**NANYANG
TECHNOLOGICAL
UNIVERSITY**

**DEVELOPMENT OF AN ENERGY-EFFICIENT EXPANDER-
COMPRESSOR UNIT FOR REFRIGERATION SYSTEMS**

YAP KEN SHAUN

SCHOOL OF MECHANICAL AND AEROSPACE ENGINEERING

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YAP KEN SHAUN

School of Mechanical and Aerospace Engineering

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Abstract

The thesis presents the development of a novel expander-compressor unit, named cross vane expander-compressor, or in short CVEC. This device is conceived with the primary motivation of integrating the function of the expander and compressor into a single simple geometry, making energy recovery and efficiency improvement economically practical.

Mathematical models have been formulated in this project to facilitate the understanding of the working physics in the ‘expander’ and ‘compressor’ of the CVEC. These include volume variations of the working chambers, kinematics of the mechanism, variation of thermodynamics properties of the working fluid, mass flow through suction and discharge ports, the response of the valve reed to pressure and centrifugal forces, the internal leakage of the working fluid through the gaps of the mating components within and across the chambers of the ‘expander’ and the ‘compressor’ and the mechanical losses of the machine. An attempt has also been made to analyse the heat transfer of the machine, following which Second Law analysis was carried out to quantify the exergy losses.

Theoretical studies were carried out with R-1234yf as the working fluid, operating at the condensing and evaporating temperatures of 54.4°C and 7.2°C, respectively. The results show that through the use of CVEC, the system’s COP was improved by 29.5% and the peak power input was reduced by 6.5% at 3000 rpm. The study also reveals that machines with a slender configuration or that with a lower rotating inertia would result in a higher mechanical efficiency. The highest mechanical efficiency of the machine was predicted to be 85.1% at slender ratio of 6.28 (i.e. expander-compressor length of 151.0 mm and outer cylinder radius of 24.1 mm).

Additionally, it was found that the major internal leakage occurs through the split bush clearances with 74.1% of the total leakage. This leakage is the main reason behind the drop in pressure in the expansion chamber of the expander and compression chamber of the compressor. The results also show that major mechanical loss is caused by the end-face friction which accounts for 61.9% of the total losses.

A prototype has been designed, fabricated and instrumented in an open-loop air-flow system. Tests were conducted from 600 to 1300 rpm. Due to severe internal leakages that occurred in

the machine, only a limited range of pressures for expander suction and compressor discharge pressure was possible, and in this case, it was from 1.1 to 1.7 bar.

The measured data were used to validate the mathematical models. It was found that the average standard deviation between the measured and predicted average torques is 10.5% and that of mass flow rates is 16.1%. Additionally, the comparison between the measured and predicted results shows that the mathematical model is capable of predicting the variation of pressure in the working chamber. As a result of severe internal leakages, the volumetric efficiencies of the machine were measured to be 3.6% to 63.6%.

With the validated mathematical model, the mechanical efficiency of the machine was predicted at the measured conditions. The results show that the mechanical efficiency ranges from 3.3% to 10.9%. Additionally, the recovery ratios were -33.9 to -6.3. The negative values suggest that the expander consumes energy instead of recovering energy.

Severe internal leakages have prevented the attainment of the theoretical potential of the CVEC. It is believed that improvement works and future modifications will bring a step closer to the achievement of the initial objective of having an energy-efficient and environmentally friendly CVEC.

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Nomenclature

A	Area [m^2]
A_{flow}	Flow area [m^2]
A_t	Tensile stress area of the fastener [m^2]
c	Damping constant [$\text{N}\cdot\text{s}/\text{m}$]
c_p	Specific heat [$\text{Jkg}^{-1}\text{K}^{-1}$]
C_d	Coefficient of discharge [-]
C_R	Reliability factor [-]
C_s	Size factor [-]
D	Diameter [m]
D_{flow}	Diameter of the flow area [m]
D_h	Hydraulic diameter [m]
E	Energy [J]/Young's modulus [Pa]
e	Distance between rotor and inner cylinder centers [m]
F	Force [N]
f	Frequency [Hz]/excitation force per length on valve reed [N/m]
f_n	Normalized mode shape of valve reed [m]
Gr	Grashof number [-]
g	Acceleration of free fall [m/s^2]
g_n	Generalised coordinate [-]
h	Specific enthalpy [J/kg]/oil film thickness [m]/convective heat transfer coefficient [$\text{Wm}^{-2}\text{K}^{-1}$]
I	Moment of inertia [$\text{kg}\cdot\text{m}^2$]/exergy destruction [J]
K_t	Stress concentration factor [-]
k	Thermal conductivity [$\text{Wm}^{-1}\text{K}^{-1}$]
l	Length [m]
M	Moment [Nm]/Mach number [-]
m	Mass [kg]
N	Design factor [-]
n	Number of fastener [-]
Nu	Nusselt number [-]
P	Instantaneous power as a function of θ_1 [W]
Pr	Prandtl number [-]
p	Pressure [Pa]/natural frequency[rad/s]
Q	Heat transfer [J]
Q'	Cooling capacity with expansion valve [J]
q	Flow rate [m^3/s]/undamped response frequency of valve reed [rad/s]
q_d	Damped response frequency of valve reed [rad/s]
R	Reliability factor [-]/flow resistance [$\text{Pa}\cdot\text{s}/\text{m}^3$]/thermal resistance

	[KW ⁻¹]
R_a	Surface roughness [m]
Ra	Rayleigh number [-]
Re	Reynolds number [-]
r	Radius [m]
S	Section modulus [m ³]
SF	Safety factor [-]
s	Specific entropy [Jkg ⁻¹ K ⁻¹]
s_y	Yield strength [Pa]
s_n	Estimated endurance strength [Pa]
s'_n	Modified endurance strength [Pa]
T	Torsion [Nm]/Temperature [°C]
t	Time [s]/thickness [m]
u	Specific internal energy [J/kg]
V	Volume [m ³]/shear force [N]
v	Velocity [m/s]
v_{sound}	Speed of sound [m/s]
W	Work [J]
w	Width [m]
X	Exergy [J]
x	Distance [m]
y	Valve lift [m]
z	Height [m]
β	Natural frequency of the valve reed [rad/s]/coefficient of volume expansion [K ⁻¹]
γ	Angle subtended by the rotor and inner cylinder [rad]
δ	Clearance [m]/characteristic length of a geometry [m]
η_s	Isentropic efficiency [%]
η_m	Mechanical efficiency [%]
η_v	Volumetric efficiency [%]
μ	Dynamic viscosity [Pa·s]/coefficient of friction [-]
ν	Kinematic viscosity [m ² /s]
ν_*	Apparent kinematic viscosity [m ² /s]
ρ	Density [kg/m ³]
σ	Stress [N/m ²]/standard deviation [-]
τ	Torque[Nm]/shear stress [N/m ²]
ζ	Viscous damping factor [-]
κ	Recovery ratio [-]
ϑ	Angle of the force [rad]
θ_1	Rotational angle of the motor shaft and outer cylinder-vane-rotor component [rad]/starting angle of the pressure curve of the

	hydrodynamic oil film in journal bearing [rad]
θ_2	Rotational angle of the inner cylinder [rad]]/ending angle of the pressure curve of the hydrodynamic oil film in journal bearing [rad]
ω_1	Angular velocity of the motor shaft and outer cylinder-vane-rotor component [rad/s]
ω_2	Angular velocity of the inner cylinder [rad/s]
α_1	Angular acceleration of the motor shaft and outer cylinder-vane-rotor component [rad/s ²]
α_2	Angular acceleration of the inner cylinder [rad/s ²]
Δh_{hs}	Difference in specific enthalpy between the discharge fluids from the expander and expansion valve [J/kg]
Δp	Pressure difference [Pa]
ε	Eccentricity ratio [-]
ϕ	Attitude angle [rad]

Subscripts

Σ	summation
∞	surrounding
<i>all</i>	allowable
<i>avg</i>	average
<i>bear</i>	bearing
<i>cv</i>	control volume
<i>com</i>	compressor
<i>cdcv</i>	compressor compression chamber
<i>cdcs</i>	between compressor compression and suction chamber
<i>cdisc</i>	compressor discharge line
<i>cscv</i>	compressor suction chamber
<i>csuct</i>	compressor suction line
<i>disc</i>	discharge
<i>dv</i>	discharge valve
<i>ef</i>	at end-face
<i>ecu</i>	expander-compressor unit
<i>exp</i>	expander
<i>edcd</i>	between expander discharge chamber and compressor compression chamber
<i>edcs</i>	between expander discharge chamber and compressor suction chamber
<i>edcv</i>	expander discharge chamber
<i>edisc</i>	expander discharge line
<i>escd</i>	between expander suction chamber and compressor compression chamber
<i>escs</i>	between expander and compressor suction chamber
<i>escv</i>	expander suction chamber
<i>esed</i>	between expander suction and discharge chamber
<i>esuct</i>	expander suction line
<i>f</i>	film/fluid/due to friction
<i>h</i>	height/hoop
<i>hs</i>	housing
<i>hscs</i>	between housing chamber and compressor suction chamber
<i>hscd</i>	between housing chamber and compressor compression chamber
<i>I</i>	due to inertia
<i>i</i>	inflow/inner
<i>ic</i>	inner cylinder
<i>icb</i>	between inner cylinder and bearing end-faces
<i>icn</i>	inner cylinder and split bush contact in the normal direction
<i>ics</i>	inner cylinder and split bush contact in the tangential direction

<i>icsb</i>	between inner cylinder and split bush
<i>irad</i>	instantaneous and radial
<i>j</i>	rotor or inner cylinder journal bearing
<i>jb</i>	journal bearing
<i>k</i>	leakage path number
<i>lb</i>	lower bearing
<i>l</i>	lower/loop
<i>lb</i>	lower bearing
<i>lef</i>	lower end-face
<i>leak</i>	leakage
<i>ls</i>	sower shaft
<i>m</i>	mechanical
<i>max</i>	maximum
<i>min</i>	minimum
<i>mx</i>	due to mixing
<i>n</i>	mode number
<i>u</i>	upper
<i>ub</i>	upper bearing
<i>uef</i>	upper end-face
<i>us</i>	upper shaft
<i>o</i>	outflow
<i>oc</i>	outer cylinder
<i>Q</i>	due to heat transfer
<i>r</i>	rotor
<i>rad</i>	radial
<i>rc</i>	rotating channel
<i>red</i>	between rotor and expander discharge chamber
<i>res</i>	between rotor and expander suction chamber
<i>s</i>	shaft
<i>sb</i>	split bush
<i>sbn</i>	radial to split bush
<i>sbs</i>	tangential to split bush
<i>sc</i>	stationary channel
<i>sg</i>	spiral groove
<i>srcd</i>	between surrounding and compressor compression chamber
<i>srcs</i>	between surrounding and compressor suction chamber
<i>suct</i>	suction
<i>th</i>	due to throttling
<i>v</i>	vane/volumetric
<i>vb</i>	between vane and bearing end-faces
<i>vn</i>	normal to vane side
<i>vs</i>	parallel to vane side

vsb	between vane side and split bush
w	wall
x	in the x -axis direction
y	in the y -axis direction

Publications Arising from this Thesis

1. K. S. Yap, K. T. Ooi, and A. Chakraborty, "A Vane Mechanism," WO/2013/162477, 2013.
2. K. S. Yap, K. T. Ooi, and A. Chakraborty, "Introduction of the novel cross vane expander-compressor unit for vapour compression cycle," presented at the International Compressor Engineering Conference, Purdue University, 2014.
3. K. S. Yap, K. T. Ooi, and A. Chakraborty, "Modelling and simulation of the dynamics of cross vane expander-compressor unit for vapour compression cycle," presented at the International Compressor Engineering Conference, Purdue University, 2014.
4. K. T. Ooi and K. S. Yap, "Design evolution: From rolling piston to revolving vane to cross-vane expander-compressor unit," presented at the 9th International Conference on Compressors and their Systems, UK, 2015.

1 Introduction

This project involves conceptualising, design, modelling and experimenting of a new refrigeration device which aims to reduce energy consumption, material wastage and promote the use of naturally occurring fluids as refrigerants and which we hope will bring about a greener and more sustainable future.

1.1. Motivations

The idea of the project was first mooted by the fact that the use of man-made refrigerants in the cooling and heating systems has caused significant environmental problems which was first uncovered in Montreal Protocol [1] in 1987. Since then, non-environmentally friendly refrigerants are slowly phased out by member nations and replaced with other more environmentally friendly refrigerants. Among them, CO₂ was first suggested to be used as replacement due to its natural existence in the atmosphere [2, 3]. However, its low critical temperature (31°C) has resulted in heat rejection process to occur at single-phase, supercritical region when the operating temperature exceeds 31°C, hence reducing the effectiveness of heat transfer. This has been overcome by the introduction of a gas cooler with a better heat transfer efficiency. Moreover, its high critical pressure (73.8 bar) has rendered CO₂ systems unfavourable as such systems are required to operate at high pressures. Therefore, a large amount of energy is wasted at the expansion valve of vapour compression systems, resulting in significant energy wastage and making the applications impractical.

To overcome this issue, the use of an expander to replace the conventional expansion valve is deemed the most promising [4]. In this method, the expander recovers the expansion power, using it to partially drive the compressor of the same system, thereby reducing the nett energy consumption. This energy recovery is particularly advantageous for CO₂ systems, as the pressure difference between the high and low side is large, which is commonly around 100 bar. Initial theoretical investigations have shown that improvement of as high as 70% in the coefficient of performance (COP) [5] could be achieved. On the other hand, the use of energy recovery expander on systems employing man-made refrigerants would obtain up to 20% of COP improvement [6]. On a global scale, the amount of energy saved would be highly significant, especially when the usage of air-conditioning system is growing dramatically due to warmer summers and increase in household incomes [7, 8]. It is estimated that the fraction of households with air conditioning will increase from 13% today to more than 70% by the

end of this century [9-13]. With many more air conditioners in use, the residential electricity consumption is expected to raise by 83% by the end of the century [14]. Without considerable improvements on the current system, energy consumptions due to air-conditioning purposes would increase year by year. Henceforth, the introduction of expander technology is foreseen to play a major part in alleviating the continual rise in building energy demand.

As beneficial as it may seem, expanders have yet to penetrate into the HVAC market. This is due to the additional cost imposed by having the extra component, i.e., expander which also makes the whole system bulkier and heavier. Particularly in applications where lightweight is an important criterion, such as automobile, the extra weight due to the expander would be counter-effective. Therefore it is proposed that the expander and compressor be combined into a single machine, where their individual functions are performed simultaneously. Such a combination not only allows for parts/components sharing, which cuts down material usage, but also reduces the mechanical losses to one machine as compared to two. This necessitates an innovative concept which allows the integration of expander and compressor into a single seamless unit, where such device would result in significant amount of energy saved annually, and making the use of CO₂ as a replacement for non-environmentally friendly refrigerants feasible.

1.2. Objectives and Scope of the Project

It is obvious that a new device which enables the following issues to be addressed would be very much beneficial:

- i. The disadvantages of using CO₂ as refrigerant that operates at high operating pressures.
- ii. The use of separate expander and compressor in a vapour compression system does not appear to be attractive economically to consumers due to high initial cost.
- iii. Material wastage as a result of separate construction of expander and compressor.
- iv. Additional energy loss as a result of more friction in two separated machines, expander and compressor.

Therefore, the main objective of this project is to invent and actualise an innovative design of an energy-efficient expander-compressor unit. To achieve this objective, the following work scope is necessary:

1. To conceive and conceptually design a highly energy-efficient expander-compressor unit.
2. To formulate mathematical model for the performance and behaviour prediction of the new design.
3. To carry out detailed industrial design and prototype fabrication of the new machine.
4. To devise a test bed for the prototype.
5. To test and measure the performance of the prototype.
6. To validate the prediction of the mathematical model.

1.3. Purpose of the Report

This thesis is submitted in accordance to the requirement of the Nanyang Technological University for its research students' PhD candidature. This thesis comprises the research study that the author has carried out during the past five years. It also includes the author's future plan for his research project.

1.4. Organisation of the Report

The thesis comprises eight chapters and seven appendices. It begins with project introduction in Chapter 1, where the motivations for the research project, objective and work scope are presented. It is then followed by Chapter 2 which reviews the available and relevant literature that includes compressor mechanisms, evolution of refrigerants, various researches on expander technology and methods of implementing the expander-compressor systems. In Chapter 3, the conceptualisation process leading to the new expander-compressor mechanism is described, in addition to its embodiment and working principle.

Chapter 4 presents the formulation of the mathematical models. In this chapter, the governing equations relating to the working chamber volume, kinematics of the mechanism, thermodynamics properties of the working fluid, mass flows through suction and discharge ports, and internal leakages within the working chambers are formulated. Moreover, the valve reed response, dynamics and friction losses are also presented. The basic simulation results are shown and discussed in Chapter 5, followed by investigations of the effect of various design parameters on the performance of the machine.

With a better understanding on the performance characteristic of the machine, a prototype is designed and fabricated. The design processes for each of the components and the assembly procedure are explained in Chapter 6. In Chapter 7, the setup of the test-bed is described. Hereafter, measurements are carried out and the measured results are compared with those from the theoretical predictions to validate the mathematical models. Last but not least, Chapter 9 sums up the findings of the research project, along with the recommendations for future work.

2 Literature Review and Patent Search

In this chapter, research publications and previous works which are relevant to the project are reviewed. These include various compressor designs, evolution of refrigerants, expander technology, as well as methods of implementing the expander-compressor system.

2.1. Existing Compressor Designs

A compressor is a mechanical device used for increasing the pressure of a compressible fluid such as air and other industrially important gases like refrigerants. It comes in a wide variety of types and sizes, each of which fulfils certain needs and represents an optimum configuration for a specific set of requirements. In general, compressors can be divided into positive displacement types and dynamic types such as centrifugal and axial compressors. A positive displacement compressor delivers a fixed volume of working fluid at high pressure while the dynamic compressor produces a low static pressure rise of the working fluid. In this project, the main emphasis is on the positive-displacement compressors in small-to-medium sized applications, such as refrigeration and air-conditioning systems. Under this classification, there are essentially two types of positive-displacement compressors: the reciprocating and the rotary.

Reciprocating compressors can be found in a wide variety of sizes and power ranges. Known to be the earliest compressor being invented, its working principle is rather straightforward. It comprises a piston which is being driven by a crankshaft as depicted in Figure 2.1. The fluid is firstly induced through the suction valve by a downward moving piston. Once it reaches the bottom dead centre, the piston moves upward and the fluid is compressed. The compression process continues until the required pressure is reached and the discharge valve opens to allow the outflow of fluid.

Figure 2.1 Physical process of single-acting reciprocating piston compressor [15]

Over the years, reciprocating compressors have proven to be highly reliable, and they are still being in production up till today. These compressors have greater flexibility in capacity and are able to handle high pressures of up to 2000 bar, where its pressure ratio is typically of the order of 4 to 7 [16]. However, due to the reciprocating motion of the piston, this type of compressors is prone to noise and vibration [17]. Additionally, its volumetric efficiency reduces due to residual volume trapped in the working chamber [18]. Furthermore, this design is bulkier as a result of the crank mechanism, and more susceptible to flow losses as it requires two valves for the suction and discharge ports. After years of progress, the efficiency of the compressor has also seemed to reach its limit [19].

Rotary compressors, on the other hand, are inherently having low noise and vibration, and are more compact and lighter; therefore they find favourability in applications where size and weight play a crucial factor.

One of the most popular rotary compressors used in room air-conditioners today is the rolling piston type compressor [20]. The schematic view of a rolling piston compressor is shown in Figure 2.2. The compressor comprises a rolling piston, a cylinder and the assembly of vane and spring. The rolling piston is mounted on a shaft eccentrically, where the shaft rotates about the centre of the cylinder. The vane protrudes out of the cylinder wall and forms a line

contact with the rolling piston. As the rolling piston rolls over the inner cylinder wall, the space between the roller, the inner cylinder wall and the vane changes, thereby forming the working chambers of the compressor. Due to its design, it has fewer components and hence high mechanical efficiency. In addition, as the residual volume can be made small, re-expansion loss is minimised and hence, the volumetric efficiency is greatly improved [21].

Figure 2.2 Physical model of rolling piston compressor [22]

Nevertheless, researchers [23] have observed that the vane jumps during the start-up process which in turn causes a loud noise, and the gap due to vane jumping contributes possible leakage paths for the working fluid [24, 25]. As the compressor is in operation, the continuous rubbing of the vane tip and vane side also cause significant losses by friction and the wear rate at this region is amplified [26]. Several mitigation methods have been proposed, but none has been found to give a complete solution or without imposing additional cost. Liu and Kosco [26] examined the tilt angle of the vane in rolling piston compressor and found that tilted vane only improves the durability and reliability of the vane and piston, whereas the efficiency is not. Takebayashi et al. [27] and Tanaka et al. [28] suggested a material and/or surface treatment of the vane to reduce its wear rate that comes with additional cost. A similar situation also occurs between the contacting surfaces of the eccentric and roller, which necessitates material hardening [29].

The other rotary compressor that appears to be very similar in geometrical configuration as the rolling piston is the sliding vane compressor. It comprises a cylinder and a rotor that has several radial or non-radial slots which allow the vane to slide. With the force of the spring or

just the centrifugal force of the vane, the vanes are pushed outward and made to be in contact with the inner wall of the cylinder as shown in Figure 2.3.

Figure 2.3 Physical model of rotary vane compressor [30]

The vane tips slide against the inner wall of the cylinder when the rotor rotates eccentrically to the cylinder, and the space between the rotor, cylinder and any two adjacent vanes forms the working chamber. The suction and discharge processes are continuous and they take place simultaneously, and thus the compressor does not require any valve. As a result, the reliability of the compressor is greatly enhanced as failure due to valve fatigue can be prevented. In addition, due to its valveless design, the rotary vane compressor has built-in volume compression ratio and the discharge pressure is predetermined by the suction process.

As compared to a single vane machine such as rolling piston type, the sliding vane has higher frictional losses [18, 31, 32] and is more costly to fabricate. Under low speed condition, chattering occurs where the vanes get detached and spring back to the inner cylinder wall. The gap created between the vane tip and inner cylinder wall during the chattering phenomenon causes leakage which reduces the volumetric efficiency of the compressor [33]. To remedy this problem, Shu et al. [34] suggested the application of a constant vane slot back pressure to ensure the contact between the vane tip and inner cylinder wall. However, this comes at the expense of higher vane tip friction and the mechanical efficiency of the compressor is compromised.

One type of the rotary compressors which is getting more popular in the last decade is the scroll compressor. It originates from a century-old concept that was invented by Léon Creux in 1905 [35]. As illustrated in Figure 2.4, there are two interfitting spiral-shaped members assembled at a relative angle of 180° from each other such that contacts are made at several points and form a series of crescent shape pockets. Often, one of the scrolls is fixed, while the other orbits eccentrically around the centre of the fixed scroll without rotating. During operation, gas is drawn from the inlet port located at the periphery of the scrolls and trapped in a pocket. As the orbiting scroll rotates, the pocket undergoes a volume reduction thereby compressing the gas while moving towards the centre of the spiral. The compressed gas is eventually discharged through the outlet port and the next cycle repeats.

Figure 2.4 Physical process of scroll compressor [36]

Nowadays, scroll compressors are found in many commercial and residential applications. Its widespread usage is second to rolling piston compressors and can be attributed to its several favourable features. First of all, its operation is smooth and quiet in nature [37] as multiple working chambers undergo compression processes simultaneously throughout the cycle, and hence, the compressor suffers from very little torque fluctuation [38]. Secondly, as no valves are required and the re-expansion volume is absent, the discharge flow is continuous which results in high volumetric efficiency over a wide range of operating conditions. Moreover,

scroll compressors are considered to be one of the most efficient compressors around as friction loss is comparatively low due to its orbital motion that results in lower relative velocity between the two scroll members.

Despite the merits stated, the scroll compressor does not yet dominate over the rolling piston compressor. It is deduced that the main reason lies in its complicated spiral profile which necessitates more precision in fabrication as compared to the simple cylindrical design of the rolling piston compressor. This implies that clearances are much harder to control in scroll compressors, and internal leakage can be significant. Studies have found two main leakage paths at the axial and radial clearances, and concluded that the former has a more dominant influence on the volumetric efficiency of the compressor [39, 40]. In order to reduce the leakages through axial clearance, it is suggested that a back-pressure chamber to be incorporated which forces the orbiting scroll to maintain contact with the fixed scroll at the scroll tips [41-44]. This nonetheless increases the friction at scroll tips, and hence a compromise between leakage and friction must be obtained. In yet another study [45], it is proposed that a thick and rigid scroll wraps be used in order to sustain the gas force and thermal distortion that would cause leakage. Likewise, the solution does not solve the problem without compromising another aspect of the compressor, i.e., the weight and size of the components in this case.

Screw compressors have a much longer history than scroll compressors. Its first patent was awarded in 1878 to a German inventor named Heinrich Krigar [46]. Though the concept has been around for over a century, it did not gain industrial application until the late 20th century. Today, they are used mainly in very high cooling load situations such as the big commercial buildings or plants. The working mechanism of which is shown in Figure 2.5. The compressor consists of two intermeshing helical rotors, in which the male lobe meshes with the corresponding female groove. Similar to sliding vane and scroll compressor, the screw compressor does not require a valve and has a built-in compression ratio. The entire operation of screw compressor can be categorised into suction, transfer, compression, and discharge phases. In the beginning, the pair of lobe and groove unmeshes. This creates a space for which the gas is drawn in until it is separated from the inlet port. The pocket of gas then move circumferentially through the threads during the transfer phase due to continuous rotation. As the process continuous, the volume of the pocket is gradually reduced and the

trapped gas is consequently compressed. In the end, the pocket of compressed gas is released through the discharge port.

Figure 2.5 Physical process of a rotary screw compressor [47]

Screw compressors are notable for their quiet and low vibration operation which is the result of the dynamically balanced rotors rotating about their axis of rotation. Although dynamic forces cause axial and radial shaft vibration, the magnitude of these vibrations is relatively low [48]. Moreover, pulsation and vibration are minimal or close to none due to the continual compression process [49], and the discharge flow is uninterrupted in the absence of valve. Its valveless design coupled with very few moving parts has also rendered this type of compressor reliable.

Similar to the scroll compressor, its complicated screw profile has made the fabrication process a challenge as good mating contact is required to ensure sealing between the working chambers. With the advancement in machining technology, the clearance can be made tighter and the performance of the compressor is enhanced dramatically. However, a tighter clearance also leads to an increase in the viscous flow losses between the male and female rotor, and the mechanical efficiency is lowered as a result [50]. Once again, it is shown that the improvement of volumetric efficiency often takes place at the expense of mechanical efficiency, or vice versa, which is the norm in all of the existing compressors. The best

solution is the complete removal of the problem areas, and this can only be achieved with a totally new and innovative design.

In 2006, a novel mechanism for a positive displacement compressor has been invented and this mechanism has fewer parts and lower frictional loss. The new design is the revolving vane (RV) compressor. It was invented by Teh and Ooi [51] to overcome some of the shortcomings facing the existing rotary type compressors. Studies have shown that frictional losses in the RV compressor are 20% lesser than that of the rolling piston compressor, and the mechanical efficiency was reported at 94.1%, typically above those of the rolling piston and scroll types [52]. In comparison with the rolling piston compressor, the leakage loss at the radial clearance was also found to be 40% lesser [53].

In 2008, a design improvement was introduced and is named the improved revolving vane (RV-I) compressor [54, 55]. In its basic form, the compressor comprises a rotor, a cylinder and a vane as shown in Figure 2.6. The rotor is placed inside the cylinder in such a way that the centre of the rotor and the centre of the cylinder are at an offset from each other. The vane is attached to the rotor and is made to slide along the slot located at the cylinder. When the rotor rotates, the vane revolves which in turn rotates the cylinder. The motion causes the space between the rotor, cylinder and vane to vary resulting in suction, compression and discharge of the working fluid. It is reported that RV-I compressor has an approximately 20% overall reduction in the frictional losses as compared to the RV compressor [56]. In agreement with the RV compressor, the RV-I compressor also exhibits a 40% reduction in leakage loss [57].

Figure 2.6 Front and side sectional view of RV-I compressor [54]

One of the drawbacks of RV-I compressor is its huge and thick cylinder as can be seen in Figure 2.6. This is required in the design to accommodate the slot for vane sliding. With this massive cylinder rotating during operation, it is no doubt that the moment of inertia is high, and the huge area ‘swept’ by the vane at the wall of the cylinder represents a huge volume that is underutilised and wastage of material.

In order to tackle the thick cylinder issue in RV compressors, the novel cross vane mechanism is introduced by the author. This mechanism was initially conceived as a compressor mechanism. However, it soon revealed its potential to integrate the function of compressor and expander into a single machine. This further benefits the refrigeration system from the energy saving point of view and will be the focus of this project.

The design and working principle of cross vane expander-compressor, as we named it, will be presented and discussed in the following chapters. Before proceeding to it, the next section presents the evolution of refrigerants and the role of expanders in green refrigeration technology.

2.2. Evolution of Refrigerants

The twentieth century was marked by both the recognition and creation of a host of environmental problems that affects the entire world. The depletion of the ozone layer and global warming are two of the most well-known environmental issues that we are currently

facing. These issues were addressed by the two famous international treaties, namely the Montreal Protocol [1] and the Kyoto Protocol [58]. The Montreal Protocol aimed at phasing out the substances that leads to the depletion of ozone layer, whereas the Kyoto Protocol aimed at combating global warming. In the same spirit with these protocols, more energy saving and environmentally friendly refrigeration systems have been investigated.

From the late 1800s until 1929, refrigerators used toxic gases such as: ammonia (NH_3), methyl chloride (CH_3Cl), and sulphur dioxide (SO_2) as refrigerants [59]. Several fatal accidents which occurred in the 1920s led three American corporations: Frigidaire, General Motors, and DuPont to launch a collaborative research to develop a safer refrigerant [60]. Their efforts led to the discovery of chlorofluorocarbon (CFC) in 1928. CFCs are odourless, non-toxic, non-flammable, and chemically inert [61]. They were widely used in compressor refrigerators and became the standard for almost all the home kitchens [59]. However, it is not until the mid-1970s, where the ozone depleting nature of CFC was identified [1]. Following this, the Montreal Protocol was proposed and the use of CFC was successfully stopped as a result. Manufacturers then use HCFCs such as R-22, as well as HFCs such as R-134a as the replacement for CFCs [62]. At present, R-134a is also getting replaced by R-1234yf, due to the former's high global warming potential [63]. Nonetheless, all these chemical substances are man-made and can only serve as temporary alternatives [64].

In search of a more environmentally friendly and non-toxic refrigerant, among others, carbon dioxide (CO_2) was proposed [2]. CO_2 was the refrigerant of choice during the early 20th century. It is natural, environmentally friendly, available abundantly, non-inflammable, non-toxic and shows an excellent thermodynamic behaviour [3]. However, due to its relatively low critical temperature, CO_2 refrigeration systems are required to operate at high pressure, which is up to five times as high as those commonly seen in current technology. As a result, the working fluid undergoes a large pressure drop during the throttling process and huge amounts of energy are lost. The energy losses which are related to the throttling process are comparatively larger than the conventional refrigeration systems. Without a significant improvement to its efficiency, the use of CO_2 systems may still be impractical from energy efficiency point of view, especially in warmer climate.

Of the many new refrigeration technologies proposed, the use of expander to replace the conventional expansion valve is the most promising [4], where the expansion process is

modified from a near isenthalpic one to a near isentropic one [65]. The recovered work from the expansion process is used to partially drive the compressor, thereby improving the coefficient of performance (COP) of the system. Because of this, some of the compressor designs which are available have been modified to be used as expanders. For example, the reciprocating piston [65-70], rotary vane [71, 72], rolling piston [73-76], revolving vane [56, 77-80], screw [81-85] and scroll [86-91] have all been modified to be used as expanders. Early investigations have shown that the installation of expanders improve the COP by more than 25% and causes the transcritical CO₂ refrigeration system to be comparably efficient to a conventional R-22 refrigeration system [92].

In the past 10 years, research on employing expanders in vapour compression systems has been gaining significant attention. In the next section, a review on the expander researches in air-conditioning, heating and other applications is presented.

2.3. Expanders in Air-conditioning, Heating and Other Applications

An expander is basically a compressor that operates in the reversed direction. Though the idea might appear simple, the conversion of a compressor to an expander is not as straightforward as it seems. There are some important issues that must be addressed when designing an expander. Firstly, expanders must be able to operate under two-phase expansion process. This is not common for compressors that usually avoid wet compression because the presence of liquid droplets can damage the compressor [93]. Secondly, some expander types (e.g., reciprocating piston, rolling piston and revolving vane) require externally controlled valves such as solenoid valves [65], electronically activated valves [5] or mechanically activated valves [67, 74, 94] at the inlet to control the beginning and ending of the suction process. And lastly, expanders must be able to handle sealing, lubrication, liquid slugging, cavitation, vibration and noise control problems which may affect the performance of the expander [95]. As long as these issues are tackled, any type of compressor can be used as an expander. Therefore, some of the currently available compressor designs have been modified to become expanders.

Li et al. [94] designed and constructed a rolling piston expander prototype, where the suction control system is made of a mechanical cam device. The experimental results showed that maximum isentropic efficiency exists at the optimal revolution speed and designed pressure ratio, and that a COP improvement of at least 10% was recorded.

Tian et al. [96] studied the expansion power recovery in CO₂ transcritical cycle using a rolling piston expander. Based on their simulation results, it was found that the system's COP increases with the evaporating temperature and the efficiency of the expander, but decreases with the outlet temperature of the gas cooler. A prototype of the rolling piston expander was also tested. It was reported that the expander was able to recover up to 14.5% of the compression work and had a maximum efficiency of 45%.

Yang et al. [97] investigated the performance of a rolling piston expander in CO₂ water-to-water heat pump system. They found that the decrease of inlet temperature and the increase of mass flow rate of cooling water into both the gas cooler and evaporator improve the performance of the system. Similarly,

Jiang et al. [98] tested the performance of a two-stage rolling piston expander in a CO₂ water-water heat pump system. From the test, the expander had an efficiency of 28% to 33% within the operational speed of 850 rpm to 1000 rpm.

Lu et al. [99] investigated the performance of a single screw expander in a compressed air refrigeration system. The expansion of compressed air has led to a temperature drop which increases with intake pressure but remains fairly constant with rotational speed. Higher adiabatic efficiency was achieved at higher intake pressure and rotational speed, where its maximum value was recorded at 65%. This also indicates that the expansion process becomes closer to an adiabatic process where its polytropic exponent increases in a similar fashion. In terms of COP, the variation was insignificant as the intake pressure increases at low rotational speed, but increases when the intake pressure increases at high rotational speed. A maximum COP of 0.9 was achieved.

