Vibration based moving inspection method to detect loose fasteners and damaged supports in a rail system

Wang, Longqi

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**Vibration Based Moving Inspection Method to
Detect Loose Fasteners and Damaged Supports
in a Rail System**

Wang Longqi

School of Civil and Environmental Engineering

2017

Vibration Based Moving Inspection Method to Detect Loose Fasteners and Damaged Supports in a Rail System

Wang Longqi

School of Civil & Environmental Engineering

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SUMMARY

The main aim of this study is to propose a workable solution which can be used to detect loosening of fasteners and degraded sleepers or blaster in view of their significant influencing on the support stiffness between the rails and the discrete supports. Since rail systems are usually in kilometers long, moving device based solutions are a more suitable choice. The following research works are carried out to develop a moving inspection method for supporting system.

Firstly, the frequency shift caused by an auxiliary mass is found to be sensitive to the stiffness reduction of the discrete supports. Further analysis shows that change of frequency shift in certain modes can be utilized to localize the supports with decreasing stiffness. These sensitive modes can be roughly determined by a dimensionless parameter. Moreover, model superposition solution of the problem shows that frequency shift is related to mode shape, which is shown to be an effective feature for detecting stiffness reduction. These findings reveal the feasibility to inspect stiffness reduction based on frequency shift caused by a moving auxiliary mass, and thus, it forms the basic principles for the solution constructed.

Secondly, frequency shift curves (FSCs) are extracted under quasi-static and moving conditions using a moving vehicle. The FSCs contain information as predicted from the analytic solutions, but they have different signal to noise ratio (SNR) at different locations and different modes. In particular, at the nodal points of one specific mode, the corresponding frequency cannot be extracted due to very low response level near the nodal points at that mode. The amplitude of the response is an indicator for the SNR of the FSC at that location, and it is used in false frequencies and low SNR frequencies removal. The FSC extraction method with false frequencies removal procedure are proposed and validated by numerical examples and experiments.

Finally, specific damage indices for rail system based on the periodic property of rail system are proposed to simplify the inspection procedure. The performance of the solution is verified by several models experimentally. The method is further extended to random excitation condition by introducing a novel frequency estimator to estimate FSC. The performance of the

method is also tested numerically and experimentally. These results show that the proposed solution is workable under various conditions and under a reasonably wide range of parameters.

A moving device based degraded supports inspection method is developed in this study. It only requires one sensor mounted on the moving device. Based on the numerical and experimental examples, it can detect localized degraded supports accurately in a rail system, irrespective of the spatial distribution of degraded supports. It is found to be more efficient and accurate than the widely used visual inspection methods, especially for loosening of fasteners and under surface degradation of sleepers and blasters. The method shows good performance under periodic impact and random excitation.

LIST OF PUBLICATIONS

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2. Wang, L., Lie, S. T., & Zhang, Y. (2016). Damage detection using frequency shift path. *Mechanical Systems and Signal Processing*, 66, 298–313.
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CHAPTER 1

INTRODUCTION

1.1 BACKGROUND

Due to its high capacity and efficiency, rail system is still one of the most important land transportation systems used in many countries. The main functional components in the rail system are the rails and the trains. Rails are installed along the routes, and they normally are supported by discrete supports as shown in Fig. 1.1.

Fig. 1.1 Typical third rail power system

The two rail tracks shown on the right side of the figure are called *running rails*, on which the trains operate. Most of the rail systems have two running rails. Normally, they are fastened by specially designed fasteners onto discrete sleepers, which are the concrete blocks underneath the running rails. They can be made of timber, concrete and other types of materials. Then, these sleepers are embedded into a compacted ballast layer (Fig. 1.2(c)) or concrete foundation (Fig. 1.1). This supporting system is important for rail tracks to retain their leveling and alignment. Once the fasteners, sleepers, ballast layer or foundation fail their function, the rail track will be unevenly supported, geometrically deteriorated or horizontally not fully constrained; the gauge which is the distance between two running rails may be unevenly spaced. In fact, several serious accidents have been reported due to this degradation in the last decades.

For example, a train derailed on February 5th 2003 is caused by a wide gauge condition, which was induced by damaged sleepers (Fig. 1.2(b)) and degraded ballast (Fig. 1.2(c)). According to the accident report (Transportation Safety Board of Canada, 2003), the widening gauge was only visible under the heavy load of a train, thus it is difficult to identify by the rail inspection personnel. Therefore, it is important to develop an effective approach to inspect the sleepers and ballasts under the running rail before serious accident happens.

(a)

(b)

(c)

Fig. 1.2 Derail accident in Canada, 2003
(a) photo of the accident, (b) damaged tie condition,
(c) fouled ballast in vicinity of derailment site

The rail shown on the left side of Fig. 1.1 is called the *third rail*, which is one type of power systems used frequently in massive rapid transit (MRT) system. Because third rail power system has many advantages such as requirement of a relatively smaller profile of tunnel and longer

service life expectancy, it is widely installed in many countries as shown in Fig. 1.3. In Singapore, except North-East line, all MRT lines are using the third rail power system.

Fig. 1.3 Map of third-rail power system usage in the world
(Data source: Wikipedia, 2016)

Fig. 1.5 shows two typical third rail mounting systems used in Singapore. They are both bottom-contact. In fact, modern third rail systems are usually side-contact or bottom-contact (Fig. 1.4), therefore, the conductor rail can be covered. The design is to prevent track workers from accidentally touching the conductor rail, and avoid contamination by falling water and leaves. It can be seen from Fig. 1.5(a) that the entire third rail and its fasteners are both covered by the protective covers. Thus, visual inspection of the third rail and its fasteners requires the cover to be removed. The conductor rail conducts current from sub-stations into metal contact blocks called *collector shoes* installed on the side of the train; then, the current is returned to the sub-stations through running rail. Therefore, the third rail is usually made of high conductivity steel, and some are covered by a layer of mild steel.

Fig. 1.6 shows the two types of third rail fasteners used in Singapore MRT lines with protective covers partially removed. For the old fastener, the claw is attached to the insulator by bolts and a spring sheet (Fig. 1.6(a)). The claw is tied with steel wires to prevent it from dropping, which had happened in the previous interruptions. Fig. 1.6(b) shows the new fasteners deployed recently where the claw is bolted to the insulator.

Fig. 1.4 Different types of shoe-third rail contact methods
(Source: Railway Technical Web Pages, 2012)

(a)

(b)

Fig. 1.5 Two types of third rail mounting systems used in Singapore MRT lines
(a) old mounting, (b) new mounting

(a)

(b)

Fig. 1.6 Fasteners used in third rail mounting systems of Singapore MRT lines
(a) fasteners in old mounting, (b) fasteners in new mounting

Train induced noise and vibration is one of the most serious environmental hazards of rail system, especially it is in urban areas. Therefore, rail tracks are usually installed on some kinds of vibration isolation bases like floating slabs, which reduce train induced noise and vibration to the environments; usually this can increase the vibration in the rail system itself. Consequently, the continuous periodic vibration from the train causes the claws and bolts in the third rail supports to loose and missing. To make it worse, the common defects such as flat wheel create as much as 10 times stronger vibration. The loose supports further weaken the mounting system and increase the vibration in the adjacent regions. Accordingly, the adjacent supports are prone to loose and the whole segment becomes unfastened. This may cause significant sag of the third rail, especially for the bottom-contact installation as used in Singapore. According to the investigation reports published in Singapore, particularly the one reported on the July 5th 2012 in The Strait Times newspaper, several MRT interruptions were caused by sagging of the third rail. This continuous faulty third rail supports caused sagging of third rail and the sagged rail damaged the contact shoes. Subsequently, the train lost its main power supply. The demonstration of the power interruption is shown in Fig. 1.7, where a claw

was dislodged, and the third rail lost its support and sagged. Finally, the collector shoe was damaged by the sagged third rail, and caused the interruption. Therefore, the integrity of the fasteners in the third rail should be inspected periodically before the claws are dislodged.

In fact, the fasteners, sleepers and ballast on the running rails are all exposed to train induced vibration as well, which causes similar problem as described in the above for the fasteners on the third rail. In ballasted tracks, the ballast needs to be refilled periodically. The fasteners of the running rails also tend to get loosed or missing (Fig. 1.8). These degradations increase the probability of derailment, and thus, they also need to be investigated and maintained.

(a)

(b)

Fig. 1.7 Demonstration of MRT interruption caused by sagging third rail
 (a) damaged segment, (b) dislodged claw, (c) damaged collector shoe
 (Source: The Strait Times, July 5th 2012)

Loosening of fasteners in the running rails and the third rail can be characterized as a problem with different parameters. In this study, sleepers, ballasts and fasteners in the running rail and fasteners and standing supports in the third rail are all called *supports* or *supporting system* in the rail system. The main function of these supports is to provide support for the rails to keep them in the right position. Unfortunately, inspection of degraded supports is not a well-solved problem. In fact, the integrity of supports is mostly relied on visual inspection by track personals in many countries (Gibert et al., 2016; Lam et al., 2012). However, most of the third rail is covered, and therefore, the supports cannot be easily inspected visually. Furthermore, the under surface degradation of sleepers and ballasts is also hard to be inspected visually. The fact that the loose bolts and degraded sleepers and ballasts have very few differences with the intact

ones, especially in the early stage, making the inspection problem even harder. Traditionally, track personals also make use of spike hammers to investigate the condition of the fasteners by impact sound. However, this requires many skilled workers and costs a lot of time.

Fig. 1.8 A fastener system with a missing right bolt

In order to solve this problem, several high-tech inspection methods are proposed. They fall into two categories. One is based on visual signal captured which can be analyzed by advanced image processing method and pattern recognition method to identify the missing supports. These methods are effective for inspection of missing sleepers and dislodged fasteners. The other is to measure the vertical displacement of the rail, since the sagging of the rail can be detected by high precision equipment, such as laser distance meter. This method requires the sagging of the rail to a certain extent, which may be too late especially for running rail supporting system. As the root of the problem is caused by degradation of material or loose fasteners, it is more effective if the faulty supports or fasteners can be inspected and reported based on mechanical properties. Therefore, inspired by the traditional impact inspection method using spike hammers, a method based on moving impacts is proposed in this dissertation.

1.2 RESEARCH OBJECTIVES

The main objectives of this study is to propose a workable solution for inspecting degradation of supports in rail system. The degradation includes material degradation of sleepers, ballast and foundation; loose fasteners in the running rails and the third rail; rail-sleeper separation or sleeper-ballast separation. Because the function of the supporting system is to keep the rails in the right position, the degraded supports generally do not provide enough

support for the rail, and thus, detection of degraded supports can be grouped as a rail support condition monitoring process. In order to accomplish the goal, it is sub-divided into the following objectives:

1. Simplify the actual problem of detecting degraded supports to a mechanics problem that can be analyzed. The results from the model can represent the behavior of the problem and the degradation of the supporting system can be reduced to a problem of identifying parameters change in the model.
2. Propose basic principles of the degraded supports detection method, determine sensitive features for degradation in supporting systems and explore selection of parameters.
3. Develop a suitable method to extract the degraded supports sensitive features. The time complexity of the method should be controlled for efficient calculation of the feature. The accuracy of the method should satisfy the requirements of degradation detection.
4. Examine the performance of the proposed method under quasi-static and moving conditions, and solve the corresponding signal processing difficulties. Performance under quasi-static condition provides the upper bound performance for the proposed method, and the signal processing is easier because of the lower noise level. Therefore, the performance of the method under quasi-static condition is verified before under moving condition.
5. Verify the proposed solution in various conditions using an experimental setup and a small-scaled model. Validations should be carried out in a wide range of parameters. Experimental validations in small-scaled models simulating different damage cases should be carried out.
6. Summarize and simplify the proposed methods. Based on the results under different conditions, simplify the proposed method by taking advantages of the special properties of a rail system.

1.3 ORIGINAL CONTRIBUTIONS

The original contributions of this study are summarized as follows:

1. The frequency shift caused by moving auxiliary mass of a beam with discrete springs and the influence of spring stiffness to the frequency shift are investigated for the first time. They provide basic principles for inspecting loosening of fasteners in a rail system.
2. A moving device based frequency extraction procedure is developed. The amplitude of the response is studied, and a damage index namely FRESH path is proposed. The stiffness reduction can be identified from FRESH curvature.
3. Important factors influencing the frequency extraction procedure under moving condition are analyzed and further verified. Spatial error and residue part from previous impacts are the most critical factor affecting the performance.
4. The amplitude component can be easily polluted by friction induce vibration and other noises, but it is found to be a good indicator of the SNR of the signal. An adaptive threshold determination procedure based on clustering method is proposed. It can be used to eliminate false frequencies and frequencies with low SNR adaptively, and it is applicable for various testing cases under different parameters.
5. The inspection procedure is simplified significantly by using the periodic property of the rail system. A novel damage index is proposed which can identify the exact location of the degraded supports accurately. Its magnitude is also found to be related to the severity of degradation. The numerical and experimental verifications show that the performance of the method is convincing under various conditions.
6. The inspection method is further extended to random excitation condition by using a new frequency estimator calculated from the ratio of MoS. Numerical and experimental examples are also presented to show the performance of the method.

1.4 REPORT OUTLINE

The thesis is organized as follows:

Chapter 2 briefly reviews past and present related research works, including inspection methods for joints and fixings, current inspection methods for supports in rail system and damage detection methods related to this study.

Chapter 3 proposes the mechanics model and analyzes the problem in details. The basic principles and parameters of the proposed method are also presented in the chapter.

Chapter 4 discusses detection of stiffness reduction by moving inspection device in a beam-like structure under quasi-static condition. A procedure is proposed to extract frequency shift curve, and the information in the amplitude of the response is discussed. A novel feature is proposed and used to provide detailed information about damage in the structure. The method is then verified numerically and experimentally.

Chapter 5 tests the procedure under moving condition. The influence of the inspection velocity and impact period is also discussed. Experimental verification under different inspection velocity and impact period is provided. It is found that under the moving condition the amplitude may be contaminated and various false frequencies may appear due to uncertainty of the contact surface and noise. A procedure to remove false frequencies based on amplitude information is proposed.

Chapter 6 proposes and verifies specifically designed inspection indices for supports in rail system.

Chapter 7 presents and verifies an extension of the method under random excitation condition.

Chapter 8 summarizes the discussions, conclusions and future works on damage detection of the supports in rail systems.

CHAPTER 2

LITERATURE REVIEW

2.1 CURRENT INSPECTION METHODS FOR SUPPORTING SYSTEMS

2.1.1 Inspection of damaged sleepers and blasters

The current available rail inspection equipment is mainly focused on properties of the rails, such as irregularity and geometry. There are very limited research on the inspection methods for sleepers and ballast or rail foundations. Visual inspection and destructive core testing are the two most frequently used methods for sleeper and ballast damage inspection (Lam et al., 2012). Core testing is accurate, but it causes serious interruption to railway operation. Visual inspection is useful for visible damage. In fact, several computer vision based methods have been proposed for detection of missing fasteners and sleepers. However, many types of damage, such as degradation of ballast under sleeper, are difficult to be detected by visual signals. Therefore, several advanced methods are proposed. Ground penetrating radar is one of the most mentioned non-destructive methods (Al-Qadi et al., 2008; Naraynan et al., 2001; Zhang et al., 2014). The basic idea is the interface between different sub-layers reflects the electromagnetic waves differently. By analyzing the changes in amplitude and time of fly, the properties such as thickness and stiffness can be identified. Vibration based methods are another type of methods used to detect the damaged sleepers. Grassie & Cox (1985) studied the dynamic response of a rail track with unsupported sleepers. The work showed potentials of instrumented hammer to inspect railway sleepers and ballast condition. Lam et al. (2012) proposed a model updating method based on vibration data to solve the problem. Then, Lam et al. (2014) extended the method by using Bayesian method to overcome the uncertainties induced by modeling error and noises.

Generally, Ground penetrating radar (GPR) is a promising method to solve the problem if the parameters can be selected wisely. Vibration based methods show great potentials, but currently they require investigating sleepers one by one. Therefore, a vibration based moving device for the problem is an interesting area to explore; this is the main objective of this study.

2.1.2 Inspection of damaged joints and fasteners

Many engineering structures are assembled from different parts by using fasteners. The failure of these fixings has an impact on the performance of the structure, and also induces safety risks to the structure. However, maintenance of the joints depends almost entirely on regular periodic manual inspection, which is not only costly and time consuming, but also unreliable. Therefore, this problem has drawn many researchers' attentions, and various methods from different perspectives are emerged. These methods are reviewed here by its basic principles.

2.1.2.1 *Visual inspection*

Because fasteners are small and they have complex geometrical shape, therefore, inspection of loose fasteners is more difficult. Moreover, the influences of the loosening usually are smaller than degradation of sleepers. Current inspection on fasteners is mainly based on visual testing, and various computer vision based methods are proposed by several research groups. Since missing or broken fasteners have different shapes, thus, shape based features can be extracted from vision signals such as photos or videos from on-board camera on a normal train or inspection vehicle. Using advanced high-speed camera, images can be captured in high speed train without motion blur. Therefore, these methods can be used under extreme high speed, and the inspection speed is only restricted by the processing speed of the algorithm.

De Ruvo et al. (2009) and Stella et al. (2002) proposed a GPU based vision system for real time detection of missing fasteners by using a neural network with a hidden layer. The feature extraction method was based on Daubechies and Haar 2D discrete wavelet transform (DWT). The method achieved to analyze video at 273 km/h for jumping phase, and 10 km/h for exhaustive phase. Xia et al. (2010) also utilized Haar DWT as the features, and then an ensemble method called adaptive boosting classifier is applied to enhance the classification results. Feng et al. (2014) built a probabilistic structure topic model for the structures of fasteners and applied the model to classify the Haar-like features. Based on processing speed, the classifier can process on fastener in about 0.07 s. Yang et al. (2011) proposed a direction field based method to classify the missing fasteners. Babenko (2009) developed an intensity based method to detect missing fasteners by optimal trade-off maximum average correlation height (OT-MACH) correlation. There are many other works making use of other image processing features, such

as edges detection (Berry et al., 2008), Gabor-transformed image (Resendiz et al., 2013), histogram of oriented gradients (HOG) (Gibert et al., 2015), support vector machine (SVM) (Gibert et al., 2015), convolutional neural network (CNN) (Giben et al., 2015) and multi-task deep learning method (Gibert et al., 2016). The general patterns of these methods are quite similar: firstly, the process needs to align the captured images, and then, applies feature extraction methods on the aligned images; finally, classification methods can be carried out to make decision on the status of the fasteners. However, the main drawback for these methods is obvious. Because these methods are based on vision features, defects like loose fasteners barely change the shape. Thus, it is very hard to detect loose fasteners based on vision features. Therefore, it is critical to explore other features to inspect loose fasteners and damaged supports.

2.1.2.2 Ultrasound testing

Wang et al. (2013a) presented a review on inspection of bolted joints using ultrasonic methods. The methods make use of several properties when ultrasound interact with the structure. The most typical method is based on acoustoelastic effect, which states that velocity of ultrasonic wave propagated along a rod depends on the axial stress. It can be represented by the following equations:

$$\begin{aligned} V_L &= V_{L0}(1 + A_L \sigma) \\ V_T &= V_{T0}(1 + A_T \sigma) \end{aligned} \quad (2.1)$$

where V_L and V_T are the longitudinal and transverse wave velocities, respectively. A_L and A_T are the acoustoelastic constants of longitudinal and transverse wave, respectively. σ is the stress.

The time of flight (TOF) method is a direct usage of above acoustoelastic effect. The derivation of TOF for axially loaded bolt can be demonstrated by Fig. 2.1.

