Three essays on fertility

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THREE ESSAYS ON FERTILITY

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SCHOOL OF SOCIAL SCIENCES

2018

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May I continue to yield to Him, trust in Him, be guided, humbled and amazed by Him, my Father in heaven; may the Holy Spirit continue to work in wondrous ways to show me His glory. May I reflect the love of the Father and His glory wherever I am and be strengthened by Him in my weakness and may this thesis bring glory to His name.

“Have I not commanded you? Be strong and courageous. Do not be frightened, and do not be dismayed, for the Lord your God is with you wherever you go.” ~ Joshua 1:9

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SUMMARY

This thesis consists of three essays on fertility.

Essay one examines the relationship between the fluctuations in fertility and housing prices. In the overlapping generations (OLG) framework with fixed land supply, nonlinear dynamics can occur. As the preference for children increases, fluctuations in fertility rate and housing price become larger; lower preferences result in their convergence to steady state. The Singapore data show that the fluctuations in birth rates and housing resale prices are large in the earlier years compared to the later years. In addition, the estimates for birth rate are significant to resale housing price.

Essay two explores the demand for housing with the demand for children under different social security systems in an OLG endogenous growth framework. Housing can be a socioeconomic good which takes up a substantial amount of one's income and competes with children for resources. It shows that for the same retirement income, the Pay-As-You-Go (PAYG) social security system results in high fertility compared to the fully funded (FF) system. Consumption for housing is also higher under the FF system. Unless the rate of return from the FF system is high enough, switching from the PAYG to the FF system tend to decrease fertility and increase growth.

Essay three examines the determinants of fertility and identifies the theory that explains the decline in fertility in Singapore. Using the Autoregressive Distributive Lag (ARDL) model, results show that literacy rate and cost of living are the two variables that decrease fertility; estimates for private consumption and real GDP per capita are not significant for all model specifications. The Value of Children (VOC) approach offers the most plausible explanation for the decline in fertility in Singapore.

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LIST OF ACRONYMS

ADF: Augmented Dickey-Fuller

AIC: Akaike Information Criterion

ARDL: Autoregressive Distributive Lag

BGP: Balance Growth Path

BTO: Built-to-Order

CPF: Central Provident Fund

CPI: Consumer Price Index

CUSUMSQ: Cumulative Sum Squared

DF-GLS: Dickey-Fuller Generalized Least Squares

DOLS: Dynamic Ordinary Least Squares

DOS: Department of Statistics

ECM: Error Correction Mechanism

FF: fully funded

FMOLS: Fully Modified Ordinary Least Squares

FPE: Final Prediction Error

GDP: Gross Domestic Product

HAC: heteroskedasticity and autocorrelation-consistent

HDB: Housing Development Board

HES: Household Expenditure Survey

HQ: Hannan-Quinn Information Criterion

IMU: Increasing marginal utility

KPSS: Kwiatkowski, Philips, Schmidt, and Shin

LBS: Lease Buyback Scheme

MSA: metropolitan statistical area

OLG: overlapping generations

OLS: Ordinary Least Squares

PAYG: Pay-As-You-Go

PP: Philip-Perron

PSID: Panel Study of Income Dynamics

RNI: rate of natural increase

SIC: Schwarz Information Criterion

TFR: total fertility rate

TY: Toda-Yamamoto

USDA: U.S. Department of Agriculture

VAR: Vector Autoregressive Model

VECM: Vector Error Correction Mechanism

VOC: Value of Children

INTRODUCTION

Around the world, fertility rates of both developed and developing countries are declining. This is a worrying trend for many of the developed countries whose fertility rates have fallen far below the population replacement rate - It is required for couples to have two children on average for the population size to remain the same.

For many of these countries, low fertility rates below the replacement rate could threaten the sustainability of their unfunded (PAYG) social security systems as this would result in the shrinking of the working population that is needed to support the ageing population. The issues surrounding social security systems are the sustainability of the system, the adequacy of the payouts for pensioners and the equity of the system in distributing resources.

I. Social Security Systems and Pension Design

A social security system is dependent on two parts: the demography of its contributors and beneficiaries and economic factors.

Demographic factors include population, age and gender composition, fertility rate, migration flow, mortality rate and life expectancy and economic factors include Gross Domestic Product (GDP), inflation rate, interest rate, labour force participation rate, unemployment rate, income and wealth inequality and the wages and earnings profiles of individuals.

Table I shows the classification of different pension systems around the world. The unfunded or the Pay-As-You-Go (PAYG) system is one whereby contributions from the working population are used to support the retired old. The fully funded (FF) system is one whereby contributions are not given out but ‘saved’ with interest returns made through the investments in one’s account. If the interest rate of return is determined or fixed by the government, it is known as the notional defined contribution system. Whether a system is a defined benefit or contribution system depends on how the retirement sum received by the pensioner is calculated. If it is based on the contributions made by a

Table I: Configurations of Social Security Systems

Type of Plan	Funded	Unfunded
Defined contribution	Traditional employer pension (example: Switzerland)	United States Australia United Kingdom
Defined benefit	U.S. Roth IRA or 401(k) Chile Latin America Australia United Kingdom	
Notional defined contribution		Italy Sweden

Note : Adapted from “A Primer on Social Security Systems and Reforms,” by C. P. Aubuchon, J. C. Conesa, and C. Garriga, 2011, Federal Reserve Bank of St. Louis Review, 93(1), p. 21. Copyright Year by "The Federal Reserve Bank of St. Louis".

person when he or she is working, it is a defined contribution plan whereby individuals each have their own accounts. If it is based on some benefit formula depending on factors such as employee’s earnings history, length of service and age, it is a defined benefit plan. In short, whether it is a funded or an unfunded system depends on where the funds are coming from and whether it is a defined contribution or benefit system depends on how the funds are given out. Some countries have combinations of systems with both PAYG and fully funded elements, etc., known as mixed systems.

Around the world, many countries have undertaken social security reforms. For example, Sweden, Italy, and Germany have switched from the defined benefit system to the notional defined contribution system. For a relatively young country like Singapore, improvements to the current system can be made. For example, Chia (2015) proposes adding an additional basic pillar to the current Central Provident Fund (CPF) system to make it more inclusive for needy elderly. It is only recently that Singapore has substantially enhanced its national health insurance scheme, now known as Medishield Life, for all Singaporeans. Whatever it is, social security is something close to the hearts of people.

In considering social security system reform, Feldstein (1997) mentions five things when transiting from the PAYG system to the FF system. They are the path of transition, the effects on savings and capital accumulation, the rate of return that each system would

earn, the risks facing both systems and the distributive effects under the new system. According to Barr and Diamond (2010), the things to consider when designing a pension system are whether it will lead to early retirement or incur government debts that are passed from one generation to the next, the redistribution of resources between the rich and the poor and across generations, the risk-sharing mechanisms and safeguards within the system and the lessons and applications from behavioural economics.

This thesis abstracts away from these issues. Instead, it focuses on how representative agents optimise their consumption decisions under different social security systems, namely the defined benefit PAYG system and the notional defined contribution FF system.

II. Fertility, Social Security, and Growth

Fertility and social security are inextricably linked. People have children as a source of income for their old age security. In agricultural and rural societies, children are primarily the means to take over the land to continue production. Today, in increasingly urbanised societies, this may no longer be relevant. People, on the contrary, are now planning and leaving bequests to their children. Cost of living is rising; it is getting more expensive to raise children in many of the developed countries. According to Becker (1960), individuals derive utility from both the quantity and quality of children. Hence, when considering the number of children to have, altruistic parents¹ may first ensure that they have enough for retirement. Next, economic growth depends on fertility or population growth rate and the type of social security while pension rate of return depends on growth (Zhang, 1995).

III. Fertility Versus Housing and Other Material Consumption

Economists propose the decline in fertility due to lifestyle consumption. Leibenstein (1975) introduces the idea of social status consumption which involves items such as "wealth, occupation, housing, education, residential location, political or military power, and hierarchical and other titles" (p. 5). With such demands, individuals prefer to have

¹Altruistic parents are those whose child's utility is part of theirs (Nerlove and Raut, 1997).

fewer children.

IV. Essays One, Two, and Three

The first essay models the effects of housing price and social security on fertility and examines the relationship between birth rates and housing prices in Singapore.

The second essay examines how two different social security systems, the PAYG and FF system, may affect housing consumption, fertility rate, and growth. In Switzerland, the total fertility rate (TFR) is 1.55 while the homeownership rate is 43.4 percent. According to Bourassa and Hoesli (2010), 83 to 90 percent of the Swiss respondents in a survey express the desire to own homes. This low homeownership rate is mainly due to the high housing prices. Comparing with Switzerland, Singapore has a low TFR of 1.2 and a high homeownership rate of 90 percent. Also, Singapore has a fully funded notional defined contribution social security system while Switzerland has a mixed system with PAYG and fully funded components.

The third essay examines the determinants of fertility in Singapore.

1 ESSAY ONE: HOUSING CONSUMPTION, FERTILITY, AND HOUSING PRICE CYCLES: CAN CHILD PREFERENCE AFFECT THE SIZE OF FLUCTUATIONS? THEORY AND EVIDENCE

1.1 Introduction

Raising children in developed countries is costly and require space. In the Fifteenth Japanese National Fertility Survey in 2015, the most frequently (56.3 percent) cited reason why couples are not realising their ideal number of children is the high cost of raising and educating children. Among the different costs of rearing children, Lino (2014) estimates housing expenditure as the largest, ranging between 30 to 33 percent.

Building upon the endogenous fertility model of Becker and Lewis (1973), this essay explores the dynamics of housing price and fertility through the demand for children, housing, and fixed land supply in the overlapping generations (OLG) framework. This essay contributes to the existing literature on fertility cycles by linking fertility and housing price together through the demand and supply for housing, demonstrating the bidirectional effect of housing price and fertility and the instability in housing price. Results show that housing price can either exhibit monotonic convergence, oscillatory convergence, flip bifurcation, or chaotic dynamics for the different preferences for children. When there is low preference for children, prices converge to the steady state. When there is high preference for children, fluctuations and non-convergence can occur. The Singapore data show that there are large fluctuations in both birth rates and public resale housing prices in the years when birth rates are high compared to the other years. It is also shown empirically that the changes in birth rates can affect the changes in public resale housing prices.

It is Malthusian theory that explains the rise and fall of land rents through changes in population before the industrial revolution (Hansen and Prescott, 2002). In the same spirit, when the technology for land supply is constant, a larger population will result in higher housing price. Land control measures and the supply of land result in the increase in housing price in many countries (Gyourko et al., 2013; Hui and Ho, 2003; Glaeser et al., 2005; Davis and Heathcote, 2007; Mayo and Sheppard, 1996). On the demand side, homeownership and fertility decisions are made simultaneously (Öst, 2012). Some literature examining the housing market include the demand for housing consumption or

services in the utility functions of individuals (Chu, 2014; Yang, 2009; Davidoff, 2006; Chen, 2010). According to Stein (1995), households will consume more housing when they are able to do so. Ioannides (1987) and Ioannides and Kan (1996) find that the *size of family*, *housing price* and wealth are key decision-making factors that lead to housing tenure choice and mobility. Among those who have linked housing price and fertility together, Day and Guest (2016) examine the effect of female wages on fertility by incorporating housing cost into the child rearing cost function. Hui et al. (2016) examine the impact of fertility rate on housing price by allowing housing wealth to be bequeathed to the next generation. The earliest work on housing and population growth are done by Mankiw and Weil (1989) and Poterba et al. (1991).

From a large data set of 44 countries, Croix and Gobbi (2017) find that population density has a negative relationship with fertility. Using U.S. state-level data from 1800 to 2000, Murphy et al. (2008) find the same relationship - baby booms coinciding with low population density states - with the explanation given to the price of space. Other than population growth, demand for housing is affected by many factors. Eichholtz and Lindenthal (2014) show that in the U.K., the demand for housing is influenced by education level, good health, income and demographics.

Fertility cycles can be explained by larger populations facing steeper competition and lower income; this, in turn, keeps the population growth in check (Easterlin, 1973). Benhabib and Nishimura (1989) show that under specific conditions on child preferences, both fertility and per capita incomes move together and endogenously oscillate each other. By showing how fertility rates in the U.S. rose initially in the 1950s when women allocate their full amount of time to child-rearing and declined subsequently in the 1960s after they have gone out to work, Day (2004) explains that a fertility recovery in the U.S. is possible. Under her postulation, if inputs for rearing children become cheaper and income continues to increase, this trend could be reversed. Butz and Ward (1979) also predict the procyclical and countercyclical effects of fertility in the U.S. by using the husband's income as a proxy for household income and the woman's wage as a proxy for the price of

children. Among OECD countries, Malmberg (2012) shows that the changes in demand for housing resulting from demographic shifts can cause booms and busts in fertility rates through housing prices.

Theoretical models are necessary to identify feedback effects between fertility and demography (Wachter, 1991). Some have modelled fluctuations in housing prices (Dieci and Westerhoff, 2013; Ma and Mu, 2007) while others, to a lesser extent, have done work on the fluctuations in fertility (Samuelson, 1976; Fanti and Gori, 2013; Chen and Li, 2013; Nishimura and Kunapongkul, 1991) and demographics (Prskawetz and Feichtinger, 1995; Day, 1983; Day et al., 1989). Dieci and Westerhoff (2013) examine the speculative price movements in housing markets around a fundamental price. Ma and Mu (2007) model the fluctuations in housing price based on the cobweb theory and extend their model to include land supply (Ma and Mu, 2008). Fanti and Gori (2013) examine the effects of social security system on fertility and find that a system with fertility incentives can lead to unstable fertility rates. Chen and Li (2013) show that the intertemporal elasticity of substitution in consumption and the effectiveness and amount of child allowances can result in the chaotic dynamics of fertility rates and Nishimura and Kunapongkul (1991) demonstrate that the income elasticity of both demand for children and marginal utility of income can determine whether the capital per capita converges to or oscillates around its steady state.

Nonlinear dynamical models can be used to explain fluctuations and trends in time series. According to Day (1992), "the fact that nonlinear models based on economic forces behave like stochastic processes is a powerful confirmation that salient features of economic data can be given intrinsic, theoretical explanations (in contrast to extrinsic, ad hoc, data-driven explanations)" (p. S21). OLG economies do demonstrate nonlinear and chaotic dynamics (De la Croix and Michel, 2002; Grandmont; 1985, Michel and de la Croix, 2000); Samuelson (1976) is the pioneer for the nonlinear dynamics of fertility cycles in a two-period OLG economy. Among models that exhibit the dynamics of cyclical movements, none to my knowledge, have examined both fertility and housing price together.

The rest of this essay is organised as follows: Section 1.2 provides the model and theoretical framework for this essay. Section 1.3 presents an empirical study based on the data in Singapore, Section 1.4 summarises the results and Section 1.5 concludes this essay.

1.2 The Model

Based on Diamond's two-period OLG framework, individuals live for three periods, as a child, a young working adult (y), and a retiree at old age (o). Individuals save part of their wages for old age. In addition, the costs of raising children and housing are incurred by the young working adult. Individuals are not altruistic in the sense that they do not derive utility from their own children's utility but only from the number of children.¹ Individuals seek to maximise their young and old age consumptions, consumption from housing services and the number of children. In this model, there is no differentiation between male and female; every individual represents a household of two adults. The economy is a small open economy and interest rate is assumed to be constant.

1.2.1 Demographics

In period t , there are N_t young individuals. Each young individual in period t determines the birth rate, n_t which gives rise to the number of young individuals, N_{t+1} in period $t + 1$. The population growth rate is given by:

$$\frac{N_{t+1}}{N_t} = n_t \tag{1.1}$$

¹See Nerlove and Raut (1997) for more details.

1.2.2 Individuals

Individual maximises the utility function with the budget constraints as follows:

$$\text{Max } U(c_t^y, c_{t+1}^o, n_t, h_t) = \ln c_t^y + \beta \ln c_{t+1}^o + \nu \ln(h_t - \underline{h}) + \delta \ln n_t \quad (1.2)$$

$$\text{s.t. } c_t^y \leq W_t - s_t - p_t h_t - \alpha n_t - (1 - \gamma)\tau W_t - \gamma\tau W_t \quad (1.3)$$

$$c_{t+1}^o \leq s_t(1 + r) + B_t + \gamma\tau(1 + g)W_t \quad (1.4)$$

where c_t^y is the consumption at young age, c_{t+1}^o is the consumption at old age, h_t is the housing consumption, $\underline{h}$ is the minimum amount of housing consumption, $\underline{h} > 0$, n_t is the fertility rate, W_t is the wage rate, s_t is the savings, p_t is the price of per unit housing,² α is the cost of rearing children, τ is the forced contribution to social security,³ g is the fixed rate of return in the fully funded notional contribution system, r is the interest rate and B_t is the Pay-As-You-Go (PAYG) pension benefit. A portion of the social security contribution, given by γ , goes to the fully funded (FF) system and the rest, given by $1 - \gamma$, goes to the PAYG system.

1.2.3 Social Security System

The social security benefit comprises of two parts, from the FF system and the fertility-related PAYG system. For the FF system, each individual is required to save the amount, $\gamma\tau W_t$ and obtains the amount, $\gamma\tau(1 + g)W_t$ in the second period. For the *fertility-related* PAYG system, the amount the individual receives in the second period is determined by the following government's balanced budget equation:

$$N_t B_t = (1 - \gamma)N_{t+1}\tau W_{t+1} \quad (1.5)$$

$$B_t = (1 - \gamma)\frac{N_{t+1}}{N_t}\tau W_{t+1} \quad (1.6)$$

$$B_t = (1 - \gamma)n_t\tau W_{t+1} \quad (1.7)$$

²In this essay, p_t depends on previous period's price, p_{t-1} , as seen in the model later.

³Similar to Hirazawa and Yakita (2009), τ is assumed to be exogenous and welfare is not maximised by the government choosing τ .

In period $t + 1$, there are N_{t+1} young individuals supporting N_t retirees. Hence, B_t is determined by fertility rate, n_t and the wage rate of the young individuals, W_{t+1} . This social security system is similar to that in Fanti and Gori (2013) which comprises of two parts: a *fertility-related PAYG* component and a *fertility-unrelated PAYG* component. The FF component is the same as the *fertility-unrelated PAYG* component when the contribution rate, $1 + g$ is equal to the population growth rate and the wage rate, W_t is equal to the next generation's wage rate, W_{t+1} .

Substituting Equation (1.7) into Equation (1.4) and simplifying Equation (1.3), the two budget constraints become:

$$c_t^y \leq W_t(1 - \tau) - s_t - p_t h_t - \alpha n_t \quad (1.8)$$

$$c_{t+1}^o \leq s_t(1 + r) + (1 - \gamma)n_t \tau W_{t+1} + \gamma \tau(1 + g)W_t \quad (1.9)$$

Assuming that only the equality signs hold for both Equations (1.8) and (1.9), the first order conditions are given by:

$$\frac{\beta(1 + r)}{c_{t+1}^o} = \frac{1}{c_t^y} \quad (1.10)$$

$$\frac{\delta}{n_t} + \frac{\beta(1 - \gamma)\tau W_{t+1}}{c_{t+1}^o} = \frac{\alpha}{c_t^y} \quad (1.11)$$

$$\frac{\nu}{h_t - \underline{h}} = \frac{p_t}{c_t^y} \quad (1.12)$$

Equation (1.10) shows the trade-off between saving for future consumption and having present consumption. Equation (1.11) shows how the optimal number of children is obtained by comparing the pension benefits in the second period and the preference for the number of children in the utility function with the cost incurred in the first period and Equation (1.12) shows the trade-off between the utility derived from housing consumption and the consumption foregone in the first period.

From the first order conditions, we derive the expressions:

$$n_t = \frac{\delta \left(W_t \left(1 - \tau + \frac{\gamma\tau(1+g)}{(1+r)} \right) - \underline{h}p_t \right)}{(1 + \beta + \nu + \delta) \left(\alpha - \frac{(1-\gamma)\tau W_{t+1}}{(1+r)} \right)} \quad (1.13)$$

$$h_t = \frac{\nu}{p_t} \frac{\left(W_t \left(1 - \tau + \frac{\gamma\tau(1+g)}{(1+r)} \right) - \underline{h}p_t \right)}{(1 + \beta + \nu + \delta)} + \underline{h} \quad (1.14)$$

$$s_t = \beta \frac{\left(W_t \left(1 - \tau + \frac{\gamma\tau(1+g)}{(1+r)} \right) - \underline{h}p_t \right)}{(1 + \beta + \nu + \delta)} - \frac{(1 - \gamma)\tau W_{t+1}}{(1 + r)} \frac{\delta \left(W_t \left(1 - \tau + \frac{\gamma\tau(1+g)}{(1+r)} \right) - \underline{h}p_t \right)}{(1 + \beta + \nu + \delta) \left(\alpha - \frac{(1-\gamma)\tau W_{t+1}}{(1+r)} \right)} - \frac{\gamma\tau(1 + g)W_t}{(1 + r)} \quad (1.15)$$

Proposition 1: Housing consumption is inelastic. Fertility decreases with the price of housing.

Proof: See Appendix. With minimum housing consumption, $\underline{h}$ set to be greater than zero, housing consumption is price inelastic. Hanushek and Quigely (1980) estimate the price elasticity of demand for housing in the U.S. to be between -0.64 to -0.45. Some others have also found it to be inelastic (Ermisch et al., 1996; Geyer, 2017; Albouy et al., 2014). Housing is a basic essential good for starting families and therefore, each household maximises its housing consumption beyond a certain minimum requirement.

Proposition 2: An increase in housing price, p_t reduces fertility, n_t , young-age consumption, c_t^y and housing consumption, h_t .

Combining and rearranging Equations (1.10), (1.11) and (1.12), we have:

$$c_t^y = \left(\alpha - \frac{(1 - \gamma)\tau W_{t+1}}{(1 + r)} \right) \frac{n_t}{\delta} \quad (1.16)$$

$$h_t = \frac{\nu c_t^y}{p_t} + \underline{h} \quad (1.17)$$

From Equation (1.13), n_t decreases with p_t and from Equation (1.16), c_t^y also decreases with p_t since n_t decreases with p_t . Although $\underline{h}$ is independent of the number of children,

h_t changes with c_t^y and n_t . From Equation (1.17), h_t decreases with p_t since the numerator of the first term decreases with p_t while the denominator increases (with p_t). Hence, a higher demand for children results in a higher demand for housing. This is also in line with the rationale that space is required for rearing children.

There are many studies on housing price and fertility. In the U.S., based on the metropolitan statistical area (MSA) data for high-density areas, Dettling and Kearney (2014) find that a \$10,000 increase in home prices results in a 0.4 percent decrease among non-homeowners. Using the Panel Study of Income Dynamics (PSID) data, Lovenheim and Mumford (2013) show that a \$100,000 increase in housing wealth increases the probability of having a child by 16 to 18 percent. Based on census data, Simon and Tamura (2009) find that higher rents lead to lower fertility. In Hong Kong, Yi and Zhang (2010) find that a 1 percent increase in housing prices results in a 0.45 percent decline in total fertility rate (TFR). Pan and Xu (2012) also conclude with the same results for China.

1.2.4 Fixed Land Supply

As in Deaton and Laroque's (2001) fixed land supply model,⁴ land is used for housing. The government administers the buying and selling of land.⁵ Individuals purchase housing in the first period and live in it for two periods. Once the lease of the land is up *after the second period*, the government takes back the land and sells it to the young population two generations later that require housing. This can be interpreted as if individuals pay their housing rents in advance (in the first period) for two periods but do not own the place. Therefore, for the young generation in period t , the total demand for land is $N_t h_t$

⁴In their model, individuals purchase land (or housing) in the first period but do not live on it until the second period; they live with their parents in the first period. In the second period, they sell that piece of land to the next generation and consumes the profit while staying on it.

⁵This model is inspired by the public housing scheme in Singapore. The homeownership rate in Singapore is close to 90 percent and 80 percent of homeowners live in public housing known as the Housing Development Board (HDB) flats. With a scarce land supply of 720 square kilometres, land supply is controlled by the government. All newly constructed flats are purchased for a period of 99 years before they are returned to the government. In addition, there is the Lease Buyback Scheme (LBS) that allows individuals to sell the remaining lease of their flats after they have passed on back to the government when they are 64 years old or older for top ups on their retirement accounts. For tractability purposes, I assume housing is purchased for two periods; after that, it is returned to the government.

and it is met by a supply, H_{t-2}^s , that is obtained from the young generation in period $t - 2$. The housing market clearing condition is

$$N_t h_t = H_{t-2}^s \quad (1.18)$$

In the next period, $t + 1$, the total demand for land is $N_{t+1} h_{t+1}$ and it is met by the supply H_{t-1}^s . Shifting the condition in Equation (1.18) forward by one period,

$$N_{t+1} h_{t+1} = H_{t-1}^s \quad (1.19)$$

Assuming that each generation inherits the *same amount of land*, we have $H_{t-1}^s = H_{t-2}^s$. From Equations (1.18) and (1.19),

$$N_{t+1} h_{t+1} = N_t h_t \quad (1.20)$$

$$n_t h_{t+1} = h_t^6 \quad (1.21)$$

As in Hirazawa and Yakita (2009), without a loss of generality, I set $W_t = W_{t+1} = w$. Substituting expressions from Equations (1.13) and (1.14) into Equation (1.21), I have the following price dynamics equation:

$$p_{t+1} = \frac{\nu A \delta (A - \underline{h} p_t)}{\underline{h} (1 + \beta + \delta) [(1 + \beta + \nu + \delta) B - \delta (A - \underline{h} p_t)] + \frac{\nu A (1 + \beta + \nu + \delta) B}{p_t}} \quad (1.22)$$

where

$$A = w \left(1 - \tau + \frac{\gamma \tau (1 + g)}{(1 + r)} \right) \quad (1.23)$$

$$B = \alpha - \frac{(1 - \gamma) \tau w}{(1 + r)} \quad (1.24)$$

Proposition 3: If $(1 + \beta + \delta) \frac{[(1 + \beta + \nu + \delta) B - \delta A]^2}{(1 + \beta + \nu + \delta) B} - 4 \delta \nu A < 0$ is satisfied, the price of housing

⁶The housing law of motion here is the same as that in Deaton and Laroque (2001). In their model, the population growth rate is assumed to be exogenous and the land purchased by each generation decreases by the population growth rate, n according to the equation, $h_t = \frac{h_0}{(1+n)^t}$.

can exhibit monotonic convergence, oscillatory convergence, flip bifurcation, or chaotic dynamics with respect to parameters δ and γ .

