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CRYSTALS**

ZHAOJU YANG

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Zhaoju Yang

Division of Physics and Applied Physics
School of Physical and Mathematical Sciences

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For my parents.

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Co-authors:

8. Fei Gao, Z. Gao, X. Shi, **Z. Yang**, X. Lin, H. Xu, J. D. Joannopoulos, M. Soljacic,

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Abstract

This thesis studies two classes of unconventional acoustic crystals.

The first class of the acoustic crystal is a two-dimensional crystal with topologically gapped band structure. Circulating flow is introduced into each unit cell to play the role of vector potential. We present a theoretical model to characterize the underlying physics – quantum Hall effect for acoustics. Through numerical calculation, we show that the nontrivial band gap emerges and the band below the gap acquires a non-zero Chern number. As a result, the non-reciprocal acoustic crystal exhibits a topologically protected one-way edge state inside the band gap.

The second class of the acoustic crystal is a three-dimensional and gapless crystal. The isolated degenerate points, which are known as type-II Weyl nodes in three-dimensional momentum space, indicate the existence of topological transition and acquire non-zero Chern number. The Weyl nodes are rather robust against perturbations and annihilate only in pairs of opposite chirality. In addition, the topological Fermi-arc-like surface states can be traced out as an analogue of Fermi arcs as in condensed matter physics. Last but not least, we demonstrate the unique features of the acoustic type-II Weyl system, such as a finite density of states, transport properties of the surface states.

Chapter 1

Introduction

The Nobel Prize in Physics 2016 was divided, one half awarded to David J. Thouless, the other half jointly to F. Duncan M. Haldane and J. Michael Kosterlitz "for theoretical discoveries of topological phase transitions and topological phases of matter".

Their theoretical discoveries illustrate in a very nice way the interplay between physics and mathematics. Here we will discuss the background of the topological phases – quantum Hall effect and Weyl physics in condensed matter physics and photonics.

1.1 Quantum Hall effect and Chern number

The study of topological properties of band structures began with the discovery of quantum Hall effect in 1980s [1]. The Hall conductance of the two-dimensional (2D) electron gas in a magnetic field is an integer multiple of a constant e^2/h , which leads to a fundamental question – what is the reason of the quantization of Hall conductance independent of sample geometry. In the pioneering work [2], D. J. Thouless, 2016 Nobel laureate in physics, and three other collaborators found the topological origin of Hall conductance. The expression of Hall conductance in the unit e^2/h is a topological invariant called Chern number or TKNN number, which is expressed as a winding

number of the Berry phase of electron wave functions around the Brillouin zone and always an integer. This integer number characterizes the topological properties of the wave functions in the band. In other words, within the scope of band theory, the insulator with a band gap can be either an ordinary band insulator or a quantum Hall state. Once one physical observable can be written as a topological invariant, it changes only discretely. Therefore, it will not respond to continuous perturbations. This explains the quantized values of Hall conductance.

Traditional phases of matter are classified by their different symmetries. However, this classification method cannot be applied to the quantum Hall states. The pioneering work of Thouless et al. offered us a new way to classify different phases of quantum matter according to their topological order.

The non-zero Chern numbers are associated with various intriguing physical phenomena. The most interesting property is the chiral edge states in quantum Hall effect, or one-way edge modes in photonic crystals – waves travel in a single direction along the edge without back-scattering, regardless of the existence of small perturbations. Now I will present a review of the relevant mathematical background.

To learn the topological invariant, we start from introducing the band structure and Bloch theorem. Generally, every crystal is an infinite and periodic structure that can be characterized by a Bravais lattice, and for each Bravais lattice we can derive the reciprocal lattice, which encapsulates the periodicity in three reciprocal vectors. The periodic potential which shares the same periodicity as the lattice can be expanded out as a Fourier series whose non-vanishing components are associated with the reciprocal

lattice vectors. From this theory, we can predict the band structure of a particular material. For electrons in a perfect crystal, there is a basis of wavefunctions with the properties: Each of these wavefunctions corresponds to an energy eigenstate. Each of these wavefunctions is a Bloch wave, meaning that this wavefunction can be written in the form: $\psi(r) = e^{ikr}u(r)$, where u has the same periodicity as the atomic structure of the crystal. This Bloch's theorem underlies the concept of band structures.

Now let us consider an insulator with N occupied bands defined by a crystal Hamiltonian $\mathcal{H}$. The Bloch wave functions can be written as $\psi_{n,k}(r) = e^{ikr}u_{n,k}(r)$, where k is the crystal momentum and n is the band index. Here $u_{n,k}(r)$ obeys the orthonormality condition and is cell-periodic eigen-function of the Bloch Hamiltonian H_k , which satisfies the relationship $H_k u_{n,k} = E_{n,k} u_{n,k}$. Through Fourier transform, we can derive the Bloch Hamiltonian in the k -momentum space from the original Hamiltonian. The eigenvectors of the Bloch Hamiltonian give the Bloch waves.

For each band n , the Chern number [3, 4] is defined as

$$C_n = \frac{1}{2\pi} \oint \oint ds \cdot \mathcal{F}(k) \quad (1.1)$$

where $\mathcal{F} = \nabla_k \times \mathcal{A}$ is Berry curvature and $\mathcal{A} = i\langle u_{n,k} | \nabla_k | u_{n,k} \rangle$ is Berry connection. The Eq. (1.1) is an area integral carried out in both momentum k space over the first Brillouin zone and real space over the unit cell. Berry connection measures the local change in the phase of wave-functions in momentum space. Similar to the vector potential and Aharonov–Bohm phase, Berry connection and Berry phase $\oint \mathcal{A} \cdot dl$ are gauge dependent, which means $u_{n,k}(r) \rightarrow e^{i\phi} u_{n,k}(r)$, whereas the Berry curvature and

Chern number are gauge-invariant. The Berry phase is defined only up to multiples of 2π . The inner product is performed in real space. The phase and flux can be connected through Stokes' theorem. The integral on a surface of the Berry curvature $\nabla_k \times \mathcal{A}$, which is also known as Chern density, is Berry flux. If the surface is a closed manifold, the boundary integral vanishes. However the indeterminacy of the boundary term modulo of 2π manifests itself in the Chern theorem, which states that the integral of the Berry curvature over a closed manifold is quantized in units of 2π . This number is the Chern number introduced above, and is essential for understanding various quantization effects. The one-dimensional (1D) Berry phase is also known as the Zak phase [5].

Figure 1.1. The first 2D Brillouin zone (a) with periodic boundary conditions is topologically equivalent to a torus (b). The Chern number can be viewed as the number of monopoles of Berry flux inside a closed 2D Brillouin zone. The blue arrows indicate Berry curvature from a positive charge.

Finally, we conclude here that in 2D systems, the Chern number becomes non-zero by breaking time-reversal (T) symmetry and preserving parity (P) symmetry [6]. This

2D quantum Hall topological phase with broken T symmetry in photonics will be the focus at the next section. The Chern number is the integral of the Berry curvature over the 2D first Brillouin zone with periodic boundary conditions. A torus is created if we connect the two pair of periodic boundaries, which indicates the first Brillouin zone is topologically equivalent to a torus as shown in Fig. 1.1. The Berry curvature is a pseudo-vector field, which is odd under T but even under P . If we take Dirac cone, which is a doubly degenerate point with linear dispersion, for elaboration, in presence of both P and T , $\mathcal{F}(k) = 0$. When either P or T symmetry is broken, the Dirac cones open and each degeneracy-lifting term contributes a Berry flux of π to each of the bulk bands. In the presence of T (P broken), $\mathcal{F}(k) = -\mathcal{F}(-k)$. The integration over the closed 2D Brillouin zone is thus zero, which means the Chern number is zero. Whereas in the presence of P (T broken), $\mathcal{F}(k) = \mathcal{F}(-k)$. The total Berry flux will become 2π and the Chern number equals one. Non-zero Chern number measures the number of monopoles (topological charges) contained within the torus as schematically shown in Fig. 1.1. In three-dimensional (3D) Brillouin zone, these charges are known as Weyl points [7].

Last but not the least, in a finite 2D system, the non-zero Chern number gives basically the relationship between the total number of edge states and the topological properties of all the bulk bands below the gap. For example, if the Chern number is 1 for a band gap, there will be one unidirectional edge state spanning across the gap. This is so called bulk-edge correspondence [3, 4].

1.2 Photonic quantum Hall effect

F. D. M. Haldane, 2016 Nobel laureate of physics, and S. Raghu proposed an analog of integer quantum Hall effect in photonic crystals in 2005 [8]. In a remarkable new direction, they predicted the existence of one-way electromagnetic modes similar to the chiral edge states. These edge modes are confined at the edge of the 2D magneto-optical photonic crystals. They acquire group velocities pointing in one direction, which is determined by the direction of the applied dc magnetic field. Due to the lack of the back-propagating modes, back-scattering is totally suppressed. This remarkable property is potentially important for creating a range of new opportunities throughout the photonic community.

Figure 1.2 shows the topological phase diagram of the 2D quantum Hall effect [6]. The top-left panel shows a projected band structure of edge modes, in which the bulk dispersions form a pair of Dirac points (shaded grey) protected by both P and T symmetry. The green and blue lines represent the flat edge dispersions similar to graphene on the opposite (top and bottom) edges. When either P or T is broken, a bandgap can be generated in the bulk but not necessarily on the edges. When T symmetry breaking is dominant, the two bulk bands split and acquire Chern numbers of ± 1 . Thus, there is one single gapless edge state localized on each of the top and bottom edges, assuming the bulk is interfaced with topologically trivial insulators. This T -breaking phase of non-zero Chern numbers is the 2D quantum Hall phase, plotted in red

in the right panel of the phase diagram.

Z. Wang et al. [9] were the first to realize the photonic analogue of quantum Hall effect at the microwave frequency range in 2009. They implemented T symmetry breaking by applying a uniform magnetic field on a gyromagnetic photonic crystal [10]–2D square-lattice yttrium-iron-garnet photonic crystal, as shown in Fig. 1.3(a). With no magnetic field, the second and third bands touch at a quadratic degenerate point. Whereas under a uniform magnetic field, anti-symmetric imaginary off-diagonal terms emerge in the magnetic permeability tensor. The quadratic degenerate point splits and a band gap is opened between second and third bands acquiring non-zero Chern numbers. The band structure of the edge mode is shown in Fig. 1.3(b). The red line between the second and third bands indicates the gapless edge state. The group velocities are positive, which means the electromagnetic waves of the edge state propagate only along one direction and are immune to back-scattering from perturbations. Figure 1.3(c) demonstrates the TM field (E_z) of the propagation of one-way mode and its topological protection against a metallic scatter. In Fig. 1.3(d), the experimental transmission shows the backwards reflection is largely suppressed.

There are several other proposals [6] of quantum Hall phase of photons in coupled resonators exhibiting an effective magnetic field as shown in Fig. 1.4. Panel (a) shows the quantum Hall effect of electrons undergoing localized cyclotron motions under a magnetic field. These proposals show that unlike electrons, photons do not carry an electric charge and therefore nearly do not interact with magnetic fields, but they can acquire phase changes in an effective way.

Firstly, Fig. 1.4(b) presents a proposal [11, 12] from Hafezi et al. The authors introduce an effective magnetic field in a 2D lattice of optical resonators by tuning the propagating and coupling phases. Each optical resonator has two modes that propagate clockwise and counter-clockwise, which are similar to the spin-up and spin-down for electrons. They are time-reversed pairs. The lengths of the coupling waveguides are carefully constructed so that the total coupling phase between resonators contributes to an effective magnetic field. For each spin, photons of opposite circulations experience opposite effective magnetic field.

Secondly, Fang et al. [13] proposes to use time-domain modulations of the coupling between two nearest-neighbor resonators. The two resonators acquiring different resonance frequencies can couple only through the time-harmonic modulation between them. The vertical coupling phases are zero and the horizontal coupling phases increase along y , therefore producing effective magnetic field. The schematic is presented in Fig. 1.4(c). As a result, the increasing of horizontal coupling phases produces an effective Aharonov-Bohm phase from a uniform magnetic field.