Fig. 2.1 Model of a bolt under axial loading (Wang et al., 2013a)

The initial length of the bolt L_1 can be divided into effective stressed length L_e and effective unstressed length L_0 . The unstressed length is assumed to remain unstressed after loading. L_e changes to $L_\sigma = L_e(1 + E^{-1})\sigma$ due to applied axial force. Therefore, the relationship between these variables can be written as

$$L_1 = L_0 + L_e \quad (2.2)$$

$$t = 2 \left(\frac{L_\sigma}{V} + \frac{L_0}{V} \right) \quad (2.3)$$

where t is the time of fly, V is the wave velocity, and the first order approximation of Eq. (2.3) can be rewritten as follows:

$$t \cong t_0 \left[\frac{L_e}{L_1} (E^{-1} + A)\sigma + 1 \right] \quad (2.4)$$

Thus, the change of t is approximately proportional to the axial load in the bolt. This formula only requires wave velocity, either longitudinal wave, or transverse wave. Thus, it is easier to apply in practice. Yasui & Kawashima (2000) proposed a method based on TOF to measure axial load using 5 MHz transverse wave and 10 MHz longitudinal wave. Hirao et al. (2001) used electromagnetic acoustic transducer to generate shear wave in the axial direction to determine the axial load. The experiments showed a good linear relationship between load and t . However, the main drawback of these methods is that the time difference of TOF is quite small, which is in the range of tens of nanoseconds. Therefore, the equipment for measuring TOF needs to have a sample rate at 1 GHz. Another problem is that t_0 is generally unknown for a fastened structure. Fortunately, this can be overcome by using two types of waves in the structure (Chaki et al., 2007). Considering Eq. (2.3) for longitudinal and transverse wave, t_0 can be written as

$$\frac{t_T}{t_L} = \frac{V_{L0}}{V_{T0}} \left[1 - \frac{L_e}{L_1} (A_T - A_L) \frac{F}{S_e} \right] \quad (2.5)$$

The TOF can also be measured from acoustic resonance frequency of the bolt. The acoustic resonant frequencies can be written as

$$f_n = \frac{2nv}{L} \quad (2.6)$$

where f_n clearly changes with stress in the bolt, and thus, it can be used to monitor the stress in the bolt. The relationship can be written as

$$\frac{\Delta f_n}{f_n} = -\left(A_L + \frac{1}{E}\right)\sigma \quad (2.7)$$

Conradi et al. (1974) and Heyman (1977) proposed several methods based on changes of resonant frequency to monitoring bolt condition using transmission oscillator ultrasonic spectrometer and reflection oscillator ultrasonic spectrometer, respectively. Generally, they can be viewed as a modified version of TOF method, and in fact, they shared similar results and drawbacks.

Another ultrasonic method is called piezoelectric active sensing. Unlike acoustoelastic effect based methods, it is based on changes of inter-facial characteristics, such as stiffness, damping and contact area. One of the most significant advantages is requirement of lower ultrasound frequency, and consequently lower cost. Yang & Chang (2006a) proposed an attenuation-based method to locate loose bolts and predict the fastened level. Ultrasonic wave attenuation across the joint was recorded. According to Hertz contact theory, within the maximum Hertzian pressure, the wave energy transmitted is proportional to the square root of the pressure. Thus, the fastened level can be determined by measured data. The theory is verified by experiments (Wang et al., 2013b; Yang & Chang, 2006b). Similarly, Lamb wave induced by piezoelectric transducer can also be used for monitoring. Doyle et al. (2010) showed experimentally that Lamb wave delay time had good linear relationship with applied torque. Several machine-learning methods were applied to active sensing monitoring process: support vector machine is used to classify Fourier amplitude at excitation frequency (Mita & Fujimoto, 2005); probabilistic neural networks were employed to classify wavelet transformed results (Park et al., 2006).

The piezoelectric impedance based methods are also used by several researchers. However, they are rather expensive due to the cost of impedance testing equipment. Park et al. (2001, 2003) tracked and monitored bolted fixings in offshore pipeline structures using this method. Wait et al. (2005) combined impedance method with Lamb wave method to analyzed bolted

systems. Generally, these methods lack theoretical background, and they are mostly based on experimental observations.

In the review by Wang et al. (2013a), a comparison shown in Table 2.1 is presented for ultrasound based methods. Most of these methods are expensive, and they are highly dependent on large sample rate and limited real time monitoring function. Moreover, they require a direct contact with the bolts, which is almost impossible for supports inspection. Therefore, they provide very useful information for designing inspection method for joints, but they can hardly be used for supports inspection in rail systems.

Table 2.1 Comparison of SHM methods for bolts

	Time of flight method	Velocity ratio method	Resonant frequency shift	Piezo active sensing	Piezo impedance
Nature of signal	Time	Velocity ratio	Frequency	Voltage	Impedance
Sample rate	High	High	High	Relatively low	Depends
Cost	High	High	High	High	High
Real time	No	No	No	Yes	Yes

2.1.2.3 Inspection methods based on dynamic properties for loose joints

Dynamic properties, such as frequency response function (FRF) and modal parameters, are another group of sensitive features studied by many researchers. Generally, the vibration based methods are viewed as *global* inspection methods. Although the spatial precision of them are poorer than ultrasound based methods, they can capture information of the structure in large scale efficiently. This feature is very useful for inspection of supports in the rail systems. Unlike ultrasound based method, the principles of these methods are quite different from each other; and thus, each of them is reviewed separately.

Lovell & Pines (1998) proposed a wave propagation method to identify the loss of torque in bolted lap joints. They also reported the FRF was susceptible to have too much uncertainty. Moreover, they stated that wave descriptions of local structural dynamics may offer advantages for evaluating the structure since global dynamic was virtually impossible to excite. This idea is especially important for inspection of rail system because they are infinite beam-like structures.

Caccese et al. (2004) applied traditional vibration based methods on the problem, and they found that the fundamental frequency did not have significant relationship with torque in the bolts. However, FRF provided some promising results.

Nichols et al. (2007) proposed a hypothesis test based method for the problem. The null hypothesis was the data generated by a linear system but observed through an instantaneous, may be nonlinear measurement function. The baseline was captured through iterative amplitude-adjusted Fourier transform. Then, the full observed data were compared with the baseline to determine whether the hypothesis is rejected or not.

Milanese et al. (2008) modelled the loosening as a formation of gap. Since response of a nonlinear system contains not only frequency components lower than cut-off frequency, but also higher ones, an output-only method based on ratio of the power content in two frequency bands was proposed. Moreover, because experimental probability density functions (PDF) are different for different conditions of joints, kurtosis was also proposed as a damage index.

Amerini et al. (2010) showed the amplitude ratio between the second harmonic and fundamental frequency can be used to inspect loosening state of joints and bolts, based on the assumption that the reaction force from bolt is a quadratic function of the displacement. Then, they investigated high-harmonics generation and sideband modulation (Amerini & Meo, 2011), which can be viewed as an extension of the previous method. The sideband modulation method utilized 100 kHz high-frequency excitation to modulate 2100 Hz low-frequency signal, and the ratio between amplitudes of sidebands and carrier frequency was the damage index for the method. However, the selection of the frequency was a difficult problem and it was not discussed. One solution to the problem is to use a wide band signal like impact instead of applying a selected low frequency signal. In fact, the impact modulation was studied by Jaques & Adams (2011) and Meyer & Adams (2015). They derived a one degree of freedom model for the problem based on a perturbation method. Inspired by the solution of the model, they examined the problem numerically and experimentally. Unlike vibration modulation methods presented by Amerini & Meo (2011), the sidebands in impact modulation were related to natural frequencies of the structure. They also found that the first three sidebands, which were somehow related to the frequencies of first three modes, were sensitive to the loose bolts.

Although researchers have tried to solve this problem through different viewpoints and the proposed methods cover a wide range of dynamic features, energy in different frequency ranges, frequencies and damping are the most frequently used properties. It should also be noted that the same property may have different performance and robustness in different studies.

Moreover, various works are also presented in the field of dynamics of structures with joints and bolts. They may be not directly related to inspection, but the problems in this field also somehow related to the inspection. These works are not reviewed here, some papers on this topic include Ahmadian & Jalali (2007), Bograd et al. (2011) and Ibrahim & Pettit (2005).

2.1.3 Discussion on current inspection methods of degraded supports

From the review presented above, there are various ideas which can be used for inspection of supports. In practice, the rail system can be in kilometers long, and the sleepers and fasteners in one city can easily exceed tens of thousands. A practical solution to monitor these vast amount of objects is to use some kind of moving inspection device, otherwise the sensor network will be extremely complex and nearly impossible to maintain. Currently, the safety of the rail supporting system are mostly relied on railroad personnel, either by walking or riding a hi-rail vehicle (Gibert et al., 2016; Lam et al., 2012). These methods are unstable and inefficient. For loose fasteners in the third rail, the inspection is more difficult because the fasteners are hidden under the cover of the third rail as shown in Fig. 1.5.

Dynamic properties such as frequencies and mode shapes are useful features for detecting degradation in sleeper or blaster damage. They can offer *global* information of the structures efficiently. Past research works carried out on loose bolts and joints also show that the dynamic properties are effective in identifying the status of bolted joints. Therefore, a moving device which can measure dynamic properties is a promising solution for inspection of degradation of supports. The previous works on moving load problem and damage detection methods based on dynamic properties, especially methods using moving device, are presented in following section.

2.2 DAMAGE DETECTION METHODS BASED ON DYNAMIC PROPERTIES

There are lots of damage detection methods based on dynamic properties. Thus, only publications related to this report are reviewed here.

2.2.1 Frequency based methods

Frequency based method is one of the earliest kinds of vibration based damage detection methods. Salawu (1997) and later Fan & Pizhong (2010) reviewed damage detection methods based on frequencies. Because measurement of frequency is cheap and easy, it draws many researchers' attention. The first damage detection method based on frequency change can be traced back to 1978, when Adams et al. (1978) presented a method for 1-D structures using longitudinal frequencies. Messina et al. (1998) proposed a damage detection method based on multiple damage location assurance criterion (MDLAC) by using sensitivity of frequencies of each mode to damage in multiple locations. Morassi (2001) used the shifts in a pair of natural frequencies as damage feature to detect single crack in a vibrating rod. They Morassi & Rollo (2001) extended the method to identify multiple cracks in simply supported beam under flexural vibrations. Kim & Stubbs (2003) proposed a single damage indicator shown in Eq. (2.8) to locate a crack in beam-like structures. Since the crack size was modeled in FEM, the method can locate the damage and assess the severity of the damage,

$$SDI_j = \left[\sum_{i=1}^{NM} \left(\frac{Z_i}{\sum_{k=1}^{NM} Z_k} - \frac{F_{ij}}{\sum_{k=1}^{NM} F_{kj}} \right)^2 \right] \quad (2.8)$$

where Z_i is the i -th eigenvalue, F_{ij} is the FEM-based modal sensitivity of i -th modal stiffness respect to the j -th element.

There are many other methods proposed based on frequency change, and they are widely used in many applications. However, their limitations are also obvious. The damage is evaluated by several *global* values, thus local information is limited in these methods. Moreover, the frequency changes usually are small and it may be polluted by environment and operational condition. A great improvement in this area is to perturb the structure, and thus the frequencies change accordingly. By analyzing the changes and values, frequency based methods may provide much detailed information about the structure instead of a *global* state. These methods usually require some auxiliary device moving along the structure, which are ideal for inspection of rail system.

2.2.2 Mode shape based methods

Mode shapes contain more local information than natural frequencies, and they are less sensitive to environmental conditions such as temperature. Therefore, many researchers proposed damage detection methods based on mode shapes.

Shi et al. (2000a) extended the damage localization method based on MDLAC (Messina et al., 1998) by using incomplete mode shape instead of natural frequencies. Pawar et al. (2006) proposed a method based on Fourier analysis of mode shapes in the spatial domain for beams with clamped boundary conditions. The Fourier coefficients were used as damage features and then neural network was applied to classify the data. Abdo & Hori (2002) presented a method based on rotation of mode shape, which was the first derivative of displacement mode shape. However, displacement mode shape is not very sensitive to small damage (Huth et al., 2005; Salawu & Williams, 1994). An enhancement of the sensitivity of mode shape is the mode shape curvature (MSC), which was proposed by Pandey et al. (1991). The MSC is defined by Eq. (2.9) as

$$\kappa_i = \frac{w_{i+1} - 2w_i + w_{i-1}}{h^2} \quad (2.9)$$

where w_i is the mode shape at i -th point and h is the spatial interval.

Ratcliffe (1997) developed a novel index called GSM to localize the damage from MSC. The idea was to calculate the difference between fitted curve and MSC

$$\delta_i = \sum_{i=0} 3a_i x^i - \kappa_i \quad (2.10)$$

where a_i is the fitting coefficients and the κ_i is MSC of i -th sensor point.

Then, GSM was applied on locating a delamination in a composite beam (Ratcliffe & Bagaria, 1998), and generalized to 2D plate-like structures (Yoon et al., 2005).

Amaravadi et al. (2001) used orthogonal wavelet transform to enhance sensitivity of MSC. Kim et al. (2006) also proposed a method for beam-like structures by using wavelet transform.

Similar to GSM, another method called Global Fitting Method (GFM) is proposed by Yoon et al. (2010). It was based on the idea that smoothed mode shape can be treated as an estimation of intact data. Thus, difference between smoothed data and original data can be used to identify damage. This idea is then extended by Zhang et al. (2013a) to plate-like structures.

A notable method proposed by Fang & Perera (2009) showed that power of mode shape (PMS) could be extracted from Fourier transforms of acceleration signals and it could be used to detect damage in linear structures. Zhang et al. (2013a) then showed that PMS can be extracted from signal captured on moving device by a similar method.

In conclusion, methods based on mode shapes have higher sensitivity. MSC is an effective enhancement for mode shape. However, it usually cannot be calculated accurately, and hence, false alarm may be found in the detection results. As a result, the error of mode shape should be controlled carefully for calculating reasonable MSC. Another drawback of mode shape based method is that measuring mode shape is more complicated than natural frequency.

2.2.3 Other vibration based methods

Many other dynamic properties are also used in damage detection. Flexibility matrix, modal strain energy and frequency response function are some of the famous ones.

Flexibility matrix based method was first proposed by Pandey & Biswas (1994). The flexibility matrix was defined as

$$\mathbf{F} = \sum_{i=1}^n \mathbf{F}_i = \sum_{i=1}^n \frac{1}{\omega_i^2} \boldsymbol{\Phi}_i \boldsymbol{\Phi}_i^T \quad (2.11)$$

where ω_i and $\boldsymbol{\Phi}_i$ are the i -th natural frequency and mode shape.

The damage index was defined as the maximum absolute value in columns of difference of intact flexibility matrix and damaged one. As similar to mode shape method, this method can locate the damage.

A curvature version of flexibility matrix based method was proposed by Zhang & Aktan (1998). The idea was to take second derivative on the flexibility matrix, and thus definition of the damage index can be written as

$$\Delta = \sum_{i=1}^n |\mathbf{F}_i^{d''} - \mathbf{F}_i''| \quad (2.12)$$

where $\mathbf{F}_i^d$ is the i -th flexibility matrix in damage status.

Modal strain energy based method was proposed by Stubbs et al. (1992) and Stubbs & Kim (1996). The method used fractional modal strain energy change (MSEC) to identify damage, which was directly related to mode shape curvature for beam-like and plate-like structures. The damage index β at location j was defined by Eq. (2.13) as

$$\beta_j = \frac{\sum_{i=1}^m (F_{ij}^* + 1)}{\sum_{i=1}^m (F_{ij} + 1)} \quad (2.13)$$

where F_{ij} and F_{ij}^* are the fractional strain energy of intact and damaged beam for i -th mode at location j .

By assuming β as a normally distributed random variable, a normalized damage index (NDI) Z at location j was defined by Eq. (2.14) as

$$Z_j = \frac{\beta_j - \bar{\beta}}{\sigma_\beta} \quad (2.14)$$

where $\bar{\beta}$ and σ_β are the mean and the standard deviation of β , respectively. A typical damage criterion can be chosen as Z_j larger than 2.

This method was further developed to localize damage by Shi et al. (1998, 2000b), and extended to plate-like structures by Cornwell et al. (1999). Clearly, the above two types of methods both use information from mode shape, and thus, they have similar drawbacks to mode shape based methods.

Frequency response function (FRF) includes enriched information about the structures, and thus it is another important feature used in damage detection. Wang et al. (1997) proposed a FRF based method to compare experimental data with FEM results. Santos et al. (2005) extended the method by using the information from sensitivity of FRF. Sampaio et al. (1999) presented a method based on curvature of FRF, which was defined by Eq. (2.15) as

$$\alpha''(\omega)_{ij} = \frac{\alpha(\omega)_{i+1,j} - 2\alpha(\omega)_{ij} + \alpha(\omega)_{i-1,j}}{h^2} \quad (2.15)$$

where α_{ij} is the FRF measured at location i for a force input at location j .

Limongelli (2010) proposed a FRF based method by examining the difference between measured FRF and interpolated FRF. Then, Dilella et al. (2015) applied the method to damage localization of bridges.

For FRF based method, the input force and response should be available at the same time. Therefore, in comparison, methods requiring only output are more convenient.

2.2.4 Vibration based damage detection method using moving device

Dynamic analysis of structures with moving load has been studied extensively by many researchers. Fryba (1999) dedicated a whole book on moving load problem in various structures. Yang et al. (2004b) presented a detailed study on the vehicle-bridge interaction problem. Ouyang (2011) presented an easy-to-follow tutorial for several moving load dynamic problems. The moving load introduces many new features to the problem, thus, it is helpful to discuss the moving load problem before proposing method based on moving device. Because problem with only moving force or moving mass is not a *real* moving load problem (Fryba, 1999), a simple problem of transverse vibration of classic beam with moving force and mass is presented here.

Assuming the moving mass does not separate from the beam during moving, the governing equation of the experimental setup can be simplified as follows:

$$\rho A \frac{\partial^2 w}{\partial t^2} + EI \frac{\partial^4 w}{\partial x^4} = - \left[N + m \frac{\partial^2 v}{\partial t^2} \right] \delta(x - ut) \quad (2.16)$$

where ρ is the density, A is the cross-sectional area of the beam, EI is the bending stiffness, $w(x, t)$ is the transverse motion, N is the excitation force, $v(t)$ is the vertical displacement of the mass, m is the mass and u is the velocity of the mass along the beam direction.

Because of the non-separation assumption, $v(t)$ can be expressed by $u(x, t)$ as follows:

$$v(t) = w(ut, t) \quad (2.17)$$

Taking derivative on both sides of Eq. (2.17), it becomes

$$\frac{dv}{dt} = \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} \quad (2.18)$$

$$\frac{d^2v}{dt^2} = \frac{\partial^2 w}{\partial t^2} + 2u \frac{\partial w}{\partial t} \frac{\partial w}{\partial x} + u^2 \frac{\partial^2 w}{\partial x^2} \quad (2.19)$$

The above equations show that both velocity and acceleration captured contain time and spatial information of the beam. The velocity is the combination of velocity and rotational angle at the contact point. The acceleration is not only the combination of acceleration and curvature, but it is also the multiplication of velocity and rotational angle.

Substituting Eq. (2.19) into Eq. (2.16), the governing equation can be represented as

$$\rho A \frac{\partial^2 w}{\partial t^2} + m \left(\frac{d^2 w}{dt^2} + 2u \frac{dw}{dt} \frac{dw}{dx} + u^2 \frac{d^2 w}{dx^2} \right) \delta(x-ut) + EI \frac{\partial^4 w}{\partial x^4} = -N \delta(x-ut) \quad (2.20)$$

It is apparent from the above equation that the model has time-varying mass and stiffness. The equation can be solved by modal expansion method as presented by Fryba (1999) and Ouyang (2011). There are two important new phenomena introduced, i.e. critical velocity and combination resonances. Both phenomena introduce extra frequencies into the system. However, it should be noted that the moving load problem introduces many extra frequencies into the response, but the original frequencies for the static mass problem appear to be in the list of natural frequencies of moving load problem. Therefore, the acceleration captured may contains extra frequencies, but it should also include information in quasi-static case. However, processing moving load signal is much more difficult due to complex nature of the data captured, and it will be discussed in the latter texts.