Taniguchi et al. [100] investigated the two-phase flow phenomena in the screw expander in a R-12 refrigeration cycle. According to their findings, flashing during the expansion process begins after bubble formation which occurs almost instantaneously. This is likely due to the vigorous mixing in the expansion chamber. Additionally, it was found that leakage becomes less severe as the operational speed increases because the direction of leakage flow is opposite to the motion of the rotor surfaces which form part of the clearances.

Yang et al. [72] experimentally investigated a double-acting rotary vane expander in the transcritical CO₂ refrigeration system. The prototype was an improved version from its original design, where springs were included in the vane slots and sealing vane was added at the sealing arc to reduce the leakage in the expander. It was found that the maximum volumetric efficiency increased from 17% to 30%, and the maximum isentropic efficiency increased from 9% to 23%. In addition, the maximum COP improvement of the expander-based refrigeration system was recorded at 14.2% under the test conditions.

In the second part of the study [101], thermodynamics and vane dynamics models were developed to investigate the major factors influencing the efficiencies of the rotary vane expander. Several factors such as leakage through various leakage paths, vane friction, and flow through the inlet and outlet ports were taken into account. The mathematical model was validated with experimental data, and showed a deviation of less than 10%. Based on the validated model, the effects of operational conditions on the expander efficiency were analysed. From the analysis, the expander obtained a maximum isentropic efficiency at the optimal pressure ratio and rotational speed.

In yet another study, Fukuta et al. [71] built a small rotary vane expander and measured its performance in a transcritical CO₂ cycle. The expander has a large leakage loss and a dominant leakage path was found at the clearance between the vane side and vane slot of the rotor. To solve the problem, they changed the way in which the vane back-pressure is supplied, and found that the improvement led to an increase in maximum total efficiency from 42% to 60%.

By having the unique rotatory cylinder design, RV compressors have shown high performance in terms of mechanical and volumetric efficiencies [52, 53]. Not surprisingly, this has sparked off a series of research on the RV expander, and Subiantoro and Ooi were the pioneers [77]. They formulated mathematical models to investigate the operational characteristics of the expander, and discovered that the only significant frictional losses were at the shaft bearing. Subsequently, they compared the design variants of RV expander, and found that RV-I expander with the rotor as the driving component was the most superior design [56]. The study also showed that RV-I expander had relatively lower frictional losses at the various friction areas, and that a mechanical efficiency of up to 96.5% can be achieved. A prototype was fabricated and tested in an open circuit using air [80]. The prototype

successfully operated up to 600 rpm and 5 barg of suction pressure. At the designed suction pressure of 3 barg, the maximum isentropic and volumetric efficiencies of the prototype were found to be 32% and 25%, respectively.

Scroll machines can also be operated as expanders and performs as well as their compressor counterpart. However, due to its complicated geometry, long operational period, mechanical design and high-speed working fluid which impinges on the scroll plates, the scroll expanders suffer from a phenomenon known as cavitation [102]. As a result, the surface of the scroll plates is prone to damage by the imploding cavitation bubbles and the life expectancy of the machine is shortened. Nonetheless, the conversion of scroll compressor to expander remains one of the most straightforward process as it requires no control valves for the suction and discharge flow. Therefore, scroll expanders have been used in various applications.

Fukuta et al. [103] made a scroll expander prototype using a mechanical element of a scroll compressor for CO₂ water heater, and measured its performance. It was found that the maximum total efficiency was 55% at 3500 rpm and the axial and radial clearances were 10 μm . Prior to this, they studied the scroll expander theoretically and concluded that axial clearance has a larger influence on the performance of the expander than radial clearance. Nonetheless both contribute to losses and certain measure is needed to improve the performance of the expander.

In an effort to minimise the leakage losses in both the axial and radial direction, Hiwata et al. [104] investigated the axial and radial forces of the scroll expander. They suggested the use of overexpansion to prevent the separation of stationary and orbiting scroll in the axial direction. The approach not only prevented the drop in volumetric efficiency, but also kept it unchanged over a wide range of operational conditions. They further analysed the radial contact force and found that maximum volumetric efficiency exists at the optimal average bearing clearance and optimal relative eccentricity.

In yet another study, Kohsokabe et al. [105] tested the performance of a scroll expander in a CO₂ system. The test results showed that the isentropic efficiency increases when the mass flow rate increases, and at its maximum of 83% at the optimal rotational speed and pressure ratio.

Apart from refrigeration systems, expanders are also used in other applications such as trilateral flash cycles and low grade heat recovery systems. Smith et al. [106] studied the performance of screw expander in trilateral flash cycles. In their investigations, they observed a large pressure drop during the initial filling process which normally occurs at constant pressure. This phenomenon is due to the acceleration of the fluid as it enters the varying and restricted space between the rotor lobes. In the multistage-expander system, an overall expander adiabatic efficiency of 82.2% was obtained. The relatively high efficiency is due to high condensing pressure which minimizes the volume ratio and increases the induced mass flow rate and hence the leakage. Moreover, the adverse effect of leakage is less severe in expander compare to that of compressor because both the mass and leakage flow are in the same direction.

The interest in low grade heat recovery systems has recently grown dramatically, and a number of new solutions have been proposed. Among these solutions, the Organic Rankine Cycle (ORC) system is the most widely used. Quoilin et al. [90] conducted an experimental study and modelled an ORC using scroll expander. Employing R-123 as the working fluid, the experiment showed that the expander isentropic efficiency ranges from 42% to 68%, where it corresponds to a maximum cycle efficiency of 7.4%. Subsequently, an optimisation study was carried out to find the optimal operational speed of the expander and the flow rate of the pump. The results showed that the optimized operational speed and flow rate are 2300 rpm and around 0.09 kg/s, respectively, and the efficiency of the ORC has been improved to a value of 9.9%.

In yet another study, Clemente et al. [88, 89] studied the performance of a bottoming ORC for the heat recovery of an internal combustion engine, where the ORC is equipped with a commercial scroll expander. Test results showed that the whole power output is improved from 18 to 19 kW, while the efficiency is increased from 34% to 36%. A comparison study was also performed between axial turbine, radial turbine, scroll expander and reciprocating piston expander, using six different working fluids in an optimised cycle. It was found that the ORC with either axial or radial turbine performed better than that of the scroll expander. In spite of this, the scroll expanders are still preferred over the dynamics expanders as it is more cost effective. The application of reciprocating piston expander is still impractical, due to its relatively large displacement.

Yun et al. [91] compared the performances of two R-245fa ORCs, one with a single scroll expander and the other one with two scroll expanders arranged in parallel. It was discovered that the shaft power and the cycle efficiency were higher in the single expander cycle than the dual expander cycle when the heat input was low.

Lastly, Zheng et al. [76] measured the performance of a rolling piston expander in a low-temperature ORC using R-245fa as the working fluid. The experimental results showed that the maximum shaft power output was 0.35 kW and the efficiency of the cycle was 5% to 6%. The steady isentropic efficiency and the maximum mechanical efficiency of the expander were 40% and 44%, respectively.

2.4. Methods of Implementing the Expander-compressor Systems

A major issue associated with any refrigeration system with power recovery from the throttling process is the way of utilising the power generated from the work recovering device. At present, there are many methods of tackling such problem, but all of them can be classified under two main categories: electrical power generation, or expander-compressor system. Both methods have advantages and disadvantages.

In electrical power generation, the expander is linked to an independent electrical generator to produce electrical power which can be used to drive the compressor of the same system. This method allows flexible control without any restriction in the wide operational conditions as the expander and compressor can be controlled independently [107]. However, the requirement of a generator incurs added cost, the need for constant speed control, and large loss in the transmission process as the power generated undergoes two additional energy transformations. The expander-compressor system would be more attractive from an efficiency perspective since transfer loss can be reduced. However, this method is more challenging to implement due to its complicated system, and would inadvertently fix the rotational speed as well as the density ratio at the inlet of both the expander and compressor. In spite of this, the concern over the ever increasing energy demand and the recent revival of CO₂ as refrigerant has altogether steered us in the direction of creating a more energy-efficient and environmentally friendly refrigeration system. For that reason, the expander-compressor system is highly favoured over electrical power generation, and has become the subject of intense research in the recent years.

Essentially, there are two ways of implementing the expander-compressor system. The first method is through the direct coupling of expander and compressor of the same or different mechanism via a connecting shaft. This method is generally less complicated as only one shaft is required to link the two machines together, and the power profile of both the expander and compressor can be matched easily to reduce the peak power of the overall system. In addition, since the expander and compressor are separated as individual machines, each of them would have its own lubrication system, and the undesirable effect of internal leakage between the expander and compressor would be minimised. However, such systems are usually bulkier and larger, relatively more expensive to implement, and comprises two sets of frictional losses from both the expander and compressor. The following are some of such expander-compressor systems found in the literature.

Quack and his other co-workers [66, 108, 109] reported the developmental program of a free piston expander-compressor unit for CO₂ transcritical cycle. A total of 12 cycles ranging from conventional isenthalpic throttling cycles to isentropic expansion cycles, with one or two stages of expansion or compression, with or without expansion work recovery for compression, and combination of throttling devices and expanders, have been proposed and compared in terms of their COP improvement over the conventional throttling cycle. The initial calculations showed that one stage expansion and compression with the recovered expansion work partially driving the compressor improves the COP by 38.7%, and that one stage expansion and two stage compression with the recovered expansion work fully driving the second stage compressor improves the COP by 64.8%. Subsequently, an expander-compressor unit with three-stage expander coupled with a compressor was built. All pistons are double acting, so there are two compressor chambers and six expander chambers in total. Since all expander valves have to be opened or closed in the end position of the piston rod, one single sleeve valve can serve all the six chambers in parallel. So far the unit has been operating with frequencies between 0.8 and 8 Hz, which translates to a maximum piston speed of 0.5 m/s. This relatively slow speed ensures that expansion across the two phase boundary occurs near thermodynamic equilibrium. Experimental results showed that COP improvement over the throttling cycle was 40%, where the mechanical efficiency, isentropic efficiency of the expander, and isentropic efficiency of the compressor were about 94%, 70% and 90%, respectively. The expander-compressor unit is now ready to be installed in commercial refrigerators.

Peng et al. and Zhang et al. from Xi'an Jiaotong University, China [67, 68, 110] developed a single acting free piston expander-compressor unit for work recovery in the transcritical CO₂ refrigeration cycle. The expander is connected to the compressor by a common shaft, whereby the compressor serves as the auxiliary compressor which operates in parallel to the main compressor. A kind of mechanical actuation, slider based control scheme was also proposed to control the suction and discharge process of the expander. Experiments were conducted on the test rig with CO₂ as the working fluid. According to the test results, a high side pressure of at least 13 MPa was required for the expander to start up when the low side pressure was 4 MPa. The high pressure difference indicates a large pressure loss in the expander, which is due to the serious gas pulsation that occurs at the expander inlet. This phenomenon should be minimised as much as possible so that the performance of the machine can be improved. The single acting model was subsequently modified to a double acting type, and tested in the air test system. It was found that the isentropic efficiency of the expander was 62%, and that an optimal frequency exists in the range of 10 to 17 Hz. Further experimental investigation of the expander-compressor unit will be conducted in CO₂ transcritical system.

Kim et al. [87] designed a scroll type expander-compressor unit and simulated its performance in two-stage compression CO₂ cycle. The compressor of the expander-compressor unit, which is also the first stage compressor, is driven by the expander via a crank shaft, and an inter-stage cooling occurs between the first and second stage compression. At the designed operational condition, the total efficiencies of the expander and compressor were 54.4% and 85.6%, respectively, and resulted in the improvement of refrigerating effect and COP by 8.6% and 23.5%, respectively. Moreover, the simulation studies also showed that COP improvement decreases as the low side pressure increases, while the effect of high side pressure was insignificant.

In yet another study conducted by Kakuda et al. [86], a prototype of the scroll expander-compressor unit was designed, fabricated and tested in a two-stage compression CO₂ cycle. The prototype consists of an expander and a compressor connected together by a shaft and an orbiting scroll plate. In the experiment, the compressor of the expander-compressor unit serves as the second stage compressor, and an intercooler exists between the first and second stage compressors. Test results showed that the use of expander-compressor unit reduces the

power consumption in the first stage compressor by 25% and improves the cycle COP by 30%.

Smith et al., Brasz et al., Stosic et al. and Kovacevic et al. from City University of UK [81-84] studied the screw expander-compressor unit. They obtained the theoretical estimates from the modified thermodynamics modelling techniques used for gas compressors, and the data was compared with the test results carried out in a R-113 refrigeration system. The analysis showed that good agreement was achievable to make reliable predictions on the performance of the screw expander. Subsequently, they constructed a twin screw expander-compressor unit for work recovery in high pressure refrigeration system. The expander-compressor unit comprises a pair of rotors, whereby the same profile is shared between the expander and compressor. By using the same profile, the rotors can be machined and ground in a single cutting operation and separated by machining a parting slot in them. Each set of rotors is then contained and sealed in a separate chamber within a single casing, thereby creating separate chambers for the expander and compressor. In the preliminary testing with R-113, the overall efficiency of the expander-compressor unit was 50% to 60%. This corresponds to an expansion efficiency of around 70% combined with a compression efficiency of around 80%. A major problem associated with any high pressure refrigeration system is the creation of large axial and radial forces on the rotors, which is mainly due to the high pressure difference between inlet and outlet ports. Therefore, some of the power transmitted through the rotors is lost in bearing friction. The combination of expander and compressor solves the problem by placing the expander inlet port and compressor outlet port in the centre of the combined unit, while placing the expander outlet port and compressor inlet port at their respective ends. Since the expander and compressor operates at equal pressure difference, the axial forces on the bearings virtually cancelled out. In addition, the pressure loads in the expander and compressor are also located almost opposite to each other near the centre of the rotors. Hence, the radial load on the bearings is reduced too. Exact calculations were carried out with Computational Continuum Mechanics to aid the design of twin screw expander-compressor unit. The results showed that the combined unit not only reduces the total bearing load but also minimizes rotor deformation, thereby reducing the critical clearances between the pair of rotors.

Some expander-compressor units have combinations of expander and compressor of different mechanisms. Matsui et al. [74, 75] built an expander-compressor unit by joining the shaft of a

two stage rolling piston expander with the shaft of a commercial scroll compressor using a coupling connector. In the two stage expander, the first stage working chambers are smaller than of the second stage, so that it can be used as a suction valve to control the amount of fluid entering the second stage. An isolation structure was also set up between the motor and the expander, so that the expander is thermally isolated from the compressor, ensuring that an optimal operational condition exist for both the expander and compressor. Additionally, a simple optimisation was performed by analysing the sensitivity of the friction loss to some performance variables. Experimentally, the optimised expander had an efficiency of 60% and improves the COP of the CO₂ refrigeration system by 6%.

Westphalen and Dieckmann [111] studied the performance of an expander-compressor unit based on scroll type expander and dual-stage rolling piston compressor in a CO₂ system. The analysis was carried out under typical military design conditions, where the initial calculations showed that COP improvement of 40% was attained when the overall expander efficiency was 70%. Subsequently, an expander-compressor prototype was fabricated by connecting the shaft of the compressor to the orbiting drive of the expander. Initial testing showed that the leakage in the expander was manageable, being limited to less than 10% of the design expander flow.

Kohsokabe et al. [73] investigated how their expander-compressor unit affects the basic operating characteristics of a CO₂ refrigeration cycle for a small capacity air-cooled chiller. Both the expander and compressor are connected together with a shaft. The expander is of scroll type, while the compressor is of rolling piston type and serve as a secondary compressor to assist the main compressor. They found that by installing the expander-compressor unit, a COP improvement of 30% was obtained when the efficiency of the machine was 57%. The result confirms that the expander-compressor unit will become a key technology in CO₂ based refrigeration system.

The other method of implementing the expander-compressor system is through the innovative use of the existing or new design to combine the function of the expander and compressor into one single mechanism. Hence, only one machine is required instead of two which indicates a compact, lighter and more cost effective system as parts can be shared between the expander and compressor. Apart from that, such a system is also predicted to have higher mechanical efficiency as it consists of one set of frictional losses from one single machine.

However, the integrated expander-compressor systems are usually more complicated as functions such as lubrication system must be considered simultaneously, and the proximity of expander and compressor means that internal leakage and heat transfer between them are unavoidable and would result in a lower COP [112]. Also, the power profiles between the expander and compressor cannot be matched easily because the power profiles are part of the inherent characteristics of the expander-compressor machine. As a result, a higher peak power is required from the motor.

2.5. Patent Search

A patent search has been carried out to study the design of the existing expander-compressor systems. It is found that most of systems are a combination of two physically separated expander and compressor of the same mechanism coupled by a common shaft. The expander and compressor can be made of the existing mechanisms, such as reciprocating piston, scroll, screw and rolling piston. The patents and their respective inventors are listed in Table 2.1.

Table 2.1 Patents based on the combination of expander and compressor of same mechanisms

Mechanism	Inventors
Free piston	Nickl [113]
Scroll	McCullough [114]
Screw	George [115] Leo [116, 117] Smith et al. [118] Tang and Brasz [119]
Sliding vane	Glen [120]

In addition, systems constructed of different expander and compressor mechanisms are found. These are shown in Table 2.2

Table 2.2 Patents based on the combination of expander and compressor of different mechanisms

Mechanism	Inventors
Scroll & rolling piston	Ogata et al. [121] Okaichi et al. [122, 123] Oyagi et al. [124] Shiotani et al [125] Takahashi et al. [126, 127] Wada et al. [128]

For expander and compressor integrated into a single machine, the patents are even scarce. Three of such designs are found as shown in Table 2.3. These systems are referred to as expander-compressor units.

Table 2.3 Patents of expander-compressor unit

Mechanism	Inventors
Sliding vane	Edwards [129, 130] Johnson et al. [131]
Integrated compressor expander apparatus	Milburn et al. [132]

Masuda's rotary fluid machine [133] is originally designed to be used as a compressor. However, further inspection reveals its potential to operate as an expander-compressor unit. With reference to Figure 2.7, the machine comprises an external and internal space labelled by the numbers 51 and 52, respectively. These two spaces are separated by the annular piston (22) that permits the working chambers of the expander and compressor to be formed. The blade (23) further separates the working chambers of the supposedly expander and compressor into suction and discharge chambers, and suction and compression chambers, respectively. Masuda's mechanism will be further analysed and compared with that of CVEC in Section 3.5, as both machines have a very similar components' shape.

Figure 2.7 Schematic diagram of the rotary fluid machine; the major components are the outer cylinder (24), the blade (23), the inner cylinder (25), the annular piston (22), the swing bush (27) and the eccentric part (35) [133]

2.6. Concluding Remarks

It is obvious that most of the available expander-compressor systems are a combination of two physically separated units connected by a common shaft. This is simply due to their straightforward construction that allows for the use of any existing compressors and expanders, and the advantages that come with having the compressor and expander separated. However, having two separated units also implies two sets of mechanical losses, and a larger and bulkier system. Therefore it is desirable to innovatively integrate both the expander and compressor into a single-seamless design that is energy-efficient, environmentally friendly and yet feasible in its applications.

3 Introduction to Cross Vane Expander-Compressor

In this chapter, a newly invented expander-compressor unit [134] will be introduced. Its conceptualisation process, working principle and detailed design will be presented.

3.1. Conceptualisation of the Mechanism

The conceptualisation process begins by attempting to improve the design of one of the simplest and most widely used mechanism in compressor industries – the rolling piston mechanism. In its basic form, the rolling piston mechanism consists of five components: an eccentric, a roller, a cylinder (or stator), a vane and a discharge valve. During operation, the roller maintains the contact with the cylinder and the vane, by rolling around the cylinder's inner wall and rubbing against the vane tip, respectively. Therefore, two working chambers are created which are bounded by the inner surface of cylinder, the outer surface of roller and the side of the vane as shown in Figure 2.2. One chamber increases in volume, while the other decreases it, resulting in the suction and compression processes, respectively. At the end of the compression process, the pressure of the working fluid increases and reaches that of the discharge pressure, the discharge valve opens and the fluid is discharged through the discharge port.

The rolling piston compressor is widely used in room air-conditioners for a few reasons. It is reliable and geometrically simple as it consists of mostly cylindrical components which lead to ease of manufacturing. However, this mechanism has a few weaknesses [22, 135]. Firstly, it has too many rubbing surfaces with high relative rubbing velocity between the components which cause high frictional losses. These losses are found at rubbing locations between

1. eccentric and inner surface of the roller,
2. roller and cylinder end-faces,
3. eccentric and cylinder end-faces,
4. outer surface of roller and inner surface of cylinder,
5. vane side and vane slot, and
6. vane tip and roller.

Secondly, the eccentric and roller are not rotating about their own axes but orbiting about the central axis of the cylinder. This results in unnecessary vibration and wobbling effects during

operation. Lastly, as the vane tip has to rub against the outer surface of the roller, the components are easily weakened, and wear and tear is inevitable.

In order to overcome these weaknesses, the revolving vane mechanism was introduced [51-53, 136-140]. The following modifications are proposed:

1. Combine eccentric and roller into a single component called rotor to eliminate the rubbing between eccentric and inner surface of the roller.
2. Make both the rotor and cylinder (and its end-faces) rotate eccentrically with respect to each other to minimise the relative rubbing velocity which reduces the end-face frictional losses. In addition, since the rotor is rotating about its own axis, the unwanted vibration and wobbling effects from the orbiting motion of eccentric and roller are eliminated.
3. Attach the vane on to the rotor as a single component and have it rotate together with the rotor. This eliminates the rubbing at vane tip and, most importantly, it also removes the dependency of the vane side-slot friction on the pressure differential across the vane.
4. Modify the vane slot to the one shown in Figure 2.6 to minimise the frictional loss at the vane side and vane slot.

Similarly, the revolving vane mechanism also has its own drawbacks. As can be seen from Figure 2.6, the relatively large rotor takes up a lot of potentially useful working space within the cylinder. This reduces the available space for the working chambers and lowers the capacity of the machine for a given weight and size. Since the rotor is relatively large and massive, the moment of inertia of the rotor is expected to be high too. Therefore a larger torque is required to rotate the rotor. In addition, the design variant which has the vane penetrating into the cylinder wall resulted in a significantly large sweep volume that is underutilized, and inadvertently makes the cylinder wall to be thicker to accommodate the length of vane that slides into the vane slot. Lastly as the vane is attached to the rotor at one end, a relatively thick vane is required to sustain the high bending moment.

In order to overcome the disadvantages of the revolving vane mechanism, the cross vane mechanism was introduced by Yap et al. [141-143]. A few modifications were proposed, these include:

1. reduce the size of the rotor relative to that of cylinder (which is renamed as inner cylinder),
2. extend the vane through and out of the cylinder wall, and
3. introduce a second cylinder called the outer cylinder that is attached to the other end of the vane and allow it to rotate about the rotating axis of the rotor and vane.

With these modifications, the components can be made more compact and the available space for working chambers have increased. More importantly, the new mechanism combines the expander and compressor into one seamless unit, called the cross vane expander-compressor or CVEC in short. This new machine is designed to replace the expansion valve and the compressor of a vapour compression refrigeration cycle. In other words, the role of the expansion valve is replaced by the ‘expander’ and the role of the compressor is replaced by the ‘compressor’ of CVEC. The ‘expander’ thus recovers the expansion energy and provides this energy ‘directly’ to the ‘compressor’ without any further energy transformation. One notable advantage of this design is that it is compact, geometrically simple and has the same total number of components as a rotary compressor. Also, as only one machine is built to provide both the functions of expander and compressor, only one set of mechanical losses is produced, which increases the mechanical efficiency further.

3.2. The Components and Their Functions

In its basic form, the CVEC consists of three main components: a) the outer cylinder-vane-rotor component, b) the inner cylinder, and c) the split bush as shown in Figure 3.1.

Figure 3.1 The basic components of CVEC: a) outer cylinder-vane-rotor component, b) inner cylinder and c) split bush

The vane is rigidly attached to the rotor and the outer cylinder as one component, where the centre axis of the rotor is made to coincide with the centre axis of the outer cylinder at C_1 . This component is named the outer cylinder-vane-rotor component. The inner cylinder is basically a hollow cylinder with a slot created at the wall which allows the split bush to be inserted. The shape of the split bush is semi-cylindrical where its thickness is reduced to allow the vane to slide through (see Figure 3.2).

3.3. The Assembly of the Machine

The outer cylinder-vane rotor component, the inner cylinder and the split bush are assembled as depicted in Figure 3.2. The components can be supported individually and concentrically on either a cantilever type or a simply supported type bearing pair as shown in Figures 3.3 and 3.4, respectively. The outer cylinder-vane-rotor component and the inner cylinder are allowed to rotate about their respective centre of circles, C_1 and C_2 separated by a distance, e given in the following equation.

$$e = r_{ic} - r_r \quad (3.1)$$

Moreover, the length of the expander-compressor unit, l_{ecu} , the thickness of the inner cylinder, t_{ic} , the radius of the outer cylinder, r_{oc} , and the length of the vane, l_v , are dependent on the radius of the rotor, r_r and the inner cylinder, r_{ic} by the following equations.

$$l_{ecu} = \frac{V_{exp}}{\pi(r_{ic}^2 - r_r^2)} \quad (3.2)$$

$$t_{ic} = \frac{V_{exp}/\pi l_{ecu} - 3r_{ic}^2 + 4r_{ic}r_r - r_r^2}{2r_{ic} - 2r_r} \quad (3.3)$$

$$r_{oc} = 2r_{ic} + t_{ic} - r_r \quad (3.4)$$

$$l_v = r_{oc} - r_r \quad (3.5)$$

where V_{exp} is the maximum volume displacement of the expander which is determined by Equation (6.5) (refer to Section 6.1).

Once the components are assembled, two contact lines are formed between the rotor and the inner surface of the inner cylinder, and between the outer surface of the inner cylinder and the inner surface of the outer cylinder. This leads to the formation of four working chambers, i.e., the suction and discharge chamber of the expander, and the suction and compression chamber of the compressor (see Figure 3.2). In the present configuration, a drive shaft is integrated concentrically with the outer cylinder-vane-rotor component which is coupled to a prime mover. During operation, the rotating vane forces the inner cylinder to rotate in the same direction as illustrated in Figure 3.5. The motion causes the volume of the suction and discharge chamber of the expander to vary, resulting in suction, expansion, and discharge of the working fluid. Similarly, the volume of the suction and compression chamber of the compressor varies. This results in suction, compression and discharge of the working fluid. Both the expander and compressor operate concurrently.

In CVEC, fluid flows in and out of the working chamber by the suction and discharge ports, respectively. The suction and discharge ports of the expander are placed at the opposite ends of the shaft to avoid communication between the suction and discharge fluid. The suction port extends along the shaft end and is inserted through the lower bearing where it is in

contact with the suction valve (see Figure 3.4). The function of the suction valve is to control the end of the suction process and the start of the expansion process. The current design is based on the blocker mechanism proposed by Subiantoro et al. [79, 80], where its opening angle is determined by the assumption of isentropic expansion. Other methods include the use of externally controlled valve such as a solenoid or pneumatic valve. The suction and discharge port of the compressor is located at the lower bearing and the outer cylinder, respectively. A reed valve is installed at the discharge port to control the end of compression process and the beginning of discharge process. It ensures the unidirectional flow of working fluid from the compression chamber to the housing chamber. The reed valve model is presented in Section 4.6.

There are many ways in which the suction and the discharge port of the expander, and compressor can be arranged in CVEC. The primary consideration for the location of these ports is their proximity to its respective working chamber. If we examine the suction and discharge chamber of the expander, they are both enclosed by the rotor, inner cylinder, lower bearing and end-face cover (when inner cylinder and outer cylinder are supported at one end by the lower bearing as shown in Figure 3.3) or upper bearing (when inner cylinder and outer cylinder are simply supported as shown in Figure 3.4). Therefore, the suction and discharge ports of the expander can be placed at these five components, either at the same or different components. For the suction and discharge chamber of the compressor, they are enclosed by the inner cylinder, outer cylinder, lower bearing, and end-face cover (when inner cylinder and outer cylinder are supported by the lower bearing as shown in Figure 3.3) or upper bearing (when inner cylinder and outer cylinder are simply supported as shown in Figure 3.4). Therefore the suction and discharge ports of the compressor can be placed at these locations.

Figure 3.2 Top sectional view of CVEC

Figure 3.3 Side sectional view of cantilever type CVEC

Figure 3.4 Side sectional view of simply supported type CVEC

3.4. The Working Principle

The expander and compressor operate concurrently as illustrated in Figure 3.5. However due to the inherent design of cross vane mechanism, the expander and compressor are half a cycle out of phase from each other. Figure 3.5a to 3.5f shows the operating processes of the expander. Figure 3.5a marks the beginning of the suction process of the current cycle, and the beginning of discharge process of the previous cycle. The suction and discharge processes continue in Figure 3.5b and 3.5c. During this time, the suction port is uncovered which allows high pressure fluid to flow into the suction chamber. The presence of the high pressure fluid inside the suction chamber and low pressure fluid inside the discharge chamber creates a pressure difference across the vane. This pressure difference creates a force that pushes the vane to move in the direction of reducing the volume of discharge chamber while increasing

the volume of suction chamber. As a result of this movement, more high pressure fluid flows into the suction chamber and at the same time more low pressure fluid is being discharged through the discharge port. At a designated point, the suction port will be covered as shown in Figure 3.5d. The closing of the suction port marks the end of the suction process and the beginning of the expansion process in the suction chamber. The selection of the angular position where the suction port closes must be appropriately designed and determined to avoid over and under expansion from occurring. Once the suction port is closed, the working fluid inside the suction chamber expands as shown in Figure 3.5e and 3.5f, while the discharge process continues as shown in Figure 3.5d, 3.5e and 3.5f. The expansion and discharge processes end in Figure 3.5a. The cycle repeats.

The compressor cycle starts at Figure 3.5e. This stage marks the beginning of the suction process of the current cycle in the suction chamber, and the beginning of compression process of the previous cycle in the compression chamber. A prime mover is attached to the drive shaft to rotate the CVEC in the direction of reducing the volume of compression chamber while increasing the volume of suction chamber, while the expansion work recovered in the expander also assists the prime mover in rotating the CVEC in the same direction. This causes the working fluid in the compression chamber to be compressed while suction fluid is continuously drawn into the suction chamber as shown in Figure 3.5f and 3.5a. At a designated point, the compressed fluid reaches the discharge pressure and the valve reed uncovers the discharge port as shown in Figure 3.5b. This marks the beginning of the discharge process whereby the high pressure fluid is forced out of the compression chamber through the discharge port as shown in Figure 3.5c and 3.5d. The suction process continues as shown in Figure 3.5b, 3.5c and 3.5d. The discharge and suction processes end in Figure 3.5e and the cycle repeats.

Figure 3.5 The operating process of CVEC

The compression work of the compressor is supported by the prime mover and the expansion work recovered by the expander. Thus, lesser work is required from the prime mover and the COP of the refrigeration system is increased. In addition, the expansion process is also assumed to be isentropic. Therefore, the discharge fluid from the expander has a lower enthalpy as compared to that of the expansion valve, and the difference, Δh_{hs} can be shown

in Figure 3.6. This contributes to an improved cooling capacity, which can be as much as 15% [5], and the COP is further increased as a result.

$$COP = \frac{Q' + \Delta h_{hs}}{W_{motor} - W_{exp}} \quad (3.6)$$

where

Q' is the cooling capacity with expansion valve (J)

Δh_{hs} is the difference in specific enthalpy between the discharge fluids from the expander and expansion valve (J)

W_{motor} is the required motor work without the expander (J)

W_{exp} is the expansion work (J)

Figure 3.6 T-s diagram of transcritical carbon dioxide cycle [92]

3.5. Comparison between Rotary Fluid Machine and Cross Vane Expander-Compressor

In Section 2.5, the rotary fluid machine (see Figure 2.7) is revealed to have the potential to operate as an expander-compressor unit. This machine is the closest in terms of design as compared to CVEC. Though similar in terms of their components' shape, the rotary fluid machine and CVEC are vastly different mechanisms. In the rotary fluid machine, the volume changes of the working chambers are the result of the wobbling motion of the outer cylinder (24), blade (23) and inner cylinder (25) with respect to the annular piston (22), which is similar to the motion of the swing piston mechanism [95, 144, 145]. The design comparison between the rotary fluid machine and CVEC is further shown in Table 3.1.

