plane momentum are excited by linearly polarized light directed along the x -direction. Their pseudospin vector undergoes Larmor precession around the total effective magnetic field, whose direction and magnitude are determined by the angle which scattered state k makes with the external magnetic field pointing along the z -direction.

4.1. Scattering induced OSHE

To understand how a single scattering with a localized disorder potential generates spin patterns, we apply a semiclassical approach, used first in the original prediction of the OSHE [83], to calculate the evolution of the pseudospin vector $\mathbf{S}(t)$ whose three components describe the different polariton spin polarizations. In particular, its components along the quantum well plane give the two linear polarization orientations while the component, S_z , along the growth axis gives the degree of circular polarization. The components of $\mathbf{S}(t)$ are analogous to the Stokes parameters describing light polarization in the realm of classical optics.

The total effective magnetic field $\mathbf{H}_{\text{eff}}$ is simply the sum of the effective magnetic field stemming from the LT-splitting and the real magnetic field $\mathbf{H}_{\text{real}}$,

$$\mathbf{H}_{\text{eff}} = \hbar\Omega_{\mathbf{k}} = \mathbf{H}_{\text{LT}} + \mathbf{H}_{\text{real}}, \quad (4.3)$$

the total Hamiltonian for the polaritons close to the edge of the lower polariton branch is

$$H = \frac{\hbar^2 k^2}{2m_P} + (\boldsymbol{\sigma} \mathbf{H}_{\text{eff}}) \quad (4.4)$$

written in k -space where m_P is the effective mass of polaritons and σ is the Pauli spin matrix vector. The total field is again represented in energy units while the real magnetic field can be described by the amount of Zeeman-splitting Δ_Z it induces between the spin up ($S_z = +1$) and spin down ($S_z = -1$) polaritons as this is a more general way, see Fig. 4.2.

Figure 4.2: The real field induced Zeeman splitting Δ_Z between left and right circularly polarized polaritons. Polaritons excited by a continuous wave pump having a definite energy E_P have different in-plane momenta for different states of circular polarization.

Taking into account the aforesaid, the in-plane part of the vector $\Omega_{\mathbf{k}}$ is expressed as

$$\Omega_{\mathbf{k},\parallel} = \frac{\Delta_{\text{LT}}}{2\hbar k^2} \begin{pmatrix} k_x^2 - k_y^2 \\ 2k_x k_y \end{pmatrix}, \quad (4.5)$$

while the momentum independent out-of-the plane component is

$$\Omega_{\mathbf{k},\perp} = \frac{\Delta_Z}{2\hbar} \hat{\mathbf{z}}. \quad (4.6)$$

To derive an equation of motion for the spin vector, one employs the Liouville-von Neumann equation for the 2x2 density matrix which is expanded in Pauli matrices with the expansion coefficients being the components of $\mathbf{S}(t)$ as discussed in the introduction. If a single state with wavevector $\mathbf{k}_0$ is being excited, one gets the equation

for the scattered state $\mathbf{k}$

$$\frac{\partial \mathbf{S}_{\mathbf{k}}(t)}{\partial t} = \mathbf{S}_{\mathbf{k}} \times \boldsymbol{\Omega}_{\mathbf{k}} + \mathbf{f}(t) - \frac{\mathbf{S}_{\mathbf{k}}}{\tau}, \quad (4.7)$$

with the initial conditions $\mathbf{S}(t) = \mathbf{0}$. In this model, the scattering from the initial state introduced at $t = 0$ to the state $\mathbf{k}$ is taken into account through the function

$$\mathbf{f}(t) = \frac{\mathbf{S}_0(t)}{\tau_1} \exp(-t/\tau), \quad (4.8)$$

where τ_1 is the disorder dependent scattering time and τ is the polariton lifetime. Here, $\mathbf{S}_0(t)$ is the time-dependent pseudospin vector of the initial state before the scattering event. It evolves according to its own equation

$$\frac{\partial \mathbf{S}_0(t)}{\partial t} = \mathbf{S}_0 \times \boldsymbol{\Omega}_0, \quad (4.9)$$

with the condition $\mathbf{S}_0(t) = S_0 \hat{\mathbf{x}}$ which means that the excited polariton has an initial S_x linear polarization. If there is no real magnetic field present, the vector $\mathbf{S}_0$ being initially parallel to $\boldsymbol{\Omega}_0$ (see Eq. 4.5), doesn't evolve with time. The application of a real magnetic field normal to the plane introduces a z component leading to a net precession of the initial vector as $[\mathbf{S}_0 \times \boldsymbol{\Omega}_0] \neq \mathbf{0}$.

