DOCTORAL THESIS

Development of a Robotic Running Foot for Footwear Testing

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Abstract

In footwear testing, the human wear participation method requires a long period and high cost to conduct the experiments and may expose the participants to injury risks. The virtual testing method, which uses computer simulations, faces significant challenges in modeling the complex geometry and realistic testing conditions. Thus, with the ability to stably continuously repeat thousands of testing cycles to reduce the testing time, a third method which uses mechanical devices and automatic systems is preferred in footwear testing. However, available systems do not possess a realistic representation of the human foot, cannot reproduce the wear conditions accurately, and do not mimic properly human running gaits.

This thesis addresses the development process of a robotic running foot (RRF) for footwear testing. The first goal was to design and manufacture a realistic robotic foot which possesses multiple powered degrees of freedom (DOFs); and has sufficient torques, speeds, and ranges of movement to mimic human running gaits and replicate wear conditions. The second goal was to develop novel control strategies for driving the foot joints to mimic human running gaits with very high loads, high frequency, and discrete loading conditions in different phases of the gait cycle.

Firstly, based on reviews of contemporary footwear testing systems, innovative robotic feet, and human foot biomechanics in running, some essential requirements for a robotic foot for footwear testing were investigated, and various possible solutions were proposed and analyzed. Then, a robotic foot with two powered joints (i.e., the ankle and metatarsophalangeal [MTP] joints) actuated by two pairs of cable-conduit mechanisms (CCMs) was constructed. The foot joints possess sufficient power to mimic human running gaits up to 14 km/h.

Secondly, three studies on torque tracking control for robotic joints actuated by the CCMs were carried out to develop the most effective strategy for the foot joints. Each study was conducted under different conditions and references. Each proposed a torque transmission model and developed an adaptive controller to compensate for the nonlinear hysteresis of the torque transmission profile. Both theoretical proof and experimental validation were carried out to evaluate their torque tracking performance. These control methods are important for applications actuated by

CCMs. They can be adopted directly if the applications require torque/force control and have available output feedback.

Finally, an overall design for an integrated footwear testing system (IFTS) was developed to mimic human running gaits. This IFTS consists of two DOFs of an upper-leg mechanism, two DOFs of the robotic foot, and two DOFs of a contact platform. The final IFTS was equipped with abundant feedbacks allows for applying effective closed-loop control. Meanwhile, many human running and walking trials of eight healthy adults were conducted to collect a reference data pool. Then, a hybrid control design for the foot joints and a motion planning algorithm for four other actuators were developed and implemented to control the IFTS mimic the ground reaction forces and human foot profile of a running trial at about 12 km/h in the reference data pool. Then, the RRF and IFTS can be used for footwear durability tests including the tests for lifespan of individual components or of the whole shoe (the weakest part). However, there are many other tests can be done by the systems such as friction (slip resistance), abrasion of the outsole, or the tests for surface performance under effects of running, walking gaits.

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Chapter 1

Introduction

1.1 Background

In sports and exercise, when athletes and runners endure large reaction forces and dynamic pressures at their feet, their footwear is the foot's protecting shield to cope with these harsh conditions. It is evident that the shoes have significant influences on the athletes' performance. Good-quality shoes can protect the feet, provide comfortable wear conditions, and improve running economy and sports performance [1-6]. The effects of footwear on athletes' feet have been widely studied and analyzed for many years [1-7], and the results of these studies can be used to develop better shoes.

During the shoe development process, a new design is assessed thoroughly by many criteria and procedures. Thus, shoe developers and manufacturers require a swift and reliable testing method to evaluate their designs quickly and accelerate the development process. Also, the footwear industry has fast grown to be one of the highest income sectors in the world [8] due to the increased demand for high-quality footwear. As a result, there is more investment and endeavor in the footwear research and development sector, then many systems have been introduced to test different shoe characteristics. Modern footwear tests include the tests of slipping resistance, shoe safety, toecap impact resistance, heel attachment and seam strength, abrasion

resistance, and wear durability. Amongst these, the durability and abrasion/wear resistance of the shoe are the two most popular and time-consuming tests.

Conventionally, there are two primary methods to assess the durability and wear resistance of a shoe. Footwear developers can either conduct some actual wear trials over several months with a group of human wearers (human participation method) or use grit paper to carry out a quick standard abrasion test in the laboratory. However, these methods can provide conflicting results, and none of them is ideal for evaluating the wear resistance/durability of the shoe [9]. Thus, many researchers and inventors have been attempting to develop a mechanical system which can simulate human gaits, reproduce the actual wear conditions, and assess a shoe automatically and realistically.

One of the most well-known footwear testing systems is the SATRA STM 528 Pedatron™, developed by SATRA (Shoe and Allied Trade Research Association) Technology Centre. It is an abrasion testing machine which mimics the biomechanical pattern of human walking gaits and assesses the shoe's durability. In this system, an imitative foot wears the shoe and "walks" on a contact platform. The machine can synchronize the movement of the foot-shoe and the rotation of the platform to alternate straight steps and turning steps. Eventually, the user evaluates the effects of the constructional factors of the footwear by measuring the amount of sole loss during the test [9]. Figure 1-1(a) shows the structure of this machine and Figure 1-1(b) illustrates a testing session on the rotating platform. This machine is a very practical footwear testing system based on mimicking the biomechanical pattern of human walking gaits. There are many other available footwear testing systems which will be reviewed thoroughly in Chapter 2.

Figure 1-1: The SATRA STM 528 Pedatron™ (a): The machine's primary structure, (b): A shoe test with the rotating contact platform.

However, there exist two principal drawbacks of those automatic footwear testing systems (including the SATRA STM 528 Pedatron™).

- i. Firstly, the imitative feet in these systems are all passive with no degree of freedom (DOF) whereas there are significant movements of the human foot joints during any gait cycle. Thus, there is a certain discrepancy between the dynamic patterns and effects generated by these systems and the human foot.
- ii. Secondly, these systems are only able to mimic human walking gaits but cannot replicate the high dynamic behaviors of the running and sprinting strides involved in many sports movements.

1.2 Motivations

Currently, there are three primary approaches to evaluate a new footwear design, which are: (i) to use mechanical devices; (ii) to utilize computer simulations; or (iii) to conduct some human wear experiments. All these methods still contain significant limitations. Firstly, the human wear participation process is too costly, time-

consuming, and may expose human subjects to injury risks. Secondly, the virtual simulations often encounter challenges in accurately modeling the testing conditions and complex geometry. Thirdly, available mechanical devices and automatic systems still lack bio-fidelity and the capability to reproduce wear conditions in different gaits realistically.

Nevertheless, a practical automatic system for footwear testing is in high demand due to the ability to continuously repeat thousands of cycles consistently and allow the users to flexibly change the wear conditions such as the subject weight, gait types, speed, and patterns. This method can save extensive time and expenditure for conducting human wear experiments. This project focuses on developing an innovative system which can overcome the limitations of the currently available systems. This Ph.D. candidature focuses on developing a robotic foot which has multiple DOF and can mimic the behaviors of the human foot in running gaits. This robotic foot will ‘wear’ the shoe and reproduce foot-shoe interactions during running in order to evaluate the shoe characteristics.

1.3 Objectives and Scope

The ultimate goal of this project is to develop a robotic footwear testing system which could assess multiple features of the shoe. However, the system primary objective is to test the shoe’s durability. This will be based on reproducing biomechanical effects of the human foot and ground conditions on the shoe during different gaits. This system should overcome the limitations of the available systems and offer some other advantages:

- i. It aims to solve the challenges of mimicking running gaits, as most sports and exercise involve running. Thus, it will be more applicable and useful in testing new footwear designs.

- ii. It should include multiple powered DOFs at the foot to simulate the foot-shoe interactions more accurately. The closer these interactions are to those by the human foot, the more accurate and reliable the testing data will be.
- iii. It should also include an upper-leg mechanism for supporting and driving the robotic foot to the correct positions and applying the body mass effects on the lower foot and shoe.
- iv. It should also feature a contact platform which can adapt to different ground conditions so that the system can assess the shoe under various conditions.
- v. Ultimately, it can equip with many in-shoe sensors that provide more information about the conditions inside the shoe such as humidity, temperature, and plantar pressure distribution. These parameters allow the users to evaluate the wear comfortability, waterproof, temperature isolation, and so forth.

Then, the scope of this Ph.D. candidature includes:

- i. Designing a robotic running foot (RRF) for footwear testing. This robotic foot must have (i) a similar shape, size, and appearance to a real human foot, (ii) multiple powered DOFs, and (iii) sufficient power, speed, and range of motion to mimic the human foot biomechanics in running.
- ii. Developing an effective torque mode control strategy for the RRF to mimic the human gaits. The controller has to drive the system to fast-response to the high frequency of the running strides. Also, the system needs to adapt to the discrete loading condition in different running phases.
- iii. Integrating the robotic foot into the upper-leg and contact platform mechanisms. The mechanical design of these two mechanisms are developed by the candidate's colleagues. However, the integrated structure, the electronic & control hardware and software for the integrated machine are developed by the candidate.
- iv. Developing an effective control algorithm for the integrated system and evaluating its ability to mimic the human biomechanics of running with experiments. To obtain essential reference data for applying to the system,

the candidate will conduct some human running experiments with a motion capture system synchronizing with a force plate.

The upper-leg and contact platform mechanisms will be developed by other team members. The integration of new in-shoe sensors and the environmental chamber development will be developed in the later stages of this project.

1.4 Applications, Significances, and Innovations

The current RRF for footwear testing possesses two powered DOFs at the ankle and metatarsophalangeal (MTP) joints and the ability to simulate the human running gaits up to 14 km/h. These two powered joints help simulate more accurately complex dynamic pressure and effects of the human foot on the shoe during gait cycles. This RRF wears the testing shoe and is integrated into an upper leg mechanism and a contact platform to form an integrated footwear testing system (IFTS). This IFTS has six powered DOFs in total and can simulate different types of gaits of the human foot-leg.

The system can run continuously repeatedly hundreds of thousands of cycles which are equivalent to the wearing distance of a prolong human wear participation testing period. After being worn on this IFTS, the shoe will be assessed its physical properties and compared with the initial values. The users can observe the test with this IFTS to identify which is the weakest part of the shoe or to measure the lifespan of each individual component such as the outsole, insole, heel counter, toe box, last, tongue, collar, or midsole. This is the current approach for footwear durability tests with this IFTS and RRF.

In addition, this RRF can integrate with various types of sensors (e.g., the temperature sensor, humidity sensor, and the in-shoe plantar pressure sensor). These sensors data can be used to evaluate many other footwear characteristics such as the cold/heat intrusion resistance, the water/moisture repellency, and especially the

accuracy of the reproduced plantar pressure in running. These are important footwear characteristics that any footwear developer desires to evaluate for their product. For instance, the water/moisture repellency, which is the ability to keep the footwear dry, makes the wearer feel more comfortable and reduces greatly the incidence of many types of foot disorder. The system can also be used to evaluate the shoe physical properties such as the shock absorption ability, slipping resistance, abrasion. All these tests can be supported by this RRF and IFTS with some certain modifications.

This is an innovative robotic system for evaluating different footwear characteristics. It presents an effective method to evaluate the running shoe and provides reliable testing results. The test results will not only help footwear developers improve their designs but also present criteria for customers to trust and base their choice of products upon. Also, it will make athletes and casual runners feel more confident in their training shoes, potentially improving their performance.

The ability to repeat continuously the cycles that this system introduces to the field will help shoe designers make the design iterations very quickly. They can quickly have a quantitative measurement of the effect of a design change to a prototype. The system stable repeatability produces more consistent results. These may change the future testing approach in footwear development as the system can replace the human wear participation method.

This system also can be used to evaluate the textile designs (e.g., the new carpet, covering system, or artificial turf) by replacing the upper layer of the contact platform with the testing surface. Then, the effects of walking, running, jumping steps with different types of shoes on the testing surface can be observed during the test. Also, their properties can be quantified and compared before and after the test. Thus, this IFTS is also a useful tool in textile and sports surface industries. In

addition, a robotic system, which can mimic human biomechanics of the lower limb in various types of gaits, can also be used for academic research.

Finally, the torque control studies in this thesis can improve the understanding of the nonlinear dynamic friction and torque transmission characteristics of the cable conduit mechanisms (CCMs) at high-loading working conditions at different frequencies. These control schemes guarantee global stability and improve the accuracy of torque/force control of the systems powered by CCMs. There is currently a lack of these reports in the literature.

1.5 Outline of the Thesis

This thesis has six chapters in total. Chapter 1 has introduced some background of the field, some weaknesses of available systems which motivated the current study, the ultimate objectives of this project and the scope of this Ph.D. candidature. It also explained some expected contributions and innovations of this research. Chapter 2 will review and discuss the strengths and weaknesses of available automatic footwear testing systems, some innovative robotic/prosthetic feet. It will also examine the biomechanics of running and walking gaits, and analyze which factors have significant effects on the footwear. Chapter 3 will explain the design process for the robotic foot including the fundamental, mechanical, electronic, and control structures. Chapter 4 will explain in depth the torque mode control for robotic joints actuated by CCMs. It includes the details of three studies and experiments. Chapter 5 will explain the principles of the whole footwear testing system. It will introduce the preliminary works and focus on discussing the design of the integrated system. It will also explain the development and experiments on mimicking human biomechanics during running. Finally, Chapter 6 will provide some discussion, conclusions, and future directions.

Chapter 2

Literature Reviews

The first part of this chapter will go through the running biomechanics of the human foot and leg in the literature, analyze and identify the critical anatomical components and biomechanical parameters that a robotic running foot should replicate. The later parts will review the available footwear testing machines and other state-of-the-art prosthetic feet which might be used to wear the shoe in footwear testing. These reviews provide a basic understanding from which to determine which features should be developed to achieve a practical and innovative robotic foot for footwear testing.

2.1 The Human Foot Biomechanics in Running

This project focuses on the running gaits because running is the dominant movement in many sports and exercise activities. In addition, it is noted that if a robotic system can replicate running gaits, it can also satisfy the requirements of mimicking walking gaits.

2.1.1 Key Structures of the Human Foot during Gait

The human foot anatomy and movements are highly sophisticated. Each foot consists of 26 bones, 33 joints, and 107 muscles, tendons and ligaments that form the rearfoot, midfoot and forefoot. This section focuses on identifying the most important anatomical structures that the robotic foot should and can possess.

2.1.1.1 The Ankle Joint

The ankle joint, supported by one of the strongest tendons of the human body, the Achilles tendon, is a critical joint of the human foot in locomotion. Therefore, most previous foot-related orthotic and prosthetic studies and products included the ankle joint but no other joints of the foot. As shown in Figure 2-1, the range of motion of the ankle joint in running is significantly dominant in the sagittal plane (around 50° [10] to 52° [11]) compared to small movements in the transversal plane (about 12° [10] to 20° [11]) and a very restricted movement in the frontal plane (about 6° [11]).

Figure 2-1: Ranges of motion of the ankle joint (the ankle joint complex) in walking [12, 13] and in running [10, 11] in all three planes of locomotion.

Other reports in walking [12, 13] also showed that the ankle joint is much more flexible in the sagittal plane than in other planes. Hence, the robotic foot focuses on actuating the ankle joint in the sagittal plane, motions in other planes may be compensated for by relative motions from the moving contact floor.

2.1.1.2 The Metatarsophalangeal (MTP) Joint

Although some studies [14, 15] reported that the MTP joint contributed only a small amount of energy in the propulsion phase of running, its potential and kinetic moments in locomotion were noted as significant in comparison to the ankle joint moments as reported in other studies [16, 17]. Among the other joints of the foot, the MTP joint has the largest range of movement [18]. The plantar pressure under the hallux and other toes is considerable when compared with other areas of the foot [19-21], and the deformation of the sole directly depends on this plantar pressure. Hence, the MTP joint of the robotic foot will be powered to attain more accurate effects of the robotic foot on the shoe.

2.1.1.3 The Midtarsal joint – Windlass mechanism

Compared to the hindfoot with the crucial role of the ankle joint and the forefoot with the flexibility of the toes, the midfoot which comprises themidtarsal joints and tarsometatarsal joints (tarsometatarsal joints are relatively rigid, and only minor movements occur in these joints) has the smallest range of movement. However, the arch of the foot absorbs a significant portion of the energy in the absorption phase and releases this energy in the propulsion phase to aid the body in moving forward [10, 22]. Ker *et al.* [22] estimated that in 100 J of the turnover energy of a 70 kg man running at 4.5 m/s, the strain energy stored in the Achilles tendon and the arch of the foot are 35 J and 17 J respectively. Recently, Song *et al.* [23] investigated the energetic cost of a foot model using a windlass mechanism in walking and proved that the windlass mechanism design could reduce this energetic cost by 15%. Hence, ultimately, the robotic foot should include the midtarsal joint and windlass mechanism to be more realistic.

2.1.2 Critical Biomechanical Factors in Running

2.1.2.1 Foot-strike Pattern

In running gaits, a cycle is usually defined as it starts when a foot contacts the ground and stops when the same foot contacts the ground again. A cycle includes two main phases: the contact phase when the foot is in contact with the ground, and the swing phase when that foot is in the air. At the initial contact, there are three types of foot landing: heel strike, midfoot strike, and forefoot strike. Depending on the shoe cushion quality, ground condition, running speed, and the runner's habit, he/she will land on his/her rearfoot (heel strike), forefoot (forefoot strike), or the whole foot (midfoot strike).

In general, when the running speed increases, runners would change the landing contact from the rearfoot to the forefoot [10, 24]. Typically, this change marks the distinction between running and sprinting [10]. Heel landing is seen in distance running where the runners are required to maintain aerobic metabolism and conserve energy to run long distances. Forefoot landing is often performed in short distance sprinting where faster speeds are demanded and the rearfoot of athletes may never contact the ground [10] during the running cycles. Figure 2-2 illustrates the position of the human foot during a gait cycle in heel strike shod running (upper panel) and in forefoot strike barefoot running (lower panel) [25].

Figure 2-2: Vertical ground reaction forces and foot positions for two types of foot strikes at 3.5 m/s of the same runner [25]. Upper panel: heel strike shod running; lower panel: forefoot strike barefoot toe-heel-toe running.

In addition, about 70% to 90% of distance runners are heel strikers, based on data from many different studies [10, 24-28]. This project focuses on testing the shoe durability in their major use, for runners and athletes in distance running and daily exercise. The speed in these running practices is considered as ‘normal speed’ and will be discussed in the later section. Thus, mimicking **heel strike** running gaits at

normal speeds is the primary goal of this project. Sprinting gaits at higher speeds are future targets.

2.1.2.2 Ground Reaction Forces

The ground reaction forces are critical parameters allowing calculation of gait kinetic variables and are also the most important factors for determining dynamic effects on the shoe. The total ground reaction force (GRF) can be divided into three component forces: the vertical GRF, the Anterior-Posterior (AP) force, and the Medial-Lateral (ML) force. As the GRFs are only active (non-trivial) in the contact phase, researchers often only concern themselves with the contact phase from the initial contact until the foot leaves the ground. During the stance phase, typical graphs of vertical GRF with the corresponding foot position of both the heel strike shod running and forefoot strike barefoot running are also portrayed in Figure 2-2 by Lieberman *et al.* [25]. The horizontal axis shows the actual running time in second while the vertical axis shows the vertical GRF normalized by the runner's body weight.

In the heel strike running gait, the first peak represents the impact of the heel and the ground in landing. The heel cushion of the shoe should aid the foot in absorbing the shock impact, and thus reducing this first peak. The second peak marks the end of the absorption sub-phase (deceleration) and the beginning of the propulsion sub-phase (acceleration). In running, the vertical GRF may increase to about three (3) times the runner's body mass at the second peak [10], and thus the foot and leg muscles must work very hard to propel the body. This causes a great challenge for any robotic systems to mimic/replicate the running gaits.

Figure 2-3 shows the graphs of other GRFs: the Anterior-Posterior (AP) force and the Medial-Lateral (ML) force in overground and treadmill running reported by Riley *et al.* [11]. The horizontal axis shows the stance phase timing in percent of the

whole gait cycle and the vertical axis shows values of the forces in percent of the runner's body weight. It is obvious that the both AP and ML forces are significantly smaller than the vertical GRF, and thus one might prioritize the vertical GRF in developing systems for mimicking the running biomechanics. For instance, in the same study [11], Riley *et al.* reported that the vertical GRF peak was about 263 %BW (percent body weight) while anterior GRF peak was 37 %BW and medial GRF peak was 11 %BW.

Figure 2-3: The Ground Reaction Forces: Anterior-Posterior (AP) and Medial-Lateral (ML) in the stance phase for overground (black lines) and treadmill (gray lines) running [11]. Forces are in percent body weight.

2.1.2.3 Foot Kinematics

The foot position during a running cycle is very different for each type of strike pattern as one can observe from Figure 2-2. The foot and toe position profiles are critical for calculating the foot joints' kinematics and dynamics. Also, these profiles are the key parameters determining the dynamic effects on each section of the shoe. The kinematic and kinetic data of the foot joint/segment are provided in detail in Chapter 5 as the results of the human running experiments, a part of this Ph.D. candidature scope. The following sections only review the general range of motion, joint angular velocity, moment and power, and the foot relative speed at the impact to determine the specifications of the robotic foot and part of the overall footwear testing system.

2.1.2.4 Foot Plantar Pressure and the Center of Pressure Profile

It is obvious that the footwear durability depends on the wear conditions including both environmental and biomechanical conditions. The environmental effects include the ground material, roughness, steepness, temperature, and humidity. The biomechanical effects on the shoe are more complex and may be represented by the varying pressure distribution (or interaction force) between the human foot and the shoe during the contact phase. The influence of the foot-shoe interaction force in the swing phase on the shoe properties is negligible. Based on the direction and magnitude of the GRF, one can observe that the vertical GRF and plantar pressure have the most significant impacts on the shoe.

According to the literature [20, 21, 29-33], the plantar pressure distribution under the foot not only varies between different areas (e.g., the heel, toes, metatarsal heads and midfoot) but also fluctuates during the stance phase of a running cycle. The shape of the foot sole causes variations in plantar pressure distribution while its biomechanical pattern results in fluctuation of this pressure during the stance phase.

Figure 2-4: Peak plantar pressure (kPa) at 10 sectional areas under the foot

Figure 2-4 shows the data of the peak plantar pressure at 10 sectional areas under the foot in a study of Schlee *et al.* [31]. The data were collected with 19 young recreational runners (body mass 70.5 ± 6.3 kg) who ran at 3.5 ± 0.1 m/s before and

after 45 minutes of running on a treadmill. The center of pressure moves substantially during the running cycle [10, 30, 32-34]. Figure 2-5 shows the average position of the center of pressure (CoP) with a rough illustrated outline of the foot of ten (10) participants running at 3.3 ± 0.41 m/s (with shoe: black lines) and about 3.27 ± 0.42 m/s (barefoot: gray lines) in a study of Becker *et al.* [33].

Figure 2-5: The average position of the center of pressure during the contact phase of running with a rough illustrated outline of the foot in [33]: (A) anterior-posterior position time series, (B) medial-lateral position time series, (C) combined position time series on a right foot outline.

2.1.3 Typical Values of the Human Foot Biomechanics in Running

2.1.3.1 Normal Speeds in Long Distance Running

There are many studies on running in the literature with diverse research purposes. Typical speeds in running-related studies range from 2.2 m/s to 4.12 m/s [10, 11, 14, 25, 34-41]. In all these studies, one can observe that the average running speed is about 3.2 - 3.3 m/s (11.5 - 12 km/h). In some specific studies [24, 42], when

researchers want to do some comparisons, they may also choose higher or lower running speeds. However, this project focuses on testing the shoe in recreational and long-distance running for the majority of runners. Although distance running speeds may range from 2.5 to 6.1 m/s, Cavanagh and Kram [41] pointed out that the steady-state of endurance running speed for recreational joggers is from 3 to 4 m/s. Bramble and Lieberman agreed with them [40]. In addition, according to Laurie Villareal [43], a professional running coach, the ‘easy run’ and ‘long distance run’ speeds for the majority of runners are equivalent to about 9.5 to 9.7 km/h. Hence, in this project, the team considers the normal speed in distance running (i.e., the normal running speed) as 9.5 to 12 km/h.

2.1.3.2 Ankle Joint Kinetics and Kinematics

There are many reports on the biomechanics of the ankle joint during running. Figure 2-6 shows the adapted data from a study of Riley *et al.* [11]. The figure plots the time series of the ankle angle, moment, and power in one running cycle. A running cycle starts when the analyzed foot contacts the ground and ends when the same foot touches the ground again. The study involved 20 young healthy subjects running at 3.84 m/s (13.8 km/h). It is noted that the maximum running speed that our team wants the footwear testing system to achieve is about 14 km/h.

In Figure 2-6, during a running cycle, the ankle dorsiflexes slightly at impact to absorb the shock and then it plantar-flexes substantially (from about $+28^\circ$ to -23°) during the propulsion phase. The ankle’s moment and power are normalized by the subject body mass. The average mass is 66.4 ± 11.2 kg. The ankle maximum moment is about 3.44 Nm/kg while its maximum power is about 16 W/kg. The contact phase is about 36% of a running cycle.

Figure 2-6: The typical angle, moment and power of the human ankle joint in the sagittal plane in running overground at 3.84 m/s (~ 13.8 km/h) [11]. On the vertical axis, positive values represent dorsiflexion while negative values represent plantarflexion. The moment and power are normalized by the body weight of the human subjects.

2.1.3.3 MTP Joint Kinetics and Kinematics

Figure 2-7 shows the MTP joint moment and net power in a study of Stefanyshyn and Nigg [14] when five (5) trained male athletes ran at 4.0 ± 0.4 m/s (~ 14.4 km/h). The subjects' average body weight was 67.4 ± 3.5 kg. During the stance phase, the MTP joint average plantar flexion moment could reach 59.9 Nm. Also, the average net power of the MTP joint was about 20.6 J during the running cycle. The potential range of movement of the MTP joint is up to 45° dorsiflexion [44]. Detailed angle profile of the MTP joint during running can be found in Section 5.2 of this thesis.

Figure 2-7: The typical moment and net power of the MTP joint in running at 4 ± 0.4 m/s in [14]. The upper panel plots the MTP joint moment while the lower panel shows its net power during the stance phase of running. The horizontal axis represents the normalized stance phase from heel strike to toe-off. The continuous lines represent the data feedback by the force plate while the dash lines represent the combined data of an insole pressure sensor and the force plate.

2.1.3.4 Foot Velocity

The foot velocity at ground contact is very important when designing the footwear testing system. More specifically, the foot horizontal and vertical velocities are required to design the actuators for driving the contact platform and the foot-shoe block to simulate the impact more accurately. Figure 2-8 shows an illustration of the foot at the contact moment in running overground at 3.6 m/s reported by Cavanagh [34]. The figure shows the vectors of absolute velocities with respect to the ground of three different points on the shoe. These points represent the position of the heel, midfoot, and forefoot. de Wit *et al.* [45] also reported the heel velocity at the touchdown and impact moments during heel strike distance running at three 3.5, 4.5,

and 5.5 m/s. The average horizontal heel velocity at touchdown of 3.5 m/s running was about 1.11 m/s which is in agreement with Cavanagh's data [34] in Figure 2-8.

Figure 2-8: The absolute velocities of three points on the shoe (representing the heel, midfoot and forefoot) with respect to the ground at ground contact in running overground at 3.6 m/s (mean data of 10 subjects) by Cavanagh [34].

2.2 State of the Art in Footwear Testing

This section reviews and discusses the three primary directions in the field of developing methods for footwear testing: the actual human-wear experiments, the computer simulations, and the mechanical testing systems. This section will focus on the available mechanical devices and automatic systems because they are the project primary targets.

2.2.1 Human Participation Method

Nowadays, footwear testing still relies on the human wear participation method, in which a group of human subjects wears the footwear during their daily life over a period of a few months, and then the mechanical properties of the shoe will be compared before and after the intervention [46-48]. For example, Hennig and his colleagues [46] at the University Essen, Germany, had done 18 years of footwear testing for 156 shoe models by using mainly the human wear methods. During 18 years from 1991 to 2009, they developed and used a testing protocol which included the material test and the biomechanical wear tests in the lab and on the field. The biomechanical wear tests were crucial in evaluating the new shoe models and required about 20 to 30 healthy subjects to participate, wear the shoes, and use them in running for a few months before the shoes could be re-assessed. The period of wearing depends on the actual running distance that the shoe was in use. Footwear testers often require a running distance of about 600 km to 1000 km to evaluate the durability of the shoe and its components. This process takes a long time to complete the required distance. For instance, the field test of Hennig's studies took about 8 to 12 weeks. In the test of multiple shoe models, the group of runners could alternately use the shoes to complete a distance of 1000 km running as in their other report [47]. In this study, the properties of three shoe models were evaluated after being worn during 50, 200, 500, and 1000 km of running at about 3.3 m/s by five runners. Another group, Heidenfelder *et al.* [48], also tested the deterioration of six running shoe models after 100, 300, and 600 km of running at about 3.5 m/s by 24 healthy runners.

These tests were conducted under the most realistic testing conditions; however, this method has a few significant disadvantages. It requires a very extended period for the experiments to complete the required working distance (e.g., from 600 km to 1000 km). This method also requires a considerable effort and expense for recruiting the

subjects and conducting the tests. Thus, it usually has limited numbers of participating subjects and experimental trials. The results are also varied due to the range of personal interest and the difficulties of measuring and controlling the environmental conditions. More importantly, human participation method is inapplicable for tests that may expose the participants to the injury risks such as the shock absorption ability test.

2.2.2 *Virtual Testing Methods*

Another approach is to use computer simulations such as the finite element method (FEM) [49-51], or the differential modeling method [52] to model the human foot and the foot-shoe interactions to evaluate the footwear's characteristics. For example, Rupérez *et al.* [51] developed a linear elastic model based on the FEM to simulate the force distribution and the deformation of materials of the shoe upper. Azariadis *et al.* [52] proposed using a virtual procedure, called as the Virtual Shoe Test Bed, to simulate the foot-shoe primary interactions which affect the performance of the shoe. Also, Tang and Hui [50] developed a virtual model of the foot motion and deformation and its pressure on the footwear during the gait motion. All these models were used to simulate the effects of the human foot on the shoe virtually so that the shoe developers can analyze their design without the need of a real physical prototype. Thus, this method can accelerate the design process. However, although the simulation tests provide swift test results and require no excessive cost, this approach has some significant limitations.

Because of the elaborate structure of the foot with many different bones, joints, tissues, ligaments, and muscles, only simplified constitutive models and material properties are used [53] in modeling the human foot. Consequently, the complex finite element analysis may consist of many potential errors. Also, researchers have to confront many challenges when modeling the complex geometry and anatomy of the human foot. Furthermore, in virtual footwear testing, more assumptions and

simplifications are made in modeling the wear conditions in different phases of the gait cycle [54].

2.2.3 Mechanical Footwear Testing Systems

The approach of using mechanical devices can overcome the limitations of other methods. It also offers significant advantages such as stable repeatability, adjustable condition parameters, and time-saving processes. Automatic footwear testing systems not only have the ability to repeat consistently over thousands of cycles continuously but also allow the users to flexibly change the wear conditions such as the subject weight, gait types, speed, and patterns. This method can save the extensive time and expenditure of conducting human wear experiments. The prior art of footwear testing devices has resulted in numerous machines which can replicate different wear conditions, from the impact, friction, and abrasion with the ground to the humidity, and temperature of the environment to evaluate the materials and design of the shoes.

2.2.3.1 Automatic Systems which mimic the Human Biomechanics

One of the first footwear testing systems was described in the US. Patent No. 2,638,776 by Aines [55]. This apparatus was developed to test the durability of the shoes by simulating the walking steps. Figure 2-9 illustrates the structure of this machine.

Figure 2-9: The structure of Aines' footwear testing system [55]

It includes eight imitative feet attached to each of four rotating wheels. These wheels are mounted at the end of four extended rods connecting with a central rotating disk. When the motor drives the disk to rotate, these wheels rotate, the shoes at the lowest positions come into contact with the floor to approximate the pressure effects of the walking feet on the testing shoes. The inner disk can continuously rotate at different speeds to simulate the wear conditions that the shoes encounter in months of actual wear, and thus the machine shortens the testing time. However, the used feet are fixed and made of live resilience rubber without any degree of freedom (DOF).

In 1978, Taub [56] disclosed his invention; a system can simulate the wear effects on a contact surface in order to evaluate its features by mimicking the human walking gait. This apparatus was developed for testing the floor covering system (e.g.,

carpeting, tile) which are affected by abrasion, friction, and pressure caused by walking feet. Figure 2-10 illustrates this device which consists of a rotating platform, which is covered by the testing carpet, and two artificial feet wearing the shoes, which are raised and lowered by the air cylinders to simulate the walking steps.

Figure 2-10: The structure of Taub's footwear testing apparatus [56]

It also includes another cylinder which can push a metal plate securely mounted at the front shoe to simulate the turning steps. Hence, Taub's device allows the users to evaluate the wear durability of the floor covering system during the straight and turning walking strides.

Another footwear testing apparatus was disclosed by the US. Patent No. 3608372 to Hovey *et al.* [57]. This device can evaluate the resistance to separation of the attachment of the shoe sole to the shoe upper by repeatedly applying forces to the shoe sole. Figure 2-11 shows its basic structure. It consists of a stress-applying mallet (18) mounted at the tip of an air cylinder and a clamp for fixing the shoe. By

actuating the air cylinder, the user can apply a distorting stress on the shoe sole at the toe end and evaluate the connection durability.

Figure 2-11: The primary structure of the shoe tester by Hovey et al. [57]

In 1978, Cohen [58] disclosed another footwear testing apparatus by the US. Patent No. 4096733. The device allows the users to evaluate the shoe sole durability by simulating the pressures on it in walking. Figure 2-12 illustrates this device's structure. It includes a rotating shaft and two extending rods which mount the shoe soles at their ends, and a treadmill simulating the contact floor. In operation, the electric motor drives the rotating shaft through a gearbox and a belt transmission mechanism. When the shaft rotates, the soles come into contact with the treadmill to simulate the walking steps.

Figure 2-12: The structure of Cohen's footwear testing device [58]

Also in 1978, Hayashi [59] disclosed his footwear testing system by the US. Patent No. 4130007. Figure 2-13 illustrates the overall structure of this machine. It includes two primary groups of linkage mechanisms. The first one is mounted on the machine stand and moves the contact platform automatically advanced and retreated towards the foot-shoe while the second one supports the footwear block in contact with the platform.

