Study on the cavity solitons in fiber lasers

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**NANYANG
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**STUDY ON THE CAVITY SOLITONS
IN FIBER LASERS**

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SCHOOL OF ELECTRICAL AND ELECTRONIC ENGINEERING

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STUDY ON THE CAVITY SOLITONS IN FIBER LASERS

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This thesis is dedicated to my parents and my wife.

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Summary

Since it was first proposed in 1960s, fiber laser has attracted consistent focus both from industry and scientific research for these decades. In industry, fiber laser is an ideal source of high power CW/pulse beam output for the fine machining. In scientific research, pulse fiber lasers can provide picosecond or femtosecond optical pulses which are needed in physics, chemistry, medical and material research. On the other hand, the pulse propagation in optical fiber is mathematically described by the nonlinear Schrödinger equation (NLSE), and pulse propagation in a fiber laser is mathematically described by the Grinzburg-Laudau Equation (GLE). Both equations are very important in nonlinear science, describing a conservative and dissipative nonlinear system, respectively. Therefore, fiber laser is also an excellent testbed for experimentally studying the nonlinear physics. As a typical nonlinear cavity, fiber laser has nonlinear properties as those externally pumped nonlinear fiber ring cavities. Moreover, without the polarization selective components, fiber laser is a vectorial cavity which has two orthogonal polarization modes. Two orthogonal polarization modes will couple together through either coherent or incoherent interaction. Recently, the temporal cavity solitons in the externally pumped nonlinear fiber ring cavities have received much attention. As a nonlinear cavity, in principle temporal cavity solitons should also can be generated in fiber laser cavity. In this thesis, I'll focus on the study of cavity solitons in fiber laser.

First of all, inspired by modulation instability induced by cavity boundary conditions in the externally pumped nonlinear fiber ring cavities, the cavity induced modulation instability (CIMI) is both theoretically and experimentally confirmed existing in a fiber ring laser. CIMI origins from the cavity detuning. It's theoretically shown that when the total cavity phase delay (linear phase delay with nonlinear phase delay) is not $2m\pi$, the non-zero cavity detuning will cause a new type of modulation instability. Compare with the traditional

modulation instability appeared in single mode fibers, CIMI can occur both in normal and anomalous dispersion regime, and the modulation frequency is related with the cavity detuning. In experiment, we studied the CIMI lasing state of fiber laser. Stable periodical pulse train emission can be achieved, whose repetition rate can be altered by either tuning the cavity or changing the light intensity. The experimental results match well with theoretical conclusions. CIMI makes the fiber laser as a promising pulse source with tunable repetition rate, without any external modulation methods.

Under the CIMI induced pulse train emission state, if increasing the light intensity, we experimentally found the pulse train will be finally shaped into dissipative solitons. Through numerical simulate the operation of fiber laser, the gain bandwidth limitation plays a key role in the shaping mechanism. Under sufficient narrow gain bandwidth limitation, the CIMI modulated CW beam can evolve into dissipative solitons. We named this kind of spontaneously generated solitons as the temporal cavity solitons in fiber laser, which distinguish from the previously reported solitons in the mode-locked fiber lasers, nevertheless they are experimentally found having similar properties. The vector temporal cavity solitons are also experimentally observed. In an anomalous dispersive fiber laser, the induced vector solitons under either coherent or incoherent coupling between two polarization modes are observed. The numerical simulation also supports the experimental conclusion. The coherently coupled vector solitons are also observed in the normal dispersive fiber laser. The coherent energy exchange between vector solitons is first time experimentally observed in normal dispersion regime.

In the vector fiber laser cavity, the temporal cavity solitons will be coupled with the light on their orthogonal polarization. Under the incoherent coupling, when the cavity birefringence is strong, an interesting phenomenon call “soliton-dark pulse pair” is experimentally observed. It can be interpreted as an extreme case of polarization domain formation, where one polarization mode are bright solitons. On the other hand, under weak birefringence and coherent coupling, a novel state of induced dark solitons is experimentally observed. Through numerical simulation, it finds that the formation of induced dark

solitons is due to the coherent coupling between soliton and a dark pulse with phase jump. Without phase jump, a bright soliton will be induced instead.

