PERFORMANCE EVALUATION AND ENHANCEMENT OF MASSIVE MIMO COMMUNICATION SYSTEMS

TRAN XUAN TUONG

School of Electrical and Electronic Engineering

A thesis submitted to Nanyang Technological University
in partial fulfillment of the requirements for the degree of
Doctor of Philosophy

2017

To my family and friends.

Acknowledgements

I would like to express my gratitude to my supervisor Prof. Teh Kah Chan for his great supervision, advice and support. It has been a wonderful experience to work with Prof. Teh for many reasons. In every discussion, Prof. Teh always keeps an open mind. His professional guidance, inspiration, and encouragement have been invaluable over the years. His belief in my skills and abilities has helped me mature as a researcher. I have learnt a lot of things from him.

I would like to acknowledge Nanyang Technological University and Singapore International Graduate Award (SINGA) Scholarship for providing the financial support during my Ph.D. study.

I would like to thank my colleagues at Communication Research Laboratory I, School of Electrical and Electronic Engineering, Nanyang Technological University, for their kind support and discussion.

Finally, I am grateful to my parents and brother for their unconditional love, encouragement and support throughout my life. I greatly appreciate my friends who made my graduate study more enjoyable.

Abstract

The increasing popularity of smart terminals and their multimedia applications, as well as social networks, leads to tremendous growth in demands for capacity and quality of service (QoS) in cellular networks. To meet these demands, multiple-input multiple-output (MIMO) wireless systems have been developed and they are now a part of current standards such as IEEE 802.11 and the fourth generation (4G) long term evolution (LTE). However, there are several challenges, i.e., spectral crisis, spectral efficiency (SE) and high energy consumption, which cannot be accommodated by current generation networks. The capacity of 4G networks with current technologies is reaching theoretical limit soon. Therefore, we focus on performance analyses and enhancement of massive MIMO system, which is considered as a potential technology for future wireless and mobile communication systems.

With massive MIMO technology, a base station (BS) is equipped with a large antenna array to serve users simultaneously over the same radio resources. The main contribution of this thesis is to investigate fundamental performance in terms of SE and energy efficiency (EE) for downlink massive MIMO systems with different precoding schemes under perfect and imperfect channel state information (CSI). In particular, in single-cell scenarios, asymptotic achievable capacities of block diagonalization (BD) precoding and BD-based with space-time block code (STBC) are analyzed. Assuming that the number of BS antennas and the number of antennas per users are large while their ratio remains bounded, the capacity expressions for these two schemes are derived. The optimal number of users and optimal length of training sequence for maximizing the SE are derived for practical implementation. For the case of single-antenna users, the SE of a massive MIMO system with signal-to-leakage-plus-noise ratio precoding scheme (SLNR-PS) is studied. By applying a realistic power consumption model, under the assumption of equal power allocation for all users, the EE maximization problem with respect to number of BS antennas, transmit power and length of training sequence is investigated. Through analysis, the optimal value of each parameter for maximizing the EE is derived when the other parameters are assumed to be known. Moreover, a closed-form expression of the

optimal length of training sequence for maximizing SE is derived at high signal-to-noise ratio (SNR) region. Based on these expressions, an alternating optimization algorithm is used to solve the EE maximization problem. It has been shown that the SLNR-PS is lower bounded by that of the zero-forcing precoding scheme (ZF-PS). The optimum EE of SLNR-PS is obtained by a massive MIMO setup with optimal values of transmit power and training sequence. If the quality of channel estimation is low, the SLNR-PS needs to use more BS antennas to achieve the optimum EE, as compared to that with better channel qualities. In addition, under the constraints of transmit power and QoS, we also obtain an energy-efficient power allocation scheme to optimize the EE. The problem of rate profile optimization is also considered while satisfying the QoS and EE target constraints.

In multicell scenarios, the pilot contamination affects significantly the performance of massive MIMO systems. This thesis also presents a scheme, which consists of a two-layer approach of optimal tilt adaptation based on the users locations and optimal power allocation based on the game theory, to minimize the effect of pilot contamination, and hence to maximize the network sum rate. The proposed scheme has low computational complexity and it can avoid a heavy signalling exchange among base stations via backhaul links. Numerical results show that the proposed scheme outperforms the conventional fixed-downtilt systems with equal power allocation and waterfilling power allocation, respectively.

A coordinated multipoint transmission for two-tier HetNets with massive MIMO technology and practical deployment is also proposed in this thesis. The small cell base stations (SBSs) are utilized outside the inner region of macrocell base stations (MBSs) to improve the performance of macrocell edge users. Based on the stochastic geometry approach, the SE and EE of the proposed system are analyzed. The EE maximization problem is formulated and solved effectively by using the alternating optimization algorithm under the constraints of QoS, density of SBS, available number of MBS antennas and MBS transmit power. The algorithm is shown to converge quickly. The impacts of the small cell density, inner region size and massive MIMO on the network performance are explicitly examined. By approximating the interference distribution by moment matching with the Gamma distribution, the coverage probability of the two-tier Hetnet is also derived. Numerical results are provided to validate the theoretical analysis. It has been shown that the proposed scheme outperforms the conventional maximum receive power association scheme. Based on the combinational use of the small cell deployment, massive MIMO and joint transmission, it has been shown that the two-tier HetNet can provide high spectral efficiency and energy efficiency.

Table of Contents

Acknowledgements	2
Abstract	3
Table of Contents	5
List of Figures	8
List of Tables	11
List of Abbreviations	12
1 Introduction	1
1.1 Massive MIMO	3
1.2 Network Densification	5
1.3 Research Motivation and Objectives	6
1.3.1 Single-Cell Scenarios	6
1.3.2 Multicell Scenarios	8
1.4 Main Contributions	10
1.5 Organization of Thesis	13
2 Background and Literature Review	15
2.1 Background Knowledge	15
2.1.1 Wireless Channels	15
2.1.2 Key Performance Indicators	17
2.1.3 Power Consumption Model	18
2.2 Precoding Schemes for Massive MIMO Systems	20
2.2.1 Single-Cell Scenarios	21
2.2.2 Multicell Scenarios	23
2.3 Three-Dimensional Beamforming	26
2.4 Massive MIMO Heterogeneous Networks	29

TABLE OF CONTENTS

3	Massive MIMO with BD Precoding Scheme in Single-Cell Systems	33
3.1	System Model Description	34
3.1.1	Perfect CSI case	35
3.1.2	Imperfect CSI case	37
3.2	Achievable Spectral Efficiency	38
3.2.1	BD Precoding Scheme	39
3.2.2	BD-Based Precoding with STBC	40
3.3	Optimal Number of Users and Optimal Length of Training Sequences	41
3.3.1	Optimal Number of Users	42
3.3.2	Optimal Lengths of Training Sequence	44
3.4	Numerical Results and Discussions	45
3.5	Summary	51
4	Massive MIMO with SLNR Precoding Scheme in Single-Cell Systems	52
4.1	System Model Description	54
4.1.1	Uplink Training	55
4.1.2	Downlink Data Transmission	56
4.2	Achievable Ergodic Rate	57
4.2.1	Imperfect CSI Case	58
4.2.2	Perfect CSI Case	62
4.3	Energy Efficiency Optimization with Imperfect CSI	64
4.3.1	Optimal Transmit Power	65
4.3.2	Optimal Number of BS Antennas	67
4.3.3	Optimal Channel Training	68
4.3.4	Alternating Optimization Algorithm	69
4.4	Energy Efficiency Optimization with Perfect CSI	71
4.4.1	Optimum P , Optimum N and Optimum Training Length	71
4.4.2	Energy-Efficient Power Allocation Scheme	74
4.5	Rate Profile Optimization	76
4.6	Numerical Results and Discussions	78
4.7	Summary	87
5	Sum-Rate Optimization for Multicell Massive MIMO Systems	89
5.1	System Model Description	90
5.2	A Two-Layer Optimization Scheme	93

TABLE OF CONTENTS

5.3	Numerical Results and Discussions	98
5.4	Summary	100
6	Massive MIMO Heterogeneous Networks	101
6.1	System Model Description	104
6.1.1	Cell Association and Coordination Scheme	106
6.1.2	Signal-to-Interference-Plus-Noise Ratio	108
6.2	Average Achievable Rate	110
6.3	Energy Efficiency	118
6.3.1	Optimal Number of MBS Antennas	120
6.3.2	Optimal MBS Transmit Power	121
6.3.3	Optimal SBS Density	121
6.3.4	Alternating Optimization Algorithm	122
6.4	Coverage Probability	123
6.5	Numerical Results and Discussions	130
6.6	Summary	138
7	Conclusion and Future Research Directions	139
7.1	Conclusion	139
7.2	Future Research Directions	141
	Appendix A	145
A.1	Proof of Theorem 1	145
A.2	Proof of Theorem 2	146
	Appendix B	148
B.1	Proof of Theorem 6	148
B.2	Proof of Theorem 7	149
	List of Publications	150
	Bibliography	152

List of Figures

1.1	A time-shifted pilot scheme for 3 groups of cells when the length of pilot sequence is 3 symbols [19].	7
1.2	An example of two-tier HetNet for future next-generation networks.	9
2.1	Classification of fading channels.	16
2.2	Block diagram of a base station.	18
2.3	An example of a base station with adaptive 3D beamforming.	26
2.4	An example of vertical sectorization with three fixed downtilting angles.	28
2.5	An illustration of possible coverage regions in a two tier HetNet.	30
2.6	Illustration of the interfering links in co-channel TDD and RTDD [60, 70].	32
3.1	Block diagram of a massive MU-MIMO downlink system.	35
3.2	Empirical eigenvalue distribution of $\mathbf{G}_k \mathbf{G}_k^H$ when $\eta = 2$	46
3.3	Spectral efficiencies versus SNR for BD-PS, BD-STBC, BD-based RZF [82] and BD-based SVD [82] when $T = 196, N = 60, K = 10, M = 3, L = 2$ and CSI is perfect.	47
3.4	Spectral efficiencies of the BD-precoding and BD-STBC schemes in the case of imperfect CSI when $T = 196, N = 60, K = 10, M = 3$, and $L = 2$	47
3.5	System SE of BD-PS versus the number of BS antennas when $T = 196, K = 10, M = 3$ and SNR = 10 dB.	48
3.6	SEs of the BD-precoding and BD-STBC schemes versus the number of users when $T = 40, M = 2, L = 2, N = 60$ for the case of perfect CSI.	49
3.7	SE comparisons of the precoding schemes for the optimum number of users K^{opt} and the case with $K = T/2$ for $M = 2, L = 2, N = 60$ and $T = 40$	50
3.8	The proposed system SE with different lengths of training sequence T_t when $T = 196, K = 10$ and $P_t = 3$ dB.	51
4.1	Illustration of the rate profile optimization for $K = 2$ users.	77

LIST OF FIGURES

4.2	The SEs versus SNR for SLNR-PS, ZF-based, MF-based, and MMSE-based precoding schemes with perfect CSI and EPA when $N = 30, K = 10, T_t = 10$ and $T = 196$	79
4.3	The SEs versus N for SLNR-PS and ZF-PS with perfect and imperfect CSI, and EPA when $K = 10, T_t = 10, T = 196$ and $\rho_d = 5$ dB. For the case of imperfect CSI, we set $\rho_t = 0$ dB and $\rho_t = 4$ dB, respectively.	80
4.4	The bounds, asymptotic limit and numerical results of the SE versus SNR for SLNR-PS with imperfect CSI and EPA when $N = 60, K = 10, T_t = 10$ and $T = 196$	81
4.5	The SE of SLNR-PS versus training length T_t when $K = 10, N = 100, \rho_d = 30$ dB, and $T = 196$ for the case of imperfect CSI.	82
4.6	The EE versus N for SLNR-PS with perfect CSI and different R_s requirements. .	83
4.7	The EE of SLNR-PS with imperfect CSI when $K = 10, \sigma_n^2 = -96$ dBm, $P_t = 10$ dBm, and $T = 196$	84
4.8	The EE of SLNR-PS versus number of iterations when $K = 10, \sigma_n^2 = -96$ dBm. .	85
4.9	Energy efficiency of SLNR-PS for both cases of EPA and OPA when $K = 10, N = 80, T_t = 10$, and $\sigma_n^2 = -96$ dBm in the case of perfect CSI.	86
4.10	Spectral efficiency of SLNR-PS versus SE for both cases of EPA and OPA when $T_t = 10$, and $\sigma_n^2 = -96$ dBm in the case of perfect CSI.	87
4.11	Plot of three-user rate region when $N = 200, P = 5$ dBm and $\eta_t = 0.3$ bits/Joule. .	88
5.1	An example of a 3-cell network for MIMO systems.	93
5.2	Comparisons of sum rate performance for different systems with $N \rightarrow \infty$ and $K = 10$	99
5.3	Sum rate comparisons for different systems with $K = 5$ and $M = 30$	100
6.1	An example of a two-tier HetNet model with one MBS.	104
6.2	An example of close-up 3×3 km view for a two-tier HetNet when $\lambda_1 = (500^2\pi)^{-1}, \lambda_2 = 5\lambda_1$ and $R = 400$ m.	105
6.3	Average achievable rates of the typical user for the proposed scheme and MRP scheme [127] with $N = 200$ and $\lambda_2 = 5\lambda_1$	130
6.4	Average achievable rates of the typical user for the proposed scheme and MRP scheme [127] with $R = 300$ m and $\lambda_2 = 5\lambda_1$	131
6.5	Coverage probability of the proposed scheme versus threshold T with $R = 200$ m, $\lambda_2 = 5\lambda_1, N = 100, P_m = 43$ dBm, $P_2 = 30$ dBm, and $\mu = 3$ dB.	133
6.6	Coverage probability versus R with $\lambda_2 = 5\lambda_1, N = 100, P_m = 43$ dBm, and $P_2 = 30$ dBm.	134
6.7	Area spectral efficiency and energy efficiency versus λ_2/λ_1 when $R = 300$ m, $P_m = 43$ dBm and $P_2 = 30$ dBm.	135

LIST OF FIGURES

- 6.8 Energy efficiency of the proposed scheme when $R = 300$ m and $P_2 = 30$ dBm. . . 136
- 6.9 Convergence of the AO algorithm for maximizing total EE with different values
of R when $P_2 = 30$ dBm. 137

List of Tables

2.1	The SINR expressions of the k th user in massive MIMO systems with linear precoders	22
3.1	The dimensionality constraints for removing the multiuser interference.	37
3.2	Algorithm to determine optimal number of users K_1^* (or K_2^*)	44
3.3	The optimal number of users	50
4.1	Bisection method for solving the rate profile maximization	77
4.2	Simulation parameters	82
5.1	Algorithm for maximizing network sum rate	98
6.1	Summary of notations	107
6.2	Bisection search algorithm for obtaining N^*	120
6.3	EE maximization with the AO algorithm	122
6.4	Simulation parameters	129

List of Abbreviations

3D	Three-dimensional
3D-MIMO	Three-dimensional multiple-input multiple-output
3G	The third generation
4G	The fourth generation
5G	The fifth generation
AC-DC	Alternating current-direct current
AEC	Area energy consumption
AO	Alternating optimization
ASE	Area spectral efficiency
BD	Block diagonalization
BD-STBC	Block diagonalization space-time block code
BS	Base station
BSA	Bisection search algorithm
CCI	Co-channel interference
CDF	Cumulative distribution function
CoMP	Coordinated multipoint
CSI	Channel state information
CTA	Coordinated tilt adaptation

DAC	Digital-to-analog converter
DC-DC	Direct current-direct current
DIRECT	DIviding RECTangle
DL	Downlink
DoF	Degree of freedom
DPC	Dirty-paper coding
DSP	Digital signal processing
EE	Energy efficiency
EED	Empirical eigenvalue distribution
EPA	Equal power allocation
ES	Exhaustive search
HetNet	Heterogeneous network
i.i.d.	Independent and identically distributed
ICI	Inter-cell interference
ICT	Information and communication technology
ITI	Inter-tier interference
JPCoMP	Joint processing coordinated multipoint
LB	Lower bound
LO	Local oscillator
LTE	Long-Term Evolution
MBS	Macrocell base station
MF	Matched-filtering
MI	Mutual information

List of Abbreviations

MIMO	Multiple-input multiple-output
mmWave	Millimeter wave
MMSE	Minimum mean-squared error
MRP	Maximum received power
MRT	Maximal ratio transmission
MU	Macrocell user
MU-MIMO	Multi-user multiple-input multiple-output
NE	Nash equilibrium
OFDM	Orthogonal frequency division multiplexing
OPA	Optimal power allocation
PA	Power amplifier
PAS	Power allocation strategy
PB	Pareto boundary
PCM	Power consumption model
PDF	Probability density function
PGFL	Probability generating functional
PPP	Poisson point process
QoS	Quality of service
RBD	Regularized block diagonalization
RF	Radio frequency
RTDD	Reserved time-division duplex
RV	Random variable
RZF	Regularized zero-forcing

List of Abbreviations

SBS	Small-cell base station
SCN	Small-cell network
SE	Spectral efficiency
SINR	Signal-to-interference-plus-noise ratio
SIR	Signal-to-interference ratio
SLNR	Signal-to-leakage-plus-noise ratio
SLNR-PS	Signal-to-leakage-plus-noise ratio precoding scheme
SNR	Signal-to-noise ratio
SP	Signal processing
SSIWA	Smoothed simultaneous iterative waterfilling algorithm
STBC	Space-time block code
SU	Small-cell user
SVD	Singular value decomposition
TDD	Time-division duplex
UB	Upper bound
UL	Uplink
ULI	Users' location information
WF	Waterfilling
ZF	Zero-forcing

Chapter 1

Introduction

One of the most successful technologies developed over the past fifty years is information and communication technology (ICT), of which wireless communication is the fastest growing segment [1]. Due to the advances of communication technologies, every aspect of human life, which includes the ability to communicate and live in both social functions and business operations, has improved significantly. Nowadays, wireless technology is an essential part of everyday life in most countries around the world. As a result, the number of wireless devices increases exponentially every year. In particular, the European Mobile Observatory reported that the annual growth of mobile broadband has been reached 92% since 2006. The Wireless World Research Forum has predicted that 7 trillion wireless devices, which run voice, data, and other applications in wireless networks, will serve around 7 billion people by 2017 [2]. The number of wireless devices connected to networks will reach 1000 times population of the world. Consequently, the demand for wireless capacity (or throughput) will always increase, however, the quantity of available spectrum will never increase. Therefore, a fundamental problem of wireless communication is how to provide an increase of capacity and achieve better reliability for given radio resources.

In wireless communications, the transmitted signals typically undergo multipath propagation

channels characterized by scattering, reflection and refraction before they reach the receiver. There are several techniques which have been proposed to improve performance of wireless communication systems. A well-known solution is to use multiple-input multiple-output (MIMO) technique, which has been well investigated and developed over the past twenty years. The key ideas of MIMO systems are the use of multiple antennas and the ability to effectively take advantage of multipath propagation channels. Therefore, MIMO systems provide a great improvement in terms of capacity and reliability by enhancing the multiplexing and diversity gains without an additional increase in bandwidth or transmit power. They are now a part of current standards including IEEE 802.11 (WiFi), 802.16 (WiMAX), as well as the third generation (3G) and the fourth generation (4G) Long-Term Evolution (LTE) cellular networks.

Although cellular networks have been designed and optimized primarily for voice and basic multimedia applications, with the increasing popularity of smart terminals and social networks, there are several challenges which cannot be accommodated by current generation networks. For most MIMO implementations, the base stations (BSs) typically employ only a few antennas, hence the corresponding improvement in capacity of the networks is reaching theoretical limit with current technologies. Specifically, 4G LTE-Advanced standard already supports as much as 8 antennas in the downlink (DL) and 4 antennas in the uplink (UL) [3]. Another challenge of current generation networks is high energy consumption. The mobile operators have reported that over 70% of the energy consumption of wireless networks is contributed by the radio transmission part [4,5]. The increase of energy consumption leads to an increase of CO₂ emission and higher operation costs. Thus, energy-efficient design should be one of the initial requirements for next-generation wireless systems in order to minimize the environmental impact of the wireless domain, as well as reduce the operation costs [4]. While spectral efficiency metric reflects the efficient utilization of the available spectrum in terms of capacity, energy efficiency metric reflects the amount of data transmission per unit energy consumed by the system. Therefore, in this thesis, we consider spectral efficiency (SE) and energy efficiency (EE) as two main key

performance indicators for analyzing and designing future wireless communication networks.

In [5], the “big three” potential technologies for the fifth generation (5G) wireless networks have been proposed and discussed. They are massive MIMO (also called as large-scale MIMO, very large antenna systems, full-dimension MIMO), network densification and millimeter wave (mmWave) communication. Although the mmWave frequency range of 30-300 GHz can offer a huge segment of spectrum (which is still underutilized), the wireless transmission at these frequencies exhibits very different propagation characteristics, consisting of strong path loss, low diffraction around obstacles, atmospheric and rain absorption, compared to the cellular frequency bands (below 6 GHz). Moreover, the mmWave transmissions require significant changes in the hardware design, including amplifiers and transceiver architectures [5]. Therefore, in this thesis, we focus on massive MIMO and combinational use of massive MIMO and deployment of small-cells for designing the 5G wireless and mobile communication networks which are discussed in the following.

1.1 Massive MIMO

Massive MIMO is a form of multi-user MIMO (MU-MIMO) systems, in which the base station is equipped with hundreds of antennas to serve tens of users simultaneously [6, 7]. With a large antenna array, massive MIMO can scale-up all the benefits of the conventional MIMO [6, 8, 9]. By using massive MIMO with time-division multiplex and frequency-division multiplex via orthogonal frequency division multiplexing (OFDM), the BS can accommodate a large number of users [6]. In the time-division duplex (TDD) protocol, the channel reciprocity can be exploited to obtain channel state information (CSI). The length of training sequence is proportional to the number of users, however it is independent of the number of BS antennas. Hence, the use of additional BS antennas does not increase the feedback overhead, and it is always advantageous to increase the number of base station antennas, even when the channel

estimation is poor and the received signal-to-interference-plus-noise ratio (SINR) is low [10].

The main advantages of massive MIMO are presented as follows:

- *Enhancing SE and EE:* By using a large number of antennas, massive MIMO system can improve multiplexing and diversity gains [7]. As a result, it can significantly increase SE and achieve good link reliability [8]. With a large antenna array, the BS can concentrate transmission power effectively with a narrow beam on small areas where the users are located. Hence, the BS can potentially reduce UL and DL transmit powers, which leads to remarkable improvement in energy efficiency [9]. This aspect provides solutions to environmental and health concerns related to mobile communications, as well as business view points.
- *Low-power and inexpensive components:* In massive MIMO systems, hundreds of low-cost amplifiers, which have output power in the milliwatt range, can be used to replace expensive ultra-linear 50 W amplifiers employed in the traditional systems [6, 8]. Hence, several expensive components can be eliminated, such as large coaxial cables.
- *Reduction of latency:* According to the beamforming techniques and law of large numbers to minimize fading dips on the radio frequency signal during operation, the fading no longer limits latency. Thus, massive MIMO is able to reduce latency significantly on the air interface [8].
- *Simplified signal processing:* When the BS has a large-scale antenna array, the effects of uncorrelated noise and small-scale fading can be eliminated with simple linear precoders and receivers [7]. Due to property of channel hardening in massive MIMO systems, the adaptive schemes for user scheduling and power control can be obtained by using the large-scale fading coefficient, instead of considering instantaneous channel realizations (i.e., both the large-scale and small-scale fading coefficients of the channel) [8]. This simplifies the

complexity of the signal processing significantly. In addition, an individual element failure of the large antenna array will not significantly affect the system performance.

Although massive MIMO has many advantages, there are a lot of research challenges which still need to be investigated. Firstly, due to the limitation of the channel coherence interval and the orthogonal pilot reuse, pilot contamination is one of the main challenges to employ massive MIMO systems in the cellular network. The issue of pilot contamination reduction will be discussed in the following Section. Secondly, new standard and designs for massive MIMO systems are required to develop because the current LTE-Advanced standard only allows as much as 8 antenna ports at the BS. Thirdly, in reality, the propagation channels can be correlated and not favourable¹. To overcome these problems, the distributed antenna systems can be applied. However, the optimized signal processing techniques are also open to research.

1.2 Network Densification

It has been shown that mobile users usually stay indoors for about 80% of time [11]. Hence, user data demands in cellular networks occur mainly in indoor areas, while the traditional cellular architecture uses an outdoor BS in the middle of a cell coverage. The indoor coverage issue is actually challenging for large buildings such as hotels, shopping malls, and enterprise and government offices, where the transmitted signals are degraded by multiple indoor surfaces. The typical number of indoor users is large, but the quality of service (QoS) delivered to them is currently low.

A traditional approach to enhance the network performance is to make the cells smaller. Due to cell shrinking, the deployment of small-cells has numerous advantages, such as improving frequency reuse across a geographic region and reducing number of users which compete for

¹The favourable propagation means that the channel vectors between the users and the BS are nearly orthogonal when the number of BS antennas is very large [7].

given radio resources at each base station [5]. In addition, the small-cells typically have lower deployment costs, smaller transmit power and hence lower energy consumption, as compared with that of macrocells. Therefore, by using high-speed optical fiber for backhaul signalling, many small-cell base stations can be effectively deployed in a certain area. It implies that the coverage and the area spectral efficiency (bit/s/Hz/m²) can be significantly increased by reducing the distance between small-cell users and small-cell base stations (SBSs). This approach has been employed in the existing wireless cellular networks including 3G and 4G LTE networks, which essentially results in a multi-tier cellular heterogeneous network (HetNet).

1.3 Research Motivation and Objectives

1.3.1 Single-Cell Scenarios

In the single-cell DL scenarios, massive MIMO systems can adopt linear precoding schemes with low-complexity for practical implementation consisting of matched-filtering (MF) (or maximal ratio transmission (MRT)), minimum mean-squared error (MMSE), zero-forcing (ZF) and regularized ZF (RZF) precoding schemes [6, 12]. In massive MIMO systems, it has been shown that the random channel vectors between users and BS are nearly orthogonal [6]. The co-channel interference (CCI) at users is minimized by using these schemes, thus a good capacity performance can be achieved. However, most of prior works in the literature focused on investigating performance of massive MIMO systems with common precoding schemes, single-antenna users, or treated the extra antennas per user as the additional autonomous users if multiple antennas were considered [13]. On the other hand, we study the performance of precoding schemes for multiple-antenna users. In this case, block diagonalization (BD), regularized BD (RBD) precoding schemes and their modified implementations (such as generalized ZF channel inversion and generalized MMSE channel inversion) can be applied [14–17]. However, the RBD precoding scheme has higher computational complexity than that of the BD precoding scheme

(BD-PS) and its rate performance approaches to that of the BD-PS at high signal-to-noise ratio (SNR) region [15]. Therefore, this motivates us to investigate the performance of massive MIMO with BD-PS and multiple-antenna users for both cases of perfect and imperfect CSI, respectively.

To avoid solving coupled problems for optimizing SINR, the ZF beamforming is usually considered in the literature. It provides solution to eliminate the CCI at each user. Unfortunately, the ZF precoding scheme is sensitive to unmodelled interferences and other sources of distortion. On the other hand, a leakage-based beamforming has been proposed to optimize the signal-to-leakage-plus-noise ratio (SLNR) for all users simultaneously [18]. The leakage is a measure of how much signal power of a desired user leaks into the other users. The leakage-based criterion results in a decoupled optimization problem which is easy to solve. Different from the zero-forcing solution, the leakage scheme does not require any restriction on the number of BS antennas [18]. It also takes into account the influence of noise when designing the beamforming vector. Hence, motivated by the above discussions, we analyze the SE and EE of massive MIMO with SLNR precoding scheme (SLNR-PS) under perfect and imperfect CSI.

Figure 1.1: A time-shifted pilot scheme for 3 groups of cells when the length of pilot sequence is 3 symbols [19].

1.3.2 Multicell Scenarios

From practical view points, the number of orthogonal pilot sequences is limited in the cellular networks. It is naturally upper bounded by the ratio of the duration of the coherence interval to the channel delay spread [8]. Hence, the available supply of orthogonal pilot sequence is easily exhausted in multicell scenarios. Due to pilot reuse in the cellular network, the channel estimate at the certain BS can be contaminated by pilots transmitted by the users in neighbouring cells. It is known as pilot contamination effect which is one of the major challenges in designing massive MIMO networks. It is a fundamental performance bottleneck, and it does not vanish even when the BS is equipped with a large number of antennas. Hence, the issue of pilot contamination reduction has been widely studied in the literature [19–23]. In particular, a time-shifted pilot scheme has been studied in [19], which uses asynchronous transmission among adjacent cells. The key idea is to divide all the cells of a cellular network into groups of cells. An example of this approach for 3 groups of cells is presented in Fig. 1.1. While users from cells in group 1 transmit their pilot sequences simultaneously, users from other two groups receive downlink data. When users in group 1 complete their pilot sequence, they start receiving downlink data and a different group starts sending pilots. To do so, it can avoid pilot contamination among users from different groups. However, it leads to mutual interferences between data and pilot. In [20], a distributed precoding scheme has been presented to reduce the inter-cell interference and the weighted sum squared error. Numerical results have been shown that the performance of the presented scheme is better than that of the conventional single-cell ZF precoding. In order to optimize the performance of users with severe pilot contamination, an adaptive pilot assignment scheme has been presented in [21]. Moreover, the blind method based on subspace partitioning in [22] can also reduce the effect of pilot contamination. This method is blind in the sense that it does not require pilot data to find the appropriate subspace. An algorithm with polynomial complexity based on singular value

decomposition (SVD) has been studied to mitigate pilot contamination without the need for coordination among cells. In [23], a coordinated approach for obtaining channel estimation has been investigated. The coordination exploits the additional second-order statistical information about the user channels. The effect of pilot contamination can be eliminated under certain conditions on the channel covariance. With a different approach, we employ three-dimensional beamforming and optimal power allocation (OPA) techniques to minimize the effect of pilot contamination for cell-edge users, and hence to maximize the network capacity.

Figure 1.2: An example of two-tier HetNet for future next-generation networks.

Despite the fact that there is no preliminary standardization for the future wireless communication systems, the combinational use of massive MIMO and HetNet has been currently considered as an effective solution to meet demands of 1000 times increase in mobile data traffic and QoS in the near future [4]. An example of a two-tier HetNet with massive MIMO is presented in Fig. 1.2. While the SBSs can be employed in indoor and cell-edge areas, the macrocell base stations (MBSs) use large number of antennas to serve outdoor users. As compared with the conventional HetNets (i.e., single-antenna BSs), the aggregate interference environment in

HetNets with massive MIMO is more heterogeneous and complicated, hence evaluating its performance is challenging. Recently, there has been a growing interest to investigate the user association problem and the performance in terms of coverage probability and area spectral efficiency for HetNets with massive MIMO in both academy and industry.

1.4 Main Contributions

The term “Massive MIMO” is commonly used to refer to base station equipped with a very large-scale antenna array in order to serve some users equipped with a finite number of antennas in the literature. However, in the thesis, it can be used for a massive MIMO setup, where the number of BS antennas and the number of antennas per users are large while their ratio remains bounded. This set up involves the traditional massive MIMO system as a specific case where the ratio set approaches infinity.

This thesis consists of two parts. Firstly, we propose some system designs of massive MIMO in single-cell scenarios. We investigate and enhance the fundamental performance in terms of SE and EE for massive MIMO in different wireless communication systems, such as massive MIMO with BD-PS, BD-based space-time block code (STBC), and SLNR-PS. Secondly, we analyze the performance of massive MIMO systems in multicell scenarios. In particular, we propose a potential approach to reduce the effect of pilot contamination based on the three-dimensional beamforming and game theory. In addition, we consider a general setup of a two-tier heterogeneous network with massive MIMO macrocell BSs and single-antenna small-cell BSs. The analytical frameworks are presented to study the aggregate interference and network performance. The major contributions of the thesis are summarized in the following:

1. We assume that the number of base station antennas and the number of antennas per users are large while their ratio is fixed as a constant. We first derive upper bounds on achievable

SEs of massive MIMO systems with BD and BD-based space-time block code (BD-STBC) under perfect and imperfect CSI, respectively. We show that BD-PS can achieve higher SE performance as compared with that of BD-STBC. This is because the achievable SE of BD-PS is proportional to the number of BS antennas, whereas the achievable SE of BD-STBC is proportional to the number antennas per user and the column size of STBC. In order to maximize the SEs of these schemes, the optimal number of users and optimal length of training sequence are analyzed for practical implementation, respectively.

2. We derive lower and upper bounds on achievable ergodic rate of massive MIMO systems with SLNR-PS under perfect and imperfect CSI conditions when the number of BS antennas N is large. By applying a realistic power consumption model, which considers the emitted power consumption, the radio frequency circuit power consumption, and the power consumption for performing digital signal processing, the EE maximization problem is formulated. We study the effects of N , transmit power P and length of training sequence T_t on the EE. The optimal value of each parameter to maximize the EE is derived when the other parameters are fixed. This provides an approach to achieve the optimum EE of SLNR-PS by employing an alternating optimization (AO) algorithm. It also provides essential insights on the dependence among three parameters and other coefficients. Numerical and theoretical results have shown that: (i) the SE of SLNR-PS is lower bounded by that of the ZF-PS; (ii) the presented algorithm converges quickly after only a few iterations and it can almost achieve the global optimal solution; (iii) the deployment of massive MIMO with the optimal transmit power and optimal training length can help to maximize the EE of SLNR-PS. If the quality of channel estimation is low, the SLNR-PS needs to use more antennas to achieve the global optimum EE, as compared to that with better channel qualities. Moreover, we also design an OPA scheme to optimize the EE of SLNR-PS while satisfying the constraints of the transmit power and QoS. By solving the rate profile optimization problem with respect to the transmit

power, QoS, and EE target, the optimal rates of individual users are obtained .

3. In order to minimize the effect of pilot contamination for multicell massive MIMO systems, we propose a scheme which includes a two-layer approach of optimal tilt adaptation and optimal power allocation. Specifically, the BSs jointly obtain a set of all optimal tilt vectors corresponding to a set of all pilot sequences based on the users' location information. With the achieved optimal tilt vectors, we then consider the sum rate optimization problem for all cells. To achieve the maximum network sum rate, we propose a scheme to obtain the optimal power allocation for all cells based on the noncooperative game theoretic approach. By using a distributed algorithm to compute the equilibrium state of the game, the proposed scheme can avoid a heavy signalling exchange among the BSs via backhaul links. Moreover, it also provides comparable performance with low complexity.
4. The performance of a two-tier massive MIMO heterogeneous network with practical deployment is investigated under shared spectrum operation. We assume that the SBSs are non-uniformly distributed. A coordinated multipoint (CoMP) transmission scheme for the MBSs and SBSs is presented to improve the performance of macrocell edge users. By using concept from stochastic geometry, a tractable approach is presented to analytically study the average achievable rate of the proposed scheme. The EE maximization problem with respect to density of SBS, available number of MBS antennas, and MBS transmit power is formulated while satisfying the QoS constraints. It is effectively solved by using an alternating optimization algorithm based on the optimal SBS density, optimal MBS transmit power, as well as optimal number of MBS antennas. The impacts of employing non-uniform small-cell deployment, the inner region size of MBS and massive MIMO on the overall network SE and EE are explicitly investigated. Moreover, by approximating the interference distribution by moment matching with the Gamma distribution, the coverage probability is derived when the two-tier HetNet is assumed to be interference

limited. This method can simplify the complexity of analysis and the expression of the coverage probability. Numerical results are provided to validate theoretical analysis. It has been shown that the proposed scheme outperforms the traditional maximum received power association scheme. Massive MIMO HetNets can be considered as a suitable approach to improve the area spectral and energy efficiencies for next-generation cellular networks.

1.5 Organization of Thesis

This thesis consists of seven chapters. In Chapter 1, we give a brief introduction to the need of ground-breaking technologies for 5G wireless and mobile communication systems in order to solve the crucial challenges, which are not accommodated by current generation networks. We also discuss the research motivation and objectives, which are investigated in the thesis. Lastly, the major contributions of the thesis are summarized.

In Chapter 2, we provide basic background knowledge to gain better understanding of further discussions in the thesis, including wireless channel, key performance indicators, and power consumption model. We conduct detailed literature review related to massive MIMO with linear precoding schemes in various wireless communication systems. We also present a brief overview of three-dimensional beamforming and heterogeneous networks, as well as their implementations.

In Chapter 3, under perfect and imperfect CSI, we analyze the performance of massive MIMO systems with BD-based precoding schemes for multiple-antenna users.

In Chapter 4, the performance analysis and enhancement of massive MIMO system with SLNR-PS are considered. The optimal solutions for the SE and EE maximization problems with perfect and imperfect CSI are provided, respectively.

In Chapter 5, we study the problem of pilot contamination. A scheme is presented to reduce the negative effect of pilot contamination in multicell massive MIMO systems, specifically, for cell-edge users.

In Chapter 6, the combinational use of massive MIMO, non-uniform small-cell deployment and joint transmission is proposed to enhance the coverage and capacity performance. We then optimize the EE with respect to the density of SBSs, number of MBS antennas and MBS transmit power while satisfying QoS constraints. The coverage probability of the proposed scheme is also investigated.

In Chapter 7, we summarize our research works, and provide several ideas and research directions for the future work.

Chapter 2

Background and Literature Review

As mentioned in Chapter 1, in the thesis, we investigate fundamental performance of massive MIMO in various wireless communication systems. In the following sections, we give a brief description of relevant background and literature review.

2.1 Background Knowledge

2.1.1 Wireless Channels

The performance of wireless communication systems depends on the propagation environments of wireless channels, which are characterized by fading, i.e., the variation of the signal amplitude over time and frequency. The fading phenomenon consists of small-scale fading and large-scale fading factors [24]. The large-scale fading is caused by path loss which depends on the distance and propagation conditions, as well as shadowing which is characterized by variation of median path loss between the transmitter and receiver, due to large objects such as mountains, vegetation and buildings. The small-scale fading refers to rapid variation of signal levels because of the destructive and constructive multipath fading effect of transmitted

signals. Moreover, the small-scale fading can be classified according to different criteria. Based on multipath delay spread, the small-scale fading is characterized by frequency-selective fading or frequency flat fading. On the other hand, based on time variation in the channel due to the Doppler spread, the small-scale fading are classified as either time variant fading or time invariant fading. These two propagation mechanisms are independent of one another. The classification of fading channel is presented in Fig. 2.1.

Figure 2.1: Classification of fading channels.

Mathematically, in a downlink MU-MIMO system, where the base station is equipped with N transmit antennas to serve its K single-antenna users, a channel vector of the k th user can be modelled as¹

$$\mathbf{g}_k = \sqrt{\beta_k} \mathbf{h}_k, \quad (2.1)$$

where $\mathbf{h}_k \in \mathbb{C}^{N \times 1}$ is the small-scale fading vector whose elements are independent and identically distributed (i.i.d.) zero-mean complex Gaussian random variables with unit variance, i.e., $\mathbf{h}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$. Parameter β_k is the large-scale fading coefficient accounting for shadowing and path loss. In [25], the role of β_k has been investigated in the uplink transmission. Moreover,

¹Note that all the notations are only applicable to this chapter.

by using the fact that the channel vectors of different users are independent and applying the law of large numbers, it is well known that [6, 7]

$$\lim_{N \rightarrow \infty} \frac{\mathbf{g}_i^H \mathbf{g}_k}{N} \xrightarrow{a.s.} 0 \quad \text{and} \quad \lim_{N \rightarrow \infty} \frac{\mathbf{g}_k^H \mathbf{g}_k}{N} \xrightarrow{a.s.} \beta_k, \quad (2.2)$$

where $\xrightarrow{a.s.}$ denotes almost convergence. In the literature, these convergence properties of channel vectors have been widely applied for evaluating the performance of massive MIMO systems.

2.1.2 Key Performance Indicators

In wireless communication systems, capacity (or throughput) indicates how much information can be transmitted for given limited spectral resources. It is defined as

$$\text{Capacity (bits/s)} = \text{Bandwidth (Hz)} \times \text{Spectral efficiency (bits/s/Hz)}. \quad (2.3)$$

The spectral efficiency is a conventional metric to evaluate the efficiency of a wireless system. It presents how efficient a limited frequency spectrum is utilized, but it does not show any insight on how efficient the energy is consumed. The achievable rate can be used instead for some communication systems, where the capacity are unknown. Another metric to evaluate the performance of the next-generation cellular networks is the energy efficiency. It is obtained as the ratio between the achievable capacity and the total power consumption (measured in Watt=Joule/s), which is expressed as [26]

$$\text{Energy efficiency (bits/Joule)} = \frac{\text{Capacity (bits/s)}}{\text{Total power consumption (Watt)}}. \quad (2.4)$$

Without loss of generality, the system is assumed to have unit bandwidth in order to simplify the notation in this thesis.

2.1.3 Power Consumption Model

In order to investigate the EE performance of a cellular network, the power consumption of the BS needs to be captured as well. This is because the BSs take the main power consumption of the network operation. In the literature, several works have assumed that the power consumption model (PCM) only consists of the emitted power consumption [13, 27]. However, since the number of antennas is large in massive MIMO systems, the effects of circuit and digital signal processing (DSP) power consumptions can contribute significantly to the total power consumption [28]. Therefore, to evaluate the practical aspects of the PCM, we study the power consumptions of all the implemented components at the BS. Note that we only consider the downlink scenarios in the thesis. Based on approaches in [28] and [29], a block diagram of the BS, which can be generalized to all BS types including macro, micro, pico and femto BSs, is shown in Fig. 2.2.

Figure 2.2: Block diagram of a base station.

Specifically, a BS consists of multiple transmitters, each of which serves one transmit antenna element. Each transmitter comprises a power amplifier (PA), a radio frequency (RF) module, a baseband engine transmitter downlink section, a direct current-direct current (DC-DC) power

supply, an active cooling system, and an alternating current-direct current (AC-DC) unit (main supply) for connection to an electrical power grid [28]. For the RF module, the circuit blocks along the signal path consist of digital-to-analog converter (DAC), filter, mixer, and frequency synthesizer [29,30]. We assume that all the antenna paths share a frequency synthesizer, i.e., a local oscillator (LO) [30]. The baseband unit performs DSP at the BS, including modulation/demodulation, signal detection (synchronization, channel estimation, equalization), as well as channel coding/decoding [28].

Let us denote P_{ant} and σ_{am} as the output power at each antenna element and the power efficiency of the PA, respectively. Due to influences of the antenna type on the power efficiency, the power consumption of the PA is defined as

$$P_{PA} = \frac{P_{ant}}{\sigma_{am}(1 - \sigma_{feed})}, \quad (2.5)$$

where σ_{feed} is the factor loss of feeder. The feeder loss for a macrocell BS is about 3 dB, while the feeder loss for a small BS type can be typically negligible. Moreover, other factor losses, which are incurred by DC-DC power supply, mains supply, and active cooling, are also considered in this PCM. We assume that the BS power consumption is linearly proportional to the number of transmitter chains. When the BS is equipped with N antenna elements, the total power consumption can be defined as

$$P_{total} = \frac{N \left(\frac{P_{ant}}{\sigma_{am}(1 - \sigma_{feed})} + p_{dac} + p_{mix} + p_{filt} \right) + p_{syn} + P_{bb}}{(1 - \sigma_{DC})(1 - \sigma_{MS})(1 - \sigma_{cool})}, \quad (2.6)$$

where $p_{dac}, p_{mix}, p_{filt}, p_{syn}, P_{bb}$ denote the power consumptions from the DAC, the mixer, the filter, the frequency synthesizer, and the baseband unit, respectively [28,31,32]. The parameters $\sigma_{cool}, \sigma_{MS}, \sigma_{DC}$ are the loss factors of active cooling system, main power supply and DC-DC

power supply, respectively. We denote

$$\omega = (1 - \sigma_{DC})(1 - \sigma_{MS})(1 - \sigma_{cool}) \quad (2.7)$$

and

$$P_c = N(p_{dac} + p_{mix} + p_{filt}) + p_{syn}. \quad (2.8)$$

Due to $P_{ant} = \frac{P}{N}$, where P is the transmit power consumption of the base station, (2.6) can be rewritten as

$$P_{total} = \frac{1}{\omega} \left(\frac{P}{\sigma_{am}(1 - \sigma_{feed})} + P_c + P_{bb} \right). \quad (2.9)$$

Note that the loss factor of active cooling is only applicable to macrocell BSs, and it can be omitted in the small BS types.