Figure 2-13: The primary structure of Hayashi's Footwear Tester [59]

It also includes another small linkage mechanism for inclining the footwear block so that the posture of foot-leg can be changed in the frontal plane. Another plus point of this machine is that it includes the movable joints at the ankle and metatarsophalangeal joints although these are all passive joints. Overall, this system can mimic the walking steps more realistically and allow the users to evaluate the footwear durability more accurately with a short testing time.

Another machine was developed to assess the durability of the shoe outsole and was disclosed by the US. Patent No. 4327572 [60]. Figure 2-14 illustrates the main structure of this system. Similar to Hayashi's machine, this footwear testing

apparatus comprises a moving abrasive surface which simulates the contact floor and an artificial foot wearing the testing shoe. In this machine, instead of the elaborate linkage mechanisms, Pitman *et al.* [60] used three cam mechanisms to apply the vertical forces to the plastic foot at the heel, ball, and toe positions to replicate the pressure pattern that the outsole encounters in actual wear conditions. Although the machine was developed to predict the shoe durability by simulating the foot motion and forces in running, the cam mechanisms limit the variation of possible force patterns. When a new reference gait pattern is applied, a new cam profile and mechanism need to be synthesized.

Figure 2-14: The primary structure of the wear tester for shoe by Pitman *et al.* [60]

Furthermore, another footwear testing machine was developed by Paquette *et al.* [61] and was disclosed by the US. Patent No. 4432223. Figure 2-15 shows the primary structure of this machine. It comprises a set of cam mechanisms, which mount a leg

pylon and an artificial foot wearing the testing shoe, and a rotational contact surface or an environmental chamber, which may be equipped with different environmental conditions. The set of cam mechanism allows the machine to duplicate the walking pattern of the knee and ankle joints, and thus with various environmental chambers and contact surfaces, the users can evaluate the footwear durability during walking in different conditions.

Figure 2-15: The footwear testing system developed by Paquette et al. [61]

Another footwear testing system was designed and disclosed by the US. Patent No. 6,581,437 to Chrystall *et al.* [62]. Figure 2-16 shows the real system's structure. It comprises a fixed artificial foot wearing the testing shoe and a Stewart motion platform. This platform comprises six linear actuators and can move in three-dimensional space with six DOFs. On the other side, the fixed foot features a force sensor for recording the impact force. When the six linear actuators activate simultaneously, the strike plate (i.e., the moving platform) comes into contact with

the fixed foot-shoe to simulate the contact phase interaction between the foot-shoe and the ground in walking or running gaits. The patent also includes the methods for transforming the foot trajectory into the platform trajectory to simulate the contact interaction. Then, the platform trajectory will be adjusted based on the recorded force data. However, the platform workspace was divided by the kinematic singularities and its “available energy for specimen loading and motion control diminishes with proximity to singularity regions” [63]. Consequently, the system was quite slow and could obtain about 3% of the velocity of human motion.

Figure 2-16: The “Automated Footwear Testing System” including a robotic 6-DOF moving platform, a prosthetic foot wearing the testing shoe, a rigid frame, and a control system [63].

Ronkainen *et al.* [64] established another system to replicate the foot-ground contact phase of human running with a 6-DOF industrial robot (iRobot, Fanuc R-2000iB/165F) mounting a foot-shoe block at the robot end effector [64]. Figure 2-17 shows the real model of this system. Although this system employs a fixed prosthetic foot, it has a broad range of movement and DOFs to drive the foot-shoe track to the human foot’s position profile.

Figure 2-17: The 6-DOF Fanuc iRobot mounts the prosthetic foot wearing the test shoe at the robot end effector [64].

Figure 2-18 shows its performance in mimicking human running gaits. It is noted that while the angles of the prosthetic foot-shoe to the floor are close to the human foot-shoe angles, the prosthetic foot lacks the flexibility at the ankle and MTP joints significantly. In addition, this system performance was also insufficient to replicate the human footstrikes in running at 3.5 m/s [64].

Figure 2-18: The performance of the iRobot system [64] in mimicking the human running gaits. The lower panels show the human foot-shoe position at the impact (v), mid-stance (vi), propulsion (vii), and pre-swing (viii) phases of a running cycle. The upper panels capture the posture of the prosthetic foot-shoe in the corresponding phases [64].

Another footwear-testing device was developed specifically for testing the durability of the shoes in skateboarding and was disclosed by the US. Patent No. 2007/0266763 A1 [65]. This device is capable of simulating the stresses on the shoe's upper caused by skateboarding tricks (e.g., ollies, kickflips, etc.). Figure 2-19 illustrates the structure of this system. It comprises a rotating disk mounting the shoe, and a supporting arm holding the skateboard deck. The footwear is kept extending outwards from the disk's plane. Also, the supporting arm allows the deck's position and angle to be adjustable before the test. When the disk rotates, it drives the shoe to come into contact with the skateboard to simulate the impact forces between the shoe and the skateboard deck. In this manner, the device approximates the forces and stresses encountered by the shoes when the wearer performs the skateboarding tricks during a few months of wearing. Then the user can evaluate the durability of the shoe in a short period of testing.

Figure 2-19: The structure of the footwear testing machine of Therrio et al. [65]

Although these systems were developed for replicating the contact interactions between the shoe and the contact surface in walking, running and other sports so that the user can evaluate the footwear durability without human participation, there exist some significant limitations in the prior art.

Firstly, the artificial feet lack bio-fidelity to replicate the foot-shoe interactions. Most of these imitative feet have no active joints despite the facts that the human foot joints have a broad range of motion, high speeds and torques in walking and running gaits, especially the ankle, and MTP joints. Moreover, the foot appearance has not been developed to be sufficiently realistic.

Secondly, only one or a restricted number of gaits can be replicated by these machines/devices due to the structure of the mechanisms (e.g., the set of cam mechanisms in [60, 61]). If another walking/running speed or gait pattern is required for the test, the developer needs to synthesize another set of cam mechanisms.

Thirdly, the controller in the above systems, if available, operates based on the position control. Fundamentally, it could be developed by applying an input profile signal, recording the output kinetics and comparing with reference data, then adjusting the input profile signal ('trial and error' method). This control process requires extensive time and effort to develop the best control signal.

Finally, the above systems still cannot realistically mimic human running gaits at normal and high speeds. Hence, there remains a long-felt need in the art for a robotic foot-leg machine that helps the users evaluate footwear durability during running at normal and high speeds.

2.2.3.2 Shoe Stress Testing Devices

Another class of footwear testing devices is the shoe sole stress test device, which fundamentally focuses on applying a predetermined force or pressure on the sole and then the user can measure the depth of penetration into the material and evaluate the durability of the shoe sole. In this class, the machines/devices do not mimic the movement of any human foot-leg joints. For example, Norton's devices, which were disclosed by the US. Patents No. 6289743 and No. 6988416 [66], are the conventional stress test devices. After a heel ram is inserted into the shoe directly above the

midsole, the device then forces the ram into the midsole and allows the user to measure the depth of penetration of the ram into the midsole and evaluate the capacity of the vertical support and the durability of the shoe.

In the standard shoe stress test, the American Society for Testing and Materials (ASTM) also stipulated the ASTM protocol F1614 (procedure A) to quantify the shock absorption ability of the footwear. If the users follow this protocol, they can drop a striker of 8.5 kg weight and diameter of 45 mm from the height of 50 mm to simulate the running impact and then evaluate the footwear absorption capability. Subsequently, Price *et al.* [67] modified this original protocol to assess the impact resistance of the sole of four different types of shoes in walking by adjusting the drop mass and drop height based on the kinetic data of human walking gaits. Figure 2-20 illustrates their testing protocol compared with a real human foot's stress on the shoe. As these stress test protocols simplify the human foot as a metal cylindrical striker dropping from a predefined height, they do not offer realistic testing conditions. If a practical footwear testing system based on mimicking the human gaits is developed, it could replace these stress test protocols.

Figure 2-20: The shoe stress test protocol modified by Price *et al.* [67]

2.3 Available Innovative Robotic Feet

In the art of robotic foot/leg development, numerous prostheses, orthoses, and bionic feet have been developed for amputees and for rehabilitation training. Recently,

researchers have developed many types of passive prostheses with flexible or movable ankle joints to assist amputees in daily life walking [68-71]. These prostheses usually primarily include an elastic component for absorbing shock in the contact phase and releasing the absorbed energy in the propulsion phase so that the user can walk with more comfortable gaits and less effort. Some particular designs may include an actuating mechanism from the upper leg [70] to provide the net energy for the ankle joint, or consist of a self-aligning mechanism supported by a hydraulic piston and cylinder assembly working as a damper [68] to improve the stabilization of standing, balance control, stair-walking and ramp-walking. However, to provide sufficient net power for the ankle joint to mimic normal walking and running gaits realistically, the powered devices offer better solutions.

In the art of powered/active robotic feet, many research groups have developed various structures and methods [72-80] using the series elastic actuators (SEA) to mimic the ankle's behavior in walking gaits. A SEA comprises a motor and gearbox in series with elastic components. With its elastic component, the SEA can simulate the compliant behaviors of the human joints and muscles. It can mimic the shock absorption and dynamic energy recycling processes of the human foot-leg. In addition, by measuring the extension/compression of the elastic element, the users can approximate the actuating force and thus control the output force without a force sensor. Also, some groups developed different types of the rotary series elastic actuator (RSEA) [80-83] to assist the human joints in locomotion and rehabilitation.

Herr and his colleagues have developed many versions of the powered ankle-foot prosthesis [84, 85] to replace the lost foot for amputees, and control methods [86, 87] to mimic human walking gaits. Figure 2-21 shows the structure of a bionic ankle-foot prosthesis developed by Herr *et al.* [88]. It includes a DC motor, the belt and ball-screw transmissions working with an in-series leaf spring and a carbon-composite foot. It also features a parallel spring to recycle the kinetic energy. They also

developed the active ankle orthosis with SEA to assist flat footed people in walking [89]. Similarly, to replace the role of both the foot and the knee, Goldfarb and his colleagues developed the transfemoral prosthesis and methods for providing a slow running ability for amputees [90]. This prosthetic foot can assist the users in upslope walking [91], stair ascending and descending [92] and even slow running [93] with a maximum speed of 2.25 m/s. Figure 2-22 shows this foot-leg prosthesis. It includes two powered DOFs at the knee and ankle joints. Two DC motors actuate these joints through two belt transmission mechanisms.

Figure 2-21: The Bionic ankle-foot prosthesis for amputees developed by Herr et al. [88]: The top right picture is the real prosthesis model while the bottom right diagram illustrates the prosthesis' structure. The left panel illustrates this bionic foot assisting an amputee in walking.

Figure 2-22: The transfemoral prosthesis for amputees developed by Goldfarb and his colleagues [92].

In these orthotic and prosthetic applications, the weight, size, and power of the system are the critical constraints because they are the human-wearable devices. In respect to a certain weight, existing motors only generate limited mechanical power for actuating the joints. As a result, although the available robotic feet successfully replace the role of the human foot in daily walking, or assist the wearer in walking and running, the candidate has not found any report of a prosthetic foot which can mimic the human running gaits at a speed higher than 8 km/h. In addition, because all intelligent and delicate components such as control circuits, strain-gauge sensors, and actuators are located on these robotic feet, there are high risks of damaging these components in high-frequency impacts. Hence, these robotic feet are not applicable for reproducing the high dynamic effects on the footwear in running, especially the foot-shoe interactions. In other words, although these systems can simulate the human foot, they are not suitable for footwear testing.

On the other hand, the development of this robotics foot and this thesis scope may benefit in rehabilitation and prostheses research. A multiple powered-DOFs robotic foot would help recreate more actual movements of the human foot and may create a smoother gait for the amputees in walking and running. This robotic foot could lay base design to develop more advanced prosthetic, bionic feet to replace the lost foot for amputees. The control methods and algorithms for the foot joints developed in this thesis would also be useful for developing torque control methods for human-assisting devices, prostheses, and bionic feet.

2.4 How the Shoe wears

A modern running shoe may have many detailed components but it comprises three primary layers: the upper, the midsole, and the outsole. The upper part is usually made from a combination of lightweight mesh and synthetic materials that are very durable, flexible and breathable. Its main parts include the heel counter, toe box, tongue, insole, lacing system, and mid-panels. [94]. The outsole part, which is usually made of rubber or composite materials [95], is the protective layer at the bottom of the shoe. The midsole is a foam layer providing cushioning and support to absorb the shock. This layer may contain extra support or cushioning devices (e.g., air unit).

Each pair of shoes have a different lifespan; however, it is reasonable to expect some structural damage, wear and tear signs after about 200 miles (~320 km) of road running [96]. After this 320 km mark, the outsole at the heel and forefoot should be wearing thin and the runners may adjust their running gaits to compensate for the outsole lost. However, many studies [46-48, 97] evaluate the worn shoe after a running distance of 500 to 1000 km. This running distance is equivalent to about two to three months of wearing for recreational runners. With an average running speed of 10 km/h, the running time varies from 50 to 100 hours. This is the reference time for the footwear testing system to simulate the wear conditions before the shoe can be

evaluated its physical properties again. If an automatic system can run continuously 24 hours/day, it takes only about 2.5 to 5 days to finish the tests.

The whole shoe and its individual components may have their different lifetime. In this project, the research teams aim to develop a system which can simulate the gait kinematic and dynamic effects on the whole shoe. The ultimate goal is to mimic the human foot behaviors and the ground condition to reproduce the same overall effects in the actual human running. However, the prioritized focuses are the plantar effects on all shoe components under the foot in the main plane of gaits, the sagittal plane. The most important wear conditions are the time series of the ground reaction forces, the center of pressure, foot and toe relative positions to the ground, the ground materials and roughness. Secondary conditions may include the environmental conditions such as temperature, humidity, and the effects caused by motions in other gait planes (i.e., the frontal and horizontal planes).

2.5 Summary

From above through literature reviews, one can observe the following shortcomings of current footwear testing methods and available automatic testing systems:

- i. The testing method using human wear participation is too costly, time-consuming, and may expose human subjects to injury risks.
- ii. The testing method using computer simulations often encounter great challenges in modeling the testing conditions and complex geometry.
- iii. Available mechanical footwear testing systems cannot replicate properly the human running gaits due to either limited power or limited speed.
- iv. The artificial foot used in these systems lacks powered DOFs and flexibility to represent the human foot complex anatomy and biomechanics in gaits.
- v. These imitative feet also lack bio-fidelity in shape and appearance to reproduce the interaction between the human foot and the shoe.

- vi. Available prosthetic and bionic feet for amputees have limited power and flexibility to be used for testing the running shoe.

Hence, an automatic footwear testing system, which can overcome these limitations, mimic the human running gaits up to 14 km/h, reproduce the dynamic effects on the footwear to evaluate it in running, is in high demand. The Ph.D. candidate's job is to develop the multiple powered DOFs robotic running foot to integrate with the upper leg and contact platform. The upper leg and contact platform are being developed by other members of the research team at the Institute for Sports Research and the School of Mechanical & Aerospace Engineering, Nanyang Technological University, Singapore.

Chapter 3

Robotic Foot Design

3.1 Introduction

This chapter will explain systematically the designing process for the robotic running foot from the working fundamental to the actual hardware.

3.1.1 *Design Requirements/Objectives*

From the overall project objectives, the Ph.D. scope (Section 1.3), and the reviews on critical anatomical and biomechanical aspects of the human foot (Section 2.1), one can observe the following prime characteristics/requirements for the robotic running foot (RRF) for footwear testing:

- i. As the human foot is very flexible and active in gaits, the robotic foot must have multiple flexible DOFs. At least two powered DOFs at the ankle and MTP joints are required for the first prototype. These powered DOFs would help the RRF simulate the kinematic and kinetic aspects of the human ankle and MTP joints in gaits.
- ii. The focus is the contact phase in distance running at normal running speeds (Section 2.1.3.1). The contact phase is when the major effects on the shoe occur. However, the RRF is required to mimic the running gaits up to 14 km/h to cope with the majority of distance running (Section 2.1.3.1).
- iii. The RRF must have a similar shape, size, and appearance to the human foot to fit properly in the testing shoe. A popular shoe/foot size can be used to design the RRF mechanical model. The scope of this Ph.D. project is to

replicate the profile of the human foot at the static no-load condition. The dynamic shape and effects during the gait cycle may be contributed by the movement and active torque of the powered joints.

- iv. The RRF must have sufficient power, speed, and range of motion to mimic the immense dynamic impacts that the human foot joints convey, especially the ankle joint. The RRF mechanical structure must also have high strength and durability to cope with the harsh conditions in running.
- v. The system must be able to continuously repeat these gait cycles for an extended period which is equivalent to a running distance of 600 km to 1000 km (as discussed in Section 2.4). This feature is important for accelerating the footwear testing process.
- vi. The RRF design must allow the team to install some in-shoe sensors to monitor the plantar pressure distribution, humidity, and temperature inside the shoe to evaluate different aspects of the testing shoe. The development of in-shoe sensors and environmental chambers is in the later stage of the project and out of the Ph.D. scope.
- vii. The system controller should allow users to change the characteristics of the reference gaits flexibly. These include the types of gait and contact (i.e., heel strike, midfoot strike, and forefoot strike), the body weight, and different running speeds up to 14 km/s. Also, the system should be able to change the stride speed during a testing process slightly to mimic fatigue effects.

3.1.2 Detailed Design Criteria/Specifications

Since the human foot anatomy is highly sophisticated, it is impossible to replicate all its movements. Thus, an ideal robotic foot should only include the critical structures of the human foot. As discussed in Section 2.1.1, the most important structures of the human foot are the ankle joint complex (the ankle joint complex rotates in all three planes of locomotion), the incline MTP joints, the midtarsal joint, and the windlass mechanism. This Ph.D. project focuses on the motions in the sagittal plane as these are the dominant motions in walking and running (Section 2.1.1.1). The ankle rotations in other planes can be compensated for by the relative movements of

the contact platform with the foot-shoe block which will be discussed in Chapter 5. In the sagittal plane, an ideal robotic foot should include the ankle joint, the first two MTP joints (assuming they have the same axis), and the windlass mechanism.

Table 3-1 shows the full specifications for designing the RRF with the powered ankle and MTP joints. With the normalized data, the aim of this project was to build a robotic foot equivalent to the foot of a 70 kg human.

Table 3-1: Design Specifications for the Robotic Foot [10, 11, 14, 18, 98, 99]

Categories	Unit	Human Typical	Human Max	Current Design (max)
Distance Running Speed	km/h	9.5-12	-	13.82[11]
Body Mass [11]	kg	66.4	-	-
Foot & Shank Mass [99]	kg	4.32	-	4.6
Popular Shoe/Foot Size	U.S.	9-12	-	9
Max Ankle Dorsiflexion [11]	deg	30	45	45
Max Ankle Plantarflexion [11]	deg	-30	-45	-45
Max Ankle Torque [11]	Nm	150	240	200
Max Ankle Power [11]	W	-	1200	2000
Max Ankle Speed [11]	deg/s	-	-590	-750
Max MTP Dorsiflexion [44]	deg	-	45	45
Max MTP Plantarflexion	deg	-	0	-15
Max MTP Torque & [14]	Nm	15	60	100
Max MTP Power	W	-	62.3	800
Max MTP Speed [5]	deg/s	-	600	720

To Integrating with the Upper Leg & Contact Platform Mechanism:

In addition to above specifications, the following requirements must be complied with in designing the RRF and the upper leg & contact platform mechanism (ULCPM) so that they can be integrated together to simulate the human gaits.

- i. The ULCPM must possess at least three powered DOFs to be able to drive the RRF in the sagittal plane: (i) the vertical DOF applies the body-mass effect on the RRF, (ii) the horizontal DOF simulates the relative movement between the foot-shoe and the ground, (iii) the rotational DOF moves the foot-shoe to the right angle in gaits.
- ii. The RRF and ULCPM must have an appropriate connecting interface. This interface must be highly rigid and stable and can convey the large dynamic force from the ‘upper body’ to the foot.
- iii. As the RRF is carried and driven by the upper leg mechanism in high-speed movement (refer to Section “2.1.3.4 Foot Velocity”), it must be compact and lightweight. If the RRF has a heavy weight, it will cause an exponential dynamic inertia and power requirement to the upper leg mechanism. The high-power actuators of the RRF may have a large size and heavy weight.
- iv. Also, as the RRF has to bear the impact with the ground, its overall structure and component must be very durable. It is suggested that all the intelligent and delicate components and sensors should not be installed on the RRF.

3.2 Possible Solutions and Analysis

There are a few types of transmission mechanisms which can mimic the compliant behaviors of the human muscles and tendons. The most modern transmissions are the series elastic actuator (SEA) and cable-conduit mechanism (CCM).

3.2.1 Series Elastic Actuators (SEA)

Figure 3-1 shows a typical structure of a SEA by Patt *et al.* [100]. It includes at least one elastic element (either axial or torsional spring) connecting the load to an actuator. The deformation of the spring can be used to derive the actuating force on the load and thus feed into the controller to close the control loop. It also has low impedance and friction [100]. Thus, the SEA can control the output force without using a delicate load cell to monitor the force feedback. For the RRF, the SEA can

be used to actuate the foot joints as many research groups have done for the ankle joint in walking (Section 2.3).

Figure 3-1: The Schematic Diagram of a Typical Series Elastic Actuator [100]

The elastic components of the SEA can simulate the contraction and extension of a muscle. They can mimic the processes of storing and releasing energy during gait cycles of the human joints and muscles. This feature allows the SEA to absorb shock during the contact phase, store kinetic energy and release it in the propulsion phase. Thus, SEAs can reduce the shock, partially recycle kinetic energy, and are often preferred in developing humanoid applications, especially prostheses, orthoses, and exoskeletons (Section 2.3).

Figure 3-2 illustrates the proposed structure for an ideal RRF in the sagittal plane by using SEAs. The foot has two powered joints (i.e., the ankle and MTP joints) and one passive joint (i.e., the midtarsal joint - MTJ). The ankle joint is controlled by the tibialis anterior (TA) muscle and a group of gastrocnemius and soleus muscles (GAS & SOL) while the toe flexor muscle (TFL) and the toe extensor muscle (TEX) control the MTP joint. Each muscle is modeled as a SEA which includes a spring in series with an actuating mechanism. The actuating mechanism is a motor in series with the ball-screw transmission. The RRF also includes the windlass mechanism for modeling the arch of the foot (formed by the hindfoot, metatarsals, midtarsal joints [MTJ] and plantar fascia [PF]). The shank mechanism has three active DOFs to drive the foot-shank block to the correct position and apply the body mass effects on the foot. Actuator no.1 moves the block horizontally;

actuator no.2 rotates the shank, and actuator no.3 moves the block up and down to represent the dynamic effects of the body mass.

Figure 3-2: The ideal model of the robotic foot in the sagittal plane.

However, this ideal model is too complex and requires lots of actuating mechanisms for the RRF. Consequently, the RRF would be really heavy and cumbersome. It is also highly challenging to design this ideal RRF to fit into the shoe. On the other hand, the shock absorption role of the plantar fascia and windlass mechanism can be compensated for by the elastic component of the SEA and an appropriate controller algorithm. The model can be simplified by removing the windlass mechanism andmidtarsal joint. Thus, in the first prototype of the integrated robotic footwear testing

system (IFTS), the RRF is developed with only two powered joints at the ankle and MTP joints. The rationale is to evaluate the design effectiveness when minimizing the complexity, weight, and dimensions of the robotic foot.

Figure 3-3 illustrates the structure of a potential simplified IFTS. Each powered joint of the RRF is controlled bidirectionally by one SEA and one parallel spring. This figure also shows another option for the shank mechanism. It may comprise two powerful rotational actuators representing the active hip and knee joints while the third active DOF is shifted to the contact platform. In this option, the contact platform can slide backward and forwards to simulate the relative interaction of the foot and the ground during gaits. However, this upper leg mechanism demands more much stronger actuators to power it. Also, if the RRF weight increase by some reasons, the required moment and power for the knee and hip joints will increase exponentially.

Figure 3-3: A simplified model of the robotic foot in the sagittal plane

Similar to the ideal model, with SEA, the actuators and transmission mechanisms are attached to the foot. This makes the robotic foot bulky. Even more cumbersome motors and transmissions are demanded for mimicking the running gaits which require very high active moments, speeds, and powers.

Furthermore, in wearable robotic prostheses, which must have lightweight and compact dimension, a single-motor SEA cannot support enough power for amputees in running. Grimmer *et al.* [72] proved that the wearable devices with single DC motor still cannot mimic the human ankle joint's behavior during high-speed gaits (over 2.6 m/s) correctly. Their studies suggested using dual motors to overcome the power challenge. Nevertheless, this solution makes the foot even more elaborate and bulkier. In fact, with a combination of a SEA and a parallel spring (shown in Figure 3-3), the robotic foot can recycle a part of the kinetic energy in the impact phase to use in the propulsion phase of the gait cycle. By these methods, researchers can reduce the required power of the motors and thus reduce the mass and bulkiness of the robotic foot. However, this combined energy is still limited and insufficient to mimic running gaits.

In footwear testing, the robotic foot is not required to be wearable. Thus, the system can bear heavier actuators. However, the size and weight of two SEAs which have adequate powers, actuating speeds and forces for the ankle and MTP joints during running are massive. They will cause huge dynamic inertial effects on the upper leg mechanism and demand a vast amount of power to drive this cumbersome foot in order to mimic running gaits. Also, the size of the SEAs causes considerable challenges when designing an identical artificial foot which fits into the testing shoe. Moreover, because the running speeds result in high frequencies and great impact forces, it is noted that all the intelligent components should be placed out of the foot. This will avoid damaging these fragile components and unintended interference to

the electronic & control systems by the impact. Also, the SEA lacks the flexibility to deal with different gait speeds.

3.2.2 *Cable-Conduit Mechanisms (CCM)*

The CCM (known as a tendon-sheath mechanism (TSM) in small scales and low loads, and a Bowden cable mechanism (BCM) in larger scales and loads) has been widely adopted and developed in many types of applications from surgical robots [101-104] to robotic hands [105, 106], wearable robots [107], and soft exosuits [108, 109]. A CCM consists of a cable/tendon which goes through a flexible conduit/sheath and connects an actuator with the robotic joint. One end of the cable/tendon attaches to a proximal pulley, which is controlled by the actuator, while its other end connects to a distal pulley, which attaches to the robotic joint. The conduit/sheath is a flexible tube which lets the cable/tendon go through and can hold its constant length so that the force/motion can be propagated from the proximal pulley to the distal pulley. Thus, these mechanisms allow designers to install the output joints away from the actuators and therefore are the preferred solutions for those systems requiring small working environments and tortuous transmission routes.

A pair of CCMs is often used to control both directions of a robotic joint. Figure 3-4 illustrates the fundamental structure of a pair of CCMs in operation. Similar to a single-CCM mechanism, the clockwise torque/motion from the motor side (i.e., the proximal pulley) is propagated to the robot side (i.e., the distal pulley) by pulling Cable 1 while the two ends of Conduit 1 are fixed. On the contrary, when the proximal pulley rotates counter-clockwise, it pulls Cable 2 and drives the robotic joint rotating counter-clockwise.

Figure 3-4: The schematic of a typical pair of cable-conduit mechanisms (CCMs)

These mechanisms are also very suitable for applications which require high payload, compact design, and small weight and inertia such as for rehabilitation [110], exoskeletons [111, 112], and ankle-foot prosthesis [113]. With CCMs, all motors, gearboxes, and strain gauge sensors can be removed from the robotic foot, the control torques/motions are propagated through the CCMs. Thus, the CCMs not only enable the possibility of increasing the joint power but also eliminate the high inertial effects caused by bulky motors and actuators on the fast-moving robotic foot. Also, removing the actuators and transmissions from the RRF frees space for adding other structures, strengthens the imitative foot, and makes it more realistic. This method also avoids the risks of damaging the electronic parts and sensors because they are placed at the proximal side of the CCMs. In addition, the cable of the CCM can be considered as a high stiffness spring and replace the elastic component of the SEA. These represent great advantages of the CCMs in developing a robotic foot for footwear testing.

Figure 3-5 shows the proposed structure of the robotic foot in the integrated footwear testing system. The foot has two powered joints (No. 4 and 5) at the ankle and MTP joints. The Achilles Tendon (AT) cable and Tibialis Anterior (TA) cable control the ankle joint, while the Toe Flexor (TFL) cable and Toe Extensor (TEX) cable actuate the MTP joint. Each pair of CCMs is controlled by a powerful motor

which is mounted on a stationary control table. This figure also shows another possible mechanism for the upper-leg mechanism. It consists of three powered DOFs (i.e., the translational motions 1 to 3). The first two actuators drive the foot-shank block horizontally and rotate it following the human foot profile. The third actuator elevates the foot-shank block and applies the body mass effects to the foot.

From this schematic, kinematic and kinetic analyses were carried out; and CAD models were designed subsequently. The next two sections will cover these processes.

Figure 3-5: The proposed structure of the robotic foot actuated by two pairs of cable-conduit mechanisms (CCMs) in attachment with the upper-leg mechanism.

3.3 Kinematics and Kinetics Analyses

This section will analyze the kinematic and kinetic correlations between different parts of the system shown in Figure 3-5. Firstly, the relationship between the ankle position and the system frame will be analyzed. The forward kinematic relation is essential for determining the position of the ankle while the inverse kinematic relation is required for calculating the profiles of the three shank actuators to drive the foot block (at the ankle joint) to the correct position. Secondly, the kinetics analysis for the foot is useful for compensating the dynamic inertial effect on the control system during a gait cycle.

Denote the horizontal and vertical translations of the ankle joint as d and h , and denote the translational motions of the three shank actuators no.1 to no.3 as s_1 , s_2 , s_3 . The foot is attached to the shank at the ankle joint and has two rotational movements at the ankle and MTP joints. Denote these rotations as θ_a and θ_m . Figure 3-6 shows the global coordinate **OXY** and a local coordinate system for each link. Each link is marked by a number (i.e., link 1 to link 7). Link 5 is the shank segment, link 6 is the foot segment, and link 7 is the toe segment. The coordinate **OXY** is fixed to the system frame as shown in Figure 3-6. Details of the local coordinates are as follows:

$B_5x_5y_5$ is fixed to the shank at the ankle joint.

$B_6x_6y_6$ is fixed to the foot segment at the MTP joint.

$B_7x_7y_7$ is fixed to the toe segment at the end of the toe.

Degrees of Freedom of the system:

$$DOF = 3(n - 1) - 2f_1 - 2f_2 = 3(8 - 1) - 2 \times 8 = 5 \quad (3-1)$$

In which: n is the number of links,

f_1 is the number of joints which have one degree of freedom,

f_2 is the number of joints which have two degrees of freedom.

Primary variables are three translational movements, s_1 , s_2 , s_3 and two rotational movements, θ_a , θ_m .

