Long-term fatigue assessment of deepwater risers in time domain

Gao, Yidan

2017

Gao, Y. (2017). Long-term fatigue assessment of deepwater risers in time domain. Doctoral thesis, Nanyang Technological University, Singapore.

<http://hdl.handle.net/10356/73044>

<https://doi.org/10.32657/10356/73044>

NANYANG
TECHNOLOGICAL
UNIVERSITY

LONG-TERM FATIGUE ASSESSMENT OF DEEPWATER RISERS IN TIME DOMAIN

YIDAN GAO

SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING

2016

LONG-TERM FATIGUE ASSESSMENT OF DEEPWATER RISERS IN TIME DOMAIN

YIDAN GAO

School of Civil and Environmental Engineering

**A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirements for the degree of
Doctor of Philosophy**

2016

ACKNOWLEDGEMENTS

I would like to express my sincere gratitude to my supervisor, Prof. Cheung Sai Hung, for his invaluable mentoring during the last two years of my PHD study and for his patience, motivation, immense knowledge sharing and especially his rigorous attitude for research. He is an excellent scientist and instructor with great passion in research as well as life. I would also like to thank him for giving me confidence, complete freedom and full support for exploring interesting findings in research areas.

I am also thankful to my previous research advisor, Prof. Low Ying Min for his professional guidance and great patience for training me to become a qualified researcher. With the help of Prof. Low, my logical thinking ability has greatly improved which is a fortune in my life. I still remember all the positive and negative comments and encouragement provided by him like a father during the first three years of my PHD study. These still move me deeply when they come to my mind.

I would also like to acknowledge the help of Prof. Wong Yiik Diew and Dr. Chai Chen for their generous support and advice.

I would also like to thank Dr. Li Fengzi and Dr. Zhang Yi for generously sharing their professional knowledge and experience with me. I would also like to share the joy of completing this thesis with fellow PHD students Shao Zhe, Queen and others under Prof. Cheung Sai Hung and friends Shi Si, Shi Yongyi, Cecilia and so on who work together in the transportation lab.

My deepest gratitude goes to my parents for their unwavering love and encouragement. My special thanks to my husband Dr. Wu Hao for helping me a lot in taking good care of our little son Wu Mingyuan and his continuous support and understanding. Without their continuous spiritual support and encouragement, I would not achieve my goal.

SUMMARY

With the increasing demand for oil and gas, offshore industry moved offshore structures towards deeper waters and severe environmental conditions. Offshore structures are subjected to many environmental loads during their service life, such as waves, winds, currents and ice. Such environmental conditions are non-stationary processes. However, in common practice, it is always assumed that these environmental actions are statistically stationary for short-term sea state, typically for three hours. Riser system is one of the most critical offshore structure component and is exposed to a large number of sea states during its life span, especially in deep water environment. Therefore, only stationary analysis for the riser system is not sufficient, and a global dynamic analysis is indispensable to obtain its response due to the fluid-structure interactions. During the global dynamic analysis, extreme response and fatigue life are two important design concerns. Extreme response predictions can be obtained based on either short-term or long-term approaches. However, fatigue predictions can only be based on long-term methods due to their accumulative characteristics.

The main objective of this study is to investigate various existing methods for their accuracy, limitations and efficiency, and to develop advanced methods in calculating the long-term fatigue damage of deep-water risers as well as in constructing a suitable statistical model for ocean environmental random variables.

The traditional way of solving long-term fatigue problem is using a wave scatter diagram. By lumping several sea states into a small number of bins, one sea state is selected as a representation of each block to ensure that the damage quantity yielded by this sea state is not lower than the original sea states. It is apparent that such block method has a lack of accuracy. Therefore, there is a need for a fast and precise method for long-term fatigue analysis especially in time domain. The first part of the author's research is to develop a new efficient approach for long-term riser fatigue analysis in time domain. This new approach allows one to simulate the wave amplitudes from a distribution different from the original one. Then, the results for many sea states can be obtained by simply altering the importance sampling weighting function. This proposed approach is enhanced by a succession of additional techniques to reduce the sampling variability. The study of this proposed new efficient approach is presented in Chapter 3.

The stochastic nature of environmental loads such as wave and wind applied on the structure is quite complicated. In the random process related structural analysis, a suitable statistical environmental condition model is essential and should be carefully developed for the prediction of long-term performance of offshore structures. As a common practice, a conditional joint distribution model is used for the statistical environmental condition, which is popularly used in the industry. However, in such model, only a linear relationship between pairs of random variables is considered. If the actual dependency is nonlinear, those traditional models no longer estimate the relationship appropriately. Chapter 4 proposes a new approach by using a classical multiple dimensional copula model to treat the joint distribution of random environmental conditions. This classical multiple copula model is slightly superior to the traditional model by model selection, and the influence of different statistical models in the long-term fatigue analysis is studied.

Due to the limitation of multiple copula type, an advanced copula model is developed in Chapter 5 to construct more flexible statistical models based on the copula concept, estimated by model selection methods. The application in the long-term fatigue analysis is also compared with traditional models and classical copula models. Compared with traditional models, the biggest advantage of multiple copula models is their ability to model nonlinear relationships among different environmental random variables.

AUTHOR'S PUBLICATIONS

Journal papers:

Gao Y., Low, Y. M. (2016), An efficient importance sampling method for long-term fatigue assessment of deepwater risers with time domain analysis, Probabilistic Engineering Mechanics, 45, 102-114.

Gao Y., Cheung, S. H. (2016a), Long-term fatigue assessment of deep-water risers based on multiple copula models, submitted to Ocean Engineering, under review.

Gao Y., Cheung, S. H. (2016b), Long-term fatigue assessment of deep-water risers based on advanced multiple copula models, to be submitted to Reliability Engineering & System Safety.

Gao Y., Chai C. and Wong Y. D. (2016), Safety impact of right-turn waiting area at signalized intersections with individual decision-making based on Fuzzy Cellular Automata, to be submitted to Accident Analysis and Prevention.

Conference papers:

Gao Y., Low, Y.M. (2014), Long-term fatigue analysis of deepwater risers in the time domain, Proceedings of the International Conference on Offshore Mechanics and Arctic Engineering, OMAE, San Francisco, United States, Volume 4B, OMAE2014-24664.

Gao Y., Cheung, S.H. (2015), Long-term fatigue analysis of risers with multiple environmental random variables in time domain, Proceedings of the International Offshore and Polar Engineering Conference, ISOPE, Kona, Big Island, United States, Volume 2015- January, Pages 228-235.

TABLE OF CONTENT

ACKNOWLEDGEMENTS	I
SUMMARY	II
AUTHOR'S PUBLICATIONS	IV
TABLE OF CONTENT	V
LIST OF FIGURES	IX
LIST OF TABLES	XI
LIST OF ABBREVIATIONS	Error! Bookmark not defined.
CHAPTER 1 INTRODUCTION	1
1.1 Background	1
1.1.1 Overview of Offshore Industry.....	1
1.1.2 Components of a Floating Structure	2
1.1.3 Design Considerations	4
1.1.4 Fatigue Analysis	6
1.1.5 Environmental Conditions	7
1.2 Research Objectives	8
1.3 Original Contributions.....	9
1.4 Organization of the Thesis	10
CHAPTER 2 LITERATURE REVIEW AND BACKGROUND THEORY	12
2.1 Introduction	12
2.2 Review of Environmental Conditions and Environmental Loads.....	12
2.2.1 Description of Ocean Waves	12
2.2.2 Environmental Parameters and Statistics	15
2.3 Review of Fatigue Analysis of Offshore Structures.....	19
2.3.1 Traditional Fatigue Design and Fatigue Analysis	19
2.3.1.1 Damage Accumulation Rule and Fatigue Safety Checks	19
2.3.1.2 Fatigue Causes of Risers.....	21
2.3.1.3 Fatigue Analysis	22
2.3.2 Frequency Domain and Time Domain Analysis	23
2.3.3 Fatigue Analysis of Flexible Risers	23
2.3.3.1 Introduction	23

2.3.3.2 Fatigue Analysis of Flexible Riser	26
2.3.3.3 Improve the Fatigue Capacity of Flexible Riser.....	28
2.4 Review of Methods for Probability Integrals	29
2.4.1 Reliability Assessment	29
2.4.2 Classification of Methods for Probability Integrals	31
2.4.3 Direct Analytical Integration and Numerical Integration.....	31
2.4.4 Monte Carlo Technique	32
2.4.4.1 Introduction	32
2.4.4.2 'Crude' Monte Carlo	33
2.4.4.3 Variance Reduction	33
2.4.4.4 Importance Sampling.....	34
2.4.4.5 Quasi-Monte Carlo	35
2.4.5 Probability Integral Methods	35
2.4.5.1 First-Order Reliability Method (FORM).....	35
2.4.5.2 Second-Order Reliability Method (SORM)	37
2.4.5.3 First Order Saddlepoint Approximation (FOSPA).....	38
2.4.5.4 Mean-Value Second Order Expansion	39
2.4.5.5 Mean-Value First Order Second Moment (MVFOSM)	40
2.4.5.6 Mean-Value First Order Saddlepoint Approximation (MVFOSA).....	40
2.4.5.7 Second-Order Reliability with First-Order Efficiency (SORM-FOE)	40
2.4.5.8 Second-Order Perturbation Approach	41
2.4.5.9 Asymptotic Approximation	41
2.4.5.10 Univariate Dimension-Reduction Method.....	42
2.4.5.11 New Point Estimates Method	43
2.4.6 Transformations.....	45
2.4.6.1 Rosenblatt Transformation	45
2.4.6.2 Nataf Transformation	47
2.4.7 Summary for different probability integral methods	47
2.5 Chapter Summary.....	48
CHAPTER 3 LONG-TERM FATIGUE ANALYSIS OF RISER SYSTEMS IN TIME	
DOMAIN	50
3.1 Introduction	50
3.2 Wave Environmental Model	51
3.3 Evaluation of Probability Integrals	53

3.4 Proposed Importance Sampling Method	54
3.4.1 Introduction of Proposed Importance Sampling Method	55
3.4.2 Challenges of this Proposed Method	58
3.4.3 Techniques.....	59
3.4.3.1 Peak Exploration Technique.....	59
3.4.3.2 Equal Variance Technique.....	60
3.4.3.3 Segmenting Simulation Time Technique	62
3.4.3.4 Combining Results from Different Applied Spectra	63
3.4.4 Curve Fitting.....	64
3.4.5 Optimization	66
3.4.5.1 Motivation	66
3.4.5.2 Formulating the Optimization Problem.....	67
3.4.5.3 Analytical Derivation of the Variance of the Weights	67
3.4.5.4 Solving the Optimization Problem	69
3.4.5.5 Error Estimation	69
3.5 Case Studies	69
3.5.1 General Description.....	69
3.5.2 Case study 1: selecting the applied spectra	72
3.5.3 Case study 2: equal response variance technique	75
3.5.4 Case study 3: combining results from different applied spectra	76
3.5.5 Case Study 4: Curve Fitting.....	78
3.5.6 Fatigue Damage Results and Comparison of Methods	81
3.6 Point Estimates Method	84
3.7 Results and Discussion.....	85
3.7.1 Results	85
3.7.2 Discussion.....	88
CHAPTER 4 MODELLING THE MULTIPLE CLASSICAL COPULA MODEL OF ENVIRONMENTAL RANDOM VARIABLES FOR THE OFFSHORE STRUCTURAL ANALYSIS	90
4.1 Introduction	90
4.2 Traditional Models for Sea State Parameters	90
4.2.1 Conditional Joint Distribution Model between H_s and T_p	91
4.2.2 Conditional Joint Distribution Model among H_s , T_p and V_w	92
4.3 Copula Theory.....	93

4.3.1 Definition and Basic Properties	94
4.3.2 Families of Copula.....	95
4.3.3 Dependence Structure of Copula	100
4.4 Comparative Study.....	101
4.4.1 Data Pre-Treatment.....	101
4.4.2 Application of 3-Dimensional Traditional Models	102
4.4.3 Application of 3-Dimensional Classical Copula Models	108
4.5 Model Selection.....	111
4.6 Comparison of Fatigue Assessment Based on Different Environment Statistical Modelling	113
4.6.1 General Description.....	113
4.6.2 Wave Amplitude Assumption	115
4.7 Results and Discussion.....	116
CHAPTER 5 ADVANCED COPULA MODEL AND THE EFFECT ON THE LONG- TERM FATIGUE ANALYSIS.....	118
5.1 Introduction	118
5.2 Advanced Copula theory	118
5.3 Application of 3-Dimensional Advanced Copula Models	120
5.3.1 Construction of First Level Pair Copula.....	120
5.3.2 Construction of Conditional Pair Copula	130
5.3.3 Conclusion and Discussion.....	132
5.4 Application to Long-term Fatigue Assessment of FPSO Risers and Comparison with Other Statistical Models	134
5.4.1 Generating Samples.....	134
5.4.2 Application of Advanced Copula Model to Long-term Fatigue Analysis.....	135
5.4.3 Results and Discussion	136
5.5 Chapter Summary.....	141
CHAPTER 6 CONCLUSION AND FUTURE WORKS.....	143
6.1 Conclusion.....	143
6.2 Recommendations of Future Works.....	145
REFERENCES	146

LIST OF FIGURES

Figure 1. 1 Different offshore platforms (NOAA, 2010).....	2
Figure 1. 2 Components of a typical FPSO (Offshore Energy Today.com).....	3
Figure 1. 3 Rigid body motions of a vessel	5
Figure 1. 4 Wave frequency and offshore structures	6
Figure 2. 1 Schematic of random waves	14
Figure 2. 2 Power spectrum density of wind speed (Chakrabarti and Subrata, 2005).....	17
Figure 2. 3 Two-slope S-N curve (ABS)	20
Figure 2. 4 Fatigue damage due to different stress range blocks.....	21
Figure 2. 5 Unbonded flexible pipe (De Sousa et al., 2012).....	24
Figure 2. 6 Cross-section of unbonded flexible pipe (Zhang et al., 2003)	25
Figure 2. 7 Pressure armor profiles (API, 2002).....	26
Figure 2. 8 Flexxtreme® hybrid unbonded flexible pipe structure (Nielsen, 2001).....	29
Figure 2. 9 Fitted cumulative distribution function based on direct sampling technique.....	33
Figure 2. 10 Linear and nonlinear limit states	37
Figure 3. 1 Contour of joint probability density function $f(H_s, T_p)$	53
Figure 3. 2 Comparison of two types of generating waves.....	57
Figure 3. 3 Relative standard error of fatigue damage for different sea states	58
Figure 3. 4 Contours of joint probability density $f(H_s, T_p)$ with path along which f is maximum for given H_s	60
Figure 3. 5 Response stress spectrum vs wave spectrum at peak wave spectrum	62
Figure 3. 6 Scatter plot of CoV_{Wd} vs CoV_W	66
Figure 3. 7 Overview of the whole system	70
Figure 3. 8 Trajectory of H_s/T_p against T_p	72
Figure 3. 9 Plot of $CoV_d H_s, T_p$ against H_s for $T_p=10.65$ s	77
Figure 3. 10 Plot of $CoV_d H_s, T_p$ against H_s for $T_p=11.15$ s	77
Figure 3. 11 Plot of various quantities against H_s for $T_p=11.15$ s	79
Figure 3. 12 Plot of various quantities against H_s for $T_p=11.4$ s	81
Figure 3. 13 1D Integral calculated for a range of T_p , with error bars using proposed method with optimization	82
Figure 3. 14 Contours of the integrand generated by using proposed method with optimization	84
Figure 4. 1 Comparison of different bivariate copulas with each marginal distribution following a standard normal distribution and the linear correlation coefficient 0.9.....	99
Figure 4. 2 Mean value of ocean parameters in different years.....	102
Figure 4. 3 Comparison of CDF of V_w obtained by two methods and raw data.....	103
Figure 4. 4 Curve fitting of (a) k vs V_w and (b) λ vs V_w based on Equations (4.9) and (4.10) for points representing V_w within interval values of 1.0.....	104
Figure 4. 5 The conditional mean spectral peak period T_{ph} vs H_s based on Equation (4.17) for points representing H_s within interval values of 0.5.....	106
Figure 4. 6 The conditional mean spectral peak period V_{wh} vs H_s based on Equation (4.18) for points representing H_s within interval values of 0.5.....	106

Figure 4. 7 Marginal parametric model fit for Hs.....	109
Figure 4. 8 Marginal parametric model fit for Tp.....	109
Figure 4. 9 Marginal parametric model fit for Vw.....	110
Figure 4. 10 Overview of the whole system	113
Figure 5. 1 Comparison of contour plot between original data and (a) Gaussian copula model, (b) T copula model, (c) Clayton copula model, (d) Frank copula model, and (e) Gumbel copula model for the pair Hs, Tp.....	124
Figure 5. 2 Comparison of contour plot between original data and (a) Gaussian copula model (b) T copula model (c) Clayton copula model (d) Frank copula model (e) Gumbel copula model for the pair Hs, Vw	127
Figure 5. 3 Comparison of contour plot between original data and (a) Gaussian copula model (b) T copula model (c) Clayton copula model (d) Frank copula model (e) Gumbel copula model for the pair Tp, Vw	129
Figure 5. 4 Scatter diagram of raw data	139
Figure 5. 5 Scatter diagram of data generated by traditional model (100000samples)	139
Figure 5. 6 Scatter diagram of data generated by 3 rd advanced copula model (100000samples)	140

LIST OF TABLES

Table 3. 1 Flexible riser characteristics	71
Table 3. 2 Mooring line characteristics.....	71
Table 3. 3 Characteristics of applied spectra (without optimization)	73
Table 3. 4 Theoretical standard deviation of Importance Sampling weight before and after optimization	74
Table 3. 5 Effect of number of wave components N on fatigue damage results	76
Table 3. 6 Comparison of different methods	83
Table 3. 7 Seven point transformation	84
Table 3. 8 Intermediate steps' value of seven point estimate method	85
Table 3. 9 Different methods comparison.....	86
Table 4. 1 Some important Archimedean Copulas	97
Table 4. 2 Regression results for curve fit of Vw	104
Table 4. 3 Regression results for points representing Vw within interval values of 1.0	105
Table 4. 4 Regression results for points representing Hs within interval values of 0.5	107
Table 4. 5 Parameters estimated for fVw, Hs, Tp	108
Table 4. 6 Results of marginal distribution model parameters estimate.....	110
Table 4. 7 Parameters estimated for Gaussian and T Copula models.....	111
Table 4. 8 Flexible Riser Characteristics	114
Table 4. 9 Mooring Line Characteristics	114
Table 4. 10 Sampling variability test for one fixed set of Hs, Tp	115
Table 4. 11 Comparison of goodness-of-fit to the data for 3 statistical models	116
Table 4. 12 Comparison of normalized fatigue damage for 3 statistical models.....	116
Table 5. 1 Comparison of parameter estimates for Hs, Tp	121
Table 5. 2 Comparison of parameter estimates for Hs, Vw	121
Table 5. 3 Comparison of parameter estimates for Tp, Vw	122
Table 5. 4 Parameter estimates and model comparisons for c12 3	132
Table 5. 5 Parameter estimates and model comparison for c23 1.....	133
Table 5. 6 Parameter estimates and model comparison for c13 2.....	134
Table 5. 7 Comparison of goodness-of-fit to three advanced copula models.....	136
Table 5. 8 Comparison of normalized fatigue damage and goodness-of-fit for all models...	137
Table 5. 9 Higer-level statistic comparison	140

LIST OF ABBREVIATIONS

The following table describes the significance of various abbreviations and acronyms used throughout the thesis. The page on which each one is defined or first used is also given.

Abbreviation	Meaning
GOM	Gulf of Mexico
FPSO	Floating, production, storage, and offloading
TLP	Tension leg platform
SPM	Single point mooring
CALM	Catenary anchor leg mooring
TTRs	Top tensioned risers
SCRs	Steel catenary risers
DOF	Degree of freedom
WF	Wave frequency
LF	Low frequency
HF	High frequency
CDF	Cumulative distribution function
PDF	Probability density function
PCC	Pair-copula construction
PM	Pierson-Mosowitz
MLM	Maximum Likelihood Model
CMA	Conditional Modelling Approach
VIV	Vortex induced vibrations
VIM	Vortex induced hull motions
FRP	Fiber-reinforced polymer
CoV	Coefficient of variation
FORM	First-Order Reliability Methods
SORM	Second-Order Reliability Methods
FOSPA	First Order Saddlepoint Approximation
CGF	Cumulant generating function
MLP	Most likelihood point
MPP	Most probable point
SMSA	Saddle-point approximation method
SMNC	Second-order method by non-central chi-square distribution
MVFOSM	Mean-Value First Order Second Moment
MVFOSA	Mean-Value First Order Saddlepoint Approximation
SORM-FOE	Second-Order Reliability with First-Order Efficiency
ISMCS	Importance Sampling Monte Carlo Simulation
IFORM	Inverse First Order Reliability Method
MSC	Monte Carlo simulation
RSE	Relative standard error
TDP	Touch down point
AIC	Akaike information criterion
BIC	Bayesian Information Criterion
CoV	Covariance

CHAPTER 1 INTRODUCTION

1.1 Background

1.1.1 Overview of Offshore Industry

An important part of the offshore industry is to support the exploration and production of hydrocarbons from the sea when the land based oil reservoirs are exhausted. In 1890s, the first offshore rigging was conducted at wharf located off the coast of Pacific Ocean. However, the first offshore exploration of oil and gas was only achieved in the middle of nineteenth century, with a 15ft (4.6m) well in the Gulf of Mexico (GOM) (Burleson, 1999). The materials of the platform evolved from wood in the very early stage to steel and concrete later. Since the first offshore platform constructed in the GOM nearly 70 years ago, the offshore industry has gone through a tremendous evolution especially in the deeper and deeper water depth and challenging hostile environments.

In the early stage, fixed offshore structures can still satisfy the requirement of oil and gas exploration in shallow water condition. However, with the increasing demand for modern development, the shallow water resources were progressively exhausted. Thus, increasingly deeper waters were considered as a promising future for offshore industry, which made fixed structures much more expensive and difficult to install. Nowadays, most of the offshore oil and gas production comes from deep or ultra-deep offshore areas, including the GOM, West Africa and Brazil, which are known as “Golden Triangle”.

In order to meet the requirement of booming oil and gas production industry, some types of innovative and cheaper alternative offshore structures appeared, such as compliant structures and floating structures. One “compliant” tower called Lena guyed tower installed in 1000 ft (305m) of water was built in 1983, with the upper truss structure which could deflect with the wave and wind forces (Chakrabarti, 2005).

Floating structure comes as an inevitable choice in the boom production trend, because of its survival capability in deep water and especially ultra-deep-water environment. There are mainly four types of floating structures: ship-shaped vessel with floating, production, storage, and offloading function (FPSO), Semi-submersible, Tension leg platform (TLP) and Spar. The main types of offshore structures are shown in Figure 1.1

Figure 1. 1 Different offshore platforms (NOAA, 2010)

FPSO is simply a ship-shaped floater which either can be purposely built or converted from an existing tanker. Compared with other floating platforms, FPSO has plenty of waterplane area and ample deck area. Big waterplane area results in large wave loading which results in its incapability of drilling which requires much stability. Ample deck area allows FPSO to have storage function. The economical characteristic is another advantage of FPSO.

A Semi-submersible has two pontoons and multiple columns, with a very small waterplane area. The main buoyancy of semi-submersible comes from components below the water surface. This distinctive feature gives semi-submersible good motion characteristics, which makes it suitable for drilling and production. The disadvantage is its incapability of storage, because of its requirement of low centre of gravity for stability. Compared with FPSO, its construction is more complicated and expensive.

TLP is a platform attached to the seabed by vertical tensioned tethers. The special structure makes TLP the most stable platform of all floating structures. Hence, it is quite suitable for drilling. TLP does not have storage function and has limitations in water depth. Another drawback of TLP is its high cost.

The classic Spar is a vertical hollow cylinder with a deep draft. Spar always has good vertical motion characteristics and storage function, both owing to its deep draft. The disadvantage is its limited number of risers that can be connected to the Spar due to its small perimeter.

1.1.2 Components of a Floating Structure

Usually a floating offshore structure has three key components: the floating platform, the mooring system and the riser system. Since the types of platforms have already been generally

introduced in the previous section, this section will be focused more on the mooring and riser systems. The mooring and riser systems of a typical FPSO are shown in Figure 1.2.

Figure 1. 2 Components of a typical FPSO (Offshore Energy Today.com)

The main function of mooring system is to keep the platform stable. The design of mooring system should consider two sides: it should make the system flexible enough to avoid excessive forces on the platform and it should be rigid enough to avoid difficulties caused by excessive offset. There are three typical types of mooring systems: catenary spread mooring system, single point mooring (SPM) system and taut spread mooring system.

Catenary spread mooring system is the most common type of mooring systems. In this type of system, a number of mooring lines are connected to the platform around the perimeter with different points. Since they are arranged at different locations of the platform, the head of the vessel is more or less constrained in such system.

In single point mooring (SPM) system, all mooring lines are attached to a single point at the platform. This system allows the vessel to weathervane its direction according to the environment, which makes environmental force minimized. Hence, this kind of mooring system provides no yaw stiffness. There are three types of SPM: turret mooring system, catenary anchor leg mooring (CALM), and single anchor leg mooring (SALM).

Taut spread mooring system is made up of synthetic fibre ropes which are more elastic and lighter than steel. As its name indicated, it is taut all the time. Taut spread mooring system is

an alternative for steel catenary systems in deep water when the increased weight of steel catenary systems imposes excessively heavy burden on the payload ability of the platform. Its force excursion is linear, which makes the design process easier to control. Moreover, the taut mooring system requires a smaller mooring radius and leaves no damage on the soil, which is another advantage of this type of mooring system.

Riser system is a vital part of a floating structure, connecting the vessel and the wells at the seabed. They have many functions: drilling, production, work-over, and water or gas injection (to maintain well pressure and power transmission). In general, there are two basic types of risers: drilling risers and production risers.

Drilling risers are working as conduits for operations from the drilling unit. Most of the time, drilling risers repeat deployment and retrieval motions during their lives and are faced with emergency disconnection problems in severe weather.

Production risers are used for production of oil and gas from seabed. Typically, there are three types of production risers: steel catenary risers (SCRs), flexible risers and top tensioned risers (TTRs).

Steel catenary riser is a kind of cost-effective riser, which is made of metal pipes hanging in catenary configuration with no intermediate buoys. With relatively high bending stiffness, they have limited compliancy and are suitable in deep water with small platform motions.

Flexible risers have relative low bending stiffness, thus large motions are allowed. They are more expensive than SCRs, but require lower costs for transportation and installation. The main disadvantages of flexible risers are that they are complex and difficult to inspect and sensitive to temperature change and internal or external pressure.

Top tensioned risers are pre-tensioned risers placed vertically. The vessels attached have very limited movement ability. Due to these special features, they can perform various functions, like production, injection, drilling and export. They are ideal style of risers for TLP and are also welcomed by Spars whose heave characteristics are very good.

1.1.3 Design Considerations

The purpose of design for offshore structures is to secure a safety level that is recognized in the professional field. Various aspects should be considered in the design of offshore structures, such as stability design and structural integrity design. Since offshore structures are always

exposed to numerous sea states over its service life rather than a determined condition, it is necessary to employ a statistical approach in the design and analysis procedure. Global dynamic analysis with statistical approaches is concerned in this work for a typical FPSO and its mooring systems.

During a global dynamic analysis, extreme load cases and fatigue are two broad areas for design criteria of offshore structures. For fixed platforms, it is not difficult to find out that the extreme load case will occur in the biggest wave height or wind speed. However, for floating structures, this may not be the case due to the frequency dependent responses. In order to investigate the floating structures, a broader range of environmental conditions are required to be considered. Fatigue is another important design consideration. Since fatigue damage is an accumulation over the serving time of the structure, it is always required to assess the fatigue on the long-term period (Barltrop, 1998). In this work, a long-term analysis is carried out for fatigue assessment of mooring and riser systems of a floating structure.

Since the local deflections within a floating structure are very small compared with the floater's overall motions, the vessel in the global analysis is always modelled as a rigid body, which is depicted in Figure 1.3.

Figure 1. 3 Rigid body motions of a vessel

The rigid body has six degrees of freedom (DOF), including three translational modes: surge, sway and heave, and three rotational modes: roll, pitch and yaw. Mooring system provides restraint to all six degrees of freedom with different extent. At the mode of heave, roll and pitch,

the vessel has hydrostatic restoring forces. While hydrostatic stiffness does not exist in the mode of surge, sway and yaw and these three modes can only be restrained by mooring systems.

Floating structures are always exposed to complex environmental loads such as wave, wind, current and ice. All responses can be categorized into three frequency regimes: wave frequency, low frequency and high frequency.

Ocean waves typically cover a frequency range from 0.25 rad/s to 1.25 rad/s, which are always avoided in the design of an offshore structure. Different from fixed structures which are always designed above the typical wave frequency (WF) range, floating structures are designed mostly below WF. Therefore, the amplitude to WF responses is normally not significant. Also, WF loading is the most critical loading in the study of wave induced loads. Figure 1.4 shows the wave frequency and frequency of offshore structures.

Figure 1. 4 Wave frequency and offshore structures

The frequency of slowly varying forces is much lower than the typical ocean wave frequency. The period of low frequency (LF) oscillations is always in the range of 100-200 seconds. Slowly varying forces originate from interactions between different wave frequencies, in wind from low frequency wind turbulence (Barltrop, 1998). Compared with WF forces, LF forces are usually very small. However, the effect of LF force on the mooring system of floating structures can be significant.

Similar to LF forces, high frequency (HF) drift forces originate from the sum of interactions of WF and have a higher frequency than WF. The amplitude of HF forces is always very small and only important when resonance occurs.

1.1.4 Fatigue Analysis

Fatigue is an important consideration in the design and analysis of offshore structures and fatigue damage is an accumulated damage over a prolonged duration from numerous sea states. During the life of an offshore structure, it will experience an infinite number of different

environmental conditions which cannot be predicted by a deterministic method. Thus, the calculation of fatigue damage should take into account all of these environmental conditions to obtain a total fatigue damage. Generally, there are mainly three different methods for fatigue analysis: deterministic method, semi-probabilistic method and spectral analysis method. The deterministic method is based on an individual wave with a given wave height and does not consider different wave periods. Hence, it is not suitable for floating structures, for which dynamic responses are significant. The semi-probabilistic method takes into account of different wave periods for each given wave height, which is the only difference compared with deterministic method and not appropriate for floating structures with consideration of dynamic responses and random waves. The spectral method is recognized as the most appropriate method for floating structures and is implemented by some standard practice and authorities (Barltrop, 1998).

Currently, the common practice to assess the fatigue damage is to divide the environmental parameter scatter diagram into a smaller number of manageable bins, and then calculate the fatigue response within each bin. It is suggested that within each bin, the fatigue damage of representative sea state should not be smaller than the original sea states (DNV-RP-F204). However, till now, there are no explicit guidelines on the effective blocking strategy as well as level of discrepancy.

1.1.5 Environmental Conditions

In the dynamic analysis of offshore structures, many environmental variables should be considered, such as winds, waves, currents and tides. Besides these, extreme environmental phenomena such as ice, earthquake, soil conditions, temperature change, fouling, and so on may also contribute to structural damage in some specific cases. It is accustomed to use physical variables of statistical features to describe environmental phenomena which can reveal extreme conditions, short-term and long-term variations (DNV-RP-C205, 2010). Environmental loads are the loads generated by environmental phenomena and can be described by environmental parameters such as significant wave height, spectrum peak period, and direction of the waves, amplitude and direction of the wind velocity, current velocity and so on. For sure, all those environmental parameters are not independent. Catastrophic consequences may occur if the correlations between environmental parameters are overlooked. Currently, the most popular and mature way to describe the relationship between environmental parameters is the joint distribution models. Joint probabilistic models for two environmental

parameters such as joint distribution of significant wave height and period and joint distribution of significant wave height and wind speed are already well studied and documented which are usually based on conditional modelling approach (DNV-RP-C205, 2010). The conditional modelling approach are made up with conditional distributions dependent on one single environmental parameter, which is difficult to take into account all possible statistical dependences among multiple random environmental variables. Hence, one methodology based on Nataf transformation is proposed to create a joint probability model which can take into account statistical dependences among all environmental parameters (Sagrilo, Lima and Papaleo, 2011). The joint probability model built up based on the Nataf transformation can take into account all statistical dependences between any two environmental parameters, which is a big step compared with conditional models. However, the correlation coefficient among variables in both Nataf based method and conditional modelling approach are only limited to linear dependency category, which cannot describe nonlinear dependency. In the second part of this thesis, a copula based probabilistic model is developed to take into account both linear and nonlinear dependencies among multiple random environmental parameters.

1.2 Research Objectives

As mentioned previously, fatigue analysis is accumulated over the whole life of offshore structures, hence, long-term consideration should be incorporated in the fatigue assessment, which imposes a great burden to computational cost especially in time domain. The main objective of this thesis is to develop efficient and accurate methods for long-term riser fatigue analysis, in conjunction with irregular wave and time domain analysis.

The evaluation of long-term performance of offshore structures requires the joint cumulative distribution function (CDF) or probability density function (PDF) of random variables for modelling the random process to characterize the sea environment. The traditional method of constructing the statistical model for the environmental variables is realized by means of conditional distribution model, which only considers the linear relationship between pairs of random variables. However, if the actual dependency among the environmental random variables is nonlinear, those traditional models may not estimate the relationship among them appropriately. An appropriate statistical model is necessary for fatigue analysis of deep-water risers with consideration of the actual nonlinear dependency, which is another important concern in this thesis. Classical multiple copula models are adopted for statistical modelling to provide a solution for such problems.

In the literature, 1-dimensional and 2-dimensional copula models have already been studied, while the high-dimensional copula models are much more complicated and rarely investigated especially for fatigue assessment in the field of offshore engineering. To model the complex environmental conditions for offshore structures, efforts are also put in developing statistical models for high dimensional long-term fatigue assessment.

1.3 Original Contributions

The original works carried out by the author in this thesis are summarized as follows:

1. A new efficient method for long-term fatigue analysis of risers in time domain is developed. In this proposed method, time series of wave is simulated as sum of sine terms with random amplitudes and phases in the time domain. To improve the computation efficiency, importance sampling strategy is used for the simulation of the wave amplitude, while the multiple sea states are obtained from one sea state by a weighing function. The proposed approach is enhanced by a succession of additional techniques to reduce the sampling variability. This method provides unbiased estimated of mean long-term fatigue damage with significant reduction of the computation cost, as compared to classical numerical method for long-term fatigue analysis in the time domain.
2. Multivariate copula-based model is introduced to assess the long-term fatigue damage of riser systems replacing the traditional conditional joint distribution model, which is believed to be the very first effort for the research of fatigue analysis for offshore riser systems. Bayesian based approach is considered in the selection of different statistical models. The result of model selection shows that the proposed copula-based approach outperforms traditional models. Moreover, the influence of different statistical models in the assessment of long-term fatigue damage of riser systems with the consideration of the nonlinear dependency is also presented. The fatigue damage differences obtained in the results suggest that the statistical model for the joint probability distribution of environmental variables employed in the long-term fatigue calculation could be improved by using the proposed copula-based models as compared to traditional models.
3. An advanced copula method is developed to overcome the limitation of classical multiple classical copula models. The purpose of this PPC is to model dependency by simple local 2-D copula blocks which are based on conditional independence. This proposed method provides a new way for generating samples which is rarely seen in

the literature in any field. In this thesis, this new type of statistical model is established and applied in the field of long-term fatigue assessment, and great potential is found through the comparison with other methods. Further development is worthy of investigation.

1.4 Organization of the Thesis

This thesis consists of six chapters, including this Introduction Chapter. This chapter gives an introduction for the background of the research topic, including the history of offshore industry, the components of floating structures, design considerations, fatigue analysis and random environmental parameters' consideration. Finally, the objectives of this research and the achievements are generally stated.

