Sub-nyquist sampling techniques for cognitive radio applications

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Sub-Nyquist Sampling Techniques for Cognitive Radio Applications

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To my family, for their unconditional love and endless support.

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Abstract

Cognitive Radio (CR) has emerged as the promising solution to overcome the limited spectral resources available to support the incessant demand for higher data throughput in today's wireless communications. CR operation exploits the underutilized spectral resources characteristics of typical radio channels by transmitting the data when a channel is found to be idle. In order to increase the probability of finding unused spectrum and therefore increase its transmission throughput, a CR will try to monitor as many channels as possible by performing spectrum sensing (SS) over a wide frequency range. However, due to the Nyquist sampling requirement, monitoring a wideband spectrum, which is termed as wideband spectrum sensing, would require very high sampling rate and is limited by existing analog-to-digital converters (ADCs) technologies. This thesis presents the study of using a sub-Nyquist sampling technique to extend the capability of CR system, primarily through the adoption of Compressive Sampling (CS) technique in CR implementation.

Compressive sampling is a signal-processing framework which enables a system to reconstruct a sparse signal that is sampled at a sub-Nyquist rate. In practice, the CS reduces the required sampling rate with the tradeoff in higher data reconstruction time cost. It hence typically limits the CS technique to off-line data processing applications, which is not possible for CR systems that require the SS process to be performed in real time. This thesis hence presents several novel approaches to overcome the existing CS limitations with the aim to minimize the time required for CS based SS operations and further enhance the performance of the CR system.

The first contribution presented in this thesis is a new CS based SS technique proposed for hybrid CR that uses the combination of underlay and interweave transmission modes. Unlike existing CS based signal processing operation, the proposed CS based technique does not require the reconstruction process. As such, it is able to achieve much lower sampling rate and greatly reduce the detection processing time compared to other known SS techniques. In addition, the proposed approach also incorporates the learned feature which can further improve the accuracy of the SS process. As a result, the proposed technique is able to achieve higher transmission throughput compared to other well known SS techniques, while operating at sub-Nyquist sampling rate without the need to use complicate ADC hardware architecture.

The second contribution presented in the thesis is a novel matrix optimization algorithm that can be incorporated into CS based CR receiver to enhance the detection and reconstruction accuracy for OFDM-based signal transmission. This is important as it is usually not feasible to implement optimal sensing matrix for CS based CR since its frontend receiver circuit is typically hardwired, and the need to remain compatible with standard digital OFDM receiver's operation. Simulation results show that the proposed approach can consistently produce smaller CS reconstruction error in term of BER under various operating conditions when comparing to existing published systems.

The third contribution of this thesis is to further extend the use of the matrix optimization algorithm to MIMO-OFDM based system, which is the dominant air interface for the latest 4G and 5G broadband wireless communications. The proposed technique enables the enhancement of CS related data transmission performance with reduced number of ADCs required at the MIMO-based receiver. Extensive simulation results have strongly confirmed the promising performance of this proposed approach.

In summary, this thesis proposed several new ideas on how the sub-Nyquist CS based technique can be adopted for CR systems that require real-time operations, without compromising the performance while at the same time reduces the complexity of the hardware circuitry required in the CR implementation.

Keywords

Cognitive radio, wideband spectrum sensing, sub-Nyquist sampling, compressive sampling, matrix optimization.

Contents

Acknowledgement	iii
Abstract	iv
List of Figures	xi
List of Tables	xv
1 Introduction	1
1.1 Background	2
1.2 Scope of Research	3
1.3 Contributions	4
1.4 Publications	6
1.5 Thesis Organization	7
2 Literature Survey	9
2.1 Cognitive Radio	9
2.1.1 Spectrum Holes	10
2.1.2 Dynamic Spectrum Access	11
2.1.3 Cognitive Radio Networks	12
2.2 Spectrum Sensing	14
2.2.1 Hypothesis Testing Model	15
2.2.2 Narrowband Spectrum Sensing	18

2.2.3	Narrowband Sensing Model	19
2.2.4	Wideband Spectrum Sensing	21
2.2.5	Wideband Sensing Model	22
2.2.6	Performance Metric: Sensing versus Throughput	25
2.3	Compressive Sampling	26
2.3.1	CS Framework	27
2.3.2	Sensing Matrix	28
2.3.3	Sampling Circuits	28
2.3.4	Mutual Coherence	30
2.3.5	Sensing Matrix Optimization	31
2.3.6	Signal Reconstruction	32
2.4	Compressive Sampling in Cognitive Radio	33
2.4.1	Overview	33
2.4.2	CS-Based OFDM Receiver	34
2.4.3	CS-Based MIMO-OFDM System	36
2.4.4	CS-Based UWB Communication for Spectrum Underlay	37
2.5	Summary	39
3	CS-Based Analog-Information Conversion	40
3.1	CS-Based Sampling Circuits	40
3.2	Random Demodulator	41
3.3	Modulated Wideband Convertor	42
3.4	Non-Uniform Sampling	45
3.4.1	System Model	45
3.5	Summary	46
4	Feature-Based Compressive Spectrum Sensing	47
4.1	Introduction	47
4.2	Sensing Frame Model	49
4.2.1	IEEE 802.22 Standard	49

4.2.2	WRAN Sensing Frame Model	49
4.2.3	Sensing Duration Trade-off	50
4.3	Sensing-Throughput Model	51
4.4	Proposed Spectrum Sensing	53
4.4.1	Sampling Model	56
4.4.2	Covariance and Eigenvectors	56
4.4.3	Proposed Spectrum Sensing	57
4.4.4	Training Procedure	60
4.4.5	Program flow of proposed SS	61
4.4.6	Sensing Frame Structure	63
4.4.7	Complexity Analysis	64
4.5	Simulation Results	65
4.5.1	Sampling Time versus Throughput	66
4.5.2	Sensing Duration versus Throughput	67
4.5.3	Interference Threshold versus Throughput	68
4.5.4	Primary Signal's Active Rate versus Throughput	69
4.5.5	Received PU's SNR versus Throughput	70
4.6	Summary	72
5	Matrix Optimized Wideband Receiver for CS-Based Cognitive Radio	73
5.1	Matrix Optimization	74
5.2	Matrix Optimized MWC-OFDM System	75
5.3	System Model and Proposed Optimization	76
5.3.1	Proposed Optimization	77
5.3.2	Reconstruction with Proposed Optimization	80
5.3.3	Discussion	81
5.4	Simulation Results	82
5.4.1	Mutual Coherence Performance	83
5.4.2	Bit Error Rate Performance	85
5.4.3	Comparisons with Conventional OFDM Receiver	88

5.5	Summary	89
6	Matrix Optimized MIMO-OFDM for CS-Based Data Reception in Cognitive Radio	90
6.1	MIMO-OFDM System	90
6.2	Proposed System	93
6.2.1	MIMO-OFDM CS based Receiver	94
6.2.2	Reconstruction with Optimized Sensing Matrix	94
6.3	Simulation results	95
6.3.1	Mutual Coherence Optimization	96
6.3.2	Reconstruction Fidelity	97
6.3.3	Scaling Performance of MIMO System	100
6.3.4	Effect of Numbers of Active Subcarriers	101
6.3.5	Comparisons with Conventional MIMO Receiver	102
6.4	Summary	103
7	Conclusion and Future Work	104
7.1	Main Contributions	104
7.2	Conclusion	105
7.3	Future Work: CS-Based Cooperative Sensing	107
7.4	Future Work: Spectrum Underlay and UWB	108
7.4.1	Related Work: UWB Positioning	110
7.5	Future Work: CS-Based Machine Learning for SS	111
	References	113

List of Figures

2.1	The frequency allocations of the radio spectrum in U.S.A, 2003 [1]. . . .	10
2.2	Spectrum usage status in Washington D.C. in 2003 [1]	11
2.3	CR opportunistically access spectrum holes [2].	12
2.4	CR (SU) coexists with PUs under the interference limitation [2].	13
2.5	CRN's functions in the PHY layer, MAC layer, and network layers [2]. .	14
2.6	<i>a)</i> Energy detector; <i>b)</i> Cyclic-stationary feature detector; <i>c)</i> Matched filtering detector.	18
2.7	Power Spectral Density (PSD) and Wavelets based SS	22
2.8	<i>a)</i> filter-bank detector; <i>b)</i> sweep-tune detector	23
2.9	Compressive sampling based SS	24
2.10	The sparsity of a sampled signal in the spectrum domain (in the left sub-figure). The time domain samples is displayed in the right sub-figure.	27
2.11	Block diagram of random demodulator (RD) circuit.	29
2.12	The MWC sampling block for analog-to-digital conversion. Its compo- nents include parallel periodic waveforms mixers, low-pass filters, sub- Nyquist ADCs.	30
2.13	System architecture of the CS based OFDM system. The communication problem of recovering the transmitted information can be modeled as a CS problem [3].	35

2.14	The analog board realizing 4 sampling channels. It consists of 3 stages: splitting analog input, mixing with 4 input periodic waveforms, and lowpass filtering [4].	35
2.15	(a) Standard MIMO-OFDM transmitter-receiver's architecture (b) CS framework based MIMO-OFDM transmitter-receiver's architecture. . . .	36
2.16	Spectrum sharing models in CR networks: (a) interweave and (b) underlay.	37
2.17	UWB signals vs narrow band signals in time and frequency domain [5].	37
2.18	The system architecture of a CS based UWB system [6]. The communication problem of recovering the transmitted information can be modeled as a CS problem.	38
3.1	Block diagram of the relationship between the spectrum of the output $y_i(n)$ and the input $X(f)$. The channels 1 and m linearly combines the original the spectrum segments around $lf_p, \bar{lf}_p, \tilde{lf}_p$ with different weights a_{il}	44
3.2	Block diagram of the random sampling ADC (RS-ADC).	45
4.1	Secondary frame structure for CR's periodic spectrum sensing	50
4.2	High-level architecture of the proposed spectrum sensing.	63
4.3	Secondary frame structure for the proposed spectrum sensing	63
4.4	SU's transmission throughput versus sampling time, with $IT = -130$ dBW, $\bar{P}_d = 0.9$, $T = 100$ ms, $\gamma_p = -15$ dB, $\gamma_s = 20$ dB	67
4.5	SU's transmission throughput versus PU interference threshold, with $\bar{P}_d = 0.9$, $T = 100$ ms, $\tau_{proposed} = 1.25$ ms, $\gamma_p = -15$ dB, $\gamma_s = 20$ dB	69
4.6	Achievable SU's transmission throughput versus PU active rate, with $\bar{P}_d = 0.9$, $T = 100$ ms, $\gamma_p = -15$ dB, $\gamma_s = 20$ dB	70
4.7	Achievable SU's transmission throughput versus primary signal's SNR, ($IT = -130$ dBW, $\bar{P}_d = 0.9$, $T = 100$ ms, $\gamma_s = 20$ dB)	71

5.1	High level architecture of (b) existing MWC and (c) proposed MWC based OFDM receivers. Both systems remain compatible with standard OFDM transmitter (a).	76
5.2	Internal struture of MWC sampling block for analog-to-digital conversion consisting of parallel periodic waveforms mixers, low-pass filters and sub-Nyquist ADCs.	77
5.3	Distribution of absolute off-diagonal entries of Gram matrices without and with P_{opt}	84
5.4	BER performance under different SNR when number of channels equals to 31 and 71, using the zero-forcing equalization method.	85
5.5	BER performance with different number of OFDM transmission at $M = 71$	86
5.6	BER performance in the existing and proposed MWC based OFDM system under different equalization methods (WF, MF and ZF) are implemented.	87
5.7	BER comparison among OFDM systems with and without the MWC.	88
6.1	High level architectures of the conventional MIMO-OFDM system	91
6.2	High level architectures of the proposed CS based MIMO-OFDM system	93
6.3	Distribution of absolute off-diagonal entries of Gram matrices without / with P_{opt} under 4×4 MIMO scheme.	96
6.4	Successful reconstruction rate (SRR) versus number of active subcarriers (K) for QPSR symbols at $\text{SNR} = 15$ dB.	98
6.5	BER versus SNR under two different MIMO scales ($2 \times 2, 4 \times 4$) with two different number of active subcarriers ($K = 32, 64$).	100
6.6	BER versus the number of active subcarriers (K) under two different MIMO scales ($2 \times 2, 4 \times 4$) with two different SNR level (10dB, 15dB).	101
6.7	BER comparison among MIMO-OFDM systems without / with CS framework ($K = 32$).	102
7.1	Proposed structure of the CS based cognitive radio's receiver	106

7.2	The structure of a cooperative spectrum sensing model with one PU, multiple SUs, and one fusion center (FC) [7]	108
7.3	UWB signals in time and frequency domain [8].	109
7.4	Block diagram of CS UWB receiver implemented by random demodulator (RD).	110
7.5	Scatter plot of energy vectors collected by two SUs in cooperative spectrum sensing where one PU is transmitting its power [9].	111

List of Tables

2.1	Comparison of existing spectrum sensing techniques	17
4.1	Notation table of vectors and matrices used for the derivation of proposed SS.	55
4.2	Comparison of existing wideband spectrum sensing techniques.	65
4.3	Achieved spectrum sensing parameter under optimal SU's transmission throughput	68
6.1	Correlation values evaluated by the Gram matrix without / with P_{opt} generated by (6.10).	97

Chapter 1

Introduction

The increasing demand for higher data rates in wireless communications in the face of limited or underutilized radio spectral resources has motivated the concept of Cognitive Radio (CR). However, for a CR to harness the spectral resources efficiently, it needs to be able to accurately sense the spectrum over as wide a frequency range as possible. But this may require a Nyquist sampling rate that is beyond many current commercially available analog-to-digital converters (ADCs). This limitation has raised the interest in exploring the feasibility of utilizing sub-Nyquist sampling techniques for the implementation of CR systems. The Compressive Sampling (CS) technique is one such promising framework that has increasingly been studied for CR applications.

The CS based data acquisition process is most suitable for a signal that can be sparsely represented in an appropriate feature domain. It typically involves two steps, including the sub-Nyquist sampling and signal reconstruction using the sub-Nyquist acquired data (or termed as compressive data). However, there are several hurdles when incorporating the CS technique in CR system. Examples of such constraints include the high reconstruction time cost which is an important factor for real time system, and the issue of non-optimal sensing matrix imposed by the hardware circuitry used in CR design. These issues, among others, continue to attract a lot of research attentions in both the academic and the industrial communities when applying CS for CR systems.

1.1 Background

Wireless communication is one of the few technologies that significantly affect lives of human beings, and its applications can be found in various aspects of our daily lives, ranging from highly commercialized cellular and satellite communication systems to television broadcast systems and personal WiFi networks. The radio spectrum, the critical media underpinning a wireless communication system, is normally pre-assigned for various wireless communications standards (e.g. Wifi and Bluetooth) or allocated to different parties (e.g. military, TV stations) for exclusive use, by national regulatory organizations such as the Federal Communications Commission (FCC) in the USA and Office of Communications (Ofcom) in the UK.

However, the radio spectrum is becoming increasingly scarce over the years due to the enormous growth in mobile wireless based spectrum demands. Meanwhile, studies indicate that there is typically a vast amount of spectrum not fully utilized all the time at most locations in the domain of time and space [10]. Measurements performed have shown that only up to 85% of the spectrum is utilized at any time instance [11]. As a result, spectrum sharing is proposed to enhance the spectrum usage efficiency by exploiting underutilized spectral resources in an opportunistic manner [12], leading to the emergence of the cognitive radio (CR).

CRs implement the spectrum sharing idea by enabling a secondary user (SU) to reuse the licensed spectrum, as long as the SU does not interfere with the incumbent primary users (PUs) operation. This dependency is guaranteed by the spectrum sensing (SS) process, where a CR sustainingly monitors the existence of available (idle) channels. However, conventional SS approach to monitoring a large number of radio channels is limited by the Nyquist sampling theory, which will require very high total sampling rate that is beyond many current commercially available analog-to-digital converters (ADCs). As such, various low-rate SS techniques have been proposed, such as sweep-tune detection and filter-bank detection [13]. These approaches tackle the wide bandwidth issue by separating the wideband SS procedure into multiple parallel tasks, each in charge of detecting narrowband signals. This approach hence reduces

the required sampling rate, but with the trade-off of higher hardware complexity and hardware cost.

An emerging and more promising approach to address the bandwidth issue is through the use of the compressive sampling (CS) [14] technique. CS is a signal-processing framework which allows a system to use sub-Nyquist sample-then-reconstruct process provided that the signal can be sparsely represented. However, CS reduces the required sampling rate at the expense of longer data reconstruction time cost, which is critical for a real-time system such as the CR. As such, there are numerous studies, by both academic and industrial communities, searching for effective ways to efficiently incorporate CS techniques for CR systems [15].

1.2 Scope of Research

The research study presented in this thesis aims to address and find effective solutions to resolve the issues of reconstruction time cost and the non-optimal sensing matrix when implementing CS framework in CR receivers. The first objective aims to develop a low rate CS based spectrum sensing (SS) technique with low real-time processing cost for CR systems. This leads to the development of a novel CS based SS approach. The approach does not need to execute the reconstruction procedure, which make it different from normal CS based signal processing operation. Thus, the approach is able to greatly reduce the detection processing time while operating at much lower sampling rate. Furthermore, the proposed approach can also be incorporated with learned information from primary users (PUs) to further enhance the detection accuracy for SS operation.

In addition to the SS process, a CS based CR also needs to eventually perform the reconstruction process to extract the data sent by the transmitter. As such, the issue of non-optimal sensing matrix that affects the performance of CR receivers in terms of communication bit error rate (BER) needs to be investigated. Specifically, if the sensing matrix, which represents the sampling operation, is not well designed, the reconstructed noise could be relatively enlarged [16] and hence degrade the BER

performance. While there are some studies [17, 18] that address such issues, they are targeted for applications such as image processing, and cannot be directly applied to hardware found in wireless communication. Hence the other main focus of this research work is to develop a new approach to optimize the sensing matrix for a hardware based sampling circuit used in wireless communication.

1.3 Contributions

The following lists the contributions presented in this thesis:

- A comprehensive model is derived to analyze the issue of sensing throughput trade-off for various well-known existing SS techniques, focusing on the accuracy of their spectrum sensing methods and their corresponding processing time cost. The model is subsequently used to compare the performance of the various SS techniques against proposed technique developed in this research work.
- A novel likelihood ratio test (LRT) is proposed for SS that incorporates feature information based on the compressed data's eigenvalues and eigenvectors extracted through a training approach on primary signals. The proposed LRT can be performed directly on the compressed observations using the learned features. Consequently, there is no need to perform the lengthy signal reconstruction typically required in CS based operations.
- A novel matrix optimization algorithm is developed, which can be incorporated into CS based CR system to minimize the mutual coherence of the hardware based sensing matrix. This algorithm enables a reduction in the communication BER for OFDM transmission based systems.
- A closed-form mathematical solution is derived specifically for CS based CR receivers to address the non-optimal sensing matrix problem. The solution can be pre-calculated and easily implemented with a digital signal processor. As such,

it can be applied to conventional CR system to minimize the mutual coherence of its sensing matrix, which enhances its spectrum detection accuracy for subsequent OFDM demodulation in the receiver.

- The proposed optimization is further extended to the CS based MIMO-OFDM systems. This approach leads to a reduction in the number of ADC required and also manages to reduce the communication error under various operating conditions when compared to the existing CS based MIMO-OFDM.

1.4 Publications

JOURNAL ARTICLES

1. Chen Hao and Vun Chan Hua, “A Feature-Based Compressive Spectrum Sensing Technique for Cognitive Radio Operation”. *Circuits, Systems and Signal Processing*, Springer (2017). pp. 1-28.
2. Chen Hao and Vun Chan Hua, “A Novel Matrix Optimization for Compressive Sampling based Sub-Nyquist OFDM Receiver in Cognitive Radio”. *Circuits, Systems and Signal Processing*, Springer. (Submitted for 2nd round review).

CONFERENCE PAPERS

1. Chen Hao and Vun Chan Hua, “A Novel Matrix Optimization for Compressive Sampling based Sub-Nyquist OFDM Receiver in Cognitive Radio.”. In the *Proceedings of IEEE 32nd URSI general assembly and scientific symposium*, 2017.
2. Chen Hao, and Vun Chan Hua, “An Efficient Compressive Spectrum Sensing Technique for Cognitive Radio System”. In the *Proceedings of IEEE Region 10 Conference (TENCON)*, 2016.
3. Chen Hao, and Vun Chan Hua, “Compressive Sensing Techniques for UWB Indoor Positioning Applications.”. In the *Conference on IEEE Consumer Electronics (ICCE)*, 2015.
4. Chen Hao, and Vun Chan Hua, “Recent Progress in Compressed Sensing based Analog-to-Digital Conversion”. In the *International Symposium on IEEE Consumer Electronics (ISCE)*, 2014.