As it is stated in frequency based damage detection methods, damage detection using moving device is a great leap for vibration based damage detection methods. Yang et al. (2004a) proposed to extract bridge frequency from signal captured from moving vehicle in 2004. It showed a balance between efficiency and accuracy. Ideally, the methods only require several pre-installed sensors on passing vehicle and it can provide spatial information within a single run through the structures. The potential of obtaining structural information cheaply and efficiently draws many other researchers' attention. Malekjafarian et al. (2015) provided a comprehensive review for bridge monitoring using this technique, which contains many related

research works. Bu et al. (2006) proposed an iterative process to identify mechanical properties from the signal. Deng & Cai (2009) suggested that the bridge parameters can be identified through an optimization process using Genetic Algorithm. González et al. (2012) also proposed to minimize error of estimated road profile to identify damping of bridge. Another damping identification method using the passing vehicle is presented by Keenahan et al. (2014), in which they presented a method involving subtraction of two axle accelerations to eliminate influence of the road profile. Kong et al. (2016, 2017) also proposed to subtract response from one sensor with a time shifted response from another sensor to eliminate driving-related frequencies and road profile influences. Then, the modal properties were extracted by FFT and STFT for damage detection. OBrien & Keenahan (2015) proposed to use a passing vehicle equipped with traffic speed deflectometer (TSD) to inspect the bridge by utilizing the cross entropy optimization algorithm. Kong et al. (2015) presented a transmissibility based method to detect damage in the bridge, in which two possible arrangements of the measurements were investigated. Moreover, mode shapes were also possible to be extracted from pass vehicles. Zhang et al. (2012) suggested the amplitude of STFT of the signal at different locations were related to mode shape square. Oshima et al. (2014) utilized SVD of the responses from several trailers to extract mode shape. OBrien & Malekjafarian (2016) proposed to use laser measurement from a passing vehicle to extract mode shape of a bridge for damage detection. Lately, Malekjafarian & OBrien (2017) combined ideas from Zhang et al. (2012) and Oshima et al. (2014) and utilized Hilbert Huang transform to estimate mode shapes of bridges. Another progress in this field is that Zhong & Oyadiji (2008) proposed to use an auxiliary mass to perturb the structure and utilize a special designed method used in rotatory machine diagnose, called spectral center correction method (SCCM), to investigate the change of frequency. The results show that it can localize the damage accurately. Yang & Oyadiji (2016, 2017a, 2017b) showed that under quasi-static condition a moving mass induced frequency shift can be utilized to detect delamination in beam-like and plate-like structures. Zhang et al. (2013b) studied the influence of auxiliary mass using modal superposition method and they showed that frequency shift is related to mode shape square, which means frequency based methods of moving signal may obtain similar accuracy and spatial resolution as mode shape based methods with the ease of only extracting frequency. Therefore, inspecting rail system with moving device is not only one of the best solutions from the viewpoint of unique feature of rail system, but it also provides more information in the signal.

However, it is also noted that previous methods based on moving device are more relied on amplitudes of the response. Because of the uncertainty of the rail system, the amplitude may be highly depended on the wheel-rail interaction. Therefore, methods using only amplitudes of response may be not enough for inspection of degraded supports. Frequencies are more resistant to noises and excitation power, and thus, they may be more suitable for the problem addressed here. Currently, the driven-by methods based on frequency are mostly quasi-static methods. Their performance under moving condition is not clear. However, as is stated before, a moving device based method is much more preferable for inspection of rail system. Therefore, a method that can be used under moving condition will be proposed in this study.

2.3 SUMMARY

In this chapter, relevant past research works closely related to this study are reviewed. Several observations can be made and they are as follows:

1. Most currently available inspection methods for supporting systems inherent mainly from ideas of visual inspection. The inspection speed can be very impressive, but the main limitation is that they are only effective for defects with clear visual difference. For loose fasteners and under surface degradation, these methods are not applicable.
2. Based on ultrasonic inspection methods, loose bolts may cause changes of wave velocity, inter-facial properties, such as contact stiffness, damping and real contact area. Moreover, dynamic properties such as frequency, FRF and harmonic response are also related, and they can possibly be applied for inspection. Accordingly, loose fasteners and degradation in sleepers or blasters can all be considered as a stiffness reduction inspection problem.
3. Recently developed in-direct damage detection methods based on moving devices for bridges show great potential. They have been successfully used in extraction of natural frequencies, mode shapes and local stiffness information, and thus, they are possible to be extended for inspection of supports in rail systems. Moreover, the advantage of using several sensors on a moving device is very attractive for an inspection method to be used on rail systems.

CHAPTER 3

DESCRIPTION OF PROBLEM AND PRINCIPLES OF PROPOSED METHOD

3.1 DYNAMIC MODEL

Inspection method based on dynamic properties is proposed in this study to solve the degraded supports. The problem is considered as an infinite beam with moving load. However, excitation of infinite beam is impossible and local excitation may provide more local information (Lovell & Pines, 1998). Therefore, a limited segment of the beam is analyzed instead of analyzing the entire infinite beam.

The two types of the easiest and cheapest types of excitation approaches are environmental excitation and impact excitation. Environmental excitation is more friendly; it requires less equipment, but the input signal and signal to noise ratio (SNR) are difficult to control. Therefore, this study focuses on impact excitation which contains a wide range of frequency components and which can be easily controlled. One can now simplify the problem as a finite beam and moving device with impact excitation. Assuming the inspection device does not separate from the beam during moving, the governing equation of the experimental setup can be written as

$$\rho A \frac{\partial^2 w}{\partial t^2} + C \frac{\partial w}{\partial t} + EI \frac{\partial^4 w}{\partial x^4} = - \left[f(t) + m \frac{\partial^2 v}{\partial t^2} \right] \delta(x - ut) \quad (3.1)$$

where ρ is the density, A is the cross-sectional area of the beam, EI is the bending stiffness, C is the damping constant per unit length, $w(x, t)$ is the transverse motion, $f(t)$ is the excitation force, $v(t)$ is the vertical displacement of the inspection device, m is the mass of the inspection device and u is the velocity of the inspection device along the beam direction.

Due to assumption of non-separation, $v(t)$ can be expressed by $u(x, t)$ as shown in Eq. (2.17). While the velocity and acceleration can be expressed as shown in Eq. (2.18) and Eq. (2.19), respectively. It is shown in these equations that both velocity and acceleration contain the time and spatial information about the beam.

Substituting Eq. (2.19) into Eq. (3.1), the governing equation can be represented as

$$\begin{aligned} \rho A \frac{\partial^2 w}{\partial t^2} + m \left(\frac{d^2 w}{dt^2} + 2u \frac{dw}{dt} \frac{dw}{dx} + u^2 \frac{d^2 w}{dx^2} \right) \delta(x-ut) + \\ C \frac{\partial w}{\partial t} + EI \frac{\partial^4 w}{\partial x^4} = -f(t) \delta(x-ut) \end{aligned} \quad (3.2)$$

It is apparent that the model has time-varying mass, damping and stiffness. In most cases, the equation does not have a close form solution. According to previous studies (Fryba, 1999; Yang et al., 2004b), this kind of problem has many extra frequencies caused by combination resonances in the response. Fortunately, the original frequencies for the static moving mass problem are in the list of natural frequencies. Therefore, if the extra frequencies are not significant, then the problem can be simplified as a quasi-static moving mass problem.

3.2 ANALYSIS OF QUASI-STATIC MOVING MASS PROBLEM

The analysis of quasi-static moving mass on a beam with elastic supports has been studied for a long time. The frequency analysis of beam with multiple elastic supports was presented by Kukla (1991) and the problem with multiple mass was studied by Kukla & Posiadala (1994) and Low (2001, 2003). The frequency sensitivity of the beam with mass was studied by using finite element method by Wang (2012). A general approach for beam-mass interactive system and elastic supports were presented by Cha (2005). In the past decades, there are many other works related to frequency analysis of interactive system consisting of a beam with several attachments. The analytical approaches used to solving this problem included assumed-modes method (Cha, 2005, 2007), the Green's function method (Hozhabrossadati et al., 2015; Kukla, 1994; Kukla & Posiadala, 1994), the transfer function method (Hosking et al., 2004) and method based on integral transform (Low, 2003; Rezaiee-Pajand & Hozhabrossadati, 2014). However, most of them considered only the beam with the same type of attachments or different types of attachments but at the same location. The general solutions proposed for this problem do not provide explicit equations to calculate the frequencies. Therefore, a direct method similar to that in Low (2003) and Rezaiee-Pajand & Hozhabrossadati (2014) is utilized to study the problem of a beam with an auxiliary mass and elastic supports.

3.2.1 Dynamic model of quasi-static moving mass problem

The dynamic model is shown in Fig. 3.1. The rail is simplified as a Euler beam; the supports are modeled as springs at location $x_{s,f}$ ($f = 1, \dots, n-1$); the sensor is modeled as a mass mounted at location x_m . The degradation of support is taken as stiffness reduction of the elastic support. For different sensor location x_m , different governing equations are formed. For the case $x_{s,i-1} < x_m < x_{s,i}$, the beam is split into $n+1$ segments, and the two boundaries are denoted as spring 0 and n . The first $i-1$ segments are between the spring $j-1$ to j ($j = 1, \dots, i-1$); the $j = i^-$ segment is between the spring $i-1$ to the sensor; the $j = i^+$ segment is between the sensor and the spring i ; and the rest segments are between the spring $j-1$ to j ($j = i+1, \dots, n$). For each segment $j = 1, \dots, i-1, i^-, i^+, i+1, \dots, n$, the governing equation can be written as

$$EI \frac{\partial^4 w_j}{\partial x^4} + \rho A \frac{\partial^2 w_j}{\partial t^2} = 0 \quad (3.3)$$

where w_j is the transverse displacement of segment j .

Fig. 3.1 Mechanical model of the problem

The compatibility conditions at each spring location can be written as

$$\begin{aligned} w_l(x, t) &= w_r(x, t) \\ w_l'(x, t) &= w_r'(x, t) \\ w_l''(x, t) &= w_r''(x, t) \\ EI w_l'''(x, t) &= EI w_r'''(x, t) + k_i w_l(x_{s,i}, t) \end{aligned} \quad (3.4)$$

where k_i is the stiffness of spring i ; w_l and w_r are the left and right segment at location i respectively as shown in Fig. 3.2(a); w' , w'' , w''' are the first, second and third partial derivatives with respect to x respectively.

Fig. 3.2 w_l and w_r in the compatibility conditions (a) for spring (b) for mass

For the sensor location, the compatibility conditions can be written as

$$\begin{aligned}
 w_l(x, t) &= w_r(x, t) \\
 w'_l(x, t) &= w'_r(x, t) \\
 w''_l(x, t) &= w''_r(x, t) \\
 EIw'''_l(x, t) &= EIw'''_r(x, t) - M\ddot{w}_l(x_m, t)
 \end{aligned} \tag{3.5}$$

where M is the mass of the sensor; w_l and w_r are the left and right segment of the mass as shown in Fig. 3.2(b) and $\ddot{w}$ is second partial derivative with respect to t .

3.2.2 Dimensionless equations

The governing equation can be transform to dimensionless form as shown in following equations

$$\frac{\partial^4 \bar{w}_j(\chi, \tau)}{\partial \chi^4} + \frac{\partial^2 \bar{w}_j(\chi, \tau)}{\partial \tau^2} = 0 \tag{3.6}$$

where Δ is the distance between two adjacent springs, $\chi = \frac{x}{\Delta}$, $\omega_b = \frac{1}{\Delta^2} \sqrt{\frac{EI}{\rho A}}$, $\tau = t\omega_b$,

$$\bar{w}_j = \frac{w_j}{\Delta}.$$

Similarly, for each spring, the compatibility conditions are written as

$$\begin{aligned}
\bar{w}_l(\chi, \tau) &= \bar{w}_r(\chi, \tau) \\
\bar{w}_l'(\chi, \tau) &= \bar{w}_r'(\chi, \tau) \\
\bar{w}_l''(\chi, \tau) &= \bar{w}_r''(\chi, \tau) \\
\bar{w}_l'''(\chi, \tau) &= \bar{w}_r'''(\chi, \tau) + \lambda_{s,i} \bar{w}_l(\chi_{s,i}, \tau)
\end{aligned} \tag{3.7}$$

where $\bar{w}_l = \frac{w_l}{\Delta}$, $\bar{w}_r = \frac{w_r}{\Delta}$, $\lambda_{s,i} = \frac{k_i \Delta^3}{EI}$, $\chi_{s,i} = \frac{x_{s,i}}{\Delta}$.

For the sensor, the compatibility conditions are written as:

$$\begin{aligned}
\bar{w}_l(\chi, \tau) &= \bar{w}_r(\chi, \tau) \\
\bar{w}_l'(\chi, \tau) &= \bar{w}_r'(\chi, \tau) \\
\bar{w}_l''(\chi, \tau) &= \bar{w}_r''(\chi, \tau) \\
\bar{w}_l'''(\chi, \tau) &= \bar{w}_r'''(\chi, \tau) - \lambda_m \ddot{\bar{w}}_l(\chi_m, \tau)
\end{aligned} \tag{3.8}$$

where $\lambda_m = \frac{M\omega_b^2 \Delta^3}{EI} = \frac{m}{\rho A \Delta}$, $\chi_m = \frac{x_m}{\Delta}$.

By writing the above equations to a dimensionless form, the number free variables of the model reduced considerably. Considering that supports are distributed evenly along the rail in practice, the spatial related parameters such as interval between two sleepers are determined by the design of the system. Therefore, the key parameters in the equations reduce to two, i.e. the mass parameter λ_m which is the ratio between the sensor mass and total mass of the beam, and the stiffness parameter λ_s which is stiffness ratio between the supports and the beam. Because mass parameter can be controlled by adjusting the auxiliary mass, the most concerned parameter for the problem is the stiffness parameter for supports with different health status.

3.2.3 Exact solution

If the stiffness of the supports is linear, the governing equation of the system can be written by Eq. 3.8, which has a well-known form of general solution as shown below

$$\bar{w}_j(\chi, \tau) = W_j(\chi)T(\tau) = W_j(\chi)(D_1 \sin(\omega\tau) + D_2 \cos(\omega\tau)) \tag{3.9}$$

where the coefficients D_1 and D_2 can be determined by the initial conditions, and they are kept identical for all the different segments.

Therefore, the problem is simplified to determining the function $W_j(\chi)$. Substituting Eq. (3.9) into Eq. (3.6), one can obtain

$$\frac{d^4 W_j}{dx^4} - \beta^4 W_j(\chi) = 0 \quad (3.10)$$

where $\beta = \sqrt{\omega}$.

The general solution of Eq. (3.10) can be written as follows

$$W_j(\chi) = C_{1,j} \sin(\beta\chi) + C_{2,j} \cos(\beta\chi) + C_{3,j} \sinh(\beta\chi) + C_{4,j} \cosh(\beta\chi) \quad (3.11)$$

where $C_{m,j}$ ($m = 1, \dots, 4$) are the coefficients for j -th segment.

In total, there are $4(n+1)$ unknown variables. Substituting the compatibility conditions of Eq. (3.7) and Eq. (3.8) into Eq. (3.11), one can obtain $4(n-1)$ independent equations for $n-1$ springs and 4 independent equations for sensor. While, substituting boundary conditions into Eq. (3.11), there are 4 independent equations. Therefore, there are $4(n+1)$ unknown variables and $4(n+1)$ independent equations. The problem can be written in matrix form as follows

$$\mathbf{M}\mathbf{c} = 0 \quad (3.12)$$

where $\mathbf{M}$ is the coefficients matrix and $\mathbf{c}$ is the vector formed by unknown variables.

In order to obtain non-trivial solutions from Eq. (3.12), the determinant of the matrix $\mathbf{M}$ should be zero, i.e.,

$$\det \mathbf{M} = 0 \quad (3.13)$$

Eq. (3.13) is the frequency equation of the model. By solving Eq. (3.13), one can obtain the natural frequencies of the system. Unfortunately, it is almost impossible to obtain a closed form solution for Eq. (3.13), either for general case, or model with several springs. Fortunately, it is shown in the later text that it is not hard to obtain a numerical solution of Eq. (3.13).

3.2.4 Influence of one degraded support

Because of the property of determinant, even for model with less than 10 springs, the frequency equation contains thousands of terms. Theoretically, the above method can solve beam with arbitrary number of springs, however, the computational costs will increase exponentially with the number of springs. In addition, degradation of one spring is a local change to the system, and thus, it should not have a strong global effect on the properties of the whole system. Therefore, it is crucial to know the difference between models with different length and the influence of one degraded support.

3.2.4.1 Differences between different length models

Three different models (Fig. 3.3) are analyzed by solving frequency equation Eq. (3.13) numerically. The stiffness parameter λ_s is set to 1600 for all the springs, and mass parameter λ_s is set to 1/5. Because the aim is to propose a moving device for inspection, the relationship between frequency and location of the sensor is first studied. The solutions are plotted in Fig. 3.4 where the results are aligned according to the middle spring. Because of the periodic pattern in the model, each frequency is split into five frequencies in a small frequency range. It is confusing to plot all of the frequency out, and thus, only the lowest frequency in each frequency group is plotted in the figure. This modification is also made on the rest figures in this section. Since the moving mass changes the model, the frequency of the system changes with respect to the location of the sensor. This change has a periodic pattern as the model. Another observation is that the model with three springs produce very close frequency results as model with five springs, except for the region near the boundaries. Model with only one spring also produces reasonably close frequency, the boundary conditions cause the differences. This may suggest that this problem can be studied with a model with only a few springs.

Fig. 3.3 Illustration of models with different number of springs

Fig. 3.4 Frequency of three models with different lengths

Then, the stiffness parameter λ_s of the middle fastener is reduced to 800, and all the other parameters are unchanged. The results are shown in Fig. 3.5. The reduction of the stiffness breaks the periodic pattern, which suggests stiffness reduction can be identified by the broken of the periodic pattern. Moreover, results from the three springs model returns almost the same frequencies as the five springs model, even in the region where the stiffness is reduced. Model with only one spring also produces similar pattern as the other two models.

Fig. 3.5 Frequency of three models with different lengths

Although results of a model with only one spring are affected slightly by the boundary conditions, it eliminates the stiffness parameters of the other intact springs which simplifies the problem significantly. The capacity of one-spring model in capturing the frequency shift caused by stiffness reduction is further demonstrated in Fig. 3.6 where the stiffness parameter of the middle spring is reduced from 1600 to 16 continuously. The frequency shown in Fig. 3.6 is when the mass is located on the top the middle spring. The frequency shift pattern from one spring model is quite similar to the other two models. Similar results are also obtained from models whose intact stiffness parameter is 8000, and stiffness parameter of the middle spring is reduced from 8000 to 80. This may suggest that the stiffness parameters of the intact springs do not influence the frequency shift significantly, and can be neglected from the analysis. Hence, the stiffness parameter of the degraded support and the mass parameter are the two critical dimensionless parameters in the problem.

Fig. 3.6 Relationship between middle spring stiffness and frequency from three models (stiffness parameter of intact springs is 1600)

Fig. 3.7 Relationship between middle spring stiffness and frequency from three models (stiffness parameter of intact springs is 8000)

3.2.4.2 Influence range of one degraded support

As discussed, the number of springs in a model have a limited influence on the frequency even when one spring has a large stiffness reduction. Another important question is the scale of a degraded support influencing the frequency shift caused by the mass.

A simply supported model with five supports shown in Fig. 3.8 are studied with three different levels of stiffness parameters. The chosen parameters are listed in Table 3.1. The stiffness of k_3 is changed to reduction level 1 and level 2 as listed in the table.

Table 3.1 Parameters of numerical model for determining minimum length

	Case 1	Case 2	Case 3
λ_s of intact fastener	8000	1600	160
λ_s reduction 1	4000	800	80
λ_s reduction 2	2000	400	40
λ_m of the sensor	1/5	1/5	1/5

Fig. 3.8 Illustration of a simply supported model with five fasteners

The frequency obtained from the mass at different locations with parameters of case 1 are shown Fig. 3.9. The stiffness of k_3 at location $\chi = 3$ is reduced to 4000 and 2000. It is shown clearly from the figure that the reduction of the stiffness mainly causes frequency decrease when the mass is in the range of $2 < \chi < 4$, which is between k_2 and k_4 as shown in Fig. 3.8. The frequency decrease outside this range is much smaller and uniform. There are five springs in this model, the frequency decrease due to stiffness change is mainly observable when the mass is in the nearest span, i.e. the span where the stiffness reduction is located. The frequencies in other regions are almost unaffected. It should also be noted that frequency decrease mostly is

concentrated in the frequency range of $8 < \beta < 11$. The other frequencies seem not affected by the reduction of stiffness at $\chi = 3$.