Proof: See Appendix.

1.2.5 A Numerical Illustration

To illustrate the result of *Proposition 3*, I assume specific values of A , B , β , ν , and $\underline{h}$.⁷ From Equation (1.22), Figure 1.1 shows the $p_{t+1} - p_t$ diagram for different values of the child preference parameter, δ . The first panel on the left shows the graph for $\delta = 0.9$, the second in the middle for $\delta = 1.3$ and the last on the right for $\delta = 1.6$. Figure 1.2 shows the sequence of 500 counts of p -values. For the top left diagram, it demonstrates monotonic convergence. For the top right, it demonstrates period doubling or flip bifurcation. For the bottom diagrams, they exhibit chaotic dynamics; as δ increases (left to right), the fluctuations become larger. Figure 1.3 gives the bifurcation diagram for different values of δ . If δ is close to 1, the steady state is reached. As the preference for children increases, housing price no longer converge to the steady state and becomes unstable. Hence, even in a steady state economy, it is possible for housing price and fertility rate to fluctuate, creating booms and busts in housing price and fertility based on the demand and supply for housing.

1.2.6 Social Security and Fertility Cycles

Based on the previous example, if we assume it to be under the pure PAYG system, i.e. $\gamma = 0$, for a given tax rate of 20 percent ($\tau = 0.2$) and rate of return of 10 percent ($r = 0.1$), w is calculated to be 37.5 and α to be 11.81. The ratio of the child rearing cost per child to the lifetime income per household is $\frac{\alpha}{w} = 0.315$, or 31.5 percent. For different values of γ , assuming that the rate of return, g is the same as interest rate, r , I examine the size of fluctuations for different social security systems.

⁷ A is chosen to be 30, B to be 5, β to be 0.9, ν to be 1, and $\underline{h}$ to be 1.

Figure 1.1

Figure 1.2

Figure 1.3

Figure 1.4

Figure 1.4 and 1.5 show two bifurcation diagrams, when $\gamma = 0.5$, under the mixed social security system, and $\gamma = 1$, under the pure FF system, respectively. As γ increases, convergence to the steady state occurs with large child preferences compared to Figure 1.3. For example, from Figure 1.4, unlike the case of the pure PAYG system, convergence to steady state still happens with preference as large as 2. This shows that by including components of the FF system, both housing price and fertility can be stabilised. Next, I examine the bifurcation diagram for all values of γ between 0 and 1 inclusive. Fixing the child preference, δ at 1.6, Figure 1.6 shows that as social security moves towards the FF system, convergence to the steady state occurs and steady state price also decreases.

Results are similar to Fanti and Gori (2013) which compares the bifurcation diagrams of the *fertility-related* PAYG system and the *fertility-unrelated* PAYG system. The fertility-related PAYG system *increases* the range of values of social security tax for which flip bifurcation and chaotic dynamics occur. Also, as social security switches more towards

Figure 1.5

Figure 1.6

the fertility-related PAYG system, chaotic dynamics occurs. Both our works demonstrate the importance of the *fertility-unrelated* social security system for stability.

1.3 Housing Prices and Birth Rates in Singapore - Empirical Evidence

Fluctuations in fertility and public housing resale price⁸ in Singapore are shown in Figures 1.7 and 1.8 respectively. Figure 1.7 shows the annual change in TFR from 1961 – 2016 and Figure 1.8 shows the quarterly change in resale public housing price from 1990 – 2016. It can be seen from the figures that the fluctuations are large in the earlier years compared to the later years. Singapore’s TFR has decreased by about 5 times from 1960 – 2016. Figure 1.9 shows the quarterly change in resale housing price, private housing price⁹ and the total births per thousand residents¹⁰ superimposed on each other. The size of the fluctuations for resale price and births per thousand residents appears to match each other closely while private housing price tends to exhibit larger fluctuations. Given the theoretical model from the previous section, in this section, I test for the effects of fertility rate on housing price and vice versa. Can fluctuations in fertility explain housing prices? Is there a link between fertility and housing price in Singapore?

1.3.1 Literature Review on Housing Prices in Singapore

Among the literature surveyed on Singapore’s housing prices, Chang et al. (2012) note that the housing returns during the boom period are determined by domestic variables - Gross Domestic Product (GDP) growth rate, volume of trade, and exchange rate - and U.S. variables - federal fund rate and external finance. In their estimation, variables affecting the housing returns during the boom period are different from those during the bust period. Others examine the determinants of housing price in the private market.

⁸Public housing refers to housing built by the government known as HDB flats. Resale price refers to the price of the HDB flats sold in the open market.

⁹Private housing price refers to the price of landed and non-landed properties such as condominiums.

¹⁰Change in total births per thousand residents is multiplied by a factor of 10 for comparison.

Figure 1.7

Figure 1.8

Figure 1.9

Lum (2002) finds that the demand and supply factors such as land sales, mortgage rate, construction cost and income are significant to housing price in the private market only in the long run while land sales and deregulatory moves by the government allowing owners of public housing to invest in private properties have impact on the housing price only in the short run. Phang and Wong (1997) find that the changes in various government policies regarding the use of compulsory savings, public homeownership and financing rules also have impact on private housing prices. Based on the Vector Error Correction Mechanism (VECM) model, Sing et al. (2006) find that both the public housing resale price and private housing price do move together in the long run although exogenous transition shocks cause them to drift apart occasionally. Changes in public housing price are also significant to changes in price in the private housing market. Using the Autoregressive Distributive Lag (ARDL) model, Ong and Sing (2002) note that both the prices have a long and short run effect on each other. Deo Bardhan et al. (2003) find that the increase in wealth or "wealth effects" from the sales of public residential homes, the decrease in real loan interest rate and the increase in *changes in public housing resale price* are variables that impact the sales of private residential homes. Lee and Ong (2005) empirically support Stein's (1995) postulation that the large swings in housing prices can cause individuals to consume more housing; they find that the swing in prices can help to ease the credit constraint of young households and increase their likelihood of housing upgrades, resulting in greater housing consumption. Tu et al. (2005) also find that housing upgrades are more likely to occur with increasing affordability.

In the following section, I examine the effects of birth rate on both private and public housing price and vice versa. In the only study on the fertility and public housing price in Singapore, Toh (2011) examine the unidirectional effect of housing price on fertility rate based on annual and partially constructed data. This empirical study differs by focusing on the *changes in housing price* or *short run* effects using the quarterly data available from 1990 – 2016. It also examines the bidirectional effects of fertility and housing price. Changes in housing price can affect housing consumption which, in turn, may affect birth rate. Examination of short run effects alone is done when no long run

Figure 1.10

cointegrating relationship exists among the variables. Firstly, it is unlikely for housing price and fertility to have a long run relationship since a country's population growth rate depends on other factors such as migration and death rates. Secondly, in examining the demand for housing, it is more relevant to look at the changes from specific demographic groups within the population that demand the bulk of housing (Lindh and Malmberg, 2008).

1.3.2 Data on Various Fertility-related Statistics

Figure 1.10 shows the annual trend of different statistics: the resident crude birth rate,¹¹ the TFR,¹² the rate of natural increase (RNI)¹³ per thousand residents obtained from

¹¹According to OCED, crude birth rate is defined as the number of live births occurring among the population of a given geographical area during a given year, per 1,000 mid-year total population of the given geographical area during the same year.

¹²For comparison, TFR is multiplied by a factor of 10.

¹³Rate of natural increase (RNI) is crude birth rate minus crude death rate.

the Department of Statistics (DOS) Singapore and by my own calculation¹⁴ and the annual births per thousand residents obtained by summing the total number of births in every quarter within a year divided by the total number of residents multiplied by a thousand. By definition, crude birth rate is larger than RNI since RNI takes into consideration crude death rate as well. The slight difference between the two RNI figures could be due to the different resident population figures used.¹⁵ The annual births per thousand residents figures are higher than the resident crude birth rate figures since the annual births figures include births by non-residents as well. TFR measures the average number of children born to a (resident) woman if she were to pass through all her childbearing years conforming to the age-specific fertility of a given year. The births per thousand residents figures do not take into consideration demographic factors such as the old-age support ratio,¹⁶ the gender ratio and the age-specific childbearing fertility. Therefore, TFR serves as a better proxy for child preferences. However, since the quarterly data for TFR are unavailable, the births per thousand residents figures are used and from Figure 1.10, their graphs do match each other closely. The quarterly births per thousand residents figures are obtained by dividing the total number of births with the total resident population multiplied by a thousand. Quarterly figures for resident population are obtained from the annual figures by linear interpolation since they are unavailable.¹⁷

1.3.3 Empirical Implementation and Methodology

To use the Vector Autoregressive (VAR) model for the Granger Causality analysis, all variables must have the order of integration 0. In other words, they must be $I(0)$ variables.

¹⁴This is done by taking the total natural increase divided by the resident population multiplied by a thousand.

¹⁵It is possible that one is obtained using the mid-year resident population and the other using the end-of-year resident population.

¹⁶Old-age support ratio refers to the number of residents aged 15 – 64 per resident aged 65 years and over.

¹⁷The actual quarterly data for resident population are only available from 2000 - 2016 (with the second quarter data missing). In order to examine whether the linearly interpolated resident population figures are suitable for use, Figure 1.11 in the Appendix compares the actual figures with the linearly interpolated ones. The graphs show that they do match each other closely.

Otherwise, it is necessary for the series to be stationary by differencing the variables. It is important to ensure that no cointegrating relationships exist among the variables so that no long relationship is lost by the differencing of the variables. If there is a cointegrating relationship, it is more appropriate to use the VECM or ARDL model to capture the long run and short run effects.

The testing procedure involves three steps. First, I test for the existence of unit roots in the series using the Augmented Dickey-Fuller (ADF) test, the Philip-Perron (PP) and the Kwiatkowski, Philips, Schmidt, and Shin (KPSS) test. Next, I use the Johansen approach to test for cointegration among the variables. If cointegration is detected, instead differencing the variables, a more suitable approach would be the Toda and Yamamoto (1995) Granger Causality test which allows for non-stationary variables to be tested regardless of whether there is cointegration among the variables. Last, I model the variables in first difference using the VAR approach. Since my main interest is to examine ‘causality’, only the Wald and MWald statistics are reported.

According to Lutkepohl (1982), a simple bivariate VAR may be subjected to omitted variable bias. Therefore, in addition to the bivariate model, a 4-variable VAR model comprising of the variables *resale housing price* (*re*), *private housing price* (*pte*), *GDP per capita* (*gdp*) and *birth rate per thousand residents* (*br*) is used. All tests are done before and after the logarithmic transformation of the variables (taking the log of the series). Definitions of the variables can be found in Table 1.6 of the Appendix.

Table 1.1: Unit Root Tests

Variable	ADF Statistic		KPSS Statistic		PP Statistic	
	Trend	No Trend	Trend	No Trend	Trend	No Trend
<i>br</i>	-0.494	-2.234	0.292***	1.039***	-5.765***	-1.62
<i>gdp</i>	-2.493	-0.486	0.210**	1.123***	-2.161	-0.156
<i>re</i>	-2.608	-1.523	0.132*	0.924***	-1.812	-1.101
<i>pte</i>	-2.336	-1.929	0.119*	0.787***	-2.02	-1.743
<i>ln br</i>	-0.603	-1.870	0.270***	1.051***	-5.587***	-1.427
<i>ln gdp</i>	-2.807	-1.551	0.0806	1.130***	-2.28	-1.746
<i>ln re</i>	-2.419	-2.222	0.134*	0.962***	-1.903	-2.065
<i>ln pte</i>	-2.532	-2.484	0.103	0.827***	-2.186	-2.273
Δbr	-4.535***	-3.874***	0.145*	0.157	-24.343***	-22.944***
Δgdp	-7.042***	-7.086***	0.0757	0.0875	-7.062***	-7.106***
Δre	-3.637**	-3.636***	0.0826	0.0848	-3.790**	-3.791***
Δpte	-5.215***	-5.194***	0.0796	0.0988	-4.110***	-4.108***
$\Delta \ln br$	-4.235***	-3.804***	0.143*	0.149	-22.551***	-21.734***
$\Delta \ln gdp$	-7.016***	-6.937***	0.0901	0.175	-6.995***	-6.960***
$\Delta \ln re$	-2.971	-2.643*	0.0966	0.214	-4.493***	-4.343***
$\Delta \ln pte$	-5.358***	-5.239***	0.0971	0.176	-4.350***	-4.388***

Δ represents first difference of variable. $\ln$ represents natural log of the variable. Lag length for ADF test is based on the Schwarz Information Criterion. For both the PP and KPSS tests, bandwidth is selected automatically according to the Newey-West (1994) procedure and Bartlett kernel is used for HAC corrected variance. *, ** and *** represent significance at 10, 5 and 1% levels, respectively.

1.4 Results

1.4.1 Unit Root Tests

Table 1.1 reports the various tests results for the variables in levels and after first differencing. Both the ADF and PP tests test the null hypothesis of a unit root while the KPSS test tests the null hypothesis of no unit root. For near unit root series, the ADF test has low power. From the table, based on all three tests, other than the variable *birth rate* (with and without taking log), results show that all the other series are stationary after first differencing, indicating that they are all $I(1)$ variables. Although the ADF tests do not reject the null hypothesis of a unit root for the $\ln re$ series after differencing, the PP tests indicate that it is stationary. Compared to the ADF test, the PP test does not assume any functional form on the error process and corrects for serial correlation and heteroskedasticity based on nonparametric methods. Since the PP tests reject the null hypothesis of a unit root, I conclude that it is an $I(1)$ series. The KPSS test (with trend) statistics indicate that birth rate is not stationary at the 10 percent level after differencing. According to Caner and Kilian (2001), the KPSS test can have size distortions for processes whose roots are close to unity. Since the null hypothesis is being rejected at the 10 percent level and both the ADF and PP tests indicate that it is an $I(1)$ variable, I take it to be an $I(1)$ variable.

1.4.2 Cointegration Tests

The maximum likelihood approach developed by Johansen (1991, 1995) is used to test for cointegration among the variables. The maximum eigenvalue and trace test statistics for the number of cointegrating vectors are presented in Tables 1.2 and 1.3. Table 1.2 shows the test results for the bivariate model with variables birth rate and resale housing price and Table 1.3 shows the results for the 4-variable model. Table 1.2 indicates that there is no cointegrating relationship in the 2-variable model while Table 1.3 indicates that there is one cointegrating relationship in the 4-variable log-specification model according

Table 1.2: Johansen Cointegration Tests

Model	Trace Statistic		Maximum Eigenvalue	
	$r = 0$	$r < \text{or} = 1$	$r = 0$	$r = 1$
br, re	12.913**	5.446	7.467**	5.446
br, pte	14.043**	4.164	9.879**	4.164
$\ln br, \ln re$	15.508**	5.91	9.598**	5.91
$\ln br, \ln pte$	14.986**	4.039	10.947**	4.039

*, ** and *** represent significance at 10, 5 and 1% levels, respectively. Optimal lag of 5 is chosen according to the Schwarz Information Criterion in the VAR modelled in levels with a maximum of 10 lags. Cointegration test is under restricted constant and linear trend specification. The 5% critical values of the trace statistic are given by 25.87211 and 12.51798 for $r = 0$ and $r = 1$ respectively. The 5% critical values for the maximum eigenvalue statistic are given by 19.38704 and 12.51798 for $r = 0$ and $r = 1$ respectively.

to the trace test. This confirms the results of both Ong and Sing (2002) and Sing et al. (2006) who find a cointegrating relationship between private housing price and public resale price using the natural logarithmic form of the variables. However, no cointegrating relationship is found before the log transformation of the variables. This may be because by ‘taking logs’, both series are compressed by the log scale, reducing any deviations between the series. The presence of cointegration is only indicated by the trace statistic and not by the maximum eigenvalue statistic.¹⁸ Since the Johansen approach is sensitive to the number of lags chosen, tests with the lag lengths based on other criteria are also done¹⁹ and the conclusion remains the same.

1.4.3 Granger Causality Tests

Unless stated otherwise, the Schwarz Information Criterion (SIC) is used to decide the number of lags chosen for the VAR model. After fitting the data into the model, serial autocorrelation must not be present in the residuals for the inference results to be valid. Table 1.4 shows the results for birth rate and resale housing price in the bivariate model.

¹⁸When 8 lags are chosen according to the Sequential Modified LR test statistic, both statistics indicate that there is 1 cointegrating vector.

¹⁹The other criteria are the Sequential Modified LR test statistic (LR), the Final Prediction Error (FPE), the Akaike Information Criterion (AIC) and the Hannan-Quinn Information Criterion (HQ).

Table 1.3: Johansen Cointegration Tests

Model	Trace Statistic			Maximum Eigenvalue Statistic		
	$r = 0$	$r < \text{or} = 1$	$r < \text{or} = 2$	$r = 0$	$r = 1$	$r = 2$
br, re, pte, gdp	54.268**	33.131	17.638	21.136**	15.493	11.4
$\ln br, \ln re, \ln pte, \ln gdp$	69.978	38.829**	20.059	31.149**	18.77	13.566

*, ** and *** represent significance at 10, 5 and 1% levels, respectively. Optimal lag is chosen according to the Schwarz Information Criterion in the VAR modelled in levels with a maximum of 10 lags. Optimal lag indicated by the Schwarz Information Criterion is 2, but there is serial autocorrelation. Hence, lag length of 5, as indicated by most other statistics is used. Cointegration test is under restricted constant and linear trend specification. The 5% critical values of the trace statistic are given by 63.8761, 42.91525 and 25.87211 for $r = 0, 1$ and 2 respectively. The 5% critical values for the maximum eigenvalue statistic are given by 32.11832, 25.82321 and 19.38704 for $r = 0, 1$ and 2 respectively.

Table 1.4: Granger Causality for br and re , br and pte

Direction of Causality		Value of Wald Statistic	
From	To	(4)	(8)
Δbr	Δre	10.390**	10.403
Δre	Δbr	5.274	7.071
Δbr	Δpte	6.564	5.759
Δpte	Δbr	3.518	9.23
$\Delta \ln br$	$\Delta \ln re$	7.839*	7.103
$\Delta \ln re$	$\Delta \ln br$	3.007	4.997
$\Delta \ln br$	$\Delta \ln pte$	6.942	5.431
$\Delta \ln pte$	$\Delta \ln br$	3.644	9.956

*, ** and *** represent significance at 10, 5 and 1% levels, respectively. Numbers in parentheses indicates the number of lags in the model. Lags 4 and 8 are chosen according to 3 criteria: Final Prediction Error (FPE), Akaike Information Criterion (AIC), and Hannan-Quinn Information Criterion (HQ). The maximum number of lags chosen is 10.

Table 1.5: Granger Causality for *br*, *re*, *pte*, *gdp*

Direction of Causality		Value of Wald Statistic		Value of MWald Statistic	
From	To	(3)	(4)	(5)	(6)
Δbr	Δre	11.021**	10.282**		
$\Delta \ln br$	$\Delta \ln re$	7.970**	7.605		
<i>br</i>	<i>re</i>			9.082*	9.681*
$\ln br$	$\ln re$			8.949*	13.903**
Δbr	Δpte	6.223	8.367*		
$\Delta \ln br$	$\Delta \ln pte$	4.921	7.046		
<i>br</i>	<i>pte</i>			7.819*	1.81
$\ln br$	$\ln pte$			5.956	1.95
Δre	Δpte	1.613	2.444		
$\Delta \ln re$	$\Delta \ln pte$	1.842	2.788		
<i>re</i>	<i>pte</i>			2.315	10.326*
$\ln re$	$\ln pte$			2.408	6.224
Δpte	Δre	3.478	3.387		
$\Delta \ln pte$	$\Delta \ln re$	3.272	3.942		
<i>pte</i>	<i>re</i>			4.398	2.906
$\ln pte$	$\ln re$			9.895**	9.646*
Δgdp	Δre	2.719	2.502		
$\Delta \ln gdp$	$\Delta \ln re$	2.243	2.897		
<i>gdp</i>	<i>re</i>			2.837	3.068
$\ln gdp$	$\ln re$			2.609	2.94
Δgdp	Δpte	13.167***	15.055***		
$\Delta \ln gdp$	$\Delta \ln pte$	10.571**	10.997**		
<i>gdp</i>	<i>pte</i>			16.123***	18.277***
$\ln gdp$	$\ln pte$			11.481**	13.988**

*, ** and *** represent significance at 10, 5 and 1% levels, respectively. Numbers in parentheses indicates the number of lags in the model. For the TY methodology, the MWald test statistics reported are based on the number in the parentheses minus 1 since the maximum integrating order among the variables is 1. The number of lags chosen are based on Final Prediction Error (FPE), Akaike Information Criterion (AIC), and Hannan-Quinn Information Criterion (HQ).

Based on the various criteria indicated in the table, the 4 and 8-lag models are chosen. Results show that under the 4-lag model, the change in *birth rate* is significant in causing resale public housing price to change but not for the 8-lag model, indicating that further lags of birth rate may not have any effect on housing price.

Table 1.5 shows the results of the 4-variable model for the variables in first difference and in levels using the Toda-Yamamoto (TY) Granger Causality approach. The main advantage of the TY test is that it does not need to presume the existence of cointegration (or lack of) based on prior tests whose null hypotheses may be rejected wrongly due to the low powers of the tests. According to the Johansen tests results in Table 1.3, the trace and the maximum eigenvalue statistic indicate mixed results. Also, since the Granger Causality tests results are highly sensitive to the number of lags used, various lag lengths based on 3 other criteria are tested: the Akaike Information Criterion (AIC), the Final Prediction Error (FPE) and the Hannan-Quinn Information Criterion (HQ). According to the SIC, 1 and 2 lags are indicated for the models with and without first differencing respectively. However, due to the presence of serial autocorrelation in the residuals, they are not chosen. Instead, models with longer lag lengths are reported. According to Table 1.5, the estimates for *birth rate* are significant to public resale housing price²⁰ and estimates for GDP per capita are significant to private housing price. In addition, a few of the other estimates also indicate that they are significant - from birth rate to private housing price and from public resale housing price to private housing price and vice versa. Since results are insignificant when birth rate and GDP per capita are dependent variables, they are not shown in the table to conserve space. Results are also consistent with the existing literature on private housing price and on the relationship between public resale price and private price.

²⁰Based on Table 1.5, for the 4-lag model, the change in log of birth rate is significant to the change in log of resale housing price at the 11 percent level.

1.5 Conclusion

Some countries like Sweden and other OECD countries have experienced fertility cycles due to changes in demand for housing and prices due to demographic factors. This could be the same for Singapore due to its limited land supply. In this essay, I simulate that relationship between housing demand, price and fertility based on a model with fixed land supply. Inspired by Deaton and Laroque (2001) and the public housing scheme in Singapore,²¹ results show that larger preferences for children generate greater fluctuations in fertility and housing price. In addition, similar to Fanti and Gori (2013), a fertility-unrelated social security system whose pension payout cannot be determined by agents choosing the number of children results in smaller fluctuations in prices and fertility.

For the housing price in Singapore, birth rate may not be seen as having an effect on housing price. However, empirical results show that the change in birth rate is significant to the change in public housing price. Firstly, although couples usually purchase their houses before having children in Singapore, their planned family size can affect the amount of housing consumption. Secondly, as lower birth rates coincide with smaller fluctuations in the change in birth rate and public resale housing price, this could be due to the smaller preferences for children²² as demonstrated by the model. Public housing is considered a necessity for all residents by the government. As such, the fundamental factors of demand and supply are more relevant to the changes in the price of public housing due to the government measures that prevent speculation and investing in the public housing resale market.

Results show that the change in GDP per capita is significant to the change in private housing price but not to the change in public housing resale price. This can be due to the many government schemes, i.e. grants and subsidies available to make public housing affordable and the policies restricting households with income beyond certain level from purchasing public housing. Also, couples have the option to purchase Built-to-Order

²¹See footnote 5.

²²Holding everything else constant, lower birth rates would imply smaller preferences for children relative to other consumption goods.

(BTO) flats directly from the government instead of buying from the resale market. Therefore, a change in GDP may not result in a large change in demand and housing resale price; on the other hand, rising income tend to cause households to upgrade their public homes to private homes due to the profits made from the increase in values of their first homes which supplement their higher incomes. Nevertheless, the two prices have *long term* spillover effects on each other possibly due to the insufficient supply of newly built BTO flats, resulting in the long run cointegrating relationship between the two variables. The models, however, do not capture the asymmetric effects of economic booms and busts on housing price.²³

It may be surprising to find that housing price does not have a significant effect on birth rate.²⁴ This could be due to several reasons. Firstly, cost of raising children involves other expenditure such as childcare and education. Secondly, it is not so easy to distinguish between the consumption of housing for parents and children²⁵ and hence, the increase in housing cost may not have a direct effect on child rearing cost. Thirdly, housing is usually purchased by couples before having children.²⁶ Cost of housing then becomes a sunk cost after making commitments to paying mortgage loans. Other future (expected) costs for raising children such as tuition, education, childcare, etc. then take precedence over housing cost and have a greater impact on the number of children. Therefore, housing costs, as large as they are, may only have secondary effects on the decline in fertility. In addition, housing prices in Singapore are not high compared to other cities such as Hong Kong. Lastly, when couples do decide to have another child after their incomes

²³In good times, people tend to upgrade their homes or increase their housing consumption. However, in bad times, people are less likely to sell or downgrade their houses unnecessarily due to the loss aversion effect.