1.5 Thesis Organization

The thesis is organized as follows:

- Chapter 2 provides a review of the background of Cognitive radio and discusses issues related to the spectrum sensing process during the cognitive radio operation. It first presents the sampling rate issue encountered in wideband spectrum sensing, followed by an overview of traditional spectrum sensing and sub-Nyquist sampling techniques. It then discusses the CS technique that can be used to address the problem, presenting its model of sampling and signal reconstruction. Some other related issues, such as mutual coherence and reconstruction delay which prevent the incorporation of CS for CR, are modelled and described as preliminary backgrounds of the proposed work in Chapter 4, 5, and 6.
- Chapter 3 presents applications of the CS technique within the analog-to-digital conversion process. These implementations typically introduce randomness in the sampling process to enable effective sub-Nyquist signal sampling, which can then be reconstructed by using techniques such as the greedy pursuit algorithms. As these CS based sampling circuits can be easily incorporated into CR, this chapter is hence regarded as establishing the preliminary implementations of the proposed work presented in Chapter 4, 5, and 6.
- Chapter 4 presents the proposed work on designing a new CS based SS for hybrid CR transmission. It innovates in utilizing the LRT on the learned feature information of the primary signal for efficient spectrum sensing, which is based directly on the compressive data and does not require any CS reconstruction process. The performance comparison is based on the sensing-throughput model along with the measurement of processing time cost, using the captured real world data (ATSC DTV signal) operating in IEEE 802.22 WRAN environment.
- Chapter 5 describes the proposed work on sensing matrix optimization for data reconstruction performance in CS based CR receiver. It first derives a closed-form

mathematical approach that can be pre-calculated, then proposes a compatible processing block which implements the approach to reduce the BER for OFDM based communications.

- Chapter 6 transplants the proposed optimization to CS based MIMO-OFDM systems, with the aims to decrease the communication error while at the same time reduces the hardware complexity of the receiver's architecture when using the CS framework for MIMO systems.
- Chapter 7 provides the conclusion of the work and results achieved in this research. It summarizes the research outputs with respect to efficient signal receiving and processing techniques for CR receiver under sub-Nyquist rate. It also identifies some future research directions, such as Ultra-Wideband (UWB) based underlay communication and machine learning techniques that could extend from the current work.

Chapter 2

Literature Survey

This chapter first reviews the background of cognitive radios and its spectrum sensing process. The sampling rate issue in wideband spectrum sensing is then discussed, followed by an overview of traditional spectrum sensing and sub-Nyquist sampling techniques. The CS framework is then presented, including its sampling models and signal reconstruction process. Several issues which impeded the incorporation of CS into CR are then described as preliminary backgrounds of the work to be presented in subsequent chapters.

2.1 Cognitive Radio

To provide a non-interfering (or orthogonal) spectrum basis for various wireless applications and services, the fixed spectrum access policy has been adopted traditionally, where each piece of spectrum can be assigned to one or more users. Only the assigned (licensed) user has the right to utilize the allocated spectrum band while other unlicensed users cannot access it, regardless of whether the band is occupied or not. With the fast growth of wireless services in the past few decades, most of the available spectrum has essentially been fully allocated in several developed countries, resulting in the spectrum scarcity problem. Figure 2.1 [1] shows an example of the frequency allocation of the radio spectrum by the regulating agency in the United States, which indicates

Figure 2.2: Spectrum usage status in Washington D.C. in 2003 [1]

2.1.2 Dynamic Spectrum Access

Based on the fact of the spectrum holes, the dynamic spectrum access (DSA) has been proposed to allow the licensed spectrum to be shared opportunistically, and utilized more efficiently [22]. In DSA, the licensed users which can access the allocated spectrum are termed as the primary users (PUs). The PUs are not exclusively granted the highest priority. Sometimes if PUs are not temporally using the spectrum, or their operation can be properly protected from interference, then unlicensed users, or termed as the secondary users (SUs), can have the access to the allocated spectrum. In essence, by dynamically utilizing the spectrum holes, DSA allows an unlicensed SU to obtain opportunities to reuse the spectrum either temporally, spectrally, or spatially without interfering with the operations of the PUs [23]. As a result, the spectrum can be re-utilized in an opportunistic manner, or shared all the time, resulting in an significant

increment of spectrum utilization efficiency.

2.1.3 Cognitive Radio Networks

SUs are required to sense the radio environmental knowledge in order to support the DSA. A radio device with capabilities to achieve these requirements is termed as the cognitive radio (CR) [24]. In detail, there are many different types of capabilities for a CR to equip. For instance, a CR must sense the presence or absence of the PUs, or predict the interference level received at primary receivers. In other words, a CR must adapt and dynamically reconfigure itself based on the radio environment.

Figure 2.3: CR opportunistically access spectrum holes [2].

Obtaining the radio environmental knowledge can be expensive and sophisticated, which involves the sensing, learning, cooperation, etc [24]. With different spectrum access models, a CR can access the allocated spectrum in different ways. In the literature, it can be noticed that there are two models for spectrum access: (1) the opportunistic spectrum access (OSA) shown in Figure 2.3, and (2) the concurrent spectrum access (CSA) shown in Figure 2.4.

In the OSA model, a CR detects the spectrum holes frequently and quickly for

an SU, i.e., spectrum allocated to some PUs but unused for a time period. Once detecting valid spectrum holes, the CR reconfigures its transmission parameters, such as the modulation type and carrier frequency, to transmit SU's data through the identified spectrum holes. Some studies also term this model as spectrum overlay [25] and interweave scheme [26].

In the CSA model, a CR coexists with a PU in an allocated spectrum as long as the CR's transmitter (Tx) limits its transmit power to an interference threshold which protects PU's operations. This model is also termed to as the spectrum underlay [1].

Figure 2.4: CR (SU) coexists with PUs under the interference limitation [2].

In practice, a cognitive radio network (CRN) will contain more than one CR with different capabilities. It can be regarded as an intelligent network that contains multiple coexisting networks, and can be classified as infrastructure-based network or ad-hoc network [27]. Building a CRN can be challenging due to the difficulties in designing multiple system components and cross-layer design, including the signal processing function in physical layer (PHY), spectrum management issue in medium access control (MAC) layer, and routing and statistical control in network-layer, etc.

Figure 2.5 [2] displays the CRN's functions in the PHY layer, MAC layer, and network layers. In the PHY layer, spectrum sensing is an important component to detect spectrum holes, while environmental learning supports the CR (SU) to gain environmental knowledge such as the channel state information. Based on these operations, DSA can be fulfilled by the CR through transceiver's optimization and reconfiguration.

Figure 2.5: CRN's functions in the PHY layer, MAC layer, and network layers [2].

2.2 Spectrum Sensing

To avoid generating any interference to PUs' operations, CR devices need to adapt to the radio environment and reconfigure its parameter to be appropriate for communication [28]. To enable dynamic adaptation of the environment, several functionalities are needed for a CR operations. These functions are spectrum sensing, spectrum sharing and spectrum mobility [24].

Among these CR functions, the spectrum sensing (SS) is responsible to sample data and monitor the valid spectrum bands. It can be considered as the fundamental ability that supports other two functions because spectrum sharing and spectrum mobility depend on the samples and detection results provided by the SS function. This conclusion can also be noted from Figure 2.5, where the spectrum sensing is the essential and fundamental technique performing in the PHY layer to identify spectrum holes, and supporting higher level service in MAC layer and network layer.

As such, SS is an important function that needs to be effectively performed, and

hence is a research topic which has attracted many research attentions in recent years. To be practical, SS procedure should be executed solely by the SU to identify the white spaces without assistance from centralized database or broadcasting [29]. Hence, signal sampling and signal processing are critical factors in SS procedure.

With the increasing demand for higher data rates in wireless communications, wide-band communication techniques where devices using multiple channels or very wide-band spectrum to transmit data are hence used. But these will impose a serious problem for CRs as the required total sampling rate would be beyond the capability provided by most commercially available ADC. As such, several sub-Nyquist sampling techniques have been studied and regarded as promising to overcome this limitation.

2.2.1 Hypothesis Testing Model

As the spectrum sensing process aims to identify the existence of PUs transmission from sampled primary signals, it can be considered as a binary classification problem, or hypothesis testing problem and be modelled as:

$$\begin{aligned}\mathcal{H}_0 : y &= w \\ \mathcal{H}_1 : y &= s + w\end{aligned}\tag{2.1}$$

where the vector y , w and s stands for the received signals, noise, and primary signals, respectively. The accuracy of the spectrum sensing can then be evaluated using the probabilities consists of (i) the detection rate $P_d = \text{Prob}(\mathcal{H}_1|y = s + w)$; and (ii) false alarm rate $P_f = \text{Prob}(\mathcal{H}_1|y = w)$.

The (4.4) describes two cases where $\mathcal{H}_0$ or $\mathcal{H}_1$ indicates the absence or presence of PU, and the following determination procedure can decide whether the hypothesis $\mathcal{H}_0$ or $\mathcal{H}_1$ is trustworthy:

$$T(y) \underset{H_1}{\overset{H_0}{\leq}} \eta\tag{2.2}$$

Specifically, the notation $T(\cdot)$ is used to represent the processing function of the SS methods, which processes sampled data (vector) y and compares it to a predetermined threshold level η . If $T(y)$ is bigger than η , the SS process can declare that the PU's signal s is present, and vice versa.

There are several spectrum sensing techniques being proposed in various studies in accordance with different $T(\cdot)$, and the most common one is the energy detection (ED). Table 2.1 summarizes the important characteristics of the various SS techniques published in the literature, based on the different factors: sensing bandwidth, required PU's information, sampling circuits and computational complexity. From the aspect of opportunistic spectrum access, a CR with wideband SS approach can increase the probability of finding unused spectrum and therefore increase its transmission throughput. Upcoming sections will present various traditional SS techniques and the corresponding $T(\cdot)$ in terms of sensing bandwidth (narrowband or wideband).

Table 2.1: Comparison of existing spectrum sensing techniques

Spectrum sensing	Detection bandwidth	Prior knowledge of PU's signal	Sampling circuits	Computational complexity
ED [30]	narrowband	none	Nyquist rate, single ADC	$O(N)$ [a]
FD [31]	narrowband	cyclic-stationary spectrum	Nyquist rate, single ADC	$O(N \log N + L_f)$ [b]
MFD [32]	narrowband	PU signal's exact template	Nyquist rate, single ADC	$O(N)$
EVD [33]	narrowband	significant leading eigenvalue	Nyquist rate, single or multiple ADCs	$O(ML + M^3)$ [c]
MS [13]	wideband	none	Nyquist rate, multiple ADCs	$O(N)$
CS[34]	wideband	sparse spectrum	sub-Nyquist rate, single ADC	$O(kmN)$ [d]

[a] N is the number of samples

[b] L_f is the frequency smoothing length; N is the number of samples.

[c] M is the smoothing factor of sample covariance. L is the length of each sample vector.

[d] k is the sparsity. m is the number of compressive samples. N is the length of samples under Nyquist rate sampling.

2.2.2 Narrowband Spectrum Sensing

The most widely used narrowband SS technique are the energy detection (ED), feature detection (FD) and the matched filter detection (MFD), where their typical processing procedures are as shown in Figure 2.6.

Figure 2.6: a) Energy detector; b) Cyclic-stationary feature detector; c) Matched filtering detector.

- **Energy Detection (ED).** The ED measures the energy of received signal to determine the existence of PU. Compared to MFD, ED requires a longer sensing time to achieve a desired performance level, but provides a low cost solution due to its implementation simplicity. The main drawback of the ED is its susceptibility to uncertainties in background noise power, especially at low signal-to-noise ratio (SNR) [35].
- **Feature Detection (FD).** If some features of the primary signal, such as its carrier frequency or modulation type are known, more sophisticated feature detectors (FD) may be employed to address this issue at the cost of increased complexity [31]. These detectors rely on spectral correlation of the built-in periodicities (features) such as carrier frequency, bit rate, and cyclic prefixes. Compared to the ED, The FD allows a CR to detect a specific primary signal buried in noise and interference [33].

- **Matched Filter Detection (MFD).** If the structure of the primary signal is known, the optimal detector in stationary Gaussian noise is the MFD. However, when more primary bands are being available for opportunistic access, the implementation cost and complexity associated with this approach will increase prohibitively since a CR will need dedicated circuitry to achieve synchrony with each type of primary licensee for coherent detection.

2.2.3 Narrowband Sensing Model

This section presents the model of the narrowband SS techniques in terms of the test statistics $T(y)$ in (2.2).

Energy Detection (ED): The ED measures the energy received during a finite time interval and compares it to a predetermined threshold. For this method, it is assumed that the primary signal does not have any known structure that could be exploited by the detector. Together with noise that is assumed to be of AWGN nature with the variance of σ , the test function of ED can then be described as:

$$T(y) = \frac{\|y\|_2^2}{\sigma^2} \underset{H_1}{\overset{H_0}{\gtrless}} \eta, \quad (2.3)$$

and its computational complexity is $O(N)$ [36]¹. While the implementation of ED is simple, it suffers from poor detection results when the noise variance is uncertain [37].

Feature Detection (FD): The FD measures the periodic patterns in man-made signals, where the patterns are related to the symbol rate, chip rate, channel code, or cyclic prefix etc. In particular, these pattern are always of second-order cyclic stationary [38]. Thus, FD try to extract the cyclic stationary characteristic through the spectral correlation from received signals (e.g. OFDM), and compare it against a predetermined threshold. Its $T(\cdot)$ can be represented as follows:

$$T(y) = \sum_l C_y(\alpha, l) e^{-j\omega l} \underset{H_1}{\overset{H_0}{\gtrless}} \eta, \quad (2.4)$$

¹ N is the number of samples which equals to the length of vector y

where the term $C_y(\alpha, l)$ is the Fourier coefficients of the autocorrelation function of the received signal under time-varying lag l and cyclic frequency α . The FFT computation for $C_y(\alpha, l)$ is the main contribution to the complexity of (2.4) processing, causing the complexity to be $O(N \log N + L_f)$ [36]¹. Compared to ED, FD exploits the known feature inside the primary signals, hence resulting in a better robustness when operating in low SNR condition, with the tradeoff of higher processing cost, longer running time and longer delay expected.

Matched Filter Detection (MFD): Knowing the features of primary signals can greatly improve the performance of detection, such as in the case of the FD. When the primary signal is completely known (e.g. noise variance, signal variance and channel coefficients), optimal SS can be achieved by using matched filter detection (MFD) [32, 39] with its

$$T(y) = \underset{H_1}{\overset{H_0}{Re(s^T y)}} \underset{H_1}{\overset{H_0}{\leq}} \eta. \quad (2.5)$$

The MF correlates the received signal y to a known template s of the primary signal, then uses their difference for signal detection, and hence its complexity equals to $O(N)$ due to the requirement of computing $s^T y$. However, the template s of the primary signal cannot always be perfectly known in practice, and hence, MF will not always be feasible.

Eigenvalue based Detection (EVD): Unlike the traditional FD which uses the second-order cyclic stationary of the primary signals, the EVD attempts to extract the feature from the time domain signal from the sample covariance matrix, which exhibits a known eigenvalue structure (e.g. when primary signals transmit through a MIMO system [33]). EVD compares the leading eigenvalue and the trace of sample covariance matrix against the pre-determined threshold to test the presence of PU as follows:

$$T(y) = \frac{\lambda_1}{\text{trace}(C_y)} = \frac{\lambda_1}{\sum_{m=1}^M \lambda_m} \underset{H_1}{\overset{H_0}{\leq}} \eta, \quad (2.6)$$

where the matrix C_y is the sample covariance matrix, and $\lambda_1, \dots, \lambda_M$ is the eigenvalues of C_y sorted in descending order. The main complexity of the EVD consists of two

¹ L_f is the frequency smoothing length; N is the number of samples that equals to the length of y

parts: computation of the C_y and the eigenvalue decomposition on C_y to get λ_m . The first part takes $O(ML)$ (addition and multiplication) operations and the second part has a complexity of $O(M^3)$, leading to a total complexity of $O(ML + M^3)$ [40]¹. Two variations of the EVD can be used; the ratio of the maximum eigenvalue to minimum eigenvalue, and the ratio of the average eigenvalue to the minimum eigenvalue.

2.2.4 Wideband Spectrum Sensing

Due to the Nyquist sampling requirement, monitoring a wideband spectrum, which is termed as wideband spectrum sensing, would require very high sampling rate and is limited by practical ADCs technologies. Compared to the narrowband techniques, wideband spectrum sensing techniques aim to sense over a wide range of frequency bandwidths of radio channels. For example, to exploit for spectral opportunities over the whole ultra-high-frequency (UHF) TV band (between 300 MHz and 3 GHz), wideband spectrum sensing techniques would be required. It should be noted that all the narrowband sensing techniques described earlier cannot be directly used for performing wideband spectrum sensing since they make a single binary decision for the whole spectrum. Hence they cannot identify individual spectral opportunities that lie within the wideband spectrum [41]. Thus, the study of sub-Nyquist techniques for SS becomes crucial and necessary for CR systems. The following lists the most common wideband SS techniques in the literature.

- Power Spectral Density (PSD) and Wavelets [42]. It uses wavelet transform to locate the discontinuities of the PSD of a multiband spectrum, and the aim of spectrum sensing is then transformed to spectral edge detection. This method is based on the fact that the PSD of a primary signal is smooth within each radio channel but exhibits discontinuities and irregularities at the boundary of two neighbour channels. However, the high sampling rate required is the bottleneck due to the requirement of the Nyquist sampling rate.

¹ M is the smoothing factor of sample covariance. L is the length of each sample vector.

- **Multichannel Sampling (MS).** It tackles the wideband sampling issue by separating the SS procedure into multiple tasks, each in charge of detecting narrowband signals, such that the required sampling rate would be reduced [41]. Based on this idea, many SS applies parallel detection strategy or iterative detection method (in the time or frequency domain), resulting in SS approaches such as the filter-bank detector (FBD) and the sweep-tune detector (STD) [13, 41]. The disadvantage of such techniques is the increasing hardware complexity and hardware cost.
- **Compressive Sampling (CS).** Different from the above method which detects the entire frequency bands separately, the compressive sampling [14] based SS treats the targeted bands integrally and detects them as a one-time task. The CS is a signal-processing framework which allows a system to sample-then-reconstruct the signal using sub-Nyquist sampling rate, with the requirement that the signal can be sparsely represented in an appropriate feature domain. CS may help the SS process by reducing the required sampling rate [34], but it is normally at the expense of long data reconstruction time cost. Hence, it typically limits the CS technique to off-line data processing applications, which is not possible for CR systems that require the SS process to be performed in real time.

Figure 2.7: Power Spectral Density (PSD) and Wavelets based SS

2.2.5 Wideband Sensing Model

This section presents the model of the wideband SS techniques in terms of the test statistics $T(y)$ in (2.2).

Power Spectral Density (PSD): PSD based SS uses wavelet transform to locate the discontinuities of the PSD of a multiband spectrum. By using a standard ADC,

[42] proposed a PSD and wavelet-based SS approach as shown in Figure 2.7. The SS is hence formulated as a spectral edge detection problem. However, the high sampling rate required is the bottleneck where the Nyquist sampling rate is needed.

Figure 2.8: a) filter-bank detector; b) sweep-tune detector

Multichannel Sampling (MS): The MS technique separates the task of wideband detection into multiple tasks, each in charge of detecting narrowband signals, such that the required sampling rate would reduce. Assume it uses the ED for each separated narrowband SS task, the detecting process function of MS can be formulated as follows:

$$T(y) = \frac{\|\sum_{l=1}^L y_l\|_2^2}{\sigma^2} \underset{H_1}{\overset{H_0}{\leq}} \eta, \quad (2.7)$$

where each y_l represents the data that collected in a narrowband. Its complexity is hence equaled to $O(N)$ which is the same as that of the ED ¹. Nowadays, most MS apply parallel detection or iterative detection in the time or frequency domain, such as the filter-bank detector (FBD) and the sweep-tune detector (STD) shown in Figure 2.8. The FBD applies parallel sampling by using a group of parallel filters and ADCs where each group samples different sub-bands of interest simultaneously. As such, the

¹ N equals to the total number of samples from all narrowband collections

required sampling rate of each channel will be proportional to the bandwidth of each sub-band, which is much lower than the Nyquist rate required of the entire bandwidth. To reduce the hardware cost, iterative detection can be used to scan the multi-band one at a time, as shown in Figure 2.8.(b), using the STD [13] technique.

Figure 2.9: Compressive sampling based SS

Compressive Sampling (CS): CS based SS reduces the required sampling rate by using the compressive sampling method [43] if the signal s has a sparse representation. (The acquisition model of CS will be described in Section 2.3, together with the implementations of the RD or MWC for CS realizations.) The signals are sampled at sub-Nyquist rate, and then reconstruct by solving it as a l_1 -norm based convex problem [44]. Standard spectrum detection procedure such as ED can then be performed using the recovered signal from CS reconstruction [34]. The threshold detection for a CS based SS method is equivalent to:

$$T(y) = \frac{\|\Psi \hat{f}\|_2^2}{\sigma^2} \underset{H_1}{\overset{H_0}{\leq}} \eta \quad (2.8)$$

The obvious drawback of this CS based SS is the long processing time incurred in performing the l_1 -minimization computation (2.11) for $\hat{f}$. For signal processing functions, such as those involve signal detection, classification, or filtering, [45] suggests that these can be potentially performed directly based on the compressive samples, without the need to reconstruct the signal. The detection function based on such approach can be expressed as:

$$T(y) = y^T (\Phi \Phi^T)^{-1} \Phi s \underset{H_1}{\overset{H_0}{\leq}} \eta \quad (2.9)$$

However, since the template s of the primary signal cannot always be perfectly known in practice, such an approach will have limited scopes in practical spectrum sensing applications.

Unlike the traditional sub-Nyquist SS techniques (such as STD and FBD) which address the sensing bandwidth issue at the expense of increasing hardware complexity and hardware cost [28], the CS solves the problem with little extra hardware cost. This is hence the preferred approach used in this thesis for sub-Nyquist sampling technique for wideband SS. In addition, it also proposes several novel approaches to overcome the existing CS limitations, with the aim to minimize the time required for CS based SS operations as well as to further enhance the performance of the CS based CR system.