2.2 Precoding Schemes for Massive MIMO Systems

MIMO systems can use both non-linear and linear precoding schemes. The non-linear precoding schemes can provide better performance (such as dirty-paper coding (DPC), Tomlinson-Harashima, vector perturbation, and lattice-aided methods), but they require higher computational complexity [33–35], as compared with the linear precoding schemes. Moreover, in massive MIMO systems, it has been shown in [6] and [7] that linear precoding schemes, such as MF and ZF, are virtually optimal. Hence, for practical implementations, massive MIMO systems can use linear precoding schemes with low complexity. Therefore, we focus on different linear precoding techniques for massive MIMO systems in this section.

2.2.1 Single-Cell Scenarios

In a single-cell system, the base station is assumed to be equipped with N transmit antennas to serve K single-antenna users. The received signal at the k th user is defined as [6]

$$y_k = \sqrt{\alpha p_k} \mathbf{g}_k^H \mathbf{w}_k s_k + \sum_{i \neq k}^K \sqrt{\alpha p_i} \mathbf{g}_k^H \mathbf{w}_i s_i + n_k, \quad (2.10)$$

where p_k , $\mathbf{w}_k$ and s_k denote transmit power, precoding vector and symbol of the k th user with $\mathbb{E}\{|s_k|^2\} = 1$, respectively, and n_k is the noise term with $\mathbb{E}\{|n_k|^2\} = \sigma^2$. Parameter α is a power normalization factor which can be defined as $\alpha = \frac{1}{\mathbb{E}\{\text{trace}(\mathbf{W}^H \mathbf{W})\}}$, where $\mathbf{W} = [\mathbf{w}_1 \dots \mathbf{w}_K] \in \mathbb{C}^{N \times K}$. The precoding matrix $\mathbf{W}$ of several linear precoding schemes, including MF (also known as the MRT), ZF, and RZF, are given by [6], [9, Eq. (19)], [12]

$$\mathbf{W} = \begin{cases} \mathbf{G}, & \text{for MF,} \\ (\mathbf{G}\mathbf{G}^H)^{-1}\mathbf{G}, & \text{for ZF,} \\ \left(\mathbf{G}\mathbf{G}^H + \frac{K}{\rho}\mathbf{I}_K\right)^{-1}\mathbf{G}, & \text{for MMSE,} \\ \left(\mathbf{G}\mathbf{G}^H + N\delta\mathbf{I}_N\right)^{-1}\mathbf{G}, & \text{for RZF,} \end{cases} \quad (2.11)$$

respectively, where $\mathbf{G} = [\mathbf{g}_1 \dots \mathbf{g}_K] \in \mathbb{C}^{N \times K}$ and $\delta > 0$ is the regularization factor which can be optimized according to the design requirements. It has been shown in [36] and [37] that, with perfect CSI, an optimal regulation factor for maximizing SINR of users is approximated as $\delta = \frac{K}{N\rho}$, where $\rho = \frac{P_{BS}}{\sigma^2}$ and P_{BS} is the BS transmit power. In this case, the RZF precoder is identical to the MMSE precoder. However, with imperfect CSI, the RZF with the optimal regularization factor is not identical to the MMSE precoder anymore [12, 38]. In addition, from (2.11), we observe that the RZF precoding scheme becomes the ZF precoding scheme when $\delta \rightarrow 0$, whereas it becomes the MF precoding scheme when $\delta \rightarrow \infty$.

Since N is very large, i.e., $N \rightarrow \infty$ and $N \gg K$, the performance analyses of MF and ZF

Table 2.1: The SINR expressions of the k th user in massive MIMO systems with linear precoders

Linear precoder	Perfect CSI	Imperfect CSI
MF [6]	$\frac{c\rho}{\rho+1}$	$\frac{(1-\sigma_e^2)c\rho}{\rho+1}$
ZF [6, 12]	$\rho(c-1)$	$\frac{(1-\sigma_e^2)\rho(c-1)}{\sigma_e^2\rho+1}$
RZF [12]	$\frac{1}{2}(\rho(c-1) + \psi - 1)$	$\frac{1}{2}(x\rho(c-1) + f(x) - 1)$ $x = \frac{1-\sigma_e^2}{1+\sigma_e^2\rho}$
$\psi = \sqrt{(c-1)^2\rho^2 + 2(1+c)\rho + 1}$ and $f(x) = \sqrt{(c-1)^2x^2\rho^2 + 2(1+c)x\rho + 1}$		

have been studied in [6, 27, 39]. Specifically, the lower bounds on the capacity of these two precoders have been derived. The ZF precoding scheme outperforms MF precoding scheme at the medium and high SNR regions, while the MF precoding scheme is better than the ZF precoding scheme at the low SNR region. Furthermore, the MF precoder may be preferable to the ZF precoder due to its robustness. It is applicable to the decentralized architecture and signal processing because it only uses itself (i.e., the estimated channel) [27]. Moreover, it can be shown that MMSE precoding scheme performs better than ZF and MF schemes over the entire range of SNRs. When N and K are large while their ratio remains bounded, i.e., $N, K \rightarrow \infty$ and $N/K = c$, the performance of the MF and ZF schemes has been analyzed in [6]. In [12], under a correlated channel model, the deterministic SINR approximations for ZF and RZF precoders have been derived, respectively. Following that, these approximations were used to solve the problem of sum rate maximization. We assume that the channel vector $\mathbf{g}_k$ has i.i.d. $\mathcal{CN}(0, 1)$ entries, $p_k = P_{BS}/K$, $\delta = K/(N\rho)$, and the parameter $\sigma_e^2 \in [0, 1]$ reflects the quality of channel estimation [6, 12]. The SINR expressions of the k th user for massive MIMO systems with these linear precoding schemes are summarized in the Table 2.1 for both cases of perfect CSI and imperfect CSI.

Moreover, in massive MIMO systems, the computational complexity of linear precoding schemes mainly lies on the inversion of large-dimensional matrices. A notable exception is the MF-based precoding scheme which has only the complexity of matrix multiplication. To overcome this problem, a matrix polynomial can be used to approximate the matrix inversion.

Therefore, based on the matrix polynomial, a linear precoding scheme has been proposed for massive MIMO systems [40]. It has been shown that the proposed scheme has lower complexity, and it can achieve comparable performance in terms of sum rate, as compared to that of the original MMSE precoding. In [41], a hybrid precoding scheme, which is based on the phase control at the radio frequency domain and the effective channel seen from baseband, has been presented to reduce complexity of ZF precoding. Furthermore, it is important to study how the phase noise affects the performance of linear precoding schemes when N is large. In [42], it has been analytically quantified as a significant reduction factor in the quality of CSI, as compared to the system without phase noise.

2.2.2 Multicell Scenarios

In multicell scenarios, the performance of cellular networks is significantly limited by the inter-cell interference (ICI) (i.e., co-channel interference), which is caused by the universal frequency reuse. In a non-cooperative cellular network, the BSs perform independently, and they simply treat the ICI as additional background noise at the users. Thus, the effect of ICI is neglected in the precoding design processes. In order to enhance the network performance, a coordinated multipoint (CoMP) transmission has been developed to deal with the ICI. Due to the level of signalling exchange among the BSs, there are two forms of coordination [43]. Firstly, with coordinated beamforming, the CSI is shared between the BSs, hence the interference generated in other cells is taken into consideration. However, each BS only transmits data to its own users. Secondly, with cooperative multicell precoding (also called as network MIMO), all the BSs fully cooperate, i.e., both the CSI and data information are shared among themselves. By this way, data can be transmitted to the user simultaneously from multiple BSs. Therefore, the cooperative multicell precoding can achieve higher capacity, but it requires a larger amount of the information exchange overhead between the BSs, as compared with the coordinated beamforming technique.

For a non-cooperative cellular network, the performance of linear precoding schemes for massive MIMO systems has been investigated in [6, 7, 9, 44, 45]. In particular, the deterministic signal-to-interference ratio (SIR) approximations for massive MIMO systems with MF and ZF precoders in TDD protocol have been derived in [6] and [7], respectively. In [9], the author employed a realistic system model which considers the impacts of imperfect CSI, pilot contamination, antenna correlation, and path loss. The asymptotic achievable rate of the MF and RZF precoders have been analyzed when $N, K \rightarrow \infty$ and $N/K = c$. Based on these expressions, the author has investigated how many antennas per user are required to obtain a given percentage of the performance limit with infinite N , as well as how many antennas are required with matched-filtering to obtain the same performance of RZF. To reduce complexity of linear precoders in multicell scenarios, in [44], the authors have proposed a method to approximate the matrix inverse for RZF precoding scheme by using a truncated polynomial expansion. With a small number of polynomial coefficients, the proposed method can provide a remarkable performance as compared to the original RZF precoder. Furthermore, in [45], the asymptotic rates of massive MIMO cellular networks with singular value decomposition and block diagonalization precoding schemes have been studied when the number of BS antennas N and the number of antennas per user M grow infinitely while $\frac{N}{KM} \rightarrow d \gg 1$. An interference-aware power allocation scheme has been presented to achieve a uniform rate over the cell.

The studies of the coordinated multicell downlink beamforming in massive MIMO systems have been presented in [46–48]. Particularly, a coordinated beamforming algorithm, which is a decentralized approach, has been proposed in [46] to obtain the multicell beamforming vectors. The design objective is to minimize the aggregate transmit power across all the BSs while satisfying the SINR constraints of the users. Based on that, the effects of imperfect CSI and pilot contamination on the network performance have been investigated. In [47], the coordinated beamforming problem of min-max power allocation in cellular networks has been investigated. The authors have shown that the optimal beamforming strategy of the proposed scheme have

an interesting nested ZF structure. In a coordinated multicell system, the problem of jointly power control and beamforming for maximizing the minimum weighted SINR of users has been studied in [48]. The optimal solution has been obtained by applying the multicell network duality and the nonlinear Perron-Frobenius theory. Generally, all coordinated beamforming algorithms presented in [46–48] require limited amount of information exchange between the BSs due to the use of channel statistics (i.e., large-scale fading coefficients), instead of using instantaneous channel realizations.

The principal feasibility of CoMP in two field testbeds, including coordinated beamforming and cooperative MIMO network, with multiple BSs and different backhaul solutions between the BSs has been discussed in [49]. Many technical challenges have been identified and partially addressed, such as backhaul traffic, synchronization and feedback design. In [50], by treating inter-cluster interference as noise at the users, the clustered cooperative multicell MIMO systems can employ joint transmission with linear ZF beamforming and consideration of per-base-station power constraints. The expression of the spectral efficiency has been provided when $N, K \rightarrow \infty$ and $N/K = c$. Moreover, it has been shown in [51] and [52] that the cooperative MIMO network with fairness scheduling can achieve the same performance limit as compared with the non-cooperative MIMO network, even when its antennas is one order of magnitude fewer than that of the non-cooperative MIMO network.

In [53], the author has proposed a distributed beamforming, which can exploit macro diversity for cell-edge users. Since the distributed beamforming cannot guarantee QoS requirements for given SINR targets, a joint power control has been developed according to two different approaches. The first approach is based on the central processing at a central unit, whereas the second approach is based on the distributed power control algorithm. In [54], with imperfect CSI and spatial channel correlations, the ergodic sum rate of the cooperative MIMO network with RZF precoding scheme has been analyzed when $N, K \rightarrow \infty$ and $N/K = c$. An optimal

regularization parameter for the RZF precoding scheme has also been derived.

2.3 Three-Dimensional Beamforming

Beamforming (or precoding) techniques have significant impacts on improving performance of a cellular network because they can concentrate the signal energy in the direction of the receiver and reduce interference in other directions at the same time. However, in conventional systems, all these techniques usually operate on the horizontal antenna pattern, whereas the vertical pattern is fixed, i.e., a fixed downtilting angle. Hence, the half-power beam width must be wide enough to cover the cell range and small enough to guarantee a high antenna gain [55]. An optimal fixed tilt for regulating power level of the ICI and improving the coverage area can be obtained by the mechanical antenna tilting, but it requires on-site visit. However, with three-

Figure 2.3: An example of a base station with adaptive 3D beamforming.

dimensional (3D) beamforming techniques, massive MIMO systems can provide fully dynamic adaptation of the vertical beam pattern per resource and per user [56]. Let us define $G_{\max}$, $\theta_{3\text{dB}}$ and $\phi_{3\text{dB}}$ as the maximum antenna gain, the half-power beam-widths in the elevation and the azimuth patterns, respectively. The 3D antenna gain of BS l at user k of the i th cell can be expressed as [57]

$$g_{ik}^l(\theta_{tilt}^l, \theta_{ik}^l, \phi_{ik}^l) = 10^{\frac{G_{max}}{10} - 1.2 \left(\frac{\theta_{ik}^l - \theta_{tilt}^l}{\theta_{3\text{dB}}} \right)^2 - 1.2 \left(\frac{\phi_{ik}^l - \phi_{BS}^l}{\phi_{3\text{dB}}} \right)^2}, \quad (2.12)$$

where θ_{tilt}^l is the tilt angle of BS l , ϕ_{BS}^l is the fixed orientation of the BS l th array boresight relative to the x -axis, θ_{ik}^l and ϕ_{ik}^l denote the incident angles connecting user k to the BS l in the elevation and the azimuth domains, respectively. The BS can effectively change antenna tilt angles in order to increase signal strength by pointing the vertical main lobe directly at any location of the user in the cell coverage area as shown in Fig. 2.3. It can reduce the ICI which leads to increase network performance. Note that, with three-sector BS, the range of possible antenna downtilt is smaller than that of the horizontal beam pattern, which can change from -60° to $+60^\circ$ for user separation. For example, when the BS has a height of 30 m and a cell radius of 500 m, it has been shown in [55] that the minimum angle towards the cell edge is 6° , and 95% of the downtilt angles needed to serve cell coverage area are below 20° .

Related to antenna downtilting, the vertical sectorization has been studied in [58–61]. Specifically, the multiple-antenna systems form two distinct beams to serve users based on their locations in the cell, i.e., a sector is vertically split into outer and inner regions. However, in [58–60], the authors have not been stated how to select the radio network parameters properly to ensure good network performance, and the amount of achievable capacity gain if the vertical sectorization is applied. In [61], based on the method of Taguchi, an optimization scheme has been presented to improve the 50%-tile and 5%-tile user rates. This scheme can jointly optimize the downtilt angles, vertical beam-widths, and transmit powers of the inner and outer beams, with

Figure 2.4: An example of vertical sectorization with three fixed downtilting angles.

consideration of the interaction amongst these network parameters. An example of the vertical sectorization with three fixed downtilting angles is shown in Fig. 2.4.

Moreover, in [62], the 3D beamforming strategies, including the user-specific beamforming, the region-partition beamforming based on vertical sectorization and the statistic-based beamforming based on the user's distribution knowledge at the BS, have been studied in a two-tier clustered user distribution model. In [63], with large-scale antenna arrays, the author has jointly designed antenna tilting and user scheduling in order to enhance achievable sum rate of a two-hop relay system. In [64], an information-theoretic channel model has been introduced. Based on this model, the authors have studied the distribution of the mutual information (MI) for the conventional MIMO systems, which have single receive antenna and multiple transmit antennas. An approximation of the MI distribution has also been derived for the systems equipped with large number of transmit antennas.

For single-cell scenarios, in [65], by optimizing the transmit antenna gain and the power allocation, a beamforming scheme has been proposed to maximize the weighted sum rate. In [57], the authors have divided cell into vertical regions by applying a finite set of downtilting angles. Based on switched antenna tilting and the location of users, a scheduling scheme has

been employed to schedule the transmission to one of the vertical regions at each time-slot in order to optimize the achievable ergodic sum rate. The ZF beamforming in the horizontal plane has been used at the BS to serve the users in each vertical region. However, in [65] and [57], the authors have only considered the case that N is small in single-cell scenarios, i.e., $N = 4$, which is not a massive MIMO set up.

2.4 Massive MIMO Heterogeneous Networks

Cellular network systems are becoming more heterogeneous due to a variety of infrastructure deployment which consists of macro, pico, and femto base stations, as well as relay stations. It results from exponential increasing mobile data traffic and high QoS demands. To capture the main characteristics of HetNets and facilitate a realistic performance evaluation, i.e., the heterogeneity in infrastructure and the irregularity in the BS locations, a random spatial model is considered as a more appropriate model for HetNets, as compared to the conventional grid models [66, 67]. In the random spatial model, the BS locations are generated by a two-dimensional point process. The simplest model is the Poisson Point Process (PPP) which is effective to demonstrate the increasing heterogeneity in the infrastructure of cellular networks [66]. By using tools from stochastic geometry, this model provides analytical framework to investigate performance metrics of the multi-tier HetNets, such as coverage probability and average rate [67]. It helps to understand the fundamental principles of network design with practical scenarios. An example of possible coverage regions in a two tier HetNet is presented in Fig. 2.5. In this example, the locations of MBSs and SBSs are modelled according to the two-dimensional homogeneous PPPs with different spatial densities. In particular, the density of SBS is assumed to be three times greater than that of MBS. The coverage region of each MBS is characterized by a standard Voronoi tessellation.

When the BSs in the two tiers share the same time-frequency resources, they can interfere

Figure 2.5: An illustration of possible coverage regions in a two tier HetNet.

with each other. Fortunately, the more antennas the MBS (or even SBS) is equipped with, the more spatial multiplexing gain can be provided onto the same radio resources. In addition, with massive MIMO technology, the BSs can also focus the transmission energy on the intended users, hence both intra- and inter-tier interference can be reduced. To meet high data traffic demands, in [68] and [69], the multi-tier Hetnets, where the massive MIMO macrocell tier is overlaid with single-antenna SBSs, have been considered in the TDD protocol. Based on stochastic geometry approach, the coverage probability and area spectral efficiency of a two-tier Hetnet have been analytically characterized in [68], whereas the spectrum and energy efficiency of the L -tier HetNets have been derived in [69].

A variant of the TDD protocol is called as the reserved TDD (RTDD) protocol, in which

the order of the UL and DL periods in one of the tiers is reversed [70]. It means that all SBSs are operating in the UL mode when the MBSs operate in the DL mode, and vice versa. Thus, the interference in the RTDD protocol is different from that of the TDD protocol. In particular, the MBSs and SBSs interfere with each other, while the macro-cell users (MUs) and small-cell users (SUs) interfere with each other. The operating principles for TDD and RTDD protocols are presented in Fig. 2.6. Since both MBS and SBSs have fixed locations, the interference channels between them are quasi-static, hence the interference subspace can be estimated accurately [70]. A precoding scheme based on the RTDD has been proposed for the two-tier HetNet [71]. The MBS can estimate the null space to the SBSs while they are operating in the DL mode. Following that, the DL data transmission of MBS can be designed into the null space of the SBSs. By this way, the MBS is able to transmit data to its own users while it does not interfere the users of small-cells.

Moreover, the user terminals can be classified into different groups due to their data rate requirements in the HetNets [72, 73]. In order to maximize the achievable sum rate, it is necessary to allocate radio resource to the BSs to serve different groups of users. Based on the game theory approach, the cell association and antenna allocation algorithms have been proposed to achieve equilibrium solution [72]. At the Nash equilibrium, the users and the BSs cannot gain higher data rate and total revenue (i.e., radio resource), respectively. The Game theory approach has also been used in [74] to solve different optimization problems, including sum rate maximization, proportional fairness scheduling, as well as joint user association and resource allocation. In [75], the cell association problem has been formulated directly as a convex network utility maximization, which can be solved efficiently by a centralized subgradient algorithm. In [76], based on the Lagrangian dual analysis, a distributed user association algorithm has been presented to achieve the optimal energy efficiency while satisfying QoS constraints at the users. Furthermore, a survey of EE resource management in heterogeneous cellular networks has been presented in [77]. It summarizes various resource management techniques which are

Figure 2.6: Illustration of the interfering links in co-channel TDD and RTDD [60, 70].

applicable for the conventional HetNets, such as EE resource allocation, intra-tier interference mitigation for improving EE performance, joint BS operation and load distribution, optimal BS switching with resource sharing, as well as EE admission control.

Recently, the dense deployment of small-cells for maximal energy efficiency has been considered in the UL scenario [78]. In the network, each BS is equipped with an antenna array. The authors have reported an interesting conclusion that the small-cells can give high EE, but the EE improvement saturates quickly with the BS density. The maximal EE is neither achieved by a pure small-cell approach, nor by a pure massive MIMO solution. It is only achieved by using a combination of massive MIMO and small-cells, i.e., the each small-cell BS should be equipped with a large number of antennas.

Chapter 3

Massive MIMO with BD Precoding Scheme in Single-Cell Systems

In the literature, various linear precoding techniques, such as BD, RBD, MMSE and their modified implementations, can be used to compose precoding matrices for the multiple-antenna users. The BD scheme is considered as a natural extension of the ZF channel inversion to the case when there are multiple antennas M at each user. In the conventional MIMO systems, the performance of the BD is limited by the number of users K and multiuser channel conditions [79–81]. In particular, it has been shown that the number of BS antennas should be no smaller than the total number of antennas of users KM for obtaining the orthogonalization process [79]. In addition, since the complete suppression of multi-user interference is achieved at the expense of noise enhancement at low and medium SNR regions, the performance of BD scheme can be influenced. However, when the BS is equipped with a large-scale antenna array, i.e., $N \gg KM$, the received user's signal can be enhanced significantly. That helps to improve the achievable rate performance. In this chapter, we will consider the BD-based precoding scheme in massive MIMO systems due to its low complexity [15].

While the performance of the conventional MIMO systems with BD-PS has been well studied in the literature [14–17, 79–81], there are only few works which investigate the performance of massive MIMO with BD-PS. Specifically, the rate performance of each single user has been studied in [45] with perfect CSI when N is large. In [82], the combination of the BD technique with other precoders, such as RZF-based and SVD-based, has been proposed to study how many users should be served via numerical search. Moreover, in order to provide a higher reliability gain over the conventional single-antenna user systems, with multiple-antenna users, the massive MIMO systems using space-time block code have been considered in [83] and [84].

In this chapter, with perfect and imperfect CSI, we derive achievable SEs of BD-PS and BD-STBC schemes, respectively, when the number of BS antennas and the number of antennas per user are very large, i.e., $N, M \rightarrow \infty$, but $\eta = \frac{N}{KM}$ is a finite number. The problems of SE maximization for these schemes are investigated with respect to the number of users and length of training sequence, respectively, while satisfying the QoS constraint. We show that the SEs of these schemes are concave functions in feasible ranges with respect to the number of users. We define an optimal value of each parameter for practical implementation. The accuracy of our analysis is validated by numerical results with different system settings.

*Notation*¹: The notations used in this paper follow usual conventions, vectors and matrices are denoted by symbols in boldface; $(\cdot)^T$, and $(\cdot)^H$ denote the transpose and conjugate transpose, respectively. $\|\cdot\|$ gives the Frobenius norm of the matrix argument. $\mathbf{X} \sim \mathcal{W}_n(m, \mathbf{\Omega})$ denotes an $n \times n$ Wishart matrix $\mathbf{X}$ with m DoFs and covariance matrix $\mathbf{\Omega}$.

3.1 System Model Description

We consider a single-cell downlink MU-MIMO system where the BS transmits signals to K independent multiple-antenna users. We assume that the BS is equipped with N transmit

¹Note that all the notations are only applicable to this chapter.

Figure 3.1: Block diagram of a massive MU-MIMO downlink system.

antennas each user has M receive antennas. Similar to [82], the channel is assumed to be not correlated. It has been shown that the BD precoding can eliminate multi-user interference with low complexity and the STBC provides the full spatial diversity with a simple decoding complexity. Therefore, we employ BD-PS and BD-STBC as shown in Fig. 3.1 for massive MIMO systems, and investigate performance of these schemes in both cases of perfect CSI and imperfect CSI.

3.1.1 Perfect CSI case

When the BD-PS is employed, the receive signal at the k th user is given by

$$\mathbf{y}_k = \sqrt{p_k} \mathbf{H}_k \mathbf{U}_k \mathbf{s}_k + \sum_{i=1, i \neq k}^K \sqrt{p_i} \mathbf{H}_k \mathbf{U}_i \mathbf{s}_i + \mathbf{n}_k, \quad (3.1)$$

where p_k is the transmit power of user k with $\sum_k^K p_k = P$, $\mathbf{s}_k$ is the data symbol vector, $\mathbf{H}_k \in \mathbb{C}^{M \times N}$ is the channel matrix whose entries are i.i.d. zero-mean complex Gaussian random variables with unit variance, and $\mathbf{n}_k$ is an additive Gaussian noise vector with $\mathcal{CN}(0, \sigma_n^2)$ elements. With BD-PS, the precoding matrix $\mathbf{U}_k$ can satisfy the constraint as $\mathbf{H}_k \mathbf{U}_i = 0$ and $\mathbf{U}_i^H \mathbf{U}_i = \mathbf{I}$ for all $k \neq i$. Following [79] and [80], $\mathbf{U}_k$ lies in the null space of the matrix $\bar{\mathbf{H}}_k$

which is obtained as

$$\bar{\mathbf{H}}_k = [\mathbf{H}_1^T \dots \mathbf{H}_{k-1}^T \mathbf{H}_{k+1}^T \dots \mathbf{H}_K^T]^T. \quad (3.2)$$

Applying the SVD on $\bar{\mathbf{H}}_k$, it can be expressed as $\bar{\mathbf{H}}_k = \bar{\mathbf{D}}_k \bar{\mathbf{\Sigma}}_k [\bar{\mathbf{V}}_k^{(1)} \bar{\mathbf{V}}_k^{(0)}]^H$. The matrix $\bar{\mathbf{V}}_k^{(0)}$ is composed of the last $N - (K - 1)M$ right-singular vectors of $\bar{\mathbf{H}}_k$. The matrix $\bar{\mathbf{V}}_k^{(0)}$ forms a unitary basis for the null space of $\bar{\mathbf{H}}_k$. The precoding matrix for user k is obtained as $\mathbf{U}_k = \bar{\mathbf{V}}_k^{(0)} \bar{\mathbf{V}}_k^{(b)}$, where the matrix $\bar{\mathbf{V}}_k^{(b)}$ is composed of the right-singular vectors of the matrix $\mathbf{H}_k \bar{\mathbf{V}}_k^{(0)}$. With BD-PS, (3.1) can be rewritten as

$$\mathbf{y}_k = \sqrt{p_k} \mathbf{H}_k \mathbf{U}_k \mathbf{s}_k + \mathbf{n}_k. \quad (3.3)$$

Hence, the ergodic rate of user k for BD-PS is defined as

$$R_k = \mathbb{E} \left\{ \log_2 \det(\mathbf{I}_M + \frac{p_k}{\sigma_n^2} \mathbf{H}_k \mathbf{U}_k \mathbf{U}_k^H \mathbf{H}_k^H) \right\}. \quad (3.4)$$

For the case of BD-STBC, the data symbol vector $\mathbf{s}_k$ of user k is sent by the adaptive processor through the STBC block to generate the STBC codewords $\mathbf{X}_k$ with a dimension of $L \times F$, where F is the STBC block length and $L \leq M$. It implies that the orthogonal STBC signal $\mathbf{X}_k$ is a linear combination of $\mathbf{s}_k$. For example, when $L = 2$, the system employs Alamouti coding, then the transmitted codeword $\mathbf{X}_k$ is a 2×2 matrix. It is then multiplied by the precoding matrix $\mathbf{U}_k$ of the BD before transmission. With condition $N \geq (K - 1)M + L$, the BD-STBC can remove the multiuser interference. This condition is different to that of the BD-PS. It is shown in Table 3.1. Following that, the received signal of user k in (3.1) can be rewritten as

$$\mathbf{Y}_k = \sqrt{p_k} \mathbf{H}_k \mathbf{U}_k \mathbf{X}_k + \mathbf{N}_k, \quad (3.5)$$

Table 3.1: The dimensionality constraints for removing the multiuser interference.

Precoding scheme	BD-PS	BD-STBC
Condition [81]	$N \geq KM$	$N \geq (K-1)M + L$

where $\mathbf{Y}_k \in \mathbb{C}^{M \times F}$ and $\mathbf{N}_k \in \mathbb{C}^{M \times F}$ has i.i.d. $\mathcal{CN}(0, \sigma_n^2)$ entries. After performing STBC decoding on the received matrix $\mathbf{Y}_k$, an equivalent equation of (3.5) is given by [85, 86]

$$\mathbf{y}_k = \sqrt{p_k} \|\mathbf{H}_{eq,k}\|_F^2 \mathbf{x}_k + \bar{\mathbf{n}}_k, \quad (3.6)$$

where $\mathbf{H}_{eq,k} = \mathbf{H}_k \mathbf{U}_k \in \mathbb{C}^{M \times L}$ and $\bar{\mathbf{n}}_k$ is the noise term after STBC decoding with a distribution $\mathcal{CN}(0, \sigma_n^2 \|\mathbf{H}_{eq,k}\|_F^2)$. Taking into account the code rate, the ergodic rate of user k for BD-STBC is obtained as [81, 85]

$$\ddot{R}_k = R_c \mathbb{E} \left\{ \log_2 \left(1 + \gamma_0 \|\mathbf{H}_{eq,k}\|_F^2 \right) \right\}, \quad (3.7)$$

where R_c is the code rate and $\gamma_0 = \frac{p_k}{LR_c \sigma_n^2}$.

3.1.2 Imperfect CSI case

We have previously assumed that the BS has full CSI knowledge which is estimated at both links based on channel reciprocity. However, the channel estimation based on a training sequence could be obtained with errors. We denote T_t, P_t and $\rho_t = \frac{P_t}{\sigma_n^2}$ as the length, the power level, and the transmit SNR of training sequence, respectively. Based on the MMSE estimation, the channel matrix $\mathbf{H}_k$ can be decomposed as [12, 87]

$$\mathbf{H}_k = \hat{\mathbf{H}}_k + \Delta \mathbf{H}_k, \quad (3.8)$$

where $\Delta \mathbf{H}_k$ is the estimation error matrix. Note that $\Delta \mathbf{H}_k$ and $\hat{\mathbf{H}}_k$ are independent of each other, and their corresponding entries are i.i.d. zero-mean complex Gaussian random variables with variance $\sigma_e^2 = \frac{1}{1+T_t \rho_t}$ and $1 - \sigma_e^2$, respectively.

The BS uses the estimated channel matrix to linearly precode the signals. In particular, the precoding matrix $\mathbf{U}_i$ is composed to be orthogonal to the estimated channel $\hat{\mathbf{H}}_k$ which satisfies $\hat{\mathbf{H}}_k \mathbf{U}_i = 0$. Thus, when the channel estimation error exists at the BS, the received signal of BD-PS at the k th user in (3.3) can be rewritten as

$$\mathbf{y}_k = \sqrt{p_k} \hat{\mathbf{H}}_k \mathbf{U}_k \mathbf{s}_k + \mathbf{n}_k^e, \quad (3.9)$$

where $\mathbf{n}_k^e = \sqrt{p_k} \sum_{i=1}^K \Delta \mathbf{H}_i \mathbf{U}_i \mathbf{s}_i + \mathbf{n}_k$. The covariance matrix of the noise vector $\mathbf{n}_k^e$ is defined as $\sigma_{n_e}^2 \mathbf{I}$, where $\sigma_{n_e}^2 = \sigma_e^2 \sum_k^K p_k + \sigma_n^2$. Thus, the corresponding ergodic capacity of user k is obtained as

$$R_{ip,k} = \mathbb{E} \left\{ \log_2 \det \left(\mathbf{I}_M + \frac{p_k}{\sigma_{n_e}^2} \hat{\mathbf{H}}_k \mathbf{U}_k \mathbf{U}_k^H \hat{\mathbf{H}}_k^H \right) \right\}. \quad (3.10)$$

For BD-STBC, the received signal at the k th user in (3.5) is rewritten as

$$\mathbf{Y}_k = \sqrt{p_k} \hat{\mathbf{H}}_k \mathbf{U}_k \mathbf{X}_k + \mathbf{N}_k^e, \quad (3.11)$$

where $\mathbf{N}_k^e = \sqrt{p_k} \sum_{i=1}^K \Delta \mathbf{H}_i \mathbf{U}_i \mathbf{X}_i + \mathbf{N}_k$ and $\mathbf{N}_k^e$ has i.i.d. $\mathcal{CN}(0, (\sigma_e^2 \sum_k^K p_k + \sigma_n^2))$ entries. After STBC decoding, the ergodic rate of user k for BD-STBC is obtained as

$$\ddot{R}_{ip,k} = R_c \mathbb{E} \left\{ \log_2 \left(1 + \bar{\gamma}_0 \|\hat{\mathbf{H}}_{eq,k}\|_F^2 \right) \right\}, \quad (3.12)$$

where $\bar{\gamma}_0 = \frac{p_k}{LR_c(\sigma_e^2 \sum_k^K p_k + \sigma_n^2)}$ and $\hat{\mathbf{H}}_{eq,k} = \hat{\mathbf{H}}_k \mathbf{U}_k$.

3.2 Achievable Spectral Efficiency

In this section, we analyze asymptotic SE of a massive MU-MIMO system when the number of BS antennas N and the number of antennas per user M are very large, i.e., $N, M \rightarrow \infty$, but $\eta = \frac{N}{KM} > 1$ and $\eta K \gg 1$.

3.2.1 BD Precoding Scheme

Let us define $\underline{\mathbf{H}}_k = \frac{1}{\sqrt{N}}\mathbf{H}_k$ as the normalized channel matrix. Similar approach as shown in Section 3.1.1 is used to determine the precoding matrix for user k which is expressed as $\underline{\mathbf{U}}_k = \underline{\mathbf{V}}_k^{(0)} \underline{\mathbf{V}}_k^{(b)}$. Therefore, the sum rate of BD-PS for the case of perfect CSI is defined as

$$\begin{aligned} R_p &= \sum_{k=1}^K \mathbb{E} \left\{ \log_2 \det \left(\mathbf{I}_M + \frac{Np_k}{\sigma_n^2} \underline{\mathbf{H}}_k \underline{\mathbf{U}}_k \underline{\mathbf{U}}_k^H \underline{\mathbf{H}}_k^H \right) \right\} \\ &= \sum_{k=1}^K \mathbb{E} \left\{ \log_2 \det \left(\mathbf{I}_M + \frac{Np_k}{\sigma_n^2} \underline{\mathbf{H}}_k \underline{\mathbf{V}}_k^{(0)} (\underline{\mathbf{V}}_k^{(0)})^H \underline{\mathbf{H}}_k^H \right) \right\} \\ &\stackrel{(q)}{=} \sum_{k=1}^K \mathbb{E} \left\{ \log_2 \det \left(\mathbf{I}_M + \frac{Np_k}{\sigma_n^2} \tilde{\Phi}_k \tilde{\Lambda}_k^2 \tilde{\Phi}_k^H \right) \right\}, \end{aligned} \quad (3.13)$$

where (q) is obtained by substituting the SVD of matrix $\mathbf{G}_k = \underline{\mathbf{H}}_k \underline{\mathbf{V}}_k^{(0)} = \tilde{\Phi}_k \tilde{\Lambda}_k (\underline{\mathbf{V}}_k^{(b)})^H$. The matrix $\tilde{\Lambda}_k = [\text{diag}(\sqrt{\lambda_1}, \sqrt{\lambda_2}, \dots, \sqrt{\lambda_M}), \mathbf{0}_{N \times (N-KM)}]$ is composed by eigenvalues $\{\lambda_m\}$ of $\mathbf{G}_k \mathbf{G}_k^H$ and $\tilde{\Phi}_k \tilde{\Phi}_k^H = \mathbf{I}$. Moreover, when $N, M \rightarrow \infty$ and $\eta K \gg 1$, the empirical eigenvalue distribution (EED) of $\mathbf{G}_k \mathbf{G}_k^H \sim \mathcal{W}_M(N - (K-1)M, \frac{1}{N} \mathbf{I}_M)$ follows the Marcenko-Pastur distribution which is expressed as [88]

$$f_\lambda(x) = \begin{cases} \frac{1}{2\pi x} \sqrt{(b - \eta K x)(\eta K x - a)}, & \frac{a}{\eta K} \leq x \leq \frac{b}{\eta K}, \\ 0, & \text{otherwise,} \end{cases} \quad (3.14)$$

where $a = (\sqrt{\eta K - K + 1} - 1)^2$ and $b = (\sqrt{\eta K - K + 1} + 1)^2$. When ηK increases, the eigenvalues $\{\lambda_m\}$ converge to deterministic values. Therefore, the matrix $\tilde{\Lambda}_k$ can be approximated as $\tilde{\Lambda}_k \approx [\sqrt{1 - \frac{K-1}{\eta K}} \mathbf{I}_M, \mathbf{0}_{N \times (N-KM)}]$ due to $\lambda_m \rightarrow \mathbb{E}\{\lambda_m\} = 1 - \frac{K-1}{\eta K}$ as ηK is large.

Without loss of generality, we assume that the noise variance is equal to 1 in order to simplify notation [6,9,13]. According to this convention, P and P_t have the interpretations of normalized transmit SNR for the downlink channel and training sequence, respectively. Since the training length T_t is used to acquire the CSI at the BS, an interval of length $T - T_t$, where T is coherence

interval, is used for the data transmission in the time-division duplex mode. Furthermore, we assume that the power levels of M parallel sub-channels at each user are equal. Under the assumption of equal power allocation (EPA), i.e., $p_k = P/K$, from (3.13), the asymptotic SE of BD-PS can be defined as

$$C_p \approx \left(1 - \frac{T_t}{T}\right) KM \log_2 \left(1 + (\eta - 1 + 1/K)P\right). \quad (3.15)$$

Similar to the approach used for the case of imperfect CSI, the system SE of BD-PS can be approximated as

$$\begin{aligned} C_{ip} &= \left(1 - \frac{T_t}{T}\right) \sum_{k=1}^K \mathbb{E} \left\{ \log_2 \det \left(\mathbf{I}_M + \frac{Np_k}{\sigma_{n_e}^2} \hat{\mathbf{H}}_k \mathbf{U}_k \mathbf{U}_k^H \hat{\mathbf{H}}_k^H \right) \right\} \\ &\stackrel{(i)}{\approx} \left(1 - \frac{T_t}{T}\right) KM \log_2 \left(1 + \frac{(1 - \sigma_e^2)(\eta - 1 + 1/K)P}{P\sigma_e^2 + 1}\right), \end{aligned} \quad (3.16)$$

where (i) is obtained based on the fact that $\hat{\mathbf{H}}_k = \frac{1}{\sqrt{N}} \hat{\mathbf{H}}_k$ and $\hat{\mathbf{H}}_k$ has i.i.d. $\mathcal{CN}(0, 1 - \sigma_e^2)$ entries.

3.2.2 BD-Based Precoding with STBC

Since $\mathbf{H}_k$ and $\mathbf{U}_k$ with $\mathbf{U}_k^H \mathbf{U}_k = \mathbf{I}$ are independent of each other, the entries of $\mathbf{H}_{eq,k} = \mathbf{H}_k \mathbf{U}_k$ share the same distribution as those of $\mathbf{H}_k$ [89]. Therefore, the probability density function (PDF) of $\|\mathbf{H}_{eq,k}\|_F^2$ is Chi-squared distributed with $2ML$ degree of free (DoF) [86]. Following that, with EPA, the average SNR of BD-STBC is given by

$$\mathbb{E} \left\{ \gamma_0 \|\mathbf{H}_{eq,k}\|_F^2 \right\} = \frac{PM}{KR_c}. \quad (3.17)$$

Based on the concavity of the $\log(\cdot)$ function and the Jensen's inequality, we have $\mathbb{E} \{\log_2(1 + x)\} \leq \log_2(1 + \mathbb{E}\{x\})$ [90]. From (3.7) and (3.17), an upper bound on the system SE of BD-STBC

can be defined as

$$\ddot{C}^{up} = \left(1 - \frac{T_t}{T}\right) K R_c \log_2 \left(1 + \frac{PM}{K R_c}\right). \quad (3.18)$$

When channel estimation error exists, the average SNR of BD-STBC can be obtained as

$$\mathbb{E}\left\{\bar{\gamma}_0 \|\hat{\mathbf{H}}_{eq,k}\|_F^2\right\} = \frac{(1 - \sigma_e^2)PM}{(\sigma_e^2 \sum_k^K p_k + 1)K R_c}. \quad (3.19)$$

With EPA, the upper bound on the system SE of BD-STBC for the case of imperfect CSI is given by

$$\ddot{C}_{ip}^{up} = \left(1 - \frac{T_t}{T}\right) K R_c \log_2 \left(1 + \frac{(1 - \sigma_e^2)PM}{(\sigma_e^2 P + 1)K R_c}\right). \quad (3.20)$$

3.3 Optimal Number of Users and Optimal Length of Training Sequences

It has been shown that an increment of number of users K can result in the degradation of sum rate of the BD technique [82]. In order to optimize the sum rate, an optimal number of users was obtained by using a numerical search method in [82]. This fact helps the system design engineers to define the optimal value of K to serve simultaneously, if there are a lot of users in each cell. In this section, we analyze the optimal length of training sequence T_t^{opt} and optimal number of users K^{opt} , which are expressed based on Lambert-W function, for maximizing the SE of the massive MU-MIMO system. We observe that the error variance σ_e^2 does not depend on K , but it depends on T_t and ρ_t . Therefore, K^{opt} will be investigated based on the system capacity for the case of perfect CSI, whereas T_t^{opt} will be investigated based on the system capacity for the case of imperfect CSI.

3.3.1 Optimal Number of Users

In order to investigate K^{opt} which maximizes SE of BD-PS and BD-STBC separately, we formulate two independent optimisation problems for C_p and $\ddot{C}^{up}$ with respect to the number of users. In order to have accurate channel estimation, we assume that $T_t = K$ [91]. Moreover, the received SINR at each user should be at least equal to the SNR threshold γ_{th} [92]. Thus, the two optimization problems can be expressed as

$$(P1) \begin{cases} \underset{K}{\text{maximize}} & C_p = \left(1 - \frac{K}{T}\right) K M \log_2 \left(1 + (\eta - 1 + 1/K)P\right) \\ \text{subject to} & (\eta - 1 + 1/K)P \geq \gamma_{th1}, \end{cases} \quad (3.21)$$

and

$$(P2) \begin{cases} \underset{K}{\text{maximize}} & \ddot{C}^{up} = \left(1 - \frac{K}{T}\right) K R_c \log_2 \left(1 + \frac{PM}{KR_c}\right) \\ \text{subject to} & \frac{PM}{KR_c} \geq \gamma_{th2}. \end{cases} \quad (3.22)$$

Based on (3.21), a tradeoff between K and T_t for the BD-PS can be obtained as

$$T_t = T \left(1 - \frac{C_p}{KM \log_2(1 + (\eta - 1 + 1/K)P)}\right). \quad (3.23)$$

Hence, for fixed values of T, K, M, η, P and C_p , we can define T_t . Similarly, a tradeoff between K and T_t for the BD-STBC is expressed as

$$T_t = T \left(1 - \frac{\ddot{C}^{up}}{KR_c \log_2(1 + \frac{PM}{KR_c})}\right). \quad (3.24)$$

In order to maximize the cost functions of problems P1 and P2 with respect to K , we need to investigate the concavity of the functions with respect to K , respectively. In particular, K_1^* is the number of users that maximizes problem P1 if the second derivative of the cost function P1 is less than zero, i.e., $\partial^2 C_p(K_1^*) < 0$, and the first derivative of the cost function P1 is equal

to zero, i.e., $\partial C_p(K_1^\star) = 0$ [92]. Similar approach is used to determine $K_2^\star$ for problem P2. The optimal solutions for P1 and P2 are presented in the following theorems.