Fig. 3.9 Frequency at different locations for case 1 model

The frequency obtained from case 2 and case 3 are shown in Fig. 3.10 and Fig. 3.11, respectively. The frequency decrease is also concentrated in range $2 < \chi < 4$. Therefore, stiffness reduction of one spring is a local change; and the influence is highly restrained in a limited range beside a uniform frequency decrease of the whole system. This means that the frequency decrease when the mass is located at different locations can localize the stiffness reduction accurately. All this results show that the degraded support is a very local change to the system when evaluated by the frequency shift curve caused by the moving mass. The frequency shift is much larger when the mass is in the nearest spans of the system. Therefore, the frequency shift curve can be used to localize the degraded supports with high accuracy.

Fig. 3.10 Frequency at different locations for case 2 model

Fig. 3.11 Frequency at different locations for case 3 model

However, unlike case 1, the frequency decrease is concentrated at different frequency ranges, i.e., $5 < \beta < 7$ for case 2, and $2 < \beta < 4$ for case 3. This suggests that for different parameters, different frequency range should be monitored. Summarizing the result shown in

Fig. 3.6, Fig. 3.7 and this sub-section, it is found that the fundamental frequency group is sensitive to the stiffness change when stiffness parameter is lower than 200; the second frequency group is sensitive to the change when stiffness parameter is between 200 to 1600. For models with stiffness parameter higher than 2000, higher mode groups should be considered.

3.2.4.3 Influence of sensor mass to the frequency

The moving mass alters the system slightly, and thus, the frequencies of the system shift. Therefore, the mass parameter is one of the key parameters. If the mass is infinitely small, the system remains the same when the mass is moving. Consequently, the frequencies of the system stay the same. If the mass is neglected, the frequencies are global values and irrelevant to the location of the sensor. On the contrary, if the mass is much larger than the beam, the beam acts as a spring and the frequency is determined by the mass.

Fig. 3.12 and Fig. 3.13 show the relationship between sensor location and the frequencies when the mass parameter of model with case 2 (Fig. 3.10) is changed to 1/50 and 1. Comparing these three figures, it can be observed that generally the frequencies shift in a similar pattern, although large mass parameter alters the shape a bit. However, the amplitude of the frequency shift varies clearly as the change of mass parameter. As the mass parameter increases, the frequencies change from a horizontal-line-like shape into a curve with distinct fluctuation. Therefore, the main influence of the mass parameter is the amplitude of the frequency shift, and this can be adjusted by attaching different auxiliary mass in practice.

Fig. 3.12 Frequencies of a simply supported model with mass parameter 1/50

Fig. 3.13 Frequencies of a simply supported model with mass parameter 1

3.2.5 Summary of findings

Based on numerical solutions of the problem presented in above, several observations can be found:

1. Frequency shift is observed when the mass is moving through the model, and the magnitude of frequency shift is mainly determined by the auxiliary mass;
2. Frequency shift is smaller when the mass is near the springs and nodal points of the specific mode;
3. If all springs have same stiffness, frequency shift curve has a periodic pattern. If stiffness of one spring reduces, the frequency shift curve also changes. The change is more obvious when the mass is in the nearest span where the spring with lower stiffness is. This indicates that the frequency shift can be used to identify the fastener with lower stiffness accurately.
4. The frequency shift is more obvious at a certain frequency range mainly determined by the stiffness parameter.

In conclusion and based on above discussions, frequency shift curve caused by a moving mass is a good candidate to identify the degraded supports accurately, and in this study, this principle is developed into a workable solution for the problem.

3.3 MODAL SUPERPOSITION SOLUTION OF THE PROBLEM

In the last section, the governing equation of static moving mass problem is solved directly. The frequency equation is difficult to solve analytically, and thus, it is solved numerically. In this section, modal superposition method is applied to obtain an approximate solution for the problem. The benefit of this method is that the solution is a closed form formula, and it is easier to interpolate.

The degradation of one support is a local change to the whole system, and in fact, in last section, the influence range of one degraded support is shown to be mainly restricted in the nearest span. Therefore, a model shown in Fig. 3.1 is a reasonable simplification of the problem. There are $n-1$ fasteners in the model. Neglecting damping and external force, the governing equation Eq. (3.1) can be rewritten as

$$\begin{aligned} \rho A \frac{\partial^2 w}{\partial t^2} + EI \frac{\partial^4 w}{\partial x^4} + \sum_{i=1}^{n-1} k_i w(x_i, t) \delta(x - x_i) \\ = -m \frac{d^2 v}{dt^2} \delta(x - ut) \end{aligned} \quad (3.14)$$

Next, modal superposition method is used to solve the problem. The displacement w can be written as

$$w(x, t) = \sum_j \varphi_j(x) q_j(t) \quad (3.15)$$

where φ_j is the j -th mode shape of the beam and $q_j(t)$ is the j -th modal coordinate.

Substituting Eq. (3.15) into Eq. (3.14), multiplying each side with φ_m and integrating over whole beam, the following is obtained

$$\begin{aligned} \rho A \int_0^l \varphi_m(x) \frac{\partial^2 \sum_j \varphi_j(x) q_j(t)}{\partial t^2} dx + EI \int_0^l \varphi_m(x) \frac{\partial^4 \sum_j \varphi_j(x) q_j(t)}{\partial x^4} dx \\ + \int_0^l \varphi_m(x) \sum_{i=1}^{n-1} k_i \sum_j \varphi_j(x_i) q_j(t) \delta(x - x_i) dx \\ = - \int_0^l m \frac{d^2 \sum_j \varphi_j(ut) q_j(t)}{dt^2} \varphi_m(x) \delta(x - ut) dx \end{aligned} \quad (3.16)$$

Due to orthogonal properties of the mode shape, the terms in Eq. (3.16) can be simplified as

$$\int_0^l \varphi_m(x) \frac{\partial^2 \sum_j \varphi_j(x) q_j(t)}{\partial t^2} dx = \ddot{q}_m(t) \Phi_m \quad (3.17)$$

$$\int_0^l \varphi_m(x) \frac{\partial^4 \sum_j \varphi_j(x) q_j(t)}{\partial x^4} dx = \Phi_m \Psi_m q_j(t) \quad (3.18)$$

$$\int_0^l \varphi_m(x) \sum_{i=1}^{n-1} k_i \sum_j \varphi_j(x_i) q_j(t) \delta(x - x_i) dx = \sum_{i=1}^{n-1} k_i \varphi_m(x_i) \sum_j \varphi_j(x_i) q_j(t) \quad (3.19)$$

$$\int_0^l m \frac{d^2 \sum_j \varphi_j(ut) q_j(t)}{dt^2} \varphi_m(x) \delta(x - ut) dx = m \varphi_m(ut) \sum_j \varphi_j(ut) \frac{d^2 q_j(t)}{dt^2} \quad (3.20)$$

where $\Phi_m = \int_0^l \varphi_m(x)^2 dx$ is an intergral constant, $\Psi_m = \frac{\rho A \omega_{bm}^2}{EI}$ is the square of the m th mode, and ω_{bm} is the m -th natural frequency of the beam.

Therefore, Eq. (3.16) can be simplified as

$$\begin{aligned} \rho A \ddot{q}_m(t) \Phi_m + EI \Phi_m \Psi_m q_j(t) + \sum_{i=1}^{n-1} k_i \varphi_m(x_i) \sum_j \varphi_j(x_i) q_j(t) \\ = -m \varphi_m(ut) \sum_j \varphi_j(ut) \frac{d^2 q_j(t)}{dt^2} \end{aligned} \quad (3.21)$$

Eq (3.21) is a second order ordinary differential equation of the m -th order modal coordinate, and thus the m -th order natural frequency can be approximated as

$$\omega_m^2 = \frac{\omega_{bm}^2 + \frac{\sum_{i=1}^{n-1} k_i \varphi_m^2(x_i)}{\rho A \Phi_m}}{1 + \frac{m \varphi_m^2(ut)}{\rho A \Phi_m}} \quad (3.22)$$

Noted that numerator of above equation is a constant, and thus the square of the m -th order natural frequency is directly related to the mode shape at the moving mass location. Therefore, the frequency shift curve may offer similar information as mode shape in inspection of structures. This is the most critical finding from this modal superposition solution.

3.4 SUMMARY

Based on literature review presented in the last chapter, dynamic model for degraded supports inspection in rail system is proposed in this chapter. The problem is modeled as a beam supported by discrete springs. Both the degradation of material in sleepers or blasters and the loosening of fasteners are modeled as stiffness reduction of the spring underneath the beam. Then, the moving load problem is simplified to a quasi-static moving mass problem. It is solving by two analytical approaches which give two different angles of the problem. The solution from frequency equation of the system shows that the certain modes of frequency shift curves caused by auxiliary mass is sensitive and able to accurately localize stiffness reduction of the springs. The solution from model superposition method provides physical meaning to the frequency shift curves, which relates mode shapes with them. Mode shapes are found to be

effective in inspecting stiffness changes. Therefore, both solutions provide theoretical insights of using frequency shift curves to inspect degradation of supporting system. The findings in this chapter provide the basic principles to design an inspection device for supporting inspection system in rail systems. Transferring these principles into a workable solution requires a great deal of effort and this is presented in following chapters.

CHAPTER 4

INSPECTION OF STIFFNESS REDUCTION BY FREQUENCY SHIFT UNDER QUASI-STATIC CONDITION

4.1 BACKGROUND OF NATURAL FREQUENCIES EXTRACTION

There are various methods to extract natural frequencies from acceleration signal. They can be roughly divided into two categories, i.e. parametric and non-parametric methods. If the model of the signal is clear, parametric methods such as auto-regression (AR) model based method and multiple signal classification (MUSIC) method, can produce more accurate results even under noisy condition. However, it is discussed in the last chapter, the captured signal is not only exposed to environmental noise, it also contains extra-frequencies from the moving load. Therefore, the model of the signal is not clear, and therefore, non-parametric methods are preferred in this study. The most commonly used non-parametric method is to convert the signal into frequency domain by Fourier transform and locate the peaks in the spectrum. Finding the peak is a common task in many scientific studies. The definition of peak is visually clear for human eyes, but it is difficult to describe in mathematical ways. A common method is first finding all the local maximums and then choosing them by using some criteria, such as amplitude or width of the maximum, gap or ratio between two maximum and signal to noise ratio (SNR).

4.2 EXTRACTION OF FREQUENCY SHIFT CURVE

A well-known fact is that although natural frequencies are the eigenvalues of the characteristic equations, not all frequencies can be obtained at all locations of the structure. This is because there is no response in a certain mode when excitation is located at a nodal point of that mode. In fact, the signal to noise ratio (SNR) drops rapidly when the excitation is near the nodal points. Therefore, the response level at the frequency, i.e. the amplitude of the spectrum, controls whether the frequency can be extracted successfully.

The amplitude of the signal in certain range of frequency contains various information about the structure. PMS (Fang & Perera, 2009) is the amplitude of spectrum under specific frequency band, and it is related to the mode shape. They proposed to use PMS as a feature to detect

damage in linear structures. Therefore, the amplitude of signal not only determines whether the frequency can be extracted, but also it contains valuable information about the structure. Based on modal superposition solution presented in Eq. (3.22), the frequency shift curve also contains information related to mode shape. Therefore, it is natural to combine the idea of PMS and frequency shift in Chapter 3, and propose to use the space curve whose projection on time-frequency plane is frequency shift curve and projection on time-amplitude plane is roughly PMS for damage detection. This space curve is called a frequency shift (FRESH) path in this study. The flowchart to extract FRESH path is shown in Fig. 4.1. It can be noticed that some new terminologies are presented in it. This is because traditional discrete Fourier transform (DFT) cannot provide sufficient precision for extraction of FRESH path. The detailed procedure on the signal processing is presented in Section 4.3. At this point, discrete time Fourier transform (DTFT) can be treated as one special method which can extract frequency and amplitude with high accuracy.

The FRESH path can be represented as

$$\text{FRESH}_{ij} = F_{ij} = (x_j, f_i(x_j), \psi_i(x_j)) \quad (4.1)$$

in which x_j is the j -th point where the auxiliary mass is located, f_i is i -th frequency extracted by DTFT and ψ_i is the corresponding amplitude from DTFT.

Fig. 4.1 Flow chart on procedure of calculating FRESH path and FRESH curvature

The benefits of FRESH curve are obvious. Since FRESH path contains information from frequency shift curve and PMS, the sensitivity to local parameter change is higher. Moreover, for general cases, sensitivity of frequency shift and sensitivity of amplitude change are not the

same. In some cases, one's sensitivity may be higher than the other. However, in either case, FRESH path can provide a reasonable sensitivity for damage detection.

4.2.1 FRESH curvature

As it is reviewed in Chapter 2, mode shape curvature is a general damage index used for damage detection. Because FRESH path contains information likes mode shape, one would like to apply the curvature strategy on the space curve, i.e. FRESH path. The FRESH path can be written in parametrical format, and the FRESH curvature can be defined as follows

$$\begin{aligned}
 FC_f(x_j) &= f_i(x_{j+1}) - 2f_i(x_j) + f_i(x_{j-1}) \\
 FC_a(x_j) &= \psi_i(x_{j+1}) - 2\psi_i(x_j) + \psi_i(x_{j-1}) \\
 FC &= w_1 |FC_f - \min(FC_f)| + w_2 |FC_a - \max(FC_a)| \\
 FC_{ang} &= \arctan\left(\frac{FC_f}{FC_a}\right) = \arctan\left(\frac{f_i(x_{j+1}) - 2f_i(x_j) + f_i(x_{j-1}))}{\psi_i(x_{j+1}) - 2\psi_i(x_j) + \psi_i(x_{j-1}))}\right)
 \end{aligned} \tag{4.2}$$

where FC_f is the frequency shift component in FRESH path, FC_a is the amplitude shift component in FRESH path, FC is the FRESH curvature and FC_{ang} is the angle representing sensitivity of frequency and amplitude, and w_1 and w_2 are weight factors.

4.2.2 Weighting method for inspection

Ideally, the weighting factors in Eq. (4.2) should be determined by the precision of frequency and amplitude component. It is known that the precision near the nodal points are lower, but the quantitative ratio between these precisions is difficult to determine in practice. One possible strategy which is used is to assume that most intact structures have smooth FRESH paths, and thus, the fluctuation level in the FRESH path can represent the quality of frequency or amplitude measurement. Consequently, a piecewise polynomial extrapolation based method is introduced here.

In this subsection, the frequency or amplitude component of FRESH path is denoted as y_i , which can be replaced by either component. The spatial coordinates are noted as x_i , which has m points in total. Then the n -th order least square (LS) fitting is the LS solutions of linear equations are given as follows

$$\mathbf{Z}\mathbf{A} = \begin{bmatrix} x_1^n & \cdots & 1 \\ \vdots & \ddots & \vdots \\ x_m^n & \cdots & 1 \end{bmatrix} \begin{bmatrix} a_0 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix} = \mathbf{Y} \quad (4.3)$$

where a_i is the coefficients of LS fitted polynomial, and m is the number of sensor points.

The method to solve the over-constrained equations can be found in many linear algebra texts. The key step is to multiply the transpose of $\mathbf{Z}$ to both sides of the equations. After the LS solutions are obtained, the value of other points can be extrapolated. Due to the instability of extrapolation, only points near to the given points are accurate. When $n=m-1$, $\mathbf{Z}$ becomes a square matrix. Thus, the equations can be solved directly, which is equivalent to polynomial interpolation.

At each point, which has previous m points, the extrapolation value is obtained by fitting previous m points in n -th order polynomial. The smoothness of the curve can be measured by the difference between extrapolation value and the measured value. Consequently, the norm of the residue in Eq. (4.4) is a reasonable index for measuring the smoothness, i.e.,

$$\text{norm}(\mathbf{R}) = \|\mathbf{A}^T \mathbf{x}_{m+1} - y_{m+1}\| \quad (4.4)$$

where $\mathbf{x}_{m+1} = [x_{m+1}^n, \dots, x_{m+1}^1, \dots, 1]^T$.

Moreover, the peak to peak value (PPV) of frequency or amplitude is another factor which should be counted. Therefore, the extrapolation residue ratio (ERR) to determine the ratio of weight factors w_1 / w_2 is defined by

$$ERR = \frac{PPV_{\text{freq}} / \text{norm}(\mathbf{R}_{\text{freq}})}{PPV_{\text{amp}} / \text{norm}(\mathbf{R}_{\text{amp}})} \quad (4.5)$$

Specially, the weight factors can be set by the following equation

$$w_i = PPV_i / \text{norm}(\mathbf{R}_i) \quad (4.6)$$

4.3 PROCEDURE TO EXTRACT FRESH PATH

Spatial resolution of a damage detection method controls its accuracy in locating the damage, and a frequency base method requires a high frequency resolution for its accuracy. Therefore, the above proposed method requires reasonably high accuracy for both spatial and frequency resolution. Fast Fourier transform (FFT) is well-known for its low time complexity, and it is the most frequently used technology to transfer time domain data into frequency domain. However, the frequency resolution of FFT is relatively low, which may introduce a maximum frequency estimation error of $\pm 0.5\Delta f$, in which Δf is the frequency resolution of FFT. Frequency resolution of DFT is related to the number of points in the transformation. High frequency resolution requires more transform data, which will increase the time complexity and memory cost of FFT. Another major limitation of FFT is the leakage effect, which causes the maximum estimate error of amplitude even worse. A useful strategy to solve leakage effect problem is by adding window to the signal.

However, if highly accurate frequency or amplitude is required, this strategy still cannot solve the problem completely. Thus, the basic formula of Fourier transform is revisited, and it is shown that it is possible to develop a procedure to extract frequencies of a signal with high degree of accuracy but with the same time complexity.

4.3.1 Relationship of CTFT and DTFT of a signal

The very first Fourier Transform is Continuous Time Fourier Transform (CTFT), which is defined by Eq. (4.7) as

$$X(f) = \int_{-\infty}^{+\infty} x(t)e^{-j2\pi ft} dt \quad (4.7)$$

where $x(t)$ is the input signal.

Normally, CTFT converts a continuous time domain data $x(t)$ into a continuous frequency domain data $X(f)$, and there is an inverse transform to reconstruct frequency domain data back into the time domain.

When input signal of CTFT is a discrete sequence, integral in CTFT reduces into summation and the summation is called DTFT in Eq. (4.8), i.e.,

$$X_D(\hat{f}) = \sum_{n=-\infty}^{+\infty} x[n]e^{-j2\pi n\hat{f}} \quad (4.8)$$

where $x[n]$ is discrete time sequence, $\hat{f}$ is a dimensionless frequency variable.

It is shown clearly in Eq. (4.8) that DTFT converts a discrete sequence into a continuous curve in frequency domain. Then, it is interesting to know how the continuous curve $X_D(\hat{f})$ is related to the frequency representation $X(f)$ of CTFT, if $x[n]$ is a sequence sampled from a continuous time signal $x(t)$ at an equal interval of T .

The sampled time domain signal $s(t)$ can be written by Eq. (4.9) as

$$s(t) = \sum_{n=-\infty}^{+\infty} x[n]\delta(t-nT) = x(t) \sum_{n=-\infty}^{+\infty} \delta(t-nT) \quad (4.9)$$

By using convolution rule of CTFT, CTFT of $s(t)$ can be obtained as

$$S(f) = X(f) * F\left(\sum_{n=-\infty}^{+\infty} \delta(t-nT)\right) \quad (4.10)$$

where F is CTFT of a signal and $*$ is convolution operation.

Note that CTFT of summation in Eq. (4.10) can be simplified as

$$\begin{aligned} F\left(\sum_{n=-\infty}^{+\infty} \delta(t-nT)\right) &= \frac{1}{T} F\left(\sum_{n=-\infty}^{+\infty} \delta\left(\frac{t}{T}-n\right)\right) \\ &= \frac{1}{T} \sum_{n=-\infty}^{+\infty} \delta\left(f-\frac{n}{T}\right) \end{aligned} \quad (4.11)$$

Thus, CTFT of $s(t)$ can be written as

$$S(f) = \frac{1}{T} \sum_{n=-\infty}^{+\infty} X\left(f-\frac{n}{T}\right) \quad (4.12)$$

As shown in Eq. (4.12), CTFT of $s(t)$ is an infinite summation of $X(f)$ with shift of n/T . However, if time domain signal band is limited in $|f| < 1/2T$, the terms in summation are not

overlapped. Thus, when $|f| < 1/2T$ the CTFT of signal $x(t)$ and the DTFT of sequence $x[t]$ has following relationship

$$X_D(\hat{f}) = S\left(\frac{\hat{f}}{T}\right) = X\left(\frac{\hat{f}}{T}\right) \quad (4.13)$$

The relationship of unit-less frequency $\hat{f}$ to actual frequency f is shown in Eq. 4.14 as

$$f = \frac{\hat{f}}{T} \quad (4.14)$$

It can be seen from Eq. (4.13) that the frequency curve in range $|f| < 1/2T$ is the same for CTFT and DTFT, if the time domain signal is a band-limited signal in $|f| < 1/2T$. This means that DTFT produces the same spectral as CTFT. Therefore, DTFT can be used as a highly accurate frequency and amplitude estimator.

