Nonlinearly enhanced vortex induced vibration energy harvesting systems : characterization and chaotic mitigation

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**NANYANG
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UNIVERSITY**

SINGAPORE

**NONLINEARLY ENHANCED VORTEX INDUCED
VIBRATION ENERGY HARVESTING SYSTEMS:
CHARACTERIZATION AND
CHAOTIC MITIGATION**

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CHARACTERIZATION AND
CHAOTIC MITIGATION**

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A thesis submitted to the Nanyang Technological
University in partial fulfilment of the requirement for
the degree of
Doctor of Philosophy

2017

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research and has not been submitted for a higher degree to any other University or Institution.

04 August, 2017

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Abstract

The world energy supply is still mainly relying on fossil fuels, which are hardly renewable and environmentally destructive. Therefore, technologies to harvest renewable energy have been developed as a solution. Among various renewable energy resources, marine energy is receiving intensive attention due to its cleanliness and worldwide availability. However, the energy supply from this resource is minor due to several critical shortcomings such as high cost of produced power, location dependence and limited working range.

As a new alternative to the current marine energy technologies, Vortex Induced Vibration (VIV) energy harvesting systems have been recently developed. The main structure of a VIV system is a cylinder elastically supported by linear springs and perpendicularly immersed into a water flow. Due to the vortex shedding effect, the induced fluid force will excite the structure and consequently lead to vibration of the structure. Utilizable energy can be produced by connecting the vibrating structure to a conversion mechanism. The prominent merits of the VIV technology compared to other renewable energy technologies are its capability to harvest energy from low velocity water flows, relatively high output efficiency and low cost of produced power.

The VIV system operates effectively at the water flow velocity that corresponds to its resonance frequency. When the flow velocity deviates from the resonance range, the effective operation is hardly maintained. This situation likely occurs since natural water flows inherently fluctuate in velocity. This challenge motivates the first objective of this thesis, which is to apply the bi-stable and hardening springs in order to widen the resonance range of the system. The investigation is carried out based upon the numerical simulation on a wake oscillator model and experimentally validated by a dedicated cyber-physical force-feedback testing platform. It is found that the bi-stable spring can widen the resonance towards the side of the low velocity flows. On a contrary, the resonance range is extended towards the side of the high velocity flows when the

system is enhanced by the hardening spring. Therefore, a combined nonlinear spring is proposed and proven to be able to extend the resonance range towards both sides of the low and high velocity flows.

Meanwhile, it is also observed that the bi-stable spring might result in chaotic responses that drastically lower the output power. Therefore, the second objective of this thesis is to quantify chaotic responses of the bi-stable VIV system through calculating the Lyapunov exponent, which is an effective tool to measure the chaotic degree. The quantification is performed in a wide range of the governing parameters to construct a comprehensive bifurcation map that presents the dependency of chaotic responses on these parameters.

As the third objective, this thesis focuses on the case where the bi-stable VIV system is designed in such the output power is significantly improved at the low velocity flows and chaotic responses occur when the flow velocity increases. Therefore, the Ott-Grebogi-Yorke (OGY) and time delay feedback controllers are designed to actively stabilize chaotic responses. Both the controllers are shown to be able to stabilize chaotic responses and improve the output power up to 73.5 % compared to that of the uncontrolled chaotic system.

This thesis is completed with three main contributions. The first contribution is the characterization of the VIV system enhanced by the hardening and bi-stable springs and the proposal of the combined nonlinear spring. The second contribution is attributed to the quantification of chaotic responses of the bi-stable VIV system. The third contribution is the design and implementation of the OGY and time delay feedback controllers to stabilize chaotic responses of the bi-stable VIV system.

List of Publications

Main findings of this thesis have been published as:

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- B.H. Huynh, and T. Tjahjowidodo, “Experimental Chaotic Quantification in Bistable Vortex Induced Vibration Systems,” *Mechanical Systems and Signal Processing*, vol. 85, pp. 1005-1019, 2017.
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- B.H. Huynh, T. Tjahjowidodo, Z. Zhong, Y. Wang, and N. Srikanth, “Chaotic Responses on Vortex Induced Vibration Systems Supported by Bistable Springs,” in *Proceedings of the International Conference on Noise and Vibration Engineering*, Leuven, Belgium, 2016, pp. 695-704.

- B.H. Huynh, T. Tjahjowidodo, Z. Zhong, Y. Wang, and N. Srikanth, “Nonlinearly Enhanced Vortex Induced Vibrations for Energy Harvesting,” in *Proceedings of the Advanced Intelligent Mechatronics IEEE International Conference*, Busan, South Korea, 2015, pp. 91-96.

Abstracts:

- B.H. Huynh, T. Tjahjowidodo, Z. Zhong, Y. Wang, and N. Srikanth, “Improving Vortex Induced Vibrations for Energy Harvesting by Nonlinearities,” in *Book of Abstracts of the 2015 Asia Clean Energy Summit*, Singapore, 2016, pp. 100.

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Abbreviations

AMI	Average Mutual Information
DNS	Direct Numerical Simulation
ERI@N	Energy Research Institute at Nanyang Technological University
FEM	Finite Element Method
FRF	Frequency Response Function
IEA	International Energy Agency
OGY	Ott-Grebogi-Yorke
PTC	Passive Turbulence Control
PTO	Power Take-Off
RMS	Root Mean Square
VIC	Vortex-In-Cell
VIV	Vortex Induced Vibration
VIVACE	Vortex Induced Vibration Aquatic Clean Energy

Chapter 1

Introduction

In the first place, this chapter explains the necessity to develop renewable energy technologies in order to alleviate the dependence on fossil fuels of the world energy supply. Next, the concept of harvesting energy from water flows by means of vortex induced vibrations (VIV) as a new potential renewable energy technology is briefly introduced. Subsequently, the main problem that limits the applicability of a VIV energy harvesting system followed by the research gap as the motivation of this thesis is demonstrated. Next, the main objectives and approach of this thesis are clarified. Finally, the organization of this thesis is presented in the chapter by chapter overview.

1.1 Background, problem statement and research gap

1.1.1 Background

A clean, sustainable and affordable energy resource is currently one of the critical global demands since world energy supply is mainly relying on fossil fuels, which are hardly renewable and environmentally destructive. Statistics of The International Energy Agency (IEA) in 2016 [1] show that despite the reduction over years, the world energy supply from fossil fuels that include coal, oil and natural gas remains at a high level, i.e. corresponding to 86.7% in 1973 and 81.1% in 2014 of the total world energy supply. Based on an estimation conducted by Shafiee and Topal [2], fossil fuels will be completely depleted after 2112. It is straightforward to explain the exhaustion of fossil fuels in the near future since it takes millions of years for the formation of fossil fuels, while they are depleted much more rapidly. Another consequence from producing and utilizing energy from fossil fuels is related to the emission of carbon dioxide (CO₂) from fuel combustion, to which the global warming phenomenon is attributed. The statistics of The IEA [1] also indicate that combustion of fossil fuels to produce energy disposes a huge amount of CO₂ annually and this amount keeps increasing continuously over years, e.g. combustion of fossil fuels disposed about 30,000 Mt of CO₂ in 2014, which has been doubled since 1973. Inevitably, other clean and sustainable energy technologies should be developed to replace fossil fuels.

As a potential alternative, technologies to harvest energy from renewable resources have been developed to relieve the dependence on fossil fuels. Qualitatively, a renewable energy resource can be distinguished from a non-renewable one due to its capability to be naturally replenished in the timescale of human being. Currently, there are seven types of renewable energy resources that include solar energy, wind energy, geothermal energy, bio-energy, hydro-energy, hydrogen energy and marine energy. In particular, marine energy is receiving intensive attention due to its abundance and ubiquitous accessibility [3]. In general, marine energy is available in three main

forms, namely wave energy, tidal energy and flow energy (energy from thermal gradient and salinity gradient can be included in the last form). Table 1.1 summarizes the current technologies and devices to harvest marine energy in these three main forms.

Table 1.1 Current technologies and devices to harvest marine energy.

Forms	Technologies and devices
Wave energy	Water column, pendulum, buoy, flap, Wave Dragon, Pelamis, point absorber...
Tidal energy	Tidal barrage (dam), tidal lagoon, tidal reef, tidal fence...
Flow energy	Watermill, water turbine, VIV energy converter...

Despite various advantages, there are numerous critical issues that lead to the very minor supply from the marine energy resource, which was less than 1.4% of the total world energy supply in 2014 [1]. Bernitsas, et al. [4] summarized the critical shortcomings of current marine energy technologies as:

- Systems to harvest wave energy, e.g. water columns, pendulums, buoys, and flaps, can only provide high output power in a limited resonant working frequency range. In addition, these systems require very robust structures to bear extremely high loads from ocean waves.
- Systems to harvest flow energy such as watermills and turbines require a flow velocity that is higher than 2 m/s to provide the acceptable output efficiency of 15 – 30%.
- Systems to harvest tidal energy such as water dams require a huge installation space, long construction period and high capital cost.
- Most of systems to harvest marine energy occupy valuable coastal estates. In addition, systems such as water dams, watermills and turbines erratically interrupt marine life.

Inevitably, in order to harvest marine energy more effectively and sustainably, current technologies require full developments and novel technologies need to be explored to overcome the aforementioned critical limitations. As a response to this demand, vortex induced vibration (VIV) systems to harvest flow energy have been recently introduced and developed. The prominent advantage of this system is its capability to harvest energy from low velocity flows, i.e. from 0.3 m/s onwards [4]. As indicated in [5], apart from a few high velocity ocean flows, most of shallow ocean flow velocities are lower than 1.5 m/s and stream/river flow velocities are lower than 1 m/s. Another significant merit of the VIV energy harvesting system is its high output efficiency, which is about 22% in the experiment and can be improved up to 37% based on the theoretical calculation [6]. Furthermore, the cost of power produced by the VIV technology is the lowest in the group of the alternative technologies [4]. In addition, the VIV energy harvesting system has been proven to be advantageous in its features related to the scalability and modularity, the produced energy density, navigation, coastal properties, marine life, maintenance, robustness, life cycle cost and design life [4].

1.1.2 Problem statement

VIV is a prevalent fluid-structure interaction phenomenon that occurs on many engineering structures that undergo fluid flows, e.g. risers, pipes, chimneys, suspended cables and mooring lines. When a fluid flow, e.g. wind and water flow, crosses over a structure, due to the flow separation, vortices are formed and alternately shed into two sides of the wake region. The vortex shedding consequently causes periodic drag and lift forces on the structure. Under the excitation of these induced forces, the structure will vibrate and the vibration is referred to as VIV. The VIV phenomenon is usually undesired and must be suppressed since its tremendous vibrational energy might destroy the engineering structures. The collapse of the Tacoma Narrows Bridge in 1940 is one of the most notorious examples of the incidents.

Nevertheless, it has also been realized that the plentiful energy from the VIV phenomenon can be harvested and converted into other utilizable forms of energy. The first energy converter based on this phenomenon was introduced in 2008 and trademarked as the VIVACE energy converter [4]. The working principle of the VIVACE converter is rather straightforward. The main component, which is a cylinder, is transversely immersed into a water flow. The cylinder is elastically supported by linear springs, where its motion is constrained to the cross-flow direction. Therefore, the structure will vibrate under the effect of the induced lift force when the flow crosses over the cylinder. Connecting the structure to a transmission mechanism to convert the kinetic energy to electrical energy allows for the utilization of the harvested energy.

A VIV energy harvesting system only operates efficiently when the VIV structure vibrates with a significantly high amplitude. Physical properties of the VIV system that include the effective mass, equivalent stiffness of the supporting springs, damping coefficient, diameter and length of the cylinder define a range of water flow velocity where the system vibrates with significantly high amplitude. This range of flow velocity is referred to as the resonance range (as illustrated in Fig. 1.1). The induced fluid force excites the structure at a frequency that is determined by the flow velocity. Therefore, the resonance range corresponds to the vicinity of the flow velocity that induces the fluid force at the natural frequency in water of the VIV structure. In short, the system must operate in the resonance range in order to offer the effective performance. If the flow velocity fluctuates beyond the resonance range, the vibrating amplitude is hardly maintained at the high level and the performance of the system deteriorates.

Meanwhile, velocity of ocean and river flows fluctuates frequently. A measurement of the actual flow velocity at the marine test bed launched by the Energy Research Institute at Nanyang Technological University (ERI@N), which is located at the Sentosa Boardwalk, Singapore, indicated that the flow velocity fluctuated in the range of 0.1 – 1.2 m/s in one-day duration [7]. Therefore,

there is a necessity of studies to widen the resonance range of a VIV energy harvesting system in order to enhance its performance and applicability.

Figure 1.1 Illustration of the resonance range of a VIV system.

1.1.3 Research gap

Since the introduction of the first VIV energy harvesting system, numerous studies have been carried out to improve its performance. Theoretical studies in [8-10] focused on analyzing effects of the physical parameters including the effective mass, damping coefficient and supporting stiffness on the energy harvesting performance of a VIV system. In addition, experimental studies in [11-17] proposed new methods to improve the vibrating amplitude and energy conversion efficiency of a VIV energy harvesting system. For example, the passive turbulence control (PTC) method with the attachment of waterproof sandpaper strips on the cylinder [11-13], and the method of attaching wires along the cylinder [14, 15] were proposed and experimented to improve the vibrating amplitude of the VIV system. However, there is a lack of studies to widen the resonance range of a VIV energy harvesting system to maintain its effective operation subjected to the fluctuation of water flow velocity.

Among various methods to improve working ranges of miniaturized vibration-based energy converters, the method of utilizing nonlinear springs is considered as the most promising method. There are two main types of nonlinearity, namely geometric nonlinearity and material nonlinearity [18]. In a system with the geometric nonlinearity, the change in the geometry of the system such as displacement will cause a nonlinear change in the system parameters such

as the body force. In a system with the material nonlinearity, the behavior of material depends on the deformation of the system. Fig. 1.2 illustrates the main types of nonlinearity and their examples. Nonlinear springs are the common nonlinear structures and characterized as the geometric nonlinearity.

Figure 1.2 Main types of nonlinearity and their examples.

Although nonlinear springs have been extensively applied in many studies (e.g. [19-22]) to widen the working range of miniaturized energy converters, such studies for a VIV energy harvesting system, which is dynamically different from miniaturized converters due to its complex fluid-structure interaction effect, are hardly found in literature. Only few studies in [23, 24] analyzed effects of hardening springs on a VIV energy harvesting system with very limited investigations of the parameters that govern the nonlinearity in the springs.

1.2 Objectives and approach

1.2.1 Objectives

Motivated by the lack of studies to apply nonlinear springs in order to widen the resonance range of a VIV energy harvesting system, this thesis is oriented by the following objectives:

- The first objective is to investigate effects of the two most potential nonlinear springs, which are hardening spring and bi-stable spring, on the resonance range of a VIV energy harvesting system. This investigation is conducted in a wide range of the parameters that govern the nonlinearity of the springs.

- A bi-stable spring at the same time poses a potential risk of introducing chaotic responses to the system. Chaotic responses result in a negative effect on the energy harvesting performance. Therefore, as the second objective, chaotic responses of the VIV system enhanced by a bi-stable spring are quantified as a function of the structural parameters in order to provide a guideline to design the system with the avoidance of chaotic responses.
- Finally, as the third objective, appropriate control strategies to actively stabilize chaotic responses of a VIV system enhanced by a bi-stable spring are designed and analyzed to confirm the applicability of a bi-stable VIV energy harvesting system.

1.2.2 Approach

In this thesis, the geometrically nonlinear restoring force functions of the nonlinear springs are applied in a wake oscillator model to dynamically model the nonlinear VIV energy harvesting system. By numerically solving this model, dynamics and output power of the system at different values of water flow velocity can be simulated to analyze effects of the springs on the energy harvesting capability. Besides, this wake oscillator model is utilized to simulate chaotic responses of the bi-stable VIV system and perform the chaotic quantification. In addition, the design and implementation of the controllers to stabilize chaotic responses of the bi-stable VIV system are also conducted on the wake oscillator model.

At the same time, experiments of the VIV system enhanced by the nonlinear springs in a real water channel are performed to validate the theoretical analyses based on the wake oscillator model. For this purpose, a cyber-physical force-feedback testing platform was designed and developed. The physical properties of the VIV system, and in particular, the nonlinear restoring force functions, can be conveniently set up in the experiments on this testing platform due to a combination of the simulation and experimental processes.

1.3 Chapter by chapter overview

Chapter 2 discusses all the main issues that will be addressed in this thesis in a general manner. Firstly, an overview on the main advantages and shortcomings of the current marine energy technologies is presented. Subsequently, the general concept and governing factors of the VIV phenomenon for energy harvesting are introduced. In addition, methodology to study the VIV phenomenon, and the state-of-the-art studies to improve the performance of a VIV energy harvesting system are also reviewed. Secondly, potentials from the hardening and bi-stable springs are analyzed. Thirdly, the necessity of a study to quantify chaotic responses of a bi-stable VIV system is explained. Finally, the discussion focuses on the potential methods to control chaotic responses of a bi-stable VIV system.

Chapter 3 firstly presents the wake oscillator to model the VIV system for theoretical analyses in this thesis. By utilizing this model, simulation of a linear VIV system as a benchmark for studies of the proposed nonlinear VIV systems is carried out. Subsequently, the chapter discusses the investigation of effects of the nonlinear springs on the energy harvesting capability of a VIV system by the simulation on the wake oscillator model. Finally, experimental validation of the theoretical investigation is performed on a cyber-physical force-feedback testing platform.

Chapter 4 presents the study to quantify chaotic responses of a VIV energy harvesting system enhanced by a bi-stable spring in a wide range of the governing parameters. Chaotic responses of the bi-stable VIV system are simulated by the wake oscillator model and quantified through the largest Lyapunov exponent. Ultimately, a series of experiments of the bi-stable VIV system was conducted on the cyber-physical force-feedback testing platform to validate the analyses from the simulation.

Chapter 5 focuses on the OGY controller and time delay feedback controller to actively stabilize chaotic responses of a bi-stable VIV system at high velocity water flows when it is designed with an adequately large bi-stable gap to

significantly improve the output power at low velocity flows. The descriptions of these controllers and their applications for the bi-stable VIV system with various validations are discussed in this chapter.

Chapter 6 summarizes the main issues addressed in this thesis and the main contributions of the thesis. Some possible recommendations of the future work are also proposed in this chapter.

Chapter 2

Literature Review

This chapter discusses the main issues that will be addressed in this thesis in a general manner. Firstly, an overview on the main advantages and shortcomings of the current marine energy technologies is presented. Subsequently, the general concept and governing factors of vortex induced vibrations (VIV) for energy harvesting are introduced. In addition, methodology to study the VIV phenomenon, and the state-of-the-art studies to improve the performance of a VIV energy harvesting system are also reviewed. Secondly, potentials from the hardening spring and bi-stable spring to improve the performance of a VIV energy harvesting system are analyzed. Thirdly, the necessity of a study to quantify chaotic responses of a bi-stable VIV system is explained. Finally, the discussion focuses on the potential control methods to stabilize chaotic responses of a bi-stable VIV system.

2.1 Overview of current marine energy technologies

In order to alleviate the extreme dependency of the world energy supply on fossil fuels, which are hardly renewable and environmentally destructive, technologies to harvest energy from renewable resources have been developed. Among various renewable energy resources, marine energy is strikingly promising since oceans and rivers cover two thirds of the earth's surface and a huge amount of kinetic energy is available due to the motion of water [25]. In general, marine energy manifests itself in three main forms, namely wave energy, tidal energy and flow energy. Current technologies and devices to harvest marine energy are developed based upon these three forms.

Devices to harvest wave energy are mainly classified based on their operational principles, which are oscillating water column, overtopping devices and attenuators. The oscillating water column devices compress and expand air due to the change in pressure of ocean waves to rotate a turbine for producing electricity [26, 27]. In an overtopping device, wave crests are captured into a reservoir. The reservoir takes the water at the level that is higher than the sea level and drives a hydro-pump to produce electricity [28]. The attenuators to harvest wave energy are the structures whose motion is induced by the wave and the energy is converted by dampening the motion [25]. In general, technologies to harvest wave energy are advantageous due to the ubiquitous availability at all times and the high predictability of ocean waves. However, these can only provide high output power in a limited resonant working frequency range. In addition, these systems require very robust structures to bear extremely high loads from ocean waves [4].

Tidal energy technologies utilize a barrage or a dam to harvest the potential energy from tides [29]. When the tide rises, water will be leaded into a huge basin behind the dam and a large amount of potential energy is stored in the basin. When the tide falls, water in the basin is released through a turbine to produce

electricity. Apart from the conventional tidal dams, new tidal energy harvesting devices have been developed. For instance, the tidal reef has been developed with the required deviation between the highest and lowest sea levels of 2 – 3 m, which is lower than that of the conventional tidal dam (5 – 10 m). The lower sea level deviation requirement will result in a less significant impact on the environment as well as more compact structures. Another alternative structure is the tidal lagoon, which does not require to be connected to the shore and can be located within the ocean. This design can minimize the effect on the environment [29]. In general, harvesting tidal energy is advantageous since this source of energy is highly predictable due to the cyclic characteristic of tides. In addition, tides are usually independent on weather conditions [29]. However, structures to harvest tidal energy require a huge installation space, long construction period and high capital cost [4].

The principle of flow energy harvesting is rather straightforward. The most common devices to harvest flow energy are watermills and water turbines. The devices are either partially or totally immersed into water flows. The kinetic energy of the flow will result in rotation of the water turbine or the wheel of the watermill, where the electricity can be produced from this rotation. The most significant merit of the flow energy technology is its high energy density due to the high density of water [30]. However, this technology is very location dependent since it requires a flow velocity that is higher than 2 m/s to provide the acceptable output efficiency of 15 – 30% [4].

Despite of several striking merits, the development of the current marine energy technologies is still limited by the aforementioned critical barriers. Therefore, there is a high demand for exploring novel technologies to harvest marine energy more efficiently and sustainably. As a response to this demand, Vortex Induced Vibration (VIV) systems to harvest flow energy have been recently introduced and developed.

2.2 Vortex Induced Vibrations (VIV) for energy harvesting

The literature study of the VIV phenomenon is very intensive and comprehensive. This section discusses the issues on the VIV phenomenon of an elastically supported rigid circular cylinder related to energy harvesting. Further specific knowledge of the VIV phenomenon can be found in [31-33].

2.2.1 General concept

VIV is a fluid-structure interaction phenomenon that is commonly observed on many engineering structures that undergo fluid flows, e.g. risers, pipes, chimneys, suspended cables and mooring lines. Fig. 2.1a describes the vortex shedding phenomenon that occurs when a fluid flow crosses over a bluff body. The flow approaching the body is separated and two shear layers are formed on two sides of the body. Due to the instability, these two shear layers roll up and vortices are formed [34]. These vortices alternately shed to two sides of the wake region and consequently cause periodic drag and lift forces on the bluff body.

Fig. 2.1b depicts a typical structure that has been prevalently utilized to study the VIV phenomenon. The bluff body in a form of a circular cylinder with the diameter of D and the length of L (oriented along the z -direction) is totally submerged into a water flow. The cylinder is elastically supported by linear springs with the equivalent stiffness of k and its motion is constrained in one degree-of-freedom, which is translation along the y -axis. The effective mass of the oscillating structure, m_{osc} , comprises the mass of the cylinder and one third of the mass of the supporting springs. The structural damping is expressed by the damping coefficient, c . In this arrangement, when the water flow crosses over the cylinder in the x -direction with the velocity of U , the structure will vibrate in the y -direction under effect of the lift force induced by the vortex shedding. This vibration is known as VIV.

Figure 2.1 (a) Formation of vortex shedding phenomenon when a fluid flow crosses over a bluff body; (b) Model of a typical VIV structure that comprises a circular cylinder elastically supported by linear springs.

The first energy converter developed based on a VIV structure was introduced in 2008 and trademarked as the VIVACE energy converter (see Fig. 2.2a) [4]. The working principle of the VIVACE energy converter is rather straightforward. Connecting the oscillating cylinder of the VIV structure to a power-take off (PTO) component via a transmission mechanism allows for the conversion of the kinetic energy harvested from the water flow by means of VIV into other utilizable forms. For example, the cylinder is connected to an electrical generator through a gear belt mechanism (see Fig. 2.2b). In this way, electrical energy can be produced from vibrations of the VIV structure.

Figure 2.2 (a) VIVACE energy converter; (b) Oscillating cylinder is connected to an electrical generator via a gear-belt mechanism (reproduced from [4]).