Figure 3-6: Coordinate system of each link of the foot-leg model

3.3.1 Kinematics Analysis for the Shank Mechanism

3.3.1.1 Forward Kinematics

From position loop equations of the shank mechanism, one has:

$$\left\{ \begin{array}{l} \tan \alpha = \frac{\mathbf{a}}{\mathbf{s}_1 - \mathbf{s}_2} \\ \mathbf{h} = \mathbf{s}_3 \cos \left(\alpha - \frac{\pi}{2} \right) \\ \mathbf{d} = \mathbf{s}_2 + \mathbf{s}_3 \sin \left(\alpha - \frac{\pi}{2} \right) \end{array} \right. \quad (3-2)$$

In which the input variables are $\mathbf{s}_1$, $\mathbf{s}_2$, and $\mathbf{s}_3$. Thus, the forward position transformation is:

$$\left\{ \begin{array}{l} \alpha = \arctan \left(\frac{\mathbf{a}}{\mathbf{s}_1 - \mathbf{s}_2} \right) \\ \mathbf{h} = \mathbf{s}_3 \sin \alpha = \frac{\mathbf{a}\mathbf{s}_3}{\sqrt{\mathbf{a}^2 + (\mathbf{s}_1 - \mathbf{s}_2)^2}} \\ \mathbf{d} = \mathbf{s}_2 - \mathbf{s}_3 \cos \alpha = \mathbf{s}_2 - \mathbf{s}_3 \frac{\mathbf{s}_1 - \mathbf{s}_2}{\sqrt{\mathbf{a}^2 + (\mathbf{s}_1 - \mathbf{s}_2)^2}} \end{array} \right. \quad (3-3)$$

Differentiating both sides of the above equations with respect to time, one has:

$$\left\{ \begin{array}{l} \dot{\alpha} = \frac{d\alpha}{dt} = \frac{d}{dt} \left(\arctan \frac{\mathbf{a}}{\mathbf{s}_1 - \mathbf{s}_2} \right) = \mathbf{a} \frac{\dot{\mathbf{s}}_2 - \dot{\mathbf{s}}_1}{\mathbf{a}^2 + (\mathbf{s}_1 - \mathbf{s}_2)^2} \\ \dot{\mathbf{h}} = \frac{d\mathbf{h}}{dt} = \frac{d}{dt} (\mathbf{s}_3 \sin \alpha) = \dot{\mathbf{s}}_3 \sin \alpha + \mathbf{s}_3 \dot{\alpha} \cos \alpha \\ \dot{\mathbf{d}} = \frac{d\mathbf{d}}{dt} = \frac{d}{dt} (\mathbf{s}_2 - \mathbf{s}_3 \cos \alpha) = \dot{\mathbf{s}}_2 - \dot{\mathbf{s}}_3 \cos \alpha + \mathbf{s}_3 \dot{\alpha} \sin \alpha \end{array} \right. \quad (3-4)$$

Substituting $\dot{\alpha}$ in the first equation to the second and third equations, and let $\mathbf{C} = \mathbf{a}^2 + (\mathbf{s}_1 - \mathbf{s}_2)^2$, one has:

$$\left\{ \begin{array}{l} \dot{\alpha} = -\frac{\mathbf{a}}{\mathbf{C}} \dot{\mathbf{s}}_1 + \frac{\mathbf{a}}{\mathbf{C}} \dot{\mathbf{s}}_2 \\ \dot{\mathbf{h}} = -\frac{\mathbf{a}\mathbf{s}_3 \cos \alpha}{\mathbf{C}} \dot{\mathbf{s}}_1 + \frac{\mathbf{a}\mathbf{s}_3 \cos \alpha}{\mathbf{C}} \dot{\mathbf{s}}_2 + \dot{\mathbf{s}}_3 \sin \alpha \\ \dot{\mathbf{d}} = -\frac{\mathbf{a}\mathbf{s}_3 \sin \alpha}{\mathbf{C}} \dot{\mathbf{s}}_1 + \left(\frac{\mathbf{a}\mathbf{s}_3 \sin \alpha}{\mathbf{C}} + 1 \right) \dot{\mathbf{s}}_2 - \dot{\mathbf{s}}_3 \cos \alpha \end{array} \right. \quad (3-5)$$

By rewriting these equations in the matrix form; one can obtain the forward velocity transformation:

$$\begin{Bmatrix} \dot{\alpha} \\ \dot{\mathbf{h}} \\ \dot{\mathbf{d}} \end{Bmatrix} = \begin{bmatrix} -\frac{\mathbf{a}}{\mathbf{C}} & \frac{\mathbf{a}}{\mathbf{C}} & 0 \\ -\frac{\mathbf{as}_3 \cos \alpha}{\mathbf{C}} & \frac{\mathbf{as}_3 \cos \alpha}{\mathbf{C}} & \sin \alpha \\ -\frac{\mathbf{as}_3 \sin \alpha}{\mathbf{C}} & \left(\frac{\mathbf{as}_3 \sin \alpha}{\mathbf{C}} + 1 \right) & -\cos \alpha \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{s}}_1 \\ \dot{\mathbf{s}}_2 \\ \dot{\mathbf{s}}_3 \end{Bmatrix} = [\mathbf{K}] \begin{Bmatrix} \dot{\mathbf{s}}_1 \\ \dot{\mathbf{s}}_2 \\ \dot{\mathbf{s}}_3 \end{Bmatrix} \quad (3-6)$$

In which, the velocity coefficient matrix is:

$$[\mathbf{K}] = \begin{bmatrix} \mathbf{K}_{\alpha 1} & \mathbf{K}_{\alpha 2} & \mathbf{K}_{\alpha 3} \\ \mathbf{K}_{h1} & \mathbf{K}_{h2} & \mathbf{K}_{h3} \\ \mathbf{K}_{d1} & \mathbf{K}_{d2} & \mathbf{K}_{d3} \end{bmatrix} = \begin{bmatrix} -\frac{\mathbf{a}}{\mathbf{C}} & \frac{\mathbf{a}}{\mathbf{C}} & 0 \\ -\frac{\mathbf{as}_3 \cos \alpha}{\mathbf{C}} & \frac{\mathbf{as}_3 \cos \alpha}{\mathbf{C}} & \sin \alpha \\ -\frac{\mathbf{as}_3 \sin \alpha}{\mathbf{C}} & \left(\frac{\mathbf{as}_3 \sin \alpha}{\mathbf{C}} + 1 \right) & -\cos \alpha \end{bmatrix} \quad (3-7)$$

Taking the differential of the velocity transformation equations, one has:

$$\frac{\mathbf{d}}{\mathbf{dt}} \begin{Bmatrix} \dot{\alpha} \\ \dot{\mathbf{h}} \\ \dot{\mathbf{d}} \end{Bmatrix} = [\mathbf{K}] \frac{\mathbf{d}}{\mathbf{dt}} \begin{Bmatrix} \dot{\mathbf{s}}_1 \\ \dot{\mathbf{s}}_2 \\ \dot{\mathbf{s}}_3 \end{Bmatrix} + \frac{\mathbf{d}}{\mathbf{dt}} ([\mathbf{K}]) \begin{Bmatrix} \dot{\mathbf{s}}_1 \\ \dot{\mathbf{s}}_2 \\ \dot{\mathbf{s}}_3 \end{Bmatrix} \quad (3-8)$$

According to the partial derivative rule, one has:

$$\begin{aligned} \frac{\mathbf{d}}{\mathbf{dt}} ([\mathbf{K}]) &= \frac{\partial}{\partial \mathbf{s}_1} ([\mathbf{K}]) \frac{\mathbf{ds}_1}{\mathbf{dt}} + \frac{\partial}{\partial \mathbf{s}_2} ([\mathbf{K}]) \frac{\mathbf{ds}_2}{\mathbf{dt}} + \frac{\partial}{\partial \mathbf{s}_3} ([\mathbf{K}]) \frac{\mathbf{ds}_3}{\mathbf{dt}} \\ &= \dot{\mathbf{s}}_1 \frac{\partial}{\partial \mathbf{s}_1} ([\mathbf{K}]) + \dot{\mathbf{s}}_2 \frac{\partial}{\partial \mathbf{s}_2} ([\mathbf{K}]) + \dot{\mathbf{s}}_3 \frac{\partial}{\partial \mathbf{s}_3} ([\mathbf{K}]) \end{aligned} \quad (3-9)$$

Thus, one can rewrite equation 3.8 as follows:

$$\begin{Bmatrix} \ddot{\alpha} \\ \ddot{\mathbf{h}} \\ \ddot{\mathbf{d}} \end{Bmatrix} = [\mathbf{K}] \begin{Bmatrix} \ddot{\mathbf{s}}_1 \\ \ddot{\mathbf{s}}_2 \\ \ddot{\mathbf{s}}_3 \end{Bmatrix} + \left(\dot{\mathbf{s}}_1 \frac{\partial}{\partial \mathbf{s}_1} ([\mathbf{K}]) + \dot{\mathbf{s}}_2 \frac{\partial}{\partial \mathbf{s}_2} ([\mathbf{K}]) + \dot{\mathbf{s}}_3 \frac{\partial}{\partial \mathbf{s}_3} ([\mathbf{K}]) \right) \begin{Bmatrix} \dot{\mathbf{s}}_1 \\ \dot{\mathbf{s}}_2 \\ \dot{\mathbf{s}}_3 \end{Bmatrix} \quad (3-10)$$

In which:

$$\begin{aligned} \frac{\partial}{\partial \mathbf{s}_1}([\mathbf{K}]) &= \begin{bmatrix} \frac{2\mathbf{a}(\mathbf{s}_1 - \mathbf{s}_2)}{\mathbf{C}^2} & -\frac{2\mathbf{a}(\mathbf{s}_1 - \mathbf{s}_2)}{\mathbf{C}^2} & 0 \\ \frac{\mathbf{a}\mathbf{s}_3}{\mathbf{C}^2}[-\mathbf{a} \sin \alpha + 2(\mathbf{s}_1 - \mathbf{s}_2) \cos \alpha] & -\frac{\mathbf{a}\mathbf{s}_3}{\mathbf{C}^2}[-\mathbf{a} \sin \alpha + 2(\mathbf{s}_1 - \mathbf{s}_2) \cos \alpha] & -\frac{\mathbf{a}}{\mathbf{C}} \cos \alpha \\ \frac{\mathbf{a}\mathbf{s}_3}{\mathbf{C}^2}[\mathbf{a} \cos \alpha + 2(\mathbf{s}_1 - \mathbf{s}_2) \sin \alpha] & -\frac{\mathbf{a}\mathbf{s}_3}{\mathbf{C}^2}[\mathbf{a} \cos \alpha + 2(\mathbf{s}_1 - \mathbf{s}_2) \sin \alpha] & -\frac{\mathbf{a}}{\mathbf{C}} \sin \alpha \end{bmatrix} \\ \\ \frac{\partial}{\partial \mathbf{s}_2}([\mathbf{K}]) &= \begin{bmatrix} -\frac{2\mathbf{a}(\mathbf{s}_1 - \mathbf{s}_2)}{\mathbf{C}^2} & \frac{2\mathbf{a}(\mathbf{s}_1 - \mathbf{s}_2)}{\mathbf{C}^2} & 0 \\ -\frac{\mathbf{a}\mathbf{s}_3}{\mathbf{C}^2}[-\mathbf{a} \sin \alpha + 2(\mathbf{s}_1 - \mathbf{s}_2) \cos \alpha] & \frac{\mathbf{a}\mathbf{s}_3}{\mathbf{C}^2}[-\mathbf{a} \sin \alpha + 2(\mathbf{s}_1 - \mathbf{s}_2) \cos \alpha] & \frac{\mathbf{a}}{\mathbf{C}} \cos \alpha \\ -\frac{\mathbf{a}\mathbf{s}_3}{\mathbf{C}^2}[\mathbf{a} \cos \alpha + 2(\mathbf{s}_1 - \mathbf{s}_2) \sin \alpha] & \frac{\mathbf{a}\mathbf{s}_3}{\mathbf{C}^2}[\mathbf{a} \cos \alpha + 2(\mathbf{s}_1 - \mathbf{s}_2) \sin \alpha] & \frac{\mathbf{a}}{\mathbf{C}} \sin \alpha \end{bmatrix} \\ \\ \frac{\partial}{\partial \mathbf{s}_3}([\mathbf{K}]) &= \begin{bmatrix} 0 & 0 & 0 \\ -\frac{\mathbf{a}}{\mathbf{C}} \cos \alpha & \frac{\mathbf{a}}{\mathbf{C}} \cos \alpha & 0 \\ -\frac{\mathbf{a}}{\mathbf{C}} \sin \alpha & \frac{\mathbf{a}}{\mathbf{C}} \sin \alpha & 0 \end{bmatrix} \end{aligned}$$

Thus, one has:

$$\begin{aligned} & \left(\dot{\mathbf{s}}_1 \frac{\partial}{\partial \mathbf{s}_1}([\mathbf{K}]) + \dot{\mathbf{s}}_2 \frac{\partial}{\partial \mathbf{s}_2}([\mathbf{K}]) + \dot{\mathbf{s}}_3 \frac{\partial}{\partial \mathbf{s}_3}([\mathbf{K}]) \right) \begin{Bmatrix} \dot{\mathbf{s}}_1 \\ \dot{\mathbf{s}}_2 \\ \dot{\mathbf{s}}_3 \end{Bmatrix} = \\ & = \begin{bmatrix} \frac{2\mathbf{a}(\mathbf{s}_1 - \mathbf{s}_2)}{\mathbf{C}^2} & 0 \\ \frac{\mathbf{a}\mathbf{s}_3}{\mathbf{C}^2}[2(\mathbf{s}_1 - \mathbf{s}_2) \cos \alpha - \mathbf{a} \sin \alpha] & -\frac{2\mathbf{a}}{\mathbf{C}} \cos \alpha \\ \frac{\mathbf{a}\mathbf{s}_3}{\mathbf{C}^2}[2(\mathbf{s}_1 - \mathbf{s}_2) \sin \alpha - \mathbf{a} \cos \alpha] & -\frac{2\mathbf{a}}{\mathbf{C}} \sin \alpha \end{bmatrix} \begin{Bmatrix} (\dot{\mathbf{s}}_1 - \dot{\mathbf{s}}_2)^2 \\ (\dot{\mathbf{s}}_1 - \dot{\mathbf{s}}_2) \dot{\mathbf{s}}_3 \end{Bmatrix} \quad (3-11) \\ & = [\partial \mathbf{K}] \begin{Bmatrix} (\dot{\mathbf{s}}_1 - \dot{\mathbf{s}}_2)^2 \\ (\dot{\mathbf{s}}_1 - \dot{\mathbf{s}}_2) \dot{\mathbf{s}}_3 \end{Bmatrix} \end{aligned}$$

In which, the derivative matrix is:

$$[\partial \mathbf{K}] = \begin{bmatrix} \frac{2\mathbf{a}(\mathbf{s}_1 - \mathbf{s}_2)}{\mathbf{C}^2} & 0 \\ \frac{\mathbf{a}\mathbf{s}_3}{\mathbf{C}^2} [2(\mathbf{s}_1 - \mathbf{s}_2) \cos \alpha - \mathbf{a} \sin \alpha] & -\frac{2\mathbf{a}}{\mathbf{C}} \cos \alpha \\ \frac{\mathbf{a}\mathbf{s}_3}{\mathbf{C}^2} [2(\mathbf{s}_1 - \mathbf{s}_2) \sin \alpha - \mathbf{a} \cos \alpha] & -\frac{2\mathbf{a}}{\mathbf{C}} \sin \alpha \end{bmatrix} \quad (3-12)$$

Finally, the forward acceleration transformation is:

$$\begin{Bmatrix} \ddot{\alpha} \\ \ddot{\mathbf{h}} \\ \ddot{\mathbf{d}} \end{Bmatrix} = [\mathbf{K}] \begin{Bmatrix} \ddot{\mathbf{s}}_1 \\ \ddot{\mathbf{s}}_2 \\ \ddot{\mathbf{s}}_3 \end{Bmatrix} + [\partial \mathbf{K}] \begin{Bmatrix} (\dot{\mathbf{s}}_1 - \dot{\mathbf{s}}_2)^2 \\ (\dot{\mathbf{s}}_1 - \dot{\mathbf{s}}_2) \dot{\mathbf{s}}_3 \end{Bmatrix} \quad (3-13)$$

3.3.1.2 Inverse Kinematics Analysis

From the forward position transformation in Eq. 3.3, one has the inverse position transformation:

$$\begin{cases} \mathbf{s}_1 = \mathbf{d} + (\mathbf{h} + \mathbf{a}) \cot \alpha \\ \mathbf{s}_2 = \mathbf{d} + \mathbf{h} \cot \alpha \\ \mathbf{s}_3 = \frac{\mathbf{h}}{\sin \alpha} \end{cases} \quad (3-14)$$

From the forward velocity transformation in Eq. 3.6, one has the inverse velocity transformation:

$$\begin{Bmatrix} \dot{\mathbf{s}}_1 \\ \dot{\mathbf{s}}_2 \\ \dot{\mathbf{s}}_3 \end{Bmatrix} = [\mathbf{K}]^{-1} \begin{Bmatrix} \dot{\alpha} \\ \dot{\mathbf{h}} \\ \dot{\mathbf{d}} \end{Bmatrix} = \begin{bmatrix} -\frac{\mathbf{C}}{\mathbf{a}} - \frac{\mathbf{s}_3}{\sin \alpha} & \cot \alpha & 1 \\ -\frac{\mathbf{s}_3}{\sin \alpha} & \cot \alpha & 1 \\ -\mathbf{s}_3 \cot \alpha & \frac{1}{\sin \alpha} & 0 \end{bmatrix} \begin{Bmatrix} \dot{\alpha} \\ \dot{\mathbf{h}} \\ \dot{\mathbf{d}} \end{Bmatrix} \quad (3-15)$$

Where $[\mathbf{K}]^{-1}$ is the inverse matrix of the matrix $[\mathbf{K}]$:

$$[\mathbf{K}]^{-1} = \begin{bmatrix} -\frac{\mathbf{C}}{\mathbf{a}} - \frac{\mathbf{s}_3}{\sin \alpha} & \cot \alpha & 1 \\ -\frac{\mathbf{s}_3}{\sin \alpha} & \cot \alpha & 1 \\ -\mathbf{s}_3 \cot \alpha & \frac{1}{\sin \alpha} & 0 \end{bmatrix} \quad (3-16)$$

From the forward acceleration transformation in Eq. 3.13, one has the inverse acceleration transformation:

$$\begin{aligned} \begin{Bmatrix} \ddot{\mathbf{s}}_1 \\ \ddot{\mathbf{s}}_2 \\ \ddot{\mathbf{s}}_3 \end{Bmatrix} &= [\mathbf{K}]^{-1} \begin{Bmatrix} \ddot{\alpha} \\ \ddot{\mathbf{h}} \\ \ddot{\mathbf{d}} \end{Bmatrix} - [\mathbf{K}]^{-1} [\partial \mathbf{K}] \begin{Bmatrix} (\dot{\mathbf{s}}_1 - \dot{\mathbf{s}}_2)^2 \\ (\dot{\mathbf{s}}_1 - \dot{\mathbf{s}}_2) \dot{\mathbf{s}}_3 \end{Bmatrix} \\ &= \begin{bmatrix} -\frac{\mathbf{C}}{\mathbf{a}} - \frac{\mathbf{s}_3}{\sin \alpha} & \cot \alpha & 1 \\ -\frac{\mathbf{s}_3}{\sin \alpha} & \cot \alpha & 1 \\ -\mathbf{s}_3 \cot \alpha & \frac{1}{\sin \alpha} & 0 \end{bmatrix} \begin{Bmatrix} \ddot{\alpha} \\ \ddot{\mathbf{h}} \\ \ddot{\mathbf{d}} \end{Bmatrix} \\ &\quad + \begin{bmatrix} \frac{2[\mathbf{a}^2 \mathbf{s}_3 \cos \alpha + \mathbf{C}(\mathbf{s}_1 - \mathbf{s}_2)]}{\mathbf{C}^2} & \frac{2\mathbf{a}}{\mathbf{C} \sin \alpha} \\ \frac{2\mathbf{a}^2 \mathbf{s}_3 \cos \alpha}{\mathbf{C}^2} & \frac{2\mathbf{a}}{\mathbf{C} \sin \alpha} \\ \frac{\mathbf{a}^2 \mathbf{s}_3}{\mathbf{C}^2} & \frac{2\mathbf{a}}{\mathbf{C} \sin \alpha} \cos \alpha \end{bmatrix} \begin{Bmatrix} (\dot{\mathbf{s}}_1 - \dot{\mathbf{s}}_2)^2 \\ (\dot{\mathbf{s}}_1 - \dot{\mathbf{s}}_2) \dot{\mathbf{s}}_3 \end{Bmatrix} \end{aligned} \quad (3-17)$$

3.3.2 Kinematics Analysis for the Foot Mechanism

The Denavit-Hartenberg notation can be adopted to analyze the kinematics of the foot. As shown in Figure 3-6, the coordinate system of the foot segment is fixed to the foot at the MTP joint while the toe segment's coordinate system is set at the end of the toe. One has the DH table for the foot in temporary “global” coordinate $\mathbf{B}_5 \mathbf{x}_5 \mathbf{y}_5$ as:

Table 3-2: The DH table for the foot

Frame No.	$\mathbf{a}_i$	α_i	$\mathbf{d}_i$	θ_i
-----------	----------------	------------	----------------	------------

6	l_f	0	0	θ_a
7	l_t	0	0	θ_m

Then, the transformation equations between the coordinate frame $\mathbf{B}_i(\mathbf{x}_i\mathbf{y}_i)$ and the coordinate frame $\mathbf{B}_{i-1}(\mathbf{x}_{i-1}\mathbf{y}_{i-1})$ are:

$$\begin{bmatrix} \mathbf{x}_{i-1} \\ \mathbf{y}_{i-1} \\ 1 \end{bmatrix} = {}^{i-1}\mathbf{T}_i \begin{bmatrix} \mathbf{x}_i \\ \mathbf{y}_i \\ 1 \end{bmatrix}, \text{ and } \begin{bmatrix} \mathbf{x}_i \\ \mathbf{y}_i \\ 1 \end{bmatrix} = {}^{i-1}\mathbf{T}_i^{-1} \begin{bmatrix} \mathbf{x}_{i-1} \\ \mathbf{y}_{i-1} \\ 1 \end{bmatrix} \quad (3-18)$$

Based on the DH table, one can find the transformation matrices from frame $\mathbf{B}_i$ to frame $\mathbf{B}_{i+1}$:

$${}^5\mathbf{T}_6 = \begin{bmatrix} \cos \theta_a & \sin \theta_a & l_f \cos \theta_a \\ -\sin \theta_a & \cos \theta_a & -l_f \sin \theta_a \\ 0 & 0 & 1 \end{bmatrix} \quad (3-19)$$

$${}^6\mathbf{T}_7 = \begin{bmatrix} \cos \theta_m & \sin \theta_m & l_t \cos \theta_m \\ -\sin \theta_m & \cos \theta_m & -l_t \sin \theta_m \\ 0 & 0 & 1 \end{bmatrix} \quad (3-20)$$

And from $\mathbf{B}_7$ to $\mathbf{B}_5$

$$\begin{aligned} {}^5\mathbf{T}_7 &= {}^5\mathbf{T}_6 {}^6\mathbf{T}_7 \\ &= \begin{bmatrix} \cos \theta_a & \sin \theta_a & l_f \cos \theta_a \\ -\sin \theta_a & \cos \theta_a & -l_f \sin \theta_a \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \theta_m & \sin \theta_m & l_t \cos \theta_m \\ -\sin \theta_m & \cos \theta_m & -l_t \sin \theta_m \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} \cos(\theta_a + \theta_m) & \sin(\theta_a + \theta_m) & l_f \cos \theta_a + l_t \cos(\theta_a + \theta_m) \\ -\sin(\theta_a + \theta_m) & \cos(\theta_a + \theta_m) & -l_f \sin \theta_a - l_t \sin(\theta_a + \theta_m) \\ 0 & 0 & 1 \end{bmatrix} \end{aligned} \quad (3-21)$$

For the foot, the forward position transformation from the toe's coordinate system to the temporary global coordinate system (i.e. the frame $\mathbf{B}_5$ fixed to the shank at the ankle joint) is:

$$\begin{aligned}
\begin{Bmatrix} \mathbf{x}_5 \\ \mathbf{y}_5 \\ 1 \end{Bmatrix} &= {}^5\mathbf{T}_7 \begin{Bmatrix} \mathbf{x}_7 \\ \mathbf{y}_7 \\ 1 \end{Bmatrix} \\
&= \begin{bmatrix} \cos(\theta_a + \theta_m) & \sin(\theta_a + \theta_m) & l_f \cos \theta_a + l_t \cos(\theta_a + \theta_m) \\ -\sin(\theta_a + \theta_m) & \cos(\theta_a + \theta_m) & -l_f \sin \theta_a - l_t \sin(\theta_a + \theta_m) \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \mathbf{x}_7 \\ \mathbf{y}_7 \\ 1 \end{Bmatrix}
\end{aligned} \tag{3-22}$$

The position of the toe's head to the ankle (i.e., the frame B_5 at the ankle joint):

$$\begin{aligned}
\begin{Bmatrix} \mathbf{x}_5 \\ \mathbf{y}_5 \end{Bmatrix} &= \begin{bmatrix} \cos(\theta_a + \theta_m) & \sin(\theta_a + \theta_m) & l_f \cos \theta_a + l_t \cos(\theta_a + \theta_m) \\ -\sin(\theta_a + \theta_m) & \cos(\theta_a + \theta_m) & -l_f \sin \theta_a - l_t \sin(\theta_a + \theta_m) \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix} \\
&= \begin{Bmatrix} l_f \cos \theta_a + l_t \cos(\theta_a + \theta_m) \\ -l_f \sin \theta_a - l_t \sin(\theta_a + \theta_m) \end{Bmatrix}
\end{aligned} \tag{3-23}$$

Then, the forward velocity transformation:

$$\begin{Bmatrix} \dot{\mathbf{x}}_5 \\ \dot{\mathbf{y}}_5 \end{Bmatrix} = \begin{bmatrix} -l_f \sin \theta_a - l_t \sin(\theta_a + \theta_m) & -l_t \sin(\theta_a + \theta_m) \\ -l_f \cos \theta_a - l_t \cos(\theta_a + \theta_m) & -l_t \cos(\theta_a + \theta_m) \end{bmatrix} \begin{Bmatrix} \dot{\theta}_a \\ \dot{\theta}_m \end{Bmatrix} = [\mathbf{J}] \begin{Bmatrix} \dot{\theta}_a \\ \dot{\theta}_m \end{Bmatrix} \tag{3-24}$$

Similar to the shank kinematics, the forward acceleration transformation of the toe's head is:

$$\begin{aligned}
\begin{Bmatrix} \ddot{\mathbf{x}}_5 \\ \ddot{\mathbf{y}}_5 \end{Bmatrix} &= [\mathbf{J}] \begin{Bmatrix} \ddot{\theta}_a \\ \ddot{\theta}_m \end{Bmatrix} + \left(\dot{\theta}_a \frac{\partial}{\partial \theta_a} ([\mathbf{J}]) + \dot{\theta}_m \frac{\partial}{\partial \theta_m} ([\mathbf{J}]) \right) \begin{Bmatrix} \dot{\theta}_a \\ \dot{\theta}_m \end{Bmatrix} \\
&= [\mathbf{J}] \begin{Bmatrix} \ddot{\theta}_a \\ \ddot{\theta}_m \end{Bmatrix} + \begin{bmatrix} -l_f \cos \theta_a & l_t \cos(\theta_a + \theta_m) \\ l_f \sin \theta_a & l_t \sin(\theta_a + \theta_m) \end{bmatrix} \begin{Bmatrix} \dot{\theta}_a^2 \\ (\dot{\theta}_a + \dot{\theta}_m)^2 \end{Bmatrix}
\end{aligned} \tag{3-25}$$

From the forward position transformation in Eq. 3.23, one has the inverse position transformation:

$$\begin{cases} \theta_a = \arctan\left(-\frac{y}{x}\right) - \arctan\left(\frac{l_t \sin \theta_m}{l_f + l_t \cos \theta_m}\right) \\ \theta_m = \pm 2 \arctan\left(\sqrt{\frac{(l_f + l_t)^2 - (x_5^2 + y_5^2)}{(x_5^2 + y_5^2) - (l_f - l_t)^2}}\right) \end{cases} \quad (3-26)$$

From the forward velocity transformation in Eq. 3.24, one has the inverse velocity transformation:

$$\begin{aligned} \begin{Bmatrix} \dot{\theta}_a \\ \dot{\theta}_m \end{Bmatrix} &= [J]^{-1} \begin{Bmatrix} \dot{x}_5 \\ \dot{y}_5 \end{Bmatrix} \\ &= \begin{bmatrix} \frac{\cos(\theta_a + \theta_m)}{l_f \sin \theta_m} & -\frac{\sin(\theta_a + \theta_m)}{l_f \sin \theta_m} \\ -\frac{l_f \cos \theta_a + l_t \cos(\theta_a + \theta_m)}{l_f l_t \sin \theta_m} & \frac{l_f \sin \theta_a + l_t \sin(\theta_a + \theta_m)}{l_f l_t \sin \theta_m} \end{bmatrix} \begin{Bmatrix} \dot{x}_5 \\ \dot{y}_5 \end{Bmatrix} \end{aligned} \quad (3-27)$$

From the forward acceleration transformation in Eq. 3.25, one has the inverse acceleration transformation:

$$\begin{aligned} \begin{Bmatrix} \ddot{\theta}_a \\ \ddot{\theta}_m \end{Bmatrix} &= [J]^{-1} \begin{Bmatrix} \ddot{x}_5 \\ \ddot{y}_5 \end{Bmatrix} - [J]^{-1} \begin{bmatrix} -l_f \cos \theta_a & l_t \cos(\theta_a + \theta_m) \\ l_f \sin \theta_a & l_t \sin(\theta_a + \theta_m) \end{bmatrix} \begin{Bmatrix} \dot{\theta}_a^2 \\ (\dot{\theta}_a + \dot{\theta}_m)^2 \end{Bmatrix} \\ &= \begin{bmatrix} \frac{\cos(\theta_a + \theta_m)}{l_f \sin \theta_m} & -\frac{\sin(\theta_a + \theta_m)}{l_f \sin \theta_m} \\ -\frac{l_f \cos \theta_a + l_t \cos(\theta_a + \theta_m)}{l_f l_t \sin \theta_m} & \frac{l_f \sin \theta_a + l_t \sin(\theta_a + \theta_m)}{l_f l_t \sin \theta_m} \end{bmatrix} \begin{Bmatrix} \ddot{x}_5 \\ \ddot{y}_5 \end{Bmatrix} \\ &\quad + \begin{bmatrix} \cot \theta_m & -\frac{l_t \cos(2\theta_a + 2\theta_m)}{l_f \sin \theta_m} \\ -\frac{l_f + l_t \cos \theta_m}{l_t \sin \theta_m} & \frac{l_f \cos(2\theta_a + \theta_m)_a + l_t \cos(2\theta_a + 2\theta_m)}{l_f \sin \theta_m} \end{bmatrix} \begin{Bmatrix} \dot{\theta}_a^2 \\ (\dot{\theta}_a + \dot{\theta}_m)^2 \end{Bmatrix} \end{aligned} \quad (3-28)$$

Also, the transformation matrix from the frame $B_5(x_5, y_5)$, which is fixed to the shank at the ankle joint, to the global coordinate **OXY** is:

$${}^0\mathbf{T}_5 = \begin{bmatrix} \cos\left(\alpha - \frac{\pi}{2}\right) & \sin\left(\alpha - \frac{\pi}{2}\right) & d \\ -\sin\left(\alpha - \frac{\pi}{2}\right) & \cos\left(\alpha - \frac{\pi}{2}\right) & h \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \sin \alpha & -\cos \alpha & d \\ \cos \alpha & \sin \alpha & h \\ 0 & 0 & 1 \end{bmatrix} \quad (3-29)$$

In which, d , h , and α are determined in Eq. 3.3. Thus, the forward position transformation from the toe to the global coordinate system is:

$$\begin{Bmatrix} \mathbf{X}_0 \\ \mathbf{Y}_0 \\ 1 \end{Bmatrix} = {}^0\mathbf{T}_5 {}^5\mathbf{T}_7 \begin{Bmatrix} \mathbf{x}_7 \\ \mathbf{y}_7 \\ 1 \end{Bmatrix} = {}^0\mathbf{T}_7 \begin{Bmatrix} \mathbf{x}_7 \\ \mathbf{y}_7 \\ 1 \end{Bmatrix} \quad (3-30)$$

$$= \begin{bmatrix} \sin(\alpha + \theta_a + \theta_m) & -\cos(\alpha + \theta_a + \theta_m) & d + l_f \sin(\alpha + \theta_a) + l_t \sin(\alpha + \theta_a + \theta_m) \\ \cos(\alpha + \theta_a + \theta_m) & \sin(\alpha + \theta_a + \theta_m) & h + l_f \cos(\alpha + \theta_a) + l_t \cos(\alpha + \theta_a + \theta_m) \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \mathbf{x}_7 \\ \mathbf{y}_7 \\ 1 \end{Bmatrix}$$

3.3.3 Kinetics Analysis for the Ankle and MTP joints

To evaluate the required torque for actuating the ankle and MTP joints, one can use the Lagrangian approach. The Lagrangian is defined as $\mathbf{L} = \mathbf{K} - \mathbf{P}$; with $\mathbf{K}$ and $\mathbf{P}$ are the kinetic energy and potential energy of the system respectively. This Lagrangian includes the Lagrangian of the shank and the foot $\mathbf{L} = \mathbf{L}_s + \mathbf{L}_f$. Moreover, $\mathbf{L}_s$ does not affect the torque of the ankle and MTP joints so one can evaluate the Lagrangian of the foot instead. The kinetic energy of the foot is:

$$\mathbf{K} = \mathbf{K}_6 + \mathbf{K}_7 = \left(\frac{\mathbf{m}_6 \mathbf{v}_{G6}^2}{2} + \frac{\mathbf{I}_{G6} \omega_6^2}{2} \right) + \left(\frac{\mathbf{m}_7 \mathbf{v}_{G7}^2}{2} + \frac{\mathbf{I}_{G7} \omega_7^2}{2} \right) \quad (3-31)$$

Where: $\mathbf{K}_6$ is the kinetic energy of the foot segment

$\mathbf{K}_7$ is the kinetic energy of the toe segment

$\mathbf{m}_6$ and $\mathbf{m}_7$ are the masses of the foot segment and toe segment

$\mathbf{v}_{G6}$ and $\mathbf{v}_{G7}$ are the velocities of the center of mass (COM) of these links

ω_6 and ω_7 are the angular velocities of these links about their COM

I_{G6} and I_{G7} are the moments of inertia of these links about the axes through their COM

Figure 3-7 shows the kinetic analysis model for the foot.