Theorem 1 *The optimal number of users K_1^{opt} for maximizing the cost function of problem P1 in (3.21) is given by*

$$K_A^{opt} = \min \left(K_1^\star, \frac{N/M + 1}{\gamma_{th1}/P + 1} \right), \quad (3.25)$$

$$K_1^\star = \left\{ k | W(\alpha(k)) = \frac{\alpha(k)}{\beta(k)}, 0 < k \leq T/2 \right\}, \quad (3.26)$$

where $W(z)$ is the Lambert-W function defined as $z = W(z)e^{W(z)}$, $\alpha(k)$ and $\beta(k)$ are obtained as $\alpha(k) = \frac{(\frac{N}{M}+1)(T-k)P}{(Tk-2k^2)}$ and $\beta(k) = 1 - P + (\frac{N}{M} + 1)\frac{P}{k}$, respectively.

Proof: Please refer to Appendix A.1. ■

Theorem 2 *The optimal number of users K_2^{opt} for maximizing the cost function of problem P2 in (3.22) is given by*

$$K_2^{opt} = \min \left(K_2^\star, \frac{PM}{\gamma_{th2}R_c} \right), \quad (3.27)$$

$$K_2^\star = \left\{ k | W(\omega(k)) = \frac{\omega(k)}{\phi(k)}, 0 < k \leq T/2 \right\}, \quad (3.28)$$

where $\omega(k)$ and $\phi(k)$ are defined as $\omega(k) = \frac{PM(T-k)}{R_c(Tk-2k^2)}$ and $\phi(k) = 1 + \frac{PM}{kR_c}$, respectively.

Proof: Please refer to Appendix A.2. ■

To facilitate understanding, we explain in more details as a simple algorithm which is summarized in Table 3.2. From (3.25), we observe that the optimum K_1^{opt} for maximizing the cost function P1 depends on the number of BS antennas N because the system SE increases as N increases. It implies that K_1^{opt} increases as N increases. On the other hand, the optimum K_2^{opt} in (3.27) does not depend on N , but it depends on M when P and T are fixed.

Table 3.2: Algorithm to determine optimal number of users K_1^* (or K_2^*)

-
1. **Input:** P, M, N, T, R_c .
 2. **For** $k := 1$ **to** $T/2$ **do**
 3. Calculate $\alpha(k), \beta(k), W(\alpha(k))$ and $\alpha(k)/W(\alpha(k))$
 (or $\omega(k), \phi(k)$ and $\omega(k)/W(\omega(k))$ for K_2^*)
 4. Calculate $x(k) = \beta(k) - \alpha(k)/W(\alpha(k))$
 (or $y(k) = \phi(k) - \omega(k)/W(\omega(k))$ for K_2^*)
 5. **If** $x(k)$ is the smallest value of set $X(k) = \{x(k), k = 1, \dots, T/2\}$, **then** let index $k = K_1^*$
 (or **if** $y(k)$ is the smallest value of set $Y(k) = \{y(k), k = 1, \dots, T/2\}$, **then** let index $k = K_2^*$)
 6. **Output:** K_1^* (or K_2^*)
-

3.3.2 Optimal Lengths of Training Sequence

From a practical view point, the transmit power of the BS is much larger than that of the users, i.e., $P \gg P_t$. We assume that P_t is finite which leads to an interference-limited system [12]. Therefore, the optimum values of T^{opt} for maximizing the system capacities ($T_{t_1}^{opt}$ for BD-PS and $T_{t_2}^{opt}$ for BD-STBC) are investigated when $P \gg P_t$.

To maximize C_{ip} in (3.16) with respect to T_{t_1} , we verify the concavity of C_{ip} with respect to T_{t_1} . By substituting $\sigma_e^2 = \frac{1}{1+T_{t_1}P_t}$ into (3.16), when $P \rightarrow \infty$, the second derivative of the C_{ip} is given by

$$\frac{\partial^2 C_{ip}}{\partial T_{t_1}^2} = -\frac{KM u}{T \ln 2} \left\{ \frac{1}{1 + uT_{t_1}} + \frac{1 + uT}{(1 + uT_{t_1})^2} \right\}, \quad (3.29)$$

where $u = (\eta - 1 + 1/K)P_t$. Thus, the function C_{ip} is concave with respect to T_{t_1} because $\frac{\partial^2 C_{ip}}{\partial T_{t_1}^2} < 0$ always holds true. By setting the first derivative of C_{ip} to be zero, we get

$$\ln(1 + uT_{t_1}) = \frac{Tu + 1}{1 + uT_{t_1}} - 1. \quad (3.30)$$

Let us define $\xi = Tu + 1$, the solution for (3.30) is obtained as

$$T_{t_1}^* = \left(\frac{\xi}{W(\xi e)} - 1 \right) \frac{1}{(\eta - 1 + 1/K)P_t}, \quad (3.31)$$

where e is Euler's number and $W(\cdot)$ is the Lambert-W function. Therefore, the optimum

training length to maximize C_{ip} under BD-PS is given by

$$T_{t_1}^{opt} = \max \left(T_{t_1}^*, K \right). \quad (3.32)$$

For BD-STBC, when P_t is finite, $P \rightarrow \infty$, the second derivative of the $\ddot{C}_{ip}^{up}$ with respect to T_{t_2} is given by

$$\frac{\partial^2 \ddot{C}_{ip}^{up}}{\partial T_{t_2}^2} = -\frac{MP_t}{T \ln 2} \left\{ \frac{1}{1 + vT_{t_1}} + \frac{1 + vT}{(1 + vT_{t_1})^2} \right\} \quad (3.33)$$

where $v = \frac{MP_t}{KR_c}$. We observe that $\frac{\partial^2 \ddot{C}_{ip}^{up}}{\partial T_{t_2}^2} < 0$ always holds true, hence the function $\ddot{C}_{ip}^{up}$ is concave with respect to T_{t_2} . By letting the first derivative of (3.20) to be zero, we have

$$\ln(1 + vT_{t_2}) = \frac{Tv + 1}{1 + vT_{t_2}} - 1. \quad (3.34)$$

From (3.34), the optimum training length $T_{t_2}^{opt}$ to maximize $\ddot{C}_{ip}^{up}$ under BD-STBC is given by

$$T_{t_2}^{opt} = \max \left(T_{t_2}^*, K \right), \quad (3.35)$$

$$T_{t_2}^* = \left(\frac{\mu}{W(\mu e)} - 1 \right) \frac{KR_c}{MP_t}, \quad (3.36)$$

$$\mu = \frac{TMP_t}{KR_c} + 1. \quad (3.37)$$

3.4 Numerical Results and Discussions

In this section, both simulated and analytical SE results of BD-PS and BD-STBC over fading channels are presented. We assume that the transmitted signals are modulated with OFDM. The parameters are chosen based on long-term evolution standards where an OFDM symbol duration is approximated as $71.4 \mu s$. We choose the channel coherence time to be $T_c = 1 ms$, then $T = 196$ symbols is the coherence interval. The simulation results are computed based on

10,000 independent channel realizations.

Figure 3.2: Empirical eigenvalue distribution of $\mathbf{G}_k \mathbf{G}_k^H$ when $\eta = 2$.

We first investigate the EED of $\mathbf{G}_k \mathbf{G}_k^H$ as shown in Fig. 3.2. As we have mentioned in Section 3.2.1, the eigenvalues of $\mathbf{G}_k \mathbf{G}_k^H$ converge to $\mathbb{E}\{\lambda_m\} = 1 - \frac{K-1}{\eta K}$ as ηK increases. In particular, with $\eta = 2$, when ηK increases from 20 to 1000, the eigenvalues of $\mathbf{G}_k \mathbf{G}_k^H$ converge to $\mathbb{E}\{\lambda_m\} \approx 0.5$. This result validates the accuracy of the matrix approximation of $\tilde{\mathbf{\Lambda}}_k$ as ηK is large.

Next we demonstrate the accuracy of the derived closed-form SE expressions of BD-PS and BD-STBC. Figure 3.3 presents the achievable spectral efficiencies for BD-PS, BD-STBC, BD-based RZF [82] and BD-based SVD [82] in the case of perfect CSI. We assume that $N = 60$, $K = 10$ and each user is equipped with $M = 3$ antennas. It is shown that the proposed BD-PS performs best due to high array gain. In Fig. 3.4 the analytical and simulated SE results of the proposed BD-PS and BD-STBC schemes are demonstrated when the CSI is imperfect. As observed from Figs. 3.3 and 3.4, the analytical results of these schemes are very close to

Figure 3.3: Spectral efficiencies versus SNR for BD-PS, BD-STBC, BD-based RZF [82] and BD-based SVD [82] when $T = 196$, $N = 60$, $K = 10$, $M = 3$, $L = 2$ and CSI is perfect.

Figure 3.4: Spectral efficiencies of the BD-precoding and BD-STBC schemes in the case of imperfect CSI when $T = 196$, $N = 60$, $K = 10$, $M = 3$, and $L = 2$.

the simulation results for both cases of perfect CSI and imperfect CSI ($\sigma_e^2 = 0.1, 0.3, 0.5$). The SE of BD-PS is proportional to N , whereas the SE of BD-STBC is proportional to M and L . In addition, N is much larger than M . Hence, the system SE of BD-PS is higher than that of BD-STBC. Figure 3.5 illustrates the SE of BD-PS when N increases for both cases of perfect CSI and imperfect CSI. When N increase, ηK is increased for fixed values of K and M , the eigenvalues are close to the deterministic value. The analytical results of BD-PS match well with the corresponding simulated results, especially at large N . Therefore, we will use these analytical SE expressions for further analysis.

Figure 3.5: System SE of BD-PS versus the number of BS antennas when $T = 196, K = 10, M = 3$ and $\text{SNR} = 10$ dB.

In Fig. 3.6, we show the capacities of BD-PS and BD-STBC, i.e., C_p and $\ddot{C}^{up}$, versus the number of users in the case of perfect CSI, respectively. We assume that the downlink SNR threshold is $\gamma_{th1} = \gamma_{th2} = 0$ dB. In order to determine the optimal numbers of users K_1^{opt} and K_2^{opt} for maximizing the cost function of the corresponding P1 and P2, we need to define the maximizers K_1^* in Theorem 1 and K_2^* in Theorem 2, respectively. The maximizers K_1^* and K_2^*

Figure 3.6: SEs of the BD-precoding and BD-STBC schemes versus the number of users when $T = 40, M = 2, L = 2, N = 60$ for the case of perfect CSI.

can be obtained from the proposed algorithm shown in Table 3.2 and they are presented in Table 3.3. Furthermore, the columns named as “Fig. 3.6” in Table 3.3 indicate the maximum points which are presented in Fig. 3.6. It has been shown in Table 3.3 that the optimal numbers of users in Theorems 1 and 2 are similar to those obtained from Fig. 3.6. We observe that the optimum K^{opt} increases as the transmit power P increases for a given parameter setting, such as $M = 2, N = 60$, and $T = 40$.

The SE comparisons of these schemes for the optimum number of users K^{opt} and the case with $K = T/2$ are shown in Fig. 3.7. Note that $K = T/2$ is greater than K^{opt} due to the conditions for the concavity of the optimization problems P1 and P2. In Fig. 3.7, the system SE for the optimum K^{opt} is always better than that of the case with $K = T/2$ for different parameter settings over the whole range of SNR.

In Fig. 3.8, we investigate the system SE for different lengths of training sequence T_t with $P_t = 3$ dB. The optimum training lengths for maximizing the system SE with BD-PS and BD-

Figure 3.7: SE comparisons of the precoding schemes for the optimum number of users K^{opt} and the case with $K = T/2$ for $M = 2, L = 2, N = 60$ and $T = 40$.

Table 3.3: The optimal number of users

Parameter settings	K_1^* for BD-PS		K_2^* for BD-STBC	
	Proposed algorithm	Fig. 3.6	Proposed algorithm	Fig. 3.6
SNR =10 dB	11	11	10	10
SNR =20 dB	14	14	15	15

STBC, i.e., $T_{t_1}^{opt}$ and $T_{t_2}^{opt}$, respectively. From Fig. 3.8, we observe that there is a small system SE loss of T^{opt} as compared to that of $T_t = K$ (it means $T_{t_1} = T_{t_2} = K$) at low SNR region. However, when SNR is greater than 11 dB, the system SE with T^{opt} outperforms that of $T_t = K$. This is because T^{opt} is typically obtained for the case $P \gg P_t$. Therefore, in order to improve the system performance, the optimum T^{opt} should be analyzed. The closed-form expressions of T^{opt} presented in Section 3.3.2, which are easily obtained from the basic system parameters, can help the system design engineers to define the optimum training length efficiently.

Figure 3.8: The proposed system SE with different lengths of training sequence T_t when $T = 196$, $K = 10$ and $P_t = 3$ dB.

3.5 Summary

In this chapter, we have investigated the SE performance of massive MIMO systems with BD-precoding and BD-STBC schemes. Although the analytical results have been obtained under a large-scale assumption, they have been shown to be tight and accurate for the large-scale and conventional MIMO systems in both cases of perfect CSI and imperfect CSI (i.e., $M = 2$ and 3 have been used to obtain the analytical and simulation results). It has been shown that BD-PS outperforms BD-STBC. In addition, we have studied the optimal users and training lengths for maximizing the SE of these schemes, respectively. The closed-form expressions of the optimum training lengths for the proposed system have also been derived for high SNR region. From the simulation results, the accuracy of our analysis on the optimum training lengths and the optimum number of users for these schemes have been successfully demonstrated.

Chapter 4

Massive MIMO with SLNR

Precoding Scheme in Single-Cell Systems

One of the main objectives in designing the optimal linear MU-MIMO precoding scheme is to optimize the SINR at each user. However, it has been shown that this problem is challenging due to its coupled nature. On the other hand, in [18], based on the concept of signal leakage, the SLNR has been introduced as an optimization metric for the linear precoder design. This metric can transform a coupled optimization problem into a completely decoupled one, such that a closed-form solution can be obtained. Specifically, in conventional MU-MIMO systems with single-antenna users, the closed-form solution of the SLNR-PS has been expressed by an eigenvector associated to the maximum eigenvalue in [18], and a regularized channel inverse shifted by random phase-shifts in [93]. For multiple-antenna users, it has been given by a generalized form of the regularized channel inversion, in which the regularization factors for each user are inversely proportional to their average signal-to-noise ratios per data streams

[94]. For massive MIMO systems, by employing the Mullen's inequality, a lower bound on the achievable SLNR has been analyzed in Ricean fading channels [95]. This result has been used to analyze achievable ergodic rate for a statistical eigenmode space-division multiple-access scheme. Moreover, a two-stage beamformer design, which includes the ZF-based inner beamforming and the SLNR-based outer beamforming, has been considered in [96]. Based on the trace quotient formulation, an iterative algorithm has been shown to obtain optimal solution.

The EE performance of massive MIMO systems with linear processing techniques has been investigated in [13] and [27], where the power consumption model only considers the emitted power consumption. In order to maximize the EE in massive MIMO systems, various resource allocation algorithms have been studied in [97–100]. Particularly, an optimal number of antennas N is selected by using sequential search or binary search algorithms in [97], and an exhaustive search method over all antennas was proposed in [98]. An iterative algorithm for joint antenna selection and power adaptation based on the exhaustive search has been proposed in [99]. However, this algorithm requires high computational complexity because it needs to obtain channel matrix inversion with large-scale dimension based on instantaneous channel realizations. In [100], the joint optimization problem of power allocation, N and pilot assignment for EE maximization in the multiuser and multicell massive MIMO network has been investigated. However, the authors did not analytically characterize the effect of changing these parameters on the optimal EE.

The reliable guidelines for EE maximization with respect to N and active users K have been addressed in [101]. In [102], the upper and lower bounds of maximum EE for MRC in the uplink and MRT in the downlink have been analyzed with respect to N , respectively, but the closed-form expressions of optimal N have not been defined. The problem of EE maximization for ZF detector with respect to (N, K) has been discussed in [103]. For a given UL SE, the optimal values of N and K have been obtained in [103] without consideration of the necessary overhead

signalling for the channel estimation, i.e., the length of training sequence. By considering the joint UL and DL, the effects of N , K and transmit power on the EE for ZF and MMSE schemes have been studied in [26]. In a realistic scenario, the quality of channel estimation can limit the potential performance of massive MIMO systems. However, most of related works only focus on the EE maximization problem for massive MIMO systems with common precoders/detectors (i.e., MF, MRC, MRT, ZF and MMSE) or perfect CSI.

In this chapter, we study the fundamental performance in terms of SE and EE for massive MIMO systems with SLNR-PS for both cases of perfect and imperfect CSI in the TDD protocol. In order to do so, we first derive bounds on achievable ergodic rate of SLNR-PS when N is large. By applying the realistic power consumption model presented in Section 2.1.3, the EE maximization problem is formulated with respect to the transmit power, number of BS antennas, as well as length of training sequence. An alternating optimization (AO) algorithm is used to solve the EE maximization problem under assumption of EPA. In addition, with perfect CSI, we present an energy-efficient power allocation scheme to optimize the EE of SLNR-PS. A rate profile optimization problem for individual users is also investigated while satisfying constraints of the transmit power, QoS, and EE target.

4.1 System Model Description

We consider a single-cell downlink MU-MIMO system. The BS is equipped with N transmit antennas to serve its K single-antenna users and we assume that $N > K$. The channel vector of the k th user is given by¹ $\mathbf{g}_k = \sqrt{\beta_k} \mathbf{h}_k$, where $\mathbf{h}_k$ is the small-scale fading vector of user k with $\mathbf{h}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$ and β_k is the large-scale fading coefficient accounting for path loss and shadowing effect. We assume that the channel reciprocity in the TDD protocol is the same for both forward and reverse links, and the channel experiences block fading. Hence, $\mathbf{h}_k$ remains

¹Note that all the notations are only applicable to this Chapter.

constant for a duration of T symbols. Based on the TDD protocol, the communication scheme includes two phases, namely uplink training and downlink data transmission. We describe these phases briefly.

4.1.1 Uplink Training

At the beginning of every coherence interval T , all K users simultaneously transmit orthogonal pilot sequences, of which the length is T_t symbols [7]. It has been shown that the pilot sequences can be defined as a $T_t \times K$ matrix $\sqrt{T_t P_t} \mathbf{\Theta}$, where P_t is the power level of the training sequence, $T_t \geq K$ and $\mathbf{\Theta}^H \mathbf{\Theta} = \mathbf{I}$. The $N \times T_t$ received pilot matrix at the BS is obtained as

$$\mathbf{Y}_{tr} = \sqrt{T_t P_t} \mathbf{G} \mathbf{\Theta}^T + \mathbf{N}, \quad (4.1)$$

where $\mathbf{G} = \mathbf{H} \mathbf{A}^{1/2}$ with $\mathbf{H} = [\mathbf{h}_1 \dots \mathbf{h}_K]$ and $\mathbf{A} = \text{diag}\{\beta_1, \dots, \beta_K\}$, and $\mathbf{N} \in \mathbb{C}^{N \times T_t}$ is the noise term matrix which has i.i.d. $\mathcal{CN}(0, \sigma_n^2)$ entries. Based on MMSE estimation, the estimated channel of $\mathbf{G}$ given $\mathbf{Y}_{tr}$ is defined as [9, 13, 104]

$$\hat{\mathbf{G}} = \frac{1}{\sqrt{T_t P_t}} \mathbf{Y}_{tr} \mathbf{\Theta}^H \bar{\mathbf{A}} = \left(\mathbf{G} + \frac{1}{\sqrt{T_t P_t}} \mathbf{N} \mathbf{\Theta}^H \right) \bar{\mathbf{A}}, \quad (4.2)$$

where $\bar{\mathbf{A}} = \left(\frac{\sigma_n^2}{T_t P_t} \mathbf{A}^{-1} + \mathbf{I}_K \right)^{-1}$. We define $\rho_t = \frac{P_t}{\sigma_n^2}$ as the transmit SNR of training sequence in the uplink channel. Since the k th diagonal element $[\mathbf{A}]_{kk} = \beta_k$, we have $\bar{\mathbf{A}} = \text{diag}\left\{ \frac{T_t \rho_t \beta_k}{1 + T_t \rho_t \beta_k} \right\}, k \in [1 \dots K]$. We denote $\hat{\mathbf{g}}$ as the k th column of $\hat{\mathbf{G}}$. From (4.2), $\hat{\mathbf{g}}$ can be shown to be distributed as $\hat{\mathbf{g}}_k \sim \mathcal{CN}(0, \sigma_{\hat{g}}^2 \mathbf{I}_N)$, where $\sigma_{\hat{g}}^2 = \frac{T_t \rho_t \beta_k^2}{1 + T_t \rho_t \beta_k}$. According to the orthogonality property of the MMSE estimate, we can decompose channel $\mathbf{g}_k$ as [9, 104]

$$\mathbf{g}_k = \hat{\mathbf{g}}_k + \mathbf{e}_k, \quad (4.3)$$

where $\mathbf{e}_k$ is the estimation error vector with $\mathbf{e}_k \sim \mathcal{CN}(0, \sigma_e^2 \mathbf{I}_N)$ and² $\sigma_e^2 = \frac{\beta_k}{1+T_t \rho_t \beta_k}$. Note that $\hat{\mathbf{g}}_k$ and $\mathbf{e}_k$ are statistically independent of each other. When the channel estimation is perfect, i.e., $\sigma_e^2 = 0$, we get $\mathbf{g}_k = \hat{\mathbf{g}}_k$ and $\sigma_{\hat{\mathbf{g}}}^2 = \beta_k$.

4.1.2 Downlink Data Transmission

The received signal at user k is given by [94]

$$y_k = \sqrt{p_k} \mathbf{g}_k^H \mathbf{w}_k s_k + \sum_{i=1, i \neq k}^K \sqrt{p_i} \mathbf{g}_k^H \mathbf{w}_i s_i + n_k, \quad (4.4)$$

where s_k is the data symbol, the beamforming vector $\mathbf{w}_k$ is obtained based on the estimated channel $\hat{\mathbf{g}}_k$ and n_k is the noise term with $n_k \sim \mathcal{CN}(0, \sigma_n^2)$. The SLNR at user k is given by [18, Eq. (52)], [105]

$$\begin{aligned} \varphi_k &= \frac{\mathbb{E}\{\mathbf{w}_k^H \mathbf{g}_k \mathbf{g}_k^H \mathbf{w}_k | \hat{\mathbf{g}}_k\}}{\mathbb{E}\{\mathbf{w}_k^H \bar{\mathbf{G}}_k \bar{\mathbf{G}}_k^H \mathbf{w}_k | \hat{\mathbf{G}}_k\} + \frac{\sigma_n^2}{p_k}} \\ &= \frac{\mathbf{w}_k^H (\hat{\mathbf{g}}_k \hat{\mathbf{g}}_k^H + \sigma_e^2 \mathbf{I}) \mathbf{w}_k}{\mathbf{w}_k^H \left(\hat{\mathbf{G}}_k \hat{\mathbf{G}}_k^H + \left(\frac{\sigma_n^2}{p_k} + (K-1)\sigma_e^2 \right) \mathbf{I} \right) \mathbf{w}_k}, \end{aligned} \quad (4.5)$$

where the expectation is conditional on the knowledge of $\hat{\mathbf{g}}_k$, $\bar{\mathbf{G}}_k = [\mathbf{g}_1, \dots, \mathbf{g}_{k-1}, \mathbf{g}_{k+1}, \dots, \mathbf{g}_K]$ and $\hat{\mathbf{G}}_k = [\hat{\mathbf{g}}_1, \dots, \hat{\mathbf{g}}_{k-1}, \hat{\mathbf{g}}_{k+1}, \dots, \hat{\mathbf{g}}_K]$. For SLNR-PS, based on [18, Eq. (11)] and [93, Eq. (6)], the optimal beamforming vector $\mathbf{w}_k$ for maximizing φ_k is given by

$$\mathbf{w}_k \propto \text{max.eigenvector}(\mathbf{B}_k), \quad (4.6)$$

$$\mathbf{B}_k = \left(\hat{\mathbf{G}}_k \hat{\mathbf{G}}_k^H + \rho_k \mathbf{I} \right)^{-1} \left(\hat{\mathbf{g}}_k \hat{\mathbf{g}}_k^H + \sigma_e^2 \mathbf{I} \right), \quad (4.7)$$

where $\rho_k = \frac{\sigma_n^2}{p_k} + (K-1)\sigma_e^2$. It can be observed that $\left(\hat{\mathbf{G}}_k \hat{\mathbf{G}}_k^H + \rho_k \mathbf{I} \right)$ is invertible because its own determinant is not zero. Specifically, we show that the minimum singular value of

²Note that the forms of the estimation error and estimated channel variances are different from that of Chapter 3 because $\mathbf{g}_k$ is assumed to be distributed as $\mathbf{g}_k \sim \mathcal{CN}(0, \beta_k \mathbf{I})$.

$(\hat{\mathbf{G}}_k \hat{\mathbf{G}}_k^H + \rho_k \mathbf{I})$ is at least greater than a non-zero value in the following Section. Let us define ω_k and ω_j as normalization factors of the precoding vectors for the k th user and the j th user such that $\mathbf{w}_k^H \mathbf{w}_k = 1$ and $\mathbf{w}_j^H \mathbf{w}_j = 1$, respectively. Similar to [93], it is assumed that $\mathbb{E}\{\omega_k^2\} = \mathbb{E}\{\omega_j^2\}$, the power of the interference plus noise can be estimated from the power of the leakage plus noise. It is expressed as

$$\sum_{i \neq k}^K p_i \|\mathbf{g}_k^H \mathbf{w}_i\|^2 + \sigma_n^2 \approx \sum_{i \neq k}^K p_i \|\mathbf{g}_i^H \mathbf{w}_k\|^2 + \sigma_n^2. \quad (4.8)$$

When the leakage power is small as compared with the noise power, this estimation is generally tight. In addition, as $N > K$, the leakage power converges to zero at low and high SNR regions [93, Eq. (19)]. Thus, the SINR of the k th user, denoted as γ_k , can be approximated by the corresponding SLNR. Furthermore, it has been shown in [18] and [93] that the resultant maximum SLNR value φ_k is the largest eigenvalue $\lambda_{\max}$ of $\mathbf{B}_k$. Therefore, under these assumptions, we have $\gamma_k \approx \varphi_k = \lambda_{\max}(\mathbf{B}_k)$. This result is used for further analysis.

4.2 Achievable Ergodic Rate

In the following, we study the achievable ergodic rates of the SLNR-PS for both cases of perfect and imperfect CSI, respectively, when N is large. We also analyze the asymptotic analysis of the achievable rates at very high SNR region. In particular, the achievable rate of the k th user is defined as [18, 94]

$$R_k = \mathbb{E}\left\{\log_2(1 + \gamma_k)\right\} \approx \mathbb{E}\left\{\log_2(1 + \lambda_{\max}(\mathbf{B}_k))\right\}. \quad (4.9)$$

4.2.1 Imperfect CSI Case

In the reality, the BS can estimate the channel state information imperfectly, then the achievable ergodic rate of this case is presented in the following theorem.

Theorem 3 *If the SLNR-PS is applied, under the imperfect CSI condition, the UB on achievable ergodic rate of the k th user is given by*

$$\hat{R}_k^{ip} = \log_2 \left(1 + v + \frac{\sigma_g^2}{\rho_k} (N - K + 1) \right), \quad (4.10)$$

where $v = \frac{K-1}{N-K+1}$.

Proof: We study the achievable rate based on (4.9). In order to obtain the largest eigenvalue λ_k of $\mathbf{B}_k$, we employ Weyl's inequality in matrix theory. In particular, if $\mathbf{C}, \mathbf{D} \in \mathbf{M}_n$ are Hermitian matrices and the eigenvalues $\lambda_i(\mathbf{C})$, $\lambda_i(\mathbf{D})$, and $\lambda_i(\mathbf{C} + \mathbf{D})$ are arranged in increasing order $\lambda_{\min} = \lambda_1 \leq \dots \leq \lambda_n = \lambda_{\max}$, for each $l = 1, \dots, n$ we have [106, Eq. (4.3.2)]

$$\lambda_l(\mathbf{C}) + \lambda_{\min}(\mathbf{D}) \leq \lambda_l(\mathbf{C} + \mathbf{D}) \leq \lambda_l(\mathbf{C}) + \lambda_{\max}(\mathbf{D}). \quad (4.11)$$

Based on (4.11), from (4.7), we have

$$\gamma_k \approx \lambda_{\max}(\mathbf{B}_k) \leq \lambda_{\max}(\mathbf{C}_k) + \lambda_{\max}(\mathbf{D}_k), \quad (4.12)$$

$$\mathbf{C}_k = \left(\hat{\mathbf{G}}_k \hat{\mathbf{G}}_k^H + \rho_k \mathbf{I} \right)^{-1} \hat{\mathbf{g}}_k \hat{\mathbf{g}}_k^H, \quad (4.13)$$

$$\mathbf{D}_k = \left(\hat{\mathbf{G}}_k \hat{\mathbf{G}}_k^H + \rho_k \mathbf{I} \right)^{-1} \sigma_e^2 \mathbf{I}, \quad (4.14)$$

where $\lambda_{\max}(\mathbf{C}_k)$ and $\lambda_{\max}(\mathbf{D}_k)$ are the corresponding maximum eigenvalues of $\mathbf{C}_k$ and $\mathbf{D}_k$, respectively. According to [94], $\lambda_{\max}(\mathbf{C}_k)$ can be obtained by solving the following characteristic

equations

$$\det \left[\lambda \mathbf{I} - \left(\hat{\mathbf{G}}_k \hat{\mathbf{G}}_k^H + \rho_k \mathbf{I} \right)^{-1} \hat{\mathbf{g}}_k \hat{\mathbf{g}}_k^H \right] = 0, \quad (4.15)$$

$$\det \left[\left(\hat{\mathbf{G}}_k \hat{\mathbf{G}}_k^H + \rho_k \mathbf{I} \right) - \left(\frac{\lambda + 1}{\lambda} \right) \hat{\mathbf{g}}_k \hat{\mathbf{g}}_k^H \right] = 0, \quad (4.16)$$

where $\hat{\mathbf{G}}_k = [\hat{\mathbf{g}}_1 \dots \hat{\mathbf{g}}_K]$. Let us define $\Phi^{-1} = \left(\hat{\mathbf{G}}_k \hat{\mathbf{G}}_k^H + \rho_k \mathbf{I} \right)$, $\ddot{\mathbf{v}} = -\left(\frac{\lambda+1}{\lambda} \right) \hat{\mathbf{g}}_k$, and $\ddot{\mathbf{u}}^H = \hat{\mathbf{g}}_k^H$.

By applying the matrix determinant lemma, i.e.,

$$\det \left(\Phi^{-1} + \ddot{\mathbf{v}} \ddot{\mathbf{u}}^H \right) = \left(1 + \ddot{\mathbf{u}}^H \Phi \ddot{\mathbf{v}} \right) \det \left(\Phi^{-1} \right), \quad (4.17)$$

after some mathematical manipulations, $\lambda_{\max}(\mathbf{C}_k)$ is obtained as [94]

$$\begin{aligned} \lambda_{\max}(\mathbf{C}_k) &= \frac{1}{1 - \hat{\mathbf{g}}_k^H (\hat{\mathbf{G}}_k \hat{\mathbf{G}}_k^H + \rho_k \mathbf{I}) \hat{\mathbf{g}}_k} - 1 \\ &= \frac{1/\rho_k}{[(\hat{\mathbf{G}}_k^H \hat{\mathbf{G}}_k + \rho_k \mathbf{I})^{-1}]_{k,k}} - 1. \end{aligned} \quad (4.18)$$

Moreover, based on the approaches in [94] and [107], (4.18) can also be decomposed as

$$\lambda_{\max}(\mathbf{C}_k) = \frac{1}{\rho_k} \hat{\mathbf{g}}_k^H \Sigma_k \hat{\mathbf{g}}_k + \hat{\mathbf{g}}_k^H \Omega_k \hat{\mathbf{g}}_k, \quad (4.19)$$

where $\Sigma_k = \mathbf{I} - \hat{\mathbf{G}}_k (\hat{\mathbf{G}}_k^H \hat{\mathbf{G}}_k)^{-1} \hat{\mathbf{G}}_k^H$ and $\Omega_k = \hat{\mathbf{G}}_k (\hat{\mathbf{G}}_k^H \hat{\mathbf{G}}_k + \rho_k \mathbf{I})^{-1} (\hat{\mathbf{G}}_k^H \hat{\mathbf{G}}_k)^{-1} \hat{\mathbf{G}}_k^H$. Next, we define the expected value of $\lambda_{\max}(\mathbf{C}_k)$. Let us denote $\theta_{zf,k} = \frac{1}{\rho_k} \hat{\mathbf{g}}_k^H \Sigma_k \hat{\mathbf{g}}_k$ and $\theta_{ad,k} = \hat{\mathbf{g}}_k^H \Omega_k \hat{\mathbf{g}}_k$.

It has been shown that $\theta_{ad,k}$ is a nondecreasing function of $1/\rho_k$; $\theta_{zf,k}$ and $\theta_{ad,k}$ are statistically independent [107]. The expected value of $\theta_{zf,k}$ is given by

$$\mathbb{E}\{\theta_{zf,k}\} \stackrel{(a_1)}{=} \mathbb{E} \left\{ \frac{1/\rho_k}{[(\hat{\mathbf{G}}_k^H \hat{\mathbf{G}}_k)^{-1}]_{kk}} \right\} \stackrel{(a_2)}{=} \frac{\sigma_{\hat{\mathbf{g}}}^2}{\rho_k} (N - K + 1), \quad (4.20)$$

where (a_1) is obtained based on the result presented in [107] and (a_2) is obtained due to³ $1/[(\mathbf{H}_k^H \mathbf{H}_k)^{-1}]_{kk} \sim \chi_{2(N-K+1)}^2$, where $\mathbf{H}_k = [\mathbf{h}_1, \dots, \mathbf{h}_K]$ [13, 107]. The challenge in deriving $\mathbb{E}\{\lambda_{\max}(\mathbf{C}_k)\}$ is to define the $\mathbb{E}\{\theta_{ad,k}\}$. We denote $\mathbf{q}_k = \mathbf{U}_k^H \hat{\mathbf{g}}_k \in \mathbb{C}^{(K-1) \times 1}$, where $\mathbf{U}_k$ is defined from the SVD of $\hat{\mathbf{G}}_k = \mathbf{U}_k \mathbf{\Lambda}_k \mathbf{V}_k^H$. Let us define a matrix $\mathbf{Q}_k \in \mathbb{C}^{N \times (K-1)}$ which has the same distribution of $\hat{\mathbf{G}}_k$, and it is independent of $\mathbf{q}_k$. Following [107, Eq. (32)], at high SNR region, $\theta_{ad,k}$ has an identical distribution of $\|\mathbf{q}_k\|^2 [(\mathbf{Q}_k^H \mathbf{Q}_k)^{-1}]_{ii}$ with $i = 1, \dots, K-1$. By using lemma 2.10 of [108], i.e., $\mathbb{E}\{\text{trace}(\mathbf{\Psi}^{-1})\} = \frac{m}{n-m}$ with $n > m$ and $\mathbb{E}\{\text{trace}(\mathbf{\Psi})\} = mn$, where $\mathbf{\Psi} \sim \mathcal{W}_m(n, \mathbf{I}_n)$ is an $m \times m$ complex Wishart matrix with n degrees of freedom, we have $\mathbb{E}\{\|\mathbf{q}_k\|^2\} = \sigma_{\hat{g}}^2(K-1)$ and

$$\begin{aligned} \mathbb{E}\{[(\mathbf{Q}_k^H \mathbf{Q}_k)^{-1}]_{ii}\} &= \frac{1}{\sigma_{\hat{g}}^2} \mathbb{E}\{[(\bar{\mathbf{H}}_k^H \bar{\mathbf{H}}_k)^{-1}]_{ii}\} \\ &= \frac{\mathbb{E}\{\text{trace}[(\bar{\mathbf{H}}_k^H \bar{\mathbf{H}}_k)^{-1}]\}}{\sigma_{\hat{g}}^2(K-1)} = \frac{1}{\sigma_{\hat{g}}^2(N-K+1)}, \end{aligned} \quad (4.21)$$

where $\bar{\mathbf{H}}_k = [\mathbf{h}_1, \dots, \mathbf{h}_{k-1}, \mathbf{h}_{k+1}, \dots, \mathbf{h}_K]$. Therefore, we get

$$\mathbb{E}\{\theta_{ad,k}\} = \mathbb{E}\{\|\mathbf{q}_k\|^2 [(\mathbf{Q}_k^H \mathbf{Q}_k)^{-1}]_{ii}\} = \frac{K-1}{N-K+1}. \quad (4.22)$$

Next, we analyze the $\lambda_{\max}$ of $\mathbf{D}_k$. From (4.11) and (4.14), we obtain

$$\begin{aligned} \lambda_{\max}(\mathbf{D}_k) &= \lambda_{\max}\left(\frac{\sigma_e^2 \mathbf{I}}{\hat{\mathbf{G}}_k^H \hat{\mathbf{G}}_k + \rho_k \mathbf{I}}\right) \\ &= \frac{\sigma_e^2}{\lambda_{\min}(\hat{\mathbf{G}}_k^H \hat{\mathbf{G}}_k + \rho_k \mathbf{I})} \leq \frac{\sigma_e^2}{\lambda_{\min}(\hat{\mathbf{G}}_k^H \hat{\mathbf{G}}_k) + \rho_k}. \end{aligned} \quad (4.23)$$

When N is very large, the eigenvalues of $\hat{\mathbf{G}}_k^H \hat{\mathbf{G}}_k$ converge to a fixed deterministic distribution which is given by the Marchenko-Pastur distribution [6]. We define $x = \frac{K-1}{N}$. Based on the results presented in [109] and [110], as $N \rightarrow \infty$, the deterministic approximation of the smallest

³A Chi-square random Z variable, denoted as χ_{2n}^2 , has probability density function $f_Z(z) = \frac{1}{\Gamma(n)} z^{n-1} e^{-z}$, $z \geq 0$ [107].

eigenvalue of $\frac{1}{N}\hat{\mathbf{G}}_k^H\hat{\mathbf{G}}_k$ converges to

$$\lambda_{\min}(\frac{1}{N}\hat{\mathbf{G}}_k^H\hat{\mathbf{G}}_k) \rightarrow (1 - \sqrt{x})^2\sigma_g^2. \quad (4.24)$$

Substituting (4.24) into (4.23), as N is very large, we obtain

$$\lambda_{\max}(\mathbf{D}_k) \leq \frac{\sigma_e^2}{(1 - \sqrt{x})^2\sigma_g^2 N + \rho_k}. \quad (4.25)$$

Applying Jensen's inequality, i.e., $\mathbb{E}\{\log_2(1 + \zeta)\} \leq \log_2(1 + \mathbb{E}\{\zeta\})$, and using results in (4.20), (4.22) and (4.25), for the case of imperfect CSI, the UB on rate of the k th user is defined as

$$\begin{aligned} \hat{R}_k^{ip} &= \log_2 \left(1 + \mathbb{E}\{\lambda_{\max}(\mathbf{B}_k)\} \right) \\ &= \log_2 \left(1 + v + \varrho + \frac{\sigma_g^2(N - K + 1)}{\rho_k} \right), \end{aligned} \quad (4.26)$$

where $\varrho = \frac{\sigma_e^2}{(1 - \sqrt{x})^2\sigma_g^2 N + \rho_k}$. We observe that $\sigma_e^2 \in [0, 1]$ is much less than N . Thus, as N is large, $\varrho \rightarrow 0$, from (4.26), we can obtain (4.10). ■

The tightness of (4.26) is presented in Figs. 4.3 and 4.4 of the Numerical Results and Discussion Section. It is shown that the expression (4.26) performs as well as the expression (4.10). The analytical results obtained by using (4.10) and (4.26) match well with the corresponding simulation results when the number of BS antennas is large.

Proposition 1 *When both the number of users K and the number of BS antennas N grow large while their ratio remains bounded, i.e., $N, K \rightarrow \infty$ and $\frac{N}{K} = c_0 > 1$, the UB on achievable rate of the k th user is defined as*

$$\hat{R}_k^{ip} = \log_2 \left(1 + \frac{1}{c_0 - 1} + \frac{\rho_d \sigma_e^2}{1 + \rho_d \sigma_g^2} (c_0 - 1) \right). \quad (4.27)$$

Proof: Let us define $\rho_d = \frac{P}{\sigma_n^2}$ as the transmit SNR in the downlink channel. From (4.10), we can obtain (4.27). ■

If $c_0 \gg 1$, the result in (4.27) becomes the exact one which is defined by employing ZF-PS in [6, Table I]. It implies that the achievable ergodic rate of SLNR-PS is lower bounded by that of ZF-PS. In addition, at very high SNR region, i.e., $\log_2(1 + \zeta) \approx \log_2(\zeta)$, from (4.10), the asymptotic analysis of achievable rate for the k th user is given by

$$\lim_{\rho_d \rightarrow \infty} \hat{R}_{\infty,k}^{ip} = \log_2 \left(v + \frac{T_t \rho_t \beta_k}{v} \right). \quad (4.28)$$

4.2.2 Perfect CSI Case

When the BS can estimate CSI perfectly, we derive the UB and lower bound (LB) on the achievable rate as the following.

Theorem 4 *If the SLNR-PS is applied, under the perfect CSI condition, the UB on achievable ergodic rate of the k th user is given by*

$$\hat{R}_k^p = \log_2 \left(1 + v + \frac{p_k}{\sigma_n^2} \beta_k (N - K + 1) \right). \quad (4.29)$$

Proof: When the CSI is perfectly estimated, i.e., $\sigma_e^2 = 0$, then $\mathbf{g}_k = \hat{\mathbf{g}}_k$ and matrix $\mathbf{B}_k$ in (4.7) is rewritten as

$$\mathbf{B}_k = \left(\bar{\mathbf{G}}_k \bar{\mathbf{G}}_k^H + \frac{\sigma_n^2}{p_k} \mathbf{I} \right)^{-1} \mathbf{g}_k \mathbf{g}_k^H. \quad (4.30)$$

By applying the similar approach, which is used to obtain $\lambda_{\max}(\mathbf{C}_k)$ in the previous Section, the largest eigenvalue λ_k of $\mathbf{B}_k$ can be defined as

$$\lambda_{\max}^p(\mathbf{B}_k) = \frac{1}{1 - \mathbf{g}_k^H (\mathbf{G}_k \mathbf{G}_k^H + \frac{\sigma_n^2}{p_k} \mathbf{I})^{-1} \mathbf{g}_k} - 1. \quad (4.31)$$

Based on results in (4.19), (4.20) and (4.22), the expected value of $\lambda_{\max}(\mathbf{B}_k)$ is given by

$$\mathbb{E}\{\lambda_{\max}^p(\mathbf{B}_k)\} = v + \frac{p_k}{\sigma_n^2} \beta_k (N - K + 1). \quad (4.32)$$

By substituting above result into (4.9), we can obtain (4.29) ■

We observe that $\hat{R}_k^p$ is monotonically increasing with the transmit power of user k , hence $\hat{R}_k^p \rightarrow \infty$ as $p_k \rightarrow \infty$. This result is different from that of the case with imperfect CSI, where the achievable rate of the k th user converges to an asymptotic limit at high SNR region. In addition, under assumption of EPA, as $N, K \rightarrow \infty$, but $\frac{N}{K} = c_0 < \infty$ and $c_0 \gg 1$, then the UB expression in (4.29) converges to

$$\hat{R}_k^p \rightarrow \log_2(1 + \rho_d \beta_k (c_0 - 1)). \quad (4.33)$$

This result is similar to the one which is obtained by using ZF-PS in [6]. Therefore, the rate of SLNR-PS converges to that of ZF-PS when SNR is high.