Table 3.1 Design comparison between the rotary fluid machine and CVEC

Mechanism	Rotary fluid machine [133]	CVEC
Minimum number of major components	<p>Total: 4</p> <ol style="list-style-type: none"> Outer cylinder (24), blade (23) and inner cylinder (25) Annular piston (22) Eccentric part (35) Swing bush (27) <p>Outer cylinder (24), blade (23) and inner cylinder (25) can be built as one single component, which is named as component 'a' for simplicity. Component 'a' and eccentric part (35) have to be built separately as they move relatively from each other.</p>	<p>Total: 3</p> <ol style="list-style-type: none"> Outer cylinder-vane-rotor component Inner cylinder Split bush <p>Outer cylinder, vane and rotor can be built as one component, as all of them rotate about the same rotational axis and at the same rotational speed.</p>
Shape of components	<p>Inner cylinder (25) must be hollow to allow the coupling of eccentric part (35).</p> <p>The driving shaft (33) is eccentrically attached to the eccentric part (35).</p>	<p>Rotor is not hollow.</p> <p>The rotor is concentric with respect to the driving shaft.</p>
Central longitudinal axes	<p>First axis location is shared by:</p> <ol style="list-style-type: none"> Central longitudinal axis of the driving shaft (33). Central longitudinal axis of 	<p>First axis location is shared by:</p> <ol style="list-style-type: none"> Central longitudinal axis of the driving shaft. Central longitudinal axis of

	<p>the annular piston (22).</p> <p>Second axis location is shared by:</p> <ol style="list-style-type: none"> 1. Central longitudinal axis of the eccentric part (35). 2. Central longitudinal axis of the outer cylinder (24). 3. Central longitudinal axis of the inner cylinder (25). <p>The second axis orbits around the first axis which is stationary.</p>	<p>the outer cylinder.</p> <p>Second axis location is located at:</p> <ol style="list-style-type: none"> 1. Central longitudinal axis of the inner cylinder. <p>Both axes remain stationary.</p>
Contact lines	<p>The first contact line between the outer surface of the inner cylinder (25) and the inner surface of the annular piston (22) orbits along the inner peripheral of the annular piston (22).</p> <p>The second contact line between the inner surface of the outer cylinder (24) and the outer surface of the annular piston (22) orbits along the outer peripheral of the annular piston (22).</p>	<p>The first contact line between the surface of the shaft and the inner surface of the inner cylinder is stationary throughout the operation.</p> <p>The second contact line between the outer surface of the inner cylinder and the inner surface of the outer cylinder is stationary throughout the operation.</p>
Annular piston (22) & inner cylinder	Annular piston (22) is stationary and does not rotate .	Inner cylinder rotates about its own central longitudinal axis.
Component 'a' & outer cylinder-vane-rotor component	Component 'a' orbits eccentrically with respect to the stationary annular piston (22) in a reciprocating manner.	Inner cylinder rotates eccentrically with respect to the rotating outer cylinder-vane-shaft assembly .
Blade (23) & vane	The blade (23) executes a swing motion about a swing center .	The vane rotates about the central longitudinal axis of the driving shaft .
Bushings (2a/b) & split bush	The bushing (2a/b) executes a swing motion about a swing center .	The split bush orbits about the central longitudinal axis of the driving shaft .
Volume variation of working chambers	Volume variation is led by the orbiting contact lines .	Volume variation is led by the leading and trailing surfaces of the rotating vane .
Dimensions	Maximum eccentricity, e (i.e. distance between the central longitudinal axis of eccentricity part (35) and that of the driving shaft(33)) has to be lesser than the radius of the eccentricity part (35), r_e .	Maximum eccentricity, e (i.e. distance between the central longitudinal axis of inner cylinder and that of the rotor) is not limited by any dimensions.
	Radius of inner surface of	Radius of inner surface of inner

	annular piston (22), r_p is dependent on the radius of the eccentric part (35), r_e , thickness of the inner cylinder (25), t_{ic} , and eccentricity, e by $r_p = r_e + t_{ic} + e$.	cylinder, r_{ic} is independent of any dimensions.
	Radius of inner surface of outer cylinder (24), r_{oc} is dependent on the eccentricity, e , radius of inner surface of the annular piston (22), r_p , and thickness of the annular piston (22), t_p by $r_{oc} = e + r_p + t_p$.	Radius of inner surface of outer cylinder, r_{oc} is dependent on the radius of inner surface of the inner cylinder, r_{ic} , thickness of the inner cylinder, t_{ic} , and radius of the rotor, r_r by $r_{oc} = 2r_{ic} + t_{ic} - r_s$.
Major friction areas	<ol style="list-style-type: none"> 1. Between driving shaft (33) and bearing. 2. Between blade (23) and bushings (2a/b). 3. Between eccentric part (35) and inner cylinder (25). 4. Between end-face of outer cylinder (24) and bearing surface. 5. Between end-face of inner cylinder (25) and bearing surface. 6. Between end-face of eccentric part (35) and bearing surface. 	<ol style="list-style-type: none"> 1. Between shaft and bearing. 2. Between inner cylinder and bearing. 3. Between vane and split bush. 4. Between end-face of outer cylinder and bearing surface. 5. Between end-face of inner cylinder and bearing surface.

Figure 3.7 illustrates the comparison of the working principle of the rotary fluid machine and CVEC in one complete revolution. Notice that rotary fluid machine has an additional component which is the eccentric part (35).

Figure 3.7 Comparison of the working principle of rotary fluid machine and CVEC

Concluding Remarks

The conceptualisation of the novel CVEC has been described in this chapter. The design has made possible the integration of expander and compressor into one single unit, preserving the advantages of a rotary machine, if not better.

4 Mathematical Models

In this study, the mathematical models describing the behaviour and performance of CVEC are (1) the geometrical model, which expresses the working chamber volume as a function of the shaft rotational angle; (2) the kinematics model, which deals with velocity and acceleration of the components; (3) the thermodynamics model, which shows the instantaneous thermodynamics properties of the working fluid; (4) the mass flow model, which formulates the mass flow rate of the working fluid through the suction and discharge ports; (5) the internal leakage model, which accounts for the leakages through the gaps of the mating components within and across the working chambers of the expander and compressor; and (6) the reed valve model, which describes the response of the valve reed to pressure and centrifugal forces. Moreover, the frictional losses at various rubbing regions, as well as the suction and discharge losses are accounted for. The instantaneous power of the machine is then computed as a function of the shaft rotational angle.

4.1. Geometrical Model

In general, the variation in thermodynamics properties of the working fluid is a consequence of volume variation. Thus, the geometrical models can be regarded as prime driver for the entire performance simulation of the expander-compressor unit. In this section, the geometrical relationship of the expander and compressor are derived. The models are developed based on the schematic diagram of CVEC presented in Figure 4.1. Point C_1 is the centre of the rotor; point C_2 is the centre of the inner cylinder; Point A is the intersection between the axis of the vane and the surface of the rotor; Point B is the intersection between the axis of the vane and the inner surface of the inner cylinder; Point C is the intersection between the axis of the vane and the middle section of the inner cylinder; Point D is the intersection between the axis of the vane and the outer surface of the inner cylinder; Point E is the intersection between the axis of the vane and the inner surface of the outer cylinder.

Figure 4.1 Schematic diagram of CVEC

4.1.1. Expander Volume

The suction chamber volume, V_{escv} and the discharge chamber volume, V_{edcv} of the expander are formulated by analysing the BC_1C_2 triangle as shown in Figure 4.2.

Figure 4.2 The BC_1C_2 triangle

Applying the cosine rule on the BC_1C_2 triangle, we have

$$r_{ic}^2 = (r_{ic} - r_r)^2 + (r_r + l_i)^2 - 2(r_{ic} - r_r)(r_r + l_i)\cos(\pi - \theta_1) \quad (4.1)$$

$$(r_r + l_i)^2 = (r_{ic} - r_r)^2 + r_{ic}^2 - 2r_{ic}(r_{ic} - r_r)\cos\theta_2 \quad (4.2)$$

where

θ_1 is the rotational angle of the outer cylinder, vane and rotor (rad)

θ_2 is the rotational angle of the inner cylinder or C_1C_2B angle (rad)

r_r is the radius of the rotor, or length C_1A (m)

r_{ic} is the radius of the inner wall of inner cylinder, or length C_2B (m)

l_i is the length of the vane measuring from the surface of the rotor to the inner surface of the inner cylinder, or length AB (m)

Performing substitutions using the trigonometric identities and a series of rearrangement, we obtain

$$l_i = -(r_{ic} - r_r)\cos\theta_1 - r_r + \sqrt{r_{ic}^2 - (r_{ic} - r_r)^2\sin^2\theta_1} \quad (4.3)$$

$$\theta_2 = \cos^{-1}\left(\frac{(r_{ic} - r_r)^2 + r_{ic}^2 - (l_i + r_r)^2}{2r_{ic}(r_{ic} - r_r)}\right) \quad (4.4)$$

The volume of the suction and discharge chamber of the expander can then be determined by

$$V_{escv} = \frac{1}{2}[r_{ic}^2\theta_2 - r_r^2\theta_1 - (l_i + r_r)(r_{ic} - r_r)\sin\theta_1 - l_i w_v]l_{ecu} \quad (4.5)$$

$$V_{edcv} = \pi(r_{ic}^2 - r_r^2)l_{ecu} - \frac{1}{2}[r_{ic}^2\theta_2 - r_r^2\theta_1 - (l_i + r_r)(r_{ic} - r_r)\sin\theta_1 + l_i w_v]l_{ecu} \quad (4.6)$$

where

w_v is the width of the vane (m)

l_{ecu} is the length of the expander-compressor unit (m)

4.1.2. Compressor Volume

The suction chamber volume, V_{scv} and the discharge chamber volume, V_{dcv} of the compressor are formulated by analysing the DC_1C_2 triangle as shown in Figure 4.3.

Figure 4.3 The DC_1C_2 triangle

Applying the cosine rule on DC_1C_2 triangle, we have

$$(r_{ic} + t_{ic})^2 = (r_{ic} - r_r)^2 + (r_r + l_{AD})^2 - 2(r_{ic} - r_r)(r_r + l_{AD})\cos(\pi - \theta_1) \quad (4.7)$$

$$(r_r + l_{AD})^2 = (r_{ic} - r_r)^2 + (r_{ic} + t_{ic})^2 - 2(r_{ic} + t_{ic})(r_{ic} - r_r)\cos\theta_3 \quad (4.8)$$

where

θ_3 is the rotational angle of the inner cylinder or C_1C_2D triangle (rad)

l_{AD} is the length of the vane measuring from the surface of the rotor to the outer surface of the inner cylinder or length AD (m)

Performing substitutions using the trigonometric identities and a series of rearrangement, we obtain

$$l_{AD} = -(r_{ic} - r_r)\cos\theta_1 - r_r + \sqrt{(r_{ic} + t_{ic})^2 - (r_{ic} - r_r)^2\sin^2\theta_1} \quad (4.9)$$

$$\theta_3 = \cos^{-1}\left(\frac{(r_{ic} - r_r)^2 + r_{ic}^2 - (l_{AD} + r_r)^2}{2r_{ic}(r_{ic} - r_r)}\right) \quad (4.10)$$

The volume of the suction and compression chamber of the compressor can then be determined by

$$V_{cscv} = \frac{1}{2}[r_{oc}^2(\theta_1 - \pi) + (\pi - \theta_3)(r_{ic} + t_{ic})^2 + (l_{AD} + r_r)(r_{ic} - r_r)\sin\theta_1 - (l_v - l_{AD})w_v]l_{ecu} \quad (4.11)$$

$$V_{cdcv} = [\pi(r_{oc}^2 - (r_{ic} + t_{ic})^2) - (l_v - l_{AD})w_v]l_{ecu} - \frac{1}{2}[r_{oc}^2(\theta_1 - \pi) + (\pi - \theta_3)(r_{ic} + t_{ic})^2 + (l_{AD} + r_r)(r_{ic} - r_r)\sin\theta_1 - (l_v - l_{AD})w_v]l_{ecu} \quad (4.12)$$

4.2. Kinematics Model

In this section, the kinematics aspect of the expander-compressor unit such as the sliding velocity and acceleration of the vane, and angular velocity and acceleration of the inner cylinder will be formulated.

4.2.1. Sliding Velocity of the Vane

The sliding velocity of the vane, $\dot{l}_i$ is formulated by taking the first derivative of Equation (4.3) with respect to time.

$$\frac{dl_i}{dt} = \omega_1(r_{ic} - r_r)\sin\theta_1 \left[1 - \frac{(r_{ic} - r_r)\cos\theta_1}{\sqrt{r_{ic}^2 - (r_{ic} - r_r)^2\sin^2\theta_1}} \right] \quad (4.13)$$

4.2.2. Sliding Acceleration of the Vane

The sliding acceleration of the vane, $\ddot{l}_v$ is formulated by taking the second derivative of Equation (4.3) with respect to time.

$$\begin{aligned} \frac{d^2 l_i}{dt^2} = & \alpha_1 (r_{ic} - r_r) \sin \theta_1 \left[1 - \frac{(r_{ic} - r_r) \cos \theta_1}{\sqrt{r_{ic}^2 - (r_{ic} - r_r)^2 \sin^2 \theta_1}} \right] \\ & + \omega_1^2 (r_{ic} - r_r)^2 \left[\frac{\cos \theta_1}{(r_{ic} - r_r)} - \frac{\cos^2 \theta_1}{\sqrt{r_{ic}^2 - (r_{ic} - r_r)^2 \sin^2 \theta_1}} \right. \\ & \left. - \frac{((r_{ic} - r_r)^2 - r_{ic}^2) \sin^2 \theta_1}{(r_{ic}^2 - (r_{ic} - r_r)^2 \sin^2 \theta_1)^{1.5}} \right] \end{aligned} \quad (4.14)$$

4.2.3. Angular Velocity of the Inner Cylinder

The angular velocity of the inner cylinder, ω_2 is formulated by observing the geometrical similarity between CVEC and RV mechanism [77].

$$\omega_2 = \omega_1 \left[\frac{2(r_r + l_i)^2}{r_{ic}^2 + (r_r + l_i)^2 - (r_{ic} - r_r)^2} \right] \quad (4.15)$$

where

ω_1 is the angular velocity of the outer cylinder, vane and rotor (rad/s).

4.2.4. Angular Acceleration of the Inner Cylinder

The angular acceleration of the inner cylinder, α_2 is formulated by differentiating Equation (4.15) with respect to time.

$$\begin{aligned} \alpha_2 = & \alpha_1 \left[\frac{2(r_r + l_i)^2}{r_{ic}^2 + (r_r + l_i)^2 - (r_{ic} - r_r)^2} \right] \\ & + \omega_1 \frac{dl_i}{dt} \left[\frac{4(r_r + l_i)(r_{ic}^2 - (r_{ic} - r_r)^2)}{(r_{ic}^2 + (r_r + l_i)^2 - (r_{ic} - r_r)^2)^2} \right] \end{aligned} \quad (4.16)$$

where

α_1 is the angular acceleration of the outer cylinder, vane and rotor (rad/s²).

4.3. Thermodynamics Model

Moran and Shapiro [93] presented an energy rate balance for a control volume based on the Law of Conservation of Energy. The equation assumes that gas properties are instantaneously propagated throughout the whole working chamber, or in other words, they are uniform and have no spatial variation. It is given as

$$\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \frac{dm_i}{dt} \left(h_i + \frac{v_i^2}{2} + gz_i \right) - \sum_o \frac{dm_o}{dt} \left(h_o + \frac{v_o^2}{2} + gz_o \right) \quad (4.17)$$

where

E_{cv} is the total energy of the control volume (J)

$\dot{Q}_{cv}$ is the net heat transfer into the control volume (W)

$\dot{W}_{cv}$ is the net work done by the control volume (W)

m_i is the mass of fluid flowing into the control volume (kg)

m_o is the mass of fluid flowing out of the control volume (kg)

h_i is the specific enthalpy of the fluid flowing into the control volume (J/kg)

h_o is the specific enthalpy of the fluid flowing out of the control volume (J/kg)

v_i is the velocity of the fluid flowing into the control volume (m/s)

v_o is the velocity of the fluid flowing out of the control volume (m/s)

z_i is the height of the fluid flowing into the control volume (m/s)

z_o is the height of the fluid flowing out of the control volume (m/s)

g is the acceleration of free fall (m/s²)

t is the time (s)

The second term on the right hand side of the equation represents work done due to volumetric change, and can be expressed as

$$\frac{dW_{cv}}{dt} = p_{cv} \frac{dV_{cv}}{dt} \quad (4.18)$$

where

p_{cv} is the pressure of the control volume (Pa)

V_{cv} is the volume of the control volume (m^3)

In addition, E_{cv} comprises the internal energy, kinetic energy, and gravitational potential energy. It can be expressed as

$$E_{cv} = m_{cv} \left(u_{cv} + \frac{v_{cv}^2}{2} + gz_{cv} \right) \quad (4.19)$$

where

m_{cv} is the mass of the control volume (kg)

u_{cv} is the specific internal energy of the control volume (J/kg)

v_{cv} is the velocity of the control volume (m/s)

z_{cv} is the height of the control volume (m)

Therefore, substituting Equations (4.18) and (4.19) into Equation (4.17), and using the product rule, we have

$$\begin{aligned} & m_{cv} \frac{d \left(u_{cv} + \frac{v_{cv}^2}{2} + gz_{cv} \right)}{dt} + \left(u_{cv} + \frac{v_{cv}^2}{2} + gz_{cv} \right) \frac{dm_{cv}}{dt} \\ &= \frac{dQ_{cv}}{dt} - p_{cv} \frac{dV_{cv}}{dt} + \sum_i \frac{dm_i}{dt} \left(h_i + \frac{v_i^2}{2} + gz_i \right) - \sum_o \frac{dm_o}{dt} \left(h_o + \frac{v_o^2}{2} + gz_o \right) \end{aligned} \quad (4.20)$$

Ignoring all the potential and kinetic energy components of the control volume and inlet/outlet flow, Equation (4.20) is simplified to

$$m_{cv} \frac{du_{cv}}{dt} + u_{cv} \frac{dm_{cv}}{dt} = \frac{dQ_{cv}}{dt} - p_{cv} \frac{dV_{cv}}{dt} + \sum_i \frac{dm_i}{dt} h_i - \sum_o \frac{dm_o}{dt} h_o \quad (4.21)$$

By the Law of Conservation of Mass, the net rate of change of mass in the control volume, $\frac{dm_{cv}}{dt}$ can be written as

$$\frac{dm_{cv}}{dt} = \sum_i \frac{dm_i}{dt} - \sum_o \frac{dm_o}{dt} \quad (4.22)$$

4.4. Mass Flow Model

In Equation (4.22), the mass flow rate through the suction port, $\frac{dm_i}{dt}$ and discharge port, $\frac{dm_o}{dt}$ of the compressor and expander are analysed as a one dimensional flow through an orifice [146] as illustrated in Figure 4.4.

Figure 4.4 The schematic of orifice flow

By applying the Law of Conservation of Energy to the orifice model and assuming isentropic flow, we have

$$\frac{dm_{i/o}}{dt} = C_d \rho_{2s} A_{i/o} \sqrt{2(h_1 - h_{2s})} \quad (4.23)$$

where

C_d is the coefficient of discharge

ρ_{2s} is the density of the fluid at the orifice when the flow is isentropic (kg/m^3)

$A_{i/o}$ is the area of the suction/discharge port (m^2)

h_1 is the specific enthalpy of the upstream fluid (J/kg)

h_{2s} is the ideal specific enthalpy of the downstream fluid (J/kg)

The orifice model accounts for the actual non-ideal flow by introducing the term known as the coefficient of discharge, C_d . This dimensionless scalar lumps both the flow loss coefficient, k and isentropic efficiency, η_s together [137], where the isentropic efficiency is defined as

$$\eta_s = \frac{h_1 - h_2}{h_1 - h_{2s}} \quad (4.24)$$

where

h_2 is the specific enthalpy of the downstream fluid (J/kg)

In the special case whereby the downstream pressure, p_2 reaches the critical pressure, the Mach number, M at the orifice becomes unity. Further reduction of the downstream pressure below the critical pressure has no effect on the fluid velocity, as the fluid has attained maximum velocity equal to the speed of sound at the orifice, v_{sound} . Such condition is known as choked flow [146], a compressible flow effect whereby the fluid velocity is choked or limited. To account for choked flow, Equation (4.23) is modified to

$$\frac{dm_{i/o}}{dt} = C_d \rho_{2s} A_{i/o} v_{sound} \quad (4.25)$$

4.5. Internal Leakage Model

An efficient expander-compressor unit depends on well-designed dimensions of the components and well-controlled clearances within components to facilitate the machine's operation in such a way that internal leakages and frictions are minimal. Clearances between

components form leakage paths that lower the efficiency of the machine by reducing the delivered flow of the working fluid, and hence the cooling capacity of the system. This requires a model that allows better understanding of the internal leakage flow characteristic and the prediction of the leakage flow rates.

The determination of internal leakage flow rate is complicated. This is due to the fact that the leakage flows may involve single or two phase fluid through clearances of various geometries. Additionally, the effective leakage areas are difficult, if not impossible to determine, because these areas may be partially blocked by lubricant and liquid refrigerant in the expander and they vary throughout the operation of the machine. For simplicity, these leakage flows are modelled as one-dimensional orifice flow and discharge coefficients are used to account for these complex flow phenomena. The leakage mass flow rates is determined by

$$\frac{dm_{leak,k}}{dt} = C_{d,k} \rho_{2s} A_{leak,k} \sqrt{2(h_1 - h_{2s})} \quad (4.26)$$

The leakage paths of CVEC are found in the following areas (see Figure 4.5), where the path numbers, k and their associated mass flow rate are described below:

- Path 1:** Leakage through expander radial clearance, $\dot{m}_{rad,exp}$
- Path 2:** Leakage through compressor radial clearance, $\dot{m}_{rad,com}$
- Path 3:** Leakage through vane-bearing end-face clearance in the expander, $\dot{m}_{vb,exp}$
- Path 4:** Leakage through vane-bearing end-face clearance in the compressor, $\dot{m}_{vb,com}$
- Path 5:** Leakage through trailing vane side-split bush clearance, $\dot{m}_{vsb,trail}$
- Path 6:** Leakage through trailing inner cylinder-split bush clearance, $\dot{m}_{icsb,trail}$
- Path 7:** Leakage through leading vane side-split bush clearance, $\dot{m}_{vsb,lead}$
- Path 8:** Leakage through leading inner cylinder-split bush clearance, $\dot{m}_{icsb,lead}$
- Path 9:** Leakage through inner cylinder-bearing end-face clearance between the suction chamber of the expander and compressor, $\dot{m}_{icb,escs}$
- Path 10:** Leakage through inner cylinder-bearing end-face clearance between the discharge chamber of the expander and compression chamber of the compressor, $\dot{m}_{icb,edcd}$

Path 11: Leakage through inner cylinder-bearing end-face clearance between the discharge chamber of the expander and suction chamber of the compressor, $\dot{m}_{icb,edcs}$

Path 12: Leakage through inner cylinder-bearing end-face clearance between the suction chamber of the expander and compression chamber of the compressor, $\dot{m}_{icb,escd}$

Figure 4.5 Leakage paths of CVEC in the a) first half of the cycle and b) second half of the cycle.

4.5.1. Leakage through Radial Clearances

The radial clearances of the expander and compressor are formed by the gap between the rotor and inner cylinder and between the inner cylinder and outer cylinder, respectively. Through these clearances, leakages occur between the working chambers of the expander (through Path 1) and compressor (through Path 2). The dynamic forces experienced by the outer cylinder-vane-rotor component and inner cylinder due to the varying pressure in the working chambers and vane forces has resulted in the variation of the radial clearances throughout the cycle. As a result, the orifice area of the radial clearance, $A_{leak,1/2}$ through which the leakages take place must be determined instantaneously.

$$A_{leak,1/2} = \delta_{irad} \times l_{ecu} \quad (4.27)$$

where the instantaneous radial clearances, δ_{irad} of the expander and compressor are obtained by

$$\delta_{irad,exp} = \delta_{rad,exp} + r_{ic} - r_r - e \quad (4.28)$$

$$\delta_{irad,com} = \delta_{rad,com} + r_{oc} - r_{ic} - t_{ic} - e \quad (4.29)$$

where e denotes the instantaneous distance (or eccentricity) between the center of the outer cylinder-vane-rotor component and center of the inner cylinder. It is expressed as

$$e = \sqrt{e_x^2 + e_y^2} \quad (4.30)$$

$$e_x = \varepsilon_r \delta_{bear,r} \sin(\vartheta_r + \phi_r) - \varepsilon_{ic} \delta_{bear,ic} \sin(\vartheta_{ic} + \phi_{ic}) \quad (4.31)$$

$$e_y = r_{ic} - r_r + \varepsilon_r \delta_{bear,r} \cos(\vartheta_r + \phi_r) - \varepsilon_{ic} \delta_{bear,ic} \sin(\vartheta_{ic} + \phi_{ic}) \quad (4.32)$$

where ϑ is the angle of the resultant force acting on the outer cylinder-vane-rotor component and inner cylinder, and the varying eccentricity, ε and attitude angle, ϕ is obtained from the analytical method of solution for a dynamically loaded journal bearing proposed by Hirani et al. [147-150] (refer to Appendix B).

$C_{d,1/2}$ is obtained by the following relationship investigated by Teo [151] for the radial clearance in RV expander.

$$C_{d,1/2} = 0.4522 \ln \delta_{irad} - 0.5451 \quad (4.33)$$

4.5.2. Leakage through End-face Clearances between the Vane and Bearing

Referring to Figure 4.5, there are two leakage paths that exist between the vane and bearing end-face clearances separating the working chambers of the expander and compressor. The former is represented by Path 3, while the latter is represented by Path 4. The area of the

leakage path, $A_{leak,3/4}$ is determined by the product of end-face clearance and the instantaneous vane exposed length to their respective working chambers, i.e.,

$$A_{leak,3} = \delta_{ef} \times l_i \quad (4.34)$$

$$A_{leak,4} = \delta_{ef} \times (l_v - l_{AD}) \quad (4.35)$$

Unlike radial clearances, the end-face clearance is assumed to be constant, while the vane exposed length varies throughout the cycle.

It is also noted that end-face clearances are comparatively smaller than the dimensions of the working chambers. Therefore, the value of $C_{d,3/4}$ can be taken as 0.995 for this case [152].

4.5.3. Leakage through Clearances between the Split Bush and Vane

The leakage flow through clearances between the split bush and vane consists of the trailing side, Path 5 and leading side, Path 7. The trailing side exists during the first half of the cycle when the vane pushes the split bush which in turn forces the inner cylinder to gain angular momentum and rotate. In the second half of the cycle, the leading side is formed because the inner cylinder now gains enough angular momentum and pushes the split bush and in turns the vane. They are akin of leakages through end-face clearances between the vane and bearing where fluid flow occurs through parallel surfaces. Hence, $C_{d,5/7}$ can be taken as 0.995.

The leakage area is assumed to remain constant in their respective half-cycle, and assigned a zero value when it falls out of it. The variation of the leakage areas can be mathematically described by

$$A_{leak,5} = \begin{cases} \delta_{sb,v} l_{ecu} & ; 0 \leq \theta_1 \leq \pi \\ 0 & ; \pi \leq \theta_1 \leq 2\pi \end{cases} \quad (4.36)$$

$$A_{leak,7} = \begin{cases} 0 & ; 0 \leq \theta_1 \leq \pi \\ \delta_{sb,v} l_{ecu} & ; \pi \leq \theta_1 \leq 2\pi \end{cases} \quad (4.37)$$

4.5.4. Leakage through Clearances between the Split Bush and Inner Cylinder Slot

Similar treatment as the leakages through Paths 5 and 7 can be given to leakages through clearances between the split bush and inner cylinder slot at the trailing side, Path 6 and leading side, Path 8. Therefore, the leakage areas are given as

$$A_{leak,6} = \begin{cases} \delta_{sb,ic} l_{ecu} & ; 0 \leq \theta_1 \leq \pi \\ 0 & ; \pi \leq \theta_1 \leq 2\pi \end{cases} \quad (4.38)$$

$$A_{leak,8} = \begin{cases} 0 & ; 0 \leq \theta_1 \leq \pi \\ \delta_{sb,ic} l_{ecu} & ; \pi \leq \theta_1 \leq 2\pi \end{cases} \quad (4.39)$$

The main difference lies in the radial profile through which the leakages take place. Since the clearances are significantly smaller than the radius of the split bush, it is assumed that leakage flow occurs through parallel surfaces, and $C_{d,6/8}$ is again taken as 0.995.

4.5.5. Leakage through End-face Clearances between the Inner Cylinder and Bearing

For fluid leakage through the clearances between the upper and lower end-faces of the inner cylinder and bearing, i.e., Paths 9, 10, 11 and 12, the leakage mass flow rate is obtained by the analytical model of homogenous leakage flow occurring radially outwards through a cylinder end-face clearance provided by Yanagisawa and Shimizu [153]. The expression of the leakage flow rate is given as

$$\frac{dm_{leak,9/10/11/12}}{dt} = \frac{\pi(\delta_{ef,u}^3 + \delta_{ef,l}^3)(p_i - p_o)}{6\bar{\nu}_* \ln(r_o/r_i)} \quad (4.40)$$

where the apparent viscosity, ν_* is represented by an average value, $\bar{\nu}_*$ as the rate of change is small within 5 to 20 bar [153]. This is also the range in which the expander-compressor unit is operated in.

Due to the unique design of CVEC, Paths 9 and 10 exist throughout the whole cycle, whereas, Paths 11 and 12 exist exclusively in the first and second half of the cycle, respectively.

4.6. Reed Valve Model

The flow of discharge fluid from the compression chamber out to the housing chamber is dependent on the response of the valve reed, as it affects the magnitude of the flow area. The reed valve is located at the outer surface of the rotating outer cylinder, where it will be subjected to centrifugal force. In order to calculate its displacement, the valve reed will be analysed as a prismatic Euler-Bernoulli beam [154], where the differential beam element will be subjected to various other external forces as shown in Figure 4.6.

Figure 4.6 Schematic diagram of an Euler-Bernoulli beam and its differential element

The rotation of the differential element is assumed to be negligible compared to translation and the angular distortion as the shear experienced by the element is small in relation to the bending deformation. This theory [154] is applicable for valve that has a length to depth ratio of more than 10, and that the valve reed's surface remains smooth after bending. The theory

also says that rotatory inertia and shear deformation effects can be ignored. Applying Newton's Second Law on the beam element along the y -axis, we have the following equation.

$$-V + (V + dV) + f(x, t)dx - c \frac{\partial y}{\partial t} dx + (r + y)\omega^2 dm = dm \frac{\partial^2 y}{\partial t^2} \quad (4.41)$$

Under the assumption that the product of the mass moment of inertia of the beam element and the angular acceleration is negligibly small, the moment equation of motion about an axis normal to x and y and passing through the right edge of the beam element is

$$Vdx - M + (M + dM) - f(x, t)dx \cdot \frac{dx}{2} + c \frac{\partial y}{\partial t} \cdot \frac{dx}{2} - (r + y)\omega^2 dm \cdot \frac{dx}{2} = 0 \quad (4.42)$$

Ignoring second-order term in dx and cancelling appropriate terms in Equation (4.42), we obtained $V = -\frac{\partial M}{\partial x}$. Substituting this equation and the relationship between bending moment and bending displacement, i.e., $M(x, t) = E_{dv}I_{dv} \frac{\partial^2 y}{\partial x^2}$ into Equation (4.41), we derived the partial differential equation for bending vibration of a beam in the form of

$$E_{dv}I_{dv} \frac{\partial^4 y}{\partial x^4} + \rho_{dv}A_{dv} \frac{\partial^2 y}{\partial t^2} + c \frac{\partial y}{\partial t} - \rho_{dv}A_{dv}\omega^2 y = \rho_{dv}A_{dv}\omega^2 r_{dv} + f(x, t) \quad (4.43)$$

It is observed that the influence of centrifugal force is on both the homogenous and non-homogenous parts of the equation. This indicates that the rotational motion of the valve not only imparts an external excitation to the component, but also intrinsically affects its dynamic characteristics. In order to obtain a solution for the beam displacement, it is necessary to consider the response of the beam under free vibration, and then followed by the effect of external excitation.

4.6.1. Valve reed Response under Free Vibration

In the case of free vibration, the non-homogenous part is not present, and we have

$$E_{dv}I_{dv}\frac{\partial^4 y}{\partial x^4} + \rho_{dv}A_{dv}\frac{\partial^2 y}{\partial t^2} + c\frac{\partial y}{\partial t} - \rho_{dv}A_{dv}\omega^2 y = 0 \quad (4.44)$$

where $I_{dv} = w_{dv}t_{dv}^3/12$ and $A_{dv} = w_{dv}t_{dv}$. The solution to such case is said to be separable in the spatial variable, x and time, t where they are independent of each other.

$$y(x, t) = f(x)g(t) \quad (4.45)$$

Substituting Equation (4.45) into Equation (4.44), and rearranging the equation, we yield the two ordinary differential equations as shown below.

$$\frac{d^4 f}{dx^4} - \beta^4 f = 0 \quad (4.46)$$

$$\frac{d^2 g}{dt^2} + 2\zeta q \frac{dg}{dt} + q^2 g = 0 \quad (4.47)$$

where $a^2 = \frac{E_{dv}I_{dv}}{\rho_{dv}A_{dv}}$, $\beta^4 = \frac{p^2}{a^2}$, $q^2 = p^2 - \omega^2$, and $2\zeta q = \frac{c}{\rho_{dv}A_{dv}}$. The rotating reed valve is in clamped-free configuration. Therefore, the boundary conditions are $f(x) = 0$ and $\frac{df(x)}{dx} = 0$ at $x = 0$; and $\frac{d^2 f(x)}{dx^2} = 0$ and $\frac{d^3 f(x)}{dx^3} = 0$ at $x = l_{dv}$.

The general solution to Equation (4.46) is

$$f(x) = A\sin\beta x + B\cos\beta x + C\sinh\beta x + D\cosh\beta x \quad (4.48)$$

Substituting the four boundary conditions into Equation (4.48) we obtain the characteristic equation as shown below.

$$\cos\beta_n l_{dv} \cosh\beta_n l_{dv} = -1 \quad (4.49)$$

The characteristic equation is transcendental and must be solved numerically. The solutions are

$$\beta_n l_{dv} = 1.87510407, 4.69409113, 7.85475744, 10.99554073, 14.13716839, \frac{(2n-1)\pi}{2}; n > 5$$

The associated eigenfunction or mode shape is expressed as

$$f_n = A_n \left[(\sin \beta_n x - \sinh \beta_n x) - \frac{\sin \beta_n l_{dv} + \sinh \beta_n l_{dv}}{\cos \beta_n l_{dv} + \cosh \beta_n l_{dv}} (\cos \beta_n x - \cosh \beta_n x) \right] \quad (4.50)$$

On the other hand, Equation (4.47) resembles viscously damped single-degree-of-freedom systems and has the initial conditions $g(0)$ and $\dot{g}(0)$, which are the initial displacement and velocity, respectively. The solutions of the generalised modal coordinate are given in the following equations, each depending on the damping factor.