Figure 3-7: Center of mass of the ankle segment and toe segment

In addition, l_{6x} and l_{6y} are the distances from the COM of the foot-link to the ankle joint along l_f and perpendicular direction of l_f ; and l_{G7} is the distance from the COM of the toe segment to the MTP joint. Then, the positions of these COMs to the global origin are:

$$\begin{cases} x_{G6} = d + l_{6x}S_{06} - l_{6y}C_{06} \\ y_{G6} = h + l_{6x}C_{06} + l_{6y}S_{06} \\ x_{G7} = d + l_fS_{06} + l_7S_{067} \\ y_{G7} = h + l_fC_{06} + l_7C_{067} \end{cases} \quad (3-32)$$

$$\text{Where: } \begin{cases} S_{06} = \sin(\alpha + \theta_a) \\ C_{06} = \cos(\alpha + \theta_a) \\ S_{067} = \sin(\alpha + \theta_a + \theta_m) \\ C_{067} = \cos(\alpha + \theta_a + \theta_m) \end{cases}$$

Then, the corresponding velocities are:

$$\begin{cases} \dot{\mathbf{x}}_{G6} = \dot{\mathbf{d}} + (l_{6x}C_{06} + l_{6y}S_{06})(\dot{\alpha} + \dot{\theta}_a) \\ \dot{\mathbf{y}}_{G6} = \dot{\mathbf{h}} + (-l_{6x}S_{06} + l_{6y}C_{06})(\dot{\alpha} + \dot{\theta}_a) \\ \dot{\mathbf{x}}_{G7} = \dot{\mathbf{d}} + l_f C_{06}(\dot{\alpha} + \dot{\theta}_a) + l_7 C_{067}(\dot{\alpha} + \dot{\theta}_a + \dot{\theta}_m) \\ \dot{\mathbf{y}}_{G7} = \dot{\mathbf{h}} - l_f S_{06}(\dot{\alpha} + \dot{\theta}_a) - l_7 S_{067}(\dot{\alpha} + \dot{\theta}_a + \dot{\theta}_m) \end{cases} \quad (3-33)$$

Thus,

$$\begin{cases} v_{G6}^2 = \dot{\mathbf{x}}_{G6}^2 + \dot{\mathbf{y}}_{G6}^2 = \dot{\mathbf{d}}^2 + \dot{\mathbf{h}}^2 + l_{6G}^2 (\dot{\alpha} + \dot{\theta}_a)^2 + \\ \quad + 2(\dot{\alpha} + \dot{\theta}_a) [\dot{\mathbf{d}}(l_{6x}C_{06} + l_{6y}S_{06}) + \dot{\mathbf{h}}(-l_{6x}S_{06} + l_{6y}C_{06})] \\ v_{G7}^2 = \dot{\mathbf{x}}_{G7}^2 + \dot{\mathbf{y}}_{G7}^2 = \dot{\mathbf{d}}^2 + \dot{\mathbf{h}}^2 + l_f^2 (\dot{\alpha} + \dot{\theta}_a)^2 + l_{7G}^2 (\dot{\alpha} + \dot{\theta}_a + \dot{\theta}_m)^2 + \\ \quad + 2l_f l_{7G} C_7 (\dot{\alpha} + \dot{\theta}_a)(\dot{\alpha} + \dot{\theta}_a + \dot{\theta}_m) + \\ \quad + 2l_f (\dot{\alpha} + \dot{\theta}_a)(\dot{\mathbf{d}}C_{06} - \dot{\mathbf{h}}S_{06}) + 2l_{7G} (\dot{\alpha} + \dot{\theta}_a + \dot{\theta}_m)(\dot{\mathbf{d}}C_{067} - \dot{\mathbf{h}}S_{067}) \end{cases} \quad (3-34)$$

Then, the kinetic energy, potential energy, and Lagrangian of these two links are:

$$\begin{cases} K = \frac{1}{2} I_{G6} (\dot{\alpha} + \dot{\theta}_a)^2 + \frac{1}{2} I_{G7} (\dot{\alpha} + \dot{\theta}_a + \dot{\theta}_m)^2 + \frac{1}{2} m_6 v_{G6}^2 + \frac{1}{2} m_7 v_{G7}^2 \\ P = m_6 g (b - y_{G6}) + m_7 g (b - y_{G7}) \\ L = K - P \end{cases} \quad (3-35)$$

Then, the required torque for the MTP joint is:

$$\begin{aligned}
\mathbf{T}_m &= \frac{d}{dt} \left(\frac{\partial \mathbf{L}}{\partial \dot{\boldsymbol{\theta}}_m} \right) - \frac{\partial \mathbf{L}}{\partial \boldsymbol{\theta}_m} \\
&= \left(\mathbf{I}_{G7} + \mathbf{m}_7 \mathbf{l}_{G7}^2 \right) (\ddot{\boldsymbol{\alpha}} + \ddot{\boldsymbol{\theta}}_a + \ddot{\boldsymbol{\theta}}_m) + \mathbf{m}_7 \mathbf{l}_f \mathbf{l}_{G7} \mathbf{C}_7 (\ddot{\boldsymbol{\alpha}} + \ddot{\boldsymbol{\theta}}_a) \\
&\quad + \mathbf{m}_7 \left[\mathbf{l}_f \mathbf{l}_{G7} \mathbf{S}_7 (\dot{\boldsymbol{\alpha}} + \dot{\boldsymbol{\theta}}_a)^2 + \mathbf{l}_{G7} \mathbf{C}_{067} \ddot{\mathbf{d}} - \mathbf{l}_{G7} \mathbf{S}_{067} \ddot{\mathbf{h}} \right] + \mathbf{m}_7 \mathbf{l}_{G7} \mathbf{g} \mathbf{S}_{067}
\end{aligned} \tag{3-36}$$

Finally, the required torque for the ankle joint is:

$$\begin{aligned}
\mathbf{T}_a &= \frac{d}{dt} \left(\frac{\partial \mathbf{L}}{\partial \dot{\boldsymbol{\theta}}_a} \right) - \frac{\partial \mathbf{L}}{\partial \boldsymbol{\theta}_a} \\
&= \left(\mathbf{I}_{G7} + \mathbf{m}_7 \mathbf{l}_f \mathbf{l}_{G7} \mathbf{C}_7 \right) (\ddot{\boldsymbol{\alpha}} + \ddot{\boldsymbol{\theta}}_a + \ddot{\boldsymbol{\theta}}_m) \\
&\quad + \left[\mathbf{I}_{G6} + \mathbf{m}_6 \mathbf{l}_{G6}^2 + \mathbf{m}_7 (\mathbf{l}_f^2 + \mathbf{l}_f \mathbf{l}_{G7} \mathbf{C}_7) \right] (\ddot{\boldsymbol{\alpha}} + \ddot{\boldsymbol{\theta}}_a) \\
&\quad + \left[\mathbf{m}_6 (\mathbf{l}_{6x} \mathbf{C}_{06} + \mathbf{l}_{6y} \mathbf{S}_{06}) + \mathbf{m}_7 (\mathbf{l}_f \mathbf{C}_{06} + \mathbf{l}_{G7} \mathbf{C}_{067}) \right] \ddot{\mathbf{d}} \\
&\quad + \left[\mathbf{m}_6 (-\mathbf{l}_{6x} \mathbf{S}_{06} + \mathbf{l}_{6y} \mathbf{C}_{06}) + \mathbf{m}_7 (-\mathbf{l}_f \mathbf{S}_{06} - \mathbf{l}_{G7} \mathbf{S}_{067}) \right] \ddot{\mathbf{h}} \\
&\quad - \mathbf{m}_7 \mathbf{l}_f \mathbf{l}_{G7} \mathbf{S}_7 \dot{\boldsymbol{\theta}}_m (2\dot{\boldsymbol{\alpha}} + 2\dot{\boldsymbol{\theta}}_a + \dot{\boldsymbol{\theta}}_m) \\
&\quad + \mathbf{m}_6 \mathbf{g} (\mathbf{l}_{6x} \mathbf{S}_{06} - \mathbf{l}_{6y} \mathbf{C}_{06}) + \mathbf{m}_7 \mathbf{g} (\mathbf{l}_f \mathbf{S}_{06} + \mathbf{l}_{G7} \mathbf{S}_{067})
\end{aligned} \tag{3-37}$$

This required torque combined with the output torque at the joint will be the reference for controlling the joint's torque.

3.4 Mechanical Design

3.4.1 The Main Mechanism

From the design specifications shown in Table 3-1 and the system schematic structure illustrated in Figure 3-5, the candidate designed the hardware and CAD model of the RRF. Its first CAD model is presented in Figure 3-8. It comprises two active joints (i.e., the ankle and MTP joints) and three segments (i.e., the toe, the foot, and the lower shank). The toe segment represents the first two toes, but the 3D printing foot tissue will cover it and represents all five toes. The lower shank segment will connect

to the upper-leg mechanism to form the IFTS. At the ankle joint, the Achilles Tendon (AT) cable and Tibialis Anterior (TA) cable connect directly to a compact ankle follower pulley, which is attached to the foot segment (by eight screws on each side), to transmit the torque/motion from the actuator side to the foot. Similarly, there is a compact MTP pulley at the MTP joint. It attaches to the toe segment and connects with the MTP actuator via the Toe Flexor (TFL) cable and Toe Extensor (TEX) cable. Each joint is featured with one high-resolution encoder to monitor its position.

Figure 3-8: The CAD model of the robotic foot

However, the maximum ankle moment in running is very large while the ankle pulley must be as compact as the human foot to fit into the shoe. This fact makes the AT cable tension increase dramatically when mimicking running gaits. High tension on the cables will cause more dynamic friction and hysteresis on the CCMs and demand much more power from the motor to actuate the joint. Thus, in the next designs, a

timing belt transmission mechanism was added into the foot structure to reduce the tension for the ankle CCMs.

Figure 3-9: The architecture of the proposed Integrated Footwear Testing System.

Figure 3-9 illustrates the proposed overall structure of the Integrated Footwear Testing System with this RRF. The CCMs of the ankle actuator do not connect directly to the ankle pulley but to an auxiliary distal pulley located on the shank segment. This pulley drives the ankle pulley via the timing belt transmission. This figure also depicts the structure of the ankle and MTP actuators. Each comprises a brushless servomotor and gearbox in series with a torque sensor, two couplings, and a proximal pulley. A high-accuracy torque sensor is used to monitor the proximal torque and enhances the joint torque tracking control performance. Also, an incremental encoder is mounted on the proximal axis to feedback the proximal angle. On the contact platform, four load cells are positioned at its four corners to monitor the vertical ground reaction force; another load cell connects the platform

horizontally to the fixed frame to measure the horizontal reaction force. Figure 3-10 shows the CAD model of the foot design with a timing belt transmission. The last column of Table 3-1 shows the characteristics of this current RRF design (except the weight), all other later modifications also guarantee these features.

Figure 3-10: The CAD model of the robotic foot with a timing belt transmission

However, although the timing belt mechanism reduces the cable tension for the CCMs and thus mitigates friction effects on the system performance, it increases the width of the shank and foot segments. As a result, a wider RRF is more difficult to fit into the testing shoe. One solution for this issue is to use an auxiliary cable transmission instead of timing belt transmission. The next CAD design, which is showed in Figure 3-11, uses this cable transmission to reduce the foot width. The same transmission ratios were used, but two steel cables replace the role of the timing belt.

Figure 3-11: The CAD model of the robotic foot with an auxiliary cable transmission

The weakness of this design is that the alloy steel cables of this auxiliary transmission are too stiff to fit around the small pulley. As a result, the RRF either requires a very large tensioning force or causes more backlash in reversing the ankle direction. In addition, with the RRF first prototype, the candidate encountered many difficulties in manually assembling the model and applying pretension for this auxiliary cable transmission. Nevertheless, the model still introduced more backlash into the control performance. Hence, the final solution is to use the timing belt mechanism with modified pulleys. The transmission ratio, strength, and size of the timing belt mechanism was calculated and its parts were selected from MISUMI extra-heavy load timing belts. Then, the pulleys were modified to reduce their depth to a minimum. The shank and foot segments also were reinforced to increase their strength and stability. Figure 3-12 shows the CAD model of this modified RRF. It also features new encoders with higher resolution. The ankle encoder is a 2RMHF-7500 (from

SCANCON, Germany) with the resolution of 7500 pulses/revolution. The MTP encoder is an SCH16F-5000 (SCANCON) with the resolution of 5000 pulses per revolution.

Figure 3-12: The final CAD model of the robotic foot with a timing belt transmission

On the other side of the CCMs, two powerful actuators are placed on a stationary control table. Figure 3-13 shows the CAD model of this control table. The 3-phase AC brushless servomotor 115U2D300BACAA130240 is used to provide adequate moments and speed to drive the ankle joint to mimic human running gaits. Its rated power is 3.27 kW at 3000 rpm. This servomotor connects to a low-backlash gearbox GBPH-1202-NP-015 (from Anaheim Automation, US) and is driven by a motor drive M701-03400100 (from Control Technique, Emersion, US). The gearbox output shaft

connects to the proximal pulley via a torque sensor block. This block comprises a T8-ECO-200Nm (provided by Interface, UK) with the measurement capacity of 200 Nm and two high-stiffness zero-backlash couplings CPSHC87-24-22 (provided by MISUMI, Japan). At the proximal pulley, a high-resolution incremental encoder (i.e., the SCH50F-4096 from SCANCON, Denmark) is used to monitor its position.

A similar structure is used for the MTP actuator. Most of components are from the same brands. A smaller brushless servomotor 075U2C300BACAA075140 (from Control Technique) connecting to a low backlash gearbox GBPN-0802-032 (from Anaheim Automation) is used to drive the MTP proximal pulley. A motor drive M701-03400031 (from Control Technique) controls this servomotor. The same encoder is mounted on the MTP proximal pulley to feedback its position. For this torque sensor block, a high-accuracy torque sensor TRS300-100Nm (from Futek, Southern California, US) with the measurement capacity of 100 Nm is installed to monitor the MTP proximal torque. Two zero-backlash couplings CPSWN65-24-18 (from MISUMI) are used to support the torque sensor.

There are two pairs of CCMs. The cables and conduits are provided by CarlStarhl GmbH, Germany. The ankle CCMs comprise two galvanized steel cables with a diameter of 2.5 mm and two flat wire spiral conduits with their inner tube. The conduit inner diameter is 3.0 mm, and the outer diameter is 6.8 mm. Its length is 1.8 m. Two cables of the MTP CCMs are of 2.0 mm diameter. The corresponding conduits have the same length (1.8 m). Their inner diameter is 2.7 mm, and the outer diameter is 6.0 mm. Each end of the cable is firmly attached to the pulley by screws. Each pair of CCMs is supported by a cable supporter. Each supporter is equipped with two hollow screws which allow the cables to go through, hold the flexible conduits, and adjust the cable pre-tension.

Figure 3-13: The CAD model of the control table

3.4.2 The Carbon-fiber Leaf Spring

The carbon-fiber leaf spring (CFLS) was added to each mechanical CAD model to reduce the shock at the impact when the foot contacts the ground. This CFLS was designed based on the reference foot size, available space between the RRF main structure and the shoe, and the reference impact in human running. It was designed to be made by binding multiple thin layers of carbon-fiber together until it reaches the required thickness. The first layer is placed on a 3D-printing plastic mold which has a similar profile with the required CFLS. Then, after applying a thin layer of appropriate adhesive, another carbon-fiber can be placed on top. With this method, one can adjust the direction of fibers to increase the tensile strength of the leaf spring in the desired direction (to mitigate the shock impact). However, at the end, the insufficient space between the main RRF structure and the shoe did not allow an

installation of a CFLS on the RRF. In addition, the CFLS must have the same thickness along its profile because it is made by binding multiple layers together. On the other hand, the human plantar profile is complex in every direction. Thus, it is highly challenging to build a strong CFLS which absorb the impact in running and can fit into a shoe to represent the human foot profile effect on the shoe.

The CFLS role was replaced by a 3D-printing foot ‘outer-tissue’. This outer-tissue was designed for the final RRF model based on 3D-printing technology. Its design and actual model will be provided in Chapter 5.

3.5 Electronic and Control Structures

Figure 3-14 illustrates the block diagram of the control system for the robotic foot. A real-time controller DS1104 (from dSPACE, Germany) is employed to control the system. Its analog to digital conversion (ADC) modules monitor the incoming signals from the torque sensors and load cells of the contact platform. It also measures the encoder positions by the encoder modules. Then, it calculates the control signals based on these feedbacks and the control algorithms. It will send the control signals to the motor drives afterward. These drives can operate in torque control mode with an internal current loop to improve torque tracking performance.

Figure 3-14: The schematic of the control system

3.6 Summary

This chapter presented the design process of the RRF for footwear testing from the project concept to the final hardware. Firstly, from the demand for a practical robotic footwear testing system and the shortcomings of available testing systems as reviewed in Chapter 2, detailed design criteria and specifications were derived and explained. Then, possible design solutions and schematics were proposed and analyzed. Cable-conduit mechanisms (CCMs), which can offer great advantages for the RRF, were adopted to power the ankle and MTP joints. After that, kinematics and kinetics analyses for the proposed IFTS were carried out to prove the design potential. Subsequently, a few CAD models were designed and analyzed, one of them was fabricated for demonstration and evaluation. Finally, a schematic of the electronic and control systems was also introduced.

Chapter 4

Torque Control for Robotic Joints Actuated by CCMs

4.1 Introduction

Since the human foot's muscles and joints convey a great amount of energy during gait cycles [10, 11, 14], contemporary robotic feet with traditional transmissions [78, 114-116] cannot afford adequate power for the joints in high dynamic gaits (e.g., high speed running gaits). Also, the robotic foot for footwear testing must possess an identical shape and size to the human foot to simulate the wear conditions realistically. Hence, with the ability to tether-transmit the force/motion to the joints from remote actuators and a compact design, CCMs have significant advantages in developing the robotic footwear testing system. CCMs not only enable the possibility of increasing power for the joints but also eliminate the high inertial effects caused by cumbersome motors and actuators on the fast-moving robotic foot.

However, the other natural characteristics of CCMs such as nonlinear friction and backlash hysteresis often limit the system performance. These drawbacks are the results of the interaction between the cables and the conduits in operation, especially when the conduit configurations randomly change along with the movement of the output joint location. In those scenarios, high accuracy tracking control results are far more challenging to accomplish. There are two primary

approaches to compensate the friction and backlash hysteresis effects of CCMs: (i) feedforward friction compensation based on model parameters and off-line identification results without any real-time feedback during the operations, and (ii) closed-loop compensation in real-time with the aid of indirect sensors (because traditional sensors such as encoders and force transducers are often not applicable for applications of CCMs).

To improve the system performance without output sensors (i.e., the off-line compensation methods), many research groups such as Tian and Wang [117], Phee *et al.* [118], Pali *et al.* [105], Chen *et al.* [119], and Wu *et al.* [120] have been developing lumped mass element methods with Coulomb friction models to characterize the force/motion transmission of the CCMs. Subsequently, inverse models and control signals were derived to compensate for the friction. Nevertheless, these approaches often require the cable-conduit configurations in advance and assume a uniform distribution of the cable pretension. These conditions are either difficult to achieve or not practical. Other limitations of these methods include the dramatic increase in computation cost when users increase the number of lumped elements and the discontinuity phenomenon in control due to the use of the static Coulomb friction model. Furthermore, by employing a set of partial derivative equations, Agrawal *et al.* [121] can describe the tendon-sheath mechanism (TSM) transmission characteristics without the assumption of constant pretension and the knowledge of sheath configuration. However, multiple model parameters, computational burden, and discontinuity are still the current limitations of this approach. Another major approach in modeling the TSM and CCM transmission characteristics is the use of either backlash models [122, 123] or backlash-like hysteresis models [124-126] to capture the transmission profile. Subsequently, inverse models and feedforward control signals are derived to compensate for the friction and backlash effects. Although the off-line compensation approaches do not require the output feedback and are thus suitable for those systems which cannot install

any sensors at the joints, they cannot cope with the dramatic changes in the transmission characteristics when the pretension and cable-conduit configurations vary in operation. Therefore, they limit the tracking control performance. Few studies on torque tracking control were found. Wu *et al.* [120] and Jeong *et al.* [127] carried out some experiments on torque tracking control but fixed configuration assumption and limited tracking performance are still their drawbacks.

Alternatively, by employing non-traditional and indirect sensors (e.g., image processing systems) to feed back the output, researchers can develop closed-loop control algorithms to enhance the position tracking performance. Recently, Do *et al.* [128-130] adopted the Bouc-Wen model [131, 132] to describe the motion transmission with displacement input and developed adaptive control schemes to cope with the changes of the conduit configuration and track the position reference. However, no force/torque control algorithm was proposed. Moreover, in the art of control theory for systems with hysteresis and nonlinearities (e.g., mechanical actuators, electromagnetic fields, and electronic relay circuits), many studies [133-137] have developed various control schemes to compensate for the backlash-like hysteresis effects. Nonetheless, few experimental validations have been conducted [137], and no force/torque control scheme was proposed.

To develop the robotic foot for footwear testing, it is crucial to control the output torque accurately at the foot joints (or the ground reaction forces and the foot profile) to realistically replicate the wear conditions that the footwear endures during the gaits. Thus, the prime objective of this chapter is to address the challenges of the output torque tracking control. A database search on Elsevier's Scopus and ISI Web of Science with keywords such as cable conduit mechanism, tendon sheath mechanism, Bowden cable mechanism, and cable-driven control found few studies on force/torque control for these mechanisms. Although the cable-driven rotary series elastic actuator (RSEA) developed by Kong *et al.* [138] operates in a

torque mode control, the authors only used the position feedbacks and derived the cable tensions to compensate for friction in four cases of cable curvature. Similarly, Lu *et al.* [139] used the derived output torque from distortion of the torsional spring, off-line estimations of the geared motor's inertia and damping, and a disturbance observer (DOB) to control the zero output torque for a human assistive joint actuated by a cable-driven RSEA.

In addition, for the system actuated by a single CCM, Zhang *et al.* [140] carried out a case study of nine low-level controllers and three high-level controllers in the torque mode control to find the best control schemes for driving an ankle exoskeleton to assist the human ankle in walking. However, this system also used feedback extension of the elastic element (i.e., the series spring) to derive the output torque and non-modeled control laws to track to the reference torque profile regardless of the nonlinear transmission characteristics of the CCM.

By installing four load cells at four corners vertically and one load cell horizontal in front of the contact platform of the robotic footwear testing system, it is possible to monitor the ground reaction forces and calculate the output torque at the foot joints. The following sections will describe the development of a few different closed-loop control schemes for enhancing the torque tracking performance of robotic joints actuated by CCMs with the availability of the output torque feedback. These control algorithms will be applied to control the robotic foot joints to mimic human running gaits and reproduce the most realistic wear conditions for footwear testing.

The three control approaches in following studies are distinguished in two categories:

a. Differences in modeling the torque transmission of a pair of CCMs:

- The first model is an asymmetric hysteresis model which is formulated from Bouc-Wen hysteresis model with the proximal torque (but not the displacement) playing as the input signal. This model is built when investigating torque transmission at a small scale.

- The second model is a symmetric hysteresis model which is formulated from Bouc-Wen hysteresis model with the proximal torque playing as the input signal. This is a simplified model of the first model.
- The third model is a multi-segments model which is formulated from a regression function and turning-point detecting function. This model is built when investigating torque transmission at a large scale.

b. Differences in the adaptive control formulation:

- The controller in the second study is a robust adaptive control design with a few σ -modifications to prevent parameter drifting phenomena. Although the robust designs introduce some residue in the final estimation error, it really helps stabilize the system.
- The adaptive control design in the first study is also a robust design with an additional variable σ -modification which helps prevent parameter drifting phenomenon and does not lose the estimation speed. There is an extra appendix which proved the new value of the residue of variable σ -modification.
- The third control design is a traditional adaptive controller.

4.2 First Torque Control Design

Torque Mode Control for a pair of Cable-Conduit Mechanisms with an Asymmetric Hysteresis Model

4.2.1 Introduction

In the early stages of this Ph.D. project with only one available powerful actuator, a 1-DOF system actuated by a pair of CCMs was set up to examine the potential performance of the foot joints' torque control designs. This system used the same set of CCMs designed for powering the ankle joint of the robotic foot. This system is illustrated in Figure 4-1 and allows the author to design and implement different types of control algorithms. It used the same ankle actuator which was described in

Section 3.4 and Figure 3-13. The CCM consists of a galvanized-strands cable with a diameter of 2.5 mm and a flexible coil conduit with an inner diameter of 3.0 mm and an outer diameter of 6.8 mm. The flexible coil is in between a cover plastic tube and an inner layer of Teflon, which helps minimize the sliding friction coefficient between the cable and conduit.

Figure 4-1: The dynamic system preceded by a hysteretic pair of CCMs

However, instead of actuating the ankle joint, this actuator powered a distal pulley connecting to two high-stiffness springs. These springs acted as the load for this control system. They were AWT24-200 provided by MISUMI, Japan. These springs' stiffness was from 7.45 N/mm to 26.48 N/mm. Also, in this experimental model, the high-accuracy torque transducer TRS300-100Nm (Futek, Southern California) with the measurement capacity of 100 Nm was installed to monitor the proximal torque. Meanwhile, two strain gauge load cells (Interface, SM-2000N) with a loading capacity of 2000 N, monitored the torque output at the distal pulley. Two

incremental encoders (SCANCON, SCH50F-4096) with a resolution of 4096 pulses/revolution, were mounted on both pulleys. Feedback signals from these encoders helped enhance the torque control performance.

In this study, an asymmetric Bouc-Wen hysteresis model was developed to portray the relationship between the input torque and output torque at the two ends of a pair of CCMs. This modified Bouc-Wen model used the proximal torque as its input signal instead of the proximal displacement. Subsequently, a robust adaptive control scheme was developed to minimize the tracking error between the desired torque and the output torque at the robotic joint.

4.2.2 System Modeling

4.2.2.1 Transmission Characteristics of a pair of CCMs

Figure 4-2 illustrates the transmission principle of a pair of CCMs and the output load with two springs AWT24-200. The dynamic system at the distal pulley can be characterized as:

$$(m\ddot{x}_d + kx_d)r + J_d\ddot{\varphi}_d + c_d\dot{\varphi}_d = \tau_d \quad (4.1)$$

where $\varphi_d, \dot{\varphi}_d, \ddot{\varphi}_d$ are the angular position, velocity, and acceleration of the distal pulley while $x_d, \dot{x}_d, \ddot{x}_d$ are corresponding displacement, velocity, and acceleration of the load cell; J_d , c_d , and τ_d are the inertia, viscous coefficient, and torque at the distal pulley; m is the mass of the loadcell and k is the spring stiffness.

Assumption 1: Pretension is applied sufficiently to the pair of CCMs to prevent any cable slacking.

Figure 4-2: The transmission principle of a pair of CCMs

Denoting T_1, T_2, T_3, T_4 as the tensions at two ends of the two cables, and F_1, F_2 as the friction on the first and second CCMs, one has:

$$T_1 - T_3 = F_1 \quad (4.2)$$

$$T_4 - T_2 = F_2. \quad (4.3)$$

Summing two sides of the above two equations and multiplying by the radius of both pulleys r , one has:

$$\tau_p(t) - \tau_d(t) = \tau(u)(t) \quad (4.4)$$

where $\tau_p(t) = r(T_1 - T_2)$ is the torque at the proximal pulley, $\tau_d(t) = r(T_3 - T_4)$ is the torque at the distal pulley, and the total friction torque $\tau(u)(t) = r(F_1 + F_2)$ is the combined effect of two friction forces on two CCMs. Each friction force can be characterized by a backlash-like hysteresis profile, which will be interpreted in the next section.

Denoting τ_s as the torque monitored by the torque sensor, one has:

$$\tau_p = \tau_s - J_p \ddot{\phi}_p - c_p \dot{\phi}_p \quad (4.5)$$

where J_p , c_p , and φ_p are the inertia, viscous effect and angular displacement of the proximal pulley and couplings.

4.2.2.2 Asymmetric Bouc-Wen model for Torque Input

In mechanical systems, the friction force which is against movement can be modeled as a backlash-like hysteresis profile [131]. There are many hysteresis models in the literature, but the Bouc-Wen model is the most widely used [132]. By the normalized Bouc-Wen model, the friction force between the cable and its corresponding conduit can be modeled as:

$$F(x)(t) = \kappa_1 x + \kappa_2 \zeta \quad (4.6)$$

where the internal state ζ is defined as:

$$\dot{\zeta} = \rho[\dot{x} - \sigma|\dot{x}||\zeta|^{n-1}\zeta + (\sigma - 1)\dot{x}|\zeta|^n] \quad (4.7)$$

where parameters κ_1 , κ_2 , ρ , σ , and n are positive constants, and x is the input displacement.

However, in this robotic foot system, the target is to control the output torque at the foot joints (i.e., the distal pulley of each pair of CCMs), and thus, the servomotor torque mode control is used as the primary control mode. Hence, this study presents a new approach which uses the torque signal as the input for the following asymmetric Bouc-Wen model:

$$\dot{\zeta} = \rho[\dot{u} - \sigma|\dot{u}||\zeta|^{n-1}\zeta + (\sigma - 1)\dot{u}|\zeta|^n] \quad (4.8)$$

where ζ is the internal state; ρ , σ , and n are positive parameters which control the shape of the hysteresis loop while $u \equiv \tau_s$ is the input torque at the proximal pulley which is monitored by the torque sensor. Then, the restoring force or friction on the pair of CCMs is:

$$\tau(u)(t) = \kappa_u(\dot{u})u + \kappa_\zeta \zeta + T_0 + u \quad (4.9)$$

where κ_z is a positive parameter, T_0 is an offset value caused by the pretension of CCMs, and the parameter $\kappa_u(u)$ can be designed as a function of input derivative $\dot{u}$ to portray the asymmetric loop. It includes a slope κ_1 when the input increases $\dot{u} > 0$ and a slope κ_2 when the input decreases $\dot{u} < 0$, ($\kappa_1, \kappa_2 > 0$)

$$\kappa_u(\dot{u}) = \kappa_1 \frac{(1 + \tanh(v\dot{u}))}{2} + \kappa_2 \frac{(1 - \tanh(v\dot{u}))}{2}. \quad (4.10)$$

In this expression, the hyperbolic tangent function $\tanh(v\dot{u})$ plays the role of a continuous *sign* function, and thus $\kappa_u(\dot{u})$ presents smooth continuous transitions of the hysteresis profile between increasing and decreasing phases of the input torque u . In addition, a positive coefficient v helps adjust the stiffness of the graph at the transitions.

4.2.2.3 Parameter Identification

From the hardware shown in Figure 4-1, some torque control experiments were carried out to evaluate the effectiveness of the proposed modified Bouc-Wen model in modeling the torque transmission. The control algorithm for these experiments can be illustrated by the block diagram in Figure 4-3. The dSPACE DS1104 controller monitors all feedback signals from the proximal and distal encoders, torque sensor, and load cells. From the proximal torque feedback, the DS1104 controller calculates the control signal using a proportional-integral-derivative (PID) loop. Subsequently, the motor drive receives this control signal from the DS1104 controller and initiates its internal motor current loop (a proportional-integral [PI] loop) to ensure the desired motor output torque at the proximal side. Both the input torque and the output torque of the pair of CCMs are recorded by the DS1104 controller and filtered by a zero-phase digital filter in MATLAB (The MathWorks Inc., Natick, MA).

Figure 4-3: The control diagram for Parameter Identification Experiments

The author established a fitness function which is the mean square error between the experimental friction torque $\tau(i)$ and the proposed model value $\hat{\tau}_{Asym}(i)$, implement the optimization algorithms and identify the set of parameters of the hysteresis model in (4.8) - (4.10),

$$f(\tau) = \frac{1}{N} \sum_{i=1}^N \left(\tau(i) - \hat{\tau}_{Asym}(i) \right)^2 \quad (4.11)$$

where N is the total number of experiment samples, and i is the sampling index. The proposed model value $\hat{\tau}_{Asym}(i)$ was approximated by the Runge-Kutta 4th order method. The objective was to seek out the set of those parameters which minimizes the fitness function. In this study, the Genetic Algorithm (GA) [141] was adopted as a global search method to roughly locate the global minimum of $f(\tau)$. Subsequently, the Nelder–Mead Simplex method was applied to refine the initial guess from GA. Both minimization methods were implemented in MATLAB.

Figure 4-4: Identification result of the asymmetric Bouc-Wen model for torque input

The identification result is shown in Figure 4-4 with the use of the following parameters values $v = 0.05$, $\rho = 0.1747$, $\sigma = 3.4548$, $n = 1.6205$, $\kappa_1 = 0.09515$, $\kappa_2 = 0.11124$, $\kappa_\zeta = 3.1147$, and $T_0 = 0.20096$. The mean square error was 0.080125 (Nm)^2 .

4.2.2.4 Torque Mode Operations of the Brushless Servomotor

The electrodynamic characteristics of the servomotor, low backlash gearbox, and servo drive can be modeled as follows:

$$\tau_s = N(\tau_m - J_m \ddot{\phi}_m - c_m \dot{\phi}_m) \quad (4.12)$$

$$\tau_m = k_\tau i_m \quad (4.13)$$

where τ_m , i_m , ϕ_m are the torque, stator current, and angular displacement of the servomotor; N is the transmission ratio of the gearbox; κ_τ is a motor coefficient; J_m , c_m are the inertia and viscous effect on the motor and gearbox shaft.

4.2.3 Control Design

The distal end system which is preceded by a pair of CCMs modeled by (4.1)-(4.5), and (4.8)-(4.10) can be rewritten as follows (note that $\mathbf{u} \equiv \tau_s$):

$$\begin{aligned} u = (mr^2 + J_d)\ddot{\phi}_d + c_d\dot{\phi}_d + kr^2\phi_d + J_p\ddot{\phi}_p + c_p\dot{\phi}_p + \kappa_u(\dot{u})(t)u + \kappa_\zeta\zeta \\ + d(t) + T_0 \end{aligned} \quad (4.14)$$

where ζ is the internal state defined by (4.8), and $d(t)$ is added to the system equations as the external disturbance. Note that the output torque fed back by the load cells is:

$$y = kr^2\phi_d. \quad (4.15)$$

We denote u_c as the designed control input signal, which will be sent to the motor drive ($u_c = \tau_m$). In addition, the correlation between motor angular displacement and proximal pulley angular displacement is $\phi_m = N\phi_d$. Thus, by substituting τ_s in (4.12) into (4.14), one has:

$$\begin{aligned} y = \beta_u N(\tau_m - J_m\ddot{\phi}_m - c_m\dot{\phi}_m) - (mr^2 + J_d)\ddot{\phi}_d - J_p\ddot{\phi}_p \\ - (c_d\dot{\phi}_d + c_p\dot{\phi}_p) - (\kappa_\zeta\zeta + d(t) + T_0) \\ = \beta_u Nu_c - \beta_u N^2 J_m\ddot{\phi}_p - \beta_u N^2 c_m\dot{\phi}_p - (mr^2 + J_d)\ddot{\phi}_d - J_p\ddot{\phi}_p \\ - (c_d\dot{\phi}_d + c_p\dot{\phi}_p) - (\kappa_\zeta\zeta + d(t) + T_0) \end{aligned} \quad (4.16)$$

where β_u denotes the control input parameter, which is:

$$\beta_u = 1 - \kappa_u = \frac{\beta_1(1 + \tanh(v\dot{u}))}{2} + \frac{\beta_2(1 - \tanh(v\dot{u}))}{2}, \quad (4.17)$$

where $\beta_1 = 1 - \kappa_1$, $\beta_2 = 1 - \kappa_2$. Letting $\xi_1 = \dot{\phi}_p$, $\xi_2 = \ddot{\phi}_p$ and denoting $D_\tau(t)$ as the total disturbance on system output including external disturbance $d(t)$, variable configuration of CCMs $\kappa_\zeta\zeta$, pretension offset torque T_0 , and the friction on the bearings of proximal and distal pulleys $c_p\dot{\phi}_p + c_d\dot{\phi}_d$, one has:

$$D_\tau(t) = c_p \dot{\phi}_p + c_d \dot{\phi}_d + \kappa_\zeta \zeta + d(t) + T_0, \quad (4.18)$$

we can rewrite (4.16) as follows:

$$\begin{aligned} y = Nu_c[\beta_1 \text{pos}(\dot{u}) + \beta_2 \text{neg}(\dot{u})] - \xi^T \theta_1 \text{pos}(\dot{u}) - \xi^T \theta_2 \text{neg}(\dot{u}) \\ - (mr^2 + J_d)x_2 - J_p \xi_2 - D_\tau(t) \end{aligned} \quad (4.19)$$

In the above equation, vector $\xi = (\xi_1, \xi_2)^T$ and $x_2 = \ddot{\phi}_d$ are state variables, $\theta_1 = (\theta_{11}, \theta_{21})^T$ and $\theta_2 = (\theta_{12}, \theta_{22})^T$ are unknown parameter vectors, in which $\theta_{11} = \beta_1 N^2 c_m$, $\theta_{12} = \beta_1 N^2 J_m$, $\theta_{21} = \beta_2 N^2 c_m$, and $\theta_{22} = \beta_2 N^2 J_m$. In addition, the sign functions were replaced by the shorter forms as follows:

$$\text{pos}(\dot{u}) = \frac{1 + \tanh(v\dot{u})}{2} \quad (4.20)$$

$$\text{neg}(\dot{u}) = \frac{1 - \tanh(v\dot{u})}{2} \quad (4.21)$$

Assumption 2: The proximal and distal pulleys' angular velocities and angular accelerations, which are derived from the feedback angular positions, are assumed bounded (i.e., $\xi_i, x_2 \in \mathcal{L}_\infty$).

Then, the torque tracking errors are defined by the following transformation:

$$e_\tau(t) = y(t) - y_r(t) \quad (4.22)$$

$$\eta = \int_0^t e_\tau(\tau) d\tau \quad (4.23)$$

$$\omega = e_\tau + \lambda \eta \quad (4.24)$$

where $y_r(t)$ is the desired reference torque, λ is an arbitrary positive coefficient.

Assumption 3: The desired torque trajectory $y_r(t)$ and its two first derivatives are known and bounded.