The instantaneous circular polarization degree is defined as

$$\rho_c(t) \equiv \frac{2S_{z,\mathbf{k}}(t)}{N_{\mathbf{k}}} \quad (4.10)$$

with $N_{\mathbf{k}}$ being the polariton population of the scattered state $\mathbf{k}$. It satisfies the rate equation

$$\frac{\partial N_{\mathbf{k}}}{\partial t} = 2 \frac{S_0}{\tau_1} e^{-t/\tau} - \frac{N_{\mathbf{k}}}{\tau}, \quad (4.11)$$

where S_0 is the modulus of $\mathbf{S}_0(t)$, yielding

$$N_{\mathbf{k}} = \frac{2S_0 t}{\tau_1} e^{-t/\tau}. \quad (4.12)$$

The precession equation, Eq.4.7, is solved by writing $\mathbf{S}'_{\mathbf{k}}(t) = \mathbf{S}_{\mathbf{k}} e^{t/\tau}$ to arrive at an easier one

$$\frac{\partial \mathbf{S}'_{\mathbf{k}}(t)}{\partial t} = \mathbf{S}'_{\mathbf{k}}(t) \times \boldsymbol{\Omega} + \frac{\mathbf{S}_0(t)}{\tau_1} = -i \hat{A} \mathbf{S}'_{\mathbf{k}}(t) + \frac{\mathbf{S}_0(t)}{\tau_1}$$

with $\hat{A}$ being a Hermitian 3x3 matrix and $\mathbf{S}'_{\mathbf{k}}(0) = \mathbf{0}$.

The eigenvalues of $\hat{A}$ are triple and real, $\omega_1, \omega_2, \omega_3$, with the corresponding eigenvectors $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$, respectively, which span the 3-D Euclidian space. Therefore, one is able to write

$$\mathbf{S}'_{\mathbf{k}}(t) = c_1(t) \mathbf{e}_1 + c_2(t) \mathbf{e}_2 + c_3(t) \mathbf{e}_3$$

and

$$\frac{\mathbf{S}_0(t)}{\tau_1} = g_1(t) \mathbf{e}_1 + g_2(t) \mathbf{e}_2 + g_3(t) \mathbf{e}_3.$$

Doing this, one ends up with three uncoupled ordinary inhomogeneous differential

equations,

$$\begin{aligned} c_j'(t) &= -i\omega_1 c_j(t) + g_j(t) \\ \Rightarrow c_j(t) &= e^{-i\omega_j t} \int_0^t e^{i\omega_j t'} g_j(t') dt' \end{aligned} \quad (4.13)$$

with $c_j = 0$ at $t = 0$, $j = 1, 2, 3$. These equations are solved in a straightforward manner and the solutions are integrated on the half-line $t \geq 0$ to give the time-averaged circular polarization value

$$\bar{\rho}_c(\phi) = 2 \frac{\int_0^\infty S_{\mathbf{k},z} dt}{\int_0^\infty N_{\mathbf{k}} dt} \quad (4.14)$$

$$= \frac{1}{\Omega^2} [\Omega_x \cos^2 \theta + \Omega_z \cos \theta \sin \theta] \Omega_z \quad (4.15)$$

$$+ \Omega_z [\Omega_x - \Omega \cos \theta] (3 \sin^2 \theta - 1) \frac{1}{2((\Omega\tau)^2 + 1)} \frac{1}{\Omega^2} \quad (4.16)$$

$$+ \cos^2 \theta \frac{\Omega_y \tau}{(\Omega\tau)^2 + 1} \quad (4.17)$$

$$+ 2 \sin \theta \frac{\Omega_y \Omega_z \tau}{(\Omega\tau)^2 + 1)^2} \frac{1}{\Omega} \quad (4.18)$$

$$- [\Omega_x \Omega_z (1 + \sin^2 \theta) + \Omega^2 \cos^3 \theta \sin \theta] \frac{(1 - \Omega^2 \tau^2)}{((\Omega\tau)^2 + 1)^2} \frac{1}{2\Omega^2}. \quad (4.19)$$

where θ is the angle between the effective magnetic field and the cavity plane, extracted from the equation

$$\tan(\theta) = \frac{\mu_B g H_{\text{real}}}{\Delta_{\text{LT}}}. \quad (4.20)$$