Utilizing the VIVACE converter to produce electricity is considered as a promising alternative to deal with low speed water flows due to its outstanding performance in many aspects. Bernitsas, et al. [4] and Raghavan [35] summarized the essential merits of the VIVACE converter as follows:

- The VIVACE converter is location-independent since it can operate effectively in low velocity water flows (from 0.3 m/s onwards).
- The VIVACE converter provides relatively high output efficiency. The output efficiency of a VIVACE energy converter, η_{VIVACE} , can be calculated as:

$$\eta_{VIVACE} = \frac{P_{VIVACE}}{P_{flow}} = \frac{P_{VIVACE}}{0.5\rho U^3 DL} \quad (2.1)$$

where P_{VIVACE} is the harvested power from the converter and P_{flow} is the input power from the flow passing the cylinder [6]. In an experiment carried out by Bernitsas, et al. [6], the VIV structure was connected to a generator and an electrical load was used to consume the harvested energy. When the system operated at the water flow velocity $U = 0.84$ m/s, the output efficiency was achieved at $\eta_{VIVACE} = 22\%$, while the upper limit efficiency was estimated at $\eta_{UL-VIVACE} = 37\%$ in theory.

- The VIVACE converter can be modularly and flexibly designed in a sense of different scales (size and number of cylinders) to produce from watts to gigawatts of electrical power.
- Cost of electricity produced by the VIVACE technology is comparable to that of the conventional technologies and other alternative technologies. Fig. 2.3 provides a comparison in the average cost of power, which is calculated based on the capital installed cost, fuel cost, operating and maintenance cost, of the conventional and alternative technologies from the data provided by the Michigan Public Service Commission [36]. It can be seen that the cost of power produced by the VIVACE technology is lowest in the group of the alternative technologies.

Figure 2.3 Comparison in the average cost of power from the conventional and alternative technologies based on the data provided by the Michigan Public Service Commission (reproduced from [4]).

Apart from the above significances, merits of the VIVACE energy converter are extended to its features related to navigation, coastal property, marine life, maintenance, robustness, life cycle cost and design life (refer to [4, 35] for detailed explanations).

2.2.2 Governing factors to harvest energy by means of VIV

The design and operation of a VIV energy harvesting system are mainly governed by four factors: (1) *vortex induced vibrations*, (2) *resonance range*, (3) *aspect ratio*, and (4) *energy conversion* [4, 35].

(1) *Vortex induced vibrations*

As discussed in Section 2.1.1, the main structure of a VIV energy harvesting system is a rigid circular cylinder elastically supported by linear springs and its motion is constrained in the cross-flow direction. Due to the vortex shedding that

occurs when a water flow crosses over the cylinder, the structure will vibrate under effect of the induced fluid force in the cross-flow direction. Therefore, kinetic energy from the flow is dissipated by vibrations of the cylinder. Dynamics of the VIV structure depicted in Fig. 2.1b can be modelled as:

$$m_{osc} \ddot{y} + c\dot{y} + ky = F_y \quad (2.2)$$

where the over dots indicate the derivatives with respect to time, t , and F_y denotes the fluid force in the cross-flow direction. It can be inferred from the above equation that the utilizable energy is the energy dissipated by the damping component. It should be noted that when an electrical load is connected to the generator, it will result in a damping effect to the system [6]. This electrical damping and damping from the mechanical components contribute to the damping coefficient, c . Therefore, the utilizable power of the vibrating system can be evaluated in a period from t_1 to t_2 as:

$$P_{utilizable} = \frac{1}{(t_2 - t_1)} \int_{t_1}^{t_2} c(\dot{y})^2 dt \quad (2.3)$$

Based on the above equation, the utilizable power of the system is dependent on the damping coefficient and the velocity of the structure. For a sinusoidal motion with a certain frequency, the peak velocity is related to the vibrating amplitude. Therefore, the VIV structure can only operate effectively to harness energy at sufficiently high damping and high vibrating amplitude.

(2) *Resonance range*

A VIV structure is characterized by a set of structural parameters that include the effective mass of the oscillating structure, m_{osc} , equivalent stiffness of the supporting springs, k , damping coefficient, c , diameter, D , and length, L , of the cylinder. For a given set of these parameters, there exists a corresponding range of water flow velocity where the structure vibrates with significantly high amplitudes. This range of water flow velocity is referred to as the resonance range.

A discussion in more details on the resonance range of a VIV structure requires an introduction of some new variables as presented in Table 2.1. Unlike other aquatic energy converters such as flaps, buoys and water columns where the linear resonance occurs sharply at the natural frequency, the resonance of a VIV system is considered as a nonlinear behavior since it occurs in a certain range around the natural frequency [4]. The resonance phenomenon of a VIV system is also known as the lock-in or synchronization phenomenon. Fig. 2.4 describes the amplitude ratio, A^* , of the VIV system with the mass ratio $m^* = 1.19$ for varying normalized water flow velocity, U^* [37]. It can be observed that the resonance range, which includes the upper and lower branches, spans from the normalized water flow velocity of 5 – 18.

Table 2.1 Definitions of the variables related to the VIV phenomenon.

Variables	Definitions
A^*	<i>Amplitude ratio</i> , $A^* = A/D$, where A is the vibrating amplitude of the cylinder
m^*	<i>Mass ratio</i> , is the effective mass of the oscillating structure, m_{osc} , normalized to mass of the water volume displaced by the cylinder, m_d , $m^* = m_{osc}/m_d = 4m_{osc}/(\pi\rho D^2 L)$.
$f_{n, water}$	<i>Natural frequency of the structure in water</i> , $f_{n, water} = [k/(m_{osc} + C_A m_d)]^{1/2}/(2\pi)$, where C_A is the fluid-added mass ratio, $C_A = 1$ for a circular cross-section of the bluff body [37].
U^*	<i>Normalized water flow velocity</i> , $U^* = U/(f_{n, water} D)$
ζ	<i>Damping ratio</i> , $\zeta = 0.5c/[k(m_{osc} + C_A m_d)]^{1/2}$
f_{St}	<i>Strouhal frequency</i> , is the vortex shedding frequency of a stationary cylinder, $f_{St} = St.U/D$, where St is the <i>Strouhal</i> number, $St = 0.2$ when the Reynolds number is in the sub-critical range, $300 < Re < 1.5 \times 10^5$ [38, 39].

The resonance occurs in distinct manners for the VIV systems with different values of the mass ratio. For the system with a high mass ratio, when the water flow velocity increases from zero until the resonance is observed, the induced fluid force excites the cylinder in the cross-flow direction at a frequency that depends on the vortex formation frequency, $f_{V, form}$. This vortex formation frequency is equal to the *Strouhal* frequency, f_{St} . When the water flow velocity keeps increasing, the frequency of the induced fluid force turns out to depend on

frequency of the periodic vortex wake-mode, $f_{V, mode}$, which is determined by the wake structure modes, e.g. 2S mode (two single vortices shed per oscillation cycle), 2P mode (two pairs of vortices shed per oscillation cycle), (P + S) mode (one pair and one single vortices shed per oscillation cycle) [37, 40]. For a high-mass ratio system, the oscillation frequency, f_{osc} , is close to the natural frequency of the system, or the frequency ratio, $f^* = f_{osc}/f_{n, water}$, is close to the unity, when the resonance takes place [33, 41, 42].

Figure 2.4 Amplitude ratio, A^* , of the VIV system with the mass ratio $m^* = 1.19$ for varying normalized water flow velocity, U^* (reproduced from [37]).

On the other hand, a low-mass ratio system vibrates at a frequency that is significantly higher than its natural frequency when the resonance occurs and the resonance range is widened. Fig. 2.5a shows the frequency ratio, f^* , in the lower branch of the VIV systems with different values of the mass ratio [37]. It can be seen that the oscillation frequency, f_{osc} , is higher than the natural frequency, $f_{n, water}$, in all cases. In addition, when the mass ratio, m^* , is decreased from 8.63 to 1.19, the deviation of the frequency ratio from the unity becomes more significant and the resonance range also becomes wider. This phenomenon was observed in many experiments [33, 43, 44].

Figure 2.5 (a) Frequency ratio, f^* , in the lower branch of the VIV systems with different values of the mass ratio; (b) Resonance range (shaded region) as a function of the mass ratio achieved by fitting the experimental data (reproduced from [37]).

For the energy harvesting purpose, a VIV system only operates efficiently in the resonance range. The variation of the water flow velocity out of the resonance range will result in a poor performance of the system since the high-amplitude vibrations are hardly maintained. From the analysis based on Fig. 2.5a, it is obvious that adjusting the effective mass of the oscillating structure can impact the resonance range. Indeed, the resonance range can be widened by lowering the mass ratio [37, 45-47]. By fitting the experimental data, Govardhan and Williamson [37] proposed an estimation of the normalized water flow velocity where the resonance ends, U_{end}^* , as a function of the mass ratio:

$$U_{end}^* \approx 9.25 \sqrt{\frac{(m^* + C_A)}{(m^* - 0.54)}} \quad (2.4)$$

The above equation results in a critical value of the mass ratio, $m_{crit}^* = 0.54$, where the resonance range is infinitely widened (see Fig. 2.5b). However, due to the physical constraint of the structure, in most of cases, it is impractical to improve the resonance range by simply lowering the mass ratio. Therefore, a more feasible approach is required to broaden the resonance range.

(3) *Aspect ratio*

The aspect ratio of the cylinder in a VIV system, L/D , has a significant role in the energy harvesting capability. Its effect on the VIV phenomenon has been thoroughly studied (e.g. [33, 48, 49]). Concisely, a high correlation length, which is the length of the cylinder corresponding to the segment where vortices shed in phase, will enhance the induced fluid force imposed on the cylinder. Consequently, higher-amplitude vibrations are achieved.

For the energy harvesting purpose, an aspect ratio from 7 – 10 is suggested for the design of a VIV system [4]. In practical scenario, the selection of the aspect ratio is also determined by other factors such as the occupied space, mass ratio and output power.

(4) *Energy conversion*

Kinetic energy of the water flow dissipated by vibrations of the cylinder must be harnessed and converted into other usable forms. This task is assigned to a PTO component. For example, in the VIVACE energy converter, the cylinder is connected to an electrical generator through a gear-belt mechanism to produce electricity. It is also feasible to convert the harvested energy into mechanical energy by hydraulics systems [4].

It has been theoretically and experimentally demonstrated in [6] that the PTO component will result in an additional constituent to the total damping coefficient of the system, c . It can be interpreted from Eq. (2.3) that the energy, which can be harvested, is proportional to the damping coefficient of the system. However, an excessively high damping coefficient will deteriorate the energy harvesting capability since the high-amplitude vibrations are hardly maintained. Therefore, the damping coefficient of the system must be rationally designed for an effective energy conversion process.

From the analyses of the governing factors, it can be concluded that a VIV energy harvesting system will operate with an optimal performance when it is designed

with sufficiently high aspect ratio and high damping coefficient while high-amplitude vibrations are maintained in a broad resonance range. This condition is considered as the optimization objective to design a VIV energy harvesting system. The appearance of the VIVACE energy converter with its various outstanding merits has boosted a new research mainstream, where studies focus on investigating effects of the structural parameters to fulfill the optimization objective.

2.2.3 Methodology

Before discussing the state-of-the-art studies to improve the performance of a VIV energy harvesting system, it is worthy to carry out a brief review on the methodology to study the VIV phenomenon. This review is oriented by the *theoretical* and *experimental studies*.

(1) *Theoretical studies*

Gabbai and Benaroya [31] classified methods to theoretically study the VIV phenomenon of an elastically supported bluff body into two main groups, namely the *semi-empirical models*, in which the flow field around the body is not taken into account, and the *numerical methods*, in which modelling the flow field is a critical task.

Semi-empirical models

The semi-empirical models are classified into three main groups, namely the *wake oscillator models*, the *sdof models*, and the *force-decomposition models*. In general, the VIV structure is modelled as an externally excited oscillator in these models. The forcing term from the fluid flow in the dynamics equation of the structural oscillator is modelled in different manners.

Numerous *wake oscillator models* have been proposed and developed (e.g. [50-59]). In these models, the vortex shedding behavior is modelled by a wake equation. The oscillator and wake equations are coupled to each other through their forcing terms to ensure the fluid-structure interaction.

The *sdof models* have been proposed in [60, 61]. Unlike the wake oscillator models, the *sdof models* include only the structural oscillator equation. The fluid forcing term is treated as an aeroelastic function that depends on the dynamical variables of the VIV structure and the *Strouhal* frequency. Several types of the forcing function were proposed based on specific applications (e.g. [62-64]).

In the *force-decomposition models*, the structural oscillator equation is also utilized to model the VIV system and the forcing term is decomposed into different components. For example, Sarpkaya [65] decomposed the fluid forcing term into a fluid damping component related to the velocity and a fluid inertia component related to the acceleration. On the other hand, Griffin [66] and Griffin and Koopmann [67] separated the fluid forcing term into an excitation component and a reaction component that includes all the forces related to motion of the structure.

The successes of the models in these three groups are all determined by the empirical coefficients and constants deducted from experiments. Therefore, they are referred to as the *semi-empirical models*.

Numerical methods

The semi-empirical models focus on dynamical behavior of the VIV structure without any analysis of the flow field around the bluff body. Numerical methods were developed to provide a closer look on the flow field in the VIV phenomenon. (It should be noted that the equations of the semi-empirical models can be numerically solved to simulate vibrations of the VIV structure. However, they are excluded from this group since the flow field is not considered).

Performing a fully numerical simulation of the VIV phenomenon is commonly an intensive task that involves several underlying steps, e.g. modelling the flow field around the bluff body, mostly by the Navier-Stoke equation, modelling motion of the structure, coupling effects of the flow and the structure, and performing data analyses [68]. Several methods were proposed and implemented

to numerically simulate the VIV phenomenon, e.g. the vortex-in-cell (VIC) method [69-71], the direct numerical simulation (DNS) method [72-74], the finite element method (FEM) [75-79]. These methods might provide more accurate results than those of the semi-empirical models. However, as a tradeoff, they are usually time-expensive due to the intensive computations.

In this thesis, a wake oscillator model is utilized to model the nonlinear VIV system since it is straightforward to include the restoring force functions of the nonlinear springs in the model. Details of this wake oscillator model will be discussed in Chapter 3.

(2) *Experimental studies*

Experimental studies of the VIV phenomenon of an elastically supported circular cylinder are carried out by three main methods, namely the *forced vibration method*, the *free vibration method* and the *combined simulation method*.

Forced vibration method

The forced vibration method has been applied in many studies (e.g. [80-82]). In this method, motion of the cylinder is forced or controlled by the sinusoidal vibration in a free stream. The induced fluid force is observed as the studied object in this case. Several contributions have been made from the studies based on this method (refer to [32, 33] for comprehensive explanations). However, the nature of the VIV phenomenon might not be truly reflected in this method since only the cylinder propels the fluid instead of the fluid-structure interaction [83].

Free vibration method

The free vibration method provides a closer look on the nature of the VIV phenomenon. In this method, the cylinder is mounted to physical springs and the system is submerged into a water flow. Therefore, the natural VIV phenomenon can be observed. As an expense, this method requires a large investment in time and cost to adjust the physical components in order to vary values of the structural

parameters, e.g. the effective mass, damping coefficient and supporting stiffness. The accessible ranges of the structural parameters are also limited. For example, it is difficult to construct a VIV system with low effective mass and low damping coefficient. Although there were some efforts to construct the VIV experimental facilities with low effective mass and low damping coefficient, e.g. by using air bearings [45-47], and by using the active negative-positive damper [84], the flexibility in structural parameters of these systems is still limited.

Combined simulation method

The disadvantages of the forced and free vibration methods led to the introduction of the combined simulation method. There are two categories of the combined simulation method, which are the force-control based method and velocity/position-control based method. The force-control based method was developed and introduced in [85, 86]. In this method, the position and velocity of the cylinder are measured in real-time and a motor is utilized in the torque control mode to provide the restoring and damping forces to the system with the corresponding supporting stiffness and damping coefficient virtually prescribed in the controller. Therefore, the expected supporting stiffness and damping coefficient can be easily imposed to the VIV system. However, this method requires an intensive identification of damping from the mechanical parts in the system [86].

The velocity/position-control based method is simpler to implement and applied prevalently (e.g. [83, 87-89]). In this method, the structural parameters that include the effective mass, damping coefficient and supporting stiffness are virtually set up in a computer-based controller, the induced fluid force is measured in real-time by a force sensor and the corresponding velocity/position of the cylinder will be calculated to actuate the cylinder by a linear actuator. As a tradeoff, the inclusion of the measured fluid force in the control loop will result in a phase-lag in the system [89].

In this thesis, a cyber-physical force-feedback testing platform is designed and developed based on the velocity/position-control method to facilitate experiments of the nonlinear VIV system. Details of this testing platform will be discussed in Chapter 3.

2.2.4 State-of-the-art

Studies to improve the performance of a VIV energy harvesting system can be generally categorized as *theoretical* and *experimental studies*.

(1) *Theoretical studies*

Theoretical studies focus on analyzing effects of the structural parameters on the energy harvesting capability of a VIV system.

In 2011, Meliga and Chomaz [8] carried out a study based on an asymptotic expansion for a VIV cylindrical structure to analyze effects of the natural frequency, $f_{n, water}$, the mass ratio, m^* , and the mass-damping parameter, $m^*\zeta$, on the energy harvesting capability. It was found that the maximum harvested energy can be obtained with an optimal set of these structural parameters.

Another parametric study presented in [9] figures out effect of the mass-damping parameter, $m^*\zeta$, on the energy harvesting efficiency of a VIV system. It concludes the optimal value of the mass-damping parameter, $m^*\zeta = 0.25$, where the efficiency of the system is maximized.

Dhanwani, et al. [10] developed a lumped parameter model to study effect of the supporting stiffness on a VIV energy harvesting system. The optimal value of stiffness can be estimated by this model with the objective of maximizing the harvested power.

(2) *Experimental studies*

Apart from the above theoretical studies, several experimental studies were performed to enhance the performance of a VIV energy harvesting system.

The passive turbulence control (PTC) method is implemented by attaching waterproof sandpaper strips along the bluff body to increase the vibrating amplitude [11-13]. This method shifts the flow from the laminar regime to the turbulence regime and can improve the vibrating amplitude to $2.9D$.

Another method, which utilizes wires attached along the cylindrical structure to widen the wake region and reinforce the induced lift force, was introduced in [14, 15]. This method can provide high energy harvesting efficiency, which is four times larger than that in the case of a smooth cylinder.

Kim, et al. [16] performed an experiment with several cylinders assembled as a grid in a water channel. The vibrating amplitudes of all the cylinders were maintained at a high level, which is from $2.2D - 2.8D$, when they were operating simultaneously. It concludes the feasibility to develop VIV energy converter farms.

Nishi, et al. [17] developed a new VIV energy converter prototype that is based on the principle of leverage aiming as an alternative for energy conversion rather than using an electrical generator. In this prototype, a linear generator that comprises a coil and a magnet is connected to the cylinder through a leverage aiming mechanism. However, this method provides low energy conversion efficiency, which is approximately 10%.

It can be seen that there are hardly significant contributions from the studies in literature in terms of widening the resonance range of a VIV energy harvesting system. The next section will discuss a potential approach to broaden the resonance range of a vibration-based energy converter, which is by introducing nonlinear springs to the system.

2.3 Enhancing a VIV energy harvesting system by nonlinear springs

Although nonlinear springs have been intensively applied in numerous studies to improve working frequency ranges of miniaturized energy converters

(e.g. [19-22]), such studies for a VIV energy harvesting system are scarce. From the literature, *hardening spring* and *bi-stable spring* are most potential to widen working ranges of vibration-based energy converters.

2.3.1 Hardening spring

An oscillator with a hardening spring has varying natural frequency that depends on the vibrating amplitude [90]. This characteristic will result in a potential way to widen the resonance range of a VIV system since the increase in the vibrating amplitude associated to the water flow velocity will also increase the natural frequency.

In 2013, Mackowski and Williamson [23] experimentally verified effect of hardening springs on a VIV system. From this study, the hardening springs represented by the cubic and quintic restoring force functions are confirmed to be able to extend the resonance range towards the high velocity water flows. However, the physical implementation of the springs is rather challenging as the parameters that govern the nonlinearity are difficult to control.

The piecewise hardening spring, which is simpler for the physical implementation than the springs represented by the cubic and quintic restoring force functions, was recently applied for the VIVACE energy converter by Ma, et al. [24]. The nonlinearity in the piecewise hardening spring is characterized by two parameters, namely the hardening range and the hardening stiffness. The parametric effect of the hardening range was thoroughly investigated in this study. However, the parametric effect of the hardening stiffness has not been taken into account.

2.3.2 Bi-stable spring

An oscillator with a bi-stable spring has become a long-history research topic due to its unique and sophisticated dynamical characteristics. When a bi-stable spring is applied in an oscillator, it will result in a double-well potential energy

characteristic as illustrated in Fig. 2.6. From the perspective of nonlinear dynamics, this system has two stable fixed points, and one unstable fixed point at the origin since it is never stabilized at this position. When the system is excited with the low excitation energy, it might vibrate within one well of the potential energy, i.e. following the trajectories (1), depending on the initial condition. This mode is referred to as the *intra-well vibrating mode*. When the energy of the excitation is sufficiently high for the system to overcome the unstable fixed point, the system will vibrate along the trajectory (2), which is referred to as the *inter-well vibrating mode*. The system can also vibrate in a more complex manner, i.e. irregularly jumping back and forth between the two potential wells, or transforming from the intra-well mode to the inter-well mode and vice versa. This mode is referred to as the *chaotic mode*.

Figure 2.6 Double-well potential energy characteristic of the system with a bi-stable spring.

In a sense of energy harvesting, the bi-stable system takes advantage most when it vibrates in the inter-well mode due to the higher amplitude and velocity than those of a linear system with the same equivalent stiffness. Effect of the bi-stable spring on performance of ambient-vibration based miniaturized energy harvesters has been intensively studied (see e.g. [20, 91, 92]). However, based on the author's knowledge, there is a lack of studies to apply the bi-stable spring for a VIV system, which dynamically differs from the miniaturized energy converters due to its strong fluid-structure interaction.

It can be concluded that effects from the hardening spring and bi-stable spring on the energy harvesting capability of a VIV system have not been comprehensively studied. In this thesis, effects of the piecewise hardening spring and the bi-stable spring on a VIV system will be thoroughly investigated in Chapter 3. In particular, the parametric effect of the factors that govern the nonlinearity in the springs is focused in this investigation.

2.4 Chaotic responses of a bi-stable VIV system

2.4.1 Occurrence of chaotic responses in a bi-stable VIV system

As mentioned in Section 2.2.2, introducing a bi-stable spring might lead to the occurrence of chaotic responses in the system. A chaotic signal is defined as a non-periodic, unpredictable and seemingly a noisy signal [93]. Apart from a bi-stable oscillator, chaotic responses might occur in any nonlinear dynamical systems and commonly in those with severe nonlinearities, such as the mechanical systems with backlash [94, 95], or those with hysteresis [96].

Chaotic responses are evidently undesired for the operation of a VIV energy harvesting system for two reasons. Firstly, the harvested power is negatively influenced since a bi-stable system only takes advantage in energy harvesting when it operates in the periodic inter-well vibrating mode. Secondly, the unpredictability in motion will lead to a difficulty in monitoring the system. Therefore, the feasibility of applying the bi-stable spring for a VIV energy harvesting system to improve its performance requires a design that can avoid chaotic responses. The prerequisite to design such a system is the knowledge in the dependency of chaotic responses on the structural parameters.

Some studies of chaotic responses of a VIV structure can be found in literature. For example, Zhao, et al. [97] studied chaotic responses of a VIV structure in a laboratory scale. Initially, the displacement and lift force were measured on a testing platform. Subsequently, the recorded displacement was prescribed to the system on a position-control testing platform and the induced lift force was

measured again. The occurrence of chaotic responses was confirmed by the significant discrepancy between the two measurements of the lift force. Perdikaris, et al. [98] studied vibrations of a VIV system in a constant flow for varying amplitude at a fixed frequency. In the case of the moderate amplitude, chaotic responses were observed and confirmed by the absence of a dominant frequency from the frequency spectrum of the lift force. Blackburn and Henderson [99] carried out a 2D numerical simulation to investigate the lock-in behavior of a cylinder excited by a steady flow. When the auto-spectrum was analyzed, chaotic responses were confirmed by their non-periodicity, and the deviation of their frequencies from the natural frequency as well as the *Strouhal* frequency.

It can be seen that studies of chaotic responses of a VIV system are limited at the confirmation of their occurrence. There is a lack of systematic studies to analyze the dependency of chaotic responses on the structural parameters. Such studies can be carried out by quantifying chaotic responses based on the *Lyapunov exponent*.