It should also be noted here that DTFT transforms infinite discrete sequence from time domain to frequency domain. However, only finite sequence will be used in practice to calculate DTFT, which may introduce high frequency component into cut signal. Thus, the final sequence may contain frequency higher than $1/2T$ which will introduce overlap between summations in Eq. (4.12). In this case, longer sequence and a suitable window are helpful to overcome this effect.

4.3.2 Time complexity of DTFT

For a n -points signal, time complexity of FFT is only $O(n \log n)$. Meanwhile, from Eq. (4.8), calculating one point in frequency curve requires n times “plus” operation and n times “multiplication” operation. Thus, time complexity for calculating one points by algorithm DTFT is $O(n)$, and the complexity of calculating n -points DTFT is n times $O(n)$ which equals to $O(n^2)$ and it is much higher than FFT. However, if only natural frequencies and corresponding amplitude is required, the time complexity can further be reduced to $O(n \log n)$, the same as FFT.

4.3.3 Procedure of using DTFT as frequency and amplitude estimators

FFT can only obtain spectral data at certain location, $f_k = k/N$; thus, the frequency-estimation accuracy is limited. However, as stated before, FFT of a signal can be calculated efficiently. Therefore, FFT as a coarse frequency detector is used which provides a rough frequency range for better estimation. A range with $5\Delta f \sim 10\Delta f$ is large enough to capture the frequency steadily with noise presented, where Δf is the frequency resolution. In this study, $5\Delta f$ is used as the width of rough frequency range. After the range is determined, maximum position and value of DTFT are obtained by using optimization algorithm. The classical Golden Mean optimization algorithm is used herein to obtain the peak value, which may obtain maximum position and value in $O(\log m)$, times where m ($m \ll n$) is the width of frequency range for optimization. Normally, optimization can converge in less than 10 iterations, which means $O(\log m) < 10$. In each iteration, DTFT at one point is calculated requiring $O(n)$ computation. Therefore, it is easy to get the total time complexity to obtain maximum position and the value is $O(n \log n) + O(n \log m) = O(n \log n)$. Since DTFT is more accurate in estimating frequency response of n -points signal, this procedure is as fast as FFT, but it is more accurate. The flowchart of this procedure is shown in Fig. 4.2.

Fig. 4.2 Flow chart of discrete time Fourier transform as a frequency and amplitude estimator

4.3.4 Numerical evaluation of using DTFT as an estimator

In this section, several simulated signals are presented to verify the proposed procedure. The first signal is a linear combination of sinusoidal signals as

$$x_1(t) = \sin(2\pi 5.5t) + 2.2 \sin(2\pi 20.25t + \pi / 4) + 1.3 \sin(2\pi 43.125t - \pi / 3) + \varepsilon(t) \quad (4.15)$$

where the first three terms are sinusoidal signals with different amplitudes and phases, and the last term $\varepsilon(t)$ is a Gaussian white noise.

This signal is sampled from $t = 0$ s with a sample frequency of 100 Hz. Before calculating FFT, a “*hanning*” window is applied to reduce leak effect. The estimation results of DTFT and FFT are shown in Table 4.1. As it is shown in the table, even with only 25 points of the signal, DTFT shows reasonable results. Both frequency and amplitude are more accurate than FFT. When the signal increases to 50 points or more, results from DTFT becomes highly accurate comparing to FFT.

In order to demonstrate the performance of the algorithm under severe noise, a noise with half the amplitude of original signal is added. The results of the algorithm with point $N=100$ and $N=500$ are shown in Table 4.1 as well. Because FFT reports many false frequencies for the case of $N=500$, some of them are neglected. It can be seen from the results DTFT reports better and more stable results under noisy condition. For the case of $N=500$, DTFT provides an even better results than FFT.

The estimation results of DTFT and FFT are shown in Table 4.2. From the table, it can be found that the frequency estimations are almost the same as the un-damping cases. For amplitude estimation, FFT gives good results for small damping case; but as the damping increases, the estimation error rises rapidly. On the contrary, DTFT always gives more accurate results.

The performance of the algorithm under the same noise sequence in Table 5.1 is also provided in Table 4.2. It is shown clearly that DTFT reports better results than FFT. For the case of $N=500$, only the frequencies near to the real frequencies are listed. Since the damping is large for high frequency component, the signal noise ratio is lower for high frequency. Thus, DTFT reports large amplitude error for high frequency.

Table 4.1 Frequency and Amplitude estimate results of FFT and DTFT for signal $x_1(t)$

Points	FFT		DTFT	
	Frequency (Hz)	Amplitude	Frequency (Hz)	Amplitude
N=25	6.25	0.8748	5.621	0.9374
	18.75	2.1108	20.235	2.1353
	43.75	1.2048	43.159	1.5121
N=50	6.0	0.9431	5.502	1.0323
	20.0	2.1350	20.250	2.1891
	44.0	1.1293	43.123	1.3053
N=100 without noise	6.0	0.8432	5.501	0.9864
	20.0	2.0932	20.250	2.2005
	43.0	1.2744	43.125	1.3181
N=100 with noise	6.0	0.5211	5.034	0.7258
	12.0*	0.5204	12.748	0.4956
	20.0	2.0072	20.187	2.1997
	38.0*	0.4041	38.063	0.3108
	44.0	0.8444	43.310	1.1215
N=500 without noise	5.6	0.8476	5.500	0.9970
	20.2	2.1088	20.250	2.1996
	43.2	1.1844	43.125	1.3027
N=500 with noise	4.6*	0.0994	5.504	0.9968
	5.6	0.8620	5.504	0.9968
	6.4*	0.0906	5.504	0.9968
	7.0*	0.1129	5.504	0.9968
	19.2*	0.1138	20.250	2.2104
	19.6*	0.1374	20.250	2.2104
	20.2	2.2159	20.250	2.2104
	44.0*	0.0644	43.117	1.3536
	44.6	0.1054	43.117	1.3536
N=1000	5.5	0.9988	5.500	0.9987
	20.3	1.866	20.250	2.1996
	43.1	1.2472	43.125	1.3040

* denotes false frequency reported from FFT

The second signal is a linear combination of sinusoidal signals with different attenuation:

$$\begin{aligned}
 x_2(t) = & \sin(2\pi 5.5t) \exp(-0.1t) + 2.2 \sin(2\pi 20.25t + \pi / 4) \exp(-t) \\
 & + 1.3 \sin(2\pi 43.125t - \pi / 3) \exp(-5t) + \varepsilon(t)
 \end{aligned}
 \tag{4.16}$$

where the first three terms are sinusoidal signals with different amplitudes, phases and attenuations, and the last term $\varepsilon(t)$ is a Gaussian white noise.

This signal is also sampled from $t = 0$ s with a sample frequency of 100 Hz. Because the signal is not a stationary signal, the energy in a unit time interval decays over time and the absolute value of the amplitude has no obvious physical meaning. Therefore, the estimated amplitude can be compared with the analytically amplitude of $x_2(t)$ without noise under certain time span and frequency. The analytic CTFT for $\sin(2\pi\omega t + \phi)\exp(-\xi t)$ can be obtained by using Maple symbolic algebra software as

$$\begin{aligned} CTFT(\sin(\omega t + \phi)\exp(-\xi t)) &= \int_0^t \sin(\omega t + \phi)\exp(-\xi t)\exp(-j\omega t) dt \\ &= \frac{e^{-t(\xi + j\omega)} \left[-\omega(1 - e^{\xi t})e^{j(\phi + t\omega)} + \xi \sin(\phi)e^{t(\xi + j\omega)} - \xi \sin(\phi + t\omega) \right]}{\xi(\xi + 2j\omega)} \end{aligned} \quad (4.17)$$

Table 4.2 Frequency and Amplitude estimate results of FFT and DTFT for signal $x_2(t)$

Points	Analytical	FFT		DTFT	
	Amplitude	Frequency (Hz)	Amplitude	Frequency (Hz)	Amplitude
N=50	0.2442	6.0	0.2302	5.503	0.2451
	0.4371	20.0	0.4194	20.250	0.4360
	0.1184	44.0	0.0920	43.123	0.1304
N=100 without noise	0.4758	6.0	0.4014	5.501	0.4714
	0.7012	20.0	0.6486	20.250	0.6974
	0.1281	43.0	0.0790	43.125	0.1416
N=100 with noise	0.2442	6.0	0.1465	5.145	0.1821
	-	12.0*	0.1034	12.730	0.0945
	0.4371	20.0	0.3936	20.199	0.4374
	-	38.0*	0.0824	37.942	0.0513
	0.1184	46.0	0.0623	46.413	0.0853
N=500 without noise	1.9673	5.6	1.6587	5.500	1.9644
	1.0969	20.2	0.6487	20.250	1.0966
	0.1290	43.2	0.0077	43.125	0.1418
N=500 with noise	1.9673	5.6	1.6940	5.505	1.9655
	1.0969	20.2	0.6611	20.286	1.1743
	0.1290	43.4	0.2811	44.206	0.3751
N=1000	3.1606	5.5	3.0809	5.500	3.1577
	1.1043	20.2	0.2881	20.250	1.1045
	0.1290	43.1	0.0020	43.125	0.1421

* denotes false frequency reported from FFT

4.4 FRESH PATH EXTRACTION FOR BEAM-LIKE STRUCTURES

4.4.1 Model set up

A simply supported beam is modeled in ABAQUS as shown in Fig. 4.3. The length l of the model is 1 m, the width w is 5 cm and the height h is 2 cm. Young's modular E and Poisson's ratio of the beam is 2×10^{11} N/m² and 0.3, respectively. A small Rayleigh damping is added, α is 25 and β is 3.5×10^{-5} . The auxiliary mass is located at $x_i = i/50$ m ($i = 1, \dots, 49$). The acceleration response of auxiliary mass is captured with a sampling frequency of 2000 Hz. The exciting force is a unit impulse with width of 0.002 s at the same location as auxiliary mass. One-second response is calculated by ABAQUS implicit dynamic module.

Fig. 4.3 Numerical model used for verification

4.4.2 Intact cases

Intact cases are calculated to verify the proposed FRESH path extracting method, and serve as references for FRESH curvature. Three intact cases are investigated with different auxiliary mass, namely 0.2 kg, 0.75 kg and 2.0 kg respectively.

The FRESH paths extracted from 2 kg case are shown in Fig. 4.4. The projections of the curve in three axis directions are plotted onto the three axis planes. Since amplitude of the dynamic response is in proportion to excitation force and the amplitude ratio between different modes is not the interest in this study, amplitude of each FRESH path is normalized to maximum amplitude of 1. It is shown clearly in Fig. 4.4 that FRESH paths extracted by DTFT are much smoother than that of FFT. Moreover, because acceleration on the node point of a mode is zero, FRESH paths are singular at these nodes. The results are shown clearly in the figure that FRESH paths extracted near the node point of the mode have a greater estimation error. Therefore, specific FERSH path cannot be used to detect damage near the node region of the mode.

From the simulation results of three different auxiliary mass models, different FRESH paths are extracted as shown in Fig. 4.5. The amplitude of FRESH path with auxiliary mass of 0.2 kg is normalized to maximum amplitude of 1 for each mode and the other FRESH paths are scaled with the same scale factor. Effect of auxiliary mass matches theoretical prediction in the last chapter, which is clearly demonstrated in Fig. 4.5. As the mass get heavier, the magnitude of both amplitude and frequency shift increase. Moreover, the shape of frequency also changes as the mass increases. It is also shown in the last chapter. Therefore, the FRESH path extraction method can successfully extract frequencies from the signal, and they are smoother than those obtained by FFT.

Fig. 4.4 First three FRESH paths extracted by discrete time Fourier transform (DTFT) and FFT

- (a) first fresh path extracted by DTFT; (b) first fresh path extracted by FFT;
(c) second fresh path extracted by DTFT; (d) second fresh path extracted by FFT;
(e) third fresh path extracted by DTFT; (f) third fresh path extracted by FFT

Fig. 4.5 FRESH path extracted from three different auxiliary mass model
(a) first FRESH path; (b) second FRESH path

The FRESH curvature of intact case is quite smooth as shown in Fig. 4.6. In the calculation, weight factors are set to values that make both FC_f and FC_a have maximum values of 1.

Fig. 4.6 FRESH curvature of intact beam

4.4.3 Single Damage case

In this section, a single damage model is studied. The damage is introduced between 26 cm and 32 cm, half of the beam height is reduced in this region. The other parameters are kept the same as in the intact case with mass of 2.0 kg. The shape of the first FRESH path is almost same as the previous case (Fig. 4.7). There is a subtle change near the damage, i.e., a fluctuation at that location. However, the FRESH curvature shown in Fig. 4.8 has a sudden change at the damage location, which is obviously different from the intact case. In the figure, intact region is colored as green while damage region is colored as red. The damage location is accurately shown as a sudden change of FRESH curvature. In the calculation, the weight factors are determined by ERR as shown in Eq. (4.5).

Fig. 4.7 FRESH path extracted from beam with single damage.

(a) first FRESH path; (b) second FRESH path

Fig. 4.8 FRESH curvature of beam with a single damage at 26 cm to 32 cm, 50% height loss, 2.0 kg auxiliary mass

4.4.4 Effects of damage range and severity

In this section, the effects of damage range and severity are investigated. Damage in different range and severity are modeled. Firstly, a smaller range damage model whose damage

is from 28 cm to 30 cm is simulated. FRESH curvature is shown in Fig. 4.9. The position where FC has a dramatic change is smaller than FC shown in Fig. 4.8, which accurately identify the damage location.

Fig. 4.9 FRESH curvature of beam with a single damage at 28 cm to 30 cm, 50% height loss, 2.0 kg auxiliary mass

Then, models with different damage severity, one with 1/4 height reduction and another with 1/10 height reduction, are introduced. FRESH curvature for each case is shown in Fig. 4.10 and 4.11, respectively. The sudden FRESH curvature change becomes fuzzy when damage severity reduces. Therefore, changes in FRESH curvature is more obvious as the damage progresses, when the change become large enough, the structure should be examined carefully for further use.

Fig. 4.10 FRESH curvature of beam with a single damage at 26 cm to 32 cm, 25% height loss, 2.0 kg auxiliary mass

Fig. 4.11 FRESH curvature of beam with a single damage at 26 cm to 32 cm, 10% height loss, 2.0 kg auxiliary mass

4.4.5 Effects of damping

In practice, damage usually causes changes in damping of the structures. In this section, the effects of different damping settings are investigated. Firstly, the damping of the entire beam is raise with the same amount for the whole beam, 5 times larger and 10 times larger. All the other parameters are kept the same as those in Section 4.3.3. The FRESH curvatures are shown in

Fig. 4.12 and 4.13, respectively. Because of damping, amplitudes of responses are smaller than those with small damping cases. In practice, if the damping of a structure is high, more energy is needed for excitation. As shown in these figures, FRESH curvature can accurately detect the damage region.

In some situation, damping in the damage region may be higher than the intact region because of internal friction. Therefore, two cases are considered: one with ten times larger damping only in the damage region and the other with five times larger damping in the intact region but ten times larger damping in the damage region. FRESH curvatures for both cases are shown in Fig. 4.14 and 4.15, respectively. In all the cases, FRESH curvatures show the accurate position where the damage is located in the structure.

Fig. 4.12 FRESH curvature of beam with a single damage at 26 cm to 32 cm, 50% height loss, 2.0 kg auxiliary mass, with five times damping

Fig. 4.13 FRESH curvature of beam with a single damage at 26 cm to 32 cm, 50% height loss, 2.0 kg auxiliary mass, with ten times damping

Fig. 4.14 FRESH curvature of beam with a single damage at 26 cm to 32 cm, 50% height loss, 2.0 kg auxiliary mass, with ten times damping in damage region

Fig. 4.15 FRESH curvature of beam with a single damage at 26 cm to 32 cm, 50% height loss, 2.0 kg auxiliary mass, with ten times damping in damage region and 5 times damping in intact region

4.4.6 Experimental verification of FRESH path algorithm

4.4.6.1 Experiment setup

In this section, two steel beams with different damage severity introduced by grinder are studied. Their dimensions are the same: 1.2 m long, 50 mm wide and 6 mm thick. The beams are fastened on an optical platform by four aluminum plates as shown in Fig. 4.16. In order to provide fixed boundary conditions, aluminum plates on each side are fastened with four bolts and only the middle 700 mm section are used for testing. The testing points are located between 60 mm from the left fastened position to 40 mm from the right fastened position. The interval between the two adjacent testing points is 30 mm. The accelerometer is mounted on top of the beam by a strong magnet as shown in Fig. 4.17. The beam is excited by the shaker assembled inside the accelerometer. The excitation force is a rising edge of a square wave. The acceleration is recorded in 4096 Hz sample rate and for 5 seconds long.

Fig. 4.16 Experimental setup to verify FRESH path based method

Fig. 4.17 Mounted accelerometer used in test set-up

Different severity of damages are introduced at the same position, away from left fasten point for 0.5 meter, with different width and height on the two beams. The smaller one has a 20 mm wide and 1.5 mm thick square shaped damage (Fig. 4.18). The larger one has a 40 mm wide and 3 mm thick parabola shaped damage (Fig. 4.19).

Fig. 4.18. Photo of smaller damage beam

Fig. 4.19. Photo of larger damage beam

4.4.6.2 Experimental results

The first mode FRESH paths of both cases are extracted from experimental data as shown in Fig. 4.20. The smaller damage beam seems to have a smoother FRESH path but the difference is not obvious. In this experiment, the amplitude provides smoother result, thus, the calculated ERR is around 1/2.

(a)

(b)

Fig. 4.20 Experimental FRESH path for both beams
(a) FRESH path for small damage beam, (b) FRESH path for larger damage beam

Then FRESH curvatures are calculated for both cases (Fig. 4.21). The locations of both damages are identified successfully. Moreover, the FRESH curvature of the larger damage beam

has a more significant sudden change which indicates that the damage is more severe. However, it still requires much more effort to relate damage severity with FRESH curvature accurately.

(a)

(b)

Fig. 4.21 Experimental FRESH curvature for both beams
(a) FRESH curvature for small damage beam, (b) FRESH curvature for larger damage beam

4.5 SUMMARY

The principles developed in Chapter 3 is developed into a procedure that can be actually be measured in practice. Based on the numerical and experimental results, the local stiffness reduction can be detected by the proposed damage index. The main findings of this chapter are as follows:

1. Frequency shift curve caused by the auxiliary mass contains local information that can be extracted for damage detection.
2. Amplitude of spectrum at the natural frequencies contains similar local information as frequency shift curve. Thus, the concept of FRESH path is proposed and the weights of frequency shift component and amplitude component is also proposed.
3. The performance of the method can be enhanced by calculating the curvature. Both the numerical and experimental verifications show good results in detecting stiffness reduction in beam-like structures. However, the derivative operation magnifies the noise as well in which the method may fail under moving condition.
4. FRESH path extraction method based on DTFT can successfully extracted frequency and amplitude components with a reasonable accuracy and efficiency. The method shows good resistance to heavy Gaussian white noise. The features are useful for the method to be applied in moving condition.

CHAPTER 5

INSPECTION OF STIFFNESS REDUCTION BY FREQUENCY SHIFT UNDER MOVING CONDITION

5.1 QUALITATIVE ANALYSIS

Before testing the performance of the method, the influencing factors should be discussed first. The solutions of Eq. (3.1) are fairly complicated; several special cases are discussed by Fryba (1999) and Yang et al. (2004b). Fortunately, for inspection problem, one does not need the responses and properties in the whole spectral domain, and the results discussed in Chapter 3 and Chapter 4 show that even the frequency shift curves of several fundamental frequencies contain enough information for inspecting the degraded supports. Moreover, due to introduction of dominated external excitation force, the responses are highly related to the excitation. Therefore, a qualitative analysis may be enough for the dynamic model as proposed in Chapter 3.