For $0 < \zeta < 1$

$$g_n(t) = e^{(-\zeta q_n t)} \left[\frac{\zeta q_n g_n(0) + \dot{g}_n(0)}{\omega_{d,n}} \sin q_{d,n} t + g_n(0) \cos q_{d,n} t \right] \quad (4.51)$$

For $\zeta = 1$

$$g_n(t) = e^{(-q_n t)} [g_n(0) + (q_n g_n(0) + \dot{g}_n(0))t] \quad (4.52)$$

For $\zeta > 1$

$$g_n(t) = e^{(-\zeta q_n t)} \left[\frac{\zeta q_n g_n(0) + \dot{g}_n(0)}{\sqrt{\zeta^2 - 1} q_n} \sinh \sqrt{\zeta^2 - 1} q_n t + g_n(0) \cosh \sqrt{\zeta^2 - 1} q_n t \right] \quad (4.53)$$

where $q_{d,n} = q_n \sqrt{1 - \zeta^2}$ and $q_n = \sqrt{p_n^2 - \omega^2}$, and p_n is basically the natural frequency for the n^{th} mode, i.e., $p_n = (\beta_n l_{dv})^2 \sqrt{\frac{E_{dv} l_{dv}}{\rho_{dv} A_{dv} l_{dv}}}$. The solution of the free vibration response in the n^{th} mode is given as

$$y_n(x, t) = f_n(x) g_n(t) \quad (4.54)$$

4.6.2. Valve Reed Response under Forced Vibration

The solution of the valve response under the effect of non-homogenous term is assumed to be in variable-separable form and by considering the principles of superposition for linear vibrations, i.e.,

$$y(x, t) = \sum_{n=1}^{\infty} f_n(x) g_n(t) \quad (4.55)$$

The natural modes are orthogonal to one another under the stated boundary conditions, where the normalised orthogonality relations for a beam and its companion orthogonality condition have the following expressions, respectively.

$$\int_0^{l_{dv}} f_s(x) f_n(x) dx = \delta_{ns} \quad (4.56)$$

$$\int_0^{l_{dv}} f_s(x) \frac{d^4 f_n(x)}{dx^4} dx = \left(\frac{p_n}{a}\right)^2 \delta_{ns} \quad (4.57)$$

where δ_{ns} is known as the Kronecker delta, having the following relation.

$$\delta_{ns} = \begin{cases} 0; & s \neq n \\ 1; & s = n \end{cases}; n, s = 1, 2, \dots$$

Substituting Equation (4.55) into Equation (4.43), dividing it by $\rho_{dv} A_{dv}$ and multiplying by $f_s(x)$, integrating it over the entire length of the valve reed, and applying the orthogonality relations, we have

$$\frac{d^2 g_n}{dt^2} + \frac{c}{\rho_{dv} A_{dv}} \frac{dg_n}{dt} + (p_n^2 - \omega^2) g_n = \int_0^{l_{dv}} f_n \omega^2 r dx + \int_0^{l_{dv}} f_n \frac{f(x, t)}{\rho_{dv} A_{dv}} dx \quad (4.58)$$

The second term on the right hand side of the equation can be rewritten as

$$\int_0^{l_{dv}} f_n \frac{f(x, t)}{\rho_{dv} A_{dv}} dx = \int_x^{x+d_{flow}} f_n \frac{f(x, t)}{\rho_{dv} A_{dv}} dx \quad (4.59)$$

where

$$f(x, t) = \frac{(p_{cdcv} - p_{cdisc}) A_{flow}}{d_{flow}} \quad (4.60)$$

Hence, we have

$$\int_0^{l_{dv}} f_n \frac{f(x, t)}{\rho_{dv} A_{dv}} dx = \int_x^{x+d_{flow}} f_n \frac{(p_{cdcv} - p_{cdisc}) A_{flow}}{\rho_{dv} A_{dv} d_{flow}} dx \quad (4.61)$$

which can be approximated to be

$$\int_x^{x+d_{flow}} f_n \frac{(p_{cdcv} - p_{cdisc}) A_{flow}}{\rho_{dv} A_{dv} d_{flow}} dx \cong \frac{f_n \left(x + \frac{d_{flow}}{2} \right) (p_{cdcv} - p_{cdisc}) A_{flow}}{\rho_{dv} A_{dv}} \quad (4.62)$$

Therefore, the final equation becomes

$$\begin{aligned} \frac{d^2 g_n}{dt^2} + 2\zeta q_n \frac{dg_n}{dt} + q_n^2 g_n \\ = \omega^2 r \int_0^{l_{dv}} f_n dx + \frac{f_n \left(x + \frac{d_{flow}}{2} \right) (p_{cdcv} - p_{cdisc}) A_{flow}}{\rho_{dv} A_{dv}} \end{aligned} \quad (4.63)$$

The above equation can be decomposed into two first ordinary differential equations which are

$$\frac{dg_n}{dt} = h_n \quad (4.64)$$

$$\frac{dh_n}{dt} = \omega^2 r \int_0^{l_{dv}} f_n dx + \frac{f_n \left(x + \frac{d_{flow}}{2} \right) (p_{cdcv} - p_{cdisc}) A_{flow}}{\rho_{dv} A_{dv}} - \frac{c}{\rho_{dv} A_{dv}} h_n - (p_n^2 - \omega^2) g_n \quad (4.65)$$

Solving the two equations above provides us the value of $g_n(t)$. Hence, the valve deflection, $y(x, t)$ is determined by the product of $g_n(t)$ and the normalised mode shape, $f_n(x)$.

4.7. Frictional Losses

In this section, the frictional power losses at the various contact regions of the expander-compressor unit are formulated. By analysing the schematic diagram of CVEC in Figure 3.2, the forces acting on the outer cylinder-vane-rotor component, inner cylinder and split bush are shown in Figures 4.7, 4.8 and 4.9, respectively.

Figure 4.7 Forces acting on the outer cylinder-vane-rotor component

The meanings of the symbols in Figure 4.7 are

$F_{x,r}$ is the x -component of the resultant force acting on the rotor (N)

$F_{y,r}$ is the y -component of the resultant force acting on the rotor (N)

$F_{com,v}$ is the resultant force acting on the side of the vane due to pressure difference between the compression and suction chamber of the compressor (N)

$F_{exp,v}$ is the resultant force acting on the side of the vane due to pressure difference between the suction and discharge chamber of the expander (N)

$F_{com,oc}$ is the resultant force acting on the outer cylinder due to pressure difference between the compression and suction chamber of the compressor (N)

$F_{exp,r}$ is the resultant force acting on the rotor due to pressure difference between the suction and discharge chamber of the expander (N)

$F_{sb,vs}$ is the friction force acting on the side of the vane due to contact with the split bush (N)

$F_{sb,vn}$ is the normal force acting on the side of the vane due to contact with the split bush (N)

$\tau_{lef,oc}$ is the resultant lower end-face friction torque acting on the outer cylinder (Nm)

$\tau_{uef,oc}$ is the resultant upper end-face friction torque acting on the outer cylinder (Nm)

Figure 4.8 Forces acting on the inner cylinder

The meanings of the symbols in Figure 4.8 are

$F_{x,ic}$ is the x -component of the resultant force acting on the inner cylinder (N)

$F_{y,ic}$ is the y -component of the resultant force acting on the inner cylinder (N)

$F_{com,ic}$ is the resultant force acting on the inner cylinder due to pressure difference between the compression and suction chamber of the compressor (N)

$F_{exp,ic}$ is the resultant force acting on the inner cylinder due to pressure difference between the suction and discharge chamber of the expander (N)

$F_{sb,ics}$ is the friction force acting on the inner cylinder due to contact with the split bush (N)

$F_{sb,icn}$ is the normal force acting on the inner cylinder due to contact with the split bush (N)

$\tau_{lef,ic}$ is the resultant lower end-face friction torque acting on the inner cylinder (Nm)

$\tau_{uef,ic}$ is the resultant upper end-face friction torque acting on the inner cylinder (Nm)

Figure 4.9 Forces acting on the split bush

The meanings of the symbols in Figure 4.9 are

$F_{v,sbs}$ is the friction force acting on the split bush due to contact with the vane (N)

$F_{v,sbn}$ is the normal force acting on the split bush due to contact with the vane (N)

$F_{ic,sbs}$ is the friction force acting on the split bush due to contact with the inner cylinder (N)

$F_{ic,sbn}$ is the normal force acting on the split bush due to contact with the inner cylinder (N)

4.7.1. Vane Side Friction

Vane side friction occurs when the vane slides along the surface of split bush during operation. With reference to Figure 4.7, the frictional loss at this region, P_{vs} is given by

$$P_{vs} = |F_{sb,vs}| \frac{dl_i}{dt} \quad (4.66)$$

The vane side friction force is related to the normal force acting on the vane side by

$$F_{sb,vs} = \mu_{vsb} F_{sb,vn} \quad (4.67)$$

where

μ_{vsb} is the coefficient of friction between the vane and the split bush

Analysing the relationship between the forces and moment arms in Figures 4.7 and 4.8, and applying Newton's Third Law, the normal force acting on the vane can be derived as

$$F_{sb,vn} = \left(\frac{I_{ic} \alpha_2}{r_{ic} + \frac{t_{ic}}{2}} \right) \cos \gamma \quad (4.68)$$

where γ is given by Equation (A.2) (refer to Appendix A). The moment of inertia of the inner cylinder, I_{ic} can be expressed as

$$I_{ic} = \frac{1}{2} m_{ic} [(2r_{ic})^2 + (2r_{ic} + 2t_{ic})^2] \quad (4.69)$$

where

m_{ic} is the mass of the inner cylinder (kg)

Substituting Equations (4.67) and (4.68) into Equation (4.66), we have

$$P_{vs} = \mu_{vsb} \left(\frac{I_{ic} \alpha_2}{r_{ic} + \frac{t_{ic}}{2}} \right) \cos \gamma \frac{dl_i}{dt} \quad (4.70)$$

4.7.2. Split Bush Friction

Split bush friction occurs due to the swivelling motion of the split bush about the hinge joint located at the inner cylinder during operation. With reference to Figure 4.9, the frictional loss at this region, P_{sb} is given by

$$P_{sb} = |\Delta\omega| r_{sb} \mu_{icsb} F_{ic,sbs} \quad (4.71)$$

where

r_{sb} is the radius of the split bush (m)

μ_{icsb} is the coefficient of friction between the split bush and the hinge joint of the inner cylinder

The relative angular velocity of the split bush with respect to the hinge joint, $\Delta\omega$ is

$$\Delta\omega = \omega_1 - \omega_2 \quad (4.72)$$

Balancing the forces acting on the split bush, and substituting Equations (4.67), (4.68) and (4.72) into Equation (4.71), we have

$$P_{sb} = |\omega_1 - \omega_2| r_{sb} \mu_{icsb} \mu_{vsb} \left(\frac{I_{ic} \alpha_2}{r_{ic} + \frac{t_{ic}}{2}} \right) \cos\gamma \quad (4.73)$$

4.7.3. End-face Friction

End-face friction occurs when two contact surfaces are in relative motion with respect to each other during operation. They can be found between: the lower end-face of the inner cylinder and the surface of the lower bearing; the lower end-face of the outer cylinder and the surface of the lower bearing; the upper end-face of the inner cylinder and outer cylinder.

The shear stress between two rubbing surfaces, $\vec{\tau}$ is given as

$$\vec{\tau} = \mu_{lub} \frac{d\vec{v}}{dz} \quad (4.74)$$

where

μ_{lub} is the dynamic viscosity of the lubricant (Pa·s)

v is the velocity (m/s)

z is the distance (m)

Assuming the distance between the two surfaces to be constant, Equation (4.74) is simplified to

$$\vec{\tau} = \mu_{lub} \frac{\Delta \vec{v}}{\Delta z} \quad (4.75)$$

The frictional power loss at the end-face, P_{ef} can then be derived using the following equation.

$$P_{ef} = \left(\int_{A_{ef}} \vec{\tau} \cdot \Delta \vec{v} dA \right) \quad (4.76)$$

Therefore, the frictional power loss between the end-face of the inner cylinder and the lower bearing, $P_{ef,iclb}$, and the end-face of the outer cylinder and the lower bearing, $P_{ef,oclb}$ are

$$P_{ef,iclb} = \frac{\mu_{lub} \pi \omega_2^2 [(r_{ic} + t_{ic})^4 - r_{ic}^4]}{2\delta_{lef}} \quad (4.77)$$

$$P_{ef,oclb} = \frac{\mu_{lub} \pi \omega_1^2 [(r_{oc} + t_{oc})^4 - r_{oc}^4]}{2\delta_{lef}} \quad (4.78)$$

where

δ_{lef} is the lower end-face clearance (m)

The frictional power loss between the upper end-face of the inner cylinder and outer cylinder, $P_{ef,icoc}$ is formulated by observing the geometrical and kinematics similarity between CVEC and RV mechanism [155].

$$P_{ef,icoc} = \frac{\mu_{lub}\pi\omega_1^2 e^2}{\delta_{uef}} ((r_{ic} + t_{ic})^2 - r_{ic}^2) + \frac{\mu_{lub}\pi(\omega_2 - \omega_1)^2}{2\delta_{uef}} ((r_{ic} + t_{ic})^4 - r_{ic}^4) \quad (4.79)$$

where

δ_{uef} is the upper end-face clearance (m)

4.7.4. Bearing Friction

The bearing pair that supports the rotor and inner cylinder is of journal bearing type. Journal bearings are used extensively in rotating machines because of their simplicity, lower wear and good damping characteristics [150]. It consists of a bearing sleeve which wraps around the journal, and lubricant that is filled in between these two components as shown in Figure 4.10.

Figure 4.10 Schematic diagram of a loaded journal bearing [148]

When the journal rotates with respect to the bearing sleeve, the lubricant is forced between the curved surfaces, thus creating an oil wedge or a hydrodynamic oil film that allows the

bearing to support the load on the journal operating at high rotational speeds. Though it may appear simple in design, the theory and operation of these bearings can be complex. It is important to understand that the rotating journal is not centred in the bearing during normal operation. This offset distance is referred to as the eccentricity of the journal bearing, and the ratio of the eccentricity and clearance of the journal bearing is defined as the eccentricity ratio, ε . Another parameter that is essential to the study of journal bearing is known as the attitude angle, ϕ which is the angle between the bearing load vector and eccentricity vector.

The variation of the eccentricity ratio and attitude angle of the journal bearing can be determined by the method proposed by Hirani et al. [147-150] (see Appendix B). In this method, the profile of the journal load acting on the rotor, F_r and inner cylinder, F_{ic} (see Figure 4.11) for the whole cycle must be determined beforehand.

Figure 4.11 Forces acting on the rotor and inner cylinder in the (a) first half and (b) second half of the cycle

With reference to Figure 4.11, the journal load components acting on the rotor is obtained from force summations in the x and y -axes, i.e.,

$$F_{r,x} = F_{com} \cos \theta_1 - F_{exp} \cos \theta_1 + F_{sb,vn} \cos \theta_1 + F_{II} \sin \frac{\theta_1}{2} \quad (4.80)$$

$$F_{r,y} = F_{com}\sin\theta_1 - F_{exp}\sin\theta_1 + F_{sb,vn}\sin\theta_1 - F_I\cos\frac{\theta_1}{2} \quad (4.81)$$

In terms of inner cylinder bearing, the journal load components are

$$F_{ic,x} = \begin{cases} -F_{II}\sin\frac{\theta_2}{2} + F_{III}\cos\frac{\theta_2}{2} - F_{IV} + F_{sb,icn}\cos\frac{\theta_2}{2}; & 0 \leq \theta_2 \leq \pi \\ -F_{II}\sin\frac{\theta_2}{2} - F_{III}\cos\frac{\theta_2}{2} + F_{IV} + F_{sb,icn}\cos\frac{\theta_2}{2}; & \pi \leq \theta_2 \leq 2\pi \end{cases} \quad (4.82)$$

$$F_{ic,y} = \begin{cases} F_{II}\cos\frac{\theta_2}{2} + F_{III}\sin\frac{\theta_2}{2} + F_{sb,icn}\sin\frac{\theta_2}{2}; & 0 \leq \theta_2 \leq \pi \\ F_{II}\sin\frac{\theta_2}{2} + F_{III}\cos\frac{\theta_2}{2} + F_{sb,icn}\sin\frac{\theta_2}{2}; & \pi \leq \theta_2 \leq 2\pi \end{cases} \quad (4.83)$$

With the history of the journal load, the eccentricity ratio and the attitude angle known, the frictional power loss can be determined by

$$P_{bear,j} = r_{bear,j}\omega_j \left(\frac{\mu_{lub}\omega_j l_{bear,j} r_j \pi}{\delta_j(1 - \varepsilon^2)^{0.5}} \left(\frac{2 + \varepsilon}{1 + \varepsilon} \right) + \frac{\delta_j \varepsilon}{2r_{bear,j}} F_j \sin\phi \right) \quad (4.84)$$

One of the important factors that govern the design of journal bearing is the prediction of minimum oil film thickness [147-150], which can be evaluated by

$$h_{min,j} = 0.5\delta_j(1 - \varepsilon) \quad (4.85)$$

During operation, the change in bearing load varies journal centre velocity and causes fluctuation in the starting angle, θ_1 and the ending angle, θ_2 of the pressure curve. This in turn causes the minimum oil film thickness and the corresponding peak pressure to vary. To avoid bearing wear, it is suggested that the surface roughness of the journal bearing to be ten times lesser than the value of the minimum oil film thickness.

4.8. Suction and Discharge Losses

It is a common phenomenon in compressor whereby the suction and discharge processes do not occur at a constant pressure, due to the presence of reed valve and the pressure losses

across the port. This occurrence is associated with the valve dynamics, as well as the restriction imposed by the valve passage areas or port flow areas. It is observed on the pressure-volume (PV) diagram in Figure 4.12a, whereby compression continues after the intended discharge pressure is reached. Similarly, during the suction process, the pressure drops below the intended suction pressure. The same phenomena also occur in the expander as shown in Figure 4.12b. The suction and discharge losses of the compressor and expander are defined as the additional work which has to be done to induce or discharge the working fluid. The instantaneous suction and discharge power losses of the compressor and expander are estimated based on

$$P_{suct/disc} = \frac{|p_{cv} - p_{intended}| \Delta v_{cv}}{\Delta t} \quad (4.86)$$

Figure 4.12 PV diagram of (a) a compressor and (b) an expander

4.9. Power of Expander-Compressor Unit

The total instantaneous power of the expander-compressor unit as a function of shaft rotational angle, θ_1 is made up of the instantaneous values of the compressor power, P_{com} ,

power due to inertial effect of the inner cylinder, $P_{inertia}$, mechanical power losses, P_{loss} and less the expander power, P_{exp} i.e.,

$$P_{total} = P_{com} - P_{exp} + P_{inertia} + P_{loss} \quad (4.87)$$

The expression of P_{com} , P_{exp} , $P_{inertia}$, and P_{loss} are given by

$$P_{com} = (p_{cdcv} - p_{cscv})(l_v - l_{AD})l_{ecu} \left(r_r + l_{AD} + \frac{l_v - l_{AD}}{2} \right) \omega_1 \quad (4.88)$$

$$P_{exp} = (p_{escv} - p_{edcv})l_i l_{ecu} \left(r_r + \frac{l_i}{2} \right) \omega_1 \quad (4.89)$$

$$P_{inertia} = I_{ic} \alpha_2 \omega_2 \quad (4.90)$$

$$P_{loss} = P_{vs} + P_{sb} + P_{ef,iclb} + P_{ef,oclb} + P_{ef,icoc} + P_{bear,r} + P_{bear,ic} + P_{suct,com} + P_{disc,com} + P_{suct,exp} + P_{disc,exp} \quad (4.91)$$

where

p_{cdcv} is the pressure of the compression chamber of the compressor (Pa)

p_{cscv} is the pressure of the suction chamber of the compressor (Pa)

p_{escv} is the pressure of the suction chamber of the expander (Pa)

p_{edcv} is the pressure of the discharge chamber of the expander (Pa)

The power input to the expander-compressor unit, P_{avg} is obtained by taking the mean value of P_{total} i.e.,

$$P_{avg} = \frac{1}{2\pi} \sum_{i=1}^n P_{total,i} \Delta\theta_{1,i} \quad (4.92)$$

4.10. Definition of Efficiencies

Since the expander and compressor are integrated as a single unit, it is more appropriate to evaluate the performance of CVEC based on the combined efficiencies.

The mechanical efficiency of the expander-compressor unit, $\eta_{m,ecu}$ can be simply defined as

$$\eta_{m,ecu} = \frac{\oint p_{com} dV - \oint p_{exp} dV}{\oint p_{com} dV - \oint p_{exp} dV + W_{loss}} \quad (4.93)$$

or

$$\eta_{m,ecu} = \frac{P_{com} - P_{exp}}{P_{com} - P_{exp} + P_{loss}} \quad (4.94)$$

It is straightforward to define volumetric efficiency for individual machines such as compressor or expander [156]. However in a combined system such as CVEC, referring to Figure 4.5, the leakage flow interaction within the working chambers of the expander and compressor of CVEC, as well as between them renders the calculation of combined volumetric efficiency less straightforward. Therefore, the definition of volumetric efficiency has to be appropriately defined. For simplicity, the volumetric efficiency of the combined system, $\eta_{v,ecu}$ is defined as the product of the respective volumetric efficiencies from the expander and compressor.

$$\eta_{v,ecu} = \frac{\dot{m}_{actual,com}}{\dot{m}_{ideal,com}} \times \frac{\dot{m}_{actual,exp}}{\dot{m}_{ideal,exp}} \quad (4.95)$$

4.11. Concluding Remarks

Equations (4.1) to (4.92) have been incorporated into a MATLAB [157] software and simulated together with REFPROP [158] routine which provides the refrigerant properties. The basic simulation results will be shown and discussed in Chapter 5.

5 Basic Simulation Results

The characteristic of CVEC is investigated by simulating it under standard operational conditions while adopting several assumptions. The simulation results are analysed and presented in this chapter.

5.1. Simulation Setup and Operational Conditions

In the earlier chapter, mathematical models describing the various aspects of CVEC were introduced. These set of equations were integrated simultaneously using the 4th order Runge-Kutta method [159]. In each integration step, the instantaneous thermodynamics properties of the working fluid are provided by NIST database using REFPROP routine [158]. Prior to the simulation, the assumptions adopted are homogenous properties of the working fluid and quasi-equilibrium process in the control volume. It is also assumed that the working fluid undergoes expansion and compression process under adiabatic condition as heat transfer is insignificant when the expander-compressor unit operates under high operational speed. Moreover, the present study assumes a constant shaft angular velocity during operation, even though the speed of the expander-compressor unit may vary slightly in actual applications.

The operational conditions of the refrigeration system are tabulated in Table 5.1. The refrigerant selected for the simulation is R-1234yf (2,3,3,3-Tetrafluoropropene) — a synthetic refrigerant co-developed by DuPont and Honeywell as a replacement for R-134a in automotive air-conditioning applications due to its relatively low global warming potential [63]. The degree of superheat, condensing and evaporating temperatures are chosen based on the ASHRAE standard testing conditions [160], which are commonly used by all the compressor and air-conditioner manufacturers. The operational speed of the expander-compressor unit is fixed (according to our assumption) at 3000 rpm, a typical design condition for commercial air-conditioners. Lastly, the cooling load is set at 5275 W, i.e., around 18000 Btu/hr. This value is applicable for a typical room air-conditioner available in the market [161].

Table 5.1 Operational conditions

Operational Conditions	
Working fluid	R-1234yf
Degree of superheat	27.8°C
Degree of subcool	0 °C

Saturated temperature of condenser	54.4°C
Saturated temperature of evaporator	7.2°C
Operational speed	3000 rpm
Dynamic viscosity of lubricant	0.0034 Pa·s [52]
Cooling load	5275 W

The major dimensions of CVEC and its physical parameters are shown in Table 5.2. This set of dimensions is chosen because it has the highest mechanical efficiency attained based on the study of the effect of machine slenderness presented in Section 5.3.1.

Table 5.2 Dimensions and physical parameters of the expander-compressor unit

Main dimensions of expander-compressor unit and its physical parameters			
Rotor radius, r_r	11.0 mm	Vane friction coeff., μ_v	0.15 [29]
Inner cylinder radius, r_{ic}	13.0 mm	Split bush friction coeff., μ_{sb}	0.15 [29]
Outer cylinder radius, r_{oc}	35.6 mm	Expander suction port dia.	10.0 mm
		Discharge coeff.	0.65 [162]
Inner cylinder thickness, t_{ic}	20.6 mm	Expander discharge port dia.	10.0 mm
		Discharge coeff.	0.65 [162]
Outer cylinder thickness, t_{oc}	5.0 mm	Compressor suction port dia.	16.0 mm
		Discharge coeff.	0.65 [162]
CVEC length, l_{ecu}	100.7 mm	Compressor discharge port dia.	16.0 mm
		Discharge coeff.	0.65 [162]
Vane length, l_v	24.6 mm	CVEC density, ρ_{ecu}	2700 kg/m ³
Dimensions of reed valve and its physical parameters			
Width, w_{dv}	15.0 mm	Dist. from rotational axis, r_{dv}	40.6 mm
Thickness, t_{dv}	0.3 mm	Density, ρ_{dv}	7800 kg/m ³
Length, l_{dv}	29.0 mm	Young's modulus, E_{dv}	200 GPa
Valve stop height, h_{dv}	5.0 mm	Damping factor, ξ_{dv}	0.1
Clearances			
Assembly expander radial clearance, $\delta_{rad,exp}$			13.0 μ m
Assembly compressor radial clearance, $\delta_{rad,com}$			13.0 μ m
Assembly upper end-face clearance, δ_{uef}			13.0 μ m
Assembly lower end-face clearance, δ_{lef}			13.0 μ m

Before the simulation begins, a step size independence test is carried out to determine the angular step size which gives converged simulation results and yet does not consume unduly much computing resources. The power input to the expander-compressor unit, P_{avg} is used as the gauging parameter, the results are shown in Figure 5.1. It is observed that P_{avg} begins to converge when the step size falls below 0.00196 rad, which is also shown in Figure 5.2. Therefore, any angle step that is below 0.00196 rad would be acceptable. In this report, all simulations shown are obtained using angular step size lower than 0.00196 rad.

Figure 5.1 Step size independence test result

Figure 5.2 Variation of pressure of the expander and compressor with different step sizes

5.2. Results and Discussions

Figure 5.3 shows the volume variation of the expander and compressor. It is interesting to point out that the expander and compressor cycles are π rad out of phase from each other due to the inherent design of CVEC mechanism. Lines ABC and DBE depict the volume variation of the suction chamber and discharge chamber of the expander, respectively. Note

that lines C'D'A'B' and B'D'A'C' represent the variation of suction and compression chamber volume of the compressor, respectively.

Figure 5.3 Volume variations of the (a) expander and (b) compressor of CVEC

The vane sliding velocity, and the angular velocity and acceleration of the outer cylinder-vane-rotor component and inner cylinder is shown in Figure 5.4.

Figure 5.4 Variations of (a) sliding velocity of the vane, (b) angular velocity and (c) angular acceleration of the outer cylinder-vane-rotor component and inner cylinder

Based on the operational conditions listed in Table 5.1, the variation of pressure, temperature and mass of the expander and compressor of CVEC are simulated as shown in Figure 5.5. The simulation is performed under perfect sealing condition.

Figure 5.5 Variation of thermodynamic properties of the working fluid in one complete cycle:
dashed lines – expander, solid lines – compressor

Figure 5.6 shows the comparisons of predicted PV diagram with and without internal leakage into consideration. It is observed in Figure 5.6a that internal leakage causes fluid pressure in the suction chamber of the expander to drop and over expand to below the intended discharge pressure. Hence in the design of the expander, anticipation has to be incorporated to deal with the over-expansion. In Figure 5.6b, internal leakage also causes the fluid pressure of the compression chamber to drop.

Figure 5.6 Pressure-volume diagram of (a) expander and (b) compressor

The variations in pressure shown in Figure 5.6 are mainly caused by leakages through the split bush clearances (i.e., leakage paths 5, 6, 7 and 8 as explained in Section 4.5), which contributes 74.1% of the total internal leakage. The variations of the internal leakage flow rate through the various leakage paths are shown Figure 5.7a.

Figure 5.7 (a) Variation of internal leakage rate through the various leakage paths and (b) variation of radial clearances of the expander or compressor

Radial clearances also contribute significantly to internal leakage, which is 18.6%. It is observed that the variation of this leakage mirrors the variation of the radial clearance; the latter is shown in Figure 5.7b. Since the stationary radial clearance for both the expander and compressor is set at the same value and that the outer cylinder and rotor are built as the same component, the same variation of the radial clearance is expected.

Figure 5.7a also shows that higher leakage is experienced in the first half of the cycle because the suction process of the expander has just begun (high pressure) and the compression process of the compressor is near its end (high pressure), and is accompanied by the larger leakage paths. At the mid-cycle, discontinuity is experienced due to the sudden change in leakage paths from trailing split bush clearances to leading split bush clearances. The pressure ‘dip’ (in dashed circle of Figure 5.7a) is due to the zero pressure difference between the discharge chamber of the expander and compression chamber of the compressor in the second half of the cycle. This leakage is caused by the leading split bush clearances.

Figure 5.8 shows the variation of power losses. In Figure 5.8a, the vane side frictional loss shows two identical arcs. Its variation is influenced by the vane sliding velocity depicted in Figure 5.4a. Also in Figure 5.8a, the split bush frictional loss is depicted with four arcs. The nature of its variation is contributed by two factors: the relative angular velocity between the split bush and the hinge joint, i.e., $|\omega_1 - \omega_2|$ (see Figure 5.4b), and the inertial effect (or the angular acceleration) of the inner cylinder (see Figure 5.4c). The power losses due to end-face friction are shown in Figure 5.8b. This form of losses are generally influenced by the square of the relative angular velocity between the contacting surfaces (refer to Figure 5.4b). An exception is made with the end-face loss between the inner and outer cylinder, whereby additional factor is contributed by the eccentricity component of CVEC. Figure 5.8c shows the frictional power loss at the shaft and inner cylinder bearings. The variations are observed to follow closely the trend of their respective angular velocities (see Figure 5.4b). Lastly, the variations of power losses due to suction and discharge flow are shown in Figure 5.8d. Its variation is affected by the fluid pressure during the suction and discharge processes of CVEC (refer to Figure 5.6).

Figure 5.8 Variations of power losses caused by (a) vane side and split bush friction, (b) end-face friction, (c) bearing friction, and (d) suction and discharge losses

In the present configuration, the mechanical efficiency of the machine is 76.2%. This is mainly caused by end-face friction, which contributes 61.9% of the total losses. This is then followed by suction and discharge losses, bearing friction, vane side friction and lastly split bush friction, where each of them contributes 19.1%, 8.7%, 6.8% and 3.5% of the total losses, respectively.

The variations of the various power components are shown in Figure 5.9. A positive sign of the total power, P_{total} indicates power consumption which occurs in the first half of the cycle, whereas a negative sign indicates power production in the second half of the cycle. Due to the recovered energy from the expander, the peak power input is reduced from 6006.3 W to 5617.5 W, i.e., a reduction of 6.5%. When the recovered power is utilised to supplement the overall power input, the COP of the refrigeration cycle increases from 4.7 to 6.1, i.e., an improvement of 29.5%.

Figure 5.9 (a) Variation of power components of CVEC and (b) comparison of the total power input with and without considering the power recovered by the expander

Table 5.3 shows the performance of CVEC and the refrigeration cycle when different working fluids are employed. The configuration of CVEC follows that of Table 5.2.

Table 5.3 Performance of CVEC and the refrigeration cycle with different working fluids

Working fluid	V_{com} (cm ³)	V_{exp} (cm ³)	$\eta_{m,ecu}$ (%)	COP w/o expander	COP w/ expander	COP improvement (%)
R-1234yf	41.86	12.26	76.2	4.7	6.1	29.5
R-134a	41.86	12.26	70.9	4.4	5.3	20.9
CO ₂	8.68	4.18	86.2	3.5	6.6	86.3

The results show that $\eta_{m,ecu}$ of CVEC is the highest in CO₂ system due to its small expander and compressor volume displacement (determined by Equations (6.4) and (6.5)) as a smaller machine leads to lower frictional losses. Additionally, the COP of the CO₂ system is improved by 86.3%, the highest compared to systems using other working fluids in Table 5.3.

5.3. Performance Variations with Design Parameters

In order to better understand the characteristics of CVEC, the effects of some design parameters on its performance are investigated in this section. Unless otherwise stated, the operational conditions and the configuration of the machine follow that in Tables 5.1 and 5.2.

5.3.1. Effect of the Slenderness of the Expander-compressor Unit

Figure 5.10 shows the variation of mechanical and volumetric efficiencies with respect to slender ratio, l_{ecu}/r_{oc} . The results show that a long and thin machine configuration gives a higher mechanical efficiency, while the volumetric efficiency drops.

Figure 5.10 Effect of slender ratio on the performance of expander-compressor unit

This is because fatter machines possess higher eccentricity, e which results in the increase of vane sliding velocity, angular velocity and acceleration of the inner cylinder, and led to higher friction. Conversely, for longer and thinner machines, the leakages at the radial and split bush clearances have been amplified due to larger leakage area at these regions. The choice of the machine configuration often depends on the constraints imposed by its applications.

5.3.2. Effect of the Inertia of Components

Figure 5.11 shows that the machine performance drops when the inertia of the inner cylinder increases, while it is unaffected by that of the outer cylinder-vane-rotor component. The former effect is due to the increase in normal vane side force required to rotate the inner cylinder which results in higher vane side and split bush friction. While the latter effect is not affected as it is rotating in constant angular velocity. Throughout the investigation, it is also observed that the varying inertia has no effect on the volumetric efficiency.

Figure 5.11 Effect of inertia of components on the performance of expander-compressor unit

The study is performed within 2000 and 8000 kg/m^3 which is the density range for metals commonly used for compressor fabrication, such as aluminium alloys (2700 kg/m^3) and various steels (7800 kg/m^3). Densities within this range are arbitrarily selected to understand the trend of mechanical efficiency of CVEC.

5.3.3. Effect of Operational Speed

A drop in mechanical efficiency is observed in Figure 5.12 as the machine operates at higher speed. Significant friction occurs because the vane sliding velocity, and the angular velocity and acceleration of the inner cylinder increase. The reversed is observed for the volumetric efficiency, as shorter time is allowed for leakages between the working chambers.

Figure 5.12 Effect of operational speed on the performance of expander-compressor unit

5.4. Comparison of Mechanical Efficiency of CVEC with Other Existing Rotary Compressors

The performance of CVEC in terms of mechanical efficiencies is compared with other existing rotary compressor mechanisms as shown in Table 5.4. Since these studies do not include the suction and discharge losses as mechanical losses, the calculation of mechanical efficiency of CVEC also excludes these losses for fair comparison.

Table 5.4 Comparison between scroll, rolling piston, revolving vane compressors, and CVEC in terms of mechanical efficiency

Mechanism	Mechanical efficiency (%)	Investigators
Scroll	80 (calculated)	Ishii et al. [38]
Scroll	92.3 – 93.4 (calculated)	Ishii et al. [163, 164]
Rolling piston	85.0 – 90.0% (calculated)	Ozu and Itami [165]
Rolling piston	94.0 (calculated)	Matsuzaka and Nagatomo [166]
Revolving vane	94.1 (calculated)	Teh and Ooi [52]
CVEC	77 – 90.3 (calculated)	Present work.

In terms of volumetric efficiency, the comparison is shown in Table 5.5.

Table 5.5 Comparison between scroll, rolling piston, revolving vane compressors, and CVEC in terms of volumetric efficiency

Mechanism	Volumetric efficiency (%)	Investigators
Scroll	76.2 (calculated)	Ishii et al. [167]
Rolling piston	90 (calculated)	Yanagisawa and Shimizu [153]
Rolling piston	94 (calculated)	Matsuzaka and Nagatomo [166]
Revolving vane	95 (calculated)	Teh and Ooi [53]
CVEC	88.3 – 97.6 (calculated)	Present work.

Results in Tables 5.4 and 5.5 show that the performance of CVEC is comparable to that of other existing rotary compressors. It is believed that optimisation of CVEC design would bring about higher efficiencies of the machine.

Concluding remarks

The results of this chapter show that the performance of CVEC is highly influenced by changes in the slenderness, split bush and radial clearances, end-face areas, suction and discharge port sizes, and less so by the inertia of the rotary components and eccentricity of the machine.

6 Design of Prototype

In this chapter, the detailed design of the CVEC prototype is presented. The components and its assembly procedure are described.

6.1. The Determination of Volume Displacement of Expander and Compressor

Table 6.1 shows the specifications of the prototype.