In Fig. 4.3, $\bar{\rho}_c(\phi)$ is plotted for a number of external magnetic field values. The lifetime of polaritons was taken to be $\tau = \hbar/\Delta_{\text{LT}}$, so that the maximum value of polarization occurs when the external magnetic field is put to zero in which case the pattern is exactly the same as the one obtained in the original proposition of the

Figure 4.3: The time-averaged circular polarization degree $\bar{\rho}_c$ plotted against the scattering angle for different sizes of the Zeeman splitting Δ_Z indicated by different colors. For vanishing external magnetic field, the polarization degree is divided into four regions of alternating signs. With increasing splitting, the pattern shifts sideways and also upwards. For very strong external magnetic field, the azimuthal separation of polarizations fades out. The plots were made for the case where $\Omega\tau = 1$.

OSHE. An increase in the field leads to the shift of the positions of the maxima and decrease of the amplitude of the oscillations, which slowly washes out the effect. The shift of the positions of the maxima corresponds to the rotation of the circular polarization degree pattern in the QW plane.

Leaving the simplistic scheme of single scattering behind, a more realistic scenario is where polaritons scatter multiple times before collapsing and emitting a photon. To study the effects of multiple scattering numerical modeling is required. Let

$$\Psi = \begin{pmatrix} \psi_+(\mathbf{r}, t) \\ \psi_-(\mathbf{r}, t) \end{pmatrix} \quad (4.21)$$

be the macroscopic polariton wavefunction in the basis where $\psi_+(\mathbf{r}, t)$ ($\psi_-(\mathbf{r}, t)$) is the right (left) circularly polarized component. The functions $\psi_+(\mathbf{r}, t)$ and $\psi_-(\mathbf{r}, t)$ allow for the determination of the Stokes parameters which give the circular polarization degree ρ_c along with the polarization degrees in the linear and diagonal components, ρ_{lin} and ρ_{diag} , respectively. Namely,

$$\rho_c = \frac{|\psi_+|^2 - |\psi_-|^2}{|\psi_+|^2 + |\psi_-|^2}, \quad (4.22)$$

$$\rho_{\text{lin}} = \frac{\psi_+^* \psi_- + \psi_-^* \psi_+}{|\psi_+|^2 + |\psi_-|^2}, \quad (4.23)$$

$$\rho_{\text{diag}} = i \frac{\psi_+^* \psi_- - \psi_-^* \psi_+}{|\psi_+|^2 + |\psi_-|^2}. \quad (4.24)$$

Taking into account external pumping and decay, the macroscopic wavefunction evolves according to Schrödinger-type equation:

$$i\hbar \frac{\partial \Psi}{\partial t} = -i\hbar \frac{\Psi}{2\tau} + \hat{H} \Psi + \hat{V}(\mathbf{r}) \Psi + \mathbf{P}(\mathbf{r}, t), \quad (4.25)$$

with the first term giving the decay, where τ is the polariton lifetime. The second term includes the free propagation of polariton on the QW plane as well as the total effective magnetic field,

$$\hat{H} = \begin{pmatrix} \frac{-\hbar^2 \nabla^2}{2m_P} + \frac{\Delta_Z}{2} & \beta \left(\frac{\partial}{\partial x} + i \frac{\partial}{\partial y} \right)^2 \\ \beta \left(\frac{\partial}{\partial x} - i \frac{\partial}{\partial y} \right)^2 & \frac{-\hbar^2 \nabla^2}{2m_P} - \frac{\Delta_Z}{2} \end{pmatrix} \quad (4.26)$$

where the diagonal terms give the kinetic energy and the real field Zeeman-splitting Δ_Z , with the off-diagonal terms corresponding to the LT-splitting. The canonical substitution $k \mapsto -i\nabla$ has been made.