5.1.1 Segments of the signal

A typical signal captured in the experiments with impact excitations is shown in Fig. 5.1. The signal is clearly divided into many segments. This is because the power of external excitation force is much larger than the other sources in the model. Consequently, the response can be treated approximately as a sequence of impact responses. Thus, it is natural to divide the signal into segments based on the excitation.

Denoting the period of the impact as Δt , the signal can be divided into a series of Δt length sub-signals. Accordingly, the beam is divided into several parts in spatial domain. In order to obtain localized information about the beam, the displacement of the moving mass $u\Delta t$ should be much smaller than the total length of the beam. Therefore, the spatial change in one segment can be neglected. Consequently, Eq. (2.19) can be rewritten as an average in the segment as

$$\frac{d^2v(t)}{dt^2} \cong \frac{\partial^2 w(ut_0, t)}{\partial t^2} + 2u \frac{\partial w(ut_0, t)}{\partial t} \frac{\partial w(ut_0, t)}{\partial x} + u^2 \frac{\partial^2 w(ut_0, t)}{\partial x^2} \quad (5.1)$$

where t_0 is a time point in the analysis segment.

Thus, the signal in each segment is a combination of localized impact response, local rotational angle and curvature information. The latter two are slow varying variables compared to the impact response. Therefore, it is possible to trade the latter two as noise terms and apply the procedure proposed in last chapter on the signal.

Fig. 5.1 Acceleration response with 1 Hz excitation and 40 mm/s moving speed

5.1.2 Qualitative analysis of the signal

According to perturbation solution of moving load problem by Fryba (1999) and Yang et al. (2004b), the model has a strong response near critical speed, and the combination resonance introduces many extra frequencies into the signal. However, it is clear that the dominated part of the signal captured in this study is the response of beam under impact excitation, since the power of excitation is dominated. Moreover, if the response is longer than Δt , the responses from previous impacts cannot decay to a lower enough level. Consequently, the current impact response is mixed with response from the previous impacts. For cases with small damping and

high frequency of excitation, this residue is even large. Therefore, for the i -th segment, the captured signal can be represented as

$$a_i(t) \cong a_{\text{beam}}(ut_i, t) + a_{\text{residue}}(ut_i, t) + a_{\text{spatial}}(ut_i, t) + a_{\text{noise}}(t) \quad (5.2)$$

where $a_i(t)$ is the captured signal, $a_{\text{beam}}(ut_i, t)$ is the acceleration at location ut_i caused by the i -th impact, $a_{\text{residue}}(t_i, t)$ is the residue acceleration from previous impacts, $a_{\text{spatial}}(ut_i, t)$ is the acceleration caused by movement of device and $a_{\text{noise}}(t)$ is the noise in the testing environment.

The above decomposition shows that quasi-static moving mass problem is a simplified version of the moving condition problem. The procedure to process quasi-static moving mass problem may be applicable for moving condition. In the next section, the factors influencing the above simplification are analyzed.

5.2 FACTORS INFLUENCING THE ANALYSIS PROCEDURE

Based on the decomposition in Eq. (5.2), the moving speed, impact force, damping, contact surface property and noise sources are the main factors that affect the analysis procedure. Each factor is discussed in detailed here.

5.2.1 Moving speed

Moving speed has several important effects. It determines the inspection velocity. The total length of inspection duration is total length of the beam divided by the moving speed. It also influences the properties of the signal directly. Higher speed tends to enhance the noise of the signal, such as friction induced noise and it also tends to cause stronger spatial influence term.

5.2.2 Excitation force

Excitation force determines the segment length Δt , and hence, it has effects on energy of the residue part, number of the data points in one analysis and spatial resolution of the procedure. Moreover, the mark-space ratio of the force determines the input frequency component, while the amplitude of the force controls the input power.

5.2.3 Damping

Damping mainly influence the residue part. Higher damping allows higher frequency of excitation force. Although damping is usually a factor which cannot be changed in the real structure, it should be considered in choosing a suitable excitation frequency.

5.2.4 Contact surface property

Contact surface properties influence the friction induced vibration. Consequently, it introduces unwanted noise into the captured signal. If possible, lubrication of the surfaces is recommended.

5.2.5 Noise sources

There are various kinds of noise sources which can influence the signal. Environmental noise caused by nearby moving part is an example. Electric noise from electrical device may also influence the signal.

5.3 PERFORMANCE OF FRESH EXTRACTION PROCEDURE

5.3.1 Experimental setup

An experimental platform as shown in Fig. 5.2 has been setup to examine the performance of the proposed procedure. The inspection device is mounted on a 1.2 m long linear motion units (LMU) with special designed connector parts as shown in Fig. 5.3. It only transfers the sliding motion along the axial direction of LMU. To isolate the electric signal from the controller of servo-motor in the LMU, the grey colour part connected to the LMU is made of polymer. The velocity of the slider is in the range of 10 mm/s to 100 mm/s due to the constraint of servo-motor in the LMU. The LMU and the investigated beam are bolted onto an optical table. The contact surface between the inspection device and the beam is lubricated to reduce friction. The inspection device attached on the beam with a 16 mm diameter magnet includes an electromagnet shaker and an accelerometer. The excitation is introduced by the embedded shaker. The force history is a 1% mark-space ratio square wave with 0.25 V amplitude. The inspection device is driven by the LMU with specified velocity, and at the same time the acceleration on the device is captured. The length between two fasteners at two ends are 800

mm. The width and thickness of the intact beam are 40 mm and 5 mm, respectively. The two fasteners on each side are bolted with four bolts with pre-loaded torque of 6 Nm.

Fig. 5.2 The experimental setup to verify moving condition verification

Fig. 5.3 The connector part between linear motion unit and sensor

5.3.2 Results from FRESH path extraction procedure

Factors discussed earlier in Section 5.2 make the signal under moving condition much more complicated than signal from quasi-static condition. The FRESH path extraction procedure is applied to a signal captured under moving condition; the results are shown in Fig. 5.4. It can be seen from this figure that frequencies cover the whole figure. Other than the natural frequencies of the system, there are mainly two sources of these frequencies. The first one is extra frequencies from moving load; while the other is environmental noises from the platform, such as mechanical vibration from LMU and electric noise from the controller of servo-motor. Fortunately, it seems that the *real* frequencies are listed in these frequencies, and if they can be

filtered out correctly, they are useful feature for inspection. Another important observation is that the amplitude component does not have a mode shape square like shape as in quasi-static cases presented in Chapter 4. For example, the largest peaks of the fundamental modes are near the 30th segments and 60th segments as shown in Fig. 5.4 as red dots, which is not in the middle of beam as shown earlier. Similar situations also happen for the other modes. This is because the signal is captured under moving condition; contact properties and noises have direct influence on the amplitude of the response. For smooth profile, the influence of the contact property does not influence the signal significantly as discussed in Chapter 4 (Martinez et al., 2016); but considering the scale effects, the contact property is not *smooth* for experiments carried out here, and thus the response levels are highly related to the property of contact surface. This problem may be solved by increasing the excitation power, and it is much preferred the excitation power is controlled carefully for the safety of the rail system. Therefore, under moving condition, amplitude component is too rough to be used for inspection and it is neglected in this study. However, it can also be observed that the *real* frequencies usually have larger amplitudes. Thus, the amplitude can be used for frequency selection. Consequently, the problem reduces to filter out unwanted peaks shown in Fig. 5.4.

Fig. 5.4 Typical results from procedure proposed earlier

5.4 FALSE FREQUENCIES ELIMINATION PROCEDURE

Eliminating false frequencies from Fig. 5.4 seems to be an easy task because the *real* frequencies tend to have larger amplitudes. However, the threshold under which the frequencies are treated as false frequencies is not easy to propose. In fact, it is unlikely to propose a threshold for all cases. Because the response near nodal points of one specific mode is low, the number of frequency peaked up by the procedure is unknown. Another property of the spectral peaks is that some of them may be extremely high and sharp. Therefore, filter methods based on a predefined threshold, SNR or wavelet transform coefficients can obtain a reasonable result for a certain test case, but applying these parameters to various working conditions may produce unpredictable results. Adjusting parameters after each test is a tedious work and requires many experiences to set a good threshold.

The reason why a quartile based threshold method, for example removing the frequencies whose amplitude is lower than 90% quantile of the whole amplitude, does not work well is that spectral analysis of the signal contains several extremely high values and their numbers are changing from case to case. Therefore, one quantile works for one case may be too high or too low for another case. A straightforward solution is to separate the peaks into two groups, i.e. peaks with extremely high amplitude and peaks with relatively low amplitude. For the latter group, the amplitudes are much even, and a quantile based threshold method is a reasonable choice. Therefore, a procedure as shown in Fig. 5.5 is proposed to solve this problem. The captured data is first feed into FRESH path extraction procedure to obtain the frequency and amplitude components of the signal. Then, the amplitude component is sent into a k-means cluster program to split into two groups. The group with higher centroid is the data points with high SNR, and thus, it is preserved. The group with lower centroid is a group contains false frequencies and low SNR data points. Thus, only value higher than the certain quantile is selected. In this study 3/4 quantile is applied. This procedure is hopeful to solve the problem because k-means cluster splits high SNR points from low SNR ones. Accordingly, the low SNR group has relatively similar amplitudes, and thus, one threshold is more likely to be applied to all the tests data. In other words, this procedure provides an adaptive threshold for each test case.

Fig. 5.5 Flowchart for frequency clean-up

The results of above procedure applied on the same signal shown in Fig. 5.4 is shown in Fig. 5.6. Most of the false frequencies are removed and some of the low SNR frequencies peaks are also removed. It is also noted that because of low SNR, frequencies near nodal points and some points without enough response level are also removed. The procedure produces reasonable results for further inspection. In the next section, it is tested under various conditions and results are used for detection of stiffness reduction.

Fig. 5.6 Results from false frequency removal procedure

5.5 PERFORMANCE ON INTACT BEAM

The fundamental frequency shift curves extracted from 1 Hz square wave excitation under 1200 mm/min, 2400 mm/min and 4800 mm/min are shown in Fig. 5.7. It can be seen from this figure that the three curves match with each other well. The total time used to extract these points are 40s, 20s and 10s for moving speed of 1200 mm/min, 2400 mm/min and 4800 mm/min,

respectively. It is much faster than manually moving sensor and testing on each point. However, the frequency shift curves are not as smooth as quasi-static cases, especially near the region of nodal points. These slight fluctuation may be amplified by the differential operation when the curvature is calculated.

Fig. 5.7 Frequency shift curves extracted from 1 Hz square wave excitation

Fig. 5.8 gives the frequency shift curves extracted from 2 Hz square wave excitation under 1200 mm/min, 2400 mm/min and 4800 mm/min, respectively. As it is shown in Fig. 5.8, the method can extract the frequencies accurately. In this case, twice as many points are generated in the same amount of time as for 1 Hz excitation. On average, one can get more than 100 frequency points in less than one minute.

Fig. 5.8 Frequency shift curves extracted from 2 Hz square wave excitation

5.6 PERFORMANCE ON BEAM WITH DAMAGE

5.6.1 Experimental setup

The beams with different severity of damages used in Chapter 4 are investigated here. The damage is shown in Fig. 4.18 and Fig. 4.19. The location of the damage is changed to locations as shown in Fig. 5.9. All the other parameters are kept the same as the intact beam case discussed in the last section.

Fig. 5.9 Illustration of damage range of the damaged beam
(a) small damaged beam, (b) large damaged beam

5.6.2 Performance under different impact frequencies

The frequency extracted from 1200 mm/min moving device with 1 Hz excitation is shown in Fig. 5.10. It can be seen from this figure that the frequency for the intact beam and the small damage beam has a similar shape. The main differences occur at the damage location. For the large damaged beam, the fundamental frequency has an obvious drop. The largest frequency drop appears at the damage location. Therefore, for the fundamental frequency shift curve, the damage information can easily be located and the severity of the damage can be qualitatively read from the magnitude of the difference between the intact beam and the damaged beam. However, the frequency shift curves are not as smooth as the quasi-static cases. This can be clearer observed from the frequency component of FRESH curvature shown in Fig. 5.11. This is because of the differential operation in the curvature calculation amplifies the fluctuation. The small estimation errors near the two ends of the beam where SNR is low causes obvious peak value in the curvature. Therefore, the differential operation should be avoided under moving condition, and frequency shift itself should be used as feature for inspection.

Fig. 5.10 Frequency extracted with 1200 mm/min moving device and 1 Hz square wave excitation

Fig. 5.11 Frequency shift curvature for 1200 mm/min and 1 Hz excitation case

For stiffness reduction inspection in beam model discussed here, the inspection method falls back into a method requiring baseline, and the stiffness reduction is located by a change of frequency shift. It will be described in the next chapter that by using properties of the rail system, the method can be a baseline-free method for rail system inspection.

As the impact frequency rising to 2 Hz, two times more points are obtained. Stiffness reduction in both beams can easily be read from the extracted frequency shift curve as shown in Fig. 5.12. Therefore, if the baseline is available, the proposed method can identify stiffness reduction accurately and the severity of the reduction is related to the amount of change in frequency.

Fig. 5.12 Frequency extracted from three beams with 1200 mm/min moving device and 2 Hz square wave excitation

5.6.3 Performance under different moving velocities

Frequency shift plot for 2400 mm/min moving velocity with 1 Hz and 2 Hz excitations are shown in Fig. 5.13 and Fig. 5.14, respectively. The location of the damage is clearly shown in these figures. Although the moving velocity increases from 1200 mm/min to 2400 mm/min, there is no obvious evidence that the frequency extraction is affected. For 1 Hz excitation case under 1200 mm/min and 2400 mm/min, results from 2400 mm/min are similar or even better than that from 1200 mm/min case. Similarly, frequency shift plot for 4800 mm/min under 2 Hz excitation is shown in Fig. 5.15. Generally, the proposed false frequency removal procedure works well under these test cases, and the frequency shift curves extracted provide some information about stiffness reduction in the beam. However, they are not as smooth as quasi-static case, and curvature will be influenced by these fluctuation due to the differential operation. Thus, frequency shift curve is suggested to be the feature for further inspection instead of curvature.

Fig. 5.13. Frequency extracted from three beams with 2400 mm/min moving device and 1 Hz square wave excitation

Fig. 5.14 Frequency extracted from three beams with 2400 mm/min moving device and 2 Hz square wave excitation

Fig. 5.15 Frequency extracted from three beams with 4800 mm/min moving device and 2 Hz square wave excitation

5.7 SUMMARY

In this chapter, the frequency shift curves are successfully extracted under moving condition with the help of false frequency removal procedure. The curves are almost invariant to the impact frequency and moving velocity of the mass in the tested cases. However, high impact frequency or moving velocity introduces more uncertainty to the system and more noise into the frequency results. It is also shown that the change of frequency shift curve is related to the stiffness reduction in the beam. However, generally the curves extracted contain more noise under moving condition, and curvature of the curves do not give good identification results. Therefore, frequency shift curve itself will be used as a feature for inspecting degraded supports in the next subsequent chapters.

CHAPTER 6

INSPECTION METHOD FOR SUPPORTS IN RAIL SYSTEM

6.1 SPECIAL ANALYSIS FOR RAIL SYSTEM

Theoretically, the method proposed in previous chapters can be applied to inspect degraded supports directly. However, properties of the rail system can also help to enhance the performance of the method and reduce the data points needed.

Normally, prior information about the inspected objects can offer valuable information. Based on the amount of information available, the inspection methods can be classified as methods with and without a baseline. Methods with a baseline are expected to produce better results, but it also requires more computational power and storage. The inspection can be evaluated by the difference between measured value and baseline value. Methods without a baseline usually transform the problem into a singular signal detection method which is based on the assumption that damage should be a low probability event, and thus, singular signal in the data may indicate some irregularities. However, evaluating whether a signal is singular also requires a baseline value, and thus, methods without a baseline usually explicitly or implicitly include a step to estimate the baseline. If the baseline is predefined, then these methods are reduced to methods with a baseline.

Considering the number of sleepers in a rail system, to obtain a baseline is a huge task to do. A practical solution is to propose a baseline-free method first, and using the results from the method as a baseline. After several iterations, the method can be converted from a baseline method to a baseline-free method.

In this study, two assumptions are made on the supporting system of a rail system. Firstly, most supports should be in intact condition. Secondly, the supports are installed nearly periodical along the rail.

6.2 PERIODIC PATTERN OF FREQUENCY SHIFT CURVE

According to results obtained in Chapter 3, the influence of one degraded support only affects the dynamic properties in a limited range. For better illustration, a specific model with

limited fasteners is considered here as shown in Fig. 6.1. The model is solved by ABAQUS , a general purpose FEM software, with the parameters listed in Table 6.1. The fundamental frequency shift curves extracted from model with $N=9$ and $N=19$ are illustrated in Fig. 6.2 and Fig. 6.3, respectively. A direct observation noted is that the frequency shift curves roughly follow a periodic pattern. The influence caused by the boundary is limited in about one gap between two sleepers, and they are reasonably small. Therefore, it is safe to assume that the frequency shift curve has a periodic pattern even if the rail is not infinitely long.

Fig. 6.1 Model of a rail system with limited length

Fig. 6.2 Fundamental frequency shift curve from model with $N=9$

Fig. 6.3 Fundamental frequency shift curve from model with $N=19$

Table 6.1 Parameters used in calculation of the model with limited length

Parameter	Description	Value
M	Mass of the auxiliary mass	0.35 kg
K	Contact stiffness between auxiliary mass and the beam	1×10^8 N/m
Δx	Gap between two sleepers	0.5 m
m	Length of the beam in unit length	0.936 kg/m
EI	Bending stiffness of the beam	32 N/m
k_i	Supporting stiffness of the springs under the beam	1×10^5 N/m
N	Number of support	9 or 19

6.3 DAMAGE INDEX FOR SUPPORTS INSPECTION

Due to the periodic property of frequency shift curve, the frequency irregularity can be expressed as a break of periodicity. Under moving condition cases, if the excitation cannot increase to a level until noise can be neglected, then the differential operation of curvature calculation will amplify the noise. Fortunately, based on results obtained in last chapter and analytical solution in Chapter 3, the frequency shift curve may be a possible feature which can be used to inspect degradation in a supporting system.

Considering that the degraded supports are always local, and they will not influence large area, the average of frequency shift curve over a period can be considered as a baseline,

$$\bar{f}_i = \frac{1}{N\Delta} \int_0^{N\Delta} f_i(x) dx = \text{mean}(f_i(x)) \quad (6.1)$$

where $f_i(x)$ is the i -th frequency when auxiliary mass is located at x , Δ is the distance between two adjacent sleepers, N is the periods in the model.

Then the damage index of i -th mode at period j demonstrated in can be proposed as

$$D_{i,j} = \frac{\bar{f}_{i,j} - \bar{f}_i}{\max(f_i)} \quad (6.2)$$

where $\bar{f}_{i,j}$ is the average of natural frequency in the j -th period

$$\bar{f}_{i,j} = \frac{1}{\Delta} \int_{j\Delta-\Delta}^{j\Delta} f_i(x) dx \quad (6.3)$$

Fig. 6.4 Illustration of the proposed damaged index

The damage index above has an outstanding feature. It is observed that frequency shift curve has several local peaks in one period for higher modes, which may cause fluctuation of damage index. However, for above damage index, the value is integrated by a step of a period, and consequently, the integrated value is an average of a period. Thus, the damage index averaged out the fluctuation in one period without sacrifice the sensitivity of detecting the break of periodicity.

6.4 NUMERICAL VERIFICATION

Several numerical cases are carried out in this section to evaluate the performance of the damage index proposed in the above. The frequency is extracted using linear perturbation procedure in ABAQUS. All parameters not specified are set the same as in Table 6.1.

6.4.1 Damage index for intact case

The damage indices for $N = 9$ and $N = 19$ under intact condition are shown in Fig. 6.5(a) and (b), respectively. Two peaks indicating the first and last supports can be observed, which is considered as the boundary effect; the damage indices of the fasteners in the middle are larger than 0.005. On the other hand, results from a model with $N = 19$ also suggest that the boundary effect is obvious. However, the damage indices of the fasteners in the middle decrease to 0.003. Therefore, the proposed damage index shows better performance if the beam is sufficiently long; fortunately, the rail track is always long enough in practice.