To derive the control signal u_c , we introduced the new virtual control signals $\bar{u}_i$, ($i = 1, 2$) as follows:

$$u_i = \hat{\alpha}_i \bar{u}_i \quad (4.25)$$

$$u_1 = N\text{pos}(\dot{u})u_c \quad (4.26)$$

$$u_2 = N\text{neg}(\dot{u})u_c \quad (4.27)$$

$$u_c = \frac{u_1 + u_2}{N} \quad (4.28)$$

where $\hat{\alpha}_i$ are the estimates of $\alpha_i = 1/\beta_i$. The parameter estimates $\hat{\alpha}_i$ are introduced to avoid dividing exceptions when the direct estimating $\hat{\beta}_i$ is near zero (i.e., the case of $u_i = \bar{u}_i/\hat{\beta}_i$). Let $\tilde{\alpha}_i$ be the estimated errors as $\tilde{\alpha}_i = \alpha_i - \hat{\alpha}_i$. From above definitions, the control term in (4.19) can be expressed in the form of new estimated parameters and virtual control signals as follows:

$$Nu_c[\beta_1\text{pos}(\dot{u}) + \beta_2\text{neg}(\dot{u})] = \beta_1 u_1 + \beta_2 u_2 \quad (4.29)$$

$$\beta_i u_i = \beta_i \hat{\alpha}_i \bar{u}_i = \beta_i (\alpha_i - \tilde{\alpha}_i) \bar{u}_i = \bar{u}_i - \beta_i \tilde{\alpha}_i \bar{u}_i, (i = 1, 2). \quad (4.30)$$

Then, the first-order derivative of the new tracking error ω defined by (4.22)-(4.24) and the closed loop system is:

$$\dot{\omega} = \dot{e}_\tau + \lambda \dot{\eta} = \dot{e}_\tau + \lambda e_\tau = \dot{e}_\tau + \lambda(y - y_r) \quad (4.31)$$

Letting $\Delta_\tau > 0$ be the bound of the total disturbance $D_\tau(t)$ (i.e., $|D_\tau| \leq \Delta_\tau$), and letting the parameter estimated errors be $\tilde{\alpha}_i = \alpha_i - \hat{\alpha}_i$, $\tilde{\theta}_{ij} = \theta_{ij} - \hat{\theta}_{ij}$, $\tilde{\Delta}_\tau = \Delta_\tau - \hat{\Delta}_\tau$ where $\hat{\alpha}_i$, $\hat{\theta}_{ij}$, and $\hat{\Delta}_\tau$ are the estimates of α_i , θ_{ij} , and Δ_τ ($i, j = 1, 2$), the author can design the control laws and update as follows:

$$u_c = \frac{\hat{\alpha}_1 \bar{u}_1 + \hat{\alpha}_2 \bar{u}_2}{N} \quad (4.32)$$

$$\begin{aligned} \bar{u}_1 = & -c_1 \omega + y_r + \xi^T \hat{\theta}_1 \text{pos}(\dot{u}) + \xi^T \hat{\theta}_2 \text{neg}(\dot{u}) + (mr^2 + J_d)x_2 + J_p \xi_2 \\ & - \tanh(\omega/\varepsilon) \hat{\Delta}_\tau - \dot{e}_\tau / \lambda \end{aligned} \quad (4.33)$$

$$\bar{u}_2 = -c_2\omega \quad (4.34)$$

$$\dot{\hat{\alpha}}_i = -\mu_i\lambda\omega\bar{u}_i - \sigma_{si}\hat{\alpha}_i \quad (4.35)$$

$$\dot{\hat{\theta}}_{i1} = -\gamma_{i1}\lambda\omega\xi_{i1}\text{pos}(\dot{u}) - \sigma_{i1}\hat{\theta}_{i1} \quad (4.36)$$

$$\dot{\hat{\theta}}_{i2} = -\gamma_{i2}\lambda\omega\xi_{i2}\text{neg}(\dot{u}) - \sigma_{i2}\hat{\theta}_{i2} \quad (4.37)$$

$$\dot{\hat{\Delta}}_\tau = \mu\lambda\omega\tanh(\omega/\varepsilon) - \sigma\hat{\Delta}_\tau \quad (4.38)$$

Where μ , μ_i , and γ_{ij} , ($i, j=1, 2$) are positive design parameters which represent the estimating coefficients, and c_i are the positive control gains. To prevent parameter drifting, we modified the parameter estimating laws with positive leakage coefficients σ_{ij} and σ [142]. Furthermore, certain knowledge about the limit of control coefficients α_{ij} is available and can be used to design the leakage switching σ -modification σ_{si} for updating α_i as follows:

$$\sigma_{si} = \begin{cases} 0 & \text{if } |\hat{\alpha}_i(t)| < M_i \\ \sigma_{0i} \left(\frac{|\hat{\alpha}_i(t)|}{M_i} - 1 \right) & \text{if } M_i \leq |\hat{\alpha}_i(t)| \leq 2M_i \\ \sigma_{0i} & \text{if } |\hat{\alpha}_i(t)| > 2M_i \end{cases} \quad (4.39)$$

M_i are chosen to be always larger than the absolute value of the real parameters ($M_i > |a_i|$), and σ_{0i} are *leakage* positive design parameters [142].

Theorem 1: Considering the dynamic system preceded by a pair of CCMs in (4.14) - (4.21) satisfying *Assumptions 1-3*, with the input torque control laws defined by (4.32)-(4.34) and parameter update laws (4.35) - (4.38), the following statements hold.

- i. The resulting closed-loop control system is globally stable in the sense that all signals are uniformly ultimately bounded.
- ii. The tracking error e_τ , which is adjustable by an appropriate set of explicit design parameters, converges to the neighborhood of zero with the bound δ defined by

$$\delta = 2 \sqrt{\frac{\sum_{i=1}^2 \frac{\beta_i \sigma_{0i}}{\mu_i} \left[\frac{\alpha_i^2}{2} + \epsilon M_i^2 \right] + \sum_{i,j=1}^2 \frac{\sigma_{ij} \theta_{ij}^2}{2 \gamma_{ij}} + \frac{\sigma \Delta_\tau^2}{2\mu} + \varsigma \epsilon \lambda \Delta_\tau}{\min(2\lambda(c_1 + c_2), \sigma_{0i}, \sigma_{ij}, \sigma)}} \quad (4.40)$$

where $\epsilon = (20 + 14\sqrt{7})/27$ and $\varsigma = 0.2785$.

Proof: Considering the Lyapunov function:

$$V_\tau = \frac{1}{2} \omega^2 + \sum_{i=1}^2 \frac{\beta_i}{2\mu_i} \tilde{\alpha}_i^2 + \sum_{i,j=1}^2 \frac{1}{2\gamma_{ij}} \tilde{\theta}_{ij}^2 + \frac{1}{2\mu} \tilde{\Delta}_\tau^2, \quad (4.41)$$

then, its first-order time derivative is:

$$\begin{aligned} \dot{V}_\tau &= \omega \dot{\omega} + \sum_{i=1}^2 \frac{\beta_i}{\mu_i} \tilde{\alpha}_i \dot{\tilde{\alpha}}_i + \sum_{i,j=1}^2 \frac{\tilde{\theta}_{ij} \dot{\tilde{\theta}}_{ij}}{\gamma_{ij}} + \frac{\tilde{\Delta}_\tau \dot{\tilde{\Delta}}_\tau}{\mu} - \sum_{i=1}^2 \frac{\beta_i \tilde{\alpha}_i \dot{\tilde{\alpha}}_i}{\mu_i} - \sum_{i,j=1}^2 \frac{\tilde{\theta}_{ij} \dot{\tilde{\theta}}_{ij}}{\gamma_{ij}} - \frac{\tilde{\Delta}_\tau \dot{\tilde{\Delta}}_\tau}{\mu} \\ &= \omega \dot{e}_\tau + \lambda \omega \left(\sum_{i=1}^2 (\bar{u}_i - \beta_i \tilde{\alpha}_i \bar{u}_i) - (mr^2 + J_d) x_2 - J_p \xi_2 \right) \\ &\quad \left(-\xi^T \theta_1 \text{pos}(\dot{u}) - \xi^T \theta_2 \text{neg}(\dot{u}) - D_\tau - y_r \right) \\ &= \omega \dot{e}_\tau + \lambda \omega \left(\bar{u}_1 + \bar{u}_2 - (mr^2 + J_d) x_2 - J_p \xi_2 - y_r \right) \\ &\quad \left(-\xi^T \hat{\theta}_1 \text{pos}(\dot{u}) - \xi^T \hat{\theta}_2 \text{neg}(\dot{u}) \right) \\ &\quad - \sum_{i=1}^2 \beta_i \tilde{\alpha}_i \left(\lambda \omega \bar{u}_i + \frac{\dot{\tilde{\alpha}}_i}{\mu_i} \right) - \sum_{i=1}^2 \tilde{\theta}_{i1} \left(\lambda \omega \xi_i \text{pos}(\dot{u}) + \frac{\dot{\tilde{\theta}}_{i1}}{\gamma_{i1}} \right) \\ &\quad - \sum_{i=1}^2 \tilde{\theta}_{i2} \left(\lambda \omega \xi_i \text{neg}(\dot{u}) + \frac{\dot{\tilde{\theta}}_{i2}}{\gamma_{i2}} \right) - \frac{\tilde{\Delta}_\tau \dot{\tilde{\Delta}}_\tau}{\mu} - \lambda \omega D_\tau \end{aligned} \quad (4.42)$$

Substituting the control and update laws (4.33) - (4.38) into (4.42), one has:

$$\begin{aligned}
\dot{V}_\tau &= -\lambda(c_1 + c_2)\omega^2 + \sum_{i=1}^2 \frac{\beta_i \sigma_{si}}{\mu_i} \tilde{\alpha}_i \hat{\alpha}_i + \sum_{i,j=1}^2 \frac{\sigma_{ij}}{\gamma_{ij}} \tilde{\theta}_{ij} \hat{\theta}_{ij} \\
&\quad - \lambda \omega \tanh(\omega/\varepsilon) (\hat{\Delta}_\tau + \tilde{\Delta}_\tau) + \frac{\sigma}{\mu} \tilde{\Delta}_\tau \hat{\Delta}_\tau - \lambda \omega D_\tau \\
&\leq -\lambda(c_1 + c_2)\omega^2 + \sum_{i=1}^2 \frac{\beta_i \sigma_{si}}{\mu_i} \tilde{\alpha}_i \hat{\alpha}_i + \sum_{i,j=1}^2 \frac{\sigma_{ij}}{\gamma_{ij}} \tilde{\theta}_{ij} \hat{\theta}_{ij} \\
&\quad + \lambda \Delta_\tau (|\omega| - \omega \tanh(\omega/\varepsilon)) + \frac{\sigma}{\mu} \tilde{\Delta}_\tau \hat{\Delta}_\tau \\
&\leq -\lambda(c_1 + c_2)\omega^2 + 0.2785\varepsilon \lambda \Delta_\tau + \sum_{i=1}^2 \frac{\beta_i \sigma_{si}}{\mu_i} \tilde{\alpha}_i \hat{\alpha}_i + \sum_{i,j=1}^2 \frac{\sigma_{ij}}{\gamma_{ij}} \tilde{\theta}_{ij} \hat{\theta}_{ij} + \frac{\sigma}{\mu} \tilde{\Delta}_\tau \hat{\Delta}_\tau
\end{aligned} \tag{4.43}$$

The second step in (4.43) was obtained based on the fact that $|D_\tau| \leq \Delta_\tau$ and therefore $-\lambda \omega D_\tau \leq \lambda |\omega| \Delta_\tau$. Moreover, by adopting a useful inequality, which was introduced by *Lemma 1* in [143], the author was able to derive the above final step. The inequality was presented as $0 \leq |z| - z \tanh(z/\varepsilon) \leq 0.2785\varepsilon$, therefore, one had $\lambda \Delta_\tau (|\omega| - \omega \tanh(\omega/\varepsilon)) \leq 0.2785\varepsilon \lambda \Delta_\tau$. Furthermore, the final term in (4.43) can be rewritten as

$$\begin{aligned}
\frac{\sigma}{\mu} \hat{\Delta}_\tau \tilde{\Delta}_\tau &= \frac{\sigma}{\mu} \tilde{\Delta}_\tau (\Delta_\tau - \tilde{\Delta}_\tau) = \frac{\sigma}{\mu} (\Delta_\tau \tilde{\Delta}_\tau - \tilde{\Delta}_\tau^2) \\
&= \frac{\sigma}{\mu} \left[-\frac{\tilde{\Delta}_\tau^2}{2} - \frac{1}{2} (\tilde{\Delta}_\tau^2 - 2\Delta_\tau \tilde{\Delta}_\tau + \Delta_\tau^2) + \frac{\Delta_\tau^2}{2} \right] \\
&= \frac{\sigma}{\mu} \left[-\frac{\tilde{\Delta}_\tau^2}{2} - \frac{1}{2} (\tilde{\Delta}_\tau - \Delta_\tau)^2 + \frac{\Delta_\tau^2}{2} \right] \leq -\sigma \left(\frac{\tilde{\Delta}_\tau^2}{2\mu} \right) + \left(\frac{\sigma \Delta_\tau^2}{2\mu} \right)
\end{aligned} \tag{4.44}$$

Similarly, one has:

$$\sum_{i,j=1}^2 \frac{\sigma_{ij}}{\gamma_{ij}} \tilde{\theta}_{ij} \hat{\theta}_{ij} \leq \sum_{i,j=1}^2 \left[-\sigma_{ij} \left(\frac{\tilde{\theta}_{ij}^2}{2\gamma_{ij}} \right) + \left(\frac{\sigma_{ij} \theta_{ij}^2}{2\gamma_{ij}} \right) \right]. \tag{4.45}$$

Finally, the third term in the final step of (4.43) can be expressed by:

$$\begin{aligned}
\sum_{i=1}^2 \frac{\beta_i \sigma_{0i}}{\mu_i} \tilde{\alpha}_i \hat{\alpha}_i &\leq \sum_{i=1}^2 \left(\frac{\beta_i \sigma_{0i}}{\mu_i} \tilde{\alpha}_i \hat{\alpha}_i + \frac{\beta_i \sigma_{0i}}{\mu_i} \epsilon M_i^2 \right) \\
&\leq - \sum_{i=1}^2 \sigma_{0i} \frac{\beta_i \tilde{\alpha}_i^2}{2\mu_i} + \sum_{i=1}^2 \frac{\beta_i \sigma_{0i}}{\mu_i} \left(\frac{\alpha_i^2}{2} + \epsilon M_i^2 \right)
\end{aligned} \tag{4.46}$$

where $\epsilon = (20 + 14\sqrt{7})/27$ (See Appendix A for proof).

Hence, the derivative of the Lyapunov function becomes:

$$\begin{aligned}
\dot{V}_\tau &\leq -\lambda(c_1 + c_2)\omega^2 - \sum_{i=1}^2 \sigma_{0i} \frac{\beta_i \tilde{\alpha}_i^2}{2\mu_i} - \sum_{i,j=1}^2 \sigma_{ij} \frac{\tilde{\theta}_{ij}^2}{2\gamma_{ij}} - \sigma \frac{\tilde{\Delta}_\tau^2}{2\mu} \\
&\quad + \sum_{i=1}^2 \frac{\beta_i \sigma_{0i}}{\mu_i} \left(\frac{\alpha_i^2}{2} + \epsilon M_i^2 \right) + \sum_{i,j=1}^2 \frac{\sigma_{ij} \theta_{ij}^2}{2\gamma_{ij}} + \frac{\sigma \Delta_\tau^2}{2\mu} + \varsigma \epsilon \lambda \Delta_\tau \\
&\leq -\rho V_\tau + \phi
\end{aligned} \tag{4.47}$$

with the positive parameter $\rho = \min(2\lambda(c_1 + c_2), \sigma_{0i}, \sigma_{ij}, \sigma)$, $\varsigma = 0.2785$, and the constant quantity $\phi = \sum_{i=1}^2 \frac{\beta_i \sigma_{0i}}{\mu_i} \left(\frac{\alpha_i^2}{2} + \epsilon M_i^2 \right) + \sum_{i,j=1}^2 \frac{\sigma_{ij} \theta_{ij}^2}{2\gamma_{ij}} + \frac{\sigma \Delta_\tau^2}{2\mu} + \varsigma \epsilon \lambda \Delta_\tau$. Hence, for $V_\tau \geq V_\tau^* = \phi/\rho$, $\dot{V}_\tau \leq 0$, which implies V_τ is non-increasing, therefore, all signals ω , $\hat{\alpha}_i$, $\hat{\theta}_{ij}$, $\hat{\Delta}_\tau$, and $\tilde{\theta}_{ij}$ ($i, j = 1, 2$) are uniformly ultimately bounded.

Furthermore, if one multiplies both sides of (4.47) by $e^{\rho t}$ before integrating the inequality, then it satisfies:

$$0 \leq \frac{1}{2} \omega^2 \leq V_\tau(t) \leq \left(V_\tau(0) - \frac{\phi}{\rho} \right) e^{-\rho t} + \frac{\phi}{\rho}. \tag{4.48}$$

where $V_\tau(0)$ can be obtained by setting $\omega(0) = e_\tau(0) = 0$ as:

$$V_\tau(0) = \sum_{i=1}^2 \frac{\beta_i}{2\mu_i} \tilde{\alpha}_i^2(0) + \sum_{i,j=1}^2 \frac{1}{2\gamma_{ij}} \tilde{\theta}_{ij}^2(0) + \frac{1}{2\mu} \tilde{\Delta}_\tau^2(0). \tag{4.49}$$

Thus, the new tracking error ω satisfies:

$$|\omega| \leq \sqrt{2 \left(V_\tau(0) - \frac{\phi}{\rho} \right) e^{-\rho t} + \frac{2\phi}{\rho}}. \quad (4.50)$$

It can be observed that there exists a time T_ω such that, for all $t > T_\omega$, one will achieve $|\omega| \leq \sqrt{2\phi/\rho}$ because when t increases, $2(V_\tau(0) - \phi/\rho)e^{-\rho t} \rightarrow 0$, which implies the filter tracking error ω converges exponentially to the residue $D_\omega = \{\omega : |\omega| \leq \delta_\omega = \sqrt{2\phi/\rho}\}$. By choosing appropriate values for those design parameters, one can achieve an δ_ω as small as desired. Furthermore, from the definition of ω in (4.22) - (4.24), one has $|\omega| = |\dot{\eta} + \lambda\eta| \leq \delta_\omega$ and thus, there are two possibilities:

Case 1: $0 \leq \dot{\eta} + \lambda\eta \leq \delta_\omega$, then by solving this inequality one has $\eta \leq (\eta_0 - \delta_\omega/\lambda)e^{-\lambda t} + \delta_\omega/\lambda$, where η_0 is the initial value of η . Hence, it can be seen that there exists $t > T_\eta > T_\omega$ such that, $\forall t > T_\eta$, $0 \leq \eta \leq \delta_\omega/\lambda$ since $(\eta_0 - \delta_\omega/\lambda)e^{-\lambda t} \rightarrow 0$ when $t > T_\eta$ and keeps increasing.

Case 2: $-\delta_\omega \leq \dot{\eta} + \lambda\eta \leq 0$, similarly, one can achieve $\eta \geq (\eta_0 + \delta_\omega/\lambda)e^{-\lambda t} - \delta_\omega/\lambda$. Thus, there exists $t > T_\eta > T_\omega$ such that, $\forall t > T_\eta$, $-\delta_\omega/\lambda \leq \eta \leq 0$ as $(\eta_0 + \delta_\omega/\lambda)e^{-\lambda t} \rightarrow 0$ when $t > T_\eta$ and keeps increasing.

Thus, for both cases, one has $|\eta| \leq \delta_\omega/\lambda$. Then, from the inequality $|\omega| = |\dot{\eta} + \lambda\eta| \geq |\dot{\eta}| - |\lambda\eta|$, one can obtain:

$$|e_\tau| = |\dot{\eta}| \leq |\lambda\eta| + |\omega| \leq \frac{\lambda\delta_\omega}{\lambda} + \delta_\omega = 2\delta_\omega = \delta, \quad (4.51)$$

which implies the second statement of Theorem 1. ■

Remark 1: Although all σ -modifications can prevent parameter drifting to infinity in aforementioned parameter estimations [142], the fixed σ -modifications cause the indefinite sign terms in $\dot{v}_\tau$ (i.e., $\sum_{i,j=1}^2 (\sigma_{ij}\tilde{\theta}_{ij}\hat{\theta}_{ij}/\gamma_{ij})$, and $\sigma\tilde{\Delta}_\tau\hat{\Delta}_\tau/\mu$ in (4.43)), might reduce the adaptation speed. In contrast, the switching σ -modifications

$\sum_{i=1}^2 \beta_i \sigma_{si} \tilde{\alpha}_i \hat{\alpha}_i / \mu_i$ only make $\dot{V}_\tau$ more negative. Indeed, the terms can be rewritten as:

$$\begin{aligned} \frac{\beta_i \sigma_{si}}{\mu_i} \tilde{\alpha}_i \hat{\alpha}_i &= \frac{\beta_i \sigma_{si}}{\mu_i} (\hat{\alpha}_i \alpha_i - \hat{\alpha}_i^2) \leq \frac{\beta_i \sigma_{si}}{\mu_i} |\hat{\alpha}_i| (|\alpha_i| - |\hat{\alpha}_i|) \\ &\leq \frac{\beta_i |\hat{\alpha}_i|}{\mu_i} \sigma_{si} [(|\alpha_i| - M_i) + (M_i - |\hat{\alpha}_i|)] \leq 0, \end{aligned} \quad (4.52)$$

where $i = 1, 2$. The final step is obtained based on the definition of σ_{si} in (4.39) that $\sigma_{si} \geq 0$, $|\alpha_i| < M_i$, and $\sigma_{si}(M_i - |\hat{\alpha}_i|) \leq 0$. Thus, with certain advantageous knowledge of control parameters α_i , adopting the switching σ -modification not only prevents parameter drifting but also sustains the adaptation speed.

4.2.4 Experiments and Results

4.2.4.1 Experimental model and different control schemes

The real experimental system is shown in Figure 4-5. All control schemes were implemented in MATLAB Simulink (The MathWorks Inc., Natick, MA), subsequently, the compiled programs were loaded into dSPACE ControlDesk program which controlled the system in real-time via the dSPACE DS1104 controller. Following these programs, the controller monitored all feedback signals from the torque sensor, load cells, and encoders to calculate control signals and send to the motor drive via 16-bit digital-to-analog conversion (DAC) modules. When receiving input signals, the motor drive actuated the mechanical system described in Figure 4-1. It operated in a bi-directional torque control mode with speed override (mode 5), which allowed a direct torque signal to be applied with a certain speed limit. In the drive's circuit, an internal high-speed proportional-integral (PI) current loop updated the current output value every 4 μ s to ensure the motor output torque was the same as the torque from the controller.

Figure 4-5: The real experimental system

Different control schemes, which are illustrated in Figure 4-3 and Figure 4-6, were implemented to compare their performance with the effectiveness of the proposed adaptive control scheme. Firstly, in the basic open-loop control scheme, the output signal was monitored for evaluating the system performance by the DS1104 controller as shown in Figure 4-3. In addition, a PID control law was installed on the proximal side with torque feedback from the torque sensor to ensure the input torque at proximal pulley as the same as the desired reference torque. Secondly, Figure 4-6(a) shows the block diagram of a closed-loop control scheme using PID control law. In this scheme, the control torque signal was calculated merely using torque feedback from two load cells. Finally, the adaptive control scheme, which is shown in Figure 4-6(b), on the other hand, uses all feedback states to online estimate and compensate for friction in the system.

Figure 4-6: Different closed-loop control schemes: a) Proportional-Integral-Derivative (PID) control scheme, and b) Robust adaptive control scheme.

4.2.4.2 Experimental Results

Figure 4-7 shows the control experiments results when using three aforementioned schemes. The left panel shows a reference signal and three real-time output torque signals fed back by two load cells and two 16-bit analog-to-digital converters in three experiments at stable states, while the right panel shows the tracking errors in the corresponding experiments.

The same reference signal was applied to the system in the three control schemes. It was the sum of two sinusoidal signals with frequencies of 0.25 Hz and $0.25\sqrt{2}$ Hz. During three control experiments, the conduit configuration was randomly changed as shown in Figure 4-1. More specifically, Figure 4-7(a) illustrates the recorded output torque when applying the reference signal of $18 + 8\sin(2\pi t \times 0.25) + 3\sin(2\pi t \times 0.25\sqrt{2})$ Nm to the proximal pulley without any friction compensation

on the CCMs, while Figure 4-7(d) shows the error between reference and output torques. On the other hand, in the closed-loop control schemes, Figure 4-7(b) and (c) portray the output tracking torques for the same reference torque with the PID and robust adaptive control laws respectively. Correspondingly, Figure 4-7(e) and (f) show their torque tracking errors.

It is evident that in the open-loop scheme, without friction compensation, although the output torque graph has the same shape as the reference signal, there is a significant error between the input and output torques with root-mean-square error (RMSE) of 2.5526 Nm. Subsequently, with output feedback, a traditional closed-loop control law, the PID control, can improve the tracking performance considerably. In the second scheme, with $K_P = 0.051$, $K_I = 0.031$, and $K_D = 0.002$, the PID control law reduces the tracking errors significantly to an RMSE of 0.83653 Nm.

Furthermore, the proposed robust adaptive control scheme can enhance the tracking performance significantly compared to two previous controllers. More specifically, with $\lambda = 3.2$, $c_1 = 3.2$, $c_2 = 3.1$, $\lambda\mu = 0.001$, $\lambda\gamma_{ij} = 0.002$, $\lambda\mu_i = 0.0032$, $\sigma_i = 0.01$, $\sigma_{0i} = 0.5$, and $M_{0i} = 2$, the robust adaptive control scheme has a torque tracking RMSE of 0.43227 Nm. Initial estimation values of the update law integrations were $\hat{\alpha}_{0i} = 1$, $\hat{\Delta}_0 = 3$, $\hat{\theta}_{ij}^0 = 0.03$.

Figure 4-7: Torque tracking performance of three aforementioned control schemes with the reference signal $18 + 8\sin(2\pi t \times 0.25) + 3\sin(2\pi t \times 0.25\sqrt{2})$ and randomly varying configuration of the conduit curve.

Figure 4-7 shows the data of stable states while includes the transient tracking performance when the control laws were initiated. Similarly to Figure 4-7, Figure 4-8(a) and (d) show reference torque, output torque and their error in the open-loop scheme. Meanwhile, Figure 4-8(b) and (c) show the torque tracking performance of the PID, and adaptive control schemes respectively while the torque tracking errors are plotted in Figure 4-8(e) and (f). It can be observed that both closed-loop schemes

result in strong tracking convergence. Torque error in the open-loop scheme, by contrast, contains large fluctuation during the experimental period.

Figure 4-8: Transient tracking performance of the three control schemes with the same reference signal $18 + 8\sin(2\pi t \times 0.25) + 3\sin(2\pi t \times 0.25\sqrt{2})$ at the beginning of each control experiment and with the randomly varying configuration of the conduit curve.

Furthermore, although the proposed robust adaptive control scheme enhances tracking performance at stable states significantly compared to the PID control

scheme, Figure 4-8(f) reveals its drawback in transient tracking performance. In the beginning, before the adaptive laws take effect, tracking performance depends on initial estimation. In this case, the PID control scheme demonstrates better results. However, after the adaptation takes effect, the adaptive control scheme shows much smaller errors.

4.2.5 Conclusion

This study introduces a new adaptive control approach with a direct torque input to enhance the torque tracking performance for a pair of CCMs, which are used in a robotic foot for footwear testing. Also, a new friction modeling method, which uses the direct torque input at the proximal pulley as the input variable for an asymmetric Bouc-Wen hysteresis model instead of a displacement input, has been introduced and used to compensate for friction.

All unknown hysteresis effect of CCMs, external disturbance, asymmetric control coefficients, friction on the motor and gearbox, and unmeasured total inertia of the motor and gearbox were estimated online with full state feedback. In the robust adaptive control scheme, the bounds of hysteresis, external disturbance, and motor-gearbox friction and inertia are not required, although theoretical bounds of asymmetric control coefficients provide advantages in stabilizing the system and avoid the estimated parameters drifting to infinity. In torque mode control, the proposed adaptive control laws can adapt efficiently to any configuration change of CCMs.

Experimental validations have been conducted to prove the effectiveness of the proposed approach. The results demonstrate its potential for torque tracking control of CCMs using an asymmetric Bouc-Wen hysteresis model with direct torque input. The next step is to employ alternative torque feedback approaches (e.g., using elastic component and calculating output torque based on the fed back displacement)

instead of traditional delicate strain gauge sensors. Moreover, the proposed control scheme will be installed on the robotic foot for footwear testing with two DOFs and a higher order of loading environment.

4.3 Second Torque Control Design

Direct Torque Control for Cable Conduit Mechanisms in the Robotic Foot for Footwear Testing

4.3.1 Introduction

In an attempt to develop a more effective controller for torque tracking of the foot joints, when a new torque sensor arrived, the experimental system was modified to achieve more accurate results. Figure 4-9 illustrates this overall system including the mechanical and control systems. The same set of ankle CCMs were used. Also, the same actuator and servo drive controlled the distal pulley via this pair of CCMs. However, the torque sensor TRS300-100Nm was installed at the distal pulley to feed back the output torque while the T8-ECO-200Nm was used to monitor the input torque at the proximal pulley. With one high-accuracy torque sensor at each side of the CCMs, this system provided more accurate feedback and could improve the control performance.

The loading mechanism comprises two high-stiffness tension springs connecting with a third pulley by two flexible loading cables. The other ends of the springs connect to a fixed frame. This frame holds the spring when it is in tension but allows it to go down when it is not so that the system does not experience any cable-spring slacking. As shown in this figure, the servomotor of the actuator is controlled by the servo drive M701-03400100A which receives analog control signals from the real-time controller dSPACE DS1104. This controller receives all feedback signals from the sensors and calculates the control signals for the servo drive. It is installed in a

computer PCI slot and sends the monitoring signals simultaneously to the Control Desk to record the signals.

Figure 4-9: The 1-DOF system actuated by a pair of cable-conduit mechanisms (CCMs)

4.3.2 System Modeling

4.3.2.1 Transmission Observation, Modeling and Parameters

Identification

Figure 4-10 shows an illustration of the transmission fundamentals of the pair of CCMs and loading mechanism without the load cells attached to the springs. Then, the distal torque satisfies the following equation:

$$\tau_d = kr^2\varphi_d + c_o\dot{\varphi}_d + (mr^2 + J_o)\ddot{\varphi}_d, \quad (4.53)$$

where φ_d , $\dot{\varphi}_d$, $\ddot{\varphi}_d$ are the angular displacement, velocity, and acceleration of the distal pulley; J_o , c_o are the total inertial and viscous coefficient of the distal pulley, couplings and loading pulley; τ_d is the distal torque; m is the mass of the spring and connectors; and k is the spring stiffness.

Figure 4-10: The transmission principle of a pair of CCMs

Denote T_1 , T_2 as the tensions at two ends of cable A and T_3 , T_4 as the tensions at two ends of cable B, then the friction between cables and conduits are

$$F_A = T_1 - T_2 \quad (4.54)$$

$$F_B = T_3 - T_4, \quad (4.55)$$

Since two pulleys have the same radius r , one can obtain the following equation from (4.54) and (4.55):

$$\tau_p(t) - \tau_d(t) = \tau_f(t), \quad (4.56)$$

where $\tau_p = r(T_1 - T_4)$ is the proximal torque, $\tau_d = r(T_2 - T_3)$ is the distal torque, and $\tau_f = r(F_A + F_B)$ represents the total friction effect on both CCMs.

To investigate the torque transmission characteristics of the pair of CCMs, the author carried out numerous observation trials. Figure 4-11 illustrates the block diagram of the observation process. From the digital to analog conversion (DAC) modules of the controller DS1104, a torque reference signal $\tau_r(t) = 0.144\sin(\pi t)$ and a maximum speed reference signal $v_{max}(t) = 0.3$ were sent to the servo drive, which was operating in torque mode control. This drive calculated the reference current $i_r(t)$ from the input signals (i.e., $\tau_r(t)$ and $v_{max}(t)$). It drove the servomotor to generate the proximal torque. All the torque and position signals at both pulleys were monitored by analog to digital conversion (ADC) modules and incremental encoder modules of the controller DS1104 and sent to the computer to record them for analysis.

Figure 4-11: The block diagram of the torque transmission investigation experiments

The recorded torque signals were filtered by a zero-phase digital filter in MATLAB to eliminate high-frequency noise. The upper panel of Figure 4-12 plots the data with respect to the recording time while the lower panel portrays the output torque $\tau_o(t)$ (measured by the distal torque sensor-TS2) with respect to the input torque $\tau_i(t)$ (measured by the proximal torque sensor-TS1). It must be noticed that $\tau_o = \tau_d - (c_d\dot{\phi}_d + J_d\ddot{\phi}_d)$ and $\tau_i = \tau_p + (c_d\dot{\phi}_d + J_d\ddot{\phi}_d)$, where J_d , c_d , J_p , c_p are the

inertial and viscous coefficients at the distal and proximal pulleys. Denoting τ_h as the total friction and hysteresis effects on the CCMs, one has $\tau_h(t) = \tau_i(t) - \tau_o(t)$.

Figure 4-12: Fitting of the observed experimental results and the simulation results (the Bouc-Wen model with torque input)

From the observed data of the torque transmission, the author proposed using the Bouc-Wen model with torque input to portray the torque transmission profile (the dash line loop in the lower panel of Figure 4-12). In this study, the author focused on stabilizing the system and used the simplified symmetric Bouc-Wen model instead of the asymmetric model as in the previous study. The torque transmission model can be rewritten as follows:

$$\tau_h(t) = \kappa_\tau \tau_i + \kappa_\eta \eta + T_0 \quad (4.57)$$

$$\tau_o = \tau_i - \tau_h = (1 - \kappa_\tau) \tau_i - \kappa_\eta \eta - T_0, \quad (4.58)$$

in which κ_τ , κ_η are positive coefficients; constant T_0 represents the effect of the initial pretension; and the internal state η can be derived by:

$$\dot{\eta} = \rho[\dot{\tau}_i - \sigma|\dot{\tau}_i||\eta|^{n-1}\eta + (\sigma - 1)\dot{\tau}_i|\eta|^n] \quad (4.59)$$

In this model, the input is the torque τ_i instead of the displacement, and $\dot{\tau}_i$ is the first time-derivative of τ_i . A similar optimization process was used to identify the parameters of this model. The best estimated parameters will minimize the following fitness function:

$$f(\tau) = \sqrt{\frac{1}{M} \sum_{i=1}^M (\tau_{om}(i) - \tau_o(i))^2} \quad (4.60)$$

where i is the sampling index, and M is the total number of recorded samples. This function is the root-mean-square error of the observed experimental output torque $\tau_o(i)$ and the estimated output torque $\tau_{om}(i)$. The value of $\tau_{om}(i)$ was approximated by using the Runge-Kutta fourth-order method and the estimated parameters, which were generated by the aforementioned optimization methods. The GA method was used to roughly locate the global minimum of the fitness function and the Nelder-Mead simplex method refined the results subsequently. The lower panel in Figure 4-12 also shows the modeled output torque τ_{om} (plotted by the continuous line) with the identified parameters: $\rho = 0.09582$, $\sigma = 98.36$, $n = 3.871$, $\beta = 0.9978$, $\kappa_\eta = 7.004$, and $T_0 = 0.05174$. The root-mean-square error is 0.6615 Nm (equivalent to 2.13 % of the range of the output torque). It is obvious that the modeled output torque matches closely with the recorded output torque, therefore, one can use the symmetric Bouc-Wen model to describe the torque transmission and compensate the hysteresis effects of the pair of CCMs.