Table 6.1 Prototype Specifications

Prototype Specifications	
Working fluid	R-134a*
Degree of superheat	27.8°C
Degree of subcool	0 °C
Saturated temperature of condenser	54.4°C
Saturated temperature of evaporator	7.2°C
Operational speed	3000 rpm
Dynamic viscosity of lubricant	0.0034 Pa·s [52]
Cooling load	5275 W

*The machine was first designed for R-134a four years ago. Now it has been replaced by R-1234yf, which has very similar properties to that of R-134a. The basic comparison of the machine characteristics between R-134a and R-1234yf are shown in Appendix E. The main reason for not using CO₂ as the refrigerant in the laboratory is to avoid possible hazards caused by the very high pressure (more than 120 bar) that CO₂ refrigerant required.

The maximum displacement volume of the compressor, V_{com} and expander, V_{exp} can be determined by assuming that the cooling load, $\dot{Q}_{load}$ equals to that of the cooling capacity, $\dot{Q}_{capacity}$ of the refrigeration system. Mathematically, this is expressed as

$$\dot{Q}_{load} = \dot{Q}_{capacity} \quad (6.1)$$

In terms of the maximum displacement volume of the compressor, $\dot{Q}_{capacity}$ is given as

$$\dot{Q}_{capacity} = V_{com} f \rho_{csuct} (h_{csuct} - h_{edisc}) \quad (6.2)$$

In terms of the maximum displacement volume of the expander, $\dot{Q}_{capacity}$ is expressed as

$$\dot{Q}_{capacity} = V_{exp} f \rho_{edisc} (h_{csuct} - h_{edisc}) \quad (6.3)$$

By substituting Equations (6.2) or (6.3) into Equation (6.1), and rearranging the equations, V_{com} and V_{exp} are determined as

$$V_{com} = \dot{Q}_{load} / f \rho_{csuct} (h_{csuct} - h_{edisc}) \quad (6.4)$$

$$V_{exp} = \dot{Q}_{load} / f \rho_{edisc} (h_{csuct} - h_{edisc}) \quad (6.5)$$

Applying Equations (6.4) and (6.5), and the prototype specifications provided in Table 6.1, V_{com} and V_{exp} are determined to be 41.86 cm³ and 12.26 cm³, respectively.

The radii of the rotor and the inner cylinder are arbitrarily chosen to be 12 mm and 15 mm, respectively. The rotor radius is designed at 12 mm so that it can accommodate the oil channel and the oil slinger that are situated at the centre of the shaft. The inner cylinder radius is designed at 15 mm so that a thickness of 30 mm of the inner cylinder wall could be obtained (by Equation (3.3)). This thickness is huge enough to accommodate the suction port of the compressor, and the suction and discharge ports of the expander. The length of the expander-compressor unit, the radius of the outer cylinder, and the length of the vane are obtained by Equations (3.2), (3.4) and (3.5), respectively. The initial dimensions of the prototype are listed in Table 6.2.

Table 6.2 Initial main dimensions of the CVEC prototype

Main Dimensions of the CVEC Prototype	
Radius of rotor, r_r	12.0 mm
Radius of inner cylinder, r_{ic}	15.0 mm
Radius of outer cylinder, r_{oc}	48.0 mm
Thickness of inner cylinder, t_{ic}	30.0 mm
Length of expander-compressor unit, l_{ecu}	48.0 mm
Length of vane, l_v	36.0 mm

The important dimension of the other parts of the prototype will be determined accordingly by design methods provided by Mott [168], and stress and deformation analyses using SolidWorks SimulationXpress 2014 [169].

6.2. Vane-rotor

In Chapter 3, the outer cylinder, the vane and the rotor is presented as a single component. However such design may pose a challenge during the fabrication process and incur a substantial cost. For prototyping purposes, the vane and rotor are made as a single component, and the outer cylinder as another component.

6.2.1. Thickness of the Vane

The vane-rotor is basically a long rod with a rigidly attached rectangular protrusion (vane) as shown in Figure 6.1.

Figure 6.1 Vane-rotor

During operation, the vane is subjected to high bending moment as it is exposed to high pressure force from the compressor and expander, and the reaction forces exerted by the outer cylinder, the split bush and the rotor. The free body diagram of the vane is shown in Figure 6.2.

Figure 6.2 Free body diagram of the vane

The forces, $F_{com,v}$, $F_{exp,v}$ and $F_{sb,vn}$ are known values. To find the values of $F_{oc,v}$ and $F_{r,v}$, two independent equations are required. The equations can be obtained from the balance of moment about the intersection between the vane and the rotor, and the balance of forces in the vertical axis. The two equations are

$$F_{oc,v}(l_o + t_{ic} + l_i) + F_{exp,v}\left(\frac{l_i}{2}\right) - F_{com,v}\left(\frac{l_o}{2} + t_{ic} + l_i\right) - F_{sb,vn}\left(\frac{t_{ic}}{2} + l_i\right) = 0 \quad (6.6)$$

$$F_{oc,v} + F_{exp,v} = F_{com,v} + F_{sb,vn} + F_{r,v} \quad (6.7)$$

A sample calculation is performed at $\theta_1 = \pi/2$ rad, where, $F_{com,v} = 166.50$ N, $F_{exp,v} = 109.33$ N, $F_{sb,vn} = 1551.20$ N, $l_o = 3.11$ mm, and $l_i = 2.70$ mm. Substituting these values into Equations (6.6) and (6.7), and solving them simultaneously, we have $F_{oc,v} = 921.50$ N and

$F_{r,v} = -686.85$ N. With these two reaction forces known, the graph of the bending moment along the length of the vane can be determined as shown in Figure 6.3.

Figure 6.3 Bending moment diagram of the vane at $\theta_1 = \pi/2$ rad

The same calculation is repeated for the entire cycle from 0 rad to 2π rad. Therefore, the maximum absolute bending moment for each angular position of the machine can be obtained as shown in Figure 6.4.

Figure 6.4 Variation of maximum bending moment of the vane

The overall maximum bending moment, $M_{v,max}$ is 13.78 Nm. $M_{v,max}$ is used to determine the thickness of the vane, w_v , based on the elastic flexure formula [170].

$$\sigma_{v,max} = \frac{M_{v,max}}{S_v} \quad (6.8)$$

where

$\sigma_{v,max}$ is the maximum stress experienced by the vane (N/m²)

S_v is the elastic section modulus of the vane (m³)

The elastic section modulus of the vane is

$$S_v = \frac{1}{6} w_v \times l_v^2 \quad (6.9)$$

Substituting Equation (6.8) into Equation (6.9) and rearranging, we have

$$w_v = \frac{6M_{v,max}}{\sigma_{v,max} l_v^2} \quad (6.10)$$

Including safety factor, SF as well as reliability factor, R into Equation (6.10), we have

$$w_{v,min} = \frac{6M_{v,max}}{\sigma_{v,max} l_v^2} \times \frac{SF}{R} \quad (6.11)$$

The prototype will be built by the material, AISI 4140, the maximum stress that can be experienced by the vane is the yield strength of the material, i.e., $\sigma_{v,max} = \sigma_{yield} = 417.1$ MPa (see Table 6.3).

Table 6.3 Mechanical properties of AISI 4140 steel [38]

Properties		Conditions	
		Temperature (°C)	Treatment
Density ($\times 10^3$ kg/m ³)	7.70 – 8.03	25	
Poisson's ratio	0.27 – 0.30	25	
Elastic modulus (GPa)	190 – 210	25	
Tensile strength (MPa)	655.0	25	Annealed at 815°C
Yield strength (MPa)	417.1	25	
Elongation (%)	25.7	25	
Reduction in area	56.9	25	
Hardness (HB)	197	25	Annealed at 815°C
Impact strength (J)	54.5	25	Annealed at 815°C

The safety factor under a typical industrial condition is 3 [168]. For a desired reliability of 99%, the reliability factor should be set at, $R = 0.81$ [168]. Putting these values into Equation (6.11), we have

$$w_{v,min} = \frac{6 \times 13.78}{(417.1 \times 10^6) \times 0.036^2} \times \frac{3}{0.81}$$

$$= 0.0005666 \text{ m or } 0.5666 \text{ mm}$$

The minimum thickness of the vane is too small for actual application. Hence, the actual thickness of the vane is set at 4 mm.

6.2.2. Radius of the Rotor and Shaft

During operation, the rotor is subjected to shear, bending and torsion. To determine the minimum radius required by the rotor and its shaft journal, the following two equations are applied. Equation (6.12) is used in cases where shearing forces are significant, whereas Equation (6.13) is used for pure bending and pure torsion cases.

$$D = \sqrt{2.94K_t(V) N/s'_n} \quad (6.12)$$

$$D = \left[\frac{32N}{\pi} \sqrt{\left[\frac{K_t M}{s'_n} \right]^2 + \frac{3}{4} \left[\frac{T}{s_y} \right]^2} \right]^{1/3} \quad (6.13)$$

where

K_t is the stress concentration factor

V is the shear force experienced by the rotor (N)

N is the design factor or the safety factor, SF

M is the bending moment experienced by the rotor (Nm)

T is the torsion experienced by the rotor (Nm)

s_y is the yield strength (Pa)

s'_n is the modified endurance strength, $s_n C_s C_R$ (Pa)

The modified endurance strength [39] is

$$s'_n = s_n C_s C_R \quad (6.14)$$

where

C_s is the size factor

C_R is the reliability factor

s_n is the estimated endurance strength (Pa)

Equation (6.12) is used to determine the minimum diameter of a shaft where shearing force, V is the only significant loading present, whereas Equation (6.13) is used to determine the minimum diameter of a shaft where bending and torsion are the significant loading present. The schematic diagram of the forces acting along the length of rotor and shaft is shown in Figure 6.5.

Figure 6.5 Schematic diagram of the rotor and shaft subjected to various forces and torque

From the free body diagram shown in Figure 6.5, the force, F_r that acts on the rotor is the resultant force caused by the pressure difference that is created between the four working chambers of CVEC. According to the simulation results, the maximum F_r is found to be 630.52 N. The reaction forces from the lower bearing, $F_{lb,s}$ and upper bearing, $F_{ub,s}$ can be obtained by considering that the forces are balanced in the y-axis, and the moments are balanced about point D. This results in two independent equations, and they are solved simultaneously as demonstrated in Section 6.2.1. $F_{lb,s}$ and $F_{ub,s}$ are 307.42 N and 323.10 N, respectively.

The rotor and shaft are also subjected to a torque from the motor, which results in torsion. The maximum torsion, $T_{s,max}$ can be obtained by Equation (6.15), where the maximum total power, $P_{total,max}$ is 4757.2 W, and the rotational speed of the shaft, ω_s is assumed to be 3000 rpm or 314.16 rad/s. Therefore, $T_{s,max}$ is calculated to be 15.14 Nm.

$$T_{s,max} = \frac{P_{total,max}}{\omega_s} \quad (6.15)$$

The lengths, l_{ls} is 40 mm, l_{ecu} is 48 mm, l_{us} is 158 mm, l_{lb} is 40 mm, and l_{ub} is 35.73 mm, which are arbitrary. The shear force, V_s , bending moment, M_s , and torsion, T_s diagram along the length of the rotor and shaft, l_l are determined and presented in Figure 6.6.

Figure 6.6 (a) Shear force, (b) bending moment and (c) torsion along the length of the rotor and shaft

With the loading profiles obtained, the minimum diameter of the shaft from point A to D can be determined. At Point A, the shear force, $V_{s,A}$, bending moment, $M_{s,A}$, and torsion, $T_{s,A}$ are 307.42 N, 0 Nm and 15.14 Nm, respectively. The minimum required diameter of the shaft is firstly determined by Equation (6.12). In this equation, $K_t = 2.5$ due to sharp fillet between the rotor and the shaft journal [168]; $s_n = 240$ MPa determined from Figure 6.7 for a machined surface condition and a tensile strength of 655 MPa [168]; $C_s = 0.8$ is an estimation from Figure 6.8 because the actual shaft diameter at this point in time is unknown [168]; $C_R =$

0.81 due to a required reliability of 0.99; $N = 3$ as the machine will be operating under a typical industrial condition.

Figure 6.7 Endurance strength versus tensile strength for wrought steel in various surface conditions [168]

Figure 6.8 Size factor for the various shaft diameter [168]

The modified endurance strength, s'_n is obtained from Equation (6.14).

$$s'_n = (240 \times 10^6) \times 0.8 \times 0.81 = 155.52 \text{ MPa}$$

The minimum diameter of the shaft at Point A, $D_{s,A}$ is then obtained from Equation (6.12).

$$D_{s,A} = \sqrt{\frac{2.94(2.5)(307.42)(3)}{155.52 \times 10^6}}$$

$$= 0.00660 \text{ m or } 6.60 \text{ mm}$$

The above calculation has to be repeated because C_s was an estimation. With the first value of $D_{s,A}$ obtained to be 6.60 mm, the new C_s can be found from Figure 6.8. The new C_s is 1.0. Therefore, the final s'_n and $D_{s,A}$ are

$$s'_n = (240 \times 10^6) \times 1.0 \times 0.81 = 194.40 \text{ MPa}$$

$$D_{s,A} = \sqrt{\frac{2.94(2.5)(307.42)(3)}{194.40 \times 10^6}}$$

$$= 0.00591 \text{ m or } 5.91 \text{ mm}$$

Next, the minimum diameter of the shaft required by Equation (6.13) is determined. The additional information that is needed is the yield strength of the material, s_y which is 417.1 MPa (refer to Table 6.3). Putting the values into Equation (6.13), we have

$$D_{s,A} = \left[\frac{32(3)}{\pi} \sqrt{\left[\frac{2.5(0)}{155.52 \times 10^6} \right]^2 + \frac{3}{4} \left[\frac{15.14}{417.1 \times 10^6} \right]^2} \right]^{1/3}$$

$$= 0.00987 \text{ m or } 9.87 \text{ mm}$$

Comparing the $D_{s,A}$ values obtained from Equations (6.12) and (6.13), it is found that the minimum shaft diameter as required by Equation (6.13) is higher than that of Equation (6.12). Therefore, the minimum required diameter of the shaft at Point A is 9.87 mm.

The same procedure is repeated for Points B, C and D. The minimum required diameter at these points are $D_{r,B} = 18.89$ mm, $D_{s,C} = 9.87$ mm and $D_{s,D} = 9.87$ mm. Hence, the overall minimum rotor and shaft diameter is 18.89 mm.

For the final prototype design, the shaft diameter is set at 20 mm. This is to accommodate the lubrication channel along the length of the shaft. The rotor diameter (at Point B) is set at 24 mm.

6.3. Outer Cylinder

The outer cylinder is basically a hollow cylinder with a discharge valve installed at its exterior. A slot can be found just beside the discharge port as shown in Figure 6.9. This feature serves as a vane slot for the vane-rotor.

6.3.1. Thickness of the Outer Cylinder Wall

The design of outer cylinder operating under high-pressure environment has to consider two modes of failure. The first failure mode occurs when the applied stress becomes too excessive that results in permanent deformation. The second failure mode occurs when the outer cylinder is subjected to extreme stress that results in wall bursting.

In our case, the first mode of failure is of higher importance than that of the second mode. The reason of which is that deformation would result in fluid leakage, and wear and tear between the rubbing components. As such, the thin walled pressure vessel method [40] is employed for the determination of the minimum wall thickness of the outer cylinder.

The loop stress, σ_l and hoop stress, σ_h of a pressure vessel are given by

$$\sigma_l = \frac{\Delta p r_i}{2t} \quad (6.16)$$

$$\sigma_h = \frac{\Delta p r_i}{t} \quad (6.17)$$

where

Δp is the pressure difference across the wall of the pressure vessel (Pa)

r_i is the inner radius of the pressure vessel (m)

t is the wall thickness of the pressure vessel (m)

By comparison, the hoop stress is greater than the loop stress. Therefore, Equation (6.17) is employed as the basis of determining the minimum wall thickness. Rearranging Equation (6.17), we have

$$t = \frac{\Delta p r_i}{\sigma_h} \quad (6.18)$$

Including the safety factor, we have

$$t_{min} = \frac{\Delta p r_i}{\sigma_h} \times SF \quad (6.19)$$

The maximum ΔP that can be experienced by the cylinder wall is when the housing chamber is at the compressor discharge pressure and when the compressor is at the suction pressure; therefore $\Delta p = (14.698 - 3.772) \times 10^5 \cong 11 \times 10^5$ Pa. As the prototype will be built with AISI 4140 steel, the maximum hoop stress that can be experienced by the cylinder wall is the yield strength of the material, i.e., $\sigma_{h,max} = \sigma_{yield} = 417.1$ MPa. The safety factor, SF under a typical industrial condition is 3. Substituting these values into Equation (6.19), the minimum wall thickness of the outer cylinder, $t_{oc,min}$ with an inner radius, r_{oc} of 48 mm is

$$\begin{aligned} t_{oc,min} &= \frac{(11 \times 10^5) \times 0.048}{417.1 \times 10^6} \times 3 \\ &= 0.0003798 \text{ m or } 0.3798 \text{ mm} \end{aligned}$$

Any wall thickness that is above $t_{oc,min}$ is safe for the outer cylinder. Therefore, t_{oc} is designed at 10 mm.

6.3.2. Balancing Features of the Outer Cylinder

The balancing features in the outer cylinder are basically drill holes as shown in Figure 6.9. These features have two functions. Firstly, it reduces the mass of the component. Secondly, it shifts the centre of gravity (CG) of the component closer to the rotational axis. The second function minimises the centrifugal force, and hence, the vibration of the machine.

Figure 6.9 Balancing features of outer cylinder

The result of introducing the balancing features on the outer cylinder is shown in Table 6.4. The balancing features reduce the centrifugal force from 70.1 N to 0.038 N at the designed operational speed.

Table 6.4 Result of introducing the balancing features on the outer cylinder

	Before	After
CG in the x -axis (mm)	0.06476	-0.00014
CG in the y -axis (mm)	0.53302	0.00026
Mass of component (kg)	1.3229	1.3087
Rotational speed (rpm)	3000	3000
Centrifugal force (N)	70.1	0.038

Note that a bracket, which serves the same purpose, is attached to the shaft of the vane-rotor outside the housing so that it would not obstruct the rotational motion of the vane-rotor.

6.3.3. Discharge Valve

The discharge valve consists of the discharge port, the valve reed and the valve stop. The valve reed and the valve stop are secured at one end as shown in Figure 6.10.

Figure 6.10 Discharge valve

The valve reed is exposed to two kinds of forces during operation: (1) force due to pressure difference between housing chamber and working chambers of the compressor, and (2) centrifugal force due to rotation of the valve reed. The valve dynamics and its response to these two forces has been analysed and presented in Section 4.6.

The dimensions and physical properties of the discharge valve are listed in Table 7.4.

6.4. Inner Cylinder

The basic construction of the inner cylinder is of a hollow cylinder with a circular slot made at the cylinder wall. The circular slot enables the split bush to be fitted so that a hinge joint can be formed between the inner cylinder and the vane.

6.4.1. Thickness of the inner cylinder wall

The minimum wall thickness of the inner cylinder is determined by the thin-walled pressure vessel method described in Section 6.3.1. The maximum ΔP that can be experienced across the wall is when the fluid pressure is at the maximum and minimum values in either the expander or compressor chamber. The maximum and minimum pressures are 15.774×10^5 Pa and 3.772×10^5 Pa, therefore $\Delta p = (15.774 - 3.772) \times 10^5 \cong 12 \times 10^5$ Pa. The material and the safety factor remains the same, and $r_{ic} = 15$ mm. Therefore, the minimum wall thickness is

$$t_{ic,min} = \frac{(12 \times 10^5) \times 0.015}{417.1 \times 10^6} \times 3$$

$$= 0.00012947 \text{ m or } 0.12947 \text{ mm}$$

In CVEC mechanism, the wall thickness of the inner cylinder is dependent on r_r , r_{ic} , V_{exp} and l_{ecu} by the relation expressed in Equation (3.3). This implies that the wall thickness of the inner cylinder is a fixed dimension which is calculated to be 30 mm, far greater than $t_{ic,min}$.

6.4.2. Balancing Features of the Inner cylinder

The balancing features of the inner cylinder are as shown in Figure 6.11.

Figure 6.11 Balancing features of inner cylinder

The result of introducing the balancing features to the inner cylinder is shown in Table 6.5. The balancing features reduce the centrifugal force from 369.1 N to 0.052 N at the designed operational speed.

Table 6.5 Result of introducing the balancing features on the inner cylinder

	Before	After
CG in the x -axis (mm)	-0.1966	0.00014
CG in the y -axis (mm)	1.2381	-0.00012
Mass of component (kg)	2.9833	2.8367
Rotational speed (rpm)	3000	3000
Centrifugal force (N)	369.1	0.052

6.4.3. Suction and Discharge Ports

The suction and discharge ports of the expander and the suction port of the compressor are located in the wall of the inner cylinder as shown in Figure 6.12. These ports are located as close to the split bush slot to prevent communication between the suction and discharge fluid of both the expander and compressor.

Figure 6.12 Expander discharge port and compressor suction port in inner cylinder

The above can also be minimised by reducing the port size and arranging them vertically with respect to the rotational axis. The total area of the smaller ports must equal to the area of the intended port by the following equation.

$$D_{port,design} = n_{port,design}^{-2} D_{port,intended} \quad (6.20)$$

The designed port diameter and their respective number of ports are shown in Table 6.6.

Table 6.6 Designed and intended port sizes

	Expander suction	Expander discharge	Compressor suction
Intended port diameter, $D_{port,intended}$ (mm)	10	10	12
Designed number of ports, $n_{port,design}$	7	3	3
Designed port diameter, $D_{port,design}$ (mm)	3.8	5.8	7

The intended port sizes are arbitrary. They have to be large enough to minimise suction and discharge losses, yet small enough to prevent communication between the suction and discharge fluid.

6.5. Upper Bearing

The main function of the upper bearing is to support the upper section of the rotor shaft journal. The bearing is designed according to the method presented in Section 4.7.4. It has a length of 35.73 mm, a diameter of 20 mm, and an estimated radial clearance of 15 μm . The minimum oil film thickness, h_{min} of the shaft bearing is then obtained as shown in Figure 6.13. It is observed that the minimum film thickness of the entire cycle is 5.6 μm . This indicates that the journal is being supported by the oil film and that there is no surface to surface contact with the bearing.

Figure 6.13 Minimum oil film thickness of the lower and upper shaft of the rotor and inner cylinder bearings

In addition, four pressure transducer ports are made at the upper bearing, which secures the pressure transducers for pressure measurement. The model that is used is the Endevco 8530b-500 piezo resistive pressure transducer, shown in Figure 6.14.

Figure 6.14 Schematic diagram of Endevco 8530b-500 model

Figure 6.15 shows the design of the pressure transducer ports. Two of the ports that communicate with the suction and discharge chamber of the expander are slanted to avoid contact with the shaft bearing. The other two ports are vertically designed. All four ports include 10-32 UNF 2A tapped hole to fasten the pressure transducers.

Figure 6.15 a) Top view showing the four miniature pressure transducer ports b) cross-sectional view showing two of the four miniature pressure transducer ports. The port on the left communicates with the expander working chamber; the port on the right communicates with the compressor working chamber.

A flange is designed as shown in Figure 6.15, which allows the upper bearing to be attached to the housing cover by twelve M4 screws. The correct orientation of the upper bearing with respect to the housing cover is ensured by the two M5 dowel pin holes.

6.6. Lower Bearing

The feature and function of the lower bearing is similar to that of upper bearing. It supports the lower section of the rotor shaft journal and the inner cylinder journal. The bearings are designed according to the method presented in Section 4.7.4. The lower shaft bearing has a

length of 40 mm and a diameter of 20 mm, whereas the inner cylinder bearing has a length of 25 mm and a diameter of 30 mm. The estimated radial clearances are 15 μm . The minimum oil film thickness of the lower shaft and inner cylinder bearings are shown in Figure 6.13. The minimum oil film thickness of the lower shaft and inner cylinder bearings are 5.6 μm and 2.7 μm , respectively. This indicates that sufficient oil film is provided for preventing surface to surface contact.

The suction valve of the expander is located as shown in Figure 6.16. Its function is to control the suction period and the onset of expansion process. The opening angle of the valve is 35°, which is obtained by assuming that fluid mass is conserved while undergoing isentropic expansion.

Figure 6.16 a) 3D view and b) top view of the lower bearing

6.7. Housing

The function of the housing is to contain all the components of CVEC. It is basically a hollow cylinder sealed at one end and opened at the other end. The top flange allows the housing cover to seal the housing with bolts and nuts, whereas the bottom flange secures the housing onto the test rig. An O-ring groove is provided to allow the placement of an O-ring to minimise leakage.

The external suction and discharge ports of the expander and the external discharge port of the compressor are attached to the housing externally as shown in Figure 6.17. They are basically tubes welded onto the wall of the housing.

Figure 6.17 Isometric view of the housing

The minimum wall thickness of the housing is 0.978 mm, which is determined by the thin-walled pressure vessel method described in Section 6.3.1. However for practical reasons, the wall thickness is set at 7 mm.

Figure 6.18 Cross-sectional view of the housing showing a) the external discharge port and b) the sight glass

Lastly, to ensure the sufficient supply of lubricant, a small sight glass is created at the wall of the housing as shown in Figure 6.18. It is made of Perspex secured in its position by a glass cover and tightened by eight M4 screws from the inside. An O-ring is placed between the glass and the wall of the housing to prevent leakage.

6.8. Housing cover

The housing cover is basically a circular plate. The 'L' shape protrusion is the external suction port of the compressor, whereas the straight protrusion is the lubricant port.

Figure 6.19 Isometric view of the housing cover

The thickness of the housing cover follows that of the wall of the housing, i.e., 7 mm. A stress analysis is performed using SolidWorks SimulationXpress 2014 [169] to ensure that the housing cover would not fail during operation. The result of the stress analysis is shown in Figure 6.20. The maximum stress experienced by the housing cover is 72.916 MPa, which is about 5.7 times smaller than the yield strength of the material, i.e., 417.1 MPa.

In Figure 6.20, green arrows represent the position of the geometry fixtures applied to the housing cover, which is at the bolt holes. Red arrows represent the direction of pressure difference across the housing cover, $\Delta p_{housing\ cover}$ which is 13.7×10^5 Pa (The surface at the bottom of the cover is exposed to the pressure of the housing chamber which is the compressor discharge pressure at 14.698×10^5 Pa; while the upper surface is exposed to the atmospheric pressure at 1.01325×10^5 Pa). Note that the material used is AISI 4140 steel.

Figure 6.20 The von-Mises stress analysis result of the housing cover

Moreover, the displacement analysis result is displayed in Figure 6.21. The result shows that the maximum deflection occurs at the tip of the lubricant port, which is about 15 μm . The deflection is not significant, and the lubricant port does not interact with any moving component. Therefore, the machine operation would not be affected.

Figure 6.21 The displacement analysis result of the housing cover

The housing cover is secured onto the housing by M5 bolts. The minimum number of bolts required has to be determined correctly to ensure that they can withstand the pressure force that is created across the housing cover. The pressure force only acts on the area of the housing cover exposed to the pressure, i.e., the area between the wall of the housing and the flange of the upper bearing. The force experienced by the housing cover is calculated by the equation below.

$$F_{housing\ cover} = \Delta p_{housing\ cover} \times \frac{\pi}{4} (D_{housing}^2 - D_{upper\ bearing\ flange}^2) \quad (6.21)$$

The pressure difference across the housing cover, $\Delta p_{housing\ cover}$ is 13.7×10^5 Pa, the diameter of the inner wall of the housing, $D_{housing}$ is 200 mm, and the diameter of the flange of the upper bearing, $D_{upper\ bearing\ flange}$ is 166 mm. Inserting these values into Equation (6.21), $F_{housing\ cover}$ is found to be 13390 N.

The allowable stress of the individual bolts, σ_{all} is

$$\sigma_{all} = \frac{F_{housing\ cover}}{n \times A_t} \quad (6.22)$$

Rearranging Equation (6.22), and including the safety factor, SF , we have

$$n = \frac{F_{housing\ cover}}{\sigma_{all} \times A_t} \times SF \quad (6.23)$$

where

σ_{all} is the allowable stress of the individual bolts which is given as $0.75 \times \sigma_{proof}$ (N/m²)

A_t is the tensile stress area of the bolt (m²)

n is the number of bolt

M5 steel bolts with metric grade of 4.8 will be used in this machine. For this type of fastener, the proof strength, σ_{proof} is 310 MPa and the tensile stress area, A_t is 16.1×10^{-6} m² [168]. Inserting these values into Equation (6.23), and using a safety factor, SF of 3, the minimum number of M5 bolts is

$$\begin{aligned} n_{min} &= \frac{13390}{(0.75 \times 310 \times 10^6)(16.1 \times 10^{-6})} \times 3 \\ &= 10.73 \\ &\cong 11 \end{aligned}$$

The minimum number of M5 bolts is found to be eleven. However, to reduce the possibility of leakage through the clearance between the housing cover and the flange of the housing, the number of M5 bolts is arbitrarily increased to sixteen instead.

6.9. Lubrication System

The lubrication system of a machine is essential in providing sufficient flow of lubricant to the bearings and the rubbing components to minimise friction and wear. It also seals the

clearances between the components to minimise internal leakage, and provides cooling of the machine while it is in operation.

Figure 6.22 shows the lubrication system of the machine, where the lubricant is assumed to flow in the paths indicated by the arrows. The lubrication system runs mechanically instead of being driven by an electrical pump.

Figure 6.22 Oil supply system of the CVEC prototype

There are two separate systems that provide lubrication at the upper and lower sections of the machine. The lubrication system includes an oil slinger (which is essentially a twisted blade), rotating channel, spiral grooves, and stationary channel. All flows are influenced by gravitational effect and wall friction along the channel. In the rotating channels, the lubricant flow is also assisted by centripetal forces; while in the spiral grooves, drag forces are induced on the lubricant by the rotating shaft.

The interaction between the lubrication elements and the flow rates at each point of the supply system can be analysed by the method proposed by Kim et al. [171, 172]. In this method, the upper and lower lubrication systems are represented by a representative electrical circuit as shown in Figures 6.23 and 6.24, respectively. The lubricant flow rates, pressure differences along each element and flow resistances are expressed by electrical currents, voltage sources and electrical resistances, respectively. The subscripts h means height, rc stands for rotating channel, sg denotes spiral groove, sc is stationary channel.

Figure 6.23 Electrical circuit network analogous to the upper lubrication system

The upper circuit has fourteen unknowns in total, i.e., $q_{0,2}$, $q_{2,3a}$, $q_{2,3b}$, $q_{3,7}$, $q_{sg\,7,8}$, $q_{jb\,7,8}$, $q_{8,10}$, $q_{8,escv}$, $q_{8,edcv}$, $q_{10,12}$, $q_{10,cscv}$, $q_{10,cdcv}$, $q_{sg\,12,0}$, and $q_{jb\,12,0}$; each of these flow rates can be found by forming fourteen independent equations and solving them simultaneously.

From Kirchoff's Current Law, we have

$$q_{0,2} = q_{2,3a} + q_{2,3b} \quad (6.24)$$

$$q_{3,7} = q_{2,3a} + q_{2,3b} \quad (6.25)$$

$$q_{3,7} = q_{sg\,7,8} + q_{g\,7,8} \quad (6.26)$$

$$q_{sg\,7,8} + q_{sc\,7,8} = q_{8,10} + q_{8,escv} + q_{8,edcv} \quad (6.27)$$

$$q_{8,10} = q_{10,12} + q_{10,cscv} + q_{10,cdcv} \quad (6.28)$$

$$q_{10,12} = q_{sg\,12,0} + q_{sc\,12,0} \quad (6.29)$$

From Kirchoff's Voltage Law, we have

$$-q_{2,3a}R_{sc\,2,3a} + q_{2,3b}R_{sc\,2,3b} = 0 \quad (6.30)$$

$$-q_{jb\,7,8}R_{jb\,7,8} + q_{sg\,7,8}R_{sg\,7,8} = \Delta p_{sg\,7,8} \quad (6.31)$$

$$-q_{8,edcv}(R_{ef\,8,edcv} + R_{jb\,8,edcv}) + q_{8,escv}(R_{ef\,8,escv} + R_{jb\,8,escv}) = p_{edcv} - p_{escv} \quad (6.32)$$

$$-q_{10,cscv}R_{ef\,10,cscv} + q_{10,cdcv}R_{ef\,10,cdcv} = p_{cscv} + \Delta p_{rc\,7,cscv} - \Delta p_{rc\,7,cd} - p_{cdcv} \quad (6.33)$$

$$-q_{jb\,12,0}R_{jb\,12,0} + q_{sg\,12,0}R_{sg\,12,0} = 0 \quad (6.34)$$

$$\begin{aligned}
& q_{2,3a}R_{g_{2,3a}} + q_{3,7}\left(R_{sc_{4,5}} + R_{sc_{5,6}}\right) + q_{sc_{7,8}}R_{sc_{7,8}} + q_{8,edcv}\left(R_{ef_{8,ed}} + R_{jb_{8,ed}}\right) \\
& = p_{csuct} + \Delta p_{h_{0,1}} - \Delta p_{h_{4,5}} + \Delta p_{oil\ slinger} + \Delta p_{rc_{5,6}} + \Delta p_{h_{6,7}} \\
& + \Delta p_{h_{8,ed}} + \Delta p_{rc_{8,ed}} - p_{edcv}
\end{aligned} \tag{6.35}$$

$$\begin{aligned}
& -q_{8,escv}\left(R_{ef_{8,escv}} + R_{jb_{8,escv}}\right) + q_{8,10}\left(R_{sc_{8,9}} + R_{sc_{9,10}}\right) + q_{10,cscv}R_{ef_{10,cscv}} \\
& = p_{escv} - \Delta p_{rc_{8,escv}} - \Delta p_{h_{8,escv}} + \Delta p_{h_{9,10}} - \Delta p_{rc_{10,cscv}} - p_{cscv}
\end{aligned} \tag{6.36}$$

$$\begin{aligned}
& -q_{10,cdcv}R_{ef_{10,cdcv}} + q_{10,12}R_{ef_{10,12}} + q_{sc_{12,0}}R_{sc_{12,0}} \\
& = p_{cdcv} + \Delta p_{rc_{10,cdcv}} + \Delta p_{rc_{10,11}} + \Delta p_{h_{11,12}} - p_{csuct}
\end{aligned} \tag{6.37}$$

Figure 6.24 Electrical circuit network analogous to the lower lubrication system

For the lower circuit, there exists sixteen unknowns, i.e., $q_{0,2}$, $q_{2,3a}$, $q_{2,3b}$, $q_{3,15}$, $q_{15,escv}$, $q_{15,edcv}$, $q_{16,escv}$, $q_{16,edcv}$, $q_{16,17}$, $q_{sg17,18}$, $q_{jb17,18}$, $q_{18,19}$, $q_{sg19,21}$, $q_{jb19,21}$, $q_{jb21,ed}$, $q_{jb19,0}$; their values can be found by the same procedure as of the upper circuit.