Theorem 5 *If a SLNR-PS is applied, under the perfect CSI condition, the lower bound on achievable ergodic rate of the k th user can be approximated as*

$$\check{R}_k^p = \log_2(1 + (\psi_k - 1)\vartheta_k), \quad (4.34)$$

where $\psi_k = \frac{(N-K+1+(K-1)\mu)^2}{N-K+1+(K-1)\theta}$ and $\vartheta_k = \frac{N-K+1+(K-1)\theta}{N-K+1+(K-1)\mu} \frac{p_k}{\sigma_n^2} \beta_k$. The values of parameters μ and θ are defined by solving the following equations

$$\begin{cases} \mu &= \frac{1}{K-1} \sum_{i=1, i \neq k}^K \frac{1}{\phi_k}, \\ \theta \left(1 + \sum_{i=1, i \neq k}^K \frac{p_k \beta_i}{\sigma_n^2 \phi_k^2}\right) &= \frac{1}{K-1} \sum_{i=1, i \neq k}^K \frac{m_k}{\phi_k^2}, \end{cases} \quad (4.35)$$

where $\phi_k = N\beta_i \frac{p_k}{\sigma_n^2} (1 - \frac{K-1}{N} + \frac{K-1}{N}\mu) + 1$ and $m_k = \frac{p_k}{\sigma_n^2} \beta_i \mu (K-1) + 1$, respectively.

Proof: We observe that (4.31) can be decomposed as

$$\lambda_{\max}^p(\mathbf{B}_k) = \frac{1}{1 - \left[\mathbf{G}_k^H \left(\mathbf{G}_k \mathbf{G}_k^H + \frac{\sigma_n^2}{p_k} \mathbf{I} \right)^{-1} \mathbf{G}_k \right]_{k,k}} - 1 = \frac{1}{\left[\left(\frac{p_k}{\sigma_n^2} \mathbf{G}_k^H \mathbf{G}_k + \mathbf{I} \right)^{-1} \right]_{k,k}} - 1. \quad (4.36)$$

By applying Jensen's inequality and the convexity of $\log_2(1 + \frac{1}{\xi})$, the LB on the achievable rate of the k th user is defined as

$$R_k = \mathbb{E} \left\{ \log_2 (1 + \lambda_{\max}^p(\mathbf{B}_k)) \right\} \geq \check{R}_k^p = \log_2 \left(1 + \left(\mathbb{E} \left\{ \frac{1}{\lambda_{\max}^p(\mathbf{B}_k)} \right\} \right)^{-1} \right). \quad (4.37)$$

Following [111], the probability density function of $\lambda_{\max}^p(\mathbf{B}_k)$ is approximated by a Gamma function, i.e., $\lambda_{\max}^p(\mathbf{B}_k) \sim \Gamma(\psi_k, \vartheta_k)$, where shape parameter ψ_k and scale parameter ϑ_k are defined in (4.34). Based on that, we can obtain (4.34). ■

Note that it is complicated to derive the closed-form LB on achievable ergodic rate of massive MIMO systems with SLNR-PS and imperfect CSI, hence it will be investigated by Monte-Carlo simulations in Section 4.6. Moreover, for the case of perfect CSI, it is more convenient to obtain (4.29) than (4.34) due to its low complexity. Therefore, the closed-form UBs in (4.10) for the case of imperfect CSI and in (4.29) for the case of perfect CSI are used for further analysis.

4.3 Energy Efficiency Optimization with Imperfect CSI

In this section, we investigate the EE optimization problem for massive MU-MIMO systems with SLNR-PS. Based on the power consumption model in (2.6) and from (4.10), an

optimization problem for maximizing EE is formulated as

$$\begin{aligned}
 & \underset{P, N, T_t}{\text{maximize}} \quad \eta_{EE} = \frac{(1 - \frac{T_t}{T})K\hat{R}_k^{ip}}{P_{total}}, \\
 & \text{subject to } P_{\min} \leq P \leq P_{\max}, \\
 & \quad K \leq N \leq N_{\max}, \\
 & \quad K \leq T_t \leq T,
 \end{aligned} \tag{4.38}$$

where the pre-log factor $(1 - \frac{T_t}{T})$ takes into account the necessary pilot overhead in the TDD protocol, $N_{\max}$ is the maximum number of antennas, $P_{\min}$ and $P_{\max}$ are the minimum and maximum transmit power levels at the BS, respectively. Next, we propose an alternative method to solve problem (4.38). In particular, we determine the optimal value of each parameter (P , N or T_t) for maximizing the EE when the other parameters are fixed under the assumption of EPA. These expressions can help engineers to efficiently design a system for maximizing the EE if the SLNR-PS is employed. Recently, similar analyses for the conventional signal processing techniques have been studied in [26, 78, 112].

4.3.1 Optimal Transmit Power

We begin by obtaining the optimal value of P for optimizing the EE when N and T_t are given. A solution for optimizing the EE means to wisely select an appropriate transmit power level to use. To define the optimal solution, we provide the following theorem which is useful in the subsequent discussions.

Theorem 6 *Consider the following optimization problem*

$$\underset{\tilde{z}_{\min} \leq \tilde{z}}{\text{maximize}} \quad \eta(\tilde{z}) = \frac{f_0 \ln \left(\frac{a\tilde{z}}{\tilde{z}+b} + c \right)}{\tilde{z} + d} \tag{4.39}$$

with constant coefficients $a, b, c, d, f_0 > 0$. The objective function $\eta(\tilde{z})$ is a quasi-concave function of $\tilde{z}$. A unique solution is given by

$$\tilde{z}^* = \left\{ \tilde{z} | W(f_1(\tilde{z})) = \frac{f_1(\tilde{z})}{f_2(\tilde{z})}, \tilde{z}_{\min} \leq \tilde{z} \leq \tilde{z}_{\max} \right\}, \quad (4.40)$$

where $f_1(\tilde{z}) = \frac{ab(\tilde{z}+d)}{(\tilde{z}+b)^2}$, $f_2(\tilde{z}) = \frac{a\tilde{z}}{\tilde{z}+b} + c$ and $W(i)$ is the Lambert- W function defined as $i = W(i)e^{W(i)}$.

Proof: Please refer to Appendix B.1. ■

The transmit power of each user is assumed to be allocated equally. We substitute $\rho_k = K\sigma_n^2/P + (K-1)\sigma_e^2$ into (4.38), and denote $c_1 = \sigma_g^2(N-K+1)$, $c_2 = (K-1)\sigma_e^2$, $\xi = \frac{1}{\omega\sigma_{am}(1-\sigma_{feed})}$, where $\omega = (1 - \sigma_{DC})(1 - \sigma_{MS})(1 - \sigma_{cool})$, and $u = (P_c + P_{bb})/\omega$. The parameters $\sigma_{am}, \sigma_{feed}, P_c, P_{bb}, \sigma_{DC}, \sigma_{MS}$, and σ_{cool} are defined in the Section 2.1.3 of Chapter 2. For given N and T_t , problem (4.38) can be rewritten as

$$\underset{P_{\min} \leq P \leq P_{\max}}{\text{maximize}} \quad \eta_{EE} = \frac{(1 - \frac{T_t}{T})K \log_2(1 + v + \frac{c_1 P}{c_2 P + K\sigma_n^2})}{\xi P + u}. \quad (4.41)$$

Based on Theorem 6, the objective function in (4.41) is quasi-concave with respect to the transmit power for the case of imperfect CSI. The optimal value of P is given by

$$P_{opt}^{ip} = \min \{P_{ip}^*, P_{\max}\}, \quad (4.42)$$

$$P_{ip}^* = \left\{ p | W(f_1(p)) = \frac{f_1(p)}{f_2(p)}, P_{\min} \leq p \leq P_{\max} \right\}, \quad (4.43)$$

where $f_1(p) = \frac{K\sigma_n^2 c_1 (p+u/\xi)}{(c_2 p + K\sigma_n^2)^2}$ and $f_2(p) = 1 + v + \frac{c_1 p}{c_2 p + K\sigma_n^2}$, respectively. Although (4.42) is not a closed-form expression, this expression can be efficiently computed by numerical methods, instead of using Monte-Carlo simulation methods.

4.3.2 Optimal Number of BS Antennas

The total power consumption of massive MIMO systems increases with N , which can significantly affect the EE. Therefore, it is important to determine the optimal number of BS antennas that maximizes the EE. Let us define $c_3 = \xi P + (P_{bb} + p_{syn})/\omega$, $c_4 = (p_{dac} + p_{mix} + p_{filt})/\omega$, $c_5 = \frac{\sigma_g^2 P}{K\sigma_n^2 + (K-1)\sigma_e^2 P}$ and $t = N - K + 1$. The parameters p_{dac} , p_{mix} , p_{filt} , and p_{syn} are also defined in Section 2.1.3 of Chapter 2. For given P and T_t , problem (4.38) can be rewritten as

$$\underset{1 \leq t \leq t_{\max}}{\text{maximize}} \eta_{EE} = \frac{(1 - \frac{T_t}{T})K \log_2 (1 + c_5 t + \frac{K-1}{t})}{c_3 + c_4(t + K - 1)}, \quad (4.44)$$

where $t_{\max} = N_{\max} - K + 1$. Moreover, it has been shown in [31] that the EE is a quasi-concave function of N . By setting the first derivative of $\eta(t)$ to be zero, we have

$$f_3(t) \ln(f_3(t)) = f_4(t), \quad (4.45)$$

where $f_3(t) = (1 + c_5 t + \frac{K-1}{t})$ and $f_4(t) = (c_5 - \frac{K-1}{t^2})(\frac{c_3}{c_4} + t + K - 1)$, respectively. The solution of (4.45) is obtained as

$$t^* = \left\{ t \mid W(f_4(t)) = \frac{f_4(t)}{f_3(t)}, 1 \leq t \leq t_{\max} \right\}. \quad (4.46)$$

Therefore, the optimal number of BS antennas for maximizing the EE is defined as

$$N_{opt}^{ip} = \min \{ \lceil t^* \rceil + K - 1, N_{\max} \}, \quad (4.47)$$

where $\lceil \cdot \rceil$ denotes the round number.

4.3.3 Optimal Channel Trainning

We observe that the training length T_t and total power consumption P_{tt} at the BS are independent. Therefore, for given N and P , the EE maximization problem (4.38) is equivalent to the SE maximization problem which is formulated as

$$\underset{K \leq T_t \leq T}{\text{maximize}} R_s = \left(1 - \frac{T_t}{T}\right) K \log_2 \left(1 + v + \frac{c_6 T_t}{(1 + T_t \rho_t \beta_k) K \sigma_n^2 + c_7}\right), \quad (4.48)$$

where $c_6 = \rho_t \beta_k^2 P(N - K + 1)$ and $c_7 = (K - 1)P\beta_k$, respectively. It can be shown that the cost function R_s is strictly concave with respect to T_t in the interval $[K, T]$, where K denotes the minimum number of training symbols, due to the orthogonality constraint of the pilot sequence. By setting $\frac{\partial R_s}{\partial T_t} = 0$, the optimal training length T_t^{opt} is defined as

$$T_t^{opt,ip} = \max \{T_t^*, K\}, \quad (4.49)$$

$$T_t^* = \left\{ j \mid W(f_5(j)) = \frac{f_5(j)}{f_6(j)}, K \leq j \leq T \right\}, \quad (4.50)$$

where $f_5(j) = \frac{(T-j)(c_6 \sigma_n^2 K + c_6 c_7)}{((1 + \rho_t \beta_k j) K \sigma_n^2 + c_7)^2}$ and $f_6(j) = 1 + v + \frac{c_6 j}{(1 + \rho_t \beta_k j) K \sigma_n^2 + c_7}$, respectively. Moreover, at high SNR region, we can derive a closed-form expression of the optimal training length for maximizing problem (4.48), which is presented in the following theorem.

Theorem 7 *Let $\rho_d \rightarrow \infty$ and ρ_t is finite, i.e. $\rho_t \ll \rho_d$, for given K, N and T , the optimum training length T_t^{opt} for maximizing the SE is given by*

$$T_t^{opt,ip} = \max \{T_t^*, K\}, \quad (4.51)$$

$$T_t^* = \frac{v}{\beta_k \rho_t} \left(\frac{c_8}{W(c_8 e)} - v \right), \quad (4.52)$$

$$c_8 = v + \frac{T \rho_t \beta_k}{v}, \quad (4.53)$$

where e is Euler's number.

Proof: Please refer to Appendix B.2. ■

For a specific case, when $\rho_d \rightarrow \infty$, $\rho_t \rightarrow 0$ and $v \rightarrow 0$ (due to the assumption $N \gg K$), applying $\ln(1 + \zeta) \approx \zeta + O(\zeta^2)$, from (4.48), we have

$$R_{s,\infty} = \left(1 - \frac{T_t}{T}\right) \frac{KT_t\rho_t\beta_k}{v \ln 2}. \quad (4.54)$$

It is clear that the optimum training length for maximizing SE in (4.54) under the SLNR precoding scheme converges to

$$\lim_{\rho_t \rightarrow 0} T_t^{opt,ip} = \frac{T}{2}. \quad (4.55)$$

This implies that T should be at least greater than $2K$. For ZF-PS, the same limit has been provided in [12] and [104].

4.3.4 Alternating Optimization Algorithm

In the previous section, the optimal values of P , N and T_t for maximizing the EE are independently obtained when the other two parameters are given. Note that the global maximal value of the EE can only be achieved by joint optimization. Since N and T_t are integers, by obtaining the optimal value of P in (4.42), we can adopt an exhaustive search on N and T_t over all reasonable pair (N, T_t) in order to achieve optimum EE. With a different approach, we employ a practical solution to optimize these parameters sequentially based on the AO algorithm with low complexity [26]. This algorithm is summarized as follows:

- Step 1: Initialize a feasible set (N, P, T_t) .
- Step 2: Update the optimal value of P according to (4.42).
- Step 3: Optimize the number of BS antennas N according to (4.47), based on P obtained in the step 2.

- Step 4: Optimize the training length T_t according to (4.49), based on the P and N obtained in the steps 2 and 3.
- Step 5: Repeat steps 2-4 until the convergence of η_{EE} in (4.38) is achieved, i.e., $|\eta_{EE}(i+1) - \eta_{EE}(i)| \leq \epsilon$.

Although a general AO algorithm is not analytically guaranteed to achieve global optimum solution, it is locally convergent according to [113, Proposition 4] and [114, Lemma 2]. In our EE maximization problem, the algorithm optimizes these parameters (N, P, T_t) sequentially until $|\eta_{EE}(i+1) - \eta_{EE}(i)| \leq \epsilon$. The local convergence of the algorithm is guaranteed for any initial set (N, P, T_t) . This is because the EE metric has a finite upper bound, and the EE metric is a quasi-concave function with respect to N, P and T_t for given constraints, respectively, hence the value of EE in the algorithm is non-decreasing in every step. It is expressed as

$$\begin{aligned} \eta_{EE}(N_{opt}^{(i),ip}, P_{opt}^{(i),ip}, T_t^{(i),opt,ip}) &\stackrel{(b_1)}{\leq} \eta_{EE}(N_{opt}^{(i),ip}, P_{opt}^{(i+1),ip}, T_t^{(i),opt,ip}) \leq \\ &\dots \leq \eta_{EE}(N_{opt}^{(i+1),ip}, P_{opt}^{(i+1),ip}, T_t^{(i+1),opt,ip}), \end{aligned} \quad (4.56)$$

where (b_1) is obtained based on the fact that the optimal solution of the sub-problem (4.41) is given by $\eta_{EE}(N_{opt}^{(i),ip}, P_{opt}^{(i+1),ip}, T_t^{(i),opt,ip}) = \max_P \eta_{EE}(N_{opt}^{(i),ip}, P_{opt}^{(i),ip}, T_t^{(i),opt,ip})$. The actual convergence speed of the algorithm depends on specific value of accuracy ϵ [114]. In order to avoid local convergence, we randomly generate several initial sets and choose the one which results in the overall maximum EE. An output of the chosen initial set via the algorithm is considered as an optimal solution, such as $(N_{opt}^{**,ip}, P_{opt}^{**,ip}, T_t^{**,opt,ip})$. Note that $N_{opt}^{**,ip}$ and $T_t^{**,opt,ip}$ are a real-valued numbers. To define integer-valued solutions of N_{opt}^{ip} and $T_t^{opt,ip}$, we only need to take floor and ceiling operations on the real-valued solutions $N_{opt}^{**,ip}$ and $T_t^{**,opt,ip}$ for achieving the maximal EE (4.38), respectively. By defining $P_{opt}^{ip} = P_{opt}^{**,ip}$, the final solution is given by $(N_{opt}^{ip}, P_{opt}^{ip}, T_t^{opt,ip})$. As compared with the Brute-force exhaustive search (ES) algorithm, nu-

merical results presented in Section 4.6 show that the AO algorithm performs well and it can almost achieve the global optimum EE.

Let us define Δ_p as a small step of transmit power level, which is used to obtain P^* in (4.43). The complexity of this algorithm to find the optimal value of P_{opt}^{ip} is on the order of $O(\lceil \frac{P_{\max} - P_{\min}}{\Delta_p} \rceil + 1)$, where the additional comparison is due to the fact that the algorithm needs to compare the two values in (4.42). Similarly, since N and T_t are integers, the complexities of the algorithm to find the optimal values of N_{opt}^{ip} in (4.47) and $T_t^{opt,ip}$ in (4.49) are on the orders of $O(N_{\max} - K + 1)$ and $O(T - K + 1)$, respectively. Hence, the total complexity of the AO algorithm is on the order of $M_1 M_2 (O(\lceil \frac{P_{\max} - P_{\min}}{\Delta_p} \rceil + 1) + O(N_{\max} - K + 1) + O(T - K + 1))$, where M_1 is the number of iterations and M_2 is the number of randomly initial sets, to obtain the final solution.

4.4 Energy Efficiency Optimization with Perfect CSI

In this section, we define the optimal values of the three parameters for maximizing the EE of SLNR-PS when the BS can estimate CSI perfectly. We also present a energy efficient power allocation scheme to maximize the EE.

4.4.1 Optimum P , Optimum N and Optimum Training Length

With perfect CSI and equal power allocation, from (4.41), the objective function of the EE maximization problem (4.38) can be rewritten as

$$\eta_{EE} = \frac{(1 - \frac{T_t}{T})K\hat{R}_k^p}{\xi P + u} = \frac{(1 - \frac{T_t}{T})K \log_2 \left(1 + v + \frac{P\beta_k}{K\sigma_n^2}(N - K + 1) \right)}{\xi P + u}. \quad (4.57)$$

It can be shown that $\frac{\partial^2 \eta_{EE}(P)}{\partial P^2} < 0$, the system EE in (4.57) is a concave function with P . By letting $\frac{\partial \eta_{EE}(P)}{\partial P} = 0$, we get

$$c_9(\xi P + u) = \xi(1 + c_9 P + v) \ln(1 + c_9 P + v), \quad (4.58)$$

where $c_9 = \frac{\beta_k}{K\sigma_n^2}(N - K + 1)$. Therefore, the optimal value of P for maximizing the EE is obtained as

$$P_{opt}^p = \min \{P_p^*, P_{\max}\}, \quad (4.59)$$

$$P_p^* = \frac{1}{c_9} \left(\frac{c_{10}e}{W(c_{10})} - v - 1 \right), \quad (4.60)$$

where P_p^* is the solution of (4.58) and $c_{10} = \frac{1}{e} \left(\frac{c_9 u}{\xi} - v - 1 \right)$. From (4.57), we observe that $\hat{R}_k^p$ and $(\xi P + u)$ are independent with T_t . Thus, the optimal length of training sequence to achieve maximal value of η_{EE} is given by $T_t^{opt,p} = K$. In addition, the optimal number of BS antennas N_{opt}^p can be defined by using the result presented in (4.47) by setting $\sigma_e^2 = 0$ and $\sigma_g^2 = \beta_k$. Similar approach is used to determine the optimum EE for massive MIMO with SLNR-PS and perfect CSI. It means that, based on the above expressions, the global optimal solution $(N_{opt}^p, P_{opt}^p, T_t^{opt,p})$ of the EE maximization problem can be obtained by applying the presented alternating optimization algorithm.

Note that the optimal value of N_{opt}^p obtained by using (4.47) with $\sigma_e^2 = 0$ and $\sigma_g^2 = \beta_k$ is not a closed-form expression. Therefore, we present another approach to determine N_{opt}^p effectively. In particular, we rewrite the sum of SE for SLNR-PS as $R_s = c_{11} \log_2 \left(1 + v + \frac{P\beta_k}{K\sigma_n^2}(N - K + 1) \right)$, where $c_{11} = (1 - \frac{T_t}{T})K$. We observe that P can be defined by obtaining the inverse function of R_s . The solution of $f^{-1}(R_s)$ can be obtained as

$$P = f^{-1}(R_s) = \frac{K2^{R_s/c_{11}} - K(1 + v)}{\frac{\beta_k}{\sigma_n^2}(N - K + 1)}. \quad (4.61)$$

Substituting (4.61) into (4.57), the trade-off function between the SE and EE can be rewritten as

$$\eta_{EE} = \frac{R_s c_{12}}{K 2^{R_s/c_{11}} - K(1+v) + c_{13}}, \quad (4.62)$$

where $c_{12} = \omega \sigma_{am} \frac{\beta_k}{\sigma_n^2} (1 - \sigma_{fed})(N - K + 1)$ and $c_{13} = (P_c + P_{bb}) \sigma_{am} \frac{\beta_k}{\sigma_n^2} (1 - \sigma_{fed})(N - K + 1)$. Based on this trade-off, we can derive closed-form expressions of the optimal SE and the optimal number of BS antennas for maximizing the EE, which are shown in the following theorems.

Theorem 8 *Under perfect CSI condition, the optimal value of SE for maximizing the EE of massive MIMO systems with SLNR-PS is obtained as*

$$R_s^{opt,p} = \frac{c_{11}}{\ln 2} \left(1 + W \left(\frac{c_{13}/K - v - 1}{e} \right) \right). \quad (4.63)$$

Proof: Since $\frac{\partial^2 \eta_{EE}(R_s)}{\partial R_s^2} < 0$, the system EE in (4.62) is a concave function. By letting $\frac{\partial \eta_{EE}(R_s)}{\partial R_s} = 0$, we have

$$\left(\frac{R_s}{c_{11}} \ln 2 - 1 \right) 2^{\frac{R_s}{c_{11}}} K = c_{13} - K(1+v). \quad (4.64)$$

After some algebraic manipulations, we can obtain the solution. ■

Theorem 9 *Under perfect CSI condition, for a massive MIMO system with a specific R_s requirement, the optimal number of BS antennas for maximizing the EE is obtained as*

$$N_{opt}^p = \min \{ \lceil t_j^* \rceil + K - 1, N_{\max} \}, \quad (4.65)$$

$$t_j^* = -\frac{1}{3c_{15}c_{17}} \left(\varpi_j \tilde{\Upsilon} + \frac{\delta_0}{\varpi_j \tilde{\Upsilon}} \right), \text{ with } j \in \{1, 2, 3\}, \quad (4.66)$$

where $\varpi_1 = 1$, $\varpi_2 = \frac{-1+i\sqrt{3}}{2}$, $\varpi_3 = \frac{-1-i\sqrt{3}}{2}$, and $\tilde{\Upsilon} = \sqrt[3]{(\delta_1 + \sqrt{\delta_1^2 - 4\delta_0^3})/2}$ with $\delta_0 = 3c_{15}c_{17}c_{19}$ and $\delta_1 = 54(c_{15}c_{17})^2 K(K-1)$.

Proof: We have denoted that $t = N - K + 1$, (4.62) can be rewritten as

$$\eta_{EE} = \frac{R_s c_{14} t^2}{c_{15} c_{17} t^3 + c_{17} c_{18} t^2 + c_{19} t - K(K-1)}, \quad (4.67)$$

where $c_{14} = \omega \sigma_{am} (1 - \sigma_{feed}) \frac{\beta_k}{\sigma_n^2}$, $c_{15} = p_{dac} + p_{mix} + p_{filt}$, $c_{16} = p_{syn} + P_{bb}$, $c_{17} = \sigma_{am} (1 - \sigma_{feed}) \frac{\beta_k}{\sigma_n^2}$, $c_{18} = (K-1)c_{15} + c_{16}$, $c_{19} = K(2^{R_s/c_{11}} - 1)$. By letting $\frac{\partial \eta_{EE}(t)}{\partial t} = 0$, we get

$$c_{15} c_{17} t^3 - c_{19} t + 2K(K-1) = 0. \quad (4.68)$$

The solutions of (4.68) are defined as results presented in (4.66). If t_j^* is real and $t_j^* > K-1$, then the optimal number of antennas is shown in (4.65). $\blacksquare$

4.4.2 Energy-Efficient Power Allocation Scheme

Herein, we present an optimal power allocation scheme to maximize the EE of SLNR-PS. The EE optimization problem under both transmit power and QoS constraints is formulated as

$$\begin{aligned} & \underset{p_k}{\text{maximize}} \quad \frac{(1 - \frac{T_t}{T}) \sum_k^K \hat{R}_k^p(p_k)}{\xi \sum_k^K p_k + u} \\ & \text{subject to} \quad (C_1) : \hat{R}_k^p(p_k) \geq C_k^{min}, (C_2) : \sum_k^K p_k \leq P. \end{aligned} \quad (4.69)$$

The objective function (4.69) is a ratio of two functions of p_k . The result in (4.69) is a fractional programming problem which is nonconvex. By applying properties of the fractional programming, the objective function is equivalent to $(1 - \frac{T_t}{T}) \sum_k^K \hat{R}_k^p(p_k) - \eta_{EE}^* (\xi \sum_k^K p_k + u)$, where η_{EE}^* is the EE when p_k is equal to the optimal value p_k^* , i.e., $\eta_{EE}^* = \frac{(1 - \frac{T_t}{T}) \sum_k^K \hat{R}_k^p(p_k^*)}{\xi \sum_k^K p_k^* + u}$ [115].

Thus, (4.69) can be rewritten as

$$\begin{aligned} & \underset{p_k}{\text{maximize}} \quad \left(1 - \frac{T_t}{T}\right) \sum_k^K \hat{R}_k^p(p_k) - \eta_{EE}^* \left(\xi \sum_k^K p_k + u\right), \\ & \text{subject to} \quad (\text{C}_1) \text{ and } (\text{C}_2). \end{aligned} \quad (4.70)$$

The optimization in (4.70) has been transformed into a convex optimization problem. The Lagrange function is obtained as

$$\begin{aligned} \mathcal{L}(p_k, \gamma, \tau) = & \left(1 - \frac{T_t}{T}\right) \sum_k^K \hat{R}_k^p(p_k) - \eta_{EE}^* \left(\xi \sum_k^K p_k + u\right) \\ & + \sum_k^K \alpha_k \left(\left(1 - \frac{T_t}{T}\right) \hat{R}_k^p(p_k) - C_k^{\min} \right) - \tau \left(\sum_k^K p_k - P \right). \end{aligned} \quad (4.71)$$

Based on the Karush-Kuhn-Tucker conditions, the OPA for the k th user is obtained as

$$p_k^* = \max \left\{ 0, \frac{(1 + \alpha_k)(1 - T_t/T)}{(\tau + \xi \eta^*) \ln 2} - \frac{(1 + v)\sigma_n^2}{\beta_k(N - K + 1)} \right\}, \quad (4.72)$$

where α and τ are Lagrange multipliers, respectively. They are updated by the gradient method which can be defined as

$$\alpha_k(i + 1) = \max \left\{ 0, \alpha_k(i) - \Delta \alpha_k \left(\left(1 - T_t/T\right) \hat{R}_k^p - C_k^{\min} \right) \right\}, \quad (4.73)$$

and

$$\tau(i + 1) = \max \left\{ 0, \tau(i) - \Delta \tau \left(P - \sum_k^K p_k \right) \right\}, \quad (4.74)$$

where $\Delta \alpha_k$ and $\Delta \tau$ are the positive iteration steps.

4.5 Rate Profile Optimization

In previous sections, we optimize the EE of SLNR-PS while considering the constraints of QoS. Herein, we would like to maximize the achievable rate of individual users with fixed values of QoS constraints and the EE target, hence the sum rate performance can be optimized. In particular, for given N and T_t , we consider the rate optimization problem for individual users based on (C₁) and (C₂), and the EE target η_t , i.e., (C₃) : $\sum_k^K \hat{R}_k \geq \eta_t(\xi \sum_k^K p_k + u)$, where $\eta_t = \frac{\eta_{EE}}{(1-T_t/T)}$. Let us define the achievable rate region $\mathcal{R}$ as a set which composes of rate-tuples for K users, i.e., [116]

$$\mathcal{R} = \left\{ \mathbf{r} \in \mathbb{R}_+^K \mid r_k = \hat{R}_k(p_k), \text{ with (C}_1\text{), (C}_2\text{), and (C}_3\text{)} \right\}. \quad (4.75)$$

We need to obtain the Pareto boundary (PB) of $\mathcal{R}$, at which it is impossible to improve a user's rate without degrading other's rate. Any rate-tuple on the PB of $\mathcal{R}$ can be achieved as a solution of the rate profile optimization problem, i.e.,

$$\begin{aligned} & \underset{\{p_k\}, R_\Sigma}{\text{maximize}} && R_\Sigma, \\ & \text{subject to} && \hat{R}_k(p_k) \geq z_k R_\Sigma, \\ & && (\text{C}_1), (\text{C}_2), \text{ and } (\text{C}_3), \forall k, \end{aligned} \quad (4.76)$$

where $R_\Sigma = \sum_k^K \hat{R}_k$ and z_k is the target ratio between the achievable rate of user k and the users' sum rate [116]. For a given rate-profile vector $\mathbf{z}$ with $\sum_1^K z_k = 1$, the optimal solution of (4.76), denoted as R_Σ^* , can be defined by applying bisection method, hence the corresponding Pareto optimal rate-tuple for user k is obtained as $z_k R_\Sigma^*$. Therefore, with different vectors $\mathbf{z}$, solving the above problem yields the complete Pareto boundary of the rate region. For example, the rate profile optimization and Pareto boundary of the rate region for $K = 2$ users are illustrated in Fig. 4.1. In addition, the bisection algorithm to determine the R_Σ^* is presented

 Figure 4.1: Illustration of the rate profile optimization for $K = 2$ users.

Table 4.1: Bisection method for solving the rate profile maximization

-
1. **Input:** Given rate profile $\mathbf{z} = \{z_1, \dots, z_K\}$ and accuracy $\varepsilon = 10^{-4}$.
 2. Given $\mathbf{r} = \{C_1^{\min}, \dots, C_K^{\min}\}$, EE target η_t , and P dBm.
 3. **Initialize:** $R_{\Sigma}^{\min} = \max \left\{ \frac{C_k^{\min}}{z_k}, \forall k \right\}$ and $R_{\Sigma}^{\max} = \sum_{k=1}^K \hat{R}_k^p(P), y = 0, R_{\Sigma}^* = 0$.
 4. **While** $(R_{\Sigma}^{\max} - R_{\Sigma}^{\min}) \geq \varepsilon$ **do**
 5. $y = (R_{\Sigma}^{\max} + R_{\Sigma}^{\min})/2$.
 6. **if** $(z_k y \geq C_k^{\min}, \forall k)$ and $\left(\sum_{k=1}^K \left(\frac{\sigma_n^2 (2^{z_k y} - v - 1)}{\beta_k (N - K + 1)} \right) \leq P \right)$
 and $\sum_{k=1}^K z_k y \geq \eta_t \left(\xi \sum_{k=1}^K \left(\frac{\sigma_n^2 (2^{z_k y} - v - 1)}{\beta_k (N - K + 1)} \right) + u \right)$ **do** $R_{\Sigma}^{\min} \leftarrow y$
 7. **else** $R_{\Sigma}^{\max} \leftarrow y$
 8. $R_{\Sigma}^* = (R_{\Sigma}^{\max} + R_{\Sigma}^{\min})/2$.
 9. **Output:** R_{Σ}^* and $R_k^* = z_k R_{\Sigma}^*, \forall k$
-

for the case of perfect CSI in the Table 4.1.

4.6 Numerical Results and Discussions

In the following, numerical results are presented to validate the theoretical analyses, to investigate the system performance with difference parameter settings, as well as to evaluate the effectiveness of the proposed algorithms. The active users are assumed to be uniformly distributed in a cell with radius $D = 288$ m and a minimum distance $D_0 = 40$ m. The distribution of the users along the radius of the cell is given by $f_D(d) = \frac{2d}{D^2 - D_0^2}$ [25]. The shadowing coefficient $\bar{\Omega}_k$ is a log-normal random variable (RV) with standard deviation σ_{sh} , i.e., the quantity $10 \log_{10}(\bar{\Omega}_k)$ has a Gaussian distribution with a mean of 0 and a standard deviation of σ_{sh} [7]. The transmitted signals are assumed to be modulated with OFDM. Based on LTE standards, an OFDM symbol duration is approximated as $71.4 \mu s$. We set the channel coherence time to be $T_c = 1$ ms, hence $T = 196$ symbols; and $K = 10, \sigma_{sh} = 8$ dB [7].

Without loss of generality, we select the noise variance to be 1 to study the SE performance for both cases of perfect CSI and imperfect of CSI [6, 9, 13]. The form of large-scale fading coefficient follows the approaches of [7] and [13], in which it is defined as $\beta_k = \bar{\Omega}_k(d_k/d_0)^{-\vartheta}$, where d_k is the distance between user k and the BS, ϑ is path loss exponent, and d_0 is a fixed reference distance. Herein, we choose $\vartheta = 3.76$ and $d_0 = 40$ m.

In particular, Figure 4.2 presents the SE bounds of SLNR-PS and the SEs of ZF-based, MF-based and MMSE-based precoding schemes (PSs) in [6] and [12], which are presented in (2.11) of Chapter 2, versus SNR for the perfect CSI case with $N = 30$ and $T_t = 10$. The curves labelled “Analytical UB (Based on Eq. (4.29))” and “Analytical LB (Based on Eq. (4.34))” refer to the corresponding upper bound and lower bound on the SE which are defined by using equations (4.29) and (4.34), respectively. As expected, in Fig. 4.2, it is shown that the SLNR-PS performs as well as the MMSE precoding scheme. In addition, the SLNR-PS performs better than ZF and MF schemes over the entire range of SNRs. At the low SNR region, MF is better than ZF, and vice versa at the medium and high SNR regions. Furthermore, the LB

Figure 4.2: The SEs versus SNR for SLNR-PS, ZF-based, MF-based, and MMSE-based precoding schemes with perfect CSI and EPA when $N = 30$, $K = 10$, $T_t = 10$ and $T = 196$.

on the SE of SLNR-PS is very close to simulated results, and the UB on the SE is very tight to the simulation when SNR is greater than 10 dB. This fact can be shown more evidently by observing Fig. 4.3 where N is increased. Clearly, as $N \geq 60$, all bounds are very tight even when the downlink SNR is low, i.e., $\rho_d = 5$ dB, as shown in Fig. 4.3.

Moreover, the SE bounds of SLNR-PS with imperfect CSI are also demonstrated in Fig. 4.3. Note that, in Fig. 4.3, the LB on the SE of SLNR-PS with imperfect CSI is obtained by Monte-Carlo simulations, whereas the LB on the SE of SLNR-PS with perfect CSI is obtained by using (4.34). The curves labelled “Analytical UB 1 (Based on Eq. (4.26))” and “Analytical UB 2 (Based on Eq. (4.10))” refer to the corresponding UBs on the SE which are obtained by using equations (4.26) and (4.10), respectively. When $\sigma_e^2 = 0$, i.e., the BS can estimate CSI perfectly, the UBs on the SE (4.26) and (4.10) are equivalent to (4.29). In this example, we set $\rho_d = 5$ dB and $T_t = 10$. For a given T_t , as ρ_t increases, the error variance σ_e^2 is decreased, hence

Figure 4.3: The SEs versus N for SLNR-PS and ZF-PS with perfect and imperfect CSI, and EPA when $K = 10, T_t = 10, T = 196$ and $\rho_d = 5$ dB. For the case of imperfect CSI, we set $\rho_t = 0$ dB and $\rho_t = 4$ dB, respectively.

the system performance is improved. This is the reason that as ρ_t increases from 0 dB to 4 dB, the SE for SLNR-PS is significantly improved as shown in Fig. 4.3. Under conditions of perfect CSI and imperfect CSI, we observe that the SE bounds of SLNR-PS are generally tight with respect to their simulation results when N is small, and they converge to the exact one when N is large. Moreover, the SE of SLNR-PS also converges to that of ZF-PS when N is large due to $\mathbb{E}\{\theta_{ad,k}\} \rightarrow 0$. Hence, the SE of SLNR-PS can be considered to be lower bounded by that of ZF-PS. On a different note, due to $\sigma_e^2 \ll N$, the “Analytical UB 2” is very close to “Analytical UB 1”, which has been mentioned in Section 4.2.1. In addition, in Fig. 4.4, as $\rho_d \rightarrow \infty$, the SE of SLNR-PS with imperfect CSI converges to an asymptotic limit due to the convergence of the achievable ergodic rate of each user, which has been shown in (4.28). This asymptotic limit strongly depends on the quality of channel estimation. If σ_e^2 is small, the asymptotic limit is high and vice versa. In conclusion, numerical results validate our analysis. The UBs on the

Figure 4.4: The bounds, asymptotic limit and numerical results of the SE versus SNR for SLNR-PS with imperfect CSI and EPA when $N = 60$, $K = 10$, $T_t = 10$ and $T = 196$.

SE based on (4.10) for the case of imperfect CSI and (4.29) for the case of perfect CSI perform well. Hence, these closed-form expressions are used in the subsequent discussions due to their low computation complexities.

In Fig. 4.5, the SE for SLNR-PS is presented as a function of the length of training sequence when CSI is imperfect. The optimum training lengths for maximizing the SE have been discussed in Section 4.3.3. In Fig. 4.5, the maximum points achieved at the optimal values of T_t are marked by a circle. As expected, the optimal value of T_t obtained through simulation matches well with that obtained by using (4.49). As ρ_t increases from 0 dB to 10 dB, the maximum SE is strictly improved.

To investigate the system EE performance, we employ a more practical channel model. In particular, the noise variance is assumed to be $\sigma_n^2 = -96$ dBm [26]. The large-scale fading coefficient β_k is generated based on the shadowing factor and the distance-dependent path

Figure 4.5: The SE of SLNR-PS versus training length T_t when $K = 10$, $N = 100$, $\rho_d = 30$ dB, and $T = 196$ for the case of imperfect CSI.

Table 4.2: Simulation parameters

Parameter	Value	Parameter	Value
Maximum transmit power $P_{\max}$ [28]	43 dBm	Noise variance σ_n^2 [26]	-96 dBm
DAC power p_{dac} [29]	15.6 mW	Mixer power p_{mix} [29]	30.3 mW
Frequency synthesizer power p_{syn} [29]	50 mW	Filter power p_{filt} [29]	20 mW
Power amplifier efficiency σ_{am} [28]	0.38	Main supply loss σ_{MS} [28]	9%
Baseband power P_{bb} [28]	29.6 W	Feeder loss σ_{feed} [28]	-3 dB
DC-DC loss σ_{DC} [28]	7.5%	Cooling loss σ_{cool} [28]	10%

loss factor $PL(d_k)$. The shadowing factor $\bar{\Omega}_k$ is also a log-normal RV with $\sigma_{sh} = 8$ dB [7]. For the macrocell BS and carrier frequency of 2 GHz, the path loss factor is given in dB by $PL(d_k) = 128.1 + 10 \log_{10}(d_k)$, where d_k is measured in km [117]. Moreover, the values of other parameters follow the settings of various works [26, 28, 29]. They are summarized in Table 4.2.

For massive MIMO system with a specific given R_s requirement, in Theorem 9, we have mentioned that the closed-form expression of the optimal number of BS antennas for maximizing EE can be obtained. In particular, the optimal numbers of N for maximizing EE are equal

Figure 4.6: The EE versus N for SLNR-PS with perfect CSI and different R_s requirements.

to $N_{opt}^p = 74, 138$, and 268 corresponding to $R_s = 120, 140$, and 160 (bits/s/Hz), respectively. The accuracy of the optimal values N_{opt}^p is demonstrated in Fig. 4.6, in which the maximum EEs achieved at the optimum values of N are marked by a circle. We observe that the N_{opt}^p is increased as the R_s requirement increases from 120 to 160 (bits/s/Hz). However, the maximums EE is decreased because the total power consumption is increased with N .

Next, we investigate the optimal values of N, P and T_t that are needed to achieve the global maximum value of the EE. Particularly, Figure 4.7 shows the EE of SLNR-PS as a function of N and P when the system has unit bandwidth, $K = 10, P_t = 10$ dBm and $T = 196$. Each point is obtained by using the optimal value of T_t in (4.49). An optimal solution obtained by the Brute-force ES algorithm, named as the global optimum EE, is also provided for comparisons. As observed from Fig. 4.7, there is a global optimum value of EE, which is equal to 1.74 (bits/Joule), at $N_{opt}^{ip} = 99$ and $P_{opt}^{ip} = 24.8$ dBm, and it can be achieved by $T_t^{opt,ip} = 10$. At this global optimum EE, the SE per user is 8.682 (bits/Hz). By applying the AO algorithm,

Figure 4.7: The EE of SLNR-PS with imperfect CSI when $K = 10$, $\sigma_n^2 = -96$ dBm, $P_t = 10$ dBm, and $T = 196$.

the iterative progression for maximizing the EE is also demonstrated in Fig. 4.7. In this example, the number of feasibly initial sets, which is randomly generated in the AO algorithm, is 10. We set $\epsilon = 10^{-5}$, the AO algorithm converges quickly after 4 iterations to achieve the global optimum EE. The convergence of the algorithm is presented with circles, whereas the globally optimal value of EE is presented with a asterisk. This fact can be easily observed in Fig. 4.8, in which the EE results for SLNR-PS with imperfect CSI (i.e., $P_t = 0$ dBm and $P_t = 10$ dBm) and perfect CSI are also presented. As expected, for a given $P_t = 0$ dBm, the AO algorithm is also able to obtain the global optimum EE numerically, which is 1.217 (bits/Joule) at $N_{opt}^{ip} = 137$, $P_{opt}^{ip} = 21.3$ dBm and $T_t^{opt,ip} = 10$. In addition, as P_t increases from 0 dBm to 10 dBm, the global optimum value of EE for SLNR-PS is significantly increased. We observe that the system with $P_t = 0$ dBm (i.e., the quality of channel estimation is low)

Figure 4.8: The EE of SLNR-PS versus number of iterations when $K = 10, \sigma_n^2 = -96$ dBm.

needs to use more antennas to achieve the global optimum EE, as compared to the system with $P_t = 10$ dBm due to better quality of channel estimation. Moreover, with perfect CSI, the global optimum EE is 2.224 (bits/Joule) at $N_{opt}^p = 80, P_{opt}^p = 29.5$ dBm and $T_t^{opt,p} = 10$. In general, we observe that the optimal energy efficient system is a massive MIMO setup because there are large number of BS antennas at the three global optimum EEs. As discussed earlier in Fig. 4.3, the UB on the achievable sum rate converges to the exact one when N is large. Hence, the optimal solution shown in Figs. 4.7 and 4.8 can be considered as the actual optimal solution for maximizing the EE of the proposed scheme. Note that this is the output by solving the optimization problem, in which the system parameters are not restricted. Therefore, we can conclude that the AO algorithm provides an effective solution to achieve the global optimum EE with a low-complexity computation and it performs well.

In Fig. 4.9, we investigate the energy-efficient power allocation scheme for the SLNR-PS with perfect CSI when $K = 10, N = 80, T_t = 10$. In this example, we set $C_k^{min} = 1$ (bit/s/Hz).

Figure 4.9: Energy efficiency of SLNR-PS for both cases of EPA and OPA when $K = 10$, $N = 80$, $T_t = 10$, and $\sigma_n^2 = -96$ dBm in the case of perfect CSI.