Thirdly, Rechtsman et al. [14] demonstrate experimentally the photonic analogue of quantum Hall effect by using optical photons in a lattice of helical waveguides as shown in Fig. 1.4(d). The paraxial approximation of Maxwell's equations results in an effective Schrodinger's equation evolving in time (z direction plays the role of time). The periodic helical modulations in z break the z -symmetry, which is equivalent to the time-domain modulation that break T symmetry. This symmetry breaking opens up band crossing points in the Floquet band structure, therefore generating a topologically

protected gapless edge modes inside a non-trivial band gap. This design of photonic crystal is tolerant of fabrication imperfections, such as variations in the lattice constant, which can enable implementation of robust waveguides. Also, photonic chiral edge states might prove useful in applications involving isolators or slow light.

Figure 1.2. Topological phase diagram of 2D quantum Hall effect. When either *parity* (P) or *time-reversal* (T) symmetry is broken, the Dirac cones, which is a doubly degenerate point with linear dispersion, open. However, the system becomes topologically nontrivial and acquires one-way edge modes in a finite 2D sample (as can be seen in upper-right and right-most panel) only if T symmetry is broken. Through P breaking, the edge states within the band gap is not topologically protected (as can be seen in lower-left panel of band structure and left-most panel). This figure was reproduced from [6].

Figure 1.3. (a) Experimental set-up. (b) Band structure of the edge modes. (c) Simulation field of the one-way edge mode and its topological protection against a scatter. (d) First measured topologically protected transmission of the edge modes at microwave frequency. This figure was reproduced from [6, 9].

Figure 1.4. Topologically protected one-way edge modes. (a) Quantum Hall phase of electrons in a magnetic field. (b) A 2D square lattice of photonic whispering-gallery resonators coupled through inter-waveguides. (c) A 2D lattice of photonic resonators coupled through time-domain modulations. (d) A 2D lattice of helical photonic waveguides breaking z symmetry. This figure was reproduced from [6].

1.3 Type-I and type-II Weyl semimetals

Weyl semimetals [15] that host isolated Weyl points in 3D momentum space have been discovered in the material TaAs [16, 17] and a double-gyroid photonic crystal [18], as a new topological phase of matter beyond topological insulators. The novel Weyl materials exhibit unusual physical properties such as open Fermi arcs [15] and chiral anomaly [19]. The new Weyl physics has drawn immediate attention in condensed matter physics as well as in photonics.

The dispersion of Weyl points is governed by the Weyl Hamiltonian [7]

$$H(k) = \sum_{i,j=x,y,z} k_i v_{ij} \sigma_j, \quad (1.2)$$

where v_{ij} and σ_j are group velocity and Pauli matrix, respectively. The existence of Weyl points is possible only if either P or T is broken and stable to weak perturbations [6, 20]. When a Weyl point is present in the 3D momentum space, it can be viewed as a topological charge – either a source or a sink of Berry curvature. The Fermi surface enclosing a Weyl point has a well-defined Chern number, which indicates the topological charge of this Weyl point. Due to the fact that the net charge must vanish in the Brillouin zone, Weyl points come up in pairs. They are stable and annihilated only in pairs of opposite chirality.

However, Soluyanov et al. recently proposed the existence of a previously missed type of Weyl fermion – Lorentz-violating type-II Weyl fermion [21], that emerges at the boundary between electron and hole pockets in a new phase of matter. It was overlooked

by Weyl because it breaks stringent Lorentz symmetry in high-energy physics. Because Lorentz invariance does not need to be respected in condensed matter physics, Soluyanov et al. found the new type of Weyl fermion by generalizing the Dirac equation. Different from a type-I Weyl point with a point-like Fermi surface proposed by Weyl, a type-II Weyl point satisfies the Hamiltonian [21]

$$H(k) = \sum_{\substack{i=x,y,z \\ j=0,x,y,z}} k_i v_{ij} \sigma_j, \quad (1.3)$$

where v_{ij} and σ_j are group velocity and Pauli matrix. It appears at the contact of electron and hole pockets in type-II Weyl semimetals. The σ_0 term tilts the cone-like spectrum, breaking Lorentz invariance of Weyl fermions in quantum field theory if the strength of the term is large enough along at least one direction. Generally speaking, the energy spectrum is $\varepsilon_{\pm}(k) = \sum_{i=x,y,z} k_i A_{i0} \pm \sqrt{\sum_{j=x,y,z} (\sum_{i=x,y,z} k_i A_{ij})^2} = T(k) \pm U(k)$ from Eqn. (1.3). The condition for a Weyl point to be of type II is that there exists a direction k , for which $T(k) > U(k)$. The comparison between type-I and type-II Weyl points are demonstrated in Fig. 1.5. The theoretical derivations of the Berry curvature associated with the Weyl points give rise to $\Omega_{i,j}(k) = \frac{1}{2|d|^3} \mathbf{d} \cdot \partial_i \mathbf{d} \times \partial_j \mathbf{d}$, where vector $\mathbf{d}$ is the coefficients of Pauli matrix.

A large number of unusual physical phenomena are associated with Weyl topological semimetals, including open Fermi arcs of the surface states, and various magnetotransport anomalies. Since it is practical to map the open arcs (difficult for other anomaly features) to bosonic systems in photonics and acoustics, here we mainly introduce the Fermi arcs [15]. Consider a curve in the surface Brillouin zone enclosing

the projection of the Weyl point, which is traversed anti-clockwise as varying the parameter $\lambda: 0 - 2\pi$ (k_λ), as shown in Fig. 1.6(a). The k_λ and k_z define the 2D surface Brillouin zone, which is topologically equivalent to a torus. Therefore the Chern number of the 2D Brillouin zone simply corresponds to the net monopole enclosed within the torus. Consider a single enclosed Weyl point, the 2D system defined in the 2D surface Brillouin zone can be viewed as a quantum Hall state with Chern number 1. For a finite 2D subsystem with a boundary, a chiral edge state is expected for the sub-2D system, as presented in Fig. 1.6(b). Consequently, each surface state crosses the zero energy (Fermi surface for simplicity) somewhere on the 2D surface Brillouin zone. Thus, at the zero energy, there is a Fermi line terminates at the Weyl points in the 2D surface Brillouin zone as demonstrated in Fig. 1.6(c). We also note that an arc starting at a Weyl point of one chirality must terminate at a Weyl point of the opposite chirality. This open arc is later well known as the “Fermi arc”.

Figure 1.5. Possible types of Weyl semimetals. (a) Type-I Weyl point with a point-like Fermi surface. (b) Type-II Weyl point appears at the contact of electron and hole pockets. The grey plane corresponds to the position of the Fermi surface. The blue and red lines

mark the edges of the hole and electron pockets. This figure was reproduced from [21].

Figure 1.6. (a) The bulk states as a function of (k_x, k_y) (and fixed k_z) exhibit a cone-like spectrum. A cylinder (red surface) which defines a 2D Brillouin zone is drawn. (b) The dispersion relationship of a chiral surface state in a 2D subsystem. (c) The intersections between the surface states and the Fermi level gives a Fermi arc connecting the Weyl points. This figure was reproduced from [15].

1.4 Weyl points in photonic crystals

The first proposal of Weyl photonics has been manifested in a double-gyroid photonic crystal [20] as shown in Fig. 1.7. Panel (a) shows the cubic unit cell of length a . Panel (b) presents the crystal in a primitive unit cell of space group 230. Consisting of triple junctions in a body-centered cubic lattice, the gyroid surface is approximated by iso-surfaces [20, 22] of

$$g(r) = \sin(2\pi x/a) \cos(2\pi y/a) + \sin(2\pi y/a) \cos(2\pi z/a) + \sin(2\pi z/a) \cos(2\pi x/a), \quad (1.4)$$

where a is the lattice constant. The double-gyroid photonic crystal are made of the two separate 3D regions enclosed by two single gyroid surfaces:

$$g(r) > 1.1 \text{ and } g(-r) > 1.1. \quad (1.5)$$

The scheme of P symmetry or T symmetry breaking lifts the threefold degeneracy at the center of the 3D Brillouin zone and leads to the different pairs of Weyl points as shown in Fig. 1.8, which are annihilated only in pairs of opposite charge. Band structures of single-gyroid and double gyroid photonic crystal are shown in Fig. 1.8(a).

In absence of P symmetry, the T symmetry is preserved and maps a Weyl point at momentum k to its inversion point $-k$ with the same chirality, there exist two other Weyl points of opposite chirality. The P symmetry is broken by putting one air sphere on one of the gyroids at the middle point of the two neighboring triple junctions shown in Fig.

1.7 (b). As presented in Fig. 1.8(b), under the P symmetry breaking perturbation, there are two pairs of Weyl points appearing along ΓH and ΓN . It should be mentioned that, there is no other state near the Weyl points' frequency.

When T symmetry is absent but P symmetry is preserved. By applying d.c. magnetic field along ΓH (y direction) to the double-gyroid photonic crystal as shown in Fig. 1.7 (a) to break T symmetry, the high-index double-gyroid material is assumed to become gyro-electric and the permittivity tensor is now

$$\varepsilon(|B|) = \begin{pmatrix} \varepsilon_{11}(|B|) & 0 & i\varepsilon_{12}(|B|) \\ 0 & \varepsilon & 0 \\ -i\varepsilon_{12}(|B|) & 0 & \varepsilon_{11}(|B|) \end{pmatrix}, \quad (1.6)$$

where $\varepsilon = 16$, and $\det(\varepsilon(|B|)) = (\varepsilon_{11}^2(|B|) - \varepsilon_{12}^2(|B|))\varepsilon = \varepsilon^3$. Note that ε_{12} is a non-zero real number when the magnetization is present. The dimensionless effective magnetic field intensity is defined as $|B| = \varepsilon_{12}/\varepsilon$. As presented in Fig. 1.8(c), under the T symmetry breaking perturbation, there is only a single pair of Weyl points appearing along the magnetization direction. The reason is that P symmetry maps a Weyl point at momentum k to $-k$ with opposite chirality.

As a result of non-zero Chern number, there are topologically protected gapless chiral surface state inside the band gap away from the Weyl points. Figure 1.9 shows the surface dispersion under P breaking perturbation along a line cut in the 2D surface Brillouin zone with non-zero Chern number. The red line indicates the existence of one-way chiral surface states. The lower panel of Fig. 1.9 demonstrates the electric field intensity of the surface state. We can easily see that the electric fields are mostly confined at the surface between topologically trivial (single gyroid) and non-trivial

(double gyroid) photonic crystals.

Two years after the theoretical proposal in 2013, Lu et al. experimentally observed the Weyl points [18] in a double-gyroid photonic crystal with P -symmetry-breaking perturbation in the micro-wave frequency range. The experimental sample is shown in Fig. 1.10(a). The measured transmission data in Fig. 1.10(b) verifies the existence of Weyl points predicated from theoretical proposal. Recently the robust surface states in a metallic-hexagonal photonic crystal were experimentally observed [23]. Besides the previous experimental observations, Noh et al. [24] has reported the measurement of the type-II Weyl points in a lattice of helical waveguides in optical frequency range.

The rich physics of Weyl points has drawn intense interests in Weyl photonic systems [18, 20, 23-29]. The Weyl points of photonics share almost the same topological properties as in condensed matter physics. At the Weyl-point frequencies, photonic Weyl materials provide angular selectivity for filtering light from any 3D incident angle. The unique density of states at the Weyl point can potentially enable devices such as high-power single-mode lasers.

Figure 1.7. The introduction of the double-gyroid photonic crystals (a) Cubic unit cell of length a . (b) Real-space geometry in body-centered cubic unit cell of space group 230. Fig. (b) was reproduced from [20].

Figure 1.8. The existence of Weyl points in double-gyroid photonic crystals with P -breaking and T -breaking perturbations, respectively. (a) Band structures of single-gyroid and double gyroid photonic crystal. There are triply degenerate points with quadratic dispersion relationship. (b) The band structure of double-gyroid photonic crystal under P -breaking perturbation. (c) The band structure under T symmetry breaking with magnetization along ΓH . The right insets show the distribution of different paired Weyl points in the Brillouin zone. This figure was reproduced from [20].