²⁴As pointed out by a referee, current year's housing price may not have an effect on birth rate; a more relevant measure may be housing price averaged over the past three years. Since quarterly prices are used in this study, the effects from the more recent quarters may not have an effect on fertility; therefore, only the later quarters (excluding the recent ones) should be used to test the hypothesis.

²⁵Duesenberry questions whether it is possible to separate the housing conditions for children and parents (Becker, 1960); hence, housing consumption for children often follow that of parents'.

²⁶Also pointed out by the referee, contrary to existing literature (Mulder and Wagner, 2001), the timing of life events in one's course of life such as getting married, having the first child or the transition to homeownership should not influence or affect the priority of one's fertility decisions. Therefore, empirical evidence should be provided for this to be justified. However, it is still a social norm for Singaporeans to consider buying a house before having children.

have risen substantially, they may not be able to do so due to the decline in fecundity of women. This is especially so when late marriages and childbirths have become the social norm. This conclusion here differs from Toh (2011) who states that housing price is to be blamed for the declining fertility in Singapore.

For the other results, the change in housing price is not significant to the change in nominal GDP per capita, the change in birth rate is not significant to the change in GDP and the change in GDP is not significant to the change in birth rate. Housing or accommodation service is only one of the components used in the computation of GDP. Due to existing open door policies on immigration, Singapore's population is still on an upward trend and it will take some time before the falling birth rates will have an impact on its population size and economic growth. Among existing literature, the evidence for the increase in income on fertility is mixed. It is also inconclusive from the results that birth rate has a significant effect on private housing price.

An important extension to this essay is to incorporate the timing of births into the model. Caucutt et al. (2002) examine the patterns of fertility timing in the U.S. by adding incentives for delays in having children from the marriage and labour markets in their model. In today's context, the timing of births can influence housing consumption since more (educated) couples are optimising the age gap between their first and second child in addition to the number of children to have and this may affect housing consumption patterns as well. In addition, one should distinguish between the preference for children for renters and homeowners due to a possible difference in (unobserved) characteristics between the two groups.²⁷

²⁷Unfortunately, due to data limitations, the effects of the timing of births on housing consumption patterns and vice versa cannot be examined. Likewise, the data on the family size of age-specific couples who own homes and those who rent are unavailable.

1.6 Appendix

Proof of *Proposition 1*:

Differentiating Equation (1.13):

$$\frac{dn_t}{dp_t} = -\frac{\delta \underline{h}}{(1 + \beta + \nu + \delta) \left(\alpha - \frac{(1-\gamma)\tau W_{t+1}}{(1+r)} \right)} < 0$$

From Equation (1.14):

$$\begin{aligned} h_t &= \frac{\nu}{p_t} \frac{\left(W_t \left(1 - \tau + \frac{\gamma\tau(1+g)}{(1+r)} \right) - \underline{h} p_t \right)}{(1 + \beta + \nu + \delta)} + \underline{h} \\ &= \frac{\nu}{p_t} \frac{W_t \left(1 - \tau + \frac{\gamma\tau(1+g)}{(1+r)} \right)}{(1 + \beta + \nu + \delta)} - \frac{\nu \underline{h}}{(1 + \beta + \nu + \delta)} + \underline{h} \end{aligned}$$

$$\begin{aligned} \frac{dh_t}{dp_t} &= -\nu \frac{W_t \left(1 - \tau + \frac{\gamma\tau(1+g)}{(1+r)} \right)}{(1 + \beta + \nu + \delta)} \frac{1}{p_t^2} \\ \frac{dh_t}{dp_t} \frac{p_t}{h_t} &= -\nu \frac{W_t \left(1 - \tau + \frac{\gamma\tau(1+g)}{(1+r)} \right)}{(1 + \beta + \nu + \delta)} \frac{1}{p_t^2} \frac{p_t}{1} / \left(\frac{\nu}{p_t} \frac{\left(W_t \left(1 - \tau + \frac{\gamma\tau(1+g)}{(1+r)} \right) - \underline{h} p_t \right)}{(1 + \beta + \nu + \delta)} + \underline{h} \right) \\ &= -\frac{\nu W_t \left(1 - \tau + \frac{\gamma\tau(1+g)}{(1+r)} \right)}{\nu \left(W_t \left(1 - \tau + \frac{\gamma\tau(1+g)}{(1+r)} \right) - \underline{h} p_t \right) + p_t (1 + \beta + \nu + \delta) \underline{h}} \\ &= -\frac{\nu W_t \left(1 - \tau + \frac{\gamma\tau(1+g)}{(1+r)} \right)}{\nu W_t \left(1 - \tau + \frac{\gamma\tau(1+g)}{(1+r)} \right) - \nu \underline{h} p_t + p_t (1 + \beta + \nu + \delta) \underline{h}} \\ &= -\frac{\nu W_t \left(1 - \tau + \frac{\gamma\tau(1+g)}{(1+r)} \right)}{\nu W_t \left(1 - \tau + \frac{\gamma\tau(1+g)}{(1+r)} \right) + p_t (1 + \beta + \delta) \underline{h}} > -1 \text{ (inelastic)} \end{aligned}$$

For $\underline{h} = 0$

$$\begin{aligned}
h_t &= \frac{\nu \left(W_t \left(1 - \tau + \frac{\gamma\tau(1+g)}{(1+r)} \right) \right)}{p_t (1 + \beta + \nu + \delta)} \\
\frac{dh_t}{dp_t} &= -\nu \frac{W_t \left(1 - \tau + \frac{\gamma\tau(1+g)}{(1+r)} \right)}{(1 + \beta + \nu + \delta)} \frac{1}{p_t^2} \\
\frac{dh_t p_t}{dp_t h} &= -\nu \frac{W_t \left(1 - \tau + \frac{\gamma\tau(1+g)}{(1+r)} \right)}{(1 + \beta + \nu + \delta)} \frac{1}{p_t^2} \frac{p_t}{1} / \frac{\nu \left(W_t \left(1 - \tau + \frac{\gamma\tau(1+g)}{(1+r)} \right) \right)}{p_t (1 + \beta + \nu + \delta)} \\
&= 1 \quad \blacksquare
\end{aligned}$$

Proof of *Proposition 3*:

From Equation (1.22), if the denominator is zero, the function has a vertical asymptote. By setting denominator to be zero, we obtain a quadratic equation and if the discriminant is less than zero, i.e., $b^2 - 4ac < 0$, no real solution exists; therefore, the denominator cannot be zero.

Setting denominator of Equation (1.22) to be zero, we have:

$$\begin{aligned}
\underline{h}(1 + \beta + \delta) [(1 + \beta + \nu + \delta) B - \delta (A - \underline{h}p_t)] + \frac{\nu A (1 + \beta + \nu + \delta) B}{p_t} &= 0 \\
\underline{h}(1 + \beta + \delta) \left[1 - \left(\frac{\delta A}{(1 + \beta + \nu + \delta) B} - \frac{\delta \underline{h}}{(1 + \beta + \nu + \delta) B^{p_t}} \right) \right] + \frac{\nu A}{p_t} &= 0 \\
\underline{h}(1 + \beta + \delta) \left[1 - \left(\frac{\delta A}{(1 + \beta + \nu + \delta) B} - \frac{\delta \underline{h}}{(1 + \beta + \nu + \delta) B^{p_t}} \right) \right] p_t + \nu A &= 0 \\
\underline{h}(1 + \beta + \delta) \left[\left(1 - \frac{\delta A}{(1 + \beta + \nu + \delta) B} \right) + \frac{\delta \underline{h}}{(1 + \beta + \nu + \delta) B^{p_t}} \right] p_t + \nu A &= 0 \\
\underline{h}(1 + \beta + \delta) \frac{\delta \underline{h}}{(1 + \beta + \nu + \delta) B^{p_t}} p_t^2 + \underline{h}(1 + \beta + \delta) \left(1 - \frac{\delta A}{(1 + \beta + \nu + \delta) B} \right) p_t + \nu A &= 0
\end{aligned}$$

Figure 1.11

Setting the discriminant of the above quadratic equation to be less than zero, we have:

$$\begin{aligned}
 & \left[\underline{h}(1 + \beta + \delta) \left(1 - \frac{\delta A}{(1 + \beta + \nu + \delta) B} \right) \right]^2 - 4 \underline{h}(1 + \beta + \delta) \frac{\delta \underline{h}}{(1 + \beta + \nu + \delta) B} \nu A < 0 \\
 & \underline{h}(1 + \beta + \delta) \left(1 - \frac{\delta A}{(1 + \beta + \nu + \delta) B} \right)^2 - 4 \frac{\delta \underline{h}}{(1 + \beta + \nu + \delta) B} \nu A < 0 \\
 & \underline{h}(1 + \beta + \delta) \left(\frac{(1 + \beta + \nu + \delta) B - \delta A}{(1 + \beta + \nu + \delta) B} \right)^2 - 4 \frac{\delta \underline{h}}{(1 + \beta + \nu + \delta) B} \nu A < 0 \\
 & \underline{h}(1 + \beta + \delta) \frac{[(1 + \beta + \nu + \delta) B - \delta A]^2}{(1 + \beta + \nu + \delta) B} - 4 \delta \underline{h} \nu A < 0 \\
 & \underline{h} \left[(1 + \beta + \delta) \frac{[(1 + \beta + \nu + \delta) B - \delta A]^2}{(1 + \beta + \nu + \delta) B} - 4 \delta \nu A \right] < 0 \\
 & (1 + \beta + \delta) \frac{[(1 + \beta + \nu + \delta) B - \delta A]^2}{(1 + \beta + \nu + \delta) B} - 4 \delta \nu A < 0 \blacksquare
 \end{aligned}$$

Table 1.6

Variables	Definitions	Period
HDB Resale Price Index	The index is based on quarterly average resale price by date of registration. The index till 3Q2014 was computed using stratification method, while that from 4Q2014 onwards is computed using the stratified hedonic regression method. 1Q2009 is adopted as the new base period with index at 100.	1990q1 - 2016q4
Total Live Births	Live births of entire population per quarter.	1986q1 - 2016q4
Private Residential Property Price Index	Index of both landed and non-landed properties. Data are computed using stratified hedonic regression method. The sum of values of transactions from Q1 2014 to Q1 2015 is used as weights to compute the index. Prior to 3Q 2016, data are compiled based on transaction prices given in contracts submitted for stamp duty payment and data provided by licensed developers on new units sold. From 3Q2016, net prices of units sold by de-licensed developers are included as well.	1975q1 - 2016q4
Resident Population	Resident population includes Singapore citizens and permanent residents.	1980 - 2016
Total Population	Total population comprises of Singapore residents & non-residents.	1960 - 2016
Nominal GDP	Expenditure of Gross Domestic Product at current market prices.	1975q1 - 2016q4

Note : All data are obtained from Department of Statistics, Singapore.

2 ESSAY TWO: HOUSING CONSUMPTION AND HAVING CHILDREN: DOES SOCIAL SECURITY INCREASE FERTILITY?

2.1 Introduction

In developed countries, a certain quality of life may affect one's decision to have children or the number of children to have. Children take time away from one's career and work and require childcare services, investments in their education and a conducive environment to be raised in. This essay studies the effects of social security system on fertility, housing consumption and growth. Using the overlapping generations (OLG) framework, individuals seek to maximise the utility derived from the number of children and housing consumption. Housing is a basic necessity to get married, start a family, and retire. Results show that housing consumption is large under the fully funded (FF) system compared to the Pay-As-You-Go (PAYG) system. With the same retirement income, the PAYG system results in higher fertility. Unless the rate of return from the fully funded system is large enough, fertility decreases and growth rate increases as social security switches from the unfunded to the funded system.

The type of social security system may be an important factor that affects housing consumption and fertility decisions. According to Mulder and Billari (2010), countries in Europe with the lowest fertility rates such as Greece, Spain, and Italy are characterised by high homeownership rates and low level of mortgage financing and "difficult homeownership, late home-leaving and lowest-low fertility in Southern European countries may all be viewed as parts of one complex system" (p. 537). Macroeconomic data show that countries with lower homeownership rates have higher fertility rates relative to other countries (Mulder and Billari, 2010; Mulder, 2006). In Switzerland, the total fertility rate (TFR) is 1.55 while homeownership rate is 43.4 percent although 83 to 90 percent of the Swiss have expressed the desire to own homes but are unable to do so due to the high housing prices (Bourassa and Hoesli, 2010). In contrast, the TFR in Singapore is lower at 1.2 while the homeownership rate is higher at 90 percent. Taiwan too has high homeownership and low fertility rates (Bourassa and Peng, 2011; Lin et al., 2016). Lin et al. (2016) note that in Taiwan, homeowners have their first child at an older age relative to renters and for families living with their parents or siblings, they became parents at a

younger age compared to families living in rented houses.

The differences between homeownership and renting and their effects on fertility and raising a family can come under four categories: cost or affordability, security, conduciveness and the wealth effect. Homeownership allows households to have more control over the quality of housing; it is shown that the quality of an owner's house tends to be better than the accommodation of a renter (Megbolugbe and Linneman, 1993). Buying houses allows homeowners to accumulate housing wealth as prices appreciate over time. However, the cost of buying is higher in the initial years compared to renting; purchasing of homes require down payments which young couples may not readily have and the availability and means to finance mortgage loans which require long-term commitment (Mulder and Wagner, 1998). Homeownership gives couples a greater sense of security¹ to start a family without the worry of getting evicted by their landlords (Saunders, 1990; Hiscock et al, 2001). However, without any consideration to sell one's home to realise the profit made from housing price appreciation or to downsize one's housing consumption in the later years, the total capital outlay of buying a home can be much larger than renting for one's entire life because of interest payments. Lastly, buying of homes allows them to be bequeathed to the next generation.

As countries develop, fertility rate decreases. According to Leibenstein (1975), it is the shift towards the desire for socioeconomic statuses due to modernisation as income increases that crowds out the desire to have children and this can be observed through the individual expenditures of households. Items that are typically classified as socioeconomic status goods are "education, housing, residential location, wealth and occupation" (p. 5). As opposed to Becker's quality-quantity trade-off models, economic growth can be the cause of the decline in fertility as people desire more socioeconomic goods.

Increasing marginal utility (IMU) goods are goods that have increasing marginal returns before a certain level of consumption after which diminishing returns sets in; they can

¹According to Vignoli et al. (2013), the feeling of security includes other aspects such as the percentage of the total household income for mortgage payments. In event of unforeseen circumstances, a high percentage may result in the forced-selling of homes while renting, on the other hand, frees up more resources for other uses.

be classified into four categories: "physical indivisible goods", "commitment goods", "lifestyle goods", and "target goods" (pp. 7-8). A target good is one "in the sense that there is some target quantity or expenditure that is especially significant for some reason and anything less than the target amount is of little utility" (p. 8). Housing may be placed in all four categories. In this essay, housing consumption is modelled as a 'target good' whereby its utility is defined *after* a minimum amount is consumed. While housing may also be an investment good, this essay examines housing only as a consumption good.

The remaining of this essay is organised as follows: Section 2.2 presents the basic framework of the model. Section 2.3 gives the steady state equilibrium results of the model. Section 2.4 examines the effects of social security taxation on growth, savings, fertility and housing consumption. Section 2.5 shows the effects of the switch from one social security system to another and Section 2.6 concludes the essay.

2.2 The Model²

2.2.1 Demographics

In period t , there are N_t young identical individuals. Each young person decides on the number of children to have which, in turn, determines the number of young individuals in the next period, $t + 1$, given by N_{t+1} . Hence, population growth rate, fertility or birth rate is given by:

$$n_t = \frac{N_{t+1}}{N_t} \quad (2.1)$$

2.2.2 Individuals

Consider a two period overlapping generations (OLG) model whereby individual lives for three periods, as a child, a young working adult (y) and a retired person (o). The working

²This is an endogenous population growth model. For a survey on endogenous population growth models, refer to Nerlove and Raut (1997).

adult makes all the decisions: the number of children to have, the amount to save and the amount of housing to consume. As such, each individual faces the following optimisation problem:

$$\text{Max } U(c_t^y, c_{t+1}^o, n_t, h_t) = \ln c_t^y + \beta \ln c_{t+1}^o + \nu \ln(h_t - \underline{h}) + \delta \ln n_t \quad (2.2)$$

$$\text{s.t } c_t^y \leq W_t(1 - \tau - q_t h_t - z_t n_t) - s_t \quad (2.3)$$

$$c_{t+1}^o \leq s_t(1 + r_{t+1}) + B_t + \gamma\tau(1 + g)W_t \quad (2.4)$$

where $U(\cdot)$ is the utility of the representative individual, c_t^y and c_{t+1}^o are the consumption of a young working adult in the first period and the consumption of a retiree in the second period respectively, h_t is the housing consumption, n_t is the fertility rate of each household and s_t is the amount of savings. All child-rearing and housing consumption costs are borne in the first period. All other symbols denote the following:

$\underline{h}$: Minimum housing consumption.

τ : Fraction of wage rate contributed to social security, $0 \leq \tau \leq 1$.

W_t : Wage rate that is endogenously determined by the previous generation.

W_{t+1} : Future wage rate that is endogenously determined by the current generation.

q_t : $\frac{p_t}{W_t}$, where p_t is the price per unit of housing consumption.

z_t : $\frac{b_t}{W_t}$, where b_t is the child-rearing cost per child.

g : Rate of return of the FF social security system.

r_{t+1} : Rate of return of capital.

γ : Proportion of social security system that is fully funded (FF), $0 \leq \gamma \leq 1$.

$1 - \gamma$: Proportion of social security system that is unfunded, Pay-As-You-Go (PAYG).

B_t : Unfunded PAYG pension payout.

In this model, the rate of return of the funded social security system, g is not given by the rate of return of capital, r_{t+1} . In many countries, pension funds are being invested in the financial markets (Sinn, 2000; Diamond and Geanakoplos, 2003). Most governments' investment horizons tend to be longer than individuals', thus allowing them to take more risks to yield higher returns.

B_t is determined by the following government balanced budget equation:

$$N_t B_t = (1 - \gamma) N_{t+1} \tau W_{t+1} \quad (2.5)$$

$$B_t = (1 - \gamma) \frac{N_{t+1}}{N_t} \tau W_{t+1} \quad (2.6)$$

$$B_t = (1 - \gamma) n_t \tau W_{t+1} \quad (2.7)$$

Substituting Equation (2.7) into Equation (2.4), the retiree's budget constraint becomes:

$$c_{t+1}^o \leq s_t(1 + r_{t+1}) + (1 - \gamma) n_t \tau W_{t+1} + \gamma \tau (1 + g) W_t \quad (2.8)$$

2.2.3 Firms

Goods are produced for both capital and consumption with the constant returns to scale concave production function under perfect competition; a single firm represents the aggregate production function F and its output is given by

$$Y_t = F(K_t, A_t L_t) = f\left(\frac{K_t}{A_t L_t}\right) A_t L_t \quad (2.9)$$

where L_t is the aggregate labour supply, K_t is the aggregate capital stock, A_t is the labour productivity and $f(\cdot)$ is the output per augmented labour. The firm maximises the profit function

$$\Pi_t = F(K_t, A_t L_t) - r_t K_t - W_t L_t \quad (2.10)$$

with respect to K_t and L_t where W_t is the wage rate and r_t is the capital rate of return. Under the constant returns to scale production function, as the firm maximises its profit

with respect to capital and labour, the following marginal product pricing conditions are derived:

$$r_t = F_1(K_t, A_t L_t) = f'(k_t) \quad (2.11)$$

$$W_t = A_t [f(k_t) - f'(k_t)k_t] \quad (2.12)$$

$$\frac{K_t}{A_t L_t} \equiv k_t \text{ and } f(k_t) \equiv F\left(\frac{K_t}{A_t L_t}, 1\right).$$

Following Grossman and Helpman (1991), Saint-Paul (1992), Grossman and Yanagawa (1993) and Romer (1986),³ the effect of capital is spilled over to labour through capital externality. The labour productivity index term, A_t , first introduced by Arrow (1962), is endogenised by assuming:

$$A_t = \frac{K_t}{m L_t} \Leftrightarrow \frac{K_t}{A_t L_t} \equiv k_t \equiv m \quad (2.13)$$

Capital per effective labour k_t is assumed to be a constant, denoted by m , and the aggregate capital K_t grows at the same rate as the effective labour, $A_t L_t$. From Equations (2.11) and (2.12), we have:

$$r_t = f'(m) = r \quad (2.14)$$

$$W_t = A_t [f(m) - f'(m)m] \quad (2.15)$$

From Equation (2.14), we observe that the capital rate of return is a constant.⁴ Denoting

$$\omega \equiv [f(m) - f'(m)m] / m \quad (2.16)$$

³For a discussion on the empirical evidence on external effects, see Caballero and Lyon (1990). Romer (1986) introduces capital externalities with the increasing returns to scale production function. In this model, the general form of productivity growth is given by $A = (\bar{K})^\psi / m$, where $\bar{K} \equiv \frac{K}{L}$ (Zhang and Zhang, 1998, p. 1230, footnote 3). If $\psi > 1$, it is an increasing returns to scale function. Setting $\psi = 1$, we have the AK_t growth model of Lucas (1988) where the steady state always exists and there is no transition dynamics or convergence to the steady state. Any change in parameters will take immediate effect.

⁴Since m is fixed, capital rate of return is an exogenous variable which is time invariant.

and combining Equations (2.13), (2.15) and (2.16), we have:

$$W_t = \omega \frac{K_t}{L_t} \quad (2.17)$$

The wage rate is proportional to the capital per worker and ω . Substituting the capital externality expression in Equation (2.13) into Equation (2.9), we have:

$$Y_t = f(m) \frac{K_t}{m} \quad (2.18)$$

The marginal social rate of return for capital is thus given by:

$$\frac{dY_t}{dK_t} = \frac{f(m)}{m} = r + \omega > r \quad (2.19)$$

Under the capital externality model, the marginal social rate of return to capital is greater than the private rate of return to capital since the effect from the capital externality is ignored. Hence, there is under demand for investment, individuals are under saving and there is social inefficiency.⁵

2.2.4 Labour Market

Labour market clears and is given by:

$$L_t = N_t \quad (2.20)$$

where labour demand of the firm equals labour supply of the young working population N_t .

⁵Refer to Wigger (2001a, pp. 264-265), Wigger (2001b, p. 58), and Wigger (2001c, p. 125) for further details.

2.2.5 Capital Market

Capital market clears and is given by:

$$K_{t+1} = N_t s_t \quad (2.21)$$

where next period's aggregate capital is supplied by the total savings of the young working population since the aggregate investment I_t equals the aggregate savings $N_t s_t$. Using $n_t = \frac{N_{t+1}}{N_t}$ and by manipulating the function above, we obtain:

$$\frac{K_{t+1}}{N_{t+1}} n_t = s_t \quad (2.22)$$

Combining Equations (2.15) and (2.16), we have $W_t = \omega m A_t$. From Equations (2.13) and (2.22),

$$A_{t+1} m n_t = s_t \quad (2.23)$$

$$\frac{W_{t+1}}{\omega} n_t = s_t \quad (2.24)$$

$$\frac{W_{t+1}}{W_t} = \omega \frac{1}{n_t} \frac{s_t}{W_t} \quad (2.25)$$

Define the labour productivity growth rate as

$$\frac{A_{t+1}}{A_t} \equiv 1 + g_t \quad (2.26)$$

Since $W_t = \omega m A_t$,

$$\frac{W_{t+1}}{W_t} = \frac{\omega m A_{t+1}}{\omega m A_t}, \quad (2.27)$$

$$\frac{W_{t+1}}{W_t} = \frac{A_{t+1}}{A_t} \quad (2.28)$$

$$\frac{W_{t+1}}{W_t} = 1 + g_t \quad (2.29)$$

From Equations (2.25) and (2.29), we have:

$$1 + g_t = \omega \frac{1}{n_t} \frac{s_t}{W_t} \quad (2.30)$$

Wage rate grows at the same rate as the productivity index or technological growth. Given that this is a capital externality model, individuals' savings decision s_t will affect capital accumulation which will, in turn, affect the rate of technological growth and next period's wage rate. From Equation (2.25), savings and fertility decisions will determine wage rate W_{t+1} and growth rate $1 + g_t$. Current wage rate is a function of capital per worker, given by $W_t = \omega \frac{K_t}{L_t}$ and $\frac{s_t}{W_t}$ is the savings rate of a young working adult.

From Equations (2.9) and (2.13),

$$\frac{Y_{t+1}}{Y_t} = \frac{K_{t+1}}{K_t} = \frac{mA_{t+1}L_{t+1}}{mA_tL_t} \quad (2.31)$$

$$\frac{Y_{t+1}}{Y_t} = \frac{K_{t+1}}{K_t} = \frac{A_{t+1}L_{t+1}}{A_tL_t} \quad (2.32)$$

$$\frac{Y_{t+1}}{Y_t} = \frac{K_{t+1}}{K_t} = (1 + g_t)n_t \quad (2.33)$$

Both capital and output changes at the rate, $(1 + g_t)n_t$. Grossman and Yanagawa (1993) show that this capital externality growth model can also be extended to “alternative engines of growth” (p. 12).