2.2.6 Performance Metric: Sensing versus Throughput

In CR network, the data throughput of the SU is the main metric used to evaluate the effectiveness of its spectrum usage.¹ In IEEE 802.22 compliant network environment, a CR periodically senses the presence of PUs, and then decide whether it should use the target channel [47]. The effect of the spectrum sensing (SS) on SU's transmission throughput, with the focus on the effect of sensing duration, was first presented in [48]. Its analysis shows that the trade-off between the sensing duration against the SU's transmission throughput performance is a convex problem.

However, in practice, the SU's transmission throughput will not only depend on the sensing duration in SS, but also on the SS algorithms, i.e. the detection accuracy (e.g. the false alarm rate) as well as the duration used to perform the spectrum detection during the SS process. As such, it is expected that different SS methods will exhibit different detection accuracy, typically with a trade-off in sampling time and processing time (more sampled data and more analysis for higher accuracy). Intuitively, a more

¹As discussed in [46] which presents the widely used performance criteria for CR networks, the throughput, interference, energy, spectrum usage, fairness and delay are considered as the widely accepted criteria. Among them, the throughput, interference, energy, and delay are suitable criteria especially to evaluate individual spectrum sensing performance in a CR.

precise sensing method executable over a shorter sensing duration (which consists of sampling time and processing time) will increase the SU's transmission throughput.

However, there is no in-depth study (based on an extensive search by the author) that analyze the effectiveness of the CS related spectrum sensing methods that take into consideration of the processing time cost required to perform the spectrum sensing, as well as the reconstruction latency. It is necessary to present a comprehensive analysis (in Chapter 4) of the sensing-throughput issue for various well-known existing SS techniques, focusing on the accuracy of their spectrum sensing methods and their corresponding processing time cost.

2.3 Compressive Sampling

There are many real world signals that exhibit sparse features and would be suitable for CS based sampling and reconstruction. Examples include images with low-rank matrix representation and wireless channels with sparse impulse response coefficients. For those signals which has no apparent sparse structures, it is also possible to derive their approximate sparsity by using orthogonal basis representation or dictionary learning algorithms. As a result, many natural and man-made signals can be approximately presented as sparse signals in the appropriate chosen basis spanned domain, and hence will be suitable to be processed based on CS techniques[49, 50, 51, 52]. For instance, in many wireless communications, the sparsity can be found in the spectrum domain by representing the transmitted signal on the Fourier basis as shown in Figure 2.10. Hence in the following discussion, it is assumed that the input signal can be sparsely represented.

Compressive sampling is a relatively recent technique that allows very efficient sub-Nyquist data sampling. The concept is first introduced by Donoho [53] and Candes, Romberg, and Tao [49] in 2006. Since then, it has become a key idea used in various areas of applied mathematics, computer science and electrical engineering. The basic principle of CS states that for signals that can be sparsely represented under a suitable

Figure 2.10: The sparsity of a sampled signal in the spectrum domain (in the left sub-figure). The time domain samples is displayed in the right sub-figure.

basis (or more generally, a frame), the signal can be sampled at sub-Nyquist rate, but can still be fully recovered by using suitable algorithms.

2.3.1 CS Framework

Assuming that x is a sparse signal in a particular chosen basis Ψ such that $x = \Psi s \in \mathbb{C}^N$, with $\|s\|_0 \ll N$ where s is the corresponding coefficients. CS theory predicts x can then be reconstructed (as $\hat{x}$) from its sub-Nyquist acquired samples y , where y consists of M undersampled observations, $y \in \mathbb{C}^M$ collected by a measurement matrix Φ based on the following expression:

$$y = \Phi x = \Phi \Psi s = A s. \quad (2.10)$$

$A = \Phi \Psi$ is hence a $M \times N$ matrix and is denoted as the sensing matrix of the CS based system. The reconstructed $\hat{x}$ can be obtained through the relation $\hat{x} = \Psi \hat{s}$, where $\hat{s}$ is normally derived by using l_1 -minimization based algorithms [54] as shown in (2.11):

$$\hat{s} = \arg \min \|s\|_1 \quad s.t. \quad y = \Phi x = \Phi \Psi s = A s. \quad (2.11)$$

$\hat{s} \in \mathbb{C}^M$ is the recovered sparse component in (2.10) that leads to $\hat{x} = \Psi \hat{s}$. Solving (2.11) is termed as finding the the solution to the basis pursuit problem [55], and is normally done by using some appropriate reconstruction algorithms such as those used in [56, 57].

2.3.2 Sensing Matrix

Sensing matrix design for real-world system is to establish the sampling circuit modelled by the A in (2.10). It is mentioned that a matrix A which satisfies the restricted isometry property (RIP) [49] condition can enable the unique reconstruction via (2.11). The RIP is defined as:

$$(1 - \delta_k)\|x\|_2 \leq \|Ax\|_2 \leq (1 + \delta_k)\|x\|_2 \quad \text{for all } k\text{-sparse } x, \quad (2.12)$$

where the A satisfy RIP of order k if there exists a $\delta_k \in (0, 1)$. Verification for RIP is not practical since testing whether a matrix satisfies RIP is NP-hard [58]. However it is possible to construct a matrix satisfying RIP by using randomness, which gives a practical way for creating a RIP sensing matrix. Another approach is to use random measurement or random matrix whose entries are independent and identically distributed variables, which is very likely to satisfy the RIP [59]. Examples of these are the Gaussian distribution matrix and Bernoulli matrix which has been shown to be suitable for CS sampling and reconstruction.

2.3.3 Sampling Circuits

Implementing the CS in hardware involves the design for real-world system and the design of sampling circuit that satisfies the sensing matrix A modelled in (2.10). It is also mentioned that a fully random sensing matrix (Gaussian or Bernoulli matrix) which satisfies the restricted RIP condition can lead to the successful CS reconstruction.

However, fully random matrix is hard to build in practice, due to the existing regular structure in orthogonal basis, or difficulty in realizing the truly random entries in hardware implementations. As such, partial randomness sensing matrices are widely studied due to its simplicity, including the random partial Fourier matrix [51], Rademacher matrix [60] and random circulant matrix [52, 61]. On the other hand, in practice, several sub-Nyquist sampling circuits such as the random demodulator (RD) [62] and the modulated wideband converter (MWC)[63] have been proposed. These hardware implementations are popular due to their relatively simple architecture, while the partial

randomness is also maintained in their circuit designs by the mixing operation with a pseudo-random sequence. Based on this approach, the required CS features have been successfully embedded in conventional sampling circuits to mitigate the requirement for high-speed ADCs for wide bandwidth signals. Examples of these are the random demodulator (RD) [60] and modulated wideband converter (MWC) [62], as shown in Figure 2.11 and Figure 2.12 respectively.

2.3.3.1 Random Demodulator (RD)

RD is well known for its simple architecture, while the MWC has a better robustness to noise compared to the RD, but requires parallel sampling circuits which is more hardware complex. These two CS based sampling circuits have become the favorite designs for CR receivers [64, 65] to alleviate both the sampling rate requirement and digital storage burden.

Figure 2.11: Block diagram of random demodulator (RD) circuit.

The RD consists of a standard modulation circuit mixed with a random sign generator, follows by an ADC that samples the data at a sub-Nyquist rate. Sparse signal acquired using CS can be reconstructed (2.11) by using standard CS reconstruction methods such as fast greedy algorithms discussed in Section 2.3.6.

2.3.3.2 Modulated Wideband Converter (MWC)

MWC [62] is an implementation that applies the CS with uniform sampling for conventional multi-band signal receivers, where the carrier's frequency is unknown. Figure

Figure 2.12: The MWC sampling block for analog-to-digital conversion. Its components include parallel periodic waveforms mixers, low-pass filters, sub-Nyquist ADCs.

2.12 depicts the architecture of the MWC, consisting of parallel sampling channels performing at sub-Nyquist rate. During the acquisition process, pseudo-random sequence $p_m(t)$ is periodically mixed (at T_p/L interval) with the input multiband signal $x(t)$. This operation shifts each channel spectrum by Δf_p ($f_p = 1/T_p$). Lowpass filters are then used to condition the mixed signal for baseband sampling via sub-Nyquist ADCs. Reconstruction of the sampled data can be considered as a two-step operation. First, it recovers the digital spectrum supports which involves the MMV solution defined in [63] using certain greedy pursuit method, e.g. SOMP [66]. The reconstructed spectrum supports are then used to recover the analog signal $\hat{x}(t)$ through D/A conversion, filtering and modulation process [63].

2.3.4 Mutual Coherence

Since it is hard to test whether a sensing matrix A satisfies RIP (an NP-hard problem), the matrix's mutual coherence is used as an alternative and practical metric for this test, although it provides weaker reconstruction guarantees than the RIP [67].

The mutual coherence $\mu(A)$ measures the highest correlation between any two

columns of the sensing matrix A . When $\mu(A)$ decreases, the sensing matrix then contains more independent columns, which leads to a lower amount of CS reconstruction error [16]. Mathematically, $\mu(A)$ can be denoted as:

$$\mu(A) = \max_{\substack{1 \leq i, j \leq n \\ i \neq j}} \frac{|A_i^T A_j|}{\|A_i\| \|A_j\|}, \quad (2.13)$$

where a lower value $\mu(A)$ provides a more accurate solution in (2.11) as described in [16]. If a minimized $\mu(A)$ is obtained, the corresponding sensing matrix A can be considered as the optimal sensing matrix. As a result, many studies [16, 18] have proposed different methods to minimize the mutual coherence of sensing matrices in wireless and image based applications. These methods, termed as the matrix optimization, aim to both decrease the coherence and to generate sensing matrices with improved CS recovery capabilities, which will be analyzed in the next section.

2.3.5 Sensing Matrix Optimization

Minimization of $\mu(A)$ has been studied in the form of sensing matrix optimization [16, 17, 18]. In these studies, given the fixed dictionary $\Psi \in \mathbb{C}^{N \times N}$, the update of the varying measurement $\Phi \in \mathbb{C}^{M \times N}$ is done by solving the following problem:

$$\min_{\Phi} \|I_N - \Psi^T \Phi^T \Phi \Psi\|_F^2 \quad (2.14)$$

However, for the circuit used in the spectrum sensing or data reception in cognitive radios, the measurement Φ is pre-designed and cannot be directly updated during the sampling operation, making the operation using (2.14) not feasible. For such cases, the optimization of mutual coherence becomes the derivation of a matrix P given the fixed $\{\Phi, \Psi\}$ such that

$$\min_P \|I_N - \Psi^T \Phi^T P^T P \Phi \Psi\|_F^2, \quad (2.15)$$

where $\Phi \Psi \triangleq A$ is the sensing matrix. As such, the mutual coherence issue, which is found in many hardware sampling circuits as well as the CR's SS circuits is crucial to effective CS implementations.

2.3.6 Signal Reconstruction

Even when the sensing matrix A is well designed, the performance is also affected by the reconstruction algorithms which aim to solve (2.11) [68]. Existing reconstruction algorithms can be categorized into several types: the convex algorithms, non-convex minimization algorithms, combinatorial algorithms and greedy algorithms [69].

Convex Algorithms: The algorithms solve an optimization problem such as the linear programming used in CS reconstruction. The number of measurements required for exact reconstruction is small but the methods are computationally complex. The basis pursuit [55], basis pursuit de-noising [55], least absolute shrinkage and selection operator [70], least angle regression (LARS) [71] are representative examples of such algorithms. Specifically for instance, solving the (2.11) is treated as solving the basis pursuit problem. when measurements are affected by noise, the minimisation problem can be changed to basis pursuit de-noising with a conic constraint to allows certain level of measurement mismatch ($\epsilon > 0$):

$$\min \|x\|_1 \quad s.t. \quad \|y - Ax\|_2 \leq \epsilon. \quad (2.16)$$

Non-convex Minimization: Many practical problems are non-convex and most non-convex problems are hard to solve exactly in a reasonable time. Then heuristic algorithms are adopted although it may not produce desired solutions. There are many algorithms proposed in the literature such as focal underdetermined system solution (FOCUSS) [72], iterative re-weighted least squares [73], sparse Bayesian learning algorithms [74], Monte-Carlo based algorithms [75].

Combinatorial Algorithms: The algorithms recover sparse signal through group testing. These algorithms are fast and efficient but require specific pattern in the sensing matrix A or measurement matrix Φ (i.e. needs to be sparse), which may not be suitable for many general CS framework. Representative algorithms are chaining pursuit [76] and heavy hitters on steroids (HHS) [77].

Greedy Algorithms: Greedy algorithms iteratively approximate the non-zero coefficients (supports) of the original signal, such as the orthogonal matching pursuit (OMP)

algorithm [78]. The basic idea is to select the columns in the sensing matrix that contribute the most to the observation y . This selection method tests the correlation values between the current columns of the sensing matrix and the residue. The OMP algorithm also has many developed versions, such as StOMP [53], regularised OMP (ROMP) [79] and Compressive Sampling MP (CoSaMp) [56].

However, most of the aforementioned methods do not allow fast execution speed while maintaining high reconstruction accuracy. Hence this typically limits the CS techniques to off-line data processing applications, and will normally need a trade-off between the speed and accuracy for real-time SS in CR systems.

2.4 Compressive Sampling in Cognitive Radio

In cognitive radio communication where many wideband signals are involved, the CS has been regarded as a potential technique for data reception by CR receivers. This section introduces some CS based wireless application in a number of common communication standards, such as IEEE 802.22 standard where the OFDM signal is considered as the main radio for communication.

2.4.1 Overview

Although several contributions exist in the literature dealing with the narrowband CR scenarios, in practice, a CR should be capable of monitoring the surrounding radio environment over a wide spectrum range in order to utilize the benefits of CR communications efficiently [15]. This environmental knowledge over a wideband spectrum helps a CR to apply adaptive resource allocation and spectrum exploitation techniques for the effective utilization of the under-utilized radio spectrum. However, due to the practical limitations on the capability of receiver hardware components, mainly ADC, its difficult to implement wideband spectrum awareness algorithms in practice. This difficulty can be alleviated by utilizing the benefits of CS.

The CS has been proven that it can be used to reduce the number of samples or the sampling rate, hence the CS based CR becomes promising solutions in the areas of CR communications covering a wide range of areas such as Spectrum Sensing (SS) [80], spectrum management [81], spectrum decision[82], and spectrum access strategies [83]. Our research is based on the CS based sub-Nyquist sampling techniques for cognitive radio, focusing on the spectrum sensing and data receiving. The upcoming sections introduce our contributions on embedding the CS into CR application, especially at CR receivers for a lower sampling rate.

2.4.2 CS-Based OFDM Receiver

Recent CR designs have adopted the orthogonal frequency division multiplexing (OFDM) modulation due to its capability to dynamically adjust its operating parameters [84], its high spectrum utilization and excellent performance over frequency selective channels [85]. In the OFDM based communications, the bandwidth of the transmission could be very large such that the ordinary sampling circuits suffer from high sampling rates, which also affects signal processing speed and system power consumption [3]. Due to its high robustness to noise, the modulated wideband converter (MWC) [63] has become the favorite design for CS based CR receivers [64, 65] to alleviate both the analog and digital processing requirements.

[3] demonstrates the wideband sub-Nyquist receiver that can sample and process wideband signals performing at sub-Nyquist rate. The prototype, referred to as the modulated wideband converter (MWC), samples multiple narrowband transmissions without knowledge of the carrier positions. As proposed in [3], Figure 2.13 shows the MWC-OFDM architecture used in the design, along with its hardware sampling circuit as shown in Figure 2.14. The system is composed of (a) OFDM transmitter, (b) MWC mixing and sampling block, (c) continuous to finite (CTF) block as proposed in [63] for support detection. (d) MWC DSP block for signal reconstruction, (e) OFDM processing block and (f) digital OFDM receiver. As a result, the specific implementation manages to reduce the rate to only 8% of the Nyquist rate.

Figure 2.13: System architecture of the CS based OFDM system. The communication problem of recovering the transmitted information can be modeled as a CS problem [3].

Figure 2.14: The analog board realizing 4 sampling channels. It consists of 3 stages: splitting analog input, mixing with 4 input periodic waveforms, and lowpass filtering [4].

2.4.3 CS-Based MIMO-OFDM System

Another potential use of CS for wireless receiver is in the application of MIMO-OFDM system. The application of MIMO-OFDM for cognitive radio operation have been proposed due to its robustness against multipath propagation and the benefits from spatial multiplexing. Taking into consideration the nature of CR network operation, it is expected that only a small number of CR users will be accessing the channels simultaneously. Under such condition, the CS framework, which is able to reduce the number of samples if the objective signals have sparse representation, can then be applied in MIMO-OFDM based CR system to reduce the number of observations needed as well as the number of analog to digital converter (ADC) based sampling circuits.

Figure 2.15: (a) Standard MIMO-OFDM transmitter-receiver's architecture (b) CS framework based MIMO-OFDM transmitter-receiver's architecture.

As demonstrated in [86, 87], incorporating the CS framework into MIMO-OFDM based CR's reception can produce a simpler hardware circuitry and reduce MIMO scale in CR receiver design. As shown in Figure 2.15, the number of receiver's sampling circuits can be reduced such that only one ADC is required.

However, those architectures do not consider the effect of the non-optimal sensing matrix on signal reception in the designs of CS-MIMO-OFDM based CR receivers. The issue of non-optimal sensing matrix, equivalent to non-optimal mutual coherence, is regarded as a very critical factor that affects the signal reconstruction performance in CS frameworks. Specifically, if the sensing matrix, which represents the sampling operation, is not well designed, the reconstructed noise could be relatively enlarged [16].

2.4.4 CS-Based UWB Communication for Spectrum Underlay

For a CR that uses spectrum underlay operation, an SU shares the spectrum by limiting its transmit power to be less than the a tolerable threshold set by the relevant regulatory authorities. Figure 2.16 illustrates the concept of spectrum underlay technology, as compared to the interweave operation where spectrum sensing is applied to detect and reuse the spectrum holes.

Figure 2.16: Spectrum sharing models in CR networks: (a) interweave and (b) underlay.

Figure 2.17: UWB signals vs narrow band signals in time and frequency domain [5].

Ultra-wide band (UWB) radio technology maintains a low powered transmissions using a very wideband as shown in Figure 2.17, and hence becomes a potential tech-

nique to support spectrum underlay scheme for cognitive radio networks [5]. However, sampling operation at its high frequency (in ranges of GHz) and wideband signals is a challenging problem in practice. As such, CS can be used to reduce the sampling rate based on the fact that space-time signals are essentially always significantly sparse as shown in Figure 2.17.

Figure 2.18: The system architecture of a CS based UWB system [6]. The communication problem of recovering the transmitted information can be modeled as a CS problem.

A typical hardware implementation of a CS based UWB communication is shown in Figure 2.18, where an incoherent filter (with pseudo-random sequence) is applied to mix with the UWB Gaussian pulses before transmission. At the receiver side, a low pass filtering follows by sub-Nyquist uniform sampling is required for the CS reconstruction. The sampling model at the receivers can be described in matrix form as $y = \Phi \Psi \theta$, where y is the output of the low-rate ADC, θ and Ψ represent the sparse bit sequence and UWB pulse generator respectively. Φ is a matrix modelling the convolution effect from the incoherent filter and channel impulse response. The receiver can then perform the CS reconstruction, and the sparse bit sequence θ can then be recovered by using CS reconstruction algorithms mentioned in Section 2.3.6.

2.5 Summary

This chapter reviews the background and presents the literature survey of the most recent work related to cognitive radio and its operation, as well as the compressive sensing techniques. It then describes the potential advantages of applying the CS techniques to CR implementation. However, incorporation of CS into CR can be impeded by some inherent shortcomings of CS, such as the long reconstruction time cost and non-optimal sensing matrix found in its hardware circuitry design. These, and other issues are hence the focus of the research work presented in this thesis.

Chapter 3

CS-Based Analog-Information Conversion

For a CR to harness the spectral resources efficiently, it needs to be able to accurately sense the spectrum over as wide a frequency range as possible. This requires a Nyquist sampling rate that is beyond many current commercially available ADCs. The CS is a promising solution since it can reduce the sampling rate as long as the target signal can be sparsely represented. This chapter presents applications of the CS technique within the ADC process. These implementations typically introduce randomness in the sampling process which enables sub-Nyquist (low-rate) sampling, and manages to reconstruct signals performed by standard CS reconstruction algorithms. These CS based sampling circuits can be incorporated into CR, and regarded as preliminary implementations of the proposed work.

3.1 CS-Based Sampling Circuits

Implementing CS in hardware for real-world system involves the design of the sampling circuit, which can be described by the sensing matrix A modelled in (2.10). It has been shown that a fully random sensing matrix (Gaussian or Bernoulli matrix) which

satisfies the restricted RIP condition can lead to the successful reconstruction in CS reconstruction [44].

However, fully random matrix is hard to achieve in practice. In other words, it's difficult to realize the truly random entries in hardware implementations. As such, partial randomness sensing matrices are proposed, which include the random partial Fourier matrix [51], Rademacher matrix [60] and random circulant matrix [52, 61]. For hard-wired implementation, several sub-Nyquist sampling circuits have been proposed, such as the random demodulator (RD) [62], the modulated wideband converter (MWC)[63], and non-uniform sampler (NUS) [88]. These hardware implementations have the advantage of relatively simple architecture, where the partial randomness can be achieved through mixing operations with pseudo-random sequence.

As a result, the CS technique can be embedded in conventional sampling circuits to mitigate the requirement for high-speed ADCs for wide bandwidth signals. These approaches are able to achieve high resolution measurement for high frequency signals using sub-Nyquist samplings, and able to reduces the storage requirement and power cost. The following provides an analysis on sensing matrix model for the widely used CS based ADCs.