The disorder is introduced via the third term in Eq 4.26 where $\hat{V}$ represents Gaussian correlated potential with both the excitonic and photonic parts being taken into consideration [108]. The values for the correlation lengths of the excitonic and photonic disorder were taken from Ref. [109], namely $\sigma_X = 0.3 \mu\text{m}$ and $\sigma_C = 1 \mu\text{m}$, respectively. The excitonic part of the potential has a depth of $A_X = 0.15 \text{ meV}$ while the photonic part has a depth of $A_C = 0.4 \text{ meV}$.

The last term in Eq. 4.25 describes the coherent optical pumping of the micro-cavity. In this case, it is a continuous wave homogeneous on the plane with linear polarization directed along the x -axis. Just as in the single scattering case, the pump has an in-plane wave vector lying along the x -axis with a wave number $k_0 = 1.56 \mu\text{m}^{-1}$ for which the LT-splitting has the value $\Delta_{\text{LT}} = 0.05 \text{ meV}$.

For times t long enough, larger than the characteristic scattering time and polariton lifetime τ , one expects to reach a steady state in Eq. 4.26. Solving the equation numerically till the intensities and the polarization degrees are stable one is able to obtain the steady state polarization patterns.

In Fig. 4.4, the steady state of the circular polarization degree ρ_c is plotted in reciprocal space as a function of the magnetic field induced Zeeman splitting Δ_Z shown above each subfigure. For vanishing real magnetic field, $\Delta_Z = 0$, the regions with positive polarization (positive ρ_c), occupy the first and third quadrants of the k -plane with negative polarization taking place on the second and fourth quadrants. The total

Figure 4.4: Numerical calculation of the evolution of the steady-state circular polarization degree in reciprocal space with increasing strength of Zeeman splitting Δ_Z as shown above each subplot. The homogeneous CW pump has an in-plane wave vector k_0 in the positive x direction and multiple scattering with a disorder potential is accounted for. It is visible how the four-leaf pattern transforms into a separation of polarization in the radial direction with increasing external magnetic field.

intensity is distributed along a circle of radius k_0 which is a result of elastic scattering. A small increase of the magnetic field leads to a small rotation of the domains until the pattern is azimuthally symmetric which is analogous to the results plotted in Fig. 4.3 using the single scattering model.

For a Zeeman splitting substantially larger than the LT-splitting, one sees that the circular polarizations become radially separated and are distributed along two concentric circles. In this limit, where the off-diagonal terms of the matrix in Eq. 4.26 are negligible, its eigenstates are those of circular polarization with Zeeman split branches. The pump of energy $\hbar\omega_P$ therefore excites the branches of opposite polarization at the wavenumbers

$$k_{\pm} = \sqrt{2m(\hbar\omega_P \mp \Delta_Z/2)\hbar^2}, \quad (4.27)$$

with the "+" ("−") corresponding to the left (right) circularly polarized light. Due to conservation of energy, the multiple scattering redistributes the polaritons of opposite polarization along circles of different radii. The separation of the circles can be estimated as

$$\Delta k \approx \Delta_Z \sqrt{\frac{m}{2\hbar^3\omega_P}} \quad (4.28)$$

which shouldn't be large in GaAs microcavities. In semimagnetic cavities, however, which have large g factors, the Zeeman splitting can be correspondingly increased [110].

4.2. OSHE with a spatially focused pump

To study the effects of an externally applied magnetic field and those of spin-orbit splitting on the OSHE generated with a spatially focused pump, a similar model as in the treatment of multiple scattering is used. In this case, however, the cavity is taken to be without disorder. In the presence of solely LT-splitting effective magnetic field, a cross-like pattern emerges around the pump spot as measured in Refs. [86] and [111].

4.2.1 External magnetic field

Let the microcavity be excited by a Gaussian pump with spotsize r_0 perpendicularly to its QW plane while the magnetic field is still applied normal to the plane along the growth direction. Taking the pump to be linearly polarized along the x -axis, the last term in Eq. 4.25 can be written as

$$\mathbf{P}(\mathbf{r}, t) = A_0 \exp\left(-\frac{iE_P t}{\hbar}\right) \exp\left(-\frac{r^2}{r_0^2}\right) \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (4.29)$$

with $E_P = \hbar\omega_P$ being the energy of the pump relative to the minimum of the lower polariton branch at $k = 0$. Solving for $\psi_{\pm}$ when the pump is switched on at time $t = 0$ the polarization degrees, or equivalently the Stokes parameters, are obtained via Eqs. 4.22-4.24.