As observed from Fig. 4.9, the SLNR-PS with OPA provides an improvement in the EE as compared to that of the EPA. When the transmit power P is greater than $P_{opt}^p = 29.5$ dBm (i.e., the EE is maximum at P_{opt}^p), the EE of SLNR-PS with EPA decreases, i.e., named as baseline in Fig. 4.9. This is because the EE is strictly quasi-concave with respect to P . Hence, it increases with $P \in [0, P_{opt}^p]$ and it decreases with $P \in [P_{opt}^p, P_{max}]$. However, by applying the energy-efficient power allocation scheme, the BS wisely selects the optimal value of P to use in order to achieve an optimal EE, instead of using the maximum available power.

In Fig. 4.10, the SEs of SLNR-PS with the assumptions of EPA and OPA are presented. Herein, the OPA is obtained by using the popular Water-Filling method. When N is not large, such as $N = 50$, the optimal power allocation algorithm can provide an improvement in the SE as compared to the SLNR-PS with EPA at the low and medium SNR regions. However, when the number of BS antennas is large, the channel hardens in most propagation environments [7]. It means that the effective channel gain become a deterministic value which is expressed as (2.2).

Figure 4.10: Spectral efficiency of SLNR-PS versus SE for both cases of EPA and OPA when $T_t = 10$, and $\sigma_n^2 = -96$ dBm in the case of perfect CSI.

Therefore, the SE improvement by using the OPA is limited. In particular, when $N = 100$ or 150 , the SE of SLNR-PS with OPA is close to that of EPA as presented in Fig. 4.10.

Finally, by solving optimization problem in (4.76) with different rate-profile vector $\mathbf{z}$, the complete Pareto boundary of $\mathcal{R}$ is illustrated in Fig. 4.11 for $K = 3$ users when $N = 200$, $C_k^{\min} = 1$ (bit/s/Hz), $P = 5$ dBm, and the EE target $\eta_t = 0.3$ (bits/Joule). The boundary looks like a box with rounded edges, and the colour bar shows the individual user rate. For example, for a given $\mathbf{z} = \{0.34; 0.15; 0.51\}$, we have $R_{\Sigma}^* = 14.62$ (bits/s/Hz), then Pareto optimal per-user rates are $R_1 = 4.97$, $R_2 = 2.19$ and $R_3 = 7.46$ (bits/s/Hz), respectively.

4.7 Summary

In this chapter, the spectral efficiency and energy efficiency of massive MIMO with SLNR-PS have been analyzed under perfect and imperfect CSI. The trade-off between SE and EE has

Figure 4.11: Plot of three-user rate region when $N = 200$, $P = 5$ dBm and $\eta_t = 0.3$ bits/Joule.

been considered. The closed-form expression of the optimal training length for maximizing SE has been derived at high SNR region. The optimal values of N , P and T_t for maximizing the EE have been defined, respectively. The AO algorithm has been adopted to achieve the optimum EE based on the optimal values of N , P and T_t . Numerical results have presented to validate our analysis. It has been show that the SLNR-PS outperforms ZF-PS. The proposed algorithm has quick convergence and it can almost achieve the global optimum energy efficiency. At this point, the optimal energy efficient system is a massive MIMO setup with optimal values of P and T_t . Moreover, if the quality of channel estimation is low, the number of BS antennas should increase to achieve the global optimum EE. By applying the energy-efficient power allocation scheme provides an improvement in the EE of SLNR-PS. Under the constraints of transmit power, QoS and EE target, the rate profile optimization problem for individual users has been formulated and solved successfully. The Pareto optimal per-user rates have been demonstrated.

Chapter 5

Sum-Rate Optimization for Multicell Massive MIMO Systems

Antenna downtilting in the three-dimensional MIMO (3D-MIMO) systems has been presented to be a potential beamforming approach to reduce ICI which leads to increase in the network sum rate [57, 117–120]. While an optimized fixed tilt for regulating power level of the ICI in the conventional systems can be done by the mechanical antenna tilting with on-site visit [119], the 3D-MIMO systems can remotely change the antenna tilt angles based on electrical tilting. With coordinated tilt adaptation (CTA), the BSs can jointly adapt tilt angles based on the locations of the active users in order to optimize the network performance. In [57] and [119], the CTA has been studied for a single-user scenario in which each BS serves only one user in its own cell, and the number of BS antennas is small, i.e., $N = 4$. In [120], the downtilt adjustment, resource assignment and power allocation have been jointly optimized based on the joint processing coordinated multipoint (JPCoMP) using dual decomposition method for MIMO OFDM systems.

In massive MIMO systems, as the BS has a large number of antennas, i.e., $N \rightarrow \infty$, the

SINR is deterministic for given users' locations and large-scale fading coefficients, named as the users' location information (ULI). The system performance significantly depends on the pilot contamination which is caused by pilot reuse in the neighbouring cells, especially at the cell-edge users. To reduce this negative effect, the blind method based on subspace partitioning, the coordinated multicell precoding, as well as the adaptive pilot allocation mechanism design, which have been mentioned in Section 1.3 of Chapter 1, have been studied in conjunction with pilot contamination [19–23].

In this chapter, we present a two-layer scheme, which consists of the optimal tilt adaptation based on the users' locations and the OPA based on the game theory, to reduce the effect of pilot contamination for cell-edge users. We extend the coordinated tilt adaptation, which has been proposed in [57] and [119], to the scenario in which multiple active users are served in each cell when $N \rightarrow \infty$. The key idea is that each interference-aware user will measure the level of ICI and feed back this information to its connected BS in each cell. Given the strategies (i.e., tilt angles and power allocations) from other BSs reflected by the ICI, each BS selfishly adjusts its strategy to maximize the sum rate for its connected users. The multicell system is said to be in competition since all BSs are competing with each other to obtain their own optimal strategies in a noncooperative game [121–124]. A distributed algorithm is presented to compute the equilibrium state of the game. The computational complexity of the proposed scheme is also provided.

5.1 System Model Description

We assume that a cellular network includes L hexagonal cells, in which the BS is equipped with a large number of N antennas. Each cell is partitioned into cell-center and cell-edge regions by radius¹ r_1 as shown in Fig. 5.1. We also assume that the users are uniformly distributed

¹Note that all the notations are only applicable to this chapter.

over the cell area. We define r_1 such that the areas of 2 regions are equal, i.e., the average numbers of users in 2 regions are equal. It is expressed as [125]

$$\frac{\pi(r_1^2 - r_0^2)}{3} = \frac{\pi(r_h^2 - r_1^2)}{3} - 2r_h^2 \left(\frac{\pi}{6} - \frac{\sqrt{3}}{4} \right), \quad (5.1)$$

where r_h is the distance from the BS to one of the hexagon vertices and r_0 is the minimum distance between the BS and the user. Thus, we have $r_1 = \sqrt{\frac{0.827r_h^2 + r_0^2}{2}}$. Note that we focus on improving the system performance when the BS is assumed to serve K single-antenna cell-edge users in this chapter. To do so, we assume that the same radio resource and K orthogonal pilot sequences are used to serve the cell-edge users among all cells. The BS height h_{BS} is assumed to be much greater than the large-scale clutter and the user height h_u . Thus, the product of a large-scale fading factor and a small-scale fading factor is adopted to describe the property of the user's channel [117]. We model the channel vector $\mathbf{h}_{ik}^l$ between the l th BS and the k th user of the i th cell as [57]

$$\mathbf{h}_{ik}^l = \sqrt{\eta_{ik}^l(\theta_{tilt}^l)} \mathbf{z}_{ik}^l, \quad (5.2)$$

where $\mathbf{z}_{ik}^l \sim \mathcal{CN}(\mathbf{0}_{N \times 1}, \mathbf{I}_{N \times N})$ is the i.i.d. small-scale fading vector. The large-scale fading factor $\eta_{ik}^l(\theta_{tilt}^l)$ is obtained as

$$\eta_{ik}^l(\theta_{tilt}^l) = \beta_{ik}^l (d_{ik}^l)^{-v} g_{ik}^l(\theta_{tilt}^l, \theta_{ik}^l, \phi_{ik}^l), \quad (5.3)$$

where β_{ik}^l is the shadowing coefficient and $(d_{ik}^l)^{-v}$ is the distance path-loss with path-loss exponent v . In this section, we do not consider any specific statistical properties of β_{ik}^l , thus β_{ik}^l is assumed to be known. The 3D antenna gain of BS l at user k of the i th cell is defined as [57]

$$g_{ik}^l(\theta_{tilt}^l, \theta_{ik}^l, \phi_{ik}^l) = 10^{\frac{G_{max}}{10} - 1.2 \left(\frac{\theta_{ik}^l - \theta_{tilt}^l}{\theta_{3dB}} \right)^2 - 1.2 \left(\frac{\phi_{ik}^l - \phi_{BS}^l}{\phi_{3dB}} \right)^2}, \quad (5.4)$$

where θ_{tilt}^l is the tilt angle of BS l , ϕ_{BS}^l is the fixed orientation of the BS l th array boresight relative to the x -axis, θ_{ik}^l and ϕ_{ik}^l denote the incident angles connecting user k to the BS l in the elevation and the azimuth domains, respectively. Since $h_u \ll h_{BS}$, we have $\theta_{ik}^l \approx \tan^{-1}(\frac{h_{BS}}{d_{ik}^l})$. The maximum antenna gain is $G_{\max}$, θ_{3dB} and ϕ_{3dB} are the half-power beam-widths in the elevation and the azimuth patterns, respectively. In the i th cell, the received signal of the k th user is obtained as [19, Eq. (5)]

$$y_{ik} = \sum_{l=1}^L \sum_{k'=1}^K \sqrt{p_{lk'} \eta_{ik}^l (\theta_{tilt}^l)} (\mathbf{h}_{ik}^l)^H \mathbf{w}_{lk'} s_{lk'} + n_{ik}, \quad (5.5)$$

where $p_{lk'}$ is transmit power, $\mathbf{w}_{lk'}$ is the precoding vector, $s_{lk'}$ is data symbol, and n_{ik} is noise term with $n_{ik} \sim \mathcal{CN}(0, \sigma_n^2)$. Without loss of generality, the noise variance is assumed to be equal to 1 in order to simplify notation [6, 9, 13, 19]. With this convention, $P_i = \sum_{k=1}^K p_{ik}$ has the interpretation of normalized transmit SNR in the downlink channel of the i th cell, hence it is dimensionless. By adopting the MF precoding, when $N \rightarrow \infty$, the SINR of the k th user in the i th cell converges to the deterministic value which is defined as [19]

$$\gamma_{ik}(\boldsymbol{\Theta}_k) = \frac{p_{ik}(\eta_{ik}^i(\theta_{tilt}^i))^2}{\sum_{l=1, l \neq i}^L p_{lk}(\eta_{ik}^l(\theta_{tilt}^l))^2}, \quad (5.6)$$

where $\boldsymbol{\Theta}_k = \{\theta_{tilt}^1, \dots, \theta_{tilt}^L\}$ is the vector of tilt angles of all BS cells. Clearly, as $N \rightarrow \infty$, the intracell interference can be negligible and only the users using the same pilot sequence in the neighbour cells can create the ICI to user k . Hence, the vector $\boldsymbol{\Theta}_k$ is considered as the tilt angles for the k th pilot sequence in the network. We formulate the problem of sum rate optimization as

$$\begin{aligned} & \underset{\mathbf{p}, \boldsymbol{\Theta}}{\text{maximize}} \quad R_s = \sum_{i=1}^L \sum_{k=1}^K \log_2 (1 + \gamma_{ik}(\boldsymbol{\Theta}_k)) \\ & \text{subject to} \quad \sum_{k=1}^K p_{ik} \leq P_i, \quad \boldsymbol{\Theta}_{\min} \leq \boldsymbol{\Theta}_k \leq \boldsymbol{\Theta}_{\max}, \end{aligned} \quad (5.7)$$

Figure 5.1: An example of a 3-cell network for MIMO systems.

where $\Theta_{\min}$ and $\Theta_{\max}$ denote the vector of minimum and maximum allowed tilt angles, respectively, $\mathbf{p} = (\mathbf{p}_1, \dots, \mathbf{p}_L)$ with $\mathbf{p}_i = \{p_{ik}\}_{k=1}^K$ denotes the set of all the users power allocation vectors of all L cells, and $\Theta = (\Theta_1, \dots, \Theta_K)$ is the set of all vectors of tilt angles for K pilot sequences. The JPCoMP method can be used to solve problem (5.7) by using the Lagrangian dual multipliers [120]. However, an cell-edge user specific downtilt is applied in [120], which does not consider the locations of the cell-edge users and the effect of fading coefficients. With a different approach, we next propose a scheme to solve (5.7) efficiently.

5.2 A Two-Layer Optimization Scheme

We solve the problem (5.7) using two main steps. We first optimize the vectors of tilt angles for each pilot sequence based on ULI when EPA is initialized in all L cells. Based on game

theory approach, we define the OPA strategies for all BSs to maximize the network sum rate.

Optimal tilt adaptation: We assume that the BS can easily acquire the ULI. It is possible because the large-scale fading factor changes slowly and it can be easily estimated by the BS, and the users are assumed to be equipped with global positioning systems in the future systems [19]. For any given ULIs, the BSs can employ the CTA to maximize their sum rate. Specifically, for the k th pilot sequence, we can formulate the following problem [57]

$$\mathbf{\Theta}_k^* = \arg \max_{\mathbf{\Theta}_{\min} \leq \mathbf{\Theta}_k \leq \mathbf{\Theta}_{\max}} \sum_{i=1}^L R_{i,k}(\mathbf{\Theta}_k), \quad (5.8)$$

where $R_{i,k}(\mathbf{\Theta}_k) = \log_2(1 + \gamma_{ik}(\mathbf{\Theta}_k))$. The problem (5.8) can be solved by applying the DIviding RECTangle (DIRECT) method presented in [57]. However, the DIRECT method performs slowly for the case of delay-sensitive scenario. By exploring the nature of the problem, to solve (5.8), we modify the effective method in [119] which considers the case that the BS is able to cancel the ICI (i.e., cooperative multicell precoding) from other cells. Moreover, in [57] and [119], the authors mainly focused on the case of single-user scenario when N is small. Herein, we obtain the optimal tilt angle for the case of multi-user scenario when N is very large. We observe that if the ICI is successfully eliminated, the optimal tilt angle for each BS is the angle of the direct pointing of main beam of the BS antennas towards its own active user location in the vertical plane, i.e., $\mathbf{\Theta}_k^* = \{\theta_{1k}^1, \dots, \theta_{Lk}^L\}$. If the ICI is available, for any given BS i , the desired signal power is decreased and the ICI at the neighbouring cells is increased if the θ_{tilt}^i decreases below θ_{ik}^i . Different from [57], for each realization of the active user location, we can initialize $\mathbf{\Theta}_k^* = \{\theta_{1k}^1, \dots, \theta_{Lk}^L\}$ instead of $\mathbf{\Theta}_k^* = \mathbf{\Theta}_{\min}$. With a step size of 1° , if the increase of each BS tilt results in an increase of the sum rate $\sum_{i=1}^L R_{i,k}(\mathbf{\Theta}_k^*)$, it is kept, otherwise, it is ignored. The optimal solution of (5.8) is found when an increase of tilt angle at any of the BSs cannot result in any increase of the sum rate anymore. It has been shown that, with step size of 1° , the obtained tilt angle can achieve reasonable performance as compared to the

solution defined by using the DIRECT method [119]. In this sub-problem, the sequence of BSs is obtained corresponding to the sorted initialization value of the tilt angles in the ascending order to avoid any bias towards a particular cell. Similar approach is used to determine the optimal vectors of the tilt angles for other pilot sequences. Thus, a set of all optimal vector tilt angles Θ^* for all K pilot sequences can be obtained based on the ULI.

Game formulation: The power allocation strategy (PAS) of the neighbouring cells can influence the ICI in the target cell. It is assumed that each user can measure the level of the ICI and feed back this information to its connected BS perfectly [121]. For given PASs from other BSs reflected by the ICI, each BS selfishly adjusts its own PAS to optimize the sum rate for its corresponding users subject to the transmit power constraint. Since each BS competes against each other by optimizing the PAS, the multicell system is said to be in a competition. Naturally, it can be considered as a strategic noncooperative game, where the BSs are the rational players [121, 123, 124]. All the channels are assumed to be changed slowly such that they are considered to be fixed during the game being played. Let us define $\Omega \triangleq \{1, \dots, L\}$ as the set of all players, $\mathcal{P}_i$ is the set of admissible PAS of player i , i.e., [124]

$$\mathcal{P}_i \triangleq \{\mathbf{p}_i \in \mathbb{R}_+^K : \sum_{k=1}^K p_{ik} \leq P_i\}, \quad (5.9)$$

and $R_i(\mathbf{p}_i, \mathbf{p}_{-i}, \Theta^*)$ is the payoff function of player i , i.e.,

$$R_i(\mathbf{p}_i, \mathbf{p}_{-i}, \Theta^*) = \sum_{k=1}^K \log_2 (1 + \gamma_{ik}(\Theta_k^*)), \quad (5.10)$$

where $\mathbf{p}_{-i} = (\mathbf{p}_l)_{l \neq i}$ denotes the PASs of all players, except the i th player. The corresponding

game can be formulated as

$$(\mathcal{G}) \begin{cases} \underset{\mathbf{p}_i}{\text{maximize}} & R_i(\mathbf{p}_i, \mathbf{p}_{-i}, \boldsymbol{\Theta}^*) \\ \text{subject to} & \mathbf{p}_i \in \mathcal{P}_i, \end{cases} \quad \text{for } \forall i \in \Omega. \quad (5.11)$$

In the game, if any player changes its PAS, each user needs to report the level of the updated ICI back to its connected player accordingly. Thus, the game $\mathcal{G}$ shows that the optimal PAS at each player is dependent on the PASs of other players. However, each player makes its decision to obtain the optimal PAS independently with others because it only uses the local information between itself and its corresponding users, i.e., the level of the ICI and the ULI. Hence, the game $\mathcal{G}$ is implemented in a fully distributed manner without any requirement of signalling exchanges among the players [121]. The competition among the players is done whenever a pure Nash equilibrium (NE) of the game $\mathcal{G}$ is approached, expressed as

$$R_i(\mathbf{p}_i^*, \mathbf{p}_{-i}^*, \boldsymbol{\Theta}^*) \geq R_i(\mathbf{p}_i, \mathbf{p}_{-i}^*, \boldsymbol{\Theta}^*), \forall \mathbf{p}_i \in \mathcal{P}_i, \forall i \in \Omega. \quad (5.12)$$

At the NE, a single player does not have the incentive to unilaterally change its PAS for given PASs of other players. The solution of $\mathcal{G}$ is always non-empty [123]. To reach the NE of $\mathcal{G}$, we adopt a smoothed simultaneous iterative waterfilling algorithm (SSIWA) which is an instance of the Jacobi scheme. The SSIWA is based on the waterfilling (WF) solution, defined as

$$p_{ik}^* = [\text{WF}_i(\mathbf{p}_{-i})]_k = \left[\frac{\sum_{l \neq i}^L p_{lk}(\eta_{ik}^l(\theta_{tilt}^{*l}))^2}{\alpha_i \ln 2} - \frac{\sum_{l \neq i}^L p_{lk}(\eta_{ik}^l(\theta_{tilt}^{*l}))^2}{(\eta_{ik}^i(\theta_{tilt}^{*i}))^2} \right]_0^{p_{ik}^{\max}}, \quad (5.13)$$

where $\text{WF}_i(\cdot)$ is waterfilling operator, $p_{ik}^{\max}$ is the maximum power allowed to be allocated to user k [122, 123], and $[x]_a^b$ is the Euclidean projection of x (defined as $[x]_a^b = a$ if $x \leq a$, $[x]_a^b = b$ if $x \geq b$, and $[x]_a^b = x$ if $a < x < b$). The value of α_i is selected to satisfy the power constraint $\sum_{k=1}^K p_{ik}^* = P_i$. In the SSIWA, at each iteration, by performing the waterfilling

solution in (5.13), each player updates its own power allocation simultaneously based on its local information (i.e., the ICI and the ULI) obtained in the previous iteration. Without any communication between the players, all players are self-enforcing to define their optimal PAS at the same time. The SSIWA can guarantee that the network converges globally to an NE, at which point the OPA for the users of each BS is defined as $\mathbf{p}_i^* = \text{WF}_i(\mathbf{p}_{-i}^*), \forall i \in \Omega$ [123]. Thanks to the Jacobi-based update, the SSIWA performs efficiently even if the number of players is large [124]. Moreover, the uniqueness of the NE of $\mathcal{G}$ can be deduced straightforwardly from the work in [124] according to the equivalence between game $\mathcal{G}$ and the variational inequality problem. The condition guaranteeing the uniqueness of the NE is that all the matrices $\Psi(k) \in \mathbb{C}^{L \times L}$ are positive definite. The entries of $\Psi(k)$ are defined as

$$[\Psi(k)]_{i,l} = \frac{(\eta_{ik}^l(\theta_{tilt}^{*l}))^2}{(\eta_{ik}^i(\theta_{tilt}^{*i}))^2}. \quad (5.14)$$

As mentioned earlier, as $N \rightarrow \infty$, the intracell interference can be negligible. Since the ICI is assumed to be significantly small (i.e., the network becomes ideal because there is no multi-user interference), the rates in (5.10) are decoupled. Therefore, the sum rate maximization problem in (5.11) provides a unique solution for each user [124]. As a result, on the physical interpretation of this condition, the Nash equilibrium is unique if the ICI among the users is significantly small [122–124].

In summary, our proposed scheme is given in Table 5.1. The tilt adaptation processes are shown from step 1 to step 12, whereas the SSIWA processes are shown in the remaining steps. In the worst case, the computational complexity of the tilt adaptation is on the order of $O(KLQ_1)$, where $Q_1 = \lceil \theta_{\max} - \min\{\theta_{ik}^i\}_{i=1}^L \rceil$ ($\lceil \cdot \rceil$ denotes the round number), that is much lower than $O(KLQ_2^L \log(Q_2))$, where $Q_2 = \lceil \theta_{\max} - \theta_{\min} \rceil$ and $Q_2 \geq Q_1$, by using the exhaustive search approach. The complexity of the SSIWA is on the order of $O(LMK^2)$, thus the total complexity of the proposed scheme is on the order of $O(LKQ_1 + LMK^2)$.

Table 5.1: Algorithm for maximizing network sum rate

Input:	$N, L, K, P_i, \eta_{ik}^i(\theta_{tilt}^i), \forall i \in \Omega$
Output:	$\Theta^*, \mathbf{p}^*, R_s^*$
1.	For $k \leftarrow 1$ to K do
2.	Set $\mathbf{p} \leftarrow \text{EPA}$; $\Theta_k^* \leftarrow \{\theta_{1k}^1, \dots, \theta_{Lk}^L\}$; $\bar{R}_k \leftarrow \sum_{i=1}^L R_{i,k}(\Theta_k^*)$
3.	$t \leftarrow 1$; id $\leftarrow$ index of sorted Θ_k^* in ascending order
4.	Repeat
5.	$c \leftarrow 0$
6.	For all $j \in \text{id}$ do
7.	$\Theta_k \leftarrow \Theta_k^*$; $\Theta_k(j) \leftarrow \Theta_k(j) + 1$; $\bar{R}_k^{tem} \leftarrow \sum_{i=1}^L R_{i,k}(\Theta_k)$
8.	If $\bar{R}_k^{tem} > \bar{R}_k$ then $\Theta_k^* \leftarrow \Theta_k$; $\bar{R}_k \leftarrow \bar{R}_k^{tem}$
9.	Else do $c \leftarrow c + 1$; $t \leftarrow (c \bmod L)$
10.	End for
11.	Until $t = 0$
12.	End for
13.	Reset $\mathbf{p}_i^{(0)} \leftarrow \text{WF}(\mathbf{p}_{-i}^{(0)})$, with any feasible power $\mathbf{p}_{-i}^{(0)} \geq 0, \forall i \in \Omega$
14.	$M \leftarrow$ number of iteration; $\tau_i \in [0, 1)$
15.	For $m \leftarrow 1$ to M do
16.	$\mathbf{p}_i^{(m+1)} \leftarrow \tau_i \mathbf{p}_i^{(m)} + (1 - \tau_i) \text{WF}_i(\mathbf{p}_{-i}^{(m)}), \forall i \in \Omega$
17.	Each user reports the level of the ICI back to its connected BS
18.	End for

5.3 Numerical Results and Discussions

The performance of the proposed scheme is studied based on the numerical results presented in this section. The values of parameters for Monte Carlo simulation are set as $\theta_{3\text{dB}} = 7^\circ$, $\phi_{3\text{dB}} = 70^\circ$, $\phi_{BS}^l = 0^\circ$, $G_{\text{max}} = 20$ dBi, $\Theta_{\text{min}} = 0^\circ \mathbf{1}$, $\Theta_{\text{max}} = 90^\circ \mathbf{1}$, $L = 3$ cells, $r_0 = 40$ m, $h_{BS} = 32$ m, $h_u = 1.5$ m, $v = 3$, and $K = 10$ cell-edge users [57]. For a given ULI, ϕ_{ik}^l is assumed to be known at the BSs. We assume that each BS has $P = 10$ dB and no mask constraint is considered (i.e., $p_{ik}^{\text{max}} = +\infty$ for $\forall i, k$); $\beta_{ik}^i = 1, \forall i \in \Omega$ and $\beta_{ik}^l \in (0, 1)$ with $l \neq i$. The inter-cell distance is $D = 500$ m, then we have $r_h = \frac{D}{2 \cos(\pi/6)} \approx 288$ m and $r_1 \approx 187$ m.

Figure 5.2 presents the comparisons of sum rate performance for different systems. The curves labelled “Cell 1 sum-rate”, “Cell 2 sum-rate”, “Cell 3 sum-rate”, and “Network sum-rate (proposed)” refer to the sum rates corresponding to cell 1, cell 2, cell 3, and the network, respectively, when the proposed scheme is applied. The curve labelled “Network sum-rate (fixed

Figure 5.2: Comparisons of sum rate performance for different systems with $N \rightarrow \infty$ and $K = 10$.

tilt + EPA)” refers to the network sum rate with fixed tilt and using EPA, named as system A, whereas the curve labelled “Network sum-rate (fixed tilt + WF)” refers to the network sum rate with fixed tilt and using classical waterfilling optimal power allocation, named as system B. In this example, we set a fixed tilt angle of 7.6° , which is defined as $\theta_{tilt}^{fix} \approx \tan^{-1}(\frac{2h_{BS}}{r_h + r_1})$. The network sum rate and the sum rate at each cell are then plotted after each iteration. Each plotted point is obtained by averaging over 100,000 independent channel realizations. Observing from Fig. 5.2, by applying the proposed scheme, the network sum rate and sum rates of cells converge very quickly after a few iterations. Statistically, the sum rates of all cells match well. As expected, the proposed scheme results in a higher network sum rate over both systems A and B. Specifically, the proposed scheme provides an improvement in network sum rate of 4.4% and 2.1%, as compared to the corresponding system A and system B, respectively. Furthermore, in Fig. 5.3, we investigate performance of the proposed scheme when N is finite. Herein, we set $M = 30$ and $K = 5$. As compared with the system A, the proposed scheme can achieve an

Figure 5.3: Sum rate comparisons for different systems with $K = 5$ and $M = 30$.

improvement in network sum rate of 2.4%, 2.9%, and 3.9% corresponding to $N = 100, 250$, and 500, respectively. These results further verify the effectiveness of the proposed scheme.

5.4 Summary

We have proposed a scheme, which consists of a two-layer approach of the optimal tilt adaptation based on the ULI and the optimal power allocation based on game theory approach, to reduce the effect of pilot contamination, and hence to maximize the network sum rate in this chapter. The proposed scheme has low computational complexity because it only uses the channel statistics (i.e., large-scale fading coefficients) when N is very large. These parameters can be effectively estimated by the BS because they change slowly. Numerical results have shown that the convergence of network sum rate is very quick after a few iterations and the proposed scheme outperforms the conventional systems. Moreover, a heavy signalling exchange among the BSs can be avoided by applying the distributed algorithm.

Chapter 6

Massive MIMO Heterogeneous Networks

The deployment of small-cells is considered as a practical solution to achieve high spectral efficiency per unit area based on spatial reuse, coverage area, and cell-edge users. The performance of conventional HetNets with single-antenna BSs has been well studied in the literature [11, 66, 126–130]. In massive MIMO HetNets, the aggregate interference environment is more heterogeneous and complicated than that of the conventional HetNets. As a result, it is challenging to evaluate the network performance. There are two main methods to investigate the performance of HetNets with massive MIMO. The first one uses the results of the random matrix theory to convert channel coefficients into deterministic values in order to simplify the analysis significantly [71, 131, 132]. The interference management in 5G reverse time-division duplexing two-tier HetNets has been studied in [71] when the number of base station antennas and the network size (macro users and small-cell access points) are assumed to be large with a fixed ratio. In [131], due to high directivity of the channel vectors in the massive MIMO systems, the inter-tier interference (ITI) coordination through spatial blanking (i.e., the macro-cell BS only concentrates transmission energy in a particular direction, whereas transmission

opportunities for its own small-cells are explored in the other directions) has been considered for sum rate improvement. In [132], an optimal beamforming vector is achieved by minimizing power consumption while satisfying QoS constraint. However, the first method is unable to consider the effects of the BS locations and the small-cell deployment scenarios on the cellular network performance. An interesting question is whether it is possible to improve network performance, such as SE and EE, by deploying small-cells, and how the density of the SBSs impacts on the network performance.

On the other hand, the second method focuses mainly on the network geometry, where both MBSs and SBSs are modelled as independent PPPs [66,126]. By using the stochastic geometry concept, the performance metrics can be defined by averaging over all network deployment scenarios and cell sizes, such as coverage probability and average rate. This method can lead to closed-form solutions for several specific choices of parameters in the conventional HetNets [127,128]. For massive MIMO heterogeneous networks, the SE and EE of L -tier HetNets have been recently investigated in [69]. When users and small-cells are assumed to be distributed as two independent PPPs, a lower bound on the uplink SE for a two-tier massive MIMO HetNet has been derived in [133]. However, the problems of SE maximization and EE maximization have not been considered in [69] and [133]. In addition, a distributed user association scheme has been presented to maximize the EE in [134].

Moreover, the performance of HetNets strongly depends on the ITI. The CoMP between MBS and its own SBSs plays an important role to limit the ITI, hence it can exploit the benefits of HetNets. In [135], under the assumption of ideal backhaul links, the operators can select n BSs with the strongest average received signal for joint data transmission. In [136], a location-aware cross-tier CoMP transmission scheme in two-tier HetNets has been presented to improve the performance of the users with high ITI due to the co-channel deployment of small-cells. Moreover, in [137], by managing the ITI to the offloaded users, an interference

nulling scheme has been proposed to improve system performance in two-tier HetNets with multi-antenna BSs.

From practical view points, due to system constraints, the small-cells can only be assigned in certain regions to improve the poor macrocell coverage areas. Hence, in [127], the authors have presented a non-uniform small-cell network (SCN) development, where the SBSs are not utilized in the region within a given distance away from any MBSs. By doing so, it has been shown that the presented scheme can achieve the same level of coverage performance as compared to the uniform SCN development, even when its SBSs is 50% less than that of the uniform SCN development. This aspect provides an ecological and economic solution to develop small-cell networks while satisfying the coverage performance. Moreover, in [128], the conventional CoMP with the non-uniform SBS deployment can improve the coverage performance by 5% for users located outside a prescribed distance from any MBSs (i.e., macrocell edge users), as compared to the maximum received power (MRP) association scheme. However, the analysis of CoMP for massive MIMO HetNets with the non-uniform SBS deployment has not been studied in the literature by using the tools from stochastic geometry. Motivated by the results in [127] and [128], we propose a novel scheme to improve coverage, SE and EE performance for next-generation cellular networks because it can enjoy the benefits of the combinational use of massive MIMO, the non-uniform SCN development, as well as the joint transmission.

In this chapter, In order to enhance the coverage and capacity, massive MIMO is assumed to be applied for the MBS, instead of considering single-antenna BS as presented in [127, 128]. Based on the stochastic geometry approach, we first analyze the average achievable rate and the area spectral efficiency for CoMP in the two-tier HetNet with massive MIMO. We then investigate the EE maximization problem under the constraints of QoS, density of SBS, available number of MBS antennas, as well as MBS transmit power. An alternating optimization algorithm is applied to obtain optimal solution of the EE optimization problem. It provides

Figure 6.1: An example of a two-tier HetNet model with one MBS.

essential insights on the relation between EE performance and these design parameters. To simplify the complexity of analysis, the interference distribution is approximated by moment matching with the Gamma distribution, which is used to analyze the coverage probability. Moreover, numerical results are provided to investigate the SE and EE for the two-tier Hetnet and the effectiveness of the algorithm.

6.1 System Model Description

We consider the downlink transmission in a two-tier HetNet. The first tier includes MBSs, while the second tier includes SBSs. We assume that the locations of MBSs and SBSs are modelled according to two-dimensional homogeneous PPPs¹ $\Phi_m \in \mathbb{R}^2$ and $\Phi_s \in \mathbb{R}^2$ with spatial

¹Note that all the notations are only applicable to this Chapter.

Figure 6.2: An example of close-up 3×3 km view for a two-tier HetNet when $\lambda_1 = (500^2\pi)^{-1}$, $\lambda_2 = 5\lambda_1$ and $R = 400$ m.

intensities λ_1 and λ_2 , respectively [129, 136]. The transmit powers of BSs are the same in each tier. We employ a non-uniform small-cell deployment scheme. In particular, we define the inner region as the union of locations within a predetermined distance R from any MBSs, i.e., $\mathcal{A}_{\text{in}} = \bigcup_{x \in \Phi_m} B(x, R)$, whereas the outer region is defined as the union of locations in which the distances to any MBSs are greater than R , i.e., $\mathcal{A}_{\text{out}} = \mathbb{R}^2 \setminus \mathcal{A}_{\text{in}}$. In order to optimize the macrocell coverage and achieve an energy-efficient solution, we only use the SBSs located in the $\mathcal{A}_{\text{out}}$, while the ones in the $\mathcal{A}_{\text{in}}$ are not activated [127, 128]. This deployment of SBSs is similar to that of Scenario I which has been presented in [127]. The example of one macrocell is presented in Fig. 6.1. An example of close-up view for the two-tier HetNet with non-uniform small-cell deployment is illustrated in Fig. 6.2. The coverage area of each MBS is bounded by the solid lines and macrocell boundaries correspond a Voronoi tessellation. The stochastic geometry is

a suitable approach to generate such random locations of the MBSs and SBSs. Moreover, we assume that the user density is much greater than that of BSs, hence there is always an active user in every SBS at each time slot and multiple active users in every MBS [138]. If there are several active users in the small-cells, multiple-access techniques can be applied at the SBSs. It is assumed that each MBS is equipped with a large number of (N) antennas and employs ZF precoding to serve at least K users simultaneously, whereas each single-antenna SBS only serves a single-antenna user at each time slot. This assumption has been used in [69, 138] and [139]. For a low density of users, the numbers of active users in different cells can be considered as independent of each other [140]. Therefore, an aggregate interference generated by the loaded BSs (i.e., the BSs have at least one connected user) and a cell association scheme can be more complicated in the two-tier HetNet with the non-uniformly small-cell development. We will consider the study of this issue as future works.

6.1.1 Cell Association and Coordination Scheme

We assume that the channels experience i.i.d. quasi-static Rayleigh fading, and the typical user o is located at the origin [126]. The standard path loss coefficient is defined as $(r)^{-\alpha}$, where α is the path loss exponent and $\alpha > 2$ [11, 127–129]. Under assumption of EPA, the long-term average receive power at the typical user associated with the l th MBS is defined as [69]

$$p_{o,l} = \frac{P_m}{K} \rho_a x_l^{-\alpha}, \quad (6.1)$$

where P_m is the transmit power of MBS, x_l is the distance between the typical user (i.e., the origin) and the l th MBS, and ρ_a is the array gain. For given K users and ZF beamforming transmission, the value of array gain is given by $\rho_a = N - K + 1$ [11, 69, 134, 139, 141]. When

Table 6.1: Summary of notations

Notation	Definition
Φ_m, Φ_s	PPPs modelling locations for MBSs and SBSs
λ_1, λ_2	Spatial densities of MBSs and SBSs
P_m, P_2	Transmit powers of MBSs and SBSs
$\Delta_m, \Delta_c, \Delta_s$	Association probabilities of the typical user in 3 cases*
τ_m, τ_c, τ_s	Average achievable rates of the typical user in 3 cases*
$P_{co}^m, P_{co}^c, P_{co}^s$	Coverage probabilities of the typical user in 3 cases*
x_l, y_j	Distances between the typical user and the l th MBS, the typical user and the j th SBS
$h_{o,l}, g_{o,j}$	Fading channel power gains between the typical user and the l th MBS, the typical user and the j th SBS
$\mathbf{h}_{k,x_l}, \mathbf{w}_{k,x_l}$	Small-scale Rayleigh channel fading and precoding vector of the k th user from the l th MBS
R, η_{EE}	Radius of inner region, energy efficiency
N, K	Number of MBS antennas, number of macrocell users
μ, T	Operation and signal-to-interference ratio thresholds
S_i, I_i	Desired signal and aggregate interference powers of the typical user in each association case*, i.e., $i \in \{m, c, s\}$

*The association cases of the typical user are given by (6.3).

the typical user associates with the j th SBS, its long-term average receive power is defined as

$$p_{o,j} = P_2 y_j^{-\alpha}, \quad (6.2)$$

where P_2 is the transmit power of SBS, y_j is the distance between the typical user and the j th SBS.

We denote MBS_0 and SBS_0 as the nearest BSs of the typical user in the first tier and the second tier. We also define x_0 and y_0 as the minimum distances between the typical user and the MBS_0 and SBS_0 , respectively. Firstly, if the typical user is located in $\mathcal{A}_{\text{in}}$ of MBS_0 , we assume that it only associates with MBS_0 . Secondly, if the typical user is located in $\mathcal{A}_{\text{out}}$, but the received signal power from SBS_0 is comparable to that from MBS_0 , thus both MBS_0 and

SBS₀ will cooperate to jointly transmit data to the typical user. This cooperation mode can improve the performance of users in the outer region. Thirdly, if the typical user is located in $\mathcal{A}_{\text{out}}$ of MBS₀ and the received signal power from SBS₀ is at least μ times greater than that from MBS₀, then the user only associates with the SBS₀. The MBS employs ZF precoding. Hence, the connected BS set of the typical user is summarized as [128]

$$BS = \begin{cases} \{\text{MBS}_0\}, & \text{if } x_0 \leq R, \\ \{\text{MBS}_0, \text{SBS}_0\}, & \text{if } x_0 > R \& \frac{P_2 y_0^{-\alpha}}{P_1 x_0^{-\alpha}} < \mu, \\ \{\text{SBS}_0\}, & \text{if } x_0 > R \& \frac{P_2 y_0^{-\alpha}}{P_1 x_0^{-\alpha}} \geq \mu, \end{cases} \quad (6.3)$$

where $P_1 = \frac{P_m}{K}(N - K + 1)$ and μ is the operation threshold with $\mu \geq 0$ dB. Note that, the coordination mode in the proposed scheme is a non-coherent joint transmission (NCJT) because only data of the typical user is shared among the MBS₀ and SBS₀, a distributed precoding vector is performed at the MBS₀, and the received signals at the typical user are non-coherently combined [142, 143]. Hence, the network overheads required by cell association and coordination scheme are limited. By using high-speed optical fiber for backhaul signalling, these requirements can be effectively accommodated.

Remark 1: For given P_m and P_2 , as $N \rightarrow \infty$, we have $\frac{P_2}{P_1} \rightarrow 0$. This implies that the typical user is only served by $\{\text{MBS}_0\}$ or $\{\text{MBS}_0, \text{SBS}_0\}$. Moreover, as $R \rightarrow \infty$, the network tends to be a conventional single-tier cellular network with only MBSs, thus it cannot enjoy the benefits of improving throughput and coverage area by employing the small-cell deployment. In this chapter, we focus on analyzing the network performance with finite values of N and R .

6.1.2 Signal-to-Interference-Plus-Noise Ratio

Next, we define the SINR of the typical user in different modes of operation. We denote $\mathbf{h}_{k,x_l}$ as the small-scale Rayleigh channel fading between the k th user and the l th MBS located

at x_l with $\mathcal{CN}(0, 1)$ entries, and $\mathbf{w}_{k,x_l}$ is assumed to be the corresponding ZF precoding vector of the k th user. If the typical user o at the origin is served by the MBS_0 , named as case 1, its received SINR is obtained as [137, 141]

$$\gamma_m = \frac{P_m |\mathbf{h}_{o,x_0}^H \mathbf{w}_{o,x_0}|^2 x_0^{-\alpha}}{K(I_m + \sigma^2)}, \quad (6.4)$$

where the noise power is σ^2 . The aggregate interference power is given by $I_m = I_{m \rightarrow m} + I_{s \rightarrow m}$, where

$$I_{m \rightarrow m} = \sum_{l \in \Phi_m \setminus \text{MBS}_0} \sum_{k=1}^K \frac{P_m}{K} |\mathbf{h}_{o,x_l}^H \mathbf{w}_{k,x_l}|^2 x_l^{-\alpha}, \quad (6.5)$$

$$I_{s \rightarrow m} = \sum_{j \in \Phi_s} P_2 |h_{o,y_j}|^2 y_j^{-\alpha}, \quad (6.6)$$

and $h_{o,y_j} \sim \mathcal{CN}(0, 1)$ is the small-scale channel fading from the j th SBS. We denote $h_{o,0} = |\mathbf{h}_{o,x_0}^H \mathbf{w}_{o,x_0}|^2$, $h_{o,l} = \sum_{k=1}^K |\mathbf{h}_{o,x_l}^H \mathbf{w}_{k,x_l}|^2$ and $g_{o,j} = |h_{o,y_j}|^2$ as the corresponding fading channel power gain between the typical user and MBS_0 , the fading interfering channel power gains between it and the $\text{MBS } l \in \Phi_m \setminus \text{MBS}_0$, as well as between it and the SBS, respectively. Under the Rayleigh fading assumption, it has been shown in [67] and [141] that the channel power distributions of both direct and interfering links follow the Gamma distribution², i.e., $h_{o,0} \sim \Gamma(N - K + 1, 1)$ and $h_{o,l} \sim \Gamma(K, 1)$. For single-input single-output transmission, there is no precoding, thus the uplink and downlink channel gains are the same. Therefore, both channel gains follow exponential distribution with unit rate parameter, hence $g_{o,j} \sim \exp(1)$ (i.e., it is similar to $\Gamma(1, 1)$ distribution). The above assumption has been commonly applied for investigating the performance of multiuser MIMO HetNets [11, 69, 137, 141].

The SINR of the typical user served by both MBS_0 and SBS_0 , named as case 2, is given

²A Gamma random variable h with shape $k > 0$ and scale $\theta > 0$, denoted as $\Gamma(k, \theta)$, has probability distribution function $f_H(h) = h^{k-1} \frac{e^{-h/\theta}}{\theta^k \Gamma(k)}$.

by [128], [135, Eq. (3)], [136]

$$\gamma_c = \frac{\left| \sqrt{\frac{P_m x_0^{-\alpha}}{K}} \mathbf{h}_{o,x_0}^H \mathbf{w}_{o,x_0} + \sqrt{P_2 y_0^{-\alpha}} h_{o,y_0} \right|^2}{I_c + \sigma^2}. \quad (6.7)$$

The aggregate interference from the BSs, which are not in the set of serving BSs, is defined as $I_c = I_{m \rightarrow m} + I_{s \rightarrow s}$, where $I_{s \rightarrow s} = \sum_{j \in \Phi_s \setminus \text{SBS}_0} P_2 g_{o,j} y_j^{-\alpha}$ and y_j is the distance from the interfering SBS $j \in \Phi_s \setminus \text{SBS}_0$ [135–137].