3.2 Random Demodulator

Random Demodulator (RD) [60] is commonly used for CS based ADCs for signal acquisition and processing system, with typical structure as shown in Section 2.3.3.1. This section analyses the sampling operation and builds the model of its sensing matrix A . In RD, one sampling channel is applied with mixture of the chipping sequence $p_c(n)$ and sub-Nyquist sampling. The operation of mixing chipping sequence $p_c(n)$ can be represented using a diagonal matrix D where the value of its non-zero entries (diagonal items) are chosen pseudo-randomly from the set $\{-1, +1\}$ as shown in (3.1):

$$D = \begin{pmatrix} \epsilon_0 & & & \\ & \epsilon_1 & & \\ & & \ddots & \\ & & & \epsilon_{n-1} \end{pmatrix}_{N \times N} \quad (3.1)$$

Next, the operation of the sampler is performed at a sub-Nyquist rate M , and assume M divides the Nyquist rate N . Each sample is the sum of N/M consecutive entries of the processed (mixed and filtered) signal [89]. The sampling operation can then be treated as an $M \times N$ matrix P , where each row has N/M successive entries beginning with its $(mN/M + 1)$ th item, where $m = 0, 1, \dots, M - 1$ refers to the column number.

$$P = \begin{pmatrix} \underbrace{1 \ 1 \ \dots \ 1}_{N/M} & & & \\ & 1 \ 1 \ \dots \ 1 & & \\ & & \ddots & \\ & & & 1 \ 1 \ \dots \ 1 \end{pmatrix}_{M \times N} \quad (3.2)$$

The observation sampled by the RD can then be modelled as:

$$y = (PD)x = (PD\Psi)s = As, \quad (3.3)$$

where $A \triangleq PD$ is the sensing matrix which represents the architecture structure shown in Figure 2.11. The reconstructed $\hat{x}$ can be obtained through the relation $\hat{x} = \Psi\hat{s}$, where $\hat{s}$ is normally derived by using l_1 -minimization based algorithms [54] as shown in (3.4):

$$\hat{s} = \arg \min \|s\|_1 \quad s.t. \quad y = PDx = As, \quad (3.4)$$

with $\hat{s} \in \mathbb{C}^M$ equivalent to the recovered sparse component in (2.10) that leads to $\hat{x} = \Psi\hat{s}$. Solving (3.4) is termed as finding the the solution to the basis pursuit problem [55], and is normally done by using some appropriate reconstruction algorithms such as those used in [56, 57, 90, 91].

3.3 Modulated Wideband Convertor

Modulated Wideband Converter (MWC) [62] is an approach that applies the CS framework for conventional multi-band signal receivers [92], where the carrier's frequency is unknown. The structure of the MWC has been introduced in Section 2.3.3.2, where each sampling channel uses a periodic sequence (at T_P interval) of $p_i(t)$ with the duration of T_P/M to mix with the input signal $x(t)$. This section analyses the sampling operation and builds the model of its sensing matrix A .

Assume that MWC samples a multi-band signal $x(t)$ to obtain the sparse spectrum $X(f)$ supported by N frequency segments (and each of these segments does not exceed B Hz). When $x(t)$ is mixed with $p_i(t)$, the spectrum $X(f)$ of $x(t)$ becomes:

$$\bar{X}_i(f) = \int_{-\infty}^{+\infty} x(t)p_i(t)e^{-j2\pi ft}dt = \sum_{l=-\infty}^{+\infty} c_{il}X(f - lf_p), \quad (3.5)$$

where the periodic sequence $p_i(t)$ can be represented by:

$$p_i(t) = \sum_{l=-\infty}^{+\infty} a_{il}e^{j\frac{2\pi}{T_p}lt}, \quad a_{il} = \frac{1}{T_p} \int_0^{T_p} p_i(t)e^{j\frac{2\pi}{T_p}lt}dt \quad (3.6)$$

Sampled at baseband by filters and ADCs, the weighted and accumulated $X(f)$ has a relationship with the discrete-time Fourier transform (DTFT) of $y_i[n]$ (the i th group of the output) as shown in (3.7):

$$Y_i(e^{j2\pi fT_s}) = \sum_{l=-L_0}^{+L_0} a_{il}X(f - lf_p) \quad (3.7)$$

where $T_s = 1/f_s$, and L_0 satisfies $2(L_0 + 1)f_p \geq f_{NYQ} + f_s$ and $2L_0 + 1 = M$.

Since (3.7) establishes the relationship between observations and input sparse signals, the measurement of the CS sensing matrix can be described as $y(f) = Az(f)$:

$$\underbrace{\begin{bmatrix} Y_1(e^{j2\pi fT_s}) \\ \vdots \\ Y_m(e^{j2\pi fT_s}) \end{bmatrix}}_{y(f)} = \underbrace{\begin{pmatrix} a_{11} & \dots & a_{1M} \\ \vdots & \ddots & \vdots \\ a_{m1} & \dots & a_{mM} \end{pmatrix}}_A \underbrace{\begin{bmatrix} X(f - L_0f_p) \\ \vdots \\ X(f + L_0f_p) \end{bmatrix}}_{z(f)} \quad (3.8)$$

where $A = \{a_{il}\}_{m \times M}$ depends on the choice of different $p_i(t)$. A popular way to generate $p_i(t)$, referred as the sign waveform generator in RD[63] is shown in Figure 2.12, which is achieved by using a shift-register structure [93].

With the expression of a_{il} and (3.8), the sampling operation of the MWC can be represented as:

$$Y_i(e^{j2\pi fT_s}) = A \cdot X(f) = HFD \cdot X(f) \quad (3.9)$$

Figure 3.1: Block diagram of the relationship between the spectrum of the output $y_i(n)$ and the input $X(f)$. The channels 1 and m linearly combines the original the spectrum segments around lf_p , $\bar{l}f_p$, $\tilde{l}f_p$ with different weights a_{il} .

Then $A = HFD$ is the sensing matrix described as follows:

$$A = \underbrace{\begin{pmatrix} a_{1,0} & \dots & a_{1,M-1} \\ \vdots & \ddots & \vdots \\ a_{m,0} & \dots & a_{m,M-1} \end{pmatrix}}_{H_{m \times M}} \underbrace{\begin{pmatrix} | & \dots & | \\ \bar{F}_{L_0} & \dots & \bar{F}_{-L_0} \\ | & \dots & | \end{pmatrix}}_{F_{M \times M}} \underbrace{\begin{pmatrix} d_0 & & \\ & \ddots & \\ & & d_{M-1} \end{pmatrix}}_{D_{M \times M}}, \quad (3.10)$$

where F is the discrete time Fourier transform (DFT) matrix and $D = \{d_l = \frac{1}{T_p} \int_0^{\frac{T_p}{M}} e^{-j\frac{2\pi}{M}lk}\}$.

Reconstruction can be achieved by (1) support detection via continuous to finite block (CTF), and (2) the signal reconstruction process. The CTF block is comprised of frame construction, and joint support reconstruction (or termed as the MMV problem) [94] that can be solved by standard CS reconstruction algorithms. Based on results of the CTF block, The signal reconstruction can then be accomplished through a direct pseudo-inverse operation.

3.4 Non-Uniform Sampling

Modulation based CS architectures such as RD and MWC use uniform samplings of the mixed input analog signals. Another variation for CS based system is the non-uniform sampling (NUS). This is based on the theory of information recovery from random samples [88] driven by a pseudo random clock that produces, on average, a sub-Nyquist sampling rate. An example of NUS implementation using multiplexers is shown in Figure 3.2 [14].

Figure 3.2: Block diagram of the random sampling ADC (RS-ADC).

There are several variations to the implementations of the CS based non-uniform sampling architectures [88, 95, 96, 97]. The implementation applies an multiplexer driven by a non-uniform clock, switching the input signal among several parallel S/H based analog queues. A low rate ADC is then used to convert the stored samples, performing at uniform intervals but operating at an average lower sampling rate.

3.4.1 System Model

When the number of measurements is reduced randomly to make the signal x under-sampled at a low average sampling rate, the behaviour of the NUS [14] based CS acquisition model can be represented in matrix form as follows:

$$\begin{bmatrix} | \\ y \\ | \end{bmatrix} = \underbrace{\begin{pmatrix} \epsilon_0 & & \\ & \ddots & \\ & & \epsilon_{n-1} \end{pmatrix}}_{D_{N \times N}} \underbrace{\begin{pmatrix} 1 & \dots & \omega^{0 \cdot (N-1)} \\ \vdots & \ddots & \vdots \\ 1 & \dots & \omega^{(N-1)(N-1)} \end{pmatrix}}_{F_{N \times N}} \begin{bmatrix} | \\ s \\ | \end{bmatrix} \quad (3.11)$$

The matrix multiplication between F and s stands for the input signal x which contains sparse spectrum s , while F is the full discrete time Fourier transform (DFT) matrix. The diagonal matrix D represents the behaviour of non-uniform sampling, where the values ϵ of diagonal items are chosen pseudo-randomly from $\{0, 1\}$.

The NUS based CS acquisition model establishes a sensing matrix A which equals to DF in (3.11). The matrix A can be regarded as random partial Fourier matrix F_T which consists of randomly chosen columns of the DFT indexed by T . [88] has shown that this sensing matrix A always guarantee a stable reconstruction of $\hat{s}$ via l_1 -minimisation using $m = O(\log(N/s))$ samples[88]. Also, greedy algorithms such as OMP and CoSaMP can also be used to achieve fast reconstruction [96].

The main problem of applying RS-ADC lies in sampling high frequency signals. Since the ADC and input MUX have inherent bandwidth limitations, which can be modelled as a low-pass filter preceding the uniform sampling, acquisitions for high frequency signals will result in a loss of the spectrum components. Besides, the high switching speed of the MUX increases noise and reduces the power efficiency.

3.5 Summary

Implementing the CS directly in hardware for real-world system is challenging because it is impractical to design a fully random sensing matrix (Gaussian or Bernoulli matrix) which satisfies the restricted RIP condition that is needed for successful CS reconstruction. As such, partial randomness sensing matrices are proposed, and several CS-ADCs are developed based on these ideas. This chapter hence analyse the operation of three CS based analog-to-digital conversion architectures, viz, RD, MWC, and NUS, and derive their sensing matrix models that will be used in subsequent discussion in this thesis.

Chapter 4

Feature-Based Compressive Spectrum Sensing

In cognitive radio systems, data throughput of the secondary user is an important performance metric used to evaluate the spectrum usage efficiency. As such, the effectiveness of the spectrum sensing process used by the secondary user, namely the spectrum sensing accuracy, its sampling time and processing time will have significant impacts on the data throughput performance. This chapter presents a novel wide-band spectrum sensing technique operating at low sub-Nyquist sampling rate that can achieve high sensing accuracy and high throughput without high computational cost. The proposed technique applies a novel likelihood ratio test on the learned feature information of the primary signal for efficient spectrum sensing, which is based directly on the compressive data collected by a sub-Nyquist sampler (i.e. without performing the CS reconstruction process).

4.1 Introduction

The effect of the spectrum sensing duration on SU's transmission throughput was first published in [48]. Results to achieve the optimal sensing time based on the ED method using the interweave transmission scheme are presented, and the analysis shows that

the trade-off between the sensing duration against the SU's transmission throughput performance is a convex problem. More recent studies on CR's transmission performance focus on the influences of the channel state information [98, 99] and energy efficiency [100].

Most of these works typically assume that SS is performed by the ED method, which is a narrowband SS approach that is noise, i.e. SNR wall problem[37], and would not adapt well to wideband signals due to the ED's Nyquist sampling requirement. As such, other SS methods have also been proposed, such as the CFD [31] and EVD [33], which primarily aim to improve SS noise robustness. Yet other SS techniques focus on tackling the bandwidth issue through the use of the FBS [41] and CS [14]. All these SS techniques have been introduced in Section 2.2.

However, the SU's transmission throughput for the transmission techniques would depend on both the detection accuracy (e.g. the false alarm rate) as well as the time cost used to perform the spectrum detection during the sensing process. As such, it is expected that different SS methods will exhibit different detection accuracy, typically with a tradeoff in sampling time and processing time (more sampled data and more analysis for higher accuracy). Intuitively, a more precise sensing method executable over a shorter sensing duration (which consists of sampling time and processing time) will increase the SU's transmission throughput. However, there is no in-depth study which analyzes the effectiveness of the various spectrum sensing methods with the consideration of processing time cost required to perform the spectrum sensing.

This chapter first analyses the sensing-throughput performance for various existing SS techniques, focusing on the accuracy of their spectrum sensing methods and their corresponding processing time cost. It then presents a novel likelihood ratio test that is based on the feature information (eigenvalues and eigenvectors) of PU's signal and the compressed observations collected by a sub-Nyquist sampler. Performance comparison of the various SS techniques and the proposed technique, in terms of SU's transmission throughput, are then discussed.

4.2 Sensing Frame Model

As earlier described, the performance of CR networks can be evaluated based on the SU's transmission throughput, which is in turn depended on the effectiveness of the spectrum sensing process. This section first presents the spectrum sharing standard IEEE 802.22, follows by the trade-off issue of the sensing-throughput problem.

4.2.1 IEEE 802.22 Standard

In December 2003, FCC identifies CR as a candidate for implementing opportunistic spectrum sharing [101]. IEEE then formed the 802.22 Working Group to develop a standard for wireless regional area networks (WRAN) [102], which is a broadband access scheme operating in unused VHF/UHF TV bands. In addition, CRs are wireless devices used in the WRAN for spectrum sharing between secondary users and the primary users (e.g. the licensed TV operators of the spectrum).

The operating principle of WRAN is based on opportunistic access to temporarily unused TV spectrum. An important objective of CR's design is to maximize the spectrum utilization of the TV channels whenever they are not used by the primary users. To do so, a CR will periodically sense the spectrum to decide whether it can use the spectrum without causing interference to the PUs. The functional requirements of IEEE 802.22 standard specifies that CR must achieve at least 90% probability of correct detection for TV signals with -116 dBm power level and above [102].

4.2.2 WRAN Sensing Frame Model

The WRAN model uses the concept of sensing frames and employs spectrum sensing approach to let SUs to sense (i.e. detect), and then use the spectrum if possible. Figure 4.1 illustrates the concept of sensing frames, where each frame occupies a constant duration of T . During operation, the CR first executes the spectrum sensing process during the initial time slot τ of each frame. It then uses the remaining duration to perform its SU's data transmission, using either the interweave scheme or underlay transmission scheme as appropriate. This is feasible in practice since real world PU's

signal typically has a periodic and known transmit duty cycle that is much longer than the SU frame[47]. As such, the PU activity remains constant during the one frame period T such that synchronization between primary and secondary transmissions is hence possible.

Figure 4.1: Secondary frame structure for CR's periodic spectrum sensing

Denoting the time duration τ as the sensing duration, the remaining duration ($T - \tau$) in each frame is then available for SU to transmit the data [48]. Furthermore, during the sensing duration τ , the SU takes time duration t_1 to collect samples and spends time duration t_2 on processing the data (e.g. calculate the energy of received samples in ED method) in order to determine the presence of PU's signal. In the ideal case, these two tasks can be performed independently (e.g. multi-core based system) such that $\tau = \max\{t_1, t_2\}$, and the remained time for data transmission is $(T - \max\{t_1, t_2\})$. On the other hand, if the two tasks are sequentially executed, e.g. the CR first samples the data, and then process data in order, then $\tau = (t_1 + t_2)$ and the time remained for data transmission is hence $(T - t_1 - t_2)$. Namely, τ is positively correlated with t_1 and t_2 , and can be denoted as $\tau = f(t_1, t_2)$.

4.2.3 Sensing Duration Trade-off

Spectrum sensing methods such as ED [37] typically tries to collect as many samples as possible (i.e. $N = t_1 f_s$ samples where f_s is the sampling rate) in order to achieve a better detection rate, which ensure that the primary communication is protected with high probability [48]. This means that a longer sampling time t_1 should be used in

order to collect more samples, which will also lead to longer processing time t_2 needed to properly detect the status of the primary signal. On the other hand, the use of more sophisticated sensing technique can produce more precise detection result, but at the expense of even longer t_2 . For instance, while CFD can achieve a higher detection accuracy than the ED, its t_2 is of the order $O(N \log N)$ [36], which is much larger than $O(N)$ of ED approach.

In summary, a longer sensing duration τ (such as increase t_1 and t_2 for collecting and processing more samples, or increase t_2 for more sophisticated detection procedure) will enable a better protection to the PU's operation. However, increasing τ will reduce the remaining duration $(T - \tau)$ for SU's data transmission. Hence, there exists a trade-off between the sensing duration τ and achievable throughput for the secondary user.

4.3 Sensing-Throughput Model

Spectrum sharing between the PU and SU is normally performed using either the interweave scheme or underlay scheme. In the interweave scheme, CR devices use the SS to detect the existence of primary signals and transmit its data using allowable full power when primary signals are not detected. In the underlay scheme, the CR devices always transmit its data at a reduced power level that will not affect the QoS standard (e.g. implemented by an interference threshold) required of the primary users. To obtain the maximum performance, hybrid transmission scheme that combines both the interweave and underlay schemes can be used [47]. The following describes operation model of the CR based on these different schemes.

For the interweave scheme, C_0 and C_1 respectively denote the SU's transmission throughput in the presence and absence of the PU. Let γ_s and γ_p be the SNRs of the received power of the SU and the PU measured at the secondary receiver respectively, which are equal to $\gamma_s = \frac{P_s}{\sigma}$ and $\gamma_p = \frac{P_p}{\sigma}$, where P_s is the received power of the SU, P_p is the interference power of the PU measured at the secondary receiver and σ is the variance of the additive white Gaussian noise (AWGN).

For the underlay scheme, C_2 is used to denote the SU's transmission throughput when operating with a reduced transmission power, and γ_{sr} is defined as the SNR of the received power of the SU when the SU is transmitting in underlay mode (i.e. using reduced power). γ_{sr} is then equivalent to $\frac{P_{red}}{\sigma}$, where P_{red} refers to the reduced power for SU's data transmission, which is limited by the interference threshold in underlay scheme.

In addition, assume all the signals are Gaussian, white and independent. The representation of C_0 , C_1 and C_2 can then be expressed as below [47], which are derived based on Shannon formula:

$$C_0 = \log_2(1 + \gamma_s) \quad (4.1)$$

$$C_1 = \log_2\left(1 + \frac{\gamma_s}{1 + \gamma_p}\right) \quad (4.2)$$

$$C_2 = \log_2\left(1 + \frac{\gamma_{sr}}{1 + \gamma_p}\right) \quad (4.3)$$

Spectrum sensing of the primary signal can be achieved by the hypothesis testing model with two hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$ denoting the presence and absence of the PU signal:

$$\begin{aligned} \mathcal{H}_0 : y &= w \\ \mathcal{H}_1 : y &= s + w \end{aligned} \quad (4.4)$$

where the vector y , w and s stands for the received signals, the AWGN, and the primary signal, respectively.

The effectiveness of the detection performance can then be evaluated using the detection rate P_f and false alarm rate P_d aforementioned in Section 2.2.1. Let $P(\mathcal{H}_1)$ denotes the probability of the PU being active, and $P(\mathcal{H}_0) = 1 - P(\mathcal{H}_1)$ as the probability of the PU being inactive. There are hence four possible situations (a) - (d) as follows for a hybrid CR to transmit its data.

- CR transmits SU's data with full power (i.e. interweave mode) (a) when the PU is inactive and there is no false alarm; (b) when the PU is active but PU presence is not detected.

- CR transmits SU's data with reduced power (i.e. underlay mode) (*c*) when the PU is inactive and there is false alarm (i.e. false positive); (*d*) when the PU is active and PU is detected (i.e. true positive).

The upper bound of SU's transmission throughput R of a hybrid CR can hence be obtained by combining the four cases as shown below:

$$R = R_a + R_b + R_c + R_d \quad (4.5)$$

with

$$R_a = \frac{T - \tau}{T} (1 - P_f) P(\mathcal{H}_0) C_0 \quad (4.6)$$

$$R_b = \frac{T - \tau}{T} P_f P(\mathcal{H}_0) C_2 \quad (4.7)$$

$$R_c = \frac{T - \tau}{T} (1 - P_d) P(\mathcal{H}_1) C_1 \quad (4.8)$$

$$R_d = \frac{T - \tau}{T} P_d P(\mathcal{H}_1) C_2 \quad (4.9)$$

where P_d , P_f are the detection rate and false alarm rate, and τ stands for the sensing duration which includes the sampling time t_1 and processing time t_2 . These relationships will be used for further analyses in the upcoming section.

4.4 Proposed Spectrum Sensing

Spectral information of PU's signal (e.g. DTV signals) is typically location dependent but time invariant [103]. As such, spectrum sensing accuracy can be improved if the PU's signal localized characteristics can be used in the detection process [104]. Therefore, a novel likelihood ratio test (LRT) can be proposed for SS that uses the learned features (eigenvalues and eigenvectors) of the PU's signal based on the compressed observations collected by a sub-Nyquist sampler. Compare to existing SS techniques

that typically use non-blind feature detections, our training based SS is more adaptable to real-world situations where the PU's signal characteristics can vary due to surrounding environmental factors. In addition, the SS can be performed completely in the CS domain, without the need to perform the computationally expensive signal reconstruction process. To obtain the maximum throughput for the SU's transmission, the hybrid transmission mode is used to switch the CR between the interweave and underlay schemes as needed. The combination of these methods enables the CR to achieve highly accurate spectrum detection, high SU's transmission throughput while operating at very low sub-Nyquist sampling rate. Table 4.4 shows a notation including all vectors and matrices used in the following derivation for the proposed SS.