Chapter 2 reviews the state-of-the-art technologies for fatigue analysis of riser systems in offshore structures using various probability integral methods. The features of different environmental conditions and environmental loads are introduced. Traditional and advanced fatigue analysis approaches are reviewed, especially for the concern of flexible risers for offshore structures in deep-water environment. A comprehensive study for the methods of probability integral is conducted, and various techniques are reviewed and discussed in this chapter.

In Chapter 3, a new efficient simulation approach for long-term fatigue analysis with the consideration of coupling effects of two random environmental parameters in time domain is presented. The results obtained through this new approach which is enhanced by a succession of additional techniques are compared against those from classical numerical method, and it is found to have higher computational time efficiency and unbiased estimation accuracy.

In Chapter 4, classical multiple copula models are constructed for probability analysis of fatigue of the risers in the offshore structures, which is believed to be the first exploration for the fatigue assessment in offshore engineering field. The developed copula models are studied and compared with traditional conditional joint distribution models. The influences of different statistical models on the fatigue damage of flexible risers of FPSO system are further studied. The major advantage of these developed copula models is that nonlinear dependency among the environmental parameters can be modelled appropriately, while the traditional model only considers linear relationship among random variables.

Based on the developed classical multiple copula models, advanced multiple copula models are constructed to apply to long-term fatigue assessment of riser systems with three environmental random variables, which is presented in Chapter 5. Comparison studies are carried out to both classical multiple copula models and traditional models presented in Chapter 3 and Chapter 4. The initial results show its great potential for improving the modelling accuracy as well as the efficiency, which is worthy of further investigation in future.

Finally, Chapter 6 summarizes all the works accomplished in this PHD research, and suggests some recommendations for future development regarding to probabilistic analysis for offshore risers in long-term consideration.

CHAPTER 2 LITERATURE REVIEW AND BACKGROUND THEORY

2.1 Introduction

This chapter reviews various theories and techniques related to the fatigue assessment of mooring and riser systems in the long-term consideration. Section 2.2 reviews the statistical feature of random environmental variables as well as mathematical models for the ocean environment parameters. Section 2.3 introduces the fatigue related analysis and design of offshore structures. Section 2.4 focuses on an in-depth literature review of probability integral methods.

2.2 Review of Environmental Conditions and Environmental Loads

Offshore structures are placed in the ocean for exploration and production. The effect of ocean environment is important, and should be considered in the design and analysis of offshore structures, especially for floating structures. The most important environmental conditions are waves, winds and currents in most part of ocean water. Besides, earthquake and tsunami waves may affect a lot in specific parts of ocean water in the world. By nature, all these environment loads are random and do not exist independently. Therefore, it is necessary to use some statistical models to analyse the effects of environmental conditions.

2.2.1 Description of Ocean Waves

Since ocean waves are the most important environmental loading mechanism on offshore structures, it is necessary to find an appropriate way to model the wave forces on an offshore structure.

There are many ways to describe ocean waves from a mathematical point of view. One general way is to separate them into deterministic approaches and probabilistic approaches. In deterministic theories, waves are assumed to be regular and their properties are invariant at all time. Waves can be divided into two types called long waves and short waves in deterministic wave theories. “Long waves” have extremely long wave periods and wavelength and are caused by tides, tsunamis, etc. Besides, long waves always have large amplitude and are slowly moving. Compared with long waves, short waves have shorter wavelength, smaller amplitudes and move faster than long waves. The offshore engineer is more concerned with wind generated “short waves”.

There are many wave theories (Faltinsen, 1990): linear waves theory, Stokes wave theory (Schwartz, 1974), Cnoidal wave theory (Wiegel, 1960; Mallery and Clark, 1972), Solitary wave theory (Sarpkaya and Isaacson, 1881), Stream function wave theory (Dean, 1965& 1970) and so on. Among them, the simplest and commonly used wave theory is the linear wave theory (DNV-RP-205, 2010). It can also be called small amplitude wave theory or Airy theory. In linear wave theory, the wave can be written as the form of a sine curve and free surface profile: $\eta = a \sin(kx - \omega t)$ where a is the wave amplitude, ω is wave frequency, k is the wave number.

Compared with deterministic models, probabilistic models for ocean waves are more appropriate, since the ocean waves are random and chaotic. It is always assumed that wave field is stationary for limited periods of time, such as three hours. The assumed stationary period is called short-term wave field. On the contrary, long-term wave field which covers long periods of time is considered as a sequence of many short-term conditions.

In a probabilistic model, ocean waves cannot be simply described by an ideal sinusoidal wave model, which are replaced by a large number of wave components generated according to certain distributions. It is common to use a linear sum of regular waves which come from the same direction to model random waves. Each wave component has its own specific amplitude a , frequency ω and phase ε and is supposed to behave based on linear wave theory. The wave elevation η can be written as:

$$\eta(x, t) = \sum_{i=1}^N a_i \cos(k_i x - \omega_i t + \varepsilon_i) \quad (2.1)$$

where N is the number of total wave components, subscript i is the indication of the i th wave component and k is the wave number which can be obtained by solving the dispersion equation $\omega^2 = gk \tanh(kd)$. d is the depth of water and g is gravity acceleration. ε_i are random phases which follow uniform distribution between 0 and 2π and are mutually independent of each other. In narrow-band fatigue damage estimation, the basic assumption is that the stress cycles (S) can be determined directly from the maxima stress (S_a). Each cycle's range is assumed to be twice the value of the maximum local stress. Moreover, the number of stress cycles per unit time can be obtained by the zero-crossing frequency of the stress response process. For narrow band gaussian process, the wave amplitude following a Rayleigh distribution (Rice, 1944; Cramer and Leadbetter, 1967; DNV, 2010).

$$f_p(a) = \frac{a}{\sigma^2} \exp\left(-\frac{a^2}{2\sigma^2}\right) \quad (2.2)$$

where $\sigma = \sqrt{\Delta\omega S_{\eta\eta}}$ for each component and $f_p(a)$ is probability density function of wave amplitude a for each wave component. On the other hand, the amplitude for each wave component can also be treated as deterministic as $\sqrt{2S_{\eta\eta}\Delta\omega}$ for each wave component, it is the mean value of random amplitude. Wave spectrum, denoted as $S_{\eta\eta}(\omega)$, is a measure of wave energy density and its distribution over a frequency range plotted the whole information discussed above. The whole process is described in Figure 2.1.

Figure 2. 1 Schematic of random waves

There are several wave spectrum models that are commonly used, such as JOSNWAP spectrum model, Pierson-Mosowitz (PM) spectrum model, Bretschneider spectrum model, ISSC spectrum model and less used Ochi-Hubble spectrum model, as listed in Table 2.1 (Chakrabarti and Subrata, 2005). In the wave spectrum in Figure 2.1, each block represents one sea state and has its own wave height and period. Besides spectrum, there are other ways that can describe random waves, like wave height sequences which are defined by statistical laws such as ARMA and Markov (Box and Jenkins, 1970).

Table 2. 1 Expressions for common spectrum models

Model	No. of parameter	Independent Parameters	Formula
Pierson-Moskowitz	1	U_ω , or ω_0	$S(\omega) = \alpha g^2 \omega^{-5} \exp(-1.25[\omega/\omega_0]^{-4})$
Modified P-M	2	H_s, ω_0	$S(\omega) = \frac{5}{16} H_s \frac{\omega_0^4}{\omega^5} \exp(-1.25[\omega/\omega_0]^{-4})$
Bretschneider	2	H_s, ω_s	$S(\omega) = 0.1687 H_s \frac{\omega_s^4}{\omega^5} \exp(-0.675[\omega/\omega_s]^{-4})$
ISSC	2	$H_s, \bar{\omega}$	$S(\omega) = 0.1107 H_s \frac{\bar{\omega}^4}{\omega^5} \exp(-0.4427[\omega/\bar{\omega}]^{-4})$
JONSWAP	5	$H_s, \omega_0, \gamma, \tau_a, \tau_b$	$S(\omega) = \bar{\alpha} g^2 \omega^{-5} \exp(-1.25[\omega/\omega_p]^{-4}) \times \gamma \exp\left[-\frac{(\omega-\omega_p)^2}{2\sigma^2 \omega_p^2}\right]$
Ochi-Hubble	6	$H_{s1}, \omega_{01}, \lambda_1, H_{s2}, \omega_{02}, \lambda_2$	$S(\omega) = \frac{1}{4} \sum_{j=1}^2 \left(\frac{4\lambda_j + 1}{4}\right)^{\lambda_j} / \Gamma(\lambda_j) \times \frac{H_{sj}^2}{\omega^{4\lambda_j+1}} \exp\left[-\left(\frac{4\lambda_j + 1}{4}\right)[\omega/\omega_{0j}]^{-4}\right]$

2.2.2 Environmental Parameters and Statistics

It is always assumed that there are no transition periods between two sea states of two short-term periods and in most cases this assumption seems to work well (Chakrabarti and Subrata, 2005).

It is always assumed that the wave field follows Gaussian distribution, which means the wave elevation at certain place at the sea surface can be described by a Gaussian random variable. In most cases, this assumption works well. But still in some special conditions, it is not always the case. It is not difficult to find ocean wave having higher crests and shallower corresponding troughs, which has already been considered in the practical design work. Different from the classical wave amplitude distribution in Equation (2.2), Jahns and Wheeler (1972) presented an empirical correction to the Rayleigh distribution of wave crests as:

$$F_{X_p}(a) = 1 - \exp\left\{-8 \frac{a^2}{h_s^2} \left[1 - \beta_1 \frac{a}{d} \left(\beta_2 - \frac{a}{d}\right)\right]\right\} \quad (a \geq 0) \quad (2.3)$$

where h_s is the significant wave height, d is the water depth, $\beta_1=4.37$ and $\beta_2=0.57$ are empirical coefficients (Haring and Heideman, 1978).

A two-parameter Weibull distribution is cited by Forristall (2000) as a short-term model for the wave crests:

$$F_{X_p}(a) = 1 - \exp\left\{-\left(\frac{a}{\alpha_F h_s}\right)^{\beta_F}\right\} \quad (a \geq 0) \quad (2.4)$$

where $\alpha_F = 0.3536 + 0.2892s_1 + 0.1060 Ur$ and $\beta_F = 2 - 1.1597s_1 + 0.0968Ur^2$, $s_1 = \frac{2\pi h_s}{gt_1^2}$, and t_1 is the mean value of wave period; $Ur = \frac{h_s}{k_1^2 d^3}$, k_1 is the wave number corresponding to t_1 (Chakrabarti and Subrata, 2005).

In the design and reliability analysis of offshore structures, it is very important to model the environmental parameters appropriately. Due to the lack of theoretical support for the choice of any model, many different environmental parameter models are proposed by different researchers.

Significant wave height H_s is the mean value of the highest one-third of wave heights in a time-series which represents a certain sea state. A lognormal distribution model was selected to describe ocean wave height in the very early years (Jaspers, 1956). Later, some researchers presented other models to describe the wave height distribution, such as Weibull distribution proposed by Battjes (1972), the mixed Lognormal-Weibull distribution presented by Haver and Nyhus (1986), Gamma distribution generalized by Ochi (1992) and Beta distribution suggested by Ferreira and Guedes Soares (1999). Equation (2.5) gives the mixed Lognormal-Weibull distribution for significant wave height:

$$f(H_s) = \begin{cases} \frac{1}{\sqrt{2\pi}\varepsilon H_s} \exp\left[-\frac{(\ln H_s - \mu_h)^2}{2\varepsilon^2}\right], & \text{if } H_s \leq \hat{\eta} \\ \frac{\beta}{\rho} \left(\frac{H_s}{\rho}\right)^{\beta-1} \exp\left[-\left(\frac{H_s}{\rho}\right)^\beta\right], & \text{if } H_s > \hat{\eta} \end{cases} \quad (2.5)$$

where $f(H_s)$ is the marginal probability density function (PDF) of H_s . In North Sea, the parameters' values are given: $\varepsilon=0.6565$, $\mu_h=0.77$, $\rho=2.691$ and $\beta=1.503$ (Haver, 2002).

Spectral peak period T_p is the wave period with the highest energy, Sagrilo and Lima (2011) described its marginal PDF by 3-parameter Weibull distribution. In most cases, spectral peak period T_p is modelled together with significant wave height H_s in a joint distribution.

Since wind speed varies with time and location, wind climate is always represented by the 10-minute mean wind speed U_{10} and the standard deviation σ_U at the height of 10 meters above the

sea surface. A 2-parameter Weibull distribution is assumed for the 10-minute mean wind speed at a certain given height z above the sea surface or the ground (Ramirez and Carta, 2005; Morgan et al., 2011; Manwell et al., 2002):

$$F_{U_{10}}(u) = 1 - \exp\left(-\left(\frac{u}{A}\right)^k\right) \quad (2.6)$$

where A is the scale parameter and k is shape parameter, both of which are site- and height-dependent. Besides, studies show that sometimes 1-parameter Weibull distribution can give a better fit for the 10-minute mean wind speed at a certain location (Celik, 2004).

Similar to random waves, short-term stationary wind conditions can be described by wind spectrums. Equation (2.7) gives the variation of wind speed with the elevation:

$$U_w(1h, z) = U_w(1h, z_R) \cdot \left(\frac{z}{z_R}\right)^{0.125} \quad (2.7)$$

where z is the elevation of the wind above the sea surface, and $z_R=10$ meters, which is set as the reference elevation. $U_w(1h, z_R)$ is the 1 hour mean wind speed at reference elevation.

A general expression for the wind spectrum about 1 hour mean wind speed is:

$$S(f) = \frac{(\sigma_\omega(z))^2}{f_p \left[1 + 1.5 \frac{f}{f_p}\right]^{\frac{5}{3}}} \quad (2.8)$$

where $S(f)$ is wind spectral energy density, f is wind frequency, f_p is wind peak frequency, and $\sigma_\omega(z)$ is the standard deviation of wind speed. Unlike wave spectrum, wind spectrum is mostly wide-banded, as shown in Figure 2.2.

Figure 2. 2 Power spectrum density of wind speed (Chakrabarti and Subrata, 2005)

Current is a common environmental phenomenon in the open sea. In the early years of offshore engineering, it was believed that current was only present in the water near the surface but disappeared below 1000-meter water depth. However, recent studies show that some classes of currents exist in deep water environment and sometimes even in ultra-deep-water region. Current is turbulent. But in the design of offshore engineering, current is usually considered time invariant. In the structural design of offshore engineering, current speed at water surface with 10-year return period is used. Gumbel distribution is usually employed to describe the marginal distribution of current velocity (Pugh, 1982; Robinson and Tawn, 1997; Sauvaget et al., 2000).

$$F_c(x) = \exp \left\{ -\exp \left(\frac{x-e}{d} \right) \right\} \quad (2.9)$$

where e and d are time and site dependent coefficients.

Besides, Sagrilo, Lima and Papaleo (2011) proposed a good fit of marginal distribution of current velocity with 2-parameter Weibull distribution. Moreover, some researchers also consider the current direction in the model (Mazumder and Mazumber, 2006).

Since the individual environmental parameter models cannot take into account the relationships among different parameters, while they are not independent of each other in the actual environment, joint distribution model is one way to solve this problem. There are many approaches for establishing a joint environmental model such as Maximum Likelihood Model (MLM) (Prince-Wright, 1995) and the Conditional Modelling Approach (CMA) (Bitner-Gregersen and Haver, 1991).

A CMA joint distribution of significant wave height and period is recommended in design of offshore structures (in DNV-RP-C205). In this model, the marginal distribution of significant wave height is described by a 3-parameter Weibull PDF (Bitner-Gregersen, 2005):

$$f_{H_s}(h) = \frac{\beta_{H_s}}{\alpha_{H_s}} \left(\frac{h-\gamma_{H_s}}{\alpha_{H_s}} \right)^{\beta_{H_s}-1} \exp \left\{ -\left(\frac{h-\gamma_{H_s}}{\alpha_{H_s}} \right)^{\beta_{H_s}} \right\} \quad (2.10)$$

and the conditional PDF of zero-crossing wave period T_z reads

$$f_{T_z|H_s}(t|h) = \frac{1}{\sigma t \sqrt{2\pi}} \exp \left\{ -\frac{(\ln t - \mu)^2}{2\sigma^2} \right\} \quad (2.11)$$

$$\mu = E[\ln T_z] = a_0 + a_1 h^{a_2}, \sigma = std[\ln T_z] = b_0 + b_1 e^{b_2 h} \quad (2.12)$$

where $\alpha_{H_s}, \beta_{H_s}, \gamma_{H_s}$ are scale parameter, shape parameter and location parameter respectively. And a_0, a_1, a_2 and b_0, b_1, b_2 are coefficients estimated from actual database.

A 2-parameter Weibull model was employed by Bitner-Gregersen and Haver (1989) to describe the mean wind speed U conditional on a given significant wave height H_s :

$$f_{U|H_s}(u|h) = k \frac{u^{k-1}}{U_c^k} \exp \left[- \left(\frac{u}{U_c} \right)^k \right] \quad (2.13)$$

where U_c and k are scale parameter and shape parameter respectively, which can be estimated from actual database. Suggested models of these two parameters are:

$$k = c_1 + c_2 h_s^{c_3}, U_c = c_4 + c_5 h_s^{c_6} \quad (2.14)$$

where the coefficients c_1, c_2, c_3 and c_4, c_5, c_6 are estimated from actual database.

For the wave direction, it is customary to assume that waves and winds are inline. Some researchers also study direction difference θ_r between waves and wind, expressed as:

$$\theta_r = \theta_{waves} - \theta_{wind} \quad (2.15)$$

where θ_{waves} and θ_{wind} are directions of wave and wind, and θ_r is assumed to follow beta distribution by Bitner-Gregersen (1996).

Besides conditional models discussed above, there are still some other reliable statistical models for wave environmental parameters, such as Nataf model and Copula model. Details of these models will be discussed in Chapters 4 and 5.

2.3 Review of Fatigue Analysis of Offshore Structures

Fatigue is caused by accumulated damage related to a large number of stress reversals and finally results in structures suffering from cracking. In offshore engineering, the purpose of fatigue design is to guarantee the offshore structures possess adequate fatigue life. There are already many well-established standards for the fatigue design of offshore structures such as DNV-RP-C203 (2011), ABS (2003), API-RP 2A-WSD (2014). In this section, general fatigue related design will be introduced, and then fatigue design of flexible risers will be specially addressed.

2.3.1 Traditional Fatigue Design and Fatigue Analysis

2.3.1.1 Damage Accumulation Rule and Fatigue Safety Checks

For most fatigue problems, the classical S-N approach is employed to determine the fatigue life of steel components of offshore structures:

$$N = K \cdot S^{-m} \quad (2.16)$$

where S is stress range, N is allowable number of cycles for the stress range, K and m are material constants depending on environmental conditions or test condition etc. Eq. (2.16) can also be expressed in another version as: $\log(N) = \log(\bar{a}) - m\log(S)$.

Compared with single-slope S-N curve, two-slope S-N curve is preferable, which is defined as:

$$N = \begin{cases} A \cdot S^{-m}, & N \leq N_Q \\ C \cdot S^{-r}, & N > N_Q \end{cases} \quad (2.17)$$

All the coefficients in the above equation can be explained in the following Figure 2.3 (ABS, 2003).

Figure 2. 3 Two-slope S-N curve (ABS)

Accumulation fatigue damage, a linear summation of the individual damage from all the stress range intervals, is carried out by Damage Accumulation Rule. The Palmgren-Miner Rule is a representative approach in calculating the cumulative fatigue damage and is mathematically expressed as Equation 2.18 and Figure 2.4:

$$D = \sum_{i=1}^J \frac{n_i}{N_i} \leq D_{allow} \quad (2.18)$$

where n_i is the number of stress cycles in stress block i , N_i is the number of stress cycles to failure at the i th constant amplitude stress range block and D_{allow} is the allowable limit defined in the design codes.

Figure 2. 4 Fatigue damage due to different stress range blocks

Fatigue Safety Check which is based on fatigue damage or fatigue life is another important definition in the fatigue assessment. In ABS (2003) codes, the structural component is considered safe when based on damage, if

$$D \cdot FDF \leq 1.0 \quad (2.19)$$

If based on life, the safe inequality is:

$$T_f \geq T \cdot FDF \quad (2.20)$$

In the above two inequalities, FDF is the abbreviation of Fatigue Design Factor (≥ 1.0), T denotes the design life (in years), D and T_f are calculated fatigue damage and calculated fatigue life.

In the DNV-PR-F204 (2010) codes, the Fatigue Safety Check is written as:

$$D_{fat} \cdot DFF \leq 1 \quad (2.21)$$

where D_{fat} is the calculated fatigue damage, DFF is the Design Fatigue Factor and has the same function as FDF in ABS codes.

2.3.1.2 Fatigue Causes of Risers

Same as other structural components, the aim of fatigue design for risers is to ensure that risers have an adequate fatigue life and retain its intended function within its fatigue life.

Generally speaking, fatigue damage in risers should consider all relevant cyclic load effects:

1. First-order wave loading and associated floater motions;
2. Second-order wave loading and associated floater motions;
3. Riser Vortex induced vibrations (VIV) due to current along the water column;
4. Vortex induced hull motions (VIM) due to loop current;
5. Thermal and pressure induced stress cycles;
6. Collisions;
7. Internal fluid slugging effects;
8. Other concept specific loading conditions, like springing motion of TLPs;
9. Fabrication and installation loads.

Among the above possible resources that may directly or indirectly cause fatigue damage of risers, the most important contributions may come from wave induced, low-frequency induced and vortex-induced stress cycles (DNV-RP-F204, 2010).

2.3.1.3 Fatigue Analysis

A general way for the calculation of fatigue damage mostly caused by wave induced and low frequency induced loadings is using block method, and it is a popular industry practice for long-term fatigue analysis. There are mainly two steps for fatigue analysis using block method: firstly, subdivide the wave environment scatter or diagram into representative blocks; then select one sea state to represent all sea states within each block. The fatigue damage is calculated by combining each selected short-term sea-state multiplied by sea state probability:

$$D_{fat} = \sum_{i=1}^{N_s} D_i P_i \quad (2.22)$$

where D_{fat} denotes the long-term fatigue damage, N_s is the number of discrete sea states in the wave environment scatter or diagram, P_i is the probability of occurrence of the i th sea state and D_i fatigue damage if the i th sea state (DNV-RP-F204, 2010). The representative short-term sea state should be carefully selected to ensure the calculated fatigue damage of which is equal or larger than all sea states generated within each block.

Fatigue damage accumulation should consider the global load effect of many states from low to moderate, which have a high probability of occurrence compared with rare extreme sea states. Moreover, the degree of nonlinearity involved is very small compared with extreme response (DNV-RP-F204, 2010).

Riser may experience VIV effect during its working time and the VIV induced fatigue damage is important for SCRs and top-tensioned risers in deep water. SCRs are very sensitive to VIV effect, especially at high current locations. Fatigue damage of high rates is generated by high frequency cyclic stress caused by high frequency vibrations of the risers due to vortex shedding. Compared with shallow water areas, risers in deep water are more sensitive to VIV influence due to several possible reasons such as current being always larger in deep water than in shallow water and longer risers being more sensitive to current effect (Bai, 2005).

2.3.2 Frequency Domain and Time Domain Analysis

The dynamic analysis of an offshore structure can be either performed in time domain analysis or frequency domain analysis. Compared with time domain analysis, frequency domain analysis is much more efficient requiring very little computational expense. However, frequency domain analysis cannot account for nonlinear effects, such as geometric nonlinearities, drag forces, second order slow drift forces on vessel and non-linearity in riser response. On the contrary, time domain analysis is more suitable in the non-linearity field at the expense of a larger amount of computational cost.

In some researchers' study, some approximate methods have been developed to balance the efficiency and accuracy. Ormberg and Larsen (1998) as well as Senra (2002) proposed a method using time domain analysis with quasi-static analysis for lines, by adding dynamic effect such as equivalent linear damping and inertia coefficients on the vessel. Low and Langley (2006) proposed a linearization scheme for the drag forces on the mooring and riser systems which makes frequency domain analysis yield good results for both first and second order response of the system.

2.3.3 Fatigue Analysis of Flexible Risers

In Chapter 1, a brief description of various types of riser systems has been given, including flexible risers. In this section, a focused literature review about fatigue analysis of flexible risers as well as detailed design requirements of flexible risers will be presented.

2.3.3.1 Introduction

Flexible risers used in offshore engineering industry can be traced back to 1970s. In the very early stage, flexible risers were only employed in mild ocean environment, such as Brazil and the Mediterranean. Later, with the development of technology, the flexible risers are used in more challenging and adverse weather conditions, like deeper water depth, higher pressure and higher temperature environment.

Flexible risers always possess low bending stiffness and high axial tensile stiffness and have composite riser wall construction. There are mainly two types of flexible risers: bonded flexible risers and unbonded flexible risers. Bonded risers have different layers bonded together through adhesives or by applying heat or pressure to fuse all layers into one construction. Bonded flexible risers are generally used in short sections such as jumpers (API, 2002).

In unbonded flexible risers, different layers are able to slip relative to each other. Unbonded flexible risers can be used for several hundred meters long and are always applied in many situations. As shown in Figure 2.5 and Figure 2.6, the cross section of an unbonded flexible riser is composed of metallic concentric layers and polymeric layers with specific functions for each layer. A typical unbonded flexible riser generally has five main components: carcass, pressure sheath, pressure armor, tensile armor and outer sheath.

Figure 2. 5 Unbonded flexible pipe (De Sousa et al., 2012)

Figure 2. 6 Cross-section of unbonded flexible pipe (Zhang et al., 2003)

The innermost layer is an interlocked metallic layer, called carcass whose main function is to provide collapse resistance to the pipe against hydrostatic pressure and crushing load. Carcass can be made up of different grades of steel, depending on the internal fluid characteristics.

Since the carcass is not leak-tight, the pressure sheath (also known as internal polymer sheath) serves as a membrane to maintain fluid integrity around it. Consideration should be paid to the design of pressure sheath due to its exposure concentrations and fluid temperature.

Pressure armor is an interlocked metallic layer to support pressure in the pipe wall caused by internal fluid pressure in the radial direction. Some types of pressure armor profiles are shown in Figure 2.7.

Figure 2. 7 Pressure armor profiles (API, 2002)

Tensile armor layers are made up of a large number of flat, round or shaped metallic wires with a high pitch around 20 degrees to 60 degrees. The tensile armor is cross-wound at an angle near 55 degrees in pairs to main a stable against the tensile load on the flexible pipe. Similar to the pressure armor, tensile armor also uses the high strength carbon steel as material to meet the requirement of “sweet” and “sour” service.

Outer sheath is an external polymer sheath, which is made up with similar material as the inner polymer sheath, works as a barrier to protect the steel layers from seawater (API, 2002).

It is always supposed that air is the only substance in the annulus of flexible risers. However, there are also water and gas in the annulus environment which may lead to corrosion of armor layers (Sheehan, 2006).

2.3.3.2 Fatigue Analysis of Flexible Riser

Special attention should be paid to flexible risers in the fatigue analysis, due to their special structural construction. Compared with rigid steel pipes, the inner structure of the flexible risers makes it more compliant with the floating structures and enhances the capability of adapting to harsh environment.

Fatigue damage is an accumulated definition based on dynamic analysis of forces and bending moments on the target object. For simple rigid pipes, such as SCRs, the stresses that induce fatigue damage are calculated by simple formulas which can be directly conducted in the global analysis. Different from SCRs, besides environmental loading and other sources of fatigue damage, the induced stresses from the flexible risers' inner layers also act on the pipe, which should also be considered in the fatigue analysis of flexible risers (De Sousa et al., 2012). So far, some researchers have proposed several methodologies for the fatigue analysis of flexible risers.

De Sousa (2012) proposed a methodology adopting a post-processor to evaluate the fatigue life of flexible risers. Based on the conventional fatigue analysis methods for rigid pipes, whose global analysis directly provide time series of stresses for each sea state, one more step is added to the proposed method. The proposed method stores the time series of tension and moments for each sea state and converts them into time series of stresses, with the help of post-processor FADFLEX (De Sousa et al., 2007; De Lemos et al., 2008). Then, a counting method like traditional Rainflow method is employed to analyse the stress cycles with the help of S-N curve and finally the fatigue damage for one sea state is obtained. Later, the classical damage accumulation rule Palmgren-Miner Rule based on linear damage hypothesis is then employed to sum all the fatigue damage of each sea state together to obtain the total fatigue damage in the long-term period. De Sousa's study shows that the friction between neighbouring layers and the different mean stress level would influence the fatigue life of flexible risers.

Another fatigue analysis method based on transfer functions includes three main steps (De la Cour, 2008). The first step is to perform global fatigue analysis to provide input to the local analysis. The second step is to transfer the data like curvature and tension variation for each element obtained from first step into local model with the purpose of accounting for the mean stress components effect. The last step is to employ an analytical tool BENDFLEX, which combines the advantages of BEND and BFLEX (Andersen et al., 2011) to work as transfer function to obtain the tensile armor stresses as a function of internal pressure, external pressure, tension and curvature. The analysis of the entire riser length under a large number of loads, as well as parameter sensitivities and the effect under an irregular model can be implemented by this program. The fatigue driving mechanisms before the implementation of fatigue analysis procedure was also studied by De la Cour (2008). In their study, detailed description of the fatigue drivers was given, which included high inter-layer contact pressure induced by severe tension and ambient hydrostatic pressure loads, corrosion fatigue concerning high strength steel

armour, structural properties of risers, riser VIV impact and second order springing effect (De la Cour, 2008).

Similar to SCRs, VIV plays an important role in the fatigue damage of flexible risers in deep water. But different from SCRs, the fatigue damage in tensile armor wires is also very important (Zhang, et al., 2011). The stress variation in wires is caused by four sources: riser tension variation, interlayer friction, interlayer friction, variation in wire normal curvature and transverse curvature when the pipe is in bending state. In Zhang's study, since the wave and surface vessel motion induced VIV is very small, the fatigue damage caused by VIV could be considered by steady current which produced curvature variation directly. There are four steps in the proposed process of VIV fatigue analysis in Zhang's study: firstly, prepare some basic information, such as properties of flexible risers, current profile, and S-N curve; secondly, dynamic global simulations of the flexible riser under current loads are implemented using Shear7 or OrcaFlex; thirdly, the curvature distribution along the flexible riser is obtained by the post-processing; finally, fatigue damage is calculated based on S-N curve (Zhang, et al., 2011).

2.3.3.3 Improve the Fatigue Capacity of Flexible Riser

In order to meet the requirement of future offshore development at deep water depth, improving the fatigue capacity of flexible riser is on demand. There are many promising methods, such as using high strength material grades for steel armor layers, replacing the conventional material for steel armor layers with new materials, updating safety factor by establishing reliability-based safety factor (Leira, et al., 2007;), optimizing the cross-section of pipe and riser system configuration, managing the fatigue integrity during its service time (Brack, et al., 2007; Chezian, 2007) and so on.

Hybrid flexible pipe is another concept which is proposed for the improvement of traditional flexible riser. An example is using fiber-reinforced polymer (FRP) composite material as an alternative to replace traditional steel materials in flexible pipes. Compared with traditional steel materials, FRP provides higher strength, lower weight, and better corrosion resistance and fatigue characteristics. However, due to its significant difference from metallic materials, it has not put into use yet. Another example is a hybrid unbonded flexible pipe concept, Flextrex® (Nielsen, 2001), as shown in figure 2.8.

Figure 2. 8 Flextreme® hybrid unbonded flexible pipe structure (Nielsen, 2001)

The new concept hybrid flexible pipe Flextreme targets at water depth more than 2000 meters, which has excellent resistance to high crushing and collapse and good tensile capacities. Their structural composition is different from that of traditional flexible pipes which have a new metallic inner armour layer and outer FRP armour layer, making the new concept hybrid flexible pipe possesses better buckling stability and good corrosion fatigue behaviour.

2.4 Review of Methods for Probability Integrals

Since there are many uncertainties in the fatigue analysis of offshore related components such as mooring and riser systems, it is necessary to employ an efficient reliability approach to solve such problems. The uncertainties may come from many sources including environmental conditions such as winds, currents and ocean waves, as well as seafloor interaction and properties of the vessel system itself.

2.4.1 Reliability Assessment

“Uncertainty” is an important definition in reliability assessment, and it is not an easy job to identify uncertainties in a complex system. One of the popular classifications of uncertainties is to divide them into two types: aleatory uncertainty and epistemic uncertainty, referring to underlying inherent uncertainties and uncertainties that might be reduced with better information, respectively.

Reliability is an inherent feature of system that cannot be achieved accidentally and can be estimated by testing until the system fails. More times to be test, better the test results will be. Therefore, there is a balance between the confidence level and the cost of testing.

Generally, reliability can be defined as a probability that a system or a component of interest that will perform its function during a specific period under given conditions. In the structural sense, reliability can be explained as the probability of safety over a given period, or as the complement of failure probability:

$$P_s = 1 - P_f = P[R > S] \quad (2.23)$$

where P_s and P_f are safety probability and failure probability respectively, R is the structural resistance and S is the load effect.

Establishing an appropriate reliability model is well known to be a challenging task. Formulating a reliability model has its own rules: first, checking the reliability data whether they are collected correctly; second, summarizing and checking up the data; third, based on the well-established understanding of failure mechanism and factors that will affect the failure, establishing a reliability model and making sure that the random variables about those factors which can control the reliability model; fourth, guaranteeing the efficiency and fairness of the estimated parameters; fifth, testing the reliability model by observations or experiments. Generally, many variables may affect the reliability of the system and it is not realistic to consider all of them. Therefore, it is important to select appropriate random variables that can meet the requirement of target reliability problem and to define them as basic random variables in the reliability analysis. Then, these basic random variables are employed to describe and characterize the behaviour and safety of a structure. Moreover, the selected basic random variables are not always independent of each other and the dependent cases would add complexities to the reliability problem. Therefore, it is required to provide appropriate probability distributions, based on the available knowledge and the basic random variables.

Let $X = [X_1, X_2, \dots, X_n]$ represent all basic variables involved in the proposed reliability model, and then any exact value set of $x = [x_1, x_2, \dots, x_n]$ represent a point in the n dimensional variable space. Since the basic random variables are not always independent of each other, the joint probability density (JPDF) function $f(x_1, x_2, \dots, x_n)$ is employed to define the probability reliability P_R :

$$P_R = \int_{x_1, \dots, x_n \in S} \dots \int f(x_1, x_2, \dots, x_n) dx_1 dx_2 \dots dx_n \quad (2.24)$$

For independent random variables, the JPDF can be expressed as:

$$f(x_1, x_2, \dots, x_n) = f(x_1)f(x_2) \dots f(x_n) \quad (2.25)$$

Insert (2.25) into (2.24):

$$P_R = \int_{x_1 x_2, \dots, x_n \in S} \dots \int f(x_1) f(x_2) \dots f(x_n) dx_1 dx_2 \dots dx_n \quad (2.26)$$

In Equation (2.26), $f(x_1)f(x_2) \dots f(x_n)$ are the marginal probability densities of the basic random variables. By integrating the probability density function over safety region S , the probability of reliability is obtained and the probability of failure is calculated by integrating the probability density function over failure region F .

2.4.2 Classification of Methods for Probability Integrals

In the last section, it is recognized that reliability approaches are considered as the most credible and consistent way to handle the fatigue problem of mooring and riser systems. In order to solve the long-term cumulative fatigue damage, multiple integral is required and the dimension of the integration depends on the number of basic random variables selected. Generally, the probability integral methods can be classified into four categories:

1. Direct analytical integration method, which is only suitable for some special cases.
2. Numerical integration method, a brute-force method which is often computationally prohibitive.
3. Sampling method, such as Monte Carlo method.
4. Other methods such as Perturbation Approach and Asymptotic Approximation which will be introduced in Section 2.4.5.

2.4.3 Direct Analytical Integration and Numerical Integration

Direct analytical integration can only be applied to very few special cases and is not able to solve general integral problems.

Direct numerical integration is very time-consuming, which has been found to give satisfactory results if each integral step is small enough with the sacrifice of the computational expense. The good results are also because the underestimation of the integrand near the mean value is compensated by the overestimation elsewhere (Dahlquist and Björck, 1974). This method is not always available due to the round-off errors and vast computational time. Furthermore, the computational time grows exponentially with dimension of the integration space.