By choosing variables n_t , s_t , and h_t , from Equations (2.2), (2.3) and (2.8) and assuming that only the equality signs hold for Equations (2.3) and (2.8), the individual's optimisation problem gives us the following first order conditions:

$$n_t : \frac{\delta}{n_t} + \frac{\beta(1 - \gamma)\tau W_{t+1}}{c_{t+1}^o} = \frac{W_t z_t}{c_t^y} \quad (2.34)$$

$$s_t : \frac{\beta(1 + r)}{c_{t+1}^o} = \frac{1}{c_t^y} \quad (2.35)$$

$$h_t : \frac{\nu}{h_t - \underline{h}} = \frac{W_t q_t}{c_t^y} \quad (2.36)$$

Equation (2.34) shows the trade-off in the utility derived from the number of children

plus the benefits obtained from the unfunded pension system in the second period versus the child rearing cost incurred or the consumption foregone because of child rearing. Equation (2.35) shows the trade-off between saving for the second period and consuming in the first period. Equation (2.36) shows the benefit derived from housing consumption versus the cost of housing incurred in the first period.

2.3 Steady State Equilibrium

Since there is no transition dynamics in this model⁶ and assuming that the fraction of the child rearing cost per child to wage rate and the fraction of the price per unit of housing consumption to wage rate do not change over time, i.e. $z_t = z$ and $q_t = q$, all the variables are on the Balance Growth Path (BGP). Replacing n_t with n , h_t with h and g_t with $\bar{g}$, we have the following equations:

$$n = \frac{\delta(1 - \tau - qh)}{z - \frac{(1-\gamma)\tau(1+\bar{g})}{\beta(1+r)} + \frac{\delta}{\omega}(1 + \bar{g}) + \delta z} \quad (2.37)$$

$$1 + \bar{g} = \frac{\beta(1 + r)(1 - \tau - qh - zn) - \gamma\tau(1 + g)}{\frac{1}{\omega}n(1 + r) + (1 - \gamma)n\tau + \frac{1}{\omega}\beta(1 + r)n} \quad (2.38)$$

$$h = \frac{\nu(1 - \tau - qh - zn - \frac{1}{\omega}(1 + \bar{g})n)}{q} + \bar{h} \quad (2.39)$$

$\bar{g}$ here refers to the growth in wage rate or technological growth while g refers to the rate of return of the funded pension system. Combining the budget Equations (2.3) and (2.8) and rearranging the terms, we have the following single budget equation:

$$c_t^y + \frac{c_{t+1}^o}{(1 + r)} + qhW_t + znW_t = (1 - \tau)W_t + (1 - \gamma)\tau\frac{n(1 + \bar{g})}{(1 + r)}W_t + \gamma\tau\frac{(1 + g)}{(1 + r)}W_t \quad (2.40)$$

The left hand side of Equation (2.40) represents the present value of the total consumption and the right hand side represents the present value of the lifetime income. We observe that the unfunded PAYG portion of the pension income grows at rate $n(1 + \bar{g})$, which is also the rate of growth for output and aggregate capital.

⁶Refer to footnote 3.

From Equations (2.37) to (2.39), I solve for the following variables in their reduced forms given by:

$$h = \frac{\nu \left(1 - \tau + \frac{\gamma\tau(1+g)}{(1+r)}\right) + (1 + \delta + \beta) qh}{(1 + \nu + \delta + \beta) q} \quad (2.41)$$

$$n = \frac{1}{z} \left[\left(\frac{\beta(1 - \gamma)\tau}{\left[\frac{1}{\omega}(1 + r) + (1 - \gamma)\tau\right]} + \delta \right) \frac{\left(1 - \tau + \frac{\gamma\tau(1+g)}{(1+r)} - \underline{h}q\right)}{(1 + \nu + \delta + \beta)} \right] \quad (2.42)$$

$$1 + \bar{g} = \frac{-\frac{1}{z} \left[\frac{\gamma\tau(1 + g)}{\left[\frac{1}{\omega}(1 + r) + (1 - \gamma)\tau\right]} \left(\frac{(1 - \gamma)\tau}{(1 + r)} \right) \right] + \beta(1 + r) \left(1 - \tau + \frac{\gamma\tau(1+g)}{(1+r)} - \underline{h}q\right) - \gamma\tau(1 + g) (1 + \nu + \delta + \beta)}{D} \quad (2.43)$$

$$\text{where } D = \frac{1}{z} \left((\delta + \beta) (1 - \gamma)\tau + \delta \left[\frac{1}{\omega}(1 + r) \right] \right) \left(1 - \tau + \frac{\gamma\tau(1 + g)}{(1 + r)} - \underline{h}q\right) - \frac{1}{z} \left[\gamma\tau(1 + g) (1 + \nu + \delta + \beta) \left(\frac{(1 - \gamma)\tau}{(1 + r)} \right) \right] \quad (2.44)$$

2.3.1 No Borrowing Constraint⁷

W_{t+1} needs to be positive for the PAYG pension income to be positive. From Equation (2.24), in order for W_{t+1} to be positive, savings s must be positive since n is positive. From Equation (2.30), the no borrowing condition can be derived by setting $(1 + \bar{g})n > 0$; the following no borrowing condition is obtained:

$$\tau < \frac{\beta(1 + r) (1 - \underline{h}q)}{\gamma(1 + g) (1 + \nu + \delta) + (1 + r)\beta} \quad (2.45)$$

For $\gamma = 0$, under the pure PAYG system, Equation (2.45) becomes:

$$\tau < 1 - \underline{h}q \quad (2.46)$$

$$1 - \tau > \underline{h}q \quad (2.47)$$

⁷For proof, refer to Appendix.

Equation (2.47) shows that the after-tax income must be greater than the cost of minimum housing consumption. For $\underline{h} = 0$, Equation (2.46) becomes:

$$\tau < 1 \quad (2.48)$$

For any tax rate less than 1, individuals will not borrow under the pure PAYG system.

For $\gamma \neq 0$, under the mixed system, the no borrowing constraint becomes less stringent since the denominator of Equation (2.45) is larger under the given condition; this is because individuals may now borrow against the FF portion of their retirement income.

In conclusion, in order for no borrowing to take place, τ cannot be too large in order to satisfy the minimum housing consumption condition.

2.3.2 Effects of $\underline{h}$ on Price Elasticity of Housing Consumption and Fertility

Proposition 1a: $\underline{h}$ must be greater than zero for the demand for housing consumption to be price inelastic, i.e. $\frac{dh}{dq} \frac{q}{h} > -1$, and for the demand for children to decrease with housing price, i.e. $\frac{dn}{dq} < 0$.

Proposition 1b: If $\underline{h}$ is a function of q and $\underline{h}$ increases with q , i.e. $\frac{dh}{dq} > 0$, h may increase (or decrease) with housing price and fertility decreases with housing price.

Proposition 1c: If $\underline{h}$ is a function of q and $\underline{h}$ is price inelastic, i.e. $\frac{dh}{dq} \frac{q}{h} > -1$, the demand for housing will also be price inelastic and the demand for children will decrease with housing price.

Proof: See Appendix. From Equations (2.41) and (2.42), we can observe that the minimum housing consumption requirement increases housing consumption and decreases fertility. If $\underline{h} = 0$, $\frac{dh}{dq} \frac{q}{h} = -1$ (unitary elastic). h is inversely proportional to price. For $\underline{h} > 0$, $\frac{dh}{dq} \frac{q}{h} > -1$.⁸ This means that for an increase in housing price relative to wage rate, demand for housing decreases less than proportionally. Hanushek and Quigely (1980)

⁸Change in q can mean either wage rate is fixed as housing price changes or housing price changes differently from wage rate.

find that the price elasticity of the demand for housing in the U.S. is between -0.64 and -0.45.

$\underline{h} > 0$ results in fertility to decrease with housing price. Under the log utility specification, when $\underline{h} = 0$, any change in housing price will result in the substitution effect exactly offsetting the income effect, resulting in no change in fertility. With $\underline{h} > 0$, as price increases, individuals will now have to consume the minimum amount of housing, $\underline{h}$ at a higher price and this reduces the income available for having children; hence, fertility decreases. Empirically, many have shown that an increase in housing price results in a decrease in fertility (Dettling and Kearney, 2014; Lovenheim and Mumford, 2013; Yi and Zhang, 2010; Pan and Xu, 2012; Hui et al., 2012). Dettling and Kearney (2014) estimate that a \$10,000 increase in price results in a 2.4% decrease in fertility among non-homeowners.

If we assume the minimum housing consumption, $\underline{h}$ to be a function of q , only when $\underline{h}$ decreases with q will h decrease with q . If $\underline{h}$ increases with q , the effect of the change in q on h is uncertain. If housing is a Veblen good, the consumption of housing increases with price. Lee and Mori (2013) show that the Veblen effect is present in certain areas of the housing market in the U.S.

2.4 Change in Social Security Taxation with the Pure PAYG and FF System

The effect of social security taxation on growth rate and fertility is examined by many in the literature (Wigger, 1999a, 1999b; Zhang and Zhang, 1998). Empirically, it is shown that social security lowers fertility (Zhang and Zhang, 2004; Boldrin et al., 2005; Ehrlich and Kim, 2007). Effects of τ are first examined for the pure PAYG system followed by the pure FF system.

2.4.1 Pure PAYG System

Setting $\gamma = 0$ in Equations (2.41) to (2.44), we have:

$$h_{payg} = \frac{\nu(1 - \tau) + (1 + \delta + \beta)q\bar{h}}{(1 + \nu + \delta + \beta)q} \quad (2.49)$$

$$n_{payg} = \frac{1}{z} \left[\left(\frac{\beta\tau}{\left[\frac{1}{\omega}(1+r) + \tau\right]} + \delta \right) \frac{(1 - \tau - \bar{h}q)}{(1 + \nu + \delta + \beta)} \right] \quad (2.50)$$

$$(1 + \bar{g})_{payg} = \frac{\beta(1+r)}{\frac{1}{z} \left((\delta + \beta)\tau + \delta \left[\frac{1}{\omega}(1+r) \right] \right)} \quad (2.51)$$

Proposition 2: Under the pure PAYG system, an increase in the contribution rate τ crowds out savings, growth, and housing consumption, i.e. $\frac{d(1+\bar{g})n}{d\tau} < 0$, $\frac{d\bar{g}}{d\tau} < 0$, $\frac{dh}{d\tau} < 0$.

For proof of $\frac{d(1+\bar{g})n}{d\tau} < 0$, see Appendix. In order for the pension income to be positive, no borrowing is allowed. We can obtain $\frac{dh}{d\tau} < 0$ and $\frac{d\bar{g}}{d\tau} < 0$ by differentiating Equations (2.49) and (2.51) respectively with respect to τ . From Equation (2.30), given that $\frac{d(1+\bar{g})n}{d\tau} < 0$, savings rate decreases with τ . As τ increases, individuals will have a larger pension income and a smaller incentive to save; the reduction in after-tax income also makes it harder for them to save. Given that $\frac{d(1+\bar{g})n}{d\tau} < 0$, if $\bar{g}$ increases with τ , n must decrease with τ . This is because if $\bar{g}$ increases with τ , the overall effect on *both* n and $\bar{g}$ must decrease with τ for $\frac{d(1+\bar{g})n}{d\tau}$ to be less than 0. Since $\bar{g}$ decreases with τ , n can either increase or decrease with τ which leads us to the next proposition.

Proposition 3: Under the pure PAYG system,

- (i) If $(1 - \bar{h}q) \frac{\beta}{1+r} < \frac{\delta}{\omega}$, an increase in contribution rate τ crowds out fertility, i.e. $\frac{dn}{d\tau} < 0$.
- (ii) if $(1 - \bar{h}q) \frac{\beta}{1+r} > \frac{\delta}{\omega}$, there is an optimal tax rate τ^* that maximises fertility.

Proof: See Appendix. Results are similar to that of Wigger (1999a).⁹ In addition to n being a pure consumption good in the first period, the increase in n also increases

⁹With parents' old-age consumption included in the utility function, Wigger shows that fertility may either increase or decrease with τ depending on the strength of preference in supporting elderly parents versus the intertemporal discount rate. If the discount factor is *smaller* than the preference for the old-age consumption of parents, fertility decreases monotonically with τ . In other words, the strength

the PAYG pension income since it depends on fertility rate n . Since the increase in τ increases the fertility-related pension income, there is a greater incentive to have children. However, it reduces the after-tax income available for child rearing in the first period. The following demonstrates the effects of τ on the second period consumption through various channels. From Equation (2.35), we know that the second period consumption increases with the first period consumption.

From Equation (2.8), the second period budget constraint is given by:

$$c_{t+1}^o = s(1+r) + n\tau W_{t+1} \quad (2.52)$$

Substituting $s = \frac{W_{t+1}}{\omega}n$ from Equation (2.24) into Equation (2.52), we have:

$$c_{t+1}^o = \frac{W_{t+1}}{\omega}n[(1+r) + \omega\tau] \quad (2.53)$$

$$\frac{c_{t+1}^o}{W_t} = \frac{W_{t+1}}{\omega W_t}n[(1+r) + \omega\tau] \quad (2.54)$$

$$\frac{c_{t+1}^o}{W_t} = (1+\bar{g})n \left[\frac{(1+r)}{\omega} + \tau \right] \quad (2.55)$$

Differentiating Equation (2.55) with respect to τ , we have:

$$\frac{d}{d\tau} \left(\frac{c_{t+1}^o}{W_t} \right) = \frac{d(1+\bar{g})n}{d\tau} \left[\frac{(1+r)}{\omega} + \tau \right] + (1+\bar{g})n \quad (2.56)$$

Since $\frac{d(1+\bar{g})n}{d\tau} < 0$, $\frac{d}{d\tau} \left(\frac{c_{t+1}^o}{W_t} \right)$ can either be positive or negative. From Equation (2.52), as τ increases, more is being transferred from the first to the second period; individuals can now save less for the same amount consumption in the second period. Decrease in savings will affect W_{t+1} negatively since $\frac{d\bar{g}}{d\tau} < 0$. However, individuals can choose to increase n to make up for the fall in W_{t+1} and mitigate the decrease in $(1+\bar{g})n$. Therefore, n can increase or decrease depending on conditions (i) or (ii) in *Proposition 3*.

Substituting $W_{t+1}n = \omega s$ from Equation (2.24) into the second term of Equation (2.52),

of altruism in children to take care of their parents is stronger than their own old-age consumption. If the discount factor is *greater* than the preference for parents' old-age consumption and the preference for parents' old-age consumption is smaller than a certain upper bound, an increase in τ can increase fertility. In this case, there exists a local maximum for which τ^* maximises fertility.

we have:

$$c_{t+1}^o = s [(1 + r) + \omega\tau] \quad (2.57)$$

Differentiating Equation (2.57) with respect to τ ,

$$\frac{dc_{t+1}^o}{d\tau} = \frac{ds}{d\tau} [(1 + r) + \omega\tau] + \omega s \quad (2.58)$$

From Equation (2.58), we observe that the first term is negative and the second term is positive. If ω and s are large, the second term will be large and second period consumption can increase (if $\omega\tau$ is not too large). If $1 + r$ is large, the first term is large, the second period consumption can decrease. A larger preference for the second period consumption, β will result in more savings, s . Therefore, for a large β , small $1 + r$ and large ω , c_{t+1}^o (and c_t^y) can increase as τ increases.

From Condition (ii) in *Proposition 3*, setting $\underline{h} = 0$, we have $\frac{\beta}{1+r} > \frac{\delta}{\omega}$. From Equations (2.24) and (2.30), ω is related to the future wage rate W_{t+1} and growth rate $1 + \bar{g}$ respectively. For the same amount of savings, the larger ω is, the larger the pension income. $1 + r$ and ω give the trade-off between the rate of return from pension income and voluntary savings and β and δ give the trade-off between the preference for the second period consumption and the number of children. From $\frac{\beta}{1+r} > \frac{\delta}{\omega}$, if β is large and δ is small, individuals prefer second period consumption to the number of children and so the (absolute) decrease in n due to the reduction in after-tax income as τ increases is smaller than the (absolute) increase in n to increase or mitigate the decrease in pension payout (and second period consumption). If $1 + r$ is small and ω is large, there is a greater incentive to increase pension income by increasing n than to increase savings since r is small to increase second period consumption. Therefore, for n to increase with τ , the condition $\frac{\beta}{1+r} > \frac{\delta}{\omega}$ must be satisfied. If $\underline{h} > 0$, individuals must now satisfy the minimum housing consumption. Since housing consumption belongs to the first period, it reduces the preference for second period consumption, β by a factor of $1 - \underline{h}q$.

In summary, if condition (ii) is satisfied, increasing τ can increase fertility up to τ^* . As

τ increases, housing consumption and savings fall and the number of children increases (decreases) for $\tau < (>) \tau^*$. Present and future consumption may increase or decrease. If condition (i) is satisfied, fertility decreases monotonically with τ . In this case, n is largest when $\tau = 0$ and from Equation (2.50), it is given by $n_{payg} = \frac{\delta(1-hq)}{z(1+\nu+\delta+\beta)}$.

2.4.2 Pure FF System

Setting $\gamma = 1$ in Equations (2.41) to (2.44) we have:

$$h_{ff} = \frac{\nu \left(1 - \tau + \frac{\tau(1+g)}{(1+r)}\right) + (1 + \delta + \beta) q h}{(1 + \nu + \delta + \beta) q} \quad (2.59)$$

$$n_{ff} = \frac{1}{z} \left[\delta \frac{\left(1 - \tau + \frac{\tau(1+g)}{(1+r)} - hq\right)}{(1 + \nu + \delta + \beta)} \right] \quad (2.60)$$

$$(1 + \bar{g})_{ff} = \frac{\beta(1+r) \left(1 - \tau + \frac{\tau(1+g)}{(1+r)} - hq\right) - \tau(1+g)(1 + \nu + \delta + \beta)}{\frac{1}{z} \delta \left[\frac{1}{\omega}(1+r)\right] \left(1 - \tau + \frac{\tau(1+g)}{(1+r)} - hq\right)} \quad (2.61)$$

Proposition 4: Under the pure FF system, an increase in the contribution rate τ crowds out savings and growth, i.e. $\frac{d(1+\bar{g})n}{d\tau} < 0$, $\frac{d\bar{g}}{d\tau} < 0$.

(i) If $g > r$, $\frac{dh}{d\tau} > 0$, $\frac{dn}{d\tau} > 0$.

(ii) If $g < r$, $\frac{dh}{d\tau} > 0$, $\frac{dn}{d\tau} < 0$.

(iii) if $g = r$, $\frac{dh}{d\tau} = 0$, $\frac{dn}{d\tau} = 0$.

Under the FF system, borrowing is allowed.¹⁰ Differentiating Equations (2.59) to (2.61), we can obtain the results. For $g > r$, individuals can *borrow* at a *lower* rate of return r and save at a higher rate of return g to increase the lifetime present value income. Hence, increasing τ up to the maximum value of 1 can increase housing consumption and the number of children. Likewise, for $g < r$, the lifetime present value income will decrease

¹⁰Under the pure FF system, pension income does not depend on growth rate, g or W_{t+1} . Individuals can optimise their utilities without caring for the next generation's income, W_{t+1} ; although borrowing is permissible, it is not beneficial to the next generation.

with τ . If $g = r$,¹¹ it does not make any difference to the lifetime present value income; hence, there will not be any effect on n , h , c_t^y and c_{t+1}^o . Also, there is no unfunded (fertility-related) pension income. Children are a mere consumption good in the first period; so, the effect of the change in τ is no different from that on housing consumption.

For both systems (FF and PAYG), compulsory savings or social security tax crowds out savings, reduces labour productivity growth rate g and output growth; the decrease in fertility (if it happens) is not sufficient to compensate for the decrease in savings, resulting in the reduction in productivity growth. From the social planner's perspective, individuals are already under saving due to the spillover effects of capital to labour (see Equation (2.19)); social security tax further reduces their savings.

Results are similar to Wigger (1999a, 2001b), Zhang and Zhang (1998) for a sufficiently high tax rate, Wigger (1999b) and Zhang (2003a) with the demand for leisure and bequest motive included except Zhang and Zhang (1995).¹² Similar to Zhang and Zhang (1995), Zhang (1995) shows that social security can increase growth, depending on whether the *fertility effect dominates the savings effect* or vice versa. In human capital externality growth models, distortionary tax that reduces fertility can result in a greater investment towards human capital (Zhang, 1995; Zhang, 2003b; Yew and Zhang, 2009; Zhang and Zhang, 2003; Kaganovich and Zilcha, 1999). Empirically, Zhang and Zhang (2004) show that social security has a positive effect on growth. However, under different demographic assumptions, Pecchenino and Utendorf (1999) show that the social security tax reduces education, growth, and welfare in most cases.

¹¹In the existing literature, the FF rate of return is given by the rate of return to capital r as the pension fund is being invested by the government in real capital K_{t+1} (Zhang, 2003a; Yoon and Talmain, 2001; Holler, 2009). Equation (2.21) then becomes $K_{t+1} = N_t (s_t + \tau W_t)$. Any change in τ will not have any effect on c_t^y, c_{t+1}^o, h and n since individuals can offset this change by changing their savings. $\frac{d\bar{g}}{d\tau}$ will also be equal to 0 since the change in investment from the change in savings will be offset by the change in government investment from the change in τ by the *same amount*; hence, there is no resultant change in capital per capita. Here, the decrease in savings will decrease the growth rate, $\bar{g}$ as shown by $\frac{d\bar{g}}{d\tau} < 0$.

¹²Zhang and Zhang (1995) says "The result that introduction of social security may induce higher economic growth seems to be novel to the literature" (p. 446).

2.4.3 Comparison of the Two Systems

Proposition 5a: Consumers will consume more housing under the pure FF system.

From Equations (2.59) and (2.49), using the two housing consumptions and subtracting one from the other, I get $h_{ff} - h_{payg} = \frac{\nu(1+g)}{1+r}\tau$. Housing consumption is always greater under the FF system because pension income is unrelated to fertility. Housing consumption depends on the *lifetime present value fertility-unrelated income*. For the FF system, there is no *fertility-related* income. For the PAYG system, housing consumption decreases with τ because the *fertility-unrelated income* decreases as τ increases. Hence, when there is social security tax, i.e. $\tau \neq 0$, the lifetime present value fertility-unrelated income is *always larger* for the FF system. When $\tau = 0$, $h_{ff} = h_{payg}$. When $\tau \neq 0$, for $g = r$, $h_{ff} - h_{payg} = \nu\tau$. For $g > r$ ($g < r$), the difference between the two consumption increases (decreases).

Proposition 5b: For same pension income, $n_{payg} > n_{ff}$. Whether fertility is greater under the pure FF or PAYG system depends on the rate of return, $1 + g$. For $1 + g > \frac{\beta(1+r)(1-\tau-hq)}{\delta[\frac{1}{\omega}(1+r)+\tau]}$, fertility is higher under the FF system.

Proof: See Appendix. If pension income is the same for the two systems, we have the following conditions:

$$n\tau W_{t+1} = \tau(1+g)W_t \quad (2.62)$$

$$n(1+\bar{g}) = (1+g) \quad (2.63)$$

$$\text{A: } n(1+\bar{g}) = 1+g \Leftrightarrow \frac{\beta(1+r)(1-\tau-hq)}{[(1+\nu+\delta+\beta)[\frac{1}{\omega}(1+r)+\tau]-\gamma\tau\beta]} = 1+g$$

$$\text{E: } n_{payg} = n_{ff} \Leftrightarrow \frac{\beta(1+r)(1-\tau-hq)}{\delta[\frac{1}{\omega}(1+r)+\tau]} = 1+g$$

Lines A and E are shown in Figure 2.1. The horizontal axis represents the values of the funded rate of return, $1 + g$. Since line A lines on the left side of line E, at line A, $1 + g$ is less than $\frac{\beta(1+r)(1-\tau-hq)}{\delta[\frac{1}{\omega}(1+r)+\tau]}$. This implies that n_{payg} is greater than n_{ff} . Hence, only when g

Figure 2.1: Lines A and E on $1+g$ diagram

is large enough (condition in *Proposition 5b*) will fertility be larger under the FF system.

2.5 Changes in Savings, Housing Consumption, Fertility and Growth with Social Security System

Proposition 6: Housing consumption increases with the FF system, $\frac{dh}{d\gamma} > 0$.

Differentiating Equation (2.41), we have $\frac{dh}{d\gamma} > 0$. Increasing γ increases the proportion of FF pension income and increases the lifetime present value fertility-unrelated income. Based on *Proposition 5a*, housing consumption increases.

Proposition 7: Whether a switch towards the fully funded system will increase n or $\bar{g}$ depends on the rate of return, $1 + g$. As $1 + g$ becomes larger and larger, savings first decreases with γ followed by growth rate, $\bar{g}$; subsequently, fertility, n increases with γ .

Proof: See Appendix. In addition to lines A and E in Figure 2.1, Figures 2.2 - 2.4 include lines B, C and D as well, given by:

B: $\frac{d(1+\bar{g})n}{d\gamma} = 0 \Leftrightarrow \frac{\beta(1+r)[1-\tau-hq]}{\left[\frac{1}{\omega}(1+r)+\tau\right](1+\nu+\delta)} = 1+g$

C: $\frac{d\bar{g}}{d\gamma} = 0$ (no reduced form for $1+g$)

D: $\frac{dn}{d\gamma} = 0 \Leftrightarrow \frac{\frac{\delta}{\omega}\beta(1+r)[1-\tau-hq]}{\frac{\delta}{\omega}(1+\nu+\delta)\left[\frac{1}{\omega}(1+r)+\tau\right]-\nu\frac{1}{(1+r)}\left(\delta+\frac{\delta}{\nu}\right)\left[\frac{1}{\omega}(1+r)+(1-\gamma)\tau\right]^2} = 1+g$

Figures 2.2, 2.3 and 2.4 show the diagrams for various assumptions of γ , i.e. $0 < \gamma < 1$, $\gamma = 1$, and $\gamma = 0$ respectively. In Figure 2.2, starting from Region I,¹³ it denotes the region where $n(1+\bar{g}) > 1+g$. In that region, $\frac{dn}{d\gamma} < 0$, $\frac{d\bar{g}}{d\gamma} > 0$, and $\frac{d(1+\bar{g})n}{d\gamma} > 0$. Crossing from the left side of line A to the right, in Region II, we have $n(1+\bar{g}) < 1+g$. It means that even though income from the FF component is greater than the PAYG component, fertility decreases with γ , growth rate, $\bar{g}$ increases with γ and there is still an overall increase in $n(1+\bar{g})$.