In the research of dark solitons, an interesting phenomenon of dark envelope emission of a fiber ring laser is discussed. When a high frequency modulation and a dark-soliton-like dark pulse coexist in a fiber laser, they will be shaped into a high repetition rate dark pulse train and a dark soliton, respectively. Due to the superposition of their optical fields, the narrow dark soliton will form a wide dark envelope embedded in the dark pulse train background. The formed dark envelope makes the narrow dark soliton easily detectable. This finding brings a new understanding on the dark soliton formation in fiber lasers, especially on their experimental detection. Based on this understanding, the formation mechanism of the experimentally observed bright-dark vector solitons also becomes clear. In a weak birefringence cavity fiber laser, as a result of the strong coherent coupling between the two orthogonal polarization components, either the phase locked bright-bright vector cavity solitons or the bright-dark vector cavity solitons will be formed, which type of the vector cavity solitons will be formed depends on the phase relationship between the solitons. The coherent polarization coupling could lead to novel types of vector solitons formation.

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List of Acronyms

Acousto-optic modulator	AOM
Cavity induced modulation instability	CIMI
Chirp pulse amplification	CPA
Continuous wave	CW
Cross phase modulation	XPM
Dispersion compensation fiber	DCF
Erbium	Er
Erbium doped fiber	EDF
Fast Fourier Transform	FFT
Four wave mixing	FWM
Full width half maximum	FWHM
Grinzburg-Laudau Equation	GLE
Group velocity dispersion	GVD
Isolator	ISO
Modulation instability	MI
Modulational polarization instability	MPI
Neodymium	Nd
Nonlinear Schrödinger Equation	NLSE
Optical spectrum analyzer	OSA

List of Acronyms

Output coupler	OC
Photonics crystal fiber	PCF
Polarization dependent loss	PDL
Polarization controller	PC
Praseodymium	Pr
Saturable absorber	SA
Self-phase modulation	SPM
Semiconductor saturable absorber mirror	SESAM
Single mode fiber	SMF
Thulium	Th
Wavelength division multiplexer	WDM
Ytterbium	Yb

1

Introduction

1.1 Background

1.1.1 Introduction to ultrashort pulse fiber lasers

Fiber lasers are the lasers whose gain medium is doped optical fibers, usually the rare-earth-ions doped fibers. The first fiber laser was made in 1960s, which used the neodymium (Nd^{3+}) doped fiber as the gain medium [1]. Nowadays, different rare-earth-ions doped fiber lasers are developed to achieve wide range of lasing wavelength. The commonly doped rare-earth-ions include erbium (Er^{3+}) [2,3], neodymium (Nd^{3+}) [4,5], ytterbium (Yb^{3+}) [6,7], thulium (Tm^{3+}) [8,9] and praseodymium (Pr^{3+}) [10,11]. Table 1.1 gives the lasing wavelength of fiber lasers with different doped rare-earth-ions. Among these, the Yb-doped fiber lasers are widely used to produce high power continuous-wave (CW) or pulse output for modern industrial applications [12]. For their important applications in modern optical fiber communications systems, Er-doped fiber lasers and amplifiers have received much attention for decades.

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Recently, the Tm-doped fiber lasers lasing around 2 μm becomes a new research focus area because of their eye-safe emission wavelength and potential applications in gas detection and medical treatment [13].