2.4.2 Chaotic quantification by the Lyapunov exponent

In nonlinear dynamics studies, the instant state of a system can be presented as a state vector in a phase space. When the system evolves in time, the state vector forms an orbit or trajectory in the phase space. For a dissipative dynamical system, the behavior will reach a steady state after the transient duration. Therefore, the trajectory will evolve within a bounded region and converge into an attractor. For a limit cycle where the behavior is periodic, a closed trajectory will be formed. On a contrary, for a chaotic response, the trajectory is non-periodic and hardly reproducible [93]. In particular, when a chaotic system is excited with two nearby initial conditions, the corresponding trajectories in the phase space will split up exponentially. This leads to the basic concept of the Lyapunov exponent to quantify the chaotic degree of a system.

If $d(t_0)$ represents distance between two state vectors of the system in a phase space that correspond to two nearby initial conditions, after a certain instance, t , their new distance after the evolution, $d(t)$, can be expressed as:

$$d(t) = d(t_0)e^{\gamma t} \quad (2.5)$$

where γ is referred to as the Lyapunov exponent that is the footprint of chaotic response if it is positive [94]. The magnitude of this quantity indicates the diverging speed of the nearby trajectories (see Fig. 2.7). Therefore, it can be utilized to measure the chaotic degree of the system.

Figure 2.7 Exponential divergence of two nearby initial states (reproduced from [18]).

It should be noted that, however, the analysis of the Lyapunov exponent is barely practical to be performed straightforwardly, i.e. exciting the system with two nearby initial conditions and observing the separation of the trajectories in the phase space. As an alternative, it is more suitable to evaluate the *largest Lyapunov exponent* that only requires the time series data of a single state variable. The fundamental procedure for the nonlinear time series analysis is comprehensively discussed in [100-102]. To elaborate, from the time series data of a single state variable, a time delay phase space, where the Lyapunov spectrum of the system in the original phase space is conserved, will be reconstructed. Then, the largest Lyapunov exponent is evaluated from the state vectors created by embedding the time series data to the reconstructed phase space.

Another issue arises when the confirmation of chaotic responses for experimental data is performed. Experimental data is usually contaminated by random noise,

which might be misinterpreted as the inherent nonlinearity of the system. Therefore, in order to avoid a bias assumption in distinguishing periodic responses from chaotic ones, a *surrogate data test* is performed. Surrogate data test, which was systematically studied and reviewed in [103, 104], is an effective stochastic tool to confirm whether the irregularity in the time series data is induced by noise (linear stochastic process) or by the inherent nonlinearity of the system.

In this thesis, a study to quantify chaotic responses of a bi-stable VIV system is performed via the largest Lyapunov exponent in a comprehensive range of the structural parameters. The calculation of the largest Lyapunov exponent is carried out based on the time series data generated from the simulation on the wake oscillator model. The results from this quantification are experimentally validated with the facilitation of the surrogate data test. Details of this quantification study will be discussed in Chapter 4.

2.5 Chaos control for a bi-stable VIV system

Chaotic responses must be eliminated from the operation of a bi-stable VIV energy harvesting system since they might deteriorate the harvested power and cause an adversity in monitoring the system. Generally, there are two approaches to get rid of chaotic responses. The first approach is based on the bifurcation map resulted from the quantification study, where the occurrence chaotic responses is presented as a function of the structural parameters. The system can be designed with the structural parameters in the non-chaotic regions on the bifurcation map.

However, there might be an inevitable scenario where the system must be designed with the structural parameters in the chaotic regions. For example, at a certain value of the parameter that governs the bi-stable effect, the harvested power of the VIV system might be significantly improved in a certain range of water flow velocity. However, when the water flow velocity fluctuates beyond this range, chaotic responses will occur. This situation leads to the second approach, which is to actively control chaotic responses whenever they occur.

Several approaches for chaos control have been proposed, e.g. the Ott-Grebogi-Yorke (OGY) method [105-107], the time delay feedback method [108, 109], the fuzzy control method [110], the nonlinear control method [111, 112], the non-feedback method [113, 114], the adaptive control method [115, 116], the proportional feedback method [117, 118] and the neural networks method [119, 120]. From the energy harvesting point of view, the control effort is desired to be as small as possible since the net harvested power of the system will be reduced by the power required to execute the control effort [121]. Therefore, the *OGY method* and the *time delay feedback method* seemingly satisfy this requirement. In addition, these methods are advantageous in other practical aspects. For instance, the design procedures of the controllers in these methods are rather straightforward and do not require prior analytical knowledge on the dynamics of the system. Furthermore, the number of state variables of the system that are required for the control feedback loop can be minimized.

The OGY method was developed based on the nature of a chaotic attractor. Firstly, a chaotic attractor is extremely sensitive to initial conditions. Secondly, unstable periodic orbits are densely embedded within a chaotic attractor. Therefore, a small perturbation in an accessible parameter of the system will be able to alter the system. If the perturbation is properly prescribed, the stabilization of the system following a desired periodic orbit is also feasible [106]. Furthermore, in a practical circumstance where the prior analytical knowledge of the system is unavailable and the measurable variables are limited, the OGY method can also be applied by utilizing the time delay coordinates [122, 123].

The time delay feedback method is also a prevalent chaos control method that was intensively applied in literature (e.g. [108, 109, 124, 125]). In this method, a control input that is constructed from a delayed signal of a system state variable is continuously applied to the system. When the delay time is chosen to be coincident to the period of a desired orbit, the system will be stabilized into that orbit.

In this thesis, the chaos controllers based on the OGY method and the time delay feedback method are designed and implemented to stabilize chaotic responses of a bi-stable VIV energy harvesting system. Chapter 5 will discuss these chaos controllers in details.

2.6 Summary

In this chapter, the main advantages and shortcomings of the current marine energy technologies have been overviewed. Subsequently, the working principle, main advantages and governing factors of a VIV energy harvesting system were discussed in details. The theoretical and experimental methods to study the VIV phenomenon were also reviewed. As an important part of Chapter 3, the methodology to study the proposed nonlinear VIV systems in this thesis, which is based on a wake oscillator model for theoretical analyses and a cyber-physical force-feedback testing platform as a tool for experimental validations, will be discussed in details.

A comprehensive discussion on the state-of-the-art studies to improve the performance of a VIV energy harvesting system was also included this chapter. It can be seen that methods to widen the resonance range of a VIV energy harvesting system have not been sufficiently studied. This research gap led to the discussion on the potential of introducing nonlinear springs to the system to widen the resonance range. Although this approach was intensively applied for miniaturized energy harvesting devices, it has not been sufficiently studied to apply for a VIV energy harvesting system. In chapter 3, effects of the hardening spring and the bi-stable spring on the energy harvesting capability of a VIV system will be thoroughly investigated.

Another important issue that is related to chaotic responses of a VIV system enhanced by a bi-stable spring was mentioned in this chapter. The importance of the knowledge in how chaotic responses depend on the structural parameters of

the system was explained. The Lyapunov exponent, an effective tool to quantify chaotic responses of a dynamical system was also introduced. Chapter 4 will focus on quantifying chaotic responses of a bi-stable VIV system via the Lyapunov exponent.

Finally, this chapter explained the necessity of controlling chaotic responses of a bi-stable VIV energy harvesting system and listed some chaos control methods proposed in literature. In Chapter 5, the design and implementation of the chaos controllers based on the OGY method and the time delay feedback method for a bi-stable VIV energy harvesting system will be systematically discussed.

Chapter 3*

System Enhancement by Nonlinear Springs

This chapter is devoted to theoretically and experimentally investigate effects of the nonlinear springs on the energy harvesting capability of a VIV system. At the first place, the wake oscillator model, which is utilized for the theoretical analyses, is introduced in details. The simulation of a linear VIV system as a benchmark for studies of the proposed nonlinear VIV systems is also discussed. Subsequently, effects of the nonlinear springs on the energy harvesting capability of a VIV system are investigated by the simulation on the wake oscillator model. Finally, the theoretical analyses from the simulation result are validated by the experiments performed on the cyber-physical testing platform with the water channel facility.

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3.1 Motivation

As discussed in Chapter 2, the output power of a VIV energy harvesting system is dependent on the vibrating amplitude. The physical properties that include the effective mass, supporting stiffness, damping coefficient and aspect ratio of the cylinder define the resonance range of a VIV structure. This resonance range corresponds to the range of water flow velocity where the system vibrates with significantly high amplitude and the output power is maximized. When the flow velocity deviates from this range, vibration is suppressed and the efficient operation is hardly maintained.

Meanwhile, natural water flows fluctuate in velocity frequently. Therefore, it is necessary to widen the resonance range of a VIV energy harvesting system in order to maintain its effective operation under the variation of water flow velocity. It should be underlined here that the VIV phenomenon is a nonlinear behavior and the resonance range of a VIV system is inherently wider than that of a linear vibrating system [4]. However, the resonance range of a VIV system is still narrow compared to the variation range in velocity of natural water flows. For example, a measurement of the actual flow velocity at the marine test bed of ERI@N, which is located at the Sentosa Boardwalk, Singapore, indicated that the flow velocity fluctuated in a wide range of 0.1 – 1.2 m/s in one-day duration [7].

Motivated by this challenge, in this chapter, the nonlinear springs that include the bi-stable spring and hardening spring are applied for the VIV energy harvesting system to investigate their effects on the resonance range.

3.2 Modelling of a VIV system by wake oscillator model

3.2.1 Background

An appropriate model is required to theoretically analyze effects of the nonlinear springs on the energy harvesting capability of a VIV system. The theoretical model should be able to dynamically model the system under varying water flow

velocity. In addition, the model should be suitable for the embedment of the restoring force functions of the nonlinear springs in the VIV system.

As discussed in Section 2.2.3, there are two approaches to theoretically study the VIV phenomenon of an elastically supported rigid circular cylinder, which are based on the numerical methods and the semi-empirical models. A fully numerical method involves modelling of the flow field around the bluff body, which is out of the scope of this thesis. As this thesis focuses on the implementation of nonlinear springs to the structure, a fully numerical model might be too tedious. Therefore, the semi-empirical models seem to be more feasible.

In a semi-empirical model, the VIV structure is modelled as an externally excited oscillator, where the restoring force functions of the nonlinear springs can be straightforwardly embedded to the system through the restoring force term of the structural oscillator equation. Among numerous proposed semi-empirical models, e.g. the wake oscillator models, the sdof models and the force-decomposition models, the wake oscillator models are most applicable since the fluid-structure interaction in the VIV phenomenon is properly reflected by the coupling of the structural and wake oscillators. Moreover, the wake oscillator models are least dependent on deductions from experimental observations [10].

Based on the author's knowledge, a semi-empirical model hardly provides an exact quantitative agreement with experimental results due to the complication of the fluid-structure interaction phenomenon. The most successful model for qualitative studies of the VIV phenomenon, e.g. the cross-flow vibrating amplitude, the occurrences of the synchronization and de-synchronization, is the wake oscillator model based on the van der Pol or classic model [31]. This model has been widely proposed and developed in many studies (e.g. [51, 54, 56-59]). In this thesis, a *modified higher-order wake oscillator* is applied for theoretical analyses of the nonlinear VIV system. Farshidianfar and Dolatabadi [55] developed this model by modifying the classic van der Pol model and combining

it with the model of Landl [53]. By a validation in [55], this model was proven to be able to capture the main characteristics of the VIV phenomenon such as the maximum vibrating amplitude at resonance, resonance range and upper branch in a wide range of structural parameters.

3.2.2 Modified higher-order wake oscillator model

(1) *Structural oscillator*

The 2D model of a VIV structure is depicted in Fig. 3.1. This model simplifies the 3D model in Fig. 2.1b. When the water flow crosses over the structure, dynamics of the structure in the cross-flow direction can be modelled as:

$$m_s \ddot{y} + r_s \dot{y} + gy = f_y \quad (3.1)$$

where m_s is the effective mass per unit length of the oscillating structure, r_s is the damping coefficient per unit length, g denotes the equivalent stiffness per unit length of the supporting springs and f_y represents the external force that excites the structure in the cross-flow direction. The above 2D equation corresponds to Eq. (2.2) that models the 3D structure.

Figure 3.1 2D model of a VIV structure.

The external force, f_y , is constituted by three components that include the induced lift force from the vortex shedding, the fluid damping force from water and the inertia force from the fluid-added mass [38]. Therefore, the external force, f_y , can be expressed as:

$$f_y = \underbrace{\frac{\rho U^2 D C_L}{2}}_{\text{lift force}} - \underbrace{\gamma \Omega \rho D^2 \dot{y}}_{\text{fluid damping force}} - \underbrace{m_f \ddot{y}}_{\text{inertia force}} \quad (3.2)$$

where: $C_L = q C_{L0}/2$ is the lift coefficient in terms of the wake variable q , where C_{L0} is the reference lift coefficient of a stationary cylinder exposed to vortex shedding; γ is the stall parameter that represents the damping effect from the fluid, $\gamma = C_D/(4\pi St)$, where C_D is the amplified drag coefficient; Ω is the angular *Strouhal* frequency, $\Omega = 2\pi f_{St} = 2\pi St.U/D$ (see Table 2.1); m_f is the fluid added-mass per unit length, $m_f = C_A \pi \rho D^2/4$ ($C_A = 1$ as explained in Table 2.1). Elaborative explanations of these quantities and parameters can be found in [38, 39, 54, 58]. By combining Eq. (3.1) and Eq. (3.2), the dynamics equation of the structural oscillator is obtained as:

$$\left(m_s + \frac{\pi \rho D^2}{4} \right) \ddot{y} + \left(r_s + \frac{\rho C_D U D}{2} \right) \dot{y} + g y = \frac{\rho U^2 D C_{L0} q}{4} \quad (3.3)$$

(2) Wake oscillator model

Classically, the vortex shedding behavior is dynamically modelled by a nonlinear equation that satisfies the van der Pol equation [58] as:

$$\ddot{q} + \varepsilon \Omega (q^2 - 1) \dot{q} + \Omega^2 q = F_{structure} \quad (3.4)$$

where: q is the wake variable that is correlated to the lift coefficient, C_L ; ε is the coefficient of nonlinearity; and $F_{structure}$ is the forcing function that represents effect from motion of the structure on the vortex shedding, it can be a function of one of the dynamical variables of the structure. By introducing new dimensionless variables, $\bar{t} = t\Omega$ and $Y = y/D$, Eq. (3.4) can be re-written as:

$$q'' + \varepsilon (q^2 - 1) q' + q = f_{structure} \quad (3.5)$$

where the over commas denote the derivatives with respect to $\bar{t}$ and $f_{structure}$ is the dimensionless form of $F_{structure}$.

Farshidianfar and Dolatabadi [55] modified the classic model by combining it with the model of Landl [53]. In the Landl's model, the wake oscillator is modelled with an addition of the fourth-order term to the damping coefficient as:

$$C_L'' + (\alpha' - \beta' C_L^2 + \lambda' C_L^4) C_L' + \Omega_r C_L = b Y' \quad (3.6)$$

where: α' , β' , λ' and b are the constants that depend on specific applications; Ω_r denotes the normalized frequency of the structure [53]. To directly correlate the above equation to the classic model, Farshidianfar and Dolatabadi [55] assumed that $\alpha' = \varepsilon$, $\beta'/\varepsilon = \beta$ and $\lambda'/\varepsilon = \lambda$. By expressing the lift force coefficient, C_L , in terms of the wake variable, q , and utilizing the general forcing term, $f_{structure}$, Eq. (3.6) can be re-written as:

$$q'' + \varepsilon(1 - \beta q^2 + \lambda q^4) q' + \Omega_r q = f_{structure} \quad (3.7)$$

Farshidianfar and Dolatabadi [55] neglected the normalized frequency, Ω_r , to ensure the similarity of the modified and classic models. Therefore, Eq. (3.7) now turns into:

$$q'' + \varepsilon(1 - \beta q^2 + \lambda q^4) q' + q = f_{structure} \quad (3.8)$$

To couple the new modified wake oscillator to the structural oscillator, the forcing term $f_{structure}$ is expressed as a linear function of either the acceleration, or velocity, or displacement of the VIV structure. The investigations in [54, 55] have shown that the expression of the forcing term $f_{structure}$ in terms of the acceleration offers the best agreement to the experimental data. Therefore, Eq. (3.8) can be re-written as:

$$q'' + \varepsilon(1 - \beta q^2 + \lambda q^4) q' + q = a Y'' \quad (3.9)$$

where a is a constant. In the dimensional form, Eq. (3.9) is expressed as:

$$\ddot{q} + \varepsilon \frac{2\pi St.U}{D} (1 - \beta q^2 + \lambda q^4) \dot{q} + \left(\frac{2\pi St.U}{D} \right)^2 q = \frac{a \ddot{y}}{D} \quad (3.10)$$

Eq. (3.3) and Eq. (3.10) are coupled to each other in a final form as:

$$\begin{cases} \left(m_s + \frac{\pi \rho D^2}{4} \right) \ddot{y} + \left(r_s + \frac{\rho C_D U D}{2} \right) \dot{y} + gy = \frac{\rho U^2 D C_{L0} q}{4} \\ \ddot{q} + \varepsilon \frac{2\pi \text{St} U}{D} (1 - \beta q^2 + \lambda q^4) \dot{q} + \left(\frac{2\pi \text{St} U}{D} \right)^2 q = \frac{a \ddot{y}}{D} \end{cases} \quad (3.11)$$

It can be seen that the coefficients C_D , C_{L0} , St , ε , β , λ and a , define a closed-form solution of the model. In this thesis, these parameters are assumed to be constants and the same values as in literature (see e.g. [54, 55]) are selected. Values of these coefficients are presented in Table 3.1.

Table 3.1 Coefficients of the modified higher-order wake oscillator model.

Coefficients	Values	Coefficients	Values
a	12	β	0.0625
C_D	2.55	ε	0.3
C_{L0}	0.3	λ	0.0005
St	0.2		

By numerically solving the system of Eqs. (3.11), vibrations of the system can be simulated at different values of the flow velocity, U . Consequently, theoretical analyses of the critical characteristics of the VIV phenomenon such as vibrating amplitude and resonance range can be performed based on the simulation data. In particular, the restoring force functions of the nonlinear springs can be simply embedded into the model via the restoring force term of the structural oscillator equation.

By using the wake oscillator model, a linear VIV system is simulated as a benchmark for the proposed nonlinearly enhanced systems. In this simulation, the mass ratio and damping ratio of the VIV structure refer to those in the experiment reported by Khalak and Williamson [47]. All the physical parameters of the VIV system in this simulation are summarized in Table 3.2.

Table 3.2 Physical parameters of linear VIV system in the benchmark simulation.

Parameters	Values
Cylinder diameter, D	0.05 m
Effective mass per unit length, m_s	21.8 kg/m
Structural damping coefficient per unit length, r_s	0.22 Ns/m ²
Supporting stiffness per unit length, g	370 N/m ²
Natural frequency in water, $f_{n, water}$	0.63 Hz

From the simulation result, the utilizable power, $P_{utilizable}$, of the system can be numerically calculated by Eq. (2.3) since the PTO component can be considered as a damping element in the system [6]. However, the analysis of this 2D model will be subsequently contrasted to the experimental analysis (based on the 3D model) for the validation purpose. Therefore, in order to simplify the comparison, the utilizable power is normalized to the damping coefficient. The normalized utilizable power is referred to as vibrational power, P^* (see Eq. (3.12)). Fig. 3.2 shows the vibrational power of the system, P^* , versus the flow velocity, U , which is numerically calculated from the simulation result.

$$P^* = \frac{1}{(t_2 - t_1)} \int_{t_1}^{t_2} (\dot{y})^2 dt \quad (3.12)$$

Figure 3.2 Vibrational power, P^* , of the system versus the flow velocity, U .

It is obvious that there exists a (narrow) range of water flow velocity where the VIV system operates effectively with high output power. Beyond this range, the system is unable to harness energy due to the low output power. Therefore, there is an evident requirement to widen the working range of the VIV energy harvesting system. A wider working range will improve the flexibility of the system to operate in natural water flows, where the flow velocity varies frequently. In the next section, the nonlinear springs will be applied for the VIV energy harvesting system to investigate their effects on the resonance range.

3.3 System enhancement by nonlinear springs

3.3.1 Bi-stable spring

Bi-stable springs have been intensively studied to apply for miniaturized vibration-based energy harvesting devices (e.g. [19-21]). However, based on the author's knowledge, in literature, there is hardly any study to apply the bi-stable spring for a VIV system, which dynamically differs from the miniaturized energy converters due to its strong fluid-structure interaction.

The spring mechanism in the left panel of Fig. 3.3 represents a bi-stable restoring force characteristic that is resulted by the nonlinear restoring force component from the springs k_2 added to the linear spring force component from the springs k_1 . By simply decomposing the restoring forces from the springs k_2 into the y -axis, Ramlan, et al. [22] derived and expressed the equivalent restoring force of this mechanism as a smooth nonlinear function that contains a negative stiffness in the vicinity of the origin. In this study, to simplify the parametric analysis, the bi-stable restoring force function is idealized as in Eq. (3.13) (see the right panel of Fig. 3.3). This idealized function conserves two vital parameters, namely the equivalent stiffness, k_0 , and the bi-stable gap, b . In order to have a direct comparison to the original linear system in Section 3.2, the same equivalent stiffness will be used for the dynamics characterization. The bi-stable gap, b , determines the degree of nonlinearity in the system.

$$F_k^b(y) = k_0[y - b \cdot \text{sign}(y)] \quad (3.13)$$

Figure 3.3 Bi-stable spring mechanism and its idealized restoring force function.

The potential wells of a VIV system enhanced by a bi-stable spring with $b = 0.2D = 1$ cm and a linear system with the same equivalent stiffness provide an insight in the physical instinct of the bi-stable system (see Fig. 3.4). Unlike the linear system that only has one stable fixed point at the origin, the bi-stable system has two stable fixed points at ± 1 cm (correspond to the gap, b) and one unstable fixed point at the origin. Due to this double-well characteristic, the system can vibrate in three different modes: (i) within one of the two stable points – *intra-well mode*; (ii) crossing over the two stable points – *inter-well mode*; and (iii) jumping back and forth between the two stable points in a less predictable and non-periodic manner – *chaotic mode*.

Figure 3.4 Potential wells of a bi-stable VIV system and a linear VIV system.

From the energy harvesting point of view, the bi-stable system takes advantage most when it vibrates in the inter-well mode since crossing over the two stable points and the unstable point means that the system results in higher kinetic energy. Due to this unique characteristic, the bi-stable system is potential to

improve the harvested power at the excitation frequency that is lower than natural frequency of the system [20, 126, 127].

To carry out a frequency response analysis of the bi-stable system, the bi-stable restoring force in Eq. (3.13) is defined per unit length to substitute for the linear restoring force term in Eq. (3.1) and the external force term, f_y , is modelled as a sinusoidal excitation force, $F \cdot \sin[(2\pi f)t]$, with varying amplitude, F , and frequency, f . Fig. 3.5 shows the Frequency Response Function (FRF) that represents the ratio of the response amplitude at the fundamental frequency and the excitation amplitude, A/F , of the bi-stable system with $b = 0.03D$ and the linear system with the same equivalent stiffness. It can be seen that, in general, the response amplitude of the bi-stable system is higher than that of the linear system.

Figure 3.5 The FRF of the bi-stable VIV system and a linear system with the same equivalent stiffness.

When the excitation amplitude is increased, intuitively, the response amplitude will also increase. This means the bi-stable gap becomes less significant compared to the response amplitude. Consequently, when the excitation amplitude is high, the FRF of the bi-stable system tends to resemble the FRF of the linear system. In other words, the lower the excitation amplitude, the farther the natural frequency shifted to the left. This agrees with the analyses in literature [20, 126, 127] saying that the bi-stable system will take advantage at lower

working frequency. When the excitation frequency increases, the response amplitude of the bi-stable system tends to become lower than that of the linear system. This effect is attributed to the adversity in maintaining the inter-well vibration when the excitation frequency is higher than the natural frequency of the system. This effect becomes more evident when the bi-stable gap is larger. In particular, when the bi-stable gap is significantly large, the inter-well vibration will be deteriorated and the system will exhibit chaotic responses, which erratically reduce the harvested power of the system. (Chaotic responses of a bi-stable VIV system will be elaborately discussed in Chapter 4.)

Subsequently, to observe the interaction of the bi-stable system with the water flow, the wake oscillator model (Eqs. (3.11)) is simulated with the substitution of the bi-stable restoring force in Eq. (3.13) defined per unit length for the linear restoring force in the structural oscillator equation. The bi-stable gap, b , is varied from $0.01D$ to $0.03D$, while other physical parameters are kept the same as those of the linear system discussed in Section 3.2 (see Table 3.2) in order to have a fair comparison. Fig. 3.6 shows the amplitude ratio, A^* , and vibrational power, P^* , of the bi-stable system in comparison to those of a linear system, against the water flow velocity, U , for various values of bi-stable gap, b . The curves are obtained by using the smoothing spline method to fit the data points in order to generalize the response trends.