4.3.2.2 Torque Mode Control of the Brushless Servomotor

Based on the electrodynamic characteristics of the brushless servomotor, the servo drive, and the zero-backlash gearbox, one can express the torque measured by the proximal torque sensor as:

$$\tau_i = N(\tau - J\ddot{\phi} - v\dot{\phi}) \quad (4.61)$$

$$\tau = k_\tau i \quad (4.62)$$

where τ is the generated motor torque, N is the gear ratio, and $-v\dot{\phi}$, $-J\ddot{\phi}$ are viscous and inertial effects on the servomotor and gearbox. The ‘Torque-Per-Amp’ coefficient k_τ contains the effect of motor flux and is provided by the motor manufacturer. More specifically, when the controller sends the final torque control signal $u_c(t)$ to the servo drive M701-03400100A, it calculates the final current reference $i_r(t)$, then the internal current proportional-integral (PI) loop (with two explicit gains k_{pi} and k_{ii}) will adjust the motor current to maintain the motor output torque as required as $\tau = cu_c$. The fundamental equations are:

$$i_r = \alpha_\tau cu_c \quad (4.63)$$

$$e_i = i_r - i \quad (4.64)$$

$$i = k_{pi}e_i + k_{ii} \int_0^t e_i(\omega) d\omega \quad (4.65)$$

The drive coefficient $\alpha_\tau = 1/k_\tau$ is calculated by the servo drive, and c is an equivalent constant between the output range of the controller and the rated maximum torque of the servomotor. However, the PI current loop of the motor drive updates its output current at an extremely high frequency (4 μ s cycle) compared to our control rate (1 ms cycle), in this study, the author simplified the current loop fundamental as:

$$\tau = Au_c \quad (4.66)$$

where A is a positive coefficient.

4.3.3 Controller Design

Taking into account the external disturbance during the control progress and denoting $y(t)$ as the feedback output torque at the distal side of the CCMs, one can write:

$$\begin{aligned} y &= (1 - \kappa_t)N(Au_c - J\ddot{\phi} - \nu\dot{\phi}) - \kappa_\eta\eta - T_0 \\ &= \alpha u_c - \vartheta_1 x_1 - \vartheta_2 x_2 - \kappa_\eta\eta - T_0 - d \end{aligned} \quad (4.67)$$

where $\alpha = (1 - \kappa_t)NA$, $\vartheta_1 = (1 - \kappa_t)N^2\nu$, $\vartheta_2 = (1 - \kappa_t)N^2J$, $x_1 = \dot{\phi}_p$, $x_2 = \ddot{\phi}_p$, and $\phi = N\phi_p$ (Note that ϕ_p is the angular displacement at the proximal pulley and ϕ is the angular displacement of the motor rotor). Denoting $e(t)$ as the torque tracking error, one has:

$$e(t) = y(t) - y_r(t) \quad (4.68)$$

Our goal is to design a closed-loop control scheme which can compensate the hysteresis effect and nonlinearities of the CCMs and enhance the torque tracking performance. This study presents the design and experimental results of a new adaptive control scheme and three conventional closed-loop control schemes for this purpose. Only the stability of the new controller design is discussed. To be able to design such controllers, the following assumptions were made: (i) no cable slacking occurs during the experiments since sufficient pretension was established to the pair of CCMs; (ii) the reference torque profile $y_r(t)$ is explicit, piece-wise continuous, and bounded; (iii) the output torque feedback $y(t)$ and its first time-derivative $\dot{y}(t)$ are bounded.

4.3.3.1 Conventional Proportional-Integral-Derivative Controller (PID)

The control signal can be derived from the tracking error $e(t)$, the accumulation of error $\int_0^t e(\omega)d\omega$, and the varying speed of error $\dot{e}$ as follows:

$$u_c(t) = -k_p e - k_i \int_0^t e(\omega)d\omega - k_d \dot{e} \quad (4.69)$$

Although this controller can give excellent results, it might experience chattering and instability due to the derivative term $-k_d \dot{e}$.

4.3.3.2 Proportional-Integral Controller (PI)

One can drop the derivative term $-k_d \dot{e}$ by defining the control signal as:

$$u_c(t) = -k_p e - k_i \int_0^t e(\omega)d\omega \quad (4.70)$$

4.3.3.3 Proportional-Integral Controller with Velocity Injection (PIV)

One also can replace the derivative term $-k_d \dot{e}$ by a speed term $-k_d \dot{\phi}_d$ as follows:

$$u_c(t) = -k_p e - k_i \int_0^t e(\omega)d\omega - k_d \dot{\phi}_d \quad (4.71)$$

where $\dot{\phi}_d$ is the angular velocity of the distal pulley.

4.3.3.4 Robust Adaptive Controller

The above controllers are none-modeled controllers, which do not compensate for the hysteresis effects and disregard the nonlinear transmission characteristics of the system. Denoting $D_h(t)$ as the total disturbance on system including the variable hysteresis effect $\kappa_\eta \eta$ on the CCMs (due to the variable configuration of the conduits), external disturbance $d(t)$, and the pretension offset torque T_0 , one has:

$$D_h = \kappa_\eta \eta + d + T_0 \quad (4.72)$$

$$y = \alpha u_c - \vartheta_1 x_1 - \vartheta_2 x_2 - D_h, \quad (4.73)$$

To develop an adaptive control scheme, another assumption was made: (iv) the angular velocity and acceleration of the proximal pulley are assumed bounded. Then, we define the variable of interest as the accumulated error z , and the filtered tracking error s as follows:

$$z = \int_{t_0}^t e(\omega) d\omega \quad (4.74)$$

$$s = \left(\frac{d}{dt} + \lambda \right) z = e + \lambda z \quad (4.75)$$

where λ is an arbitrary positive constant. The integral lower limit t_0 can be chosen as the initial moment of the control process or the moment when e is close to zero. To derive the control signal u_c , we introduced the new virtual control signals $\bar{u}_c$ as follows:

$$u_c = \hat{\beta} \bar{u}_c \quad (4.76)$$

where $\hat{\beta}$ is the estimate of the new control coefficient $\beta = 1/\alpha$. Let $\tilde{\beta}$ be the estimated error of β as $\tilde{\beta} = \beta - \hat{\beta}$. From above definitions, the control term in (4.73) can be expressed in terms of the new virtual control signal $\bar{u}_c$ as follows:

$$\alpha u_c = \alpha \hat{\beta} \bar{u}_c = \alpha(\beta - \tilde{\beta}) \bar{u}_c = \bar{u}_c - \alpha \tilde{\beta} \bar{u}_c \quad (4.77)$$

Then, the first time-derivatives of the accumulated error z and the new tracking error s defined by (4.68), (4.74), (4.75) and the closed-loop system are:

$$\dot{z} = e = s - \lambda z \quad (4.78)$$

$$\begin{aligned} \dot{s} &= \dot{e} + \lambda e = \dot{e} + \lambda(y - y_r) \\ &= \dot{e} + \lambda(\bar{u}_c - \alpha \tilde{\beta} \bar{u}_c - \vartheta_1 x_1 - \vartheta_2 x_2 - D_h - y_r) \end{aligned} \quad (4.79)$$

Denote $\Delta_h > 0$ as the bound of the system disturbance $D_h(t)$ (i.e., $|D_h| \leq \Delta_h$), and denote $\hat{\vartheta}_i$ and $\hat{\Delta}_h$ as the estimates of parameters ϑ_i and Δ_h , one has the

corresponding estimated errors as $\tilde{\vartheta}_i = \vartheta_i - \hat{\vartheta}_i$, $\tilde{\Delta}_h = \Delta_h - \hat{\Delta}_h$. Then, the control and update laws can be designed as follows:

$$\bar{u}_c = -ks + y_r - \frac{z + \dot{e}}{\lambda} + \hat{\vartheta}_1 x_1 + \hat{\vartheta}_2 x_2 - \hat{\Delta}_h \tanh(s/\epsilon) \quad (4.80)$$

$$\dot{\hat{\beta}} = -\gamma \lambda s \bar{u}_c - \sigma_\alpha \hat{\beta} \quad (4.81)$$

$$\dot{\hat{\Delta}}_h = \mu \lambda s \tanh(s/\epsilon) - \sigma \hat{\Delta}_h \quad (4.82)$$

$$\dot{\hat{\vartheta}}_i = -\mu_i \lambda s x_i - \sigma_i \hat{\vartheta}_i \quad (4.83)$$

where the arbitrary positive coefficient k denotes the control gain; and the positive parameters γ, μ, μ_i ($i = 1, 2$) represent the estimating coefficients. In addition, the leakage terms [142] introduced by the positive leakage coefficients σ_α, σ , and σ_i are added to the parameter estimated laws (4.81)-(4.83) to prevent parameter drifting. Moreover, the tangent hyperbolic function $\tanh(s/\epsilon)$ is used to replace the $\text{sign}(s)$ function as a smooth sign function, where ϵ is a small positive design constant.

Theorem 2: Consider the dynamic system, which is modeled by (4.58), (4.59), (4.72), (4.73) and satisfies the assumptions (i)-(iv), with the control and update laws (4.80)-(4.83), the following statements hold.

- i. All signals of the resulting closed-loop system are uniformly ultimately bounded.
- ii. The filtered tracking error s and accumulated error z converge to a neighborhood of zero with the bound defined by $\delta = \sqrt{2\phi/\theta}$, where $\phi = \alpha \sigma_\alpha \beta^2 / 2\gamma + \sigma \Delta_h^2 / 2\mu + \sum_{i=1}^2 (\sigma_i \vartheta_i^2 / 2\mu_i) + 0.2785\epsilon \lambda \Delta_h$ and $\theta = \min(2\lambda, 2k\lambda, \sigma_1, \sigma_2, \sigma, \sigma_\alpha)$.
- iii. The tracking error converges to a neighborhood of zero with the bound of $(1 + \lambda)\delta$.

Proof: Consider the Lyapunov function

$$V = \frac{1}{2}s^2 + \frac{1}{2}z^2 + \sum_{i=1}^2 \frac{1}{2\mu_i} \tilde{\vartheta}_i^2 + \frac{1}{2\gamma} \alpha \tilde{\beta}^2 + \frac{1}{2\mu} \tilde{\Delta}_h^2 \quad (4.84)$$

then, substitute (4.79), (4.80) into to the first time-derivative of the Lyapunov function, one has:

$$\begin{aligned}
\dot{V} &= s\dot{s} + z\dot{z} + \sum_{i=1}^2 \frac{\tilde{\vartheta}_i \dot{\tilde{\vartheta}}_i}{\mu_i} + \frac{\alpha \tilde{\beta} \dot{\tilde{\beta}}}{\gamma} + \frac{\tilde{\Delta}_h \dot{\tilde{\Delta}}_h}{\mu} \\
&= s\dot{s} + z(s - \lambda z) + \sum_{i=1}^2 \frac{\tilde{\vartheta}_i \dot{\tilde{\vartheta}}_i}{\mu_i} + \frac{\alpha \tilde{\beta} \dot{\tilde{\beta}}}{\gamma} + \frac{\tilde{\Delta}_h \dot{\tilde{\Delta}}_h}{\mu} \\
&= -\lambda z^2 + s[z + \dot{e} + \lambda(\bar{u}_c - \alpha \tilde{\beta} \bar{u}_c - \vartheta_1 x_1 - \vartheta_2 x_2 - D_h - y_r)] \\
&\quad - \sum_{i=1}^2 \frac{\tilde{\vartheta}_i \dot{\tilde{\vartheta}}_i}{\mu_i} - \frac{\alpha \tilde{\beta} \dot{\tilde{\beta}}}{\gamma} - \frac{\tilde{\Delta}_h \dot{\tilde{\Delta}}_h}{\mu} \\
&= -\lambda z^2 + \lambda s \left[\bar{u}_c + \frac{z + \dot{e}}{\lambda} - \hat{\vartheta}_1 x_1 - \hat{\vartheta}_2 x_2 - y_r \right] - \frac{\tilde{\Delta}_h \dot{\tilde{\Delta}}_h}{\mu} - \lambda s D_h \\
&\quad - \sum_{i=1}^2 \tilde{\vartheta}_i \left(\lambda s x_i + \frac{\dot{\tilde{\vartheta}}_i}{\mu_i} \right) - \alpha \tilde{\beta} \left(\lambda s \bar{u}_c + \frac{\dot{\tilde{\beta}}}{\gamma} \right)
\end{aligned} \tag{4.85}$$

Substituting the control and update laws (4.80)-(4.83) into (4.85), one has:

$$\begin{aligned}
\dot{V} &= -\lambda z^2 + \lambda s \left[-ks - \hat{\Delta}_h \tanh\left(\frac{s}{\epsilon}\right) \right] + \sum_{i=1}^2 \frac{\sigma_i \hat{\vartheta}_i \tilde{\vartheta}_i}{\mu_i} + \frac{\alpha \sigma_\alpha \hat{\beta} \tilde{\beta}}{\gamma} \\
&\quad - \tilde{\Delta}_h \frac{\mu \lambda s \tanh\left(\frac{s}{\epsilon}\right) - \sigma \hat{\Delta}_h}{\mu} - \lambda s D_h \\
&\leq -\lambda z^2 - k\lambda s^2 + \sum_{i=1}^2 \frac{\sigma_i \hat{\vartheta}_i \tilde{\vartheta}_i}{\mu_i} + \alpha \frac{\sigma_\alpha \hat{\beta} \tilde{\beta}}{\gamma} + \frac{\sigma \hat{\Delta}_h \tilde{\Delta}_h}{\mu} \\
&\quad - \lambda s (\hat{\Delta}_h + \tilde{\Delta}_h) \tanh\left(\frac{s}{\epsilon}\right) + \lambda |s| \Delta_h
\end{aligned} \tag{4.86}$$

The inequality (4.86) was obtained from the fact that $|D_h| \leq \Delta_h$, and therefore $-\lambda s D_h \leq \lambda |s| \Delta_h$. Subsequently, by adopting the inequality $0 \leq |s| - s \tanh(s/\epsilon) \leq 0.2785\epsilon$, which was introduced by *Lemma 1* in [143], one has $0 \leq \lambda \Delta_h [|s| - s \tanh(s/\epsilon)] \leq 0.2785\epsilon \lambda \Delta_h$. Hence, the inequality (4.86) becomes:

$$\dot{V} \leq -\lambda z^2 - k\lambda s^2 + \frac{\alpha \sigma_\alpha}{\gamma} \hat{\beta} \tilde{\beta} + \frac{\sigma}{\mu} \hat{\Delta}_h \tilde{\Delta}_h + \sum_{i=1}^2 \frac{\sigma_i}{\mu_i} \hat{\vartheta}_i \tilde{\vartheta}_i + 0.2785\epsilon \lambda \Delta_h \tag{4.87}$$

In addition, the first leakage term in (4.87) can be rewritten as follows:

$$\begin{aligned}
\frac{\alpha\sigma_\alpha}{\gamma}\hat{\beta}\tilde{\beta} &= \frac{\alpha\sigma_\alpha}{\gamma}\tilde{\beta}(\beta - \tilde{\beta}) = \frac{\alpha\sigma_\alpha}{\gamma}(\beta\tilde{\beta} - \tilde{\beta}^2) \\
&= \frac{\alpha\sigma_\alpha}{\gamma}\left[-\frac{\tilde{\beta}^2}{2} - \frac{1}{2}(\tilde{\beta}^2 - 2\beta\tilde{\beta} + \beta^2) + \frac{\beta^2}{2}\right] \\
&= \frac{\alpha\sigma_\alpha}{\gamma}\left[-\frac{\tilde{\beta}^2}{2} - \frac{1}{2}(\tilde{\beta} - \beta)^2 + \frac{\beta^2}{2}\right] \leq -\sigma_\alpha\frac{\alpha\tilde{\beta}^2}{2\gamma} + \frac{\alpha\sigma_\alpha\beta^2}{2\gamma}
\end{aligned} \tag{4.88}$$

Similarly, one can obtain:

$$\frac{\sigma}{\mu}\hat{\Delta}_h\tilde{\Delta}_h \leq -\sigma\frac{\tilde{\Delta}_h^2}{2\mu} + \frac{\sigma\Delta_h^2}{2\mu} \tag{4.89}$$

$$\sum_{i=1}^2 \frac{\sigma_i}{\mu_i}\hat{\vartheta}_i\tilde{\vartheta}_i \leq -\sum_{i=1}^2 \sigma_i\frac{\tilde{\vartheta}_i^2}{2\mu_i} + \sum_{i=1}^2 \frac{\sigma_i\vartheta_i^2}{2\mu_i} \tag{4.90}$$

Hence, the time derivative of the Lyapunov function becomes:

$$\begin{aligned}
\dot{V} &\leq -2\lambda\frac{z^2}{2} - 2k\lambda\frac{s^2}{2} - \sum_{i=1}^2 \sigma_i\frac{\tilde{\vartheta}_i^2}{2\mu_i} - \sigma_\alpha\frac{\alpha\tilde{\beta}^2}{2\gamma} - \sigma\frac{\tilde{\Delta}_h^2}{2\mu} + \frac{\alpha\sigma_\alpha\beta^2}{2\gamma} + \frac{\sigma\Delta_h^2}{2\mu} \\
&\quad + \sum_{i=1}^2 \frac{\sigma_i\vartheta_i^2}{2\mu_i} + 0.2785\epsilon\lambda\Delta_h \\
&\leq -\theta V + \phi
\end{aligned} \tag{4.91}$$

if one denotes $\theta = \min(2\lambda, 2k\lambda, \sigma_1, \sigma_2, \sigma, \sigma_\alpha)$ and the constant quantity $\phi = \alpha\sigma_\alpha\beta^2/2\gamma + \sigma\Delta_h^2/2\mu + \sum_{i=1}^2(\sigma_i\vartheta_i^2/2\mu_i) + 0.2785\epsilon\lambda\Delta_h$. Hence, when $V \geq V^* = \phi/\theta$, $\dot{V} \leq 0$, which implies V is a non-increasing function, therefore, all signals z , s , $\tilde{\Delta}_h$, $\hat{\Delta}_h$, $\tilde{\alpha}$, $\hat{\alpha}$ and $\tilde{\vartheta}_i$, $\hat{\vartheta}_i$ ($i = 1, 2$) are bounded. This combines with assumptions (ii)-(iv) and control laws (4.76), (4.80) to make the control signals $\bar{u}_c$ and u_c be bounded. Then, (4.81)-(4.83) imply that $\dot{\beta}$, $\dot{\Delta}_h$, $\dot{\vartheta}_i$ are bounded.

Furthermore, multiplying both sides of (4.91) with $e^{\theta t}$ and then integrating the inequality, one can obtain:

$$0 \leq \frac{1}{2} z^2 \leq V(t) \leq \left[V(t_0) - \frac{\phi}{\theta} \right] e^{-\theta(t-t_0)} + \frac{\phi}{\theta} \quad (4.92)$$

in which $V(t_0)$ is obtained at the initial moment t_0 when the errors are zero:

$$V(t_0) = \frac{1}{2\gamma} \alpha \tilde{\beta}^2(t_0) + \frac{1}{2\mu} \tilde{\Delta}_h^2(t_0) + \sum_{i=1}^2 \frac{1}{2\mu_i} \tilde{\theta}_i^2(t_0) \quad (4.93)$$

Thus, one has:

$$|z| \leq \sqrt{2 \left[V(t_0) - \frac{\phi}{\theta} \right] e^{-\theta(t-t_0)} + \frac{2\phi}{\theta}} \quad (4.94)$$

When t increases, $[V(t_0) - \phi/\theta]e^{-\theta(t-t_0)} \rightarrow 0$, implying that the accumulated error z converges to the proximity of zero with the bound δ as $|z| \leq \delta = \sqrt{2\phi/\theta}$.

Using a similar analysis procedure, one can obtain the bound of the filtered tracking error as $|s| \leq \delta = \sqrt{2\phi/\theta}$. This implies statements (ii). Then, from the inequality $|s| = |e + \lambda z| \geq |e| - |\lambda z|$, one can prove statement (iii) as:

$$|e| \leq |e + \lambda z| + \lambda |z| = |s| + \lambda |z| \leq \delta + \lambda \delta = (1 + \lambda) \delta \quad (4.95)$$

Since the bounds are independent of t_0 , all signals are uniformly bounded and statement (i) holds. ■

Remark 2: The leakage terms in the adaptive laws (4.81)-(4.83) prevent the estimated parameters drifting to infinity and the use of the $\tanh(\cdot)$ function helps the system avoid chattering. Therefore, they assist the system in achieving robustness. On the other hand, they also introduce a residue to the tracking error. Thus, to achieve smaller errors, one may choose tiny values for σ_i , σ , σ_α , and ϵ . However, too small leakage terms cannot adapt to a steep increase of the estimated values. If certain knowledge of the parameter limit is available, one can employ the switching σ -modification leakage terms [142] to improve the control performance.

Remark 3: The noise effects contained in the time derivative of error $\dot{\mathbf{e}}$ can be minimized by increasing the error accumulation gain λ . In addition, the higher values of the control and update gains k , λ , γ , μ , μ_i can mitigate the effect of the initial estimated errors and increase the adaptation speed. However, the large gains can cause chattering and high amplitude tracking errors. In practice, one can choose reasonable values of k and λ with respect to the reference $\mathbf{y}_r$ and the amplitude of $\dot{\mathbf{e}}$, subsequently, one should fix these gains to adjust the estimating gains.

4.3.4 Experiment Setup and Validation

4.3.4.1 Experimental Procedure

Figure 4-13 shows the real experimental model and Figure 4-14 sketches the general block diagram for all four mentioned control schemes. All aforementioned control algorithms were implemented and compiled in the MATLAB-Simulink environment. The resulting programs were then loaded into the dSPACE controller's software and hardware (i.e., the Control Desk and the DS1104 board). In operation, the controller DS1104 receives the feedback signals from the distal torque sensor and encoders to calculate control signals. Two control signals are sent to the servo drive via 16-bit-DAC modules. The torque signal commands the drive and motor to generate the required torque while the maximum speed signal is used to limit the rotating speed of the motor.

Figure 4-13: The real experimental system

In addition, while the PI and PID controllers merely use the output torque feedback to calculate the control signals, the PIV controller also utilizes the angular velocity of the distal pulley (fed back by the distal encoder). On the other hand, the adaptive control law employs the rotating speed and acceleration of the proximal pulley to estimate and compensate the viscous and inertial effects of the actuator.

Figure 4-14: The common block diagram for all closed-loop torque tracking control schemes

Two types of experiments were conducted: control with fixed and randomly varying configurations of the CCMs to evaluate the performance of the aforementioned schemes. A combination of two sinusoidal signals (Signal A has an amplitude of 10 Nm and a frequency of 0.25 Hz and Signal B has an amplitude of 3 Nm and a frequency of $0.2\sqrt{3}$ Hz) was used as the torque reference $y_r(t)$.

4.3.4.2 Experimental Results

Figure 4-15 shows the results of the control experiments with fixed cable-conduit configurations using the four control schemes. Specifically, the torque tracking data of the PI, PID, PIV, and adaptive control schemes are plotted respectively from the top. For each scheme, the left panel shows the same reference signal $25 + 10 \sin(2\pi t \times 0.25) + 3 \sin(2\pi t \times 0.2\sqrt{3})$ (Nm) and the corresponding output torque (Nm) while the middle panel plots the torque tracking error (Nm) at the same scale with the left panel. On the other hand, the right panel plots the corresponding torques (Nm) at the proximal and distal pulleys at a larger scale. The proximal torque has much larger values, and its graph fluctuates more wildly than the distal torque. All horizontal axes represent the trial time in seconds. The chosen parameters of the conventional controllers are shown in Table 4-1 while the designed parameters of the adaptive controller are shown in

Table 4-2. The initial guesses for parameter integrations are $\hat{\Delta}_{h0} = 7$, $\hat{\beta}_0 = 0$, and $\hat{\vartheta}_{i0} = 0$.

Table 4-1: The designed parameters for PI, PID, and PIV controllers

Designed Parameters	k_p	k_i	k_d
PI Controller	30	18	—
PID Controller	30	18	2
PIV Controller	30	18	10

Table 4-2: Designed Parameters for the Adaptive Controller

Parameters	k	λ	$\lambda\mu$	$\lambda\gamma$	$\lambda\mu_i$	σ	σ_α	σ_i	ϵ
Values	24	8	0.05	0.002	0.004	0.01	0.01	0.01	0.05

Figure 4-15: The experimental results of four closed-loop control schemes for the same reference signal $25 + 10\sin(2\pi t \times 0.25) + 3\sin(2\pi t \times 0.2\sqrt{3})$ (Nm) when the cable-conduit configurations were fixed. The tracking performance of the PI, PID, PIV and adaptive control schemes are shown respectively from top to bottom. The left panels plot the reference and output torques, the middle panels show the corresponding tracking errors and the right panels compare the torque values at the proximal and distal pulleys.

It is evident that the torque tracking error of the adaptive controller has much smaller amplitudes compared to the other tracking errors. The root-mean-square errors (RMSE) of the PI, PID, and PIV control trials are approximately twice as big as the adaptive compensation trials. Quantitatively, Table 4-3 shows the RMSE of six typical trials of four control schemes in both fixed and varying configuration conditions.

The average, standard deviation and percentage of the maximum reference of these errors are also calculated and summarized. For the fixed configuration of CCMs, the adaptive scheme has an RMSE of 0.3027 ± 0.0081 Nm, which is approximate 0.8% of the maximum reference, while the RMSEs of the PI, PID, and PIV tracking trials are 0.5991 ± 0.0154 Nm, 0.6093 ± 0.0164 Nm, and 0.532 ± 0.0134 Nm respectively. Also, the graph of the adaptive compensation error is very consistent while the others have significant fluctuations, especially when the system reverses its direction. This is because the adaptive scheme, which is aided by the model-based compensation term $-\hat{\Delta} \tanh(s/\epsilon)$, can mitigate the hysteresis effects and keep the output torque tracking closely during sudden reverses of the reference signal.

In addition, it can also be observed that the PI controller, which does not include the damping term $-k_d \dot{e}$, shows more significant chattering responses than the PID and PIV controllers. Furthermore, the PIV controller shows better tracking performance compared to the PID controller. This results from the fact that the PID controller uses the derivative of the torque tracking error for the damping term while the PIV controller takes the angular speed of the output pulley to compensate for the fluctuation. The output torque, which is fed back by the ADC module, usually contains more noise than the output angular displacement, which is monitored by the incremental encoder. Consequently, the damper composed by the output angular speed works more efficiently. Thus, the average RMSE of the PIV

control tracking trials is slightly smaller than the PI and PID control trials (about 0.5 Nm compared to about 0.6 Nm).

Other control trials were carried out with varying configurations of the CCMs to validate the effectiveness of the designed control schemes in coping with the natural change of the cable conduit configuration. To be able to compare the results with the fixed CCMs' configuration trials, the same controllers' parameters were used for torque tracking with the same reference signal. However, during the experimental trials, the route and curvature of the CCMs were wildly and randomly changed.

Figure 4-16 shows the results of these varying configuration experiments. The left panels show the output torques for the same reference signal $25 + 10 \sin(2\pi t \times 0.25) + 3\sin(2\pi t \times 0.2\sqrt{3})$ (Nm) of the PI, PID, PIV, and adaptive control schemes respectively; the right panels show the corresponding tracking errors. The quantitative results of these experimental trials are also shown in the second part of Table 4-3. It can be observed that similar results were recorded. The adaptive control scheme still shows more consistently effective capability in output torque tracking. The data in Table 4-3 also demonstrate that tracking errors of trials with varying CCM configurations are slightly larger than those with the fixed CCM configurations.

Figure 4-16: The experimental results of four closed-loop control schemes for the same reference signal $25 + 10\sin(2\pi t \times 0.25) + 3\sin(2\pi t \times 0.2\sqrt{3})$ (Nm) when the cable-conduit configurations were varying during the trials. The tracking performance of the PI, PID, PIV, and adaptive control schemes are shown respectively from top to bottom. The left panels plot the reference and output torques and the middle panels show the corresponding tracking errors.

Table 4-3: The root-mean-square error (RMSE) of the experimental trials of four control schemes in both fixed and varying configurations of the CCMs. The results of six typical trials for each scheme are described.

	Fixed Configuration				Varying Configuration			
Trials	PI	PID	PIV	Adaptive	PI	PID	PIV	Adaptive
1	0.5967	0.6323	0.5550	0.2937	0.6368	0.6333	0.5375	0.3005
2	0.5856	0.5960	0.5212	0.2915	0.6054	0.5912	0.5468	0.3112
3	0.6087	0.6058	0.5296	0.3045	0.6071	0.6052	0.5164	0.3242
4	0.5942	0.6271	0.5403	0.3084	0.5905	0.6262	0.5577	0.3213
5	0.6250	0.5942	0.5243	0.3091	0.5901	0.6187	0.5269	0.3247
6	0.5845	0.6004	0.5213	0.3092	0.5902	0.6090	0.5084	0.3350
Avg	0.5991	0.6093	0.532	0.3027	0.6034	0.6139	0.5323	0.3195
SD	0.0154	0.0164	0.0134	0.0081	0.0182	0.0153	0.0186	0.012
% of Max	1.5766	1.6034	1.4	0.7966	1.5879	1.6155	1.4008	0.8408

To evaluate the transient performance, the recorded signals were plotted in a shorter time interval (12 seconds) in the initial stage of the control processes in Figure 4-17. All control schemes were initiated at the same moment of the same interval of the reference signal $25 + 10\sin(2\pi t \times 0.25) + 3\sin(2\pi t \times 0.2\sqrt{3})$ (Nm).

It is clear that the robust adaptive control scheme converges more slowly due to the effects of initial estimates, but its torque tracking performance was enhanced significantly after the adaptation took effect. It can be observed that the PI controller, which is not supported by a damping component performed more poorly in coping with fluctuations while PID and PIV which are backed by a damping term can extinguish the fluctuations effectively. Three conventional control schemes performed similarly at the initiation. At that moment, adaptive control had larger fluctuation because the parameters had not converged to their real values yet. However, after the adaptation took effects, the adaptive control showed significantly

better performance with consistently smaller tracking error as shown in the right panels.

Figure 4-17: Transient performance of the four closed-loop control schemes for the same reference signal $25 + 10\sin(2\pi t \times 0.25) + 3\sin(2\pi t \times 0.2\sqrt{3})$ (Nm) when the control trials initiate. The results of the PI, PID, PIV, and adaptive schemes are shown respectively from top to bottom. The left panels show the reference, output and proximal torques, the right panels plot the corresponding tracking errors.

4.3.5 Discussion and Conclusion

This study proposed a new approach for output torque tracking control for the systems preceded by CCMs. With the availability of torque feedback, four closed-loop control schemes were designed and implemented to control a 1-DOF system actuated by a pair of CCMs. Direct torque signals were generated by the controllers

and sent to a servo drive operating in torque control modes to adjust the motor output torque.

Although all control schemes performed effectively, the new adaptive control scheme utilized the Bouc-Wen hysteresis model to estimate and compensate for the bound of hysteresis and disturbance effects and therefore provided better results. The Bouc-Wen model used the proximal torque as the input signal of the hysteresis profile instead of the proximal displacement as in the original Bouc-Wen model and other modeling approaches. By expressing the output torque in terms of the input torque, this model enabled the possibility for a new adaptive controller to estimate and compensate for the disturbance and hysteresis effects on the CCMs. It also estimated and compensated for the dynamic viscous and inertial effects on the high-speed rotating rotor and the gearbox. This adaptive compensation scheme did not require in advance the value of the model parameters. All unknown parameters such as the bound of total disturbance and hysteresis effects, the viscous and inertial coefficients of the motor and gearbox, and the control gain were online estimated during the control trials.

The stability of the new control scheme was discussed in detail and experiments were carried out to validate its effectiveness. It significantly enhanced the torque tracking performance for the system preceded by a pair of CCMs in both fixed and varying configurations of the CCMs.

For future developments, experimental validation with full-scale loading (compared to the robotic footwear testing system) will be conducted. Subsequently, these control schemes will be developed and implemented for controlling the robotic foot for footwear testing with the availability of output torque feedback. The footwear testing system is equipped with multiple load cells at the contact floor to monitor the ground reaction forces and calculate the real-time torque of the foot joints.

4.4 Third Torque Control Design

Improving Torque Control Performance for Systems driven by Cable-Conduit Mechanisms with a Multi-Segments Model

4.4.1 Introduction

Although in the two previous studies, the author was able to design and validate some effective control schemes for torque tracking for robotic joints actuated by CCMs, the reference torque signals were relatively small compared to the real human ankle torque during walking and running gaits. This limitation was caused by the loading capacity of the two springs. In this study, a 1-DOF system with two powerful servomotors was set up (one motor acts as the load), to increase the load capacity and thus a higher reference torque can be applied. Figure 4-18 illustrates the structure of this system while Figure 4-19 shows the signal flow diagram of the general program for torque transmission investigation. It comprises two actuators of the footwear testing system described in Section 3.4 and a pair of CCMs connecting them. This is the same pair of CCMs designed for actuating the ankle joint. The bigger actuator controls the system while the smaller acts as a dynamic load. As shown in Figure 4-18 and Figure 4-19, the real-time controller dSPACE DS1104 sends two control signals to each servo drive. The first signal is the real-time command torque and the second signal is the maximum speed signal that limits the speed of the servomotor. These motors work independently.

Figure 4-18. The 1-DOF system actuated by a pair of cable-conduit mechanisms (CCMs)

Figure 4-19. Signal flow diagram of the system working in free motion

The load generated by the smaller actuator simulates a physical spring-damper system. The reaction torque was defined as

$$u_{small} = k\varphi + d\dot{\varphi} \quad (4.96)$$

in which φ and $\dot{\varphi}$ are the angular position and velocity of the distal pulley, u_{small} is the control signal sent to the small servo drive, k and d represent the spring and damper coefficients.