Table 4.1: Notation table of vectors and matrices used for the derivation of proposed SS.

Symbol	The Meaning of the Symbol
$s \in \mathbb{R}^N$	Sample vector of primary signal $s(t)$. It's collected under Nyquist sampling rate, during t_0
$w \in \mathbb{R}^N$	Sample vector of Gaussian noise $w(t)$. It's collected under Nyquist sampling rate, during t_0
$A \in \mathbb{R}^{L \times N}$	Sensing matrix in CS framework
$y \in \mathbb{R}^L$	Sample vector collected by CS based sampler
$y_m \in \mathbb{R}^L$	m -shifted vector from the y
$s_m \in \mathbb{R}^N$	m -shifted vector from the s
$C_y \in \mathbb{R}^{L \times L}$	Sample covariance matrix, equals to $\frac{1}{M} \sum_{m=1}^M y_m y_m^T$
$C_s \in \mathbb{R}^{L \times L}$	Sample covariance matrix, equals to $\frac{1}{M} \sum_{m=1}^M A s_m (A s_m)^T$
$C_w \in \mathbb{R}^{L \times L}$	Sample covariance matrix from $\frac{1}{M} \sum_{m=1}^M A w_m (A w_m)^T$
$R_A \in \mathbb{R}^{L \times L}$	Matrix defined as $A A^T$
$\phi_{y,l} \in \mathbb{R}^L$	Eigenvectors from eigen-decomposition of C_y
$\phi_{s,l} \in \mathbb{R}^L$	Eigenvectors from eigen-decomposition of C_s
$\phi_{a,l} \in \mathbb{R}^L$	Eigenvectors from eigen-decomposition of R_A
$C_s^1 \in \mathbb{R}^{L \times L}$	Simplified sample covariance matrix which is approximately equals C_s when leading eigenvalue structure exists. It equals to $\lambda_{s,1} \phi_{s,1} \phi_{s,1}^T$

4.4.1 Sampling Model

Consider a zero-mean primary signal $s(t)$ mixed with AWGN $w(t)$ that is received by a CR's receiver. A conventional Nyquist rate ADC with f_s sampling frequency and sampling period $t_0 = N/f_s$ will acquire N samples from the $s(t)$ and $w(t)$ respectively, which are denoted as $s = [s[1], \dots, s[N]]^T$ and $w = [w[1], \dots, w[N]]^T$.

On the other hand, our proposed SS system employs the random demodulator (RD, see Figure 2.11) in the SU's receiver. As such, the ADC can operate at a sub-Nyquist sampling rate $(\frac{L}{N}f_s)$, where L refers to the number of samples collected over the duration t_0 and is denoted as $y = [y[1], y[2], \dots, y[L]]^T$. (As $L < N$, $(\frac{L}{N})$ is the compression ratio.) The relationship between the s , w and y can be described as a projection through the sensing matrix A under the following hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$ [60]:

$$\begin{aligned}\mathcal{H}_0 : y &= Aw, \\ \mathcal{H}_1 : y &= A(s + w).\end{aligned}\tag{4.10}$$

4.4.2 Covariance and Eigenvectors

Now assuming that over the duration of a sensing frame T (see Figure 4.1), the proposed SS obtains $(M + L - 1)$ samples during the sensing duration τ . The overall samples y are now organised as Γ_y , where $\Gamma_y = \{y_1, y_2, \dots, y_M\}$ and each y_m is the m -shifted vector from the y :

$$y_m = [y[m], y[m+1], \dots, y[m+L-1]]^T.\tag{4.11}$$

As such, $y_m \sim \mathcal{N}(0, C_y)$ where C_y is the covariance matrix with the following form:

$$C_y = \frac{1}{M} \sum_{m=1}^M y_m y_m^T.\tag{4.12}$$

Performing the eigen-decomposition of C_y leads to

$$C_y = \Phi_y \Lambda_y \Phi_y^T = \sum_{l=1}^L \lambda_{y,l} \phi_{y,l} \phi_{y,l}^T.\tag{4.13}$$

The eigenvectors and eigenvalues of C_y are hence equal to $\{\phi_{y,l}\}$ and $\{\lambda_{y,l}\}$ respectively, with $\lambda_{y,1} \geq \lambda_{y,2} \geq \dots \geq \lambda_{y,L}$ assumed.

4.4.3 Proposed Spectrum Sensing

Since the received primary signal s and noise w are uncorrelated, the distribution of received signal vector y_m under $\mathcal{H}_0$ and $\mathcal{H}_1$ of (4.10) can now be expressed as

$$\begin{aligned}\mathcal{H}_0 : y_m &\sim \mathcal{N}(0, \sigma^2 C_w I), \\ \mathcal{H}_1 : y_m &\sim \mathcal{N}(0, C_s + \sigma^2 C_w I),\end{aligned}\tag{4.14}$$

where σ stands for the variance of w , and

$$\begin{aligned}C_s &= \frac{1}{M} \sum_{m=1}^M (As_m)(As_m)^T, \\ C_w &= \frac{1}{M} \sum_{m=1}^M (Aw_m)(Aw_m)^T = \sigma^2 AA^T.\end{aligned}\tag{4.15}$$

In (4.15), s_m and w_m are the m -shifted vector from the s and w . For instance, $s_m = [s[m], s[m+1], \dots, s[m+N-1]]^T$. Detecting the presence of s can then be based on the probability of $\mathcal{H}_1$ given the observation Γ_y in one frame, which is derived as follows.

Assume $\{y_m\}$ are identical and independent, the probability $p(\Gamma_y|\mathcal{H}_1)$ is then equal to

$$\begin{aligned}p(\Gamma_y|\mathcal{H}_1) &= \prod_{m=1}^M p(y_m|\mathcal{H}_1) \\ &= \prod_{m=1}^M \frac{1}{(2\pi)^{\frac{L}{2}} \det^{\frac{1}{2}}(C_s + C_w)} \exp \left[-\frac{1}{2} y_m^T (C_s + C_w)^{-1} y_m \right].\end{aligned}\tag{4.16}$$

Representing $C_w = \sigma^2 AA^T$ as $\sigma^2 R_A$, and considering

$$\begin{aligned}\det(C_s + \sigma^2 R_A) &= \det(\sigma^2 R_A) \times \det(I + C_s(\sigma^2 R_A)^{-1} I) \\ &= \prod_{l=1}^L (\sigma^2 \lambda_{a,l}) \prod_{l=1}^L \left(1 + \frac{\lambda_{s,l}}{\sigma^2 \lambda_{a,l}}\right) = \prod_{l=1}^L (\sigma^2 \lambda_{a,l} + \lambda_{s,l}),\end{aligned}\tag{4.17}$$

where $\{\lambda_{a,l}\}$ and $\{\lambda_{s,l}\}$ are the eigenvalues obtained from the eigen-decompositions of R_A and C_s respectively. With $\phi_{a,l}^T \phi_{a,l} = 1$, the logarithm of $p(\Gamma_y|\mathcal{H}_1)$ in (4.16) is then

equal to:

$$\begin{aligned}
 \ln p(\Gamma_y|\mathcal{H}_1) &= -\frac{1}{2} \sum_{m=1}^M \sum_{l=1}^L \frac{y_m^T y_m}{\phi_{s,l} \lambda_{s,l} \phi_{s,l}^T + \phi_{a,l} \lambda_{a,l} \phi_{a,l}^T \sigma^2} \\
 &\quad - \frac{LM}{2} \ln 2\pi - \frac{M}{2} \sum_{l=1}^L \ln(\lambda_{a,l} \sigma^2 + \lambda_{s,l}) \\
 &= -\frac{1}{2} \sum_{m=1}^M \sum_{l=1}^L \frac{(\phi_{a,l}^T y_m)^2}{\phi_{a,l}^T \phi_{s,l} \lambda_{s,l} \phi_{s,l}^T \phi_{a,l} + \lambda_{a,l} \sigma^2} \\
 &\quad - \frac{LM}{2} \ln 2\pi - \frac{M}{2} \sum_{l=1}^L \ln(\lambda_{a,l} \sigma^2 + \lambda_{s,l}).
 \end{aligned} \tag{4.18}$$

Similarly,

$$\ln p(\Gamma_y|\mathcal{H}_0) = -\frac{LM}{2} \ln 2\pi - \frac{M}{2} \sum_{l=1}^L \ln(\lambda_{a,l} \sigma^2) - \frac{1}{2} \sum_{m=1}^M \sum_{l=1}^L \frac{(\phi_{a,l}^T y_m)^2}{\lambda_{a,l} \sigma^2}. \tag{4.19}$$

The difference between $\ln p(\Gamma_y|\mathcal{H}_1)$ and $\ln p(\Gamma_y|\mathcal{H}_0)$ can then be regarded as the test function to identify the presence of primary signal:

$$T(y) = \ln p(\Gamma_y|\mathcal{H}_1) - \ln p(\Gamma_y|\mathcal{H}_0) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\geq}} \eta \tag{4.20}$$

However, it will be costly to calculate (4.20) directly due to the high computational complexity in (4.18) and (4.19). But the computation speed can be greatly increased if the eigenvalue $\lambda_{s,l}$ has the property $\lambda_{s,1} \gg \lambda_{s,2} = \dots = \lambda_{s,L} = \epsilon$ where ϵ is a small number close to zero, in this case, $C_s \approx C_s^1 \triangleq \lambda_{s,1} \phi_{s,1} \phi_{s,1}^T$.

As a matter of fact, the leading eigenvalue structure often exists in primary signals, e.g. OFDM based digital television signals [105]. Hence (4.18) and (4.19) can be expressed as:

$$\begin{aligned}
 \ln p(\Gamma_y|\mathcal{H}_1) &= -\frac{M}{2} \left[\frac{\phi_{a,1}^T C_y \phi_{a,1}}{\phi_{a,1}^T C_s^1 \phi_{a,1} + \lambda_{a,1} \sigma^2} + \frac{\frac{1}{M} \sum_{m=1}^M y_m^T y_m - \phi_{a,1}^T C_y \phi_{a,1}}{\epsilon \sigma^2} \right] \\
 &\quad - \frac{LM}{2} \ln 2\pi - \frac{M}{2} [\ln(\lambda_{s,1} + \lambda_{a,1} \sigma^2) + (L-1) \ln(\epsilon \sigma^2)]
 \end{aligned} \tag{4.21}$$

and

$$\ln p(\Gamma_y|\mathcal{H}_0) = -\frac{M}{2} \left[\frac{\phi_{a,1}^T C_y \phi_{a,1}}{\lambda_{a,1} \sigma^2} + \frac{\frac{1}{M} \sum_{m=1}^M y_m^T y_m - \phi_{a,1}^T C_y \phi_{a,1}}{\epsilon \sigma^2} \right] - \frac{LM}{2} \ln 2\pi - \frac{M}{2} [\ln(\lambda_{a,1} \sigma^2) + (L-1) \ln(\epsilon \sigma^2)]. \quad (4.22)$$

If the unknown σ can be found, the test function (4.20) for the proposed spectrum sensing can also be found. Here the maximum likelihood estimation (MLE) can be applied to obtain the unknown σ .

To do so, the differential expression of the (4.21) and (4.22) is derived, leading to

$$\begin{aligned} \frac{\partial \ln p(\Gamma_y|\sigma^2, \mathcal{H}_1)}{\partial \sigma^2} &= -\frac{M}{2} \left[\frac{\lambda_{a,1}}{\lambda_{s,1} + \lambda_{a,1} \sigma^2} + \frac{(L-1)}{\sigma^2} \right] \\ &- \frac{M}{2} \left[-\frac{\phi_{a,1}^T C_y \phi_{a,1} \lambda_{a,1}}{(\phi_{a,1} C_s^1 \phi_{a,1} + \lambda_{a,1} \sigma^2)^2} - \frac{\frac{1}{M} \sum_{m=1}^M y_m^T y_m - \phi_{a,1}^T C_y \phi_{a,1}}{\epsilon (\sigma^2)^2} \right] \end{aligned} \quad (4.23)$$

and

$$\begin{aligned} \frac{\partial \ln p(\Gamma_y|\sigma^2, \mathcal{H}_0)}{\partial \sigma^2} &= -\frac{M}{2} \left[\frac{1}{\sigma^2} + \frac{(L-1)}{\sigma^2} \right] \\ &- \frac{M}{2} \left[-\frac{\phi_{a,1}^T C_y \phi_{a,1}}{\lambda_{a,1} (\sigma^2)^2} - \frac{\frac{1}{M} \sum_{m=1}^M y_m^T y_m - \phi_{a,1}^T C_y \phi_{a,1}}{\epsilon (\sigma^2)^2} \right] \end{aligned} \quad (4.24)$$

Setting $\frac{\partial \ln p(\Gamma_y|\sigma^2, \mathcal{H}_0)}{\partial \sigma^2} = 0$ in (4.24), it then provides the MLE of σ^2 under $\mathcal{H}_0$:

$$\hat{\sigma}_0^2 = \frac{\phi_{a,1}^T C_y \phi_{a,1}}{\lambda_{a,1} L} + \frac{\sum_{m=1}^M y_m^T y_m - \phi_{a,1}^T C_y \phi_{a,1}}{\epsilon L} \quad (4.25)$$

Similarly, setting $\frac{\partial \ln p(\Gamma_y|\sigma^2, \mathcal{H}_1)}{\partial \sigma^2} = 0$ in (4.23), and with $\frac{\partial \ln p(\Gamma_y|\lambda_{a,1}, \sigma^2, \mathcal{H}_1)}{\partial \lambda_{a,1}} = -\frac{M}{2} \left(\frac{\sigma^2}{\lambda_{s,1} + \lambda_{a,1} \sigma^2} - \frac{\sigma^2 \phi_{a,1}^T C_y \phi_{a,1}}{(\phi_{a,1} C_s^1 \phi_{a,1} + \lambda_{a,1} \sigma^2)^2} \right) = 0$, it has the MLE of σ^2 under $\mathcal{H}_1$:

$$\hat{\sigma}_1^2 = \frac{\frac{1}{M} \sum_{m=1}^M y_m^T y_m - \phi_{a,1}^T C_y \phi_{a,1}}{\epsilon (L-1)} \quad (4.26)$$

Substitute the estimated $\hat{\sigma}_0^2$ and $\hat{\sigma}_1^2$ back to the (4.22) and (4.21), the updated logarithm likelihood function becomes:

$$\begin{aligned} \ln p(\Gamma_y|\mathcal{H}_1) = & -\frac{LM}{2} \ln 2\pi - \frac{M}{2} [\ln(\lambda_{s,1} + \lambda_{a,1}\sigma_1^2) + (L-1) \ln(\epsilon\sigma_1^2)] \\ & - \frac{M}{2} \left[\frac{\phi_{a,1}^T C_s^1 \phi_{a,1} + \lambda_{a,1}\sigma_1^2}{\lambda_{s,1} + \lambda_{a,1}\sigma_1^2} + (L-1) \right] \end{aligned} \quad (4.27)$$

and

$$\ln p(\Gamma_y|\mathcal{H}_0) = -\frac{LM}{2} \ln 2\pi - \frac{M}{2} [\ln(\lambda_{a,1}\sigma_0^2) + (L-1) \ln(\epsilon\sigma_0^2) + L] \quad (4.28)$$

With (4.27) and (4.28), the test function for the proposed spectrum sensing (4.29) becomes:

$$T(y) = \ln \frac{\lambda_{a,1}\sigma_0^2}{\lambda_{s,1} + \lambda_{a,1}\sigma_1^2} + (L-1) \ln \frac{\sigma_0^2}{\sigma_1^2} + \frac{\lambda_{s,1} - \phi_{a,1}^T C_s^1 \phi_{a,1}}{\lambda_{s,1} + \lambda_{a,1}\sigma_1^2} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta \quad (4.29)$$

with

$$\begin{aligned} \hat{\sigma}_0^2 &= \frac{\phi_{a,1}^T C_y \phi_{a,1}}{\lambda_{a,1}L} + \frac{\sum_{m=1}^M y_m^T y_m - \phi_{a,1}^T C_y \phi_{a,1}}{\epsilon L}, \\ \hat{\sigma}_1^2 &= \frac{\frac{1}{M} \sum_{m=1}^M y_m^T y_m - \phi_{a,1}^T C_y \phi_{a,1}}{\epsilon(L-1)}, \end{aligned} \quad (4.30)$$

To compute $T(y)$, the values of $\lambda_{a,1}$, $\phi_{a,1}$, σ_0^2 , σ_1^2 , $\lambda_{s,1}$ and C_s^1 are needed, which can be obtained as follows. $\lambda_{a,1}$ and $\phi_{a,1}$ can be pre-calculated (before spectrum detection operation) based on the eigen-decomposition of AA^T , since the sensing matrix A is known a priori (based on the fixed architecture of the RD in Figure 2.11). σ_0^2 and σ_1^2 can be obtained using $\frac{1}{M} \sum_{m=1}^M y_m^T y_m$ which is based on the (real-time) compressive samples y_m . Computation of C_s^1 and $\lambda_{s,1}$ requires the feature information ($\lambda_{s,1}$ and $\phi_{s,1}$) of the primary signals, which can also be pre-calculated using a training approach as described in the next section.

4.4.4 Training Procedure

In order to execute the proposed SS based on (4.29), a training process is first used to learn (i.e. extract) the signal features, $\lambda_{s,1}$ and $\phi_{s,1}$. The training is based on

the approach proposed in [106]. It is an efficient learning technique based on the observation that $\phi_{s,1}$ of the non-white wide-sense stationary signal is typically stable over time (while $\phi_{w,1}$, the leading eigenvector of the sample covariance of the white noise, is random.) The flow of the training approach is shown in Algorithm 1 below, which basically try to ascertain the stability of $\phi_{s,1}$ in neighbouring frames based on a “similarity” parameter $\rho_{i,i+1}$ defined as follows:

$$\rho_{i,i+1} = \max_{h=1,2,\dots,L} \left| \sum_{k=1}^L \phi_{s,1}^i[k] \phi_{s,1}^{i+1}[k+h] \right|, \quad (4.31)$$

where $\phi_{s,1}^i[k]$ represents the k th element in $\phi_{s,1}$ in the i th frame.

Algorithm 1 Training for Features $\lambda_{s,1}$ and $\phi_{s,1}$ [106]

Input: Group of samples $\Gamma_{s,i}$ and $\Gamma_{s,i+1}$.

Output: Feature $\lambda_{s,1}$ and $\phi_{s,1}$.

- 1: $i \leftarrow 0, \rho_{i,i+1} \leftarrow 0$.
 - 2: **while** $\rho_{i,i+1} < T_e$ **do** ▷ similarity is not high enough (feature isn't stable)
 - 3: Extract $\phi_{s,j}^i$ and $\phi_{s,j}^{i+1}$ from $\Gamma_{s,i}$ and $\Gamma_{s,i+1}$ using (4.13).
 - 4: Update $\rho_{i,i+1}$ between the two features using (4.31).
 - 5: $i \leftarrow i + 1$.
 - 6: **end while**
 - 7: $\phi_{s,1} \leftarrow \phi_{s,1}^i, \lambda_{s,1} \leftarrow \lambda_{s,1}^i, C_s^1 \leftarrow \lambda_{s,1} \phi_{s,1} \phi_{s,1}^T$.
-

During the training process, $\lambda_{s,1}$ and $\phi_{s,1}$ are considered to be successfully obtained when $\rho_{i,i+1}$ exceeds the predetermined threshold value T_e . The information can then be used in equation (4.29) for the proposed SS technique, which is presented in the next section.

4.4.5 Program flow of proposed SS

With $\lambda_{s,1}$ and $\phi_{s,1}$ obtained through the training procedure, the spectrum detection test function (4.29) can be executed, using the pre-determined $\lambda_{a,1}$, $\phi_{a,1}$ and the real-time calculated σ_0^2 and σ_1^2 from compressive samples. Execution of this test function

can be implemented based on the program flow shown in Algorithm 2, which includes an initialization for the training procedure.

Algorithm 2 Proposed Spectrum Sensing with Learning Features

Input: Sensing matrix A , real-time compressive samples y_m

Output: Hypothesis boolean result $\mathcal{H} = \mathcal{H}_0$ or $\mathcal{H}_1$. ($\mathcal{H}_1$ indicates PU is present)

Initialize: (a) Calculate $\lambda_{s,1}$, $\phi_{s,1}$ and C_s^1 using Algorithm 1.

(b) Compute $\lambda_{a,1}$, $\phi_{a,1}$ using the eigen-decomposition of AA^T .

1: Compute $C_y \leftarrow \frac{1}{M} \sum_{m=1}^M y_m y_m^T$.

2: Calculate σ_0^2 and σ_1^2 using (4.30), depending on y_m , C_y , $\lambda_{a,1}$ and $\phi_{a,1}$.

3: Calculate $T(y)$ using (4.29), depending on $\lambda_{a,1}$, $\phi_{a,1}$, σ_0^2 , σ_1^2 , $\lambda_{s,1}$ and C_s^1 .

4: **if** $T(y) \leq \eta$ **then**

5: $\mathcal{H} = \mathcal{H}_0$ $\triangleright T(y) \leq \eta$, it indicates PU is absent.

6: **else**

7: $\mathcal{H} = \mathcal{H}_1$ $\triangleright T(y) > \eta$, it indicates PU is present.