Figure 3.6 Amplitude ratio, A^* , and vibrational power, P^* , of the bi-stable system for varying bi-stable gap, b , together with the linear system, versus the water flow velocity, U .

It can be seen that the bi-stable spring is able to extend the working range at the low velocity flows. However, it can easily be seen that beyond the resonant point, the harvested power of the bi-stable system is lower than that of the linear system. This trend becomes more obvious with the increase of the bi-stable gap, b . This result agrees with the frequency response analysis.

At this point, it can be concluded that a bi-stable spring can improve the performance of a VIV energy harvesting system in the lower operating range.

3.3.2 Hardening spring

A system with a hardening spring has varying natural frequency that depends on the vibrating amplitude. This characteristic will result in a potential way to widen the resonance range of the VIV system since the increase of the vibrating amplitude associated to the water flow velocity will also increase the natural frequency.

Mackowski and Williamson [23] recently verified effect of hardening springs on a VIV system experimentally. From this study, the hardening springs represented by the cubic and quintic functions, are confirmed to be able to offer higher output power at high velocity water flows. However, the physical implementation of the springs is rather challenging as the parameters that govern the nonlinearity are not straightforward to control.

Fig. 3.7 presents a piecewise hardening spring mechanism and its restoring force function. The hardening nonlinearity is governed by two parameters, namely the hardening gap, h , and the hardening stiffness, k_h . The restoring force is therefore defined as:

$$F_k(y) = k_0 y + F_k^h(y) \quad (3.14)$$

where:

$$F_k^h(y) = \begin{cases} k_h[y - h \cdot \text{sign}(y)], & |y| \geq h \\ 0, & |y| < h \end{cases}$$

Figure 3.7 Piecewise hardening spring mechanism and its restoring force function.

Fig. 3.8 illustrates the potential wells of a hardening VIV system with $h = 0.6D = 3$ cm and $k_h = 0.75k_0$, a bi-stable VIV system with $b = 0.2D = 1$ cm, and a linear system with the same equivalent stiffness. It can be observed that the potential energy of the hardening system is higher than that of the linear system when the displacement exceeds the defined hardening gap (± 3 cm). This trend reflects the hardening effect of the system.

Figure 3.8 Potential wells of the hardening, bi-stable and linear VIV systems.

To analyze the frequency response of the hardening system, a simulation is conducted by prescribing a sinusoidal force as the excitation to the system with varying amplitude and frequency. The FRF of the hardening system with $h = 0.5D$ and $k_h = 0.75k_0$ is derived and shown in Fig. 3.9.

Figure 3.9 The FRF of the hardening system with $h = 0.5D$ and $k_h = 0.75k_0$.

In general, since the stiffness of the hardening system is enhanced when the response exceeds the hardening gap, the resonance range is shifted towards the right side at the high excitation amplitudes. Due to this characteristic, the increase in the flow velocity, which corresponds to the increase in the magnitude and frequency of the induced fluid force, will improve the response amplitude and output power. As a tradeoff, since the resonance range of the hardening system is pushed towards to the higher frequency side, the amplitude and output power will decrease at lower frequencies.

To simulate response of the hardening VIV system under the impact of the induced fluid force, the hardening restoring force in Eq. (3.14) defined per unit length is substituted for the linear restoring force in the structural oscillator of the wake oscillator model (Eqs. (3.11)) to solve for the displacement, $y(t)$. The simulation is carried out for varying governing parameters (the hardening gap, h , is varied from $0.25D$ to $0.75D$, and the hardening stiffness, k_h , is varied from $0.5k_0$ to k_0) to investigate the parametric effect. All other physical parameters are also chosen similar to those of the linear system discussed in Section 3.2 (see Table 3.2). Figs. 3.10a-b illustrate the amplitude ratio, A^* , and vibrational power, P^* , of the linear system and the hardening system, versus the water flow velocity, U , for different values of the governing parameters h and k_h , respectively.

In general, at low operating frequencies, the hardening system harvests less power than the linear system does. However, beyond the resonant point, the harvested power of the hardening system is strikingly improved. In addition, the resonance range of the hardening system is widened towards the side of the high operating frequencies. This tendency is more obvious with the decrease of the hardening gap, h , and the increase of the hardening stiffness, k_h .

This result is well in line with the observation from the FRF analysis. In conclusion, a hardening spring is able to improve the output power and widen the resonance range of a VIV energy harvesting system towards the high velocity flows.

Figure 3.10 Amplitude ratio, A^* , and vibrational power, P^* , of the linear system and the hardening system, versus the water flow velocity, U : (a) Varying hardening gap, h ; (b) Varying hardening stiffness, k_h .

3.3.3 Combined nonlinear spring

Based on the observations of the nonlinear systems in the previous sections, it can be concluded that there are two distinct advantages of the two systems, i.e. (i) the bi-stable spring improves the working range on the low velocity side, however, it sacrifices the efficiency at high velocity flows, and (ii) the hardening spring enhances the harvested power at high velocity flows but less power can be harvested in the low working range. In order to optimize the performance, intuitively, a combination of the bi-stable spring and hardening spring will take both advantages of these two springs. Fig. 3.11 illustrates the restoring force function of the combined nonlinear spring, which can be mathematically presented as:

$$F_k(y) = F_k^b(y) + F_k^h(y) \quad (3.15)$$

where:

$$F_k^b(y) = k_0[y - b \cdot \text{sign}(y)]$$

$$F_k^h(y) = \begin{cases} k_h[y - h \cdot \text{sign}(y)], & |y| \geq h \\ 0, & |y| < h \end{cases}$$

Figure 3.11 Restoring force function of the combined nonlinear spring.

Fig. 3.12 illustrates a conceptual design of a spring mechanism that results in the combined nonlinear restoring force characteristic. As can be seen in the upper panel, the horizontal springs provide the bi-stable characteristic while the vertical

springs result in the hardening effect in the equivalent restoring force function. The lower panel demonstrates the connection of the spring mechanism to one end of the cylinder. The same mechanism will be connected to the other end of the cylinder.

Figure 3.12 Conceptual design of the combined nonlinear spring mechanism and its connection to the cylinder.

Fig. 3.13 shows the potential wells of the combined nonlinear system ($b = 0.2D = 1$ cm, $h = 0.6D = 3$ cm and $k_h = 0.75k_0$), the hardening system ($h = 0.6D = 3$ cm and $k_h = 0.75k_0$), the bi-stable system ($b = 0.2D = 1$ cm) and the linear system with the same equivalent stiffness. The combined nonlinear system incorporates all the nonlinear features, namely the double-well characteristic and the hardening effect. Therefore, it is expected that the combined nonlinear spring will enhance the performance of the system at both low and high working frequencies.

Figure 3.13 Potential wells of the combined nonlinear system, the hardening system, the bi-stable system and the linear system.

Fig. 3.14 illustrates the FRF of a combined nonlinear system with $b = 0.02D$, $h = 0.5D$ and $k_h = 0.75k_0$, for varying excitation amplitude and frequency. It can be observed that, conceptually, the system takes advantages of both the nonlinear features, i.e. enhancing the response amplitude in the high frequency range due to the hardening feature, while improving the response amplitude in the low frequency range because of the bi-stable feature.

Figure 3.14 The FRF of a combined nonlinear system with $b = 0.02D$, $h = 0.5D$ and $k_h = 0.75k_0$.

The combined nonlinear VIV system is subsequently simulated by incorporating the wake oscillator model. The moderate values of the parameters ($b = 0.02D$, $h = 0.5D$ and $k_h = 0.75k_0$) are used in the simulation. Fig. 3.15 presents the amplitude ratio, A^* , and vibrational power, P^* , as functions of water flow velocity, U .

Figure 3.15 Amplitude ratio, A^* , and vibrational power, P^* , of the system with the combined nonlinear spring versus the water flow velocity, U .

It can be seen that the combined nonlinear spring can significantly improve the harvested power and widen the resonance range on both sides of the low and high velocity flows. However, there still exists a distinct limited range, where the nonlinear system harvests less power than that of the linear system. This limited range is attributed to the resonance range that is shifted towards the high working frequencies due to the hardening effect, while the bi-stable effect is insufficient to compensate for this reduction in the output power.

3.4 Experimental validation by cyber-physical force-feedback testing platform

3.4.1 Cyber-physical force-feedback testing platform

To experimentally validate theoretical analyses based on the wake oscillator model, experiments of a VIV system in a real water channel facility need to be designed and carried out. As discussed in Section 2.2.3, experiments on the VIV phenomenon of an elastically supported circular cylinder are performed mostly by the forced vibration method, the free vibration method and the combined

simulation method. The forced vibration method is obviously inapplicable in this case since the fluid-structure interaction of the nonlinear VIV system and the water flow might not be truly reflected. On the other hand, it would be excessively time and cost-expensive to design and fabricate different setups of physical springs with the prescribed nonlinear restoring force functions for the experiments based on the free vibration method. Therefore, the combined simulation method is most appropriate in this case since the nonlinear springs can be easily appended to the system by virtually prescribing their restoring force functions in the simulation process.

There are two methods to design a combined simulation VIV experiment, namely the force-control based method and the velocity/position-control based method. The former requires an intensive identification of damping from the mechanical components in the system [86]. Therefore, a cyber-physical force-feedback testing platform is designed and developed based on the velocity/position-control method in this thesis.

The cyber-physical force-feedback testing platform is developed based on the principle of combining simulation and experimental processes. Several experimental facilities with this principle were proposed and developed to study the VIV phenomenon (see e.g. [83, 87-89]). In this testing platform, the VIV structure is simulated as an externally excited oscillator in a computer-based controller, where structural parameters such as effective mass, damping coefficient and supporting stiffness are explicitly defined. A physical cylinder is immersed across a water flow, and the induced fluid force that excites the cylinder in the cross-flow direction is measured in real-time. The real-time force signal is delivered to the controller as the external excitation of the simulated oscillator. By solving the dynamics equation with the prescribed structural parameters and the real-time measured fluid force, through a linear actuator, the controller drives the cylinder to the position that corresponds to the calculated displacement in the cross-flow direction. By iterating this loop at a sufficiently high frequency, the VIV phenomenon is observed.

By using the simulated structure, the nonlinear restoring force functions can be easily prescribed through the computer-based controller. This feature is a prominent advantage of the cyber-physical force-feedback testing platform. In addition, the testing platform will also minimize any possible structural uncertainties in order to produce more conclusive results. Please also note that the main intention of experimenting on this testing platform is not to accurately model the real scenario of VIV, but more to demonstrate the feasible application of nonlinear springs as well as analyze their effects.

Fig. 3.16 describes the physical components of the testing platform. The cylinder as the VIV body is submerged into a water channel along the z -direction. The cylinder is connected to a belt-drive actuator through a force sensor. When the water flow passes over the cylinder in the x -direction, the force sensor measures the induced fluid force (in the y -direction) and feeds the real-time signal to the computer-based controller where the structural parameters are defined. At the controller, the corresponding structural displacement in the y -direction is calculated based on the governing equation (Eq. (3.20)) and subsequently fed to the belt-drive actuator to generate the corresponding motion in the cross-flow direction of the cylinder. As a result, the cylinder will vibrate under the excitation of the fluid force.

Figure 3.16 Conceptual design of the physical system in the cyber-physical force-feedback testing platform.

To construct the control law at the controller, the dynamics equation of the VIV structure (Eq. (2.2)) is re-written here:

$$m_{osc} \ddot{y} + c\dot{y} + ky = F_y \quad (3.16)$$

The values of the structural parameters m_{osc} , c and k will be defined at the controller. It should be noted that the linear restoring force term $F_{restoring} = ky$ in Eq. (3.16) will become a nonlinear function of the displacement, y , in the case of embedding nonlinear springs to the system.

Due to the inertial effect, the force measured by the sensor, $F_{measured}$, comprises the fluid force in the cross-flow direction, F_y , and the inertial force from the cylinder and the supporting structure that correspond to a physical mass, $m_{physical}$. This relation can be expressed as:

$$F_{measured} = F_y + F_{inertial} \quad (3.17)$$

The inertial force is related to the acceleration of the system as:

$$F_{inertial} = -m_{physical} \ddot{y} \quad (3.18)$$

Therefore, the fluid force in the cross-flow direction is obtained as:

$$F_y = F_{measured} + m_{physical} \ddot{y} \quad (3.19)$$

By substituting Eq. (3.19) into Eq. (3.16), the governing equation of the controller is obtained as:

$$m_{osc} \ddot{y} + c\dot{y} + ky = F_{measured} + m_{physical} \ddot{y} \quad (3.20)$$

It can be seen in Eq. (3.20) that the physical mass, $m_{physical}$, must be lower than the effective mass of the oscillating structure, m_{osc} , to avoid the unstable motion. In addition, selecting a low physical mass will minimize the inertial effect, and in turn, will simplify the simulation of the effective mass. In this design, a hollow acrylic cylinder and thin aluminum plates for the supports are utilized to minimize the physical mass.

The control law based on Eq. (3.20) is programmed in the Simulink environment (Matlab R2012a). Subsequently, the control process is implemented by the dSPACE DS1103 PPC controller and the dSPACE ControlDesk 3.7.4 software. The control strategy of the testing platform is summarized by the diagram in Fig. 3.17.

Figure 3.17 Control strategy of the cyber-physical force-feedback testing platform.

Fig. 3.18 presents the physical components of the testing platform together with the circulating water channel facility. In this testing platform, a 400 W AC servo motor (Delta ECMA-C10604F8) controlled by an AC servo drive (Delta ASD-A2-0421-L) is connected to a belt-drive actuator (Toyo M80M-S-X102-400-L-T40-C4) with the repeatability of ± 0.08 mm to produce the linear motion of the cylinder. The cylinder is connected to the belt-drive actuator through a 1-axis compression/tension load cell (Futek LSB302) to measure the induced fluid force. When the force signal is sent to the controller, a low-pass Butterworth filter with the 30 Hz cutoff frequency is used. The actual displacement of the cylinder is measured by an encoder that is incorporated in the servo motor with the resolution of 8192 pulse/rev. Through a step-response test, it is found that the time constant of the system is around 30 ms.

The experiments are facilitated by a circulating water channel at the Maritime Research Centre, Nanyang Technological University. This water channel with the testing section of $0.31 \times 0.45 \times 5$ m (W $\times$ H $\times$ L) is able to produce the water flow

velocity from 0.02 – 0.7 m/s. The uniformity of the flow is confirmed within 1.5% of the free stream velocity across the testing section and the turbulence intensity is below 2% in the free stream.

Figure 3.18 Physical components of the cyber-physical force-feedback testing platform with the circulating water channel located at the Maritime Research Centre, Nanyang Technological University. An experiment of the linear VIV system is also carried out on the testing platform as the benchmark for experimental studies of the proposed nonlinear VIV systems. In this experiment, the diameter of the cylinder is kept the same as that in the simulation on the wake oscillator model ($D = 0.05$ m), and the length of the cylinder is designed at $L = 0.235$ m to match the dimension of the water channel. The effective mass, damping coefficient and supporting stiffness of the VIV system, which are defined at the computer-based controller, are also kept consistent to those in the simulation. The natural frequency of the structure in water, $f_{n, water}$, is confirmed by a free decay test in water with an error of 0.5% compared to the calculated value. All the physical properties of the VIV system in this experiment are summarized in Table 3.3.

Table 3.3 Physical parameters of linear VIV system in the benchmark experiment.

Parameters	Values	Parameters	Values
Cylinder diameter, D	0.05 m	Structural damping coefficient, c	0.052 Ns/m
Cylinder length, L	0.235 m	Supporting stiffness, k	87 N/m
Effective mass, m_{osc}	5.1 kg	Natural frequency in water, $f_{n, water}$	0.63 Hz
Physical mass, $m_{physical}$	0.6 kg		

Unlike the simulation data where motion of the system is purely periodic and the vibrating amplitude can be straightforwardly determined, the experimental data is contaminated by random noise. Therefore, the vibrating amplitude in the experiment is statistically calculated based on the root mean square (RMS) value of the displacement. Fig. 3.19 presents the vibrating amplitude ratio, A^* , of the cylinder and the vibrational power, P^* , versus the normalized water flow velocity, U^* , from the simulation on the wake oscillator model and the experiment conducted on the testing platform.

Figure 3.19 Amplitude ratio, A^* , and vibrational power, P^* , versus the normalized water flow velocity, U^* , from the simulation and experiment.

From the experimental result, the resonance, where the vibrating amplitude is significantly magnified, takes place around the normalized velocity of 5, which is comparable to the simulation result and the experimental result presented by Khalak and Williamson [47] (see Fig. 3.20). However, there is a noticeable discrepancy in the vibrating amplitude and the width of the resonance range in the present experiment compared to the referred experiment. This discrepancy is attributed to the different aspect ratio, L/D , of the cylinder used in the present experiment, where the ratio is equal to 4.7 compared to 10 in the referred experiment. The lower the aspect ratio, the lower and the narrower the vibrating amplitude and the resonance range of the system [128, 129]. As can be seen in the right panel of Fig. 3.19, the output power from the experiment is lower than

the simulated power. This is also attributed to the low aspect ratio of the cylinder used in the experiment.

At this point, the resulted experimental observation will be used as a benchmark for the experimental studies of the proposed nonlinear VIV systems that will be discussed in the subsequent sections.

Figure 3.20 Maximum amplitude ratio, A^*_{max} , versus normalized water flow velocity, U^* , of the VIV system with $m^* = 10.1$ and $m^*\zeta = 0.013$ (reproduced from [47]).

3.4.2 Validation of bi-stable spring

The bi-stable restoring force in Eq. (3.13) is substituted for the linear restoring force in Eq. (3.20) to configure the bi-stable VIV system at the computer-based controller. In this experiment, the system is investigated for different bi-stable gap values, while other physical properties are kept the same to those of the linear system in the benchmarking experiment in Section 3.4.1 (see Table 3.3) in order to have an apple-to-apple comparison result. Fig. 3.21 shows the amplitude ratio, A^* , and vibrational power, P^* , of the bi-stable system with various bi-stable gap values, b , versus the water flow velocity, U .

Figure 3.21 Amplitude ratio, A^* , and vibrational power, P^* , of the bi-stable system versus the water flow velocity, U , with varying bi-stable gap, b .

It can be seen that on the low operating side, the bi-stable system harvests more power than that of the linear system. However, beyond the peak point, the bi-stable system harvests less power than the linear system does. This trend becomes more evident when the bi-stable gap, b , is increased. This result is consistent to the simulation result obtained by using the wake oscillator model, where it can also be observed that the bi-stable spring improves the performance of a VIV system at the low operating frequencies.

3.4.3 Validation of hardening spring

To experimentally investigate response of the hardening system, the hardening restoring force in Eq. (3.14) is substituted for the linear restoring force in Eq. (3.20) with various values of the governing parameters h and k_h . All other physical properties are kept the same as those in the benchmarking experiment in Section 3.4.1 (see Table 3.3). Figs. 3.22a-b depict the amplitude ratio, A^* , and vibrational power, P^* , of the hardening system versus the water flow velocity, U , for varying hardening gap, h , and hardening stiffness, k_h , respectively.

In both cases of varying h and k_h , it is shown that the hardening system harvests less power than that of the linear system at the low frequencies in the resonant regime. Nevertheless, the hardening system significantly improves the harvested power when the water flow velocity is increased in the resonant regime. The

improvement becomes more evident with the decrease of the hardening gap, h . However, less power can be harvested in the low working range when the hardening gap is decreased. This gives a contradictive effect to the bi-stable spring.

Figure 3.22 Amplitude ratio, A^* , and vibrational power, P^* , of the system with hardening spring versus the water flow velocity, U : (a) Varying hardening gap, h ; (b) Varying hardening stiffness, k_h .

Meanwhile, when the hardening stiffness, k_h , is increased, the harvested power will also increase in the later regime of the working range. On the other hand, the reduction in output power in the prior regime of the resonance range is more significant for a system hardened by a higher stiffness. This result agrees to the simulation result based on the wake oscillator model.

There is another noticeable observation from the experimental result that was not captured in the simulation. The hardening spring extends the resonance range until the vibrating amplitude drops within the hardening gap and the system

returns to behave like a linear system. For instance, as can be seen in the left panel of Fig. 3.22a, the resonance range of the hardening system with $h = 0.25D$ is widened to the water velocity of 0.23 m/s, where the vibrating amplitude decreases to $0.25D$. From this observation, it can be concluded that the hardening gap, h , has the dominant effect on extending the resonance range.

In conclusion, it is experimentally confirmed that effect of the hardening spring in a VIV system is enhancing the harvested power in the high operating regime.

3.4.4 Validation of combined nonlinear spring

To experimentally validate effect of the combined nonlinear spring, the combined nonlinear restoring force in Eq. (3.15) is substituted for the linear restoring force in Eq. (3.20) with varying governing parameters h and k_h . The bi-stable gap, b , is kept constant at $0.02D$ to maintain the moderate bi-stable effect. All other physical properties are also kept the same as those of the benchmarking system in Section 3.4.1 (see Table 3.3). Figs. 3.23a-b show the amplitude ratio, A^* , and vibrational power, P^* , of the combined nonlinear system versus the water flow velocity, U , for varying hardening gap, h , and hardening stiffness, k_h , respectively.

In general, the behavior of the combined nonlinear system in this experiment is consistent to the simulation result based on the wake oscillator model. Owing to the advantages of both the nonlinear springs, the system is capable of improving the harvested power at the low operating frequencies and in the later regime of the resonance range. Similar to the hardening system, the effective regime in the resonance range, where the combined nonlinear system improves the harvested power, is also determined by the hardening gap, h .

It is also found that the combined nonlinear system harvests less power than the linear system does towards the end of the resonance range. This observation is more evident in the experiment than as found in the simulation. This drop in power is attributed to the bi-stable effect. Therefore, unlike the hardening system,

in which the capability to broaden the resonance range is only dependent on the hardening gap, this capability of the combined nonlinear system is also determined by the bi-stable gap, b .

Figure 3.23 Amplitude ratio, A^* , and vibrational power, P^* , of the system with the combined nonlinear spring versus the water flow velocity, U , from the experiment: (a) Varying hardening gap, h ; (b) Varying hardening stiffness, k_h .

In conclusion, the combined nonlinear spring is potential to enhance the performance of a VIV energy harvesting system subjected to an optimal selection of the bi-stable gap. An optimal bi-stable gap will maintain the inter-well vibration in the whole operating range and minimize the bi-stable effect around the end point of the resonance range.

3.5 Summary

In this chapter, the theoretical model of a VIV system was discussed in details. In this model, the structural oscillator equation that models the VIV structure is

coupled to the modified higher-order wake oscillator that simulates the vortex shedding behavior.

The cyber-physical force-feedback testing platform that was designed and developed for experiments of the VIV system in a real water channel facility was also thoroughly described. The testing platform is working by combining the simulation process at the computer-based controller and the real-time measurement of the induced fluid force to produce the physical vibration of the VIV structure.

By employing the wake oscillator model and the cyber-physical force-feedback testing platform, effects of the bi-stable spring and hardening spring on the performance of a VIV energy harvesting system were numerically and experimentally investigated for various governing parameters. The experimental result shows a good qualitative agreement with the simulation result. The bi-stable spring is able to improve the harvested power at the low operating frequencies, while the hardening spring improves the harvested power at the high operating frequencies.

To further exploit the advantages of both the bi-stable spring and hardening spring, a combined nonlinear spring is proposed. The simulation and experimental results show that this combined nonlinear spring is potential to improve the performance of the system by extending the overall operating range.

The performance of the VIV system enhanced by the nonlinear springs is strongly dependent on the governing parameters that determine the degree of nonlinearity. In particular, when the bi-stable spring is embedded in a VIV system, the bi-stable gap must be properly selected to maintain the highest-energy vibration orbit, which is the inter-well orbit, and avoid chaotic responses. In the next chapter, chaotic responses of a bi-stable VIV system will be thoroughly studied.

Chapter 4****Quantifying Chaotic Responses of Bi-stable Vortex Induced Vibration Energy Harvesting Systems**

In this chapter, chaotic responses of a VIV energy harvesting system enhanced by a bi-stable spring are quantified in a wide range of the governing parameters to analyze their dependency on these parameters. Chaotic responses of the bi-stable VIV system are simulated by the wake oscillator model and quantified via the largest Lyapunov exponent. Subsequently, a series of experiments is carried out on the cyber-physical force-feedback testing platform to validate the analyses from the simulation. The main challenge in dealing with experimental data is to distinguish chaotic responses from the noise-contaminated periodic ones as noise will smear out the regularity of periodic responses. To overcome this issue, a surrogate data test is used in order to check the hypothesis for the presence of chaotic behavior.