Table 1.1 Central lasing wavelength of fiber laser with different doped gain fibers

Doped Element	Neodymium	Ytterbium	Praseodymium	Erbium	Thulium
Central Lasing Wavelength (μm)	1.06	1.03	1.3	1.55	2

Compared with other types of lasers, such as solid state lasers or dye lasers, fiber lasers have the following advantages:

1. In a fiber laser because optical fiber serves a light waveguide and light is confined inside the fibers, so a fiber laser is less possible to suffer from beam misalignment, which is a big issue in other lasers. The wave-guide properties of fibers also ensure good laser beam quality and laser operation stability.
2. As the gain medium is a doped fiber, a fiber laser is more ease to achieve high output power. In addition, power scaling of the lasers also benefits from the fiber output, using the fiber beam combiner. More important, optical fiber has large ratio of surface to volume, it can minimize the thermal effects in high power operation.
3. As optical fiber can be bent or coiled, a fiber laser can be made much more compact than other lasers with comparable output power.
4. The optical components used in fiber lasers are compatible with those used in fiber commutations systems, so the manufacturing costs are significantly reduced due to the mass production.

Among the industry applications and scientific research of fiber lasers, generation of ultrashort pulses is one of the key tasks. In industry, high energy ultrashort pulses are used to machine sub-micrometer or nanometer scale structures. In scientific area, ultrashort pulses are the enabling tools driving the research of physical, chemical and material science. It is the ultrashort pulses that enable direct observation of various fast processes in nature and the studies

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of matter under extreme conditions. In terms of ultrashort pulse generation and manipulation, the intrinsic properties of fiber lasers make them an ideal system for the purpose. It's well known that the group velocity dispersion (GVD) of materials has major influence on the shaping of ultrashort pulses. Fiber lasers consist of silica optical fibers whose GVD parameter can be shifted by changing the doping or fiber design. Therefore, one can design fiber lasers to operate near the zero average cavity GVD, which is the desired dispersion regime for ultrashort pulse generation [14]. The technique is known as the dispersion management, which is also widely used in the optical fiber communications systems [15]. Another factor affects the generation of ultrashort pulses is power and nonlinearity. As light propagation in optical fibers is confined inside a small core area, whose diameter is usually several micrometers (9 μm for the SMF-28 optical fiber) it's easy to achieve high light intensity in fiber lasers. The high intensity results in high nonlinearity. Sometimes excess nonlinearity is bad for the shaping of ultrashort pulses, such as the wave breaking [16]. In this case one should control the strength of nonlinearity, e.g. by using the large mode area fibers. On the other hand, the enhancement of fiber nonlinearity could help to achieve ultra-narrow pulses through the high order soliton effect [17].

The generation of ultrashort pulses in fiber lasers is conventionally mainly based on the "mode locking" method. The basis of the technique is to induce a fixed phase relationship between the lasing longitudinal modes of a laser. The coherent superposition of the modes then forms a train of the mode locked pulses whose repetition rate is determined by the cavity length. There are two types of mode locking method: active mode locking and passive mode locking. In active mode locking an active modulation device is inserted in the laser cavity. The modulation device can either modulate the amplitude or phase of the light. The most commonly used active mode-lockers are the standing wave acousto-optic modulators (AOM). When driven with an electrical signal at suitable frequency, an AOM produces a sinusoidal amplitude modulation to the light in the fiber laser cavity. The modulated CW beam then evolves into a

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periodical mode locked pulse train. Fig 1.1 is a typical schematic of an active mode locked fiber laser [18].

Figure 1.1. Schematic of an active mode locked fiber laser. (After [18], © OSA)

For passive mode locking, literally the active modulation device is replaced by a passive one. The most commonly used passive mode locking method is using a saturable absorber (SA). Saturable absorbers have a special transmittance which is proportional to the light intensity: high light intensity results in high transmittance. Imaging that a pulse passing through a saturable absorber, the lead and tail parts of the pulse will suffer greater loss than the peak part, so the pulse will become narrower. The actual effect of a saturable absorber is to introduce an amplitude modulation to the light in a fiber laser. Fig. 1.2 shows the relationship of transmittance vs. light intensity for a saturable absorber and its effect on the pulse shaping [19]. The commonly used saturable absorbers include the semiconductor saturable absorber mirror (SESAM) and some other new two-dimensional materials, such as graphene [20,21] and black phosphor [22]. Fig. 1.3 shows a schematic of a graphene mode locked fiber laser.