8: **end if**

During the initialization procedure, parameters $\lambda_{s,1}$, $\phi_{s,1}$, $\lambda_{a,1}$ and $\phi_{a,1}$ are determined and stored. These are then used with the compressive samples y_m to compute the test function $T(y)$ in real-time. Spectrum detection (4.29) is then performed by comparing $T(y)$ against the pre-determined threshold η , where η is set empirically such that at least 90% probability of correct detection can be achieved, equivalent to $P_d \geq 0.9$ [102].

Figure 4.2 shows the high-level architecture of the proposed SS. The compressive samples y_m are collected by the RD in real-time operating at sub-Nyquist rate, while the LRT executes the spectrum detection algorithm and generates the detection result, using the pre-calculated parameters obtained during the initialization.

Figure 4.2: High-level architecture of the proposed spectrum sensing.

4.4.6 Sensing Frame Structure

Figure 4.3 shows the frame structure used in the proposed SS approach. It includes an initialization phase followed by the conventional sensing frames, each of duration T that can be used for real-time data transmission.

Figure 4.3: Secondary frame structure for the proposed spectrum sensing

During the initialization phase, a CR armed with the proposed SS will first calculate $\lambda_{a,1}$ and $\phi_{a,1}$ that lasts time duration τ_0 . It then starts the training procedure to learn the signal features, which will take $k\tau$ duration (i.e. multiples of the normal sensing frames duration, with $k \geq 2$ as indicated in Algorithm 1). Once all these parameters are established, the CR data transmission can commence as per normal. As such, the initialization phase will only occur once at the beginning, and will not affect the

data throughput during normal operation. For example, the target primary signal encountered in IEEE 802.22 environment, the VHF/UHF TV signal normally contains stable localized characteristics (location dependent but time invariant) [103] such that re-training is not needed. On the other hand, the initialization procedure (specifically the feature learning process) can be triggered if the performance is found to have deteriorated drastically during operation.

4.4.7 Complexity Analysis

The complexity of the proposed SS can be determined as follows. The initialization phase involves the calculation of sample covariance matrix and eigenvalue decomposition for the pre-stored parameters. The computational cost during this phase is of $O(ML + M^3)$ since it has the same complexity as that of EVD [40]. During data transmission operation, the real-time calculation of σ_0^2 and σ_1^2 contributes the most to the computational cost, with $\frac{1}{M} \sum_{m=1}^M y_m^T y_m$ setting the upper bounds of the computational complexity to be of $O(ML)$.

Assuming that the initialization doesn't need to be repeated, the processing time (termed as t_2 in Section 2.2) of the proposed SS is hence bounded by the real-time computational cost $O(ML)$, which is related to the achievable transmission throughput R for secondary user as indicated in (4.5).

The complexity of our proposed SS technique is included in Table 2.1 that also summarizes the complexity of all SS techniques analyzed earlier. Compared to the fastest SS technique which is based on ED that takes a processing time $t_{2(ED)} = O(N)$ to complete the spectrum detection, the proposed SS takes $O(ML)$. Both of these will be of similar complexity order when M is empirically small, e.g. $M < 20$ as shown in [40], which also happens to be the case for the proposed SS technique.

Table 4.2 summarizes the processing cost t_2 for wideband spectrum sensings which affect the SU's transmission throughput in term of computational complexity. These will be used to benchmark against our proposed SS technique to be presented in the upcoming section. Therefore, to further verify the correctness of our theoretical analysis,

simulation tests are hence performed, with the results presented in next section.

Table 4.2: Comparison of existing wideband spectrum sensing techniques.

Performance	Wideband Spectrum Sensing		
	Multi-band ED [13]	CS [34]	Proposed
Time Cost	$O(N)$ [a]	$O(kmN)$ [d]	$O(ML)$ [c]
Hardware Cost	multiple ADCs, mixing circuits	single ADCs, mixing circuit	single ADCs, mixing circuit
Sampling Rate	Nyquist	Sub-Nyquist	Sub-Nyquist

[a] N is the number of samples.

[b] k is the sparsity. m is the number of compressive samples. N is the length of samples under Nyquist rate sampling.

[c] M is the smoothing factor of sample covariance. L is the length of each sample vector under sub-Nyquist rate.

4.5 Simulation Results

This section presents the simulation results to validate the correctness of the proposed technique, and compares SU's transmission throughput against other SS techniques described earlier, viz, the ED, CFD, EVD and CS(with reconstruction follows by ED), all operating using the hybrid scheme. (MF and FBS are not included, since MF is not really practical for real-world application while FBS can be regarded as a multi-band version of the ED method.)

Data used in the simulation are based on the samples of real world ATSC DTV signals [103], which are collected in Washington D.C, USA. The sampling rate of the vestigial sideband (VSB) DTV signal at the receiver is 10.762 MHz [107]. (i.e. over-sampling factor of 2 compared to the DTV bandwidth). As suggested in [108], the data are first filtered and then re-sampled at 6MHz for simulation tests. (For the proposed

CS based sensing technique, the sampling is actually performed at an optimal rate of 1MHz which is determined empirically). The multipath channel and the SNR of the received signal are unknown. As such, AWGN is added to obtain various SNR levels in our simulation tests.

In the simulation tests, receiver's sensitivity is set at -116 dBm for sensing DTV signals with 90% probability of positive detection [109]. Setting of other relevant parameters are as used in [48]: Frame duration $T = 100$ ms, $P(\mathcal{H}_1) = 0.2$ and the target detection rate $\overline{P_d} = 0.9$. Sampling rate of 6MHz is used by non-CS related SS techniques, while both the CS based techniques use 1MHz as mentioned earlier.

It is also assumed that there is a fixed channel attenuation of 10 dB for the channel between the SU and the primary receiver as used in [110]. It also assumes that during the spectrum sensing duration, the task of data collection and the task of data processing are independent such that both tasks can be executed in parallel, i.e. the ideal case. Thus the sensing duration $\tau = \max\{t_1, t_2\}$, and the remaining time, $(T - \tau)$ is for SU's data transmission. The simulation is performed in Matlab running on a 3.5 GHz processor (Intel Xeon CPU E5-1650).

4.5.1 Sampling Time versus Throughput

To avoid interfering with the PUs' operations, each SS technique is required to achieve minimum 90% probability of detection [109]. As such, sufficient sampling time t_1 must be used to collect the number of samples required. However, while bigger number of samples would improve the detection accuracy, it also requires longer processing time t_2 , and correspondingly reduces the remaining time for SU transmission. As such, this is equivalent to solve an optimization problem (i.e. trade-off issue), finding the optimal sampling time t_1 to achieve the maximum SU's transmission throughput.

Figure 4.4 presents the results that relate the sampling time versus SU's transmission throughput for the different SS techniques (with $IT = -130$ dBW, $\overline{P_d} = 0.9$, $T = 100$ ms, $\gamma_p = -15$ dB, $\gamma_s = 20$ dB). It can be observed that each SS technique requires different sampling time in order to achieve their individual optimum SU's

Figure 4.4: SU's transmission throughput versus sampling time, with $IT = -130$ dBW, $\overline{P_d} = 0.9$, $T = 100$ ms, $\gamma_p = -15$ dB, $\gamma_s = 20$ dB

transmission throughput. For instance, the results show that ED-based CR reaches its maximum attainable transmission throughput when its sampling time is 1.75 ms, while the proposed CS based hybrid transmission CR achieve its optimum throughput at the duration of 1.25 ms. A point to note is while Figure 4.4 shows the throughput performance against the sampling duration t_1 (i.e., against the number of sampled data, as being normally done in most SS related publications), the actual throughput will also depend on the processing time t_2 , which is typically assumed to be negligible in most studies, but is not necessary so as will be shown in the next section.

4.5.2 Sensing Duration versus Throughput

Table 4.3 further presents the detail of the individual spectrum sensing parameters (P_d , P_f , t_1 , t_2 and sampling characteristic) when each SS is operating at its optimal data

throughput.

Table 4.3: Achieved spectrum sensing parameter under optimal SU's transmission throughput

	Detection Rate	False Alarm Rate	Number of Samples	Sampling Time (ms)	Process Time (ms)	Sampling Rate (MHz)	System Throughput (bits/sec/Hz)
Proposed	91.5%	1.0%	1250	1.25	0.6	1	5.793
ED	91.8%	2.5%	7500	1.75	0.17	6	5.677
CFD	92.0%	0.9%	3000	0.5	16.0	6	5.049
EVD	89.0%	3%	7500	1.75	14.4	6	4.738
CS	91.3%	13%	1000	1.0	17.3	1	2.893

Compared to the EVD, CFD and CS techniques, the proposed method exhibits very good false alarm rate and requires much shorter sensing duration (calculated based on either $\max\{t_1, t_2\}$ or $(t_1 + t_2)$), which hence allow more time for SU's data transmission. Compared to the next best performance technique based on ED, which has shorter processing time t_2 but longer sampling time t_1 , our proposed method uses lower sampling rate over shorter sampling duration, hence lower number of sampled data. It also exhibits lower false alarm rate, such that it can correctly transmit at full power (i.e. interweave scheme) more frequently. Combination of all these factors enables the proposed technique to exhibit the best transmission throughput performance as shown in Table 4.3.

4.5.3 Interference Threshold versus Throughput

Another factor that can affect the SU's transmission throughput is the setting of the PU's interference threshold (IT) level. A higher IT setting indicates that the PU can tolerate a higher level of interference by CR transmitting in underlay scheme. As such, the secondary transmission power during underlay operation can be correspondingly

increased, which increases the overall data throughput. Figure 4.5 illustrates the SU's transmission throughput against different PU's IT settings for various SS techniques (with $\overline{P_d} = 0.9$, $T = 100$ ms, $\gamma_p = -15$ dB, $\gamma_s = 20$ dB). The tests are again performed at the optimal sampling duration for each SS technique.

Figure 4.5: SU's transmission throughput versus PU interference threshold, with $\overline{P_d} = 0.9$, $T = 100$ ms, $\tau_{proposed} = 1.25$ ms, $\gamma_p = -15$ dB, $\gamma_s = 20$ dB

As expected, as the PU interference threshold is set higher, the throughput increases for all CRs since their underlay transmission power can be set at a higher level. The SU's transmission throughput for our proposed technique is again consistently higher than other SS techniques for all settings of PU interference thresholds.

4.5.4 Primary Signal's Active Rate versus Throughput

Above results are generated based on the assumption that the probability that a PU (or its primary signal) is active is fixed at 20%, i.e. $P(H_1) = 0.2$. However, when

PU is more active, SU will have less opportunity to transmit at full power using the interweave scheme, and hence causes a reduction in SU's transmission throughput. Figure 4.6 presents the results when the PU's active rate is varied. As expected, higher PU active rate causes the throughput to decrease proportionally.

Figure 4.6: Achievable SU's transmission throughput versus PU active rate, with $\overline{P_d} = 0.9$, $T = 100$ ms, $\gamma_p = -15$ dB, $\gamma_s = 20$ dB

As the PU active rate does not affect the detection accuracy as well as the processing time of SS, the performance of the achievable SU's transmission throughput of all techniques will remain similar, while the proposed approach is always better than the others under all PU active rates.

4.5.5 Received PU's SNR versus Throughput

To display the effect of the SNR on SU's throughput performance of the various SS techniques, different amount of white noise is added to the data for each simulation

test. Figure 4.7 presents the impact of different levels of SNR (γ_p) in the primary signal on the performance of the various SS techniques. The tests are performed at the optimal sampling duration for each SS techniques as shown in Figure 4.4 (e.g. ED is tested at the $t_1 = 1.75$).

Figure 4.7: Achievable SU's transmission throughput versus primary signal's SNR, ($IT = -130$ dBW, $\overline{P_d} = 0.9$, $T = 100$ ms, $\gamma_s = 20$ dB)

As expected, all SS techniques exhibit a monotonic increase in their achievable SU's transmission throughput with higher SNR in the received primary signal. It can be also noted that, the proposed SS technique consistently achieves better throughput performance under all the conditions, indicating a better noise robustness. Compared to the other feature-based SS, such as the EVD and CFD, the proposed SS technique is able to achieve better performance due to its better accuracy and computational efficiency. Compared to non-feature-based SS techniques, such as the ED and CS, the proposed SS technique is consistently more robust to noise, in particular when operating

under low SNR condition.

4.6 Summary

This chapter presents a comprehensive analysis of the various SS techniques and provides their corresponding sensing-throughput comparison based on detection accuracy and the processing time. A novel feature-based SS technique using likelihood ratio test is then proposed. By detecting the eigenvectors of the primary signal during the SS, the proposed technique can exhibit good P_d and P_f values with short processing duration under noisy condition. Simulation is based on the captured ATSC DTV data, operating in IEEE 802.22 WRAN environment. Compared to other well-known spectrum sensing techniques, results have shown that the proposed technique, operating at 0.17 time of the Nyquist sampling rate, can achieve a better SU's transmission throughput due to its higher detection accuracy and shorter spectrum sensing duration.

Chapter 5

Matrix Optimized Wideband Receiver for CS-Based Cognitive Radio

The modulated wideband converter is the most commonly adopted technique for implementing sub-Nyquist compressive sampling based wireless receiver to reduce the analog and digital processing complexity when detecting wideband spectrum for cognitive radio systems. However, the issue of non-optimal mutual coherence, which leads to a higher receiving bit error rate, has not been considered in existing compressive sampling based cognitive radio studies. Furthermore, existing theoretical compressive sampling based solutions cannot be directly applied because typical modulated wideband converter based designs use fixed parameters that cannot be easily updated during its sampling operations. This chapter hence presents a novel matrix optimization which can be incorporated into modulated wideband converter in cognitive radios.

5.1 Matrix Optimization

For system such as the MWC-based receiver which collects data using multiple sampling channels, the reconstruction can be solved by deriving $\hat{S} \in \mathbb{C}^{N \times N}$ [111] as follows:

$$\hat{S} = \arg \min \|S\|_{2,1} \quad s.t. \quad Y = \Phi\Psi S = AS, \quad (5.1)$$

where Φ is an $M \times L$ measurement matrix representing the sampling structure (i.e. the MWC); M refers to the number of sampling channels; $Y \in \mathbb{C}^{M \times L}$ refers to the M groups of observations (L samples in each group); $\|\cdot\|_{2,1}$ denotes the sum of the Euclidean norms of the columns of the matrix. The $\Psi \in \mathbb{C}^{L \times N}$ is the dictionary matrix comprised of Fourier matrix with selection indices [111]. The $M \times N$ matrix A , which equals to $\Phi\Psi$, is denoted as the sensing matrix of the MWC, representing the model of the overall sampling operation.

Solving (5.1) is known as the multiple measurement vector (MMV) problem in the literature of CS framework. It can be performed using the continuous-to-finite block [63], which transforms MMV into single measurement vector (SMV) problem, and then using traditional CS based reconstruction, i.e. greedy pursuit based solution [66, 94], which can be adopted to reconstruct the $\hat{S}$ in (5.1).

Although the CS can enables an efficient reduction in sampling rate, it has been noted that a non-optimal sensing matrix (or non-optimal mutual coherence) is one of the most important factors that negatively affects the reconstruction performance. This optimality is often measured by the mutual coherence $\mu(A)$ which is defined as the maximum correlation value between any two columns of the sensing matrix A .

A smaller $\mu(A)$ indicates that the sensing matrix contains more independent columns, which can lead to a lower level of CS reconstruction error [16]. Minimization of $\mu(A)$ has been studied in the form of sensing matrix optimization [16, 17, 18].

However, for the MWC circuit used in the OFDM based CS-CR receiver, the issue of mutual coherence has always been neglected in existing designs [3, 86, 112]. An important reason is due to the fact that Φ and Ψ are pre-determined such that the

sensing matrix A cannot be directly updated during the sampling operation. The conventional approach based on solving (2.14) [16, 18] is hence not feasible.

Since the A cannot be directly updated due to the hardware constraints, a possible solution could be generating an external projection matrix P and optimizing the $\{PA\}$, where P can be fully controlled and updated at any time (i.e. processed in digital domain). In this paper, we propose an algorithm to optimize the mutual coherence by finding a matrix P for fixed A such that

$$\min_P \|I_N - A^T P^T P A\|_F^2 \text{ s.t. } A = \Phi\Psi. \quad (5.2)$$

The solution of this problem returns the matrix $P_{opt} \in \mathbb{C}^{M \times M}$ that minimizes the $\mu(P_{opt}A)$, where the $\{P_{opt}A\}$ can be regarded as the updated equivalent sensing matrix.

5.2 Matrix Optimized MWC-OFDM System

OFDM has been widely used in modern wireless applications such as digital television broadcasting and 4G as well as the latest 5G mobile communications. It uses a number of closely spaced orthogonal subcarrier signals of equal bandwidth to transfer data on several parallel data streams (channels), with each subcarrier modulated with a conventional modulation scheme. In practice, due to the large numbers of idle subcarriers in an OFDM system at some time instance [86], the spectrum usage characteristic could be considered as sparse. This matches well with the CS framework that allows efficient sub-Nyquist sampling for sparse signal, enabling a reduction in the sampling rate and/or the number of samples used in an OFDM system.

As such, CS framework has been adopted for OFDM based CR receivers to reduce the required number of samples as well as to lower energy cost [3, 86, 112]. To do so, a CS based sampling circuit, the modulated wideband converter (MWC) [63], has been used for OFDM reception as shown in Figure 5.1.

Figure 5.1 shows the high level architectures of existing and proposed MWC based OFDM communication systems, with both systems remain compatible with a standard OFDM transmitter (a). The main difference between the two systems is that, the

Figure 5.1: High level architecture of (b) existing MWC and (c) proposed MWC based OFDM receivers. Both systems remain compatible with standard OFDM transmitter (a).

proposed MWC system (b) incorporates an additional module to perform the mutual coherence minimization in Section 5.3.1.

5.3 System Model and Proposed Optimization

Figure 5.2 shows the circuit structure of the MWC, which consists of parallel sampling channels running at a certain sub-Nyquist rate. Assuming that the spectrum usage could be considered as sparse such that the received OFDM signal $x(t)$ contains sparse spectrum $X(f)$ in a given channel [86]) at receivers.

During the acquisition process, pseudo-random sequence $p_m(t)$ is mixed periodically (at T_p/L interval) with the $x(t)$. $p_m(t)$ consists of sequences that are used to mix the incoming signals with pseudo-random $\{-1, +1\}$ values, which is performed to achieve higher orthogonality and independence needed for higher reconstruction accuracy. As

such, the mixing operation shifts each channel spectrum by Δf_p ($f_p = 1/T_p$) [63]. Lowpass filters are then used to prepare the mixed signal for baseband sampling via sub-Nyquist ADCs. Assume that the input multiband signal $x(t)$ contains sparse spectrum $X(f)$, the sensing matrix model of the sampling circuits in MWC can be described:

Figure 5.2: Internal struture of MWC sampling block for analog-to-digital conversion consisting of parallel periodic waveforms mixers, low-pass filters and sub-Nyquist ADCs.

$$A = \underbrace{\begin{bmatrix} p_{1,0} & \cdots & p_{1,L-1} \\ \vdots & \ddots & \vdots \\ p_{M,0} & \cdots & p_{M,L-1} \end{bmatrix}}_{P_{M \times L}} \underbrace{\begin{bmatrix} \bar{F}_{N_0} & \cdots & \bar{F}_{-N_0} \\ \vdots & \ddots & \vdots \end{bmatrix}}_{F_{L \times N}} \underbrace{\begin{bmatrix} d_0 & \cdots & d_{N-1} \end{bmatrix}}_{D_{N \times N}}, \quad (5.3)$$

where F is the discrete Fourier transform (DFT) matrix constructed by $\bar{F}_{N_i} = [e^{-j0 \cdot i}, \dots, e^{-j\frac{2\pi}{L}(L-1) \cdot i}]$ and $D = \{d_n = \frac{1}{T_p} \int_0^{\frac{T_p}{L}} e^{-j\frac{2\pi}{T_p} nt} dt\}$. As a result, the sensing matrix for the proposed system can be modelled as $A = PFD$, which can be optimized to improve the reconstruction performance as will be presented in the next section.

5.3.1 Proposed Optimization

The performance of CS reconstruction can be enhanced by minimizing $\mu(A)$, which is represented by the absolute off-diagonal entries of Gram matrix ($A^T A$) [17]. Since A represents the sampling circuit of the MWC and cannot be directly updated during

sampling operation, an alternative solution is to find an optimum matrix P such that

$$\min_P \|I_N - (PA)^T PA\|_F^2, \text{ s.t. } A = PFD. \quad (5.4)$$

P in (5.4) can be found by using the following Lemma 1 [17] to directly provide the solution.

Lemma 1: Let $P \in \mathbb{C}^{M \times M}$ with full rank and $A \in \mathbb{C}^{M \times N}$ with $A = U_A [\Lambda_A \ 0] V_A^T$ an singular value decomposition (SVD) of A , where $\Lambda_A = \text{diag}(\lambda_1, \dots, \lambda_{\bar{M}}) > 0$, with $\bar{M} \leq M$ assumed ($\bar{M}$ is the number of non-zero eigenvalues). Let

$$P_{opt} \triangleq U [I_M \ 0] \begin{bmatrix} V_1^T \Lambda_A^{-1} \\ 0 \end{bmatrix} U_A^T, \quad (5.5)$$

where both U and V_1 are arbitrary orthonormal matrices of proper dimension. Then P_{opt} yields the solution to the problem defined by (5.4).