The polarization degree patterns ρ_c , ρ_{lin} and ρ_{diag} , obtained from the steady state solution of Eq. 4.25, are plotted in Figs. 4.5, 4.6 and 4.7, respectively.

Figure 4.5: The steady-state circular polarization degree ρ_c plotted for different magnitudes of the Zeeman splitting Δ_Z . The excitation pump has a Gaussian profile of width $3 \mu\text{m}$ and is of linear polarization. The OSHE cross for $\Delta_Z = 0$ is transformed into a spiral with increasing magnetic field. If Δ_Z is much larger than the TE-TM splitting of polaritons, their spin is aligned along the external magnetic field.

When there is no magnetic field present, the circular polarization degree ρ_c ex-

hibits the four-leaf pattern associated with the OSHE. Increasing the magnetic field slightly results in a pattern resembling a legged cross or a gammadion. For higher magnetic field, the legs of the cross begin to spiral around the pump spot position at $\mathbf{r} = 0$ and the pseudospin becomes aligned with the field. This effect goes hand in hand with a decrease in polarization degree maxima.

As the pump is linearly polarized, the pattern of the linear polarization degree is seen to conserve this polarization along the horizontal and vertical lines. Increasing the magnetic field intensity by a small value results in a rounded rectangular form with corners on the Cartesian axes inside which ρ_{lin} has an opposite sign compared to the value outside of it. For high values of the Zeeman splitting Δ_Z , concentric rings appear with centers at the origin. The radii of the rings decreases with increasing Δ_Z .

The diagonal polarization degree shows a similar dependence on the external magnetic field with a rectangle forming at low Δ_Z separating ρ_{diag} of opposite signs, see Fig. 4.7. At high magnetic field intensities, concentric rings are formed with the similar radii as the ones of ρ_{lin} .

The transformation of the zero magnetic field patterns can be explained in the pseudospin picture. As polaritons move away from the pumping spot in all directions, their pseudospin precesses about the total effective magnetic field given in Eq. 4.3. Setting $\Delta_Z = 0$, this effective magnetic field lies on the QW (XY) plane and results in the four-leaf pattern of the OSHE where the circular polarization degree is of opposite sign in adjacent quadrants of the plane. In the other limit, when the magnetic

Figure 4.6: The steady-state linear polarization degree ρ_{lin} plotted for different magnitudes of the Zeeman splitting Δ_Z . The excitation pump has a Gaussian profile of width $3 \mu\text{m}$ and is of linear polarization. Increasing the field results in the formation of rings centered at the pumping spot. The difference in the radii of the rings decreases with increasing external magnetic field which corresponds to an increase in precession frequency of the polariton pseudospin vector around the total effective magnetic field.

Figure 4.7: The steady-state diagonal polarization degree ρ_{diag} plotted for different magnitudes of the Zeeman splitting Δ_Z . The excitation pump has a Gaussian profile of width $3 \mu\text{m}$ and is of linear polarization. Increasing the field results in the formation of rings centered at the pumping spot. The difference in the radii of the rings decreases with increasing external magnetic field which corresponds to an increase in precession frequency of the polariton pseudospin vector around the total effective magnetic field.

field induced Zeeman splitting is much larger than the LT-splitting, the total effective magnetic field is oriented along the z -axis, that is normal to the plane. As the optical pump is linearly polarized, the pseudospin vector $\mathbf{S}$ is at a right angle with the total effective magnetic field everywhere on the plane. As polaritons propagate outwards, $\mathbf{S}$ rotates with the same frequency, regardless of direction, which is linearly proportional to the magnetic field strength resulting in the observed circles. The increase in the real magnetic field makes the circles in the plots for ρ_{lin} and ρ_{diag} shrink and their density grow.