The two sets of linear equations from the upper and lower circuits are solved by arranging the equations in matrix form and solving them using MATLAB [157] software. Simulation shows that the design of the lubrication system with the use of a twisted blade, 0.15 mm wide

and 0.15 mm depth spiral groove and 3 mm diameter stationary channel provides sufficient flow of lubricant throughout the machine.

6.10. Prototype and Assembly

The main dimensions of the prototype are presented in Table 7.4. The final product of the prototype is treated using aqueous alkaline nitrate bath (for 10 to 15 minutes at temperature ranging from 130°C to 150°C) to form black oxide coatings to achieve corrosion resistance [173]. In addition, the moving components are built with sliding fit tolerance to ensure smooth rotational and sliding motion, and the surface roughness, R_a of the prototype is produced at 0.6 μm .

The final product of the prototype components and its assembly procedure are presented and described in the following steps:

1. The lower bearing is assembled onto the housing.

Figure 6.25 The lower bearing is assembled onto the housing

2. The inner cylinder is assembled onto the lower bearing.

Figure 6.26 The inner cylinder is assembled onto the lower bearing

3. The outer cylinder is assembled onto the lower bearing.

Figure 6.27 The outer cylinder is assembled onto the lower bearing

4. The vane-rotor is inserted into the lower bearing, and the vane is fitted into the vane slot at the outer cylinder.

Figure 6.28 The vane-rotor is inserted into the lower bearing, and the vane is fitted into the vane slot at the outer cylinder

5. The first split bush is slotted between the vane and the inner cylinder on one side, followed by the second split bush on the other side.

Figure 6.29 The split bush is slotted between the vane and the inner cylinder

6. The upper bearing is assembled onto the housing cover by M4 screws.

Figure 6.30 The upper bearing is assembled onto the housing cover by M4 screws

7. The dowel pin is inserted into the dowel pin holes at the flange of the housing.

Figure 6.31 The dowel pin is inserted into the dowel pin holes at the flange of the housing

8. The housing cover is assembled to the housing. Before that, an O-ring is placed in the O-ring groove. The shaft of the vane-rotor is rotated to ensure the correct orientation.

Figure 6.32 The housing cover is assembled to the housing and the shaft of the vane-rotor is rotated to ensure the correct orientation

9. The housing cover together with the upper bearing is removed. The lower bearing is secured to the housing by the final tightening of the M4 screws.

Figure 6.33 The lower bearing is secured to the housing by the final tightening of the M4 screws

10. The housing cover is re-assembled to the housing, and tightened by M5 bolts, nuts and washers.

Figure 6.34 Final assembly of the prototype

6.11. Concluding Remarks

The design process and description of the important features of the various components of CVEC prototype has been presented in this chapter. The design ultimately led to its fabrication, which is the first of its kind.

7 Experimental Study

In this chapter, experimental findings from the measurements of CVEC prototype are presented. The gathered data from the measurements are used to facilitate the validation of the mathematical model.

7.1. The Experimental Setup

7.1.1. Overview of the Experimental Setup

Referring to the work scope in Chapter 1, the initial plan of the experiment was to have the prototype tested in a closed refrigeration circuit as shown in Figure 7.1. The test rig is designed based on ASHRAE's standard operational conditions for an R-134a system (see Figure 7.2). It consists of a condenser, an evaporator, an accumulator, a receiver, charging ports, safety valve, as well as various measuring devices installed at various position of the test rig for pressure, temperature, flow rate, angular position, torque and operational speed measurements.

Figure 7.1 Closed-loop measurement setup

Figure 7.2 Actual test bed

The CVEC prototype is driven by an electric motor (refer to Appendix F.1) as shown in Figure 7.3 and its speed is adjustable via a frequency controller.

Figure 7.3 The prototype in the test bed

Of noteworthy is the construction of the evaporator (see Figure 7.4a). The component is basically sandwiched between two electric heating plates of the size 450×450×12 mm. The heating power of each plates is 3 kW which is controllable by the heater controller as shown in Figure 7.4b. Additionally, both the evaporator and the heating plates are insulated by K-Flex ECO sheets (of 20 mm in thickness and having a thermal conductivity of $k = 0.0001T + 0.038$ within the temperature range of -165°C and 120°C [174]) so that heat transfer to the surrounding can be minimised. It is assumed that all the heat generated is used to evaporate the working fluid that flows through the evaporator coil during steady state. In other words, the heating power is equivalent to the cooling load of the system, and this value (together with the torque and speed data obtained from the torque meter and encoder) provides the measurement for the system's COP.

(a)

(b)

Figure 7.4 (a) Evaporator with built-in heater and (b) heater controller

At the suction and discharge lines of both the expander and compressor of the CVEC, as well as inlet and outlet of the evaporator and condenser, pressure transducers and thermocouples are installed to provide pressure and temperature measurement of the fluid. The thermocouples are of type-T having a gauge size of $\varnothing 4.8$ mm; and the pressure transducers are WIKA pressure transmitter, model A-10. Additionally, four miniature pressure transducers are inserted into the prototype to measure the pressure of the working chambers (refer to Section 6.5). These pressure transducers are Endevco piezo resistive pressure transducer, model 8530B-500. At the discharge line of the expander and compressor, TRICOR Coriolis flow meters are installed for mass flow rate measurement.

Between the prototype and the electric motor, an encoder and a torque meter are mounted along the shaft axis. The encoder consists of a magnetic sensor, SIKO MSK210 used in conjunction with a magnetic ring, SIKO MR200 secured onto the drive shaft to determine the angular position and speed of the shaft. The torque meter provides measurement for the shaft torque, and the model selected is Futek TRS605. Lastly, The lubricant used in the prototype is Shell Refrigeration Oil S4 FR-F 68 [175] (see Appendix G.1).

The specifications of the measuring devices are summarised in Table 7.1 (see Appendix F.2).

Table 7.1 Specifications of the various measuring devices

Measurement devices	Specifications
SIKO differential incremental encoder	4096 pulses per revolution Accuracy: $\pm 0.10^\circ$ Repeatability: ± 1 increment(s)
Type-T thermocouple	Accuracy: $\pm 0.5^\circ\text{C}$
WIKA pressure transmitter (model A-10)	Linearity: $\pm 0.50\%$ of full scale output (F.S.O.) Repeatability: $\pm 0.10\%$ (F.S.O.) Operating pressure range: 0 to 25 bar Operating temperature range: 0 to 80°C Response frequency: Dependent on module
Endevco piezo resistive pressure transducer (model 8530B-500)	Linearity: $\pm 0.20\%$ (F.S.O.) Repeatability: $\pm 0.10\%$ (F.S.O.) Operating pressure range: 0 to 34.5 bar Operating temperature range: -54 to 121°C Response frequency: 1000 kHz
Futek torque sensor (model TRS605)	Repeatability: $\pm 0.20\%$ (F.S.O.) Linearity: $\pm 0.20\%$ (F.S.O.) Maximum torque: 50 N·m Maximum rotation speed: 7000 rpm Response frequency: 30 kHz
Tricor Coriolis flow meter (model TCM 325)	Accuracy: $\pm 0.10\%$ (O.R.) Repeatability: $\pm 0.10\%$ (O.R.) Turndown ratio: 100:1 Operating pressure range: maximum 200 bar Operating temperature range: -40 to 100°C Response frequency: 0.5 to 10 kHz Mass flow rate: 3 to 300 kg/hr
Gilmont correlated variable-area flowmeter	Accuracy: $\pm 0.2\%$ of full-scale reading or ± 1 of scale division, whichever is greater Mass flow rate: 2 to 90 litre/min Maximum operating pressure: 4.1 bar Maximum operating temperature: 65°C Float type: glass Fluid: air

However, as testing begins with R-134a, severe leakages are found between the suction and discharge line of the expander. As a result, the expanded fluid of the expander reverts back to its suction condition before it could flow through the evaporator and into the suction chamber of the compressor. There is basically no refrigerating effect being produced and the COP of the system could not be measured. It is believed that the deficiency is caused by the pre-opening of the suction valve and inappropriate positioning of the suction and discharge port which cause fluid communication between the expanded and suction fluid by the end of the cycle. In order to control the high and low side pressure of the system, the original closed-

loop system had to be modified to become an open-loop air-flow circuit as shown in Figure 7.5.

Figure 7.5 Open-loop measurement setup

In the current setup, pressurised air is supplied by an external air compressor to the expander, which enables the suction pressure to be controlled and varied. The expanded air is then discharged to the atmosphere. For the compressor, suction pressure is maintained at the atmospheric pressure, while the compressed air is discharged to a 300 cm³ air receiver and released to the atmosphere by a manual flow regulating valve. Such arrangement sufficiently dampens the pulsation phenomena, caused by the intermittent discharge of the compressor, to facilitate the accurate measurement of flow rate.

Figure 7.6 Air receiver with manual flow regulating valve and variable area flow meter

Since the working fluid has now been changed to air, its density is relatively lower than of R-134a and falls out of the measurement range of the Coriolis flow meter. Therefore, the Coriolis flow meter is now replaced by Gilmont variable-area flow meter for volumetric flow rate measurement of air (see Figure 7.6). Its correlated flow table is provided in Appendix F.2. Additionally, a pair of thermocouple and pressure transducer is mounted before the flow meter for flow rates adjustment.

7.1.2. Calibration of Measuring Devices

Before any tests were carried out, all the measuring devices were calibrated at either the laboratories or by the suppliers. The purpose of calibration is to ensure the accurate indication or output signal of the instrument when it is used.

The thermocouples are calibrated by comparing their values with a platinum resistance thermometer (PRT). The data of the thermocouples are captured using the DAQ system and LabVIEW, whereas the temperature of the PRT is displayed by the Fluke 1529 Chub-E4.

Both the thermocouples and the PRT probe are immersed in the oil bath where its temperature is varied from the highest to the lowest set point temperature, at 100°C, 75°C, 50°C, 25°C, and 0°C. The temperature range is chosen such that it covers the operating range

of the system. The calibration data for one of the thermocouples is shown in Figure 7.7. The remainder of the results are included in Appendix H.1.

Figure 7.7 Set point temperature-DAQ reading relationship for a Type-T thermocouple (TC-T-1)

For WIKA pressure transmitters and Endevco miniature piezoresistive pressure transducers, both were calibrated by the deadweight calibrator. The calibration starts at low pressure to high pressure by adding weights at a 2 bar interval, and the results are recorded at each point. Once the highest pressure is reached, the process is repeated from the high pressure back to the low pressure. The calibration range is 1 to 23 barg for WIKA pressure transmitters, and 1 to 25 barg for the Endevco miniature piezoresistive pressure transducers. Both range falls within the operational condition of the system. The calibration data for one of each of these two pressure sensors are shown in Figures 7.8 and 7.9. The remainder of the results are included in Appendix H.2 and H.3.

Figure 7.8 Pressure-current relationship for a WIKA pressure transmitter with the serial number: 1105TGHH

Figure 7.9 Pressure-current relationship for an Endevco piezoresistive pressure transducer with the serial number: 29399

The calibration of the torque meter and the two mass flow meters are provided by the suppliers. The calibration certificates can be found in Appendix H.4 and H.5.

7.1.3. Shaft Alignment

The prototype is driven by an electric motor in the vertical arrangement. To prevent damage and increase the life span of these rotary machineries, both the shaft of the prototype and the motor has to be aligned within an acceptable limit of tolerance. Failure to do so would result in excessive vibration, noise, friction, and temperature rise in couplings and bearings. In extreme cases, it would even cause premature bearing, coupling, shaft and seal failure. Moreover, the encoder and the torque meter are placed in between the prototype and the motor for position, speed and torque measurement. Any misalignment would also result in sensor failure due to excessive vibration. The alignment procedure was performed on the prototype and the motor as illustrated in Figure 7.10.

Figure 7.10 Shaft alignment using laser alignment tools and shims

There are three types of shaft misalignment: angular, parallel and the combination of these two misalignments. They can be minimised by investigating the degree of misalignment using any shaft alignment tools, and correcting them by adjusting the position of the components in the plane perpendicular to the rotating axis and adding shims at strategic locations. The alignment tools used in this procedure is the Easy-Laser E530.

The alignment procedure has resulted in the improvement presented in Table 7.2. The starting position is arbitrary. For shaft rotating at 3000 rpm, the result falls out of the acceptable limit according to the alignment standards proposed by Piotrowski [176]. The safe limit in which the prototype can be operated is up to 900 rpm as shown in Table 7.3.

Table 7.2 Shaft alignment results

Position	Before procedure		After procedure	
	Angular misalignment (/100 mm)	Radial offset misalignment (mm)	Angular misalignment (/100 mm)	Radial offset misalignment (mm)
3 o'clock	0.18	0.92	-0.04	-0.04
6 o'clock	-0.30	2.05	-0.14	-0.02

Table 7.3 Misalignment limits for rotating shafts by Piotrowski [176]

Shaft speed (rpm)	Angular misalignment (/100 mm)		Radial offset misalignment (mm)	
	Excellent	Acceptable	Excellent	Acceptable
3600	0.03	0.05	0.0254	0.0508
1800	0.05	0.07	0.0508	0.1016
1200	0.07	0.10	0.0762	0.1524
900	0.10	0.15	0.1016	0.2032

Due to the constraint imposed by the bracket that supports the motor, any further improvement is costly and challenging to achieve, and the desired results may not be achievable. Therefore, the procedure is terminated at the current result. To prevent damage to the prototype, testing will begin at 600 rpm and gradually increases to its limit before vibration becomes too severe. In the next section, test results obtained from the prototype measurement are presented and discussed.

7.1.4. Experimental Results and Discussions

The prototype has been operated from 600 rpm to 1300 rpm. Vibration is observed during the prototype testing, particularly as the operational speed increases. The maximum speed in which the prototype can be operated is around 1300 rpm. Beyond this, vibration becomes too severe for accurate measurements.

The suction pressure of the expander, p_{esuct} and discharge pressure of the expander, p_{disc} are regulated by a pressure control valve installed before the outlet of the external compressor. In theory, p_{esuct} and p_{disc} are equal. Therefore during testing, high pressurised air is supplied by the same external compressor to the expander and to the air receiver

connected to the outlet of the compressor. Unless otherwise stated, the suction and discharge pressure refers to p_{esuct} and p_{cdisc} , respectively. The suction pressure of the compressor, p_{csuct} is at 1 atm.

Due to severe internal leakage, the compressor could only compress the working fluid from 1 atm to a maximum pressure of 1.69 bar that is achieved with an operational speed of 1064 rpm. The pressure difference of 0.69 bar is comparatively lower than the designed pressure difference of approximately 12 bar with R-134a as the working fluid. At such a low speed and low discharge pressure condition, the load experienced by the bearings would be low. Under this condition, it is highly likely that viscous friction between the shaft and the sleeve prevails, instead of hydrodynamic lubrication. Therefore, the viscous friction condition is employed for the friction calculation between the shaft and the bearing [22]. The expression of the frictional loss is given by

$$P_{bear,static,j} = \omega_j \frac{2\pi\omega_j l_{bear,j} r_{bear,j}}{\delta_j^4} \quad (7.1)$$

Table 7.4 shows the major dimensions and physical properties of the CVEC prototype. The coefficients of discharge, C_d are chosen based on the best fit with the measured results. C_d values are not the same because different amount of lubricant exists at the suction and discharge ports which further impedes fluid flow. The lubricant viscosity is 0.0168 Pa·s at the expected lubricant temperature of around 70°C (See Appendix G.2).

Table 7.4 Dimensions and physical properties of the CVEC prototype

Main dimensions of expander-compressor unit and its physical parameters			
Rotor radius, r_r	12.0 mm	Vane friction coeff., μ_v	0.15 [29]
Inner cylinder radius, r_{ic}	15.0 mm	Split bush friction coeff., μ_{sb}	0.15 [29]
Outer cylinder radius, r_{oc}	48.0 mm	Expander suction port dia.	10.0 mm
		Discharge coeff.	0.1
Inner cylinder thickness, t_{ic}	30.0 mm	Expander discharge port dia.	10.0 mm
		Discharge coeff.	0.1
Outer cylinder thickness, t_{oc}	10.0 mm	Compressor suction port dia.	12.0 mm
		Discharge coeff.	0.9
CVEC length, l_{ecu}	48.0 mm	Compressor discharge port dia.	12.0 mm
		Discharge coeff.	0.9
Vane length, l_v	36.0 mm	CVEC density, ρ_{ecu}	7800 kg/m ³
Vane width, w_v	4.0 mm	Lubricant dynamic viscosity, μ_{lub}	0.0168 Pa·s

Dimensions of reed valve and its physical parameters			
Width, w_{dv}	15.0 mm	Dist. from rotational axis, r_{dv}	58.0 mm
Thickness, t_{dv}	0.3 mm	Density, ρ_{dv}	7800 kg/m ³
Length, l_{dv}	34.0 mm	Young's modulus, E_{dv}	200 GPa
Valve stop height, h_{dv}	5.0 mm	Damping factor, ξ_{dv}	0.1
Clearances			
Average expander radial clearance, $\delta_{rad,exp}$			29.5 μ m
Average compressor radial clearance, $\delta_{rad,com}$			43 μ m
Average shaft journal bearing radial clearance, $\delta_{rad,bear,s}$			15.0 μ m
Average inner cylinder journal bearing radial clearance, $\delta_{rad,bear,ic}$			37.5 μ m
Total inner cylinder end-face clearance, $\delta_{ef,ic}$			25.0 μ m
Lower clearance			7 μ m
Total outer cylinder end-face clearance, $\delta_{ef,oc}$			46.0 μ m
Lower clearance			7 μ m
Total vane end-face clearance, $\delta_{ef,v}$			52.0 μ m
Lower clearance			7 μ m

Figure 7.11 shows the average torque, τ_{avg} of the prototype measured at various operational conditions. As the operational speed increases, the average torque increases, which is mainly due to the increase in frictional losses. In addition, it is also observed that the average torque becomes higher when the discharge pressure increases. The torque difference is contributed by the increased in compression work required to compress the working fluid to a higher pressure less the expansion work recovered by the expander.

Figure 7.11 Average torque of CVEC at various operational conditions

Comparisons between the measured and predicted results for various operational conditions are shown in Figure 7.12. For clarity purposes, the comparisons are presented individually for different suction/discharge pressure. By Equation (7.2), the average standard deviation between the measured and predicted results is 10.5%.

$$\sigma = \sqrt{\frac{\sum (x_{predicted} - x_{measured})^2}{\sum x_{measured}^2}} \quad (7.2)$$

Figure 7.12 Comparison of measured and predicted average torque for various operational conditions: points – measured, lines – predicted

The average power of the machine is obtained by the product of average torque and its corresponding operational speed. The results are presented in Figure 7.13, where the variation of average power increases with respect to operational speed.

Figure 7.13 Comparison of measured and predicted average power for various operational conditions: points – measured, lines – predicted

The predicted average torque which is required for compressing the fluid in the compressor, τ_{com} and that of which is recovered by fluid expansion in the expander, τ_{exp} are shown in Figure 7.14. Due to the low suction pressure of the expander, the ‘recovered’ expansion work drops to negative within the tested conditions. In other words, instead of providing energy recovery which reduces the power input of the machine, the fluid expansion in the expander now consumes power. However, as the suction pressure increases, τ_{exp} becomes more ‘positive’, indicating that above certain suction pressure, energy recovery is possible.

Figure 7.14 (a) Average torque required for compressing the fluid in the compressor and (b) average torque recovered by fluid expansion in the expander

Another observation which can be made in Figure 7.14, is the decrease in gradient of τ_{com} and the increase in gradient of τ_{exp} as the suction/discharge pressure increases. The reduced gradient of τ_{com} at higher discharge pressure would mean a lower energy consumption as operational speed increases. Likewise, the increase in gradient of τ_{exp} signifies higher energy recovery as the operational speed increases at higher discharge pressure. It is deduced that such occurrence lies in the reduced internal leakage as the operational speed increases. The effect is especially significant at higher suction/discharge pressure, whereby the influence of operational speed on the internal leakage is more significant.

As it is obvious that the expander now becomes a ‘burden’ to the machine, a new definition of the mechanical efficiency, $\eta_{m,com}$ is given in the form of

$$\eta_{m,com} = \frac{P_{com}}{P_{com} - P_{exp} + P_{loss}} \quad (7.3)$$

The above equation represents the mechanical efficiency of the expander-compressor unit that operates as a compressor. From Figure 7.15a, it can be seen that $\eta_{m,com}$ are greater at higher discharge pressure, because the compression work, which is also the useful work, becomes more significant. Furthermore, $\eta_{m,com}$ is also observed to increase as the operational speed reduces, due to lesser frictional losses. The predicted mechanical efficiency ranges

from 3.3% to 10.9% within the tested conditions. This indicates that more than 90% of the power input is used for overcoming the friction of the machine.

Figure 7.15 (a) predicted mechanical efficiency and (b) recovery ratio of the expander-compressor unit in various operational conditions

The recovery ratio, κ_{ecu} is defined as τ_{exp}/τ_{com} . For the same reason given, κ_{ecu} values are negative and are observed to increase at higher suction/discharge pressure (see Figure 7.15b), indicating its potential in energy recovery increases. κ_{ecu} ranges from -33.9 to -6.3 within the tested conditions.

Figure 7.16 shows the comparison between the measured and predicted pressure variation of the expander and compressor. Due to the four locations of the miniature pressure transducer ports, the measurement ranges of the expander and compressor are limited to within 53° to 307° and 42° to 318° , respectively. The choice of these four transducer locations is constrained by the design.

Figure 7.16 Comparison of measured and predicted variation of pressure of CVEC at suction pressure of expander = 1.48 bar, discharge pressure of compressor = 1.48 bar and operational speed = 824 rpm: lines – measured, dashed lines – predicted (w/o in-chamber heat transfer, and leakage between compressor working chambers and housing chamber)

In Figure 7.16, the pressure variation of the expander shows a relatively good agreement between the measured and predicted values. However, it is not the case for the compressor, where a large discrepancy is observed for the fluid pressure during the compression process. It is deduced that the large discrepancy is due to the severe leakage between the housing chamber and working chambers of the compressor. The radial clearance in this region is measured to be 27 μm .

In Section 4.5, the internal leakage model presented is limited to within the working chambers of the machine. Leakages between the working chambers of the compressor and housing chamber can be significant and must be included in the analysis. Through observation, it is discovered that leakage paths exist in the radial clearance of the outer cylinder and bearing by axial flow, i.e., Paths 13 and 14 in Figure 7.17. The leakage rate is determined based on the orifice flow model, where the orifice flow area is given as

$$A_{leak,13/14} = \delta_{rad,oc} \times 2\pi r_{oc} \times \theta_{hscs/hscd} \quad (7.4)$$

$\theta_{hscs/hscd}$ denotes the exposed angle between the housing chamber and the suction/compression chamber of the compressor, respectively.

Figure 7.17 Leakage paths of CVEC in the a) first half of the cycle and b) second half of the cycle with inclusion of leakage paths between working chambers of compressor and housing chamber

Upon the inclusion of leakage effect between the housing chamber and working chambers of the compressor, the discrepancy in the compressor reduces as shown in Figure 7.18. It is predicted that leakage in this region contributed 88.2% of the total leakage loss.

Figure 7.18 Corrected variation of pressure of the compressor with the inclusion of leakage effect between the housing chamber and working chambers of the compressor: lines – measured, dashed lines – predicted (w/o in-chamber heat transfer)

The comparisons of measured and predicted variation of pressure of CVEC for operational speed at 931 rpm and 1030 rpm are also shown in Figures 7.19 and 7.20, respectively. In these figures, suction and discharge losses are observable due to the small opening area (10 mm) of the suction and discharge ports of the expander, whereas not in compressor due to the sufficient port sizes (12 mm).

Furthermore, discrepancies are observed between the measured and predicted fluid pressure in the expansion process of the expander and becomes apparent towards the end of fluid expansion. Similar observation can be made with the compression process of the compressor, but with the agreement improved towards the end of fluid compression. The reason for these pressure differences is deduced to lie in the absence of heat transfer model in the prediction model. An attempt has been made to improve the accuracy of the mathematical model with a heat transfer model described in Appendix C, following which Second Law analysis was carried out to quantify the exergy losses in Appendix D.

In addition, the predicted pressure pulsations during the discharge process of the compressor are observed to have lower amplitude with that of the measured ones. It was later realised that it could be due to the rubbing of the side of the valve reed and the valve slot. This friction makes the valve harder to open which results in higher pressure build-up before the fluid is discharged.

Figure 7.19 Comparison of measured and predicted variation of pressure of CVEC at suction pressure of expander = 1.48 bar, discharge pressure of compressor = 1.49 bar and operational speed = 931 rpm: lines – measured, dashed lines – predicted (w/o in-chamber heat transfer)

Figure 7.20 Comparison of measured and predicted variation of pressure of CVEC at suction pressure of expander = 1.48 bar, discharge pressure of compressor = 1.49 bar and operational speed = 1030 rpm: lines – measured, dashed lines – predicted (w/o in-chamber heat transfer)

Figure 7.21 shows the comparison between the measured and predicted mass flow rates, $\dot{m}_{com}$ of the compressor at various operational conditions. The average standard deviation between the measured and predicted values is 16.1%. Significant reduction in mass flow rate is shown in this figure when the discharge pressure varies marginally for all operational speeds shown. This could be due to large leakage paths which cause significant leakages. The figure also shows that at higher speeds, the mass flow rate is higher. This is because at higher speeds, the leakage is significantly reduced due to a shorter cycle time. The mass flow rates of the expander are not measured as they are assumed to be equal to that of the compressor.

Figure 7.21 Comparison of measured and predicted mass flow rates of compressor at various operational conditions: points – measured, lines – predicted

The measured volumetric efficiencies of the compressor part of the machine, $\eta_{v,com}$ for the various operational conditions are shown in Figure 7.22. The definition of $\eta_{v,com}$ is defined as

$$\eta_{v,com} = \frac{\dot{m}_{com}}{\dot{m}_{com,ideal}} \times 100\% \quad (7.5)$$

where $\dot{m}_{ideal,com}$ is given as

$$\dot{m}_{com,ideal} = V_{com} \times \rho_{csuct} \times \frac{\omega_1}{2\pi} \quad (7.6)$$

$\eta_{v,com}$ is observed to increase at lower discharge pressure due to lesser pressure differential for leakage to occur. Moreover, for a given discharge pressure, $\eta_{v,com}$ is higher at higher

operational speed due to reduced leakage. The measured $\eta_{v,com}$ ranges from 3.6% to 63.6% within the tested conditions.

Figure 7.22 The volumetric efficiency of the compressor at various operational conditions

7.2. Concluding Remarks

Measurements have been carried out to validate the mathematical models and to evaluate the performance of CVEC. It was found that the average standard deviation between the measured and predicted average torques is 10.5%. Using the measured operational conditions, the mechanical efficiencies were predicted to be 3.3% to 10.9%, while the predicted recovery ratios range from -33.9 to -6.3. The negative values of the recovery ratio suggest that the expander consumes energy instead of providing energy recovery. Additionally, the mass flow rates were measured and compared with the predicted results. The average standard deviation is 16.1%, while the measured volumetric efficiencies range from 3.6% to 63.6%. Lastly, the comparison between the measured and predicted results shows that the mathematical model is capable of predicting the variation of pressure in the working chamber.

8 Conclusions and Recommendations for Future Work

In this final chapter of the thesis, conclusions about the author's research project and recommendations for future work are presented.

8.1. Conclusions

In this project, a new design of expander-compressor unit named cross vane expander-compressor (CVEC) has been conceived. The uniqueness of this mechanism lies in its seamless integration of expander and compressor into a simple single body. Two cylinders are mounted slightly eccentrically with respect to each other in a common housing, and driven by a single shaft which performs the expansion and compression simultaneously.

Mathematical models describing the volume variation of the expander and compressor of CVEC, kinematics of the machine, variation of thermodynamics properties of the working fluid, suction and discharge mass flow rates, valve reed response, internal leakage and mechanical losses of the machine were formulated to analyse the operational characteristic and performance of the new machine. These mathematical models were simulated with MATLAB software and REFPROP routine. The latter provides the thermodynamics properties of the working fluid.

Following this, a CVEC prototype was designed and fabricated, which is the first of its kind. The main dimensions of the prototype are: $r_r = 12.0$ mm, $r_{ic} = 15.0$ mm, $r_{oc} = 48.0$ mm, $t_{ic} = 30.0$ mm, $t_{oc} = 10.0$ mm and $l_{ecu} = 48.0$ mm. The prototype is made of AISI 4140 steel and surface treated to obtain the black oxide coating for corrosion resistance.

A test rig is built to measure the performance of the CVEC prototype. It comprises the major components of a refrigeration cycle such as a condenser, an evaporator, an accumulator, a receiver, and an electric motor that drives the prototype. While measurements were attempted in the closed-loop refrigeration system, severe leakages were encountered between the suction and discharge line of the expander. As a result, the expanded fluid of the expander reverts back to its suction condition before it could flow through the evaporator. There is no refrigerating effect being produced and the cycle COP could not be measured. This deficiency is believed to be caused by the inappropriate positioning of the suction and discharge port and the pre-opening of the suction valve which led to undesirable

communication between the expanded and suction fluid by the end of the cycle. As a result, the original closed-loop system had to be modified to become an open-loop air-flow circuit.

The average torque, speed, variation of chamber pressure and volumetric flow rate of the machine were measured in this modified circuit. The test data were then used to validate the mathematical models and evaluate the performance of CVEC.

The following conclusions regarding the CVEC design, mathematical model and experimental study are drawn:

8.1.1. Design

- The CVEC design consists of three major components: the outer cylinder-vane-rotor component, the inner cylinder, and the split bush.
- The novel features include:
 - The existence of four working chambers, two of which for the expander and the other two for the compressor. This allows the function of expander and compressor to be performed in one single unit, permitting fluid compression and expansion energy recovery to be accomplished simultaneously.
 - Such a design reduces the number of major components (three with CVEC) and the mechanical losses to one machine compared with systems that use a separated expander and a compressor.
 - The components of CVEC are also of simple cylindrical shape. This allows for greater ease of manufacturing.
 - Furthermore, all the components rotate during shaft rotation. This reduces the relative velocity between the rubbing components, and hence reducing the frictional loss.
 - Lastly, the design reduces the size of the rotor relative to the outer cylinder radius. This is not possible with the revolving vane and rolling piston compressors as it is an inherent nature of their design.

8.1.2. Mathematical Model

- Significant leakage is found at the split bush clearances separating the expander and compressor working chambers. The leakage in this region contributes 74.1% of the total leakage. As a result, lower fluid pressure in both the suction chamber of the

expander and compression chamber of the compressor are observed. The next major leakage occurs at the radial clearances of the machine which constitutes 18.6%.

- Additionally, the major mechanical loss comes from end-face friction which constitutes 61.9% of the total losses, and followed by suction and discharge losses which accounts for 19.1%.
- The use of recovered work from the expander has resulted in COP improvement of the system by 29.5% and reduces the peak power input by 6.5%.
- It is revealed that machines with a slender configuration or that with a lower rotating inertia would result in a higher mechanical efficiency.
- The highest mechanical efficiency of the machine was predicted to be 85.1% at slender ratio of 6.28 (i.e. $l_{ecu} = 151.0$ mm and $r_{oc} = 24.1$ mm).
- However, a short and fat machine shows a higher volumetric efficiency.
- Results also show that the mechanical and volumetric efficiencies of CVEC are comparable to that of other existing rotary compressors. It is believed that optimisation of CVEC design would bring about higher efficiencies of the machine.

8.1.3. Experiment

- Tests have been carried out successfully from 600 rpm to 1300 rpm in the range of suction/discharge pressure from 1.1 bar to 1.7 bar, where the suction and discharge pressures were measured to within 1.4% of discrepancy.
- The operational speed was limited to 1300 rpm due to severe vibration of the prototype.
- The discharge pressure of the compressor was achievable up to 1.7 bar due to severe internal leakage that occurs in the compressor. As a result, the suction pressure of the expander was also limited to 1.7 bar.
- Comparisons show that the average standard deviation between the measured and predicted average torques is 10.5%.
- Using the measured operational conditions, the predicted mechanical efficiency of the machine ranges from 3.3% to 10.9%. This suggests that the useful compression work is small at the low discharge pressure.

- Moreover, the low suction pressure causes the expansion process to consume energy rather than to recover energy. This is shown by the negative predicted recovery ratios ranging from -33.9 to -6.3 within the tested conditions.
- Comparison between the measured and predicted results shows that the mathematical model is capable of predicting the variation of pressure in the working chamber. The discrepancies are deduced to be caused by the heat transfer effect and friction between the reed valve and valve slot.
- The average standard deviation between the measured and predicted mass flow rates is 16.1%.
- The volumetric efficiencies were measured to be 3.6% to 63.6%.
- Severe leakage is predicted between the housing chamber and working chambers of the compressor, which accounts for 88.2% of the total leakages.

8.2. Recommendations for Future work

In this section, future work which can be carried out to improve the performance of the machine in terms of design, mathematical model and measurement is proposed:

- The opening angle of the expander suction valve is fixed in the current design. This has prevented other working fluid to be used optimally because of the different suction fluid condition. An automatic pressure driven valve such as those commonly used in compressor discharge may be employed to solve this issue.
- An attempt has been made to model the in-chamber heat transfer in the machine which is presented in Appendix C. A more comprehensive model should consider actual flow phenomena occur in the working chambers such as Taylor-Coutte flow in the housing chamber [177-182].
- The components temperature should be modelled by lumped thermal conductance method [183] or any commercially available computational code.
- A more comprehensive simulation should include two phase flow [184] in the energy consideration to account for interaction between the liquid and gas phase in the working chamber of the expander.
- The severe vibration encountered by the prototype has ceased its operation before 1300 rpm. Therefore, a shaft alignment between the shaft of the CVEC and the main driver (motor) should be further improved.

- The location of the opening of the suction valve was mistakenly referred to θ_1 instead of θ_2 and this has resulted in the pre-opening of the suction valve. In future design, θ_2 should be used as the basis for determining the opening angle of the suction valve because of its location at the inner cylinder.
- A future prototype should be manufactured with tighter tolerance to reduce internal leakage; in particular the radial clearances between the outer cylinder and the lower bearing which must be less than the current value of 27 μm . Results in Section 7.1.4 showed that this internal leakage is predicted to cause 88.2% of the total leakage loss. Its effect can be seen by comparing Figure 7.16b and Figure 7.18.
- Currently because of the location of the pressure transducers in the working chamber, certain angular position of the pressure signal in the working chamber were not able to be recorded. In future measurement exercise, thought has to be given to the positioning of the pressure transducers such that the pressure variation for a complete cycle could be obtained.
- With the improvement suggested in this section for prototype design and measurement, the test should be carried out using a close-loop refrigeration cycle to appropriately test the function of the CVEC and its expected improvement in the refrigeration cycle.
- The improvement in cycle COP or any energy saving as a result of using CVEC as opposed to a conventional refrigeration cycle that uses expansion valve with a compressor must be measured and compared. Figure 8.1 shows a possible test circuit using CVEC for COP measurement.