If the typical user is served by the SBS_0 , named as case 3, its received SINR can be defined as [128, 137, 141]

$$\gamma_s = \frac{P_2 |h_{o,y_0}|^2 y_0^{-\alpha}}{I_s + \sigma^2}, \quad (6.8)$$

where $g_{o,0} = |h_{o,y_0}|^2 \sim \exp(1)$ is the fading channel power gain between the typical user and SBS_0 and the aggregate interference $I_s = I_{m \rightarrow s} + I_{s \rightarrow s}$, where $I_{m \rightarrow s} = \sum_{l \in \Phi_m} P_m h_{o,l} x_l^{-\alpha} / K$.

6.2 Average Achievable Rate

In this section, we propose a tractable approach to analyze the average achievable rate of the two-tier HetNet with massive MIMO and non-uniformly small-cell deployment. We define $X_m, \mathbf{Z}_c = (X, Y)$, and Y_s as the corresponding distances between the typical user and its serving BS(s), i.e., $BS = \{\text{MBS}_0\}$, $BS = \{\text{MBS}_0, \text{SBS}_0\}$, and $BS = \{\text{SBS}_0\}$, respectively. The probabilities that the typical user is served by $\{\text{MBS}_0\}$, $\{\text{MBS}_0, \text{SBS}_0\}$, and $\{\text{SBS}_0\}$ are given by [127, 128]

$$\Delta_m = 1 - \exp(-\pi \lambda_1 R^2), \quad (6.9)$$

$$\Delta_c = (\lambda_1 / \lambda'_1) \exp(-\pi \lambda'_1 R^2), \quad (6.10)$$

$$\Delta_s = \exp(-\pi \lambda_1 R^2) - \Delta_c, \quad (6.11)$$

respectively. The probability density functions (PDFs) of X_m , $\mathbf{Z}_c$, and Y_s are defined as [127, 128]

$$f_{X_m}(x) = \frac{2\pi\lambda_1 x}{\Delta_m} \exp(-\pi\lambda_1 x^2), x \in (0, R), \quad (6.12)$$

$$f_{\mathbf{Z}_c}(x, y) = \frac{4\pi^2\lambda_1\lambda_2 xy}{\Delta_c} \exp(-\pi\lambda_1 x^2 - \pi\lambda_2 y^2),$$

$$x > R \& y > x \left(\frac{P_2}{\mu P_1} \right)^{1/\alpha}, \quad (6.13)$$

$$f_{Y_s}(y) = \begin{cases} \frac{2\pi\lambda_2 y}{\Delta_s} \exp(-\pi\lambda_2 y^2 - \pi\lambda_1 R^2), y \leq R \left(\frac{P_2}{\mu P_1} \right)^{1/\alpha}, \\ \frac{2\pi\lambda_2 y}{\Delta_s} \exp(-\pi\lambda_2' y^2), y > R \left(\frac{P_2}{\mu P_1} \right)^{1/\alpha}, \end{cases} \quad (6.14)$$

where $\lambda_1' = \lambda_1 + \lambda_2 \left(\frac{P_2}{\mu P_1} \right)^{2/\alpha}$ and $\lambda_2' = \lambda_2 + \lambda_1 \left(\frac{\mu P_1}{P_2} \right)^{2/\alpha}$, respectively.

When massive MIMO is adopted at the MBS, by using the concept of stochastic geometry, a conventional method for evaluating the achievable rate on the macro-cell is to use the moment generating functions (MGFs) [144]. However, this technique requires a high complexity which consists of several integrations [138, 144, 145]. To simplify the analysis, a lower bound on the achievable rate has been recently derived in [69] and [139]. On the other hand, we present a tractable approximation of the achievable rate according to Lemma 1 of [146]. In particular, if both $\Omega = \sum_{i=1}^q \Omega_i$ and $\Phi = \sum_{j=1}^p \Phi_j$ are sums of non-negative random variables (RVs), then we have [146, Lemma 1]

$$\mathbb{E} \left\{ \log_2 \left(1 + \frac{\Omega}{\Phi} \right) \right\} \simeq \left\{ \log_2 \left(1 + \frac{\mathbb{E}\{\Omega\}}{\mathbb{E}\{\Phi\}} \right) \right\}. \quad (6.15)$$

Note that (6.15) does not require both Ω and Φ to be independent of each other. This approximation is appropriate to massive MIMO systems because it converges to the exact one when q and p are large. Based on the random theory matrix, this method has been adopted to analyze the achievable rates for linear massive MIMO precoders in the presence of phase noise [147], multi-cell massive MIMO systems with downlink training and pilot contamination precoding

scheme [148], and massive MIMO systems with low-resolution analog-digital converter [149]. Herein, we apply it in a different network topology. Based on concept from stochastic geometry, the achievable rates for the three association cases are shown in the following lemmas.

Lemma 1 *When the typical user is served by MBS_0 , an approximation of the achievable rate is defined as*

$$\tau_m \simeq \log_2 \left(1 + \frac{P_1}{\Delta_m} \int_0^R \frac{2\pi\lambda_1 x^{1-\alpha} \exp(-\pi\lambda_1 x^2) dx}{\frac{2\pi\lambda_1 P_m x^{2-\alpha}}{\alpha-2} + \frac{2\pi\lambda_2 P_2 R^{2-\alpha}}{\alpha-2} + \sigma^2} \right). \quad (6.16)$$

Proof: Based on Lemma A.3 of [150], from (6.4), it can be shown that $\mathbf{h}_{o,x_0}^H \mathbf{w}_{o,x_0}$ is equivalent to another vector of dimension $N - K + 1$ (i.e., denoted as $\mathbf{h}_o$) with i.i.d. $\mathcal{CN}(0, 1)$ entries, hence $\mathbf{h}_o^H \mathbf{h}_o$ is a sum of $N - K + 1$ independent elements. Note that Lemma A.3 also holds for I_m in the denominator of (6.4). Hence, by using (6.15), we approximate the achievable rate of the typical user as [147, Eq. (21)], [148, Eq. (31)], [149, Eq. (14)]

$$\tau_m \simeq \log_2 \left(1 + \frac{\mathbb{E}\{P_m |\mathbf{h}_{o,x_0}^H \mathbf{w}_{o,x_0}|^2 x_0^{-\alpha} / K\}}{\mathbb{E}\{I_m\} + \sigma^2} \right), \quad (6.17)$$

where $\mathbb{E}\{I_m\} = \mathbb{E}\{I_{m \rightarrow m}\} + \mathbb{E}\{I_{s \rightarrow m}\}$ is an expected value of the aggregate interference. In this case, the interference of MBSs $I_{m \rightarrow m}$ comes from all the MBSs which are located outside the circle $B(0, x)$. It is given by

$$\begin{aligned} \mathbb{E}\{I_{m \rightarrow m}\} &= \mathbb{E}_{\Phi_m, h_{o,l}} \left\{ \sum_{l \in \Phi_m \setminus MBS_0} \frac{P_m}{K} P_m h_{o,l} x_l^{-\alpha} | x_0 = x \right\} \\ &= \frac{P_m}{K} \mathbb{E}\{h_{o,l}\} \mathbb{E}_{\Phi_m} \left\{ \sum_{l \in \Phi_m} x_l^{-\alpha} \mathbf{1}(x_l \neq x) | x_0 = x \right\} \\ &\stackrel{(a_1)}{=} P_m \mathbb{E}_{\Phi_m} \left\{ \sum_{l \in \Phi_m} x_l^{-\alpha} \mathbf{1}(x_l > x) | x_0 = x \right\} \\ &\stackrel{(a_2)}{=} 2\pi\lambda_1 P_m \int_x^\infty u^{-\alpha} u du = \frac{2\pi\lambda_1 P_m x^{2-\alpha}}{\alpha-2}, \end{aligned} \quad (6.18)$$

where (a_1) is obtained due to $h_{o,l} \sim \Gamma(K, 1)$, $\mathbf{1}(\cdot)$ is the indicator function and (a_2) is defined by applying Campbell's Formula with polar coordinates [66]. Moreover, the SBSs are distributed according to homogeneous PPP with the density λ_2 , but we only use the SBSs in the outer region. Hence, the small cell tier process Φ_s becomes a Poisson Hole Process (PHP) with the density $\lambda_{\text{PHP}} = \lambda_2 \exp(-\pi\lambda_1 R^2)$ which makes performance analysis more challenging. In the literature, there are four main approaches to characterize the interference of PHP model [151, 152]. The first approach is to ignore the holes and approximate the PHP by the baseline PPP with the density λ_2 . The second approach is to approximate the PHP by a PPP with the density λ_{PHP} . Note that that the PPP in the second approach is independently thinned due to $\lambda_{\text{PHP}} < \lambda_2$ which distorts the local neighbouring SBSs around the typical node. On the other hand, the PPP in the first approach still preserves the local neighbouring SBSs in the outer region. Therefore, it has been shown that the first approach can provide a tighter approximation of the interference [151]. Furthermore, the third approach is to approximate the PHP by a Poisson Cluster Process (PCP) (such as Thomas or Matérn Cluster Processes) based on matching the first and second order statistics [152]. The difference between the PHP and the PCP is the higher-order statistics. As compared with these three approaches, the fourth one is the most fitting-based approach, where the holes are dissolved in such a way that the PHP is reduced to an equivalent non-homogeneous PHP. Thus, the upper and lower bounds on the PHP interference can be analyzed [151]. However, the third and fourth approaches require high computational complexities. To achieve the tractable analysis and reasonable performance, we use the first approach to characterize the interference of SBSs in this paper. Specifically, we assume that the small cell interference $I_{s \rightarrow m}$ approximately comes from whole region outside the circle $B(0, R)$, thus we can obtain [127, Eq. (34)], [128, Eq. (18)]

$$\begin{aligned}
 \mathbb{E}\{I_{s \rightarrow m}\} &= \mathbb{E}_{\Phi_s, g_{o,j}} \left\{ \sum_{j \in \Phi_s} P_2 g_{o,j} y_j^{-\alpha} \right\} \\
 &\approx P_2 \mathbb{E}\{g_{o,j}\} \mathbb{E}_{\Phi_s} \left\{ \sum_{l \in \Phi_s} y_l^{-\alpha} \mathbf{1}(y_l > R) \right\} = \frac{2\pi\lambda_2 P_2 R^{2-\alpha}}{\alpha - 2}.
 \end{aligned} \tag{6.19}$$

Similar assumption was applied and accurately proved in [127] and [128], in which the coverage probabilities of the conventional MIMO HetNets with the non-uniform small cell deployment were analyzed. Moreover, since $h_{o,0} = |\mathbf{h}_{o,x_0}^H \mathbf{w}_{o,x_0}|^2 \sim \Gamma(N - K + 1, 1)$, we have $\mathbb{E}\{h_{o,0}\} = N - K + 1$. By substituting (6.12), (6.18) and (6.19) into (6.17), we can obtain (6.16). ■

Note that the expression (6.16) is simpler to implement than the achievable rate [153, Eq. (26)], which is defined by using the complicated Fàa di Bruno's formula. The tightness of (6.16) will be shown in the Numerical Results and Discussions Section.

Lemma 2 *When the typical user is served by both MBS₀ and SBS₀, an approximation of the achievable rate is defined as*

$$\tau_c \simeq \log_2 \left(1 + \int_R^\infty \int_{x(\frac{P_2}{\mu P_1})^{1/\alpha}}^\infty \frac{(P_1 x^{-\alpha} + P_2 y^{-\alpha}) 4\pi^2 \lambda_1 \lambda_2 x y}{\frac{2\pi\lambda_1 P_m x^{2-\alpha}}{\alpha-2} + \frac{2\pi\lambda_2 P_2 y^{2-\alpha}}{\alpha-2} + \sigma^2} \frac{1}{\Delta_c} \exp(-\pi\lambda_1 x^2 - \pi\lambda_2 y^2) dy dx \right). \tag{6.20}$$

Proof: Similar to the proof of Lemma 1, when $BS = \{\text{MBS}_0, \text{SBS}_0\}$, the interference of MBSs $I_{m \rightarrow m}$ comes from the outside circles $B(0, x)$, while the interference of SBSs $I_{s \rightarrow s}$ approximately comes from the whole region outside the circle $B(0, y)$ [127, 128]. Hence, an expected value of I_c in the denominator of (6.7) is approximated as

$$\begin{aligned}
 \mathbb{E}\{I_c\} &= \mathbb{E} \left\{ \sum_{l \in \Phi_m \setminus \text{MBS}_0} \frac{P_m}{K} h_{o,l} x_l^{-\alpha} \right\} + \mathbb{E} \left\{ \sum_{j \in \Phi_s \setminus \text{SBS}_0} P_2 g_{o,j} y_j^{-\alpha} \right\} \\
 &\approx \frac{2\pi\lambda_1 P_m x^{2-\alpha}}{\alpha - 2} + \frac{2\pi\lambda_2 P_2 y^{2-\alpha}}{\alpha - 2},
 \end{aligned} \tag{6.21}$$

while an expected value of the nominator of (6.7) with respect to the fading channel power gain is defined as $\mathbb{E}_{x,y}\{|\sqrt{P_m x_0^{-\alpha}/K} \mathbf{h}_{o,x_0}^H \mathbf{w}_{o,x_0} + \sqrt{P_2 y_0^{-\alpha}} h_{o,y_0}|^2\} = P_1 x^{-\alpha} + P_2 y^{-\alpha}$. By combining these results with (6.13) and (6.15), we can define (6.20). $\blacksquare$

Lemma 3 *When the typical user is served by SBS₀, the achievable rate is given by*

$$\begin{aligned} \tau_s = & \frac{1}{\ln 2} \int_0^{R(\frac{P_2}{\mu P_1})^{1/\alpha}} \int_0^\infty \frac{2\pi\lambda_2 y}{\Delta_s} \exp(-\pi\lambda_2 y^2 - \pi\lambda_1 R^2) \Psi(R, t, y) \Omega(t, y) dt dy \\ & + \frac{1}{\ln 2} \int_{R(\frac{P_2}{\mu P_1})^{1/\alpha}}^\infty \int_0^\infty \frac{2\pi\lambda_2 y}{\Delta_s} \exp(-\pi\lambda_2 y^2) \Psi\left(y\left(\frac{\mu P_1}{P_2}\right)^{1/\alpha}, t, y\right) \Omega(t, y) dt dy, \end{aligned} \quad (6.22)$$

where $\Psi(\beta, t, y)$ is defined as

$$\begin{aligned} \Psi(\beta, t, y) = & \exp\left(-\frac{(e^t - 1)y^\alpha \sigma^2}{P_2}\right) \exp\left(-2\pi\lambda_1 \sum_{k=1}^K \binom{K}{k} \frac{z^k \beta^{2-k\alpha}}{k\alpha - 2}\right) \\ & \times {}_2F_1\left(K, k - \frac{2}{\alpha}, k - \frac{2}{\alpha} + 1, -z\beta^{-\alpha}\right). \end{aligned} \quad (6.23)$$

Proof: We do not use (6.15) to approximate the achievable rate of the typical user because it cannot perform well in this case. Hence, we define the achievable rate as

$$\tau_s = \mathbb{E}\{\log_2(1 + \gamma_s)\} = \frac{1}{\ln 2} \int_0^\infty \mathbb{E}_{\gamma_s}\{\ln(1 + \gamma_s(y))\} f_{Y_s}(y) dy. \quad (6.24)$$

Since $\mathbb{E}\{U\} = \int_0^\infty \Pr[U > u] du$ for $U > 0$, we define

$$\begin{aligned} \mathbb{E}_{\gamma_s}\{\ln(1 + \gamma_s(y))\} &= \int_0^\infty \Pr[\ln(1 + \gamma_s(y)) > t] dt \\ &= \int_0^\infty \Pr\left[g_{o,0} > \frac{(e^t - 1)y^\alpha}{P_2} (I_{m \rightarrow s} + I_{s \rightarrow s} + \sigma^2)\right] dt \\ &\stackrel{(b)}{=} \int_0^\infty \exp\left(-\frac{(e^t - 1)y^\alpha \sigma^2}{P_2}\right) \mathbb{E}\left\{e^{-\frac{(e^t - 1)y^\alpha}{P_2} I_{m \rightarrow s}}\right\} \\ &\quad \times \mathbb{E}\left\{e^{-\frac{(e^t - 1)y^\alpha}{P_2} I_{s \rightarrow s}}\right\} dt, \end{aligned} \quad (6.25)$$

where (b) is obtained due to $g_{o,0} \sim \exp(1)$. We define $z = \frac{(e^t-1)y^\alpha P_m}{P_2 K}$. By using the Laplace transform method, we have

$$\begin{aligned}
 \mathbb{E}\left\{e^{-\frac{(e^t-1)y^\alpha}{P_2}I_{m \rightarrow s}}\right\} &= \mathbb{E}\left\{e^{-z \sum_{l \in \Phi_m} h_{o,l} x_l^{-\alpha}}\right\} \\
 &\stackrel{(c_1)}{=} \mathbb{E}_{\Phi_m, h_{o,l}} \left\{ \prod_{l \in \Phi_m} e^{-z h_{o,l} x_l^{-\alpha}} \right\} = \mathbb{E}_{\Phi_m} \left\{ \prod_{l \in \Phi_m} \mathcal{L}_{h_{o,l}}(z x_l^{-\alpha}) \right\} \\
 &\stackrel{(c_2)}{=} \exp\left(-\lambda_1 \int_{\mathbb{R}^2} (1 - \mathcal{L}_{h_{o,l}}(zu^{-\alpha})) du\right) \\
 &\stackrel{(c_3)}{=} \exp\left(-2\pi\lambda_1 \int_{x_{\min}}^{\infty} \left(1 - \frac{1}{(1+zu^{-\alpha})^K}\right) u du\right) \\
 &\stackrel{(c_4)}{=} \exp\left(-2\pi\lambda_1 \sum_{k=1}^K \binom{K}{k} \int_{x_{\min}}^{\infty} \frac{(zu^{-\alpha})^k}{(1+zu^{-\alpha})^K} u du\right) \\
 &\stackrel{(c_5)}{=} \exp\left(-2\pi\lambda_1 \sum_{k=1}^K \binom{K}{k} \frac{z^k x_{\min}^{2-k\alpha}}{k\alpha-2} {}_2F_1\left(K, k - \frac{2}{\alpha}, k - \frac{2}{\alpha} + 1, -zx_{\min}^{-\alpha}\right)\right),
 \end{aligned} \tag{6.26}$$

where (c_1) is obtained based on the fact that the channel power levels and the BS locations are independent, (c_2) is defined by applying the probability generating functional (PGFL) of PPP [126], (c_3) results from the Laplace transform of $h_{o,l} \sim (K, 1)$, $x_{\min}$ is the minimum distance between the interfering MBS and the typical user, (c_4) results from applying Binomial expansion, and (c_5) is obtained by using Eq. (3.194.1) of [90], where ${}_2F_1(\cdot, \cdot, \cdot, \cdot)$ is the Gaussian hypergeometric function [90, Eq. (9.142)]. Moreover, we define $\varphi = (e^t - 1)y^\alpha$. Similarly, it has been shown in [127] that the interference of SBSs approximately comes from all the SBSs outside the circle $B(0, y)$. We can approximate the Laplace transform (6.25) as [127, Eq.

(46)], [128, Eq. (17)]

$$\begin{aligned}
 \mathbb{E}\left\{e^{-\frac{(e^t-1)y^\alpha}{P_2}I_{s \rightarrow s}}\right\} &= \mathbb{E}_{\Phi_s, g_{o,j}} \left\{ \prod_{j \in \Phi_s \setminus \text{SBS}_0} e^{-\varphi g_{o,j} y_j^{-\alpha}} \right\} \\
 &= \mathbb{E}_{\Phi_s} \left\{ \prod_{j \in \Phi_s \setminus \text{SBS}_0} \mathbb{E}_g \{ e^{-\varphi g y_j^{-\alpha}} \} \right\} \\
 &\stackrel{(e_1)}{=} \mathbb{E}_{\Phi_s} \left\{ \prod_{j \in \Phi_s \setminus \text{SBS}_0} \frac{1}{1 + \varphi y_j^{-\alpha}} \right\} \\
 &\approx \exp \left(-2\pi\lambda_2 \int_{y_{\min}}^{\infty} \left(1 - \frac{1}{1 + \varphi u^{-\alpha}}\right) u du \right) \\
 &\stackrel{(e_2)}{=} \exp \left(-\frac{2\pi\lambda_2}{\alpha - 2} \varphi y_{\min}^{2-\alpha} {}_2F_1 \left(1, 1 - \frac{2}{\alpha}, 2 - \frac{2}{\alpha}, -\frac{\varphi}{y_{\min}^\alpha}\right) \right) \\
 &\stackrel{(e_3)}{=} \exp \left(\frac{2\pi\lambda_2 y^2 (e^t - 1)}{2 - \alpha} {}_2F_1 \left(1, 1 - \frac{2}{\alpha}, 2 - \frac{2}{\alpha}, 1 - e^t\right) \right) = \Omega(t, y). \quad (6.27)
 \end{aligned}$$

where (e_1) is obtained due to $g \sim \exp(1)$, $y_{\min}$ is the minimum distance between the interfering SBS $j \in \Phi_s \setminus \text{SBS}_0$ and the typical user, (e_2) results from adopting the approach provided in [129], (e_3) results from replacing $y_{\min} = y$ and $\varphi = (e^t - 1)y^\alpha$. In addition, if $y \leq R(\frac{P_2}{\mu P_1})^{1/\alpha}$, the interference of MBSs comes from all the MBSs outside the circle $B(0, R)$, and if $y > R(\frac{P_2}{\mu P_1})^{1/\alpha}$, the macro interference is from all MBSs outside the circle $B(0, y(\frac{\mu P_1}{P_2})^{1/\alpha})$ [127, 128]. Substituting (6.25), (6.26) and (6.27) into (6.24), we obtain (6.22). $\blacksquare$

When $\alpha = 4$, due to $\int_a^\infty \frac{1}{1+v^2} dv = \frac{\pi}{2} - \arctan(a)$, (6.27) can be expressed through elementary functions such as $\Omega(t, y) = \exp(-\pi\lambda_2 y^2 \sqrt{e^t - 1} (\frac{\pi}{2} - \arctan(\frac{1}{\sqrt{e^t - 1}})))$. Similarly, when $K = 1$ and $\alpha = 4$, (6.23) is simplified as $\Psi(\beta, t, y) = \exp(-(e^t - 1)y^4 \sigma^2 / P_2) \exp(-\pi\lambda_1 y^2 \sqrt{(e^t - 1)P_m / P_2} (\frac{\pi}{2} - \arctan(\beta^2 y^{-2} / \sqrt{(e^t - 1)P_m / P_2})))$. These results help to simplify the expression (6.22) for this specific case.

By using the law of total expectation and the three Lemmas 1 to 3, an average achievable rate of the typical user in the two-tier HetNet with massive MIMO can be approximated as

$$C \simeq \Delta_m \tau_m + \Delta_c \tau_c + \Delta_s \tau_s. \quad (6.28)$$

Note that (6.28) can be efficiently computed by numerical methods. Thus, obtaining analytical results for studying network performance is more effective than using Monte Carlo methods which rely on a large number of realizations to define their results.

Remark 2: As observed from Lemmas 1 to 3, adding more antennas at the MBS has no effect on the existing interference environment, but it is beneficial at the achievable rate.

6.3 Energy Efficiency

In order to study the EE performance of HetNets with massive MIMO, the total power consumption needs to be captured as well. The power consumption model at the MBS in each channel use is expressed as [28, 31]

$$P_{MBS} = \frac{1}{\omega_1} \left(\frac{P_m}{\sigma_{am}(1 - \sigma_{feed})} + P_c + P_{bb} \right) + P_{bh}, \quad (6.29)$$

where $\omega_1 = (1 - \sigma_{DC})(1 - \sigma_{MS})(1 - \sigma_{cool})$ and σ_{am} is the power amplifier efficiency. P_c is the circuit power consumption, i.e., $P_c = N(p_{dac} + p_{mix} + p_{filt}) + p_{syn}$, where $p_{dac}, p_{mix}, p_{filt}, p_{syn}$, $\sigma_{feed}, \sigma_{DC}, \sigma_{MS}, \sigma_{cool}$ are defined in Section 2.1.3 of Chapter I, respectively. We assume that P_{bh} is the backhaul power consumption for signalling exchange between a MBS and its SBSs, and as well as core network. Moreover, it has been shown in [28] that the loss factor of active cooling σ_{cool} and the feeder loss σ_{feed} for the SBS can be typically negligible. From (6.29), the power consumption of the single-antenna SBS is defined as

$$P_{SBS} = \frac{1}{\omega_2} \left(\frac{P_2}{\sigma_{am}} + p_{dac} + p_{mix} + p_{filt} + p_{syn} + P_{bb} \right), \quad (6.30)$$

where $\omega_2 = (1 - \sigma_{DC})(1 - \sigma_{MS})$.

Without loss of generality, we assume that the system has unit bandwidth in order to simplify

notation [31]. The theoretical analysis is studied at the typical user which is located at the origin [126]. The probabilities that the typical user is served by MBS_0 , $\{\text{MBS}_0, \text{SBS}_0\}$, and SBS_0 are defined as Δ_m, Δ_c , and Δ_s , respectively. For given K users associated with each MBS, the number of users served by only MBS_0 is given by $K_1 = \frac{K\Delta_m}{\Delta_m + \Delta_c}$, while the number of users served by both MBS_0 and SBS_0 is given by $K_2 = \frac{K\Delta_c}{\Delta_m + \Delta_c}$. In every macrocell, the average number of SBSs located in the outer region is defined as $\frac{\lambda_2 \Pr[o \in \mathcal{A}_{out}]}{\lambda_1} = \frac{\lambda_2(\Delta_c + \Delta_s)}{\lambda_1}$ [127, 136]. Since each SBS is assumed to serve a user, then the number of users served by the SBSs is defined as $K_3 = \frac{\lambda_2(\Delta_c + \Delta_s)}{\lambda_1} - K_2$ in every macrocell. Hence, the area spectral efficiency (ASE) (in bits/s/Hz/m²) of the whole network is obtained as

$$C_{ASE} \simeq \lambda_1(K_1\tau_m + K_2\tau_c + K_3\tau_s). \quad (6.31)$$

We also assume that the MBS and SBS contribute main power consumption in the downlink [31, 69]. Thus, we formulate an optimization problem for maximizing EE as [26, 130, 154]

$$\begin{aligned} & \underset{P_m, N, \lambda_2}{\text{maximize}} \quad \eta_{EE} = \frac{C_{ASE}}{P_{AEC}}, \\ & \text{subject to} \quad (\text{C}_1) : K \leq N \leq N_{\max}, (\text{C}_2) : 0 < P_m \leq P_m^{\max}, \\ & \quad (\text{C}_3) : \lambda_2^{\min} \leq \lambda_2 \leq \lambda_2^{\max}, (\text{C}_4) : C_{ASE} \geq C_{ASE}^{\min}, \\ & \quad (\text{C}_5) : \tau_i \geq \tau_i^{\min}, \forall i \in \{m, c, s\} \end{aligned} \quad (6.32)$$

where $P_{AEC} = \lambda_1 P_{MBS} + \lambda_2(\Delta_c + \Delta_s)P_{SBS}$ is the area energy consumption (AEC) (in Joule/s/m²), C_{ASE} is defined in (6.31), P_m is the maximum MBS transmit power, $\tau_i^{\min}$ is the minimum rate requirement, $N_{\max}$ is the available number of MBS antennas. $C_{ASE}^{\min}$ is the minimum requirement of the ASE, $\lambda_2^{\min}$ and $\lambda_2^{\max}$ are the minimum and the maximum SBS densities, respectively. The minimum SBS density should be $\lambda_2^{\min} = \lambda_1(K_2 + 1)/(\Delta_c + \Delta_s)$ due to $K_3 \geq 1$. Next, we present an alternative method to solve problem (6.32). Firstly, we define the optimal value of

Table 6.2: Bisection search algorithm for obtaining N^*

-
1. **Input:** Given $N_{\min} = K$, $N_{\max} = N_{\text{avai}}$, and accuracy ϵ .
 2. **Initialize:** $\eta_{EE}^{\min} = \eta_{EE}(N_{\min})$ and $\eta_{EE}^{\max} = \eta_{EE}(N_{\max})$.
 3. **while** $|\eta_{EE}^{\max} - \eta_{EE}^{\min}| > \epsilon$ **or** $(N_{\max} - N_{\min}) > 1$
 4. $N_{\text{temp}} = (N_{\max} + N_{\min})/2$ and $\eta_{EE}^{\text{temp}} = \eta_{EE}(N_{\text{temp}})$.
 5. **if** (6.33) is feasible **and** $\eta_{EE}(N_{\text{temp}} + 1) \geq \eta_{EE}(N_{\text{temp}})$
 do $N_{\min} \leftarrow N_{\text{temp}}$ and $\eta_{EE}^{\min} \leftarrow \eta_{EE}^{\text{temp}}$.
 6. **else** $N_{\max} \leftarrow N_{\text{temp}}$ and $\eta_{EE}^{\max} \leftarrow \eta_{EE}^{\text{temp}}$.
 7. **return** $N^* = (N_{\max} + N_{\min})/2$.
 8. **Output:** $N^{**} = \min(N^*, N_{\text{avai}})$ and $\eta_{EE}^{\text{opt}} = \eta_{EE}(N^{**})$.
-

each parameter (N, P_m or λ_2) for maximizing the EE when the other parameters are fixed as the following.

6.3.1 Optimal Number of MBS Antennas

The total power consumption of massive MIMO systems increases with N , which can significantly affect the EE. Therefore, it is important to determine the optimal number of MBS antennas that maximizes the EE. For given P_m and λ_2 , the problem of EE optimization (6.32) can be rewritten as

$$\underset{N}{\text{maximize}} \quad \eta_{EE} \quad \text{subject to} \quad (\text{C}_1) \text{ and } (\text{C}_5). \quad (6.33)$$

To solve problem (6.33), we adopt the bisection search algorithm (BSA). First, we initialize $N_{\min}$ and $N_{\max}$, then we can determine $\eta_{EE}^{\min} = \eta_{EE}(N_{\min})$ and $\eta_{EE}^{\max} = \eta_{EE}(N_{\max})$, respectively. We define $N_{\text{temp}} = (N_{\max} + N_{\min})/2$ at every iteration, then we obtain $\eta_{EE}(N_{\text{temp}} + 1)$ and $\eta_{EE}(N_{\text{temp}})$. If the conditions in (6.33) are satisfied, and if $\eta_{EE}(N_{\text{temp}} + 1) \geq \eta_{EE}(N_{\text{temp}})$, the algorithm updates $N_{\min} = N_{\text{temp}}$, otherwise $N_{\max} = N_{\text{temp}}$. The algorithm will be terminated whenever $|\eta_{EE}^{\max} - \eta_{EE}^{\min}| \leq \epsilon$ or $(N_{\max} - N_{\min}) = 1$ is satisfied. Following that, the maximum overall EE is $\eta_{EE}^{\text{opt}} = \eta_{EE}(N^{**})$ at the optimum value of the number of MBS antennas $N^{**} = \min(N^*, N_{\text{avai}})$, where $N^* = (N_{\max} + N_{\min})/2$. The above steps of the BSA, named as algorithm 1, are summarized in Table 6.2. In the worst case, the computational complexity

of the algorithm 1 is on the order of $O(\log_2(N - K) + 1)$ to find the optimal number of MBS antennas. The additional comparison in the algorithm is due to the fact that the algorithm needs to compare the two consecutive values in the array, as compared with the conventional bisection search algorithm.

6.3.2 Optimal MBS Transmit Power

For given N and λ_2 , a solution for maximizing the EE means to wisely select a good transmit power level to use. The problem of EE optimization with respect to P_m is rewritten as

$$\underset{P_m}{\text{maximize}} \quad \eta_{EE} \quad \text{subject to} \quad (\text{C}_2) \text{ and } (\text{C}_5). \quad (6.34)$$

Following the similar approach, we modify the bisection search algorithm to solve problem (6.34) by initializing $P_m^{\min} = 0$ dBm. All other steps are almost the same. When we use a step of 0.5 dBm for EE comparisons, in the worst case, the complexity of the modified algorithm is on the order of $O(\log_2(2P_m^{\max}) + 1)$ to find the optimal values of P_m .

6.3.3 Optimal SBS Density

In the network with a certain MBS density, it is feasible to improve the area spectral efficiency performance by adopting high small-cell density [155]. However, the interference from the second tier and its power consumption will be increased. The goal of the following optimization problem is to maximize the EE while satisfying the demand of sum rate for the network with respect to SBS density. Thus, it can be formulated as

$$\underset{\lambda_2}{\text{maximize}} \quad \eta_{EE} \quad \text{subject to} \quad (\text{C}_3), (\text{C}_4) \text{ and } (\text{C}_5). \quad (6.35)$$

Table 6.3: EE maximization with the AO algorithm

1. Initialize: Randomly feasible sets (N, P_m, λ_2) and given accuracy ϵ
2. Repeat
3. Obtain N by using algorithm 1,
4. Obtain P_m by using modified algorithm 1, based on N defined in the step 3 and given λ_2 ,
5. Obtain λ_2 by using modified algorithm 1, based on P_m and N defined in steps 3 and 4.
6. Until $ \eta_{EE}(i+1) - \eta_{EE}(i) \leq \epsilon$ (or convergence)
7. Output: $(N^{opt}, P_m^{opt}, \lambda_2^{opt})$

To solve this problem, we also modify the algorithm 1. Particularly, we define κ as the small step for EE comparisons. If the conditions in (6.35) are satisfied, and if $\eta_{EE}(\lambda_2^{temp} + \kappa) \geq \eta_{EE}(\lambda_2^{temp})$, where $\lambda_2^{temp} = (\lambda_2^{\max} + \lambda_2^{\min})/2$, the algorithm updates $\lambda_2^{\min} = \lambda_2^{temp}$, otherwise $\lambda_2^{\max} = \lambda_2^{temp}$. The modified algorithm will return the optimal value of λ_2 when $|\eta_{EE}^{\max}(\lambda_2^{\max}) - \eta_{EE}^{\min}(\lambda_2^{\min})| \leq \epsilon$. We define $J = \lceil (\lambda_2^{\max} - \lambda_2^{\min})/\kappa \rceil$ as the number of elements in the array. In the worst case, the complexity of the modified algorithm is on the order of $O(\log_2(J) + 1)$ to define the optimal λ_2 .

6.3.4 Alternating Optimization Algorithm

In the previous section, the optimal values of N, P_m and λ_2 for maximizing the EE are obtained independently for given parameters. However, the global maximal value of the EE can only be achieved by joint optimization. In the following, we adopt a standard AO algorithm as shown in Table 6.3 to achieve an optimum EE. This method is a practical solution because it has low complexity and it has been widely used in literature [26,112]. According to [113, Proposition 4] and [114, Lemma 2], the AO algorithm is locally convergent. In our EE maximization problem, the algorithm optimizes these parameters (N, P_m, λ_2) sequentially until $|\eta_{EE}(i+1) - \eta_{EE}(i)| \leq \epsilon$. It has been shown that the EE metric is a quasi-concave function of N in [31] and [134], transmit power in [26] and [156], and SBS density in [157] for given constraints, respectively. Therefore, the achievable value of EE in the AO algorithm is non-decreasing in

every step, such as

$$\eta_{EE}(N^{(i)}, P_m^{(i)}, \lambda_2^{(i)}) \stackrel{(e_5)}{\leq} \eta_{EE}(N^{(i+1)}, P_m^{(i)}, \lambda_2^{(i)}) \leq \dots \leq \eta_{EE}(N^{(i+1)}, P_m^{(i+1)}, \lambda_2^{(i+1)}), \quad (6.36)$$

where (e_5) is due to the fact that $\eta_{EE}(N^{(i+1)}, P_m^{(i)}, \lambda_2) = \max_N \eta_{EE}(N^{(i)}, P_m^{(i)}, \lambda_2^{(i)})$ is the optimal solution of N for the sub-problem (6.33). Moreover, the EE metric has a finite upper bound. Hence, the local convergence of the algorithm is guaranteed for any initially feasible set (N, P_m, λ_2) . To avoid local convergence, we randomly generate several initial sets and choose the one which results in the overall maximum EE. An output of the chosen initial set via the algorithm is considered as an optimal solution, such as $(N^{**}, P_m^{**}, \lambda_2^{**})$. Note that N^{**} is a real-valued number. To define an integer-valued solution of N^{opt} , we only need to take floor and ceiling operations on the real-valued solution N^{**} for achieving the maximal EE (6.32). By defining $P_m^{opt} = P_m^{**}$ and $\lambda_2^{opt} = \lambda_2^{**}$, the final solution is given by $(N^{opt}, P_m^{opt}, \lambda_2^{opt})$. The actual convergence speed of the algorithm depends on specific value of accuracy ϵ [114]. As compared with the Brute-force ES algorithm, it has been shown in Section 6.5 that the AO algorithm performs well and it is almost able to obtain the global optimum EE. In the worst case, the total computational complexity of the AO algorithm is on the order of $d_1 d_2 (O(\log_2(N - K) + 1) + O(\log_2(2P_m^{\max}) + 1) + O(\log_2(J) + 1))$, where d_1 is the number of initial sets and d_2 is the number of iterations, to achieve an optimum EE.

6.4 Coverage Probability

We study coverage probability of the proposed scheme in this section. The coverage probability significantly depends on the different kinds of cell associations. It has been shown in [11] and [67] that the heterogeneous cellular networks are typically interference-limited. Hence, the noise term in the SINR expressions can be negligible for further analysis, i.e.,

$\gamma_i \rightarrow \frac{S_i}{I_i}, \forall i = \{m, c, s\}$ (named as signal-to-interference ratio (SIR)), where S_i is the desired signal of the typical user for each case of associations [11, 67, 157]. Note that SIR is a useful qualification for evaluating performance in HetNets, especially the performance at the cell-edge areas. In each association case, the coverage probability of the typical user is given by [127, Eq. (7)], [129, Theorem 1] [11, 158]

$$P_{co}^i = \int_0^\infty \int_0^\infty \mathbb{P}(\gamma_i(x, y) > T) f_{XY}(x, y) dy dx, \quad (6.37)$$

$$\mathbb{P}(\gamma_i(x, y) > T) = \int_0^\infty F_{S_i}(vT) f_{I_i}(v) dv, \forall i = \{m, c, s\}, \quad (6.38)$$

where $f_{XY}(x, y) \in \{f_{X_m}(x), f_{\mathbf{Z}_c}(x, y), f_{Y_s}(y)\}$, F_{S_i} is the cumulative distribution function (CDF) of S_i . For the HetNets with massive MIMO, it is challenging to obtain the closed-form distribution of the aggregate interference $f_{I_i}(\cdot)$ and conditional coverage probability $\mathbb{P}(\gamma_i(x, y) > T)$. The coverage probability expressions for these networks are usually very complex, i.e., they have been expressed in terms of high-order derivatives of composite functions in [68], or Faà di Bruno's formula in [153]. Moreover, it has been shown in [159] that the interference statistics is well characterized by the Gamma distribution. Due to the tractability of Gamma distribution, many complicated distributions have been approximated by the Gamma distributions in the literature [11, 142, 150, 159–162]. Specifically, according to the second-order moment matching technique [11, Lemma 3], the approximate Gamma distribution $\Gamma(k_i, \theta_i)$ for a particular distribution with mean φ and variance δ^2 will have the values of shape $k_i = \frac{\varphi^2}{\delta^2}$ and scale $\theta_i = \frac{\delta^2}{\varphi}$. The Gamma approximation of interference was used to study the ergodic sum rates of the network MIMO transmission [150] and the MIMO distributed antenna systems [160, 161], as well as the coverage probability of the conventional HetNets [142, 162]. Hence, we propose to approximate the interference distribution in the proposed scheme as a Gamma distribution. As compared with prior works [142, 150, 161, 162], the aggregate interference of our scheme is more

complicated. We first approximate the interference of each tier as a Gamma distributed RV with parameter k_i and θ_i according to Lemma 3 of [11]. As a result, the aggregate interference is now a sum of 2 Gamma distributed RVs. Applying sum of the second-order moment match in [11, Lemma 5], the aggregate interference can be approximated as a Gamma RV³ $\Upsilon \sim \Gamma(k, \theta)$, where $k = \frac{(\sum_i k_i \theta_i)^2}{\sum_i k_i \theta_i^2}$ and $\theta = \frac{\sum_i k_i \theta_i^2}{\sum_i k_i \theta_i}$. By doing so, we can simplify the complexity of the coverage probability expression. Numerical results presented in the following section show that this approach can provide reasonable performance. We start by obtaining $P_{co}^m, P_{co}^c, P_{co}^s$, that the typical user is served by $\{\text{MBS}_0\}$, $\{\text{MBS}_0, \text{SBS}_0\}$, and $\{\text{SBS}_0\}$, respectively.