(a)

(b)

Fig. 6.5 Damage index when all supports are intact
 (a) damage index for 5 m model, (b) damage index for 10 m model

6.4.2 Slight decrease of local support stiffness

The situation of support with slight degradation is studied here. They can be roughly classified into two cases: fastener is partial loose and the ballast is damaged as shown in Fig. 6.6 and Fig. 6.7, respectively. They are regarded as identical and the corresponding dynamic model is shown in Fig. 6.8 where the damage is simulated as the decrease in stiffness of the fifth spring. In this example, k_5 decreases from 1×10^5 N/m to 5×10^4 N/m.

Fig. 6.6 The situation showing the fastener is loose

Fig. 6.7 The situation showing the ballast is damaged

Fig. 6.8 The dynamic model of slight decrease of local support stiffness

The fundamental frequency shift curve is shown in Fig. 6.9(a). It can be seen clearly that the fundamental frequency of the interactive system changes periodically. At the fifth support having a lower stiffness, the frequency shift pattern is different: the frequency changes sharply near the support whereas the frequency changes smoothly near the other undamaged supports and moreover, the frequency near the damaged support drops slightly more. When applying the proposed damaged index, it observed from Fig. 6.9(b) that the damaged support is clearly indicated by a sharp peak, which is red while others are green. Because the damage in this example is slight, hence, there are still two peaks indicating the first and last supports due to the boundary effect.

(a) Fundamental frequency shift curve

(b) Damage index

Fig. 6.9 Results of model with slight decreases of local support stiffness

6.4.3 Large decrease of local support stiffness

The supports with larger stiffness decrease is also investigated here. They are regarded as the cases with larger damage as shown in Fig. 6.6 and Fig. 6.7, respectively. The damage is also simulated as the decrease of stiffness of the fifth spring from 1×10^5 N/m to 2.5×10^4 N/m. The fundamental frequency shift curve is shown in Fig. 6.10(a). It can be seen clearly that the frequency near the damaged support drops significantly than other locations. The damage indices as shown in Fig. 6.10(b) have an obvious peak for the damaged supports. This suggests that the damage index is related to the magnitude of the stiffness decrease, and it is effective in evaluating the change of stiffness.

(a) Fundamental frequency shift curve

(b) Damage index

Fig. 6.10 Results of model with larger decreases of local support stiffness

6.4.4 Missing of local support

Finally, the two most serious situations are considered herein: One presents that the fastener is missing, and there is a small gap between the bottom surface of rail and the sleeper (Fig. 6.11), the other one indicates that the ballast is severely damaged and the sleeper is unsupported (Fig. 6.12). For these two situations, the dynamic models are regarded as the same (Fig. 6.13), where the spring is removed completely.

Fig. 6.11 The situation showing the fastener is missing

Fig. 6.12 The situation showing the sleeper is unsupported

Fig. 6.13 The finite element model of missing local support

The numerical results including the fundamental natural frequency and damage index are presented in Fig. 6.14(a) and (b), respectively. It is found from Fig. 6.14(a) that the fundamental frequency drops suddenly when the auxiliary mass just passes through the fourth support. It looks like the continuous beam splits into three parts: part one is from the left end to the fourth support, part two is from the fourth support to the sixth support and part three is from the sixth support to the right end. For the first and third parts, the frequency shift pattern is almost identical to the intact case, while the second part has a quite different performance. In fact, the second part performs like a separate beam with a length of 2 m, which has elastic supports at the two ends; and the auxiliary mass induced frequency drops to nearly 10 Hz. For these two situations where the stiffness of the damaged supports is ignored, the auxiliary mass induced frequency shift can be used directly to identify the damaged support. When applying the proposed damage index, a peak which has a higher value than 0.4 can be observed. The damage indices corresponding to undamaged supports also have higher values than those in other cases, which is because the dramatic decrease of the frequency near the damaged support significantly reduces the baseline $\bar{f}_i$. This can be eliminated when the rail track is quite long and most of the supports are in intact condition as assumed previously.

(a) Fundamental frequency shift curve

(b) Damage index

Fig. 6.14 Results of model with missing local support

6.5 QUASI-STATIC EXPERIMENTAL VERIFICATION

Several experimental verifications on several models are designed and carried out to verify the proposed method.

6.5.1 Quasi-static experimental setup

As is discussed in previous chapters, performance of the method under quasi-static condition is tested first. A shaker Wilcoxon Model F3/Z602WA with mass of 0.3 kg regarded as auxiliary mass is mounted on the top surface of the beam with a magnetic base. Between any two adjacent neighboring supports, twelve testing points are labeled on which the accelerometer is positioned. Because the shaker which is placed on one of the testing points contains a coaxial accelerometer internally, it generates a pulse impact to excite the beam and the corresponding acceleration is recorded accordingly.

Fig. 6.15 Experimental setup for quasi-static condition

There are totally five experimental cases carried out in this study shown in Fig. 6.16: one is undamaged case and the other three are damaged cases. For the first case, all supports are undamaged and each fastener is fastened by a torque of 8 Nm; for Cases 2, the fourth pair of fasteners are loosened to 2 Nm, respectively; for Case 3, the fasteners are totally loosened; for Case 4, the piece of wooden layer is replaced by a soft rubber to simulate the damaged ballast; for Case 5, the fourth pair of fasteners are removed, and there is actually no support at the damaged area.

(a) Case 1

(b) Cases 2

(c) Case 3

(c) Case 4

(d) Case 5

Fig. 6.16 Experimental cases carried out in this study

6.5.2 Quasi-static experimental results

When all supports are undamaged, the first order auxiliary mass induced frequency shift is shown in Fig. 6.17. It is seen that the fundamental frequency shift shows almost perfect periodicity which coincides with the numerical example very well. Fig. 6.17(b) presents the damage indices of the six supports which have smaller values and no obvious peak indicating damaged support can be observed.

For the second case with the fourth pair of fasteners is loosen to 2Nm, it is observed from Fig. 6.17(b) that the fundamental frequency shift curve produces regular periodicity as well but the frequency drops slightly when the auxiliary mass is placed near the damaged support. The damaged support is identified when the proposed damaged index is applied (Fig. 6.17(d)). Because the beam length used in the experiment is quite limited, the boundary effect is more obvious than that in the numerical examples.

For the third case, the fourth pair of fasteners is totally loose, and the upper parts are also removed as shown in Fig. 6.16(c). It can be seen from Fig. 6.17(e) that the frequency shift curve is quite similar to the pattern shown in Fig. 6.10 which produces the fundamental frequency shift where local support stiffness decreases severely. Therefore, it can be concluded that the simplified dynamic model is promising. The damage indices shown in Fig. 6.17(f) indicate clearly the location of the loosening. The damage index of the fourth fastener is much larger than that of Case2, which indicates the damage is more severe.

In the third case, the wooden layer under the fourth sleeper is replaced by a soft rubber layer to simulate the damaged ballast. The dynamic model is considered the same as the previous case but with more severe damage. It can be seen from Fig. 6.17 (g) that the fundamental natural frequency drops clearly when the auxiliary mass passes through the third pair of fasteners; it is also quite similar to the pattern shown in Fig. 6.10. The damage index of the fourth support is also larger than that of Case 2.

For the last case where the fourth support is removed to simulate missing fastener and the unsupported sleeper, the experimental results coincide with the numerical examples quite well. The periodicity of the first order auxiliary mass induced frequency shift is broken at the damaged support by emerging two intervals into one as seen in Fig. 6.17(i) clearly. The damage

index is larger than that in the previous case. This observation also proves that the proposed damage index is capable of evaluating the damage severity. At the same time, the boundary effect is eliminated in this case because the damage is sufficiently severe.

(a) Fundamental frequency shift for Case 1

(b) Damage index for Case 1

(c) Fundamental frequency shift for Case 2

(d) Damage index for Case 2

(e) Fundamental frequency shift for Case 3

(f) Damage index for Case 3

(g) Fundamental frequency shift for Case 4

(h) Damage index for Case 4

(i) Fundamental frequency shift for Case 5

(j) Damage index for Case 5

Fig. 6.17 Results from quasi-static experimental setup

6.6 MOVING CONDITION VERIFICATION

Based on findings in Chapter 5, the frequency extraction procedure has been extended for moving condition. Therefore, applying the damage index proposed in Section 6.3 to moving condition is straight forward. In this section, a small scale third rail model with the mounting systems shown in Fig. 1.5 is tested for inspection of loose third rail fasteners.

6.6.1 Moving condition experimental setup

The experimental setup shown in Fig. 6.18 is similar to the setup used for moving condition verification in Chapter 5 (Fig. 5.2 and Fig. 5.3). The main difference between them are the model changed from a simple beam to a small scaled third rail model as shown in Fig. 6.18. The small scaled third rail is mounted using old mounting-like (Fig. 1.5) fasteners as shown in Fig. 6.19. The total length of the third rail is 1.8 m, the cross section is roughly 1/5 of the third rail as shown in Fig. 6.21. The other setup is kept the same as in Chapter 4.

Fig. 6.18 Experimental setup for moving condition verification

As small size bolts of the fasteners need to be used in the experiment, the applied torque is hard to be controlled. Moreover, there are only five supports in the model, and thus, only the support in the middle is changed through the following three cases. Case 1: intact case; Case 2: the bolt on the top is released; Case 3: the entire fastener is removed.

Fig. 6.19 Small scale model of old mounting for the third rail

Fig. 6.20 Small scale model of new mounting for the third rail

Fig. 6.21 Dimensions of the cross-section of the small scale third rail

6.6.2 Special treatment for frequency shift curve under moving condition

Due to noises under moving condition, the frequency shift curves extracted are not as smooth as those under quasi-static condition shown in last chapter. Moreover, the false frequency removal procedure also removes low SNR frequencies, and thus, the frequency shift curves obtained lack information in some range, especially near the nodal points. Therefore, they need to be processed before the damage index in Eq. (6.2) is calculated.

The idea of damage index is to calculate the difference of the averages in a period. The problem is the response near the supports are low, and thus, frequencies near them are more likely to be removed by frequency removal procedure. Therefore, only part of the rail in between two supports which is demonstrated in Fig. 6.22 are included in calculating $\bar{f}_{i,j}$. If there are frequency points missing in these selected segments, interpolation method can be applied.

Fig. 6.22 Illustration of the experiment setup

Due to the limited length of the third rail, if the damage index is calculated as defined in Eq. (6.2), there are only five indices and three of them are affected by one degraded support. This would not be a problem in real situation because the rails are much longer and the number of supports are huge. However, for experimental setup discussed here, an additional assumption is that the left half in one support interval is symmetric to the right half, for example, segment 1 and segment 2 in Fig. 6.22. Thus, the calculation interval of the damage index can be adjusted to half a period, i.e. 8 segments as shown in Fig. 6.22.

6.6.3 Moving condition experimental results

The fundamental frequency shift curve from 20 mm/s with 1 Hz impact with amplitude of 0.2 V is shown in Fig. 6.23. The original frequencies are plotted as a square point. Because there are frequencies missing from the results, they are interpolated by cubic spline as the circle points in Fig. 6.23. It is shown in the figure that the frequencies extrapolated by cubic spline near the supports location are not accurate, and they should be excluded from calculating damage index as discussed in Fig. 6.22.

Fig. 6.23 Frequency shift curve extracted from 20 mm/s with 1 Hz impact condition

The damage indices for the three cases are shown in Fig. 6.24. For the intact case, the damage indices are quite low as shown in Fig. 6.24(a). For Case 2, the release of the top bolts causes the damage indices to increase in the range of 0.5 m to 1.5 m, i.e. segment 3 to segment 6 as shown in Fig. 6.22. Ideally, only damage indices segment 4 and 5 should be raised. However, in fact, frequencies in one span such as segments 3 and 4 are usually continuous, therefore, indices of segment 3 and 6 are also higher than other regions. For Case 3, the release of the bolts at the back changes the model completely. Thus, the damage indices increase dramatically. As the same as for Case 2, the damage indices rise in the range of segment 3 to segment 6. These results suggest that the loosening of the bolts at the back of the mounting is more dangerous, which is also true in practice. This is because loosening of the bolts at the back unconstrains the third rail completely, but loosening of the bolts on the top only unconstrains the third rail partially. This is explained by Fig. 6.25 where the back part of the mounting includes a support unit as shown in the figure with a rectangle. Considering the third rail is hanged by the fasteners, this support unit can provide partial support when the third rail is constrained by other fasteners in the installation location.

(a) Case 1

(b) Case 2

(c) Case 3

Fig. 6.24 Damage indices for old mounting

Fig. 6.25 Illustration of the back part of mounting

Similar results are also obtained from small scaled third rail model with new mountings (Fig. 6.20). There are also three cases being studied. Case1 is the intact case; Case 2 releases the bolts at the front part; and Case 3 releases the bolts at the back part. The results are plotted

in Fig. 6.26. They are almost the same as in Fig. 6.24. Therefore, there is no obvious difference between these two kinds of mounting for the method proposed.

(a) Case 1

(b) Case 2

(c) Case 3

Fig. 6.26 Damage indices for new mounting

6.7 DISCUSSION OF SEVERAL DEGRADED SUPPORTS

As discussed in Chapter 3 and in this chapter, the frequency shift based method can be used to identify the degradation of one support accurately. The detection of more than one degraded supports should be easier. Because the experimental setup contains very limited supports, the performance of the method is verified by *numerical* examples. The model in Section 6.4 with $N=9$ is adopted here. The spring in the middle and the adjacent spring on its right are changed according to the stiffness settings used in Section 6.4. The frequency shift curve when k_5 and k_6 decreases from 1×10^5 N/m to 5×10^4 N/m is shown in Fig. 6.27. Comparing it with Fig. 6.9, the frequencies are found to be quite similar. The minor difference is in the range of 2.5m to 3.5m. This difference can be read from the damage indices shown in Fig. 6.28(a). The damage index at 3.0m is also increased to a similar level as the index at 2.5m. The damage indices for the other two cases are also shown in Fig. 6.28. It is shown clearly that the proposed method works well for the model with two continuous degraded supports as predicted.

Fig. 6.27 Frequency shift of model with slight decrease of stiffness

(a) Slight decrease case

(b) Large decrease case

(c) Missing case

Fig. 6.28 Damage indices of model with two continuous degraded supports from numerical simulations

To complete the discussion about multiple degraded supports, the above model is also investigated when the stiffness change happens in springs k_5 and k_7 . This model is interesting because it is the case with the closest distance between two non-adjacent degraded supports. If the method can differentiate the two degraded supports separately, then it can certainly distinguish degraded supports with interval larger than one interval, i.e., the method is able to produce correct results for any distribution of degraded supports.

The results of this case are shown in Fig. 6.29. For Case 1, it is shown clearly that the two degraded supports have two separated peaks. If the boundary condition effects are neglected, the damage indices are much higher than the others. The performance of the method is almost the same as the results of single degraded support shown in Fig. 6.9. Similar observations can be made for Case 2. For Case3, the damage index at 3.0m also increases to a very high level. This is because the two supports close to it are both missing, and the damage index is affected significantly. This false alarm can be removed easily by examining the frequency since the frequency of a span with length of four intervals is nearly four times lower than a span with length of two intervals. Based on these results, the method can identify the degraded supports accurately even if the degraded supports are only separated by one intact support. Therefore, the proposed method can produce reasonable results in detecting multiple degraded supports irrespective whether they are continuous or spreading along the rail system.

(a) Slight decrease case

(b) Large decrease case

(c) Missing case

Fig. 6.29 Damage indices of model with two degraded supports

6.8 SUMMARY

Based on the two assumptions, i.e. most supports are in intact condition and in periodic pattern, a damage index specifically designed for supports in a rail system is proposed in this chapter. The average of frequency can serve as a baseline, and the difference between the frequency and the baseline is a representative value of the damage. Numerical studies and experimental verifications under quasi-static and moving conditions are carried out, and the results show that the proposed method is sensitive to degraded supports. The performance of the method for continuous and close degraded supports is also discussed. The results show that the method can identify the degraded supports accurately irrespective of the distribution of the degraded supports.

CHAPTER 7

INSPECTION METHOD FOR SUPPORTS IN RAIL SYSTEM UNDER RANDOM EXCITATION

7.1 RESPONSE FROM RANDOM EXCITATION AND DAMAGE INDEX

In previous chapters, the periodic impact excitation is applied to the structure ensuring the impact response is large enough for frequency extraction. The aim of this chapter is to extend the method to the random excitation condition. The most important difference between response under random excitation and periodic impact excitation is the SNR of the signal and the amount of false frequencies in the signal. For signal under moving condition with external excitation, the adaptive threshold determining procedure proposed in Chapter 5 shows decent performance in removing the false frequencies. It is based on the assumption that “true” frequency has stronger response than false frequencies. However, under random excitation the variation of energy in different frequency range may cause “strong” response at a false frequency. This will fail the method discussed in Chapter 5. Therefore, extracting the “correct” frequencies from the signal may be a difficult task under random excitation condition. Consequently, a damage index which does not require frequency explicitly is preferable in this case, and thus, it is proposed in this study.

Inspired by the Power of Mode Shape (PMS) defined by (Fang & Perera, 2009), which is an integration of spectrum in a frequency range. The idea is extended to define the following integration called the n -th Moment of Spectrum (MoS) here

$$M_n(x, f_{\text{low}}, f_{\text{high}}) = \int_{f_{\text{low}}}^{f_{\text{high}}} f^n |S(f, x)| df \quad (7.1)$$

where $|S(f, x)|$ is the amplitude of the spectrum, f_{low} and f_{high} are the lower and upper bound of frequency ranges, $x = vt$ is the location of the passing train and v is the velocity of the passing train. It can be noted that PMS is a special case when $n = 0$, and it can be considered as the energy of the response in the integration frequency range.

The ideal spectrum from an impact response is a series of delta functions, and each peak indicates one vibration mode. However, when the structure suffers random excitation, the spectrum usually becomes several dominated frequency bands. This is also the reason that the resonance frequencies cannot be extracted accurately in practice. Considering the fact that if the integration range contains only one frequency f_i , the frequency can be represented as the ratio of M_1 and M_0 , i.e.

$$f_i^* = \frac{\int_{f_{i,low}}^{f_{i,high}} f |S(f)| df}{\int_{f_{i,low}}^{f_{i,high}} |S(f)| df} = \frac{\int_{f_{i,low}}^{f_{i,high}} f \delta(f - f_i) df}{\int_{f_{i,low}}^{f_{i,high}} \delta(f - f_i) df} = f_i \quad (7.2)$$

Therefore, by extending above equation to response from random excitation, an approximate estimation of the resonance frequency of the interactive system when the train passes x can be proposed as

$$f_i^*(x) = \frac{M_1(x, f_{i,low}, f_{i,high})}{M_0(x, f_{i,low}, f_{i,high})} = \frac{\int_{f_{i,low}}^{f_{i,high}} f |S(f)| df}{\int_{f_{i,low}}^{f_{i,high}} |S(f)| df} \quad (7.3)$$

where $f_{i,low}$ and $f_{i,high}$ are the lower and upper bound of the i -th frequency band and they should be selected to contains only the i -th resonance frequency, i.e.

$$\begin{aligned} f_{i-1} &< f_{i,low} < f_i \\ f_i &< f_{i,high} < f_{i+1} \end{aligned} \quad (7.4)$$

where f_{i-1} , f_i and f_{i+1} are the $(i-1)$ -th, i -th and $(i+1)$ -th resonance frequency of the interactive system, respectively.

There are two advantages of estimator defined in Eq. (7.2). First, it only requires a rough frequency range other than the exact frequency location; then, it eliminates the influence of the excitation energy by dividing the 0-th MoS, and thus, it is can be used for non-constant energy excitation condition. Based on this estimator, a damage index similar to the one described in Chapter 6 can be proposed as shown in Fig. 7.1

$$D_j^* = \frac{f_{i,j}^* - \bar{f}_i^*}{\max(f_i^*(x))} \quad (7.5)$$

$$f_{i,j}^* = \frac{1}{\Delta} \int_{(j-1/2)\Delta}^{(j+1/2)\Delta} f_i^*(x) dx \quad (7.6)$$

$$\bar{f}_i^* = \frac{1}{N\Delta} \int_0^{N\Delta} f_i^*(x) dx \quad (7.7)$$

where Δ is the interval between two adjacent springs, N is the number of intervals, i indicates the i -th mode.