Figure 8.1 Closed-loop measurement setup using CVEC while having the expansion valve (TXV) bypassed: dashed arrow indicates flow direction

Figure 8.2 shows that the same circuit can be used to measure the COP of a conventional refrigeration cycle which uses expansion valve. In this case, the ‘expander’ portion of the CVEC has been bypassed.

Figure 8.2 Closed-loop measurement setup using the expansion valve (TXV) while having the ‘expander’ portion of CVEC bypassed: dashed arrow indicates flow direction

- Measurement should be performed using CO₂ as a working fluid to fully realise the use of CVEC in refrigeration systems.
- Last but not least, to design a better performing CVEC, the validated mathematical model can be coupled with a computerised optimisation algorithm [22] to search for a set of design dimensions within the prescribed constraints which produces a best performed CVEC for a given predefined objective function. The objective function in this case is most likely the highest cycle COP or the most energy-efficient CVEC.

8.3. Final remarks

Throughout the course of this research project, a lot of learning has been attained by the author. Even though the proposed device has not been able to achieve its full operational capability, it is believed that the valuable insights and knowledge gained from this study will serve as a stepping stone for the next researcher. While the developmental path of this new invention is an extensive and a challenging one, the fruit of this research will be greatly

rewarding and worthy of the cause and effort. With this final remark, the thesis has reached its end.

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Appendices

Appendix A: Formulation of Angle γ

The angle subtended by the inner cylinder and shaft, γ are formulated by analysing the CC_1C_2 triangle as shown in Figure A.1.

Figure A.1 The CC_1C_2 triangle

Applying the cosine rule on CC_1C_2 triangle, and performing substitutions using the trigonometric identities and a series of rearrangement, we obtain

$$l_{AC} = -(r_{ic} - r_r)\cos\theta_1 - r_r + \sqrt{\left(r_{ic} + \frac{t_{ic}}{2}\right)^2 - (r_{ic} - r_r)^2\sin^2\theta_1} \quad (\text{A.1})$$

where

l_{AC} is the length of the vane measuring from the surface of the shaft to the middle section of the inner cylinder, or length AC (m)

The angle subtended by the inner cylinder and shaft is then formulated as

$$\gamma = \begin{cases} \cos^{-1} \left(\frac{\left(r_{ic} + \frac{t_{ic}}{2}\right)^2 + (r_r + l_{AC})^2 - (r_{ic} - r_r)^2}{2 \left(r_{ic} + \frac{t_{ic}}{2}\right) (r_r + l_{AC})} \right); & 0 \leq \theta_1 \leq \pi \\ -\cos^{-1} \left(\frac{\left(r_{ic} + \frac{t_{ic}}{2}\right)^2 + (r_r + l_{AC})^2 - (r_{ic} - r_r)^2}{2 \left(r_{ic} + \frac{t_{ic}}{2}\right) (r_r + l_{AC})} \right); & \pi \leq \theta_1 \leq 2\pi \end{cases} \quad (\text{A.2})$$

Appendix B: Determination of the Eccentricity and Attitude Angle of the Journal Bearing

The procedure begins by guessing the values of the eccentricity ratio, ε , the rate of change of the eccentricity ratio, $\dot{\varepsilon}$, the attitude angle, ϕ , and the rate of change of the attitude angle, $\dot{\phi}$. With these guessed values, the range of angles (from θ_1 to θ_2) where the bearing pressure is positive can be determined by

$$\theta_1 = \tan^{-1} \left(\frac{\dot{\varepsilon}}{\varepsilon(0.5(\omega_j - \omega_{bear,j}) - \dot{\phi})} \right) \quad (B.1)$$

$$\theta_2 = \theta_1 + \left(\frac{0.5(\omega_j - \omega_{bear,j}) - \dot{\phi}}{|0.5(\omega_j - \omega_{bear,j}) - \dot{\phi}|} \right) \pi \quad (B.2)$$

The corrected $\dot{\varepsilon}$ and $\dot{\phi}$ can then be obtained by

$$\dot{\varepsilon} = \frac{F_j \delta_j^2}{6\mu l_{bear,j} r_{bear,j}^3} \left(\frac{A_3 \sin \phi + A_2 \cos \phi}{A_2^2 - A_1 A_3} \right) \quad (B.3)$$

$$\dot{\phi} = \frac{F_j \delta_j^2}{6\mu l_{bear,j} r_{bear,j}^3 \varepsilon} \left(\frac{A_1 \sin \phi + A_2 \cos \phi}{A_2^2 - A_1 A_3} \right) + \frac{\omega_j + \omega_{bear,j}}{2} \quad (B.4)$$

where

$$A_1 = \int_{\theta_1}^{\theta_2} A \cos^2 \theta d\theta \quad (B.5)$$

$$A_2 = \int_{\theta_1}^{\theta_2} A \cos \theta \sin \theta d\theta \quad (B.6)$$

$$A_3 = \int_{\theta_1}^{\theta_2} A \sin^2 \theta d\theta \quad (B.7)$$

$$A = \left(1 - \frac{C}{B} \log \left(\frac{B + 1/2}{B - 1/2} \right) \right) \left(\frac{1 + h}{h^2} \right) \quad (\text{B.8})$$

$$B = \sqrt{C + 1/4} \quad (\text{B.9})$$

$$C = \frac{h(1 + h)}{(l_{bear,j}/r_{bear,j})^2} \quad (\text{B.10})$$

$$h = \delta_j(1 - \varepsilon \cos \theta) \quad (\text{B.11})$$

Applying $\dot{\varepsilon}$ and $\dot{\phi}$ to Equations B.12 and B.13, the eccentricity ratio and attitude angle of the next time step are obtained.

$$\varepsilon_{t+1} = \varepsilon_t + \dot{\varepsilon}_t \Delta t \quad (\text{B.12})$$

$$\phi_{t+1} = \phi_t + \dot{\phi}_t \Delta t \quad (\text{B.13})$$

The procedure is repeated until the last bearing load data is determined. As the bearing load is cyclical, the eccentricity ratio and attitude angle of the first and last data must be the same, i.e., $\varepsilon_1 = \varepsilon_{last}$ and $\phi_1 = \phi_{last}$. This requirement governs the convergence criteria. If they are not acceptably close, the iteration must be continued with ε_{last} and ϕ_{last} to be used as the new guessed values for the next iteration. The procedure continues until the convergence criteria are met.

Appendix C: Formulation of Heat Transfer Model

A highly simplified in-chamber heat transfer model has been formulated in an attempt to improve the agreement between the measured and predicted results in Chapter 7.

The paths in which heat transfer occur are illustrated in Figure C.1, where the description of the path numbers and their associated heat transfer rate are specified below:

Path 1: Heat transfer rate between the rotor and the suction chamber of the expander,

$$\dot{Q}_{res}$$

Path 2: Heat transfer rate between the rotor and the discharge chamber of the expander, $\dot{Q}_{red}$

Path 3: Heat transfer rate between the suction and discharge chamber of the expander,

$$\dot{Q}_{esed}$$

Path 4: Heat transfer rate between the suction chamber of the expander and the suction chamber of the compressor, $\dot{Q}_{escs}$

Path 5: Heat transfer rate between the discharge chamber of the expander and the compression chamber of the compressor, $\dot{Q}_{edcd}$

Path 6: Heat transfer rate between the compression and suction chamber of the compressor, $\dot{Q}_{cdcs}$

Path 7: Heat transfer rate between the suction chamber of the compressor and the surrounding of the expander-compressor unit, $\dot{Q}_{srcs}$

Path 8: Heat transfer rate between the compression chamber of the compressor and the surrounding of the expander-compressor unit, $\dot{Q}_{srcd}$

Path 9: Heat transfer rate between the discharge chamber of the expander and the suction chamber of the compressor, $\dot{Q}_{edcs}$

Path 10: Heat transfer rate between the suction chamber of the expander and the compression chamber of the compressor, $\dot{Q}_{escd}$

Figure C.1 Heat transfer paths of CVEC in the a) first half of the cycle and b) second half of the cycle.

The components of CVEC are assumed to reach a constant temperature after a few rounds of operation due to the relatively high conductivity of the material used (AISI 4140 steel). The

mean temperature of each component, tabulated in Table C.1, is estimated by considering the fluid temperature in which it is in contact with.

Table C.1 The estimated mean temperature of solid components of CVEC

Estimated Mean Temperature	
surrounding, T_{∞}	30°C
Rotor, T_r	50°C
Inner cylinder, T_{ic}	60°C
Outer cylinder, T_{oc}	70°C
Vane, T_v	60°C
Split bush, T_{sb}	60°C
End-faces, T_{ef}	60°C
Rotor bearing, $T_{bear,r}$	50°C
Inner cylinder bearing, $T_{bear,ic}$	60°C
Housing, T_{hs}	60°C

However, it is noted that the temperature of every component should be determined from measurement or at least predicted using a lumped thermal conductance method, which have been shown to give a reasonably good prediction [183].

C.1. Heat Transfer between Working Chambers through the Inner Cylinder

In this machine, there are four types of pathway in which heat transfer occur. The first type of pathway exists when heat is transferred from one working chamber to another working chamber through the rotating inner cylinder, i.e., $\dot{Q}_{escs}$, $\dot{Q}_{edcd}$, $\dot{Q}_{edcs}$, and $\dot{Q}_{escd}$ that flow along Paths 4, 5, 9, and 10, respectively. The process begins with convection, followed by conduction, and lastly convection as shown in Figure C.2. Due to the inherent nature of CVEC, Path 9 only exists in the first half of the cycle, whereas Path 10 exists in the second half of the cycle.

Figure C.2 Rotating conducting wall with convective heat transfer on both sides

In order to calculate the rate of heat transfer in a complex system, the concept of equivalent thermal circuit is employed. Assuming $T_{\infty,1}$ is greater than $T_{\infty,2}$, the one dimensional heat transfer rate is expressed by

$$\dot{Q} = \frac{T_{\infty,1} - T_{\infty,2}}{R_{total}} \quad (C.1)$$

where

R_{total} is the total thermal resistance of the complex system (KW^{-1})

Since the conduction and convection paths are in series, the total thermal resistance of the first pathway, $R_{total,I}$ is derived as

$$R_{total,I} = \frac{1}{h_1 2\pi r_1 l} + \frac{\ln(r_2/r_1)}{2\pi k_w l} + \frac{1}{h_2 2\pi r_2 l} \quad (C.2)$$

The correct selection of the Nusselt number, Nu correlation is crucial to the calculation of the heat transfer rate. This parameter is equal to the dimensionless temperature gradient at the surface, where it gives a measure of the convection heat transfer that occurs in that area. Determination of the value of Nu provides the local convection coefficient, h through the relation given by Equation (C.3). All fluid properties are evaluated at the film temperature, T_f which is the arithmetic mean of the temperature at the surface of the solid boundary wall, T_w and the free-stream temperature, T_∞ .

$$Nu = \frac{h \times \delta}{k_f} \quad (C.3)$$

where

k_f is the thermal conductivity of the fluid evaluated at the film temperature ($\text{Wm}^{-1}\text{K}^{-1}$)

δ is the characteristic length of the geometry (m)

In the current case, the forced convective heat transfer is modelled according to the method proposed by Tan and Ooi [137]. In their study, they found that Liu and Zhou's correlation [185] gives the best prediction amongst the other researchers, where the deviation from the measurement results were as small as 1.3% at 1000 rpm (see Figure C.3) [137]

Figure C.3 Measured and predicted pressure variations in compression chamber of RV compressor at 1000 rpm [137]. The predicted results were based on Liu and Zhou [185], Adair et al. [186] and Annand et al. [187] correlations

The correlation [185] is expressed by Equation (C.4); Nu is taken to be an average value for the heat transfer between the fluid and the surface of the working chamber which is bounded by the cylindrical wall, the piston surface and the end-face.

$$Nu = 0.75Re^{0.8}Pr^{0.6} \quad (C.4)$$

The Reynolds number, Re and the Prandtl number, Pr are given by Equations (C.5) and (C.6), respectively.

$$Re = \frac{\rho_f v_f D_h}{\mu_f} \quad (C.5)$$

$$Pr = \frac{c_{p,f} \mu_f}{k_f} \quad (C.6)$$

where

ρ_f is the density of the fluid evaluated at the film temperature (kg/m^3)

$c_{p,f}$ is the specific heat of the fluid evaluated at the film temperature ($\text{Jkg}^{-1}\text{K}^{-1}$)

μ_f is the dynamic viscosity of the fluid evaluated at the film temperature ($\text{Pa}\cdot\text{s}$)

As CVEC design is similar to that of RV compressor in terms of the components' shape and the volume variation of the working chamber, the characteristic of fluid flow is assumed to be similar as well, which in turn approximates the movement of fluid induced by a reciprocating piston. Hence, the same correlation will be employed in all the working chambers of CVEC.

Assuming that the flow of the working fluid on both sides of the conducting wall is due to vane movement as well as the rotational motion of the surfaces that encloses it with no slip boundary condition, the characteristic velocity, v_f of the working fluid can be taken as the

average of the tangential velocity of the two rotating surfaces modelled as a trapezoidal profile [137]. Therefore, the characteristic velocity of the fluid inside the expander, $v_{f,exp}$ and compressor, $v_{f,com}$ are approximated by Equations (C.7) and (C.8), respectively.

$$v_{f,exp} = \omega_1 \left(r_r + \frac{l_i}{2} \right) \quad (C.7)$$

$$v_{f,com} = \omega_1 \left(r_r + \frac{l_v + l_{AD}}{2} \right) \quad (C.8)$$

By definition, the equivalent hydraulic diameter is the quadruple of the ratio between the cross-sectional area as seen by the flow and the wetted perimeter of that cross-section. Therefore, the cross-sectional area as seen by the fluid inside the working chamber can be regarded as the area of the protruded part of the sliding vane, where it is exposed to the working fluid [137]. As a result, the equivalent hydraulic diameter of the fluid flow in the expander, $D_{h,exp}$ and compressor, $D_{h,com}$ are expressed by Equations (C.9) and (C.10), respectively.

$$D_{h,exp} = \frac{2l_i l_{ecu}}{l_i + l_{ecu}} \quad (C.9)$$

$$D_{h,com} = \frac{2(l_v - l_{AD}) l_{ecu}}{(l_v - l_{AD}) + l_{ecu}} \quad (C.10)$$

The second mode of heat transfer involves conductive heat flow through the inner cylinder which is modelled as one dimensional heat conduction through a cylindrical wall with no heat generation and constant thermal conductivity, k_w .

C.2. Heat Transfer between Working Chambers through the Sliding Vane

In the second type of pathway, heat transfer $\dot{Q}_{esed}$ and $\dot{Q}_{cdcs}$ occur along Paths 3 and 6, respectively. The solid component in this case is the sliding vane as shown in Figure C.4. Similar to the first pathway, heat transfer process occurs by convection, followed by conduction and lastly by convection.

Figure C.4 Plane wall with convective heat transfer on both sides

The total thermal resistance of the second pathway, $R_{total,II}$ can be written as

$$R_{total,II} = \frac{1}{h_3 A} + \frac{t}{k_w A} + \frac{1}{h_4 A} \quad (C.11)$$

The conductive heat transfer is modelled as one dimensional heat conduction through a plane wall, while the convective heat transfer is assumed to be like that of between the fluid and the reciprocating piston by reasons given in Section C.1. The correlation provided by Liu and Zhou [185] is used.

C.3. Heat Transfer between the Working Chamber of the Compressor, the Housing Chamber and the Surrounding of the Expander-compressor Unit

The third heat transfer pathway for $\dot{Q}_{srcs}$ (Path 7) and $\dot{Q}_{srcd}$ (Path 8) has an added complexity to the first and second pathway where the process occurs through two conducting walls with convective heat transfer on both sides as shown in Figure C.5. The conducting walls are the rotating outer cylinder wall and the stationary housing wall.

Figure C.5 Two conducting walls with convective heat transfer on both sides

The total thermal resistance of the third pathway, $R_{total,III}$ can be written as

$$R_{total,III} = \frac{1}{h_5 2\pi r_5 l} + \frac{\ln(r_6/r_5)}{2\pi k_w l} + \frac{1}{h_{6-7} 2\pi r_6 l} + \frac{\ln(r_8/r_7)}{2\pi k_w l} + \frac{1}{h_8 2\pi r_8 l} \quad (C.12)$$

The heat transfer at the outer wall of the housing is modelled as free convection over a vertical cylinder, as the surrounding air is assumed to be in stationary state. The vertical cylinder of height, l can be treated as a vertical plate when the following relation is satisfied [188],

$$D \geq \frac{35l}{Gr^{\frac{1}{4}}} \quad (C.13)$$

where the correlation of a vertical plate is

$$Nu = \left\{ 0.825 + \frac{0.387Ra^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}} \right\}^2 \quad (C.14)$$

The Rayleigh number, Ra which is the product of Grashof number, Gr and Prandtl number, Pr is given by

$$Ra = Gr \cdot Pr \quad (C.15)$$

and the Grashof, Gr is

$$Gr = \frac{g\beta(T_w - T_\infty)\delta^3}{\nu^2} \quad (C.16)$$

where

β is the coefficient of volume expansion (K^{-1})

T_w is the temperature at the surface of the wall (K)

T_∞ is the temperature of the fluid sufficiently far from the surface (K)

ν is the kinematic viscosity of the fluid (m^2/s)

All fluid properties are evaluated at the film temperature, T_f and the characteristic length, δ is the height of the cylinder.

For the heat transfer between the inner wall of the outer cylinder and the working fluid in the compressor, the correlation given by Liu and Zhou [185] is used, and the characteristic velocity and hydraulic diameter is approximated by Equations (C.8) and (C.10), respectively.

The heat transfer between the rotating outer cylinder and the stationary housing, where fluid motion is induced by the rotation of the outer cylinder, corresponds to flow dynamics in a vertical annulus with a rotating inner cylinder which is termed the Taylor-Coutte flow [179]. The heat transfer in this flow condition – with the inner cylinder being heated – has been studied by various researchers. Ball et al. performed measurements by varying the radius

ratio. The proposed correlation is not applicable for other working fluid as their research was conducted with air only. Aoki et al. [177] and Tachibana et al. [180, 181] included the effect of Prandtl number, Pr in their correlation by conducting tests with various fluids such as air, water, isobutyl alcohol and spindle oil. However, the range of Taylor number, Ta cannot be applied for our case which has a relatively high Ta value.

Therefore, the heat transfer process is assumed to be limited to between the outer cylinder wall and the working chambers of the compressor. The heat transfer rate in this pathway can be obtained by adapting the method used for that of between the working chambers of the expander and the rotating rotor (see Section C.4).

C.4. Heat Transfer between the Working Chamber and the Rotating Rotor

The fourth and last heat transfer occurs between the surface of the rotating rotor and the working fluid in the expander, i.e., $\dot{Q}_{res}$ and $\dot{Q}_{red}$ along the Paths 1 and 2 as shown in Figure C.6.

Figure C.6 Convective heat transfer from the solid rotor to its surrounding

This process only involves convective heat transfer, where Liu and Zhou's correlation [185] is used, and the characteristic velocity and hydraulic diameter is approximated by equations C.7 and C.9, respectively. The total thermal resistance, $R_{total,IV}$ is expressed as

$$R_{total,IV} = \frac{1}{h_9 2\pi r_9 l} \quad (C.17)$$

C.5. Results and Discussion

The predicted variation of pressure of the machine with the inclusion of in-chamber heat transfer model are compared with the measured results as shown in Figures C.7 to C.9.

Figure C.7 Comparison of measured and predicted variation of pressure of CVEC at suction pressure of expander = 1.48 bar, discharge pressure of compressor = 1.48 bar and operational speed = 824 rpm: lines – measured, dashed lines – predicted (w/ in-chamber heat transfer)

Figure C.8 Comparison of measured and predicted variation of pressure of CVEC at suction pressure of expander = 1.48 bar, discharge pressure of compressor = 1.49 and operational speed = 931 rpm: lines – measured, dashed lines – predicted (w/ in-chamber heat transfer)

Figure C.9 Comparison of measured and predicted variation of pressure of CVEC at suction pressure of expander = 1.48 bar, discharge pressure of compressor = 1.49 bar and operational speed = 1030 rpm: lines – measured, dashed lines – predicted (w/ in-chamber heat transfer)

It is noticed that heat transfer model improves the pressure prediction in the expander, but not that of the compressor, in fact the changes are almost unnoticeable. In future study, a more comprehensive model that includes components' temperature estimation by lumped thermal conductance method [183] and the inclusion of Taylor-Coutte Flow [177-182] should be proposed.

Appendix D: Second Law Analysis

The second law analysis enables all types of losses to be compared in the same basis under exergy loss. It reveals the machine's performance in comparison to its best possible performance in a given operational condition, which is not possible with first law analysis. This section studies the second law efficiency of the expander-compressor unit by exploring the various areas in which the supplied exergy could be destroyed. The second law efficiency of a machine is defined by

$$\eta_{II} = 1 - \frac{\dot{X}_{destroyed}}{\dot{X}_{supplied}} \quad (D.1)$$

For our case, $X_{supplied}$ is the total exergy that is supplied to the machine by the electric motor, whereas $X_{destroyed}$ represents the total exergy that is destroyed in the machine. According McGovern and Harte [189], the exergy destruction is due to these four defect areas: friction, irreversible heat transfer, fluid throttling, and irreversible fluid mixing.

The exergy destruction due to heat transfer from a solid component at temperature, T_j to the working fluid at temperature, T is given by

$$\dot{I}_{Q,j} = \dot{Q}_j T_\infty \left(\frac{1}{T} - \frac{1}{T_j} \right) \quad (D.2)$$

The direction of $\dot{Q}_j$ is taken to be from the solid component to the working fluid, where T_j is higher than T . It is reversed, if T is higher than T_j . Therefore, the sign of $\dot{I}_{Q,j}$ will always be positive, regardless of the direction of heat transfer. Note that T_∞ is the temperature of environment.

The working fluid flows into and out of the working chambers through the suction and discharge ports. This process causes another type of irreversibility called adiabatic throttling. The expression of exergy destruction rate is given by

$$\dot{I}_{i,th} = \dot{m}_i T_\infty (s'_i - s_i) \quad (D.3)$$

where s'_i is the specific entropy of the working fluid after throttling, which is evaluated at the pressure and enthalpy of the bulk fluid at downstream. The enthalpy of the downstream fluid is assumed to be the same as the upstream fluid.

After the throttling process, the working fluid which is at temperature, T'_i mixes with the bulk fluid at temperature, T . The difference in temperature between the incoming fluid and the bulk fluid causes irreversibility during mixing, which is expressed as

$$\dot{I}_{mx} = \dot{m}_i T_\infty \left((s - s'_i) - \frac{(h - h'_i)}{T} \right) \quad (D.4)$$

The last form of irreversibility that occurs in the machine is due to friction. The exergy destruction rate associated with the friction work that is dissipated as heat transfer to the rubbing components at temperature, T_j is

$$\dot{I}_f = P_{loss} \frac{T_\infty}{T_j} \quad (D.5)$$

With the derivation of these four types of irreversibility, the total instantaneous rate of exergy destruction is

$$\dot{I} = \sum \dot{I}_Q + \sum \dot{I}_{th} + \sum \dot{I}_{mx} + \sum \dot{I}_f \quad (D.6)$$

where $\dot{I}$ is essentially the value for $\dot{X}_{destroyed}$.

Figure D.1 shows the breakdown of exergy destruction due to fluid mixing, throttling, heat transfer, and friction loss in CVEC prototype. The operational condition is chosen based on the highest predicted mechanical efficiency of the prototype obtained within the tested conditions, i.e., p_{cdisc} and $p_{esuct} = 1.69$ bar, and $\omega_1 = 709$ rpm. The second law efficiency of the machine is 3.6%.

Figure D.1 Breakdown of exergy destruction (a) due to fluid mixing, throttling, heat transfer, and friction, and (b) the major contributors of exergy loss due to friction

It is found that friction contributes the most in exergy destruction with 85.4% of the overall destruction. Within this amount, end-face friction between inner cylinder and bearing and outer cylinder and bearing contribute the most, i.e., 51.5% and 44.9%, respectively. The reason for such a high end-face friction is due to the fat configuration of the machine which resulted in the increase of relative velocity between the rotating cylinders and the stationary bearings. Furthermore, the rubbing surface of the inner cylinder is exceptionally huge, due to its thick cylinder wall, i.e., 30 mm.

To reduce the exergy loss due to friction, it is proposed that a more slender machine to be built, so that the inner cylinder end-face could be minimised and that the eccentricity of the machine could be made smaller. A smaller eccentricity means that the relative velocity between the end-faces is reduced which leads to lower end-face friction. Additionally, a circular slot can be introduced on the inner cylinder and outer cylinder end-faces to reduce the rubbing contact with the bearings, if the thick cylinder wall is unavoidable

Appendix E: Comparison of Variation of Thermodynamics Properties between R-1234yf and R-134a

Figure E.1 Comparison of pressure variation: lines – R-1234yf, dashed lines – R-134a

Figure E.2 Comparison of temperature variation: lines – R-1234yf, dashed lines – R-134a

Figure E.3 Comparison of mass variation: lines – R-1234yf, dashed lines – R-134a

Appendix F: Technical Specifications of Experimental Devices

F.1. Technical Data Sheet of ABB Motor

Figure F.1 Technical data sheet of ABB motor

F.2. Technical Specifications of the Measuring Devices

Pressure transmitter for general industrial applications Model A-10

WIKA data sheet PE 81.60

Applications

- Machine building
- Machine tools
- Measurement and control technology
- Hydraulics and pneumatics
- Pumps and compressors

Special features

- Measuring ranges from 0 ... 1 to 0 ... 600 bar
- Non-linearity 0.25 % or 0.5 %
- Output 4 ... 20 mA, DC 0 ... 10 V, DC 0 ... 5 V and others
- Electrical connection: Angular connector form A and C, circular connector M12 x 1, cable outlet 2 m
- Process connection G 1/4 A DIN 3852-E, 1/4 NPT and others

Pressure transmitter model A-10

Description

The model A-10 pressure transmitter for general industrial applications is not only notable for its compact design, but it also offers excellent quality at an extremely competitive price.

The user can choose between a non-linearity of 0.25 % and 0.5 %. A free test certificate provides information on the measuring points recorded during manufacture.

The model A-10 is set up for worldwide use through the international cULus and GOST certification. The various pressure units and process connections required for particular operating conditions are available at short notice.

Figure F.2 Technical Specifications of the WIKA pressure transmitter (model A-10): page

Measuring ranges

Relative pressure									
bar	Measuring range	0 ... 1	0 ... 1.6	0 ... 2.5	0 ... 4	0 ... 6	0 ... 10 ¹⁾	0 ... 16 ¹⁾	0 ... 25 ¹⁾
	Overpressure limit	2	3.2	5	8	12	20	32	50
	Measuring range	0 ... 40	0 ... 60	0 ... 100	0 ... 160	0 ... 250	0 ... 400	0 ... 600	
	Overpressure limit	80	120	200	320	500	800	1,200	
psi	Measuring range	0 ... 15	0 ... 25	0 ... 30	0 ... 50	0 ... 100	0 ... 160 ¹⁾	0 ... 200 ¹⁾	0 ... 300 ¹⁾
	Overpressure limit	30	60	60	100	200	290	400	600
	Measuring range	0 ... 500	0 ... 1,000	0 ... 1,500	0 ... 2,000	0 ... 3,000	0 ... 5,000	0 ... 10,000	
	Overpressure limit	1,000	1,740	2,900	4,000	6,000	10,000	17,400	
Absolute pressure									
bar	Measuring range	0 ... 1	0 ... 1.6	0 ... 2.5	0 ... 4	0 ... 6	0 ... 10	0 ... 16	0 ... 25
	Overpressure limit	2	3.2	5	8	12	20	32	50
psi	Measuring range	0 ... 15	0 ... 25	0 ... 30	0 ... 50	0 ... 100	0 ... 150	0 ... 200	0 ... 300
	Overpressure limit	30	60	60	100	200	290	400	600
Vacuum and +/- measuring range									
bar	Measuring range	-1 ... 0	-1 ... +0.6	-1 ... +1.5	-1 ... +3	-1 ... +5			
	Overpressure limit	2	3.2	5	8	12			
	Measuring range	-1 ... +9	-1 ... +15	-1 ... +24					
	Overpressure limit	20	32	50					
psi	Measuring range	-30 inHG ... 0	-30 inHG ... +15	-30 inHG ... +30	-30 inHG ... +60	-30 inHG ... +100			
	Overpressure limit	30	60	60	150	250			
	Measuring range	-30 inHG ... +160	-30 inHG ... +200	-30 inHG ... +300					
	Overpressure limit	350	450	600					

1) If the medium water is measured, a higher overpressure limit is recommended.

The given measuring ranges are also available in kg/cm², MPa and kPa.
Other measuring ranges available on request

Vacuum resistance
Yes

Output signal

Signal type	Signal
Current (2-wire)	4 ... 20 mA
Voltage (3-wire)	DC 0 ... 10 V
	DC 0 ... 5 V
	DC 1 ... 5 V
	DC 0.5 ... 4.5 V
Ratiometric (3-wire)	DC 0.5 ... 4.5 V

Other output signals available on request

Depending on the signal type the following loads apply:

Signal type	Load in Ω
Current (2-wire)	$\leq (\text{power supply} - 8 \text{ V}) / 0.02 \text{ A}$
Voltage (3-wire)	$> \text{maximum output signal} / 1 \text{ mA}$
Ratiometric (3-wire)	$> 4.5 \text{ k}$

Figure F.3 Technical Specifications of the WIKA pressure transmitter (model A-10): page

Voltage supply

Power supply

Output signal	Power supply	
	Standard	Option
4 ... 20 mA	DC 8 ... 30 V	DC 8 ... 35 V ¹⁾
DC 0 ... 10 V	DC 14 ... 30 V	DC 14 ... 35 V
DC 0 ... 5 V	DC 8 ... 30 V	DC 8 ... 35 V
DC 1 ... 5 V	DC 8 ... 30 V	DC 8 ... 35 V
DC 0.5 ... 4.5 V	DC 8 ... 30 V	DC 8 ... 35 V
DC 0.5 ... 4.5 V ratiometric	DC 5 V $\pm$ 10 %	-

¹⁾ Not possible with non-linearity 0.25 % BFSL

The power supply for the pressure transmitter must be made via an energy-limited electrical circuit in accordance with section 9.3 of UL/EN/IEC 61010-1, or an LPS to UL/EN/IEC 60950-1, or class 2 in accordance with UL1310/UL1585 (NEC or CEC). The power supply must be suitable for operation above 2,000 m should the pressure transmitter be used at this altitude.

Total current consumption

Signal type	Total current consumption
Current (2-wire)	Signal current, max. 25 mA
Voltage (3-wire)	8 mA
Ratiometric (3-wire)	8 mA

Accuracy

Optionally the model A-10 is available with an improved non-linearity. Depending on the selected non-linearity the following values result:

	Standard	Option
Non-linearity per BFSL (IEC 61298-2)	$\leq \pm 0.5$ % of span	$\leq \pm 0.25$ % of span
Measuring deviation of the zero signal	Typical: $\leq \pm 0.5$ % of span Maximum: $\leq \pm 0.8$ % of span	Typical: $\leq \pm 0.15$ % of span Maximum: $\leq \pm 0.4$ % of span
Accuracy at room temperature ¹⁾	$\leq \pm 1$ % of span	$\leq \pm 0.5$ % of span $\leq \pm 0.6$ % of span (at DC 0 ... 5 V)

¹⁾ Including non-linearity, hysteresis, zero offset and end value deviation (corresponds to measured error per IEC 61298-2), calibrated in vertical mounting position with process connection facing downwards

Non-repeatability

≤ 0.1 % of span

Signal noise

$\leq \pm 0.3$ % of span

Temperature error at 0 ... 80 °C

- Typical: 1 % of span
- Maximum: 2.5 % of span

Long-term drift

$\leq \pm 0.1$ % of span

Figure F.4 Technical Specifications of the WIKA pressure transmitter (model A-10): page

Model 8530B Piezoresistive pressure transducer

Features

- 200 to 1000 psia
- Absolute reference

Description

The Endevco® model 8530B is a miniature, high sensitivity piezoresistive pressure transducer for measuring absolute pressure. The volume behind the diaphragm is evacuated and glass sealed to provide an absolute pressure reference. Full scale output is 300 mV with high overload capability and high frequency response. It is available in ranges from 200 psia to 1000 psia. See model 8530C for lower pressure ranges.

Endevco pressure transducers feature an active four-arm strain gage bridge diffused into a sculptured silicon diaphragm for maximum sensitivity and wideband frequency response. Self-contained hybrid temperature compensation provides stable performance over the temperature range of 0°F to 200°F [-18°C to +93°C]. Endevco transducers also feature excellent linearity and provide stable performance over the temperature range of 0°F to 200°F [-18°C to +93°C]. Endevco transducers also feature excellent linearity (even to 3X range), high shock resistance, and high stability during temperature transients.

The model 8530B is available with metric M5X0.8 mounting threads as 8530B-XXM5 on special orders. Also available with integral four-pin connector as model 8530B-XXM37. The standard cable length is 30 inches. Alternate lengths are available in quantity by using part number 8530B-XXX-ZZZ where "XXX" is the pressure range and "ZZZ" is the cable length in inches

Endevco model 136 three-channel system, model 4428A or 4430A signal conditioner, or OASIS 2000 computer-controlled system are recommended as signal conditioner and power supply.