In a special case such as:

$$G(x) = a_1x_1 + a_2x_2 + \dots + a_nx_n \quad (2.27)$$

where $G(x)$ is the limit state function, which returns a negative value under system failure conditions and a positive value when the system is stable; a_i is the i th constants; x_i is the i th random variable ($i = 1, 2, \dots, n$). The multiple dimensional integral in Equation (2.24) is quite complex and computationally time consuming. Stevenson and Moses (1970) states that it is possible to reduce the multi-dimensional integral into a series of one dimensional integral, which proves to be still complex due to extensive numerical work. In order to simplify the integral, divergence theorems of Stokes and Gauss are invoked to transfer two or three dimensional integral to one or two dimensional integral respectively (Shinozuka, 1983).

Since the direct numerical integration costs too much computational time especially when the integral dimension is large, it is not very popular for solving large integral problems. Instead, Monte Carlo and other similar methods are developed, which are more superior in terms of robustness and accuracy.

2.4.4 Monte Carlo Technique

2.4.4.1 Introduction

Monte Carlo method is based on repeated random sampling to obtain numerical results. Monte Carlo methods are generally used in three different types of problems: optimization, numerical integration and generation of samples according to a probability distribution.

In structural reliability analysis, Monte Carlo simulation is to sample each random variable X_i randomly to give a sample value x_i . For instance, if the limit state is violated, $G(x) \leq 0$, then the structure is defined as failure. The failure probability is approximated by Monte Carlo simulation as:

$$P_f \approx \frac{n(G(x_i) \leq 0)}{N} \quad (2.28)$$

where N is the total number of times the experiment is repeated and $n(G(x_i) \leq 0)$ is the trial number satisfied $G(x_i) \leq 0$.

Monte Carlo method is based on the known probabilistic properties, which can be solved by many times of sampling and data analysis.

Compared with direct numerical integration, Monte Carlo methods are superior when the number of simulations is less than the number of integration points required by numerical

methods (Melchers, 1999). Moreover, the efficiency of Monte Carlo simulation depends on the coefficient of variation (CoV) instead of integral dimension. Therefore, the superiority of Monte Carlo methods is more obvious when the integral dimension is large and can give relative precise result when the number of simulation experiments is large enough.

2.4.4.2 'Crude' Monte Carlo

'Crude' Monte Carlo simulation is the simplest Monte Carlo approach which generates samples directly. Equation (2.28) uses 'crude' Monte Carlo method and provides a direct estimated failure probability P_f . The 'crude' Monte Carlo can be expressed as:

$$P_f \approx J_1 = \frac{1}{N} \sum_{j=1}^N I[G(x_j) \leq 0] \quad (2.29)$$

where I is indicator function and $I[\]=1$, if $[\]$ is true, else $I[\]=0$. P_f is an unbiased estimator of J . The direct sampling technique may be shown in Figure 2.9.

Figure 2. 9 Fitted cumulative distribution function based on direct sampling technique

It can be seen from Figure 2.9 that only a small range of $F_G(g)$ is of interest and the structure is safe at most time. The region at $G(\) \leq 0$ (the left-hand tail in Figure 2.9) is a more interesting region which represents the failure region. Therefore, the direct sampling technique is not efficient, and searching for a method to improve the sampling efficiency attracts many researchers' attention (Melchers, 1999).

2.4.4.3 Variance Reduction

Based on the central limit theorem, the right part of Equation (2.29) can be considered as the sum of independent sampling function following normal distribution. When N approaches infinity, the expectation of J_1 is:

$$E(J_1) = \sum_{i=1}^N \frac{1}{N} E[I(G \leq 0)] = E[I(G \leq 0)] \quad (2.30)$$

It should be indicated that the estimator in Eq. (2.30) is unbiased.

The variance of J_1 is:

$$\sigma_{J_1}^2 = \sum_{i=1}^N \frac{1}{N^2} \text{var}[I(G \leq 0)] = \frac{\sigma_{I(G \leq 0)}^2}{N} \quad (2.31)$$

From the above equations, both the standard deviation decreases in proportion to $N^{-1/2}$. The slow convergence makes the ‘cruel’ Monte Carlo not so efficient. ‘Variance reduction’ techniques are proposed to solve the slow convergence problem. The essence of ‘variance reduction’ techniques is to reduce σ_I^2 . Additional information is required to implement the ‘variance reduction’ techniques. One general way of reduction strategy is to limit the simulation to more “interesting” regions. Various ‘variance reduction’ strategies can be referred to references (Rubinstein, 1981; Warner and Kabaila, 1986).

2.4.4.4 Importance Sampling

Importance sampling is a classical ‘variance reduction’ technique which is commonly used. Equation (2.24) can be rewritten with the indicator function $I(\cdot)$:

$$J = \int \cdots \int I[G(x) \leq 0] \frac{f_x(x)}{h_v(x)} h_v(x) dx \quad (2.32)$$

where $h_v(\cdot)$ is the ‘importance sampling’ PDF.

According to the definition of expected value (First Moment),

$$E(X) \equiv \mu_x = \int_{-\infty}^{+\infty} x f_x(x) dx \approx \sum_i x_i p_x(x_i) \quad (2.33)$$

Equation (2.32) can be rewritten in the expected value form:

$$J = E \left\{ I[G(v) \leq 0] \frac{f_x(v)}{h_v(v)} \right\} = E \left(\frac{If}{h} \right) \quad (2.34)$$

Compared with Equation (2.30), $I(\cdot)$ is replaced by $I(\cdot) \frac{f}{h}$. And v is the random vector with PDF $h_v(v)$. Then the unbiased estimated value of J can be rewritten as:

$$P_f \approx J_2 = \frac{1}{N} \sum_{j=1}^N \left\{ I[G(\hat{v}_j) \leq 0] \frac{f_x(\hat{v}_j)}{h_v(\hat{v}_j)} \right\} \quad (2.35)$$

From the above equation, it is easy to find that the distribution of $h_v(\cdot)$ governs the samples' distribution. The most challenging task in importance sampling method is to choose an efficient importance sampling density $h_v(\cdot)$. The essence of importance sampling is to reduce the variance in J_2 in Equation (2.35). A perfect choice of $h_v(\cdot)$ would produce zero variance, which means $h_v(\cdot)$ equals to the density of $I[G(\hat{v}_j) \leq 0]f_x(\cdot)$. On the contrary, the variance can also be increased if the $h_v(\cdot)$ is poorly selected (Kahn, 1956).

2.4.4.5 Quasi-Monte Carlo

The only difference between Quasi-Monte Carlo and classical Monte Carlo exists in the way how sample points are chosen. Different from classical Monte Carlo method which is based on pseudorandom number sequence, Quasi-Monte Carlo method generates samples based on low-discrepancy sequence, such as Halton sequence, Sobol sequence and Faure sequence.

The pros and cons of Quasi-Monte Carlo method have been studied by many researchers in the past (Morokoff and Caflisch, 1995; Caflisch, 1994; Sarkar, 1987). Compared with standard Monte Carlo method, the superiority of Quasi-Monte Carlo is not obvious especially in high dimensions or for not smooth integrals (Morokoff and Caflisch, 1995). Quasi-Monte Carlo shows a slight advantage in some special conditions.

Since the advantage of Quasi-Monte Carlo method is not so apparent, standard Monte Carlo method is still preferred by most researchers and engineers.

2.4.5 Probability Integral Methods

This section introduces various existing methods for probability integrals. The suitability of each method will to some extent depend on the characteristics of the integrand. For instance, First-Order Reliability Methods is only suitable for linearized problems. For nonlinear problems or even more complex problems, other methods are required. The followings are the detailed discussion of different probability integral methods and their suitability in solving different problems.

2.4.5.1 First-Order Reliability Method (FORM)

First-Order Reliability Method (FORM) is a common reliability method which is easy to implement. As its name suggests, the FORM is based on a first-order Taylor series approximation (Achintya, 2000).

In FORM, its performance function is linearized at the design point θ^* . The design point is located on the limit state surface at the minimum distance from the origin. R and S are resistance and load effect respectively. The most important function for FORM is to solve the reliability problem of structure's failure:

$$P_f = P(R < S) = \int \cdots \int_{Z < 0} f_{\theta}(\theta_1, \theta_2, \dots, \theta_n) d\theta_1 d\theta_2 \cdots d\theta_n \quad (2.36)$$

where $\theta_1, \theta_2, \dots, \theta_n$ are random variables distributed according to $f_{\theta}(\theta_1, \theta_2, \dots, \theta_n)$.

Then failure event occurs when $Z < 0$. Suppose $z = G(\theta)$ is normally distributed, then the reliability index and failure probability are:

$$\beta = \frac{\mu_z}{\sigma_z} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \quad (2.37)$$

$$P_f = \Phi(-\beta) \quad (2.38)$$

The Taylor expansion of the performance about the design point is:

$$Z = g(\theta^*) + \sum_{i=1}^n \frac{\partial g}{\partial \theta_i} (\theta_i - \theta_i^*) + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \frac{\partial^2 g}{\partial \theta_i \partial \theta_j} (\theta_i - \theta_i^*) (\theta_j - \theta_j^*) + \cdots \quad (2.39)$$

where derivatives are evaluated at the design point $(\theta_1^*, \theta_2^*, \dots, \theta_n^*)$, and θ_i^* is the value of θ_i at the design point. Keeping the linear terms, the mean and variance of z are:

$$\mu_z \approx g(\theta_1^*, \theta_2^*, \dots, \theta_n^*) \quad (2.40)$$

$$\sigma_z^2 \approx \sum_{i=1}^n \sum_{j=1}^n \frac{\partial g}{\partial \theta_i} \frac{\partial g}{\partial \theta_j} CoV(\theta_i, \theta_j) \quad (2.41)$$

where $CoV(\theta_i, \theta_j)$ is the covariance of θ_i and θ_j .

In the implementation of FORM evaluation, the most critical step is to find the design point. Usually, the solutions of limit state equations during the iteration are required for searching design point (Rackwitz, 1976). Alternatively, a Newton-type recursive formula is proposed to find the design point instead of using limit state equations (Rackwitz and Fiessler, 1978).

In the above discussion, the FORM can be easily carried out assuming the variables are normally distributed. However, most practical distributions are not normal distributions. In order to solve such problems, transformations can be employed to transform other random variables into normal space, such as Rosenblatt transformation (Madsen et al., 1986).

Since FORM is based on the first order approximation, it is mostly suitable for linearized problems. For nonlinear problems, significant error may occur due to neglecting the higher order terms.

2.4.5.2 Second-Order Reliability Method (SORM)

As mentioned in the above, FORM can no longer give good solutions when the limit state function is nonlinear. Hence, a higher order approximation for the failure probability computation is required. As shown in Figure 2.10, one linear and one nonlinear limit states with the same design point are demonstrated. The result shows that the failure domains differ a lot. The FORM ignores the curvature of the nonlinear limit state which has been improved by Second-Order Reliability Method (SORM).

Figure 2. 10 Linear and nonlinear limit states

The Taylor expansion of a second-order nonlinear function $g(\theta_1, \theta_2, \dots, \theta_n)$ at the design point is:

$$g(\theta_1, \theta_2, \dots, \theta_n) = g(\theta_1^*, \theta_2^*, \dots, \theta_n^*) + \sum_{i=1}^n (\theta_i - \theta_i^*) \frac{\partial g}{\partial \theta_i} + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n (\theta_i - \theta_i^*) (\theta_j - \theta_j^*) \frac{\partial^2 g}{\partial \theta_i \partial \theta_j} + \dots \quad (2.42)$$

where $(\theta_1^*, \theta_2^*, \dots, \theta_n^*)$ are design points. Compared with FORM which ignores the terms beyond the first-order, SORM ignores the terms beyond the second-order (Haldar, 2000).

SORM method has improved the accuracy of reliability evaluation by considering the curvature of limit state function.

Besides linear limit state function and second-order nonlinear limit state function, a number of more complex approximations for limit state functions have been proposed, which include the use of piecewise spherical sectors (Veneziano, 1974), the use of quadratic expressions centred about the checking point (Fiessler et al., 1979; Horne and Price, 1977) and the use of (linear) tangents at various predefined locations on $g(y) = 0$ (Berthelley and Rackwitz, 1979). For those complex limit state functions, First-Order Reliability and Second-Order Reliability method may not be appropriate.

2.4.5.3 First Order Saddlepoint Approximation (FOSPA)

Daniels (Daniels, 1954) was the first researcher who introduced the saddlepoint approximations in statistical application. Saddlepoint Approximation is an efficient method to estimate the statistical characteristics of a random variable such as the PDF and mean.

The PDF of a random variable y can be approximated by saddlepoint approximation as follows.

The moment generating function of y is defined as:

$$M_y(t) = \int_{-\infty}^{+\infty} e^{ty} f_Y(y) dy \quad (2.43)$$

Let $-c_1 < t < c_2$, and $0 \leq c_1 \leq \infty$, $0 \leq c_2 \leq \infty$, but $c_1 + c_2 > 0$.

The cumulant generating function (CGF) of y is defined as the natural logarithm of $M_y(t)$:

$$K_y(t) = \log\{M_y(t)\} \quad (2.44)$$

By using the inverse Fourier formula, the density function $f_Y(y)$ can be obtained:

$$f_Y(y) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} M(it) e^{-ity} dt = \frac{1}{2\pi} \int_{-i\infty}^{+i\infty} \exp\{K(t) - ty\} dt \quad (2.45)$$

If n random variables are independent and identically distributed, the PDF of y can be written as:

$$f_n(y) \cong \frac{1}{2\pi} \left[\frac{n}{K_Y''(t_s)} \right]^{\frac{1}{2}} e^{n[K(t_s) - t_s y]} \quad (2.46)$$

where $K_Y''(\)$ is the second order derivative of the CGF of random variable y , and t_s is the saddle point.

Since the CGF of a random variable is the central idea of the FOSPA, the condition of CGF may influence the result a lot. Details of CGF can be found in references (Du and Sudjianto, 2004; Huang and Du, 2008).

FOSPA was proposed by Du and Sudjianto, which can generate reliability approximation more accurately than FORM at the expense of ‘unclear’. In FOSPA, the limit state function is linearized at the so-called most likelihood point (MLP) in the original space. The saddlepoint approximation can be directly applied if the CGF of random variable X is available. If the CGF of random variable X is not available, it is required to transform X to the CGF available distribution before the implementation of saddlepoint approximation.

Compared with FORM, FOSPA can be linearized at the original space instead of only in standard normal space and the process of searching for MLP is faster than most probable point (MPP) in most cases. At the same computational effort, FOSPA can be more accurate than FORM.

In order to improve the accuracy, the second or higher order of saddlepoint approximation can be considered. The most critical problem exists in how to calculate the CGF of y with the known CGF of x and the relationship between x and y when the order is higher.

2.4.5.4 Mean-Value Second Order Expansion

After the discussions about the FORM, SORM and FOSPA, it is found that all of them require the searching for MPP or MLP and both the FORM and FOSPA linearize the limit-state function at MPP or MLP respectively. The SORM improves the FORM with higher accuracy at relatively lower efficiency. Liu and Peng (2012) proposed a new method called mean-value second-order expansion to improve the characteristics of the above three. This new method avoids the process of searching for MPP or MLP, and its limit-state function is decomposed into one dimensional function which then expands at the mean value to the second order. After the above procedure, there are two branch methods: one is called second-order saddle-point approximation method (SMSA) and the other is named second-order method by non-central chi-square distribution (SMNC). The details of saddle point approximation can be referred to Daniels (1954) and Lugannani and Rice (1980). If the saddle-point does not exist, SMNC would be used then and the cumulative distribution function (CDF) of the limit-state function will be approximated by non-centrally chi-square distribution or normal distribution (Liu and

Peng, 2012). The Mean-Value Second-Order method has been proved to be more precise and efficient than the three methods discussed previously.

2.4.5.5 Mean-Value First Order Second Moment (MVFOSM)

Besides FORM and FOSA, MVFOSM is another method based on first order Taylor expansion. In MVFOSM, the performance function is linearized at the mean values instead of MPP. Similar to FORM, MVFOSM requires transformation from the original distribution space to standard normal space. Since it avoids the process of searching for MPP, MVFOSM is much more efficient than FORM. An obvious deficiency of MVFOSM is that it only uses the first two moments of random variables instead of the information of the complete distribution (Haldar and Mahadevan, 2001; Youn and Choi, 2004).

2.4.5.6 Mean-Value First Order Saddlepoint Approximation (MVFOSA)

Mean-value first order saddlepoint approximation is to calculate the CDF and PDF by saddlepoint approximation. The first step of MVFOSA is to linearize the performance function at the mean values of random variables. Then, the CGF of the performance function is evaluated. Finally, the CDF and PDF are estimated. More detailed information about CGF can be referred to Refs (Huang and Du, 2008; Bain and Engelhardt, 1991; Johnson and Kotz, 1970). Compared with MVFOSM, MVFOSA is more accurate due to the use of complete distribution information instead of only the first two moments of random variables.

2.4.5.7 Second-Order Reliability with First-Order Efficiency (SORM-FOE)

Since the FORM is efficient but may not be accurate for linear limit-state functions and SORM is more accurate but with less efficiency, Zhang and Du (2010) proposed a new method which contains both second-order's accuracy and first-order's efficiency.

SORM-FOE first searches for the MPP with FORM, and then decomposes the limit-state functions at MPP into univariate functions which are approximated into a quadratic version. CGF of the limit-state function is then available and then with the help of saddlepoint approximation, the probability of failure can be obtained.

Compared with FORM, SORM-FOE has higher accuracy but slightly less efficiency. The slight additional computational cost is due to the gradient calculation in MPP searching. However, if the univariate dimension reduction cannot approximate the limit-state function accurately, big error may appear.

2.4.5.8 Second-Order Perturbation Approach

Second-order perturbation approach requires very little computational cost for the approximation of integral. It is based on a second-order Taylor expansion about the mean of θ , which can be expressed as:

$$I = h(\bar{\theta}) + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n T_{ij}(\theta) V_{ij} \quad (2.47)$$

where I is the multidimensional integral; $T_{ij}(\theta) = \frac{\partial^2 h(\theta)}{\partial \theta_i \partial \theta_j}$ is the element of Hessian matrix, and $V_{ij} = E[(\theta_i - \bar{\theta}_i)(\theta_j - \bar{\theta}_j)]$ is the element of covariance matrix (Papadimitriou, et al., 1997).

This approximation approach only works well for very limited cases (Koyluoglu, 1995). Even for some low level of uncertainty cases, it can still give unsatisfactory results (Singh, 1980; Papadimitriou et al., 1995). Another deficiency of this method is that it only considers its mean value instead of its real distribution.

2.4.5.9 Asymptotic Approximation

In general, a multidimensional integral can be written in the form:

$$I = \int_{\Theta} h(\theta) p(\theta) d\theta \quad (2.48)$$

where $h(\theta)$ and $p(\theta)$ are smooth functions when $\theta \in \Theta$, $p(\theta)$ is PDF function, Θ is sub region of $\mathfrak{R}^n$. In very rare cases, Equation (2.48) can be solved analytically. Numerical methods could give accurate result with costly computational expense. Asymptotic approximation gives an analytical approximation for the integral in Equation (2.48).

Asymptotic approximation is more complicated and accurate than the perturbation method. The integral of asymptotic approximation is based on an expansion of its natural logarithm about the point which locates at a single maximum of the integrand (Papadimitriou, et al., 1997):

$$I = \int_{\Theta} \exp[l(\theta)] d\theta \quad (2.49)$$

where

$$l(\theta) = \ln h(\theta) + \ln p(\theta) \quad (2.50)$$

Suppose θ^* is the local maximum inside Θ , and is also the global maximum over Θ . Then by explaining $l(\theta)$ at its maximum point θ^* , the following can be obtained:

$$p \frac{\partial h}{\partial \theta_i} + h \frac{\partial p}{\partial \theta_i} = 0, \quad i = 1, 2, \dots, n \quad (2.51)$$

Equation (2.51) is satisfied at $\theta^* \in \Theta$.

Then Equation (2.49) can be rewritten as:

$$I = \int_{\Theta} \exp \left[l(\theta^*) - \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n L_{ij}(\theta^*) (\theta_i - \theta_i^*) (\theta_j - \theta_j^*) + E(\theta) \right] d\theta \quad (2.52)$$

where

$$L_{ij}(\theta) = - \frac{\partial^2 l(\theta)}{\partial \theta_i \partial \theta_j} \quad (2.53)$$

Then Equation (2.52) can be rewritten as:

$$I = h(\theta^*) p(\theta^*) \int_{\Theta} \exp \left[- \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n L_{ij}(\theta^*) (\theta_i - \theta_i^*) (\theta_j - \theta_j^*) \right] \cdot \exp[E(\theta)] d\theta \quad (2.54)$$

By introducing Laplace's method into the above integral (Bleistein and Handelsman, 1986):

$$I(\theta^*) \sim (2\pi)^{n/2} h(\theta^*) p(\theta^*) \frac{1}{\sqrt{\det[L(\theta^*)]}} \quad (2.55)$$

If there is more than one local maximum in Θ , say $\theta_j^*, j = 1, \dots, r$. The proposed method can be modified by summing all asymptotic contributions:

$$I = \sum_{j=1}^r I(\theta_j^*) \quad (2.56)$$

where r is the number of local maximum.

In asymptotic approximation, the most critical and computationally expensive task is to search for the maximum points θ_j^* . Local maximum method such as Modified-Newton method can be used for searching for one single local maximum. If there is more than one local maximum, some complicated optimization methods can be used such as homotopy and relaxation technique (Yang and Beck, 1997).

2.4.5.10 Univariate Dimension-Reduction Method

Univariate dimension-reduction method is a simple and efficient approach for evaluating a multi-dimensional integral. It makes the computational process more efficient by decomposing a multi-dimensional response function into multiple one-dimensional functions (Rahman and Xu, 2004).

Suppose n independent variables are $\theta_1, \theta_2, \dots, \theta_n$, then a multi-dimensional integral can be decomposed into several one-dimensional integral:

$$\int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} y(\theta_1, \dots, \theta_n) d\theta_1 \dots d\theta_n \approx \sum_{j=1}^n \int y(\mu_1, \dots, \mu_{i-1}, \theta_i, \mu_{i+1}, \dots, \mu_n) d\theta_i - (n-1)y(\mu_1, \dots, \mu_n) \quad (2.57)$$

where each term of the summation is a one-dimensional integral regarding to the j th random variable. $\mu_1, \dots, \mu_j, \dots, \mu_n$ are the mean value of all random variables. $y(\mu_1, \dots, \mu_n)$ is a deterministic function (Acar, et al., 2010).

The above function is only valid for independent random variables. When some random variables are dependent, it is necessary to transform them to independent ones.

The univariate dimension reduction method makes moderate to large number of random variables' integral problems much more efficient. And this method does not require the computation of any partial derivatives. The approximated value of the original integral has dropped a higher dimensional integration residual, which are required to be negligibly small.

Since the proposed integral usually contains dependent random variables and non-symmetric domains, before applying univariate dimension-reduction method, it is necessary to transfer the initial integral domain to a symmetric domain and transform all random variables to independent ones.

2.4.5.11 New Point Estimates Method

Point Estimate Method is one type of moment methods. In general, in moment methods, the moments of performance functions are obtained using point estimates in standard normal space (Zhao and Ono, 2001).

Point estimate method uses some weighted average of some points to obtain the k th central moment. These points are chosen considering the following equation:

$$\sum_{j=1}^m P_j (x_j - \mu_x)^k = M_{kx} \quad (2.58)$$

where $x_j, j = 1, 2, \dots, m$, represent estimating points and $P_j, j = 1, 2, \dots, m$ are weights correspondingly. This method does not require derivatives for estimating the first few moments of a function of random variables and can be applied to implicit functions of random variables (Rosenblueth, 1975).

According to the literature, expressions for the two-point estimate (Rosenblueth, 1975) and three-point estimate (Gorman, 1980) were derived. A three-point estimate is expressed as:

$$\begin{aligned} x_- &= \mu_x - \frac{\sigma_x}{2}(\theta - \alpha_{3x}); P_- = \frac{1}{2} \left(\frac{1 + \alpha_{3x}/\theta}{\alpha_{4x} - \alpha_{3x}^2} \right) \\ x_0 &= \mu_x; P_0 = 1 - \frac{1}{\alpha_{4x} - \alpha_{3x}^2} \\ x_+ &= \mu_x + \frac{\sigma_x}{2}(\theta + \alpha_{3x}); P_+ = \frac{1}{2} \left(\frac{1 - \alpha_{3x}/\theta}{\alpha_{4x} - \alpha_{3x}^2} \right) \end{aligned} \quad (2.59)$$

where $\theta = (4\alpha_{4x} - 3\alpha_{3x}^2)^{1/2}$; x_-, x_0, x_+ are three estimating points and P_-, P_0, P_+ are their weights correspondingly; α_{3x} and α_{4x} are skewness and kurtosis. The k th central moment of $y = y(x)$ can be obtained by:

$$\mu_y = P_- y(x_-) + P_0 y(x_0) + P_+ y(x_+) \quad (2.60)$$

$$M_{ky} = P_- (y(x_-) - \mu_y)^k + P_0 (y(x_0) - \mu_y)^k + P_+ (y(x_+) - \mu_y)^k \quad (2.61)$$

When the standard deviation is very large, for some special functions such as lognormal distribution, some estimating points might be placed outside the proposed region by using the point estimate method. A new point estimate method was developed to solve this weakness by transforming the original space into standard normal space and increasing the number of estimating points (Hohenbichler and Rackwitz, 1981).

After obtaining the estimating points and corresponding weights in the standard normal space, the k th central moment of $y = y(x)$ can be calculated as:

$$\mu_y = \sum_{j=1}^m P_j y[T^{-1}(u_j)] \quad (2.62)$$

$$M_{ky} = \sum_{j=1}^m P_j (y[T^{-1}(u_j)] - \mu_y)^k \quad (2.63)$$

where $u_1, u_2, \dots, u_m$ are estimating points in the standard normal space and $P_1, P_2, \dots, P_m$ are the corresponding weights. $T^{-1}(u_j)$ is the inverse Rosenblatt transformation, which will be introduced in Section 2.4.6.1 (Zhao, Y., Ono, T., 2000; Zhao, Y., Ono, T., 2001).

One can obtain the estimating points and the corresponding weights by:

$$u_i = \sqrt{2}x_i; P_i = \frac{\omega_i}{\sqrt{\pi}} \quad (2.64)$$

where x_i and $\omega_i = \exp(-x^2)$ are the abscissas and weights for Hermite integration (Abramowitz and Stegun, 1972).

For a five-point estimate, the estimating points can be chosen as:

$$\begin{aligned} u_0 = 0; P_0 = \frac{8}{15}; u_{1+} = -u_{1-} = 1.3556262; P_1 = 0.2220759; \\ u_{2+} = -u_{2-} = 2.8569700; P_2 = 1.12574 \times 10^{-2} \end{aligned} \quad (2.65)$$

For a seven-point estimate, the estimating points can be chosen as:

$$\begin{aligned} u_0 = 0; P_0 = 16/35; u_{1+} = -u_{1-} = 1.1544054; P_1 = 0.2401233; \\ u_{2+} = -u_{2-} = 2.3667594; P_2 = 3.07571 \times 10^{-2}; \\ u_{3+} = -u_{3-} = 3.7504397; P_3 = 5.48269 \times 10^{-4} \end{aligned} \quad (2.66)$$

When a function contains N random variables, $Z = g(\boldsymbol{\theta})$, $\boldsymbol{\theta} = \theta_1, \theta_2, \dots, \theta_n$, (n is the number of random variables) using new point estimate method, $g(\boldsymbol{\theta})$ can be approximated as:

$$g'(\boldsymbol{\theta}) = \sum_{i=1}^n (g_i - g_\mu) + g_\mu \quad (2.67)$$

where $g_i = g[T^{-1}(\mathbf{U}_i)]$, and $\mathbf{U}_i$ indicates u_i is the only random variable with others set to mean values transformed to standard normal space. g_i is a function that has only one random variable u_i . Since $\mathbf{U} = T(\boldsymbol{\theta})$ are independent, g_i are also independent. Then we obtain:

$$\mu_g = \sum_{i=1}^n (\mu_i - g_\mu) + g_\mu \quad (2.68)$$

where g_μ is the function evaluated at the mean of all the random variables and μ_i is the mean of g_i , which can be calculated by the point estimate of one random variable function (Zhao, Y., Ono, T., 2000).

2.4.6 Transformations

In some integral approaches, random variables are required to be transformed to certain type of distributions such as standard normal distribution. In this section, two general transformation methods are introduced.

2.4.6.1 Rosenblatt Transformation

Rosenblatt transformation is an efficient tool to transform a dependent random vector $X = \{X_1, X_2, \dots, X_n\}$ to an independent uniformly distributed random vector $R = \{R_1, R_2, \dots, R_n\}$ (Rosenblatt, 1952). By applying the Rosenblatt transformation twice, it can transform a group of dependent non-normal distributed random variables to equivalent independent standard normal random variables.

The detailed transformation process can be expressed by:

$$\begin{aligned}
 F_1(x_1) &= r_1 = F_1(u_1) \\
 F_2(x_2|x_1) &= r_2 = F_2(u_2|u_1) \\
 &\cdot \\
 &\cdot \\
 &\cdot \\
 F_n(x_n|x_1, \dots, x_{n-1}) &= r_n = F_n(u_n|u_1, \dots, u_{n-1})
 \end{aligned} \tag{2.69}$$

where F_i is shorthand for the conditional cumulative distribution function $F_{X_i|X_{i-1}, \dots, X_1}(\cdot)$; $\mathbf{X}$ is a vector of correlated random variables; $\mathbf{U}$ is a vector of uncorrelated random variables which are standard normal distributed. Equation (2.69) can be written as:

$$\begin{aligned}
 x_1 &= F_1^{-1}[\Phi(u_1)] \\
 x_2 &= F_2^{-1}[\Phi(u_2)|x_1] \\
 &\cdot \\
 &\cdot \\
 &\cdot \\
 x_n &= F_n^{-1}[\Phi(u_n|x_1, \dots, x_{n-1})]
 \end{aligned} \tag{2.70}$$

It is not always very smooth to do the transformation, even if the $F_i(\cdot)$ is in a simple form, the inverse function may be required to be done numerically (Melchers, 1999). In practical situations, there is another difficulty that there are $n!$ possible ways in the writing of Eq. (2.69), depending on the numbering adopted for the variables in $\mathbf{X}$. Furthermore, there are also $n!$ possible ways of conditioning the X_i in Eq. (2.69) (Rosenblatt, 1952; Rubinstein, 1981).

2.4.6.2 Nataf Transformation

Nataf transformation is to transform a correlated random vector $X = (X_1, \dots, X_n)$ to standardized normal random variables $Y = (Y_1, \dots, Y_n)$, for which the marginal cumulative distribution functions $F_{x_i}(\cdot)$, $i = 1, \dots, n$ and the correlation coefficient matrix $\mathbf{P} = \{\rho_{ij}\}$ are known (Melchers, 1999). The basic transformation expression is:

$$Y_i = \Phi^{-1}[F_{x_i}(X_i)], \quad i = 1, \dots, n \quad (2.71)$$

where Φ is standard normal CDF. $\mathbf{Y} = (Y_1, \dots, Y_n)$ is n -dimensional standard normal with JPDPF $\Phi_n(\mathbf{y}, \mathbf{P}')$ having zero means, unit standard deviations and correlation matrix $\mathbf{P}' = \{\rho_{ij}'\}$. The correlation matrix $\mathbf{P}' = \{\rho_{ij}'\}$ can be obtained from the known $\mathbf{P} = \{\rho_{ij}\}$ iteratively from Eq. (2.72):

$$\rho_{ij} = \frac{\text{Cov}[X_i, X_j]}{\sigma_{X_i} \sigma_{X_j}} = E[Z_i Z_j] = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} z_i z_j \varphi_2(y_i, y_j; \rho_{ij}') dy_i dy_j \quad (2.72)$$

where $Z_i = (X_i - \mu_{X_i})/\sigma_{X_i}$.

According to the usual rules for the transformation of random variables, the approximate JPDPF $f_x(\cdot)$ in $\mathbf{x}$ space is (Nataf, 1962):

$$f_x(\mathbf{x}) = \Phi_n(\mathbf{y}, \mathbf{P}') \cdot |\mathbf{J}| \quad (2.73)$$

where $\mathbf{J}$ is Jacobian matrix, the element $j_{ij} = \frac{\partial y_i}{\partial x_j}$, and

$$|\mathbf{J}| = \frac{\partial(y_1, \dots, y_n)}{\partial(x_1, \dots, x_n)} = \frac{f_{x_1}(x_1) \cdot f_{x_2}(x_2) \cdots f_{x_n}(x_n)}{\Phi(y_1) \Phi(y_2) \cdots \Phi(y_n)} \quad (2.74)$$

Compared with Rosenblatt transformation, Nataf transformation is superior only when the marginal cumulative distribution functions $F_{x_i}(\cdot)$, $i = 1, \dots, n$ and the correlation matrix $\mathbf{P} = \{\rho_{ij}\}$ are available instead of the joint cumulative distribution function $F_X(\mathbf{X})$. The conditional distributions are necessary condition for applying Rosenblatt transformation (Melchers, 1999).

2.4.7 Summary for different probability integral methods

Section 2.4 gives a detailed and in-depth literature review about the probability integral methods. Generally, probability integral methods can roughly be categorized into three types: sampling based-method, moment matching method and MPP based method.

Sampling based-method includes direct Monte Carlo method, importance sampling, Latin hypercube sampling and so on. Sampling based-method are flexible to implement and can give accurate result if the number of sampling simulations is large enough. Given enough simulations, they are robust and can be set as a benchmark for other approximate integral methods. However, since they require sufficient number of simulations to obtain accuracy, they are not efficient for many engineering problems where high reliability and expensive computational performance functions are involved.

Moment matching methods, such as mean-value first order second moment (MVFOSM) and point estimate method, are more efficient but less accurate methods, compared with the first type. This kind of method approximates the full distribution by fitting the first few moments. Thus, the accuracy of such methods would have a large discount.

Most probable point (MPP)-based method exists as a good balance between efficiency and accuracy. First order reliability method (FORM), second order reliability method (SORM), and first order saddlepoint approximation (FOSA) are all typical MPP-based methods. In this type of methods, the performance function is approximated with Taylor expansion at the most probable point (MPP), which are always required to be numerically determined. Compared with the above two types of integral methods, MPP-based methods have relative satisfactory accuracy and moderate computational expense.

Besides the above important characteristics of probability integral methods, the number of dimensions of integral is another critical issue which requires serious consideration. For instance, the efficiency of numerical simulation has much to do with the number of integral dimensions; instead, the efficiency of Monte Carlo integral method depends on the coefficient of variation (CoV) of the performance function instead of the dimension.

Based on the solid ground about probability integral methods established so far, it is quite important to find or create a method which can balance the accuracy and efficiency as well as be suitable for the proposed integral problem of long-term fatigue damage. A case study about long-term fatigue assessment of mooring and riser systems is implemented in the next chapter, which is worth attempting before a more sophisticated analysis being conducted in the future.

2.5 Chapter Summary

This Chapter presents a comprehensive literature review of the theories and methods for the fatigue assessment of offshore risers. Section 2.2 reviews the environmental conditions and

loads of offshore structures. In this section, some mathematical models about ocean waves are summarized and environmental parameters and relative statistics are discussed. Section 2.3 is focused on the review of fatigue analysis of offshore structures, which includes the traditional fatigue design and analysis, comparisons of frequency domain and time domain analysis, and finally gives an in-depth review about fatigue analysis of flexible risers which will be studied in the following sections. Section 2.4 gives an in-depth literature review about methods for probability integrals.