Figure 1.2. Transmittance vs. light intensity for a saturable absorber (SA), and illustration of its effect on pulse shaping.(After [19], © WILEY)

Figure 1.3. Laser configuration of a graphene mode locked fiber laser. WDM: Wavelength division multiplexer; EDF: Erbium doped fiber; SMF: Single mode fiber(After [20], © WILEY)

1.1.2 Pulse shaping and optical soliton formation in fiber lasers

The ultrashort pulses in fiber laser cavity can be shaped into special optical pulses: optical solitons. In physics, a soliton stands for a particle-like localized wave either in spatial or temporal domain, which is formed due to the interaction between diffraction/dispersion and nonlinearity. It exists in a wide range of nonlinear physical system from the fluid, plasma physics to optical, biological and atmospheric systems [23–27]. The optical solitons have attracted many research attentions in these decades because of their potential applications in high speed optical fiber communications and signal processing. Hasegawa *et al.* firstly theoretically predicted the existence of solitons in optical fibers in

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1973 [24], and Mollenauer *et al.* experimentally demonstrated the propagation of optical bright solitons in the anomalous dispersion fibers in 1980 [25].

The formation of optical soliton in optical fiber lasers is the result of interaction of dispersion and nonlinearity. When a pulse propagating in optical fiber, the different frequency components in the pulse will suffer from the group velocity dispersion (GVD), which results in different frequency components travel at different group velocity in the optical fiber. This will naturally lead to the pulse spreading. Fig. 1.4 shows a brief description of pulse spreading resulting from group velocity dispersion in different dispersion regime [28].

Figure 1.4. Pulse spreading resulting from the group velocity dispersion. (k_2 : parameter of group velocity dispersion; $k_2 < 0$: anomalous dispersion; $k_2 > 0$: normal dispersion) (After [28], © Elsevier)

Another key effect affects the pulse propagation is fiber nonlinearity. As discussed previously, as the light is confined inside a micrometer-scale core area of optical fiber, the light intensity can be very high and fiber nonlinearity becomes very strong, especially for the ultrashort optical pulses. In optical fibers, the self-phase modulation (SPM) is a main manifestation of the nonlinearity. In physics, SPM will induce frequency chirp to the pulse and also result the pulse spreading. Similarly, a brief description of SPM leading pulse spreading is shown in Fig.1.5 [28].

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Figure 1.5. (a) Time dependence of the incident pulse. (b) Change in instantaneous frequency of the transmitted pulse. (c) Experimental arrangement to observe self-phase modulation. (After [28], © Elsevier)

From the above two figures, we can see that when a pulse propagating in an anomalous dispersion regime, the pulse spreading effects caused by GVD and SPM just opposite. If they can balance with each other, the pulse will keep its shape unchanged in the optical fiber. We call this kind of particle-like pulses optical bright solitons. On the other hand, in normal dispersion regime, there is another kind of solitons called dark soliton [29]. Contrasted to the solitons in anomalous dispersion regime which are optical pulses, a dark soliton is an intensity dip on the CW background which also won't spread during the propagation. A more detailed physical and mathematical discussion about the pulse propagation, effects of GVD and SPM, and properties of optical solitons will be given in Chapter 2.