After crossing to the right side of line B, in Region III, $(1+\bar{g})n$ and savings now decrease with γ . The increase in growth rate, $\bar{g}$ is unable to result in the increase in the PAYG

¹³Differentiating right side of Equation (2.40) with respect to γ , we can find the optimal proportion that maximises the lifetime present value income, given by $\gamma^* = \frac{\frac{d(1+\bar{g})n}{d\gamma} - (n(1+\bar{g}) - 1 + g)}{\frac{d(1+\bar{g})n}{d\gamma}}$. For region I, γ^* lies between 0 and 1. For region II and the rest, γ^* is greater than 1. Hence in region II, the pure FF system ($\gamma = 1$) maximises the lifetime income.

Regions I, II, III, IV, V and VI

Figure 2.3: $1+g$ diagram for FF system

Regions I, II, III, IV, V and VI

Figure 2.4: $1+g$ diagram for PAYG system

pension income with γ . Even though savings now decrease with γ , the *trade-off between lower savings and lower fertility* still results in higher capital per capita and the increase in growth rate, $\bar{g}$. After crossing to the right side of line C, Region IV is where $\bar{g}$ decreases with γ ; here, both $\bar{g}$ and n decrease with γ . Next, in Region V, n increases with γ .

Lines A, C, and D depend on the value of γ . Hence, their positions shift for Figures 2.3 and 2.4. In Figures 2.3 and 2.4, I include line E from Figure 2.1. Regions to the left of line E are where the PAYG system results in higher fertility than the FF system and regions to the right are the opposite. As the rate of return, $1 + g$ increases from the left to the right on the horizontal axis, savings first decrease with γ , followed by growth rate, $\bar{g}$ and lastly, fertility, n increases with γ .

To summarise, housing consumption depends only on the present value of the lifetime fertility-unrelated income while fertility, on the other hand, depends on *both the funded and unfunded components of the retirement income*. A larger population growth rate will result in a larger unfunded PAYG rate of return. As the PAYG system switches to the FF system, fertility decreases (Regions I-IV, Figure 2.2). This should decrease the unfunded PAYG rate of return, but it is offset by the increase in savings and growth rate, $\bar{g}$ (Regions I and II, Figure 2.2). Savings fall when individuals no longer need to save as much as before (Regions III - V, Figure 2.2). Since fertility depends on both the funded and unfunded components of the pension system, fertility increases only when the funded rate of return is large enough (Region V, Figure 2.2).

2.6 Conclusion

Housing consumption depends on individuals' lifetime present value fertility-unrelated incomes. In Singapore, many couples utilise their mandatory social security contributions to pay for the down payments and mortgage loans of their homes, resulting in the high homeownership rates. However, too much may have been spent on housing consumption, leaving too little for having children. This essay gives a possible explanation why countries

with high homeownership rates have low birth rates.

Homeownership can result in unintended bequests¹⁴ which lead to low welfare (Feldstein, 1990). In Singapore, one option is to have shorter lease houses sold at cheaper prices. However, this may not be popular among the current generation who still hold certain view towards homeownership, marriage and family due to the existing framing effects (Samuelson & Zeckhauser, 1988) of what they are exposed to and where they are at.¹⁵

This essay supports the negative relationship between social spending and homeownership and the positive relationship between income inequality and homeownership (Kemeny, 1980; Kemeny, 1981; Kemeny, 1992; Castles, 1998). In ‘asset based’ welfare states with low tax and social spending, people keep housing assets as a form of private insurance (Conley and Gifford, 2006). The redistributive effects of social spending can result in an overall improvement in welfare and efficiency by reducing wealth inequality; this may also reduce housing prices instead of having a ‘winner-take-all’ housing market (Conley and Gifford, 2006; Hui et al., 2016; Gyourko et al., 2013).

By switching more towards the unfunded PAYG system, social security can increase fertility. By including a fertility-related social security component, having more social insurance and increasing social spending, one may increase fertility.

Lastly, taxation should not only be used to increase the number of children but also the quality of children. Having child-rearing subsidies may increase fertility (Yasuoka and Goto, 2011; Hirazawa and Yakita, 2009; Van Groezen et al., 2003), but it is equally important to invest in training and education to enhance the human capital and productivity growth of a country (Cremer et al., 2011; Yew and Zhang, 2013; Zhang, 1997).

An extension to this essay would be to compare the welfare of the two social security

¹⁴Unintended bequests are bequests that are left accidentally or unintentionally to their children.

¹⁵According to (Mulder and Wagner (2001), the culture where you cannot get married until you can afford to own a house is more widespread in countries with high homeownership rates. In Singapore, houses with leases shorter than 99 years may not be popular as some may feel that the purchase of these houses does not qualify one as ‘owning’ a house; this ‘shorter lease’ is somewhere in between the duration of owning and renting. Unless the majority feels that it is alright not to leave any housing bequest to their children as in other countries and cultures, this may not take off.

systems.¹⁶

¹⁶We can solve for conditions of welfare increasing or decreasing with γ (like in Section 2.5) or whether welfare is larger (or smaller) under the FF ($\gamma = 1$) or the PAYG system ($\gamma = 0$). The welfare of an economy is given by the sum of utility of every individual (young and old) in *each period*. The welfare of each person is determined by his or her *lifetime* utility.

2.7 Appendix

Proof of No Borrowing Constraint:

From Equation (2.77) below, setting $(1 + \bar{g})n > 0$,

$$\begin{aligned}
\frac{\beta(1+r)(1-\tau) - \underline{h}q\beta(1+r) - \gamma\tau(1+g)(1+v+\delta)}{(1+v+\delta+\beta) \left[\frac{1}{\omega}(1+r) + (1-\gamma)\tau \right]} &> 0 \\
\beta(1+r)(1-\tau) - \underline{h}q\beta(1+r) - \gamma\tau(1+g)(1+v+\delta) &> 0 \\
\beta(1+r) - \tau\beta(1+r) - \underline{h}q\beta(1+r) - \gamma\tau(1+g)(1+v+\delta) &> 0 \\
\beta(1+r) - \underline{h}q\beta(1+r) - \tau(\beta(1+r) + \gamma(1+g)(1+v+\delta)) &> 0 \\
\frac{\beta(1+r) - \underline{h}q\beta(1+r)}{\beta(1+r) + \gamma(1+g)(1+v+\delta)} &> \tau \quad \blacksquare
\end{aligned}$$

Proof of *Proposition 1a*:

From Equations (2.41) and (2.42),

$$\begin{aligned}
\frac{dh}{dq} &= \frac{\nu(1-\tau) + \nu \frac{\gamma\tau(1+g)}{(1+r)}}{(1+v+\delta+\beta)q^2}(-1) \\
\frac{dh}{dq} \frac{1}{h} &= \frac{\nu(1-\tau) + \nu \frac{\gamma\tau(1+g)}{(1+r)}}{(1+v+\delta+\beta)q^2}(-1) \frac{(1+v+\delta+\beta)q}{\nu(1-\tau) + \nu \frac{\gamma\tau(1+g)}{(1+r)} + (1+\delta+\beta)qh} \\
\frac{dh}{dq} \frac{q}{h} &= -\frac{\nu(1-\tau) + \nu \frac{\gamma\tau(1+g)}{(1+r)}}{\nu(1-\tau) + \nu \frac{\gamma\tau(1+g)}{(1+r)} + (1+\delta+\beta)qh} > -1 \\
\frac{dn}{dq} &= -\frac{1}{z} \left(\frac{\beta(1-\gamma)\tau}{\left[\frac{1}{\omega}(1+r) + (1-\gamma)\tau \right]} + \delta \right) \frac{h}{(1+v+\delta+\beta)} < 0 \quad \blacksquare
\end{aligned}$$

Proof of *Proposition 1b*:

If $\underline{h}$ is a function of q , differentiating Equation (2.41), we have:

$$\frac{dh}{dq} = -\frac{\nu(1-\tau) + \nu \frac{\gamma\tau(1+g)}{(1+r)}}{(1+\nu+\delta+\beta)q^2} + \frac{(1+\delta+\beta)}{(1+\nu+\delta+\beta)} \frac{dh}{dq}$$

If $\frac{dh}{dq} > 0$, sign of $\frac{dh}{dq}$ is uncertain.

Differentiating Equation (2.42), we have:

$$\frac{dn}{dq} = -\frac{1}{z} \left(\frac{\beta(1-\gamma)\tau}{\left[\frac{1}{\omega}(1+r) + (1-\gamma)\tau\right]} + \delta \right) \frac{1}{(1+\nu+\delta+\beta)} \left(\frac{dh}{dq} q + h \right) \quad (2.64)$$

If $\frac{dh}{dq} > 0$, $\frac{dn}{dq} < 0$. ■

Proof of *Proposition 1c*:

$$\begin{aligned} \frac{dh}{dq} \frac{1}{h} &= \left(-\frac{\nu(1-\tau) + \nu \frac{\gamma\tau(1+g)}{(1+r)}}{(1+\nu+\delta+\beta)q^2} + \frac{(1+\delta+\beta)}{(1+\nu+\delta+\beta)} \frac{dh}{dq} \right) \frac{(1+\nu+\delta+\beta)q}{\nu(1-\tau) + \nu \frac{\gamma\tau(1+g)}{(1+r)} + (1+\delta+\beta)qh} \\ &= \left(-\frac{\nu(1-\tau) + \nu \frac{\gamma\tau(1+g)}{(1+r)}}{q} + \frac{q(1+\delta+\beta)}{1} \frac{dh}{dq} \right) \frac{1}{\nu(1-\tau) + \nu \frac{\gamma\tau(1+g)}{(1+r)} + (1+\delta+\beta)qh} \\ &= -\frac{1}{q} \left(\frac{\nu(1-\tau) + \nu \frac{\gamma\tau(1+g)}{(1+r)}}{1} - \frac{q^2(1+\delta+\beta)}{1} \frac{dh}{dq} \right) \frac{1}{\nu(1-\tau) + \nu \frac{\gamma\tau(1+g)}{(1+r)} + (1+\delta+\beta)qh} \\ \frac{dh}{dq} \frac{q}{h} &= - \left(\frac{\nu(1-\tau) + \nu \frac{\gamma\tau(1+g)}{(1+r)} - q^2(1+\delta+\beta) \frac{dh}{dq}}{\nu(1-\tau) + \nu \frac{\gamma\tau(1+g)}{(1+r)} + (1+\delta+\beta)qh} \right) \end{aligned} \quad (2.65)$$

From Equation (2.65),

$$\begin{aligned} \frac{dh}{dq} \frac{q}{h} &> -1 \\ - \left(\frac{\nu(1-\tau) + \nu \frac{\gamma\tau(1+g)}{(1+r)} - q^2(1+\delta+\beta) \frac{dh}{dq}}{\nu(1-\tau) + \nu \frac{\gamma\tau(1+g)}{(1+r)} + (1+\delta+\beta)qh} \right) &> -1 \\ \left(\frac{\nu(1-\tau) + \nu \frac{\gamma\tau(1+g)}{(1+r)} - q^2(1+\delta+\beta) \frac{dh}{dq}}{\nu(1-\tau) + \nu \frac{\gamma\tau(1+g)}{(1+r)} + (1+\delta+\beta)qh} \right) &< 1 \\ -q^2(1+\delta+\beta) \frac{dh}{dq} &< (1+\delta+\beta)qh \\ -q \frac{dh}{dq} &< h \\ -\frac{q}{h} \frac{dh}{dq} &< 1 \\ \frac{q}{h} \frac{dh}{dq} &> -1 \end{aligned}$$

From Equation (2.64),

$$\frac{q}{h} \frac{dh}{dq} > -1 \Leftrightarrow \frac{dh}{dq} q + h > 0, \text{ therefore, } \frac{dn}{dq} < 0 \quad \blacksquare$$

Proof of $\frac{d(1+\bar{g})n}{d\tau} < 0$ in *Proposition 2*:

From Equation (2.74) below, setting $\gamma = 0$,

$$(1 + \bar{g})n = \frac{\beta(1+r)^{\frac{1}{\nu}}(qh - qh)}{\left[\frac{1}{\omega}(1+r) + \tau\right]} \quad (2.66)$$

From Equation (2.49), setting $\gamma = 0$,

$$\frac{dh}{d\tau} = \frac{\nu(-1)}{(1+v+\delta+\beta)q} \quad (2.67)$$

Differentiating Equation (2.66),

$$\frac{d(1 + \bar{g})n}{d\tau} = \frac{\beta(1+r)^{\frac{1}{\nu}}(qh - qh)}{\left[\frac{1}{\omega}(1+r) + \tau\right]^2}(-1) + \frac{1}{\left[\frac{1}{\omega}(1+r) + \tau\right]} \beta(1+r)^{\frac{1}{\nu}} \left(q \frac{dh}{d\tau}\right) \quad (2.68)$$

Substituting Equation (2.67) into Equation (2.68),

$$\begin{aligned} \frac{d(1 + \bar{g})n}{d\tau} &= \frac{\beta(1+r)^{\frac{1}{\nu}}(qh - qh)}{\left[\frac{1}{\omega}(1+r) + \tau\right]^2}(-1) + \frac{1}{\left[\frac{1}{\omega}(1+r) + \tau\right]} \beta(1+r)^{\frac{1}{\nu}} q \frac{\nu(-1)}{(1+v+\delta+\beta)q} \\ &= \frac{\beta(1+r)^{\frac{1}{\nu}}(qh - qh)}{\left[\frac{1}{\omega}(1+r) + \tau\right]^2}(-1) + \frac{1}{\left[\frac{1}{\omega}(1+r) + \tau\right]} \beta(1+r)^{\frac{1}{\nu}} \frac{(-1)}{(1+v+\delta+\beta)} < 0 \quad \blacksquare \end{aligned}$$

Proof of *Proposition 3*:

From Equation (2.50),

$$\begin{aligned}
zn &= \frac{1}{\left[\frac{1}{\omega}(1+r) + \tau\right]} \left[\left(\frac{(\delta + \beta)\tau + \delta \left[\frac{1}{\omega}(1+r)\right]}{1} \right) \frac{(1 - \tau - \underline{h}q)}{(1 + \nu + \delta + \beta)} \right] \\
z \frac{dn}{d\tau} &= \frac{1}{\left[\frac{1}{\omega}(1+r) + \tau\right]} \left[\left(\frac{(\delta + \beta)\tau + \delta \left[\frac{1}{\omega}(1+r)\right]}{1} \right) \frac{-1}{(1 + \nu + \delta + \beta)} + \frac{(1 - \tau - \underline{h}q)}{(1 + \nu + \delta + \beta)} (\delta + \beta) \right] \\
&\quad + \left[\left(\frac{(\delta + \beta)\tau + \delta \left[\frac{1}{\omega}(1+r)\right]}{1} \right) \frac{(1 - \tau - \underline{h}q)}{(1 + \nu + \delta + \beta)} \right] (-1) \frac{1}{\left[\frac{1}{\omega}(1+r) + \tau\right]^2} \\
z \frac{dn}{d\tau} &= \frac{1}{\left[\frac{1}{\omega}(1+r) + \tau\right]} \left[\frac{(1 - \tau - \underline{h}q)}{(1 + \nu + \delta + \beta)} (\delta + \beta) - \left(\frac{(\delta + \beta)\tau + \delta \left[\frac{1}{\omega}(1+r)\right]}{(1 + \nu + \delta + \beta)} \right) \right] \\
&\quad - \frac{1}{\left[\frac{1}{\omega}(1+r) + \tau\right]} \left[\left(\frac{(\delta + \beta)\tau + \delta \left[\frac{1}{\omega}(1+r)\right]}{\left[\frac{1}{\omega}(1+r) + \tau\right]} \right) \frac{(1 - \tau - \underline{h}q)}{(1 + \nu + \delta + \beta)} \right] \tag{2.69}
\end{aligned}$$

Setting $\frac{dn}{d\tau}$ equals to zero, Equation (2.69) becomes:

$$\begin{aligned}
0 &= \frac{(1 - \tau - \underline{h}q)}{(1 + \nu + \delta + \beta)} (\delta + \beta) - \left(\frac{(\delta + \beta)\tau + \delta \left[\frac{1}{\omega}(1+r)\right]}{(1 + \nu + \delta + \beta)} \right) \\
&\quad - \left(\frac{(\delta + \beta)\tau + \delta \left[\frac{1}{\omega}(1+r)\right]}{\left[\frac{1}{\omega}(1+r) + \tau\right]} \right) \frac{(1 - \tau - \underline{h}q)}{(1 + \nu + \delta + \beta)} \tag{2.70}
\end{aligned}$$

Simplifying Equation (2.70), we have:

$$(1 - \underline{h}q) - \frac{\delta}{\beta} \frac{1}{\omega} (1 + r) - 2 \left(\frac{\delta + \beta}{\beta} \right) \tau - \left(\frac{(\delta + \beta)}{\left[\frac{1}{\omega}(1+r)\right] \beta} \right) \tau^2 = 0 \tag{2.71}$$

Equation (2.71) is a quadratic equation. In order for an optimal tax rate, τ^* to exist, it depends on the constant term, $(1 - \underline{h}q) - \frac{\delta}{\beta} \frac{1}{\omega} (1 + r)$. If $(1 - \underline{h}q) - \frac{\delta}{\beta} \frac{1}{\omega} (1 + r) > 0$, a positive solution exists. Otherwise, no solution exists and the tax rate τ must be zero to achieve maximum fertility, given that $\frac{dn}{d\tau} < 0$ for all values of τ . ■

Proof of *Proposition 5b*:

From Equation (2.39),

$$\begin{aligned}
\frac{\nu(1 - qh - zn - \tau) - \nu \frac{1}{\omega}(1 + \bar{g})n}{q} + \underline{h} &= h \\
\nu(1 - qh - zn - \tau) &= q(h - \underline{h}) + \nu \frac{1}{\omega}(1 + \bar{g})n \\
(1 - qh - zn - \tau) &= \frac{1}{\nu} \left(q(h - \underline{h}) + \nu \frac{1}{\omega}(1 + \bar{g})n \right) \quad (2.72)
\end{aligned}$$

From Equation (2.38),

$$\begin{aligned}
(1 + \bar{g}) &= \frac{\beta(1 + r)(1 - qh - zn - \tau) - \gamma\tau(1 + g)}{\frac{1}{\omega}n(1 + r) + (1 - \gamma)n\tau + \frac{1}{\omega}\beta(1 + r)n} \\
(1 + \bar{g})n \left[\frac{1}{\omega}(1 + r) + (1 - \gamma)\tau + \frac{1}{\omega}\beta(1 + r) \right] &= \beta(1 + r)(1 - qh - zn - \tau) \\
&\quad - \gamma\tau(1 + g) \quad (2.73)
\end{aligned}$$

Substitute Equation (2.72) into right hand side of Equation (2.73), we have:

$$\begin{aligned}
(1 + \bar{g})n \left[\frac{1}{\omega}(1 + r) + (1 - \gamma)\tau + \frac{1}{\omega}\beta(1 + r) \right] &= \beta(1 + r) \left(\frac{1}{\nu} \left(q(h - \underline{h}) + \nu \frac{1}{\omega}(1 + \bar{g})n \right) \right) \\
&\quad - \gamma\tau(1 + g) \\
(1 + \bar{g})n \left[\frac{1}{\omega}(1 + r) + (1 - \gamma)\tau \right] &= \beta(1 + r) \frac{1}{\nu} q(h - \underline{h}) - \gamma\tau(1 + g) \\
(1 + \bar{g})n &= \frac{\beta(1 + r) \frac{1}{\nu} q(h - \underline{h}) - \gamma\tau(1 + g)}{\left[\frac{1}{\omega}(1 + r) + (1 - \gamma)\tau \right]} \quad (2.74)
\end{aligned}$$

From Equation (2.41),

$$\begin{aligned}
h &= \frac{\nu(1 - \tau) + \nu \frac{\gamma\tau(1+g)}{(1+r)} + (1 + \delta + \beta)qh}{(1 + \nu + \delta + \beta)q} \\
h - \underline{h} &= \frac{\nu(1 - \tau) + \nu \frac{\gamma\tau(1+g)}{(1+r)} + (1 + \delta + \beta)qh}{(1 + \nu + \delta + \beta)q} - \underline{h} \\
h - \underline{h} &= \frac{\nu(1 - \tau) + \nu \frac{\gamma\tau(1+g)}{(1+r)} + (1 + \delta + \beta)qh - \underline{h}(1 + \nu + \delta + \beta)q}{(1 + \nu + \delta + \beta)q} \\
h - \underline{h} &= \frac{\nu(1 - \tau) + \nu \frac{\gamma\tau(1+g)}{(1+r)} - hvq}{(1 + \nu + \delta + \beta)q} \\
q(h - \underline{h}) &= \frac{\nu(1 - \tau) + \nu \frac{\gamma\tau(1+g)}{(1+r)} - hvq}{(1 + \nu + \delta + \beta)} \quad (2.75)
\end{aligned}$$

Substituting Equation (2.75) into the Equation (2.74), we have:

$$\begin{aligned}
(1 + \bar{g})n &= \frac{\beta(1+r) \frac{1}{\nu} \frac{\nu(1-\tau) + \nu \frac{\gamma\tau(1+g)}{(1+r)} - \underline{h}qv}{\left[\frac{1}{\omega}(1+r_{t+1}) + (1-\gamma)\tau\right]} - \gamma\tau(1+g) \\
(1 + \bar{g})n &= \frac{\beta(1+r)(1-\tau) + \gamma\tau(1+g)\beta - \underline{h}q\beta(1+r)}{(1+\nu+\delta+\beta) \left[\frac{1}{\omega}(1+r) + (1-\gamma)\tau\right]} - \frac{\gamma\tau(1+g)}{\left[\frac{1}{\omega}(1+r) + (1-\gamma)\tau\right]}
\end{aligned} \tag{2.76}$$

$$(1 + \bar{g})n = \frac{\beta(1+r)(1-\tau) - \underline{h}q\beta(1+r) - \gamma\tau(1+g)(1+v+\delta)}{(1+v+\delta+\beta) \left[\frac{1}{\omega}(1+r) + (1-\gamma)\tau\right]} \tag{2.77}$$

Set $(1 + \bar{g})n = 1 + g$ in Equation (2.76).

line A:

$$\begin{aligned}
(1 + \bar{g})n &= 1 + g \\
\frac{\beta(1+r)(1-\tau) + \gamma\tau(1+g)\beta - \underline{h}q\beta(1+r)}{(1+\nu+\delta+\beta) \left[\frac{1}{\omega}(1+r) + (1-\gamma)\tau\right]} - \frac{\gamma\tau(1+g)}{\left[\frac{1}{\omega}(1+r) + (1-\gamma)\tau\right]} &= 1 + g \\
&\vdots \\
\frac{\beta(1+r)(1-\tau - \underline{h}q)}{[(1+\nu+\delta+\beta) \left[\frac{1}{\omega}(1+r) + \tau\right] - \gamma\tau\beta]} &= 1 + g
\end{aligned}$$

From Equations (2.50) and (2.60), set $n_{payg} = n_{ff}$.

line E:

$$\begin{aligned}
n_{payg} &= n_{ff} \\
\left(\frac{\beta\tau}{\left[\frac{1}{\omega}(1+r) + \tau\right]} + \delta \right) \frac{(1 - \tau - \underline{h}q)}{(1 + \nu + \delta + \beta)} &= \delta \frac{\left(1 - \tau + \frac{\tau(1+g)}{(1+r)} - \underline{h}p_t\right)}{(1 + \nu + \delta + \beta)} \\
\left(\frac{\beta\tau}{\left[\frac{1}{\omega}(1+r) + \tau\right]} + \delta \right) (1 - \tau - \underline{h}q) &= \delta \left(1 - \tau + \frac{\tau(1+g)}{(1+r)} - \underline{h}q\right) \\
\left(\frac{\beta\tau}{\left[\frac{1}{\omega}(1+r) + \tau\right]} + \delta \right) (1 - \tau - \underline{h}q) &= \delta (1 - \tau - \underline{h}q) + \frac{\tau(1+g)}{(1+r)}\delta \\
\left(\frac{\beta\tau}{\left[\frac{1}{\omega}(1+r) + \tau\right]} + \delta \right) (1 - \tau - \underline{h}q) - \delta (1 - \tau - \underline{h}q) &= \frac{\tau(1+g)}{(1+r_{t+1})}\delta \\
\left(\frac{\beta\tau}{\left[\frac{1}{\omega}(1+r) + \tau\right]} \right) (1 - \tau - \underline{h}q) &= \frac{\tau(1+g)}{(1+r_{t+1})}\delta \\
\frac{\beta(1 - \tau - \underline{h}q)}{\left[\frac{1}{\omega}(1+r) + \tau\right]} &= \frac{(1+g)}{(1+r)}\delta \\
\frac{\beta(1 - \tau - \underline{h}q)(1+r)}{\delta \left[\frac{1}{\omega}(1+r) + \tau\right]} &= 1 + g \quad \blacksquare
\end{aligned}$$