From the literature reviewed in this chapter, research gaps are identified and the research objectives in this thesis are found, as:

1. A new approach is proposed and studied in Chapter 3 to solve the integral problem of long-term fatigue damage in time domain which can balance the accuracy and efficiency.
2. Since the environmental conditions of offshore structures are random, it is very important to build an appropriate statistical distribution model to model the environmental parameters with the internal dependent relationships considered appropriately. In the current stage, joint distribution models are the most popular which are already documented well in the existing codes. However, they are only limited to considering linear correlation between pairs of random variables. The second part of this thesis is trying to find a more robust method to construct a more appropriate statistical model of environment variables which may take into account the nonlinear relationships as well as linear relationships, and finally improve the accuracy of industry problems such as the fatigue assessment of flexible risers in the long-term period, which is presented in Chapter 4, multi-dimensional copula model is developed.
3. Further to the classical multi-dimensional copula model studied in Chapter 4, an advanced copula model with more flexible characteristics is presented in Chapter 5.

CHAPTER 3 LONG-TERM FATIGUE ANALYSIS OF RISER SYSTEMS IN TIME DOMAIN

3.1 Introduction

Increasing demand for oil and gas resulted in offshore industry moving towards deeper water and harsh environment explorations. Offshore structures are subjected to many environmental conditions such as waves, winds, currents and ice, which vary geographically and in time (E.M., Bitner-Gregersen, 2014). Even though these environmental actions are not stationary processes in the long run, statistical properties of these periods for short-time around 3 to 6 hours' duration could be considered approximately stationary, which could be described as sea state (Vazquez-Hernandez, 2006). Ultimate Limit State and Fatigue Limit State are two important design criteria for riser pipes (DNV, 2010, Dynamic Risers). The Ultimate Limit State corresponds to extreme response prediction which could be assessed by either short-term or long-term method. On the other side, the assessment of Fatigue Limit State should always be considered by long-term method (DNV, 2010, Dynamic Risers; Campbell, 1999). As stated in the previous, the long-term approach accounts for all possible sea states during the service life of the structure, whereas the short-term method only considers one or several critical storms (Naess and Moan, 2005).

Since the fatigue damage in the long run is accumulated over a long duration from many sea states, and all the relevant cyclic load effects such as first order wave effects, second order floater motions and vortex induced vibrations should be taken into consideration in the dynamic analysis of the riser system, it is quite computational expensive to realize especially in time domain. Compared with frequency domain, time domain analysis can solve transient, non-stationary and nonlinear problems instead of just steady state, stationary ones. The most disadvantages for time domain analysis is its time consuming which would further compound the computational difficulty in the long-term fatigue analysis of deep-water riser systems.

The traditional way for fatigue damage is to subdivide the wave environment scatter diagram into several representative blocks. One single sea state is selected to represent all sea states within each block. Then the total long-term fatigue damage could be obtained based on the selected sea-states instead of all possible sea-states (DNV, 2010, Riser Fatigue). In the above traditional approach, the short-term fatigue conditions should be chosen carefully in order to make sure that they are representative. However, there are no detailed law or research work on the selection strategy and the precision of the fatigue assessment still cannot be guaranteed.

Long-term problems are essentially multi-dimensional integration which can be solved accurately by brute force approaches. It attracted many researchers to find methods that can balance the efficiency and accuracy. Till now, there are many researchers study the long-term extreme response problems: In the early 1993, Farnes and Moan (1993) employed Response Surface Approach to study the nonlinear flexible riser system. Grime and Langley (2008) considered the pseudo-asymptotic integration method in frequency domain; Vazquez-Hernandez (2011) used the Monte Carlo based approach with an interpolation scheme and Sagrilo et al. (2011) proposed a combination of the Inverse First Order Reliability Method (IFORM) and an Importance Sampling Monte Carlo Simulation (ISMCS) approach. However, only few attempts are reported for the long-term assessment of fatigue damage in recent years. Low and Cheung (2012) adopted a multi-peaked third-order asymptotic approximation to solve the fatigue problem with two random variables: significant wave height H_s and spectral peak period T_p in the frequency domain and obtained good results. Currently, there seems to be no precedent studies for the fatigue assessment of mooring and riser systems in real time domain.

In this chapter, a novel and efficient simulation method is presented to solve the long-term fatigue problems of flexible risers in coupled global dynamic analysis in time domain. An irregular wave elevation is always simulated as the sum of regular wave components with random variables and phase angles in time series. The essence of the new method is based on some characteristics of importance sampling, to simulate wave amplitudes from a distribution different from the original one by simply altering the weighting function. With the exploration of this new approach, some additional techniques are also developed to reduce the sampling variability. Since the proposed importance sampling is not an approximate method, there is no bias in the predicted long-term fatigue damage.

Most of the content in this chapter is presented in a published journal paper (Gao and Low, 2016).

3.2 Wave Environmental Model

It is generally assumed that sea surface is statistically stationary for a short duration like 3 hours. During the short-term sea state, wave condition could be described by a set of environmental parameters, such as significant wave height H_s and spectral peak period T_p . Significant wave height H_s is the average of the highest one-third of wave height in that period. Spectral peak period T_p is defined as the inverse of the frequency at the maximum value of the wave energy spectrum. Short-term stationary random wave environment may be characterized by a wave

spectrum $S_{\eta\eta}(\omega)$, which is the power spectral density of vertical sea surface displacement (DNV, 2010, Environmental Conditions and Environmental Loads). JONSWAP spectrum is adopted here to model the wave environment of the proposed problem. The JONSWAP spectrum could be expressed as follows:

$$S_{\eta\eta}(\omega) = \hat{\alpha} g^2 \omega^{-5} \exp \left[-\frac{4}{5} \left(\frac{\omega}{\omega_p} \right)^{-4} \right] \gamma^{\exp \left[-\frac{(\omega - \omega_p)^2}{2 \hat{\alpha}^2 \omega_p^2} \right]} \quad (3.1)$$

where γ is the shape parameter and

$$\hat{\alpha} = 5.061 \left(\frac{H_s^2}{T_p^4} \right) (1 - 0.287 \ln \gamma), \quad \omega_p = \frac{2\pi}{T_p}, \quad \sigma = \begin{cases} \sigma_a, & \text{if } \omega \leq \omega_p \\ \sigma_b, & \text{if } \omega > \omega_p \end{cases} \quad (3.2)$$

Average values for the above JONSWAP experiment data are: $\gamma = 3.3, \sigma_a = 0.07, \sigma_b = 0.09$ (DNV, 2010, Environmental Conditions and Environmental Loads).

In the long-term analysis, two environmental random variables are considered in this work: significant wave height H_s and spectral peak period T_p . In the past, the statistical environmental model of these two random variables had been studied by many researchers such as Haver and Nyhus (1986) and Bitner-Gregersen (1991). As recommended by Naess and Moan (2005), in this work, Haver and Nyhus's probabilistic model is adopted (Haver, 2002):

$$f_{H_s}(h_s) = \begin{cases} \frac{1}{\sqrt{2\pi} \alpha h_s} \exp \left\{ -\frac{(\ln h_s - \theta)^2}{2\alpha^2} \right\} & h_s \leq \eta \\ \frac{\beta}{\rho} \left(\frac{h_s}{\rho} \right)^{\beta-1} \exp \left\{ -\left(\frac{h_s}{\rho} \right)^\beta \right\} & h_s > \eta \end{cases} \quad (3.3)$$

$$f_{T_p|H_s}(t_p|h_s) = \frac{1}{\sqrt{2\pi} \sigma t_p} \exp \left\{ -\frac{(\ln t_p - \mu)^2}{2\sigma^2} \right\} \quad (3.4)$$

where $\mu = a_1 + a_2 h_s^{a_3}$, $\sigma^2 = b_1 + b_2 \exp\{-b_3 h_s\}$.

The parameters for the above probability model have been cited by Haver (2002) for the North Sea: $\alpha = 0.6565, \theta = 0.77, \eta = 2.90, \beta = 2.691, \rho = 1.503, a_1 = 1.134, a_2 = 0.892, a_3 = 0.225, b_1 = 0.005, b_2 = 0.120, b_3 = 0.455$ (Nygaard et al., 2000).

Then the JPDP of H_s and T_p could be written:

$$f_{H_s T_p}(h_s, t_p) = f_{H_s}(h_s) f_{T_p|H_s}(t_p|h_s) \quad (3.5)$$

Figure 3.1 indicates the contour of JPDF of H_s and T_p .

Figure 3. 1 Contour of joint probability density function $f(H_s, T_p)$

3.3 Evaluation of Probability Integrals

In this study, estimated long-term fatigue damage $\hat{D}$ could be expressed as the accumulation over a duration T :

$$\hat{D} = T \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \bar{d}(H_s, T_p) f(H_s, T_p) dH_s dT_p \quad (3.6)$$

In Equation (3.6), $\bar{d}(H_s, T_p)$ represents the fatigue damage for a fixed H_s and T_p . H_s and T_p are the only environmental variables considered in this work. $f(H_s, T_p)$ is the JPDF discussed in the above section. T is the total time considered for the fatigue damage (Low and Cheung, 2012).

The two-dimensional integration is the essential problem in this study. Many existing methods for the evaluation of probability integrals are reviewed here. In general, there are four main types of probability integral methods: (1) Direct analytical integration method; (2) Numerical integration method; (3) Sampling methods; and (4) Other methods. Direct analytical integration

method can only be applied to some special problems. Direct numerical integration can give satisfactory results with the expense of high computational costs, especially when the dimension of integral is large (Dahlquist, 1974). Moreover, the numerical integration result is not always good due to round off errors. And with the dimension increases, the computational time grows exponentially. Monte Carlo method is a robust method which relies on repeated random sampling. The efficiency of Monte Carlo simulation (MSC) depends on the CoV instead of integral dimension (Melchers, 1999). The Monte Carlo method could be expressed as:

$$g(\hat{\theta}) \approx \frac{1}{N} \sum_{j=1}^N f(\theta^{(j)}), \quad j = 1, 2, \dots, N \quad (3.7)$$

In Equation (3.7), $\theta^{(j)}$ ($j = 1, 2, \dots, N$) are samples simulated from its distribution $f(\theta)$. The right hand of Equation (3.7) will be close to the left hand when N approaches infinity.

Importance sampling is a classical ‘variance reduction’ technique for ‘crude’ Monte Carlo method. The Importance sampling method is trying to reduce sampling uncertainty by limiting the simulation to ‘interesting regions’. The detail implementation is to replace the original PDF $f(\theta)$ with ‘importance sampling’ PDF $h(\theta)$. Eq. (3.7) could be rewritten as:

$$g(\hat{\theta}) \approx \frac{1}{N} \sum_{j=1}^N g(\theta^{(j)}) \frac{f(\theta^{(j)})}{h(\theta^{(j)})}, \quad j = 1, 2, \dots, N \quad (3.8)$$

Besides the above classic methods, there are many approaches which could solve such probability integral methods, such as perturbation approach, asymptotic approximation, univariate dimension-reduction method and moment methods (Papadimitriou, 1995; Rahman, 2004; Rosenblueth, 1975; Zhao and Ono, 2001). Details can be found in Chapter 2.

3.4 Proposed Importance Sampling Method

For the long-term fatigue integral problem in Equation (3.6), some existing probability methods can be used, such as Moment method. Seven-point estimate method (Zhao and One, 2000) was employed here to calculate the two-dimensional integration in Equation (3.6) and the result is compared with brutal Monte Carlo approach, which is set as a benchmark with enough sampling simulations. The function calls used by seven-point estimate method is 13 compared with 3198 which is used by Monte Carlo method. However, the precision is less optimistic with up to 49% error compared with Monte Carlo result due to the highly nonlinear characteristic

of the problem. Therefore, it is quite important to explore a method to solve this problem which can improve the precision with acceptable cost.

3.4.1 Introduction of Proposed Importance Sampling Method

For most research exploration, it is a general assumption that, wave field follows Gaussian distribution, which has been proved to approximately close to reality (DNV, 2012).

A real sea state is more satisfactory to be described by irregular random wave model instead of deterministic model. It is required to simulate time series of $\eta(t)$ in time domain analysis. The wave elevation $\eta(t)$ can be described by a summation of sinusoidal wave components which can be stated as:

$$\eta(t) = \sum_{i=1}^N a_i \cos(\omega_i t + \varepsilon_i) \quad (3.9)$$

where N is the total number of wave components; ω_i is the wave frequency for i th wave components; ε_i are mutually independent random phases uniformly distributed between 0 and 2π ; a_i is the random wave amplitude which could be supposed to follow a Rayleigh distribution with mean square value $E[a_i^2] = S_{\eta\eta}(\omega_i)\Delta\omega_i$. $S_{\eta\eta}$ is the wave spectrum and $\Delta\omega_i = \omega_i - \omega_{i-1}$ is the frequency interval. The Rayleigh distribution of a_i could be expressed as (DNV, 2010, Environmental conditions and environmental loads; Low, 2011; Naess and Moan, 2005):

$$f_P(a_i) = \frac{a_i}{\sigma_i^2} \exp\left(-\frac{a_i^2}{2\sigma_i^2}\right) \quad (3.10)$$

where $\sigma_i^2 = S_{\eta\eta}(\omega_i)\Delta\omega_i$ is the variance of i th wave component, σ_i is the scale parameter of Rayleigh distribution.

The traditional way of simulating time series of an irregular wave elevation is by simulating a sum of regular wave components with random amplitudes and phase angles, where each wave component is simulated from its own distribution. In the long term, since all possible sea storms should be considered, if all the waves are generated by their own distributions, it requires numerous computational time to complete the dynamic simulations in all sea states especially in time domain.

The proposed Importance Sampling Method comes from the inspiration of traditional importance sampling method which is reviewed in the previous section with the classical Equation (3.8). The purpose here is not to reduce sampling variability, but rather to explore a flexibility to simulate from a distribution instead of the original one.

The main equation for the proposed importance sampling method could be expressed as:

$$\bar{d}(\mathbf{S}_T) = \frac{1}{J} \sum_{j=1}^J d(\boldsymbol{\theta}_j) W(\boldsymbol{\theta}_j) \quad (3.11)$$

With the weighting function

$$W(\boldsymbol{\theta}_j) = \frac{f(\boldsymbol{\theta}_j|\mathbf{S}_T)}{f(\boldsymbol{\theta}_j|\mathbf{S}_A)} \quad (3.12)$$

where $\bar{d}(\mathbf{S}_T)$ is the estimated fatigue damage pertaining to the target spectrum. $\mathbf{S} = [H_s \ T_p]^T$ characterizes the parameters of a wave spectrum. $\mathbf{S}_T$ indicates ‘target spectrum’, while $\mathbf{S}_A$ indicates ‘applied spectrum’. In the present context, samples $\boldsymbol{\theta}_j$ are simulated from the density of applied spectrum $h(\boldsymbol{\theta}) = f(\boldsymbol{\theta}|\mathbf{S}_A)$. $\boldsymbol{\theta} = [a_1, a_2, \dots, a_N, \varepsilon_1, \varepsilon_2, \dots, \varepsilon_N]$, including wave amplitude a_i and phase ε_i , $i=1, 2, \dots, N$; N is the number of wave spectrum components. In what follows, ‘applied spectrum’ means the wave spectrum which represents the sea state that are simulated by its own spectrum and distribution, while ‘target spectrum’ represents the sea state which is supposed to be solved indirectly by the known data of ‘used spectrum’.

The essence of the proposed method is to fix the applied spectrum $\mathbf{S}_A$, but vary the target spectrum $\mathbf{S}_T$ according to the integration point. Hence, simulation for only one sea state is required and fatigue damage for other sea states can be obtained by altering the weighting function as shown in equation (3.12).

Figure 3.2 is the comparison of traditional method and proposed Importance Sampling method for generating waves.

Figure 3. 2 Comparison of two types of generating waves

In Figure 3.2, $\sigma_1^{(i)}, \sigma_2^{(i)}, \dots, \sigma_k^{(i)}$ is the scale parameter for k wave spectrum components for each wave spectrum, k is the number of wave components, $i = 1, 2, \dots, n$ is the number of wave spectrums and D_i , $i = 1, 2, \dots, n$ is fatigue damage in i th sea states. D_{used} indicates fatigue damage pertaining to the applied spectrum.

It is noted that all elements in vector $\boldsymbol{\theta} = [a_1, a_2, \dots, a_N, \varepsilon_1, \varepsilon_2, \dots, \varepsilon_N]$ are independent with phases angles uniformly distributed between 0 and 2π . Then the distribution of $\boldsymbol{\theta}$ conditioned on $\mathbf{S}$ can be written as:

$$f(\boldsymbol{\theta}|\mathbf{S}) = f(a_1|\mathbf{S})f(a_2|\mathbf{S}) \dots f(a_n|\mathbf{S})(1/2\pi)^N \quad (3.13)$$

where

$$f(a_i|\mathbf{S}) = \frac{a_i}{\sigma_i^2(\mathbf{S})} \exp\left(-\frac{a_i^2}{2\sigma_i^2(\mathbf{S})}\right), \sigma_i^2(\mathbf{S}) = \mathbf{S}_{\eta\eta}(\omega_i, \mathbf{S})\Delta\omega_i \quad (3.14)$$

By inserting Equation (3.13) and (3.14) into Equation (3.12), the weighting function can be expressed as:

$$W(\theta_j) = W_1(\theta_j)W_2(\theta_j) \cdots W_N(\theta_j) \quad (3.15)$$

$$W_i = \frac{f(a_i|S_T)}{f(a_i|S_A)} = \frac{\sigma_i^2(S_A)}{\sigma_i^2(S_T)} \exp \left[\frac{a_i^2}{2} \left(\frac{1}{\sigma_i^2(S_A)} - \frac{1}{\sigma_i^2(S_T)} \right) \right] \quad (3.16)$$

where $j = 1, 2, \dots, J$ is the number of simulation and $i = 1, 2, \dots, N$ is the number of wave components for each wave spectrum.

3.4.2 Challenges of this Proposed Method

A preliminary study was conducted before deep exploration of this proposed Importance Sampling Method, which is shown in Figure 3.3. Details of the riser system will be introduced in section 3.5 case studies.

Figure 3. 3 Relative standard error of fatigue damage for different sea states

In this study, JONSWAP spectrum is employed to describe each short-term stationary irregular sea state and each point in Figure 3.3 represent one sea state described by significant wave height H_s (x axis with unit meter) and spectrum peak period T_p (y axis with unit second). Each pair of (H_s, T_p) could represent one short term sea state. The total 21 circles in Figure 3.3 representing 21 short-term sea states are selected to implement the preliminary study. The bigger circle in the centre is employed as the ‘applied spectrum’ and other 20 circles represent ‘target spectrums’. Then, each wave spectrum is divided into 89 components with equal $\Delta\omega$ in the same way. The fatigue damage of a proposed offshore structural component in ‘applied spectrum’ sea state is obtained by Monte Carlo simulation based on the mean of 200

simulations with 5 min duration for each. Other mean and relative standard error (RSE) of fatigue damage of proposed structural component is calculated by the proposed method using Equation (3.11). In Figure 3.3, it is observed that the further the target spectrum from the applied spectrum, the larger the RSE is. And the RSE values are unacceptably large at some locations far from the applied spectrum. Therefore, the challenge exists on how to reduce the RSE of the estimated fatigue damage and finally improve the precision of the proposed method.

3.4.3 Techniques

Based on the challenges proposed in the previous section, several techniques are developed to solve them.

3.4.3.1 Peak Exploration Technique

From the above preliminary study, it is observed that the sampling variability tends to increase when the distance from the applied spectrum increases. Hence, it is naturally to reason that the applied spectrum should be carefully chosen, better near the location with maximum contribution to the long-term fatigue damage. Suppose $S^* = [H_s^*, T_p^*]$ is the point that would maximize $\bar{d}(H_s, T_p)f(H_s, T_p)$ in Equation (3.6). It is an optimization problem to search for S^* , which requires iteration and simulation. An approximate analytical procedure is made by Low and Cheung (2012) in order to improve the efficiency of the optimization procedure.

One dimensional problem is considered when T_p is fixed. Then the objective is to find a proper $H_s = H'_s(T_p)$ which may maximize the value of $J(\) = \bar{d}(H_s, T_p)f(H_s, T_p)$. The sea state for fixed T_p with maximum $J(\)$ will be called “one-dimensional peak spectrum”. If the influences of H_s and T_p can be approximately decoupled, then it can be assumed that:

$$\bar{d}(H_s, T_p) = \alpha(T_p)H_s^\beta \quad (3.17)$$

where $\beta = m\chi$ depends on the relative dominance of the linear and quadratic load components with χ ranging from 1(linear) to 2(quadratic). Then the value of $H_s = H'_s(T_p)$ according to the maximum value of $f(H_s|T_p)f(T_p)\alpha(T_p)H_s^{m\chi}$ can be obtained by plotting $Y(H_s) = f(H_s|T_p)f(T_p)\alpha(T_p)H_s^{m\chi}$ against H_s , as stated by Low and Cheung (2012). In this process, $\alpha(T_p)$ has no influence since T_p is considered as constant.

Two-dimensional case involving H_s and T_p can be extended from the above procedure (Low and Cheung, 2012). Figure 3.4 shows the path tracking the maximum $f(H_s, T_p)$ for given H_s .

Before the tracking of S^* , the assumed value of χ should be approximately defined. Since there is no requirement of exact peak in this study, this approximated method can match this task. However, this global peak only corresponds to one applied spectrum, i.e. S_A^* . In actual process, more than one applied spectra for different T_p value can be considered, which may be solved by the aforementioned one-dimensional peak tracking method.

Figure 3. 4 Contours of joint probability density $f(H_s, T_p)$ with path along which $f()$ is maximum for given H_s

3.4.3.2 Equal Variance Technique

Since $\mathbf{W} = W_1 * W_2 * \dots * W_N$, $W_i = \frac{f(a_i|\mathbf{S}_T)}{h(a_i|\mathbf{S}_A)}$ $i = 1, 2, \dots, N$, the Equation (3.11) could be rewritten as:

$$\bar{d}(\mathbf{S}_T) = \frac{1}{J} \sum_{j=1}^J d(\boldsymbol{\theta}_j) W_1(\theta_j) W_2(\theta_j) \dots W_N(\theta_j) \quad (3.18)$$

Different from conventional MCS whose RSE comes directly from the samples' $d(\boldsymbol{\theta}_j)$, the RSE of importance sampling method comes from $d(\boldsymbol{\theta}_j)W(\boldsymbol{\theta}_j)$. The best condition for the variance reduction is when $d(\boldsymbol{\theta})$ and $W(\boldsymbol{\theta})$ are negatively correlated. Since $h(\boldsymbol{\theta}) = f(\boldsymbol{\theta}|\mathbf{S}_A)$

is not purposely selected for the trend of $d(\theta_j)W(\theta_j)$, it is reasonably supposed that the dependence between $d(\theta)$ and $W(\theta)$ are very weak. Then the target is to reduce the CoV of $d(\theta)$ and $W(\theta)$. Since $d(\theta)$ is a black box which is unknown before simulation, minimizing the uncertainty of $W(\theta)$ would be the task.

Take the log on both side of Equation (3.12) and get the variance, we can obtain

$$\text{var}(\ln W) = \text{var}(\ln W_1) + \text{var}(\ln W_2) + \cdots \text{var}(\ln W_N) \quad (3.19)$$

Assume W_i are all independent, then $\text{var}(\ln W_i)$ are all independent too. Therefore, Equation (3.19) would get a larger $\text{var}(\ln W)$ when N is larger, which means that wave spectrum are divided into more wave components. In this deduction, fewer wave components would result in a smaller $\text{var}(\ln W)$. Therefore, it is desirable to divide the wave spectrum as few as possible. Also, it is important to divide the wave spectrum wisely, so that the effectiveness of each of the wave component can be maximized. The most common way is to divide the wave spectrum equally with the same $\Delta\omega_i$. However, it is the stress spectrum instead of wave spectrum that is related to the fatigue damage. Therefore, in this work $\Delta\omega_i$ is appropriately chosen to follow the principle of equal variance in the stress spectrum. Since the stress spectrum is priori unknown, following several steps are proposed to retrieve the stress spectrum. Firstly, the applied wave spectrum is divided into many components with equal $\Delta\omega_i$ and a relative short time simulation is performed (800s). Secondly, the stress spectrum is retrieved from time history by standard procedures. Thirdly, the applied wave spectrum is divided into N components according to the principle of equal variance in the stress spectrum. If there are more than one applied spectrum, the procedure should be conducted for each applied spectrum.

Figure 3.5 is the normalized wave spectrum and accordingly response stress spectrum at the approximated peak point (obtained by peak exploration technique). The normalization is conducted in the way that both the stress spectrum and the wave spectrum are divided by the maximum spectrum value respectively.

Figure 3. 5 Response stress spectrum vs wave spectrum at peak wave spectrum

(Based on 3000s)

From Figure 3.5, it can be observed that not all the frequency range contributes equally to the response stress which causes the fatigue damage. Moreover, the contribution below frequency of 0.4 rad/s and above 0.8m/s is rather small which could be neglected.

Detail implementations of Equal variance technique and final efficient plan for the division of wave components will be discussed in the following case study section.

3.4.3.3 Segmenting Simulation Time Technique

In the implementation of proposed Importance Sampling Method, suppose n minutes are defined for each simulation period. If T is the total simulation time for each used spectrum, the number of Monte Carlo samplings would be T/n , which means segmenting total simulation time T into T/n n minutes 'simulation instead of simulating T period time at one time. The reason is explained as follows: In the traditional time domain analysis, the uncertainty mainly depends on T and the RSE is proportional to $1/\sqrt{T}$, if stress cycles are supposed to be independent. In a tradition way, it is natural to let T long enough to obtain a smaller RSE.

However, in the proposed importance sampling method, the RSE is related to $d(\boldsymbol{\theta})W(\boldsymbol{\theta})$. The increasing of T can only reduce the uncertainty of $d(\boldsymbol{\theta})$ with no impact on $W(\boldsymbol{\theta})$. On the contrary, the increase of T/n would have a direct effect on the reduction of the RSE of $W(\boldsymbol{\theta})$ which is proportional to $1/\sqrt{T/n}$.

3.4.3.4 Combining Results from Different Applied Spectra

In order to cover the wide variety of sea states, several applied spectra are employed. For each applied spectrum, there is a applied spectrum that may result in the smallest uncertainty in the damage prediction, which does not mean other applied spectra are useless. Instead, the prediction from other applied spectra may still more or less have some improvement in the damage prediction, which would be better than the damage predicted by only one applied spectrum. Suppose the predicted damage can be taken from the weighted average of K applied spectra, that is

$$\hat{d}(\mathbf{S}_T) = \sum_{k=1}^K \lambda_k \widehat{d}_k(\mathbf{S}_T) \quad (3.20)$$

where $\widehat{d}_k(\mathbf{S}_T)$ is the fatigue damage estimated from k th applied spectrum and λ_k is the weight of k th applied spectrum which satisfy $\sum \lambda_k = 1$. The weights are selected in the principle of minimizing the variance of $\hat{d}(\mathbf{S}_T)$. The optimal weights are calculated by the inverse variance weighting method which is popular applied in the meta-analysis (Hartung et al., 2008), and the expression is:

$$\lambda_k = \frac{1/\text{var}(\widehat{d}_k(\mathbf{S}_T))}{\sum_{n=1}^K 1/\text{var}(\widehat{d}_n(\mathbf{S}_T))} \quad (3.21)$$

The variance of the weighted damage is:

$$\text{var}(\hat{d}(\mathbf{S}_T)) = \frac{1}{\sum_{n=1}^K 1/\text{var}(\widehat{d}_n(\mathbf{S}_T))} \quad (3.22)$$

It can be easily discovered that all samples are uncorrelated, because simulations from different applied spectra are independent. In reality, $\text{var}(\hat{d}(\mathbf{S}_T))$ is estimated by sample variance, which shows large sampling variability. Since all $\widehat{d}_k(\mathbf{S}_T)$ have the same theoretical mean $\overline{d}_k$. From above deduction, Equation (3.21) can be written as:

$$\lambda_k = \frac{1/\text{CoV}^2[\widehat{d}_k(\mathbf{S}_T)]}{\sum_{n=1}^K 1/\text{CoV}^2[\widehat{d}_n(\mathbf{S}_T)]} \quad (3.23)$$

where the theoretical CoV of $\hat{d}(\mathbf{S}_T)$ can be expressed as:

$$\text{CoV}[\hat{d}(\mathbf{S}_T)] = \frac{1}{\sum_{n=1}^K 1/\text{CoV}[\hat{d}_n(\mathbf{S}_T)]} \quad (3.24)$$

where the sample CoV is estimated in practise. Compared with variance, CoV is more robust in the statistical tool to estimate sampling variability, because any error in the standard deviation is compensated by the mean.

3.4.4 Curve Fitting

The proposed importance sampling method can be used to estimate the fatigue damage from all target spectra and hence can calculate the long-term fatigue damage accordingly. Moreover, in order to improve the performance of the proposed method, curve fitting is employed. Curve fitting is applied to fixed T_p cases to identify the trend of H_s among the noise produced by sampling variability. As stated in section 3.4.3.1, fatigue damage is fitted to the equation:

$$\hat{d}(H_s) = \alpha H_s^\beta \quad (\text{With constant } T_p) \quad (3.25)$$

Take the log on both sides of Equation (3.25) and obtained:

$$\ln \hat{d} = \ln \alpha + \beta \ln H_s \quad (3.26)$$

Then linear regression problem is solved by least squares method.

It is acknowledged that the precision of estimated results is different for all ‘target spectrums’ with the same ‘applied spectrum’ by proposed method, and the closer the ‘target spectrum’ next to the ‘applied spectrum’, the better the result is. A straightforward method obtained by weighted least squares regression (Montgomery et al., 2012) with the weight of $1/\text{var}[\ln \hat{d}(H_s)]$ can be applied to try to improve the performance of curve fitting result. However, the result is very poor. Therefore, a deeper exploration is conducted to try to obtain a better result.

Since the objective of this research work is to estimate the integral $\int f(H_s) \hat{d}(H_s) dH_s$, as a natural reasoning the residuals should be related to the integrand $f(H_s) \hat{d}(H_s)$. The regression problem can be stated as:

$$f \hat{d} = f \alpha H_s^\beta + \varepsilon, \omega = \frac{1}{\text{var}(f \hat{d})} = \frac{1}{f^2 \text{var}(\hat{d})} \quad (3.27)$$

In following equations, ε and ω indicate residual and weight respectively. For simplicity intention, $f \equiv f(H_s)$ and $\hat{d} \equiv \hat{d}(H_s)$. Equation (3.27) which is a nonlinear weighted regression problem seeks to minimize the weighted residual as stated:

$$\omega \varepsilon^2 = \frac{(fd - f\alpha H_s^\beta)^2}{f^2 \text{var}(\hat{d})} = \frac{(\hat{d} - \alpha H_s^\beta)^2}{\text{var}(\hat{d})} \quad (3.28)$$

An equivalent expression is:

$$\hat{d} = \alpha H_s^\beta + \varepsilon, \omega = \frac{1}{\text{var}(\hat{d})} \quad (3.29)$$

Both Equation (3.28) and (3.29) yield the same $\omega \varepsilon^2$.

Compared with nonlinear regression which requires iteration and whose convergence cannot be assured, linear regression is preferred. A transformation is conducted by taking the log on both sides of Equation (3.29):

$$\ln \hat{d} = \ln \alpha + \beta \ln H_s + \Delta \quad (3.30)$$

where Δ is the residual of linear regression and Equation (3.30) is equivalent to

$$\hat{d} = \alpha H_s^\beta e^\Delta \quad (3.31)$$

For small Δ , the above equation can be approximately written as:

$$\hat{d} = \alpha H_s^\beta (1 + \Delta) \quad (3.32)$$

Hence, Δ can be interpreted as a fractional error and be applied to both $\hat{d}$ and $f\hat{d}$. Then the proper weight in the modified linear regression problem can be written as:

$$\omega = \frac{(f\hat{d})^2}{\text{var}(\ln \hat{d})} \quad (3.33)$$

It should be explained that $\hat{d}$ in above equation is the mean of J samples, $\hat{d} = \frac{d_1 + d_2 + \dots + d_J}{J}$.

Therefore, the CoV of $\hat{d}$ can be formulated as:

$$\text{CoV}(\hat{d}) = \frac{\text{CoV}(d_1, d_2, \dots, d_J)}{\sqrt{J}} \quad (3.34)$$

In order to estimate $\text{var}(\ln \hat{d})$, $\hat{d}$ is assumed to be approximately following lognormal distribution, which is the sum of identically positive distributed variables. From the central

limit theorem, it is obtained that $\hat{d}$ will follow a normal distribution when J approaches infinity. However, for finite number of J , $\hat{d}$ is approximately following a lognormal distribution (Mouri, 2013). The conclusion is: $\ln \hat{d}$ follows a normal distribution and $\text{var}(\ln \hat{d}) = \ln[\text{CoV}^2(\hat{d}) + 1]$.

3.4.5 Optimization

3.4.5.1 Motivation

Till now, each applied spectrum is assumed to belong to a real sea state in the wave scatter diagram and 1 D peak for each T_p is always chosen as applied spectrum. However, it is not obligatory that applied spectrum should be actual real sea state, which means for each wave component, $h(a_i)$ can be independently assigned and together constituting an “imaginary spectrum” which does not belong to any sea state.

In ideal condition, the principle for selection of $h(a_i)$ should minimize the uncertainty of the fatigue damage result over the range of T_p for which the applied spectrum is responsible, the process of which requires the knowledge of $W(\mathbf{X})d(\mathbf{X})$. Figure 3.6 shows a scatter plot of $\text{CoV}[W(\mathbf{X})d(\mathbf{X})]$ against $\text{CoV}[W(\mathbf{X})]$.

Figure 3. 6 Scatter plot of $\text{CoV}(Wd)$ vs $\text{CoV}(W)$

The plot in Figure 3.6 is obtained by extracting data from some samples which will be detailed in section 3.5. It is apparent that the two set of data $\text{CoV}(Wd)$ and $\text{CoV}(W)$ are highly correlated with correlation coefficient 0.9564. The result makes sense that $W(\mathbf{X})$ and $d(\mathbf{X})$ are supposed to be largely independent and therefore larger uncertainty in $W(\mathbf{X})$ would correlate with larger uncertainty in $W(\mathbf{X})d(\mathbf{X})$.

The findings from Figure 3.6 implicates that the CoV of $W(\mathbf{X})$ can be explored as a representation of the CoV in $W(\mathbf{X})d(\mathbf{X})$. It is a valuable resource that the statistical properties of $W(\mathbf{X})$ can be determined before simulation, while the characteristics of $d(\mathbf{X})$ are unknown before simulation.

3.4.5.2 Formulating the Optimization Problem

It is supposed that a particular applied spectrum is designed to cover a certain range of T_p and 1D peak target spectra $\mathbf{S}_{T1}, \mathbf{S}_{T2}$, etc within this range. Let $V_k = \text{var}(W^{(k)})$ be the variance of W for k th target spectrum. In the original proposed model, the applied spectrum is chosen as one of the target spectrum, whose corresponding $V_k = 0$. The performance of this target spectrum is very well compared to other target spectra further away from the applied spectrum. The principle of the optimization is to construct an imaginary applied spectrum, which may improve the worst performing target spectra with some expense of the better performing ones, and finally lead to an overall improvement.

In the optimization problem, an imaginary applied spectrum is constructed to minimizes the value of V_{max} :

$$V_{max} = \max(V_1, V_2, \dots, V_k) \quad (3.35)$$

Theoretically, any type of distribution can be assigned to $h(a_i)$ ($i = 1, 2, \dots, N$). In this work, as actual wave spectrum, Rayleigh distribution is adopted for simplicity. Thus, V_k depends on the Rayleigh parameters of the applied spectrum as well as the k th target spectrum. Since the Rayleigh parameter σ_{ki} of k th target spectrum is fixed, V_k mostly varies with the iteration of Rayleigh parameter σ_{Ai} of applied spectrum.