Case 1: As $BS = \{\text{MBS}_0\}$, the first-order moment of $I_{m \rightarrow m}$ from (6.4) is defined in (6.18) and the-second order moment of $I_{m \rightarrow m}$ is given by

$$\begin{aligned} \mathbb{E}\{I_{m \rightarrow m}^2\} &= 2\pi\lambda_1 \mathbb{E}\{h_{o,l}^2\} \int_x^\infty \left(\frac{P_m}{K}\right)^2 u^{-2\alpha} u du \\ &= \frac{2\pi\lambda_1(K+1)P_m^2 x^{2-2\alpha}}{K(2\alpha-2)}. \end{aligned} \quad (6.39)$$

Thus, the variance of $I_{m \rightarrow m}$ is defined as

$$\mathbb{V}\{I_{m \rightarrow m}\} = \frac{2\pi\lambda_1(K+1)P_m^2 x^{2-2\alpha}}{K(2\alpha-2)} - \left(\frac{2\pi\lambda_1 P_m x^{2-\alpha}}{\alpha-2}\right)^2. \quad (6.40)$$

According to Lemma 3 of [11], $I_{m \rightarrow m}$ is approximated as a Gamma RV (denoted as $\Upsilon_{mm} \sim \Gamma(k_{mm}, \theta_{mm})$), which has the shape and scale parameters given by

$$k_{mm} = \frac{\mathbb{E}^2\{I_{m \rightarrow m}\}}{\mathbb{V}\{I_{m \rightarrow m}\}} = \frac{4\pi\lambda_1 K(\alpha-1)x^{2(2-\alpha)}}{(K+1)(\alpha-2)^2 x^{2-2\alpha} - 4\pi\lambda_1 K(\alpha-1)x^{2(2-\alpha)}}, \quad (6.41)$$

$$\theta_{mm} = \frac{\mathbb{V}\{I_{m \rightarrow m}\}}{\mathbb{E}\{I_{m \rightarrow m}\}} = \frac{P_m(K+1)(\alpha-2)x^{-\alpha}}{2K(\alpha-1)} - \frac{2\pi\lambda_1 P_m x^{2-\alpha}}{\alpha-2}, \quad (6.42)$$

respectively. Moreover, as mentioned earlier, the small-cell interference $I_{s \rightarrow m}$ approximately

³This Gamma RV has the same first and second order moments as those of the aggregate interference [11].

comes from whole region outside the circle $B(0, R)$, which has been accurately proved in [127] and [128]. Following a similar approach, the interference term $I_{s \rightarrow m}$ is approximated as a Gamma distributed RV (denoted as $\mathcal{I}_{sm} \sim \Gamma(k_{sm}, \theta_{sm})$) with parameters [11, 127, 128]

$$k_{sm} = \frac{2\pi\lambda_2(\alpha-1)R^{2(2-\alpha)}}{(\alpha-2)^2R^{2-2\alpha} - 2\pi\lambda_2(\alpha-1)R^{2(2-\alpha)}}, \quad (6.43)$$

$$\theta_{sm} = \frac{(\alpha-2)P_2R^{-\alpha}}{\alpha-1} - 2\pi\lambda_2P_2\frac{R^{2-\alpha}}{\alpha-2}. \quad (6.44)$$

Therefore, according to Lemma 5 of [11], the sum of interference $I_m = I_{m \rightarrow m} + I_{s \rightarrow m}$ in (6.4) is approximated as the Gamma distributed RV with parameters

$$k_{I_m} = \frac{(k_{mm}\theta_{mm} + k_{sm}\theta_{sm})^2}{k_{mm}\theta_{mm}^2 + k_{sm}\theta_{sm}^2}, \quad (6.45)$$

$$\theta_{I_m} = \frac{k_{mm}\theta_{mm}^2 + k_{sm}\theta_{sm}^2}{k_{mm}\theta_{mm} + k_{sm}\theta_{sm}}. \quad (6.46)$$

It means that the distribution of the aggregate interference I_m is approximated as $\Gamma(k_{I_m}, \theta_{I_m})$ which is given by

$$f_{I_m}(v) = v^{k_{I_m}-1} \frac{e^{-v/\theta_{I_m}}}{\theta_{I_m}^{k_{I_m}} \Gamma(k_{I_m})}. \quad (6.47)$$

Moreover, as $h_{o,0} \sim \Gamma(N-K+1, 1)$, based on the scaling property of Gamma RVs, we have [11, 141]

$$S_m = \frac{P_m}{K} h_{o,0} x_0^{-\alpha} \sim \Gamma\left(N-K+1, \frac{P_m}{K} x_0^{-\alpha}\right). \quad (6.48)$$

By replacing x_0 by x , from (6.48), the CDF of S_m is given by

$$F_{S_m}(u) = \frac{\gamma(k_m, u\theta_m)}{\Gamma(k_m)}, \quad (6.49)$$

where $k_m = N-K+1$, $\theta_m = \frac{P_m}{K} x^{-\alpha}$, and $\gamma(a, b) = \int_0^b t^{a-1} e^{-t} dt$ is the lower incomplete gamma function. By substituting (6.12), (6.47), and (6.49) into (6.37), after some manipulations, the

coverage probability of the typical user served by MBS_0 is given by [11]

$$P_{co}^m = \int_0^R \frac{2\pi\lambda_1 x \psi_m(x)}{\Delta_m} \exp(-\pi\lambda_1 x^2) dx, \quad (6.50)$$

$$\psi_m(x) = \frac{\Gamma(k_m + k_{I_m})}{\Gamma(k_m)\Gamma(k_{I_m} + 1)} \left(\frac{\theta_m}{T\theta_{I_m}} \right)^{k_{I_m}} {}_2F_1\left(k_{I_m}, k_m + k_{I_m}, k_{I_m} + 1, -\frac{\theta_m}{T\theta_{I_m}}\right). \quad (6.51)$$

Case 2: A similar approach is used to obtain Gamma approximation of the aggregate interference for the typical user served by both MBS_0 and SBS_0 . Hence, the sum interference I_c in (6.7) is approximated as a Gamma RV $\Gamma(k_{I_c}, \theta_{I_c})$ with

$$k_{I_c} = \frac{(k_{mm}\theta_{mm} + k_{ss}\theta_{ss})^2}{k_{mm}\theta_{mm}^2 + k_{ss}\theta_{ss}^2}, \quad (6.52)$$

$$\theta_{I_c} = \frac{k_{mm}\theta_{mm}^2 + k_{ss}\theta_{ss}^2}{k_{mm}\theta_{mm} + k_{ss}\theta_{ss}}, \quad (6.53)$$

$$k_{ss} = \frac{2\pi\lambda_2(\alpha - 1)y^{2(2-\alpha)}}{(\alpha - 2)^2y^{2-2\alpha} - 2\pi\lambda_2(\alpha - 1)y^{2(2-\alpha)}}, \quad (6.54)$$

$$\theta_{ss} = \frac{(\alpha - 2)P_2y^{-\alpha}}{\alpha - 1} - \frac{2\pi\lambda_2P_2y^{2-\alpha}}{\alpha - 2}, \quad (6.55)$$

where k_{mm} and θ_{mm} are defined in (6.41) and (6.42), respectively. Next, we obtain the CDF of the desired signal S_c . Due to the NCJT mode, from (6.7), the desired signal S_c can be rewritten as [142, Appendix A], [143, Eq. (3)]

$$S_c \triangleq \frac{P_m}{K} h_{o,0} x_0^{-\alpha} + P_2 g_{o,0} y_0^{-\alpha}, \quad (6.56)$$

where $h_{o,0}$ and $g_{o,0}$ are defined in (6.4) and (6.8), respectively. We replace x_0 and y_0 by x and y , respectively. Since $g_{o,0} \sim \Gamma(1, 1)$, then $P_2 g_{o,0} y^{-\alpha} \sim \Gamma(1, P_2 y^{-\alpha})$ due to the scaling property of Gamma RVs [11]. Based on [11, Lemma 5], from (6.7) and (6.48), S_c is approximated as a

Gamma distributed RV $\Gamma(k_c, \theta_c)$ with parameters

$$k_c = \frac{\left((N - K + 1)P_m x^{-\alpha}/K + P_2 y^{-\alpha}\right)^2}{(N - K + 1)(P_m x^{-\alpha}/K)^2 + (P_2 y^{-\alpha})^2}, \quad (6.57)$$

$$\theta_c = \frac{(N - K + 1)(P_m x^{-\alpha}/K)^2 + (P_2 y^{-\alpha})^2}{(N - K + 1)P_m x^{-\alpha}/K + P_2 y^{-\alpha}}. \quad (6.58)$$

Therefore, the CDF of S_c is given by $F_{S_c}(u) = \frac{\gamma(k_c, u\theta_c)}{\Gamma(k_c)}$. Similarly, from (6.38), the conditional coverage probability of case 2 is obtained as [11]

$$\mathbb{P}(\gamma_c(x, y) > T) = \frac{\Gamma(k_c + k_{I_c})}{\Gamma(k_c)\Gamma(k_{I_c} + 1)} \left(\frac{\theta_c}{T\theta_{I_c}}\right)^{k_{I_c}} {}_2F_1\left(k_{I_c}, k_c + k_{I_c}, k_{I_c} + 1, -\frac{\theta_c}{T\theta_{I_c}}\right) = \psi_c(x, y). \quad (6.59)$$

By substituting (6.14) and (6.59) into (6.37), the coverage probability of case 2 is given by

$$P_{co}^c = \int_R^\infty \int_{x(\frac{P_2}{\mu P_1})^{1/\alpha}}^\infty \frac{4\pi^2 \lambda_1 \lambda_2 x y}{\Delta_c} \psi_c(x, y) \exp(-\pi \lambda_1 x^2 - \pi \lambda_2 y^2) dy dx. \quad (6.60)$$

Case 3: Similarly, the sum interference I_s in (6.8) is approximated as a Gamma RV $\Gamma(k_{I_s}, \theta_{I_s})$ with parameters

$$k_{I_c} = \frac{(k_{ms}\theta_{ms} + k_{ss}\theta_{ss})^2}{k_{ms}\theta_{ms}^2 + k_{ss}\theta_{ss}^2}, \quad (6.61)$$

$$\theta_{I_c} = \frac{k_{ms}\theta_{ms}^2 + k_{ss}\theta_{ss}^2}{k_{ms}\theta_{ms} + k_{ss}\theta_{ss}}, \quad (6.62)$$

$$k_{ms} = \frac{4\pi \lambda_1 K(\alpha - 1)\varpi^{2(2-\alpha)}}{(K + 1)(\alpha - 2)^2 \varpi^{2-2\alpha} - 4\pi \lambda_1 K(\alpha - 1)\varpi^{2(2-\alpha)}}, \quad (6.63)$$

$$\theta_{ms} = \frac{P_m(K + 1)(\alpha - 2)\varpi^{-\alpha}}{2K(\alpha - 1)} - \frac{2\pi \lambda_1 P_m \varpi^{2-\alpha}}{\alpha - 2}, \quad (6.64)$$

where k_{ss} and θ_{ss} are defined in (6.54) and (6.55), respectively. Moreover, the CDF of the desired signal S_s is given by

$$F_{S_s}(u) = \gamma(1, u\theta_s), \quad (6.65)$$

where $\theta_s = P_2 y^{-\alpha}$ due to $S_s = P_2 g_{o,0} y^{-\alpha} \sim \Gamma(1, P_2 y^{-\alpha})$. Therefore, the conditional coverage probability of case 3 is defined as

$$\mathbb{P}(\gamma_s(x, y) > T) = \left(\frac{\theta_s}{T\theta_{I_s}} \right)^{k_{I_s}} {}_2F_1\left(k_{I_s}, k_{I_s} + 1, k_{I_s} + 1, -\frac{\theta_s}{T\theta_{I_s}}\right) = \psi_s(\varpi, y). \quad (6.66)$$

By substituting (6.13) and (6.66) into (6.37), the coverage probability of case 3 is given by

$$\begin{aligned} P_{co}^s &= \int_0^{R(\frac{P_2}{\mu P_1})^{1/\alpha}} \frac{2\pi\lambda_2 y}{\Delta_s} \psi_s(R, y) e^{-\pi\lambda_2 y^2 - \pi\lambda_1 R^2} dy \\ &+ \int_{R(\frac{P_2}{\mu P_1})^{1/\alpha}}^{\infty} \frac{2\pi\lambda_2 y}{\Delta_s} \psi_s\left(y\left(\frac{\mu P_1}{P_2}\right)^{1/\alpha}, y\right) e^{-\pi\lambda_2' y^2} dy. \end{aligned} \quad (6.67)$$

Finally, by applying the law of total probability, the coverage probability of the presented scheme is obtained as

$$P_{co} = \Delta_m P_{co}^m + \Delta_c P_{co}^c + \Delta_s P_{co}^s, \quad (6.68)$$

where P_{co}^m, P_{co}^c and P_{co}^s are defined in (6.50), (6.60), and (6.67), respectively. Note that, similar to Remark 2, adding more antennas at the MBS is also beneficial at the coverage probability due to increasing the received power at the typical user, while it does not change the aggregate interference environment.

Table 6.4: Simulation parameters

Parameter	Value	Parameter	MBS	SBS
Max. MBS transmit power $P_m^{\max}$	43 dBm	Baseband power P_{bb} [28]	29.6 W	3 W
DAC power p_{dac} [31]	15.6 mW	Feeder loss σ_{feed} [28]	-3 dB	0
Freq. synthesizer power p_{syn} [31]	50 mW	DC-DC loss σ_{DC} [28]	7.5%	9%
Backhaul power P_{bb}	5 W	Cooling loss σ_{cool} [28]	10%	0
Path loss exponent α [26]	3.76	Main supply loss σ_{MS} [28]	9%	11%
Max. available MBS antennas	800	PA efficiency σ_{am} [28]	0.38	0.067
SBS transmit power P_2	30 dBm	Noise variance σ_n^2 [128]	-104 dBm	
Filter power p_{filt} [31]	20 mW	Mixer power p_{mix} [31]	30.3 mW	
Operation threshold μ	3 dB	MBS density λ_1 [128]	$(500^2\pi)^{-1}$	

6.5 Numerical Results and Discussions

In this section, numerical results are presented to verify the accuracy of our theoretical analysis and to investigate the network performance with difference parameter settings. The simulation area is a circular region with radius of 10 km. The values of simulation parameters follow the settings of various works [26,28,31,128]. They are summarized in Table 6.4. Similar to [163], we assume that $\lambda_u = 60\lambda_2$, the MBS can select $K = 10$ users to serve simultaneously.

Figure 6.3: Average achievable rates of the typical user for the proposed scheme and MRP scheme [127] with $N = 200$ and $\lambda_2 = 5\lambda_1$.

In Fig. 6.3, the average achievable rates of the typical user for the proposed HetNet and the conventional maximum receive power (MRP) association scheme [127] versus radius of the inner region R are presented. Each plotted point of the simulation results is obtained by averaging over 10,000 independent channel realizations. The average rates of the typical user in the inner region and outer region are defined as $\Delta_m\tau_m$ and $\Delta_c\tau_c + \Delta_s\tau_s$, respectively, where τ_m , τ_c , and τ_s are given in (6.16), (6.20), and (6.22), while the overall average of the typical user is defined

Figure 6.4: Average achievable rates of the typical user for the proposed scheme and MRP scheme [127] with $R = 300$ m and $\lambda_2 = 5\lambda_1$.

in (6.28). For the MRP scheme, the typical user will associate with the i th tier that provides the maximal average receive power [127, Eq. (1)]. The same scenario of the non-uniform SCN deployment is applied for both the proposed scheme and MRP scheme. In Fig. 6.3, when R increases, the typical user rate in the inner region is increased, while the typical user rate in the outer region is decreased. This can be explained by the fact that as R increases, the probability that a user is associated with MBS_0 Δ_m is increased, while the probability that the user is associated with $\{\text{MBS}_0, \text{SBS}_0\}$ Δ_c is decreased. As R increases from 50 to 450 m, the overall average rate of the typical user increases slowly [128]. This is due to the fact that the increasing rate of case 1 with Δ_m is faster than the decreasing sum rates of cases 2 and 3 in this range. The optimal choice of inner region radius R will be discussed later. Nevertheless, the rate approximation of the proposed scheme is quite close to the simulation result, which validates the theoretical analysis.

Furthermore, in Fig. 6.3. in the inner region, the typical user rate of the proposed scheme

is the same as that of the MRP scheme when R is not large, i.e., $R \leq 300$ m. When $R > 300$ m, the achievable rate of the MRP scheme is slightly higher than that of the proposed scheme. This is because the typical user always associates with the MBS_0 in the proposed scheme even when it is far away (i.e., R increases), while it should associate with better SBSs located in the outer region as in the MRP scheme. However, in the outer region, the proposed scheme provides much higher average rate than that of the MRP scheme because the typical user can enjoy the performance benefit in the coordination mode. For the overall system, from Fig. 6.3, we observe that the proposed scheme outperforms the MRP scheme. This fact can be shown more evidently by observing Fig. 6.4 where the number of BS antennas N is increased from 20 to 620. On a different note, from Fig. 6.4, the rate approximation as shown in (6.28) is close to the simulation results even when N is small, and it converges to the exact one when N is large, such as $N \geq 400$. These results verify the accuracy of our analysis by applying the rate approximation given in (6.15). Moreover, the increase in the number of MBS antennas can improve the data rate, and the increased rate of the proposed scheme is mainly contributed by the MBS. Therefore, it can be considered that the MBS with a large-scale antenna array has potential to provide higher network capacity.

Fig. 6.5 shows the coverage probabilities of the three association cases and the overall system. The curves of coverage probability for cases 1, 2, and 3 are obtained as $\Delta_m P_{co}^m$, $\Delta_c P_{co}^c$, and $\Delta_s P_{co}^s$, respectively, whereas the curves labelled “Overall system” is defined as (6.68). Due to high antenna gain and transmit power from the MBS, the coverage probability of case 1 is always higher than that of case 3 over the entire range of threshold T . Due to advances of joint transmission and high association probability, the average coverage probability of case 2 is higher than that of cases 1 and 3 at low and medium threshold T regime. However, as the typical user is far away from the MBS_0 (i.e., $x > R = 200$ m in this example), its received signal has high path loss attenuation. Moreover, in case 2, the received signal power from SBS_0 is at most $\mu = 3$ dB (i.e., μ is the operation threshold) greater than that from MBS_0 . Hence, the high

Figure 6.5: Coverage probability of the proposed scheme versus threshold T with $R = 200$ m, $\lambda_2 = 5\lambda_1$, $N = 100$, $P_m = 43$ dBm, $P_2 = 30$ dBm, and $\mu = 3$ dB.

QoS delivered to the typical user in case 2 can be low. As a result, for case 2, the probability that the typical user receives a high value of threshold T is low. At high SIR regime, i.e., $T \geq 13$ dB, due to smaller path loss attenuation from the MBS_0 , the average coverage probability of case 1 is higher than that of case 2. Note that the typical user is far away from the MBS_0 in cases 2 and 3. Since the received signal power from SBS_0 in case 3 is at least $\mu = 3$ dB greater than that from MBS_0 , then the average coverage probability of case 3 can be slightly greater than that of case 2 at high SIR regime. On the physical interpretation, it means that when the typical user is far away from both MBS and SBS, it cannot receive a high average power level of the desired signal. In reality, the typical user is considered as the cell-edge user. Hence, we can conclude that the coordination mode can be used to improve significantly the coverage and capacity for cell-edge users which do not require high QoS. However, to support high QoS for the users, it is necessary to make the cells smaller (i.e., to increase the number of BSs) or to increase the transmit power of BSs. Based on the ecological and economic perspective, the

Figure 6.6: Coverage probability versus R with $\lambda_2 = 5\lambda_1$, $N = 100$, $P_m = 43$ dBm, and $P_2 = 30$ dBm.

operators can build an appropriate network deployment. Generally, it has been shown that the analytical results of the three cases and overall system are close to that of their simulated results, respectively. It validates our analysis.

The network performance is affected by choosing different values of R as presented in Fig. 6.3. To optimize the network performance, an appropriate value of R can be defined by different criteria, such as based on the single-user throughput performance in [127], and based on the same achievable rate of users in the inner region (named as inner users) as compared with the MRP association scheme in [128]. Herein, we use coverage probability based on the SIR threshold T . In particular, the coverage probability of the proposed scheme P_{co} versus R with different thresholds T is shown in Fig. 6.6. In order to achieve high coverage performance, i.e., low threshold $T = 5$ (dB), the mobile operator can choose $R = 800$ m. As R is large, the sum rate of inner users in case 1 is high due to high Δ_m . However, the inner users, which are located far away from MBS, may have low SINR in the presented scheme. It is a possible

Figure 6.7: Area spectral efficiency and energy efficiency versus λ_2/λ_1 when $R = 300$ m, $P_m = 43$ dBm and $P_2 = 30$ dBm.

choice for users with low rate requirements. For given medium coverage performance and users with medium rate requirements, i.e., $T = 10$ (dB), $R = 600$ m can be chosen. For a given high SIR threshold, i.e., $T = 20$ dB, a near-optimal value of P_{co} can be achieved with $R = 300$ m. Hence, for the given network parameters, based on different QoS requirements, the operator can choose an appropriate value of R . We use $R = 300$ m for the next discussion.

In Fig. 6.7, we study the effect of different density ratio of the MBS's density to the SBS's density (i.e., λ_2/λ_1) on the network performance in terms of the ASE defined as (6.31) and the EE defined as (6.32) when $K = 5$, $R = 300$ m, $P_m = 43$ dBm and $P_2 = 30$ dBm. For a given λ_1 , as λ_2 increases, the number of SBSs is increased. Hence, the ASE is increased due to reducing distance between the typical user and SBSs. However, the power consumption of the whole network increases with λ_2 . As presented in Fig. 6.7, there exists a maximizer λ_2^* that the EE increases with λ_2 when $\lambda_2 \leq \lambda_2^*$ and decreases with λ_2 when $\lambda_2 \geq \lambda_2^*$. This means that the EE is a quasi-concave function of λ_2 . Moreover, increasing the number of antennas N at the MBS can increase the ASE and change the maximizer λ_2^* . The effects of these parameters on the optimal value of EE are clearly shown in the following.

Figure 6.8: Energy efficiency of the proposed scheme when $R = 300$ m and $P_2 = 30$ dBm.

Finally, we obtain the optimal values of N , P_m and λ_2 that are needed to achieve the maximum EE. Particularly, Fig. 6.8 shows the EE of the proposed HetNet as a function of N and P_m when the system has unit bandwidth, $R = 300$ m, and $P_2 = 30$ dBm. Each point is obtained by using the optimal value of λ_2 as presented in Section 6.3.3. In this example, we set $N_{\min} = 10$, $\lambda_2^{\max} = 9 \times 10^{-5}$, $\delta = 10^{-8}$, $\epsilon = 10^{-6}$, $\tau_i^{\min} = 0.5$ bits/s/Hz, and $C_{ASE}^{\min} = C_{ASE}(\tau_i^{\min}, \lambda_2^{\min})$, $\forall i \in \Theta$. An optimal solution obtained by the Brute-force ES algorithm, named as the global optimum EE, is also provided for comparisons. As observed from Fig. 6.8, there is a global optimum value of EE, which is equal to 0.358 (bits/Joule) at $N^{opt} = 428$, $P_m^{opt} = 15$ dBm, and $\lambda_2^{opt} = 2.325 \times 10^{-5} \text{ m}^{-2}$. The number of feasible initial sets, which is randomly generated in the AO algorithm, is 10. We observe that the algorithm converges very quickly after 4 iterations and the iterative progression for maximizing the EE is demonstrated in Fig. 6.8. The convergence of the algorithm is presented with squares, whereas

Figure 6.9: Convergence of the AO algorithm for maximizing total EE with different values of R when $P_2 = 30$ dBm.

the global optimum EE is presented with an asterisk. This fact can be explicitly observed in Fig. 6.9. Moreover, Fig. 6.9 also presents the EE results when $R = 200$ m. As expected, the AO algorithm also converges quickly. In this case, the global maximum value of the EE is 0.338 (bits/Joule) at $N^{opt} = 446$, $P_m^{opt} = 8$ dBm, and $\lambda_2^{opt} = 2.325 \times 10^{-5} \text{ m}^{-2}$. As $R = 200$ m, the proposed scheme provide lower EE than that of $R = 300$ m because it offers lower average achievable rate for the typical user as shown in Fig. 6.3. In general, we observe that the optimal energy efficient system is a massive MIMO setup, i.e., N^{opt} is large. As discussed earlier in Fig. 6.4, the rate approximation of the typical user converges to the exact one when N is large. Hence, the optimal solution shown in Figs. 6.8 and 6.9 can be considered as the actual optimal solution for maximizing the EE of the proposed scheme. Note that this is the output by solving the optimization problem, in which the system parameters are not restricted. In general, the AO algorithm provides an effective solution to achieve the global optimum EE. This algorithm has low complexity and it performs well.

6.6 Summary

In order to improve the coverage and capacity, the CoMP in the two-tier HetNet with massive MIMO and non-uniform small-cell deployment has been proposed. The tractable rate approximation of the proposed scheme has been derived. The EE maximization problems have been formulated, and it has been effectively solved by using the AO algorithm. In order to obtain insightful design guidelines, the impacts of massive MIMO, SBS density, and the inner region size of MBS on the network performance have been examined. The coverage probability of the HetNet has been analyzed by applying the Gamma approximation of the aggregate interference. Numerical results have shown that the proposed scheme outperforms the conventional MRP scheme. The proposed scheme can be used to enhance the performance of cell-edge users which do not require high QoS. In addition, by increasing the number of MBS antennas, the achievable rate and coverage probability can be increased without affecting the existing aggregate interference environment in the network. Based on the combinational use of small cell deployment and massive MIMO, the two-tier HetNet can achieve high spectral efficiency and energy efficiency.

Chapter 7

Conclusion and Future Research Directions

7.1 Conclusion

Massive MIMO is one of the “big three” potential key technologies to meet high data traffic demands for 5G wireless and mobile communication systems [8]. It offers huge benefits in terms of capacity, link reliability and robustness. Linear processing techniques have been considered as practical solutions for massive MIMO systems due to their low complexity and good performance. In this thesis, the fundamental performance in terms of SE and EE for various downlink massive MIMO systems have been analyzed in the TDD protocol. In single-cell scenarios, since the achievable SEs of massive MIMO systems with MF, ZF, RZF precoding schemes and single-antenna users have been studied in literature [6, 7, 12, 27], herein, we have derived tight lower and upper bounds on the achievable SE of massive MIMO systems with SLNR-PS under perfect and imperfect CSI. Different from ZF-PS solution, the SLNR-based scheme takes into account the influence of noise when determining the beamforming vectors for all users. Hence, it has been shown that the SLNR-PS outperforms the ZF-PS. Moreover, based

on the realistic power consumption model, the EE maximization problem have been formulated and solved successfully by applying an alternating optimization algorithm. It has been shown that the proposed algorithm has low complexity and quick convergence. The global optimum EE of SLNR-PS can be achieved by a massive MIMO setup with the optimal transmit power and optimal length of training sequence. A similar conclusion has been reported for ZF and MMSE precoding schemes in [26]. The rate profile optimization problem for individual users has also been considered under the constraints of QoS, transmit power and EE target. The Pareto boundary of the achievable rate region, which composes of optimal per-user rates, has been presented successfully.

For multiple-antenna users, the massive MIMO systems with BD-PS and BD-STBC can be applied. The upper bounds on the achievable SE of these schemes have been derived, respectively, when the number of BS antennas and the number of antennas per users are large while their ratio remains bounded. It has been shown that the SEs of these schemes are concave functions with respect to the number of users in the feasible range. The optimal number of users and optimal length of training sequence for maximizing the corresponding SEs of these schemes have been analytically investigated, respectively.

Pilot contamination is a major bottleneck of massive MIMO systems according to pilot reuse in multicell scenarios. When the number of BS antennas is large, the SINR of mobile user converges to a deterministic value, which depends on users' location and large-scale fading coefficient. Based on this fact, we have proposed a scheme based on vertical beamforming and optimal power allocation to minimize the effect of pilot contamination for 3D-MIMO systems in the cellular networks. Numerical results have shown that the presented scheme can achieve remarkable performance and it has low complexity.

The future 5G wireless communication systems will be dense and heterogeneous due to the deployment of small-cells and various BS types [155]. To characterize the heterogeneity in the

network architecture, the locations of BSs in the multi-tier HetNets can be modelled according to the two-dimensional Poisson Point Processes with different spatial densities. In this thesis, we have also considered a two-tier HetNet with massive MIMO and practical deployment. The MBS has been assumed to be employed massive MIMO technology, the single-antenna SBSs have been assumed to be non-uniformly distributed. A coordinated multipoint transmission between MBS and its own SBSs has been proposed to improve the coverage probability and capacity. Under shared spectrum operation, the tractable approximation of average achievable rate for the proposed scheme has been derived, and it is then used to investigate the EE maximization problem. Based on that, we have studied the effects of the number of MBS antennas, MBS transmit power, SBS density and inner region size of MBS on the overall EE. The coverage probability of the two-tier HetNet has also been analytically investigated. It has been shown that the combinational use of the massive MIMO technology, small-cell deployment and joint transmission is a suitable approach to achieve high SE and EE performance for future cellular networks.

7.2 Future Research Directions

In the thesis, we have analytically studied the performance of massive MIMO in various wireless communication systems. However, there are still some challenging issues that need to be further investigated. As far as future work is concerned, we identify a few potential problems in the following

1. *5G Wireless HetNet based on mmWave and massive MIMO*: It has been shown that massive MIMO for cellular bands (below 6 GHz) and millimeter bands (30-300 GHz) are two possible branches of the same tree, where the cellular bands have been commonly studied in the literature. Although we focus on massive MIMO and the combinational use of massive MIMO and development of small-cells in this thesis, for future research

directions, there are several reasons that we will consider to apply the mmWave communication for 5G wireless HetNet [164, 165]. Firstly, the mmWave frequency range can offer a huge segment of spectrum that is still underutilized, which possesses many exciting research opportunities [8, 129]. Secondly, as the mmWaves have an extremely short wavelength, it becomes possible to pack a large number of antenna elements into a small area, which consequently helps realize massive MIMO at both the BSs (MBS and SBS) and user terminal. A massive hybrid array consisting of multiple analog subarrays, with each subarray having its digital processing chain has been proposed in [166]. It is an attractive solution for future mmWave cellular communications. Thirdly, in realistic scenarios, mmWave frequencies are used for outdoor point-to-point backhaul links or for supporting indoor high-speed wireless applications such as high resolution multimedia streaming [167, 168]. These frequencies have already been standardized for short-range services in IEEE 802.11ad. Therefore, in the 5G cellular architecture, massive MIMO and mmWave technologies can be adopted in different parts and for different communication purposes. Specifically, we assume that both MBSs and SBSs are equipped with large-scale antenna arrays. The MBSs use the cellular bands for long-range outdoor communications, whereas the SBSs use mmWave bands for short-range outdoor communications. The combinational use of massive MIMO, network densification and mmWave technology is expected to improve the network performance significantly. However, various challenges in this network architecture need to be investigated. These challenges include network planning, backhaul design, as well as radio resource and interference management (such as traffic scheduling, cell association and mobility management, channel allocation, power control, and QoS requirements).

2. *Hardware impairments:* Although a massive MIMO system provides remarkable benefits, it is faced with many research challenges that require a lot of efforts to be made, such as hardware impairments. It has been shown in [102] and [169] that the hardware effects

can lead to channel estimation error which limits the system performance. Moreover, the low-cost components can be used to build massive MIMO systems [8]. It implies that hardware imperfections are larger, such as in-phase (I)/quadrature (Q) imbalance and phase noise [42]. It has been shown that low-cost and energy-efficient analog-to-digital (or digital-to-analog) converters yield higher levels of quantization noise [8]. Thus, the effect of these factors on the performance of massive MIMO systems should be analyzed.

3. *Optimized signal processing:* To support high-speed wireless applications for the 5G wireless networks, the signal processing at the massive MIMO base stations should be simple and effective. This is because the BS needs to generate huge amounts of baseband data which are required to process effectively in real time [8]. It is well-known that the BS can adopt the linear precoders/detectors, whose complexity mainly relies on the inversion of large-dimensional matrices. Several researches have been carried on about low-complexity approximate matrix inversion using polynomial expansion [40, 44]. In particular, the approximated solutions of the matrix inversion for MMSE and RZF precoding schemes have been presented in [40] and [44], respectively. For massive MIMO detection, a diagonal band Newton iteration method has been recently discussed in [170] to approximate the matrix inversion. Moreover, in order to minimize both complexity and cost of hardware, the number of RF chains is assumed to be finite with $M \ll N$, where N is the number of BS antennas. For given RF chain constraints M , the two-stage precoding scheme, which employs phase only control at the RF domain and a low-dimensional baseband precoding (such as ZF or MMSE) based on the effective channel seen from baseband, has been studied to reduce the complexity of baseband signal processing significantly [41, 171, 172]. The dimension of the baseband precoding is now equal to M , instead of N in the conventional massive MIMO systems. It has been shown in [171] that the proposed scheme provides comparable performance as compared with the conventional massive MIMO systems. This is because the performance loss due to limiting the number of RF chains is typically neg-

ligible when N is large [171]. A similar approach has been applied for the uplink massive MIMO systems in [173]. However, the prior works in [41, 171–173] only focused on the case of perfect CSI. It is interesting to investigate the system performance when the BS estimates CSI imperfectly. In addition, more optimized methods with low complexity and fast convergence are required to improve the performance of massive MIMO systems in the UL and/or DL scenarios.

4. Due to the limitation in space, a practical implementation of the large-scale antenna array maybe need to reduce the inter-element spacing which adversely affects the system performance due to the spatial and temporal channel correlation. Recently, a channel correlation modeling and its application to massive MIMO channel feedback reduction have been presented in [174]. In [175], based on the channel statistics from the dynamic and analytical channel spatial correlation models, the authors have designed efficient pilot beam patterns for frequency-division duplex (FDD) massive MIMO systems. In [176], the performance of massive MIMO systems over the spatially and temporally correlated Rician fading channel has been investigated under a high-speed railway scenario. The impacts of the velocity, Rician factor, and SNR have been discussed. Note that the existing works have mainly focused on analyzing the system performance at the cellular frequency bands. To reduce the effect of the channel correlation, effective beamforming schemes operating at both low and high frequency bands are also open to research in the low and medium SNR regions.

Appendix A

A.1 Proof of Theorem 1

The first derivative of the cost function P1 in (3.21) with respect to K is given by

$$\frac{\partial C_p}{\partial K} = \underbrace{\left(1 - \frac{2K}{T}\right) M \log_2 \left(a_1 + \frac{b_1}{K}\right)}_{d_1(K)} + \underbrace{\frac{M b_1 (\frac{1}{T} - \frac{1}{K})}{\ln 2 (a_1 + \frac{b_1}{K})}}_{d_2(K)}, \quad (\text{A.1})$$

where $a_1 = 1 - P$ and $b_1 = (N/M + 1)P$. From (3.21), we observe that the maximum number of users K is equal to T . From (A.1), $d_1(K)$ is positive in the interval $[0, T/2]$ and negative in the interval $(T/2, T]$. In addition, $d_2(K)$ is negative in the interval $[0, T]$, thus (A.1) will always be negative in the interval $(T/2, T]$. It implies that C_p is a monotonically decreasing function for $T/2 < K \leq T$. Then we need to investigate the concavity of (3.21) in the interval $[0, T/2]$.

The second derivative of the cost function P1 is given by

$$\frac{\partial^2 C_p}{\partial K^2} = \frac{M}{T \ln 2} \left\{ \underbrace{-2 \ln \left(a_1 + \frac{b_1}{K}\right)}_{\tau_1(K)} + \underbrace{\frac{b_1 (2K^2 a_1 + 3K b_1 - b_1 T)}{K^3 (a_1 + \frac{b_1}{K})^2}}_{\tau_2(K)} \right\}. \quad (\text{A.2})$$

We observe that $\frac{\partial^2 C_p}{\partial K^2}$ is monotonically increasing in the interval $(0, T/2)$ because $\tau_1(K)$ and $\tau_2(K)$ are monotonically increasing functions in the interval $(0, T/2)$. From (A.2), we obtain

$\lim_{K \rightarrow 0} \frac{\partial^2 C_p}{\partial K^2} < 0$. Since $K = T/2$, the negative region of $\partial^2 C_p(T/2)$ is given by

$$-\ln(a_1 + \frac{2b_1}{T}) + \frac{2a_1b_1/T + 2b_1^2/T^2}{(a_1 + \frac{2b_1}{T})^2} < 0. \quad (\text{A.3})$$

Let us define $t_1 = a_1 + \frac{2b_1}{T}$, (A.3) can be rewritten as

$$2t_1^2 \ln t_1 - t_1^2 + a_1^2 > 0. \quad (\text{A.4})$$

Equation (A.4) always holds true. Thus, C_p in (3.21) is concave in the interval $[0, T/2]$. By letting $\frac{\partial C_p}{\partial K} = 0$, the maximiser K_1^* can be obtained by applying the solution for the Lambert-W function $W(z)$ [177]. From γ_{th1} in (3.21), we get $K \leq \frac{N/M+1}{\gamma_{th1}/P+1}$. Hence, the proof is complete.

A.2 Proof of Theorem 2

The proof of Theorem 2 is similar to the proof of Theorem 1. The first derivative of the cost function P2 in (3.22) with respect to K is given by

$$\frac{\partial \check{C}^{up}}{\partial K} = \left(1 - \frac{2K}{T}\right) R_c \log_2 \left(1 + \frac{v}{K}\right) + \frac{v R_c \left(\frac{1}{T} - \frac{1}{K}\right)}{\ln 2 \left(1 + \frac{v}{K}\right)}, \quad (\text{A.5})$$

where $v = \frac{PM}{R_c}$. The second derivative of the cost function P2 is obtained as

$$\frac{\partial^2 \check{C}^{up}}{\partial K^2} = \underbrace{\frac{-2R_c}{T} \log_2 \left(1 + \frac{v}{K}\right)}_{g_1(K)} + \underbrace{\frac{R_c v (2K^2 + 3Kv - vT)}{T \ln 2 K (K + v)^2}}_{g_2(K)}. \quad (\text{A.6})$$

From (A.6), we can get $\lim_{K \rightarrow 0} \frac{\partial^2 \check{C}^{up}}{\partial K^2} < 0$. Since $K = T/2$, the negative region of $\partial^2 \check{C}^{up}(T/2)$ is given by

$$-\frac{2R_c}{T} \log_2 \left(1 + \frac{2v}{T}\right) + \frac{R_c v (T + v)}{T \ln 2 (T/2)^2 (1 + 2v/T)^2} < 0. \quad (\text{A.7})$$

Let us define $t_2 = 1 + \frac{2v}{T}$, (A.7) always holds true since $2t_2^2 \ln t_2 - t_2^2 + 1 > 0$. Thus, the function $\ddot{C}^{up}$ is concave in the interval $[0, T/2]$. By letting $\frac{\partial \ddot{C}^{up}}{\partial K} = 0$, the maximiser K_2^* can be obtained. From γ_{th2} in (3.22), we get $K \leq \frac{PM}{\gamma_{th2}R_c}$. Therefore, the proof is complete.

Appendix B

B.1 Proof of Theorem 6

Based on the fraction programming and generalized concavity theories, the property of the energy-efficient function for a general case, i.e., $\eta(\zeta) = \frac{Y_1(\zeta)}{Y_2(\zeta)}$ with $Y_1(\zeta) : \mathcal{C} \subseteq \mathbb{R}^n \rightarrow \mathbb{R}$ and $Y_2(\zeta) : \mathcal{C} \subseteq \mathbb{R}^n \rightarrow \mathbb{R}_+$, has been recently studied in [178]. However, the optimal solution of $\eta(\zeta)$ has not been shown. As a specific case, we investigate property of the objective function $\eta(\tilde{z})$ in (4.39) in the following. Let us define the upper level sets $S_{\phi_{th}} = \{\tilde{z} \geq \tilde{z}_{\min} | \eta(\tilde{z}) \geq \phi_{th}\}$. In order to prove that the objective function $\eta(\tilde{z})$ is quasi-concave, we need to prove the sets $S_{\phi_{th}}$ of $\eta(\tilde{z})$ are convex for any $\phi_{th} \in \mathbb{R}$ [179, Section 3.4]. Particularly, we observe that if $\phi_{th} \leq 0$, this set is empty, hence it is convex. If $\phi_{th} > 0$, this set is non-empty, $S_{\phi_{th}}$ can be rewritten as

$$S_{\phi_{th}} = \left\{ \tilde{z} \geq \tilde{z}_{\min} | \phi_{th}(\tilde{z} + d) - f_0 \ln \left(\frac{a\tilde{z}}{\tilde{z} + b} + c \right) \leq 0 \right\}. \quad (\text{B.1})$$

We denote $L(\tilde{z}) = \phi_{th}(\tilde{z} + d) - f_0 \ln \left(\frac{a\tilde{z}}{\tilde{z} + b} + c \right)$. We observe that $L(\tilde{z})$ is strictly convex with respect to $\tilde{z}$ because its Hessian is positive definite. Thus, $S_{\phi_{th}}$ is also strictly convex when $\phi_{th} > 0$. It means that $\eta(\tilde{z})$ is a quasi-concave function of $\tilde{z}$.

If there exists a point $\tilde{z}^*$ such that $\frac{\partial \eta(\tilde{z}^*)}{\partial \tilde{z}} = 0$, then $\tilde{z}^*$ is the global maximizer due to property

of the quasi-concavity. By setting the first derivative of $\eta(\tilde{z})$ to be zero, we get

$$\left(\frac{a\tilde{z}}{\tilde{z}+b}+c\right)\ln\left(\frac{a\tilde{z}}{\tilde{z}+b}+c\right)=\frac{ab(\tilde{z}+d)}{(\tilde{z}+b)^2}. \quad (\text{B.2})$$

By using the solution for the Lambert-W function, the optimal value of $\tilde{z}$ is obtained. Therefore, the proof is complete.

B.2 Proof of Theorem 7

At high SNR regime, i.e. $\rho_d \rightarrow \infty$, and ρ_t is finite, from (4.28), the objective function R_s in (4.48) is obtained as

$$R_s = \left(1 - \frac{T_t}{T}\right) K \log_2 \left(v + \frac{T_t \rho_t \beta_k}{v}\right). \quad (\text{B.3})$$

By setting $\frac{\partial R_s}{\partial T_t} = 0$, we have

$$\ln \left(v + T_t \rho_t \beta_k / v\right) = \frac{v + T \rho_t \beta_k / v}{v + T_t \rho_t \beta_k / v} - 1. \quad (\text{B.4})$$

Let us define $\tilde{y}(T_t) = \frac{c_8}{v + T_t \rho_t \beta_k / v}$, where c_8 is defined in (4.53). From (B.4), we get

$$c_8 e = \tilde{y}(T_t) e^{\tilde{y}(T_t)}. \quad (\text{B.5})$$

By applying the solution for the Lambert-W function, after some algebraic manipulations, the optimal training length T_t^* is defined, which completes the proof.

List of Publications

- [1] T. X. Tran, S. Luo, and K. C. Teh, “Millimeter wave massive MIMO networks with energy harvesting,” Submitted to the *IEEE Commun. Lett.*, under review.
- [2] S. Luo, T. X. Tran, K. C. Teh, and K. H. Li, “Adaptive spatial modulation for uplink mmWave communication systems,” *IEEE Commun. Lett.*, doi: 10.1109/LCOMM.2017.2724556.
- [3] T. X. Tran and K. C. Teh, “Performance analysis of massive MIMO systems with SLNR-based precoding under imperfect CSI,” Submitted to the *IEEE System Journal*, under review.
- [4] T. X. Tran and K. C. Teh, “Performance analysis of two-tier HetNets with massive MIMO and non-uniform small cell deployment,” Submitted to the *IEEE Trans. Veh. Technol.*, under revision.
- [5] T. X. Tran and K. C. Teh, “Network sum-rate optimization for multicell massive MIMO downlink based on coordinated tilt adaptation and game theory approach,” *IEEE Wireless Commun. Lett.*, vol. 5, no. 1, pp. 64-67, Feb. 2016.
- [6] T. X. Tran and K. C. Teh, “Energy and spectral efficiency of leakage-based precoding for large-scale MU-MIMO systems,” *IEEE Commun. Lett.*, vol. 19, no. 11, pp. 2041-2044, Sept. 2015.

LIST OF PUBLICATIONS

- [7] T. X. Tran and K. C. Teh, “Performance analysis of massive multiuser MIMO systems with block diagonalisation,” *IET Commun.*, vol. 10, no. 7, pp. 832- 838, Jan. 2016.
- [8] T. X. Tran and K. C. Teh, “Error probability analysis of a novel adaptive beamforming receiver for large-scale multiple-input multiple-output communication system,” *IET Commun.*, vol. 9, no. 2, pp. 219- 299, Jan. 2015.
- [9] T. X. Tran and K. C. Teh, “A novel beamforming receiver with adaptive detection for large-scale MIMO communication system,” in *Proc. IEEE Wireless Commun. Netw. Conf. (WCNC)*, New Orleans, LA, Mar. 2015, pp. 405-410.

Bibliography

- [1] A. Goldsmith, *Wireless Communications*, Cambridge, U.K.: Cambridge Univ. Press, 2005.
- [2] L. Sorensen and K. E. Skouby, *User Scenarios 2020*, Jul. 2009. [Online]. Available: <http://www.wwrf.ch/files/wwrf/content/files/publications/outlook/Outlook4.pdf>.
- [3] E. Dahlman, S. Parkvall, J. Skold, and P. Beming, *3G Evolution HSPA and LTE for Mobile Broadband*, New York: Academic, 2008.
- [4] C. X. Wang et al., “Cellular architecture and key technologies for 5G wireless communication networks,” *IEEE Commun. Mag.*, vol. 52, no. 2, pp. 122-129, Feb. 2014.
- [5] J. G. Andrews, S. Buzzi, W. Choi, S. V. Hanly, A. Lozano, A. C. K. Soong, and J. Zhang, “What will 5G be?,” *IEEE J. Sel. Areas Commun.*, vol. 32, no. 6, pp. 1065-1082, Jun. 2014.
- [6] F. Rusek, D. Persson, B. K. Lau, E. G. Larsson, O. Edfors, F. Tufvesson, and T. L. Marzetta, “Scaling up MIMO: Opportunities and challenges with very large arrays,” *IEEE Signal Process. Mag.*, vol. 30, no. 1, pp. 40-46, Jan. 2013.
- [7] T. L. Marzetta, “Noncooperative cellular wireless with unlimited numbers of base station antennas,” *IEEE Trans. Wireless Commun.*, vol. 9, no. 11, pp. 3590-3600, Nov. 2010.
- [8] E. G. Larsson, F. Tufvesson, O. Edfors, and T. L. Marzetta, “Massive MIMO for next generation wireless systems,” *IEEE Commun. Mag.*, vol. 52, no. 2, pp. 186-195, Feb. 2014.