Proof of *Proposition 7*:

line B:

Differentiating Equation (2.77):

$$\begin{aligned}
\frac{d(1 + \bar{g})n}{d\gamma} &= \frac{(1 + v + \delta + \beta) \left[\frac{1}{\omega}(1+r) + (1 - \gamma)\tau\right] [-\tau(1+g)(1+v+\delta)]}{(1 + v + \delta + \beta)^2 \left[\frac{1}{\omega}(1+r) + (1 - \gamma)\tau\right]^2} \\
&\quad - \frac{(\beta(1+r)(1-\tau) - \beta(1+r)\underline{h}q)(1+v+\delta+\beta)(-1)\tau}{(1 + v + \delta + \beta)^2 \left[\frac{1}{\omega}(1+r) + (1 - \gamma)\tau\right]^2} \\
&\quad + \frac{\gamma\tau(1+g)(1+v+\delta)(1+v+\delta+\beta)(-1)\tau}{(1 + v + \delta + \beta)^2 \left[\frac{1}{\omega}(1+r) + (1 - \gamma)\tau\right]^2}
\end{aligned} \tag{2.78}$$

Setting $\frac{d(1+\bar{g})n}{d\gamma}$ in Equation (2.78) to be zero:

$$\begin{aligned}
0 &= (1 + \nu + \delta + \beta) \left[\frac{1}{\omega}(1 + r) + (1 - \gamma)\tau \right] [-\tau(1 + g)(1 + \nu + \delta)] \\
&\quad - (\beta(1 + r)(1 - \tau) - \beta(1 + r)\underline{h}q)(1 + \nu + \delta + \beta)(-1)\tau \\
&\quad + \gamma\tau(1 + g)(1 + \nu + \delta)(1 + \nu + \delta + \beta)(-1)\tau \\
0 &= \left[\frac{1}{\omega}(1 + r) + (1 - \gamma)\tau \right] [-\tau(1 + g)(1 + \nu + \delta)] \\
&\quad + (\beta(1 + r)(1 - \tau) - \beta(1 + r)\underline{h}q - \gamma\tau(1 + g)(1 + \nu + \delta))\tau \\
\beta(1 + r)[(1 - \tau) - \underline{h}q] &= \left[\frac{1}{\omega}(1 + r) + (1 - \gamma)\tau \right] (1 + g)(1 + \nu + \delta) + \gamma\tau(1 + g)(1 + \nu + \delta) \\
\beta(1 + r)[(1 - \tau) - \underline{h}q] &= \left[\frac{1}{\omega}(1 + r) + (1 - \gamma)\tau + \gamma\tau \right] (1 + g)(1 + \nu + \delta) \\
\beta(1 + r)[(1 - \tau) - \underline{h}q] &= \left[\frac{1}{\omega}(1 + r_{t+1}) + \tau \right] (1 + g)(1 + \nu + \delta) \\
1 + g &= \frac{\beta(1 + r)[(1 - \tau) - \underline{h}q]}{\left[\frac{1}{\omega}(1 + r) + \tau \right] (1 + \nu + \delta)}
\end{aligned}$$

line D:

From Equation (2.74),

$$(1 + \bar{g})n \left[\frac{1}{\omega}(1 + r) + (1 - \gamma)\tau \right] = \beta(1 + r) \frac{1}{\nu} (qh - q\bar{h}) - \gamma\tau(1 + g) \quad (2.79)$$

Differentiating Equation (2.79) with respect to γ :

$$\begin{aligned}
&-(1 + \bar{g})n\tau + (1 + \bar{g}) \left[\frac{1}{\omega}(1 + r) + (1 - \gamma)\tau \right] \frac{dn}{d\gamma} + \left[\frac{1}{\omega}(1 + r) + (1 - \gamma)\tau \right] n \frac{dg}{d\gamma} \\
&= \beta(1 + r) \frac{1}{\nu} q \frac{dh}{d\gamma} - \tau(1 + g) \\
&\quad (1 + \bar{g}) \left[\frac{1}{\omega}(1 + r) + (1 - \gamma)\tau \right] \frac{dn}{d\gamma} + \left[\frac{1}{\omega}(1 + r) + (1 - \gamma)\tau \right] n \frac{dg}{d\gamma} \\
&= \beta(1 + r) \frac{1}{\nu} q \frac{dh}{d\gamma} - \tau(1 + g) + (1 + \bar{g})n\tau
\end{aligned} \quad (2.80)$$

From Equation (2.37):

$$\begin{aligned}
n \left(z - \frac{(1-\gamma)\tau(1+\bar{g})}{(1+r)} + \frac{\delta}{\omega}(1+\bar{g}) + \delta z \right) &= \delta(1 - qh - \tau) \\
n \left(z(1+\delta) - \frac{(1-\gamma)\tau(1+\bar{g})}{(1+r)} + \frac{\delta}{\omega}(1+\bar{g}) \right) &= \delta(1 - qh - \tau) \\
(1+\bar{g})n \left(\frac{\delta}{\omega} - \frac{(1-\gamma)\tau}{(1+r)} \right) &= \delta(1 - qh - \tau) - z(1+\delta)n \\
(1+\bar{g})n \left(\frac{\delta}{\omega} - \frac{(1-\gamma)\tau}{(1+r)} \right) &= \delta(1 - qh - \tau - zn) - zn \quad (2.81)
\end{aligned}$$

Substitute Equation (2.72) into right hand side of Equation (2.81),

$$\begin{aligned}
(1+\bar{g})n \left(\frac{\delta}{\omega} - \frac{(1-\gamma)\tau}{(1+r)} \right) &= \delta \frac{1}{\nu} \left(qh - q\bar{h} + \nu \frac{1}{\omega}(1+\bar{g})n \right) - zn \\
(1+\bar{g})n \left(\frac{\delta}{\omega} - \frac{(1-\gamma)\tau}{(1+r)} \right) &= \delta \frac{1}{\nu} q(h - \bar{h}) + \delta \frac{1}{\nu} \nu \frac{1}{\omega}(1+\bar{g})n - zn \\
(1+\bar{g})n \left(\frac{\delta}{\omega} - \frac{(1-\gamma)\tau}{(1+r)} \right) - \delta \frac{1}{\nu} \nu \frac{1}{\omega}(1+\bar{g})n &= \delta \frac{1}{\nu} q(h - \bar{h}) - zn \\
(1+\bar{g})n \left(\frac{\delta}{\omega} - \frac{(1-\gamma)\tau}{(1+r)} \right) - \delta \frac{1}{\omega}(1+\bar{g})n &= \delta \frac{1}{\nu} q(h - \bar{h}) - zn \\
(1+\bar{g})n \left(-\frac{(1-\gamma)\tau}{(1+r)} \right) &= \delta \frac{1}{\nu} q(h - \bar{h}) - zn \\
(1+\bar{g})n \left(\frac{(1-\gamma)\tau}{(1+r)} \right) &= \alpha n - \delta \frac{1}{\nu} q(h - \bar{h}) \\
\delta \frac{1}{\nu} q(h - \bar{h}) &= \alpha n - (1+\bar{g})n \left(\frac{(1-\gamma)\tau}{(1+r)} \right) \quad (2.82)
\end{aligned}$$

From Equation (2.37),

$$\begin{aligned}
\delta(1 - qh - \tau) &= n \left(z - \frac{(1-\gamma)\tau(1+\bar{g})}{(1+r)} + \frac{\delta}{\omega}(1+\bar{g}) + \delta z \right) \\
\delta(1 - qh - \tau) &= nz - n \frac{(1-\gamma)\tau(1+\bar{g})}{(1+r)} + \frac{\delta}{\omega}(1+\bar{g})n + \delta zn \\
\delta(1 - qh - \tau) &= nz - (1+\bar{g})n \frac{(1-\gamma)\tau}{(1+r)} + \frac{\delta}{\omega}(1+\bar{g})n + \delta zn \quad (2.83)
\end{aligned}$$

Substitute Equation (2.82) in right hand side of Equation (2.83),

$$\delta(1 - qh - \tau) = \delta \frac{1}{\nu} q(h - \bar{h}) + \frac{\delta}{\omega}(1+\bar{g})n + \delta zn \quad (2.84)$$

Differentiating Equation (2.84) with respect to γ , we get:

$$\frac{\delta}{\omega} n \frac{d\bar{g}}{d\gamma} + \frac{dn}{d\gamma} \left(\frac{\delta}{\omega} (1 + \bar{g}) + \delta z \right) = -q \frac{dh}{d\gamma} \left(\delta + \delta \frac{1}{\nu} \right) \quad (2.85)$$

From Equation (2.80) and Equation (2.85),

$$\begin{aligned} & \begin{pmatrix} \frac{\delta}{\omega} (1 + \bar{g}) + \delta z & \frac{\delta}{\omega} n \\ (1 + \bar{g}) \left[\frac{1}{\omega} (1 + r) + (1 - \gamma) \tau \right] & \left[\frac{1}{\omega} (1 + r) + (1 - \gamma) \tau \right] n \end{pmatrix} \begin{pmatrix} \frac{dn}{d\gamma} \\ \frac{d\bar{g}}{d\gamma} \end{pmatrix} \\ &= \begin{pmatrix} -q \left(\delta + \frac{\delta}{\nu} \right) \frac{dh}{d\gamma} \\ \beta(1 + r) \frac{1}{\nu} q \frac{dh}{d\gamma} - \tau(1 + g) + (1 + \bar{g}) n \tau \end{pmatrix} \end{aligned}$$

Determinant of the above 2 x 2 linear system is given by:

$$\begin{aligned} & \left(\frac{\delta}{\omega} (1 + \bar{g}) + \delta z \right) \left[\frac{1}{\omega} (1 + r) + (1 - \gamma) \tau \right] n - \frac{\delta}{\omega} n (1 + \bar{g}) \left[\frac{1}{\omega} (1 + r) + (1 - \gamma) \tau \right] \\ &= \frac{\delta}{\omega} (1 + \bar{g}) \left[\frac{1}{\omega} (1 + r) + (1 - \gamma) \tau \right] n + \delta z \left[\frac{1}{\omega} (1 + r) + (1 - \gamma) \tau \right] n \\ & \quad - \frac{\delta}{\omega} n (1 + \bar{g}) \left[\frac{1}{\omega} (1 + r) + (1 - \gamma) \tau \right] \\ &= \delta z \left[\frac{1}{\omega} (1 + r) + (1 - \gamma) \tau \right] n \end{aligned}$$

Using Cramer's rule, we have the following:

$$\begin{aligned} \frac{d\bar{g}}{d\gamma} &= \frac{1}{\delta z \left[\frac{1}{\omega} (1 + r) + (1 - \gamma) \tau \right] n} \left(\frac{\delta}{\omega} (1 + \bar{g}) + \delta z \right) \left[\beta(1 + r) \frac{1}{\nu} q \frac{dh}{d\gamma} + \tau(1 + \bar{g}) n - \tau(1 + g) \right] \\ & \quad - \frac{1}{\delta z \left[\frac{1}{\omega} (1 + r) + (1 - \gamma) \tau \right] n} \left(-q \frac{dh}{d\gamma} \left(\delta + \frac{\delta}{\nu} \right) \right) (1 + \bar{g}) \left[\frac{1}{\omega} (1 + r) + (1 - \gamma) \tau \right] \quad (2.86) \end{aligned}$$

$$\begin{aligned} \frac{dn}{d\gamma} &= \frac{1}{\delta z \left[\frac{1}{\omega} (1 + r) + (1 - \gamma) \tau \right]} \left[-q \frac{dh}{d\gamma} \left(\delta + \frac{\delta}{\nu} \right) \left[\frac{1}{\omega} (1 + r) + (1 - \gamma) \tau \right] \right] \\ & \quad - \frac{1}{\delta z \left[\frac{1}{\omega} (1 + r) + (1 - \gamma) \tau \right]} \frac{\delta}{\omega} \left[\beta(1 + r) \frac{1}{\nu} q \frac{dh}{d\gamma} + \tau(1 + \bar{g}) n - \tau(1 + g) \right] \quad (2.87) \end{aligned}$$

$$\begin{aligned} \frac{dn}{d\gamma} \left(\delta z \left[\frac{1}{\omega}(1+r) + (1-\gamma)\tau \right] \right) &= -q \frac{dh}{d\gamma} \left(\delta + \frac{\delta}{\nu} \right) \left[\frac{1}{\omega}(1+r) + (1-\gamma)\tau \right] \\ &\quad - \frac{\delta}{\omega} \left[\beta(1+r) \frac{1}{\nu} q \frac{dh}{d\gamma} + \tau(1+\bar{g})n - \tau(1+g) \right] \end{aligned} \quad (2.88)$$

Differentiating Equation (2.41),

$$\frac{dh}{d\gamma} = \frac{\nu \frac{\tau(1+g)}{(1+r)}}{(1+\nu+\delta+\beta)q} \quad (2.89)$$

Substitute Equation (2.89) and Equation (2.77) into Equation (2.88):

$$\begin{aligned} \frac{dn}{d\gamma} \left(\delta z \left[\frac{1}{\omega}(1+r) + (1-\gamma)\tau \right] \right) &= -q \frac{\nu \frac{\tau(1+g)}{(1+r)}}{(1+\nu+\delta+\beta)q} \left(\delta + \frac{\delta}{\nu} \right) \left[\frac{1}{\omega}(1+r) + (1-\gamma)\tau \right] \\ &\quad - \frac{\delta}{\omega} \left[\beta(1+r) \frac{\frac{\tau(1+g)}{(1+r)}}{(1+\nu+\delta+\beta)} - \tau(1+g) \right] \\ &\quad - \frac{\delta}{\omega} \tau \frac{\beta(1+r)(1-\tau) - \beta(1+r)hq - \gamma\tau(1+g)(1+\nu+\delta)}{(1+\nu+\delta+\beta) \left[\frac{1}{\omega}(1+r) + (1-\gamma)\tau \right]} \end{aligned} \quad (2.90)$$

From Equation (2.90), setting $\frac{dn}{d\gamma} = 0$, we have:

$$\begin{aligned} 0 &= -\frac{\nu \frac{\tau(1+g)}{(1+r)}}{(1+\nu+\delta+\beta)} \left(\delta + \frac{\delta}{\nu} \right) \left[\frac{1}{\omega}(1+r) + (1-\gamma)\tau \right] \\ &\quad - \frac{\delta}{\omega} \left[\beta(1+r) \frac{\frac{\tau(1+g)}{(1+r)}}{(1+\nu+\delta+\beta)} - \tau(1+g) \right] \\ &\quad - \frac{\delta}{\omega} \tau \frac{\beta(1+r)(1-\tau) - \beta(1+r)hq - \gamma\tau(1+g)(1+\nu+\delta)}{(1+\nu+\delta+\beta) \left[\frac{1}{\omega}(1+r) + (1-\gamma)\tau \right]} \end{aligned} \quad (2.91)$$

Simplifying Equation (2.91), we have:

$$1+g = \frac{\frac{\delta}{\omega}\beta(1+r)[(1-\tau)-hq]}{\left(\frac{\delta}{\omega}(1+\nu+\delta) \left[\frac{1}{\omega}(1+r) + \tau \right] - \nu \frac{1}{(1+r)} \left(\delta + \frac{\delta}{\nu} \right) \left[\frac{1}{\omega}(1+r) + (1-\gamma)\tau \right]^2 \right)} \quad \blacksquare$$

3 ESSAY THREE: AN EMPIRICAL STUDY ON THE DETERMINANTS OF FERTILITY IN SIN- GAPORE

3.1 Introduction

In this essay, I explore two important factors, namely the expenditure of private consumption and the cost of living, in the decline in fertility in Singapore and seek to reconcile the results with existing theories on fertility.

According to Lindert (1978), rising cost of children is the main reason for the decline in fertility in the U.S. since the 1900s. Others examine the trade-off in quality and quantity for children but find little or mixed evidence for it (Doepke, 2015). Based on the Autoregressive Distributive Lag (ARDL) model, results show that Consumer Price Index (CPI) and literacy rate are the two variables significant in causing the decline in fertility in Singapore.

Most developed countries face low fertility rates, resulting in the shrinking of a young and vibrant workforce and the increase in the old age dependency ratio. This can lead to the dilution of a country's national identity and culture due to an over reliance on immigrants and the incorporation of new citizens. Currently, Singapore's total fertility rate (TFR) is at 1.2, far below the replacement rate of 2.1. As such, it is important to understand the cause of this decline.

Among the empirical studies done on fertility, some have included income variables such as real Gross Domestic Product (GDP) per capita but not cost variables such as price (Narayan, 2006; Cigno and Rosati, 1992; Masih and Masih, 2000; Narayan and Peng, 2006) while others have included cost variables but not income variables (Yi and Zhang, 2010). In this study, I include price, GDP and private consumption expenditure in my model. These variables typically represent the constraints and preferences of a household; from them, one may be able to infer how households make decisions regarding having children. In many developed countries today, factors such as infant mortality and life expectancy may no longer be relevant as to why people choose to have fewer babies since low infant mortalities and high life expectancies are the norms. Rather, factors such as rising cost of living, old age security, social norms, expectations on children and the

decision-making process of couples in having children can be more important.

Subramaniam et al. (2015) examine Singapore's fertility rate using variables such as infant mortality rate, GDP and female participation rate and show that none of the estimates are significant in the long run. Female participation rate is likely a consequence of rising education level and literacy rate among females in Singapore.

The rest of this essay is organised as follows: Section 3.2 gives the relevant background knowledge for the Becker's quality-quantity trade-off theory. Section 3.3 examines the relationship between education and fertility. Section 3.4 summarises many of the other theories and determinants of fertility. Section 3.5 gives an overview of Singapore's macro-economic data and the changes in Singapore's fertility policies over the years. Section 3.6 gives the empirical model and methodology of this study, Section 3.7 presents the results and Section 3.8 concludes this essay.

3.2 Becker's Quality-Quantity Trade-off Model

The economic approach¹ to the demand for children is to consider them as consumption goods. This approach differs from those by demographers and sociologists by assuming that the preference for children is constant over time (Doepke, 2015). In the economic model, the increase in the price of children decreases the demand for children while the effect of the change in income on the demand for children is ambiguous (Hotz et al, 1997, p. 293).

Becker (1960) introduces quality of children, defined as real expenditure per child, as one of the criteria when deciding the number of children to have. He concludes that fertility is determined by "income, *child costs*, knowledge of contraception, uncertainty in the production of children, and tastes" (p. 231). A formal analysis by Becker and Lewis (1973) show that the quality and quantity of children are more closely related relative to other goods because the greater the number of children, the higher the marginal cost or

¹This is also known as the Chicago model, the New Home Economics Theory or the Economic Theory of Fertility (ETF).

shadow price of child quality and vice versa. Subsequently, time cost (Becker, 1965) and other extensions are added to the model. Based on Becker's work on social interactions (1974), Becker and Tomes (1976) extend the model to include endowment factors such as genes or inherent ability, public education, i.e. elements not within the control or choice of parents that may affect child quality. After that, endogenous fertility growth models are developed. The first endogenous fertility growth model is introduced by Becker and Barro (1988). Based on Becker et al. (1990), growth is linked to human capital formation (Zhang, 2003b). The human capital model is first and formally developed by Becker and Tomes (1976).

3.2.1 Taste for Children: Exogenous or Endogenous?

Leibenstein (1975),² in support for his earlier work (Leibenstein, 1957), critiques the Becker model, "If a population has homogenous tastes (i.e., not dependent on status or "lifestyle"), it is implausible for the cost of children, especially those attributed to "quality" or mother's time, to account for the inverse relation between family size and income level, given the larger income differentials that exist and the income growth that occurs in development countries" (p. 3). He introduces the concept of social status based consumption which involves the consumption of items like "wealth, occupation, housing, education, residential location, political or military power, and hierarchical and other titles" (p. 5). Under different economic circumstances and social standings, choices made by the *same person* will be different and people will have different preferences for children.

²In terms of choices, Duesenberry comments that Leibenstein's approach is "more sociological" compared to Becker's (Becker, 1960, p. 233). Subsequently, Becker accounts for the changes in 'taste' or preference in his other works (Stigler and Becker, 1977; Becker, 1996; Becker and Mulligan, 1997). See Easterlin's theories in Section 3.4 as well.

3.2.2 Quality of Children

While Becker seeks to maximise child quality through real expenditure per child, often the production function of child quality is more complicated, due to the difficulties in separating the different effects such as family influences and education (Hanushek 1992, p. 85). Human capital, used as a measure of child quality, is determined by parents' IQ and education, quality and quantity of time and good inputs and child's IQ and education; the income produced by the child measures his or her quality (Leibowitz, 1974). Robinson (1987) examines the time cost of children and the relative time cost between husband and wife. According to Guryan et al. (2008), time spent on children can be classified into four categories: "basic, educational, recreational and travel" (p. 24). These categories can be extended to the expenditure on children as well.

3.2.3 Expenditure and Cost of Children

Duesenberry questions the separation of children and parental consumption: "But in many respects the standard of living of the children is mechanically linked to that of the parents. Is it possible to have crowded housing conditions for the children and uncrowded conditions for the parents" (Becker, 1960, p. 234)? Hotz et al. (1997) highlight the problems in defining the expenditures of children; they include identifying the expenditures that belong to them and the separation of expenditures into real expenditures and prices (p. 298). Being fully aware of these issues, Lindert (1978) estimates the cost of a child using an "index of relative cost rather than an absolute cost" (p. 15).

In estimating the cost of children, Browning (1992) surveys two methods to calculate the cost of children: according to the needs of children and according to the actual expenditure. By the method of equivalent scales, income is deflated to compare households with different income and consumption patterns. An indirect method is to regress household expenditure against the number of children and other variables. From the U.S. Department of Agriculture (USDA) data, Lino (2014) computes the expenditures for ma-

jor budget components spent directly on children until they reached the age of 17 but excludes children's college expenses and the opportunity costs of parents. Folbre (2008) estimates the public and private costs of children and highlights some of the problems in the existing work using the method of equivalent scales.

Quality must be controlled in examining the cost of children. Helburn and Howes (1996) show that the variation of prices in child care centres can be due to the difference in the quality of centres.

3.3 Education and Fertility

While fertility may be reduced through education by increasing the quality of children (Leibowitz, 1974), there are other ways by which education may affect fertility.³

Female education affects fertility indirectly, directly and jointly with other exogenous variables (Kasarda, 1979). Education affects fertility indirectly by increasing the prospects of women working outside and their opportunity cost of having children, allowing them to find alternative satisfaction or means to such satisfaction through financial remuneration; this results in the increase in female labour force participation rates, delays in marriages, the lowering of the complete fertility rate and the reduction in the need and desire to depend on their children for old age. Education also reduces the perceived economic utility derived from having children due to the ability to assess the costs and benefits of having children better, increases the exposure to issues concerning family planning and existing social norms through the mass media, improves the communication and bargaining power in women and reduces infant and child mortality. The direct effects of education are those that cause women to prefer smaller families through social psychological factors based on values, attitudes and sentiments. Joint effects are those that interact with other exogenous factors such as religion, urban-rural areas and husband's education.

According to Narayan and Peng (2007), all the reasons for the decline in fertility through

³See Cochrane (1979) for a comprehensive review on fertility and education.

education can be classified under "the taste hypothesis, the cost-of-fertility-control hypothesis and the cost-of-time hypothesis" (p. 267). Bratti (2003) includes two other factors by which fertility is affected by education: the duration of marriage and non-marital fertility.

3.4 Other Theories and Determinants of Fertility

According to Masih and Masih (2000), all reasons for the decline in fertility can be classified under two hypotheses: the structural hypothesis and ideational hypothesis.

Determinants that are due to external factors such as infant mortality, social economic status, life expectancy, social security, wealth and income all fall under the structural hypothesis. This includes theories like those of Becker and Leibenstein (pp. 1618-1619). Shenk et al. (2013) classify some of them into three groups of models: the risk and mortality models, the economic and investment models, and the cultural transmission models.

Ideational hypothesis refers to anything that can cause a change in ideas, perceptions, values, attitudes, mindsets or culture. Hence, mass education, communication of government policies and preferences, organised family planning and social networks all fall under this hypothesis. While there are mutual dependencies between the two hypotheses, what researchers are interested in is the emphasis - which is more important and whether there is a "prerequisite" for one in order for the other to take place during a fertility transition (p. 1618).

Easterlin (1961) contributes to the theories of fertility with factors such as unemployment and cohort size. Some of his other work complements Becker's by formulating the taste for children with factors such as religion, culture, material aspirations, relative income of spouses, relative income of different generations and one's childhood material consumption. (Easterlin, 1969, 1973, 1976, 1978a, 1978b).

According to Easterlin (1976), there are three concepts of fertility: desired fertility, nat-

ural fertility, and optimal fertility. Desired fertility refers to the number of children a couple wants. Natural fertility refers to the number of children a couple can have, and optimal fertility refers to the actual number of children a couple has after being subjected to various preferences, constraints and scientific advancements. As a country develops, natural fertility has risen but both desired and optimal fertility have fallen.

In contrast to static models of fertility, life cycle models focus on the lifetime consumption instead of a single period or current consumption of individuals; this introduces factors such as savings rate, risk preferences and commitment levels, social security, timing of fertility, delays in marriages, health care, longevity, and income uncertainty (Caucutt et al., 2002; Schmidt, 2008; Nugent, 1985; Zhang and Zhang, 2005; Cochrane, 1988; Sommer, 2016; Cordoba and Ripoll, 2016). Some others examine the effects of inequality and status competition on fertility (Shenk et al., 2016; Cordoba et al., 2016).