3.4.5.3 Analytical Derivation of the Variance of the Weights

The weighting function is expressed as $W = W_1 W_2 \dots W_N$ (as indicated in Equation (3.15)), with which each component W_i is independent of each other. The variance of W can be expressed (Goodman, 1962):

$$\text{var}(W) = \text{var}(W_1 \cdots W_N) = \prod_i^N (\text{var}(W_i) + \bar{W}_i^2) - \prod_i^N \bar{W}_i^2 \quad (3.36)$$

where

$$\begin{aligned} \bar{W}_i &= \int_0^\infty \frac{f(a_i)}{h(a_i)} h(a_i) da_i \\ \therefore \bar{W}_i &= \int_0^\infty f(a_i) da_i = 1 \end{aligned} \quad (3.37)$$

In the above equation, the explanation for the second line is that $f(a_i)$ is a PDF and the integral is unity.

Then, it is considered that $\text{var}(W_i) = E[W_i^2] - \bar{W}_i^2$, and

$$E[W_i^2] = \int_0^\infty \frac{f^2(a)}{h^2(a)} h(a) da = \int_0^\infty \frac{f^2(a)}{h(a)} da \quad (3.38)$$

In the above equation a_i is replaced by a for a notational convenience. By inserting Equation (3.14) into Equation (3.38):

$$\begin{aligned} E[W_i^2] &= \int_0^\infty \frac{[(a/\sigma_{Ti}^2) \exp(-a^2/2\sigma_{Ti}^2)]^2}{(a/\sigma_{Ai}^2) \exp(-a^2/2\sigma_{Ai}^2)} da \\ &= \int_0^\infty \frac{\sigma_{Ai}^2}{\sigma_{Ti}^4} a \exp(-ca^2) da, c = \frac{1}{\sigma_{Ti}^2} - \frac{1}{2\sigma_{Ai}^2} \end{aligned} \quad (3.39)$$

In the above equation, the exponential term should decay in order to make sure the integral be finite, requiring that $c > 0$. By introducing the ratio $r_i = \frac{\sigma_{Ti}^2}{\sigma_{Ai}^2}$, the requirement can be rewritten as $r_i < 2$. There are two interesting findings from the above equation. 1st, it is explicit that when $r_i \geq 2$, $\text{var}(W_i)$ is infinite. 2nd, when the $\text{var}(W_i)$ is finite, σ_{Ti}^2 has an upper bound but does not has a lower bound.

Suppose $r_i < 2$ is already provided, the following deduction can be obtained:

$$\text{var}(W_i) = E[W_i^2] - 1 = \frac{r_i^2}{2r_i - 1} - 1 \quad (3.40)$$

It is a fascinating finding that $\text{var}(W_i)$ only depends on r_i instead of σ_{Ti}^2 and σ_{Ai}^2 . Moreover, as expected when $r_i = 1$, $\text{var}(W_i) = 0$. By combining Equation (3.36), (3.37) and (3.40), Equation (3.41) is obtained:

$$\text{var}(W) = \prod_i^N \left(\frac{r_i^2}{2r_i-1} \right) - 1 \quad (3.41)$$

Equation (3.41) gives a new result to authors' knowledge.

3.4.5.4 Solving the Optimization Problem

This section mainly treats the multi-dimensional optimization problem and gives a detail introduction about the implementation process. The optimization problem stated in Equation (3.35) is solved by the aid of Equation (3.41). However, it is a challenging task to solve the multi-dimensional optimization problem in order to find the optimum σ_{Ai} ($i = 1, 2, \dots, N$). The optimization is conducted for each σ_{Ai} in turn instead of simultaneously. At the beginning, a real spectrum is employed as the initial condition of σ_{Ai} . In the first iteration, all other σ_{Ai} are fixed and $\min(V_{max})$ is solved by varying σ_{A1} . The second step is to find the $\min(V_{max})$ by varying σ_{A2} with other σ_{Ai} being kept constant, and so on until the optimization is conducted by varying σ_{AN} . After that, the second round of optimization continues starting with σ_{A1} , and the iteration repeats until V_{max} does not change any longer. The total procedure is easy to implement and rather robust, which guarantees a local optimum.

3.4.5.5 Error Estimation

For a single estimate of long-term damage rate $\overline{d_{LT}}$, no less important is to quantify the error of this result. Thus, bootstrapping method is employed for error estimation (Efron and Tibshirani, 1994). The procedure of bootstrapping method here is summarized as follows: 1st, for each applied spectrum, a new set of $d(\tilde{x}_j)$ samples ($j = 1$ to J) is created from the original set of results via sampling with replacement. 2nd, a new estimate of $\overline{d_{LT}}$ is obtained by the algorithm of the proposed method. Since the dynamic simulations are not involved in the procedure, the whole process can be fully automated and not time consuming. The procedure is repeated until enough samples of $\overline{d_{LT}}$ for statistical inference obtained. RSE will be used as a measure of sampling variability in this work.

3.5 Case Studies

3.5.1 General Description

A turret floating, production, storage and offloading (FPSO) in a water depth of 500m was selected for the current study. The floating system consists of three components: one vessel, two unbonded flexible risers and four mooring lines (Figure 3.7). The wave environment is described by the JONSWAP spectrum. Dynamic analysis was performed in the global coupled

model in time domain. The commercial software OrcaFlex Dynamics (Version 9.5a) was employed to implement the time domain analysis and subsequent rainflow counting analysis. For unbonded flexible risers, the approach based on stress factors is adopted. Especially, the fatigue stress is calculated as a weighted sum of the tensile and bending contributions (Orcina Ltd, 2012). The random waves approach the FPSO from the bow. Figure 3.7 is overview of the whole system.

Figure 3. 7 Overview of the whole system

The environmental loads were always assumed to come from the same direction in order to simplify the study. As this is the first trial of this proposed Importance Sampling Method and the second-order slowly varying response requires rather long simulation time, low frequency response will be neglected to expedite the research and it will be considered in the future work. In this research work, three methods for long-term fatigue assessment will be compared: proposed method without optimization, proposed method with optimization and direct numerical integration. For direct numerical integration, $\Delta H_s = 0.5m$ by $\Delta T_p = 0.25s$ is chosen as grid size and the range of integration is $0 \leq H_s \leq 16m, 7.15 \leq T_p \leq 16.15s$, which is already computationally intensive and hence a finer mesh is not considered. On the contrary, the mesh size has no effect on the computation simulation demands for the proposed importance sampling method and the increment $\Delta H_s = 0.1m$ by $\Delta T_p = 0.1s$ is used in the proposed method.

The characteristics of the mooring lines and flexible risers are shown in Table 3.1 and Table 3.2 respectively.

Table 3. 1 Flexible riser characteristics

Parameter	Value	Parameter	Value
Outer Diameter(m)	3.56×10^{-1}	Wall Thickness(m)	0.102
Mass per Unit Length(kg/m)	1.84×10^2	Bending stiffness(N*m ²)	1.25×10^5
Poisson Ratio	5.00×10^{-1}	Axial stiffness(N)	7.11×10^8
Added mass coefficient	1.00	Total Length(m)	0.60×10^3

Table 3. 2 Mooring line characteristics

Parameter	Value	Parameter	Value
Outer Diameter(m)	5.76×10^{-1}	Axial stiffness(N)	9.38×10^9
Mass per Unit Length(kg/m)	2.03×10^3	Poisson Ratio	5.00×10^{-1}
Drag coefficient	1.00	Added mass coefficient	1.00
Bending stiffness(N)	0	Total Length(m)	0.75×10^3

The length of the FPSO is 300 meters and the mass is 13.20×10^6 kg. Since the two locations at the top of the riser and the seabed at touch down point (TDP) are considered as the most challenge for fatigue design and the TDP is very complex due to many reasons such as pipe-soil interaction (Li and Low, 2012), the specific point near the top end of the flexible riser is chosen as the study point and the fatigue assessment is conducted at 5 meter from top end of the risers. S-N curve is used for the fatigue damage calculation and the parameters of S-N curve could be chosen from reference (DNV, 2011, Fatigue Design of Offshore Steel Structures). The S-N parameters are taken as $m=3$ and $\log \bar{a}=11.3$.

3.5.2 Case study 1: selecting the applied spectra

Peak Exploration Technique (discussed in section 3.4.3.1) is employed here to estimate $H'_s(T_p)$ for a range of T_p , with trajectory shown in Figure 3.8.

Figure 3. 8 Trajectory of $H'_s(T_p)$ against T_p

The nonlinearity parameter χ is assumed to be equal to 1, because only the first-order responses are considered. As discussed before, the peak $H'_s(T_p)$ does not need to be exact. For more nonlinear applications such as fatigue damage at the touchdown zone, which is not considered in this work, it may be worth obtaining a better estimate of χ by regular wave analysis of different wave heights. After the Peak Exploration Analysis, the global peak S^* is obtained with $H_s = 5.2m$, $T_p = 11.15s$. All the fatigue damage appeared hereafter are normalized by dividing the exact damage by damage at the global peak S^* .

Before optimization, all applied spectra are selected at the “1D peak spectra”, which lie on the trajectory in Figure 3.8. In this work, a total of 10 applied spectra are employed with uniform

T_p increment 1s. All applied spectra are shown in table 3.3, who are also indicated in Figure 3.8. It is explicit that all applied spectra are characterized by the JONSWAP spectra.

Table 3. 3 Characteristics of applied spectra (without optimization)

No.	T_p (s)	H_s (m)	Required no. of samples, N_k	N_k used (minimum 20)
1	7.15	1.098	0.004	20
2	8.15	1.825	0.680	20
3	9.15	2.742	27.70	28
4	10.15	3.860	132.3	133
5	11.15	5.200	250.0	250
6	12.15	6.800	113.3	114
7	13.15	8.659	28.70	28
8	14.15	10.74	3.300	20
9	15.15	13.03	0.250	20
10	16.15	15.51	0.010	20
				Total=653

In the optimization, $\text{var}(W)$ is allowed to be minimized at each 1D peak. It is found to be less effective to use a uniform spectrum to cover a wider range of frequencies, due to the too much disparity between the applied and target spectra at the critical 1D peaks which will result in excessive sampling variability.

During optimization, T_p increment is not fixed but determined by trial and error. As the beginning of optimization, 1s is set as the default increment for T_p . After optimization, increment will be adjusted according to the variance V_{max} (See Eq. (3.35)), such as that if the optimized variance V_{max} is too large, a smaller increment will then be considered. The process of the optimization is recorded in Table 3.4.

Table 3. 4 Theoretical standard deviation of Importance Sampling weight before and after optimization

Applied spectrum			Target spectrum		Standard deviations of weight	
Notation	$T_p(s)$	$H_s(m)$	$T_p(s)$	$H_s(m)$	Before optimization	After optimization
S_{A1}	7.65	1.44	7.15	1.1	2.436	2.351
			7.65	1.44	0	1.329
			8.15	1.83	2.262	2.351
S_{A2}	8.4	2.04	8.15	1.83	1.492	1.372
			8.4	2.04	0	1.123
			8.65	2.26	1.169	1.372
S_{A3}	9.15	2.74	8.65	2.26	1.677	1.530
			9.15	2.74	0	0.888
			9.65	3.28	1.394	1.534
S_{A4}	10.15	3.86	9.65	3.28	1.430	1.289
			10.15	3.86	0	0.578
			10.65	4.51	1.126	1.286
S_{A5}	11.15	5.2	10.65	4.51	1.078	1.164
			11.15	5.2	0	0.522
			11.65	5.99	1.303	1.164
S_{A6}	12.15	6.8	11.65	5.99	0.545	0.392
			12.15	6.8	0	0.130
			12.65	7.71	0.710	0.393
S_{A7}	14.4	11.29	12.65	7.71	0.856	1.091
			14.4	11.29	0	0.811
			16.15	15.51	1.519	1.091

Take the peak applied spectrum S_A^* as an example, which covers three target spectra called S_{T1} , S_{T2} and S_{T3} corresponding to $T_p=10.65s$, $11.15s$ and $11.65s$. Before optimization, the theoretical standard deviations of the importance sampling weights are $\sqrt{V_1} = 1.078$, $\sqrt{V_2} = 0$,

and $\sqrt{V_3} = 1.303$. After optimization, the largest $\sqrt{V_3}$ reduces at the expense of other two increase slightly.

For the global peak applied spectrum $\mathbf{S}_A^*$, if the number of samplings required is set as 200, then it makes sense that to use a smaller N for applied spectra further from $\mathbf{S}^*$ is appropriate. The method for selecting the number of samplings for each applied spectrum will be elaborated in Section 3.5.6.

3.5.3 Case study 2: equal response variance technique

A short period of simulation time 800s is performed for each applied spectrum, which is divided into 500 components from 0.2 to 1.2 rad/s with equal $\Delta\omega_i$ (See Section 3.4.3.2). Since the purpose is to quantify the fatigue damage instead of characterizing the response frequencies, the wave amplitudes are modelled in a deterministic way as $a_i = \sqrt{2S_{\eta\eta}(\omega_i)\Delta\omega_i}$ to smooth the noise in the stress spectrum. Figure 3.5 gives the wave spectrum and the stress spectrum at the global peak $\mathbf{S}^*$ with the curves normalized to facilitate a comparison on the same plot. It can be seen that the wave spectrum covers a much wider frequency range than the response. Therefore, it makes sense to divide the wave spectrum according to the response to maximize the effectiveness of each wave component.

A mini-study is conducted to compare the fatigue damage from two approaches for discretizing the wave spectrum, i.e. equal response variance and equal frequency interval. The purpose of this mini-study is to exemplify certain characteristics, which is not a formally part of the proposed method. The comparison conducted between different numbers of wave components N . Each case includes 200 simulations with 5 minutes' duration for each simulation. Random amplitude scheme is used here in the purpose of evaluating the fatigue damage.

Table 3.5 gives the effect of number of wave components and different discretion scheme on the fatigue damage results. All damage shown is normalized damage as explained in previous sections. From the results in Table 3.5, it can be obtained that for equal frequency interval discretization, the mean damage is sensitive to the number of wave components and only converges to “correct” result when N is large enough. On the contrary, the mean damage from equal response variance are much more stable even for very small wave components such as $N = 10$. It can be concluded from this mini-study that equal response variance is an efficient way for discretizing the wave spectrum and can get an accurate result by a comparatively small N . In the light of above findings, N is selected as 10 in the following studies. The RSE in Table

3.5 is calculated in the equation: $RSE = \frac{\sigma}{\mu\sqrt{200}}$, where σ and μ are standard deviation and mean value for each type of wave components' dividing.

Table 3. 5 Effect of number of wave components N on fatigue damage results

N	Equal response variance		Equal frequency interval	
	Mean damage	RSE	Mean damage	RSE
10	0.935	0.038	0.709	0.082
14	0.937	0.038	0.883	0.066
20	0.953	0.038	1.052	0.059
40	1.034	0.034	1.029	0.038
120	1	0.04	0.991	0.039

3.5.4 Case study 3: combining results from different applied spectra

This section gives the case study to implement the principles described in Section 3.4.3.4. Even though in theory, all applied spectra combined may give a best result, it is found that only some of applied spectra which are closest to an arbitrary target spectrum are effective. Following procedure is to simply the calculations. If a target spectrum whose T_p is between k th applied spectrum $\mathbf{S}_{Ak}$ and $(k+1)$ th applied spectrum $\mathbf{S}_{A(k+1)}$, then only these two applied spectra are employed in the combination. Then if T_p coincides with one applied spectrum $\mathbf{S}_{Ak}$, three applied spectra $\mathbf{S}_{A(k-1)}$, $\mathbf{S}_{Ak}$ and $\mathbf{S}_{A(k+1)}$ are considered in the combination.

Figure 3.9 and 3.10 give the illustration of the benefits of combination. In Fig. 3.9, the target spectrum is taken at $T_p=10.65$ s, which lies between $\mathbf{S}_{A4}$ and $\mathbf{S}_{A5}$ (before optimization). The plots of $\text{CoV}[\hat{d}(H_s, T_p)]$ against H_s for three cases are shown in Fig. 3.9. It is apparent that the CoV due to the combination spectra of $\mathbf{S}_{A4}$ and $\mathbf{S}_{A5}$ is the lowest among three. In addition, Fig. 3.10 shows the target spectrum with $T_p=11.15$ s, corresponding applied spectrum $\mathbf{S}_{A5}$. In this case, three applied spectra are combined, namely $\mathbf{S}_{A4}$, $\mathbf{S}_{A5}$ and $\mathbf{S}_{A6}$. Similar to Fig. 3.9, Fig. 3.10 compares $\text{CoV}[\hat{d}(H_s, T_p)]$ for four scenarios. The results also show that the combination is advantageous.

Figure 3. 9 Plot of $\text{CoV}[\hat{d}(H_s, T_p)]$ against H_s for $T_p=10.65$ s

Figure 3. 10 Plot of $\text{CoV}[\hat{d}(H_s, T_p)]$ against H_s for $T_p=11.15$ s

3.5.5 Case Study 4: Curve Fitting

The curve fitting is conducted by the modified weighted linear regression method developed in Section 3.4.4. The normalized mean damage rate $\hat{d}(H_s, T_p)$ and the integrand $\hat{d}(H_s, T_p)f(H_s, T_p)$ for varying H_s with $T_p=11.15$ s are plotted in Figures 3.11(a)-(b). The three methods mentioned in Section 3.5.1 are compared. The normalized curve fitting weights with optimization and without optimization are shown in Fig. 3.11(c). Since only the relative values matter, the normalization makes sense. As expected, the largest weight occurs at the vicinity of the applied spectrum.

(a) $\hat{d}(H_s, T_p)$

(b) $\hat{d}(H_S, T_p)f(H_S, T_p)$

(c) Curve-Fitting Weights

Figure 3. 11 Plot of various quantities against H_S for $T_p=11.15$ s

(a) $\hat{d}(H_S, T_p)$; (b) $\hat{d}(H_S, T_p)f(H_S, T_p)$ and (c) curve-fitting weights

From Fig. 3.11(a), compared with numerical method which is set as benchmark, the proposed method does not seem good, especially when H_s is large. Furthermore, the optimization makes the results even slightly worse. It makes sense that optimization compromises the accuracy at the applied spectrum to improve the accuracy at other places. In Fig. 3.11(b), the integrand curve produced by proposed method shows a good agreement with the curves obtained by numerical method. The area under this integrand is important, which is given by the 1D integral $I_{1D}(T_p) = \int_0^\infty \hat{d}(H_s, T_p) f(H_s, T_p) dH_s$. As indicated in Fig. 3.11(b), the areas under three different methods are in good agreement. It can be under the explanation that the overestimation of the integrand for smaller H_s is compensated by the underestimation by the larger H_s . As a whole, the curve fitting strategy can be considered as a success, whose weights are properly calculated to make the error in the 1D integral minimum (not the damage). Moreover, it is pointed out that the disparity appeared in Fig. 3.11(b) is not a systematic error which may be different for different set of samples. Take $T_p=11.4s$ as another example, which is between two applied spectra. Fig. 3.12(a)-(b) gives the plot of $\hat{d}(H_s, T_p)$ and $\hat{d}(H_s, T_p)f(H_s, T_p)$, respectively. The comments for the above example remain relevant expect for that, the optimization improves the accuracy of the proposed method.

(a) $\hat{d}(H_s, T_p)$

$$(b) \hat{d}(H_s, T_p) f(H_s, T_p)$$

Figure 3. 12 Plot of various quantities against H_s for $T_p=11.4$ s

3.5.6 Fatigue Damage Results and Comparison of Methods

The 1D integral after optimization calculated for a range of T_p is plotted in Fig. 3.13. It should be noted that the proposed importance sampling is not an approximate method which produces an unbiased estimation of the mean damage, which can be considered just as a different simulation technique compared to conventional Monte Carlo. Even though the disparity between the methods is informative to some extent, too much attention should not be paid to this disparity, because the present results are specific to a particular set of samples. Instead, sampling variability is much more important, which can be estimated straightforwardly by bootstrapping (200 samplings are considered here). The standard error of the 1D integrals calculated using bootstrapping are depicted by error bars in Fig. 3.13.

Figure 3. 13 1D Integral calculated for a range of T_p , with error bars using proposed method with optimization

As discussed in Section 3.5.3, $N = 200$ is used for peak applied spectrum $\mathbf{S}_A^*$ and smaller N values are considered for other applied spectra. The law of selecting N is to make the standard errors of $\hat{d}(\mathbf{S}_A)$ equal for all applied spectra, for instance, $\frac{SSD[\hat{d}(\mathbf{S}_{Ak})]}{\sqrt{N_k}} = SSD[\hat{d}(\mathbf{S}_A^*)]/\sqrt{200}$, where SSD indicates the sample standard deviation, N_k is the value of N for the k th applied spectrum. Since $SSD[\hat{d}(\mathbf{S}_{Ak})]$ is known prior, a minimum initial value $N_k=20$ is set. Then, more simulations are conducted until the above principle is satisfied, and $SSD[\hat{d}(\mathbf{S}_{Ak})]$ is updated after each simulation.

The comparison study results among the three methods are listed in Table 3.6, which gives the following results: mean value of normalized fatigue damage $\bar{d}_{LT}$, total number of samples N_{tot} for 5-min simulations required, and the RSE of $\bar{d}_{LT}$ evaluated by bootstrapping. It is important to indicate that $\bar{d}_{LT}$ should not be compared at face value due to inherent sampling variability in all methods. Also, the numerical integration may be biased due to discretization errors.

Table 3. 6 Comparison of different methods

	Numerical integration	Proposed method (no optimization)	Proposed method (with optimization)
Normalized damage $\bar{d}_{LT}$	0.189	0.179	0.182
No. of samples N_{tot}	592000	653	585
$RSE(\bar{d}_{LT})$	1.81×10^{-2}	4.51×10^{-2}	3.71×10^{-2}
$N_{tot}RSE^2(\bar{d}_{LT})$	1.95	1.33	0.81

Due to the difference in sampling variability, it is not meaningful to compare the single value N_{tot} . Since the $RSE(\bar{d}_{LT})$ is proportional to $1/\sqrt{N_{tot}}$, the parameter $N_{tot}RSE^2(\bar{d}_{LT})$ is calculated to measure the computational effect, considering the sampling variability. From the results shown in Table 3.6, the proposed method has a great advantage in the computational efficiency compared with numerical integration. In the overall effectiveness measured by $N_{tot}RSE^2(\bar{d}_{LT})$, the optimized proposed method shows the most advantages among the three. For the classical numerical integration method, N_{tot} is required to be extremely large in order to cover the relevant integration domain and a relatively coarse grid may lead to biased result. As a contrast, the discretization for the proposed method can be made very fine without additional dynamic analysis, and the RSE value is also acceptable.

Finally, the contours of the integrand generated by the proposed method with optimization are depicted in Fig. 3.14.

Figure 3. 14 Contours of the integrand generated by using proposed method with optimization

3.6 Point Estimates Method

The detail introduction of point estimates method is discussed in section 2.4.5.11. In this section, a seven-point estimate method is employed to solve the two-dimensional random variable's integral problem stated in this chapter. Table 3.7 shows the transformations from standard normal space to original random variables H_s and T_p space.

Table 3. 7 Seven-point transformation

No	1	2	3	4	5	6	7
	u_0	u_{1+}	u_{1-}	u_{2+}	u_{2-}	u_{3+}	u_{3-}
value	0	1.154	-1.154	2.367	-2.367	3.750	-3.750
$\Phi(u)$	0.500	0.876	0.124	0.991	0.009	0.999	8.83×10^{-5}
H_s	2.160	4.389	1.012	7.550	0.457	11.895	0.184
T_p	8.978	11.619	6.937	15.234	5.291	20.752	3.884
P	0.457	0.240	0.240	0.031	0.031	0.548×10^{-3}	0.548×10^{-3}

In the above table, $\Phi(u)$ is the cumulative distribution function (CDF) of $u_0, u_{1+}, u_{1-}, u_{2+}, u_{2-}, u_{3-}, u_{3+}$ in standard normal space, P is the corresponding weight. Table 3.8 is the fatigue damage of intermediate steps of seven-point estimate method.

Table 3. 8 Intermediate steps' value of seven-point estimate method

$T_p = mean$							
H_s	i = 1	i = 2	i = 3	i = 4	i = 5	i = 6	i = 7
g_i	2.66E-11	2.23E-10	2.73E-12	1.14E-09	2.52E-13	4.49E-09	1.65E-14
$H_s = mean$							
T_p	i = 1	i = 2	i = 3	i = 4	i = 5	i = 6	i = 7
g_i	2.66E-11	2.43E-11	1.16E-12	5.15E-12	1.19E-15	1.10E-12	1.65E-14
P	0.457143	0.240123	-0.24012	0.030757	-0.03076	0.000548	-0.00055

The above table is the process of calculating the g_i in Equation (2.67) in section 2.4.5.11. g_μ is the fatigue value evaluated at the mean of all variables H_s and T_p .

According to Equation (2.67), the final total fatigue damage can be obtained as:

$$g'(\theta) = \sum_{i=1}^n (g_i - g_\mu) + g_\mu \approx 9.59 \times 10^{-11}$$

The total fatigue value proposed by point estimates method will be normalized by dividing by the result obtained by global peak value of fatigue damage at $(T_p = 11.15s, H_s = 5.2m)$ by numerical method in the next section.

3.7 Results and Discussion

3.7.1 Results

Table 3.9 is the comparison of different methods for the total fatigue damage calculated by Equation (3.6).

Table 3. 9 Different methods comparison

method used	numerical	seven-point estimate	proposed method (10components)	
			before opt	after opt
Normalized damage	0.189	0.233	0.179	0.182
samplings number required (function calls)	592000	2600	653	585
dividing components time (function calls)	0	0	30	21
total simulation required(function calls) N_{tot}	592000	2600	683	606
$RSE(\bar{d}_{LT})$	1.81×10^{-3}	1.12×10^{-1}	4.51×10^{-2}	3.71×10^{-2}
$N_{tot} RSE^2(\bar{d}_{LT})$	1.953	32.60	1.391	0.806

- 1) Each function call is a duration of 300s simulation time.
- 2) The point increment calculated by numerical methods is $H_s(0 \sim 16m, \Delta H_s = 0.2m)$ $T_p(7.15 \sim 16.15s, \Delta T_p = 0.25s)$.
- 3) The point increment calculated by proposed methods is $H_s(0 \sim 16m, \Delta H_s = 0.1m)$ $T_p(7.15 \sim 16.15s, \Delta T_p = 0.1s)$.
- 4) For the point estimated method, 13 pairs of H_s and T_p are simulated with each based on 200 samplings (function calls).
- 5) For numerical method, each pair of H_s and T_p wave spectrum is based on 200 samplings (function calls).
- 6) The bootstrapping is based on 200 re-do –samplings.
- 7) The total fatigue damage in the above table is normalized by divided by the result obtained by global peak value of fatigue damage at $(T_p=11.15s, H_s=5.2m)$ by numerical method.

The last line $N_{tot} RSE^2(\bar{d}_{LT})$ is used as a parameter to compare the computation efforts required by different models to achieve a similar level of accuracy. It is chosen to combine the time parameter and sampling variability parameter together to get an integrated index for comparison convenience.

This comparison assumes that $RSE(\overline{d_{LT}}) \propto 1/\sqrt{N_{tot}}$. For example, for the direct numerical integration method to achieve the same RSE as the proposed method after optimization, the number of numerical simulations (function calls, dynamic analyses of the offshore platform) required is given as follows:

$$\frac{RSE(before)}{RSE(after)} = \frac{\frac{1}{\sqrt{N_{tot(b)}}}}{\frac{1}{\sqrt{N_{tot(a)}}}}$$

$$\frac{0.00181}{0.0371} = \frac{\frac{1}{\sqrt{592000}}}{\frac{1}{\sqrt{N_{tot(a)}}}}$$

$$N_{tot(a)} \approx 1409$$

Therefore, to get the same RSE as the proposed method after optimization, the direct numerical integration method needs 1409 simulations which are more than twice those required by the proposed method after optimization. However, even with the same sampling variability, the result obtained by the direct numerical integration is still biased while the result obtained by the proposed method after optimization is unbiased. Below gives the explanation:

Since for this research, there are in total 4 random variables: $H_s, T_p, a, \text{ and } \varepsilon$, if direct numerical integration method is used, two random variables H_s and T_p are divided into 7 segments and the other two random variables are divided into 6 segments, then the total numerical simulation calls required would be $7^2 \times 6^2 = 1764$ (a bit higher than 1409, for integral convenience). This means that the point increment for the direct numerical integration method would be: $\Delta H_s = 2.29m, \Delta T_p = 1.29s$ (with the previous range unchanged). The Figure 3.14 shows the contours of the integrand generated by the proposed method with optimization. From Figure 3.14, it is obvious that the above increment for the direct numerical integration method is too coarse to achieve an accurate, stable and unbiased result. The main practical difficulty for the direct numerical integration method lies in deciding suitable values for $H_{max}, T_{max}, \Delta H_s \text{ and } \Delta T_p$. For the direct numerical integration method, the mesh size should be fine enough to capture the peaks and any local fluctuations. Also, the upper limits should be imposed that the truncated region has negligible contribution.

On the contrary, the proposed method is unbiased and the grid size can be reduced without requiring additional dynamic simulations, which can avoid the discretization error problem always caused by the coarse direct numerical integration method.

Therefore, $N_{tot}RSE^2(\overline{d_{LT}})$ is one of the important metrics indicating the computation effort required by a numerical method in long-term fatigue analysis to achieve a similar level of accuracy in the results as the other methods. More than twice the computational saving of the proposed method compared with the direct numerical integration method is only one of the advantages of the proposed approach. Another strength of the newly proposed method is that it allows the analyst to determine the required simulation time to achieve an unbiased, stable with a certain acceptable level of accuracy.

3.7.2 Discussion

Long-term fatigue assessment of FPSO risers is computational expensive as it involves a double integral over the $H_s - T_p$ space and each integration point requires a dynamic analysis. It is rather challenging if the dynamic analysis is performed in the time domain which is always required due to system nonlinearities.

This chapter proposed a novel efficient method for the long-term fatigue analysis with time domain simulations. The time history of the wave elevation is expressed as a sum of regular waves with random amplitudes and phase angles. By invoking the importance sampling technique, the wave amplitudes can be obtained by simulating from a distribution different from the actual one. Then, the results for multiple sea states can be obtained by simply altering the weighting function instead of simulating each sea state which requires significantly large amount of computational time. This concept is very powerful potentially. However, there are difficulties when conducting the idea in its original form as discussed in Section 3.4.2. In order to solve the unacceptably large sampling variability problem, a succession of techniques are developed, including: 1st, choosing the applied wave spectra at the peak, which is introduced in Section 3.4.3.1; 2nd, dividing the wave spectrum by equal response variance for the purpose of maximizing each wave component; 3rd, combining the results from multiple applied wave spectra as detailed in Section 3.5.4; 4th, curve fitting technique based on a modified weighted linear regression scheme; and 5th, an optimization technique by selecting Rayleigh parameters for the wave amplitudes of applied spectra.

The proposed importance sampling method is illustrated on a turret-moored FPSO vessel moored with two flexible risers. For comparison purposes, direct numerical integration method and seven-point estimate method are considered. It is emphasized again that the proposed method is not an approximate method, but offers an alternative means for simulation. Therefore, the result obtained by proposed method is unbiased and it remains to calculate the standard error, which is achieved by bootstrapping technique. From all case studies and relevant results, an explicit advantage of the proposed method is demonstrated: computational efficiency, which is significantly prior to direct numerical integration, with the consideration of sampling variability. There is another benefit of the proposed method that the size of grid can be reduced without additional dynamic simulation, which may avoid the problem of discretization error that is always caused by coarse numerical method.

CHAPTER 4 MODELLING THE MULTIPLE CLASSICAL COPULA MODEL OF ENVIRONMENTAL RANDOM VARIABLES FOR THE OFFSHORE STRUCTURAL ANALYSIS

4.1 Introduction

In the previous chapter, an innovative approach for the long-term fatigue analysis of flexible risers in time domain is discussed and achieves good efficiency and accuracy.

This chapter mainly discusses the modelling of ocean environmental multivariate random variables experienced by offshore structures. The joint distribution model employed in the previous chapter is based on the traditional method (Haver, 2002) which is a joint conditional model. All the aforementioned joint distribution models only considered the linear relationship between pairs of random variables (Sagrilo, Lima and Papaleo, 2011; Haver, 2002), which may not be valid to the case where nonlinear relationship among random variables is important. Copula model is introduced in this chapter to study the dependence among random variables which can model both linear and nonlinear dependence among different variables. If the actual dependency between certain random variables is nonlinear, the commonly used traditional model considering nonlinear dependence is inadequate (Guedes Soares et al., 2001; Vanem, 2011). Till now, nonlinear correlation considered statistical models are not widely applied in engineering practice and are even seldom studied in offshore engineering field, which may be due to the lack of criteria or the difficulty of selecting the appropriate models, etc.

In this chapter, a comparative study will be conducted between the classical multiple copula model and traditional conditional joint distribution model. The biggest advantage of classical multiple copula models is its ability in treating nonlinear relationship among pairs of random variables. Later the effect of different statistical models on the long-term fatigue assessment result of mooring and riser systems will be analysed.

The materials in this chapter are presented in the journal paper (Gao and Cheung, 2016a) which has been submitted for review.

4.2 Traditional Models for Sea State Parameters

Among many probabilistic models in the review, conditional joint distribution model is widely adopted by the industry application. In this section, some popular traditional bivariate models and tri-variate models will be introduced.

4.2.1 Conditional Joint Distribution Model between H_s and T_p

The most popular joint distribution model in the offshore engineering field is the joint distribution model with two environmental random variables: significant wave height H_s and spectral peak period T_p which are the key factors for the construction of wave spectrum. In the literature, different distribution models have been proposed, all of which involve the marginal distribution of H_s and conditional distributions of T_p . Then the JPDP of the two random variables can be written as $f(H_s, T_p) = f(H_s)f(T_p|H_s)$.

In DNV design code (DNV, 2012), H_s is assumed to follow two-parameter Weibull distribution, while T_p is considered to follow a lognormal distribution conditioned on H_s . The expression is:

$$f(H_s) = \frac{k}{\lambda} \left(\frac{H_s}{\lambda}\right)^{k-1} \exp\left[-\left(\frac{H_s}{\lambda}\right)^k\right] \quad (4.1)$$

$$f(T_p|H_s) = \frac{1}{\sqrt{2\pi}\sigma T_p} \exp\left\{-\frac{(\ln T_p - \mu)^2}{2\sigma^2}\right\} \quad (4.2)$$

where parameters μ and σ can be expressed as a function of H_s ,

$$\mu = E[\ln T_p] = a_1 + a_2 h^{a_3} \quad (4.3)$$

$$\sigma^2 = \text{Var}[\ln T_p] = b_1 + b_2 \exp(b_3 h) \quad (4.4)$$

In the above equations, parameters k, λ can be determined by the fitting of the data set of H_s , $a_1, a_2, a_3, b_1, b_2, b_3$ are coefficients which can be obtained by the conditional model $f(T_p|H_s)$ fit with H_s fixed at several values of H_s , such as $\Delta H_s = 0.5s$.

Haver and Nyhus (1986) proposed a classical model that is popular in related research field (Naess and Moan, 2005). The classical model only differed from the DNV design codes model in the expression of the marginal distribution of H_s , which used piecewise functions:

$$f(H_s) = \begin{cases} \frac{1}{H_s \sigma \sqrt{2\pi}} \exp\left[-\frac{(\ln H_s - \mu)^2}{2\sigma^2}\right] & \text{if } H_s \leq \hat{\eta} \\ \frac{k}{\lambda} \left(\frac{H_s}{\lambda}\right)^{k-1} \exp\left[-\left(\frac{H_s}{\lambda}\right)^k\right] & \text{if } H_s > \hat{\eta} \end{cases} \quad (4.5)$$

where the parameters σ, μ, k, λ could be determined by fitting data set of H_s .

Besides, Ochi (1992) also introduced a bivariate lognormal distribution with marginal H_s fitted as lognormal distribution.