**Main findings of this chapter have been published as:

- B.H. Huynh, and T. Tjahjowidodo, "Experimental Chaotic Quantification in Bistable Vortex Induced Vibration Systems," *Mechanical Systems and Signal Processing*, vol. 85, pp. 1005-1019, 2017.
- B.H. Huynh, T. Tjahjowidodo, Z. Zhong, Y. Wang, and N. Srikanth, "Chaotic Responses on Vortex Induced Vibration Systems Supported by Bistable Springs," in *Proceedings of the International Conference on Noise and Vibration Engineering*, Leuven, Belgium, 2016, pp. 695-704.

4.1 Motivation – Effect of chaotic responses on the energy harvesting performance of a bi-stable VIV system

As discussed in Chapter 3, when a bi-stable spring is applied in an oscillator, it will result in a double-well potential energy characteristic as illustrated in Fig. 3.4. From the perspective of nonlinear dynamics, this system has two stable fixed points and one unstable fixed point at the origin. Depending on the excitation energy level, the bi-stable oscillator might vibrate in three different modes. At a low excitation energy level, the system vibrates within one of the potential wells. This mode is referred to as the *intra-well mode*. When the excitation energy is sufficiently high, the system will overcome the unstable point and vibrate in the *inter-well mode*. The system can also vibrate in a more complex manner, i.e. irregularly jumping back and forth between the two potential wells, or transforming from the intra-well mode to the inter-well mode and vice versa. This mode is referred to as the *chaotic mode*.

In chapter 3, periodic inter-well vibrations were observed on the bi-stable VIV system with a small bi-stable gap, b , which is from $0.01D - 0.03D$. This chapter is extending the discussion to the cases where chaotic responses occur. A chaotic signal is defined as a non-periodic, unpredictable and seemingly a noisy signal [93]. An effective method to study the chaotic response of a nonlinear system is to quantify it by the Lyapunov exponent, which is a familiar quantifier to measure the chaotic degree of a dynamical system [94, 130]. By correlating this quantifier to the structural parameters, the dependency of chaotic responses on these parameters can be analyzed and subsequently used as the primary reference to design the system with the avoidance of chaotic responses.

Studies of chaotic responses of VIV systems, e.g. [97-99], are limited at the confirmation of their occurrence. There is a lack of comprehensive studies to analyze the dependency of chaotic responses on the structural parameters of a VIV system. Therefore, this chapter will focus on quantifying chaotic responses of the bi-stable VIV system in a wide range of the structural parameters.

Dynamics of a VIV system enhanced by a bi-stable spring can be modelled as:

$$m\ddot{y} + c\dot{y} + k_0[y - b \cdot \text{sign}(y)] = F_y(t) \quad (4.1)$$

where m is sum of the effective mass and fluid-added mass, c denotes the damping coefficient, k_0 is the equivalent stiffness, b represents the bi-stable gap (see Fig. 3.3) and $F_y(t)$ comprises the induced lift force and fluid damping force. Based on Eq. (4.1), the fundamental parameters of a bi-stable VIV system are m , c , k_0 , and b . A dimensional analysis is performed to minimize the number of parameters for the parametric quantification of chaotic responses. Some non-dimensional parameters are introduced, namely: $p = y/b$, $\omega_0^2 = k_0/m$, $\bar{t} = \omega_0 t$, $b^* = b/D$, and $\zeta = 0.5c/(k_0 m)^{1/2}$. By substituting these parameters to Eq. (4.1), its dimensionless form can be obtained as:

$$p'' + 2\zeta p' + p - \text{sign}(p) = \psi F_y\left(\frac{\bar{t}}{\omega_0}\right) \quad (4.2)$$

where $\psi = 1/(k_0 b)$ and the over commas denote the derivatives with respect to $\bar{t}$. The system turns out to be governed by two main parameters ζ and ψ . It should be noted that ψ depends on k_0 and b . Since k_0 determines the natural frequency, b , or its dimensionless form, b^* , will be varied to investigate effect of ψ .

The wake oscillator model is also utilized to simulate chaotic responses of the bi-stable VIV system. The physical properties of the VIV system discussed in Chapter 3 (see Table 3.2) are also utilized in this analysis. The damping ratio of this system is resulted at $\zeta = \zeta_0 = 0.0012$. Fig. 4.1 shows the vibrational power, P^* , versus water flow velocity, U , of the bi-stable system with varying bi-stable gap and a linear system with the same equivalent stiffness. It can be observed that the bi-stable system with a small bi-stable gap, $b^* = 0.02$, exhibits a minor effect on improving the output power at the low velocity flows. When the bi-stable gap is increased to $b^* = 0.2$, the improvement in the output power at the low velocity flows becomes noticeable due to the high-amplitude vibration in the inter-well mode. However, the power suddenly drops when the flow velocity is increased (beyond 0.16 m/s). At a glance, it is suspected that the drop in power

is attributed to chaotic responses that occur at the corresponding water flow velocities.

Figure 4.1 Vibrational power, P^* , versus flow velocity, U , of a bi-stable system with varying bi-stable gap, b^* , and a linear system with the same equivalent stiffness.

The occurrence of chaotic responses in the bi-stable VIV system also depends on the damping ratio, ζ . Fig. 4.2 shows the time series displacement of the cylinder and the phase plots when the damping ratio is varied. At a low damping ratio, $\zeta = 5\zeta_0$, the response of the system is chaotic as indicated in Fig. 4.2a. However, when the damping ratio is increased to $\zeta = 80\zeta_0$, the chaotic effect is diminished and the system vibrates periodically (see Fig. 4.2b).

Evidently, to significantly improve the output power, the bi-stable VIV system should be designed with a sufficiently large bi-stable gap. Unfortunately, increasing the bi-stable gap is suspected to result in chaotic responses. Therefore, chaotic responses must be taken into account when a bi-stable spring is introduced to the VIV system to improve the output power. Besides, a high damping ratio seems to be able to mitigate chaotic responses of the bi-stable VIV system. To confirm these observations, hereafter, the study will focus on parametrically quantifying chaotic responses of the bi-stable VIV system to map the occurrence of chaotic responses as a function of the governing parameters b^* and ζ .

Figure 4.2 (a) Chaotic response of the system at $U = 0.158$ m/s, $b^* = 0.4$, $\zeta = 5\zeta_0$; (b) Periodic response of the system at $U = 0.158$ m/s, $b^* = 0.4$, $\zeta = 80\zeta_0$.

4.2 Parametric quantification of chaotic responses

4.2.1 Chaotic quantification by the Lyapunov exponent

As discussed in Chapter 2, in nonlinear dynamics, behavior of a system is analyzed in a phase space, where its dynamical state at a time instance is represented by a state point or vector, and a trajectory or orbit will be formed when the state vector evolves in time. For a system that behaves periodically, the trajectory is closed and reproducible after every cycle. On a contrary, when the chaotic response occurs, a non-reproducible trajectory is formed in a bounded region. In particular, chaos in a dynamics system can be explicitly identified and measured by the quantity Lyapunov exponent.

The Lyapunov exponent, γ , in Eq. (2.5) is defined as the rate of divergence of two nearby state vectors in a phase space. A positive value of this quantity is a proof of the chaotic response and its magnitude measures the chaotic level of the system. However, it has to be highlighted that calculating the Lyapunov exponent

by observing the separation of the trajectories initiated from two nearby initial points in a phase space is barely practical. For example, it is almost impossible to observe the dynamics of a VIV system from two nearby initial conditions in an experimental circumstance. Moreover, measuring all state variables might be a tedious process. Therefore, the practical calculation is adapted to the estimation of the *largest Lyapunov exponent*. The estimation is carried out by observing the separation of the orbits initiated from two nearby state vectors in a time delay phase space. This time delay phase space is reconstructed from the time series of a single state variable, which is chosen to be displacement of the cylinder, $y(t)$, in this case.

The largest Lyapunov exponent is one of the components in the Lyapunov spectrum of a dynamical system [131]. The Lyapunov spectrum, $(\gamma_1, \gamma_2, \gamma_3, \dots, \gamma_N)$, contains the individual Lyapunov exponent components of an N -dimensional dynamical system. Each component indicates the rate of expansion or contraction of the attractor in the corresponding phase space direction. The Lyapunov exponent components are ordered as $\gamma_1 \geq \gamma_2 \geq \gamma_3 \geq \dots \geq \gamma_N$, where the component γ_1 is the largest Lyapunov exponent. A positive value of the largest Lyapunov exponent is the confirmation of chaotic response and can also be used as the chaotic quantifier. Hereafter, the largest Lyapunov exponent is denoted by γ to simplify the discussion.

The procedure to estimate the largest Lyapunov exponent from a time series is systematically presented in [100-102] and widely applied for various nonlinear systems. First, from the selected time series data, a so-called time delay phase space is properly constructed to ensure that all the dynamics characteristics including the Lyapunov spectrum of the system in the inherent phase space are preserved. Then, by embedding the time series data to the new phase space, the state vectors that can represent the dynamics of the system are constituted accordingly. Finally, the largest Lyapunov exponent is estimated based on the embedded state vectors.

4.2.2 Reconstruction of time delay phase space

In this study, the analysis is carried out based on the time series displacement of the cylinder, $y(t)$. In the time delay phase space, the dynamical state of the system is represented by the state vector, $\mathbf{y}(t)$, which is formed as:

$$\mathbf{y}(t) = [y(t), y(t + t_d), y(t + 2t_d), \dots, y(t + (d - 1)t_d)] \quad (4.3)$$

where t_d is the time delay and d is the embedding dimension of the reconstructed phase space. Therefore, appropriate selections of t_d and d are required to reconstruct the time delay phase space.

(1) Selection of time delay, t_d

The time delay, t_d , is selected based on the minimum correlation between the elements of the state vector. For a linear system, the auto-correlation technique is usually utilized to serve this task. However, it is suggested in [100, 132, 133] that the Average Mutual Information (AMI) is more appropriate for nonlinear systems. In general, the AMI function is analogous to the auto-correlation function of the linear data series. The AMI function is calculated based upon the probability density of the data measurement at a certain point of time. Therefore, the AMI function is intuitively more suitable for a chaotic data series due to its probabilistic nature [134]. Zduniak and Lusakowski [135] estimated the time delay for the phase space reconstruction of a chaotic system, i.e. the self-generated current oscillations in a semi-insulating Gallium Arsenide wafer, by both the auto-correlation and AMI functions. It was observed that the estimation based on the AMI function was highly repeatable for different measured data series. In other words, the AMI function provides more reliable estimation of the time delay for the chaotic data series. This method also provides good performance for analyses of many other nonlinear systems (e.g. [94, 130]).

Assuming that $y(t)$ and $y(t + t_d)$ are the two measurements of displacement at t and $(t + t_d)$, respectively, their mutual information is defined as the amount of information (bits) of one measurement that can be learned from the other. If $P[y(t)]$ is the probability density of the measurement $y(t)$ and $P[y(t), y(t + t_d)]$ is the joint

probability density of the measurements $y(t)$ and $y(t + t_d)$, the AMI between these two measurements, $I(t_d)$, is calculated as:

$$I(t_d) = \sum_{y(t), y(t+t_d)} P[y(t), y(t+t_d)] \log_2 \frac{P[y(t), y(t+t_d)]}{P[y(t)]P[y(t+t_d)]} \quad (4.4)$$

The measurements $y(t)$ and $y(t + t_d)$ are expected to be independent on each other or the AMI tends to be zero when t_d increases. Therefore, the value of t_d , where the AMI function reaches its first local minimum or the two measurements $y(t)$ and $y(t + t_d)$ are least dependent on each other, is suggested to be the time delay for the phase space reconstruction.

Fig. 4.3 illustrates the AMI function from the time series displacement of the bi-stable VIV system simulated at $U = 0.158$ m/s, $\zeta = \zeta_0$ and $b^* = 0.25$. The time delay, t_d , is chosen to be 0.41 s, where the AMI function reaches its first local minimum.

Figure 4.3 AMI function, $I(t_d)$, of the time series displacement of the cylinder simulated at $U = 0.158$ m/s, $\zeta = \zeta_0$ and $b^* = 0.25$.

(2) Selection of embedding dimension, d

After the time delay, t_d , is determined, the embedding dimension, d , is selected to complete the phase space reconstruction. The embedding dimension, d , determines the number of elements in a state vector, or the dimensionality of the reconstructed phase space. Basically, the embedding dimension must be sufficiently high to totally unfold the attractor in the reconstructed phase space.

In this study, the method proposed in [136] is applied to select the embedding dimension. To elaborate the method, $\mathbf{y}(d)$ and $\mathbf{y}(d + 1)$ are defined as the state vectors in the reconstructed phase spaces with the embedding dimensions d and $(d + 1)$, respectively. The Euclidian distance between each point and its nearest neighbor in the corresponding phase space is calculated. Then, the ratio of this distance of the point $\mathbf{y}(d + 1)$ to that of the point $\mathbf{y}(d)$ is defined as the embedding error when the embedding dimension is increased from d to $(d + 1)$. Then, the mean value, $E(d)$, of the embedding errors corresponding to all the reference points is calculated. Finally, the embedding dimension is selected based on the ratio $E_1(d)$ defined as:

$$E_1(d) = E(d + 1)/E(d) \quad (4.5)$$

The ratio $E_1(d)$ indicates the change of $E(d)$ when the embedding dimension is increased from d to $(d + 1)$. The dimension $(d_0 + 1)$ is selected as the embedding dimension if the ratio $E_1(d)$ starts to saturate at the dimension d_0 . Fig. 4.4 illustrates the ratio $E_1(d)$ as a function of the embedding dimension from the time series displacement of the bi-stable VIV system simulated at $U = 0.158$ m/s, $\zeta = \zeta_0$ and $b^* = 0.25$. This ratio saturates when the embedding dimension is larger than 3. Therefore, the embedding dimension $d = 4$ is selected in this case.

Figure 4.4 The ratio $E_1(d)$ as a function of the embedding dimension from the time series displacement of the cylinder simulated at $U = 0.158$ m/s, $\zeta = \zeta_0$ and $b^* = 0.25$.

4.2.3 Estimating the largest Lyapunov exponent

After the time delay and the embedding dimension are selected, the state vectors of the system in the time delay phase space are reconstructed by embedding the time series data into the space with respect to Eq. (4.3). In this study, the largest Lyapunov exponent, γ , is estimated from the embedded state vectors by the direct method described in [102], which was developed based on the original method proposed in [131]. In this method, the largest Lyapunov exponent is estimated from the prediction error, $e(\Delta t)$, which is calculated as:

$$e(\Delta t) = \frac{1}{M} \sum_{i=1}^M \log_2 \left(\frac{\|y_i(t + \Delta t) - y_i^m(t + \Delta t)\|}{\|y_i(t) - y_i^m(t)\|} \right) \quad (4.6)$$

where $y_i(t)$ is a state vector at the time instance t , and $y_i^m(t)$ is its nearest neighbor; M is the number of reference state vectors; $\|y_i(t) - y_i^m(t)\|$ denotes the Euclidian distance between the vectors $y_i(t)$ and $y_i^m(t)$. The calculation in Eq. (4.6) can be interpreted as follows. First, the calculation is started by specifying a state vector $y_i(t)$. Subsequently, the Euclidian distance between this vector and its nearest neighbor is repeatedly calculated for every increment of Δt while the vectors evolve in the phase space. At the same time, the ratio of the evolving distance to the original distance is also calculated. This ratio, which is referred to as the prediction error, $e(\Delta t)$, is plotted as a function of the evolution time, Δt , on a logarithmic scale. To ensure the estimation accuracy, the prediction error is calculated for all the reference vectors and the average value is utilized to estimate the largest Lyapunov exponent.

The variation of the prediction error function, $e(\Delta t)$, follows three phases. The first phase is the transient phase where the trajectory of the nearest neighbor approaches the direction of the largest Lyapunov exponent. In the second phase, the Euclidian distance between the reference vector and its nearest neighbor increases exponentially with respect to $2^{\gamma \Delta t}$ until the valid range for the linear

approximation of the orbit around the reference point is exceeded. In the last phase, the Euclidian distance increases with a rate that is slower than the exponential rate and then, decreases due to the folding effect of the attractor. When the second phase lasts sufficiently, the slope of the linear segment on the prediction error plot provides a reasonable estimation of the largest Lyapunov exponent, γ .

From the perspective of the information theory, the largest Lyapunov exponent indicates the speed of creating or destroying information of the system process [137]. Therefore, the exponent is presented in bits of information per unit of time (bit/s). For example, the largest Lyapunov exponent of a dynamical system is estimated at $\gamma = 2$ bit/s. It means, when a state vector is specified with an accuracy of 20 bits, it is impossible to predict the system response after 10 s [20 bits / 2 (bit/s)]. It is obvious that the higher the largest Lyapunov exponent, the lower predictability for the future state of the system, or the higher chaotic level of the system. Fig. 4.5 illustrates the prediction error function, $e(\Delta t)$, from the time series displacement of the cylinder simulated at $U = 0.158$ m/s, $\zeta = \zeta_0$ and $b^* = 0.25$. In this case, the largest Lyapunov exponent is estimated at $\gamma = 0.7018$ bit/s.

Figure 4.5 Prediction error function, $e(\Delta t)$, from the time series displacement of the cylinder simulated at $U = 0.158$ m/s, $\zeta = \zeta_0$ and $b^* = 0.25$.

4.2.4 Results and discussions

The time series displacement of the cylinder is simulated for a wide range of the governing parameters that include the damping ratio, ζ , and bi-stable gap, b^* . Subsequently, the largest Lyapunov exponent, γ , is calculated correspondingly. Fig. 4.6 presents the largest Lyapunov exponent as a function of ζ and b^* for the water flow velocity of 0.158 m/s, which corresponds to the resonant velocity estimated from the vortex shedding frequency. The contour plot is constructed from 4941 data points, where each point is evaluated from 5000 s simulated data with the 0.002 s sampling time (but only data in the last 2000 s is used to calculate the largest Lyapunov exponent to ensure the exclusion of the transient response).

Figure 4.6 Largest Lyapunov exponent, γ (bit/s), of the system excited by the water flow with velocity of 0.158 m/s as a function of damping ratio, ζ , and bi-stable gap, b^* .

It can be observed that the chaotic degree decreases and subsequently diminishes when the damping ratio is increased. For the damping ratio that is higher than $70\zeta_0$, chaotic responses become trivial. It can also be seen that when the damping ratio increases, chaotic responses are more likely to occur at small bi-stable gap until it diminishes at high damping ratio.

On the other hand, when the bi-stable gap is small, the system tends to result in regular responses. This can be easily understood as the system resembles the linear system. When the bi-stable gap is increased, chaotic responses occur and the chaotic degree also increases together with the increment of the bi-stable gap. However, until a certain bi-stable gap is reached, the chaotic degree starts to decrease gradually. As the bi-stable gap is further increased, the system tends to vibrate in the periodic intra-well mode since the vibrational energy is insufficient for the system to overcome the unstable fixed point.

Fig. 4.7 depicts the bifurcation diagrams of the corresponding system for varying damping ratio and bi-stable gap. The diagrams are introduced as functions of the water flow velocity, U , where each simulation is performed for 20,000 s at the 0.002 s sampling time, but only data in the last 10,000 s is used to ensure the exclusion of the transient part. The regions with chaotic responses are indicated by the infinite numbers of amplitudes on the bifurcation diagrams.

In general, at low velocity flows, the system behaves regularly. In particular, at the very low velocity flows, the system initially stays stationary as no resonance occurs. The system starts to vibrate periodically with the increasing amplitudes when the flow velocity is increased due to the resonance phenomenon. Until a certain velocity is reached, the system starts to vibrate chaotically. Subsequently, when the flow velocity keeps increasing, the system behaves regularly again before vibrations are totally suppressed at the high velocity flows. For example, in the case of $\zeta = 50\zeta_0$, chaotic responses occur in the flow velocity range of 0.136 m/s – 0.191 m/s, and the system becomes completely regular again beyond this range (however, it is noted that within the chaotic range, there also exist some regions where the system exhibits regular responses, e.g. from 0.169 m/s – 0.186 m/s). The abrupt transitions (sudden jumps in amplitudes) between periodic and chaotic responses in the bifurcation sequence are attributed to the discontinuity of the proposed spring force in the system.

Figure 4.7 Bifurcation diagrams presented by the vibrating amplitude as a function of water flow velocity, U , for varying damping ratio, ζ , and bi-stable gap, b^* .

It can be observed from Fig. 4.7a that increasing the damping ratio is capable of diminishing chaotic responses. Meanwhile, effect of the bi-stable gap can be

studied from Fig. 4.7b. When the bi-stable gap is increased, the chaotic regions extend. This concludes that chaotic responses have a higher chance to occur in the system with a larger bi-stable gap. However, when the bi-stable gap is further increased, chaotic responses are less likely to occur as indicated by the contraction of the chaotic regions in the case of $b^* = 0.6$. These observations from the bifurcation diagrams tally with the analyses based on the contour plot in Fig. 4.6.

Fig. 4.8 shows the Poincaré maps, which illustrate the intersections of the orbits in the phase space with a lower-dimensional subspace. The maps are generated by simulating the system in the duration of 200,000 s after the exclusion of the transient response in order to have sufficient intersection points in the phase space.

Fig. 4.8a illustrates the variation of the Poincaré map with the increase of the damping ratio. At the low damping ratio, $\zeta = \zeta_0$, the response of the system is chaotic as indicated by the fractal structure of the map. When the damping ratio is increased to $\zeta = 70\zeta_0$, the chaotic response is eliminated as indicated by the finite number of points on the map, instead of a fractal structure. On the other hand, regular responses become chaotic when the bi-stable gap is increased and turn into regular responses again when the bi-stable gap exceeds a certain value (see Fig. 4.8b). These observations from the Poincaré maps are also in line with the analyses based on the contour plot in Fig. 4.6 and the bifurcation diagrams in Fig. 4.7.

In practice, a high damping ratio to mitigate chaotic responses of the bi-stable VIV energy harvesting system can be attained by damping of the PTO component. Meanwhile, designing the system with either a small or very large bi-stable gap can also be useful to avoid chaotic responses. However, the bi-stable system with a very large bi-stable gap tends to vibrate in the intra-well mode dominantly. Thus, the technical merit of enhancing the output power through introducing a bi-stable spring will not be achieved.

Figure 4.8 Poincaré maps of the system for varying damping ratio, ζ , and bi-stable gap, b^* .

4.3 Experimental validation

To verify the existence of chaotic responses in a bi-stable VIV system and qualitatively validate the quantification result from the simulated data, a series of experiments was carried out on the cyber-physical force-feedback testing platform described in Chapter 3. Physical properties of the VIV system in this experimental study are kept the same to those of the system discussed in Chapter 3 (see Table 3.3).

An initial experiment was carried out to qualitatively validate effect of chaotic responses on the energy harvesting performance of the bi-stable VIV system. Fig. 4.9 presents the vibrational power, P^* , versus the water flow velocity, U , of a bi-stable system with varying bi-stable gap and a linear system with the same equivalent stiffness from this experiment.

It can be observed that the bi-stable VIV system configured by a small bi-stable gap, $b^* = 0.02$, provides a minor improvement in the output power at the low speed flows. When the bi-stable gap, b^* , is increased to 0.2, the improvement in the output power at the low speed flows becomes striking. However, when the flow velocity exceeds the value of 0.14 m/s, chaotic responses occur and erratically deteriorate the output power. The experimental result is well in line with the simulation result based on the wake oscillator model (see Fig. 4.1).

Figure 4.9 Vibrational power, P^* , versus flow velocity, U , of a bi-stable system with varying bi-stable gap, b^* , and a linear system with the same equivalent stiffness.

Subsequently, in order to qualitatively validate the quantification of the simulated chaotic responses, nine experimental cases were performed with various governing parameters that include the damping ratio, ζ , bi-stable gap, b^* , and water flow velocity, U , as listed in Table 4.1. The data series of the displacement in each case was continuously recorded in about two hours in order to ensure the reliability of the calculation of the Lyapunov exponent.

Table 4.1 Experimental cases with different parameters.

Case no.	ζ	b^*	U (m/s)
1	$\zeta_0 (= 0.0012)$	0.1	0.13
2	ζ_0	0.1	0.18
3	ζ_0	0.1	0.20
4	ζ_0	0.2	0.13
5	ζ_0	0.2	0.18
6	ζ_0	0.2	0.20
7	$20\zeta_0$	0.1	0.13
8	$20\zeta_0$	0.1	0.18
9	$20\zeta_0$	0.1	0.20

4.3.1 Surrogate data test

Fig. 4.10 presents the time series displacement of the cylinder in Case 3 and Case 6. At a glance, one might see a regular response in Case 3 and (more) irregular response in Case 6. Even though the regular behavior is observed in Case 3, a slight irregularity, which is potentially attributed to random noise, is also observed. Therefore, in order to avoid a bias assumption in distinguishing periodic responses from chaotic ones, a surrogate data testing procedure is performed to all sets of experimental data.