Another prominent theory is the value of children (VOC) approach. Also known as the supply side of children model, this approach states that the preference for children is derived from their net values, i.e. benefits minus costs. The resultant or actual fertility is determined by the parents' ability and desired or preferred number of children (Thomson, 2015). Based on the Becker model, the cost of children belongs to the budget constraint and does not have any influence on the preference for children while the benefit is the total utility derived from children.

3.5 Macroeconomic Data and Fertility Policies between 1960 and 2015

3.5.1 Fertility Policies Between 1960 and 2015

The fertility stance of the Singapore government can be divided into four phases.⁴

Between 1966 – 1982, just after its independence in 1965, to facilitate growth and de-

⁴See Wong and Yeoh (2003) for more details.

velopment, Singapore adopted several anti-natal policies to control birth rates. Those policies include financial disincentives such as higher delivery fees for the second child onwards, the lack of tax exemption from the third child onwards and the reduced chances of getting subsidised housing. Abortion and voluntary sterilization also became legal.

A partial switch in position came in 1983. Prime Minister Mr Lee Kuan Yew noted that the census data for 1980 "showed that women with little or no formal schooling had an average of 3.5 children, while university-educated women had 1.6 children" (Lee et al., 1991, p. 66). During the 'Eugenics Phase' between 1983 – 1986, the Singapore government tried to encourage female university graduates to marry other graduates and have three children or more by giving them better tax benefits and their children priority to enter primary school. To discourage the less educated from having children, \$10,000 was given to all women without an 'O' Level qualification who underwent sterilisation.

The pro-natal period started with the slogan 'Have Three Or More Children If You Can Afford It' in 1987. Some of those the benefits include up to 4 years of no pay leave for childcare, childcare subsidies, special tax rebates and priority in admission to primary schools for all families with three or more children regardless of education levels. No longer was there a focus on only those with higher academic qualifications to have more children.

In 2001, the government introduced the Baby Bonus Scheme. For the first time, money was given out to encourage couples to have children and there had been four revisions ever since, in 2004, 2008, 2012, and 2015, with several enhancements made to the cash entitlements and tax reliefs. To encourage couples to save for their children in their children's accounts, a one-for-one government matching scheme called the Child Development Account was introduced in 2016.

3.5.2 Overview of Data and Analysis of Structural Breaks

The annual time series dataset available for this study is from 1960 - 2015.⁵ Details and definitions of all the variables can be found in the Appendix, Table 3.10.

Graph of TFR, savings rate, private consumption and rate of private consumption

Figure 3.1

Figure 3.2

Figure 3.3

Figure 3.4

Figures 3.1, 3.2, 3.3 and 3.4 show the total fertility rate (TFR), the savings rate, the real private consumption expenditure per resident and the rate of private consumption expenditure for Singapore.⁶ From Figure 3.1, there is an increase in fertility in 1976, 1988, 2000 and 2012, during the ‘dragon years’ according to the lunar calendar. Fertility rates also increased after the introduction of the pro-natal policies in 1987. However, it continued to decrease subsequently and hit the low point of 1986 in 2001. Savings rate and real private expenditure consumption have been steadily rising while the rate of consumption has been falling.

According to the Bai and Perron (1998) endogenous multiple structural break test and

⁵Only the Consumer Price Index (CPI) series starts one year later from 1961.

⁶Private consumption expenditure comprises of items such as food, clothing, housing and utilities, transport, recreation, education, etc. Rate of private consumption is the real private consumption expenditure divided by the real Gross Domestic Product (GDP). Savings rate is the nominal gross domestic savings divided by the nominal GDP. More details can be found in Table 3.10 of Appendix.

Table 3.1: Bai and Perron Multiple Break Tests

Variable	Number of Break(s)	Schwarz Criterion	Estimated Break Date(s)
TFR	3	-2.0086	1968, 1976, 2001
	4	-1.8915	1968, 1976, 1984, 2001
CPI	5	3.4729	1973, 1981, 1991, 1999, 2008

Note: Schwarz Criterion for 3 breaks is the lowest for TFR, followed by 4 breaks. Schwarz Criterion for 5 breaks is the lowest for CPI.

Figure 3.5**Figure 3.6**

the use of the minimum Schwarz Information Criterion (SIC) as proposed by Liu et al. (1997), there are 3 structural breaks, in 1968, 1976 and 2001, for TFR, as shown in Table 3.1. The second lowest SIC score indicates 4 structural breaks with an additional break in 1984. The breaks coincide with the start of the anti-natal period in 1966 and the introduction of the baby bonus scheme in 2001. 1976 is the auspicious year of the dragon and the additional structural break in 1984 coincides closely with the ‘Eugenics Phase’ that started in 1983.

Figures 3.5 and 3.6 show the time series for both CPI and literacy rate.⁷ In Figure 3.5, there is a structural break in 1973 (also identified by the structural break test in Table 3.1). This is due to the global oil crisis which caused inflation rate to rise from 2 percent

⁷Literacy rate refers to the proportion of resident population aged 15 years and over with the ability to read with understanding.

in 1972 to 20 and 22 percent in 1973 and 1974 respectively. Figure 3.6 shows a smooth trend in Singapore's literacy rate increasing over the years.

3.6 Empirical Model and Methodology

3.6.1 Model Specification

The following equation models the long run relationship between fertility, price level, income, expenditure and literacy rate:

$$\ln fert(t) = \ln cpi(t) + \ln lit(t) + \ln gdp(t) + \ln con(t) + dum87(t) + dum01(t) \quad (3.1)$$

$\ln fert$ refers to the log of TFR, $\ln cpi$ refers to the log of CPI, $\ln con$ refers to the log of real private consumption expenditure per resident, $\ln lit$ refers to the log of literacy rate and $\ln gdp$ refers to the log of real GDP per resident.⁸

As in Subramaniam et al. (2015), the dummy variable $dum87$ represents the pro-natal era (between 1987 – 2001) and $dum01$ represents the period when the Baby Bonus Scheme is in place. Although the period from 2001 – 2015 falls under the pro-natal era as well, two *mutually exclusive* time dummies are used to distinguish the two periods and allow for two estimates instead of one. $dum87$ indicates '1' from 1987 – 2000 and '0' otherwise and $dum01$ indicates '1' from 2001 – 2015 and '0' otherwise. As the education level of the general population rises over time, the pro-natal policies may not have an effect on the general population compared to the earlier and less educated population.

Masih and Masih (2000) mention that female secondary education serves as a proxy for the ideational hypothesis and real income per capita and female labour force participation serve as proxies for the structural hypothesis. However, changes in female education level can also result in changes in income level (structural hypothesis). In this model,

⁸Per resident differs from per capita since one refers to the resident population and the other refers to the total population. All series are representative of the total resident population instead of the total population.

female labour force participation rate is excluded due to the lack of data. This, however, can be proxied by education level or literacy rate⁹ since it is likely an outcome of female education. Masih and Masih's model specification also includes the consumption of contraceptive (sterilization) which is also not available in the data.

According to Becker (1960), real private consumption per child is the measure of the child quality. However, the real private consumption expenditure variable does not only represent the real private consumption per child; it also represents the consumption of parents which according to Leibenstein and Easterlin can also lead to the decline in fertility. Since it is difficult to identify the expenditures for children, this variable is used as a proxy for personal consumption per person (child or parent).

Other variables used as controls are included in the following model:

$$\begin{aligned} \ln fert(t) = & \ln gdp(t) + \ln cpi(t) + \ln con(t) + \ln lit(t) + dum87(t) + dum01(t) \\ & + \ln mr(t) + \ln old(t) + sr(t) + \ln life(t) \end{aligned} \quad (3.2)$$

$\ln mr$ refers to the log of infant mortality rate, sr refers to the savings rate, $\ln old$ refers to the log of old age dependency ratio and $\ln life$ refers to the log of life expectancy.¹⁰

Since it takes nine months to conceive a child, there is at least a nine months lag from the time a couple decides to have a child. Hence, one period lag regressors are also used and different time periods are used to examine the stability of the coefficients.

3.6.2 Empirical Methodology

The Autoregressive Distributive Lag (ARDL) model is the single equation framework used in this analysis and there are several advantages to it.

⁹As the general population becomes more literate or educated, people are becoming more aware of the social norm of family size. Comparing the time series for literacy rate and average years of schooling (available only from 1980 – 2015), average years of schooling has a steep increase.

¹⁰In this essay, we are only interested in the long run relationship. Therefore, it is not necessary to control for certain years in the lunar calendar such as the year of the 'dragon' or 'tiger' as they will not have any long run effect or permanent shock on fertility.

Firstly, it allows the variables to be tested for cointegration *directly* within the unrestricted Error Correction Mechanism (ECM) framework using the bounds cointegration test (Pesaran et al., 2001). In the bounds cointegration test, all the regressors can be a mixture of $I(1)$ and $I(0)$ variables. Hence, pretesting for unit roots is not required. Pretesting may falsely conclude whether a series is an $I(1)$ variable for near unit root cases where the power of the unit root tests is low. However, none of the variables can be $I(2)$ processes. As opposed to methods such as the Engle-Granger test that exclude the short run dynamics (if they are present), the ARDL bounds test does not force the short run variables into the error terms. The ARDL bounds test allows for dummies variables¹¹ to be included in the cointegrating relationship for testing (Pesaran and Pesaran, 2009, p. 483). While the Vector Error Correction Mechanism (VECM) model also allows dummy variables to be included in the model, it does not allow them to be part of the cointegration equation (Johansen et al., 2000).

The F - and Wald statistics of the bounds test have nonstandard limiting distributions. As such, critical values are computed through simulations and they differ for small sample sizes (Narayan, 2004a, 2004b; Narayan, 2005); the bounds test does not need to depend on asymptotic assumptions which may not hold for finite sample sizes. Narayan (2005) computes the critical values for various sample sizes and they depend on a number of factors: the number of regressors, the sample size, the presence of a trend or intercept term and the number of dummy variables in the model. Microfit 5.0 provides the stimulated critical values based on 20,000 replications. As compared to other popular tests such as the Engle and Granger (1987) and Johansen and Juselius (1990) multi-cointegration tests, the bounds test has better small sample properties (Narayan and Smyth, 2005). Many studies show that the small sample properties of the trace and likelihood ratio tests (Johansen, 1988, 1996) differ from their asymptotic properties (Cheung and Lai, 1993; Toda, 1995); hence, small sample correction is proposed for the cointegration rank test (Johansen, 2002).

¹¹Because of the presence of dummy variables in the cointegrating relationship, other recommended tests *in addition* to the ARDL bounds tests as the ECM t -test (Banerjee et al., 1998) or the bounds t -test (Pesaran et al., 2001) cannot be done due to the lack of critical values.

Next, the ARDL approach models the dynamics of the effects of the explanatory variables by including the lag dependent variable as one of the regressors. For most variables, their effects do not happen immediately. Hence, by including the lag dependent variables, it allows the effects of the *independent* variables to be distributed over time. ARDL model can also be transformed into the ECM model to examine the long and short run effects of the model.

Lastly, the standard t -tests can be used to test the significance of the variables. Pesaran and Shin (1998) show that the test statistic in the ARDL model has an asymptotic normal distribution.¹²

3.7 Empirical Results

3.7.1 Unit Root Tests

To ensure that none of the variables are $I(2)$ variables, unit root tests (with and without trend) are done. If a series is stationary after taking the first difference, it cannot be an $I(2)$ process. The tests used are the Dickey-Fuller Generalized Least Squares (DF-GLS), the Philips-Perron (PP) and the Kwiatkowski, Philips, Schmidt, and Shin (KPSS). The PP test has low power if the process is stationary but has a root close to 1. Therefore, both the stationary (KPSS) and unit root tests (DF-GLS and PP) are applied to the series. Elliott et al. (1996) show that the DF-GLS test has greater power compared to the Augmented Dickey-Fuller (ADF) test.

From Table 3.2, all tests clearly indicate that the variables are $I(1)$ series¹³ except for $\ln lit$. $\ln lit$ fails all three tests after first differencing, indicating that it may be an $I(2)$

¹²This is under the condition that the coefficient of the lag dependent variable is between -1 and 1.

¹³In the presence of structural break(s), unit root tests may wrongly conclude a series to be an $I(1)$ variable when it is an $I(0)$ process. Hence, unit root tests with the allowance of one structural break (Perron, 1989; Vogelsang and Perron, 1998) are done for both $\ln fert$ and $\ln cpi$ because of the structural breaks that are identified previously. The tests indicate both series to be $I(1)$ processes.

Table 3.2: Unit Root Tests

Variable	DF-GLS Statistic		PP Statistic		KPSS Statistic	
	Trend	No Trend	Trend	No Trend	Trend	No Trend
$\ln fert$	-0.707	0.934	-1.597	-2.986	0.212**	0.887***
$\ln cpi$	-1.878	0.154	-1.316	-1.209	0.215**	0.964***
$\ln gdp$	-0.631	1.008	-0.772	-1.808	0.245***	1.02***
$\ln con$	-1.188	1.874	-1.219	-1.073	0.192**	1.02***
$\ln lit$	-1.66	0.398	-4.653***	-14.451***	0.256***	0.925***
$\ln lit - \ln cpi$	-3.623**	-0.188	-2.361	-0.275	0.125*	0.957***
$\Delta \ln fert$	-5.573***	-4.395***	-7.583***	-6.991***	0.0958	0.422*
$\Delta \ln cpi$	-4.897***	-2.656***	-4.094**	-4.036***	0.0741	0.18
$\Delta \ln gdp$	-5.223***	-4.749***	-5.844***	-5.557***	0.0593	0.329
$\Delta \ln con$	-5.893***	-4.922***	-5.364***	-5.361***	0.0792	0.177
$\Delta \ln lit$	-1.200	0.488	-1.826	-0.822	0.222***	0.89***
$\Delta(\ln lit - \ln cpi)$	-4.852***	-2.438**	-4.067**	-4.134***	0.0804	0.0813

Notes: Δ represents first difference of variable. Number of lags for DF-GLS model is chosen with minimum SIC criteria. PP statistic reported here is the Z_τ statistic. Number of lags are chosen according to $\text{int}\{4(T/100)^{(2/9)}\}$ where T is sample size, and int represents integer value. For KPSS model, automatic bandwidth selection procedure proposed by Newey and West (1994) as described by Hobijn et al. (1998, 7) is used. *, ** and *** represent significance at 10, 5 and 1% levels, respectively.

process.¹⁴ Before differencing, the PP statistics show that $\ln lit$ is stationary.¹⁵ From Figure 3.6, since $\ln lit$ has a smooth graph, it is possible for $\ln lit$ to have a quadratic trend (Gonzalo and Lee, 1998).¹⁶

For any two series with different orders of integration, the difference of the two series will have the higher order of integration of the two series. For example, if $\ln cpi$ is an $I(1)$ or $I(0)$ series and $\ln lit$ is an $I(2)$ series, the difference of the two must be an $I(2)$ process. As in Kurita (2013), here, I take the difference between $\ln lit$ and $\ln cpi$. Figure 3.7 shows the plot of $\ln lit - \ln cpi$ and Table 3.2 includes the test results for $\ln lit - \ln cpi$. Since $\ln lit - \ln cpi$ is indicated as an $I(1)$ series according to the tests, $\ln lit$ cannot be an $I(2)$ series.¹⁷

¹⁴For examples of $I(2)$ series, see Gonzalo and Lee (1998). To test whether a series is an $I(2)$ process, see Haldrup (1998).

¹⁵For the PP test, two statistics, Z_ρ and Z_τ are reported by the statistical software package Stata. Before differencing, Z_ρ indicates that $\ln lit$ has a unit root while Z_τ indicates that $\ln lit$ is stationary. After differencing, both show that $\Delta \ln lit$ is non-stationary. The one reported in Table 3.2 is Z_τ .

¹⁶Gonzalo and Lee note that it is hard to differentiate “between an $I(2)$ series and $I(1)$ series with drift” (pp. 139-140). If a series has a quadratic trend, the linear deterministic trend will remain after first differencing.

¹⁷In the univariate model, when $\ln lit$ is regressed against its own lag and the quadratic trend, the coefficient is estimated to be 1.01. Hence, to include the quadratic trend seems appropriate for the series.

Figure 3.7

3.7.2 Main Results

All variables except savings rate are in the logarithmic form. Real private consumption and real GDP per resident shall be referred to as private consumption and GDP respectively. Table 3.3 shows the results for the long run estimates when the control variables are included, and Table 3.4 gives the results when the one period lag regressors are used. Table 3.3 shows that only literacy rate and CPI are significant at the 1 and 10 percent levels respectively for all the regressions. GDP and private consumption are not significant for any of the regressions. From Table 3.4, literacy rate is also significant at the 1 percent level for all the regressions, CPI is significant for all except regression (4),¹⁸ and GDP and private consumption are significant for some regressions. From the two tables, the coefficient estimates for literacy rate and CPI and the two dummy variables are relatively stable compared to those for GDP and private consumption; their absolute values are large for Table 3.4 compared to Table 3.3 for regressions (1) to (5).

Since all the regressions show that there is cointegration according to the ARDL bounds

¹⁸The absolute value of the estimate for CPI in regression (4) is small compared to the other regressions. When savings rate is included as a control variable in regressions (4) and (5), the absolute values of the estimates for literacy rate are also smaller.

test, the long run coefficient estimates are super-consistent. From the two tables, regressions (7) and (8) show that omitting either private consumption or GDP from the model will result in their estimates to become different from their own estimates in the other regressions. This suggests that either the two variables are highly collinear or omitting one results in the omitted variable finite sample bias. In regression (8) of Table 3.3, the coefficient estimate of private consumption is positive which is a wrong sign. The absolute values of the estimates for both GDP and private consumption are very small for regressions (7) and (8) of Table 3.4.

The significances of the estimates for GDP and private consumption in Table 3.4 are due to the large absolute values compared to Table 3.3. Dummy 1987 is significant at the 1 percent level for all regressions in both tables, indicating that the government's pro-fertility stance has worked in increasing the number of birth rates in Singapore. However, Dummy 2001 is not significant for all the regressions in Table 3.3 and some in Table 3.4. Perhaps, either the monetary incentives are not sufficient or they are ineffective in encouraging couples to have more children (Botev, 2015).

Since none of the controls from Tables 3.3 and 3.4 are significant, a more parsimonious model is examined with just the following variables: literacy rate, CPI, GDP, private consumption and the two dummy variables.¹⁹ After excluding the controls, regressions (1) - (5) and (6) - (10) of Table 3.5 gives the estimates of the "no lag" and one period lag regressors respectively. Coefficient estimates for literacy rate and CPI are relatively stable and close to -2.8 and -0.20 respectively. Comparing the coefficient estimate of private consumption in regression (9) and its estimate in regression (8) of Table 3.4, the estimate in Table 3.5 is large, increasing by 61.7 percent, and is close to its other estimates in Table 3.3 and Table 3.5. This demonstrates the possibility of multicollinearity effects when the control variables are added in Table 3.4. Literacy rate is significant at the 1 percent level for all regressions and CPI is also significant for most regressions.²⁰

¹⁹Since the estimates for life expectancy are only significant in Table 3.4, I have excluded it as well.

²⁰Except for regressions (4) and (9), the rest of its estimates are significant. The p-value for regression (4) is 0.115 or 11.5 percent. The absolute value of the estimate in regression (9) is small compared to the others and regression (9) does not pass the functional form test.

Table 3.3: Long Run Estimates with No-lag Regressors

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
<i>I. Cointegration test</i>								
ARDL bounds test	10.031**	7.014**	8.057**	7.411**	8.512**	8.634**	8.182**	8.035**
<i>II. ARDL estimates</i>								
Literacy rate	-2.947 (.000)***	-2.956 (.000)***	-2.955 (.000)***	-2.952 (.000)***	-2.974 (.000)***	-3.100 (.000)***	-3.015 (.000)***	-3.040 (.000)***
CPI	-0.301 (.042)**	-0.283 (.080)*	-0.394 (.040)**	-0.309 (.061)*	-0.402 (.038)**	-0.413 (.029)**	-0.402 (.037)**	-0.401 (.040)**
GDP	0.333 (.128)	0.184 (.498)	0.223 (.382)	0.333 (.216)	0.224 (.445)		0.125 (.357)	
Private consumption	-0.248 (.389)	-0.161 (.625)	-0.134 (.698)	-0.259 (.450)	-0.158 (.648)			0.111 (.518)
Dummy 1987	0.236 (.000)***	0.246 (.000)***	0.219 (.000)***	0.235 (.000)***	0.220 (.000)***	0.235 (.000)***	0.211 (.000)***	0.210 (.000)***
Dummy 2001	0.0829 (.370)	0.0625 (.520)	0.0401 (.708)	.0811 (.412)	0.0409 (.709)	0.0195 (.827)	0.0238 (.806)	0.0231 (.830)
Infant mortality rate		-0.0239 (.798)	0.0243 (.809)	-0.0117 (.901)		0.00824 (.921)	0.0360 (.705)	0.0324 (.749)
Old age dependency ratio		0.197 (.360)				0.277 (.085)*		
Life expectancy			1.249 (.345)		1.154 (.358)	1.573 (.185)	1.410 (.257)	1.659 (.179)
Savings rate				0.0133 (.980)	0.0475 (.931)	0.214 (.637)	0.0938 (.856)	0.227 (.636)
Intercept	13.749 (.000)***	14.009 (.000)***	8.8208 (.114)	13.926 (.000)***	9.583 (.051)*	8.466 (.100)*	8.098 (.126)	7.316 (.178)
<i>III. Diagnostic tests</i>								
LM test for serial correlation	0.0451 (.832)	0.0297 (.863)	0.438 (.508)	0.0425 (.837)	0.349 (.555)	0.674 (.412)	0.323 (.570)	0.234 (.629)
Ramsey's RESET test	0.345 (.557)	1.564 (.211)	0.564 (.453)	0.345 (.557)	0.431 (.512)	0.0000552 (.994)	0.708 (.400)	0.689 (.406)
Normality test	0.0122 (.994)	0.0293 (.985)	0.110 (.947)	0.0102 (.995)	0.0884 (.957)	0.0253 (.987)	0.277 (.870)	0.310 (.856)
Heteroskedasticity test	0.306 (.580)	0.301 (.583)	0.437 (.509)	0.336 (.562)	0.482 (.487)	0.526 (.468)	0.355 (.551)	0.395 (.530)

Notes: All variables are in logarithmic form except for the dummy variables and savings rate. The optimal lag structure for ARDL model is chosen using the minimum SIC criterion. Test statistics for the bounds test are compared with the bounds critical values. Number in parentheses are p-values. *, ** and *** represent significance at 10, 5 and 1% levels, respectively. Finite sample critical values of bounds test are computed by stochastic simulations using 20,000 replications. Only 90 and 95% bounds critical values are reported by Microfit 5.0.

Table 3.4: Long Run Estimates with Lag Regressors

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
<i>I. Cointegration test</i>								
ARDL bounds test	11.773**	6.290**	9.264**	6.732**	9.715**	9.693**	9.704**	9.959**
<i>II. ARDL estimates</i>								
Literacy rate	-2.778 (.000)***	-2.672 (.000)***	-2.700 (.000)***	-2.343 (.000)***	-2.387 (.000)***	-2.646 (.000)***	-2.576 (.000)***	-2.506 (.000)***
CPI	-0.287 (.007)***	-0.225 (.070)*	-0.329 (.011)**	-0.167 (.170)	-0.290 (.026)**	-0.322 (.014)**	-0.321 (.018)**	-0.301 (.031)**
GDP	0.475 (.002)***	0.492 (.032)**	0.252 (.161)	0.629 (.004)***	0.445 (.040)**		0.0437 (.663)	
Private consumption	-0.677 (.002)***	-0.637 (.021)**	-0.383 (.122)	-0.768 (.007)***	-0.582 (.026)**			-0.0742 (.571)
Dummy 1987	0.286 (.000)***	0.264 (.000)***	0.241 (.000)***	0.260 (.000)***	0.236 (.000)***	0.208 (.000)***	0.199 (.000)***	0.219 (.000)***
Dummy 2001	0.250 (.001)***	0.181 (.023)**	0.113 (.130)	0.147 (.054)*	0.0923 (.223)	0.00823 (.897)	0.0167 (.810)	0.0577 (.462)
Infant mortality rate		-0.0491 (.483)	0.00650 (.925)	-0.0286 (.681)		0.0686 (.262)	0.0782 (.261)	0.0438 (.559)
Old age dependency ratio		-0.0811 (.688)				0.143 (.287)		
Life expectancy			1.794 (.049)**		1.618 (.056)*	2.503 (.004)***	2.486 (.007)***	2.603 (.005)***
Savings rate				-0.722 (.102)	-0.633 (.124)	-0.332 (.338)	-0.364 (.351)	-0.255 (.489)
Intercept	15.582 (.000)***	14.508 (.000)***	7.143 (.064)*	12.743 (.000)***	6.538 (.049)**	2.495 (.507)	2.123 (.586)	2.379 (.554)
<i>III. Diagnostic tests</i>								
LM test for serial								
correlation	0.120 (.729)	1.408 (.235)	0.000179 (.989)	0.374 (.541)	0.229 (.632)	0.745 (.388)	0.443 (.506)	0.172 (.679)
Ramsey's RESET test	1.592 (.207)	6.040 (.014)**	1.648 (.199)	5.568 (.018)**	2.057 (.151)	0.535 (.465)	0.0999 (.752)	0.528 (.467)
Normality test	0.864 (.649)	1.870 (.393)	1.902 (.386)	4.088 (.130)	3.989 (.136)	1.779 (.411)	1.530 (.465)	1.407 (.495)
Heteroskedasticity test	0.0479 (.827)	0.104 (.747)	0.458 (.499)	0.292 (.589)	0.615 (.433)	1.506 (.220)	1.700 (.192)	1.455 (.228)

Notes: All variables are in logarithmic form except for the dummy variables and savings rate. The optimal lag structure for ARDL model is chosen to ensure no serial correlation in the residuals. Test statistics for the bounds test are compared with the bounds critical values. Number in parentheses are p-values. *, ** and *** represent significance at 10, 5 and 1% levels, respectively. Finite sample critical values of bounds test are computed by stochastic simulations using 20,000 replications. Only 90 and 95% bounds critical values are reported by Microfit 5.0.