4.2.2 Conditional Joint Distribution Model among H_s , T_p and V_w

The above bivariate conditional models only consider the relationship between significant wave height H_s and spectral peak period T_p . Johannessen (2001) suggested a joint probabilistic model of wind and waves, including 1-hour mean wind speed at 10m above sea level V_w , significant wave height H_s , and spectral peak period T_p . The joint probabilistic model contains three parts: marginal distribution for the wind speed $f(V_w)$, conditional distribution of H_s for given V_w , $f(H_s|V_w)$ and conditional distribution of T_p , for given H_s and V_w $f(T_p|H_s, V_w)$. Then the joint PDF can be expressed as:

$$f(V_w, H_s, T_p) = f(V_w)f(H_s|V_w)f(T_p|H_s, V_w) \quad (4.6)$$

The marginal distribution of 1-hour mean wind speed at 10m is described by two-parameter Weibull distribution:

$$f(V_w) = \frac{k_w}{\lambda_w} \left(\frac{V_w}{\lambda_w} \right)^{k_w-1} \exp \left[- \left(\frac{V_w}{\lambda_w} \right)^{k_w} \right] \quad (4.7)$$

where the parameters k_w and λ_w are determined by curve fitting of data for V_w .

The conditioned distribution of H_s for given V_w is also described by 2-parameter Weibull distribution, which can be expressed as:

$$f(H_s|V_w) = \frac{k_h}{\lambda_h} \left(\frac{H_s}{\lambda_h} \right)^{k_h-1} \exp \left[- \left(\frac{H_s}{\lambda_h} \right)^{k_h} \right] \quad (4.8)$$

$$k_h = a_1 + a_2 V_w \quad (4.9)$$

$$\lambda_h = b_1 + b_2 V_w^{b_3} \quad (4.10)$$

where parameters a_1, a_2, b_1, b_2, b_3 can be estimated by regression analysis with k_h, λ_h obtained within each V_w range. Moreover, based on the characteristics of Weibull distribution, the parameters k_h and λ_h can be further employed to calculate the mean and standard deviation of H_s given wind speed V_w (represent each designed V_w range) with the following expressions:

$$E(H_s) = \lambda_h \Gamma \left(\frac{1}{k_h} + 1 \right) \quad (4.11)$$

$$\sigma(H_s) = \lambda_h \left[\Gamma \left(\frac{2}{k_h} + 1 \right) - \Gamma^2 \left(\frac{1}{k_h} + 1 \right) \right]^{0.5} \quad (4.12)$$

The conditioned distribution of T_p for given H_s and V_w is described by a log-normal distribution expressed as (Johannessen et al., 2001; Dong et al., 2011):

$$f(T_p | V_w, H_s) = \frac{1}{T_p \sigma_{\ln(T_p)} \sqrt{2\pi}} \cdot \exp \left[-\frac{1}{2} \left(\frac{\ln(T_p) - \mu_{\ln(T_p)}}{\sigma_{\ln(T_p)}} \right)^2 \right] \quad (4.13)$$

where $\mu_{\ln(T_p)}$ and $\sigma_{\ln(T_p)}$ are mean and standard deviation of the associated normal distribution respectively, which can be calculated as:

$$\mu_{\ln(T_p)} = \ln \left(\frac{\mu_{T_p}}{\sqrt{1 + v_{T_p}^2}} \right) \quad (4.14)$$

$$\sigma_{\ln(T_p)}^2 = \ln [v_{T_p}^2 + 1] \quad (4.15)$$

where $v_{T_p} = \sigma_{T_p} / \mu_{T_p}$, σ_{T_p} and μ_{T_p} are standard deviation and mean value of T_p , respectively.

In order to decide how T_p varies with H_s and V_w , the following expression is proposed by Johannessen (2001):

$$\mu_{T_p} = \overline{T_p}(h) \left[1 + \theta \cdot \left(\frac{V_w - \overline{V_w}(h)}{\overline{V_w}(h)} \right)^\gamma \right] \quad (4.16)$$

where $\overline{T_p}(h)$ is the mean value of spectral period for a given H_s , $\overline{V_w}(h)$ is the mean value of wind speed for a given H_s . $\overline{T_p}(h)$ and $\overline{V_w}(h)$ can be fitted by the following expression:

$$\overline{T_p}(h) = c_1 + c_2 \cdot H_s^{c_3} \quad (4.17)$$

$$\overline{V_w}(h) = d_1 + d_2 \cdot H_s^{d_3} \quad (4.18)$$

In the above equations (4.14) ~ (4.18), the coefficients $c_1, c_2, c_3, d_1, d_2, d_3$ can be determined by fitting easily, while θ, γ and σ_{T_p} can be obtained by the methods proposed by Johannessen (2001).

4.3 Copula Theory

The word ‘copula’ originates from the Latin noun, which means “link” or “lie” (Cassell’s Latin Dictionary) and was used as grammar. Sklar (1959) was the first one employing copula in a

mathematical or statistical sense and from the original meaning of this word, the basic function of copula is to describe the dependence between random variables. As stated by Fisher (1997), copula is popular in probability and statistics for two main reasons: scale free measures of dependence can be studied and bivariate as well as multivariate distributions can be constructed by one-dimensional distribution function with simulation views.

Till now, copula theory has been popular in the terrain of quantitative finance (Longin and Solnik, 2001; Ang and Chen, 2002; Cooke et al., 2011; Aas et al., 2009). Quite recently, this concept also captures the attention of researchers in the fields of transportation (Thompson et al., 2011), offshore engineering (Zhang et al., 2015) and reliability engineering etc.

However, the study of copulas and their application in the field of probability, statistics and stochastic process are still in the preliminary stage with many open problems to be solved.

4.3.1 Definition and Basic Properties

In general definition, the concept of copula can be interpreted as functions that link multivariate distribution functions to one dimensional marginal distribution functions. The mathematical definition is given as follows:

Suppose $(X_1, X_2, \dots, X_d)$ is a random vector with continuous marginal distributions $F_i(x) = P[X_i \leq x], i = 1, 2, \dots, d$. Then let the random vector $(U_1, U_2, \dots, U_d)$ have uniformly distributed marginals:

$$(U_1, U_2, \dots, U_d) = (F_1(X_1), F_2(X_2), \dots, F_d(X_d)) \quad (4.19)$$

Then the copula of $(X_1, X_2, \dots, X_d)$ is defined as:

$$C(u_1, u_2, \dots, u_d) = P[U_1 \leq u_1, U_2 \leq u_2, \dots, U_d \leq u_d] \quad (4.20)$$

In the above Equation, all the information in the dependence structure among $(X_1, X_2, \dots, X_d)$ is included in the copula $C(u_1, u_2, \dots, u_d)$ and all the marginal distribution related information is contained in $F_i, i = 1, 2, \dots, d$.

A d -dimensional copula $C: [0,1]^d \rightarrow [0,1]$ has following basic properties:

$$C(u_1, u_2, \dots, u_{i-1}, 0, u_{i+1}, \dots, u_d) = 0 \quad (4.21)$$

The copula equals to zero when any of the argument is zero.

$$C(1, \dots, 1, u, 1, \dots, 1) = u \quad (4.22)$$

The copula equals to u if all arguments are 1 except one is u .

The third property is that copula is d -increasing. Take bivariate copula for instance,

$$C(u_2, v_2) - C(u_2, v_1) - C(u_1, v_2) + C(u_1, v_1) \geq 0 \quad (4.23)$$

For all $0 \leq u_1 \leq u_2 \leq 1$ and $0 \leq v_1 \leq v_2 \leq 1$.

Sklar' Theorem

Let H be the joint distribution function with margins $X_1, \dots, X_d$ and there is a copula C :

$$H(x_1, \dots, x_d) = C(F_1(x_1), \dots, F_d(x_d)) \quad (4.24)$$

where $F_i(x_i) = P[X_i \leq x]$.

Then the density of the multivariate distributions $f(x_1, \dots, x_d)$ can be expressed as:

$$f(x_1, \dots, x_d) = c(F_1(x_1), \dots, F_d(x_d)) \cdot f_1(x_1) \cdot \dots \cdot f_d(x_d) \quad (4.25)$$

where c is the density of the copula and $f_i(x_i)$, ($i = 1, \dots, d$) is the PDF of each marginal distribution. The copula density c can be obtained as the derivation of n th partial derivative of copula function C :

$$c(F_1(x_1), \dots, F_n(x_n)) = \frac{\partial^n (C(F_1(x_1), \dots, F_n(x_n)))}{\partial F_1(x_1) \dots \partial F_n(x_n)} \quad (4.26)$$

In this theorem, it is stated that if $F_1, \dots, F_d$ are continuous, then the copula C is unique. On the other hand, if the copula C is defined, and $F_1, \dots, F_d$ are marginal distribution functions, then $C(F_1(x_1), \dots, F_d(x_d))$ defines a d -dimensional cumulative distribution function.

4.3.2 Families of Copula

In this section, several families of copula will be introduced. As discussed by Nelson (2006), each family or class of copula has its own characteristic and has its own advantage and disadvantage which will be suitable for certain types of data.

Gaussian Copula

Gaussian copula is quite popular in risk analysis field (Renard and Lang, 2007) and is constructed from a multivariate normal distribution. For the application of Gaussian copula, all variables should be transformed to standard normal distribution space from the original distributions.

A d -dimensional Gaussian Copula can be expressed:

$$C_R^{Gauss}(u) = \Phi_R(\Phi^{-1}(u_1), \dots, \Phi^{-1}(u_d)) \quad (4.27)$$

where $R \in \mathbb{R}^{d \times d}$ is the correlation matrix, Φ^{-1} is the inverse CDF of a standard normal distribution, and Φ_R is the joint CDF of a multivariate normal distribution with zero mean and covariance matrix R .

The density of Gaussian Copula can be expressed as:

$$c_R^{Gauss}(u) = \frac{1}{\sqrt{|R|}} \exp\left(-\frac{1}{2} \mathbf{x}^T (R^{-1} - \mathbf{I}) \mathbf{x}\right) \quad (4.28)$$

where $x_i = \Phi^{-1}(u_i)$, ($i = 1, \dots, d$), $\mathbf{I} \in \mathbb{R}^{d \times d}$ is the identity matrix and $|\cdot|$ denotes the determinant.

Gaussian copula belongs to elliptical copula family and its distributions are radially symmetric, without upper or lower tail dependence. In Gaussian copula, the same linear dependency covers all data within the domain. Therefore, it may not be suitable to model the structural dependence that is not consistent over the entire domain (Lebrun and Dutfoy, 2009).

Archimedean Copula

Archimedean Copula was first studied in the development of probabilistic view of triangle inequality (Schweizer, 1991). Compared with triangular copulas which do not have closed form of expressions, all commonly encountered Archimedean copulas have explicit formula and are superior to treat asymmetrical problems (Embrechts et al., 2001).

An Archimedean copula can be expressed as:

$$C(u_1, \dots, u_d; \theta) = \psi^{[-1]}(\psi(u_1; \theta) + \dots + \psi(u_d; \theta); \theta) \quad (4.29)$$

where $\psi: [0,1] \times \Theta \rightarrow [0, \infty)$ is a strictly decreasing convex function with $\psi(1; \theta) = 0$. θ is a parameter within the parameter space Θ . ψ is the generator function and $\psi^{[-1]}(t; \theta) = 0$ for

all $t \geq \psi(0; \theta)$. Following table 4.1 gives the most prominent bivariate Archimedean copulas:

Table 4. 1 Some important Archimedean Copulas

Copula	Bivariate Formula	Generator Function	Parameter θ
Ali-Mikhail-Haq	$\frac{uv}{1 - \theta(1-u)(1-v)}$	$\ln \left[\frac{1 - \theta(1-t)}{t} \right]$	$\theta \in [-1, 1)$
Clayton	$[\max\{u^{-\theta} + v^{-\theta} - 1; 0\}]^{-1/\theta}$	$\frac{1}{\theta}(t^{-\theta} - 1)$	$\theta \in [-1, \infty) \setminus \{0\}$
Frank	$-\frac{1}{\theta} \ln \left[1 + \frac{(\exp(-\theta u) - 1)(\exp(-\theta v) - 1)}{\exp(-\theta) - 1} \right]$	$-\ln \left(\frac{\exp(-\theta t) - 1}{\exp(-\theta) - 1} \right)$	$\theta \in \mathbb{R} \setminus \{0\}$
Gumbel	$\exp \left[- \left((-\ln(u))^\theta + (-\ln(v))^\theta \right)^{1/\theta} \right]$	$(-\ln(t))^\theta$	$\theta \in [1, \infty)$
Independence	uv	$-\ln(t)$	
Joe	$1 - [(1-u)^\theta + (1-v)^\theta - (1-u)^\theta(1-v)^\theta]^{1/\theta}$	$-\ln(1 - (1-t)^\theta)$	$\theta \in [1, \infty)$

Figure 4.1 gives the comparison of different bivariate copulas with each marginal distribution following a standard normal distribution and the linear correlation coefficient 0.9.

Figure 4. 1 Comparison of different bivariate copulas with each marginal distribution following a standard normal distribution and the linear correlation coefficient 0.9

From Figure 4.1, it can be easily found that Clayton Copula is more suitable in describing data with strong lower tail dependence, while Gumbel Copula is good at describing data with stronger correlations at higher values. Frank Copula is more appropriate at modelling data with weak dependence at tails.

4.3.3 Dependence Structure of Copula

Pearson's product-moment correlation

There are various ways to discuss and measure the dependence among random variables, such as the most popular Pearson's product-moment correlation which measures the linear dependence between random variables. The formula for Pearson's correlation coefficient between two random variables X_1 and X_2 is:

$$\rho_{X_1 X_2} = \frac{CoV(X_1, X_2)}{\sigma_{X_1} \sigma_{X_2}} \quad (4.30)$$

where $CoV(X_1, X_2) = E[(X_1 - E(X_1))(X_2 - E(X_2))]$, is the covariance between X_1 and X_2 . σ_{X_1} and σ_{X_2} are the standard deviation of X_1 and X_2 , respectively, and $E()$ is the expectation function.

Pearson's correlation coefficient is commonly used in most multivariate analysis and it can be easily estimated from the data. However, it can only measure linear correlation and is not invariant to strictly increasing nonlinear transformations. Therefore, rank correlation coefficient may be more appropriate to certain problems.

The most widely known scale-invariant measures of association are called Kendall's tau and Spearman's rho.

Concordance

For a pair of random variables $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$, if larger values of one pair tend to be associated with larger values of the other pair, that is if $x_i > x_j$ and $y_i > y_j$ or smaller values associated with smaller values of the other pair, such as if both $x_i < x_j$ and $y_i < y_j$, then the pair of random variables are called to be concordant. Else, they are discordant.

Kendall's tau

Kendall's tau is a measure of rank correlation, developed by Maurice Kendall in 1938 (Kendall, 1938). The expression for Kendall's tau τ is:

$$\tau = \frac{(\text{number of concordant pairs}) - (\text{number of discordant pairs})}{\frac{1}{2}n(n-1)} \quad (4.31)$$

where n is the number of total pairs.

Spearman's rho

Similar to Kendall's tau, Spearman's rho is also a nonparametric measure of statistical dependence between two variables. ρ can be obtained by:

$$\rho = 1 - \frac{6 \sum d_i^2}{n(n^2-1)} \quad (4.32)$$

where $d_i = x_i - y_i$ is the difference between ranks and n is the number of total pairs. (Wikipedia).

4.4 Comparative Study

In this section, the copula approach of modelling the same multivariate ocean data is compared with the traditional method. The detailed procedure of construction of the multivariate model by two different methods will be stated and some model selection techniques will be discussed in the next section.

4.4.1 Data Pre-Treatment

The data collected for this research is obtained from National Data Buoy Centre. The location of data collected is in the east coast of Brazil in the Atlantic (7.91°S, 30.49°W) and the water depth near the location is 1500m. The data set collected is from year 1973 to 2014, covering 42 years span of hourly records with several years' data lost. The ocean parameters collected and being used in this section include: significant wave height H_s (m), spectral peak period T_p (s) and wind speed V_w (m/s). Since the direction parameters such as wave direction and wind direction data are mostly missing, the study in this section does not include direction parameters.

Before applying the multivariate data into statistical analysis, some pre-treatment should be done. Figure 4.2 shows the mean value of ocean parameters H_s , T_p and V_w in different years.

Figure 4. 2 Mean value of ocean parameters in different years

From Figure 4.2, the mean value of ocean parameters is relative stable for each year, the details are:

$$\frac{CoV(\mu_{H_s})}{\sqrt{N}} = \frac{\sigma_{\mu_{H_s}}}{E(H_s) * \sqrt{N}} = 0.0201 \quad (4.33)$$

$$\frac{CoV(\mu_{T_p})}{\sqrt{N}} = \frac{\sigma_{\mu_{T_p}}}{E(T_p) * \sqrt{N}} = 0.0065 \quad (4.34)$$

$$\frac{CoV(\mu_{v_w})}{\sqrt{N}} = \frac{\sigma_{\mu_{v_w}}}{E(V_w) * \sqrt{N}} = 0.0161 \quad (4.35)$$

where μ_{H_s} , μ_{T_p} and μ_{v_w} are the mean value of H_s , T_p and V_w within each year, respectively. N is the total number of years calculated. From the above analysis, the parameters within the 42 years are quite stable.

4.4.2 Application of 3-Dimensional Traditional Models

Based on the data extracted, three-dimensional conditional model discussed in Section 4.2 is applied. As introduced in Equation (4.6) to Equation (4.18), many parameters should be estimated in the construction of the three-dimensional conditional models. If all parameters are

decided by the maximum likelihood method, it would be rather complicated and difficult, especially when the data set is very large. Hence, the procedure for constructing the conditional model includes three main steps as the three multiplier factors in Equation (4.6).

The first step is to define the marginal distribution of 1-hour mean wind speed at 10m $f(V_w)$ which is described by two-parameter Weibull distribution. This step can be either decided by maximum likelihood method or regression analysis. Figure 4.3 shows the CDF of V_w obtained by maximum likelihood method and regression method compared with those constructed by raw data. The results show that CDF obtained by both two methods matches very well with the original data. The shape and scale parameters estimated by maximum likelihood and regression analysis are $k_l = 2.132, \lambda_l = 7.486$ and $k_r = 2.184, \lambda_r = 7.516$ respectively. The parameters' value obtained by maximum likelihood is employed here to continue the next step of parameter estimation.

Figure 4. 3 Comparison of CDF of V_w obtained by two methods and raw data

Table 4. 2 Regression results for curve fit of V_w

Regression fitting equation: $1-\exp(-(x/a)^b)$
General model: $f(x) = 1-\exp(-(x/a)^b)$
Coefficients (with 95% confidence bounds): $a = 7.516 (7.512, 7.521)$ $b = 2.184 (2.18, 2.188)$
Goodness of fit: SSE: 0.01236 R-square: 0.9999 Adjusted R-square: 0.9999 RMSE: 0.003519

The next step is to estimate parameters for the conditional distribution of wave height H_s for given wind speed V_w denoted by $f(H_s|V_w)$, which is also described by 2-parameter Weibull distribution. It is supposed that if the domain of V_w is divided into small intervals from all covered values, the conditional 2-parameter Weibull distribution is almost constant for each local V_w within the same small V_w interval. Then all H_s data are categorised into wind velocity V_w bands of 1.0m/s, indicated by their middle-point values, i.e. data falling between wind velocity of 5 and 6 m/s will be classified as 5.5 m/s. Figure 4.4 shows a curve fitting for the 2-parameter Weibull parameters of H_s conditioned on V_w . Table 4.3 gives the regression results for points representing V_w within interval values of 1.0.

Figure 4. 4 Curve fitting of (a) k vs V_w and (b) λ vs V_w based on Equations (4.9) and (4.10) for points representing V_w within interval values of 1.0

Table 4. 3 Regression results for points representing V_w within interval values of 1.0

Regression fitting equation: $k = a_1 + a_2 * V_w$	Regression fitting equation: $\lambda = b_1 + b_2 * V_w^{b_3}$
Coefficients (with 95% confidence bounds): $a_1 = 2.226 (1.819, 2.37)$ $a_2 = 0.085 (0.08735, 0.1269)$	Coefficients (with 95% confidence bounds): $b_1 = 1.157 (0.7673, 1.426)$ $b_2 = 0.034 (0.01262, 0.06137)$ $b_3 = 1.734 (1.502, 1.908)$
Goodness of fit: SSE: 2.614 R-square: 0.8513 Adjusted R-square: 0.8446 RMSE: 0.3447	Goodness of fit: SSE: 1.955 R-square: 0.9898 Adjusted R-square: 0.9889 RMSE: 0.2981

It should be pointed out that points in Figure 4.4 are mid-points within an interval of 1.0 m, and the points span from 0 m to 25 m in the domain of V_w . As a whole, the deviation from the fitted line is small especially in part (b). Furthermore, the number of points within each interval (in other words, the length of the interval for each local V_w) that are used in the fit would influence the estimation of the model parameters. Therefore, there is a trade-off between the accuracy in the fitting regression and the accuracy in the statistical parameters. Several experiments by changing the interval distance show that an interval of 1.0 m would obtain an appropriate result measured by maximum likelihood method.

The third step is to estimate the parameters for the conditional distribution of spectral peak period T_p for given wind speed V_w and significant wave height H_s . The conditional distribution $f(T_p|H_s, V_w)$ is described by a log-normal distribution as defined in Equation (4.13). As stated by Johannessen et al. (2001), a function was proposed to describe the trending how T_p vary with V_w and H_s , as shown in Equation (4.16). Similar to the second step, the significant wave height H_s is divided into small intervals from its whole domain. All H_s within each small interval are given the same value of the middle point. Then the conditional parameters of T_p and V_w are supposed to be constant as expressed in Equation (4.17) and (4.18). The range of the H_s interval is set to be 0.5 and is found appropriate after several experiments. Figures (4.5)

and (4.6) show a nonlinear fit for the mean value of T_p conditioned on H_s and mean value of V_w conditioned on H_s , respectively.

Figure 4. 5 The conditional mean spectral peak period $\overline{T_p}(h)$ vs H_s based on Equation (4.17) for points representing H_s within interval values of 0.5

Figure 4. 6 The conditional mean spectral peak period $\overline{V_w}(h)$ vs H_s based on Equation (4.18) for points representing H_s within interval values of 0.5

The results of fitted values are shown in table 4.4.

Table 4. 4 Regression results for points representing H_s within interval values of 0.5

Regression fitting equation: $\overline{T_p}(h) = c_1 + c_2 * H_s^{c_3}$	Regression fitting equation: $\overline{V_w}(h) = d_1 + d_2 * H_s^{d_3}$
Coefficients (with 95% confidence bounds): $c_1 = 6.117 (4.164, 8.071)$ $c_2 = 1.460 (-0.06685, 2.986)$ $c_3 = 0.699 (0.3448, 1.054)$	Coefficients (with 95% confidence bounds): $d_1 = 1.735 (-1.14, 4.61)$ $d_2 = 3.202 (1.072, 5.331)$ $d_3 = 0.764 (0.531, 0.9963)$
Goodness of fit: SSE: 6.057 R-square: 0.937 Adjusted R-square: 0.9304 RMSE: 0.5646	Goodness of fit: SSE: 17.04 R-square: 0.9735 Adjusted R-square: 0.9708 RMSE: 0.9471

From Figures 4.5, 4.6 and table 4.4, it can be observed that the fitting is rather good except some “jumps” at the higher tails.

Equation (4.16) can be rewritten as:

$$\frac{\mu_{T_p} - \overline{T_p}(h)}{\overline{T_p}(h)} = \theta \cdot \left(\frac{V_w - \overline{V_w}(h)}{\overline{V_w}(h)} \right)^\gamma \quad (4.36)$$

As discussed by Johannessen et al. (2001) and Dong et al. (2011), it is not difficult to find out that the normalized spectral period $\frac{\mu_{T_p} - \overline{T_p}(h)}{\overline{T_p}(h)}$ is almost linearly correlated to the normalized wind speed $\left(\frac{V_w - \overline{V_w}(h)}{\overline{V_w}(h)} \right)$, which indicates that γ is close to 1.

The standard deviation σ_{T_p} in Equation (4.15) of the conditional distribution of spectral peak period T_p for given wind speed and wave height is proposed based on experiences (Johannessen, 2001) and is expressed as:

$$\sigma_{T_p} = \left[-1.7 * 10^{-3} + 0.259 * \exp(-0.113 * H_s) \right] * \mu_{T_p} \quad (4.37)$$

The final step is to estimate the value of parameter $\theta = 0.008338$ by optimization of maximum the log likelihood of $f(V_w, H_s, T_p) = f(V_w)f(H_s|V_w)f(T_p|H_s, V_w)$.

Table 4.5 gives the parameters recommended for the use in the joint wind and wave distribution.

Table 4. 5 Parameters estimated for $f(V_w, H_s, T_p)$

Wind distribution (2-parameter Weibull distribution)	Shape parameter $k=2.132$ Scale parameter $\lambda=7.486$
Conditional distribution H_s conditioned on V_w (2-parameter Weibull distribution)	Shape parameter: $k = 2.226 + 0.085 * V_w$ Scale parameter: $\lambda = 1.157 + 0.034 * V_w^{1.734}$
Conditional distribution T_p conditioned on H_s and V_w (lognormal distribution)	Mean value of associated normal distribution $\mu_{\ln(T_p)} = \ln \left(\frac{\mu_{T_p}}{\sqrt{1 + v_{T_p}^2}} \right)$ Standard deviation of associated normal distribution $\sigma_{\ln(T_p)} = \sqrt{\ln [v_{T_p}^2 + 1]}$ where $v_{T_p} = \sigma_{T_p} / \mu_{T_p}$
Where $\mu_{T_p} = (6.117 + 1.46 * H_s^{0.6995}) * \left[1 - 0.008338 * \left(\frac{V_w - (1.735 + 3.202 * H_s^{0.7637})}{1.735 + 3.202 * H_s^{0.7637}} \right) \right]$ and $\sigma_{T_p} = [-1.7 * 10^{-3} + 0.259 * \exp(-0.113 * H_s)] * \mu_{T_p}$	

4.4.3 Application of 3-Dimensional Classical Copula Models

Unlike the conditional joint distribution approach discussed above, the copula approach requires the determination of marginal distributions for all variables. In order to search for the best marginal distribution model for each variable, three theoretically available and most frequently used marginal distribution models are adopted for all variables H_s, T_p and V_w

(Jaspers, 1956; Battjes, 1972; Ochi, 2011). Figures 4.7-4.9 show the three models' fit for the three variables. The parameters of models are estimated by maximum likelihood method and the results of each model for H_s , T_p and V_w are summarized in Table 4.6.

Figure 4. 7 Marginal parametric model fit for H_s

Figure 4. 8 Marginal parametric model fit for T_p

Figure 4. 9 Marginal parametric model fit for V_w

Table 4. 6 Results of marginal distribution model parameters estimate

	H_s	T_p	V_w
Lognormal Model PDF: $f(x \mu, \sigma) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln x - \mu)^2}{2\sigma^2}\right)$	$\mu = 0.489$ $\sigma = 0.485$ Loglikelihood= -217998 ^a	$\mu = 2.054$ $\sigma = 0.261$ Loglikelihood= -392130 ^a	$\mu = 1.738$ $\sigma = 0.638$ Loglikelihood= -498738
Gamma Model PDF: $f(x k, \theta) = \frac{1}{\Gamma(k)\theta^k} x^{k-1} e^{-\frac{x}{\theta}}$	$k = 4.234$ $\theta = 0.435$ Loglikelihood= -225695	$k = 14.961$ $\theta = 0.539$ Loglikelihood= -392564	$k = 3.361$ $\theta = 1.977$ Loglikelihood= -478934
Weibull Model PDF: $f(x k, \lambda) = \frac{k}{\lambda} \left(\frac{x}{\lambda}\right)^{k-1} \exp\left[-\left(\frac{x}{\lambda}\right)^k\right]$	$\lambda = 2.09$ $k = 1.969$ Loglikelihood= -239582	$\lambda = 8.875$ $k = 3.931$ Loglikelihood= -404218	$\lambda = 7.486$ $k = 2.132$ Loglikelihood= -472987 ^a

^a indicates the best model with maximum log-likelihood value.

From Figures 4.7-4.9 and table 4.6, the selected models for significant wave height H_s , spectral period T_p and wind speed V_w are Lognormal Model, Lognormal Model and Weibull Model respectively.

After marginal distribution models for three variables are selected, the next step is to construct three-dimensional copula models. There are many selection options for two-dimensional copula models, such as Gaussian, Gumbel, Clayton and Frank (Nelson, 2006). For multiple dimensional copulas, the selection space is quite limited. In this subsection, Gaussian Copula and T Copula models are chosen in this work. The law of maximizing the pseudo-likelihood function value is employed to estimate the corresponding parameters for each copula model. Table 4.7 gives the parameters estimated for the two three-dimensional copula models.

Table 4. 7 Parameters estimated for Gaussian and T Copula models

Copula families: (H_s, T_p, V_w)	Parameters estimate	Total log-likelihood
Gaussian Copula	$\theta = \begin{bmatrix} 1 & 0.3196 & 0.6426 \\ 0.3196 & 1 & -0.0489 \\ 0.6426 & -0.0489 & 1 \end{bmatrix}$	-1012080
T Copula	$\theta = \begin{bmatrix} 1 & 0.3092 & 0.6528 \\ 0.3092 & 1 & -0.0568 \\ 0.6528 & -0.0568 & 1 \end{bmatrix}$ Degrees of freedom: 22.126	-1011140

4.5 Model Selection

From the above discussion, traditional 3D model as well as three-dimensional classical Gaussian and T copula statistical models are constructed based on data collection. The corresponding parameter values in each statistical copula model and each constructed marginal distribution are estimated based on the maximization of the marginal distribution likelihood function separately for each variable and copula likelihood function, which can be expressed as:

$$\hat{\theta}_{MLE} = \begin{cases} \max_{\theta_1 \in \Theta} \sum_{j=1}^m \ln c(F_1(x_{1j}), \dots, F_n(x_{nj})) \\ \max_{\theta_2 \in \Theta} \sum_{j=1}^m \sum_{i=1}^n \ln f_i(x_{ij}) \end{cases} \quad (4.38)$$

where n is the number of copula dimension and m is the number of samples collected in the database for study. Θ includes all parameters in both marginal and copula functions. θ_1 and θ_2

are parameters in copula function and marginal distribution function, respectively (Genest and Favre, 2007).

With different statistical models constructed already, the next step is to select models by model selection methods. In the literature, Akaike information criterion (AIC) (Akaike, 1974) and Bayesian Information Criterion (BIC) (Schwarz, 1978) are very popular in model selection. The best-fit copula can be determined by the minimum value of the AIC and BIC, which can be expressed as:

$$AIC = -2l(k) + 2k \quad (4.39)$$

$$BIC = -2l(k) + k \ln(m) \quad (4.40)$$

where k is the number of parameters, $l(k)$ is the maximum log-likelihood which can be written as:

$$l(k) = \sum_{j=1}^m \ln c(F_1(x_{1j}), \dots, F_n(x_{nj})) + \sum_{j=1}^m \sum_{i=1}^n \ln f_i(x_{ij}) \quad (4.41)$$

where $f_i(x_{ij})$ is the marginal PDF of j th sample's i th variable. n is the number of copula dimension and m is the number of samples collected.

For AIC model, the first log-likelihood function part is proportional to the number of samples m , while the penalty term is proportional to the number of parameters. The first term will dominate when the sample number is large.

BIC appears as another independent version, which was developed by Akaike (1976) and Schwarz (1978). The penalty term increases with the number of sample compared with AIC.

Besides AIC and BIC, model selection using dynamic data from some system is an active area of research for a long-time history in many fields such as civil and mechanical engineering (Gersch et al., 1976; Beck, 1978; Beck and Katafygiotis, 1998; Sanayei et al., 1999; Vanik et al., 2000). Beck and Yuen (2004) proposed a Bayesian approach based on their probabilities conditional on the response data, which finally provides a quantitative expression of a principle for selecting the most plausible class of modes. Moreover, when the number of sample is large, the BIC value agrees with the leading order terms of the logarithm of the evidence of the Bayesian approach (Beck and Yuen, 2004).

In this work, the total number of data collected is 184212. AIC and BIC are adopted to test the quality of the potential models. The results will be presented and discussed in section 4.7.

4.6 Comparison of Fatigue Assessment Based on Different Environment Statistical Modelling

4.6.1 General Description

Similar to the FPSO model described in Chapter 3, a turret is installed to connect all mooring lines and flexible risers, as seen in Figure 4.10. 1500-meter water depth is selected in this study. The whole system consists of one vessel, two flexible risers and four mooring lines connected at the turret. JONSWAP spectrum is chosen to describe the wave environment with parameters: significant wave height H_s and spectral peak period T_p . According the discussion in the above sections of this chapter, besides H_s and T_p , 1-hour mean wind speed at 10m above sea level V_w is also considered as the third environmental random variable in the proposed statistical models.

Figure 4. 10 Overview of the whole system

Similar to Chapter 3, OrcaFlex Dynamics (Version 9.5a) is employed to implement the time domain dynamic coupled analysis and subsequent rainflow counting analysis. It is assumed that all wave, wind and current loads come from the same direction.

The length of the FPSO is 300 meters long and the mass of that is 13.20×10^6 kg. The specific point near the top end of the flexible riser is chosen as the study point. S-N curve is used for the fatigue damage calculation and the parameters of S-N curve can be chosen from reference (DNV, 2011, Fatigue Design of Offshore Steel Structures). The characteristics of the mooring lines and flexible risers are shown in Table 4.8 and Table 4.9, respectively.

Table 4. 8 Flexible Riser Characteristics

Parameter	Value	Parameter	Value
Outer Diameter(m)	3.56×10^{-1}	Wall Thickness(m)	0.102
Mass per Unit Length(kg/m)	1.84×10^2	Bending stiffness(N*m ²)	1.25×10^5
Poisson Ratio	5.00×10^{-1}	Axial stiffness(N)	7.11×10^8
Added mass coefficient	1.00	Total Length(m)	1.75×10^3

Table 4. 9 Mooring Line Characteristics

Parameter	Value	Parameter	Value
Outer Diameter(m)	5.76×10^{-1}	Axial stiffness(N)	9.38×10^9
Mass per Unit Length(kg/m)	2.03×10^3	Poisson Ratio	5.00×10^{-1}
Drag coefficient	1.00	Added mass coefficient	1.00
Bending stiffness(N)	0	Total Length(m)	2.00×10^3

Classical Monte Carlo method is employed to generate samples for all the above three statistical models. For classical copula models, the generation step can be divided into two steps: 1st step is to generate CDF of each random variable according to their copula relation functions; 2nd step is to transfer each CDF to their original data.

4.6.2 Wave Amplitude Assumption

In the work of this chapter, the amplitude for each wave component is treated as deterministic as $\sqrt{2S_{\eta\eta}\Delta\omega}$ for each wave component, which is also the mean value of random amplitude as discussed in section 2.2.1. Compared with Rayleigh distribution based random wave amplitude assumption, the sampling variability of time history generated by each pair of significant wave height H_s and spectral peak period T_p is much smaller. Table 4.10 is the sampling variability test of three different assumptions:

Table 4. 10 Sampling variability test for one fixed set of (H_s, T_p)

Normalized fatigue damage based on different assumptions				
different assumptions	N=5		N=200	
	Mean	CoV/sqrt(N)	Mean	CoV/sqrt(N)
1.random amplitude and phase	0.903	0.164	1.003	0.038
2.fixed amplitude and random phase	0.978	0.037	1	0.008
3.random amplitude and fixed phase	0.852	0.103	0.819	0.032

The mean value of fatigue damage in table 4.10 is normalized by dividing by the absolute mean value of fatigue damage of 200 samplings according to the 2nd assumption in Table 4.10. The first assumption of random amplitude and phase is the most precise one with biggest sampling variability which requires biggest number of samples to get a stable mean value. The second assumption of fixed amplitude and random phase is the most popular in engineering field with less sampling variability but quite convenient and realistic in use. The third assumption is seldom used and most probability due to the unprecise result is shown in table 4.10.

In this work in this chapter, the second assumption is employed. Hence, for each pair of (H_s, T_p) , the fatigue damage is calculated as the mean value of 5 samples with different phases, and each sampling lasts for 5 minutes similar to chapter 3.