Figure 4.10 Time series displacement of the cylinder in Case 3 and Case 6.

Surrogate data test, which was systematically studied and reviewed in [103, 104], is an effective stochastic tool to confirm whether the irregularity in the time series data is induced by noise (linear stochastic process) or by the inherent nonlinearity of the system. In this method, the surrogate data is

generated by using a linear process that is consistent with a specific process defined in a null hypothesis. The surrogate data sets are defined to mimic only the linear properties of the original data. Subsequently, a discriminating statistic is calculated for both the original data and the surrogate data. Stochastically, if the discriminating statistic of the original data passes the hypothesis test, the irregularity in the data is confirmed to be attributed to the linear stochastic process, otherwise it is confirmed as a product of the nonlinear process.

In this study, the surrogate data is produced by the Fourier Transform algorithm and the discriminating statistic is chosen to be the correlation dimension of the time series data. The correlation dimension, D_2 , is the most common discriminating statistic for a surrogate data test. This quantity measures the dimensionality of a space occupied by a number of random points and can be calculated as the limit of the correlation integral [100]:

$$D_2 = \lim_{\varphi \rightarrow 0} \frac{\log|C(2, \varphi)|}{2\log|\varphi|} \quad (4.7)$$

where the function $C(2, \varphi)$ counts all the points within the vicinity φ of each other. Details on the calculation of the correlation dimension can be found in [93, 100, 101]. The Fourier Transform algorithm and correlation dimension were successfully applied in the surrogate data tests for various nonlinear mechanical systems (e.g. [94, 130]).

For each experimental case, the correlation dimension is calculated for the original data and 50 sets of the corresponding surrogate data. The calculations for all cases are implemented in the time delay phase spaces reconstructed by the same embedding dimension. Fig. 4.11 presents the correlation dimensions of the nine original data sets together with the mean values of the correlation dimensions of the corresponding surrogate data sets with the 3σ error margins.

Evidently, the responses in Case 5, 6, 8 and 9 are concluded as results of the nonlinear processes, i.e. chaotic systems. The responses in the remaining cases

support the null hypothesis. Therefore, their irregularity is attributed to the linear stochastic processes.

Figure 4.11 Correlation dimensions of the nine original data sets and their corresponding surrogate data sets.

4.3.2 Results and discussions

A similar procedure as presented in Section 4.2 is applied to estimate the largest Lyapunov exponents of Cases 5, 6, 8 and 9 that exhibit chaotic responses. Fig. 4.12 shows the AMI functions to estimate the time delay. It can be observed that the AMI functions reach their first local minima at around 0.4 s, which is comparable to the time delay estimated for the simulated data.

Figure 4.12 AMI functions of time series displacement of cylinder in chaotic cases.

Subsequently, Fig. 4.13 depicts the ratio $E_1(d)$ that is utilized to determine the embedding dimensions to reconstruct the time delay phase spaces for the experimental data. It can be seen that the qualified embedding dimensions for these cases are 5 or 6, while an embedding dimension of 4 is sufficient for the simulated data. This can be understood as for noise-contaminated data, a higher embedding dimension is required to completely unfold the attractor [94].

Figure 4.13 The ratio $E_1(d)$ of the time series displacement of the cylinder in the chaotic cases.

Based on the estimated time delay and embedding dimensions, the largest Lyapunov exponents of the chaotic cases are calculated in the reconstructed phase spaces and listed in Table 4.2 (the numbers in the brackets indicate the case numbers listed in Table 4.1). In line with the simulation result, the largest Lyapunov exponent that indicates the chaotic degree decreases when the damping ratio is increased. The same trend is observed with the decrease of the bi-stable gap.

In order to further validate the result, two additional cases are utilized. The first case (Case 10 with $\zeta = \zeta_0$, $b^* = 0.2$ and $U = 0.12$ m/s) aims to re-validate the observation at a low velocity flow. The purpose of the second case (Case 11 with $\zeta = \zeta_0$, $b^* = 0.4$ and $U = 0.18$ m/s) is to validate the existence of chaotic response at a different bi-stable gap (see Table 4.3).

Table 4.2 Estimated largest Lyapunov exponents, γ (bit/s), of the chaotic cases.

Parameters		$U = 0.13$ m/s		$U = 0.18$ m/s		$U = 0.20$ m/s	
$\zeta = \zeta_0$	$b^* = 0.1$	(1)	—	(2)	—	(3)	—
$\zeta = \zeta_0$	$b^* = 0.2$	(4)	—	(5)	1.240	(6)	1.137
$\zeta = 20\zeta_0$	$b^* = 0.1$	(7)	—	(8)	1.119	(9)	0.845

Table 4.3 Estimated largest Lyapunov exponents, γ (bit/s), of the additional cases.

Case no.	Parameters			γ (bit/s)
10	$\zeta = \zeta_0$	$b^* = 0.2$	$U = 0.12$ m/s	—
11	$\zeta = \zeta_0$	$b^* = 0.4$	$U = 0.18$ m/s	0.983

By observing the sequence of Cases 10, 4, 5 and 6, it can be concluded that when the damping ratio and bi-stable gap are fixed, the system response is regular at low velocity flows (Case 10 and 4). When water flow velocity is increased, chaotic response is built up in the system (Case 5). However, it also implies that the chaotic degree decreases when the water flow velocity is further increased (Case 6).

Similarly, by observing the series of Cases 2, 5 and 11, where the damping ratio and water flow velocity are fixed, it can be seen that the increase of the bi-stable gap will result in chaotic response (from Case 2 to Case 5). However, when the bi-stable gap is further extended, the chaotic degree will reduce as indicated in Case 11.

The commonality in the observations from the simulation and experimental results confirms the existence of chaotic responses in a bi-stable VIV system depending on the variation of the governing parameters. This information can be used as the primary knowledge to design a bi-stable VIV energy harvesting system with the optimum output power, while the negative effect of chaotic responses is evaded.

4.4 Summary

In this chapter, chaotic responses of a VIV energy harvesting system enhanced by a bi-stable spring were quantified in a wide range of the governing parameters

that include the damping ratio and bi-stable gap. Chaotic responses of the bi-stable VIV system were simulated on the wake oscillator model and subsequently quantified via the largest Lyapunov exponent that was calculated in the reconstructed time delay phase space.

In order to validate the observations based on the simulation result, a series of experiments was carried out on the cyber-physical force-feedback testing platform. However, in the experimental cases, the quantification of chaotic responses cannot be carried out in a straightforward manner due to the presence of noise in the measurement. There is no clear distinction in the responses, whether the irregularity is resulted by the linear stochastic process (noise) or, indeed, by the nonlinear process. The chaotic response as a result of the nonlinear process is confirmed by performing the surrogate data test with the correlation dimension applied as the discriminating statistic.

Based on the experimental result, it is confirmed that there exist some ranges of the governing parameters where chaotic responses occur. This finding agrees with the observation from the simulation result. The bi-stable spring in a VIV energy harvesting system, on one side, is considered as a potential approach to enhance the output power. However, the presence of chaotic responses will limit the performance of the system. Therefore, the bifurcation information, which essentially shows the map of the chaotic regions, can be utilized as a primary reference to design a bi-stable VIV energy harvesting system.

Apart from using the bifurcation map, chaotic responses can also be mitigated by some active control strategies. In the next chapter, appropriate strategies to actively control chaotic responses of a bi-stable VIV energy harvesting system will be introduced.

Chapter 5*****Controlling Chaotic Responses of Bi-stable Vortex Induced Vibration Energy Harvesting Systems**

This chapter focuses on the cases where a bi-stable VIV system is designed with an adequately large bi-stable gap to obtain a noticeable improvement in the output power at low velocity flows. However, a large bi-stable gap might lead to the occurrence of chaotic responses, which drastically lower the output power and result in an adversity in monitoring the system operation. Therefore, appropriate control strategies are required to actively stabilize chaotic responses. Among many proposed chaos control methods, the OGY method and time delay feedback method are selected since they require low control efforts, thus, minimum power is needed to drive the control input. The descriptions of these methods and their applications for the bi-stable VIV system will be thoroughly discussed in this chapter.

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- B.H. Huynh, T. Tjahjowidodo, Z. Zhong, Y. Wang, and N. Srikanth, "Comparative Study of Full State Variables and Time-delay Coordinates OGY Chaos Control for Bi-stable Vortex Induced Vibration Energy Harvesters," in *Proceedings of the Advanced Intelligent Mechatronics IEEE International Conference*, Munich, Germany, 2017, pp. 1544-1549.

5.1 Motivation

Theoretical and experimental analyses of the bi-stable VIV energy harvesting system in Chapter 4 have provided detailed information on the bifurcation from periodic to chaotic responses and vice versa as a function of the bi-stable gap and damping ratio. According to this information, chaotic responses can be avoided when the system is designed with a bi-stable gap that is smaller than $0.06D$ (see Fig. 4.6). However, the desired effect on improving the utilizable power becomes minor in the case of a small bi-stable gap (see Fig. 4.1). On the other hand, if the bi-stable gap is increased, chaotic responses might appear and result in a significant reduction in the output power at high velocity flows. Even though at an excessively large bi-stable gap, i.e. larger than $0.5D$, chaotic responses are less likely to occur, the inter-well vibration is hardly maintained and the improvement in the output power by means of introducing the bi-stable spring will not be acquired. As an alternative, the bifurcation map indicates that a high damping ratio is also able to eliminate chaotic responses. Unfortunately, an excessively high damping ratio can diminish the vibrating amplitude and lower the harvested energy.

Therefore, this chapter is focusing on the cases where the bi-stable system is designed with a significant bi-stable gap to acquire a major improvement in the output power at low velocity flows. At the same time, appropriate controllers will be designed and implemented to stabilize chaotic responses that occur at high velocity flows. In this way, the reduction in the output power caused by chaotic responses will be alleviated. In other words, the performance of the bi-stable VIV system at high velocity flows will be compromised and its merit at low velocity flows is conserved.

Various methods for chaos control have been proposed, e.g. the Ott-Grebogi-Yorke (OGY) method [105-107], the time delay feedback method [108, 109], the fuzzy control method [110], the nonlinear control method

[111, 112], the non-feedback method [113, 114], the adaptive control method [115, 116], the proportional feedback method [117, 118] and the neural networks method [119, 120]. For the energy harvesting application, it is expected to keep the control effort as low as possible since the net harvested power will be reduced by the power required to execute the control effort [121]. Therefore, the *OGY method* and the *time delay feedback method* are considered as the suitable solutions since they only require low control efforts. Furthermore, from practical point of view, these methods are beneficial in various aspects. For instance, the design and implementation strategies of the controller in these methods are relatively straightforward and no prior analytical knowledge on the dynamics of the system is required. In addition, the number of the state variables of the system that are required for the control feedback loop can be minimized. In the next sections, the applications of these methods for a bi-stable VIV energy harvesting system will be discussed in details.

5.2 Chaos control by the OGY method

5.2.1 Method description

The OGY method was first introduced in [106] to confirm the feasibility of controlling discrete chaotic dynamical systems and subsequently extended to apply for continuous dynamical systems. The principle of this method is based on the nature of a chaotic attractor. Elaborately, a chaotic attractor is extremely sensitive to initial conditions and unstable periodic orbits are densely embedded within it. Therefore, an appropriate small perturbation in an accessible system parameter will be able to stabilize the system into a desired periodic orbit. Detailed discussions on the OGY method and its applications can be found in [138, 139].

To describe the OGY method with a correlation to the bi-stable VIV system, the dynamics model is recalled as:

$$\begin{cases} \left(m_s + \frac{\pi \rho D^2}{4} \right) \ddot{y} + \left(r_s + \frac{\rho C_D U D}{2} \right) \dot{y} + g[y - b \cdot \text{sign}(y)] = \frac{\rho U^2 D C_{L0} q}{4} \\ \dot{q} + \varepsilon \frac{2\pi \text{St} U}{D} (1 - \beta q^2 + \lambda q^4) \dot{q} + \left(\frac{2\pi \text{St} U}{D} \right)^2 q = \frac{a \ddot{y}}{D} \end{cases} \quad (5.1)$$

Since the OGY controller is constructed in the state space, the bi-stable VIV model is transformed into the state-space form by introducing a state vector as:

$$\mathbf{x} = [x_1 \quad x_2 \quad x_3 \quad x_4]^T = [y \quad \dot{y} \quad q \quad \dot{q}]^T \quad (5.2)$$

By substituting Eq. (5.2) into the system of Eqs. (5.1), the state-space form of the bi-stable VIV model is obtained as:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \left\{ \frac{\rho U^2 D C_{L0}}{4} x_3 - \left(r_s + \frac{\rho C_D U D}{2} \right) x_2 - g[x_1 - b \cdot \text{sign}(x_1)] \right\} \\ &\quad \times \left(\frac{4}{4m_s + \pi \rho D^2} \right) \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \frac{a}{D} \left\{ \frac{\rho U^2 D C_{L0}}{4} x_3 - \left(r_s + \frac{\rho C_D U D}{2} \right) x_2 - g[x_1 - b \cdot \text{sign}(x_1)] \right\} \\ &\quad \times \left(\frac{4}{4m_s + \pi \rho D^2} \right) - \varepsilon \left(\frac{2\pi \text{St} U}{D} \right) (1 - \beta x_3^2 + \lambda x_3^4) x_4 - \left(\frac{2\pi \text{St} U}{D} \right)^2 x_3 \end{aligned} \quad (5.3)$$

The OGY controller was first introduced for a discrete-time N -dimensional dynamical system defined as:

$$\mathbf{x}_{n+1} = \mathbf{F}(\mathbf{x}_n, p_n) \quad (5.4)$$

where $\mathbf{x}_n \in \mathbf{R}^N$, $\mathbf{F}$ denotes a smooth vector function and p_n is an accessible parameter of the system that can be externally regulated. It can be seen that the bi-stable VIV system in Eqs. (5.3) has $N = 4$. Assuming that at a nominal value of the accessible parameter, $p_n = \bar{p}$, the system $\mathbf{F}(\mathbf{x}_n, \bar{p})$ manifests itself as a chaotic attractor, the objective of the controller is to perturb the parameter p_n so that the dynamics of the system converges to a desired periodic orbit of the attractor for most of the initial conditions in the basin of attraction. To avoid large

control efforts, the perturbation in the regulated parameter is constrained in a narrow range, δ , as:

$$|p_n - \bar{p}| < \delta \quad (5.5)$$

To apply the controller for a continuous-time dynamical system, this system will be discretized following Eq. (5.4). The discretization can be conducted by utilizing a Poincaré map that is constructed at a certain period. The Poincaré map is basically the intersection of the orbit in the state space to a lower dimensional (hyper-)plane.

When a continuous-time dynamical system is discretized at a certain period, a period-one orbit (the orbit that corresponds to a single point on the Poincaré section that repeats itself in every cycle) can be assumed as a fixed point, $\mathbf{x}_*(\bar{p})$, of the discrete map $\mathbf{F}$ that satisfies the following condition:

$$\mathbf{x}_{n+1}(\bar{p}) = \mathbf{x}_n(\bar{p}) = \mathbf{x}_*(\bar{p}) \quad (5.6)$$

Similarly, a period- M orbit corresponds to a fixed point, $\mathbf{x}_*(\bar{p})$, that satisfies:

$$\mathbf{x}_{n+M}(\bar{p}) = \mathbf{x}_n(\bar{p}) = \mathbf{x}_*(\bar{p}) \quad (5.7)$$

From the perspective of energy harvesting, a bi-stable VIV system takes advantage most when it operates in the inter-well vibrating mode, where the trajectory mostly corresponds to the period-one orbit. Therefore, the desired periodic orbit in this case corresponds to the fixed point of the discrete map as reflected in Eq. (5.6).

If the dynamics of the system is within a defined neighborhood of the fixed point with the accessible parameter closed to the nominal value, $\bar{p}$, the map can be linearized as:

$$\mathbf{x}_{n+1} - \mathbf{x}_*(\bar{p}) = \mathbf{A}[\mathbf{x}_n - \mathbf{x}_*(\bar{p})] + \mathbf{B}(p_n - \bar{p}) \quad (5.8)$$

where $\mathbf{A}$ denotes the $N \times N$ Jacobian matrix and $\mathbf{B}$ is an $N \times 1$ matrix that represents the effect of the regulated parameter p_n or the sensitivity matrix. The matrices $\mathbf{A}$ and $\mathbf{B}$ are evaluated at $\mathbf{x}_n = \mathbf{x}_*$ and $p_n = \bar{p}$ as:

$$\begin{aligned} \mathbf{A} &= \frac{\partial}{\partial \mathbf{x}_n} \mathbf{F}(\mathbf{x}_*, \bar{p}) \\ \mathbf{B} &= \frac{\partial}{\partial p_n} \mathbf{F}(\mathbf{x}_*, \bar{p}) \end{aligned} \quad (5.9)$$

A linear correlation between the perturbation in the regulated parameter, $(p_n - \bar{p})$, and the state vector, $\mathbf{x}_n$, can be assumed as:

$$p_n - \bar{p} = -\mathbf{K}^T [\mathbf{x}_n - \mathbf{x}_*(\bar{p})] \quad (5.10)$$

Therefore, by substituting Eq. (5.10) to Eq. (5.8), the following relationship is obtained:

$$\mathbf{x}_{n+1} - \mathbf{x}_*(\bar{p}) = (\mathbf{A} - \mathbf{B}\mathbf{K}^T) [\mathbf{x}_n - \mathbf{x}_*(\bar{p})] \quad (5.11)$$

If the magnitudes of the eigenvalues of the matrix $(\mathbf{A} - \mathbf{B}\mathbf{K}^T)$ are smaller than the unity, the matrix is asymptotically stable and the fixed point is also confirmed to be stable.

The matrix $\mathbf{K}^T$ can be solved by using the pole placement technique through the Ackermann's method [140] with the prerequisite of a full rank ($= N$) controllability matrix, $\mathbf{C}$, which is an $N \times N$ matrix defined as:

$$\mathbf{C} = [\mathbf{B} : \mathbf{A}\mathbf{B} : \mathbf{A}^2\mathbf{B} : \dots : \mathbf{A}^{N-1}\mathbf{B}] \quad (5.12)$$

Since the fixed point $\mathbf{x}_*(\bar{p})$ is unstable, the matrix $\mathbf{A}$ is characterized by N_u unstable eigenvalues and N_s stable eigenvalues, where $N_u + N_s = N$. To achieve the super-stability in the pole placement technique, the N_s poles of the matrix $\mathbf{A}$ are retained, while the remaining N_u poles are set to zero.

By referring to Eqs. (5.5) and (5.10), it can be immediately seen that:

$$|\mathbf{K}^T [\mathbf{x}_n - \mathbf{x}_*(\bar{p})]| < \delta \quad (5.13)$$

This means, the control effort or the perturbation in the regulated parameter, $(p_n - \bar{p})$, is enabled only when the trajectory of the system is within the neighborhood $2\delta/|\mathbf{K}^T|$ of the fixed point in the $\mathbf{R}^N$ space. Otherwise, the regulated parameter remains as its nominal value, $\bar{p}$.

(1) Construction of Poincaré map for an autonomous continuous system

In order to implement the OGY controller for a continuous-time system, the first and foremost step is to discretize the system through a Poincaré map. Based on the constructed Poincaré map, relevant information that includes the fixed point, the matrices $\mathbf{A}$ and $\mathbf{B}$ can be acquired. For a periodically driven system, there exists a natural Poincaré map that corresponds to the driving period. Therefore, the system can be simply discretized at this driving period (see e.g. [121, 124, 141]). However, the bi-stable VIV system in Eqs. (5.3) is an autonomous system where an explicit-time-dependent driving term does not exist. Therefore, the period to discretize the system must be estimated.

In this study, the period to discretize the system, T , is estimated by the method discussed in [142], which is based on the intersection of the trajectory at an $(N - 1)$ -dimensional hyperplane in the $\mathbf{R}^N$ space. Elaborately, the system of Eqs. (5.3) is simulated at a very fine fixed time step for sufficiently long duration to gather a comprehensive map. When the representative state space trajectory is achieved, the state vector $\mathbf{x}$ is sampled every time the trajectory intersects an $(N - 1)$ -dimensional hyperplane. Obviously, in the case of a well-behaved system, the trajectory will intersect the hyperplane at the same position. However, in the case of a chaotic attractor, the piercing points on the hyperplane will be scattered within a closed area. Therefore, if there exist two consecutive state vectors $\mathbf{x}(t_1)$ and $\mathbf{x}(t_2)$ on the intersecting hyperplane, whose the Euclidian distance is smaller than a certain threshold, the period is estimated as $T = (t_2 - t_1)$. If there are many pairs of consecutive state vectors that satisfy this condition, the estimation of the period T is averaged.

To maximize the harvested energy, the inter-well period-one orbit is desired. Therefore, in order to ensure that the orbit caters for the inter-well vibration, intuitively, the state vector $\mathbf{x}$ is sampled every time the displacement or the element x_1 crosses the unstable fixed point of the potential well.

(2) *Estimation of the fixed point*

After the period T to discretize the system is estimated, the next step in the designing procedure is to estimate the fixed point of the discrete map. Since a period-one orbit is desired, the fixed point $\mathbf{x}_*(\bar{p})$ that satisfies Eq. (5.6) must be found. Such a fixed point can be estimated by the recurrence method proposed in [143].

Since a period-one orbit is assumed to be embedded in the chaotic attractor, a state vector $\mathbf{x}_n(\bar{p})$ that is nearby to this orbit will remain in the neighborhood of this orbit in, at least, one period of the orbit. Therefore, the state vectors $\mathbf{x}_n(\bar{p})$ and $\mathbf{x}_{n+1}(\bar{p})$ that are within the vicinity of each other can be considered as a reasonable estimation of the fixed point $\mathbf{x}_*(\bar{p})$. In order to estimate this fixed point, first, a large number of state vectors or data points are generated by integrating the system (Eqs. (5.3)) at the estimated period T for certain duration. Next, the state vectors $\mathbf{x}_n(\bar{p})$ that satisfy Eq. (5.14) are searched.

$$\|\mathbf{x}_{n+1}(\bar{p}) - \mathbf{x}_n(\bar{p})\| < \kappa \quad (5.14)$$

where κ defines a vicinity or threshold in which the state vector is assumed to remain the same after one period. The state vectors that satisfy Eq. (5.14) are called recurrent state vectors. The longest series of state vectors, $[\mathbf{x}_n(\bar{p}), \mathbf{x}_{n+1}(\bar{p}), \mathbf{x}_{n+2}(\bar{p}), \mathbf{x}_{n+3}(\bar{p}), \dots]$, where all the elements are the recurrent state vectors, has the highest possibility to represent the desired fixed point. Therefore, to estimate the fixed point $\mathbf{x}_*(\bar{p})$, the state vectors in this series and all other neighboring recurrent state vectors within the vicinity of κ are averaged.

(3) *Estimation of matrices A and B*

Based on the fixed point $\mathbf{x}_*(\bar{p})$ estimated by the recurrence method, the matrices A and B in Eq. (5.8) can be estimated by the least square regression method as discussed in [144]. To estimate the matrix A , the set of state vectors that was used

to estimate the fixed point can be reused. First, the pairs of state vectors $[\mathbf{x}_n(\bar{p}), \mathbf{x}_{n+1}(\bar{p})]$ that satisfy the following equation are searched:

$$\begin{aligned} \|\mathbf{x}_n(\bar{p}) - \mathbf{x}_*(\bar{p})\| &< \chi \\ \|\mathbf{x}_{n+1}(\bar{p}) - \mathbf{x}_*(\bar{p})\| &< \chi \end{aligned} \quad (5.15)$$

where χ defines a vicinity or threshold in which the linearization in Eq. (6.8) is valid. The linear fitting for the function $[\mathbf{x}_{n+1}(\bar{p}) - \mathbf{x}_*(\bar{p})] = f[\mathbf{x}_n(\bar{p}) - \mathbf{x}_*(\bar{p})]$ can be expressed as:

$$\begin{aligned} x_{n+1,1}(\bar{p}) - x_{*,1}(\bar{p}) &= a_{11}[x_{n,1}(\bar{p}) - x_{*,1}(\bar{p})] + a_{12}[x_{n,2}(\bar{p}) - x_{*,2}(\bar{p})] + \dots \\ &\quad + a_{1N}[x_{n,N}(\bar{p}) - x_{*,N}(\bar{p})] + c_1 \\ x_{n+1,2}(\bar{p}) - x_{*,2}(\bar{p}) &= a_{21}[x_{n,1}(\bar{p}) - x_{*,1}(\bar{p})] + a_{22}[x_{n,2}(\bar{p}) - x_{*,2}(\bar{p})] + \dots \\ &\quad + a_{2N}[x_{n,N}(\bar{p}) - x_{*,N}(\bar{p})] + c_2 \\ &\vdots \\ x_{n+1,N}(\bar{p}) - x_{*,N}(\bar{p}) &= a_{N1}[x_{n,1}(\bar{p}) - x_{*,1}(\bar{p})] + a_{N2}[x_{n,2}(\bar{p}) - x_{*,2}(\bar{p})] + \dots \\ &\quad + a_{NN}[x_{n,N}(\bar{p}) - x_{*,N}(\bar{p})] + c_N \end{aligned} \quad (5.16)$$

The matrix $\mathbf{A}$ is therefore defined as:

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1N} \\ a_{21} & a_{22} & \dots & a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1} & a_{N2} & \dots & a_{NN} \end{bmatrix} \quad (5.17)$$

By collecting a sufficient number of pairs of the state vectors $[\mathbf{x}_n(\bar{p}), \mathbf{x}_{n+1}(\bar{p})]$ that satisfy Eq. (5.15), the matrix $\mathbf{A}$ can be calculated by the least square regression method. It should be noted that the matrix $[c_1 \ c_2 \dots c_N]^T$ in Eq. (5.16) denotes an offset that can be neglected if the estimation of the fixed point is sufficiently accurate.