Figure 1.9. The surface dispersion under P symmetry breaking only along a line cut in a 2D surface Brillouin zone with non-zero Chern number and enclosing unpaired Weyl points. The dispersion of the surface state acquires only negative group velocity within the band gap. The bottom panel demonstrates the electric field intensity of the surface state. Clearly we can see that the electric field is localized at the interface between the topologically trivial and non-trivial crystals. This figure was reproduced from [20].

Figure 1.10. (a) Experimental sample of the double-gyroid photonic crystal. (b) Measured transmission data and theoretical band structure. The experimental transmission indicates the existence of the Weyl point around 11.3 GHz, which shows good agreement with the theoretical band structure. This figure was reproduced from [18].

1.5 Motivation and Outlook

Acoustic waves in fluids has many applications in our daily life, including medical imaging, acoustic sonar etc. Acoustic technologies frequently develop using shared concepts with optics such as crystal and meta-media. The sonic crystals and acoustic metamaterial developed in the last two decades may have novel applications in acoustic isolators, super-lens, cloaking etc. It is thus valuable to introduce new concepts like topology into the system to explore the topological manipulations of acoustic waves.

When we started to focus on the topological aspects of the band structures, we were partly attracted by the mathematical beauty. Later on, we were driven by the promising applications such as one-way propagation, fault-tolerant signal processing et al. Following the development of topological phases in condensed matter and photonics, as schematically shown in Fig. 1.11, our general motivation is to bridge the gap between topology and acoustics. In the following, I will briefly introduce the motivation and main result in Part I and II, respectively.

Figure 1.11. The concept of topology and electronics, photonics and acoustics.

First, based on the previous introductions, we can see the concept of topology has been introduced into electronic and photonic systems. Therefore, an open question remains elusive, whether the concept of topology can be implemented in the traditional acoustic systems. In Part I, we will show that for the first time we map the quantum Hall effect into an acoustic crystal by introducing circulating flow.

Second, as the observation of the type-I Weyl points in the condensed matter TaAs and a double-gyroid photonic crystal, Weyl physics has drawn intense attention. However, the newly developed Lorentz-violating type-II Weyl fermions, which are missed in Weyl's original prediction, still stay isolated from bosonic systems such as photonics and acoustics. In Part II, we propose a 3D acoustic crystal hosting type-II Weyl points. A structure is constructed, in a simple way, from stacking 1D building blocks – dimerized chains.

It is valuable to explore the topological manipulations of acoustic waves, given the promising applications of topological states in electronic and photonic systems (such as fault-tolerant quantum computations, optical multiplexing, et al.). For instance, sound waves are guided in a single direction around the surface of a region and ignore imperfections that would scatter the sound in an ordinary material. If it can be realized, such a system may find applications in many acoustic technologies, such as one-way waveguides, soundproofing and sonar stealth systems.

The field of topological insulators is now developing rapidly in condensed matter physics as well as in bosonic systems, such as photonics [6, 27, 30-40], acoustics [41,

42] and mechanics [43].

Part I

Two-dimensional acoustic quantum Hall effect

The manipulation of acoustic waves in fluids has tremendous applications, including those in everyday life. Acoustic technologies have frequently developed in tandem with optics, using shared concepts such as wave-guiding and meta-media. It is thus noteworthy that an entirely novel class of topological edge states in a photonic quantum Hall state, has recently been demonstrated. Haldane (one of the winners of 2016 Nobel laureates in physics) and Raghu predicted that a similar phenomenon can arise in the context of classical electromagnetism, which was subsequently borne out by experiments on microwave-scale magneto-optic photonic crystals and other photonic devices. These are inspired by electronic edge states occurring in topological insulators, and possess a promising property – the ability to propagate in a single direction along an edge without back-scattering, regardless of the defects or disorder.

Here in Part I, we first develop a theoretical model of 2D topological acoustics [44], and propose a scheme for realizing topological edge states in a non-reciprocal acoustic structure containing circulating fluids as shown in Chapter 2. The property of topologically protected one-way acoustic wave propagation is demonstrated in Chapter 3, which does not occur in ordinary acoustic devices and may have novel applications for acoustic isolators, modulators and transducers.

Chapter 2

Non-reciprocal acoustic crystals

2.1 Non-reciprocal acoustic crystals

Acoustic wave in fluid is an oscillatory motion with small amplitude in a compressible fluid [45, 46]. It has no intrinsic spin and does not respond to magnetic fields and its reciprocal transmission is directly associated to the symmetry of physics laws under time reversal, which in other word indicates the lack of unidirectional control.

Figure 2.1. (a) Triangular acoustic lattice with lattice constant a . $a=0.2$ m in the following calculation. (b) The unit cell containing a central metal rod of radius $r_1 = 0.2a$, surrounded by an anticlockwise circulating fluid flow (flow direction indicated by red arrows) in a cylinder region of radius $r_2 = 0.4a$. [44]

In order to realize topological band theory in acoustics, we begin with a spatially periodic medium, and introduce a mechanism that breaks T symmetry. A periodic acoustic medium, sometimes called a ‘phononic crystal’ [47], is commonly realized by engineering a structure whose acoustic properties (elastic moduli and/or mass density) vary periodically on a scale comparable to the acoustic wavelength. As for T symmetry breaking, although traditional acoustic devices lack an efficient mechanism to accomplish this propose, a recent breakthrough [48] has shown that strong T -breaking can be achieved in a ‘meta-atom’ containing a ring of circulating fluid. Although these developments have direct device applications as acoustic diode [49] and acoustic circulator [48], they do not have the topological protection against defects possessed by the topological edge states. We utilize the design concept by incorporating circulating fluid elements into an acoustic crystal structure. As shall be seen later, the resulting acoustic band structure is topologically nontrivial, supports the non-reciprocal propagations of acoustic waves and maps theoretically onto an integer quantum Hall gas – the simplest version of a 2D topological insulator.

The proposed acoustic structure is shown in Fig. 2.1(a). Fig. 2.1(b) shows the unit cell with circulating air flow. It is a triangular lattice of lattice constant a , where each unit cell consists of a rigid solid cylinder (e.g. a metal cylinder) with radius r_1 , surrounded by a cylindrical rotating-fluid-filled region of radius r_2 . The rest of the unit cell with radius bigger than r_2 consists of a stationary fluid, separated from the fluid in the cylindrical region by a thin impedance-matched layer at radius r_2 . (This layer can be

achieved using a thin sheet of solid material that is permeable to sound) The central cylinder rotates along its axis with angular speed Ω , which produces a circulatory flow in the surrounding fluid. (We will not consider the possibility of vortices like Taylor vortex [45] caused by large Ω in experiment because we here focus on 2D model and Taylor vortex does not contribute an effective flux through x - y plane.) We assume that fluid velocity is much slower than the speed of sound (Mach number, which is defined as v/c , is less than 0.3). The motion of the fluid can be described by a circulating ‘Couette flow’ distribution [45]; the velocity field points in the azimuthal direction, with component $v_\theta = -\frac{\Omega r_1^2}{r_2^2 - r_1^2}r + \frac{\Omega r_1^2 r_2^2}{r_2^2 - r_1^2} \frac{1}{r}$, where r is measured from the origin at the axis of the cylinder. This angular velocity is equal to Ω at radius $r=r_1$, and zero at radius $r=r_2$.

2.2 Governing equation

In the previously designed non-reciprocal acoustic crystal, the propagation of sound waves in the presence of such a steady-state non-homogenous velocity background is described in Refs. [50-52]. Assuming that the viscosity and heat flow are negligible, we can start from three independent equations – Euler, continuity and state equations in terms of acoustic disturbances of p , ρ and v : $\rho_0 \partial_t v + \nabla p = 0$, $\partial_t \rho + \rho_0 \nabla \cdot v = 0$ and $p = c_0^2 \rho$. Derived from the above three equations, we can arrive at the sound master equation:

$$\frac{1}{\rho} \nabla \cdot \rho \nabla \phi - (\partial_t + \vec{v}_0 \cdot \nabla) \frac{1}{c^2} (\partial_t + \vec{v}_0 \cdot \nabla) \phi = 0,$$

(2.1)

where ρ is the fluid density, c is the speed of sound, and $\vec{v}_0$ is the background fluid velocity.

The relation between velocity potential ϕ and sound pressure p is $p = \rho(\partial_t + \vec{v}_0 \cdot \nabla)\phi$. We take the surface of each cylinder as an impenetrable hard boundary by setting $\vec{n} \cdot \nabla\phi = 0$ where $\vec{n}$ is the surface normal vector.

It should be mentioned that, the Eq. (2.1) is a linearized-approximated equation. The latter simulation results may have small deviations if we adopt other wave models as recently shown in Ref. [53, 54]. However, the physical results are robust and reliable. We can explore the above differential equation analytically through the plane wave expansion method and obtain the effective Hamiltonian in vicinity of the high symmetry points K (K') at the corners of the Brillouin zone. The details of the mathematical manipulation can be found in Appendix A. Without circulating flow, the dispersion of the two lowest bands at the corners of the Brillouin zone exhibits the Dirac cone spectrum. However, when the circulating air flow in each unit cell is introduced, band gap opens because of the T symmetry breaking.

We restrict our attention to time-harmonic solutions with frequency ω and neglect second order terms as $|\vec{v}_0/c|^2 \ll 1$. With a change of variables $\Psi = \sqrt{\rho}\phi$ the master equation can be rewritten as [10]

$$[(\nabla - i\vec{A}_{eff})^2 + V(x, y)]\Psi = 0, \quad (2.2)$$

where the effective vector and scalar potentials are

$$\vec{A}_{eff} = -\frac{\omega \vec{v}_0(x, y)}{c^2}$$

$$V(x, y) = -\frac{1}{4} |\nabla \ln \rho|^2 - \frac{1}{2} \nabla^2 \ln \rho + \frac{\omega^2}{c^2}.$$

Evidently Eq. (2.2) maps onto the Schrodinger equation for a spin-less charged quantum particle in non-uniform vector and scalar potentials. The details of the derivation can be found in the Appendix B. For non-zero Ω , the inner boundary of the Couette flow contributes positive effective magnetic flux, and the rest of the Couette flow contributes negative effective magnetic flux; the net magnetic flux, integrated over the entire unit cell, is zero. The acoustic system thus behaves like a ‘zero field quantum Hall’ system [55] and is periodic in the unit cell.

It is worth mentioning that a similar approach to construct an effective magnetic vector potential for classical wave propagation has been discussed by Berry and colleagues [56]. These authors showed that an irrotational (‘bathtub’) fluid vortex exhibits a classical wavefront dislocation effect, analogous to the Aharanov-Bohm effect [57]. Here we advance this insight by applying the flow model to an acoustic crystal context, so that the effective magnetic vector potential gives rise to a topologically nontrivial acoustic band structure.

2.3 Introduction and generation of the weak form

Apart from exploring Eq. (2.1) analytically as shown in Appendix A and analyzing the physical pictures qualitatively, we need to quantitatively characterize the physical

properties of the acoustic crystal we introduced in section 2.1. To solve the complex Eq. (2.1) numerically, we can resort to the finite element method – commercial software COMSOL Multiphysics, weak form PDE (physics interfaces). In general, the commercial software collects all the equations and boundary conditions formulated by the physics interfaces into a large system of partial derivative equations and boundary conditions. COMSOL Multiphysics then solves the system by using a weak formulation. The mathematical weak form can give us direct access to the terms of the weak equation and provide maximum freedom in defining finite element problems. Therefore, I provide a theoretical background to the weak form in COMSOL Multiphysics [58] in this section and generate the weak form for our acoustic model.