BIBLIOGRAPHY

- [9] J. Hoydis, S. T. Brink, and M. Debbah, “Massive MIMO in the UL/DL of cellular networks: How many antennas do we need?” *IEEE J. Sel. Areas Commun.*, vol. 31, no. 2, pp. 160-171, Feb. 2013.
- [10] T. L. Marzetta, “How much training is required for multiuser MIMO?” in *Proc. IEEE Asilomar Conf. on Signals, Systems and Computers (ACSSC)*, Pacific Grove, CA, US, Nov. 2006, pp. 359-363.
- [11] R. W. Health, M. Kountouris, and T. Bai, “Modeling heterogeneous network interference using Poisson point processes,” *IEEE Trans. Signal Process.*, vol. 61, no. 16, pp. 4114-4126, Aug. 2013.
- [12] S. Wagner, R. Couillet, M. Debbah, and D. T. M. Slock, “Large system analysis of linear precoding in correlated MISO broadcast channels under limited feedback,” *IEEE Trans. Inf. Theory*, vol. 58, no. 7, pp. 4509-4537, Jul. 2012.
- [13] H. Q. Ngo, E. G. Larsson, and T. L. Marzetta, “Energy and spectral efficiency of very large multiuser MIMO systems,” *IEEE Trans. Commun.*, vol. 61, no. 4, pp. 1436-1449, Apr. 2013.
- [14] K. Zu and R. C. Lamare, “Low-complexity lattice reduction-aided regularized block diagonalization for MU-MIMO systems,” *IEEE Commun. Lett.*, vol. 16, no. 6, pp. 925-928, Apr. 2012.
- [15] K. Zu, R. C. Lamare, and M. Haardt, “Generalized design of low-complexity block diagonalization type precoding algorithms for multiuser MIMO systems,” *IEEE Trans. Commun.*, vol. 61, no. 10, pp. 4232-4242, Sept. 2013.
- [16] M. H. A. Khan, K. M. Cho, M. H. Lee, and J. G. Chung, “A simple block diagonal precoding for multi-user MIMO broadcast channels,” *EURASIP J. Wireless Commun. Netw.*, Article id 1687-1499, 2014.

BIBLIOGRAPHY

- [17] D. H. N. Nguyen, H. L. Nguyen, and T. N. Le, "Block diagonalization precoding in a multiuser multicell MIMO system: Competition and coordination," *IEEE Trans. Commun.*, vol. 14, no. 2, pp. 968-981, Jan. 2014.
- [18] M. Sadek, A. Tarighat, and A. H. Sayed, "A leakage-based precoding scheme for downlink multi-user MIMO channels," *IEEE Trans. Wireless Commun.*, vol. 6, no. 5, pp. 1711-1721, May 2007.
- [19] F. Fernandes, A. Ashikhmin, and T. L. Marzetta, "Inter-cell interference in noncooperative TDD large scale antenna systems," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 2, pp. 191-201, Feb. 2013.
- [20] J. Jose, A. Ashikhmin, T. L. Marzetta, and S. Vishwanath, "Pilot contamination and precoding in multi-cell TDD systems," *IEEE Trans. Wireless Commun.*, vol. 10, pp. 2640-2651, Aug. 2011.
- [21] X. Zhu, Z. Wang, L. Dai, and C. Qian, "Smart pilot assignment for massive MIMO," *IEEE Commun. Lett.*, vol. 19, no. 9, pp. 1644-1647, Sept. 2015.
- [22] R. Muller, L. Cottatellucci, and M. Vehkaperä, "Blind pilot decontamination," *IEEE J. Sel. Top. Signal Process.*, vol. 8, no. 5, pp. 773-786, Oct. 2014.
- [23] H. Yin, D. Gesbert, M. Filippou, and Y. Liu, "A coordinated approach to channel estimation in large-scale multiple-antenna systems," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 2, pp. 264-273, Feb. 2013.
- [24] Y. S. Cho, J. Kim, W. Y. Yang, and C. G. Kang, *MIMO-OFDM Wireless Communications with MATLAB*. Singapore: John Wiley and Sons (Asia), 2010.
- [25] A. Yang, Z. He, C. Xing, Z. Fei, and J. Kuang, "The role of large-scale fading in uplink massive MIMO systems," *IEEE Trans. Veh. Technol.*, vol. 65, no. 1, pp. 477-483, Jan. 2016.

BIBLIOGRAPHY

- [26] E. Björnson, L. Sanguinetti, J. Hoydis, and M. Debbah, “Optimal design of energy-efficient multi-user MIMO systems: Is massive MIMO the answer?,” *IEEE Trans. Wireless Commun.*, vol. 14, no. 6, pp. 3059-3075, Jun. 2015.
- [27] H. Yang and T. L. Marzetta, “Performance of conjugate and zero forcing beamforming in large-scale antenna system,” *IEEE J. Sel. Areas Commun.*, vol. 31, no. 2, pp. 172-179, Feb. 2013.
- [28] G. Auer, V. Giannini, C. Desset, I. Godor, P. Skillermark, M. Olsson, M. A. Imran, D. Sabella, M. J. Gonzalez, O. Blume, and A. Fehske, “How much energy is needed to run a wireless network?,” *IEEE Trans. Wireless Commun.*, vol. 18, no. 5, pp. 40-49, Oct. 2011.
- [29] H. Kim, C. B. Chae, G. D. Veciana, and J. R. W. Heath, “A cross-layer approach to energy efficiency for adaptive MIMO systems exploiting spare capacity,” *IEEE Trans. Wireless Commun.*, vol. 8, no. 8, pp. 4264-4275, Aug. 2009.
- [30] S. Cui, A. J. Goldsmith, and A. Bahai, “Energy-efficiency of MIMO and cooperative MIMO techniques in sensor networks,” *IEEE J. Sel. Areas Commun.*, vol. 22, no. 6, pp. 1089-1098, Aug. 2004.
- [31] D. Ha, K. Lee, and J. Kang, “Energy efficiency analysis with circuit power consumption in massive MIMO systems,” in *Proc. IEEE Pers. Indoor and Mobile Radio Commun. (PIMRC)*, London, UK, Sept. 2013, pp. 938-942.
- [32] J. Xu and L. Qiu, “Energy efficient optimization for MIMO broadcast channels,” *IEEE Trans. Wireless Commun.*, vol. 12, no. 2, pp. 690-701, Feb. 2013.
- [33] M. Costa, “Writing on dirty paper,” *IEEE Trans. Inf. Theory*, vol. 29, no. 3, pp. 439-441, May 1983.
- [34] B. M. Hochwald, C. B. Peel, and A. L. Swindlehurst, “A vector-perturbation technique for

BIBLIOGRAPHY

- near-capacity multiantenna multiuser communication-Part II: Perturbation,” *IEEE Trans. Commun.*, vol. 53, no. 3, pp. 537-544, Mar. 2005.
- [35] C. Windpassinger, R. F. H. Fischer, and J. B. Huber, “Lattice-reductionaided broadcast precoding,” *IEEE Trans. Commun.*, vol. 52, no. 12, pp. 2057-2060, Dec. 2004.
- [36] M. Joham, K. Kusume, M. H. Gzara, and W. Utschick, “Transmit Wiener filter for the downlink of TDD DS-CDMA systems,” in *Proc. IEEE 7th Symp. Spread-Spectrum Technol.*, Prague, Czech Republic, Sept. 2002, pp. 9-13.
- [37] C. B. Peel, B. M. Hochwald, and A. L. Swindlehurst, “A vector-perturbation technique for near-capacity multiantenna communication-Part I: Channel inversion and regularization,” *IEEE Trans. Commun.*, vol. 53, no. 1, pp. 195-202, Jan. 2005.
- [38] A. D. Dabbagh and D. J. Love, “Multiple antenna MMSE based downlink precoding with quantized feedback or channel mismatch,” *IEEE Trans. Commun.*, vol. 56, no. 11, pp. 1859-1868, Nov. 2008.
- [39] H. Q. Ngo, E. G. Larsson, and T. L. Marzetta, “Massive MU-MIMO downlink TDD systems with linear precoding and downlink pilots,” in *Proc. Allerton Conf. Commun., Control, and Computing*, Urbana-Champaign, Illinois, Oct. 2013, pp. 293-298.
- [40] S. Zarei, W. Gerstacker, R. R. Muller, and R. Schober, “Low-complexity linear precoding for downlink large-scale MIMO systems,” in *Proc. IEEE Pers. Indoor and Mobile Radio Commun. (PIMRC)*, London, Sept. 2013, pp. 1119-1124.
- [41] L. Liang, W. Xu, and X. Dong, “Low-complexity hybrid precoding in massive multiuser MIMO systems,” *IEEE Wireless Commun. Lett.*, vol. 3, no. 6, pp. 653-656, Dec. 2014.
- [42] R. Krishnan, M. R. Khanzadi, N. Krishnan, Y. Wu, A. G. I. Amat, T. Eriksson, and R. Schober, “Linear massive MIMO precoders in the presence of phase noise: A large-scale analysis,” *IEEE Trans. Veh. Technol.*, vol. 65, no. 5, pp. 3057-3071, May 2016.

BIBLIOGRAPHY

- [43] R. Zakhour and S. V. Hanly, "Base station cooperation on the downlink: Large system analysis," *IEEE Trans. Inf. Theory*, vol. 58, no. 4, pp. 2079-2106, Apr. 2012.
- [44] A. Kammoun, A. Muller, E. Björnson, and M. Debbah, "Linear precoding based on polynomial expansion: Large-scale multi-cell MIMO systems," *IEEE J. Sel. Top. Signal Process.*, vol. 8, no. 5, pp. 861-875, Oct. 2014.
- [45] Z. Liu and L. Dai, "Asymptotic per-user rate analysis of downlink MIMO cellular networks with linear precoding," *IEEE Trans. Wireless Commun.*, vol. 11, no. 12, pp. 4536-4548, Dec. 2012.
- [46] S. Lakshminaryana, M. Debbah, and M. Assaad, "Coordinated multicell beamforming for massive MIMO: A random matrix approach," *IEEE Trans. Inf. Theory*, vol. 61, no. 6, pp. 3387-3412, Jun. 2015.
- [47] R. Zakhour and S. V. Hanly, "Min-max power allocation in cellular networks with coordinated beamforming," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 2, pp. 287-302, Feb. 2013.
- [48] Y. Huang, C. W. Tan, and B. D. Rao, "Joint beamforming and power control in coordinated multicell: Max-min duality, effective network and large system transition," *IEEE Trans. Wireless Commun.*, vol. 12, no. 6, pp. 1536-1276, Jun. 2013.
- [49] R. Irmer et al., "Coordinated multipoint: Concepts, performance, and field trial results," *IEEE Commun. Mag.*, vol. 49, no. 2, pp. 102-111, Feb. 2011.
- [50] H. Huh, A. M. Tulino, and G. Caire, "Network MIMO with linear zero-forcing beamforming: Large system analysis, impact of channel estimation, and reduced-complexity scheduling," *IEEE Trans. Inf. Theory*, vol. 58, no. 5, pp. 2911-2934, May 2012.
- [51] H. Huh, S. H. Moon, Y. T. Kim, I. Lee, and G. Caire, "Multi-cell MIMO downlink with cell

BIBLIOGRAPHY

- cooperation and fair scheduling: A large system limit analysis,” *IEEE Trans. Inf. Theory*, vol. 57, no. 12, pp. 7771-7786, Dec. 2011.
- [52] H. Huh, G. Caire, H. C. Papadopoulos, and S. A. Ramprasad, “Achieving “massive MIMO” spectral efficiency with a not-so-large number of antennas,” *IEEE Trans. Wireless Commun.*, vol. 11, no. 9, pp. 3226-3239, Sept. 2012.
- [53] J. Choi, “Distributed beamforming for macro diversity and power control with large arrays in spatial correlated channels,” *IEEE Trans. Wireless Commun.*, vol. 14, no. 4, pp. 1871-1881, Apr. 2015.
- [54] J. Zhang, C. K. Wen, S. Jin, X. Gao, and K. K. Wong, “Large system analysis of cooperative multi-cell downlink transmission via regularized channel inversion with imperfect CSIT,” *IEEE Trans. Wireless Commun.*, vol. 12, no. 10, pp. 4801-4813, Oct. 2013.
- [55] H. Halbauer, S. Saur, J. Koppenborg, and C. Hoek, “3D beamforming: Performance improvement for cellular networks,” *Bell Labs Technic. J.*, vol. 18, no. 2, pp. 37-56, Sept. 2013.
- [56] Y. H. Nam, B. L. Ng, K. Sayana, Y. Li, J. Zhang, Y. Kim, and J. Lee, “Full-dimension MIMO (FD-MIMO) for next generation cellular technology,” *IEEE Commun. Mag.*, vol. 51, no. 6, pp. 172-179, Jun. 2013.
- [57] N. Seifi, M. Coldrey, and M. Viberg, “Throughput optimization for MISO interference channels via coordinated user-specific tilting,” *IEEE Commun. Lett.*, vol. 16, no. 8, pp. 1248-1251, Aug. 2012.
- [58] O. N. C. Yilmaz, S. Hamalainen, and J. Hamalainen, “System level analysis of vertical sectorization for 3GPP LTE,” in *Proc. IEEE Int. Symp. Wireless Commun. Syst. (SWCS)*, Siena, Italy, Sep. 2009, pp. 453-457.

BIBLIOGRAPHY

- [59] Y. Fu, J. Wang, Z. Zhao, L. Dai, and H. Yang, "Analysis of vertical sectorization for HSPA on a system level: Capacity and coverage," in *Proc. IEEE Veh. Technol. Conf. (VTC)*, Quebec, Canada, Sep. 2012, pp. 1-5.
- [60] K. Zheng, L. Zhao, J. Mei, B. Shao, W. Xiang, and L. Hanzo, "Survey of large-scale MIMO systems," *IEEE Commun. Surveys Tuts.*, vol. 17, no. 3, pp. 1738-1760, 3rd Quart. 2015.
- [61] S. E. Nai, Z. Lei, S. H. Wong, and Y. H. Chew, "Optimizing radio network parameters for vertical sectorization via Taguchi's method," *IEEE Trans. Veh. Technol.*, vol. 65, no. 2, pp. 860-869, Mar. 2015.
- [62] Z. Zhang, K. C. Teh, and K. H. Li, "Study of 3D beamforming strategies in cellular networks with clustered user distribution," *IEEE Trans. Veh. Technol.*, doi: 10.1109/TVT.2016.2543259.
- [63] H. Liu, H. Gao, C. Zhang, and T. Lv, "Low-complexity joint antenna tilting and user scheduling for large-scale ZF relaying," *IEEE Signal Process. Lett.*, vol. 22, no. 3, pp. 361-365, Mar. 2015.
- [64] Q. U. A. Nadeem, A. Kammoun, M. Debbah, and M. S. Alouini, "3D massive MIMO systems: Modeling and performance analysis," *IEEE Trans. Wireless Commun.*, vol. 14, no. 12, pp. 6926-6939, Dec. 2015.
- [65] W. Lee, S. R. Lee, H. B. Kong, S. Lee, and I. Lee, "Downlink vertical beamforming designs for active antenna systems," *IEEE Trans. Commun.*, vol. 62, no. 6, pp. 1897-1907, Apr. 2014.
- [66] M. Haenggi, *Stochastic Geometry for Wireless Networks*. Cambridge, U.K.: Cambridge Univ. Press, 2012.
- [67] H. S. Dhillon, M. Kountouris, and J. G. Andrews, "Downlink MIMO HetNets: Modeling,

- ordering results and performance analysis,” *IEEE Trans. Wireless Commun.*, vol. 12, no. 10, pp. 5208-5222, Oct. 2013.
- [68] M. Kountouris and N. Pappas, “HetNets and massive MIMO: Modeling, potential gains, and performance analysis,” in *Proc. IEEE APS Top. Conf. Wireless Commun.*, Torino, Italy, Sep. 2013, pp. 1319-1322.
- [69] A. He, L. Wang, M. Elkashlan, Y. Chen, and K. K. Wong, “Spectrum and energy efficiency in massive MIMO enabled HetNets: A stochastic geometry approach,” *IEEE Commun. Lett.*, vol. 19, no. 12, pp. 2294-2297, Oct. 2015.
- [70] J. Hoydis, K. Hosseini, S. ten Brink, and M. Debbah, “Making smart use of excess antennas: Massive MIMO, small cells, and TDD,” *Bell Labs Technic. J.*, vol. 18, no. 2, pp. 5-21, Aug. 2013.
- [71] L. Sanguinetti, A. L. Moustakas, and M. Debbah, “Interference management in 5G reverse TDD HetNets with wireless backhaul: A large system analysis,” *IEEE J. Sel. Areas Commun.*, vol. 33, no. 6, pp. 1187-1200, Mar. 2015.
- [72] P. Wang, W. Song, D. Niyato, and Y. Xiao, “QoS-aware cell association in 5G heterogeneous networks with massive MIMO,” *IEEE Network*, vol. 29, no. 6, pp. 76-82, Nov. 2015.
- [73] G. Bacci, E. V. Belmega, P. Mertikopoulos, and L. Sanguinetti, “Energy-aware competitive power allocation for heterogeneous networks under QoS constraints,” *IEEE Trans. Wireless Commun.*, vol. 14, no. 9, pp. 4728-4742, Sept. 2015.
- [74] Y. Xu and S. Mao, “User association in massive MIMO HetNets,” *IEEE Systems J.*, doi: 10.1109/JSYST.2015.2475702.
- [75] D. Bethanabhotla, O. Y. Bursalioglu, H. C. Papadopoulos, and G. Caire, “Optimal user-

BIBLIOGRAPHY

- cell association for massive MIMO wireless networks,” *IEEE Trans. Wireless Commun.*, vol. 15, no. 3, pp. 1835-1850, Mar. 2016.
- [76] D. Liu, L. Wang, Y. Chen, T. Zhang, K. K. Chai, and M. ElKashlan, “Distributed energy efficient fair user association in massive MIMO enabled HetNets,” *IEEE Commun. Lett.*, vol. 19, no. 10, pp. 1770-1773, Oct. 2015.
- [77] J. B. Rao and A. O. Fapojuwo, “A survey of energy efficient resource management techniques for multicell cellular networks,” *IEEE Commun. Surveys Tuts.*, vol. 16, no. 1, pp. 154-180, 1st Quart. 2015.
- [78] E. Björnson, L. Sanguinetti, and M. Kountouris, “Deploying dense networks for maximal energy efficiency: Small cells meet massive MIMO,” *IEEE J. Sel. Areas Commun.*, vol. 34, no. 4, pp. 832-847, Apr. 2016.
- [79] Q. H. Spencer, A. L. Swindlehurst, and M. Haardt, “Zero-forcing methods for downlink spatial multiplexing in multiuser MIMO channels,” *IEEE Trans. Signal Process.*, vol. 52, no. 2, pp. 461-471, Feb. 2004.
- [80] H. Sung, S. R. Lee, and I. Lee, “Generalized channel inversion methods for multiuser MIMO systems’,” *IEEE Trans. Commun.*, vol. 57, no. 11, pp. 3489-3499, Nov. 2009.
- [81] D. Lee and K. Kim, “Reliability comparison of opportunistic scheduling and BD-precoding in downlink MIMO systems with multiple users,” *IEEE Trans. Wireless Commun.*, vol. 9, no. 3, pp. 964-969, Mar. 2010.
- [82] W. Ding and T. Lv, “Multiuser MIMO using block diagonalization: How many users should be served?” in *Proc. IEEE Global Telecommun. Conf. (GLOBECOM)*, Atlanta, US, Dec. 2013, pp. 4210-4215.
- [83] R. Hicheri, N. Hajri, N. Youssef, M. Patzold, and T. Kawabata, “Statistical analysis of the

BIBLIOGRAPHY

- channel capacity outage intervals in massive MIMO systems with OSTBC over Rayleigh fading channels,” in *Proc. IEEE Vehic. Technol. Conf. (VTC)*, Glasgow, May 2015, pp. 1-5.
- [84] Y. Zhou, C. Zhong, and Z. Zhang, “Performance comparison of different transmission schemes in uplink massive MIMO systems with dual-antenna users,” in *Proc. IEEE Int. Conf. Wireless Commun. and Signal Process. (WCSP)*, Oct. 2015, pp. 15-17.
- [85] H. Zhang and T. A. Gulliver, “Capacity and error probability analysis for orthogonal space-time block codes over fading channels,” *IEEE Trans. Wireless Commun.*, vol. 4, no. 2, pp. 808-819, Mar. 2005.
- [86] K. S. Ahn, R. W. Heath, and H. K. Baik, “Shannon capacity and symbol error rate of space-time block codes in MIMO Rayleigh channels with channel estimation error,” *IEEE Trans. Wireless Commun.*, vol. 7, no. 1, pp. 324-333, Jan. 2008.
- [87] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*, Englewood Cliffs, NJ, 1st edn, 1993.
- [88] V. Marcenko and L. Pastur, “Distribution of eigenvalues for some sets of random matrices,” *Math USSR Sbornik*, vol. 1, no. 4, pp. 457-483, 1967.
- [89] R. Chen, J. G. Andrews, and R. W. Heath, “Multiuser space-time block coded MIMO system with downlink precoding,” in *Proc. IEEE Int. Conf. Commun. (ICC)*, Jun. 2004, pp. 2689-2693.
- [90] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*. Orlando, FL: Academic, 1980.
- [91] B. Hassibi and B. M. Hochwald, “How much training is needed in multiple-antenna wireless links?,” *IEEE Trans. Inf. Theory*, vol. 49, no. 4, pp. 951-963, Apr. 2003.

BIBLIOGRAPHY

- [92] M. Jung, Y. Kim, J. Lee, and S. Choi, "Optimal number of users in zero-forcing based multiuser MIMO systems with large number of antennas," *IEEE J. Commun. and Networks*, vol. 15, no. 4, pp. 362-369, Aug. 2013.
- [93] P. Patcharamaneepakorn, S. Armour, and A. Doufexi, "On the equivalence between SLNR and MMSE precoding schemes with single-antenna receivers," *IEEE Commun. Lett.*, vol. 16, no. 7, pp. 1034-1037, Jul. 2012.
- [94] P. Patcharamaneepakorn, A. Doufexi, and S. Armour, "Equivalent expressions and performance analysis of SLNR precoding schemes: A generalization to multi-antenna receivers," *IEEE Commun. Lett.*, vol. 16, no. 7, pp. 1034-1037, Jul. 2013.
- [95] W. Tan, S. Jin, J. Wang, and M. Matthaiou, "Achievable sum-rate of multiuser massive MIMO downlink in Ricean fading channels," in *Proc. IEEE Int. Conf. Commun. (ICC)*, London, 2015, pp. 1453-1458.
- [96] D. Kim, G. Lee, and Y. Sung, "Two-stage beamformer design for massive MIMO downlink by trace quotient formulation," *IEEE Trans. Commun.*, vol. 63, no. 6, pp. 2200-2211, Jun. 2015.
- [97] H. Li and M. Debbah, "Energy efficiency of large-scale multiple antenna systems with transmit antenna selection," *IEEE Trans. Commun.*, vol. 62, no. 2, pp. 638-647, Feb. 2014.
- [98] C. Jiang and L. J. Cimini, "Antenna selection for energy-efficient MIMO transmission," *IEEE Wireless Commun. Lett.*, vol. 1, no. 6, pp. 577-580, Dec. 2012.
- [99] X. Zhou, B. Bai, and W. Chen, "An iterative algorithm for joint antenna selection and power adaptation in energy efficient MIMO," in *Proc. IEEE Int. Conf. Commun. (ICC)*, Sydney, AUS, pp. 3812-3816, Jun. 2014.
- [100] T. M. Nguyen, V. H. Nguyen, and L. B. Le, "Resource allocation optimization in multi-

BIBLIOGRAPHY

- user multi-cell massive MIMO networks considering pilot contamination,” *IEEE Access*, vol. 3, pp. 1272-1287, Aug. 2015.
- [101] S. Tombaz, A. Vastberg, and J. Zander, “Energy- and cost-efficient ultra-high-capacity wireless access,” *IEEE Wireless Commun. Mag.*, vol. 18, no. 5, pp. 18-24, 2011.
- [102] E. Björnson, J. Hoydis, M. Kountouris, and M. Debbah, “Massive MIMO systems with non-ideal hardware: Energy efficiency, estimation and capacity limits,” *IEEE Trans. Inf. Theory*, vol. 60, no. 11, pp. 7112-7139, Nov. 2014.
- [103] S. K. Mohammed, “Impact of transceiver power consumption on the energy efficiency of zero-forcing detector in massive MIMO systems,” *IEEE Trans. Commun.*, vol. 62, no. 11, pp. 3874-3890, Nov. 2014.
- [104] J. Jose, A. Ashikhmin, P. Whiting, and S. Vishwanath, “Channel estimation and linear precoding in multiuser multiple-antenna TDD systems,” *IEEE Trans. Veh. Technol.*, vol. 60, no. 5, pp. 2102-2116, Jun. 2011.
- [105] P. Patcharamaneepakorn, A. Doufexi, and S. M. D. Armour, “Weighted sum capacity maximization using a modified leakage-based transmit filter design,” *IEEE Trans. Veh. Technol.*, vol. 62, no. 3, pp. 1177-1188, Mar. 2013.
- [106] R. A. Horn and C. R. Johnson, *Matrix Analysis*. Cambridge, U.K.: Cambridge Univ. Press, 1985.
- [107] Y. Jiang, M. K. Varanasi, and J. Li, “Performance analysis of ZF and MMSE equalizers for MIMO systems: An in-depth study of the high- SNR regime,” *IEEE Trans. Inf. Theory*, vol. 57, no. 4, pp. 2008-2026, Apr. 2011.
- [108] A. M. Tulino and S. Verd, “Random matrix theory and wireless communications,” *Foundations Trends Commun. Inf. Theory*, vol. 1, no. 1, pp. 1-182, Jun. 2004.

BIBLIOGRAPHY

- [109] J. W. Silverstein, "The smallest eigenvalue of a large dimensional Wishart matrix," *Ann. Probab.*, vol. 13, no. 4, pp. 1364-1368, Nov. 1985.
- [110] Z. D. Bai and Y. Q. Yin. "Limit of the smallest eigenvalue of a large dimensional sample covariance matrix," *Ann. Probab.*, vol 21, no. 3, pp. 1275-1294, Jul. 1993.
- [111] P. Li, D. Paul, R. Narasimhan, and J. Cioffi, "On the distribution of SINR for the MMSE MIMO receiver and performance analysis," *IEEE Trans. Inf. Theory*, vol. 52, no. 1, pp. 271-286, Jan. 2006.
- [112] Y. Huang and L. Qiu, "On the energy efficiency-spectral efficiency tradeoff in random beamforming," *IEEE Wireless Commun. Lett.*, vol. 3, no. 5, pp. 461-464, Oct. 2014.
- [113] L. Grippo and M. Sciandrone, "On the convergence of the block nonlinear Gauss-Seidel method under convex constraints," *Oper. Res. Lett.*, vol. 26, pp. 127-136, 2000.
- [114] J. C. Bezdek and R. J. Hathaway, "Convergence of alternating optimization," *Neural, Parallel & Scientific Comput.*, vol. 11, pp. 351-368, 2003.
- [115] D. W. K. Ng, E. S. Lo, and R. Schober, "Energy-efficient resource allocation for secure OFDMA systems," *IEEE Trans. Veh. Technol.*, vol. 61, no. 6, pp. 2572-2585, Jul. 2012.
- [116] R. Mochaourab, P. Cao, and E. Jorswieck, "Alternating rate profile optimization in single stream MIMO interference channels," *IEEE Signal Process. Lett.*, vol. 21, no. 2, pp. 221-224, Feb. 2014.
- [117] 3GPP TR 36.814 V9.0, "Further advancements for E-UTRA physical layer aspects," Tech. Rep., Mar. 2010.
- [118] J. Choi, "Massive MIMO with joint power control," *IEEE Wireless Commun. Lett.*, vol. 3, no. 4, pp. 329-332, Aug. 2014.

BIBLIOGRAPHY

- [119] N. Seifi, J. Zhang, R. W. Heath, T. Svensson, and M. Coldrey, "Coordinated 3D beamforming for interference management in cellular networks," *IEEE Trans. Wireless Commun.*, vol. 13, no. 10, pp. 5396-5410, Oct. 2014.
- [120] W. Zhang, Y. Wang, F. Peng, and Y. Yuan, "Interference coordination with vertical beamforming in 3D MIMO-OFDMA networks," *IEEE Commun. Lett.*, vol. 18, no. 1, pp. 34-37, Jan. 2014.
- [121] D. H. N. Nguyen, H. L. Nguyen, and T. N. Le, "Block-diagonalization precoding in a multiuser multicell MIMO system: Competition and coordination," *IEEE Trans. Signal Process.*, vol. 13, no. 2, pp. 968-981, Feb. 2014.
- [122] G. Scutari, D. P. Palomar, and S. Barbarossa, "Optimal linear precoding strategies for wideband noncooperative systems based on game theory-Parts I: Nash Equilibria," *IEEE Trans. Signal Process.*, vol. 56, no. 3, pp. 1230-1249, Mar. 2008.
- [123] G. Scutari, D. P. Palomar, and S. Barbarossa, "Optimal linear precoding strategies for wideband noncooperative systems based on game theory-Parts II: Algorithms," *IEEE Trans. Signal Process.*, vol. 56, no. 3, pp. 1250-1267, Mar. 2008.
- [124] G. Scutari, D. P. Palomar, F. Facchinei, and J. S. Pang, "Convex optimization, game theory, and variational inequality theory," *IEEE Signal Process. Mag.*, vol. 27, no. 3, pp. 35-39, May 2010.
- [125] N. Seifi, M. Coldrey, and T. Svensson, "Throughput optimization in MU-MIMO systems via exploiting BS antenna tilt," in *Proc. IEEE Global Telecommun. Conf. (GLOBECOM)*, Anaheim, CA, pp. 653-657, Dec. 2012.
- [126] D. Stoyan, W. S. Kendall, and J. Mecke, *Stochastic Geometry and Its Applications*, 2nd ed. John Wiley and Sons, 1995.

BIBLIOGRAPHY

- [127] H. Wang, X. Zhou, and M. C. Reed, "Coverage and throughput analysis with a non-uniform small cell deployment," *IEEE Trans. Signal Process.*, vol. 13, no. 4, pp. 2047-2059 Apr. 2014.
- [128] H. Wu, X. Tao, J. Xu, and N. Li, "Coverage analysis for CoMP in two-tiers HetNets with nonuniformly deployed femtocells," *IEEE Commun. Lett.*, vol. 19, no. 9, pp. 1600-1603, Sept. 2015.
- [129] J. G. Andrews, F. Baccelli, and R. K. Ganti, "A tractable approach to coverage and rate in cellular networks," *IEEE Trans. Commun.*, vol. 59, no. 11, pp. 3122-3134, Nov. 2011.
- [130] Y. S. Soh, T. Q. S. Quek, M. Kountouris, and H. Shin, "Energy efficient heterogeneous cellular networks," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 5, pp. 840-850, May 2013.
- [131] A. Adhikary, H. S. Dhillon, and G. Caire, "Massive-MIMO meets HetNet: Interference coordination through spatial blanking," *IEEE J. Sel. Areas Commun.*, vol. 33, no. 6, pp. 1171-1186, Mar. 2015.
- [132] E. Björnson, M. Kountouris, and M. Debbah, "Massive MIMO and small cells: Improving energy efficiency by optimal soft-cell coordination," in *Proc. 20th Int. Conf. Telecommun.*, May 2013, pp. 1-5.
- [133] W. Liu, S. Jin, C. K. Wen, M. Matthaiou, and X. You, "A tractable approach to uplink spectral efficiency of two-tier massive MIMO cellular HetNets," *IEEE Commun. Lett.*, vol. 20, no. 2, pp. 348-351, Feb. 2016.
- [134] D. Liu, L. Wang, Y. Chen, T. Zhang, K. K. Chai, and M. Elkashlan, "Distributed energy efficient fair user association in massive MIMO enabled hetnets," *IEEE Commun. Lett.*, vol. 19, no. 10, pp. 1770-1773, Oct. 2015.
- [135] G. Nigam, P. Minero, and M. Haenggi, "Coordinated multipoint joint transmission in

BIBLIOGRAPHY

- heterogeneous networks,” *IEEE Trans. Wireless Commun.*, vol. 62, no. 11, pp. 4134-4146, Nov. 2014.
- [136] A. H. Sakr and E. Hossain, “Location-aware cross-tier coordinated multipoint transmission in two-tier cellular networks,” *IEEE Trans. Wireless Commun.*, vol. 13, no. 11, pp. 6311-6325, Nov. 2014.
- [137] Y. Wu, Y. Cui, and B. Clerckx, “Analysis and optimization of inter-tier interference coordination in downlink multi-antenna HetNets with offloading,” *IEEE Trans. Wireless Commun.*, vol. 14, no. 12, pp. 6550-6564, Dec. 2015.
- [138] H. H. Yang, G. Geraci, and T. Q. S. Quek, “Energy-efficient design of MIMO heterogeneous networks with wireless backhaul,” *IEEE Trans. Wireless Commun.*, vol. 15, no. 7, pp. 4914-4927, Jul. 2016.
- [139] Y. Zhu, L. Wang, K. K. Wong, S. Jin, and Z. Zheng, “Wireless power transfer in massive MIMO-aided HetNets with user association,” *IEEE Trans. Commun.*, vol. 64, no. 10, pp. 4181-4195, Oct. 2016.
- [140] C. Li, J. Zhang, M. Haenggi, and K. B. Letaief, “User-centric intercell interference nulling for downlink small cell networks,” *IEEE Trans. Commun.*, vol. 63, no. 4, pp. 1419-1431, Apr. 2015.
- [141] K. Hosseini, W. Yu, and R. S. Adve, “Large-scale MIMO versus network MIMO for multicell interference mitigation,” *IEEE J. Sel. Areas Commun.*, vol. 8, no. 5, pp. 930-941, Oct. 2014.
- [142] R. Tanbourgi, S. Singh, J. G. Andrews and F. K. Jondral, “A tractable model for noncoherent joint-transmission base station cooperation,” *IEEE Trans. Wireless Commun.*, vol. 13, no. 9, pp. 4959-4973, Sept. 2014.

BIBLIOGRAPHY

- [143] Q. Li, R. Q. Hu, Y. Qian, and G. Wu, "Intracell cooperation and resource allocation in a heterogeneous network with relays," *IEEE Trans. Veh. Technol.*, vol. 62, no. 4, pp. 1770-1784, May 2013.
- [144] K. Hamdi, "A useful lemma for capacity analysis of fading interference channels," *IEEE Trans. Commun.*, vol. 58, no. 2, pp. 411-416, Feb. 2010.
- [145] A. Shojaeifard, K. A. Hamdi, E. Alsusa, D. K. C. So, and J. Tang, "Exact SINR statistics in the presence of heterogeneous interferers," *IEEE Trans. Inf. Theory*, vol. 61, no. 12, pp. 6759-6773, Dec. 2015.
- [146] Q. Zhang, S. Jin, K. K. Wong, H. B. Zhu, and M. Matthaiou, "Power scaling of uplink massive MIMO systems with arbitrary-rank channel means," *IEEE J. Sel. Topics Signal Process.*, vol. 8, no. 5, pp. 966-981, Oct. 2014.
- [147] R. Krishnan, M. R. Khanzadi, N. Krishnan, Y. Wu, A. G. Amat, T. Eriksson, and R. Schober, "Linear massive MIMO precoders in the presence of phase noise: A large-scale analysis," *IEEE Trans. Veh. Technol.*, vol. 65, no. 5, pp. 3057-3071, May 2016.
- [148] J. Zuo, J. Zhang, C. Yuen, W. Jiang, and W. Luo, "Multicell multiuser massive MIMO transmission with downlink training and pilot contamination precoding," *IEEE Trans. Veh. Technol.*, vol. 65, no. 8, pp. 6301-6314, Aug. 2016.
- [149] L. Fan, S. Jin, C. K. Wen, and H. Zhang, "Uplink achievable rate for massive MIMO systems with low-resolution ADC," *IEEE Commun. Lett.*, vol. 19, no. 12, pp. 2186-2189, Dec. 2015.
- [150] N. Seifi, R. W. Heath, M. Coldrey and T. Svensson, "Adaptive multicell 3-D beamforming in multiantenna cellular networks," *IEEE Trans. Veh. Technol.*, vol. 65, no. 8, pp. 6217-6231, Aug. 2016.

BIBLIOGRAPHY

- [151] Z. Yazdanshenasan, H. S. Dhillon, M. Afshang, and P. H. J. Chong, "Poisson hole process: Theory and applications to wireless networks," *IEEE Trans. Wireless Commun.*, vol. 15, no. 11, pp. 7531-7546, Nov. 2016.
- [152] C. H. Lee and M. Haenggi, "Interference and outage in Poisson cognitive networks," *IEEE Trans. Wireless Commun.*, vol. 11, no. 4, pp. 1392-1401, Apr. 2012.
- [153] A. He, L. Wang, Y. Chen, M. Elkashlan, and K. K. Wong, "Massive MIMO in K-tier heterogeneous cellular networks: Coverage and rate," in *Proc. IEEE GLOBECOM*, San Diego, CA, 2015, pp. 1-6.
- [154] J. B. Rao and A. O. Fapojuwo, "Analysis of spectrum efficiency and energy efficiency of heterogeneous wireless networks with intra-/inter-RAT offloading," *IEEE Trans. Veh. Technol.*, vol. 18, no. 5, pp. 3120-3139, Jul. 2015.
- [155] V. Jungnickel, K. Manolakis, W. Zirwas, B. Panzner, V. Braun, M. Lossow, M. Sternad, R. Apelfrjd, and Tommy Svensson, "The role of small cells, coordinated multipoint, and massive MIMO in 5G," *IEEE Commun. Mag.*, vol. 52, no. 5, pp. 44-51, May 2014.
- [156] H. Pervaiz, L. Musavian, Q. Ni, and Z. Ding, "Energy and spectrum efficient transmission techniques under QoS constraints toward green heterogeneous networks," *IEEE Access*, vol. 3, pp. 1655-1671, Sept. 2015.
- [157] L. Li, M. Peng, C. Yang, and Y. Wu, "Optimization of base station density for high energy efficient cellular networks with sleeping strategies," *IEEE Trans. Veh. Technol.*, doi: 10.1109/TVT.2015.2483374.
- [158] H.S. Jo, Y. J. Sang, P. Xia, and J. G. Andrews, "Heterogeneous cellular networks with flexible cell association: A comprehensive downlink SINR analysis," *IEEE Trans. Wireless Commun.*, vol. 11, no. 10, pp. 3484-3495, Oct. 2012.

BIBLIOGRAPHY

- [159] M. Haenggi and R. K. Ganti, "Interference in large wireless networks," *Found. Trends Netw.*, vol. 3, no. 2, pp. 127-248, Feb. 2009.
- [160] K. T. Hemachandra and N. C. Beaulieu, "A moment matching technique to estimate the achievable ergodic rate for limited feedback distributed antenna systems in the presence of out-of-cell interference," *IEEE Commun. Lett.*, vol. 17, no. 7, pp. 1360-1363, Jul. 2013.
- [161] R. W. Heath Jr, T. Wu, Y. H. Kwon, and A. C. K. Soong, "Multiuser MIMO in distributed antenna systems with out-of-cell interference," *IEEE Trans. Signal Process.*, vol. 59, no. 10, pp. 4885-4899, Oct. 2011.
- [162] G. Geraci, H. S. Dhillon, J. G. Andrews, J. Yuan, and I. B. Collings, "Physical layer security in downlink multi-antenna cellular networks," *IEEE Trans. Commun.*, vol. 62, no. 6, pp. 2006-2021, Jun. 2014.
- [163] T. Bai and R. W. Heath, "Analyzing uplink SINR and rate in massive MIMO systems using stochastic geometry," *IEEE Trans. Commun.*, vol. 64, no. 11, pp. 4592-4606, Nov. 2016.
- [164] A. L. Swindlehurst, E. Ayanoglu, P. Heydari, and F. Capolino, "Millimeter-wave massive MIMO: The next wireless revolution?," *IEEE Wireless Commun.*, vol. 52, no. 9, pp. 56-62, Sept. 2014.
- [165] T. E. Bogale and L. B. Le, "Massive MIMO and mmWave for 5G wireless HetNet: Potential benefits and challenges," *IEEE Veh. Technol. Mag.*, vol. 11, no. 1, pp. 64-75, Mar. 2016.
- [166] J. A. Zhang, X. Huang, V. Dyadyuk, and Y. J. Guo, "Massive hybrid antenna array for millimeter-wave cellular communications," *IEEE Wireless Commun.*, vol. 22, no. 1, pp. 79-87, Feb. 2015.

BIBLIOGRAPHY

- [167] S. Hur, T. Kim, D. J. Love, J. V. Krogmeier, T. A. Thomas, and A. Ghosh, “Millimeter wave beamforming for wireless backhaul and access in small cell networks,” *IEEE Trans. Commun.*, vol. 61, no. 10, pp. 4391-4403, October 2013.
- [168] Z. Gao, L. Dai, D. Mi, Z. Wang, M. A. Imran, and M. Z. Shaker, “mmWave massive-MIMO-based wireless backhaul for the 5G ultra-dense network,” *IEEE Wireless Commun.*, vol. 22, no. 5, pp. 13-21, Oct. 2015.
- [169] E. Björnson, M. Matthaiou, and M. Debbah, “Massive MIMO with non-ideal arbitrary arrays: Hardware scaling laws and circuit-aware design,” *IEEE Trans. Wireless Commun.*, vol. 14, no. 8, pp. 4353-4368, Aug. 2015.
- [170] C. Tang, C. Liu, L. Yuan, and Z. Xing, “High precision low complexity matrix inversion based on Newton iteration for data detection in the massive MIMO,” *IEEE Commun. Lett.*, vol. 20, no. 3, pp. 490-493, Mar. 2016.
- [171] A. Liu and V. Lau, “Phase only RF precoding for massive MIMO systems with limited RF chains,” *IEEE Trans. Signal Process.*, vol. 62, no. 17, pp. 4505-4515, Sept. 2014.
- [172] R. Zi, X. Ge, J. Thompson, C. X. Wang, H. Wang, and T. Han, “Energy efficiency optimization of 5G radio frequency chain systems,” *IEEE J. Sel. Areas Commun.*, vol. 34, no. 4, pp. 758-771, Apr. 2016.
- [173] A. Alkhateeb, Y. H. Nam, J. Zhang and R. W. Heath, “Massive MIMO combining with switches,” *IEEE Wireless Commun. Lett.*, vol. 5, no. 3, pp. 232-235, Jun. 2016.
- [174] J. Joung, E. Kurniawan, and S. Sun, “Channel correlation modeling and its application to massive MIMO channel feedback reduction,” *IEEE Trans. Veh. Technol.*, vol. 66, no. 5, pp. 3787-3797, May 2017.
- [175] S. Noh, M. D. Zoltowski, Y. Sung, and D. J. Love, “Pilot beam pattern design for channel

BIBLIOGRAPHY

- estimation in massive MIMO systems,” *IEEE J. Sel. Topics Signal Process.*, vol. 8, no. 5, pp. 787-801, Oct. 2014.
- [176] Y. Cui and X. Fang, “Performance analysis of massive spatial modulation MIMO in high-speed railway,” *IEEE Trans. Veh. Technol.*, vol. 65, no. 11, pp. 8925-8932, Nov. 2016.
- [177] R. M. Corless, G. H. Gonnet, D. E. G. Hare, D. J. Jeffrey, and D. E. Knuth, “On the lambert w-function,” *Adv. Comput. Math.*, vol. 5, no. 1, pp. 329-359, Dec. 1996.
- [178] A. Zappone and E. Jorswieck, “Energy efficiency in wireless networks via fractional programming theory,” *Foundations Trends Commun. Inf. Theory*, vol. 11, no. 3, pp. 185-396, Jun. 2015.
- [179] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.