4.7 Results and Discussion

The AIC and BIC model selection statistics as well as log-likelihood function values are presented in Table 4.11 for different statistical models. The results shown in table 4.11 shows that the traditional conditional joint distribution approach gives the largest AIC and BIC value, while the 3D T copula models give the lowest AIC and BIC which indicates that they are the best models. From the three statistical models, the two copula models show a bit superior to the traditional model.

Table 4. 11 Comparison of goodness-of-fit to the data for 3 statistical models

(H_s, T_p, V_w) , based on 184212 samples	Total log-likelihood	No of parameter	AIC	BIC
3D Gaussian Copula	-1012080	9	2024179	2024208
3D T Copula*	-1011140	10	2022300	2022333
Traditional model	-1032000	17	2064033	2064089

(Notes: The lowest AIC or BIC value indicates the best model, highlighted by “*”.)

Table 4.12 gives the comparison results of normalized long-term fatigue damage. The value of fatigue damage shown in Table 4.12 is normalized by dividing the fatigue damage value of T copula. Suppose 3D T copula model is the best model as estimated by model selection, there seems to be around 28% difference between the traditional conditional joint distribution model and best fit T copula model when they extract data from the same environmental history data set.

Table 4. 12 Comparison of normalized fatigue damage for 3 statistical models

Model type	normalized fatigue damage, N=2000	
	Mean	CoV/sqrt(N)
traditional	0.722	0.087
T Copula*	1	0.084
Gaussian Copula	0.950	0.096

There are significant differences in the construction of the traditional conditional distribution model and the multiple copula model. The key advantage of the multiple copula model is that it considers the relationship beyond linear relationship and it can estimate the relationship among all multiple random variables instead of only two. The three steps of the traditional models' parameters' estimation are not always or even not the best fit by the maximum likelihood method which also has limitations which may be the reason for the less fit of the traditional conditional model.

The multiple copula model shows great priority in the construction of statistical models and the differences shown in the fatigue damage result indicate that the traditional conditional model might be less precise even though is commonly.

Besides all the advantages of the multiple copula models, there are still some limitations that should be considered in future works: for instance, breaking wave limit may cause the unlikelihood of a large value of H_s accompanied by a small value of T_p which can be considered in the traditional model but not in the copula model.

CHAPTER 5 ADVANCED COPULA MODEL AND THE EFFECT ON THE LONG-TERM FATIGUE ANALYSIS

5.1 Introduction

In the above chapter, classical 3-dimensional copula is employed to construct the statistical models for the environmental random variables H_s , T_p and V_w , and is compared with 3-dimensional traditional methods of conditional distribution model. Based on model selection, classical copula model is slightly superior to the traditional conditional model, which shows the advantage of copula model. However, since the selection of parametric copulas is very limited when the dimension is high, only Gaussian copula and T copula are employed in the work in the previous chapter for convenience. Moreover, both Gaussian copula and T copula are very restrictive and cannot model asymmetry and heavy tails (Brechmann and Schepsmeier, 2013). Yan (2006) pointed out that the joint distribution may not be multivariate normal, even if all the marginal distributions are normal.

To overcome the limitations in the above chapter, pair-copula construction (PCC) is employed in this chapter to construct more statistical multivariate models in copula concept and the models are estimated by model selection. Furthermore, the application of PPC models in the long-term fatigue effect are also compared with the traditional statistical model in this chapter.

The content in this chapter is presented in the journal paper (Gao and Cheung, 2016b) which will be submitted for peer review.

5.2 Advanced Copula theory

Pair-copula construction (PCC) was first put forward by Joe (1996), and was further explored and studied by Bedford and Cooke (2001, 2002), Kurowicka and Cooke (2006) and Aas et al (2009) etc. This probabilistic construction method gives a new way of solving multivariate highly dependent models by decomposing a multivariate density into a number of pair copulas, which in turn extend the bivariate copulas to higher dimensions. Hence, the limitation of higher dimensional copula selection can be alleviated by the large number of parametric bivariate copulas.

The essence of this PCC is to model dependency by simple local 2-D copula blocks which are based on conditional independence. The detailed construction procedures are stated in the following:

Consider a random vector $(X_1, X_2, \dots, X_d)$, the JPDP $f(x_1, x_2, \dots, x_d)$ can be expressed in the way of:

$$f(x_1, x_2, \dots, x_d) = f_d(x_d) \cdot f(x_{d-1}|x_d) \cdot f(x_{d-2}|x_{d-1}, x_d) \cdots f(x_1|x_2, \dots, x_d) \quad (5.1)$$

According to the property of copula, the JPDP $f(x_1, x_2, \dots, x_d)$ can also be expressed as:

$$f(x_1, x_2, \dots, x_d) = c_{1\dots d}\{F_1(x_1), F_2(x_2) \cdots, F_d(x_d)\} \cdot f_1(x_1)f_2(x_2) \cdots f_d(x_d) \quad (5.2)$$

Where $c_{1\dots d}$ is the density of the copula $C_{1\dots d}$.

When $d=3$, the JPDP $f_{123}(x_1, x_2, x_3)$ is factorised as:

$$f_{123}(x_1, x_2, x_3) = f_3(x_3)f_{2|3}(x_2|x_3)f_{1|23}(x_1|x_2, x_3) \quad (5.3)$$

where $f_{2|3}(x_2|x_3)$ is

$$f_{2|3}(x_2|x_3) = \frac{f_{23}(x_2, x_3)}{f_3(x_3)} = \frac{c_{23}(F_2(x_2), F_3(x_3))f_2(x_2)f_3(x_3)}{f_3(x_3)} = c_{23}(F_2(x_2), F_3(x_3))f_2(x_2) \quad (5.4)$$

Moreover, we can have the $f_{1|23}(x_1|x_2, x_3)$ as

$$\begin{aligned} f_{1|23}(x_1|x_2, x_3) &= \frac{f_{12|3}(x_1, x_2|x_3)}{f_{2|3}(x_2|x_3)} \\ &= \frac{c_{12|3}(F_{1|3}(x_1|x_3), F_{2|3}(x_2|x_3); x_3)f_{1|3}(x_1|x_3)f_{2|3}(x_2|x_3)}{f_{2|3}(x_2|x_3)} \\ &= c_{12|3}(F_{1|3}(x_1|x_3), F_{2|3}(x_2|x_3); x_3)f_{1|3}(x_1|x_3) \\ &= c_{12|3}(F_{1|3}(x_1|x_3), F_{2|3}(x_2|x_3); x_3)c_{13}(F_1(x_1), F_3(x_3))f_1(x_1) \end{aligned} \quad (5.5)$$

By inserting Eq. (5.5) into Eq. (5.3), we obtain

$$\begin{aligned} f_{123}(x_1, x_2, x_3) &= f_1(x_1)f_2(x_2)f_3(x_3) \cdot c_{13}(F_1(x_1), F_3(x_3))c_{23}(F_2(x_2), F_3(x_3)) \\ &\quad \cdot c_{12|3}(F_{1|3}(x_1|x_3), F_{2|3}(x_2|x_3); x_3) \end{aligned} \quad (5.6)$$

Eq (5.6) is a full PPC expansion of a three-dimensional JPDP $f_{123}(x_1, x_2, x_3)$ and it is one out of three possible decompositions in three dimensions. Generally, the copula density $c_{12|3}$ depends on x_3 by $F_{1|3}(x_1|x_3)$, $F_{2|3}(x_2|x_3)$ as well as direct x_3 . In application, it is always

assumed that the pair-copulas are independent of the conditioning variables such as x_3 in Eq (5.6), except through the conditional distributions such as $F_{1|3}(x_1|x_3)$ and $F_{2|3}(x_2|x_3)$. Based on this assumption, Eq (5.6) can be rewritten as:

$$\begin{aligned} f_{123}(x_1, x_2, x_3) &= f_1(x_1)f_2(x_2)f_3(x_3) \cdot c_{13}(F_1(x_1), F_3(x_3))c_{23}(F_2(x_2), F_3(x_3)) \\ &\cdot c_{12|3}(F_{1|3}(x_1|x_3), F_{2|3}(x_2|x_3)) \end{aligned} \quad (5.7)$$

For higher dimensional problems, the number of possible decompositions grows fast with the number of dimension. For instance, a five-dimensional JPDP can have 240 different PPCs (Haff et al., 2009).

5.3 Application of 3-Dimensional Advanced Copula Models

Similar to three-dimensional classical copula models constructed in Chapter 4, advanced PPC models with three environmental random variables: significant wave height H_s , spectral peak period T_p and 1 hour mean wind speed at 10m above sea level V_w will be built in this section.

An appropriate PCC model should in principle considers three main factors: First, the different selection of factorisation should be considered, which is in total 3 options for 3-dimensional pair-copula model; Second, the choice of pair-copula types should be decided for each pair-copula combination of any two random variables; Third, the copula parameters should be estimated based on some laws.

5.3.1 Construction of First Level Pair Copula

In this section, the best-fitting for each type of pair copula for any two of the random variables is studied by maximum log-likelihood method, which is based on the best marginal distribution estimated in section 4.4.3. There are in total 3 types of pair copula with 3 random variables (H_s, T_p) , (H_s, V_w) and (T_p, V_w) . In the following paragraphs of this chapter, H_s , T_p and V_w will be indicated as 1, 2 and 3 in equations or symbols. In this study, five well known copulas are considered: Gaussian copula, T copula, Clayton copula, Frank copula and Gumbel copula. Tables 5.1, 5.2 and 5.3 are comparisons of parameter estimates and goodness-of-fit to the data for different copula models for three pair of random variables (H_s, T_p) , (H_s, V_w) and (T_p, V_w) respectively. Moreover, Figures 5.1, 5.2 and 5.3 are the comparisons of contour plot between the original data and 5 types of copula models for the pairs (H_s, T_p) , (H_s, V_w) and (T_p, V_w) , respectively.

Table 5. 1 Comparison of parameter estimates for (H_s, T_p)

(H_s, T_p) C ₁₂ , based on 184212 samples					
Copula family:	parameter estimate	log-likelihood	No of parameter	AIC	BIC
Gaussian*	0.319631	-600202.887	5	1200416	1200466
t	0.319633	-600202.889	6	1200418	1200479
	degrees of freedom parameter=3334598.0				
Clayton	0.316121	-605053.600	5	1210117	1210168
Frank	1.651515	-602927.998	5	1205866	1205917
Gumbel	1.196710	-601909.695	5	1203829	1203880

Table 5. 2 Comparison of parameter estimates for (H_s, V_w)

(H_s, V_w) C ₁₃ , based on 184212 samples					
Copula family:	parameter estimate	log-likelihood	No of parameter	AIC	BIC
Gaussian	0.642643	-641927.807	5	1283866	1283916
t	0.649528	-640573.902	6	1281160	1281221
	degrees of freedom parameter=11.2				
Clayton	0.813928	-667262.085	5	1334534	1334585
Frank	5.078260	-642238.995	5	1284488	1284539
Gumbel*	1.801941	-635702.377	5	1271415	1271465

Table 5. 3 Comparison of parameter estimates for (T_p, V_w)

$(T_p, V_w)C_{23}$, based on 184212 samples					
Copula family:	parameter estimate	log-likelihood	No of parameter	AIC	BIC
Gaussian	-0.0489	-864901.580	5	1729813	1729864
t	-0.0485	-864901.570	6	1729815	1729876
	degrees of freedom parameter=3.8064x10 ⁶				
Clayton	1.4500E-06	-865121.652	5	1730253	1730304
Frank*	-0.4487	-864602.305	5	1729215	1729265
Gumbel	1.0000	-865121.631	5	1730253	1730304

(Notes: the lowest AIC and BIC value indicates the best model, highlighted by “*”)

(a) Comparison of original data and Gaussian copula model for the pair (H_s, T_p)

(b) Comparison of original data and T copula model for the pair (H_s, T_p)

(c) Comparison of original data and Clayton copula model for the pair (H_s, T_p)

(d) Comparison of original data and Frank copula model for the pair (H_s, T_p)

(e) Comparison of original data and Gumbel copula model for the pair (H_s, T_p)

Figure 5. 1 Comparison of contour plot between original data and (a) Gaussian copula model, (b) T copula model, (c) Clayton copula model, (d) Frank copula model, and (e) Gumbel copula model for the pair (H_s, T_p)

(a) Comparison of original data and Gaussian copula model for the pair (H_s, V_w)

(b) Comparison of original data and T copula model for the pair (H_s, V_w)

(c) Comparison of original data and Clayton copula model for the pair (H_s, V_w)

(d) Comparison of original data and Frank copula model for the pair (H_s, V_w)

(e) Comparison of original data and Gumbel copula model for the pair (H_s, V_w)

Figure 5. 2 Comparison of contour plot between original data and (a) Gaussian copula model (b) T copula model (c) Clayton copula model (d) Frank copula model (e) Gumbel copula model for the pair (H_s, V_w)

(a) Comparison of original data and Gaussian copula model for the pair (T_p, V_w)

(b) Comparison of original data and T copula model for the pair (T_p, V_w)

(c) Comparison of original data and Clayton copula model for the pair (T_p, V_w)

(d) Comparison of original data and Frank copula model for the pair (T_p, V_w)

(e) Comparison of original data and Gumbel copula model for the pair (T_p, V_w)

Figure 5. 3 Comparison of contour plot between original data and (a) Gaussian copula model (b) T copula model (c) Clayton copula model (d) Frank copula model (e) Gumbel copula model for the pair (T_p, V_w)

From Tables 5.1, 5.2 and 5.3, it can be seen that Gaussian copula, Gumbel copula and Frank copula are best fit copulas measured by AIC and BIC for random variable pairs (H_s, T_p) , (H_s, V_w) and (T_p, V_w) respectively. A general illustration of different copula models for three random variable pairs can be seen from the contour plots by comparing the fitted models against the empirical data for (H_s, T_p) , (H_s, V_w) and (T_p, V_w) in Figures 5.1, 5.2 and 5.3. Since differences in the value of model selection estimator AIC and BIC are not so significant for different copula models for any of the three pair, accordingly the illustration differences are also very small, as expected. Anyway, the best fit copula models with lowest AIC and BIC value in this study are chosen to implement the next step of the PCC model construction.

5.3.2 Construction of Conditional Pair Copula

The next step is to solve the last component in the Equation (5.7), which is $c_{12|3}(F_{1|3}(x_1|x_3), F_{2|3}(x_2|x_3))$. According to Joe (1996), marginal conditional distributions $F(x|\mathbf{v})$ can be written as:

$$F(x|\mathbf{v}) = \frac{\partial C_{x, \mathbf{v}_j | \mathbf{v}_{-j}} \{F(x|\mathbf{v}_{-j}), F(\mathbf{v}_j|\mathbf{v}_{-j})\}}{\partial F(\mathbf{v}_j|\mathbf{v}_{-j})} \quad (5.8)$$

where $\mathbf{v}$ is a d -dimensional vector, $\mathbf{v}_j$ is one arbitrary chosen component of $\mathbf{v}$, and $\mathbf{v}_{-j}$ is the $\mathbf{v}$ -vector which excludes $\mathbf{v}_j$.

When $\mathbf{v}$ is univariate, Equation (5.8) can be written as:

$$F(x|\mathbf{v}) = \frac{\partial C_{xv} \{F(x), F(\mathbf{v})\}}{\partial F(\mathbf{v})} \quad (5.9)$$

Therefore, $F_{1|3}(x_1|x_3)$ and $F_{2|3}(x_2|x_3)$ in Equation (5.7) can be expressed as:

$$F_{1|3}(x_1|x_3) = \frac{\partial C_{13}(F_1(x_1), F_3(x_3))}{\partial F_3(x_3)} \quad (5.10)$$

$$F_{2|3}(x_2|x_3) = \frac{\partial C_{23}(F_2(x_2), F_3(x_3))}{\partial F_3(x_3)} \quad (5.11)$$

The solution of $F_{1|3}(x_1|x_3)$ and $F_{2|3}(x_2|x_3)$ are similar problems which involve partial derivative of a two-dimensional copula. The two-dimensional copula can be any one of the copula type selected. As discussed in the previous section, Gaussian copula, Gumbel copula and Frank copula are estimated as the best fit copula for pairs (H_s, T_p) , (H_s, V_w) and (T_p, V_w)

respectively. Therefore, only Gaussian copula, Gumbel copula and Frank copula are considered in the derivation.

The decomposition in Equation (5.7) is taken as an example to deduce the construction process, especially the last component $c_{12|3}(F_{1|3}(x_1|x_3), F_{2|3}(x_2|x_3))$. The main idea is that solving the components $F_{1|3}(x_1|x_3)$ and $F_{2|3}(x_2|x_3)$ first and then constructing the two-dimensional copulas, similar to the previous work. After that, a best fit two-dimensional copula will be selected according to model selection techniques.

As indicated in Equation (5.10), $F_{1|3}(x_1|x_3) = \frac{\partial C_{13}(F_1(x_1), F_3(x_3))}{\partial F_3(x_3)}$, and Gumbel copula is estimated as the best fit copula for C_{13} , then two-dimensional Gumbel copula is chosen and partial derived by $u_3 = F_3(x_3)$. The detail deduction is:

$$\begin{aligned} \frac{\partial C_{13}(u_1, u_3)}{\partial u_3} &= \frac{\partial \left(\exp \left(-((-\ln(u_1))^\theta + (-\ln(u_3))^\theta) \right)^{\frac{1}{\theta}} \right)}{\partial u_3} \\ &= C_{13}(u_1, u_3) \left[(-\ln(u_1))^\theta + (-\ln(u_3))^\theta \right]^{\frac{1}{\theta}-1} (-\ln(u_3))^{\theta-1} \frac{1}{u_3} \end{aligned} \quad (5.12)$$

where $C_{13}(u_1, u_3) = \exp \left(-((-\ln(u_1))^\theta + (-\ln(u_3))^\theta) \right)^{1/\theta}$, and θ is the parameter estimated according to pseudo-likelihood method as discussed in Chapter 4. In this study, $\theta = 5.078260$ according to Table 5.2.

Similarly, $F_{2|3}(x_2|x_3) = \frac{\partial C_{23}(F_2(x_2), F_3(x_3))}{\partial F_3(x_3)}$, and Frank copula is estimated as the best fit copula for C_{23} . Then,

$$\begin{aligned} \frac{\partial C_{23}(u_2, u_3)}{\partial u_3} &= \frac{\partial \left(-\frac{1}{\theta} \log \left[1 + \frac{(\exp(-\theta u_2) - 1)(\exp(-\theta u_3) - 1)}{\exp(-\theta) - 1} \right] \right)}{\partial u_3} \\ &= \frac{(\exp(-\theta u_2) - 1) \exp(-\theta u_3)}{\exp(-\theta) - 1 + (\exp(-\theta u_2) - 1)(\exp(-\theta u_3) - 1)} \end{aligned} \quad (5.13)$$

where $C_{23}(u_2, u_3) = -\frac{1}{\theta} \log \left[1 + \frac{(\exp(-\theta u_2) - 1)(\exp(-\theta u_3) - 1)}{\exp(-\theta) - 1} \right]$, and θ is the parameter estimated according to pseudo-likelihood method as discussed in Chapter 4. In this study, $\theta = -0.4487$ according to Table 5.3.

Similar to the construction process as C_{12} , C_{13} and C_{23} , $C_{12|3}$ is constructed based on already calculated $F_{1|3}(x_1|x_3)$ and $F_{2|3}(x_2|x_3)$. Table 5.4 gives the results of the $C_{12|3}$ constructed by different copula models.

Table 5. 4 Parameter estimates and model comparisons for $c_{12|3}$

$c_{12 3}$, based on 184212 samples					
Copula family:	parameter estimate	log-likelihood of copula density	No of parameter	AIC	BIC
Gaussian	0.47191	23204	9	-46389	-46298
t*	0.47194	23480	10	-46939	-46838
	degrees of freedom parameter=2 0.04284				
Clayton	0.62800	15458	9	-30898	-30808
Frank	2.98336	20250	9	-40481	-40390
Gumbel	1.38492	21759	9	-43500	-43409

(Notes: the lowest AIC and BIC value indicates the best model, highlighted by “*”)

From Table 5.4, it is obtained that t copula is the best fit copula for $c_{12|3}(F_{1|3}(x_1|x_3), F_{2|3}(x_2|x_3))$ according to both AIC and BIC. The log-likelihood of copula density of Gaussian copula is quite close to t copula but with one less number of parameter, and in the end it has a bit larger AIC and BIC compared with t copula. Clayton copula with the highest AIC and BIC is considered as the worst fit copula for $c_{12|3}(F_{1|3}(x_1|x_3), F_{2|3}(x_2|x_3))$.

5.3.3 Conclusion and Discussion

The whole estimation for PPC expansion in Equation (5.7) has three main components: 1, Marginal distribution for each individual random variable $f_1(x_1)f_2(x_2)f_3(x_3)$; 2, two dimensional copula $c_{13}(F_1(x_1), F_3(x_3))c_{23}(F_2(x_2), F_3(x_3))$; 3, two dimensional conditional copula $c_{12|3}(F_{1|3}(x_1|x_3), F_{2|3}(x_2|x_3))$. All the three components are estimated in section 4.4.3, section 5.3.1 and section 5.3.2 respectively. The results of the intermediate steps for the three components are shown in Table 4.6, Table 5.1~5.3 and Table 5.4.

Besides the type of PPC expansion in Equation (5.7), there are another two types of PPC expansion for the joint three-dimensional random variables H_s, T_p and V_w , written in Equation (5.14) and Equation (5.15):

$$f_{123}(x_1, x_2, x_3) = f_{231}(x_2, x_3, x_1) = f_2(x_2)f_3(x_3)f_1(x_1)c_{21}(F_2(x_2), F_1(x_1))c_{31}(F_3(x_3), F_1(x_1))c_{23|1}(F_{2|1}(x_2|x_1), F_{3|1}(x_3|x_1)) \quad (5.14)$$

$$f_{123}(x_1, x_2, x_3) = f_{132}(x_1, x_3, x_2) = f_1(x_1)f_3(x_3)f_2(x_2)c_{12}(F_1(x_1), F_2(x_2))c_{32}(F_3(x_3), F_2(x_2))c_{13|2}(F_{1|2}(x_1|x_2), F_{3|2}(x_3|x_2)) \quad (5.15)$$

Similar to the PPC expansion type in Equation (5.7), the estimation of first and second components for PPC expansion in Equation (5.14) and (5.15) are listed in Table 4.6 and Table 5.1~5.3. The parameter estimation results for the third component of the above PPC expansions are shown in Table (5.5) and Table (5.6).

Table 5. 5 Parameter estimates and model comparison for $c_{23|1}$

C _{23 1} , based on 184212 samples					
Copula family:	parameter estimate	log-likelihood of copula density	No of parameter	AIC	BIC
Gaussian	-0.388	15037	9	-30055	-29964
t	-0.395	15222	10	-30424	-30322
	degrees of freedom parameter=25.253				
Clayton	1.450E-06	-0.08721	9	18.174	109.29
Frank*	-2.597	15848	9	-31679	-31588
Gumbel	1.000	-0.12784	9	18.256	109.37

(Notes: the lowest AIC and BIC value indicates the best model, highlighted by “*”)

Table 5. 6 Parameter estimates and model comparison for $c_{13|2}$

$C_{13 2}$, based on 184212 samples					
Copula family:	parameter estimate	log-likelihood of copula density	No of parameter	AIC	BIC
Gaussian	0.694	60400	9	-120783	-120692
t	0.706	62637	10	-125255	-125153
	degrees of freedom parameter=7.910				
Clayton	0.837	26920	9	-53822	-53731
Frank	6.115	63241	9	-126464	-126373
Gumbel*	2.078	74378	9	-148738	-148647

(Notes: the lowest AIC and BIC value indicates the best model, highlighted by “*”)

According to Table 5.5 and 5.6, Frank copula and Gumbel copula are the best fit copula models for $c_{23|1}$ and $c_{13|2}$ with both lowest AIC and BIC values. For both $c_{23|1}$ and $c_{13|2}$, Clayton copula provides the worst fit model, which might be due to the significant difference between the model construction data and the characteristic of Clayton copula which is good at describing data at strong left tail dependence. The large difference in copula modelling also indicates that the selection of copula type should be cautious and there are some limitations in some special copula types in the modelling of certain data, such as Clayton copula for modelling $c_{23|1}$.

5.4 Application to Long-term Fatigue Assessment of FPSO Risers and Comparison with Other Statistical Models

For the ease of comparison, the exact same FPSO model system in Chapter 4 is employed in this section to analyse the long-term fatigue performance of flexibles risers under the advance copula statistical model with three environmental random variables and the results are compared with the traditional models and classical copula models.

5.4.1 Generating Samples

For the advanced copula models, the sample generating is not as straight forward as classical copula models which have two generating steps: 1st is to generate 3-dimensional random marginal distribution vector on the unit cubic $[0,1]^3$ according to copula relationship designed;

2nd is to inverse transfer function to obtain environmental random variables from their CDF obtained from 1st step according to their marginal distributions estimated and designed. For the advanced copula models, the sample generation process could be treated as the inverse process of model construction. The first type of PPC expansion is taken as an example to deduce the sample generating procedure in the following paragraph.

Since $f_{123}(x_1, x_2, x_3) = f_3(x_3)f_{2|3}(x_2|x_3)f_{1|23}(x_1|x_2, x_3)$ according to Equation (5.3), there are three steps in generating these three random variables. The 1st step is to generate n random variables U_3^* from uniform distribution $U [0,1]$, and then inversely transfer them back to random variables X_3^* according to marginal distribution $f_3(x_3)$. The 2nd step is to generate n random variables U_2^* from uniform distribution $U [0,1]$, and then find a X_2^* to satisfy Equation (5.16)

$$U_2^* = \int_0^{X_2^*} f_{2|3}(x_2|x_3^*)dx_2 = F_{2|3}(x_2|x_3^*) \quad (5.16)$$

$$\text{where } f_{2|3}(x_2|x_3^*) = \frac{f(x_2, x_3^*)}{f_3(x_3^*)} = \frac{c_{23}(F_2(x_2), F_3(x_3^*)) * f_2(x_2)f_3(x_3^*)}{f_3(x_3^*)} = c_{23}(F_2(x_2), F_3(x_3^*)) * f_2(x_2)$$

The 3rd step is to generate n random variables U_1^* from uniform distribution $U [0,1]$, and then find a proper X_1^* to satisfy Equation (5.17)

$$U_1^* = \int_0^{X_1^*} f_{1|2,3}(x_1|x_2^*, x_3^*)dx_1 \quad (5.17)$$

$$\text{where } f_{1|2,3} = c_{12|3}(F_{1|3}(x_1|x_3^*), F_{2|3}(x_2^*|x_3^*)) * f_{1|3}(x_1|x_3^*);$$

$$\text{where } f_{1|3}(x_1|x_3^*) = c_{13}(F_1(x_1), F_3(x_3^*)) * f_1(x_1) \quad ; \quad F_{2|3}(x_2^*|x_3^*) = \int_0^{x_2^*} f_{2|3}(x_2|x_3^*)dx_2 = \int_0^{x_2^*} c_{23}(F_2(x_2), F_3(x_3^*)) * f_2(x_2)dx_2; F_{1|3}(x_1|x_3^*) = \int_0^{x_1^*} c_{13}(F_1(x_1), F_3(x_3^*)) * f_1(x_1)dx_1.$$

In the above deduction procedure, the sign “*” indicates a known constant value under it.

Similar to the above derivation for the 1st type of PPC expansion, other two types of PPC expansions are following the same way.

5.4.2 Application of Advanced Copula Model to Long-term Fatigue Analysis

After n groups of samples (H_s, T_p, V_w) generated by the generating method deduced in the above section, FPSO system built in Orcasflex software is employed to conduct the simulation and to extract the long-term fatigue damage of flexible risers. For the convenience of

comparison, the same FPSO model system and wave amplitude assumption are used in this chapter similar to Section 4.6. The only difference exists in the input data set of three environmental random variables.

5.4.3 Results and Discussion

The total log-likelihood function values, AIC statistics and BIC statistics for the advanced copula models based on three types of PPC expansions are presented in Table 5.7.

Table 5. 7 Comparison of goodness-of-fit to three advanced copula models

PPC type	PPC expansion	total log-likelihood	no of parameters	AIC	BIC
1 st	$f_{123}=f_1f_2f_3*C_{13}C_{23}*C_{12 3}$	-1003834	26	2007720	2007983
2 nd	$f_{123}=f_1f_2f_3*C_{21}C_{31}*C_{23 1}$	-1002058	25	2004167	2004420
3 ^{rd*}	$f_{123}=f_1f_2f_3*C_{12}C_{32}*C_{13 2}$	-998297	25	1996644	1996897

(Notes: the lowest AIC and BIC value indicates the best model, highlighted by “*”)

Table 5.7 gives the whole assessment of the advanced copula models including three main components discussed in section 5.3. Take the 1st PPC expansion for example, the number of parameters include three parts: $26=2*3+(5+5)+10$, in which the first part “ $2*3$ ” indicates the number of parameters for the marginal distributions for “ $f_1f_2f_3$ ” as shown and selected in Table 4.6, the second part “ $(5+5)$ ” indicates the number of parameters for the two dimensional classical part “ $C_{13}C_{23}$ ” as shown and selected in Table 5.2 and Table 5.3, the third part “10” indicates the number of parameters for the conditional two dimensional copula “ $C_{12|3}$ ” as shown and selected in Table 5.4. The 3rd type of PPC expansion based advanced copula model with the lowest AIC and BIC value is supposed to be the best fit statistical model among the three.

Table 5.8 gives the final results of the comparison of model fit and normalized long-term fatigue damage estimation. The normalization is conducted by dividing the fatigue damage at the best fit model which is the 3rd type of advanced 3D copula model. The absolute fatigue damage is shown in the bracket next to the normalized one.

Table 5. 8 Comparison of normalized fatigue damage and goodness-of-fit for all models

(H_s, T_p, V_w) , based on 184212 samples		Total log-likelihood ($\times 10^6$)	No of parameters	AIC	BIC	Fatigue Damage ($N=3000$)	
						Mean	$\text{CoV}/\sqrt{N}$
traditional method	conditional joint distribution	-1.032	17	2064033	2064089	0.564 (4.69×10^{-10})	0.068
classical 3D copula	Gaussian	-1.012	9	2024179	2024208	0.669 (5.57×10^{-10})	0.071
	T	-1.011	10	2022300	2022333	0.703 (5.86×10^{-10})	0.065
advanced 3D copula	1 st type	-1.004	26	2007720	2007805	0.875 (7.29×10^{-10})	0.083
	2 nd type	-1.002	25	2004167	2004248	0.645 (5.37×10^{-10})	0.060
	3 rd type*	-0.998	25	1996644	1996725	1 ($\times 10^{-10}$)	0.057

(Notes: The lowest AIC or BIC value indicates the best model, highlighted by “*”).

Firstly, model selection methods AIC and BIC are selected to compare different statistical models. Akaike information criterion (AIC) is used to measure the relative quality of statistical models for a given set of data. It is a trade-off between the goodness of fit of the model and complexity of the model. As the equation shows:

$$AIC = 2k - 2\ln(\hat{L})$$

where k is the number of estimated parameters in the model, $\hat{L}$ is maximum value of the likelihood function, $\hat{L} = P(x|\hat{\theta}, M)$, where $\hat{\theta}$ are the parameter values that maximize the likelihood function. Bayesian information criterion (BIC) is a model selection criterion among a finite set of models. Compared with AIC, the penalty term is larger, which includes the consideration of the number of samples, n . The expression for BIC is:

$$BIC = \ln(n)k - 2\ln(\hat{L})$$

Let P_{PCC3} be the probability that the 3rd advanced PCC copula model is the best model for explaining the data. It is well known that the probability is approximately proportional to $\exp(-BIC)$. Thus the probabilities of the other 5 statistical models being the best $P_{tra}, P_{class_g}, P_{class_t}, P_{PCC1}, P_{PCC2}$, respectively are given as follows:

$$P_{tra} + P_{class_g} + P_{class_t} + P_{PCC1} + P_{PCC2} + P_{PCC3} = 1 \quad (5.18)$$

$$\frac{P_{tra}}{P_{PCC3}} = \frac{\hat{L}_{tra}}{\hat{L}_{PCC3}} = \frac{e^{-2064089}}{e^{-1996725}} \quad (5.19)$$

$$\frac{P_{class_g}}{P_{PCC3}} = \frac{\hat{L}_{class_g}}{\hat{L}_{PCC3}} = \frac{e^{-2024208}}{e^{-1996725}} \quad (5.20)$$

$$\frac{P_{class_t}}{P_{PCC3}} = \frac{\hat{L}_{class_t}}{\hat{L}_{PCC3}} = \frac{e^{-2022333}}{e^{-1996725}} \quad (5.21)$$

$$\frac{P_{PCC1}}{P_{PCC3}} = \frac{\hat{L}_{PCC1}}{\hat{L}_{PCC3}} = \frac{e^{-2007805}}{e^{-1996725}} \quad (5.22)$$

$$\frac{P_{PCC2}}{P_{PCC3}} = \frac{\hat{L}_{PCC2}}{\hat{L}_{PCC3}} = \frac{e^{-2004248}}{e^{-1996725}} \quad (5.23)$$

From the above calculation, the probabilities for the listed statistical models to be the best models are: $P_{tra} \approx 0, P_{class_g} \approx 0, P_{class_t} \approx 0, P_{PCC1} \approx 0, P_{PCC2} \approx 0, P_{PCC3} \approx 1$.

Therefore, the probability for the 3rd advanced copula model to be the best model among those models that are considered in this study is almost 100%.

Besides the model selection metrics AIC and BIC, the 3D scatter diagram of samples generated by the traditional model and the 3rd advanced copula model can also be compared with the raw data to see which model captures the raw data the best.

Figure 5. 4 Scatter diagram of raw data

Figure 5. 5 Scatter diagram of data generated by the traditional model (100000samples)

Figure 5. 6 Scatter diagram of data generated by the 3rd advanced copula model (100000samples)

From Figures 5.4-5.6, it can be seen that the 3rd advanced copula model proposed in this study captures the characteristics of the raw data better than the traditional model.

The statistics of the raw data, the samples obtained by the traditional method and those obtained by the proposed 3rd advanced copula model are shown as follows:

Table 5. 9 Higher-level statistic comparison

mean of	raw data	traditional method	3rd advanced copula model
H_s	1.84	1.95	1.81
T_p	8.09	8.25	8.07
V_w	6.80	6.65	6.64
$H_s * T_p$	15.59	16.11	15.20
$H_s * V_w$	14.82	15.44	14.07
$T_p * V_w$	54.73	54.86	53.12
$H_s * T_p^2$	141.21	137.95	138.17
$H_s * V_w^2$	138.00	146.72	139.69
$T_p * V_w^2$	442.77	453.96	445.89
$T_p * H_s^2$	39.86	41.09	37.96
$V_w * H_s^2$	40.85	46.00	40.71
$V_w * T_p^2$	458.27	470.11	459.26
$H_s * T_p * V_w$	123.50	127.27	121.77

It can be observed from the above table that the proposed 3rd advanced copula model fits the marginal distribution statistics of H_s and T_p much better than the traditional model. Since the traditional model fits the marginal distribution of V_w first and then the conditional relationship and finally obtains the joint distribution, the fitting of V_w is much better than other two random variables. All the other statistical values of the samples obtained from the 3rd advanced copula model are much closer to the raw data than those obtained from the traditional model.

In conclusion, the proposed 3rd advanced copula model is more accurate and realistic than the traditional model.

The above fatigue damage is based on 3000 simulations *5mins for each method.

It is supposed that fatigue damage is $\propto$ time, 20 years' fatigue damage calculated by the traditional method is approximately:

$$D_{tra(20)} = 20 * 365 * 24 * 60/5 * D_{tra} = 10^{-3} \quad (5.24)$$

It shows that this model is too conservative. However, in the offshore engineering industry, people are more likely to design structures less conservatively for economic consideration. Hence, the result that fatigue damage considering the 3rd type of the proposed advanced copula model is twice the value considering the traditional model is still meaningful.