4.2.2 Spin-orbit coupling

In quantum wells grown along the (110)-direction, the energy dispersion of excitons having spin projections $\pm$ on the z -axis is expressed as

$$E_{X,\pm} = E_{X0} + \frac{\hbar^2 k^2}{2m_X} \pm \gamma_X k_x, \quad (4.30)$$

where $k^2 = k_x^2 + k_y^2$ with k_x (k_y) being the components of the wavevector along the x (y) direction. Here, E_{X0} is the energy of stationary excitons (with $\mathbf{k} = 0$), $m_X = m_e + m_{hh}$ is the total exciton mass where m_e and m_{hh} are the effective masses of the electrons and heavy holes, respectively and the reduced SO-splitting constant is given by

$$\gamma_X = \frac{\gamma_e m_e + \gamma_{hh} m_{hh}}{m_e + m_{hh}} \quad (4.31)$$

with γ_e (γ_{hh}) being the electron (heavy hole) SO coupling parameter. In this basic model, the free carrier SO coupling is assumed to affect the exciton energies only via the last term of Eq. 4.30, that is to say, it doesn't affect the internal energy structure of excitons. The exciton SO coupling splits the parabolic energy dispersion into two branches displaced along the k_x axis in reciprocal space, see Fig. 4.8.

The minimum of the energy spectrum of excitons having spin projection $+1$ is shifted by $\gamma_X m_X / \hbar^2$ along the $-k_x$ direction with the parabola of the spin -1 excitons shifted by the same but positive amount.

Pumping the (110)-oriented quantum well at the $k = 0$ point in resonance with the

Figure 4.8: The splitting of the free exciton branch as a result of spin-orbit coupling in (110)-grown quantum wells. Due to the splitting, excitons with anti-parallel spin projections (± 1) on the growth axis propagate in different directions once excited with a pump having no in-plane momentum.

excitonic transition generates excitons with opposite spin projections, the respective populations being dependent on the polarization of the normally incident optical field. A right handed circularly polarized light creates excitons where the majority of the population is spin $+1$ polarized while light with left handed polarization generates excitons with spin -1 majority. If the light is linearly polarized, the two spin-split branches are excited at equal rates.

Excitons, with spin projections ± 1 , have opposite group velocities $v_x^\pm = \pm \gamma_X / \hbar$ due to the SO coupling and therefore propagate in opposite directions once excited and are separated by a distance $\delta l = 2|v_g^\pm|t$ after a time t . For the exciton recombination time $\tau = 10$ ps and SO coupling parameter $\gamma_X = 100$ meV Å, one can thus estimate a separation for the helicity of the emitted light of order $\delta l \approx 0.3 \mu\text{m}$.

How does this kind of free carrier SO coupling affect the polarization pattern of polaritons when such a quantum well is placed inside a microcavity? For the purpose of studying polariton spin currents generated by SO coupling, a model Hamiltonian,

similar to the one in Eq. 4.26, can be written in reciprocal space as

$$\hat{H} = \begin{pmatrix} \frac{\hbar^2 k^2}{2m_P} + \gamma_P k_x & \beta(k_x - ik_y)^2 \\ \beta(k_x + ik_y)^2 & \frac{\hbar^2 k^2}{2m_P} - \gamma_P k_x \end{pmatrix} \quad (4.32)$$

with $\gamma_P = X\gamma_X$ being the polariton SO coupling parameter where X is the excitonic Hopfield coefficient and β is the LT-splitting constant.

To obtain the polarization degrees in Eqs. 4.22-4.24, the Schrödinger-type Eq. 4.25 is solved numerically in the circular polarization basis where $\hat{V} = 0$ and the normally incident pump being given by Eq. 4.29 which resonantly excites the lower polariton brach at the energy E_P and has a spot radius r_0 . The optical field of the pump is linearly polarized along the x -axis.

Here, the model paramaters are taken to be those of standard GaAs/AlGaAs planar microcavities: polartion life time $\tau = 10$ ps, Rabi splitting $V_R = 5$ meV, detuning of cavity mode relative to the heavy-hole exciton energy at $k = 0$ is $\Delta E_0 = -5$ meV and the cavity photon LT-splitting $\beta_C = \hbar^2(1/m_C^{\text{TM}} - 1/m_C^{\text{TE}})/4$, where m_C^{TE} and m_C^{TM} are the in-plane effective masses of the transverse-electric and transverse-magnetic photonic modes of the cavity, respectively. Here $m_C = 5 \times 10^{-5}m_0$ where m_0 is the vacuum electron rest mass. The energy of the pump relative to the lower polariton band edge is taken to be $E_P = 1$ meV.