Figure F.5 Technical Specifications of the Endevco piezoresistive pressure transducer (model 8530B-500): page 1/2

Model 8530B Piezoresistive pressure transducer

Endevco

Specifications

	Units	-200	-500	-1000
Range	psia	0-200	0-500	0-1000
Sensitivity [1]	mV/psi	1.5 ±0.5	0.6 ±0.2	0.3 ±0.1
Combined: non-linearity, non repeatability, pressure hysteresis [2]	% FSO RSS max	0.50	0.50	1.0
Non-linearity, independent	% FSO typ	0.2	0.2	0.2
Non-repeatability	% FSO typ	0.1	0.1	0.1
Pressure hysteresis	% FSO typ	0.1	0.1	0.1
Zero measurand output [3]	mV max	±10	±10	±10
Zero shift after 3X over range	±% 3X FSO max (typ)	0.2 (0.05)	0.2 (0.05)	
after 2.5X over range	±% 2.5X FSO max (typ)			0.2 (0.1)
Thermal zero shift				
From 0 to 200°F [-18°C to +93°C]	±% FSO max	3	3	3
Thermal sensitivity shift				
From 0 to 200°F [-18°C to +93°C]	±% max	4	4	4
Resonant frequency	Hz	750 000	1 000 000	> 1 000 000
Non-linearity at 3X over range	% 3X FSO	1.5	2.0	2.0 @ % 2.5X FSO
Thermal transient response per ISA-S37.10, PARA. 6.7, procedure I	psi / °F	0.02	0.02	0.04
	psi / °C	0.04	0.04	0.07
Photoflash response [4]	equiv psi	5	10	20
Warm-up time [5]	ms	1	1	1
Acceleration sensitivity	equiv. psi/g	0.0003	0.0002	0.0002
Burst pressure (diaphragm)	psia min	800	2000	4000
Case pressure [6]	psia min	1000	5000	5000
Electrical				
Full scale output	300 ±100 mV at 10.0 Vdc			
Supply voltage [7]	10.0 Vdc recommended, 18 Vdc maximum			
Electrical configuration	Active four-arm piezoresistive bridge			
Polarity	Positive output for increasing pressure			
Resistance				
Input	2000 ±800 ohms			
Output	1600 ±500 ohms			
Isolation	100 megohms minimum at 50 Volts, leads to case, leads to shield, shield to case			
Noise	5 microvolts rms typical, DC to 50 000 Hz; 50 microvolts rms maximum, DC to 50 000 Hz			
Mechanical				
Case, material	Stainless steel (17-4 PH CRES)			
Cable, integral	Four conductor No. 32 AWG Teflon insulated leads, braided shield, silicone jacket			
Dead volume port (+)	0.0003 cubic inches (0.005 cc)			
Mounting/torque	10-32 UNF-2A threaded case 0.438 inch [11.12 mm] long / 15 ±5 lbf-in (1.7 ±0.6 Nm)			
Weight	2.3 grams (cable weighs 9 grams/meter)			
Environmental				
Media [8] [9]	Clean dry gas. Contact factory for protective modifications for other media.			
Temperature [10]	-65°F to +250°F [-54°C to +121°C]			
Vibration	1000 g pk			
Acceleration	1000 g			
Shock	20 000 g, 100 microsecond haversine pulse			
Humidity	Isolation resistance greater than 100 megohms at 50 V when tested per MIL-STD-202E, method 103B, test condition B			

Calibration data [11]

Data supplied for all parameters in Certified Performance section. Optional calibrations available for all parameters in Typical Performance section

Included accessories

EHR93 O-ring, Viton

Optional accessories

EHR96 O-ring, Fluorosilicone
24328-3 4 conductor shielded cable, white

Notes:

- 1 psi = 6.895 kPa = 0.069 bar
2. FSO [Full Scale Output] is defined as transducer output change from 0 to + full scale pressure.
3. Zero Measurand Output (ZMO) is the transducer output with 0 psia applied.
4. Per ISA-S37.10, Para. 6.7, Proc. II.
5. Warm-up time is defined as elapsed time from excitation voltage "turn on" until the transducer output is within ±1% of reading accuracy.
6. Case pressure identifies media containment pressure in the event of diaphragm rupture.

7. Use of excitation voltages other than 10.0 Vdc requires manufacture and calibration at that voltage since thermal errors increase with high excitation voltages.
8. Internal seals are epoxy and are compatible with clean dry gas media. Media in measurand port is exposed to CRES, Parylene C, epoxy and the Viton O-ring. Not suitable for use with high pH or low pH liquids, long term exposure to water, or exposure to solvents which may attack epoxies.
9. O-ring, Parker 5-125, compound V747-75 (Viton®) is supplied unless otherwise specified on purchase order. Fluorosilicone O-ring, for leak tight operation below 0°F is available on special order.
10. Units can be compensated over any 200°F (93°C) span from -65°F to +250°F [-54°C to +121°C] on special order.
11. Maintain high levels of precision and accuracy using Endevco's factory calibration services. Call Endevco's inside sales force at 800-982-6732 for recommended intervals, pricing and turn around time for these services as well as for quotations on our standard products.

Continued product improvement necessitates that Endevco reserve the right to modify these specifications without notice. Endevco maintains a program of constant surveillance over all products to ensure a high level of reliability. This program includes attention to reliability factors during product design, the support of stringent Quality Control requirements, and compulsory corrective action procedures. These measures, together with conservative specifications, have made the name Endevco synonymous with reliability.

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800/982-6732 • 949/472-8181 fax 949/461-7221 • www.endevco.com • Email: applications@endevco.com

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Figure F.6 Technical Specifications of the Endevco piezoresistive pressure transducer (model 8530B-500): page 2/2

Correlated Flow Table

Fluid Air
Density 0.00120000 grams/ml
Viscosity 0.01812000 cp
Tube Size 5 & 250
Float Type Glass/Stainless-Steel
Serial No. 55044
Date Jun 19, 2007
 Measured & flowing at 1.00 atm and 70 °F
 R at 0 = 1.31; R at 50 = 13.58; R at 100 = 26.41

FLOW L/MIN			FLOW L/MIN			FLOW L/MIN			FLOW L/MIN		
GLASS	SCALE	S.S.	GLASS	SCALE	S.S.	GLASS	SCALE	S.S.	GLASS	SCALE	S.S.
1.999	1	4.587	20.74	26	39.00	41.94	51	76.54	65.62	76	118.6
2.589	2	5.803	21.55	27	40.43	42.84	52	78.13	66.62	77	120.3
3.202	3	7.043	22.36	28	41.87	43.74	53	79.73	67.62	78	122.1
3.870	4	8.390	23.17	29	43.30	44.65	54	81.34	68.62	79	123.9
4.574	5	9.700	23.99	30	44.75	45.56	55	82.96	69.62	80	125.7
5.259	6	10.97	24.80	31	46.20	46.48	56	84.58	70.63	81	127.5
5.907	7	12.21	25.62	32	47.65	47.40	57	86.21	71.64	82	129.3
6.642	8	13.61	26.45	33	49.11	48.32	58	87.85	72.65	83	131.1
7.385	9	15.02	27.28	34	50.58	49.25	59	89.50	73.67	84	132.9
8.133	10	16.43	28.11	35	52.05	50.18	60	91.15	74.69	85	134.7
8.886	11	17.84	28.94	36	53.53	51.12	61	92.81	75.71	86	136.5
9.647	12	19.24	29.78	37	55.01	52.06	62	94.48	76.73	87	138.4
10.43	13	20.65	30.62	38	56.50	53.01	63	96.15	77.76	88	140.2
11.21	14	22.05	31.47	39	58.00	53.95	64	97.84	78.79	89	142.0
12.00	15	23.47	32.32	40	59.51	54.91	65	99.53	79.82	90	143.9
12.79	16	24.87	33.18	41	61.02	55.86	66	101.2	80.85	91	145.7
13.58	17	26.28	34.03	42	62.54	56.82	67	102.9	81.89	92	147.6
14.37	18	27.68	34.90	43	64.07	57.79	68	104.6	82.93	93	149.4
15.16	19	29.09	35.76	44	65.60	58.75	69	106.4	83.97	94	151.3
15.95	20	30.50	36.63	45	67.14	59.72	70	108.1	85.01	95	153.1
16.75	21	31.91	37.51	46	68.69	60.70	71	109.8	86.05	96	155.0
17.54	22	33.32	38.38	47	70.24	61.68	72	111.5	87.10	97	156.9
18.34	23	34.74	39.27	48	71.81	62.66	73	113.3	88.14	98	158.7
19.14	24	36.16	40.15	49	73.38	63.64	74	115.0	89.19	99	160.6
19.94	25	37.58	41.04	50	74.96	64.63	75	116.8	90.24	100	162.5

Figure F.7 Correlated flow table of the variable-area flow meter

Magnetic ring MR200

Incrementally coded ring with permanently connected flange, pole length 2 mm

Profile

- Easy hollow-shaft mounting
- Rotary encoder system with IP67 protection category (in combination with MSK210)
- Up to 2000 pulses/revolution (8000 increments)
- With reference point as an option

Mechanical data

Feature	Technical data	Additional information
Flange	aluminum	

■ Table of dimensions

Poles	50	64	100
Diameter D (mm)	32.3	41.2	64.14
Circumference (mm)	100	130	200
Speed [min ⁻¹]	≤15000	≤11500	≤7500

System data

Feature	Technical data	Additional information
Pole length	2 mm	
System accuracy	±0.1°	
Measuring range	360°	

Figure F.8 Technical specifications of the SIKO magnetic ring MR200: page 1/2

Magnetic ring MR200

Incrementally coded ring with permanently connected flange, pole length 2 mm

■ Pulses/revolution

Pole number	50	64	100
Sensor scaling factor	20	1000	1280
	16	800	1024
	10	500	640
	8	400	512
	5	250	320
	4	200	256
	1	50	64

The table applies to the combination of MR200 with MSK210

Ambient conditions

Feature	Technical data	Additional information
Ambient temperature	0 ... 60 °C	50, 64 pole number
	-20 ... 70 °C	100 pole number
Storage temperature	-20 ... 70 °C	
Relative humidity	100 %	condensation admissible
Protection category	IP67	EN 60529

Order

■ Ordering table

Feature	Ordering data	Specification	Additional information
Pole number	50	A	50 poles
	64		64 poles
	100		100 poles
			others on request
Bore/diameter	20	B	ø20 mm
			others on request
Type of fastening	MING	C	hub thread
	ONG		without hub thread
Reference point	0	D	without
	M		with

Figure F.9 Technical specifications of the SIKO magnetic ring MR200: page 2/2

Magnetic sensor MSK210 rotativ

Compact sensor, incremental, digital interface, scaling factor 64

Profile

- max. resolution 0.045° with MR200 and MBR200 (100 poles)
- Repeat accuracy of ±1 increment
- Works with MR200 magnetic ring, MBR200 magnetic tape ring
- Reading distance ≤ 0.8 mm
- Max. 4600 pulses/revolution with MBR200 (230 poles)

Mechanical data

Feature	Technical data	Additional information
Housing	green plastic	
Sensor/ring reading distance	0.1 ... 0.8 mm	Q, I reference signals
	0.1 ... 0.4 mm	R reference signal
Cable sheath	PUR suitable for drag-chain use	4-wire Ø4.4-Ø4.4 mm; 5, 6, 8-wire Ø5.4-Ø4.4 mm

Figure F.10 Technical specifications of the SIKO magnetic sensor MSK210: page 1/2

Magnetic sensor MSK210 rotativ

Compact sensor, incremental, digital interface, scaling factor 64

Electrical data

Feature	Technical data	Additional information
Operating voltage	24 V DC $\pm 20\%$ 5 V DC $\pm 5\%$	reverse polarity protected no reverse polarity protection
Current consumption	< 20 mA < 7.5 mA	at 24 V DC, off-load loaded
Output circuit	PP, LD (RS422), TTL	PP only with 24 V
Output signals	A, A/, B, B/, I, I/, R, R/	quadrature signal
Output signal level high	$> U_B - 2.5$ V > 2.5 V > 2.4 V	PP LD TTL
Output signal level low	< 0.8 V < 0.5 V < 0.4 V	PP LD TTL
Jitter	$< 15\%$	0.5 mm reading distance
Pulsewidth of reference signal	1 increment(s)	
Real-time requirement	speed-proportional signal output	
Type of connection	open cable end plug connector D-Sub	7/8-pole 9-pole

Signal pattern

! The logical condition of signals A and B is not defined in reference to the index signal I or the reference signal R. It can deviate from the signal form.

System data

Feature	Technical data	Additional information
Scaling factor	1, 5, 10, 20, 64	
System accuracy	$\pm 0.1^\circ$	
Repeat accuracy	± 1 increment(s)	
Measuring range	∞	
Circumferential speed	≤ 25 m/s	≤ 2 m/s referencing speed

Ambient conditions

Feature	Technical data	Additional information
Ambient temperature	$-10 \dots 70^\circ\text{C}$	
Storage temperature	$-30 \dots 80^\circ\text{C}$	
Relative humidity	100 %	condensation admissible
EMC	EN 61000-6-2 EN 61000-6-4	interference resistance / immission emitted interference / emission
Protection category	IP67	EN 60529
Shock resistance	500 m/s^2 , 11 ms	EN 60068-2-27
Vibration resistance	100 m/s^2 , 5 ... 150 Hz	EN 60068-2-6

Figure F.11 Technical specifications of the SIKO magnetic sensor MSK210: page 2/2

Figure F.12 Technical specifications of Futek torque sensor (model TRS605)

Tricor - Mass Flow Meter

Technical Data - TCM Transducer

	TCM0325	TCM0650	TCM1550	TCM3100	TCM5500	TCM7900	TCM28k	TCM65k
Flow range (kg/h)	3 to 300	6 to 600	15 to 1500	30 to 3000	55 to 5500	79 to 7900	280 to 28 000	650 to 65 000
Flow range (lb/min)	0.11 to 11	0.22 to 22	0.55 to 55	1.1 to 110	2.0 to 200	2.9 to 290	10 to 1030	24 to 2390
Flow range (l/h)	3 to 300	6 to 600	15 to 1500	30 to 3000	55 to 5500	79 to 7900	280 to 28 000	650 to 65 000
Basic Accuracy (% of flow reading)	0.1							
Zero Stability (% of full scale)	0.01							
Zero Drift (% f.s. per °C)	0.001							
Repeatability (% of flow)	0.1							
Density measuring range	0 - 4,500 kg/m³							
Density accuracy	± 0.002 kg/ltr							
Temperature accuracy	±1°C ±0.5% of reading							
Process and Ambient								
Process connections	female thread 1/2" adaptors for flanges, diaphrag and tri-clamp				flanges EN1092, ANSI B16.5, DIN2512			
Max. pressure	200 bar				350 bar	100 bar		
Max. pressure (Option)	350 bar							
Pressure Drop at max. flow H ₂ O	ask KEM							
Operating Density range	500 - 2,500 kg/m³ (standard) , 5 - 4500 kg/m³ (upon request)							
Process temperature	-40 ... +100°C (standard) / -100 ... +150°C (upon request)							
Ambient temperature	-40 ... +70°C							
Storage temperature	-40 ... +100°C							
Electr. connections remote	screw type terminals							
Electr. connections compact.	none (internally connected to the electronics)							
Ingress Protection	IP65 (IP67 on request)							
General								
Tube arrangement	2 serial	2 parallel	2 serial	2 parallel	2 parallel	2 parallel	2 parallel	2 parallel
Tube inner diameter	4mm	4mm	8mm	8mm	7mm	9mm	16mm	28mm
Tube material	stainless steel DIN 1.4404							
Housing material	stainless steel DIN 1.4301							
Dimensions	see drawings							

Figure F.13 Technical specifications of Tricor Coriolis flow meter (TCM 325)

F.3. Technical Specifications of NI Modules

Data acquisition (DAQ) is the process of sampling signals that measures electrical or physical phenomenon and converting the collected samples into digital numeric values that can be manipulated by a computer. A DAQ system comprises the required measuring devices, DAQ measurement hardware, and a computer with programmable software such as LabVIEW. It has to be able to receive and capture all the different types of output signals generated from the measuring devices. In the experiment, the different types of signal output generated by the various measurement devices are summarised in Table F.1.

Table F.1 Output signal of the measuring devices

Qty.	Measurement Devices	Type of Output Signal
2	Tricor Coriolis flow meter (model TCM 325)	4 to 20 mA current
8	Type-T thermocouple	Millivolts
1	Futek torque sensor (model TRS605)	± 5 Vdc
1	SIKO Encoder (model MR200 and MSK210)	Impulse (TTL)
8	WIKA pressure transmitter (model A-10)	4 to 20 mA current
4	Endevco piezoresistive pressure transducer (model 8530B-500)	Unamplified voltage signals from an active four-arm bridge 300 mV $\pm$ 100 mV at 10 Vdc

The chosen DAQ chassis is the cDAQ-9188 of the National Instruments. In order to receive and capture all the outputs specified in Table F.1, the following modules are added to the chassis:

1. NI 9208 with 16 channels for current output signals
2. NI 9214 with 16 channels for thermocouples output signals
3. NI 9220 with 16 channels for voltage output signals
4. NI 9411 with 6 channels for impulse output signals
5. NI 9237 with 4 channels for output signals from active four-arm bridges

Module NI 9208 is selected to capture the current outputs from the Tricor Coriolis flow meter and the WIKA pressure transmitter. Module NI 9214 is used to capture the outputs from thermocouples. Module NI 9220 is chosen to capture the voltage outputs from the Futek torque sensor, whereas Module NI 9411 is used to capture impulse outputs from the SIKO encoder. The last module NI 9237 will be used to capture the outputs from the Endevco

piezoresistive pressure transducer. These modules will be installed on the DAQ chassis which in turn connected to the computer. The technical specifications of these five modules are provided in Figures F.14 to F.18.

Last Revised: 2014-11-06 07:14:23.0

NI 9208

16-Channel Current Input Module

- 16 current inputs (± 21.5 mA)
- High-resolution mode with 50/60 Hz rejection
- 500 S/s sample rate (high-speed mode)
- Voltage supply pins for external power routing (2 A/30 V maximum)
- -40 to 70 °C
- 60 VDC, CAT I bank isolation (1,000 Vrms withstand)
- Optional NI 9933 connectivity kit shown in image on right
- 24-bit resolution

Figure F.14 Technical specifications of NI 9208 module

Last Revised: 2014-09-24 23:07:09.0

NI 9214

High-Accuracy Thermocouple Module

- Multiple CJC sensors (cold-junction compensation)
- Isothermal terminal block (included) for measurement accuracy up to 0.45 °C
- 250 Vrms channel-to-earth ground safety isolation
- 24-bit ADC for up to 0.02 °C measurement sensitivity
- Autozero channel for offset error compensation
- J, K, T, E, N, B, R, and S types supported

Figure F.15 Technical specifications of NI 9214 module

Last Revised: 2014-11-06 07:14:53.0

NI 9220

± 10 V, Simultaneous Analog Input, 100 kS/s, 16 Ch Module

- 16 differential channels, 100 kS/s per channel sample rate
- ± 10 V measurement range, 16-bit resolution
- ± 30 V overvoltage protection
- 250 Vrms channel-to-earth, CAT II (spring-terminal), or 60 VDC channel-to-earth, CAT I (D-SUB), isolation
- 36-position spring-terminal or 37-pin D-SUB connectors available
- -40 °C to 70 °C operating, 5 g vibration, 50 g shock

Figure F.16 Technical specifications of NI 9220 module

Last Revised: 2014-11-06 07:14:26.0

NI 9411

± 5 to 24 V, Differential Digital Input, 6 Ch Module

- 6-channel, 500 ns digital input
- ± 5 V to 24 V, differential/single-ended digital input
- Compatible with NI CompactDAQ counters
- 60 VDC, CAT I isolation
- Industry-standard 15-pin D-SUB connector
- -40 °C to 70 °C operating, 5 g vibration, 50 g shock

Figure F.17 Technical specifications of NI 9411 module

NI 9237

± 25 mV/V, Bridge Analog Input, 50 kS/s/ch, 4 Ch Module

- Built-in full- and half-bridge measurements
- 120 and 350 Ω quarter-bridge completion via NI 9944/45 accessory kits
- 24-bit resolution on four simultaneous inputs sampled at up to 50 kS/s/ch
- Compatible with TEDS sensors
- Up to 10 V programmable excitation
- Connection for external excitation supply for specific levels
- 1,000 Vrms transient isolation for safety
- RJ50 (10P10C) or D-Sub connector options

Figure F.18 Technical specifications of NI 9237 module

Appendix G: Specifications of Lubrication Oil

G.1. Technical Data Sheet of the Lubrication Oil

Technical Data Sheet

Previous Name: Shell Clavus Oil R

Shell Refrigeration Oil S4 FR-F 68

*Reliable Performance
For use with HFC Refrigerants*

Advanced Synthetic Refrigerator Compressor Lubricant

Shell Refrigeration Oil S4 FR-F is a synthetic refrigeration lubricant with a polyol ester base fluid. It has been developed for use with R134a and other HFC refrigerants.

DESIGNED TO MEET CHALLENGES

Performance, Features & Benefits

- Extended maintenance intervals**
Shell Refrigeration Oil S4 FR-F has excellent high temperature and oxidation stability providing long service life even where high compressor discharge temperatures are found.
- Wear protection**
Shell Refrigeration Oil S4 FR-F has been designed to minimise the wear in bearings and pistons – the most critical parts of a reciprocating compressor.

Main Applications

- Refrigerator Compressors**
Shell Refrigeration Oil S4 FR-F is recommended for use in open, semi-open and hermetic compressors in domestic, commercial and industrial refrigeration systems. It can be used in both rotary and reciprocating compressor types.
- Refrigerant Compatibility**
Shell Refrigeration Oil S4 FR-F is recommended for use with R134a and other types of HFC refrigerant.
- Seal Compability**
Shell Refrigeration Oil S4 FR-F is compatible with all sealing materials commonly used with HFC refrigerant systems.

Specifications, Approvals & Recommendations

- Shell Refrigeration Oil S4 FR-F meets the requirements of DIN 51503 KD.

For a full listing of equipment approvals and recommendations, please consult your local Shell Technical Helpdesk.

Typical Physical Characteristics

Properties	Method	Refrigeration Oil S4 FR-F 68
ISO Viscosity Grade	ISO 3448	68
Refrigerator Oil Group	DIN 51503	KD
Kinematic Viscosity @40°C mm ² /s	ISO 3104	66
Kinematic Viscosity @100°C mm ² /s	ISO 3104	8.8
Density @15°C kg/m ³	ISO 12185	991
Flash Point (COC) °C	ISO 2592	>230
Pour Point °C	ISO 3016	-42
Neutralisation Number mg KOH/g	ASTM D664 (TAN)	<0.06
Characteristics when used with R134a - Flocc-point °C	DIN 51351	<-30
Characteristics when used with R134a - Refrigerant Stability @250°C	DIN 51393	>96
Miscibility		Miscible over the whole range of typical refrigeration temperatures

These characteristics are typical of current production. Whilst future production will conform to Shell's specification, variations in these characteristics may occur.

Figure G.1 Technical data sheet of the lubrication oil

G.2. Viscosity – Temperature relation of the Lubrication Oil

The viscosity – temperature chart is provided by the supplier, who also ensured that the relation is also suitable to be used for the lubrication oil supplied.

Figure G.2 Viscosity – temperature diagram of the lubrication oil

Appendix H: Calibration Data and Certificates of Various Measuring Devices

H.1. Calibration Data of Type-T Thermocouples

Figure H.1 Calibration data for type-T thermocouple (TC-T-2)

Figure H.2 Calibration data for type-T thermocouple (TC-T-3)

Figure H.3 Calibration data for type-T thermocouple (TC-T-4)

Figure H.4 Calibration data for type-T thermocouple (TC-T-5)

Figure H.5 Calibration data for type-T thermocouple (TC-T-6)

Figure H.6 Calibration data for type-T thermocouple (TC-T-7)

Figure H.7 Calibration data for type-T thermocouple (TC-T-8)

H.2. Calibration Data for WIKA Pressure Transmitters

Figure H.8 Pressure-current relationship for a WIKA pressure transmitter with the serial number: 1105TGHB

Figure H.9 Pressure-current relationship for a WIKA pressure transmitter with the serial number: 1105TGH9

Figure H.10 Pressure-current relationship for a WIKA pressure transmitter with the serial number: 1105TGHE

Figure H.11 Pressure-current relationship for a WIKA pressure transmitter with the serial number: 1105TGHG

Figure H.12 Pressure-current relationship for a WIKA pressure transmitter with the serial number: 1105TGHC

Figure H.13 Pressure-current relationship for a WIKA pressure transmitter with the serial number: 1105TGHA

Figure H.14 Pressure-current relationship for a WIKA pressure transmitter with the serial number: 1105TGHF

Figure H.15 Pressure-current relationship for a WIKA pressure transmitter with the serial number: 1105TGHD

Figure H.16 Pressure-current relationship for a WIKA pressure transmitter with the serial number: 1105TGH8

H.3. Calibration data of Endevco piezoresistive pressure transducers

Figure H.17 Pressure-current relationship for an Endevco piezoresistive pressure transducer with the serial number: 29407

Figure H.18 Pressure-current relationship for an Endevco piezoresistive pressure transducer with the serial number: 29410

Figure H.19 Pressure-current relationship for an Endevco piezoresistive pressure transducer with the serial number: 29412

Figure H.20 Pressure-current relationship for an Endevco piezoresistive pressure transducer with the serial number: 29408

H.4. Calibration certificate of Futek TRS605 torque sensor (S/N: 562435)

FUTEK ADVANCED SENSOR TECHNOLOGY, INC.	10 Thomas, Irvine, CA 92618 USA Tel: (949) 465-0900 Fax: (949) 465-0905 Toll Free: (800) 23 FUTEK
--	--

Certificate of Calibration

Certificate Number: **1403170022**

Sensor Info:	
S/N: 562435	Model: TRS605
Item #: FSH02057	Capacity: 50 N-m
Description: TRS605, 50 N-m, Non-Contact Shaft to Shaft Rotary Torque Sensor w/ Encoder, Material - 17-4 PH S.S., 16mm Shaft w/ 5mm Feather Keyway, 12 Pin Binder Receptacle, 09-0331-90-12	

Calibration Procedure OP1000

CALIBRATION EQUIPMENT USED

Digital Multimeter:
HP Model: Agilent 34401A, S/N: US36134723

Dead Weight(s):
1-50 lb, Traceability No: 679504
5 lb, Traceability No: 679505

Torque Equipment:
Model: 2500-152-10 in, S/N: 041, Traceability No: M71302-5

This certifies that the following sensor has been calibrated using equipment traceable to NIST. Supporting documentation relative to traceability is on file and is available for examination upon request. This certificate shall not be reproduced except in full, without the written approval of FUTEK.

Calibration Technician: Fernando Aguilar	Re-Calibration Date: One Year After Issue Date
Issue Date: 3/17/2014	

Page 1 of 3

Sensor Solution Source
Load - Torque - Pressure - Multi Axis - Calibration - Instruments - Software
www.futek.com

Figure H.21 Calibration certificate of Futek TRS605 torque sensor (S/N: 562435): page 1/3

Certificate Number: **1403170022**

Single Channel Item

CALIBRATION DATA

Test Temp: 76 °F (24 °C)

Relative Humidity: 46 %

Excitation: 19.98 Vdc

TorqueCW

Load (N-m)	Output (Vdc)	Non-Lin. Error (% R.O.)
0.0	0.000	0.000
11.3	1.128	-0.022
22.6	2.256	-0.044
33.9	3.386	-0.026
45.2	4.514	-0.048
50.0	4.996	0.000
0.0	0.007

TorqueCCW

Load (N-m)	Output (Vdc)	Non-Lin. Error (% R.O.)
0.0	0.000	0.000
11.3	-1.128	-0.008
22.6	-2.257	0.003
33.9	-3.386	0.015
45.2	-4.515	0.027
50.0	-4.993	0.000
0.0	0.005

Page 2 of 3

Figure H.22 Calibration certificate of Futek TRS605 torque sensor (S/N: 562435): page 2/3

Certificate Number: **1403170022**

Single Channel Item

SHUNT CALIBRATION

Direction	Shunt Value (K Ω)	Shunt Connection	Output Value (Vdc)	Equivalent Load (N-m)
TorqueCW	Internal	Pin A to K	4.999	50.0
TorqueCCW	Internal	Pin A to K	4.999	50.1

Page 3 of 3

Figure H.23 Calibration certificate of Futek TRS605 torque sensor (S/N: 562435): page 3/3

H.5. Calibration certificate of Tricor TCM 325 Coriolis flow meter (S/N: 01084224)

www.kem-kueppers.com
info@kem-kueppers.com

 KÜPPERS ELEKTROMECHANIK GMBH

Kalibrierprotokoll / Calibration Record

Kunde / Customer
Auftragsnr. / Order no.
Datum / Date
Prüfer / Examiner
Gehäusenr. / Case no.
Gebertyp / Typ of Flowmeter
Seriennr. / Serial no.

Eastern Energy Services Pte Ltd
2240370
24.02.2014
Jan Speich
9815
TCM 0325-FK-SGSS-FSDS
01084224

Hiermit bestätigen wir, daß die Werkskalibrierstände einer Prüfmittelüberwachung gem. DIN EN ISO 9001 unterliegen.
Die Überwachung erfolgt durch regelmäßige Vergleichsmessungen mit DKD-kalibrierten Normalen. Durch dieses Verfahren ist eine direkte Rückführbarkeit der Meßergebnisse auf nationale Normale gewährleistet.

Dieses Protokoll wurde maschinell erstellt und ist ohne Stempel und Unterschrift gültig.

*We hereby confirm that our calibration rigs are subjected to control of inspection, measuring and test equipment in line with DIN EN ISO 9001.
The inspection, which is carried out at regular intervals, consists of comparative measurements, i. e. calibrations results of our rigs are compared with calibration references of our DKD laboratory approved of by the PTB as per DIN EN 45001 (The PTB is the Federal Institute of Physics and Metrology). The inspection ensures that calibration results have a valid relationship to nationally recognized standards.
This record is machine-made and valid without stamp and signature.*

Meßbereich von Q_{max}
Measuring range Q_{max}
Meßbereich von Q_{min}
Measuring range Q_{min}
KalibriemEDIUM
Calibration medium

300 kg/h

3 kg/h

Wasser / Water

Nr. No.	m _{ref} kg/h	m kg/h	Fehler / Error %	V (Rohr/Tube) m/s	Analogsignal mA
1	270,17	270,01	-0,06	6,00	16,40
2	210,38	210,33	-0,02	4,67	15,22
3	150,11	150,12	0,00	3,33	12,01
4	90,22	90,22	0,00	2,00	8,81
5	30,21	30,20	-0,05	0,67	5,61

Genauigkeitskurve / Accuracy - Curve

KEM Küppers Elektromechanik GmbH | Liebigstraße 5 | D-85757 Karlsfeld | tel +49 8131 5 93 91 - 0 | fax +49 8131 9 26 04

zertifiziert gemäß DIN EN ISO 9001 certified

Figure H.24 Calibration certificate of Tricor TCM 325 Coriolis flow meter (S/N: 01084224):

Einstellblatt

24.02.2014

Auftrags.-Nr.:	2240370
Typ	TCM 0325-FK-SGSS-FSDS
Ser.-Nr.: (Typenschild)	01084224
Gehäuse-Nr.:	9815
Software Main/Display:	V2C56
Hardware Rev.:	1.1
Ex - Board	nein
Interface	RS 485

Mtr Variable	3,150129	g/s ²
PWM-1 4 mA	-	counts
PWM-1 20 mA	-	counts
PWM-2 4 mA	56714	counts
PWM-2 20 mA	20490	counts
Switch Volt	-	mV
Sensor	79 / 81	mV / mV
Frequenz / Massestrom	1000 / 5	Hz / kg/min

Calibrate AIR	Freq. in Hz	153,1096	Temp. in °C	25,91	Dichte in kg/m ³	1,18
Calibrate WATER	Freq. in Hz	142,6818	Temp. in °C	26,38	Dichte in kg/m ³	996,29

Mpkt.	Massestrom kg/min	NORMAL Impulse	Prüfling Impulse	Zeit sec.
1	4,5	74286	54443	60,489
2	3,5	57767	42352	60,409
3	2,5	41226	30232	60,418
4	1,5	24775	18168	60,412
5	0,5	8299	6083	60,428
6				
7				
8				

Figure H.25 Calibration certificate of Tricor TCM 325 Coriolis flow meter (S/N: 01084224):

Kalibrierprotokoll / Calibration Record

Kunde / Customer
Auftragsnr. / Order no.
Datum / Date
Prüfer / Examiner
Gehäusenr. / Case no.
Gebertyp / Typ of Flowmeter
Seriennr. / Serial no.

Eastern Energy Services Pte Ltd
2240370
24.02.2014
Jan Speich
9816
TCM 0325-FK-SGSS-FSDS
01085224

Meßbereich von Q_{max}
Measuring range Q_{max}
Meßbereich von Q_{min}
Measuring range Q_{min}
Kalibriermedium
Calibration medium

300 kg/h
3 kg/h
Wasser / Water

Hiermit bestätigen wir, daß die Werkkalibrierstände einer Prüfmittelüberwachung gem. DIN EN ISO 9001 unterliegen.
Die Überwachung erfolgt durch regelmäßige Vergleichsmessungen mit DKD-kalibrierten Normale.
Durch dieses Verfahren ist eine direkte Rückführbarkeit der Meßergebnisse auf nationale Normale gewährleistet.
Dieses Protokoll wurde maschinell erstellt und ist ohne Stempel und Unterschrift gültig.
We hereby confirm that our calibration rigs are subjected to control of inspection, measuring and test equipment in line with DIN EN ISO 9001.
The inspection, which is carried out at regular intervals, consists of comparative measurements, i. e. calibrations, results of our rigs are compared with calibration references of our DKD laboratory approved of by the PTB as per DIN EN 45001 (The PTB is the Federal Institute of Physics and Metrology). The inspection ensures that calibration results have a valid relationship to nationally recognized standards.
This record is machine-made and valid without stamp and signature.

Nr. No.	m _{ref} kg/h	m kg/h	Fehler / Error %	V (Rohr/Tube) m/s	Analogsignal mA
1	270,10	269,92	-0,07	5,99	18,40
2	210,27	210,19	-0,04	4,67	15,21
3	150,14	150,12	-0,01	3,33	12,01
4	90,16	90,16	-0,01	2,00	8,81
5	30,23	30,22	-0,02	0,67	5,61

Figure H.26 Calibration certificate of Tricor TCM 325 Coriolis flow meter (S/N: 01085224):

Einstellblatt

24.02.2014

Auftrags.-Nr.:	2240370
Typ	TCM 0325-FK-SGSS-FSDS
Ser.-Nr.: (Typenschild)	01085224
Gehäuse-Nr.:	9816
Software Main/Display:	V2C56
Hardware Rev.:	1.1
Ex - Board	nein
Interface	RS 485

Mtr Variable	3,181012	g/s^2
PWM-1 4 mA	-	counts
PWM-1 20 mA	-	counts
PWM-2 4 mA	56600	counts
PWM-2 20 mA	29172	counts
Switch Volt	-	mV
Sensor	80 / 80	mV / mV
Frequenz / Massestrom	1000 / 5	Hz / kg/min

Calibrate AIR	Freq. in Hz	153,2324	Temp. in °C	25,93	Dichte in kg/m³	1,18
Calibrate WATER	Freq. in Hz	142,7709	Temp. in °C	22,07	Dichte in kg/m³	996,08

Mpkt.	Massestrom kg/min	NORMAL Impulse	Prüfling Impulse	Zeit sec.
1	4,5	74162	54347	60,404
2	3,5	57815	42381	60,489
3	2,5	41309	30289	60,527
4	1,5	24794	18181	60,496
5	0,5	8309	6092	60,463
6				
7				
8				

Figure H.27 Calibration certificate of Tricor TCM 325 Coriolis flow meter (S/N: 01085224):