4.4.2 System Modeling

4.4.2.1 CCM Torque Transmission Observation

In this study, a higher-torque signal was applied to the system to investigate the torque transmission at a high scale similar to the human ankle's moment during walking and running gaits. Since the ankle's moment is one-sided, a positive torque signal was chosen as the reference. It was a combination of two sinusoidal signals as $\tau_r(t) = 48 + 38\sin(0.4\pi t) + 10\sin(0.4\sqrt{2}\pi t)$. Then, the author implemented a free motion control scheme as illustrated in Figure 4-19 on the experimental system shown in Figure 4-18 and recorded all position and torque signals to analyze the torque transmission. The output torque was monitored by the Futek torque sensor TRS300-100Nm at the distal pulley while the input torque was fed back by the Interface torque sensor T8-ECO-200Nm. The current configuration of the conduit is a half-circle route (with a radius of approximately 0.573 m). Figure 4-20 shows the graphs of two typical trials of the distal torque versus the proximal torque when applying this reference signal to the bigger servo drive, and letting the smaller actuator react to the control torque with equation (4.96).

Figure 4-20: Experimental data of two typical control trials on the system described in Figure 4-18 and the signal flow diagram shown in Figure 4-19.

4.4.2.2 System Modeling and Parameters Identification

As shown in Figure 4-20, the profile of the output torque versus the input torque is significantly asymmetric between the loading (phases 1 and 2) and unloading (phases 3 and 4) phases. The arrows indicate the signal directions during the observation trial. It can be observed that the torque transmission profile is far wider than the motion transmission profile. The backlash phenomenon in the motion transmission can be reduced significantly by increasing the cable pretension. On the contrary, higher cable pretension only widens the torque transmission loop. At this scale, the Bouc-Wen model is no longer effective in capturing this profile. The loop can be approximated as four linear segments. Each segment includes two

characteristic coefficients (i.e., a_i and b_i ($i = 2, 4$)) and a start point decided by the signs of the proximal torque u and its derivative $\dot{u}$ as follows:

$$\tau = [\tau_1 \text{sg}(u_l - u) + (a_2 u - b_2) \text{sg}(u - u_l)] \text{sg}(\dot{u}) + [\tau_3 \text{sg}(u - u_h) + (a_4 u + b_4) \text{sg}(u_h - u)] \text{sg}(-\dot{u}) \quad (4.97)$$

$$\tau_1 = a_4 u_d + b_4 \quad (4.98)$$

$$\tau_3 = a_2 u_a - b_2 \quad (4.99)$$

in which $u_l = (y_1 + b_2)/a_2$, $u_h = (y_3 - b_4)/a_4$, $u_d = u_{d0} + (u - u_{d0}) \text{sg}(-\dot{u})$, and $u_a = u_{a0} + (u - u_{a0}) \text{sg}(\dot{u})$. The $\text{sg}(\cdot)$ functions is defined as:

$$\text{sg}(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x \leq 0 \end{cases} \quad (4.100)$$

From the recorded data, the author set up a similar identification process to the one in previous studies to determine these parameters. With the data in Figure 4-20b, the identification results are: $a_2 = 0.7833$, $b_2 = 2.2673$, $a_4 = 1.1700$, $b_4 = 6.8232$. Figure 4-21 plots the original and identified loops. The dash-dot line represents the recorded data, and the continuous line is reconstructed from the identified parameters of the proposed model.

Figure 4-21. Model Identification Result

4.4.2.3 Torque Mode Control for the AC Brushless Servomotors

As analyzed in the previous study (Section 4.3.2.2), since the PI current loop of the motor drive updates its output current at a very high frequency (4 μ s cycle) compared to the control rate (1 ms period), one can simplify the current loop fundamental as

$$u = Mu_c \quad (4.101)$$

4.4.3 Control Design

In this study, the candidate has designed an adaptive controller for estimating online the parameters of the torque transmission profile and compensate for the hysteresis effects to enhance the torque tracking performance. He was able to prove the stability of this controller. This section explains the control procedure.

Adaptive Control Design

Because the proximal torque u is available and its first time-derivative $\dot{u}$ can be derived with a minimum delay, one can replace them with the state variables x_1 and x_2 , and rewrite the output torque at the distal pulley as:

$$\begin{aligned} \tau = & \left[\tau_1 \text{sg}(x_l - x_1) + (a_2 Mu_c - b_2) \text{sg}(x_1 - x_l) \right] \text{sg}(x_2) \\ & + \left[\tau_3 \text{sg}(x_1 - x_h) + (a_4 Mu_c + b_4) \text{sg}(x_h - x_1) \right] \text{sg}(-x_2) \end{aligned} \quad (4.102)$$

in which u_c is the control signal, and state variables x_h, x_l replace for u_h, u_l . To design this controller, the author made the same set of reasonable assumptions: (i) sufficient pretension is applied to the pair of CCMs to prevent any cable slacking; (ii) the reference torque signal $\tau_r(t)$ is prior known, piece-wise continuous, and bounded; (iii) the feedback output torque $\tau(t)$ and its first time-derivative $\dot{\tau}(t)$ are bounded.

Denoting $e(t)$ as the torque tracking error, one has

$$e(t) = \tau(t) - \tau_r(t) \quad (4.103)$$

We defined the new filter tracking errors as follows

$$n = \int_0^t e(\tau) d\tau \quad (4.104)$$

$$w = e + \lambda n \quad (4.105)$$

where λ is an arbitrary positive constant. To calculate the control signal u_c , one can divide it into four new control signals corresponding to the four phases as follows:

$$u_1 = u_c \text{sg}(x_2) \text{sg}(x_l - x_1) \quad (4.106)$$

$$u_2 = u_c \text{sg}(x_2) \text{sg}(x_1 - x_l) \quad (4.107)$$

$$u_3 = u_c \text{sg}(-x_2) \text{sg}(x_1 - x_h) \quad (4.108)$$

$$u_4 = u_c \text{sg}(-x_2) \text{sg}(x_h - x_1) \quad (4.109)$$

Then, one has:

$$u_c = u_1 + u_2 + u_3 + u_4 \quad (4.110)$$

In above equations, when function $\text{sg}(x_l - x_1) = 1$, then $\text{sg}(x_1 - x_l) = 0$, and vice versa; $\text{sg}(\tau_p - U_h) = 1$ $\text{sg}(x_1 - x_h) = 1$ when $\text{sg}(x_h - x_1) = 0$, and vice versa; $\text{sg}(x_2) = 1$ $\text{sg}(\dot{\tau}_p) = 1$ when $\text{sg}(-\dot{\tau}_p) = 0$ $\text{sg}(-x_2) = 0$ and vice versa. With the stable feedback from the proximal torque sensor, one can assume the two state variables $x_1 = \tau_p$ and $x_2 = \dot{\tau}_p$ are available. The author also introduced the virtual control signals $\bar{u}_i$ as $u_i = \hat{\beta}_i \bar{u}_i$, where $\hat{\beta}_i$ are the estimates of the new control coefficients $\beta_i = 1/M a_i$. Let $\tilde{\beta}_i$ be the estimated errors as $\tilde{\beta}_i = \beta_i - \hat{\beta}_i$ $\tilde{\beta}_i = \beta_i - \hat{\beta}_i$. Then, one can rewrite the control term in (4.102) in terms of the new virtual control signals $\bar{u}_i$ $\bar{u}_i$ as:

$$a_i M u_i = a_i M \hat{\beta}_i \bar{u}_i = a_i M (\beta_i - \tilde{\beta}_i) \bar{u}_i = \bar{u}_i - a_i M \tilde{\beta}_i \bar{u}_i \quad (4.111)$$

Then, the first time-derivative of the accumulated error n and the new tracking error w defined by (4.103) - (4.105) and the closed-loop system are:

$$\dot{n} = e = w - \lambda n \quad (4.112)$$

$$\dot{w} = \dot{e} + \lambda e = \dot{e} + \lambda(\tau - \tau_r) \quad (4.113)$$

Then, the control and update laws can be designed as follows:

$$u_c = u_2 + u_4 = \hat{\beta}_2 \bar{u}_2 + \hat{\beta}_4 \bar{u}_4 \quad (4.114)$$

$$\bar{u}_2 = \left[\tau_r - cw - \frac{n + \dot{e}}{\lambda} - \tau_1 \text{sg}(x_l - x_1) + \hat{b}_2 \text{sg}(x_1 - x_l) \right] \text{sg}(x_2) \quad (4.115)$$

$$\bar{u}_4 = \left[\tau_r - cw - \frac{n + \dot{e}}{\lambda} - \tau_3 \text{sg}(x_1 - x_h) - \hat{b}_4 \text{sg}(x_h - x_1) \right] \text{sg}(-x_2) \quad (4.116)$$

$$\dot{\hat{\beta}}_i = -\gamma_i \lambda w \bar{u}_i \quad (4.117)$$

$$\dot{\hat{b}}_2 = -\mu_2 \lambda w \text{sg}(x_2) \text{sg}(x_1 - x_l) \quad (4.118)$$

$$\dot{\hat{b}}_4 = \mu_4 \lambda w \text{sg}(-x_2) \text{sg}(x_h - x_1) \quad (4.119)$$

where $\tau_1 = a_4^* x_d + b_4^*$, $x_d = x_{d0} + (x_1 - x_{d0}) \text{sg}(-x_2)$, $\tau_3 = a_2^* x_a - b_2^*$, $x_a = x_{a0} + (x_1 - x_{a0}) \text{sg}(x_2)$, $x_l = (\tau_1 + b_2^*)/a_2^*$, and $x_h = (\tau_3 - b_4^*)/a_4^*$. In addition, a_2^* , b_2^* , a_4^* , and b_4^* , are the values of the corresponding coefficients were identified in Section 4.4.2.2, in which the arbitrary positive coefficient c denotes the control gain; and the positive parameters γ_i and μ_i ($i=1, 2$) represent the estimating coefficients.

Theorem 3: Consider the dynamic system modeled by (4.102) and satisfies the assumptions (i)-(iii), with the control and update laws (4.114) - (4.119), the following statements hold:

- i. All signals of the resulting closed-loop system are uniformly bounded.
- ii. The tracking errors asymptotically converge to zero.

Proof: See Appendix B.

4.4.4 Experimental Validation

A traditional PID controller was also designed to compare its performance with the performance of the proposed adaptive controller. With a signal flow diagram similar to the free motion diagram shown in Figure 4-19, the author implemented both controllers in Matlab-Simulink and loaded them into the DS1104 controller. A combination of two sinusoidal signals $\tau_r(t) = 45 + 38\sin(0.4\pi t) + 7\sin(0.4\sqrt{2}\pi t)$ was used as the reference torque. These designed control signals were loaded into the first actuator (i.e., the motor drive M701-03400100 and the servomotor 115U2D300) to power the CCMs. Simultaneously, the second actuator was dynamically reacting (followed equation 4.96) against the torque/motion propagated from the first actuator. However, dynamic load generated by the second actuator was too sensitive to the control torque and the interference, it kept the system fluctuating during the control trials. Consequently, both control schemes on this setup could not achieve good accuracy. Then, the candidate applied these control designs into the system shown in Figure 4-13 and Figure 4-14 to evaluate the effectiveness of the proposed adaptive control method. Within the capacity of the loading springs, he chose the signal $\tau_r(t) = 22 + 7\sin(0.4\pi t) + 3\sin(0.4\sqrt{2}\pi t)$ as the reference.

Figure 4-22 shows the performance of these control schemes in torque tracking control when the CCMs configuration is fixed. The left panel plots the tracking performance of the PID controller and its error while the right panel shows the results of the proposed adaptive controller. In the top graphs, the dashed line represents the reference signal; the continuous line is feedback torque at the distal pulley, and the dotted line is the proximal feedback torque. Both controllers demonstrated good ability to stabilize the system and achieve high tracking accuracy. However, as expected, the adaptive control method, which estimated the torque transmission parameters and compensated the hysteresis effects, resulted in

much smaller and consistent tracking errors, especially when the system reversed its direction. Quantitatively, as shown in Table 4-4, which shows the root-mean-square error (RMSE) of six control trials for each control scheme, the average error of the PID controller (0.3134 Nm) is about 31% more than the adaptive controller's average error (0.2397 Nm).

Figure 4-22: Torque tracking performance of the PID controller (left panel) and the proposed adaptive controller (right panel).

Table 4-4. The root-mean-square error (RMSE) of the experimental trials of the adaptive and PID control schemes.

	Fixed Configuration		Variable Configuration	
Trials	PID	Adaptive	PID	Adaptive
1	0.3038	0.2437	0.3419	0.2388
2	0.3057	0.2459	0.3384	0.2372
3	0.3065	0.2414	0.3381	0.3073
4	0.3098	0.2321	0.3597	0.2570
5	0.3408	0.2438	0.3460	0.2366
6	0.3138	0.2313	-	-
Average	0.3134	0.2397	0.3448	0.2554
SD	0.0139	0.0064	0.0089	0.0302

Furthermore, the candidate also carried out some experimental control trials with these controllers when the configuration of the CCMs was varying. He randomly fluctuated the CCMs' route to change the setup and let the system control the output torque. Figure 4-23 shows the torque tracking results of one control trial for each control scheme with variable CCMs' configuration. The adaptive control still showed more accurate results. However, the tracking errors were slightly larger than those errors in the fixed CCM's configuration experiments. As shown in Table 4-4, the average error of the PID control increased by about 10% (from 0.3134 Nm to 0.3448 Nm), and the error of adaptive control increased by around 6.5% (from 0.2397 Nm to 0.2554 Nm).

Figure 4-23: Torque tracking performance of the PID controller (left panel) and the designed adaptive controller (right panel) with varying configuration of the CCMs.

It is also apparent that the proximal torque in the adaptive control scheme fluctuated significantly more than in the PID control scheme. This phenomenon was caused by the estimation and compensation processes. It led to more small-chattering in the error graph of the adaptive controller. Nevertheless, the adaptive control error was still lower than the PID control error.

4.4.5 Conclusion

In this study, the new experimental model setup with two powerful 3-phase brushless servomotors allowed the author to investigate the torque transmission characteristics of the pair of CCMs at high torque/force loading. Different to the motion transmission, the torque transmission profile is significantly asymmetric and highly broad. A four-linear-segment model was developed based on an asymmetric sign function (i.e., $sg(.)$ function) and a recursive algorithm (to detect the turning points) to capture the torque transmission profile.

With this torque transmission model, the author was able to design an adaptive controller to estimate its parameters and compensate for the hysteresis effects. Although the candidate did not have a suitable facility to implement the control design with a large-scale reference torque, he managed to validate the performance of the designed controller by applying it to the system with two high stiffness springs, which was described in the previous study. Thus, a smaller torque reference signal was used. Then, good torque tracking results were achieved with both PID and designed adaptive controllers. The adaptive control which takes into account the torque transmission characteristics estimates the model parameters and compensates for the hysteresis effects, demonstrated better tracking performance than a traditional closed-loop PID controller.

In the integrated footwear testing system (IFTS), the high loading capacity of the contact platform will allow the high torque reference to be applied and the proposed control design to be evaluated properly.

Chapter 5

Mimicking Human Running Gaits with the Integrated System

5.1 Introduction

The RRF for footwear testing was initially built to integrate with an upper leg mechanism which replicates the effects of the human leg on the foot during gait. This chapter will present how to mimic the critical biomechanical aspects of the human foot in running by using the RRF and the integrated footwear testing system (IFTS). Firstly, this chapter will introduce the human running experiments and data which will be used as the reference for controlling the IFTS and comparing its data to. The second part will describe the mechanical, electronics & control structures of the IFTS, it will focus on the tasks done by the candidate. The later parts will present the control design process and experiments for mimicking the human running gait at 12 km/h with the IFTS. **It is noted that the original hardware design of the upper-leg and contact platform mechanism is not the candidate's work.** When he took over the hardware, he redesigned the contact platform at the feedback system and developed the electronic & control architecture for the whole system. After that, he was able to control the IFTS to mimic human running gaits.

5.2 Human Running Experiments

5.2.1 Purpose

The goal of this study was to collect kinematic and kinetic data of the lower limbs in normal distance running gaits at three different speeds (8, 10, and 12 km/h). The collected data shall be used as the reference data for controlling the IFTS to mimic the dynamic behaviors of the human foot-leg in running gaits. Based on these biomechanics data, the candidate will synthesize an appropriate profile for the upper leg mechanism and design a suitable control strategy for the robotic foot. Eventually, the output dynamic behaviors of the IFTS will be compared to the collected data to evaluate the system performance.

5.2.2 Method

5.2.2.1 Subjects

Eight (8) young subjects (two females) were recruited from students and researchers of the Biomechanics Group at the School of Sport, Exercise and Health Sciences, Loughborough University. Subjects were free of chronic musculoskeletal pathology and had no running-related injury within the last 6 months. The Ethics Approvals Sub-Committee of Loughborough University approved the experimental protocol and written informed consent was obtained from each subject before conducting his/her experimental trials. The subjects were 29.6 ± 3.4 years old, had an average mass of 63.2 ± 15.1 kg, and were 1.71 ± 0.10 m in height.

5.2.2.2 Procedure

All experiments were conducted in the Biomechanics Lab, School of Sport, Exercise and Health Sciences, Loughborough University. All trials were overground running on an approximately 20 m runway. The subjects made practice runs until they and

the researchers agreed that they had accommodated the required running gaits and speeds. They were asked to run with heel strike at three different speeds (i.e., 8 km/h, 10 km/h, and 12 km/h). Each subject was required to complete 10 successful trials for each speed. A trial was successful when the subject landed on the force plate with his/her heel and the running speed was within $\pm 15\%$ of the target speed. Two timing gates (i.e., the SmartSpeed Timing System from Fusion Sport Europe, Berkshire, United Kingdom) were used to provide feedback on the running speed immediately after each trial so that subjects could adjust their speed to fulfill the requirements.

All subjects wore the same type of running shoes (model M1400BL2, New Balance, USA) with their comfortable size. They also wore the same set of reflective markers, and the positions of these markers on each subject was unchanged for all his/her running trials. This set of 49 reflective markers allowed the subsequent calculation of 3D kinematics of all the main joints of the body. For comparison, three (3) subjects were also asked to complete another 10 walking trials at their preferred walking speed.

5.2.2.3 Marker Model

Figure 5-1: Position of 49 reflective markers on human subject and model in VICON

The set of 49 reflective markers were placed at critical positions on each subject's body, allowing three-dimensional whole body kinematics to be calculated. Figure 5-1 shows a subject wearing the set of 49 markers and the reconstructed segmental body model based on the markers' position. For the lower limbs, six markers were attached to each foot comprising markers on the toe, heel, medial ankle and MTP joints, lateral ankle and MTP joints. In addition, two markers were placed on each medial and lateral knee joint, and one was attached to the greater trochanter of each leg. Four markers were placed on the pelvis, on the left and right anterior and posterior superior iliac spines.

5.2.2.4 Measurement System

Kinematic data were recorded by a VICON Motion Capture system with 18 VICON MX13 cameras operating at 250 Hz (Vicon Peak, Lake Forest, CA, USA). For each running trial, the system tracked the positions of all markers, then the time histories of these marker positions were used to derive whole body kinematics. GRFs were

measured by an AMTI OR6 series force plate was positioned in the middle of the runway. The force plate was operating at 1000 Hz and synchronized with the VICON Motion Capture system.

5.2.2.5 Analysis

After the experiments, the recorded data were processed in the Vicon Nexus 1.7 software. Each trial was labeled with the 49-markers set. Then, marker trajectories and GRF data were filtered to eliminate high-frequency noise. Matlab programs were developed to analyze all recorded data. Positions of the ankle, MTP, knee, and hip joint centers were derived from recorded positions of the marker set. Subsequently, all joint angles, angular velocities, and angular accelerations were calculated. For deriving kinetic data, the body mass and anthropometric data of each subject were measured to calculate the segmental inertia parameters of all body segments using Yeadon's (1990) Human Inertia Model [144-147]. Then, a 2D inverse dynamics process was carried out to calculate the lower limbs' angular moments and powers for each running trial.

5.2.3 Results

Each experimental trial was analyzed by the same procedure as described in the previous section. Figure 5-2 and Figure 5-3 show the analyzed kinematic and kinetic data of the ankle joint of 12 successful trials at the average running speed (10 km/h) of a male subject. The users of the IFTS can use this data pool to test the shoe with gaits incorporating natural human variability.

Figure 5-2: The angle and angular velocity of the ankle joint of subject no.7 (male) running at approximately 10 km/h (average speed).

Figure 5-3: The ankle joint moment of subject no.7 (male) running at approximately 10 km/h (average speed). Top panel: Graphs of the ankle moment vs. running time; Bottom panel: Graphs of the ankle's moment vs. ankle angle.

Figure 5-4: The angle and angular velocity of the ankle joint of subject no.7 (male) running at approximately 12 km/h (fast speed).

Figure 5-5: The ankle joint moment of subject no.7 (male) running at approximately 12 km/h (fast speed). Top panel: Graphs of the ankle moment vs. running time; Bottom panel: Graphs of the ankle's moment vs. ankle angle.

Figure 5-6: The MTP joint biomechanics data vs. time of the same 21 trials of subject no.7 (male) running at approximately 12 km/h (fast speed). Top left panel: MTP angle; Middle panel: MTP angular velocity; Bottom panel: MTP moment.

Figure 5-7: The profiles of the center of pressure (CoP) vs. time of the 20 trials of subject no.7 (male) running at 12 km/h: Top panel: Medial-Lateral profile normalized by the ankle center at contact; middle panel: Anterior-Posterior profile normalized by the ankle center at contact; Bottom panel: combined profile in both directions.

5.3 The Integrated Footwear Testing System (IFTS)

5.3.1 Mechanical Structure

Currently, the IFTS comprises three main groups: the upper leg mechanism, the robotic foot, and the contact platform. The schematic diagram of this IFTS is shown in Figure 5-8. The system consists of six powered DOFs in total. The actual hardware of the IFTS is captured in Figure 5-9.

Figure 5-8: The Current Structure of the Integrated Footwear Testing System (IFTS)

The upper-leg mechanism is controlled by two linear actuators. The vertical actuator (i.e., actuator no.1) elevates the shank-foot block up and down and applies the required vertical force on this block. The sagittal actuator (i.e., actuator no.2) rotates the shank-foot block around the pivot point P . The contact platform has

two more DOFs. The horizontal actuator (i.e., actuator no.3) moves the platform backward and forwards while a DC motor and timing belt mechanism rotate this platform in the frontal plane on a half-circular sliding mechanism. Two others powered DOFs are on the RRF as introduced in Chapter 3.

Figure 5-9: The real Model of the Integrated Footwear Testing System (IFTS)

As illustrated in Figure 5-8, the robotic foot has three main segments: the shank, foot, and toe segments. It connects to the upper-leg mechanism at two rotational joints P and R . The structure of the robotic foot and its control actuators were described in Section 3.4 and Figure 3-13. However, the actual model of the upper leg mechanism is different with the previous structure shown in Figure 3-9. The robotic foot has been also modified (compared to the CAD model in Figure 3-12) to assemble with the upper leg mechanism. Figure 5-10 shows the foot final design for integrating into the IFTS. It includes two more joints to connect to the vertical and sagittal actuators. At the pivot joint P , a strong steel alloy 20 mm diameter shaft was used to connect the RRF and the vertical actuator. The high body-weight force from the upper-leg mechanism is conveyed through this pivot joint to the RRF. A smaller shaft was used at the rotating joint to connect the RRF to the sagittal

actuator. When this actuator contracts/extends, it pulls/pushes this shaft and thus rotates the RRF about the pivot joint P.

Figure 5-10: The final design of the RRF for assembling to the integrated system

On the final RRF design, a 3D-printing foot model covers the foot main segments and toes structure. This 3D-printing foot (subsequently referred as the foot-tissue) has been developed based on a CT (computerized axial tomography) scan foot model [148] (size US 9). The foot-tissue is divided into three parts for assembling around the foot-toes main structure. While Figure 5-10 shows the RRF with transparent foot-tissue to display the inside main structure, Figure 5-11 shows the outer shape of these 3D-printing parts. Also, Figure 5-12 shows more details of the inside profiles of the left foot and toes parts when removing the right foot part.

Figure 5-11: The CAD model of the 3D-printing parts in the RRF

Figure 5-12: The CAD model of the RRF without the right foot

Figure 5-13 shows the pictures of the real RRF integrated into the IFTS. The foot-tissue was printed out with ABS material with SLA technology. Total model mass is about 4.6 kg. This mass is comparable with an average weight of the calf and foot segment of a 66-70 kg male adult [99].

Figure 5-13: The real Robotic Foot in the Integrated Footwear Testing System

Figure 5-14 captures a closer view of the robotic foot wearing the shoe in the IFTS. This figure also shows the structure of the contact platform with the modified force feedback system. Three load cells are installed between two metal plates to measure the ground reaction force more accurately. In the original design, the vertical load cell was placed at the tip of the vertical actuator to measure the vertical force. Thus, the feedback vertical reaction force was affected by the dynamic inertia of the shank-foot block and the friction between this block and the machine frame. On the contrary, the new contact platform, which is equipped with three load cells (named as load cells A, B, and C) at three predetermined locations, can measure the vertical reaction force without these influences. Hence, the feedback results must be more accurate. In addition, with the three load cells installed at three predetermined points, one can triangulate the center of pressure (CoP) under the shoe sole. With the availability of the CoP, the author can calculate the foot joints' moments and control the joints' output torques with a closed-loop controller. Thus, this design enables a whole new solution: closed-loop torque tracking control, which should enhance the gait mimicking performance significantly compared to the open-loop control methods.

Figure 5-14: The real Robotic Foot wearing the Shoe in the Integrated Footwear Testing System

Moreover, Figure 5-14 also shows more details of the frontal rotation mechanism. The contact platform includes a horizontal translation (3), and a frontal rotation (4). When Linear Actuator 31 extends/retracts its length, it pushes/pulls the contact platform horizontally to simulate the relative anterior-posterior movement between the shoe and the ground in gaits. All of the three actuators are the electromechanical drives ESBF-BS-63-100-25P supplied by Festo (Esslingen am Neckar, Germany).

The lower layer of the contact platform can slide horizontally with two sliding tracks connecting to four sliding units which are fixed on the platform carriage. This carriage is supported by four rollers which are placed at its four corners and can roll on two half-circular rail tracks. The platform carriage is driven by a DC servomotor & gearbox 41 and a special timing belt mechanism which connects directly to the platform carriage. When Actuator 41 rotates the timing pulley, it also rotates the platform carriage through the timing belt. The platform rotation is used to simulate

the relative inversion/eversion and effects in the frontal plane. The motion and effects in the transverse plane are neglected in this current design of the IFTS and may be included in the future developments.

5.3.2 Electronic & Control System

The electronic & control system for the robotic foot was described in Section 3.5 and in the torque control studies of Chapter 4.

The electronic & control system of the upper leg mechanism and contact platform was redesigned from National Instrument hardware + LabVIEW to dSPACE hardware + Simulink. The original control system used the computer as the central controller and a data acquisition chassis (the NI cDAQ-9184) to connect with peripheral devices (i.e., the sensors and motor drives). Four peripheral modules were integrated into this chassis. One module NI-9264 was for sending analog control signals to the motor drives. One module NI-9237 was for receiving analog feedback signals from the load cells. Two modules NI-9401 were for digital communication. Since all control algorithms were implemented in LabVIEW and ran on the PC, the programs had to share the resource with Windows and thus there were always delays in executing the commands. In other words, the cDAQ system cannot interact with the electronic hardware in real-time.

The new controller dSPACE DS1104, on the other hand, is a real-time controller. It operates on its own resources and interacts with the other peripheral devices instantly. Figure 5-15 illustrates the signal flow diagram of the electronic & control system for the IFTS. The controller DS1104 monitors all the feedback signals from the sensors/amplifiers. It executes the control algorithms and sends out all necessary control signals to the motor drives. The system possesses six active DOFs powered by six actuators which can be divided into three blocks: the shank, the foot, and the contact platform.

Figure 5-15: The architecture of the Electronic & Control system for the Integrated Footwear Testing System. (ADC_i : the i^{th} Analog to Digital Conversion module; DAC_i : the i^{th} Digital to Analog Conversion module; DO_i : the i^{th} Digital Output).

For the foot block, the ankle motor drive (Control Technique M701-03400100) and the MTP motor drive (Control Technique M701-03400031) operate in torque mode control. After executing the control algorithms, the controller DS1104 (subsequently referred to as ‘DS1104’) sends out two analog signals to each drive to control its output torque. The first signal is the command torque; the second signal is the maximum speed that the motor is allowed to reach. These signals are sent to the motor drives via modules DAC1 (Digital to Analog Conversion no.1) to DAC4. At the same time, module ADC5 (Analog to Digital Conversion no.5) helps the

controller monitor the ankle's proximal torque while two modules ENC1 (Encoder Interface no.1) and ENC2 measure the ankle and MTP joints' angles.

For the shank block, two Festo motor drives CMMP-AS-C5-3A-M3 (subsequently referred to as the 'Festo drive') control the vertical and sagittal actuators in the closed-loop position control mode. Each actuator is equipped with its own high-resolution encoder/resolver to ensure the required output position. In this system, each Festo drive is controlled by two digital command signals and one analog signal. The analog signal is the reference position that the motor must reach. One digital signal chooses the set of speeds, accelerations, and decelerations; the other commands the drive to start positioning following the analog reference. DS1104 sends these analog signals via modules DAC5 and DAC6. The digital signals are generated by modules DO1 (Digital Output no.1) and DO2. Subsequently, an isolation circuit (not shown in above diagram) raises these digital signals' voltage to 24VDC to connect with the Festo drive.

In the new design, the linear actuator's actual position is also extracted from each Festo drive to calculate the actual absolute position of the shank-foot block in real time. Different to the open-loop control, these position values are essential for applying the inverse dynamics method to derive the joints' kinematics and kinetics in real time and control them accordingly. The actual positions of the vertical and sagittal actuators are monitored by modules ADC6 and ADC7.

For the contact platform block, another Festo drive receives the command signals from module DAC7, DO1, and DO2 to control the horizontal actuator while a Maxon EPOS2-70/10 receives the position signal from module DAC8 and controls the Maxon frontal actuator. The actual position of the horizontal actuator is monitored by module ADC8. Also, the reaction force signals from four load cells (i.e., three vertical load cells A, B, C between two contact plates and one horizontal load cell) are amplified and sent to DS1104 by four Interface DCA amplifiers.

$$y_V^A = \overline{AH} = \frac{(F_B + F_C)\overline{AO}}{F_V} \quad (5.3)$$

In which x_V^A and y_V^A are the position values of the center of pressure V in the coordinate system Axy . These values will be used to calculate the joints' dynamics during the contact phase.

5.4 Control Design for the IFTS

The current design of this IFTS, which has six flexible powered-DOFs, abundant sensors monitoring all positions and reaction forces, and a controller interacting with peripherals in real-time, allows the authors to design many types of control strategies to reproduce more realistic wear conditions for shoe testing. The current RRF has two powered ankle & MTP joints and replicates the outer shape profile of a human foot; an artificial skin can also be used to cover the 3D-printing foot-tissue to enhance the foot bio-fidelity. Then, the primary objective in replicating the wear conditions is to mimic the interaction forces/pressure between the shoe and its working environment (i.e., the foot and the contact ground). Many biomechanical parameters need to be tracked to mimic the actual effects on the footwear, such as the GRFs profile, the joints moments and powers, the position profiles of the foot, the shank, the ankle and MTP joints, and the profile of CoP. This study began developing the control strategy for the IFTS by tracking the pattern of the GRFs, the foot and toe segments. If the correct position of the foot-shoe block is achieved when the right GRF occurs, then other gait parameters such as the CoP, the joints moments and powers must be the same as well. Then, realistic wear interactions are accomplished. This section describes a study of mimicking the GRF and angles of foot and toe segments with the IFTS.

5.4.1 High-Level Control

The high-level control strategy is illustrated in Figure 5-17. From the human running experimental data pool, the data of a typical trial was extracted to be the reference for the control processes. Meanwhile, an open-loop control program was developed to measure the outer dimensions of the testing shoe. Based on these dimensions and the biomechanics data (i.e., the shank, foot, toe angles and GRF profiles), the program calculated the variable compressions of the hindfoot and the forefoot. Then the motion planning for three linear actuators of the shank and contact platform mechanisms were executed.

After that, the program initiated the running cycle, in which the linear actuators were running in the position profile control mode and the foot was operating in closed-loop position control with feedforward torque compensation. During the cycle, all positions and reaction forces were monitored and compared with the human reference data. If the errors are acceptable, the program will run a new cycle with the same control signals. Otherwise, it will generate a new shoe compression ratio and re-estimate the shoe compression and update the control loop.

Figure 5-17: The flowchart of the high-level control algorithm for the IFTS

The next sections will explain the motion planning algorithm for the Festo linear actuators of the upper leg mechanism and the contact platform, and the control strategy for the robotic foot in this IFTS.

5.4.2 Motion Planning for the Linear Actuators

In the frontal plane, currently, the sixth actuator (i.e., the Maxon actuator for powering the frontal rotation of the contact platform) is operating in a firm position control mode to keep the platform stand still against the vertical forces. This section will focus on motion planning for the vertical, sagittal, and horizontal actuators. Figure 5-18 shows the geometric model of the IFTS for motion planning for the three linear actuators. All following equations are derived based on this model.

Figure 5-18: Analysis model for motion planning for the upper leg mechanism and the contact platform in the sagittal plane.

In the sagittal plane, denote α as the angle between the shank segment and the vertical axis (subsequently referred to as the ‘shank angle’); ϕ as the angle between the shoe sole and the transverse plane (subsequently referred to as the ‘foot angle’).

These angle profiles were derived from the human running experiments and calibrated with the geometrical data in the static session.

Denote H and M as the center contact points between the contact plate and the shoe at the heel and the foot's ball (under the MTP joint). One can divide the contact phase of running into two sub-phases: the impact and propulsion phases. In the impact phase, the heel is fixed to the floor while in the propulsion phase, the contact platform and the ball are attached together. Thus, in the impact phase, the actuators' profile is derived from the interaction between the shoe heel and the contact plate while the second half of these profiles are synthesized from the shoe ball interaction with the platform in the propulsion phase.