To estimate the matrix $\mathbf{B}$ that represents effect of the regulated parameter p_n , another set of state vectors is generated. The system of Eqs. (5.3) is integrated at the period T for certain duration with the maximum perturbation, δ , activated intermittently, i.e. the maximum perturbation is enabled at the step n^{th} and disabled at the step $(n+1)^{\text{th}}$ to form a pair of state vectors $[\mathbf{x}_n(\bar{p}), \mathbf{x}_{n+1}(\bar{p} + \delta)]$. In order to ensure the validity of the linearization in Eq. (5.8), the pairs of the state vectors that satisfy the following equation are explored:

$$\begin{aligned} \|\mathbf{x}_n(\bar{p}) - \mathbf{x}_*(\bar{p})\| &< \chi \\ \|\mathbf{x}_{n+1}(\bar{p} + \delta) - \mathbf{x}_*(\bar{p})\| &< \chi \end{aligned} \quad (5.18)$$

Then, based on the linearization in Eq. (5.8), the matrix $\mathbf{B}$ can be calculated as:

$$\mathbf{B} = \begin{bmatrix} x_{n+1,1}(\bar{p} + \delta) - x_{*,1}(\bar{p}) \\ x_{n+1,2}(\bar{p} + \delta) - x_{*,2}(\bar{p}) \\ \vdots \\ x_{n+1,N}(\bar{p} + \delta) - x_{*,N}(\bar{p}) \end{bmatrix} - \mathbf{A} \begin{bmatrix} x_{n,1}(\bar{p}) - x_{*,1}(\bar{p}) \\ x_{n,2}(\bar{p}) - x_{*,2}(\bar{p}) \\ \vdots \\ x_{n,N}(\bar{p}) - x_{*,N}(\bar{p}) \end{bmatrix} \frac{1}{\delta} \quad (5.19)$$

Many pairs of state vectors that satisfy Eq. (5.18) provide the averaged calculation of the matrix $\mathbf{B}$.

In order to simplify the selections of the thresholds κ and χ to estimate the discretizing period T , the fixed point $\mathbf{x}_*(\bar{p})$, the matrices $\mathbf{A}$ and $\mathbf{B}$, the Euclidian distance of two state vectors, e.g. $\mathbf{x}_n$ and $\mathbf{x}_{n+1}$, is normalized to the distance ranges of the corresponding state elements as:

$$\|\mathbf{x}_{n+1} - \mathbf{x}_n\| = \sqrt{\sum_{i=1}^N \left[\frac{x_{n+1,i} - x_{n,i}}{\max(x_i) - \min(x_i)} \right]^2} \quad (5.20)$$

It should be noted that the estimation accuracy of the period T , the fixed point $\mathbf{x}_*(\bar{p})$, the matrices $\mathbf{A}$ and $\mathbf{B}$ is dependent on the thresholds κ and χ and the number of state vectors in the data sets. Therefore, a trial and error attempt might be required regarding the selections of the thresholds and the number of state vectors to ensure the performance of the OGY controller.

5.2.2 Application of the OGY controller for the bi-stable VIV system

With respect to the procedure described in Section 5.2.1, the OGY controller is designed and applied to stabilize chaotic responses of the bi-stable VIV system. The bi-stable VIV system is configured with a sufficiently large bi-stable gap, b , where the output power is noticeably improved at the low velocity flows and chaotic responses occur with the increase of the flow velocity (see Fig. 5.1).

Figure 5.1 Utilizable power, $P_{utilizable}$, of the bi-stable system with $b = 0.075D$ and the linear system with the same equivalent stiffness (upper), and the bifurcation map of the bi-stable system (lower), as functions of water flow velocity, U .

A high damping coefficient (per unit length), r_s , which represents the total damping effect from the electrical load and the mechanical components, is

imposed on the system. The OGY controller is designed for the system operating at the flow velocity of 0.16 m/s, where the chaotic response occurs as indicated by the phase plot in Fig. 5.2. All the physical parameters of the system are summarized in Table 5.1.

The design of the OGY controller requires an appropriate selection of a structural parameter, p_n , which can be externally manipulated for the control action. In this case, the regulated parameter can be chosen among the effective mass of the oscillating structure, the spring parameters and the damping coefficient. From the practical point of view, since the electrical load contributes to the total damping of the system [6], therefore, the damping coefficient, r_s , is selected as the perturbed parameter of the controller, $p_n = r_s$. Manipulating the electrical load also brings an additional advantage as it will involve the minimum physical interference to the VIV structure.

Figure 5.2 Chaotic response of the bi-stable VIV system at $U = 0.16$ m/s.

Table 5.1 Parameters of the chaotic bi-stable VIV system.

Parameters	Values	Parameters	Values
Cylinder diameter, D	0.05 m	Natural frequency, $f_{n, \text{water}}$	0.7 Hz
Effective mass/unit length, m_s	19.8 kg/m	Bi-stable gap, b	$0.075D$
Damping coefficient/unit length, $\bar{r}_s$	17.8 Ns/m ²	Water flow velocity, U	0.16 m/s

Estimation of the discretizing period T

The model is simulated at a fixed time step, $\Delta t = 0.5$ ms, and data points are collected for long duration (200,000 s) in order to have a representative chaotic time series. Subsequently, the discretizing period is estimated by intersecting the chaotic attractor to a subspace hyperplane section (Poincaré). Fig. 5.3 shows the corresponding time series displacement of the cylinder. From the intersecting section, the discretizing period is estimated at $T = 1.6575$ s.

Figure 5.3 Time series displacement of the cylinder (evaluated at fixed time step, $\Delta t = 0.5$ ms).

Estimation of the fixed point

To estimate the fixed point, $\mathbf{x}_*(\bar{p})$, through the recurrence method, 2,000,000 data points on the Poincaré section are generated. Fig. 5.4 shows the location of the estimated fixed point, which is obtained from the longest series of the recurrent points within the neighboring threshold $\kappa = 0.0003$ on the Poincaré section. The left panel of Fig. 5.4 illustrates the magnified Poincaré section that contains the estimated fixed point and its neighbors. The estimated fixed point, which subsequently will be used as the control target, is identified at $\mathbf{x}_* = \mathbf{x}_{target} = [0.0040 \ -0.0340 \ -13.1434 \ 16.0563]^T$.

Figure 5.4 Location of the estimated fixed point on the Poincaré section.

Estimation of the Jacobian matrix A

From the same data set, the pairs of state vectors that satisfy Eq. (5.15) are explored. By applying the least square regression method for Eq. (5.16), the Jacobian matrix A is estimated at:

$$A = \begin{bmatrix} 0.4284 & 0.0970 & -4.1671 \times 10^{-4} & -8.1153 \times 10^{-5} \\ 2.8297 & 0.7412 & 7.6704 \times 10^{-4} & 1.4834 \times 10^{-4} \\ 27.8646 & 19.3578 & 0.4271 & 0.0817 \\ -501.1878 & -102.9648 & 0.7780 & 0.7398 \end{bmatrix}$$

Calculation of the matrix B

By selecting the damping coefficient, r_s , as the regulated parameter, p_n , another data set (about 2,000,000 data points) is generated by introducing the maximum perturbation $\delta = \delta r_{s \max}$ intermittently. By searching the pairs of state vectors that satisfy Eq. (6.18), the matrix B is calculated with respect to Eq. (5.19) as:

$$B = [3.7430 \times 10^{-6} \ 0.0013 \ 0.0528 \ 0.0919]^T$$

Controllability and control gain matrix K^T

The control gain matrix, K^T , is derived by respecting the controllability condition (Eq. (5.12)) and implementing the Ackermann's method. The matrix K^T is, therefore, obtained as:

$$K^T = [4.3016 \times 10^3 \ 1.0298 \times 10^3 \ -2.1660 \ -0.8316]$$

Implementation of the controller

The control action is implemented by perturbing the damping coefficient, r_s , with respect to Eq. (5.10) when the condition in Eq. (5.13) is satisfied. Fig. 5.5 shows the response of the system with the implementation of the controller. The maximum perturbation is set at $\delta = \delta r_{s \max} = \pm 5\%$ of the nominal value, $\bar{r}_s$. The displacement of the system, x_1 , at the discrete steps is shown in the upper panel and the perturbation in damping, $(r_s - \bar{r}_s) \times 100 / \bar{r}_s$, is shown in the lower panel of Fig. 5.5.

Figure 5.5 Response of the system operating at $U = 0.16$ m/s with the implementation of the controller: values of x_1 at discrete steps (upper); perturbation in damping at discrete steps (lower).

The initial condition $\mathbf{x}_0 = [0 \ 0 \ 0 \ 50]^T$ is arbitrarily selected within the basin of attraction. After wandering within the attractor in about 785 s, the orbit

approaches the vicinity of the estimated fixed point and the condition in Eq. (5.13) is achieved. Therefore, the perturbation in damping is enabled as the control effort. After the transient response decays in about 140 s, the system reaches the steady state and the period-one inter-well trajectory is achieved as illustrated by the time series in Fig. 5.6 and the phase plot in Fig. 5.7.

At the steady state, the fixed point is achieved at $\mathbf{x}_{achieved} = [0.0040 \ -0.0341 \ -13.1603 \ 15.9788]^T$ with an error of 0.4% compared to the targeting fixed point and the perturbation in damping becomes constant at $\delta r_s = -0.4\%$. This result is practically beneficial since for a constant perturbation in damping, the net harvested power will not be affected by the control effort.

Figure 5.6 Time series displacement when the OGY chaos controller is implemented.

Figure 5.7 Phase portraits of the uncontrolled and controlled systems.

(1) Reducing duration to enable control effort

As shown in Fig. 5.6, the system requires about 785 s to approach the fixed point from the initial condition and enable the control effort. In order to improve the speed, the maximum perturbation in the regulated parameter can be set at a larger value to cover a wider searching area of the fixed point for the system to enter. To demonstrate effect of this strategy, the controller with a larger maximum allowable perturbation in damping, $\delta r_{s \max} = \pm 10\%$, is implemented and the response of the system can be observed in Fig. 5.8. It can be seen that the new controller also demonstrates a satisfactory performance, where the response is completely stabilized once the system reaches the steady state. In particular, the new controller results in a fast response, where it only takes about 330 s before the system becomes stable. This response is much faster than that of the system controlled with $\delta r_{s \max} = \pm 5\%$, where it takes about 785 s to stabilize the system (see Fig. 5.6). However, the increase in the maximum perturbation range, δ , results in a larger perturbation for the system to approach the desired fixed point (see the lower panels of Figs. 5.5 and 5.8).

Furthermore, selecting a larger maximum perturbation range, δ , might lead to a chance of having false control efforts. It can be observed from the lower panel of Fig. 5.8 that at $t \approx 240$ s, the system suggests a false control effort when the state vector is at $\mathbf{x} = [-0.0040 \ 0.0345 \ 13.8690 \ -10.0855]^T$, which is relatively far from the targeting fixed point. In this case, the system will not result in a stable state, whilst such a false control effort does not take place when the system is controlled with $\delta r_{s \max} = \pm 5\%$. Therefore, the maximum allowable perturbation in the regulated parameter, δ , must be carefully chosen. The system requires long duration to approach the desired fixed point when a small value of δ is utilized. On the other hand, a high value of δ might result in false control efforts. In addition, the large perturbation in the regulated parameter might lead to a significant change in the structural properties and a potential instability to the system.

Figure 5.8 Response of the system controlled with $\delta r_{s \max} = \pm 10\%$: values of x_1 at discrete steps (upper); perturbation in damping at discrete steps (lower).

(2) Verification at different water flow velocity

This subsection presents the implementation of the OGY controller at a different water flow velocity, $U = 0.165$ m/s. This analysis is performed to further validate performance of the controlled system since in the real operating condition, the water flow velocity might fluctuate. The discretizing period T , the fixed point x_s , the matrices A , B and K^T in this case are estimated as:

$$\begin{aligned}
 T &= 1.6155 \text{ s} \\
 x_s &= \mathbf{x}_{target} = [0.0047 \ -0.0366 \ -13.4119 \ 15.1856]^T \\
 A &= \begin{bmatrix} 0.4195 & 0.0863 & -6.7565 \times 10^{-4} & -5.6066 \times 10^{-5} \\ -0.6016 & -0.1185 & 0.0067 & -4.1032 \times 10^{-4} \\ -150.4246 & -27.0696 & 0.8205 & 0.0377 \\ -1.0234 \times 10^3 & -215.6480 & 3.8386 & 0.2772 \end{bmatrix} \\
 B &= [7.7214 \times 10^{-5} \ 0.0011 \ 0.0443 \ -0.0082]^T \\
 K^T &= [5.9726 \times 10^3 \ 1.1836 \times 10^3 \ -16.0066 \ -1.8959]
 \end{aligned}$$

Figure 5.9 Response of the controlled system operating at $U = 0.165$ m/s: values of x_1 at discrete steps (upper); perturbation in damping at discrete steps (lower).

The maximum perturbation in the regulated parameter is also re-selected to accommodate to the variation in the nature of the chaotic attractor when the water flow velocity is varied. A trial and error iteration results in the maximum allowable perturbation in damping at $\delta r_{s\max} = \pm 20\%$. The response of the system with the implementation of the controller is shown in Fig. 5.9. When the steady state is reached, the perturbation in damping is constant at $\delta r_s = -13.6\%$. The period-one inter-well orbit is achieved with the steady state error of 5.7%. It should be noted that this error is larger than that of the previous case ($U = 0.16$ m/s). This difference in the steady state errors can be explained by the difference in the estimation accuracies of the fixed points in two cases. In the case of $U = 0.16$ m/s, the fixed point is defined with the neighboring threshold $\kappa = 0.0003$, while due to the chaotic nature in this case, the threshold is increased to $\kappa = 0.02$ in order to find the fixed point. A higher neighboring threshold, κ ,

will result in lower estimation accuracy of the fixed point, the matrices $\mathbf{A}$ and $\mathbf{B}$. As a consequence, a higher steady state error is resulted. This concludes that the performance of the controller is dependent on the estimation accuracy of the fixed point.

(3) Verification at different system parameters

To further validate the applicability of this method, the OGY controller is implemented for the system with different values of the structural parameters, i.e. bi-stable gap, $b = 0.1D$, and damping coefficient, $r_s = 16.8 \text{ Ns/m}^2$. At the flow velocity of 0.13 m/s , the response of this system is chaotic. By implementing the same procedure presented in Section 5.2.1, the discretizing period T , the fixed point $\mathbf{x}_s$, the matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{K}^T$ are estimated as:

$$\begin{aligned}
 T &= 2.0202 \text{ s} \\
 \mathbf{x}_s &= \mathbf{x}_{target} = [0.0028 \ -0.0290 \ -13.9599 \ 10.7525]^T \\
 \mathbf{A} &= \begin{bmatrix} 1.2852 & 0.2129 & 5.5180 \times 10^{-4} & -1.1132 \times 10^{-4} \\ 1.8886 & -0.0048 & 0.0050 & -2.1266 \times 10^{-5} \\ 75.6520 & -11.0386 & 0.7307 & 0.0480 \\ -1.2560 \times 10^3 & -284.0342 & 1.9948 & 0.3513 \end{bmatrix} \\
 \mathbf{B} &= [4.0032 \times 10^{-4} \ 0.0016 \ 0.1296 \ -0.0632]^T \\
 \mathbf{K}^T &= [2.1858 \times 10^3 \ 277.4965 \ 2.5571 \ -0.0960]
 \end{aligned}$$

The response of the system with the implementation of the controller is shown in Fig. 5.10. The maximum allowable perturbation in damping is set at $\delta r_{s \max} = \pm 20\%$. When the steady state is reached, the system is stabilized with the constant perturbation in damping at $\delta r_s = -9.2\%$ and the steady state error of 5.9% . In particular, it only takes about 110 s to enable the control effort and additional 90 s for the system to completely reach the steady state. This validation concludes the applicability of the controller for different systems with different structural parameters, e.g. bi-stable gap and damping coefficient.

Figure 5.10 Response of the system at different structural parameters with the implementation of the controller: values of x_1 at discrete steps (upper); perturbation in damping at discrete steps (lower).

5.2.3 Energy harvesting performance of the controlled system

Table 5.2 demonstrates effect of the controller in terms of the utilizable power that is calculated by Eq. (2.3) with the substitution of the damping coefficient per unit length, r_s . In all cases, the utilizable power is significantly improved when the OGY controller is applied for the system. Particularly, in the case of $U = 0.165$ m/s, the output power of the controlled system is improved to 73.5% compared to that of the uncontrolled system. This noticeable improvement can be understood since the controller stabilizes the system into the period-one inter-well orbit, which corresponds to the highest-energy orbit of a bi-stable system.

Table 5.2 Comparison in the utilizable power (mW/m) of the uncontrolled and controlled systems at different structural parameters and water flow velocity, U .

Parameters Utilizable power (mW/m)	Case 1	Case 2	Case 3
	$U = 0.16$ m/s	$U = 0.165$ m/s	$U = 0.13$ m/s
	$r_s = 17.8$ Ns/m ²	$r_s = 17.8$ Ns/m ²	$r_s = 16.8$ Ns/m ²
	$b = 0.075D$	$b = 0.075D$	$b = 0.1D$
Uncontrolled system (chaotic), P_U	10.7	9.8	7.0
Controlled system (periodic), P_C	13.0	17.0	8.5
Improvement, $(P_C - P_U) \times 100/P_U$	21.5%	73.5%	21.4%

The control effort is implemented by perturbing the damping coefficient (through perturbing the electrical load in the practical operation) and a certain amount of power will be required to enable the control effort. This amount of power is maintained at a low level by the prescribed maximum perturbation, δ . More importantly, the fruit of the proposed controller lies when the steady state is reached, the perturbation in damping will be kept constant, saying that the system damping or electrical load remains constant. Consequently, no additional power is required for the control input. The improvement in the output power of the system with the OGY controller shows a promising opportunity for practical applications of the bi-stable VIV energy harvesting system.

It should be noted that in some cases, e.g. the cases of $U = 0.16$ m/s and $U = 0.165$ m/s, the utilizable power of the controlled system might still be lower than that of the system with a linear spring. However, the negative effect from chaotic responses of the bi-stable system at high velocity flows is minimized by the OGY controller. This can be considered as a compromise when a bi-stable spring is applied to improve the performance of the VIV system at low velocity flows.

5.2.4 Real-time experimental validation

An experimental validation was carried out to confirm the feasibility of the OGY controller in real-time applications. The experiment was conducted on the same the testing platform described in Chapter 3. However, to isolate the uncertainty in the water flow produced by the water channel, the wake oscillator model was

still utilized to simulate the fluid force that excites the virtual bi-stable VIV structure.

The (virtual) bi-stable VIV system in Eqs. (5.3) with the OGY controller is simulated by the testing platform in real-time. The displacement of the cylinder, y or x_1 in its state-space form, is prescribed to actuate the cylinder in real-time. The control strategy is summarized in Fig. 5.11. The structural parameters used in this experimental validation are the same as those in the case of $U = 0.16$ m/s (see Table 5.1). The estimated information that includes the discretizing period T , the fixed point $\mathbf{x}_*(\bar{p})$, the matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{K}^T$ obtained from the previous analysis is also utilized in this experiment. The control strategy is programmed in the Simulink environment (Matlab R2012a) with the fourth-order Runge-Kutta solver at the fixed time step of 0.5 ms and implemented through the dSPACE 1103 controller. A PID controller is used to position the cylinder on the testing platform.

Figure 5.11 Real-time OGY control strategy implemented on the testing platform.

Fig. 5.12 shows the time series displacement of the cylinder (upper panel) and the perturbation in damping (lower panel) when the OGY controller is implemented in real-time. It can be seen that the controller is able to stabilize the system to the period-one inter-well vibration. When the maximum allowable perturbation is set at $\delta r_{s \max} = \pm 20\%$, it takes about 85 s for the system to approach the desired fixed point and additional 65 s to settle the transient response before the system is completely stabilized. Once the steady state is reached, the perturbation in damping remains constant at $\delta r_s = -9.3\%$ and the system is stabilized with a steady state error of 4.3%. In conclusion, the success in stabilizing the system of the OGY controller confirms its feasibility in real-time applications.

Figure 5.12 Time series displacement of cylinder (upper) and perturbation in damping (lower) when the OGY controller is implemented in real-time.

5.2.5 The OGY controller with time delay coordinates

The design and implementation of the OGY controller require the measurement of all the state variables of the system. Unfortunately, measuring all state variables of a dynamical system in a real-time circumstance might be a tedious and costly process. As a solution, the time delay coordinates that require only the time series data of a single state variable can also be used as an alternative to the full-state variable OGY controller [122, 123]. This method is equivalent to designing and implementing the OGY controller in the reconstructed time delay phase space that was discussed in Chapter 4. This section will discuss the method of using the time delay coordinates for the OGY controller and its application for the bi-stable VIV system.

(1) Method description

Assuming that in the real-time environment, only a single state variable, $s(t)$, of the system can be measured, a state vector of the system, $\mathbf{x}_n$, at the instance t_n can be reconstructed from this state variable based on the time delay coordinates as:

$$\mathbf{x}_n = [s(t_n), s(t_n - t_d), s(t_n - 2t_d), \dots, s(t_n - (d-1)t_d)]^T \quad (5.21)$$

where t_d is the time delay and d is the embedding dimension.

When the time delay coordinates are utilized, the variation in the manipulated parameter leads to an alteration of the map in Eq. (5.4). To be specific, since the state vector $\mathbf{x}_n$ comprises the elements $s(t_n - it_d)$ for $i = 0, 1, 2, \dots, (d-1)$, the system state at the step $(n+1)^{\text{th}}$ is determined not only by the value of the manipulated parameter at the step n^{th} , p_n , but also by its former values in the span $[t_n - (d-1)t_d] \leq t \leq t_n$. Therefore, the system is re-defined by a new map as:

$$\mathbf{x}_{n+1} = \mathbf{G}(\mathbf{x}_n, p_n, p_{n-1}, p_{n-2}, \dots, p_{n-w}) \quad (5.22)$$

where w is defined as the smallest integer that satisfies the following condition:

$$(d-1)t_d < w(t_{n+1} - t_n) \quad (5.23)$$

where $(t_{n+1} - t_n) = T$ when a continuous system is discretized at the period T . The fixed point $\mathbf{x}_*(\bar{p})$ that corresponds to the period-one orbit is also defined as in

Eq. (5.6). The linearization of the map for a state vector in the vicinity of the fixed point at a value of the parameter p_n closed to $\bar{p}$ is now re-written as:

$$\begin{aligned} \mathbf{x}_{n+1} - \mathbf{x}_*(\bar{p}) &= \mathbf{A}[\mathbf{x}_n - \mathbf{x}_*(\bar{p})] + \mathbf{B}^1(p_n - \bar{p}) \\ &\quad + \mathbf{B}^2(p_{n-1} - \bar{p}) + \dots + \mathbf{B}^{w+1}(p_{n-w} - \bar{p}) \end{aligned} \quad (5.24)$$

where:

$$\begin{aligned} \mathbf{A} &= \frac{\partial}{\partial \mathbf{x}_n} \mathbf{G}(\mathbf{x}_n, p_n, p_{n-1}, p_{n-2}, \dots, p_{n-w}) \\ \mathbf{B}^j &= \frac{\partial}{\partial p_{n-(j-1)}} \mathbf{G}(\mathbf{x}_n, p_n, p_{n-1}, p_{n-2}, \dots, p_{n-w}) \end{aligned} \quad (5.25)$$

with $1 \leq j \leq (w+1)$. To construct the control law in a lumped form, the state vector is re-defined as:

$$\mathbf{X}_n = [\mathbf{x}_n^T \quad p_{n-1} \quad p_{n-2} \quad \dots \quad p_{n-w}]^T \quad (5.26)$$

Consequently, the period-one fixed point, the Jacobian and sensitivity matrices, and the linearization in Eq. (5.24) are also re-defined as:

$$\mathbf{X}_*(\bar{p}) = [\mathbf{x}_*(\bar{p})^T \quad \bar{p} \quad \bar{p} \quad \dots \quad \bar{p}]^T \quad (5.27)$$

$$\tilde{\mathbf{A}} = \begin{bmatrix} \mathbf{A} & \mathbf{B}^2 & \mathbf{B}^3 & \dots & \mathbf{B}^w & \mathbf{B}^{w+1} \\ \mathbf{0} & 0 & 0 & \dots & 0 & 0 \\ \mathbf{0} & 1 & 0 & \dots & 0 & 0 \\ \mathbf{0} & 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & 0 & 0 & \dots & 1 & 0 \end{bmatrix} \quad (5.28)$$

$$\tilde{\mathbf{B}} = [(\mathbf{B}^1)^T \quad 1 \quad 0 \quad 0 \quad \dots \quad 0]^T \quad (5.29)$$

$$\mathbf{X}_{n+1} - \mathbf{X}_*(\bar{p}) = \tilde{\mathbf{A}}[\mathbf{X}_n - \mathbf{X}_*(\bar{p})] + \tilde{\mathbf{B}}(p_n - \bar{p}) \quad (5.30)$$

With the new defined state vector, fixed point, matrices and linearization, the design and implementation of the controller can be continued by following the same procedure described in Section 5.2.1. The same methods discussed in Section 5.2.1 can also be used to estimate the discretizing period T , the period-one fixed point $\mathbf{x}_*(\bar{p})$, the matrices $\mathbf{A}$ and $\mathbf{B}^j$. When the data points are

generated to estimate the matrices $\mathbf{B}^j$ with $1 \leq j \leq (w + 1)$, the maximum perturbation in the manipulated parameter, δ , is intermittently enabled at each $(w + 1)^{\text{th}}$ step.