First, I show a simple example – the conversion of a general formula to the weak form. Consider a partial derivative equation with a single dependent variable, u , in two space dimensions:

$$\nabla \cdot \Gamma = F, \text{ in domain } \Omega. \quad (2.3)$$

The functions Γ and F in general may be functions of both the dependent variable u itself and its time derivative. Now let v be an arbitrary function on Ω , and call it the test function (v should of course belong to a suitably chosen well-behaved class of functions, V). Multiplying the partial derivative equation with this function and integrating leads to

$$\int_{\Omega} v \nabla \cdot \Gamma dA = \int_{\Omega} v F dA, \quad (2.4)$$

where dA is the area element. We can use Gauss' formula to integrate by parts and arrive at

$$\int_{\partial\Omega} v \Gamma \cdot n ds - \int_{\Omega} \nabla v \cdot \Gamma dA = \int_{\Omega} v F dA. \quad (2.5)$$

where ds is the length element. Therefore when we apply the Neumann boundary condition:

$$-n \cdot \Gamma = G + \frac{\partial R}{\partial u} \mu, \quad (2.6)$$

we can obtain the equation below:

$$0 = \int_{\Omega} (\nabla v \cdot \Gamma + v F) dA + \int_{\partial\Omega} v (G + \frac{\partial R}{\partial u} \mu) ds. \quad (2.7)$$

Together with the Dirichlet condition, this is a weak reformulation of the original partial derivative equation problem. The requirement shows that the previous weak formula should hold for all test functions v . One can reverse the steps of the derivation to show that if the functions u and μ satisfy the weak formula, then they also satisfy the original formula. However, this holds true only if the solutions and coefficients are smooth enough. For example, in the case of discontinuities in material properties, one can have a solution of the weak formulation, however the strong formula then has no sense. The names weak and strong come from the difference: the weak formulation is a weaker condition on the solution than the strong formula. An advantage of this weak formulation is that it needs less regularity of Γ . This is vital in the finite element method.

By introducing the boundary conditions on the test functions

$$v \frac{\partial R}{\partial u} = 0, \text{ on } \partial\Omega.$$

(2.8)

Then the weak reformulation becomes the following function

$$\begin{cases} 0 = \int_{\Omega} (\nabla v \cdot \Gamma + vF) dA + \int_{\partial\Omega} vG ds. \\ 0 = R_m \text{ on boundary } \partial\Omega \end{cases}.$$

(2.9)

The Eq. (2.9) holds true for all test function v meeting the boundary condition. Such a formula arises if one have a variational principle. For example, to find the function u that minimizes the energy of a physical system on the condition of the constraints $0 = R_m$. If the energy is given like an integral of an expression involving the function u , thus the stationarity condition on the solution is accurately the weak formula as shown above. Because variational principles are more fundamental than the corresponding partial derivative equation, the weak form is often more natural than the strong forms.

After the introduction of the weak form in COMSOL Multiphysics, we try to convert our acoustic Eq. (2.1) to a weak formula. Later on, we use the commercial software to simulate the acoustic crystal and numerically calculate the dispersions and acoustic pressure fields.

Following the similar path shown above, we start from Eq. (2.1) and apply the Gauss' formula. Finally, the weak reformulation of the Eq. (2.1) is

$$\begin{aligned} & dA[-\rho * ((\partial_{t,t}u + \partial_x v * \partial_{x,t}u + \partial_y v * \partial_{y,t}u) * \frac{test(u)}{c_0^2} + \\ 0 = \int_{\Omega} & test(\partial_x u) * \left(\partial_x u - (\partial_t u + \partial_x v * \partial_x u + \partial_y v * \partial_y u) * \frac{\partial_x v}{c_0^2} \right) + \\ & test(\partial_y u) * (\partial_y u - (\partial_t u + \partial_x v * \partial_x u + \partial_y v * \partial_y u) * \frac{\partial_y v}{c_0^2}))] \end{aligned}$$

(2.10)

where u means the test function and ∂_i indicates the derivative of a function with respect to i , where $i = x, y, t$. Several boundary conditions for different physical characterizations correspond to different weak form on the boundaries. For example, Floquet (periodic) boundary condition is used to calculate the band structures. Sound hard boundary and scattering boundary condition are applied to mimic the physical experiment for topologically protected edge states.

In the next Chapter, we technically import the Eq. (2.10) into the COMSOL Multiphysics (the weak form physics interface), choose different boundaries and numerically calculate the results.

Chapter 3

One-way edge modes

3.1 Topological band structure

We can calculate the acoustic band structures by using the finite-element commercial software COMSOL Multiphysics. For simplicity, we assume the background fluid involved is air. The results, with $\Omega = 0$ and $\Omega \neq 0$, are shown in Fig. 3.1(a). For $\Omega = 0$ [red curves in Fig. 3.1(a)], the acoustic band structure exhibits a pair of Dirac points at the corner of the hexagonal Brillouin zone, at frequency $\omega_0 = 0.577 \times 2\pi c_a/a$ (992 Hz), where c_a is the sound velocity in air.

For $\Omega \neq 0$ the circulating air flow produces a dramatic change in the band structure [blue curves in Fig. 3.1(a)]. Here, we set the angular velocity of the inner rods to be $\Omega = 2\pi \times 400 \text{ rad/s}$ (which means 400 resolutions per second, achievable with miniature electric motors). The Dirac point degeneracies are lifted, producing a finite complete bandgap. The frequency splitting at the zone corners as a function of Ω , is plotted in Fig. 3.1(b). The ratio of the operating frequency to the bandgap, which is an estimate for the penetration depth of the topological edge states in units of the lattice constant, is on the order of $\omega/\delta\omega \approx 10$ for the range of angular velocities plotted here.

Figure 3.1. (a) Band structures of the acoustic lattice without the circulating fluid flow (red curves; $\Omega = 2\pi \times 0 \text{ rad/s}$) and with fluid flow (blue curves; $\Omega = 2\pi \times 400 \text{ rad/s}$). In the gapped band structure, the bands have Chern number ± 1 (blue labels). Left inset: enlarged view of Dirac cone. Right lower inset: the first Brillouin zone. (b) Frequency splitting as a function of the angular velocity of the cylinder in each unit cell. The degeneracy at the Dirac point with frequency $\omega_0 = 0.577 \times 2\pi c_a/a$ (992 Hz) is removed for $\Omega \neq 0$. [44]

3.2 Calculation of Chern number

To verify the existence of topological band structure, we need to calculate the topological invariants of the bands below the band gap. As it can be seen in Fig. 3.1(a), there is one band (lowest) below the band gap. In the first chapter, we have already shown that for a 2D system, an electronic band can be characterized by a topological invariant – Chern number [2-4]. We simply adopt this characterization and calculate the Chern number for the acoustic band. The Chern number of the n th acoustic band can be defined as a function of Berry connection $\mathcal{A} = i\langle u_{n,k} | \nabla_k | u_{n,k} \rangle$, where the function $u_{n,k}(r)$ obeys the orthonormality condition and is cell-periodic eigen-function of the Bloch Hamiltonian. Eq. (1.1) (Chern number) is an area integral carried out in both momentum k space over the first Brillouin zone and real space over the unit cell. Note that Berry connection and Berry phase are gauge dependent, whereas the Berry curvature and Chern number are gauge-invariant. The phase and flux can be connected through Stokes' theorem. The integral on a surface of the Berry curvature $\nabla_k \times \mathcal{A}$ is Berry flux. The integral of the Berry curvature over a closed manifold is quantized in units of 2π , which is so called Chern number.

For detailed calculations, we used the method described in Ref. [59]. By exporting the Bloch wave functions calculated from finite-element method, we can numerically manifest that the lowest two bands in Fig. 3.1(a), lifted by the T symmetry breaking, have Chern numbers of ± 1 , as indicated by the blue numbers.

3.3 One-way edge modes

The principle of bulk-edge correspondence [60] then predicts that, for a finite acoustic crystal, the gap between these two bands is spanned by unidirectional acoustic edge states, analogous to the electronic edge states occurring in the quantum Hall effect. To confirm the existence of these topologically-protected acoustic edge states, we numerically calculate the band structure for a 20×1 super-cell [61] (a ribbon that is 20-unit-cell wide in y direction and infinite along x direction). As shown in Fig. 3.2(a), for $\Omega = 2\pi \times 400 \text{ rad/s}$ the bandgap contains two sets of edge states, which are confined to opposite edges of the ribbon and have opposite group velocities.

Figs. 3.2(b-c) show the propagation of these edge states in a finite (34×14) lattice. In these simulations, the upper edge of the acoustic crystal is enclosed by a sound-impermeable hard boundary (e.g., a flat metal surface), in order to prevent sound waves from leaking into the upper half space; absorbing boundary conditions are applied to the sides. A point sound source with mid-gap frequency ω_0 is placed near the upper boundary. For $\Omega = 2\pi \times 400 \text{ rad/s}$, this excites a unidirectional edge state which propagates to the left along the interface [Fig. 3.2(b)]. If the sign of angular velocity is reversed, the edge state would be directed to the right (not plotted). The field distribution for a reduced angular velocity $\Omega = 2\pi \times 200 \text{ rad/s}$ [Fig. 3.2(c)] shows an edge state with a longer penetration depth because of a narrower bandgap. Note that the group velocity flips sign within a very small frequency range near the bulk states. This behavior is dependent on boundary details and does not violate the bulk-edge

correspondence principle.

Due to the lack of backward-propagating edge modes, the presence of disorder cannot cause backscattering. Fig. 3.3(a) shows an acoustic cavity located along the interface; the incident wave flows through the cavity, and excites localized resonances within the cavity, but does not backscatter. Fig. 3.3(b) shows a Z-shape bend connecting two parallel surfaces at different y ; again, the acoustic edge states are fully transmitted across the bend. Finally, Fig. 2.5(c) shows a 180-degree bend which allows acoustic edge states to be guided from the top of

the sample to the bottom of the sample. Note that the top and bottom boundaries are called ‘zigzag’ shape boundaries. Whereas the left boundary in this sample is an ‘armchair’ boundary, which supports one-way edge states with different dispersion relations.

In addition, we numerically calculate the dispersion relation of a 20×1 super-cell (a ribbon that is 20-unit-cell wide in x direction and infinite along y direction). In this case, the armchair shape of the edges can still support the topologically protected one-way surface modes, as shown in Fig. 3.4(a). For $\Omega = 2\pi \times 400 \text{ rad/s}$, within the bandgap there are two edge states corresponding to right and left edges of the ribbon, which have positive and negative group velocities, respectively. The schematic in Fig. 3.4(b) shows the propagation style of the edge states shown in panel (a).

Figure 3.2. (a) Dispersion of the one-way acoustic edge states (red curves) occurring in a finite strip of the acoustic lattice, for $\Omega = 2\pi \times 400$ rad/s, which means 400 resolutions per second. The left and right red curves correspond to edge states localized at the bottom and top of the strip. (b-c) The normalized acoustic pressure p for a left-propagating acoustic edge state at frequency $\omega_0 = 0.577 \times 2\pi c_a/a$ (992 Hz) for $\Omega = 2\pi \times 400$ rad/s (b) and $\Omega = 2\pi \times 200$ rad/s (c). [44]

Figure 3.3. Demonstration of the robustness of acoustic one-way edge states against disorder. Topological protection requires the acoustic waves to be fully transmitted through an acoustic cavity (a), a Z-shape bend along the interface (b) and a 180-degree bend (c). The operating frequency is $\omega_0 = 0.577 \times 2\pi c_a/a$ (992 Hz) and $\Omega = 2\pi \times 400 \text{ rad/s}$. [44]

Figure 3.4. (a) The dispersion of the edge states projected along x direction. Note that in Fig. 3.2, the band structure projects along y direction. There are two edge states propagating along one direction corresponding to opposite edges. (b) The schematic shows the propagation style of the edge states shown in panel (a).

3.4 Three more circulating distributions

To perform the numerical calculations of our model, we use the velocity distribution of Couette flow as the effective vector potential. To further verify our results, we also use the irrotational-vortex model [56] as the background velocity distribution. The circulating air flow is curl-free with velocity components $v_r = 0$ and $v_\theta = \frac{K_c}{r}$, where r is measured in localized coordinates centered at each hexagonal unit cell, θ is the azimuthal angle in each unit cell, and K_c is the strength of the vortex. Then, we adopt several further operations of the velocity distribution – Abrupt truncation, Cellular and Muffin-tin method (latter two are classical approximated methods in solid state physics [62]).