After finding the steady state solutions of Eq. 4.25, the polarization degrees are deduced. In Fig. 4.9, ρ_c , ρ_{lin} and ρ_{diag} are plotted for two different values of the exciton

SO coupling parameter γ_X and the pump spot size $r_0 = 20 \mu\text{m}$. For the lower value of $\gamma_X = 50 \text{ meV } \text{\AA}$, represented by the left column of subfigures, a slight conversion to oppositely oriented circular polarization occurs both sides of the pump spot along the horizontal. This observation is consistent with the picture drawn in Fig. 4.8 where polaritons of opposite spin move in opposite directions.

Increasing the SO coupling parameter to $\gamma_X = 100 \text{ meV } \text{\AA}$, results in higher absolute values of group velocities, so that polaritons are able to propagate further before decaying. This explains the sharper distinction of positive and negative domains of ρ_c in the top right corner subplot in Fig. 4.9. In both cases, the pump region remains neutral in the circular polarization degree as the density of newly excited polaritons having $\rho_c = 0$ is overwhelmingly larger than that of polaritons with rotated pseudospins.

The linear and diagonal polarization degrees are shown in the second and third row of Fig. 4.9, respectively. The linear polarization of polaritons is directly inherited from the pump so that the pumping region is fully linearly polarized. The diagonal polarization degree stems from the polariton pseudospin precession about the total effective magnetic field generated by the LT and SO-splitting. In Eq. 4.6, the component of the magnetic field normal to the QW plane is given by the SO coupling term $\gamma_P k_x / 2\hbar$ instead of the real field induced Zeeman splitting. A pump with a radius of r_0 excites polaritons having wavenumber

$$k \approx 1/r_0, \quad (4.33)$$

Figure 4.9: Numerical simulations of the three polarization degree patterns in the steady state for different strength of the spin-orbit coupling γ_X and pump spot-size $r_0 = 20 \mu\text{m}$. In the panels to the left, $\gamma_X = 50 \text{ meV } \text{\AA}$, while for the panels to the right, $\gamma_X = 100 \text{ meV } \text{\AA}$.

courtesy of the Heisenberg uncertainty, which for finite r_0 is not zero. Increasing the SO splitting leads to pseudospin rotation about an effective magnetic field whose linear-in- $\mathbf{k}$ dependence grows relative to the LT-splitting odd-in- $\mathbf{k}$ and leads to the

observed change in the ρ_{diag} pattern which becomes slightly asymmetric, see bottom row of Fig. 4.9.

Figure 4.10: The circular polarization degree calculated for different spot-sizes of the pump r_0 . The pump is linearly polarized along the x -direction and the spin-orbit coupling is given by $\gamma_X = 100 \text{ meV } \text{\AA}$. The central area is mostly linearly polarized as a result of the pump polarization.

In Fig. 4.10, the circular polarization degree is plotted as a function of pump radius r_0 , as indicated above each subplot. As the pump spot decreases, the polariton wavenumber increases so that the LT-splitting takes over the SO coupling term. For large pump spot sizes, the asymmetric spin currents give way to the cross-like pattern of the OSHE.

Therefore, when the pump spot-size is small, polaritons with high wavevectors are excited. The SO coupling effects lose importance and the observed pattern in this case are in essence controlled by the LT-splitting which is the cause for the OSHE. Nevertheless, to see how a relatively strong SO coupling parameter can alter the OSHE polarization patterns, appropriate calculations are shown in Fig. 4.11.

The subplots on the left are made with zero SO coupling while $\gamma_X = 100 \text{ meV \AA}$ for the right column. Turning on the SO coupling leads to evident asymmetry in the circular polarization degree ρ_{circ} while its effect on the linear and diagonal polarization degrees is less dramatic. It nominally squeezes the linear polarization pattern and provides a small but noticeable asymmetry for the diagonal polarization degree ρ_{diag} .

Figure 4.11: Numerical simulations of the three polarization degree patterns in the steady state for different strength of the spin-orbit coupling γ_X and pump spot-size $r_0 = 2 \mu\text{m}$. In the panels to the left, $\gamma_X = 0 \text{ meV \AA}$, while for the panels to the right, $\gamma_X = 100 \text{ meV \AA}$.