This Lemma has been proven by [17], suggesting that the optimal solution P_{opt} can be obtained based on the SVD of the sensing matrix A . However, the equiangular tight frame (ETF) of the Gram matrix should also be considered in order to achieve the minimal mutual coherence restricted by the Welch bound [113]. As a result, (5.4) can be solved by (5.6):

$$\begin{aligned} \min_P \|PA - A_{etf}\|_F^2 \\ \text{s.t. } P = U [I_M \ 0] \begin{bmatrix} V_1^T \Lambda_A^{-1} \\ 0 \end{bmatrix} U_A^T, \end{aligned} \quad (5.6)$$

where the above objective function aims to restrict the ETF of the updated Gram matrix, and the constraint function presents the solution provided by the Lemma 1. Using the A_{etf} provided by [113], the final solution which minimizes the mutual coherence is given by our proposed Theorem as follows.

Theorem 1: Let $K = \Lambda_A^{-1} U_A A A_{etf}^T$ and $K \triangleq U_K \Sigma_K V_K$ be an SVD of the K . Let

$$P_{opt} \triangleq V_K U_K^T \Lambda_A^{-1} U_A^T, \quad (5.7)$$

Then P_{opt} yields the solution to the optimal sensing matrix problem defined by (5.6), or equivalently (5.4).

Proof: The target optimization problem in (5.6) can be represented as:

$$\min_{U, V_1} \|U \begin{bmatrix} I_M & 0 \end{bmatrix} \begin{bmatrix} V_1^T \Lambda_A^{-1} \\ 0 \end{bmatrix} U_A^T A - A_{\text{etf}}\|_F^2, \quad (5.8)$$

Denote $V_0 \triangleq UV_1^T$, $U_0 \triangleq \Lambda_A^{-1} U_A^T A$, and $\eta \triangleq \min_{V_0} \{\|V_0 U_0 - A_{\text{etf}}\|_F^2\}$. Consider

$$\eta = \text{tr}(U_0^T U_0) + \text{tr}(A_{\text{etf}}^T A_{\text{etf}}) - 2\text{tr}(V_0 U_0 A_{\text{etf}}^T), \quad (5.9)$$

the target optimization problem as shown in (5.8) is equivalent to:

$$\max_{V_0} \text{tr}(V_0 U_0 A_{\text{etf}}^T). \quad (5.10)$$

Denote $U_0 A_{\text{etf}}^T \triangleq K$ and the SVD of $K \triangleq U_K \Sigma_K V_K^T$. The target optimization problem can be represented as:

$$\max_{V_0} \text{tr}(V_0 K) = \max_{V_0} \text{tr}(\Sigma_K V_K^T V_0 U_K) = \max_{V_0} \text{tr}(\Sigma_K Q), \quad (5.11)$$

where $Q \triangleq V_K^T V_0 U_K \triangleq \{q_{ij}\}$. Consider the upper boundary of the target optimization problem represented as follows:

$$\max_{V_0} \text{tr}(\Sigma_K Q) = \sum_{i=1}^N \sigma_i q_{ii} \leq \sum_{i=1}^N \sigma_i, \quad (5.12)$$

its maximum value can be reached if and only if Q becomes the identity matrix (i.e. $q_{ii} = 1, \forall i$). In this case, the optimum V_0 for the target optimization problem can be expressed as:

$$V_{0(\text{opt})} = V_K U_K^T. \quad (5.13)$$

Consider $P_{\text{opt}} A \equiv V_{0(\text{opt})} U_0$, the expression for P_{opt} can be expressed as:

$$P_{\text{opt}} \triangleq V_K U_K^T \Lambda_A^{-1} U_A^T, \quad (5.14)$$

which is the solution presented in (5.7) shown in Theorem 1. Q.E.D.

Since K and A_{etf} can be deduced from the SVD of A , Theorem 1 suggests that the optimal solution P_{opt} can be obtained as long as the sensing matrix model A is known a priori. This hence enables the optimization of the sensing matrix through mutual coherence minimization.

5.3.2 Reconstruction with Proposed Optimization

Using $P_{opt} = V_K U_K^T \Lambda_A^{-1} U_A^T$ as derived in (5.7) above, the CS reconstruction for $\hat{S}$ based on the MMV problem (5.1) becomes:

$$\hat{S} = \arg \min \|S\|_{2,1} \quad s.t. \quad Y_{opt} = P_{opt}Y = P_{opt}AS = A_{opt}S, \quad (5.15)$$

where $\hat{S}$ denotes the recovered digital spectrum supports, and Y_{opt} refers to the projected samples.

To implement this solution, P_{opt} is required to be calculated first, which involves the computation for A_{eff} , the SVD for V_K, U_K^T , the SVD for Λ_A^{-1}, U_A^T and the matrix multiplication for generating P_{opt} .

The algorithm for A_{eff} has been shown in [113], where its complexity is bounded by (a) the SVD of Gram matrix, which is $O(\min\{MN^2, M^2N\})$ [114], and (b) the matrix multiplication for updating the Gram matrix, which is $O(MN^2)$. Thus, the computational cost for A_{eff} is $O(\min\{MN^2, M^2N\})$. Similarly, the complexity of the SVD for $V_K, U_K^T, \Lambda_A^{-1}, U_A^T$ can be regarded as $O(\min\{MN^2, M^2N\})$. Besides, the complexity of the matrix multiplication for $P_{opt} = V_K \times U_K^T \times \Lambda_A^{-1} \times U_A^T$ is $O(MN^2)$. Therefore, the total complexity of the proposed optimization can be bounded as $O(\min\{MN^2, M^2N\})$.

In practice, the proposed optimization can be pre-calculated (and pre-stored) in the initialization step without any updating during the sampling period since the sensing matrix model is fixed. As such, this proposed optimization only takes a real-time computation on matrix multiplication for $P_{opt} \times A$ with a computational cost of $O(M^2N)$, which can be easily implemented by a digital signal processor.

After the pre-calculation of P_{opt} , solution for (5.15) can be considered as solving a MMV problem. This can be performed based on the continuous-to-finite block [63] and become a single measurement vector problem by joint sparsity. Greedy pursuits based solutions such as the simultaneous orthogonal matching pursuit (SOMP) [66, 94] can then be adopted to calculate $\hat{S}$ [63].

Once the $\hat{S}$ value is found, the inverse DTFT transforms of spectrum slices $Z \in$

$\mathbb{C}^{N \times L}$ can be obtained as follows:

$$Z_S = A_S^\dagger P_{opt} Y \quad s.t. \quad z_i[n] = 0, \quad i \notin S, \quad (5.16)$$

where $Z = [z_1[n]^T, \dots, z_L[n]^T]$ is the reconstructed spectrum slices. This solution, or linear equalization, is known as zero-forcing (ZF). Other well-known solutions, such as the matched filtering (MF) and Wiener filtering (WF) applied in [3] will also be examined later in Section 5.4. Finally, the reconstructed spectrum slices are used for the subsequent FFT and QAM demodulation to get the recovered OFDM symbols.

In conclusion, the entire program flow of the abovementioned reconstruction steps can be summarized as follows:

1. MWC based receiver collects groups of observations Y .
2. Calculate A_{eff} using [113].
3. Calculate K by $\Lambda_A^{-1} U_A A A_{eff}^T$, and compute its SVD coefficients U_K, Σ_K, V_K .
4. Generate the optimization matrix P_{opt} by $V_K U_K^T \Lambda_A^{-1} U_A^T$ as shown in (5.7).
5. Reconstruction $\hat{S}$ by $\{\arg \min \|S\|_{2,1} \quad s.t. \quad Y_{opt} = P_{opt} Y = P_{opt} A S\}$.
6. Update $Z_S \leftarrow \{A_S^\dagger Y_{opt} \quad s.t. \quad z_i[n] = 0, \quad i \notin S\}$.
7. FFT on Z_S : $z \leftarrow fft(Z_S)$.
8. Get recovered bits by QAM decoding: $\hat{b} \leftarrow demod(z)$.

5.3.3 Discussion

Once the OFDM symbols are recovered, the system performance can be evaluated terms of bit error rate (BER). Theoretically, the BER can be modelled by (a) the bit error probability (P_e) of a standard OFDM system under (b) the effect of CS reconstruction error. For the proposed system where 16-QAM OFDM is applied, P_e can be expressed as [115]:

$$P_e = \frac{3}{8} \text{erfc}\left(\sqrt{\frac{2E_b}{5N_0}}\right), \quad (5.17)$$

where the $\text{erfc}(\cdot)$ is the complementary error function, E_b is the energy per bit, N_0 is the noise power spectral density. When CS framework (MWC) is applied and the corresponding reconstruction error is generated, the E_b/N_0 is affected and becomes variant [116]:

$$\left\| \frac{E_b}{N_0} - \frac{\hat{E}_b}{N_0} \right\|_2 \leq \frac{1}{\sqrt{1 - \mu \cdot (K - 1)}}, \quad (5.18)$$

where the $\mu \in (0, 1)$ is the mutual coherence of the sensing matrix, $K \in (1, N]$ is the number of non-zero input bits. Finally, combining (5.17) and (5.18), the upper boundary of BER for the proposed system can be expressed as:

$$P_e \leq \frac{3}{8} \text{erfc} \left(\sqrt{\frac{2E_b}{5N_0 \left(1 + \frac{1}{\sqrt{1 - \mu \cdot (K - 1)}}\right)}} \right). \quad (5.19)$$

Equation (5.19) not only presents the upper boundary of the BER performance, but also indicates the fact that BER (P_e) can be decreased by minimizing the mutual coherence (μ), which is the target of the proposed optimization presented in this section. In the next section, the BER performance will be analysed based on simulation results to further verify the effectiveness of the proposed optimization.

5.4 Simulation Results

This section presents the simulation results and analysis on the BER performance of a MWC OFDM based CS-CR system incorporated with the proposed optimization technique, using a broadcast 4G LTE input signal operating under the same conditions used in [3]. The OFDM carriers are assumed to be within the range of $f_{min} = 0.7\text{GHz}$ to $f_{max} = 2.1\text{GHz}$ ($BW = 1.4\text{GHz}$) with a typical channel bandwidth $B = 20\text{MHz}$. The MWC sampling rate is selected to be $f_s = f_p = B$, which is the minimal possible rate according to [63]. The signal is intentionally corrupted in the transmission by additive white Gaussian noise.

5.4.1 Mutual Coherence Performance

The effectiveness of the proposed matrix optimization (in the proposed Theorem) can be observed via the amount of reduction in the mutual coherence, which is equal to the maximum correlation value between any two columns of A [43]. The correlation values can be calculated based on the absolute off-diagonal entries of Gram matrix, which is defined as $\{(P_{opt}A)^T(P_{opt}A)\}$ if the proposed optimization (i.e. P_{opt}) is armed, or defined as $\{A^T A\}$ if not. The distribution of such entry's value is shown in Figure 5.3.

Figure 5.3.(a) shows the distribution of values of absolute off-diagonal entries when MWC applies $M = 31$ sampling channels. From the comparison it can be noted that the maximum value in the histogram, equivalent to the mutual coherence, decreases from 0.565 to 0.315 after the optimization. Figure 5.3.(b) further demonstrates this effect when the number of sampling channels increases to 71, and it can be seen that the proposed optimization manages to reduce the mutual coherence, i.e. maximum value in the histogram, from 0.425 to 0.255. Hence it can be inferred that the proposed technique generates more centralized elements but with a lower maximum value, equivalent to a reduction in mutual coherence. As such, It will expect to obtain better system performance, as will be demonstarted below.

(a) Distribution of absolute off-diagonal entries of Gram matrices without and with P_{opt} for MWC under 31 sampling channels.

(b) Distribution of absolute off-diagonal entries of Gram matrices without and with P_{opt} for MWC under 71 sampling channels.

Figure 5.3: Distribution of absolute off-diagonal entries of Gram matrices without and with P_{opt}

5.4.2 Bit Error Rate Performance

This subsection presents the receiver BER when operating in a noisy environment, with a different number of sampling channels and using different equalization methods. The BER is measured based on the number of erroneous symbol bits measured at receiver when compared to the transmitted symbol bits. The total number of transmitted symbol bits is set to 10^6 empirically.

Figure 5.4: BER performance under different SNR when number of channels equals to 31 and 71, using the zero-forcing equalization method.

Figure 5.4 shows the BER performance as a function of SNR under two different number of sampling channels (M), based on the zero-forcing (ZF) equalization. This figure denotes the curves 'MWC-OPT-OFDM BER' and 'MWC-OFDM BER' as the BER performance with and without the proposed optimization added, respectively. First, it can be seen that the BER is inversely proportional to SNR for both systems. More importantly, the system using the proposed optimization technique (i.e. the proposed system) is able to reduce the BER over all the SNR levels for the two different

Figure 5.5: BER performance with different number of OFDM transmission at $M = 71$.

number of sampling channels used. For instance, at $\text{SNR} = 14\text{dB}$, the optimized system is able to reduce the BER by 9% for $M = 31$, and by 13% for $M = 71$.

Figure 5.5 further compares the BER of the proposed and existing MWC based OFDM system as a function of SNR for two OFDM transmissions. As can be seen, when the number of established OFDM transmissions decreases (from 16 to 1), the BER in both systems reduce. As before, system incorporated with proposed optimization exhibits better BER performance over all the SNR level for the two different number of OFDM transmission tested. These results (in Figure 5.4 and 5.5) hence confirm that the proposed system can provide a better communication performance.

Figure 5.6: BER performance in the existing and proposed MWC based OFDM system under different equalization methods (WF, MF and ZF) are implemented.

Figure 5.6 compares the BER of the proposed and existing MWC based OFDM system using different equalization methods. In addition to ZF, another two well-known equalization methods, the matched filtering (MF) and Wiener filtering (WF) [3] are examined in this test with the sampling channel set at $M = 71$. It can be noted that the ZF method provides the lowest BER over the SNR range for both systems. Also, these results show that the proposed system is able to consistently achieve a stable reconstruction performance under different SNR levels with different equalization methods.

Figure 5.7: BER comparison among OFDM systems with and without the MWC.

5.4.3 Comparisons with Conventional OFDM Receiver

The performance of the proposed system against the conventional (i.e. non-CS based) OFDM system is next performed. The BER performance of the conventional OFDM system, the proposed system and existing MWC-OFDM system are shown in Figure 5.7, with 71 sampling channels and implemented using ZF. As shown in the figure, the BER performance of standard OFDM is the best among the three. But it can be seen that the BER performance of the proposed system is much closer to the conventional (standard) system than the non-optimized MWC-OFDM CS based system. This result hence clearly illustrates the trade-off in CS based receiver design. While the CS technique reduces the sampling rate needed by the cognitive radio's receivers, it increases the bit error rate and thus reduces the signal reconstruction reliability. However, by incorporating the proposed matrix optimization approach, it is able to reduce significantly the performance gap.

5.5 Summary

This chapter presents a novel matrix optimization algorithm that can be easily incorporated into existing MWC based OFDM CS-CR receiver to reduce the signal reconstruction error. The algorithm can be pre-calculated without incurring additional computational load, and is compatible with the conventional digital OFDM receiver. Results show that the proposed optimization can consistently reduce the mutual coherence in the sensing matrix, leading to a reduction in communication BER under various operating conditions. In addition, the proposed optimization manages to greatly reduce the BER performance gap between the CS and conventional non-CS based OFDM receivers while providing the benefit of much lower sampling rate.

Chapter 6

Matrix Optimized MIMO-OFDM for CS-Based Data Reception in Cognitive Radio

Most recent studies have proposed the use of MIMO-OFDM for cognitive radio operation together with the compressive sampling technique to simplify the hardware complexity of the circuit implementation. This chapter presents a novel digital processing approach that can further enhance the signal transmission performance in compressive sampling based MIMO-OFDM cognitive radio systems.

6.1 MIMO-OFDM System

Figure 6.1 shows the architectural structure of a MIMO-OFDM based system with N_t and N_r number of transmit and receive antennas respectively, and N_f subcarriers in each channel. The n th output of the IFFT on the i th antenna can then be expressed as

$$x_i(n) = (1/\sqrt{N_f}) \sum_{k=0}^{N_f-1} b_i(k) e^{j2\pi nk/N_f}, \quad (6.1)$$

Figure 6.1: High level architectures of the conventional MIMO-OFDM system

where $b_i(k)$ is the complex modulated signals on i th antenna. During operation, whenever a user tries to access a MIMO-OFDM channel, some of the N_f subcarriers would be assigned to the user for the duration of usage.

The transmitted signal could also be described in a vector-matrix format as $x_i = F_{N_f}^{-1}b_i$, where $F_{N_f}^{-1}$ is the matrix representing the IFFT. Assuming that there are L multipath between i th transmit antenna and the j th receive antenna, then the channel impulse response (CIR) can be expressed as $h_{i,j} = [h_{i,j}^0 \cdots h_{i,j}^{L-1}]^T$. For the subsequent simulation tests to be shown later, the values of $h_{i,j}$ are selected based on the channel model specify in IEEE 802.22 standard [102], which is also widely adopted for CR networks performance studies.

The complex baseband equivalent received signal is then equal to the transmitted signal convoluted by the CIR. When cyclic prefix is added before each transmit vector x_i , the received signals on j th receive antenna from i th transmit antenna can be expressed in vector-matrix format as $y_{i,j} = H_{i,j}x_i$, where $H_{i,j}$ refers to the operation of

cyclic convolution with $h_{i,j}$ as shown in (6.2):

$$H_{i,j} = \begin{pmatrix} h_{i,j}^0 & 0 & \dots & 0 & h_{i,j}^{L-1} & \dots & h_{i,j}^1 \\ \vdots & h_{i,j}^0 & \ddots & \vdots & 0 & \ddots & \vdots \\ h_{i,j}^{L-1} & \vdots & \ddots & 0 & \vdots & \ddots & h_{i,j}^{L-1} \\ 0 & h_{i,j}^{L-1} & \dots & h_{i,j}^0 & 0 & \vdots & 0 \\ \vdots & 0 & \ddots & \vdots & h_{i,j}^0 & 0 & \vdots \\ \vdots & \vdots & \ddots & \ddots & \vdots & \ddots & 0 \\ 0 & 0 & \dots & 0 & h_{i,j}^{L-1} & \dots & h_{i,j}^0 \end{pmatrix}. \quad (6.2)$$

Now consider the case where the signals transmitted from all N_t antennas are received by j th antenna. The received data can then be expressed by $y_j = \sum_{i=1}^{N_t} y_{i,j}$ ($j = 1, \dots, N_r$). Thus, for each receive antenna, the transmitted signals can be written in a vector-matrix format:

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{N_r} \end{bmatrix} = \underbrace{\begin{bmatrix} H_{1,1} & \dots & H_{N_t,1} \\ H_{1,2} & \ddots & H_{N_t,2} \\ \vdots & \vdots & \vdots \\ H_{1,N_r} & \dots & H_{N_t,N_r} \end{bmatrix}}_{H=H_{\{i,j\}}} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{N_t} \end{bmatrix} + V, \quad (6.3)$$

where V stands for the corresponding additive noise. For convenience, the equation can be expressed as

$$y_r = Hx + V, \quad (6.4)$$

where the vector $y_r \triangleq [y_1 \ y_2 \ \dots \ y_{N_r}]^T$, and H stands for the channel matrix in (6.3).

In the conventional MIMO-OFDM system, instead of solving (6.4), the signal reconstruction will be done by first applying FFT to transform the received signals to Fourier domain. Let h'_k be the channel gain matrix for the symbols on k th subcarrier, whose element, h'_{ij} , is the gain between j th transmit antenna and i th receive antenna. Then the conventional MIMO detector demodulates the transmitted symbols by solving the following equations:

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{N_r} \end{bmatrix} = \begin{bmatrix} h'_{11,k} & \dots & h'_{N_t1,k} \\ \vdots & \ddots & \vdots \\ h'_{1N_r,k} & \dots & h'_{N_tN_r,k} \end{bmatrix} \begin{bmatrix} b'_1(k) \\ b'_2(k) \\ \vdots \\ b'_{N_t}(k) \end{bmatrix} + V', \quad (6.5)$$

where V' is the corresponding additive noise on the subcarrier, $y'_1(k)$ is the signals after FFT on the k th subcarrier. An exhaustive search on all possibilities of $b(k)$ could find $\hat{b}(k)$, which minimizes $\|y'_1(k) - h'_k b(k)\|$. This solution $\hat{b}(k)$ is also called maximum likelihood (ML) solution for MIMO detection. Instead of using the ML for signal reconstruction in standard MIMO-OFDM system, our proposed CS-MIMO-OFDM system has a different (simpler) sampling circuit and thus faces a different reconstruction issue.

6.2 Proposed System

As mentioned before, as the subcarriers in OFDM systems are not always fully utilized all the time [86], it means that there always exist large numbers of idle subcarriers at any time instance. This sparsity in spectrum usage characteristic matches well with the CS framework that allows efficient sub-Nyquist sampling for the spectrum. This feature is particularly useful for MIMO-OFDM system as it enables a further reduction in the required number of samples as well as the number of sampling ADC circuits[86, 87]. In this work, the proposed system only requires one ADC at receiver as shown in Figure 6.2.

Figure 6.2: High level architectures of the proposed CS based MIMO-OFDM system

6.2.1 MIMO-OFDM CS based Receiver

For CS-MIMO-OFDM system shown in Figure 6.2, the received signals are first mixed with random sequences, then summed together before it is sampled by a single ADC. The sampled signal can then be modelled as :

$$y = [D_1 D_2 \cdots D_{N_r}] [y_1 \ y_2 \ \cdots \ y_{N_r}]^T, \quad (6.6)$$

where $D_j \triangleq \text{diag}(d_j)$ is a diagonal matrix with the random binary number d_j on its diagonal, due to the mixing operation performed at j th receive antenna.