As shown in Fig. 3.5, three methods take different operations of effective vector potential within each Wigner-Seitz primitive cell. Blue curve is Abrupt truncation method. We manually truncated vector potential at radius $r_t = 0.48a$. This will no doubt result in spatial discontinuity of pressure field due to the relationship $p = \rho(\partial_t + \vec{v}_0 \cdot \nabla)\phi$. But according to our results, the discontinuity is very small on the condition of low Mach number. Black curve represents the Cellular method. The potential adding six nearest flow potentials is extended to the boundary of a hexagonal unit cell. The summation of six nearest flow potentials will make slight change to the potential distribution near the boundary and nearly no influence near the center. This method in our model does not take complicated flow near the boundary into account for practical reason, which leads to a discontinuous derivative of velocity potential whenever the

boundary between two cells is crossed. Red curve indicates Muffin-tin approximation with a smoothed function between $r_2 = 0.4a$ and $r_t = 0.48a$, which makes the flow potential decrease to zero continuously. It should be mentioned that the latter two methods ensure the continuous derivative of Bloch waves when the Bloch functions cross the boundaries of unit cell.

Let us consider the irrotational vortex in terms of its circulation $\Gamma_c = 2\pi K_c = 2\pi \times 4$ for all the calculations. It is related to the effective magnetic flux as

$$\phi_{eff} = \oint A_{eff} dl = \frac{\omega}{c^2} \Gamma_c. \quad (3.2)$$

The results of the band structures calculated from the above three methods are shown in Fig. 3.6. The parameters used here are the same with the parameters shown in Chapter 2. We can see that the band gaps opened are almost the same for the three different operations of the velocity profile. Most importantly, the ratio of the operating frequency to the bandgap is on the order of $\omega/\delta\omega \approx 10$, which is similar to the result from Coutte flow. We conclude here that the small different operations of the velocity distribution do not change the band structure dramatically.

The calculated dispersions of the edge modes from above three numerical approximated methods are shown in Fig. 3.7 and 3.8. Fig. 3.7 is calculated through applying Dirichlet boundary conditions for the top and bottom edges and Fig. 3.8 is calculated by applying sound hard boundary conditions for the top and bottom edge. According to the numerical results, the dispersion relations of bulk states are the same for all kinds of boundary conditions. But the dispersion relations of edge modes are

slightly different for different boundary conditions (Dirichlet and sound hard conditions). However the fact that one interface can only support single one-way edge mode does not change, because of the non-zero Chern number ± 1 associated with the bands across the topological band gap.

Figure 3.5. Different operations of effective vector potential. Inset shows the Wigner-Seitz primitive unit cell of the acoustic crystal. Parameters: $r_1 = 0.2a$, $r_2 = 0.4a$ and $r_3 = 0.5a$. Blue curve is Abrupt truncation method, which truncates the air flow at radius $r_t = 0.48a$. Black curve represents the Cellular method. The overall flow potential takes six nearest flow potentials into consideration. Red curve indicates Muffin-tin approximation, which makes the flow potential decrease to zero continuously.

Figure 3.6. Band structures of an acoustic crystal with T symmetry breaking. The degeneracy of Dirac points at the corners of the Brillouin zone is lifted. The results of the three numerical methods show good agreements with each other. Blue, black and red curves correspond to Abrupt truncation method, Cellular method and Muffin-tin approximation, respectively.

Figure 3.7. Dispersion relations of zigzag edge modes of a ribbon-shape acoustic crystal. Dirichlet boundary conditions for the top and bottom edge are used. The results of the three numerical methods are almost the same. The dispersions of the surface states acquire only positive (negative) group velocity within the band gap for one (another) edge. Blue, black and red curves correspond to Abrupt truncation method, Cellular method and Muffin-tin approximation, respectively.

Figure 3.8. Dispersion relation of zigzag edge modes of a ribbon-shape acoustic crystal. Sound hard boundary conditions for the top and bottom edge are applied. The results of the three numerical methods are almost the same with the results in Fig. 3.7. Blue, black and red curves correspond to Abrupt truncation method, Cellular method and Muffin-tin approximation, respectively.

3.5 Conclusion

In conclusion, we numerically calculate topological band structure and the dispersions of the edge states in the non-reciprocal acoustic crystal. We also demonstrate the presence of topologically protected one-way acoustic edge states.

We need to point out that similar effects can be achieved with alternative designs featuring circulatory fluid velocity distributions; e.g., having azimuthally-directed fans in each unit cell [48], or stirring with a rotating disc on the top plate [52]. The effect could be tunable in frequency ranges by appropriately scaling down lattice constant or practically operating at higher band gaps with larger Chern numbers. Acoustic devices based on these topological properties may be useful for invisibility from sonar detection, one-way signal processing regardless of disorders, acoustic isolator, which will greatly broaden our interest in military, medical and industrial applications.

We also want to point out that several groups have reported topological vibrational modes in mechanical lattices [43, 63-68]. The present acoustic system, by contrast, involves acoustic waves in continuous fluid media, which is considerably more relevant for the existing acoustic technologies.

Part II

Three-dimensional type-II Weyl acoustics

The discovery of 3D Weyl semimetals hosting isolated Weyl nodes has drawn remarkable attention in condensed matter physics as well as in photonics. Recently, type-II Weyl fermions have been proposed [21] and observed [69-72] in condensed-matter systems with both the band structure and Fermi-arc surface states. The type-II Weyl node appears at the contact of electron and hole pockets and exhibits a strongly tilted cone spectrum with non-vanishing density of states, in contrast to a point-like Fermi surface at a type-I Weyl node with vanishing density of states. The type-II Weyl fermions were in fact missed by Weyl because of their violation of the fundamental Lorentz symmetry in quantum field theory. All type-I Weyl fermions are adiabatically and topologically equivalent to Weyl's original prediction of Lorentz-invariant Weyl fermions, but type-II Weyl fermions cannot be adiabatically connected back to Weyl's solution. However, the acoustic type-II Weyl nodes has not been explored so far.

In Part II, we first design the 1D dimerized chain – mimicking the well-known Su-Schrieffer-Heeger (SSH) model [73] in Chapter 4. Then in Chapter 5, we build the 3D acoustic type-II Weyl nodes by stacking the building blocks of dimerized chains.

Chapter 4

Acoustic dimerized chain

Note that a 1D acoustic topological phase has been realized in a 1D phononic crystal [74], but not with resonators. Therefore, it cannot be precisely described by the SSH tight-binding model due to the presence of long-range couplings. It is thus interesting to investigate if a 1D chain consisting of resonators, each of which supports a single-resonance mode, can realize 1D acoustic topological phase corresponding to SSH model.

4.1 Reducing the resonator model to tight-binding Hamiltonian

In the section, we start with constructing a simple 1D tight-binding model which consists of acoustic resonators only coupled to its nearest-neighbor resonator through one coupling waveguide. Generally, this method can be extended to 2D and 3D acoustic resonator systems, which will be directly applied in the next Chapter

Assuming there is an infinitely long 1D resonator chain as shown schematically in the upper panel of Fig. 4.1(a) – two resonators per unit cell. The filled (open) circle indicates A (B) type resonator. The l th resonator mode satisfies the following coupled-mode equations [11, 14, 61]:

$$i\partial_t a_l = \sum_{\langle m \rangle} \kappa_1 b_m \text{ and } i\partial_t b_l = \sum_{\langle m \rangle} \kappa_2 a_m, \quad (4.1)$$

where a (b) is the amplitude of resonator mode for type A (B) resonator. The summation $\langle m \rangle$ is taken over the nearest-neighbor resonator. The hopping strength can be obtained from above coupled-mode equations as $\kappa_1 = -\omega_n \int \langle b_l | a_l \rangle dV$ and $\kappa_2 = -\omega_n \int \langle a_{l+1} | b_l \rangle dV$, where the integration is taken over the volume of the coupling waveguide. The above coupled-mode equations can be viewed as a tight-binding eigenvalue problem $H\psi = E\psi$.

In particular, for the two-band model of 1D acoustic dimerized chain, the construction is shown in lower panel of Fig. 4.1(a). The left and right nearest-neighbor (NN) hopping strengths of A type resonator are $t + \delta t$ and $t - \delta t$, respectively. The dispersion from coupled-mode equations is the same as the Hamiltonian $H = \sum_l [(t + \delta t)a_l^\dagger b_l + (t - \delta t)b_l^\dagger a_{l+1} + h.c.]$, where a (b) and $a^\dagger$ ($b^\dagger$) are the annihilation and creation operators on the sub-lattice sites, t and δt are the nearest hopping and the tuning strength. They can be tuned by changing the radius (equivalently changes V) of the cylindrical coupling waveguide. Furthermore we transfer the Hamiltonian into k -space by performing Fourier transformation and setting zero energy offset between two sites, we can arrive at the SSH model and obtain the Bloch Hamiltonian $H(k)$ for the 1D acoustic resonator system:

$$H_1(k) = 2t\cos(k_x a)\sigma_x - 2\delta t\sin(k_x a)\sigma_y. \quad (4.2)$$

Figure 4.1. (a) The schematic of a dimerized chain and one unit-cell of the acoustic structure. (b) Three band structures with $\delta w = 0.3w$, $\delta w = 0$ and $\delta w = -0.3w$. The band gap closes at the middle panel, which indicates the existence of the topological transition. [75]

4.2 Acoustic dimerized chain consisting of resonators

Now, we step into the details of the acoustic structure and characterize its physical properties, since the Hamiltonian Eq. (4.2) can be implemented in an acoustic dimerized chain. One unit cell of the dimerized chain consists of two resonators, connected by two coupling waveguides with different radii, as shown in the lower part of Fig. 4.1(a). The periodic boundary condition is applied to the left and right surfaces. Other surfaces (marked with blue color) of the unit cell are treated as hard boundaries for sound. The distance between two nearest resonators is $a = 0.1$ m. The radius and height of the cylinder (resonator) is $r = 0.4a$ and $h = 0.8a$. For dimerization, we apply modulation of $\delta w = 0.3w$ to the original radius of coupling waveguide $w = 0.26r$. We thus have $w + \delta w$ for one radius of the coupling waveguide, and $w - \delta w$ for the other, as shown in the lower part of Fig. 4.1(a).

Since there are two atoms in one unit cell, hereafter we only consider the two-band model with two lowest acoustic eigen modes, whose pressure field patterns calculated from finite-element commercial software COMSOL Multiphysics are almost single valued in each acoustic resonator. By choosing three values of modulation $\delta w = 0.3w, 0, -0.3w$, we arrive at three band structures by solving acoustic wave equation in the first Brillouin zone as shown in Fig. 4.1(b). The closing of bandgap at $\delta w = 0$ (black curve) indicates the existence of topological phase transition. For the lower bands of three cases in Fig. 4.1(b), we can characterize their topological properties by

calculating the topological invariant – Zak phase [5, 74, 76, 77] $\varphi_{Zak} = i \int_{-\pi/2a}^{\pi/2a} \langle u_{n,k} | \nabla_k | u_{n,k} \rangle dk$. The results are $-\pi/2$, 0 and $\pi/2$ for $\delta w > 0$, $\delta w = 0$ and $\delta w < 0$, respectively. Note that the Zak phase of each dimerization is a gauge dependent value, but the difference between the Zak phases of two dimerized configurations with $\delta w > 0$ and $\delta w < 0$, which is $\Delta\varphi_{Zak} = \varphi_{Zak2} - \varphi_{Zak1} = \pi$ in our acoustic model, is topologically defined. Because the topological property of a bandgap is determined by the summation of Zak phases of all bands below the gap, the two dimerizations in Fig. 4.1(b) (red and blue curves) are topologically distinct to each other.

The above topologically nontrivial phases in acoustic resonators ensure the existence of interface states between two configurations of dimerized lattices with different Zak phases. Hereafter we cut and connect the two chains through their mirror centers for physical reasons [78]. The interface dispersions demonstrated in Fig. 4.2(a) are the results from numerical simulation. For the left panel, we apply $\delta w = 0.3w$ and $\delta w = -0.3w$ on two sides of an interface. For the right panel, $\delta w = 0.3w$ and $\delta w = 0.1w$ are applied. There is an interface state, predicated, locating inside the bandgap in the left panel of Fig. 4.2(a), as highlighted by the red line. The acoustic pressure field pattern of the interface state is shown in Fig. 4.2(b). The green arrow points to the interface between two topologically distinct structures. We can clearly see that the amplitude of the acoustic wave decays rapidly into the bulk on both sides of the interface.