5.5 Chapter Summary

Advance copula which is based on pair-copula constructions (PCC) becomes more and more popular for the construction of multivariate distributions, mostly due to its simple structure and high flexibility. Traditional conditional distribution model is constructed more from a physical aspect and based on experience in some steps as discussed in Section 4.4.2. The result of model

selection method AIC and BIC both show that copula models including advanced copula models and classical copula models are all superior to the traditional models and the results of their applications in the long-term fatigue assessment show some differences. Even though some publications on the PCC have appeared in the financial field (Fisher et al., 2007; Chollete and Valdesogo, 2008), it has seldom been seen in the long-term fatigue application in the offshore field. In other works, the work in this chapter is the first trial of PCC in the construction of statistical modelling of ocean environmental random variables and the application of this model in the long-term fatigue assessment of flexible risers in the offshore field. This work shows a great potential of constructing statistical models which are more flexible and are able to capture the nonlinear relationship among different random variables. In future works, more environmental random variables can be considered, which requires higher dimensional PCC construction.

CHAPTER 6 CONCLUSION AND FUTURE WORKS

6.1 Conclusion

The main objective of this study is to propose a framework for the reliable assessment of fatigue damage of FPSO's mooring and riser systems under a set of ocean environmental random variables. This study focuses on three main parts: the 1st part is to explore an efficient and precise method to assess the long-term fatigue damage, the 2nd part is to establish an appropriate statistical environmental model as a set up before simulation by constructing multiple classical copula models, the 3rd part is to investigate a type of more flexible advanced copula models to overcome the limitation of classical copula models, and finally to achieve a more robust statistical model.

With the studies carried out in this work, the contents presented in this thesis can be summarized as follows:

(1) A new efficient approach for long-term riser fatigue analysis is established.

A floating structure is always exposed to many sea conditions during its service life. Due to the mechanical characteristics of fatigue damage, the long-term fatigue analysis should be considered, which causes a great of computer load especially when it is performed in the time domain. In Chapter 3, a new efficient simulation method is proposed for long-term riser fatigue analysis in time domain. This proposed approach is based on importance sampling concept but not for the purpose of variance reduction. It is widely accepted that time series of an irregular wave elevation can be simulated as a sum of regular wave components with random amplitudes and phases. The essence of the newly proposed method is that it allows one to simulate the wave amplitudes from a distribution different from the original one, which means the dynamic analysis for many sea states can be replaced by altering the importance sampling weighting function. This proposed method is enhanced by a set of additional techniques for the purpose of reducing the sampling variability. It should be pointed out that the fatigue damage estimation given by the new importance sampling method is not biased. Moreover, the standard error is estimated by bootstrapping technique. This proposed method shows a significant reduction of the computation cost, which can be concluded as a more efficient method compared with classical numerical method.

(2) Multiple classical copula models are constructed for fatigue assessment with consideration of the nonlinear dependency among different random variables.

In the long-term fatigue analysis, the joint statistical model of ocean environmental random variables is required to be carefully set up. The traditional model commonly used in the industry is the conditional joint distribution model which only considers the linear relationship between random variables, such as $f(H_s, T_p)$ employed in Chapter 3. However, the traditional conditional joint distribution model may not be appropriate if the actual dependency is nonlinear. In Chapter 4, multiple classical copula models are constructed to capture the random process of environmental effect in the long-term fatigue assessment of deep water risers. By employing several model selection methods, the multiple copula model shows superiority to traditional models according to the results shown in Chapter 4.

(3) Advanced multiple copula models are developed which are more flexible than the classical copula models.

In Chapter 4, classical multiple copula models are found to be superior to traditional conditional models based on model selection. However, due to the limitations of multidimensional copula type, only Gaussian copula and T copula are employed. In Chapter 5, pair-copula construction (PCC) is employed to construct more flexible statistical models. The essence of this probabilistic construction method is that it solves multivariate highly dependent models by decomposing a multivariate density into numbers of pair copulas. Therefore, the limitation of higher dimensional copula selection is alleviated by a large number of bivariate copulas choices. Details of this construction procedures are listed in Chapter 5. Moreover, till now, many PPC works only study the construction of multiple copula models, while the generating of samplings based on PPC models is seldom studied. As the third part of this thesis work, the advanced multiple copula models based on PPC are built up and samples generated by PPC models are employed to take part in the risers' fatigue assessment. Finally the statistical modelling effect on the fatigue assessment of deep water risers is studied. From the differences of the fatigue assessment results obtained due to different statistical models, one conclusion is that the result obtained by traditional model can be reconsidered in the precision aspect, since the model selection shows that the advanced copula models are superior to classical copula models and traditional model.

6.2 Recommendations of Future Works

The probability analysis for the long-term fatigue assessment is a challenging research topic that requires extensive exploration. The works conducted in this thesis provide some possible solutions to improve the simulation efficiency as well the accuracy for the long-term fatigue assessment in time domain. Based on that, the author believes that the research for the probability analysis of long-term fatigue can be further extended as follows:

1. Since the new efficient approach for the long-term fatigue damage calculation discussed in Chapter 3 only considers two environmental random variables: significant wave height H_s and spectrum peak period T_p , some more advanced approaches such as adaptive response surface method or advanced Gaussian process can be tried to solve this problem which may take into account more ocean environmental random variables such as mean wind velocity at 10 m above the mean still water level V_w and wave direction θ .
2. In Chapter 5, three environmental random variables are incorporated in the constructed advanced multiple copula model. Theoretically, the advanced multiple copula model can deal with more random variables besides H_s , T_p and V_w with an increase of complex decomposition items. It would be closer to real applications that more environmental random variables can be included in the simulation, and is worthy of future investigation.
3. Moreover, in the expansion of PPC work, from Equation (5.6) to Equation (5.7), an assumption is made that the pair-copulas are independent of the conditioning variables, except through the conditional distributions, which in fact can be re-considered by removing the hypothesis. In the 3-dimensional case, it is written as Equation (5.6), within which the last x_3 will not be neglected: $f_{123}(x_1, x_2, x_3) = f_1(x_1)f_2(x_2)f_3(x_3) \cdot c_{13}(F_1(x_1), F_3(x_3))c_{23}(F_2(x_2), F_3(x_3)) \cdot c_{12|3}(F_{1|3}(x_1|x_3), F_{2|3}(x_2|x_3); x_3)$.

The problem above can be solved by inserting a Fuzzy logic to treat the parameters as an extension of probability solution. Furthermore, as an extension of solution to this problem in the field of offshore engineering, further studies can be applied to solve probability problem in other fields, such as transportation safety.

REFERENCES

- Aas, K., Czado, C. and Bakken, H. (2009), "Pair-copula constructions of multiple dependence", *Insurance: Mathematics and Economics*, vol. 44(2), pp. 182-198.
- Acar, E., Rais-Rohani, M. and Eamon, C. D. (2010), "Reliability estimation using univariate dimension reduction and extended generalized lambda distribution", *International Journal of Reliability and Safety*, *International Journal of Reliability and Safety*, vol. 4(2-3), pp. 166-187.
- Achintya, H. (2000), *Probability, Reliability, and Statistical Methods in Engineering Design*, John Wiley & Sons, Inc., pp. 181-226.
- Akaike, H. (1974), "A new look at the statistical model identification", *IEEE Transactions on Automatic Control*, vol. 19(6), pp. 716-723.
- Akaike, H. (1976), "On entropy maximization principle", *Applications of statistics*, P.R. Krishnaiah, ed., North-Holland, Amsterdam, The Netherlands, pp. 27-41.
- American Bureau of Shipping (ABS). (2003), *Guide for the Fatigue Assessment of Offshore Structures*
- American Petroleum Institute (API). (2014), *API-RP 2A-WSD Planning, Designing, and Constructing Fixed Offshore Platforms- Working Stress Design*.
- American Petroleum Institute (API). (2002), *Recommended Practice for Flexible Pipe*.
- Andersen, M., Berg, A. and Saevik, S. (2001), "Development of an Optical Monitoring System for Flexible Risers", *Proceedings of OTC Conference*, Houston, Texas, USA, 13201.
- Ang, A. and Chen, J. (2002), "Asymmetric correlations of equity portfolios", *Journal of Financial Economics*, vol. 63(3), pp. 443-494.
- Bai, Y. and Bai, Q. (2005), *Subsea Pipeline and Risers*, Elsevier.
- Bain, L. and Engelhardt, M. (1991), *Introduction to probability and mathematical statistics (Duxbury classic series)*, 2nd edition, Thomson Learning.
- Barltrop, N. C. P. (1998), *Floating structures: a guide for design and analysis*, London: CMPT, c1998.
- Battjes, J. A. (1972), "Long term wave height distributions at seven stations around the British Isles", *Deutsche Hydrographische Zeitschrift*, vol. 25, pp. 179-189.
- Beck, J. L. (1978), "Determining models of structures from earthquake records", *Technical Rep. No. EERL 78-01*, California Institute of Technology, Earthquake Engineering Research Laboratory, Pasadena, Calif.

- Beck, J. L. and Katafygiotis, L. S. (1998), "Updating models and their uncertainties. I: Bayesian statistical framework", *Journal of Engineering Mechanics*, vol. 124(4), pp. 455-461.
- Beck, J. L. and Yuen, K. (2004), "Model Selection Using Response Measurements: Bayesian Probabilistic Approach", *Journal of Engineering Mechanics*, vol. 130(2), pp. 192-203.
- Bedford, T. and Cooke, R. (2001), "Probabilistic density decomposition for conditionally dependent random variables modelled by vines", *Annals of mathematics and Artificial Intelligence*, vol. 32, pp. 245-268.
- Bedford, T. and Cooke, R. (2002), "Vines-a new graphical model for dependent random variables", *Annals of Statistics*, vol. 30(4), pp. 1031-1068.
- Berthellamy, J. and Rackwitz, R. (1979), "Multiple Point Checking in System Reliability", Research Report, SFB 96, Technical University, Munich.
- Bitner-Gregersen, E. M. and Haver, S. (1989), "Joint Long Term Description of Environmental Parameters for Structural Response Calculation", *Proc. 2nd Int. Workshop on Wave Hindcasting and Forecasting*, Vancouver, B.C. Canada, April 25-28.
- Bitner-Gregersen, E. M. and Haver, S. (1991), "Joint Environmental Model for Reliability Calculations", *Proceedings of the 1st International Offshore and Polar Engineering Conference*, Edinburgh, United Kingdom, August 11-15.
- Bitner-Gregersen, E. M. (1996), "Distribution of Multidirectional Environmental Effects", *Proceedings of 15th International Conference of Offshore Mechanics and Arctic Engineering; OMAE 1996*; Florence, Italy.
- Bitner-Gregersen, E. M. (2005), "Joint Probabilistic Description for Combined Seas", *Proceedings of 24th International Conference on Offshore Mechanics and Arctic Engineering (OMAE 2005)*, June 12-17, 2005, Halkidiki, Greece OMAE 2005-67382.
- Bleistein, N. and Handelsman, R. (1986), "Asymptotic expansions for integrals", Dover Publications, Inc., New York, N.Y.
- Box G. E. P. and Jenkins G. M. (1970), *Time series analysis forecasting and control*, Holden Day, San Francisco.
- Brack, M., Mendes, S. R. A., Campos, R. C. L. and De Souza, L. A. and L. (2007), "An Integrity management Strategy based on Flexible Risers conceived to be Self-Monitored", *Proceedings of the 26th International Conference on Offshore Mechanics and Arctic Engineering, OMAE*, San Diego, California, USA, 29576.
- Brayton, R., Director, S., Hachtel, G. and Vidigal, L. (1979), "A New Algorithm for Statistical Circuit Design Based on Quasi-Newton Methods and Function Splitting", *IEEE Trans. Circuits and Systems*, vol. CAS-26, pp. 784-794, Sept. 1979.

- Brechmann, E. C. and Schepsmeier, U. (2013), "Modeling Dependence with C- and D- Vine Copulas: The R Package CDVing", *Journal of Statistical Software*, Vol. 52(3).
- Burleson, Clyde. W. (1999), "Deep challenge: the true epic story of our quest for energy beneath the sea", Gulf Publishing Company, Houston, Texas.
- Campbell, M. (1999), "The complexities of fatigue analysis for deepwater risers", In: *proceedings of the Deepwater Pipeline Conference*, New Orleans.
- Celik, A. N. (2004), "A statistical analysis of wind power density based on the Weibull and Rayleigh models at the southern region of Turkey", *Renew Energy*, vol. 29 (4), pp. 593-604.
- Chakrabarti, S. (2005), *Handbook of offshore engineering*. Elsevier Science.
- Chezhian, M., Mørk, K., Lespinasse, P., Farrant, T. and Søreide, M. (2007), "Riser Integrity Management-Recent Advances in the Deepwater Industry Practice", *Offshore Technology Conference, OTC*, Houston, Texas, USA, 18904.
- Chollete, L., Heinen, A., and Valdesogo, A. (2008), "Modeling international financial returns with a multivariate regime switching copula", MPRA paper 8114, University of Munich, Department of Economics.
- Cooke, R. M., Joe, H., Aas, K., Kurowicka, D. and Joe, H. (2011), "Dependence Modeling Vine Copula Handbook", *World Scientific*. pp. 37-72.
- Cramer, H. and Leadbetter, M. R. (1967), *Stationary and related stochastic processes*, New York: John Wiley.
- Dahlquist, G. and Björck, A. (1974), *Numerical Methods*, Prentice-Hall, Englewood Cliffs, NJ.
- Daniels, H. E. (1954), "Saddlepoint Approximations in Statistics", *The Annals of Mathematical Statistics*, Vol. 25, No. 4(Dec., 1954), pp. 631-650.
- Dean, R. G. (1965), "Stream function representation of nonlinear ocean waves", *Journal of Geophysical Research*, Vol. 70, No. 8.
- Dean, R. G. (1970), "Relative validities of water wave theories", *Journal of the Waterways and Harbours Div., ASCE*.
- De la Cour, D., Kristensen, C. and Nielsen, N. J. R. (2008), "Managing Fatigue in Deepwater Flexible Risers", *Proceedings of Offshore Technology Conference*, Houston, Texas, USA, 19443.
- De Lemos, C. A. D., De Sousa, F. J. M. and De Sousa, J. R. M. (2008), "Flexible riser fatigue re-evaluation to extend the service life", *Proceedings of the 27th International Conference in Offshore Mechanics and Arctic Engineering*, Oslo, Norway, 28145.

- De Sousa, J. R. M., De Sousa, F. J. M., De Siqueira, M. Q., Sagrilo, L. V. S. and De Lemos, C. A. D. (2007), "Parametric study of flexible risers service life", Rio Pipeline Conference and Exposition, Rio de Janeiro, Brazil, IBP1462_07.
- De Sousa, J. R. M., De Sousa, F. J. M., De Siqueira, M. Q., Sagrilo, L. V. S. and De Lemos, C. A. D. (2012), "A Theoretical Approach to Predict the Fatigue Life of Flexible Pipes", Journal of Applied Mathematics, Vol. 2012, Article ID 983819, 29 pages.
- Det Norske Veritas. (2010), DNV-RP-C205 Environmental conditions and Environmental loads.
- Det Norske Veritas. (2012), DNV-RP-C205 Environmental conditions and Environmental loads.
- Det Norske Veritas. (2011), DNV-RP-C203 Fatigue Design of Offshore Steel Structures.
- Det Norske Veritas. (2010), DNV-RP-F204 Riser Fatigue.
- Det Norske Veritas. (2010), DNV-OS-F201 Dynamic Risers.
- Dong, W., Moan, T. and Gao Z. (2011), "Long-term fatigue analysis of multi-planar tubular joints for jacket-type offshore wind turbine in time domain", Engineering Structures 33(2011) 2002-2014.
- Du, X. and Sudjianto, A. (2004), "First-Order Saddlepoint Approximation for Reliability Analysis", AIAA Journal, Vol. 42, No. 6.
- Efron B. and Tibshirani R. J. (1994), "An introduction to the Bootstrap", Chapman and Hall/CRC.
- Elzbieta M. Bitner-Gregersen, (2014), "Recent development of ocean environmental description with focus on uncertainties", Ocean Engineering, Vol. 86, pp. 26-46.
- Embrechts, P., Lindskog, F. and McNeil, A. (2001), "Modelling Dependence with Copulas and Applications to Risk Management", Department of Mathematics ETHZ, CH-8092 Zürich, Switzerland.
- Faltinsen O. M. (1990), Sea loads on ships and offshore structures. Cambridge University Press.
- Farnes, K., Moan, T., (1993), "Extreme Response of a Flexible Riser System Using a Complete Nonlinear Long-Term Approach", In: proceedings of the Third International Offshore and Polar Engineering Conference", Singapore.
- Ferreira, A. and Guedes Soares, C. (1999), "Modelling the Long-Term Distribution of Significant Wave Height with the Beta and Gamma Models", Ocean Engineering, vol. 26(8), pp. 713-725.

- Fiessler, B., Neumann, H-J. and Rackwitz, R. (1979), "Quadratic limit states in structural reliability", *Journal of Engineering Mechanics*, ASCE, vol. 105(4), pp. 661-676.
- Fisher NI (1997), Copulas. In: Kotz S, Read CB, Banks DL (eds) *Encyclopedia of Statistical Sciences*, Update Vol 1. Wiley, New York, pp. 159-163.
- Fisher, M., Köck, C., Schlüter, S., Weigert, F. (2007), "Multivariate copula models at work: Outperforming the 'desert island copula'?" Tech. Rep. 79, Universität Erlangen-Nürnberg, Lehrstuhl für Statistik und Ökonometrie.
- Forristall, G. Z. (2000), "Wave crest distributions: observations and second-order theory", *Journal of Physical Oceanography*, vol. 30, pp. 1931-1943.
- Frees, E. W. and Valdez, E. A. (1998), "Understanding relationships using copulas", *N. Am. Actuar. J.*, vol. 2(1), pp. 1-25.
- Gao Y. and Low, Y. M. (2016), "An efficient importance sampling method for long-term fatigue assessment of deepwater risers with time domain analysis", *Probabilistic Engineering Mechanics*, vol. 45, pp. 102-114.
- Genest, C. and Favre, A. (2007), "Everything you always wanted to know about copula modelling but were afraid to ask", *Journal of Hydrologic Engineering*, vol. 12(4), pp. 347-368.
- Gersch, W., Taoka, G. T., and Liu, R. (1976), "Structural system parameter estimation by two-stage least squares method", *J. Eng. Mech.*, vol. 102(5), pp. 883-899.
- Goodman L. A. (1962), "The variance of the product of K random variables", *J Amer Statist Assoc*, vol. 57(297), pp. 54-60.
- Grime, A. J., Langley, R. S., (2008), "Lifetime reliability based design of an offshore vessel mooring", *Applied Ocean Research*, vol. 30, pp. 221-234.
- Guedes Soares, C. and Scotto, M. G. (2001), "Modelling uncertainty in long-term predictions of significant wave height", *Ocean Engineering*, vol. 28, pp. 329-342.
- Haff, I. H., Aas, K. and Frigessi, A. (2010), "On the simplified pair-copula construction—simply useful or too simplistic?" *Journal of Multivariate Analysis*, Vol. 101(5), pp. 1296-1310.
- Haldar, A. and Mahadevan, S. (2001), *Probability, reliability, and statistical methods in engineering design*, Wiley.
- Haring, R. E. and Heideman, J. C. (1978), "Gulf of Mexico rare wave return periods", proceeding of the Offshore Technology Conference, Number OTC 3230, Houston, Texas.
- Hartung J., Knapp G. and Sinha B. K. (2008), *Statistical meta-analysis with applications*. John Wiley & Sons.

- Haver, S., Nyhus, K. A. (1986), "A wave climate description for long term response calculations", In: Proceedings of the 9th International Conference on Offshore Mechanics and Arctic Engineering, vol. 4, pp. 27-34.
- Haver, S. (2002), "On the prediction of extreme wave crest heights", In: Proceedings of the 7th International Workshop on Wave Hindcasting and Forecasting, Meteorological Service of Canada, Banff, Canada, Environment Canada.
- Horne, M. R. and Price, P. H. (1977), Commentary on the level 2 procedure, "Rationalization of Safety and Serviceability Factors in Structural Codes", Report No. 63, Construction Industry Research and Information Association, London, pp. 209-226.
- Huang, B. and Du, X. (2008), "Probabilistic uncertainty analysis by mean-value first order Saddlepoint Approximation", Reliability Engineering and System Safety, vol. 93, pp. 325-336.
- Jahns, H. O. and Wheeler, J. D. (1972), "Long term wave probabilities based on hindcasting of severe storms", Proceedings of the Offshore Technology Conference, Number OTC 1590, Houston, Texas.
- Jaspers, N. H. (1956), "Statistical distribution patterns of ocean waves and of wave induced stresses and motions with engineering applications", Trans Soc Naval ArchMar Eng, vol. 64, pp. 375-432.
- Joe, H. (1996), "Distributions with Fixed Marginals and Related Topics", IMS, Hayward, CA, Ch. Families of m -variate distributions with given margins and $m(m-1)/2$ dependence parameters.
- Joe, H. (1996), "Families of m -variate distributions with given margins and $m(m-1)/2$ bivariate dependence parameters. In L. Rüschendorf and B. Schweizer and M.D. Taylor (Ed.), *Distributions with Fixed Marginals and Related Topics*.
- Johannessen, K., Meling, T. S., and Haver S. (2001), "Joint Distribution for Wind and Waves in the Northern North Sea", Proceedings of the Eleventh International Offshore and Polar Engineering Conference, Stavanger, Norway, June 17-22, 2001.
- Johnson, N. and Kotz, S. (1970), Continuous univariate distribution, Wiley, New York.
- Kahn, H. (1956), "Use of different Monte Carlo sampling techniques", Proc. Symp. on Monte Carlo Methods, H. A. Meyer (Ed), John Wiley & Sons, New York, 149-190.
- Kendall, M. (1938), "A New Measure of Rank Correlation", Biometrika, vol. 30(1-2), pp. 81-89.
- Koçluoglu, H. U. (1995), "Stochastic response and reliability analyses of structures with random properties subject to stationary random excitation", Rep., Dept. of Civ. Engrg, and Operations Res., Princeton Univ., Princeton, N.J.

- Kurowicka, D. and Cooke, R. (2006), "Uncertainty Analysis with High Dimensional Dependence Modelling", Wiley, New York.
- Lebrun, R. and Dutfoy, A. (2009), "A generalization of the Nataf transformation to distributions with elliptical copula", *Probabilistic Engineering Mechanics*, vol. 24, pp. 172-178.
- Leira, B. J., Lgland, R. T., Ye, N. and Passano, E. A. (2007), "Fatigue Safety Factors for the Tensile and Pressure Armours of Flexible Risers", *Proceedings of the 26th International Conference on Offshore Mechanics and Arctic Engineering, OMAE*, San Diego, California, USA, 27486.
- Li F. and Low, Y. M. (2012), "Fatigue reliability analysis of a steel catenary riser at the touchdown point incorporating soil model uncertainties", *Applied Ocean Research*, vol. 38, pp. 100-110.
- Liu, D. and Peng, Y. (2012), "Reliability Analysis by Mean-Value Second-Order Expansion", *Journal of Mechanical Design*, June, 2012, vol. 134, 061005-1~8.
- Longin, F. and Solnik, B. (2001), "Extreme correlation of international equity markets", *Journal of Finance*, vol. 56(2), pp. 649-676.
- Low, Y. M., (2011), "Variance of the fatigue damage due to a Gaussian narrowband process", *Structural Safety*, vol. 34, pp. 381-389.
- Low Y. M. and Cheung, S. H. (2012), "On the long-term fatigue assessment of mooring and riser systems", *Ocean Engineering*, vol. 53, pp. 60-71.
- Low, Y. M. and Langly, R. S. (2006), "Time and frequency domain coupled analysis of deepwater floating production systems", *Applied Ocean Research*, vol. 28, pp.371-385.
- Lugannani, R. and Rice, S. O. (1980), "Saddle Approximation for the Distribution of the Sum of Independent Random Variables", *Adv. Appl. Probab.*, vol. 12, pp. 475-490.
- Madsen, H. O., Krenk, S. and Lind, N. C. (1986), *Methods of structural safety*, Prenticce Hall, New Jersey.
- Mallery, G. O. and Clark, G. C. (1972), "Cnoidal wave theory for application to offshore structural design", *OTC paper No. 1614*.
- Manwell, J. F., McGowan, J. G. and Rogers, A. L. (2002), *Wind energy explained: theory, design and application*.
- Mazumder, R. and Mazumber, B. S. (2006), "Statistical characterization of circulation patterns and direction of turbulent flow over a waveform structure", *Environmetrics*, vol. 17(5), pp. 417-434.
- Melchers, R. E. (1999), *Structural Reliability Analysis and Prediction*, 2nd edition, John Wiley&Sons, Inc., Baffins Lane, Chichester.

- Miles, J. W., (1954), On structural fatigue under random loading. *Journal of the Aeronautical Sciences*, vol. 21(11), pp. 753-762.
- Montgomery D. C., Peck E. A. and Vining G. G. (2012), “Introduction to linear regression analysis”, 5th ed. Wiley.
- Morgan, E. C., Lackner, M. and Vogel, R. M. (2011), “Probability distributions for offshore wind speeds”, *Energy Conversion and Management*, vol. 52(1), pp. 15-26.
- Morokoff, W. J. and Caflisch, R. E. (1994), “Quasi-Random Sequences and their discrepancies”, *Siam Journal on Scientific Computing*, vol. 15(6), pp. 1251-1279.
- Morokoff, W. J. and Caflisch, R. E. (1995), “Quasi-Monte Carlo Integration”, *Journal of Computational Physics*, vol. 122, pp. 218-230.
- Mouri H. (2013), “Long-normal distribution from a process that is not multiplicative but is additive”, *Physical Review E*, vol. 88(4): 042124.
- Naess, A. and Moan, T. (2005), “Probabilistic design of offshore structures”, Ed. In: Chakrabarti, S.K. (Ed.), *Handbook of Offshore Engineering*, 1. Elsevier, Amsterdam, pp. 197-255.
- Nataf, A. (1962), “Determination des Distribution dont les Marges sont Donnees”, *Comptes Rendus de l’ Academie des Sciences*, vol. 225, pp. 42-43.
- Nelson, R. B. (2006), “An Introduction to Copula”, Springer, New York.
- Nielsen N.-J. R., Glejbøl, K. and Kulakov, V. (2001), “Innovative Un-bonded Flexible Pipe Design with CFRP/metal Armour”, *Deep Offshore Technology Conference*, DOT, Rio de Janeiro, Brazil.
- NOAA Ocean Explorer (2010), Types of offshore oil and gas structures. National oceanic and Atmospheric Administration.
- Nygaard, E., Johannessen, K., (2000), “Metoccean Design Criteria for Kvitebjørn”, Statoil Report, C193-KVB-N-FD-0001, Rev. date: 2000-12-14, Stavanger.
- Ochi, K. (1992), “New approach for estimating the severest sea state from statistical data”, *Coastal Engineering Conference*, pp. 512-525.
- Ochi, K. (2011), “New approach for estimating the severest sea state from statistical data”, *Coast Eng Conf*, vol. 1(23).
- Orcina Ltd. (2012), Orcaflex manual ver 9.5a.
- Ormberg, H. and Larsen, K. (1998), “Coupled analysis of floater motion and mooring dynamics for a turret-moored ship”, *Applied Ocean Research*, vol. 20(1-2), pp.55-67.

- Papadimitriou, C., Katafygiotis, L. S. and Beck, J. L. (1995), "Approximate analysis of response variability of uncertain linear systems", *Probabilistic Engineering Mechanics*, vol. 10(4), pp. 251-264.
- Papadimitriou, C., Beck, J. L. and Katafygiotis, L. S. (1997), "Asymptotic expansions for reliability and moments of uncertain systems", *Journal of Engineering Mechanics*, vol. 123(12), pp. 1219-1229.
- Prince-Wright, R. (1995), "Maximum Likelihood Models of Joint Environmental Data for TLP Design for TLP Design", *OMAE, Safety and Reliability*, volume II, ASME.
- Pugh, D. T. (1982), "Estimating extreme currents by combining tidal and surge probabilities", *Ocean Engineering*, vol. 9(4), pp. 361-372.
- Rackwitz, R. (1976), *Practical Probabilistic Approach to Design*, bulletin no. 112, Comite European du Beton, Paris, France.
- Rackwitz, R. and Fiessler, B. (1978), "Structural Reliability Under Combined Random Load Sequences", *Computers and Structures*, vol. 9, No. 5, pp. 484-494.
- Ramirez, P. and Carta, J. A. (2005), "Influence of the data sampling interval in the estimation of the parameters of the Weibull wind speed probability density distribution: a case study", *Energy Conversion Management*, vol. 46(15-16), pp. 2419-2438.
- Rahman, S. and Xu, H. (2004), "A univariate dimension-reduction method for multi-dimensional integration in stochastic mechanics", *Probabilistic Engineering Mechanics*, vol. 19, pp. 393-408.
- Renard, B. and Lang, M. (2007), "Use of a Gaussian copula for multivariate extreme value analysis: some case studies in hydrology", *Advances in Water Resources*, vol. 30, pp. 897-912.
- Rice SO. (1944), "Mathematical analysis of random noise", *Bell System Technical Journal*, vol. 29, pp: 282-332.
- Robinson, M. E. and Tawn, J. A. (1997), "Statistics for extreme sea currents", *Applied Statistics-Journal of Royal Statistical Society Series C*, vol. 46(2), pp. 183-205.
- Rosenblatt, M. (1952), "Remarks on a multivariate transformation", *Ann. Math. Stat.*, vol. 23, pp. 470-472.
- Rosenblueth, E., (1975), "Point estimates for probability moments", *Proc., Nat. Acad. Of Sci.*, vol. 72(10), pp. 3812-3814.
- Rubinstein, R. Y. (1981), *Simulation and the Monte Carlo Method*, John Wiley & Sons, New York.
- Sagrilo, L. V. S., Lima, E. C. P. and Papaleo, A. (2011), "A joint probability model for environmental parameters", *Journal of Offshore Mechanics and Arctic Engineering*, vol. 133, 031605.1-7.

- Sanayei, M., McClain, J. A. S., Wadia-Fascetti, S. and Santini, E. M. (1999), "Parameter estimation incorporating model data and boundary conditions", *J. Struct. Eng.*, vol. 125(9), pp. 1048-1055.
- Sarkar, P. K. and Prasad, M. A. (1987), "A comparative-study of pseudo and Quasi Random sequences for the solution of integral-equations", *Journal of Computational Physics*, vol. 68(1), pp. 66-88.
- Sarpkaya, T. and Isaacson, M. (1981), "Mechanics of Offshore Structures", Van Nostrand Reinhold Company.
- Sauvaget, P., David, E. and Guedes Soares, C. (2000), "Modelling Tidal Currents on the Coast of Portugal", *Coastal Engineering*, vol. 40(4), pp. 393-409.
- Schwarz G. (1978), "Estimating the dimension of a model", *Ann Stat*, vol. 6(2), pp. 461-464.
- Schwartz, L. (1974). "Computer extension and analytical continuation of Stokes' expansion for gravity waves". *Journal of Fluid Mechanics*. Vol. 62, pp. 553-578.
- Schweizer B. (1991). "Thirty years of copulas", In: Dall' Aglio G, Kotz S, Salinetti G(eds) *Advances in Probability Distributions with Given Marginals*. Kluwer, Dordrecht, pp. 13-50.
- Senra, S. F., Correa, F. N., Jacob, B. P., Mourelle, M. M. and Masetti, I. Q. (2002), "Towards the integration of analysis and design of mooring systems and risers: Part I- Studies on a Semisubmersible Platform", 21st International Conference on Offshore Mechanics and Arctic Engineering, Oslo, Norway, June 23-28, 28046.
- Sheehan, J. M. (2006), "Characterizing the Wave Environment in the Fatigue Analysis of Flexible Risers", *Journal of Offshore Mechanics and Arctic Engineering*, vol. 128, pp. 108-118.
- Shinozuka, M. (1983), "Basic analysis of structural safety", *Journal of Structural Division, ASCE*, vol. 109(3), pp. 721-740.
- Singh, M. P. (1980), "Seismic response of structures with random parameters", *Proc., 7th World Conf. On Earthquake Engrg*, vol. 6, pp. 705-712.
- Sklar, A(1959), *Fonctions de répartition à n dimensions et leurs marges*, *Publ. Inst. Statist. Univ. Paris*, vol. 8, pp. 229-231.
- Stevenson, J. and Moses, F. (1970), "Reliability analysis of frame structures", *Journal of Structural Division, ASCE*, vol. 96 (11), pp. 2409-2427.
- Thompson, D. and Kilgore, R. (2011), "Estimating Joint Flow Probabilities at Stream Confluences using Copulas", *Transportation Research Record*, vol. 2262, pp. 200-206.
- Vazquez-Hernandez, A. O., (2011), "Long-term response analysis of FPSO mooring systems", *Applied Ocean Research*, vol. 33, pp. 375-383.

- Vandenberghe, S., Verhoest, N. E. C., Onof, C. and De Baets, B. (2011), "A comparative copula-based bivariate frequency analysis of observed and simulated storm event: A case study on Bartlett_Lewis modelled rainfall", *Water Resources Research*, 47: W07529 DOI: 10.1029/2009WR008388.2011.
- Vanik, M. W., Beck, J. L. and Au, S. K. (2000), "A Bayesian probabilistic approach to structural health monitoring", *J. Eng. Mech.*, vol. 126(7), pp. 738-745.
- Vazquez-Hernandez, A. O. (2006), "Reliability-based comparative study for mooring lines design criteria", *Applied Ocean Research*, vol. 28, pp. 398-406.
- Veneziano, D. (1974), "Contributions to Second Moment Reliability", Research Report No. R74-33, Department of Civil Engineering, Cambridge, MA.
- Warner, R. F. and Kabaila, A. P. (1968), "Monte Carlo study of structural safety", *J. Structural Div.*, ASCE, vol. 94 (12), pp. 2847-2859.
- Wiegel, R. L. (1960), "A presentation of cnoidal wave theory for practical application", *Journal of Fluid Mechanics*, vol. 7, No. 18.
- Wikipedia.
(https://en.wikipedia.org/wiki/Spearman%27s_rank_correlation_coefficient#cite_note-myers2003-3).
- Yan, J. (2006), "Modeling with Copulas and Engineering Applications", *Springer Handbook of engineering statistics*, Springer, London, pp. 973-990.
- Yang, C. M. and Beck, J. L. (1997), "Generalized trajectory methods for finding multiple extrema and roots of function, *Journal of Optimization Theory and Applications*, vol. 97(1), pp. 211-227.
- Youn, B. and Choi, K. (2004), "Selecting probabilistic approaches for reliability-based design optimization", *AIAA J*, vol. 42(1), pp. 124-131.
- Zhang, Y., Chen, B., Qiu, L., Hill, T. and Case, M. (2003), "State of Art Analytical Tools Improve Optimization of Unbonded Flexible Pipes for Deepwater Environments", *OTC*, 15196.
- Zhang, J. and Du, X. (2010), "A Second-Order Reliability Method with First-Order Efficiency", *Journal of Mechanical Design*, vol. 132(10), 101006-(1-8).
- Zhang Y., Beer, M. and Quek S.T. (2015), "Long-term performance assessment and design of offshore structures", *Computers and Structures*, vol. 154 (2015), pp. 101-115.
- Zhang, Y. and Tan, X. (2011), "An investigation of fatigue damage due to VIV in flexible risers with different methodologies", *Proceeding of ASME 2011 30th International Conference in Offshore Mechanics and Arctic Engineering*, Rotterdam, OMAE, the Netherlands, 49036.
- Zhao, Y. and One, T. (2000), "New Point Estimates for Probability Moments", *Journal of Engineering Mechanics*, vol. 126, No. 4, April.

Zhao, Y., Ono, T. (2001), “Moment methods for structural reliability”, *Structural Safety*, 23:47-7.