Fig. 7.1 Illustration of damage index by using the fundamental vibration mode

7.2 NUMERICAL VERIFICATIONS

Numerical verifications have been carried out by ABAQUS, the general purpose FEM software and the model is shown in Fig. 7.2. The mass-spring system has a mass of 2 kg and stiffness of 1×10^{10} N/m. The rail track is modeled as a simply supported Euler beam with a length of 20 m. It has a cross-sectional area of 5×10^{-3} m² and bending stiffness of 1.15×10^6 Nm². It is supported by 9 evenly distributed springs and the distance between two neighboring springs is 2 m. The support in the middle, i.e. k_5 , is degraded, while others have identical stiffness of 2×10^7 N/m. The velocity of the moving mass-spring system is 0.5 m/s. Gaussian white noise is applied to excite the mass-spring system and its standard deviation is 1. The sampling frequency is 1000 Hz. The parameters for spectrograms are Hanning window, the overlap is 50%, the number of data points in each block of FFT is 500, and therefore, the frequency resolution is 2 Hz.

Fig. 7.2 Illustration of the finite element model

7.2.1 Detection of damaged support under random excitation

Because the stiffness of the undamaged support is selected as 2×10^7 N/m, the fundamental frequency is more sensitive to the reduction of stiffness at the damaged support as shown in the analyses carried out in Section 3.2.4. Therefore, only the fundamental frequency is used. The spectrograms of the signal on the acceleration of the moving mass-spring system are shown in Fig. 7.3. Fig. 7.3(a) shows the spectrogram of dynamic response when the supports are intact. Although the periodic frequency shift pattern can be observed, it is still difficult to extract the frequency accurately because various peaks can be found near the resonance frequency. For the case that the fifth support is damaged and the stiffness decreases by 50%, the pattern of the spectrogram changes slightly (Fig. 7.3(b)), and the frequency drops slightly when the train passes through the damaged support. As the damage is getting more severe and the stiffness of the fifth support decreases by three quarters, drop of frequency can be seen and peaks of frequency near 50 Hz are obvious (Fig. 7.3(c)). Fig. 7.3(d) shows the spectrogram for the case that the stiffness of the fifth support decrease to 10%; it is observed that the spectrum pattern changes significantly when the mass-spring system passes through the fifth support. The fundamental resonance frequency of the interactive system decreases to nearly 30 Hz when the mass is just above the damaged support, while it remains nearly 70 Hz when the train passes through the undamaged supports.

(a) Intact case

(b) The fifth support suffers 50% stiffness reduction

(c) The fifth support suffers 75% stiffness reduction

(d) The fifth support suffers 90% stiffness reduction

Fig. 7.3 The spectrogram of acceleration of the passing train for the cases of fifth support having different stiffness reductions

The frequency range for calculating the approximate estimation of the resonance frequency is selected from 10 Hz to 100 Hz because the fundamental resonance frequency of the interactive system fall within this range as shown in Fig. 7.3. Fig. 7.4 presents the approximate estimation of frequency based on Eq. (7.3). For the undamaged case, periodic frequency shift is shown although it is not as smooth as that plotted in Fig. 6.2, which is because the beam is subjected to random excitation. Once the fifth support is damaged and the stiffness decreases, the estimated fundamental resonance frequency drops clearly when the train is passing over the damaged support, while it remains stable and periodic if the train passes through the undamaged supports. Moreover, it is observed that the frequency drops more heavily as the damage severity increases, which coincides with the observation found in Chapter 6.

Fig. 7.4 Approximate estimation of frequency for different cases

Fig. 7.5 shows the damage index proposed in Eq. (7.5), and it performs well in indicating the damaged supports. For the healthy case, the damage indices at all the nine supports are almost zero and no peak indicating the degraded support can be found. For the damaged cases, the damage index at the fifth support is clearly larger than those at undamaged supports. Moreover, the value of the damage index also presents the severity of the damage. When the stiffness decreases by 50%, 75%, 90%, the corresponding damage index increases from 0.05 to 0.12 and 0.17, respectively.

Fig. 7.5 Damage index for different cases

7.2.2 Effect of different moving velocity

As is discussed in Chapter 5, moving velocity is a key factor in the inspection method. Generally, if the velocity is lower, the inspection will be better and vice versa. The effect of different moving velocity on the performance of the proposed damage index is investigated through numerical examples. Four different velocities of 0.5 m/s, 1 m/s 2 m/s and 8 m/s are applied. The stiffness of the fifth support is assumed to decrease by 75% for all the four cases.

Fig. 7.6 Approximate estimation of frequency for different cases

Fig. 7.6 shows the approximate estimation of frequency with different velocities. It is observed that the estimated frequencies are quite close even the speed increases from 0.5 m/s to 8 m/s, especially when the train passes through the damaged supports. It may be concluded that the method performs pretty well under different velocity. Fig. 7.7 presents the damage index for the cases with different train velocities, and it is also observed that the proposed damage index performs reasonably well even the train is passing through the track at speed of 8 m/s.

Fig. 7.7 Damage indices for different cases

7.2.3 Effect of stiffness on undamaged supports

As analyzed in Chapter 3, the foundation stiffness plays an important role in selecting a suitable vibration mode for detecting damaged support. A validation analysis is performed here. Except for the intact stiffness is changed from 2×10^7 N/m to 2×10^8 N/m, the other parameters are kept the same. The case investigated here is with 75% stiffness reduction. Because the natural frequencies of the interactive system are increased due to the increase of support stiffness, the frequency ranges used in calculating MoS are also changed accordingly. In particular, $f_{1,\text{low}} = 10\text{Hz}$, $f_{1,\text{high}} = 10\text{Hz}$ and $f_{2,\text{low}} = 10\text{Hz}$, $f_{2,\text{high}} = 10\text{Hz}$ are selected for $k_i = 2 \times 10^7$ N/m cases; $f_{1,\text{low}} = 80\text{Hz}$, $f_{1,\text{high}} = 150\text{Hz}$ and $f_{2,\text{low}} = 150\text{Hz}$, $f_{2,\text{high}} = 250\text{Hz}$ are selected for $k_i = 2 \times 10^8$ N/m cases.

Fig. 7.8(a) and (b) show the approximate estimation of the fundamental and second order resonance frequencies when the supports stiffness are weaker ($\lambda_s=138.7$). It is observed that f_1^* drops significantly when the mass is over the degraded support while f_2^* only fluctuates periodically. Fig. 7.8(c) and (d) show f_1^* and f_2^* when the supports stiffness are stronger ($\lambda_s=1387$). It is found on the contrary that f_1^* fluctuates periodically as the train passes through the track, while f_2^* drops clearly when the train passes over the damaged support. This observation coincides with the analysis carried out in Chapter 3 quite well: for the weaker supports the fundamental resonance frequency is more sensitive to the reduction of damaged support, the second order natural frequency plays a more important role in identifying the damaged support if the supports are stronger.

Fig. 7.8 Approximate estimation of frequency for different cases

(a) f_1^* ; $k_1=2 \times 10^7$ N/m (b) f_2^* ; $k_1=2 \times 10^7$ N/m (c) f_1^* ; $k_1=2 \times 10^8$ N/m (d) f_2^* ; $k_1=2 \times 10^8$ N/m

The same observations can also be obtained from Fig. 7.9, which show the damage indices based on the first two vibration modes of the cases with different support stiffness. When the supports of the track are weaker, the damage index based on the fundamental frequency

indicating the damaged support is as high as 0.12, while that based on the second vibration mode is almost zero. The results are just contrary if the supports are stronger. The second vibration mode produces a higher damage index at the degraded location, while the fundamental mode provides almost flat damage indices.

Fig. 7.9 Damage index for different cases
(a) $k_i = 2 \times 10^7$ N/m case, (b) $k_i = 2 \times 10^8$ N/m case

7.2.4 Numerical verifications on a half bogie model

A coupled ‘half-bogie’ vehicle model is investigated in this section to show the performance of the method for a typical running rail system. The parameters of the vehicle and rail are extracted from Quirke et al. (2016) as shown in Fig. 7.10. The parameters of the rail are shown in Table 7.1. The length of the beam is 54.5 m, i.e., 100 intervals between sleepers. The 19th spring from left side of the model is reduced to 50%, 25% and 10% in different simulation cases. The velocity of the train is set to be 5 m/s, 10 m/s and 20 m/s. It takes 109 ms, 54.5 ms and 27.25 ms for a train to pass an interval of sleepers. The acceleration of the wheel is extracted for analysis.

Fig. 7.10 Half-bogie model and its parameters

Table 7.1 Parameters of rail system in the simulations

Property	Symbol	Value	Unit
Elastic modulus of rail	E_r	2.059×10^{11}	Pa
Cross area of rail	A_r	7.69×10^{-3}	m^2
Rail second moment of area	I_r	3.217×10^{-5}	m^4
Rail mass per unit length	μ_r	60.64	kg/m
Sleeper spacing	L_s	0.545	m

The estimation of frequencies for 5 m/s cases are shown in Fig. 7.11(a). Under this velocity, the method works well. Because the total number of supports in the model is too large, the damage indices in the range of 1030 are shown in Fig. 7.11(b). It is clearly shown in the figure, the stiffness reduction can be located at the degraded location.

(a) Estimated frequencies

(b) Damage indices

Fig. 7.11 Results from 5 m/s cases (109 ms per segment)

If the velocity of moving device is increased to 10 m/s, the estimation of frequency becomes very noisy. This is because of the uncertainty of the excitation and the short length of analysis segment (54.5 ms of the signal). The estimated frequencies are shown in Fig. 7.12(a), and the corresponding damage indices are shown in Fig. 7.12(b). There is no obvious signal appears in the analysis results.

(a) Estimated frequencies

(b) Damage indices

Fig. 7.12 Results from 10 m/s cases (54.5 ms per segment)

However, this difficulty can be easily overcome by increasing the length of the analysis segment. This can be achieved by overlapping analysis segments. In following text, the length of the analysis segment is increased to $3 \times 54.5 \text{ ms} = 163.5 \text{ ms}$, and overlap between two segments is $2 \times 54.5 \text{ ms} = 109 \text{ ms}$. The frequency estimations of the signal with these parameters are shown in Fig. 7.13(a), and the corresponding damage indices are shown in Fig. 7.13(b). The stiffness reduction locations are clearly demonstrated in the figure. However, the spatial resolution of the method is decreased, due to the increase of the length of analysis segment.

(a) Estimated frequencies

(b) Damage indices

Fig. 7.13 Results from 10 m/s cases (163.5 ms per segment)

Furthermore, the velocity of the moving device is increased to 20 m/s. The same trick used in 10 m/s cases is also applied here. The length of analysis segments is $3 \times 27.25 \text{ ms} = 81.75 \text{ ms}$, and the overlap between two analysis segments is $2 \times 27.25 \text{ ms} = 54.5 \text{ ms}$. The frequency estimations are shown in Fig. 7.14(a). The location where stiffness reduction located is clearly shown in the figure, but the frequency estimations are also not as steady as in previous cases. The damage indices are shown in Fig. 7.14(b). The stiffness reduction can be easily located from the figure.

(a) Estimated frequencies

(b) Damage indices

Fig. 7.14 Results from 20 m/s cases (81.75 ms per segment)

7.3 EXPERIMENTAL STUDY

7.3.1 Experimental setup

Fig. 7.15 shows the experimental setup, while Fig. 7.16 shows the corresponding analytical model. The small scaled running rail model is investigated here. The rail has a length of 1.6 m, cross-sectional area of 0.0003 m^2 , bending stiffness of 5253 Nm^2 and density of 7800 kg/m^3 . It

is supported by five plastic plates which are evenly distributed. Under the plastic plate there is a small piece of plywood to perform acting as ballast. For each plate, which is regarded as sleeper in this study, the contact area of the scaled rail is $2.6 \times 3.0 \text{ cm}^2$. Considering that the elastic modules of the plywood is quite small, the support stiffness is small, and therefore, the dimensionless stiffness λ_s is considerable small. Therefore, the fundamental vibration mode should be more sensitive to the degraded support, and this is the focus of in this experiment. A Wilcoxon Model F3/Z602WA shaker of mass 0.3 kg with a magnetic base is used to simulate the passing train, and hence, to generate the random excitation. There is a coaxial accelerometer embedded in the shaker which can measure the vertical acceleration as it passes through the rail track. A linear guide controls the motion of the shaker, and two different velocities of 20 mm/s and 80 mm/s are used in this study.

Fig. 7.15 Experimental setup showing the shaker, rail and linear motion unit

Fig. 7.16 Analytical model of the experiment test

Four scenarios are considered in the experiments (Fig. 7.17): the first one is the undamaged case that all supports are fastened with uniform torque of 8 Nm; the second one has a degraded support (the third one) and the torque of corresponding pair of fasteners reduces to 4 Nm; the torque further reduces to 0 Nm in the third one; the last one loses a pair of fasteners and a sleeper.

Fig. 7.17 Illustration of four damage scenarios

7.3.2 Experimental results

Fig. 7.18 shows the power spectrogram of the acceleration of the passing shaker of the four cases where the velocity is 20 mm/s. The sampling frequency is 8192 Hz. The following parameters for spectrogram calculation are: Hanning window, the overlap is 50%, the number of data points used in each block of FFT is 4096, and therefore, the frequency resolution is 2 Hz.

For Case 1 in which the supports are undamaged, periodic frequency shift can be observed clearly, and the fundamental frequency of the system is nearly 650 Hz. It drops slightly when the shaker passes over the third support in the second case, and it further drops in the third case when the damage is more severe. Finally, it drops significantly to 200 Hz when the third support is removed in the fourth case. For the first three cases, $f_{1,low} = 500\text{Hz}$ and $f_{1,high} = 750\text{Hz}$ are selected so that the fundamental natural frequency is included. However, $f_{1,low} = 100\text{Hz}$ and $f_{1,high} = 750\text{Hz}$ are chosen for the final case because the fundamental frequency drops to nearly 500 Hz when the shaker passes over the damaged support.

(a) Intact case

(b) Partial loose case

(c) Total loose case

(d) Missing case

Fig. 7.18 The acceleration spectrograms of the passing shaker for the four damage cases

The damage indexes are calculated for all four cases with three different velocities. Due to limitation of rail track length, only five supports are used. Hence, the same technique as used in Section 6.6.3 is also applied here, i.e. the interval for calculating damage index is adjusted as shown in Fig. 7.19. Then, there should be 8 damage indices along the rail track. If the damage index is calculated based on Eq. (7.5) and only five damage indices are obtained, boundary effect will significantly influence the performance of the proposed method.

Fig. 7.19 Interval for calculating damage index in experiment

Fig. 7.20 shows the damage index when the velocity is only 20 mm/s. The damage index is always small and near 0 for the undamaged case. The third to sixth damage indices become larger for Case 2 because the middle support loses stiffness slightly and they further increase

for Case 3. Finally, they become quite large when the middle support is removed. Fig. 7.20 shows the damage index when the velocity increases to 80 mm/s. It is similar to Fig. 7.20 (a) which validates the proposed method very well even the train speed is high.

(a)

(b)

Fig. 7.20 Damage index for four cases with velocities (a) 20 mm/s and (b) 80 mm/s

7.4 SUMMARY

The method proposed in the previous chapters is extended to random excitation. Because the frequencies are difficult to be extracted accurately, a frequency estimator inspired by PMS is proposed in this chapter. Then, a similar damage index is applied for detecting degraded supports. Numerical examples and experimental verifications are both presented to evaluate the performance of the method under random excitation. Different degraded severity and moving velocities are studied. Generally, the damage index performs well in all these conditions. Therefore, the proposed method may be used under random excitation or environmental excitation. However, it should also be noticed the frequency estimator is much fluctuated than frequencies extracted from impact excitation. It is noted that the results suffer more uncertainty, and the impact excitation solution is also suggested if applicable.

CHAPTER 8

CONCLUSIONS AND FUTURE WORKS

8.1 SUMMARY AND CONCLUSIONS

The main objective of this research work is to develop a workable solution for inspection of degraded supports in a rail system. Based on current inspection techniques and previous research studies, a moving inspection device is an optimal choice, and thus is proposed in this study. The main findings are summarized as follows:

1. Degraded sleepers and loosening fasteners reduces the contact stiffness between rail and supporting system causing the decrease of natural frequencies. However, natural frequencies are global parameters which are not related to spatial parameter, and thus it cannot be used to detect localized degraded supports.

2. Frequency shift caused by auxiliary mass at different locations along the rail is related to mode shapes which contain local information and they are shown to be sensitive to stiffness reduction. Further analysis shows that degradation of supports influence the shape of frequency shift curves. Furthermore, changes of some frequency shift curves are mainly located in the nearest span where degraded support is located. Hence, they can be a useful feature to locate the degraded support accurately. Moreover, sensitive modes and optimal inspection locations are also provided by the analytical solutions.

3. Frequency shift curves (FSC) can be extracted from responses captured from the moving auxiliary mass. Under quasi-static condition, the process can be viewed as a modal analysis procedure where input and output are captured at the same location if the influence of the mass is neglected. Thus, the response level near a natural frequency is roughly proportional to the mode shape square. Therefore, both FSCs and the amplitudes offer localized information that can be used to inspect the structures. In this study, a damage index namely FRESH path is proposed to detect stiffness reduction. However, amplitude component of FRESH path proportional to mode shape square also implies that FSCs are curves with varying SNR, and SNR near nodal points are very low. Consequently, confidence interval of points in FSCs are

quite different, and using frequency points at a location near nodal points requires further attentions.

4. Under moving condition, amplitude component of FRESH path is not proportional to mode shape square due to uncertainty of contact properties between the moving device and rail, and environmental noises. However, frequency component is much stable and resistant to noises. In fact, wide band noises such as friction induced vibration, do not influence the frequency captured at that location. It can be treated as an extra-input source. Therefore, amplitude component is not an ideal feature for inspection under moving condition, but it is a good indicator of the SNR. An adaptive threshold determination procedure using cluster method and quantile is proposed based on this finding.

5. Periodic property of rail system is a useful feature used to simplify the inspection solution. A damage index based on average of frequency in one period of rail system is proposed. It is more resistant to noise. The damage index is found to be able to localize the degraded support accurately, and its magnitude is related to the severity of the degraded support.

6. Frequencies are difficult to be extracted accurately under random excitation. The ratio between the first and zero-th MoS is a frequency estimator can be calculated without providing an accurate frequency. It is found to be a reasonable replacement of frequency shift curve under random excitation.

8.2 FUTURE WORKS

This study has accomplished the objectives. However, several works can also be further extended and preliminary ideas for the future work are suggested:

1. A traditional accelerometer is used in this study. However, non-contact sensor is another possible choice to explore (Martinez et al., 2016; OBrien et al., 2016). Lasorvibrometer paired with source separation methods is a promising solution. The advantages are obvious: the interference with the structure can be minimized; electrical isolation and installation procedure are much simpler.

2. As shown in the study, periodic property is a useful tool to simplify the inspection procedure and enhance noise resistance. Multiple sensor scheme is another interesting way to

explore (Kong et al., 2015; OBrien & Keenahan, 2015). The signals can be used to remove noise or enhance the frequency extraction procedure.

3. Experimental studies conducted in this study using small scale models with very limited length. Experiments carried out on-site in a real rail system is very crucial for determining the parameters and performance of the method.

4. The proposed method can be applied to other types of structures and the results obtained from the study can be a good starting point for vibration based method for fasteners in other civil engineering infrastructures.

5. Frequency component and amplitude component discussed in FRESH path are strong tools for detecting stiffness reduction of structures. Due to the vulnerable of rail system, the excitation power must be carefully controlled. The amplitude component is heavily influenced by noises. Therefore, frequency component is the focus of this study. However, amplitude component is also a useful feature for damage detection. Infrastructures like bridges have very low natural frequency, and thus the frequency shift may be easily contaminated by frequency estimation error. However, the low frequency mode usually have large amplitude, and thus the amplitude component may play a more positive role in inspection, which can be further studied.

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