Figure 4.2. (a) Topologically non-trivial and trivial band structures for the interface between two connected chains. The red line indicates the topological interface state. (b) The acoustic field of the interface state. The green arrow points to the interface. Hereafter, the blue (red) color represents negative (positive) acoustic pressure (p) in the colorbar. [75]

4.3 Dirac nodes in a dimerized square lattice

Utilizing these 1D dimerized chains shown in last section as building blocks, we can construct 2D Dirac nodes by stacking these 1D dimerized chains in a staggered way. In the following, we first design the theoretical Hamiltonian for predicting the existence of 2D Dirac nodes and later 3D Weyl nodes for simplicity. Then we construct the acoustic structure through following the parameters of the Hamiltonian.

We find that the 2D Dirac nodes can be constructed by the Bloch Hamiltonian of a 2D dimerized acoustic lattice shown below:

$$H_2(k) = [2t_x \cos(k_x a) + 2t_y \cos(k_y a)]\sigma_x - 2\delta t_x \sin(k_x a)\sigma_y, \quad (4.3)$$

where t_x (t_y) is the hopping strength along x (y) direction, and δt_x is the modulation of the hopping strength along x direction. Following the above tight-binding model, we set the unit cell of the acoustic lattice as shown in Fig. 4.3(a). The right inset is the schematic of 2D lattice whose unit cell is enclosed by green dashed lines. Through tuning the coupling strength, we find that there are two linear degenerate points in the first Brillouin zone if $|t_x| < |t_y|$, no degenerate points (trivial band gap) if $|t_x| > |t_y|$, and quadratic degenerate points [79] in the corners of 2D Brillouin zone if $|t_x| = |t_y|$. Here we choose $t_x = -1$, $t_y = -2$ and $\delta t_x = -0.5$. The band structure in the 2D momentum space (k_x, k_y) , as shown in Fig. 4.3(b), can be calculated from the Hamiltonian Eq. (4.3), as shown in Fig. 4.3(c). There are two isolated degenerate points located at $(0, \pm 2/3)$ in units of π/a in the first 2D Brillouin zone, enclosed by blue

lines in Fig. 4.3(b).

For acoustic structure, the lattice constant and parameters of the resonator (radius and height) are the same with those in 1D dimerized chain. Similar to 1D dimerized chains, the modulation $\delta w_x = 0.3w_x$, where $w_x = 0.26r$, is applied to the coupling waveguides along x direction, whose radii are $w_x \pm \delta w_x$, respectively. Coupling waveguides along y direction with radius $w_y = 2w_x$ connect these 1D dimerized chains. For this real acoustic structure, the band structure along high symmetry lines in the first Brillouin zone is shown in Fig. 4.3(d). It can be seen that there are two degenerate points (2D Dirac nodes) at frequency $0.21 \times 2\pi c/a$ (718.05 Hz), where c is the speed of sound, located at $(k_x, k_y) = (0, +0.61)$ and $(k_x, k_y) = (0, -0.61)$ in units of π/a within high symmetry lines $M_2\Gamma$ and ΓM_3 .

After expanding the Hamiltonian Eq. (4.3) by substituting $k_x = k_{x0}/a + \delta k_x/a$ and $k_y = k_{y0}/a + \delta k_y/a$ around the degenerate points (k_{x0}, k_{y0}) and keeping the first order term, we arrive at:

$$h_2(\delta k) = -2t_y \sin(k_{y0}) \delta k_y \sigma_x - 2\delta t_x \delta k_x \sigma_y, \quad (4.4)$$

which meets the form of $H(k) = \sum_{i,j=x,y} k_i v_{ij} \sigma_j$, where v_{ij} and σ_j are group velocity and Pauli matrix, respectively. Here the group velocity matrix is $v_{ij} =$

$$\begin{pmatrix} 0 & -2\delta t_x \\ -2t_y \sin(k_{y0}) & 0 \end{pmatrix}$$

($i, j=x, y$). From chirality definition $w = \text{sgn}[\det(v_{ij})]$, we obtain that $w = -1$ ($w = 1$) for the degenerate point in $M_2\Gamma$ (ΓM_3), as indicated by red numbers in Fig. 4.3(d). These degenerate points can also be regarded as topological charges. The

topological invariant is the winding number, as shown in Ref. [80],

$$w = 1/2\pi \oint dk [\hat{d}_x \nabla \hat{d}_y - \hat{d}_y \nabla \hat{d}_x], \quad (4.5)$$

where $\hat{d}_{x,y}$ are the normalized coefficients of different components of Pauli matrix from Hamiltonian Eq. (4.3). After performing loop integral around 2D degenerate points, we find $w = -1$ ($w = 1$) for degenerate point on $M_2\Gamma$ (ΓM_3), which are consistent with the results from chirality analysis.

Here the 2D acoustic lattice has flat edge states similar to those in graphene [81]. We investigate the 2D acoustic structure that is finite in the x - y direction having a width of 15.5 unit cells and is infinite in the $x+y$ direction, as plotted in Fig. 4.4(b). The 2D Dirac nodes are projected onto the $k_x - k_y$ direction with good quantum number $k_{//} = (k_x + k_y)/\sqrt{2}$. The Dirac nodes with opposite chirality do not meet with each other. Fig. 4.4(a) shows the band structure of the finite acoustic system. The red curves with degeneracy of two indicate the nearly flat edge states connecting two projected Dirac nodes with opposite chirality. Note that the little derivation from a perfectly flat dispersion is a result of the real acoustic structure, mesh settings and boundary conditions adopted. The acoustic fields are shown in Fig. 4.4(b). The acoustic waves of degenerate edge states almost do not propagate due to nearly zero group velocity. Note that when the finite ribbon consists of 16 unit cells (odd number of sites), there will be a single nondegenerate edge state that traverses the Brillouin zone.

Figure 4.3. (a) The schematic of the 2D dimerized lattice and one unit-cell of the acoustic structure. (b) The first Brillouin zone enclosed by the blue lines. Green and black dots are locations of the Dirac nodes. (c) The band structure calculated from the tight-binding model shows the existence of the accidental degeneracy. (d) The band structure of the acoustic lattice from numerical simulation. Red numbers indicate the chirality. There exists the accidental degeneracy. [75]

Figure 4.4. (a) The band structure for a finite acoustic lattice. The red curves indicate the flat edge states, which means the nearly zero group velocity. (b) The acoustic fields of the edge states. Acoustic waves are localized at the edges of the sample. [75]

Chapter 5

Acoustic type-II Weyl nodes from stacking dimerized chains

Following the development of Weyl physics in condensed matter physics [15-17] and photonics [18, 23], in acoustics, 3D Weyl nodes were first proposed theoretically by applying on-site unequal coupling as shown in Fig. 5.1(a) or chiral coupling on a graphite structure [82] as shown in Fig. 5.1(c). Fig. 5.1(b, d) present the distributions of the Weyl nodes corresponding to the two methods and their dispersion as a function of k_z at the Brillouin zone corners. The Weyl nodes associated with the structure shown in Fig. 5.1(a) [Fig. 5.1(b)] is type-II (type-I) Weyl nodes. However, by that time, the classification of type-I and type-II Weyl fermions has not been officially addressed in condensed matter physics and photonics. Consequently, the proposal based on the graphite structure did not distinguish type-I and type-II Weyl Hamiltonians and only acoustic topological surface states for type-I Weyl nodes was further studied. In this Chapter, on a platform of stacked dimerized chains of acoustic resonators, we construct acoustic type-II Weyl nodes following the explicit type-II Weyl Hamiltonian [21]. Unique features of this acoustic type-II Weyl system include the distinct finite density of states and transport properties of topological surface states. In a certain momentum space direction, the bands of the surface states have the same sign of velocity, which is determined by the tilting direction of type-II Weyl nodes rather than their chirality

dictated by the Chern number. Because of the existence of an incomplete bandgap, the acoustic waves of the surface states can be scattered by defects and penetrate into the bulk, and thus do not exhibit the same robust propagation as demonstrated in Ref. [82].

Figure 5.1. Acoustic Weyl points in 3D acoustic lattices stacked with 2D acoustic honeycomb lattices consisting of resonators. (a, b) Method 1: on-site unequal coupling. (c, d) Method 2: chiral coupling. (a, c) Acoustic structure and construction schematic. (b, d) Weyl nodes distribution and dispersion as a function of k_z at the first Brillouin zone corners. This figure was reproduced from [82].

We follow the line of thought in the previous proposal, and adopt 1D resonator chains as building blocks. Firstly, since Weyl nodes are 3D extensions of 3D Dirac nodes, we first construct 2D Dirac nodes by stacking 1D dimerized chains. Secondly, in view of the difficulty of achieving T -symmetry breaking in topological acoustics, we introduce P -symmetry breaking when stacking 1D resonator chains.

5.1 Type-II Weyl nodes in a dimerized cubic lattice

Finally, by stacking the 2D dimerized lattice as in section 3.2 along the z direction with periodicity a and tuning the coupling strength, we can construct a Bloch Hamiltonian for the 3D dimerized lattice:

$$H_3(k) = d_0 I + d_x \sigma_x + d_y \sigma_y + d_z \sigma_z, \quad (5.1)$$

where $d_x = 2t_x \cos(k_x a) + 2t_y \cos(k_y a)$, $d_y = -2\delta t_x \sin(k_x a)$, $d_z = t_{z1} \cos(k_z a) - t_{z2} \cos(k_z a)$, $d_0 = t_{z1} \cos(k_z a) + t_{z2} \cos(k_z a)$ and I is the 2×2 identity matrix. The parameter t_x (t_y, t_z) is the hopping strength along x (y, z) direction. Note that the first term in Eq. (5.1) plays the role of tilting the cone-like spectrum. With a strongly tilted cone spectrum, this Hamiltonian satisfies the condition of recently proposed type-II Weyl Hamiltonian [21] $H(k) = \sum_{j=0,x,y,z} k_j v_{ij} \sigma_j$.

Hereafter we choose $t_x = -1$, $t_y = -2$, $t_{z1} = -1$, $t_{z2} = -2$ and $\delta t_x = -0.5$, as schematically shown in Fig. 5.2(a). The Hamiltonian breaks P -symmetry and respects T -symmetry. We calculate the band structure as shown in Fig. 5.2(b) with $k_z = 0.5$ in $k_x - k_y$ Brillouin zone plane. Four linear degenerate points locate at $(0, \pm 2/3, \pm 1/2)$ in units of π/a in the 3D first Brillouin zone. Typically, we plot in Fig. 5.2(c) the cone spectrum in the vicinity of the degenerate point $(0, 2/3, 1/2)$ in $k_x - k_y$ Brillouin zone plane. It can be seen that the cone spectrum indeed has been strongly tilted. Since the group velocities along z direction near the degenerate point are $2t_{z1} \sin(k_{z0})$ and $2t_{z2} \sin(k_{z0})$ where k_{z0} is the location of degenerate point, the

two bands acquire the same sign of group velocity.

Fig. 5.2(a) shows one unit-cell of the 3D acoustic structure. The inset presents the schematic of the 3D dimerized lattice. The radii of the coupling waveguides along the z direction are $w_{z1} = w_x + 2\delta w_x$ for the A resonator and $w_{z2} = w_x$ for the B resonator. The other parameters are the same as in previous 1D and 2D acoustic structures. The band structures in Brillouin zone planes $(k_x, k_y, 0)$ and $(k_x, k_y, 0.51)$ are shown in Fig. 5.3(a-b), which reveal a band gap at $k_z = 0$, and two degenerate points with frequency $0.26 \times 2\pi c/a$ (900.10 Hz) at $k_z = 0.51$. In the left part of Fig. 5.3(c), by sweeping k_z at $(k_x, k_y) = (0, 0.61)$, we get the band structure with a degenerate point at $k_z = 0.51$. It can be seen that the two bands have the same sign of group velocity. Therefore, there are four acoustic type-II Weyl nodes that locate at $(0, \pm 0.61, \pm 0.51)$.