Denoting h_d and m_d as the variable distances from the ankle joint A to shoe heel and the shoe 'ball' (the lowest part under the MTP joint), one has:

$$h_d = hk_1F_z \quad (5.4)$$

$$m_d = mk_2F_z \quad (5.5)$$

In which h and m are the original values of these distances obtained from the shoe measurement program. F_z is the vertical ground reaction force. k_1 and k_2 are the estimated stiffness of the shoe sole at the heel and the ball. Then the estimate height of the ankle joint A from the contact plate is:

$$z_{A1} = h_d \sin(\epsilon) \quad (5.6)$$

$$z_{A2} = m_d \sin(\sigma) \quad (5.7)$$

In which, z_{A1} and z_{A2} are the ankle height in the impact and propulsion phases. The angle ϵ and σ are derived from the foot-shoe geometry as follows:

$$\epsilon = \widehat{AHM} + \varphi \quad (5.8)$$

$$\sigma = \widehat{HMA} - \varphi \quad (5.9)$$

The transition between the impact and propulsion phases is when the foot angle $\varphi = 0$ (when the foot plantar flexes from positive angle to negative angle). Notice that transition, one can combine $\mathbf{z}_{A1}$ and $\mathbf{z}_{A2}$ to form the complete height profile of the ankle joint $\mathbf{z}_A$. Then, based on the diagram in Figure 5-18, one can derive the following equations:

$$\theta = 270 - (\hat{A} - \alpha) \quad (5.10)$$

$$z_A = z_P + \overline{PA} * \sin(\theta) \quad (5.11)$$

$$s_V = h_0 - z_P \quad (5.12)$$

$$\beta = \theta - 90 - \hat{P} \quad (5.13)$$

$$\overline{QP} = \overline{QP_0} + s_V \quad (5.14)$$

$$\overline{GP} = \sqrt{\overline{QG} * \overline{QG} + \overline{QP} * \overline{QP}} \quad (5.15)$$

$$\gamma = \tan^{-1}(\overline{QG}, \overline{QP}) \quad (5.16)$$

$$\varphi = \beta + \gamma \quad (5.17)$$

$$l_S = \sqrt{\overline{GP} * \overline{GP} + \overline{RP} * \overline{RP} - 2\overline{GP} * \overline{RP} * \cos(\varphi)} \quad (5.18)$$

$$s_S = l_S - l_{S0} \quad (5.19)$$

In which s_V and s_S are the required extensions of the vertical and sagittal actuators. Then, a negative value of these extensions means the actuator is in contraction and vice versa. The geometric angles $\hat{A}$ and $\hat{P}$ are constant values of the robotic foot design. P is the position of the pivot point connecting the shank segment and the vertical actuator. R is the connecting point between the shank and the sagittal actuator rod. Then, $\overline{RP}$ represents the distance between R and P ; a similar concept is applied for other distances $\overline{GP}$, $\overline{QP}$, $\overline{QG}$ and $\overline{PA}$. Also, when the system is at the zero position ($s_V = s_S = 0$), l_{S0} , h_0 , and $\overline{QP_0}$ are the initial length of the sagittal actuator, the initial height of the pivot point P from the coordinate O , and the

initial distance from the pivot point P to the image point of G on the vertical axis Oz .

Furthermore, the motion profile of the horizontal actuator was planned from the profiles of both the shoe heel (H) and ‘ball’ (M) as previously mentioned. The y -position of H and M in the global coordinate system $Oxyz$ can be derived as:

$$y_A = PA * \cos(\theta) \quad (5.20)$$

$$y_H = y_A - h_d \cos(\epsilon) \quad (5.21)$$

$$y_M = y_A + m_d \cos(\sigma) \quad (5.22)$$

Then the horizontal translational movements of H and M are:

$$s_1 = y_H - y_H^0 \quad (5.23)$$

$$s_2 = y_M - y_M^* \quad (5.24)$$

In which y_H^0 is the y -coordinate of H at the impact moment (when the heel starts touching the contact plate) and y_M^* is the y -coordinate of M at the impact-propulsion phase transition. Finally, the horizontal actuator profile is

$$s_H = \begin{cases} s_1, & \text{if } \varphi > 0 \\ s_2 + s_1^*, & \text{if } \varphi \leq 0. \end{cases} \quad (5.25)$$

5.4.3 Control Strategy for the Robotic Foot in the IFTS

At the current stage, a combination of a closed-loop position control and a feedforward torque control was designed for the ankle and MTP joints. The next step is to use closed-loop torque control for each joint to replicate the GRF profile.

The control signal for the ankle joint is:

$$u_A(t) = -k_{p1}e_\varphi(t) - k_{i1} \int_0^t e_\varphi(\omega) d\omega - k_{d1}\dot{e}_\varphi(t) + \kappa_1(t)\tau_{r1}(t) \quad (5.26)$$

And the control signal for the MTP joint is:

$$u_M(t) = -k_{p2}e_M(t) - k_{i2} \int_0^t e_M(\omega)d\omega - k_{d2}\dot{e}_M(t) + \kappa_2(t)\tau_{r2}(t) \quad (5.27)$$

In which $e_\varphi(t) = \varphi(t) - \varphi_r(t)$ is the tracking error between the human foot angle $\varphi_r(t)$ (the reference signal) and the robotic foot angle (to the contact plate); $e_M(t) = \varphi_m(t) - \varphi_{mr}(t)$ is the tracking error between the reference MTP angle and the feedback MTP angle. The feedforward signal $\tau_{FFi}(t) = \kappa_i(t)\tau_{ri}(t)$ is composed from the reference joint moment $\tau_{ri}(t)$ of the corresponding joint in running and a designed variable parameter $\kappa_i(t)$.

Moreover, an additional proportional-integral control signal $-k_{p1A}e_\varphi(t) - k_{i1A} \int_0^t e_\varphi(\omega)d\omega$ is added to the ankle control signal in (5.26) during the contact phase to increase the reaction torque of the ankle joint.

5.4.4 Experiments and Results

With the IFTS described in Section 5.3, the control strategies designed in the previous sections (i.e., sections 5.4.1, 5.4.2, and 5.4.3) were applied to synchronously control all six actuators to replicate the GRFz profile and the foot position in the sagittal plane. From the human running experiments described in Section 5.2, the data of a successful trial of subject no.7 (male) running at the speed of 12 km/h was chosen as the reference for the IFTS to mimic. When this running trial was recorded, the subject was about 67 kg in mass, 172 cm in height, and was wearing a pair of New Balance running shoes of size UK 8.5 (equivalent to the size US 9 of the robotic foot and the current shoe). The maximum vertical GRFz in this trial is about 1750 N.

The system is able to mimic this running trial at 50% of the real speed (with the same forces and ranges of movements). In other words, the same reference data profiles are loaded into the system two times slower than the original period; all other values are intact. Figure 5-19 shows the GRFz replicating performance of the

IFTS and the proposed control strategies at 50% of the real speed (12 km/h). Similarly, Figure 5-20 plots the data when the IFTS mimics the same running trial at full speed (12 km/h).

Figure 5-19: The vertical GRF mimicking result when the IFTS ran at 50% speed of a real running trial at 12 km/h

In both figures, the red line represents the reference GRFz extracted from the human running trial while the blue line portrays the feedback GRFz measured by the set of three load cells of the contact platform. In both experiments, the IFTS ran smoothly with stable performances. Recorded video of these tests can be downloaded at <https://goo.gl/cSkF6J>. However, there are significant differences in GRFz replicating performances between two tests. At 50% of full speed, the system can adapt well to the jumps in the GRFz reference. It creates two distinct peaks. The first peak is at the impact and the second peak simulates the vertical GRF in the propulsion phase. Overall, during the running cycle, the feedback (i.e., the blue dotted line) tracks closely to the reference (i.e., the red line). The root-mean-square error of the GRFz mimicking results at this speed is $RMSE = 107\text{ N} \sim 6.1\%$ of the GRFz magnitude.

Figure 5-20: The vertical GRF mimicking result when the IFTS ran at a running speed of 12 km/h.

However, at the full speed of 12 km/h, the IFTS system cannot achieve a good GRFz mimicking performance. The primary cause of this limitation is the low speed of the vertical linear actuator. Despite operating at its maximum speed (400 mm/s), it cannot provide enough speed to cause the sufficient reaction force between the contact platform and the shoe in time. There are long delays between the reference and the output GRFz at this speed. Also, there is insufficient time for the vertical actuator to push the foot-shoe to reach the maximum reference GRFz. The solution to this issue is to replace the gear reduction of the vertical linear actuator to increase its speed.

Figure 5-22 to Figure 5-26 plot other biomechanical aspects of the mimicking results when the IFTS ran at 50% speed of 12 km/h. Figure 5-21 shows the foot angle profile of the RRF versus the reference human foot angle data in running. In this current control strategy, replicating simultaneously the vertical GRF and foot angle is critical for reproducing the dynamic effects on the shoe. While the GRF is guaranteed, the ‘right’ foot position would cause a ‘correct’ pressure/effect on the footwear. The terms ‘right’ & ‘correct’ are referred/compared to the human data.

Also, Figure 5-22 plots the Foot Angle profile and the vertical GRF profile in the same plot for more visual comparison.

Figure 5-21: The Foot Angle mimicking result when the IFTS ran at 50% speed of a real running trial at 12 km/h.

Figure 5-22: Correlation between the vertical GRF and the foot angle when the IFTS ran at 50% speed of the same running trial at 12 km/h.

In the frontal plane, the DC actuator rotates the contact platform to simulate the inversion/eversion effects on the shoe. As the force/moment is more important than

the angle profile in simulating the dynamic effects, the frontal moment is chosen as the control target for this frontal actuator. Figure 5-23 plots the Frontal Moment mimicking performance in the same running trial at 50% speed of 12 km/h.

Figure 5-23: The frontal moment mimicking result when the IFTS ran at 50% speed of a real running trial at 12 km/h

Figure 5-24: The graph of horizontal GRF when the IFTS ran at 50% speed of a real running trial at 12 km/h.

While Figure 5-24 plots the anterior-posterior GRF, Figure 5-25 and Figure 5-26 show the profile of the center of pressure in both directions: medial-lateral profile in Figure 5-25 and anterior-posterior profile in Figure 5-26.

Figure 5-25: The Medial-Lateral profile of the Center of Pressure when the IFTS ran at 50% speed of a real running trial at 12 km/h

Figure 5-26: The Anterior-Posterior profile of the Center of Pressure when the IFTS ran at 50% speed of a real running trial at 12 km/h.

Although the mimicking performance of most of above gait parameters is quite acceptable (the feedback data are close to the reference data), replicating the horizontal GRF and CoP profiles are the more challenging to achieve. The medial-lateral CoP profile depends on the frontal motion caused by the frontal actuator while this actuator is used mainly to control the frontal moment. As current linear actuators have limited range of motion, the toe segment is not really active in mimicking the running gait. Thus, there is a major difference between the human and feedback anterior-posterior CoP profiles at the end of each cycle.

These limitations are primarily caused by current linear actuators which have limited speed and range of motion. The next section explains the capacity of the RRF in mimicking the trajectory of the foot joints during running.

5.5 Mimicking the Foot Joints' Profile in Running

Some experiments on position control for the foot joints were also carried out to prove that the limited speed of the IFTS is not caused by the robotic foot. As mentioned in the previous section, the current IFTS does not have sufficient speed to mimic human running gaits at 12 km/h. This limitation is caused by the limited speed of the vertical linear actuator (maximum 400 mm/s). On the other hand, the robotic foot was designed with excessive torque and speed as described in Chapter 3. The robotic foot joints not only can reach the maximum speeds of the real human foot joints in running but they also can track accurately these real joints' profiles. Figure 5-27 and Figure 5-28 plots the control performance of the robotic foot joints to a real reference foot profile of subject no.7 in running at 12 km/h.

Figure 5-27: The mimicking performance of the Ankle joint to a real reference ankle profile of subject no.7 in running at 12 km/h. The RMSE is 1.1437° .

Figure 5-28: The mimicking performance of the MTP joint to a real reference MTP profile of subject no.7 in running at 12 km/h. The RMSE is 0.9702° .

These reference signals (i.e., the profiles of the ankle and MTP joints) were extracted from the data pool and simplified their swing phase interval. Each foot joint was controlled by a closed-loop position controller combined with a feedforward torque profile. These feedforward control signals were synthesized based on the joints' angle profile during the running cycle. The control design process is similar to the process described in Section 5.4.3.

Figure 5-27 shows the result of a typical control trial for the ankle joint tracking to the real ankle profile of subject no.7 in running at 12 km/h. The root-mean-square error between the reference signal and the feedback ankle angle is 1.1437° . Similarly, Figure 5-28 plots the result of a typical control trial for the MTP joint following the real MTP profile of subject no.7 in running at 12 km/h; and the RMSE is 0.9702° .

5.6 Applications of the IFTS

The primary target test of the RRF and IFTS is the durability test. This includes the durability of the whole shoe (to determine which is the weakest part) or of each individual component such as the outsole, insole, heel counter, toe box, last, tongue, collar, midsole, or lacing system. These components may have different lifespans and have their own criteria for evaluating whether they are spoiled. For instance, the outsole lifespan ends when it loses a significant part of its shape but the durability of the last depends on its strength to hold its integrity against high shear forces during gaits. It depends on the client intention that which tests should be conducted.

- i. Firstly, the system can run a fixed number of cycles which is equivalent to 600 km to 1000 km of wearing and then the physical properties of the shoe will be compared before and after the intervention. These physical properties may include the remained shape of the outsole/insole, the shock absorption ability, bending stiffness, or slip resistance.
- ii. Secondly, the IFTS can run until one specific component loses its integrity or a significant part of its shape (e.g., the outsole material lost due to abrasion with the contact surface/ground) and then the IFTS counts for the total cycles to calculate for its lifespan. The lifespan of the shoe weakest component of the shoe can be used to indicate the whole shoe durability.
- iii. Finally, in each type of test, the IFTS mimics a pre-installed running gait while adapting to different shoe models. The system duty is to reproduce the closest wear condition to the real effects on the shoe in human running gaits. Thus, the system will try to self-adjust its operating profiles to adapt to new shoe models and generate the required wear conditions. The key required

wear conditions (or parameters) are the GRFs profile, the foot & toes angle profiles, and the CoP profile, which were discussed in Section 5.4.

Moreover, the IFTS can be used to evaluate physical properties of the shoe. These applications include the tests for slip resistance (friction) following the ASTM F2913-11 protocol [149], the abrasion ability by measuring the outsole material lost, the shock absorption ability or cushioning of the soles. With options to replace different surfaces such as wooden surface, concrete, asphalt road, or metals, the IFTS can evaluate the shoe slip resistance and abrasion against these surfaces.

Recently, the ASTM F2913-11 protocol has been adopted and programmed for the IFTS to simulate and evaluate the friction (slipping resistance) between the shoe and contact surface. These friction tests include the evaluation of slipping resistance in three different slip conditions: (i) forward heel slip (at 7° shoe heel - ground angle), (ii) backward forepart slip (at 7° front shoe - ground angle), and (iii) forward flat slip (at flat shoe - ground position). These tests were done with aluminum, wood, and concrete surfaces.

Figure 5-29: A forward heel slipping resistance test for a Nike-Airmax shoe model against the aluminum contact surface following the ASTM F2913-11 protocol.

Figure 5-29 plots the data of a friction test of a Nike-Airmax shoe against an aluminum surface following the ASTM F2913-11 at the forward heel slip condition. The blue line represents the vertical GRF, the red line is the horizontal force, and the green line plots the coefficient of friction values during the test. Followed the protocol, the slipping friction between the provided shoe model and aluminum surface is about 0.82 when the ‘static’ friction is about 0.88. An asphalt road surface is being produced for the future tests.

Other applications of the IFTS may include all above tests for surfaces and covering systems such as carpets, artificial turfs, and court floors. A sample of these covering surfaces can be set up on the contact platform of the IFTS. Then, walking, running, jogging or other sports gaits can be applied to the IFTS to simulate real human gaits and evaluate the performance of these surface with different types of footwear.

Furthermore, the RRF can be developed with more in-shoe sensors to monitor the plantar pressure, humidity, and temperature to test the shoe breathability or water/heat transfer resistance. With more modifications with an environmental chamber, the IFTS can be used to evaluate the footwear under different weather conditions.

5.7 Summary

This chapter has introduced the IFTS from the principle architecture, mechanical structure and design, electronic & control hardware design, to the designed control strategies for it to mimic the human running gaits. It has also presented the mimicking results of the IFTS following a reference human data set from the human running experiments. At the current state, the IFTS can simulate a running gait at 12 km/h by two times slower than the real gait (all data values are intact but they are loaded twice slower). At this speed, most of the critical gait parameters are

replicated with certain accuracy, especially the vertical GRF and foot angle profiles.

However, the following limitations need more attention in the future stages:

- The IFTS, currently, does not include the effects in the transverse plane.
- Limited range of motion and speed of current linear actuators constraint the IFTS performance in mimicking the running gaits at 12 km/h.
- The vertical and horizontal actuators' range of motion also prevents the toes segment interacting with the ground as the same as the human toes do.
- The mimicking results of anterior-posterior GRF and medial-lateral CoP profiles are limited as these variables are influenced by many factors.
- In-shoe sensors have not been installed to monitor and feedback the plantar pressure for evaluation and control development.
- The appearance and elasticity of the foot tissue and skin have not been focused and imitated in this current foot.

Chapter 6

Conclusions and Future Directions

6.1 Conclusions

6.1.1 *Summary of Contributions and Novelty*

A Realistic Robotic Running Foot and Effective Torque Control Strategies for System Actuated by CCMs

The main contributions of this Ph.D. work include:

- i. Developing a new RRF for footwear testing to replicate the wear conditions in running to evaluate the running shoes. This is the first footwear testing system which possesses multiple powered DOFs at the foot and ability to mimic human running gaits.
- ii. Introducing and evaluating three new torque mode control strategies for the robotic foot joints actuated by cable-conduit mechanisms. Each strategy includes a method for torque transmission modeling and a development of adaptive torque tracking control.
- iii. Integrating the RRF into the upper-leg and contact platform mechanisms, and developing a new electronic system and control algorithms for the IFTS to mimic a human running gait at 12 km/h. The RRF joints have excessive torque and speed to adapt to this running speed. Also, the reference data pool of human running gaits, which were collected with a 3D motion capture system synchronizing with a force plate, can be used for future development of the IFTS.

6.1.2 Solutions for a Robotic Running Foot for Footwear Testing

The first part of this thesis shows an investigation on the art of footwear testing systems and available innovative robotic feet. Then, based on the review on the human foot biomechanics in running, various solutions for an RRF for footwear testing were proposed and analyzed. Eventually, a realistic robotic foot was developed, evaluated and assembled with other mechanisms to form the IFTS. It has a similar weight and the same shape and size as a real human foot. Currently, the RRF possesses two powered DOFs at two critical joints (i.e., the ankle and MTP joints). Each joint is actuated by one powerful 3-phases AC brushless servomotor via a pair of flexible CCMs. With this current design, both RRF joints have sufficient torque and speed to mimic the reference moment and angular velocity of the human foot joints in running.

6.1.3 Torque Control for the Robotic Joint actuated by CCMs

One critical requirement for the IFTS is to mimic the reaction force and its center of pressure between the foot-shoe and the ground during a running cycle. Thus, some modeling and control strategies were developed to enhance the torque (force-related) tracking performance for the foot joints actuated by the CCMs.

Different to the motion transmission of the CCMs, the torque transmission loop is very broad, asymmetric and cannot be mitigated by increasing the cables' pretension. In this Ph.D. project, some experimental models were set up to observe, analyze and develop mathematics equations to describe the torque transmission profile. The modified Bouc-Wen hysteresis models with a proximal torque input proved their ability to portray the torque transmission loop with relatively low stimulating torque. In very high loading conditions, a four-segment linear model with the aid of an asymmetric *sign* function and a recursive algorithm for detecting the turning points was developed to capture the torque transmission loop.

Although many studies have discussed intensively methods to improve the position control accuracy of a robotic joint actuated by CCMs (or TSMs), there is a lack of studies on torque control, especially for enhancing the torque tracking performance of the system actuated by the CCMs. From these transmission models, the candidate has developed three new adaptive control schemes to estimate and compensate for the hysteresis effects on the CCMs to improve the torque tracking control. These control schemes use the accumulated tracking error as the variable of interest instead of the error itself and add a portion of this accumulated error directly into the control signal to stabilize the system.

One controller uses the hyperbolic *tangent* function and σ -modification leakage terms to improve the smooth transitions between the loading and unloading phases and avoid the parameter drifting phenomenon during the estimation process. This control scheme (i.e., the second study described in Section 4.3) does not require any knowledge of the hysteresis loop or the bound of the unknown parameters. Another robust adaptive control scheme (the controller described in Section 4.2) uses the theoretical bounds of the asymmetric control coefficients to adopt the switching σ -modification leakage terms to improve the control performance and system stability. The last adaptive controller (the scheme described in Section 4.4) exploits the identified parameters of the torque transmission profile to stabilize the system instead of using the robust algorithms.

6.1.4 Mimicking Human Running Gaits with the IFTS

From the original hardware of the upper leg mechanism, the candidate modified the contact platform and the mechanical system to integrate the robotic foot into this system. A new electronics & control system was also designed to control the IFTS to mimic the human running gaits in real time. Many trials and sessions of human running experiments were carried out to collect reference data for future development of the IFTS. Control strategies for the robotic foot joints and motion

planning for the upper leg mechanism and the contact platform have been designed and implemented. The IFTS was driven to mimic a 12 km/h running trial. The system can run with smooth and stable profiles. However, due to the limited speed of the vertical linear actuator, currently, the upper leg mechanism cannot push the foot-shoe block to reach the required vertical GRF at full speed. When the same reference profiles are loaded at 50% of the real speed, the system can reproduce the vertical GRF and foot angle profiles with acceptable accuracy. This limitation will be solved with a new gearbox for the vertical actuator and a longer cylinder for the horizontal actuator.

6.2 Future Directions

6.2.1 *For the Integrated Footwear Testing System*

Firstly, a new set of linear actuators must be installed for the IFTS. The vertical actuator needs to be faster to push the foot-shoe block towards the contact plate to achieve the required GRFz in time. Also, the travel range of the vertical and horizontal actuators must be sufficient to simulate the correct relative movement between the foot and the ground in running. These are essential requirements to develop the IFTS.

Secondly, when the new hardware system is established, the evaluation of the proposed control strategies can be carried out to control the IFTS to mimic more accurately the GRF and the position of the foot-shoe block at full speed (12 km/h). Other control strategies for the IFTS (such as using closed-loop control for all actuators; or using closed-loop torque control for the foot joints based on the monitored GRF, CoP and the inverse dynamic method to calculate the feedback joints' torque) can be developed and evaluated.

Thirdly, a promising direction is to develop the online learning algorithms for the current control strategy to tune the control signals after every running cycle. The

new design of the electronics & control system allows the controller DS1104 to monitor all necessary ground reaction forces, positions of the foot joints and the shank to derive all other parameters of a running cycle. Then DS1104 can compare these feedback parameters with the reference data in real time. Thus, the controller can update the control signal very efficiently.

Finally, in the next stage of developing the IFTS, this machine can be developed to mimic the human foot movements in other sports and gaits such as walking, jogging, sprinting, tennis, football and so forth. The shoe can be evaluated under many different wear conditions and gaits.

6.2.2 For the Robotic Foot

All proposed control schemes for the robotic joint actuated by a pair of CCMs can be assessed with high torque on the robotic foot joints in the IFTS because this system is equipped with a strong contact platform. This contact platform also possesses four high-accuracy load cells to feedback the GRFs and CoP to derive the joints' moments in real time. This feature allows the user to develop a highly accurate torque control for the foot joints.

The location of the three vertical load cells on the contact platform must be calibrated accurately, and a suitable filter needs to be developed to improve the accuracy of the feedback torque of the foot joints. Only then can the torque control scheme be evaluated accurately. In addition, multi-material 3D printing technology can be used to develop the foot tissue with variable hardness and artificial skin can be used to cover this foot tissue to mimic the human foot appearance and elasticity. Also, an in-shoe plantar pressure sensor can be installed to monitor and feedback the plantar pressure for evaluation and control development.

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Appendices

A. PROOF OF INEQUALITY 4.46

Consider the two variables function

$$f(\hat{\alpha}_i, \alpha_i) = \frac{\beta_i \sigma_{si}}{\mu_i} \tilde{\alpha}_i \hat{\alpha}_i - \frac{\beta_i \sigma_{si}}{\mu_i} \tilde{\alpha}_i \hat{\alpha}_i = \frac{\beta_i (\sigma_{si} - \sigma_{0i})}{\mu_i} (\alpha_i - \hat{\alpha}_i) \hat{\alpha}_i. \quad (\text{A.1})$$

From the definition of σ_{si} in (4.39), there are three possibilities:

Case 1: If $|\hat{\alpha}_i| < M_i$, then $\sigma_{si} = 0$, and one has

$$\begin{aligned} f(\hat{\alpha}_i, \alpha_i) &= -\frac{\beta_i \sigma_{0i}}{\mu_i} (\alpha_i - \hat{\alpha}_i) \hat{\alpha}_i = \frac{\beta_i \sigma_{0i}}{\mu_i} (\hat{\alpha}_i^2 - \hat{\alpha}_i \alpha_i) \\ &\leq \frac{\beta_i \sigma_{0i}}{\mu_i} (\hat{\alpha}_i^2 + |\hat{\alpha}_i| |\alpha_i|) \leq \frac{\beta_i \sigma_{0i}}{\mu_i} 2M_i^2. \end{aligned} \quad (\text{A.2})$$

The final inequality was obtained by the definition of M_i : $M_i > |\alpha_i|$.

Case 2: If $|\hat{\alpha}_i| > 2M_i$, then $\sigma_{si} = \sigma_{0i}$, and thus $f = 0$.

Case 3: If $M_i \leq |\hat{\alpha}_i| \leq 2M_i$, then $\sigma_{si} = \sigma_{0i}(|\hat{\alpha}_i|/M_i - 1)$, one has

$$f(\hat{\alpha}_i, \alpha_i) = \frac{\beta_i (\sigma_{si} - \sigma_{0i})}{\mu_i} (\alpha_i - \hat{\alpha}_i) \hat{\alpha}_i = \frac{\beta_i \sigma_{0i}}{\mu_i} \left(2 - \frac{|\hat{\alpha}_i|}{M_i}\right) (\hat{\alpha}_i^2 - \hat{\alpha}_i \alpha_i). \quad (\text{A.3})$$

Then, the derivative of function $f(\hat{\alpha}_i, \alpha_i)$ respect to α_i is

$$f'_{\alpha_i} = \frac{\beta_i \sigma_{0i}}{\mu_i} \left(\frac{|\hat{\alpha}_i| \hat{\alpha}_i}{M_i} - 2\hat{\alpha}_i \right). \quad (\text{A.4})$$

Consider two small cases of $\hat{\alpha}_i$ as follows:

Case 3i: if $M_i \leq \hat{\alpha}_i \leq 2M_i$, then $|\hat{\alpha}_i| = \hat{\alpha}_i$, and $f'_{\alpha_i} \leq 0$. Hence, $f(\hat{\alpha}_i, \alpha_i)$ is a non-increasing function respect to α_i on the given interval, and thus to find maximum of $f(\hat{\alpha}_i, \alpha_i)$, we set $\alpha_i = -M_i$ as

$$f(\hat{\alpha}_i, -M_i) = \frac{\beta_i \sigma_{0i}}{\mu_i} \left(-\frac{\hat{\alpha}_i^3}{M_i} + \hat{\alpha}_i^2 + 2\hat{\alpha}_i M_i \right). \quad (\text{A.5})$$

Then, its derivative respect to $\hat{\alpha}_i$ is

$$f'_{\hat{\alpha}_i} = \frac{\beta_i \sigma_{0i}}{\mu_i} \left(-\frac{3\hat{\alpha}_i^2}{M_i} + 2\hat{\alpha}_i + 2M_i \right) \quad (\text{A.6})$$

Solving the equation $f'_{\hat{\alpha}_i} = 0$, we find the position where f maximizes, $\hat{\alpha}_i = M_i(1 + \sqrt{7})/3$ and thus one has

$$f_{\max} = \frac{2\beta_i \sigma_{0i}(10 + 7\sqrt{7})M_i^2}{27\mu_i} = \frac{\beta_i \sigma_{0i}}{\mu_i} \epsilon M_i^2 \quad (\text{A.7})$$

where $\epsilon = (20 + 14\sqrt{7})/27$.

Case 3ii: if $-2M_i \leq \hat{\alpha}_i \leq -M_i$, then $|\hat{\alpha}_i| = -\hat{\alpha}_i$, and $f'_{\alpha_i} \geq 0$. Hence, $f(\hat{\alpha}_i, \alpha_i)$ is a non-decreasing function respect to α_i on the given interval, and then to find its maximum, we set $\alpha_i = M_i$ as

$$f(\hat{\alpha}_i, M_i) = \frac{\beta_i \sigma_{0i}}{\mu_i} \left(\frac{\hat{\alpha}_i^3}{M_i} + \hat{\alpha}_i^2 - 2\hat{\alpha}_i M_i \right). \quad (\text{A.8})$$

Thus, its derivative respect to $\hat{\alpha}_i$ is

$$f'_{\hat{\alpha}_i} = \frac{\beta_i \sigma_{0i}}{\mu_i} \left(\frac{3\hat{\alpha}_i^2}{M_i} + 2\hat{\alpha}_i - 2M_i \right) \quad (\text{A.9})$$

Solving the equation $f'_{\hat{\alpha}_i} = 0$, we find the position where f maximizes, $\hat{\alpha}_i = -M_i(1 + \sqrt{7})/3$, and thus one has

$$f_{\max} = \frac{2\beta_i \sigma_{0i}(10 + 7\sqrt{7})M_i^2}{27\mu_i} = \frac{\beta_i \sigma_{0i}}{\mu_i} \epsilon M_i^2. \quad (\text{A.10})$$

where $\epsilon = (20 + 14\sqrt{7})/27$.

In all cases, one has

$$f \leq \frac{\beta_i \sigma_{0i}}{\mu_i} \epsilon M_i^2 \quad (\text{A.11})$$

and thus,

$$\begin{aligned} \sum_{i=1}^2 \frac{\beta_i \sigma_{0i}}{\mu_i} \tilde{a}_i \hat{a}_i &\leq \sum_{i=1}^2 \left(\frac{\beta_i \sigma_{0i}}{\mu_i} \tilde{a}_i \hat{a}_i + \frac{\beta_i \sigma_{0i}}{\mu_i} \epsilon M_i^2 \right) \\ &\leq \sum_{i=1}^2 \frac{\beta_i \sigma_{0i}}{\mu_i} \left(-\frac{\tilde{a}_i^2}{2} + \frac{\alpha_i^2}{2} + \epsilon M_i^2 \right) \\ &\leq -\sum_{i=1}^2 \sigma_{0i} \frac{\beta_i \tilde{a}_i^2}{2\mu_i} + \sum_{i=1}^2 \frac{\beta_i \sigma_{0i}}{\mu_i} \left(\frac{\alpha_i^2}{2} + \epsilon M_i^2 \right) \end{aligned} \quad (\text{A.12})$$

B. PROOF OF THEOREM 3

Consider the Lyapunov function

$$V = \frac{1}{2} w^2 + \frac{1}{2} n^2 + \sum_{i=2,4} \left(\frac{a_i M \tilde{\beta}_i^2}{2\gamma_i} + \frac{\tilde{b}_i^2}{2\mu_i} \right)$$

Then its first time-derivative is

$$\begin{aligned}
\dot{V} &= w\dot{w} + n\dot{n} + \sum_{i=2,4} \left(\frac{a_i M \tilde{\beta}_i \dot{\tilde{\beta}}_i}{\gamma_i} + \frac{\tilde{b}_i \dot{\tilde{b}}_i}{\mu_i} \right) \\
&= w(\dot{e} + \lambda e) + n(w - \lambda n) - \sum_{i=2,4} \left(\frac{a_i M \tilde{\beta}_i \dot{\tilde{\beta}}_i}{\gamma_i} + \frac{\tilde{b}_i \dot{\tilde{b}}_i}{\mu_i} \right) \\
&= -\lambda n^2 + \lambda w \left(\frac{n + \dot{e}}{\lambda} + \tau - \tau_r \right) - \sum_{i=2,4} \left(\frac{a_i M \tilde{\beta}_i \dot{\tilde{\beta}}_i}{\gamma_i} + \frac{\tilde{b}_i \dot{\tilde{b}}_i}{\mu_i} \right) \\
&= -\lambda n^2 - \lambda c w^2 - \sum_{i=2,4} \left(\frac{a_i M \tilde{\beta}_i \dot{\tilde{\beta}}_i}{\gamma_i} + \frac{\tilde{b}_i \dot{\tilde{b}}_i}{\mu_i} \right) \\
&\quad + \lambda w \left\{ \begin{aligned} &\frac{n + \dot{e}}{\lambda} + c w - \tau_r + u_2 + u_4 \\ &+ [\tau_1 \text{sg}(x_l - x_1) - b_2 \text{sg}(x_1 - x_l)] \text{sg}(x_2) \\ &+ [\tau_3 \text{sg}(x_1 - x_h) + b_4 \text{sg}(x_h - x_1)] \text{sg}(-x_2) \end{aligned} \right\} \\
&= -\lambda n^2 - \lambda c w^2 - \sum_{i=2,4} \left(\frac{\dot{\tilde{\beta}}_i}{\gamma_i} + \lambda w \bar{u}_i \right) a_i M \tilde{\beta}_i \\
&\quad - \tilde{b}_2 \left[\frac{\dot{\tilde{b}}_2}{\mu_2} + \lambda w \text{sg}(x_2) \text{sg}(x_1 - x_l) \right] - \tilde{b}_4 \left[\frac{\dot{\tilde{b}}_4}{\mu_4} - \lambda w \text{sg}(-x_2) \text{sg}(x_h - x_1) \right] \\
&\quad + \lambda w \left\{ \begin{aligned} &\left(\frac{n + \dot{e}}{\lambda} + c w - \tau_r \right) [\text{sg}(x_2) + \text{sg}(-x_2)] + \bar{u}_2 + \bar{u}_4 \\ &+ [\tau_1 \text{sg}(x_l - x_1) - \hat{b}_2 \text{sg}(x_1 - x_l)] \text{sg}(x_2) \\ &+ [\tau_3 \text{sg}(x_1 - x_h) + \hat{b}_4 \text{sg}(x_h - x_1)] \text{sg}(-x_2) \end{aligned} \right\}
\end{aligned}$$

Substitute the control and update laws (4.116) - (4.121) into the above equation, one has

$$\dot{V} = -\lambda n^2 - \lambda c w^2 \leq 0; \forall t \geq 0; \forall n, w \in \Re$$

The derivative of Lyapunov function is non-positive in the above equation implies that V is non-increasing and therefore all signals $w, n, \hat{b}_i, \tilde{b}_i, \hat{\beta}_i, \tilde{\beta}_i$ ($i=2, 4$) are globally uniformly ultimately bounded. Furthermore, by applying LaSalle-Yoshizawa theorem in [150], it is guaranteed that w and n converge to zero as $t \rightarrow \infty$.

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