Table 3.5: Long Run Estimates

	No-lag Regressors			Lag Regressors						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
<i>I. Cointegration test</i>										
ARDL bounds test	20.065**	17.246**	11.793**	10.944**	10.031**	17.676**	12.717**	9.409**	10.737**	11.773**
<i>II. ARDL estimates</i>										
Literacy rate	-2.884*** (.000)	-2.822*** (.000)	-2.979*** (.000)	-2.866*** (.000)	-2.947*** (.000)	-2.794*** (.000)	-2.750*** (.000)	-2.819*** (.000)	-2.619*** (.000)	-2.778*** (.000)
CPI		-0.244* (.068)	-0.295** (.044)	-0.218 (.115)	-0.301** (.042)		-0.195* (.058)	0.229* (.060)	0.144 (.215)	-0.287*** (.007)
Dummy 1987	0.245*** (.000)	0.253*** (.000)	0.212*** (.000)	0.220*** (.000)	0.236*** (.000)	0.227*** (.000)	0.235*** (.000)	0.224*** (.000)	0.267*** (.000)	0.286*** (.000)
Dummy 2001	0.0949** (.026)	0.144*** (.002)	0.0332 (.644)	0.0475 (.601)	0.0829 (.370)	0.0897** (.011)	0.127*** (.000)	0.102* (.092)	0.199** (.016)	0.250*** (.001)
GDP			0.169* (.096)		0.333 (.128)			0.0422 (.606)		0.475*** (.002)
Private consumption				0.145 (.290)	-0.248 (.389)				-0.12 (.319)	-0.677*** (.002)
Intercept	13.258*** (.000)	14.073*** (.000)	13.202*** (.000)	12.736*** (.000)	13.749*** (.000)	12.853*** (.000)	13.530*** (.000)	13.545*** (.000)	13.863*** (.000)	15.582*** (.000)
<i>III. Diagnostic tests</i>										
LM test for serial correlation	0.0306 (.861)	0.289 (.591)	0.0262 (.871)	0.157 (.692)	0.0451 (.832)	1.140 (.286)	1.334 (.248)	1.295 (.255)	0.945 (.331)	0.120 (.729)
Ramsey's RESET test	0.922 (.337)	0.612 (.434)	0.288 (.591)	0.999 (.318)	0.345 (.557)	0.0286 (.866)	3.415* (.065)	3.096* (.078)	5.671** (.017)	1.592 (.207)
Normality test	0.130 (.937)	0.167 (.920)	0.231 (.891)	0.106 (.948)	0.0122 (.994)	0.157 (.925)	0.948 (.622)	0.951 (.621)	0.848 (.655)	0.864 (.649)
Heteroskedasticity test	0.0404 (.841)	0.118 (.732)	0.108 (.742)	0.108 (.743)	0.306 (.580)	0.0957 (.757)	0.404 (.525)	0.323 (.570)	0.442 (.506)	0.0479 (.827)

Note: See Tables 3.3 and 3.4.

3.7.3 Multicollinearity and Stability of Coefficient Estimates

The trade-off in including more variables is the increase in the size of variance versus the decrease in the bias of the estimates under finite sample size. Multicollinearity is often associated with the loss of power of statistical tests, resulting in the under rejection of the null hypothesis because of the larger variance estimates (Greene, 1990; Johnston, 1984). However, Mela and Kopalle (2002) show that under certain conditions, multicollinearity may also reduce the variance estimates. Hendry (1986) notes that omitting a highly collinear variable when it is supposed to be included in the model may render other variables insignificant; therefore, it is better to start with a more general specification (pp. 207-208).

According to Taylor (2012), in examining the effects of multicollinearity, focusing on the ‘correlated-ness’ of regressors alone is insufficient; one should also look at the overall fit of the model. Through Monte Carlo simulation, based on a t -value of 2, he establishes a ‘rule of thumb’ whereby the largest r^2 value obtained from all the regressions among the regressors should not be larger than the r^2 value of the overall model for multicollinearity to not pose a problem. Taylor’s ‘rule of thumb’ suggests that regressions (5) and (10) in Table 3.5 may have multicollinearity problems when both GDP and private consumption are included in the model.²¹

I compare the coefficient estimates in Table 3.5 with those from three other methods: the Fully Modified Ordinary Least Squares (FMOLS), the Dynamic Ordinary Least Squares (DOLS) and the Ordinary Least Squares (OLS). Only the inference results of the FMOLS and DOLS estimates are reported in the table.²²

According to Table 3.6, in terms of the absolute values for CPI, the FMOLS estimates are small for regressions (7) and (9) and the DOLS estimate is small for regression (9)

²¹The Ordinary Least Squares (OLS) method without the lag dependent regressor is used for this test.

²²The OLS estimates have bias and non-normal asymptotic distributions even when their estimates are super-consistent (Stock, 1987). Therefore, inferences cannot be done with the standard t -test. The ARDL, DOLS and FMOLS estimators all have asymptotic normal distribution. For small sample size, increasing the number of lead and lag (difference) terms in the DOLS model results in the coefficient estimates to become unstable.

Table 3.6: Long Run Estimates with No-lag Regressors

	(1)	(2)	(3)	(4)	(5)
Literacy rate					
OLS	-2.849	-2.508	-2.713	-2.604	-2.713
FMOLS	-2.872*** (.000)	-2.700*** (.000)	-2.805*** (.000)	-2.754*** (.000)	-2.768*** (.000)
DOLS	-2.753*** (.000)	-2.787*** (.000)	-2.939*** (.000)	-2.902*** (.000)	-2.973*** (.000)
CPI					
OLS		-0.187	-0.31	-0.244	-0.309
FMOLS		-0.137* (.083)	-0.282*** (.002)	-0.188* (.056)	-0.269*** (.002)
DOLS		-0.212** (.020)	-0.350*** (.004)	-0.236** (.044)	-0.366*** (.003)
Dummy 1987					
OLS	0.194	0.227	0.194	0.204	0.216
FMOLS	0.211*** (.000)	0.245*** (.000)	0.221*** (.000)	0.227*** (.000)	0.254*** (.000)
DOLS	0.215*** (.000)	0.236*** (.000)	0.204*** (.000)	0.212*** (.000)	0.237*** (.000)
Dummy 2001					
OLS	0.0615	0.119	0.0477	0.0688	0.0942
FMOLS	0.0740** (.019)	0.130*** (.000)	0.0721 (.112)	0.0910 (.164)	0.151*** (.007)
DOLS	0.0841*** (.004)	0.138*** (.000)	0.0636 (.236)	0.0850 (.231)	0.154** (.034)
GDP					
OLS			0.121		0.279
FMOLS			0.110* (.079)		0.319** (.013)
DOLS			0.129* (.082)		0.383** (.017)
Private consumption					
OLS				0.0852	-0.237
FMOLS				0.0653 (.505)	-0.337** (.050)
DOLS				0.0847 (.422)	-0.400* (.074)

Notes : All variables are in logarithmic form except for dummy variables. For DOLS model, only the first differenced of variables (excluding dummies) without the leads and lags terms are included due to small sample size. For FMOLS method, the Bartlett kernel is specified. Bandwidth is selected according to the Andrews (1991) method. Coefficient estimates for the intercept term are omitted to save space.

Table 3.6: Long Run Estimates with Lag Regressors

	(6)	(7)	(8)	(9)	(10)
Literacy rate					
OLS	-2.77	-2.515	-2.631	-2.461	-2.64
FMOLS	-2.833*** (.000)	-2.720*** (.000)	-2.876*** (.000)	-2.754*** (.000)	-2.803*** (.000)
DOLS	-2.775*** (.000)	-2.738*** (.000)	-2.859*** (.000)	-2.718*** (.000)	-2.862*** (.000)
CPI					
OLS		-0.145	-0.211	-0.113	-0.209
FMOLS		-0.0790 (.326)	-0.189** (.041)	-0.0643 (.509)	-0.204** (.020)
DOLS		-0.192** (.033)	-0.186 (.123)	-0.0978 (.392)	-0.251** (.025)
Dummy 1987					
OLS	0.212	0.240	0.221	0.253	0.270
FMOLS	0.218*** (.000)	0.234*** (.000)	0.197*** (.000)	0.231*** (.000)	0.244*** (.000)
DOLS	0.224*** (.000)	0.231*** (.000)	0.218*** (.000)	0.253*** (.000)	0.282*** (.000)
Dummy 2001					
OLS	0.0738	0.119	0.0797	0.147	0.184
FMOLS	0.0843*** (.004)	0.113*** (.000)	0.0448 (.338)	0.116** (.072)	0.140** (.012)
DOLS	0.0882*** (.001)	0.122*** (.000)	0.0953* (.076)	0.182*** (.008)	0.259*** (.000)
GDP					
OLS			0.0671		0.428
FMOLS			0.108* (.096)		0.411*** (.002)
DOLS			0.0380 (.610)		0.519*** (.000)
Private consumption					
OLS				-0.0473	-0.537
FMOLS				-0.00369 (.970)	-0.461*** (.008)
DOLS				-0.106 (.299)	-0.743*** (.000)

compared to its other estimates. The FMOLS method gives a different estimate for GDP in regression (8) which is similar to all its other estimates in regression (3). Taking only the negative values into consideration, the estimates for private consumption seem rather unstable, ranging from -0.004 to -0.7. Estimates for private consumption are only significant for regressions (5) and (10) when both GDP and private consumption are included and the OLS estimates for literacy rate have small absolute values compared to the other methods for all regressions except regression (1). GDP is significant when its coefficient estimate is -0.11 for both regressions (3) and (8). As for all the other estimates not mentioned, Table 3.6 shows that all the four methods yield similar estimates.

Table 3.7 shows the ARDL and OLS estimates for literacy rate and CPI obtained using the data for different time periods and Figure 3.8 shows the test for stability of the parameters given by the CUSUMSQ statistic. Results and discussion are given in the Appendix.

3.7.4 Reverse Causality and Adequacy of Model

Due to the possibility of the existence of multiple cointegrating relationships among variables, Hall et al. (1990) and Bhaskara Rao (2007) recommend the use of the Johansen (1988) maximum likelihood multi-cointegration estimation procedure based on the VECM model to verify the results from the single equation model. However, the tests and inferences depend on asymptotic properties which may not be suitable for small sample sizes. According to Muscatelli and Hurn (1992), "*both the Engle-Granger²³ and Johansen procedure are explicitly multivariate, in the sense that the existence of an error-correction formulation for all the variables involved is postulated.*" (p. 29). According to Pesaran and Pesaran (2009), all the variables are to be treated as dependent variables to test for alternative cointegrating specifications within the ARDL framework. Results are summarised in Table 3.8 in the Appendix. According to the table, most other specifications do not pass the cointegration bounds test except when life expectancy and literacy rate

²³The ARDL approach, like the Engle-Granger procedure, is a single equation model.

are dependent variables. However, they do not pass some of the other diagnostic tests, namely the Lagrange Multiplier test of autocorrelation. The presence of serial correlation in the error terms can be problematic to the cointegration F -statistics. Results show that by changing the long run forcing or dependent variables, none of the other models can be adequately specified.

3.8 Conclusion

By incorporating measures of taste and cost into his model, Lindert (1978) examines the formation of taste for children and concludes that cost is the main reason for the decline in fertility in the U.S. Based on the lessons from behavioural and institutional economics, Folbre (2008) emphasises on the imperfect choices made, the uncertainty in outcomes, the determinants of the preferences for children, the influences from peers, culture and social norms, the (cost of) commitment and responsibility to children as opposed to the cost of children as investment or quality goods, the bargaining power in husbands and wives in having children and the decision to maximise children's potential, choices in life and capabilities instead of their human capital. In support for their work, this essay shows the importance of the education level of the population and the cost of children on fertility.

Results show that literacy rate and price are the two variables that contributed to the decline in fertility in Singapore. While it is possible for GDP and private consumption to be significant as well, including them in the regressions may cause multicollinearity problems.²⁴ Using India's time series data from 1965 – 1991 and based on the VECM model, Masih and Masih (2000) find that the estimates for female education and the use of contraceptives are significant but not for female participation rate and real income per capita. Similarly, Narayan and Peng (2006) find that real per capita income is not

²⁴I restrict the coefficients of GDP and private consumption to be the same by dividing private consumption with GDP to obtain private consumption per GDP. The results are found in Table 3.9 in the Appendix. The estimate for private consumption per GDP is significant for the model without any control variable but is insignificant when a control variable (except infant mortality rate) is included.

significant in Japan using the time series data from 1950 – 2000. The positive coefficient estimates of GDP obtained here imply that children are normal goods (Becker, 1960; Lo, 2012; Narayan, 2006). As opposed to some studies (Ehrlich and Lui, 1991; Zhang and Zhang, 2005), the estimates for life expectancy are also positive. Life expectancy should have the same directional effects as infant mortality rate. As infant mortality rate decreases, fewer babies are needed to ensure that some survive; therefore, infant mortality rate increases with fertility.²⁵ Likewise, as life expectancy increases, the return from every dollar invested in them is higher; therefore, it makes more sense to have more children. For savings rate, the signs of the coefficient estimates are mixed - positive when the "no lag" regressors are used and negative when the one period lag regressors are used. As fertility decreases, savings should increase since couples can now save more and have a greater need to do so with fewer or no children to take care of them.

All of Leibenstein's, Becker's and Easterlin's theories imply that private consumption has an important role to play in fertility decline. However, results show that price is more important than private consumption. A study in Singapore (Ng et al., 2012) shows that 58 percent²⁶ of those surveyed cite cost as the most important factor in deciding the number of children to have, followed by commitment of time and effort. The factor with the greatest impact on the decision to have children is cost of childcare followed by children's education. The study also finds that the average desired number of children is 1.69 which is much higher than the current TFR.

According to the VOC approach, the preference for children is determined by their costs and benefits. While the intangible benefits such as the joy of having children may still be there, the (expected) tangible benefits have fallen. Economically speaking, people may not see it as worthwhile to have kids compared to other opportunities such as investing, starting a business or saving for old age. With more opportunities presenting themselves in an increasingly globalised world, women are also becoming more driven and entrepre-

²⁵For the estimates of infant mortality rate, some are positive and some are negative. This should not be a problem as all the estimates have very small absolute values and a slight change in their estimates can cause them to change signs.

²⁶The rest of the 42 percent is made up of the following: 20 percent for commitment, 3 percent for career, 5 percent for childcare support and 14 percent for others.

neurial, choosing their businesses and careers over settling down to start families as they become more educated.

Most literature only considers one aspect of social security on fertility, namely social security has replaced the need for children for old age security (Hohm, 1975). However, it is possible that due to the *inadequacy* of the payouts of existing social security systems, people who do not wish to depend on their children may choose to have fewer or no children in order to save more for old age as they feel that they cannot afford to retire well given today's cost of living. Hence, having insufficient or no social security benefits may also lead to the decrease in fertility.

Due to the lack of data, this study abstracts from other factors such as the average age of marriage, the proportion of singles and the use of contraceptives. An extension would be to explore other *cost related measures* such income inequality and the competitiveness and stress levels of children.

3.9 Appendix

Table 3.7: Long Run Estimates with No-lag Regressors

	1960 - 1986		1960 - 1990		1960 - 2000		1960 - 2015	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
<i>I. Cointegration test</i>								
ARDL bounds test	5.370*	3.615 [†]	8.355**	7.759**	15.015**	12.953**	12.063**	18.034**
<i>II. Coefficient estimates</i>								
Literacy rate								
OLS	-2.303	-2.851	-2.307	-2.848	-2.347	-2.851	-2.687	-2.733
ARDL	-2.643***	-2.852***	-2.619***	-2.873***	-2.586***	-2.890***	-2.887***	-2.682***
	(.000)	(.000)	(.000)	(.000)	(.000)	(.000)	(.000)	(.000)
CPI								
OLS	-0.29		-0.287		-0.266		-0.0273	
ARDL	-0.514**		-0.476**		-0.475***		0.0624	
	(.022)		(.029)		(.007)		(.585)	
Intercept								
OLS	11.902	13.156	11.904	13.144	11.999	13.154	12.572	12.662
ARDL	14.450***	13.079***	14.177***	13.197***	14.017***	13.279***	13.058***	12.406***
	(.000)	(.000)	(.000)	(.000)	(.000)	(.000)	(.000)	(.000)
Dummy 1987								
OLS			0.219	0.190	0.240	0.194	0.157	0.156
ARDL			0.305***	0.311***	0.271***	0.253***	0.191***	0.190***
			(.000)	(.001)	(.000)	(.000)	(.000)	(.000)
<i>III. Diagnostic tests for ARDL model</i>								
LM test for serial correlation	0.296	0.558	3.386*	0.0765	2.714*	0.0270	0.732	0.842
	(.586)	(.455)	(.066)	(.782)	(.100)	(.870)	(.392)	(.359)
Ramsey's RESET test	0.236	0.109	0.0896	0.627	0.00246	1.719	9.358***	2.590
	(.627)	(.741)	(.765)	(.428)	(.960)	(.190)	(.002)	(.108)
Normality test	2.026	1.409	0.528	0.418	0.0334	0.0639	0.860	0.736
	(.363)	(.494)	(.768)	(.812)	(.983)	(.969)	(.651)	(.692)
Heteroskedasticity test	0.300	1.453	1.093	1.309	0.158	0.210	0.167	0.135
	(.584)	(.228)	(.296)	(.253)	(.691)	(.647)	(.683)	(.713)

Notes : See Table 3.3. For regressions (1) and (2), no dummy variable is included because time period is before the policy switch.

[†]F value as reported by Microfit 5.0 is insignificant, but when the exact same model is run using Eviews and Stata, they reported much larger values (12.585 and 13.084 respectively) and are significant at 5% level.

I compare the above results with that of regression (2) in Table 3.5; the coefficient estimates for literacy rate and CPI are close to -2.8 and -0.2 respectively. From regression (2) in Table 3.7, both the ARDL and OLS estimates are -2.85. When CPI is included in regression (1), the ARDL estimate for literacy rate is close to -2.8 compared to the OLS estimate of -2.3. The estimate of CPI by OLS, however, is closer to -0.2. The estimates for literacy and CPI in regressions (3) - (6) are similar to those in regressions (1) and (2).

Figure 3.8

For regressions (7) and (8), I exclude the variable Dummy 2001 since it is insignificant for all regressions in Table 3.3. The ARDL and OLS estimates for literacy rate are close to -2.8. Both the estimates for CPI in regression (7), however, are very small; the ARDL estimate has a wrong sign. Regression (7) also fails the functional form test.

Figure 3.8 shows that there is no instability in the coefficient estimates based on regression (2) of Table 3.5 since the CUSUMSQ statistic is within the 5 percent critical bounds of parameter stability.

Table 3.8. Bounds Cointegration Tests

	Test Statistic			
	LM Test for serial correlation	RESET test	Normality test	Heteroskedasticity test
$F(\text{gdp} \mid \text{lit}, \text{fert}, \text{con}, \text{sr}, \text{mr}, \text{dum87}, \text{dum01}) = 1.207$	1.369 (.242)	4.797 (.029)**	13.856 (.001)***	0.332 (.564)
$F(\text{sr} \mid \text{fert}, \text{cpi}, \text{gdp}, \text{lit}, \text{dum87}, \text{dum01}) = 4.223$	10.311 (.001)***	0.0807 (.776)	2.603 (.272)	0.313 (.576)
$F(\text{con} \mid \text{sr}, \text{gdp}, \text{lit}, \text{fert}, \text{mr}, \text{dum87}, \text{dum01}) = 1.175$	3.188 (.074)*	1.902 (.168)	1.147 (.564)	0.0515 (.820)
$F(\text{mr} \mid \text{con}, \text{sr}, \text{gdp}, \text{lit}, \text{fert}, \text{dum87}, \text{dum01}) = 3.128$	1.189 (.276)	1.244 (.265)	0.269 (.874)	0.0197 (.888)
$F(\text{old} \mid \text{con}, \text{sr}, \text{gdp}, \text{lit}, \text{fert}, \text{dum87}, \text{dum01}) = 3.288$	2.203 (.138)	3.765 (.052)*	8.511 (.014)**	2.090 (.148)
$F(\text{cpi} \mid \text{fert}, \text{con}, \text{gdp}, \text{mr}, \text{lit}, \text{dum87}, \text{dum01}) = 3.939$	6.634 (.010)***	5.138 (.023)**	44.461 (.000)***	5.134 (.023)**
$F(\text{lit} \mid \text{fert}, \text{gdp}, \text{sr}, \text{con}, \text{mr}, \text{dum87}, \text{dum01}) = 43.653**$	19.074 (.000)***	27.015 (.000)***	0.0417 (.979)	6.674 (.010)***
$F(\text{life} \mid \text{fert}, \text{cpi}, \text{lit}, \text{gdp}, \text{dum87}, \text{dum01}) = 5.276**$	13.504 (.000)***	0.245 (.620)	2.899 (.235)	3.072 (.080)*

Notes: All variables are in logs except savings rate and the dummy variables. *, ** and *** represent significance at 10, 5 and 1% levels, respectively.

Table 3.9: Long Run Estimates with No-lag Regressors

	(1)	(2)	(3)	(4)	(5)	(6)
<i>I. Cointegration test</i>						
ARDL bounds test	12.803**	12.768**	10.596**	11.772**	10.235**	10.433**
<i>II. ARDL estimates</i>						
Literacy rate	-2.854 (.000)***	-3.135 (.000)***	-2.854 (.000)***	-2.880 (.000)***	-2.884 (.000)***	-2.879 (.000)***
CPI	-0.262 (.047)**	-0.394 (.010)***	-0.262 (.052)*	-0.378 (.038)**	-0.298 (.061)*	-0.254 (.066)*
Consumption per GDP	-0.371 (.080)*	-0.138 (.591)	-0.371 (.153)	-0.259 (.285)	-0.366 (.091)*	-0.416 (.177)
Dummy 1987	0.260 (.000)***	0.251 (.000)***	0.260 (.000)***	0.236 (.000)***	0.248 (.000)***	0.267 (.000)***
Dummy 2001	0.133 (.003)***	0.0882 (.111)	0.133 (.008)***	0.0737 (.315)	0.107 (.157)	0.151 (.126)
Old-age dependency ratio		0.301 (.133)				
Savings rate			-0.00130 (.998)			
Life expectancy				1.243 (.313)		
Infant mortality rate					-0.0339 (.664)	
Population						-0.0474 (.836)
Intercept	13.9358 (.000)**	15.3344 (.000)***	13.9330 (.000)***	9.2907 (.051)*	14.2897 (.000)***	14.6847 (.000)***
<i>III. Diagnostic tests</i>						
LM test for serial						
correlation	0.168 (.682)	2.171 (.141)	0.169 (.681)	0.546 (.460)	0.109 (.741)	0.387 (.534)
Ramsey's RESET test	0.675 (.411)	0.0101 (.920)	0.676 (.411)	0.114 (.735)	0.491 (.484)	0.801 (.371)
Normality test	0.0809 (.960)	0.198 (.906)	0.0810 (.960)	0.000165 (1.00)	0.0393 (.981)	0.134 (.935)
Heteroskedasticity test	0.499 (.480)	0.00174 (.967)	0.499 (.480)	0.632 (.427)	0.501 (.479)	0.516 (.473)

Notes : See Table 3.3. Consumption per GDP is real private consumption divided by GDP.

Table 3.10	Definitions and Construction of Variables
Literacy Rate	Proportion of resident population aged 15 years and over with the ability to read with understanding.
Consumer Price Index	Average price changes for a fixed basket of consumption goods and services commonly purchased by resident households over time with base year set at 100 for year 2014. Base year is changed to 2010 to match the reference year prices of real GDP and private consumption per resident. The weighting pattern is derived from the expenditure values collected from the 2012/13 Household Expenditure Survey (HES) and updated to 2014 values. It excludes non-consumption expenditure such as loan repayments, income taxes, purchase of houses, shares and other financial assets.
Real Gross Domestic Product Per Resident	Expenditure in gross domestic product at 2010 market prices divided by country's total resident population.
Total Fertility Rate	The average number of live-births each resident female would have during her reproductive years if she were to experience the age-specific fertility rates prevailing during the period.
Gross Domestic Saving	Gross Domestic Product - Private & Government Consumption Expenditure + Statistical Discrepancy.
Savings Rate	Gross Domestic Saving divided by Gross Domestic Product.
Life Expectancy	Average number of additional years which a person at birth could expect to live for assuming given age-specific mortality rates throughout a person's life.
Infant Mortality Rate	Number of deaths of persons under 1 year of age per 1,000 live-births.
Old Age Dependency Ratio	Residents aged 65 years & over per hundred residents aged 20-64 years.
Real Private Consumption Per Resident	Includes the following items: food, alcohol, clothing, housing and utilities, household furnishing and maintenance, health, transport, communication, recreation, food serving services, accommodation services, miscellaneous goods & services, residents' expenditure abroad and non-residents' expenditure locally.
Total Population	Singapore residents & non-residents.
Resident Population	Data for 1970 and from 1980 onwards refer to Singapore residents (citizens and permanent residents). Data prior to 1980 (except 1970) refer to total population.
<i>Note:</i> All data and definitions are obtained from the Department of Statistics, Singapore.	

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