One significant distinction between the type-I and type-II Weyl nodes appears in the density of states [21]. As shown in Fig. 5.3(c), the density of states acquires finite values for type-II Weyl nodes due to the presence of unbounded two-band pockets, which will be further discussed.

Figure 5.2. (a) The schematic of the 3D dimerized lattice and one unit-cell of the acoustic structure. (b-c) The band structures calculated from the tight-binding model in the first Brillouin zone plane with $k_z = \pi/2a$ (b) and around the Weyl node with $k_x = 0$ (c). There exist 3D isolated degenerate points, which means the Weyl points. Note that the Weyl cone is tilted. [75]

Figure 5.3. (a-c) The band structures along the high symmetry lines in 2D Brillouin zone planes. The black (green) dot indicates the Weyl node with positive (negative) chirality. (c) Left: the band structure as a function of k_z with fixed $(k_x, k_y) = (0, 0.61)$ in units of π/a . Right: non-vanishing density of states, which indicates this Weyl point is type-II Weyl point. [75]

5.2 Chirality of the Weyl nodes

Because the Chern number is not changed by the first term [21] of the Hamiltonian Eq. (5.1), we expand the Pauli-matrix components around the Weyl node by substituting $k_x = k_{x0}/a + \delta k_x/a$, $k_y = k_{y0}/a + \delta k_y/a$ and $k_z = k_{z0}/a + \delta k_z/a$ around the Weyl nodes (k_{x0}, k_{y0}, k_{z0}) and keeping the first-order terms and Pauli-matrix terms, we have:

$$\begin{aligned} h_3(\delta k) = & -2t_y \sin(k_{y0}) \delta k_y \sigma_x - 2\delta t_x \delta k_x \sigma_y - (t_{z1} \sin(k_{z0}) \delta k_z \\ & - t_{z2} \sin(k_{z0}) \delta k_z) \sigma_z \end{aligned} \quad (5.2)$$

which meets the expression of $H(k) = \sum_{i,j=x,y,z} k_i v_{ij} \sigma_j$, where v_{ij} ($i, j=x, y, z$) and σ_j are group velocity and Pauli matrix, respectively. In the 3D momentum space, the Weyl nodes are topological monopoles of quantized Berry flux characterized by the chirality $c = \text{sgn}[\det(v_{ij})]$, or the topological invariant—Chern number. Here the group velocity matrix from Eqn. (3.5) is:

$$v_{ij} = \begin{pmatrix} 0 & -2\delta t_x & 0 \\ -2t_y \sin(k_{y0}) & 0 & 0 \\ 0 & 0 & -t_{z1} \sin(k_{z0}) + t_{z2} \sin(k_{z0}) \end{pmatrix}. \quad (5.3)$$

Therefore the chirality is $c = 1$ ($c = -1$) for $(0, 2/3, 1/2)$ and $(0, -2/3, -1/2)$ [$(0, 2/3, -1/2)$ and $(0, -2/3, 1/2)$], which are indicated by black “+” (green “-”) in the momentum plane $(0, k_y, k_z)$ as shown in Fig. 5.4(a). From above equations, an analytical expression for Berry curvature around the Weyl point $(0, 2/3, -1/2)$ is

$$\left(k_x \frac{2\sqrt{3}\delta t_x t_y (t_{z2}-t_{z1})}{(4(\delta t_x k_x)^2 + 3(t_y k_y)^2 + 4(k_z(t_{z1}-t_{z2}))^2)^{3/2}}, k_y \frac{2\sqrt{3}\delta t_x t_y (t_{z2}-t_{z1})}{(4(\delta t_x k_x)^2 + 3(t_y k_y)^2 + 4(k_z(t_{z1}-t_{z2}))^2)^{3/2}}, k_z \frac{2\sqrt{3}\delta t_x t_y (t_{z2}-t_{z1})}{(4(\delta t_x k_x)^2 + 3(t_y k_y)^2 + 4(k_z(t_{z1}-t_{z2}))^2)^{3/2}}\right).$$

Also, we adopt the method from Ref. [21] and calculate the Berry phases of the two bands over a closed sphere of radius $r = 0.05 \times 2\pi/a$, presented in the left panel of Fig. 5.4(b), centered at a 3D degenerate node $(0, 2/3, 1/2)$. We discretize the sphere into closed loops in terms of polar angle θ (0 to π), as indicated by horizontal black circles. For each closed loop θ_i , the Berry phase is numerically calculated by using the discretized formula [83, 84]:

$$\gamma_n = -lm \ln \prod_j \langle u_{n,k_j} | u_{n,k_{j+1}} \rangle, \quad (5.4)$$

where k_j represents j th point at closed loop θ_i .

After manipulating the eigenvectors calculated theoretically, as shown in the mid part of Fig. 5.4(b), we plot the Berry phases as a function of polar angle θ . The red (blue) dotted line represents the first (second) band. The Berry phase of the lower (upper) band changes from 0 to 2π (2π to 0), which verifies that the crossing point in 3D momentum space is a Weyl node with Chern number of 1. The right inset shows the Berry curvature around the Weyl node. The same calculation can be applied to identifying the charges of other Weyl nodes.

Figure 5.4. (a) The distribution of type-II Weyl nodes in 3D first Brillouin zone. Black “+” (green “-”) indicates the positive (negative) chirality. (b) Left panel: A sphere in momentum space enclosing one Weyl point. The radius of the sphere is $0.05 \times 2\pi/a$. Right panel: Berry phase and Berry curvature around the Weyl point $(0, 2/3, 1/2)$. [75]

5.3 Fermi-arc-like surface state

The nonzero Chern numbers imply the existence of topological surface states. We investigate the 3D acoustic structure that is finite in the x - y direction and infinite in the $x+y$ and z directions. In this case, the Weyl nodes are projected along the $k_x - k_y$ direction, indicated by black and green dotted points in Fig. 5.5(d), with good quantum numbers $k_{//} = (k_x + k_y)/\sqrt{2}$ and k_z . Fig. 5.5(a-b) show the projected band structures with fixed $k_{//} = 1/\sqrt{2}$ [Fig. 5.5(a)] and $k_{//} = 0$ [Fig. 5.5(b)] in units of π/a , as indicated by “Cut 1” and “Cut 2” in Fig. 5.5(d). In Fig. 5.5(a), two surface states (red and green curves), corresponding to the two opposite surfaces, are located in an incomplete bandgap and both acquireing positive group velocity. In Fig. 5.5(b), no surface states show up. As presented in Fig. 5.5(c), the upper (lower) acoustic field corresponds to the surface state of green (red) curve in Fig. 5.5(a). We also plot the sound intensity $I = p\mathbf{v}$ (p is sound pressure and $\mathbf{v}$ is the velocity) as grey arrows, whose length represents the amplitude of sound intensity. Both the two surface states propagate along the z direction, which is consistent with the positive group velocity in Fig. 5.5(a).

To demonstrate the acoustic open “Fermi arcs,” we trace out the trajectories of surface states at frequency $0.26 \times 2\pi c/a$ (900.1 Hz) in the 2D Brillouin zone $(k_{//}, k_z)$, as indicated by red dotted points in Fig. 5.5(d). Here we consider a semi-infinite system and thus only the surface states localized at one surface [red curve in Fig. 5.5(a)] are included. These trajectories indeed connect two pairs of type-II Weyl nodes, as an analog

of open Fermi arcs in type-II Weyl semimetals [21].

Figure 5.5. (a-b) The band structures with $k_{||} = 1/\sqrt{2}$ (a) and $k_{||} = 0$ (b). The red and green curves indicate the surface states. (c) The acoustic fields of the surface states in panel (a). The acoustic fields are localized at the edges. The grey arrows represent the sound intensity. (d) Red dotted points indicate the trajectories of the “Fermi Arc” in the 2D surface Brillouin zone. Black dots (green dots) indicate Weyl points with positive (negative) chirality. [75]

5.4 Distinct features

Firstly, one significant distinction between the type-I and type-II Weyl nodes appears in the density of states. The equation of density of states is expressed as $g(\omega) = \frac{V}{(2\pi)^3} \int \delta(\omega - \omega(k)) d^3k$, where V is the volume of contributed momentum box in the vicinity of the Weyl node. For type-I Weyl nodes, the density of states vanishes at the frequency of Weyl nodes. However, the density of states acquires finite values for type-II Weyl nodes due to the presence of unbounded two-band pockets. We retrieve the parameters from fitting the data of bands with the Eq. (5.1) and plot the density of states that arises due to the type-II Weyl node in the right part of Fig. 5.3(c). The contribution of the other iso-frequency surface to the density of states is not included. The peak indicates the location of the type-II Weyl node.

Secondly, as presented in Fig. 5.5(a), the positive group velocity of the surface states is determined by the strong tilted cone spectrum of type-II Weyl nodes. For $k_z > 0$ with fixed $k_{//} = 1/\sqrt{2}$, the two surface states localized at the opposite surfaces of the system both acquire group velocities of the same positive sign. In contrast, in the previously demonstrated topological surface states [82] of acoustic type-I system, their propagation direction is surface-dependent: if the surface states on one surface propagate in one direction, those on the opposite surface should propagate in the opposite direction. This distinction is schematically illustrated in Fig. 5.6(a).

Thirdly, another distinction is that the surface states of type-II Weyl nodes stay in an incomplete bandgap. By putting sound sources with frequency $0.26 \times 2\pi c/a$ (900.1

Hz) at the surface of the acoustic type-II Weyl system, we can study the propagation features of the surface states. As shown in Fig. 5.6(b), we consider a structure that is finite in the x - y direction. Under the condition of single-frequency excitation, the surface states as well as the bulk states can both be excited by the sound source, as a consequence of the incomplete bandgap associated with type-II Weyl node. At the two bottom corners, the surface states will get scattered into the bulk because of the existence of backscattering modes. Therefore, they do not have the same robustness (scattering-immunity against defects) as demonstrated in Ref. [82], but the existence of open “Fermi arcs” connecting Weyl nodes in the momentum space is robust.

Figure 5.6. (a) The distinct diagram of the propagating directions of surface states between type-I and type-II Weyl systems. The arrows indicate the sound energy flow direction. (b) Excitations of acoustic waves in the finite systems. Two sound sources are put at opposite surfaces. Sound hard boundary conditions are applied to the left, right and bottom boundaries. The front and back boundaries acquire periodic boundary condition with $k_{//} = 1/\sqrt{2}$. The upper surfaces serve as plane wave radiation boundary conditions as indicated by black arrows, which do not reflect acoustic waves. [75]

5.5 Conclusion

The above results demonstrate the feasibility of constructing acoustic type-II Weyl nodes by stacking 1D dimerized chains of acoustic resonators. The unique features of acoustic type-II Weyl system, such as the finite density of states and transport properties of surface states, are demonstrated. The Fermi-arc-like surface states can be traced out as an analog of Fermi arcs in recently demonstrated type-II Weyl semimetals. The stacking method shown in this work provides an approach of constructing topological phases at different dimensions with the same building blocks, and may be extended to other systems including cold atoms [76, 85] and photonics [6, 27].

Summary and future work

In this thesis, I have studied two types of acoustic crystals in two dimension and three dimension, respectively. I extend the quantum Hall effect into acoustics and explore the distinct features of the type-II Weyl nodes in a 3D acoustic lattice.

Firstly, the time-reversal symmetry can be broken in an acoustic crystal by incorporating circulating air flow. We can see that the unconventional acoustic crystal exhibits a topological band gap from opening the Dirac cone. The band below the gap acquires a non-zero Chern number, which ensures the existence of the topologically protected one-way acoustic edge state.

Secondly, we construct the acoustic version of type-II Weyl Hamiltonian by stacking one-dimensional dimerized chains of acoustic resonators. This acoustic type-II Weyl system exhibits distinct features in finite density of states and unique transport properties of Fermi-arc-like surface states. In a certain momentum space direction, the velocity of these surface states are determined by the tilting direction of the type-II Weyl nodes, rather than the chirality dictated by the Chern number.