Conclusions

In the original work comprising this thesis, the following has been achieved. The strong coupling of a 2DEG to a dressing electromagnetic field has been shown to result in a set of such unexpected transport phenomena as a giant increase of conductivity, oscillating behavior of conductivity versus intensity of dressing field, and suppression of weak localization effects.

In addition, the dressing field is able to switch the spin relaxation mechanism in the 2DEG between the cases corresponding to the well-known Elliott-Yafet and D'yakonov-Perel' regimes. As a result, the spin properties of the 2DEG can be tuned by a high-frequency electromagnetic field. Particularly, the irradiation of 2DEG by the dressing field results in increasing the spin relaxation time. The study is important since only low-frequency (stationary in particular) magnetic and electric fields were previously considered as a tool to control the spin properties of solids.

In relation to graphene, a circularly polarized dressing field monotonically decreases the isotropic conductivity, whereas a linearly polarized dressing field results in the oscillating behavior of the conductivity and its giant anisotropy. The dressing field

has been shown to modify the magneto-optic properties of graphene, in contrast to the case of conventional conductors with the parabolic dispersion of electrons. In particular, it shifts increases the inter Landau level absorption peaks and shifts their position, the magnitude depending on the indices of the levels involved. Since graphene serves as a basis for nanoelectronic devices, the developed theory opens a way for optical control of their characteristics.

The effect of an applied magnetic field on the optical spin Hall effect was studied in a disordered semiconductor microcavity excited by homogenous laser beam and clean cavity excited by localized laser spot. It was shown that in both cases magnetic field dramatically changes the pattern of spin currents. For the disordered cavity the most basic single scattering semiclassical approximation predicts suppression of spin currents in a strong magnetic field. Accounting for multiple scattering leads to the transition for azimuthal to radial separation of circular polarizations in the reciprocal space. For tightly focused laser excitation the magnetic field leads to the warping of the pattern of circular polarization in real space, changing it from cross-like to gammadion-like. It also causes the formation of a series of concentric linearly polarized rings in real space.

The pattern of spin currents in (110)-grown quantum wells was calculated where spin-orbit splitting causes polaritons with opposite spin projections to flow in the opposite directions, resulting in an asymmetric spin separation in real space. For narrow light beams, when polaritons populate states with large wave vectors, the longitudinal-transverse splitting determines the polariton spin dynamics which results

in the formation of the OSHE symmetric spin texture.

List of publications

Relevant to this thesis

- [A1] S. Morina, O. V. Kibis, A. A. Pervishko, and I. A. Shelykh, “Transport properties of a two-dimensional electron gas dressed by light”, Phys. Rev. B **91**, 155312 (2015).
- [A2] A. A. Pervishko, O. V. Kibis, S. Morina, and I. A. Shelykh, “Control of spin dynamics in a two-dimensional electron gas by electromagnetic dressing”, Phys. Rev. B **91** 205403 (2015).
- [A3] K. Kristinsson, O. V. Kibis, S. Morina, and I. A. Shelykh, “Control of electronic transport in graphene by electromagnetic dressing”, Sci. Rep. **6**, 20082 (2016).
- [A4] O. V. Kibis, S. Morina, K. Dini, and I. A. Shelykh, “Magnetoelectronic properties of graphene dressed by a high-frequency field”, Phys. Rev. B **93**, 115420 (2016).
- [A5] S. Morina, T. C. H. Liew, and I. A. Shelykh, “Magnetic field control of the optical spin Hall effect”, Phys. Rev. B **88**, 035311 (2013).

- [A6] V. Shahnazaryan, S. Morina, S. A. Tarasenko, and I. A. Shelykh, “Spin currents of exciton polaritons in microcavities with (110)-oriented quantum wells”, Phys. Rev. B **92**, 155305 (2015).

Other publications

- [B1] C. Antón, S. Morina, T. Gao, P. S. Eldridge, T. C. H. Liew, M. D. Martín, Z. Hatzopoulos, P. G. Savvidis, I. A. Shelykh, and L. Viña, “Optical control of spin textures in quasi-one-dimensional polariton condensates”, Phys. Rev. B **91**, 075305 (2015).

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