It should be noted that to construct the time delay state vectors from the measured state variable, the embedding dimension, d , and the time delay, t_d , in Eq. (5.21) must be properly chosen. Typically, for the chaos control application, the embedding dimension, d , is chosen to be equal to N (state-space dimensionality of the system) [107]. The time delay, t_d , can be estimated by the AMI function as discussed in Chapter 4.

(2) *Application of the OGY controller by the time delay coordinates for the bi-stable VIV system*

The bi-stable VIV system discussed in Section 5.2.2 (see Table 5.2, Case 3) is re-utilized to demonstrate the performance of the OGY controller based on the time delay coordinates.

To apply the algorithm for the bi-stable VIV system, it is assumed that only displacement of the cylinder, $y(t)$, can be measured in the real-time operation. In practice, displacement of the cylinder can be measured by various techniques, e.g. by using position transducers. The time delay state vector at t_n is defined as:

$$\mathbf{x}_n = [y(t_n), y(t_n - t_d), y(t_n - 2t_d), \dots, y(t_n - (d - 1)t_d)]^T \quad (5.31)$$

Since the system is 4-dimensional in the state space, the embedding dimension is chosen at $d = N = 4$, and the time delay, t_d , is chosen at $t_d = 0.488$ s based on the AMI function (see Fig. 5.13). The system is discretized at the period $T = 2.0202$ s. Therefore, the condition in Eq. (5.23) is satisfied with $w = 1$. Consequently, the fixed point $\mathbf{x}_*(\bar{p})$, the matrices $\mathbf{A}$, $\mathbf{B}^1$, and $\mathbf{B}^2$ are accordingly estimated in the time delay space as:

$$T = 2.0202 \text{ s}$$

$$\mathbf{x}_* = \mathbf{x}_{target} = [0.0028 \ 0.0108 \ -0.0019 \ -0.0111]^T$$

$$\mathbf{A} = \begin{bmatrix} 0.9514 & -0.9849 & 0.1483 & 0.0893 \\ 0.1462 & 1.9035 & 0.3016 & -0.4138 \\ -0.8749 & 0.3563 & -0.2469 & 0.1000 \\ -0.4855 & -0.9266 & -0.0657 & 0.0575 \end{bmatrix}$$

$$\mathbf{B}^1 = [3.8365 \times 10^{-4} \ -5.8131 \times 10^{-4} \ -9.9558 \times 10^{-5} \ 1.1189 \times 10^{-4}]^T$$

$$\mathbf{B}^2 = [3.3200 \times 10^{-4} \ -3.7186 \times 10^{-4} \ -1.7646 \times 10^{-4} \ 1.3951 \times 10^{-4}]^T$$

$$\mathbf{K}^T = [-281.6379 \ -2.6452 \times 10^3 \ -383.3011 \ 521.8829 \ 0.5069]$$

Next, the new state vector, $\mathbf{X}_n$, fixed point, $\mathbf{X}_*(\bar{p})$, matrices $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{B}}$ are defined by Eqs. (5.26)-(5.29). With these new quantities, the controller can be designed and implemented by following the same procedure in Section 5.2.1. The maximum perturbation in damping is prescribed at $\delta r_{s \max} = \pm 20\%$.

Fig. 5.14 shows the displacement of the system (upper panel) and the perturbation in damping, δr_s , (lower panel) when the controller is implemented. In general, the performance of the controller with the time delay coordinates is comparable to that of the controller with the full state variables (see Fig. 5.10). It takes about 110 s to enable the control effort and additional 90 s before the inter-well period-one orbit is achieved in the steady state. A constant perturbation in damping is acquired at $\delta r_s = -8.1\%$. The controller results in a steady state error of 8.4%. This error is larger than the error of 5.9% that is resulted by the OGY controller using the full state variables with the same prescribed maximum perturbation in damping, $\delta r_{s \max}$.

It can be confirmed that when the time delay coordinates are utilized, the controller shows a comparable performance to the case of using the full state variables except with a larger steady state error. However, the stabilization of the system into the inter-well period-one orbit is still ensured. Therefore, the time delay coordinates can be utilized as an alternative when the measurement of all state variables is cumbersome and costly. This method enhances the applicability of the OGY controller for a bi-stable VIV system. As an expense, more quantities need to be estimated in this method, e.g. matrices $\mathbf{B}^j$ and time delay t_d .

Figure 5.13 Estimation of the time delay, t_d , based on the AMI function.

Figure 5.14 Displacement of the cylinder (upper) and perturbation in damping (lower) at the discrete steps when the controller based on time delay coordinates is applied for the system.

5.3 Chaos control by the time delay feedback method

The OGY controller was successfully designed and applied to stabilize chaotic responses of the bi-stable VIV system. It is realized that the design procedure of the OGY controller requires the prior knowledge of the discretizing period, fixed point, Jacobian matrix and sensitivity matrix. This section investigates the time delay feedback method as an alternative, where only the period of the desired unstable orbit is required to design and implement the controller.

5.3.1 Method description

The time delay feedback method was first introduced in [108] for a continuous dynamical system defined as:

$$\dot{y} = P(v, \mathbf{x}) + u(t), \quad \dot{\mathbf{x}} = \mathbf{Q}(v, \mathbf{x}) \quad (5.32)$$

where $v(t)$ is a state variable or output variable that can be measured in real-time and the vector $\mathbf{x}$ comprises the remaining state variables of the system. It should be highlighted that the functions P and $\mathbf{Q}$ might be unknown, while $u(t)$ is the external control input that is constructed as:

$$u(t) = K[v(t - \tau) - v(t)] \quad (5.33)$$

where $v(t - \tau)$ is the delayed signal of the output $v(t)$ with the time delay τ . If the time delay τ is chosen to be coincident to the period T of the desired unstable periodic orbit, $\tau = T$, an appropriate selection of the gain K will allow for the stabilization of the system into the desired orbit. The blocks diagram in Fig. 5.15 summaries the strategy of this control method.

Figure 5.15 Control blocks diagram of the time delay feedback method.

Socolar, et al. [145] extended this original time delay feedback method by using data from many previous states of the system to accommodate the applications with high-period orbits. In this study, only the original method is applied since the period-one orbit of the bi-stable VIV system is desired.

5.3.2 Design and implementation of the time delay feedback controller for the bi-stable VIV system

A state variable of the system must be chosen as the output variable, $v(t)$, to construct the control input, $u(t)$. If this state variable is chosen among the displacement and velocity of the VIV structure, an actuator will be required to execute the control input. The additional actuator might lead to a significant physical interference to the system. In addition, the net harvested energy might be noticeably affected by the energy consumed by the actuator.

Therefore, in order to design the time delay feedback controller, the bi-stable VIV structure is connected to an electrical generator through a rack-gear mechanism and a resistance component is utilized as the load of the overall system. In this way, the control input can be designed from the new resulted state variable of the load circuit with the minimum physical interference to the VIV structure. Fig. 5.16 depicts the connection of the VIV structure to the generator and the load circuit.

Figure 5.16 (a) Connection of the VIV structure to the generator; (b) Load circuit.

Assuming that θ is the angular displacement of the shaft of the generator caused by vibration of the cylinder, the angular velocity of the shaft is defined as:

$$\dot{\theta} = \frac{\dot{y}}{r} \quad (5.34)$$

where r is radius of the gear to connect the shaft. Assuming that a self-excited generator is utilized, the armature or induced voltage from rotation of the shaft is defined as:

$$E = K_E \dot{\theta} = \frac{K_E \dot{y}}{r} \quad (5.35)$$

where K_E is the generator constant. The induced armature current, i_a , results in a torque, T_{gen} , in the generator defined as:

$$T_{gen} = K_M i_a \quad (5.36)$$

where K_M is the torque constant. This torque results in a force on the VIV structure that is opposite to the induced fluid force. This force from the generator, F_{gen} , is defined as:

$$F_{gen} = \frac{K_M i_a}{r} \quad (5.37)$$

Combining the wake oscillator model applied for the 3D VIV structure and the Kirchhoff voltage law applied on the load circuit, the overall system is modelled as:

$$\left(m_{osc} + \frac{\pi \rho D^2 L}{4} \right) \ddot{y} + \left(c + \frac{\rho C_D U D L}{2} \right) \dot{y} + k[y - b \cdot \text{sign}(y)] = \frac{\rho U^2 D L C_{L0} q}{4} - \frac{K_M}{r} i_a \quad (5.38)$$

$$\ddot{q} + \varepsilon \left(\frac{2\pi \text{St} U}{D} \right) (1 - \beta q^2 + \lambda q^4) \dot{q} + \left(\frac{2\pi \text{St} U}{D} \right)^2 q = \frac{a}{D} \ddot{y} \quad (5.39)$$

$$\dot{i}_a = \frac{K_E}{r L_a} \dot{y} - \frac{(R_a + R_L)}{L_a} i_a \quad (5.40)$$

where L_a is the armature inductance, R_a is the armature resistance and R_L is the load resistance. Effects of the inertia mass from the gear and rotor of the generator

and frictional losses on the rack and the gear are assumed to be negligible in this model.

The armature current is chosen as the output variable, $v(t) = i_a(t)$. Therefore, the control input $u(t)$ is constructed from the delayed armature current, $i_a(t - \tau)$, as:

$$u(t) = K[i_a(t - \tau) - i_a(t)] \quad (5.41)$$

Subsequently, the control input is applied to the system through Eq. (5.40) as:

$$\dot{i}_a = \frac{K_E}{rL_a} \dot{y} - \frac{(R_a + R_L)}{L_a} i_a + K[i_a(t - \tau) - i_a(t)] \quad (5.42)$$

Therefore, to physically implement the control input, an input voltage ΔV from an external source will be applied to the circuit as:

$$\Delta V = L_a K[i_a(t - \tau) - i_a(t)] \quad (5.43)$$

The controlled system model (Eqs. (5.38), (5.39) and (5.42)) is simulated in order to demonstrate the performance of the time delay feedback controller. A set of physical properties that results in chaotic response of the system is chosen as shown in Table 5.3. The cylinder is scaled up to a larger size that is compatible to the generator. The specifications of the generator are chosen accordingly to the Portescap DC motor 35GLT2R82-426E.1. The time delay, τ , is chosen to be equal to the period T of the inter-well period-one orbit. This period T is also estimated by the method of intersecting the chaotic orbit in the phase space to a lower dimensional hyperplane as discussed in Section 5.2.

Table 5.3 Parameters in the simulation of the time delay feedback controller.

Parameters	Values	Parameters	Values
Mass ratio, m^*	5	Radius of gear, r	0.005 m
Diameter of cylinder, D	0.1 m	Generator constant, k_E	0.157 V/rad s ⁻¹
Length of cylinder, L	1 m	Torque constant, k_M	0.157 Nm/A
Damping ratio, ζ	0.002	Armature inductance, L_a	0.0017 H
Natural frequency, $f_{n, water}$	1 Hz	Armature resistance, R_a	9.5 Ω
Water flow velocity, U	0.41 m/s	Load resistance, R_L	50 Ω
Bi-stable gap, b	0.25D	Estimated time delay, τ	1.3939 s

Fig. 5.17 shows the bifurcation map of the controlled system as a function of the control gain K . It can be observed that the system is stabilized into the period-one orbit with the control gain K of about 15,000 onwards. Fig. 5.18 depicts the time series displacement of the cylinder when the system is controlled with the gain $K = 15,000$. It can be seen that the period-one vibration is achieved after the transient duration of about 100 s.

Figure 5.17 Bifurcation map of controlled system as a function of control gain K .

Figure 5.18 Displacement of the cylinder when the system is controlled.

Fig. 5.19 shows the control input voltage, ΔV , and the output voltage on the load, V_L . Unlike the OGY controller, where the perturbation in the system parameter

as the control effort remains constant when the system is stabilized, the control input voltage, ΔV , must be continuously applied to the system in the time delay feedback method. Therefore, the net harvested power will be affected by the power required for the control input.

Figure 5.19 Control input voltage, ΔV , and output voltage on load, V_L .

The average harvested power or the power consumed by the load, P_L , and the average power required for the control input, $P_{control}$, can be calculated as:

$$P_L = i_{a,RMS} V_{L,RMS} \quad (5.44)$$

$$P_{control} = i_{a,RMS} \Delta V_{RMS} \quad (5.45)$$

Table 5.4 compares the net harvested power of the uncontrolled and controlled systems. It can be seen that when the system is controlled, the harvested power can be significantly improved to 52.3%. This result is comparable to the performance of the OGY controller. The only shortcoming of the time delay feedback controller compared to the OGY controller is the control input that must be continuously applied to the system, thus, the net harvested power is affected. However, only the period of the desired orbit needs to be estimated to design the controller. This feature is a prominent advantage of the time delay feedback controller.

Table 5.4 Comparison in the net harvested power, P_{net} (mW), of the uncontrolled and controlled systems.

Power (mW) \ Systems	Uncontrolled system	Controlled system
Power consumed by load, P_L	325	540
Power required by control input, $P_{control}$	0	45
Net harvested power, $P_{net} = P_L - P_{control}$	325	495
Improvement of controlled system compared to uncontrolled system	52.3%	

5.4 Summary

The OGY controller and time delay feedback controller were designed and implemented to stabilize chaotic responses of the bi-stable VIV system. Both the controllers are able to stabilize chaotic responses into the desired inter-well period-one orbit, which is the highest-energy orbit of a bi-stable system. In comparison to the uncontrolled chaotic system, the controlled system offers a significant improvement in the output power. This result opens a potential window for practical applications of the bi-stable VIV energy harvesting system. Both the OGY controller and the time delay feedback controller have their advantages and shortcomings. Therefore, based on practical applications, a suitable controller can be chosen among them.

The OGY controller

The prominent advantage of the OGY controller is the constant perturbation in the regulated system parameter achieved at the steady state, thus, no power is further required for the control effort. The applicability of the OGY controller is concreted by the real-time experimental validation and the utilization of the time delay coordinates instead of the full state variables.

However, the success and performance of the OGY controller are essentially dependent on the estimation accuracy of the discretizing period, the fixed point, the Jacobian and sensitivity matrices. In addition, the maximum allowable perturbation in the regulated parameter must be rationally chosen for the system

to approach the desired fixed point in reasonable duration and to avoid the potential instability to the system caused by the large control efforts in the transient duration.

The time delay feedback controller

The time delay feedback controller is advantageous compared to the OGY controller in terms of the estimated information, where only the period of the desired orbit needs to be estimated. As an expense, the time delay feedback controller requires an external source to supply the control input voltage, thus, the net harvested power is lowered by the power required for the control input.

Chapter 6

General Conclusions, Contributions and Future Work

This chapter concludes the thesis with an overview on the main findings and how they can be applied in practical circumstances. Subsequently, the contributions that tally with the objectives of this thesis are highlighted. Ultimately, some recommendations of the future work are discussed.

6.1 General conclusions

This thesis essentially deals with modelling, characterization and control of the VIV energy harvesting system enhanced by the nonlinear springs, namely bi-stable spring and hardening spring. To theoretically model the nonlinear VIV system, the nonlinear springs are represented by the geometrically nonlinear restoring force functions and applied in a wake oscillator model. At the same time, a cyber-physical force-feedback testing platform was developed to experimentally validate the theoretical analyses based on the wake oscillator model. The physical properties of the VIV system in the experiments including the nonlinear restoring force functions are defined on the testing platform by combining the experimental and simulation processes.

After the theoretical and experimental models were developed, characteristics of the bi-stable and hardening springs on the energy harvesting capability of the VIV system were investigated in a wide range of the governing parameters. Subsequently, it is observed that a bi-stable spring might result in chaotic responses that are undesired for the operation of the system. Therefore, a parametric quantification of chaotic responses was comprehensively carried out to identify the ranges of the system parameters where chaotic responses will occur. Ultimately, appropriate control strategies were applied to actively stabilize chaotic responses of the bi-stable VIV energy harvesting system.

The findings of this thesis are believed to significantly contribute to the understanding in dynamics characteristics of the nonlinear springs and how they can be practically applied to improve the performance of a VIV energy harvesting system. In the following, the individual findings are elaborated.

Parametric characterization of nonlinear VIV systems

Based on the numerical and experimental investigations performed on the wake oscillator model and the cyber-physical force-feedback testing platform, it is indicated that a bi-stable spring, in which the nonlinearity is governed by the

bi-stable gap parameter, is able to improve the output power and widen the resonance range towards the side of low velocity flows. However, the bi-stable system harvests less power than the linear system does when the flow velocity increases. This effect is more obvious with the increase of the bi-stable gap.

On the other hand, a hardening spring, in which the nonlinearity is controlled by the hardening stiffness and hardening gap parameters, can significantly improve the output power and widen the resonance range towards the side of high velocity flows. However, the output power of the hardening system at low velocity flows is lower than that of the linear system. This trend is more significant when the hardening stiffness is increased and the hardening gap is decreased.

Motivated by the advantages and negative effects of the bi-stable and hardening springs, a combined nonlinear spring was proposed. The simulation and experimental results show that this combined nonlinear spring is able to extend the resonance range towards both sides of low and high velocity flows.

The design of the combined nonlinear spring might be more complicated than the individual bi-stable or hardening springs. Therefore, based on the water flow velocity profile of the installation location, the most suitable nonlinear spring can be selected among the bi-stable, hardening and combined springs.

Parametric quantification of chaotic responses

The bi-stable VIV energy harvesting system is further studied since there is a chance for the occurrence of chaotic responses that will drastically deteriorate the output power. A chaotic quantification via the Lyapunov exponent was performed to identify the ranges of the governing parameters in which chaotic responses will occur and how the chaotic degree evolves subjected the variation of these parameters. The parameters of interest in this quantification study are the bi-stable gap and damping ratio, which were evaluated from the non-dimensional form of the system model.

Based on the quantification result, it is observed that chaotic responses tend to occur on the system with a large bi-stable gap. However, when the bi-stable gap is excessively large, chaotic responses are less likely to occur. For the energy harvesting purpose, the system with an excessively large bi-stable gap is unfavorable since the inter-well periodic vibration is hardly maintained.

In addition, it is found that chaotic responses can also be eliminated by a high damping component imposed on the system. However, the damping parameter of the system must be properly designed since an excessively high damping value will suppress the vibration and lower the output power.

Design and validation of the controllers to stabilize chaotic responses

In the cases where the bi-stable VIV system is designed with a noticeably large bi-stable gap to achieve a major improvement in the output power at low velocity flows, chaotic responses will occur when the flow velocity increases. Therefore, the OGY controller and time delay feedback controller were designed to actively stabilize the chaotic responses. When the system is stabilized into the period-one inter-well orbit by these controllers, the output power is significantly improved.

In the OGY method, the control effort is executed by a small perturbation in the damping coefficient, which can be conducted by perturbing the electrical load. When the system is completely stabilized by the OGY controller, the net harvested power is not affected by the power required for the control effort since the perturbation in damping as the control effort becomes constant. This feature can be considered as a noticeable merit of the OGY controller. A real-time experiment of the designed OGY controller was successfully performed on the cyber-physical testing platform to confirm its applicability in a real-time context. Last but not least, the OGY controller based on the time delay coordinates, which requires only the measurement of a single state variable, was also successfully designed and implemented for the system. This effort further

consolidates the applicability of the OGY controller since in a practical context, measuring all the state variables of the system might be tedious and expensive.

In the time delay feedback method, a control input voltage is continuously applied to the system, thus, the net harvested power is lowered by the power required for the control input. However, the time delay method manifests its advantage in terms of the minimum information that needs to be estimated to design the controller.

6.2 Contributions

By fulfilling all the objectives that have been identified in Chapter 1, main contributions of this thesis are summarized as:

- The first contribution is the characterization of the VIV system enhanced by the hardening and bi-stable springs. Effects of these two springs on the energy harvesting capability of the system were investigated in a wide range of the governing parameters. Both the springs show their advantages as well as negative effects. A new combined nonlinear spring was proposed to obtain the advantages of both the hardening and bi-stable springs, while their negative effects are minimized.
- The second contribution is attributed to the study to quantify chaotic responses of the bi-stable VIV system. It should be highlighted that such a study to quantify chaotic responses of a VIV system has never been carried out in literature. Based on this study, the bifurcation map that shows the dependency of chaotic responses on the bi-stable gap and damping ratio can be used as the primary knowledge to design a bi-stable VIV energy harvesting system.
- The third contribution is the design and implementation of the OGY controller and time delay feedback controller to stabilize chaotic responses of the bi-stable VIV energy harvesting system. Detailed guidelines to design these controllers

are provided and their successful implementations consolidate the applicability of the bi-stable VIV energy harvesting system.

In addition, the novelty of the thesis is attributed to the first exploration comprehensive investigation on the implementation of nonlinear springs on a VIV energy harvesting system. In particular, such a study to quantify and control chaotic responses of a bi-stable VIV energy harvesting has never been carried out in literature. The findings of the thesis are practically promising to enhance the capability in power extraction of the VIV energy harvesting system.

6.3 Future work

Although the potential of applying the nonlinear springs to improve the performance of a VIV energy harvesting system was well demonstrated by the research activities of this thesis, some further tasks are required to realize the application. As the recommended future work, these tasks are explained as:

- The capability to widen the resonance range of the bi-stable, hardening and combined nonlinear springs is dependent on the governing parameters that include the bi-stable gap, hardening gap and hardening stiffness. Therefore, an optimization study of these parameters is required in order to provide an optimal nonlinear spring that is most suitable to a given water flow velocity profile.
- A study to design the physical models of the nonlinear springs proposed in this thesis is required. These physical springs must be compatible to the overall design of the energy harvesting system, the constraint of the overall effective mass of the system and other practical requirements.
- Since the performance of the time delay feedback controller was only verified by the simulation on the wake oscillator model, an experimental validation of the controller in real-time is required.
- In order to utilize the OGY controller and time delay feedback controller in practical applications, chaotic responses of the system must be investigated at different water flow velocities to design the controllers accordingly. Subsequently,

the systems to measure the real-time water flow velocity and output variables must be developed. For the OGY controller, a mechanism to trigger the perturbation in the electrical load as the control effort is required. For the time delay feedback controller, an external source to supply the control input voltage needs to be designed.

- Both the OGY and time delay feedback controllers still have their own shortcomings. For example, the design procedure of the OGY controller requires the estimation of various parameters. On the other hand, the time delay feedback controller requires a continuous input voltage applied to the system. Therefore, it is recommended to explore and apply other existing chaos control methods in literature for the bi-stable VIV energy harvesting system.
- Chaotic responses might also occur in the combined nonlinear VIV energy harvesting system. Therefore, it is necessary to expand the chaotic quantification and control studies for the combined nonlinear VIV system.

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