The sampled data can then be represented as:

$$\begin{aligned} y &= \underbrace{[D_1 \cdots D_{N_r}]}_D H \underbrace{\begin{bmatrix} F_{N_f}^{-1} & 0 & \cdots & 0 \\ 0 & F_{N_f}^{-1} & \vdots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & \cdots & 0 & F_{N_f}^{-1} \end{bmatrix}}_{F^{-1}} \underbrace{\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_{N_r} \end{bmatrix}}_b + DV \\ &= DHF^{-1}b + DV \\ &= Ab + V^*, \end{aligned} \quad (6.7)$$

where D models the mixing and addition operations before the ADC, F^{-1} is the block matrix consisting of $F_{N_f}^{-1}$ and V^* is the equivalent noise. Symbol b stands for concatenation of OFDM symbols, which is sparse when the subcarriers are rarely occupied. The sensing matrix for the proposed system can hence be expressed as:

$$A = DHF^{-1}, \quad (6.8)$$

which can be optimized to improve the reconstruction performance based on the same approach described in Section 5.3.1 in Chapter 5.

6.2.2 Reconstruction with Optimized Sensing Matrix

Existing CS-MIMO-OFDM systems do not consider the effect of the non-optimal sensing matrix on signal reception at CR receivers. The issue of non-optimal sensing matrix, equivalent to non-optimal mutual coherence, is regarded as a very critical factor

that affects the signal reconstruction performance in CS frameworks. Specifically, if the sensing matrix, which represents the sampling operation, is not well designed, the reconstructed noise could be relatively enlarged [16].

The CS based reconstruction performance can be enhanced by minimizing the mutual coherence of the sensing matrix A . This is equivalent to finding an optimum matrix P that solves the following equation with $A = DHF^{-1}$ obtained in (6.8):

$$\min_P \|I_N - (PA)^T PA\|_F^2, \text{ s.t. } A = DHF^{-1} \quad (6.9)$$

and the solution P_{opt} can be found by the Theorem provided in Section 5.3.1, where the P_{opt} can be described as:

$$P_{opt} \triangleq V_K U_K^T \Lambda_A^{-1} U_A^T, \quad (6.10)$$

where $K = \Lambda_A^{-1} U_A A A_{etf}^T$ and $K \triangleq U_K \Sigma_K V_K$ be an SVD of the K .

Using P_{opt} obtained in (6.10), the CS based reconstruction for OFDM symbols $\hat{b}$ becomes:

$$\hat{b} = \arg \min \|b\|_1 \quad \text{s.t.} \quad y_{opt} = P_{opt} y = P_{opt} DHF^{-1} b = P_{opt} A b, \quad (6.11)$$

Since the system structure (i.e. sensing matrix A) is known a priori, P_{opt} can be pre-calculated and applied directly on the sampled raw data y . The equivalent sampling operation ($P_{opt} A$) then leads to minimum mutual coherence for optimum signal reconstruction performance for $\hat{b}$. In this chapter, the widely used approximate message passing (AMP) algorithm [57] is used to solve this l_1 -minimization problem defined as (6.11), with results presented in next section.

6.3 Simulation results

As before, the simulations are performed for an environment compliant with IEEE 802.22 standard [102], using the WRAN channel model [117, 118] with the channel impulse response perfectly known. Performance for two different MIMO schemes, a 2×2 MIMO and a 4×4 MIMO are evaluated for QPSK modulated system with

$N_f = 256$ OFDM subcarriers in the given channel. For each scheme, simulations are performed for different numbers of active OFDM subcarriers as well as different SNR levels. Performance is evaluated based on the communication BER, for systems with and without the proposed matrix optimization [86], denoted as Xu et. al, 2015 in the following analysis.

6.3.1 Mutual Coherence Optimization

Figure 6.3: Distribution of absolute off-diagonal entries of Gram matrices without / with P_{opt} under 4×4 MIMO scheme.

As described in Chapter 5, the effectiveness of the proposed matrix optimization can be observed via the amount of reduction in the mutual coherence. It can be calculated based on the absolute off-diagonal entries of Gram matrix, i.e., $\{(P_{opt}A)^T(P_{opt}A)\}$ if the proposed optimization (i.e. P_{opt}) is armed, or $\{A^T A\}$ if not. The distribution of such entry's value are presented in Table 6.1 as well as in Figure 6.3.

Table 6.1: Correlation values evaluated by the Gram matrix without / with P_{opt} generated by (6.10).

Gram Matrices	Correlation Values		
	median	mean	max
$A^T A$	0.189	0.203	0.564
$(P_{opt}A)^T(P_{opt}A)$	0.233	0.217	0.449

Table 6.1 shows the median value, mean value, and maximum value of correlation between any two columns of the sensing matrix, where the maximum value refers to the mutual coherence of sensing matrix. Obviously, the P_{opt} is able to reduce the mutual coherence, with a lower maximum correlation value 0.449 (against 0.564), equivalent to a reduction of 20.39% in mutual coherence. Figure 6.3 shows the distribution of correlation values, where $\mu(A)$ stands for the original mutual coherence and $\mu(P_{opt}A)$ refers to the minimized mutual coherence. It can be seen that the proposed method leads to a shorter tail with more centralized elements, leading to a lower maximum values and lower mutual coherence. The result is obviously consistent with that shown in Table 6.1, presenting a reduction of 20.39% in mutual coherence.

6.3.2 Reconstruction Fidelity

SRR is defined as ratio of the number of correctly reconstructed symbols (at the receiver) over the total number of symbols transmitted. Based on the reconstruction algorithm presented in Section 6.2.2, Figure 6.4 shows the general trend of the SRR variation to the occupancy of the OFDM subcarriers for QPSK modulated symbols with SNR of 15dB. As expected, SRR degrades with increasing number of occupied subcarriers (K), while MIMO system of larger scale (e.g. 4×4 as against 3×3) is more resilient to the increase in occupied subcarrier.

Results in Figure 6.4 clearly show that our proposed system consistently provides a higher reconstruction rate against the existing CS based system operating under all K levels and different N_f (total number of subcarriers), for all the 2×2 , 3×3 and

(a) SRR versus K when $N_f = 256$ and $\text{SNR} = 15$ dB (b) SRR versus K when $N_f = 512$ and $\text{SNR} = 15$ dB

Figure 6.4: Successful reconstruction rate (SRR) versus number of active subcarriers (K) for QPSR symbols at $\text{SNR} = 15$ dB.

4×4 MIMO systems. For instance, when 12.5% subcarriers (32 out of 256) is occupied, the SRR in the proposed and existing CS-MIMO-OFDM system with 2×2 MIMO scheme reach 94.9% and 91.1% respectively, showing an increase of 4.2% in SRR after processing the proposed optimization. This result also holds when the scale of MIMO structure increases, where SRR increase in all systems. For example, for the 4×4 MIMO system, 99.5% SRR can be achieved in the proposed system compared to 98.1% in the existing CS-MIMO-OFDM system. This better performance remain the same when the total number of subcarriers N_f increases to 512, with the proposed system keeping its advantage in SRR.

6.3.3 Scaling Performance of MIMO System

Figure 6.5 presents the BER versus SNR performance for different MIMO scales (2×2 , 4×4) with two different number of active subcarriers ($K = 32, 64$). As expected, a higher SNR provides a better communication environment which reduces the BER. Besides, enlarging the MIMO scale, or reducing the number of active subcarriers can also reduce the BER [86]. Figure 6.5 also clearly shows that, for the same K , our proposed system reconstructs signals with a higher reliability in terms of a lower BER for both the 2×2 and 4×4 MIMO scales (e.g. 0.0178 versus 0.0229, 0.002 versus 0.003 respectively, when $K = 64$ and $\text{SNR} = 10\text{dB}$). This result still holds when the two systems keep the same MIMO scales for both $K = 32$ and $K = 64$.

Figure 6.5: BER versus SNR under two different MIMO scales (2×2 , 4×4) with two different number of active subcarriers ($K = 32, 64$).

6.3.4 Effect of Numbers of Active Subcarriers

To examine the impact of the occupied OFDM subcarriers, the BER over different numbers of active subcarriers is simulated. Figure 6.6 presents the BER versus the number of active subcarriers (K) under two different MIMO scales (2×2 , 4×4) with two different SNR level (10dB, 15dB). As expected, a higher K leads to a worse CS reconstruction environment for data communication, which increases the BER [43]. However, enlarging the MIMO scale or enhancing the SNR level can decrease the BER [86]. Again, it can be observed that, for the same SNR level, the proposed system reconstructs signals with a higher reliability in terms of a lower BER for both the 2×2 and 4×4 MIMO scales (e.g. 0.005 versus 0.008, 0.001 versus 0.002 respectively, when $\text{SNR} = 15\text{dB}$ and $K = 56$). These results hence shown that our proposed technique is able to recover the received signal with higher reliability, reflected in the lower BER achieved regardless of SNR, K and the MIMO scales.

Figure 6.6: BER versus the number of active subcarriers (K) under two different MIMO scales (2×2 , 4×4) with two different SNR level (10dB, 15dB).

6.3.5 Comparisons with Conventional MIMO Receiver

Figure 6.7 compares the performance of our proposed system versus the conventional (non-CS based) MIMO-OFDM system for both 2×2 and 4×4 MIMO setup, with 32 occupied subcarriers. As expected, a higher SNR provides a better communication environment which reduces the BER, while enlarging the MIMO scale can also reduce the BER [86]. Figure 6.7 also clearly shows that, for the same MIMO scale, the conventional MIMO-OFDM system generates the best performance in terms of BER, while the proposed system provides the second best BER result (e.g. 0.0004 versus 0.0006 with $\text{SNR} = 13\text{dB}$ and a 4×4 MIMO scales). Meanwhile, the existing CS-MIMO-OFDM system produces the worse reconstruction performance in comparison. In other words, the BER performance of the proposed system is much closer to the one exhibited by the conventional MIMO-OFDM system.

Figure 6.7: BER comparison among MIMO-OFDM systems without / with CS framework ($K = 32$).

6.4 Summary

This chapter presents a novel digital processing approach that can enhance the signal transmission performance for CS based MIMO-OFDM cognitive radio. The novel approach is based on minimizing the mutual coherence in the CS based sensing matrix of the CR receiver. Simulation results under various operating conditions show that the proposed approach can indeed consistently reduce the communication BER when compared to existing CS based MIMO-OFDM system, while achieving a simpler hardware circuitry (one ADC) compared to the conventional MIMO-OFDM system.

Chapter 7

Conclusion and Future Work

In order to increase the probability of finding unused spectrum and therefore increase its transmission throughput, a cognitive radio (CR) will need to monitor as many channels as possible by performing spectrum sensing over a wide frequency range. However, this would require very high sampling rate which is limited by existing ADCs technologies. This thesis hence presents the study of using sub-Nyquist sampling techniques to overcome this limitation and at the same time do not compromising the performance of the CR system, primarily through the adoption of compressive sampling (CS) technique in CR implementation. Conventionally, the CS reduces the required sampling rate with the tradeoff in higher data reconstruction time cost, which hence typically limits the CS technique to off-line data processing based applications. But this will not be feasible for CR systems which require the data processing to be performed in real time. This thesis hence presents several novel approaches to overcome the existing CS limitations with the aim to minimize the time required for CS based SS operations as well as optimizing the CS techniques to enhance the performance of the CR system.

7.1 Main Contributions

The first main contribution presented in this thesis is a new CS based SS technique proposed for hybrid CR that uses the combination of underlay and interweave transmission

modes. Unlike existing CS based signal processing operation, the proposed CS based technique does not require the reconstruction process. As such, it is able to achieve much lower sampling rate and greatly reduce the detection processing time compared to other known SS techniques. In addition, the proposed approach also incorporates the learned feature which can further improve the accuracy of the SS process. As a result, the proposed technique is able to achieve higher transmission throughput compared to other well known SS techniques, while operating at sub-Nyquist sampling rate without the need to use complicate ADC hardware architecture.

The second main contribution presented in the thesis is a novel matrix optimization algorithm that can be incorporated into CS based CR receiver to enhance the detection and reconstruction accuracy for OFDM-based signal transmission. This is important as it is usually not feasible to implement optimal sensing matrix for CS based CR since its frontend receiver circuit is typically hardwired, and the need to remain compatible with standard digital OFDM receiver's operation. Simulation results show that the proposed approach can consistently produce smaller CS reconstruction error in term of BER under various operating conditions when comparing to existing published systems.

The third main contribution of this thesis is to further extend the use of the matrix optimization algorithm to MIMO-OFDM based system, which is the dominant air interface for the latest 4G and 5G broadband wireless communications. The proposed technique enables the enhancement of CS related data transmission performance with reduced number of ADCs required at the MIMO-based receiver. Extensive simulation results have strongly supported the promising performance of this proposed approach.

7.2 Conclusion

The proposed contributions overcome the shortcomings of embedding CS such as the long-term reconstruction time cost and non-optimal sensing matrix in hardware design, and manage to minimize the time required for CS based SS operations and further enhance the performance of the CR system. These solutions cover the problems in spectrum sensing and data reception. In particular, they focus on incorporating the

Figure 7.1: Proposed structure of the CS based cognitive radio's receiver

CS into CR transceivers for (1) signal detection (spectrum sensing, discussed in Chapter 4) and (2) signal reconstruction (discussed in Chapter 5 and 6). These work lead to a novel CR transceiver structure which is shown in Figure 7.1.

The figure presents the architecture of the novel CS based cognitive radio's transceiver with the integration of the proposed solutions in the thesis. It can be seen that the CS based sampling circuits are applied and shared by multiple subsequent blocks. This CS sampling circuit first mixes and downsamples the input signals which is operating at a very low sub-Nyquist sampling rate. Then data is then passed to two subsequent blocks for spectrum sensing and data reception (relies on the CS based reconstruction) respectively.

For spectrum sensing, the proposed feature-based likelihood ratio test (in Chapter 4) is able to greatly reduce the detection processing time with very low rate sampled data. It can also be used with primary users that incorporate their learned information in their transmission signals to further enhance the SS detection accuracy. As a result, the data throughput when transmitting to other secondary users (SUs) can be further enhanced.

For data reception, the proposed matrix optimization (in Chapter 5) can be adopted to improve the CS reconstruction accuracy so as to reduce the transmission error in communication. As a result, the data received from other SUs can be obtained with lower BER as over wide range of SNR conditions. The use of the matrix optimization

can also be applied to the CS based MIMO-OFDM system which also enhances the performance in data reception (as discussed in Chapter 6).

In summary, this thesis presents several new ideas on how the sub-Nyquist CS based technique can be adopted for CR systems that require real-time operations and high data reception accuracy. The proposed ideas lead to the novel CR transceiver architecture that can achieve excellent performance in the data reception and data transmission operations.

7.3 Future Work: CS-Based Cooperative Sensing

When the CS is performed locally at individual CRs to scan the spectrum, the local SS technique cannot always achieve a high performance due to noise uncertainty, effect of multipath fading, and hidden terminal problem [80]. The detection performance can be enhanced through the collaboration of spectrum sensing in multiple CRs, where the detection accuracy will be increased by spatial diversity and shared sensing information [119]. In cooperative spectrum sensing (CSS), CRs first send the collected raw data to a fusion center (FC). Alternatively, each CR can individually perform local SS and then report the test statistics $T(y)$ or binary decision to the FC. Then the FC detects the presence of the PU based on its merged information. Figure 7.2 shows a classic CSS model with one PU, N SUs (CRs), and one FC. Each CR monitors the frequency band of interest, and reports a message to the FC by reporting channels.

The challenge of wideband spectrum sensing still exists for the cooperative SS. As discussed, monitoring a wideband for SS brings high signal-acquisition costs due to limitations of current ADC technology. Then it's reasonable to adopt the proposed CS based SS (in Chapter 4) for the cooperative SS. At individual CRs, the CS can be applied to reduce sampling rate and data storage costs. Thus, CRs can perform the proposed CS based SS individually to sample the data at a low rate, and can make a better decision for the spectrum sharing. The proposed technique can also greatly reduce the detection processing time while sustaining much lower sampling rate.

Figure 7.2: The structure of a cooperative spectrum sensing model with one PU, multiple SUs, and one fusion center (FC) [7]

The CS technique can also be adopted to solve issues in data fusion. In the case when FC collects full test statistics from all CRs to make centralized sensing decisions, it may lead to high communication costs and make the entire network vulnerable to node failure. Then CS can be used at each CR node to reduce the number of samples, so as to largely decrease the total data size in transmission to FC. For each CR, it uses the CS-ADC to downsample the data, then transmit it to the FC. The FC reconstructs the data and then make the decision of spectrum reuse for SUs. As a result, CS helps the FC to shorten the data size in communication (reporting channels). It can also be expected that, our proposed CS based SS will provide better performance in terms of the trade-off performance between detection accuracy and processing time as concluded in Chapter 4.

7.4 Future Work: Spectrum Underlay and UWB

In cognitive radio networks, SUs coexist with PUs by sharing the allocated spectrum without interfering with the primary users' communication. Apart from the spectrum sensing based method in OSA model (interweave) which enables the cognitive radio to detect and avoid interference to PUs, the spectrum underlay is another possible

solution for spectrum sharing.

In the spectrum underlay, an SU can consistently use the licensed band as long as its power doesn't generate a large amount of noise to PUs. In this case, an SU control its transmission power at a low level such that the total interference to PUs are tolerable. This spectrum underlay scheme is display in Figure 2.16.

Ultra Wideband (UWB) radio uses a ultra wide band frequency to transmit its data at a low power, as shown in Figure 7.3 [8]. Then it's reasonable to implement UWB for the spectrum underlay in cognitive radio. However, as UWB radio has a wide transmission bandwidth (in ranges of GHz) [8], sampling at very high frequency over such a wideband becomes a challenging problem in practice.

Figure 7.3: UWB signals in time and frequency domain [8].

Then it's reasonable to adopt CS to reduce the sampling rate, based on the fact that space-time UWB signals are essentially always sparse as shown in Figure 7.3. The CS based UWB receivers can be designed based on the CS-ADC, i.e. the RD discussed in Chapter 3, where a pseudo-random sequence is mixed with the received UWB pulses and sampled by sub-Nyquist uniform sampling ADC. After sampling, the receiver can perform standard CS reconstruction to recover the transmitted sparse bit sequence.

If CS based UWB communication for spectrum underlay can be achieved, the CS based spectrum sharing can then be implemented in an more efficient manner, where both interweave scheme or underlay scheme are applied simultaneously. Specifically, CR devices armed with the proposed CS based SS can detect spectrum holes and reuse them in a full power. Meanwhile, for the spectrum underlay, the CR devices can always transmit its data in UWB radio at a reduced power level that will never exceed an interference threshold required of PUs.

7.4.1 Related Work: UWB Positioning

The CS based UWB can be further extended for accurate position tracking application, since the UWB is a very suitable communication technique for high data rate short-range communication, i.e. indoor positioning, where a target UWB transmitter is moving in an area while other UWB receivers are surrounded (with fixed locations) and detect the position of the target UWB transmitter. The transmitter periodically broadcasts Gaussian shaped pulse through a multipath channel, and receivers detect signals for the time of arrival (TOA) based positioning calculation. Since the geometrical difference leads to different TOA, the received signals at the different receivers are collected and calculated [120].

Figure 7.4: Block diagram of CS UWB receiver implemented by random demodulator (RD).

My work adopts CS technique to UWB positioning systems as shown in Figure 7.4. The hardware implementation of these CS based UWB receivers typically uses the CS based ADC, i.e. the RD at the receivers. The CS technique enables a significant reduction in the receiver's sampling rate compared to the conventional Nyquist rate. It is also shown that this will also further improve the SNR of the reconstructed signal, before it is forward to the subsequent stage that performs the time of arrival (TOA) based positioning algorithm. As such, the introduction of CS not only reduces the ADC's sampling rate, but also increases the performance of the positioning system.

7.5 Future Work: CS-Based Machine Learning for SS

The machine learning techniques are often used for classification. In these techniques, a feature vector is first extracted from a pattern, and then used to guide the classifier to categorize the pattern into a definite class. Since that the spectrum sensing is a binary hypothesis testing problem which can be considered as a binary classification problem, some traditional machine learning techniques (e.g. logic regression and support vector machine) can be applied for the SS task.

Figure 7.5: Scatter plot of energy vectors collected by two SUs in cooperative spectrum sensing where one PU is transmitting its power [9].

In [9], the author proposed a cooperative spectrum sensing schemes based on machine learning techniques. In this paper, individual CR measures the energy of sampled data locally, and treat it as a feature vector. Then support vector machine categorizes the vectors into one of two classes, referring the presence and absence of spectrum holes (where these two classes are termed as the channel available class and channel unavailable class respectively). This machine learning-based SS technique are capable to implicitly learn the topology of the PU and CRN or the channel effects, without

the need for prior knowledge about the environment for optimization. The SS technique describe more optimized decision boundary on the feature space, rather than a linear threshold used in traditional CSS techniques, which results in a better detection performance.

Considering the wideband sensing issue, it's also reasonable to adopt the proposed CS based SS with the machine learning technique for the cooperative spectrum sensing. If the proposed CS based SS (in Chapter 4) can be applied, the system can overcome the limitation of sensing bandwidth. The new system can first generates the special detection statistic $T(y)$ in (4.29) according to the proposed CS based SS, and then regarded it as a new feature. Combining this new feature with the traditional features such as those extracted from the energy of sampled data or the eigenvectors in the sample covariance matrix, the new CSS technique is expected to enhance the detection accuracy further meanwhile maintain the low-rate sampling ability.

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