It has been about two years since the first topological acoustic paper was published. The field named “topological acoustics [44, 53, 75, 86-89]” or even bigger category “topological bosonics [27, 43, 90, 91]” has drawn a lot of attention in both theoretical proposals and experimental realizations. Most importantly, there is still much work that remains to be explored.

In the next, I suggest some of the potential projects for future work.

Firstly, we can introduce strain engineering into a two-dimensional acoustic structure in order to form a uniform effective magnetic field for airborne acoustic wave propagation. Landau levels in the energy spectrum can be formed near the Dirac cone region. We have built an experimental setup to verify the existence of acoustic Landau levels with an acoustic measurement.

Secondly, inspired by the recent progress in higher-dimensional topological phases in condensed matter physics and photonics, I try to build a two-dimensional square lattice consisting of compressibility-modulated acoustic resonators to investigate the four-dimensional quantum Hall effect. By modulating the resonators in one dimension, we can realize one-dimensional Harper model, which the on-site potential is modulated. Through adding the modulation along the second direction, we can implement 4D quantum Hall effect for acoustic waves, which is associated with nonzero second Chern numbers.

Thirdly, we can experimentally demonstrate valley-polarized kink states in valley photonic crystals, designed from a four-band model. When the valley pseudospin is conserved, we show that the kink states exhibit perfect out-coupling efficiency into directional beams, through the intersection between the internal domain wall and the external edge separating the valley photonic crystals from ambient space. Interestingly, the valley Hall effect of four-band model can be mapped into phononic metamaterials for elastic waves.

Appendix A

Plane wave expansion method

In a triangular phononic crystal with air flow, master equation is given as:

$$\frac{1}{\rho} \nabla \cdot \rho \nabla \phi - (\partial_t + \vec{v}_0 \cdot \nabla) \frac{1}{c^2} (\partial_t + \vec{v}_0 \cdot \nabla) \phi = 0, \quad (\text{A1})$$

where ρ and c are the spatially-dependent density and sound velocity of air, respectively. The velocity field has two components: $v_r = 0$ and $v_\theta = -\frac{\Omega r_1^2}{r_2^2 - r_1^2} r + \frac{\Omega r_1^2 r_2^2}{r_2^2 - r_1^2} \frac{1}{r}$, where r is measured in localized coordinates centered at each hexagonal unit cell, θ is the azimuthal angle in each unit cell.

To find solutions of Eq. (A1), we neglect terms of second order $|\vec{v}_0/c|^2 \ll 1$, and keep the first order, we have

$$\frac{c^2}{\rho} \nabla \cdot \rho \nabla \phi - 2\partial_t \vec{v}_0 \cdot \nabla \phi = \partial_t^2 \phi, \quad (\text{A2})$$

Then we perform the Plane Wave Expansion Method by expanding the velocity potential ϕ and spatial parameters in terms of plane waves,

$$\begin{aligned} \phi(r, q) &= \sum_G \phi_G e^{i(q+G) \cdot r}, \quad \frac{c^2}{\rho}(r) = \sum_G \rho'_G e^{iG \cdot r}, \quad \rho(r) = \sum_G \rho_G e^{iG \cdot r}, \\ v_x(r) &= \sum_G v_{xG} e^{iG \cdot r}, \quad v_y(r) = \sum_G v_{yG} e^{iG \cdot r}. \end{aligned} \quad (\text{A3})$$

By substituting equation (A3) into (A2), we obtain the linear equation for the Fourier component of the velocity potential,

$$\sum_{G'} \sum_{G''} \{-\rho'_{G''-G'} \rho'_{G'-G} [(q_x + G_x)(q_x + G'_x) + (q_y + G_y)(q_y + G'_y)]$$

$$-2\omega[v_{xG''-G}(q_x + G_x) + v_{yG''-G}(q_y + G_y)]\phi_G = -\omega^2\phi_G. \quad (\text{A4})$$

Numerically we can obtain the eigen frequencies and eigen states of our phononic crystal by solving above matrix.

We now analysis the Dirac cones in the vicinity of K point. In the situation we truncate the plane wave basis to the three nearest Γ points. And the three equal length reciprocal lattice vectors $\mathbf{K} + \mathbf{G}_i$ each rotated $2\pi/3$ with respect to one another. $\mathbf{K}$ is a vector pointing from Γ point to K point and $i = 0, 1, 2$ indexes the three reciprocal lattice vectors. Then in the vicinity of the K point $q_x = |\mathbf{K}| + \delta k_x$ and $q_y = \delta k_y$, where the length of $\mathbf{K}$ equals to $4\pi/(3a)$, a 3×3 Hermitian equation can be obtained. Performing the transformation $U = 1/\sqrt{3} [1, 1, 1; 1, \eta^2, \eta; 1, \eta, \eta^2]$, where $\eta = \exp(i2\pi/3)$, we can get effective Hamiltonian of the two Dirac bands by eliminating the singlet,

$$\begin{bmatrix} d(\delta\mathbf{k}, \mathbf{v}_G, \mathbf{v}'_G) & v_D(\delta k_x - i\delta k_y) + \kappa_A - i\kappa_B \\ v_D(\delta k_x + i\delta k_y) + \kappa_A + i\kappa_B & d(\delta\mathbf{k}, \mathbf{v}_G, \mathbf{v}'_G) \end{bmatrix} \phi = \Omega \phi, \quad (\text{A5})$$

where

$$\begin{aligned} d(\delta\mathbf{k}, \mathbf{v}_G, \mathbf{v}'_G) &= \omega[(v_{xG0} - v_{xG1})\delta k_x + (v_{yG0} - v_{yG1})\delta k_y], \\ v_D &= K\rho'_{Gi}(\rho_{G0} - \rho_{G1}), \\ \kappa_A &= \frac{\omega K}{2}[v_{xG1} - v_{xG0}], \\ \kappa_B &= \frac{\omega K}{2}[v_{yG1} - v_{yG0}], \\ \Omega &= \omega^2 + \frac{K^2}{2}\rho'_{Gi}(2\rho_{G0} + \rho_{G1}). \end{aligned} \quad (\text{A6})$$

And $i=0, 1, 2$ indicates the summation of three components.

We can rewritten equation (A5) as

$$H(k) = d(k)I + (v_D \delta k_x + \kappa_A) \sigma_x + (v_D \delta k_y + \kappa_B) \sigma_y. \quad (\text{A7})$$

where I is 2×2 identity matrix. Under transformation $\sigma_i \rightarrow -\sigma_i$ and $k_i \rightarrow -k_i$, we know that $T^\dagger H(k) T \neq H(-k)$, which indicates that the time-reversal symmetry is broken.

As we can see that (A7) will be mapped to massless Dirac-like equation if there is no air flow. And the linear dispersion is,

$$\omega = \omega_D \pm v_D |\delta \mathbf{k}|. \quad (\text{A8})$$

The flow induced the first term $d(k)$ as shown in Eq. (A7), which is small when the circulation is less than $\Gamma_c = 2\pi \times 4$, can shift the global energy at the Dirac point. The terms κ_A and κ_B changing the momentum can be simply understood as Doppler effect [3]. Using the expression for the Dirac point, we find the dispersion relation of the Dirac bands is now given by

$$\omega = \omega'_D \pm \sqrt{|v_D \delta k_x + \kappa_A|^2 + |v_D \delta k_y + \kappa_B|^2} \quad (\text{A9})$$

The degeneracy is lifted and the band gap at Dirac point is $|\kappa| = \sqrt{|\kappa_A|^2 + |\kappa_B|^2}$ determined by reciprocal components of flow velocity. Similar method can be applied to K' point.

Appendix B

Zero-energy time-independent Schrodinger-type equation

To map the triangular acoustic crystal with circulating air flow to a two-dimensional quantum Hall system, we start from the governing master equation Eq. (A1) in the main text

$$\frac{1}{\rho} \nabla \cdot \rho \nabla \phi - (\partial_t + \vec{v}_0 \cdot \nabla) \frac{1}{c^2} (\partial_t + \vec{v}_0 \cdot \nabla) \phi = 0. \quad (\text{B1})$$

Using the fact in our model that $\vec{v}_0 \cdot (\nabla \frac{1}{c^2}) \partial_t \phi = 0$, $\frac{1}{c^2} (\nabla \cdot \vec{v}_0) \partial_t \phi = 0$, we have

$$\nabla \cdot \left(\frac{\vec{v}_0}{c^2} \partial_t \phi \right) = \frac{\vec{v}_0}{c^2} \cdot \nabla \partial_t \phi = \vec{v}_0 \cdot \nabla \left(\frac{1}{c^2} \partial_t \phi \right). \quad (\text{B2})$$

Thus master equation can be rewritten as:

$$\nabla \cdot \nabla \phi + \frac{1}{\rho} \nabla \rho \cdot \nabla \phi - 2 \frac{\partial_t}{c^2} \vec{v}_0 \cdot \nabla \phi - \frac{\partial_t^2}{c^2} \phi - \vec{v}_0 \cdot \nabla \left(\frac{1}{c^2} \vec{v}_0 \cdot \nabla \phi \right) = 0. \quad (\text{B3})$$

And then we take time dependent term $\exp(-i\omega t)$ and let the vector potential be

$\vec{A}_{eff} = -\frac{\omega \vec{v}_0(x,y)}{c^2}$, Eq. (B3) is now

$$\nabla \cdot \nabla \phi + \frac{1}{\rho} \nabla \rho \cdot \nabla \phi - 2i \vec{A}_{eff} \cdot \nabla \phi - \frac{\partial_t^2}{c^2} \phi - \vec{v}_0 \cdot \nabla \left(\frac{1}{c^2} \vec{v}_0 \cdot \nabla \phi \right) = 0. \quad (\text{B4})$$

Expressing first three terms in Eqn. (B4) in terms of $\Psi = \phi \sqrt{\rho}$, we have

$$\sqrt{\rho} \nabla^2 \frac{1}{\sqrt{\rho}} \Psi = [\nabla^2 - \frac{1}{2} \nabla^2 \ln \rho + \frac{1}{4} |\nabla \ln \rho|^2 - \frac{1}{\rho} \nabla \rho \cdot \nabla] \Psi, \quad (\text{B5})$$

$$\sqrt{\rho} \frac{1}{\rho} \nabla \rho \cdot \nabla \frac{1}{\sqrt{\rho}} \Psi = \left[\frac{1}{\rho} \nabla \rho \cdot \nabla - \frac{1}{2} |\nabla \ln \rho|^2 \right] \Psi \quad (\text{B6})$$

$$-\sqrt{\rho} 2i \vec{A}_{eff} \cdot \nabla \frac{1}{\sqrt{\rho}} \Psi = [-2i \vec{A}_{eff} \cdot \nabla + i \vec{A}_{eff} \cdot \frac{1}{\rho} \nabla \rho] \Psi \quad (\text{B7})$$

In the main text, we study our acoustic system under the condition of low Mach number, which means $|\vec{v}_0/c| \leq 0.3$. Therefore by neglecting second order terms ($|\vec{v}_0/c|^2 \ll 1$) and substituting Eqn. (B5, B6, B7) into Eq. (B4), we finally arrive at the Schrodinger type equation

$$\left[(\nabla - i \vec{A}_{eff})^2 + V(x, y) \right] \Psi = 0, \quad (\text{B8})$$

where

$$\vec{A}_{eff} = -\frac{\omega \vec{v}_0(x, y)}{c^2} \quad (\text{B9})$$

$$V(x, y) = -\frac{1}{4} |\nabla \ln \rho|^2 - \frac{1}{2} \nabla^2 \ln \rho + \frac{\omega^2}{c^2}. \quad (\text{B10})$$

The Eqn. (B8) is the equation for zero-energy wave functions in periodic vector and scalar potentials, which can map our model to a similar problem in a quantum Hall system. We should note that we simplify our formulas by neglecting the second order perturbation, but the vector potential is also a function of frequency. Therefore the formulas work well for our acoustic system operating at the low frequency energy bands. However when we consider the system in much high frequency region, the term $|\vec{A}_{eff}|^2$ contributing to scalar potential cannot be neglected any more in the above derivation

procedures.

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