Due to the similar fiber environment, optical solitons naturally exist in fiber lasers. However, there is gain (doped gain fiber) and loss (propagation loss and laser output) in a fiber laser. A fiber laser is natively a dissipative system, so it is anticipated that the solitons formed in a fiber laser is much more complicated. The solitons formed in a fiber laser are called as dissipative solitons [30]. Unlike the solitons in optical fibers which is a conservative system, the formation of dissipative solitons is due to the mutual interaction between

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dispersion and nonlinearity, the gain, gain bandwidth limitation and losses. Dissipative solitons can exist in both anomalous and normal dispersion regimes [30–33]. However, in anomalous dispersion regime, the soliton shaping process is dominated by the co-effect of dispersion and nonlinearity (SPM) [34]. Additionally, when a fiber laser is in a steady state operation, the cavity losses are always balanced by the gain. In case where the gain bandwidth limitation is ignorable, the situation comes back to the NLSE. It means that the properties of the solitons in a fiber laser are similar like those of the solitons formed in an anomalous dispersive fiber.

The optical soliton formation in fiber lasers was first observed in the mode-locked erbium-doped fiber lasers operating in the anomalous dispersion regime [35,36]. In the normal dispersion regime, the dark soliton formation in fiber lasers has also been well theoretically and experimentally confirmed [37–41]. Besides, in normal dispersive gain fiber, when the gain bandwidth limitation can be neglected, there is another type of pulses called self-similar pulses or similaritons. The same as solitons [19,34,42–44], a self-similar pulse is also wave-breaking-free. Unlike the solitons keeping pulse width unchanged, when propagating in gain fiber, a self-similar pulse will be amplified and broaden but always keep its initial parabolic pulse shaped. This novel feature makes a self-similar fiber laser promising to produce high power output [45]. Figure 1.6 gives an illustration of self-similar pulse evolution in optical fibers [22].

Figure 1.6. Evolutions of self-similar pulse in optical fiber. Left: ordinary single mode fiber; Right: gain fiber. (After [19], © WILEY)

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In summary, the pulse shaping and soliton formation in fiber lasers are mainly determined by the cavity dispersion. In anomalous dispersion, pulse shaping is dominated by co-effect of dispersion and nonlinearity, optical bright solitons can be generated; in normal dispersion regime, dark solitons, dissipative solitons and self-similar pulses/similaritons can be formed under different conditions.

1.1.3 Nonlinear cavity properties of fiber lasers

When one talks about single mode fibers, “single mode” means there is only one fundamental transverse mode that can propagate inside the optical fiber. However, in practice a single-mode fiber actually supports two orthogonally polarized modes, owing to the birefringence introduced by imperfection or mechanical twisting/bending of the optical fiber. Thus, if there are no polarization dependent components inside the cavity to enforce the scalar laser operation, a fiber laser can emit light polarizing along the two orthogonal polarization directions. We call such a fiber laser cavity as a quasi-vector cavity. Since a fiber cavity has strong nonlinearity, the light along the two orthogonal polarization directions will couple with each other through the cross phase modulation (XPM) and four wave mixing (FWM) [46,47]. Cross-phase modulation can occur between different wavelengths and different polarizations. No phase match condition is required. Such a coupling is also regarded as the incoherent coupling. On the other hand, the FWM between two polarization modes requires phase matching. It only can happen at very weak birefringence condition. In a quasi-vector fiber laser cavity, in addition to the scalar solitons, these are solitons that have only one polarization component, vector solitons, these are solitons that have two coupled polarization components, can also be formed. Depending on the strength of cavity birefringence and the coupling mechanism between two polarization modes, various types of vector solitons are theoretically predicted and experimentally observed, such as polarization locked vector solitons [48,49], polarization rotation vector solitons [48,50], and group velocity locked vector solitons [51]. Under suitable conditions even high-

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order phase locked vector solitons and the XPM induced vector solitons were experimentally revealed [17, 52]. So far the vector solitons are coupled bright-bright solitons. There is other kind of vector solitons known as the bright-dark vector solitons, where a dark soliton along one polarization direction is coupled to a bright soliton polarized along the orthogonal polarization. One is the XPM paired bright-dark vector solitons, which is based on the incoherent coupling between two polarization modes [53]. To form this vector solitons, very tough requirement on the group velocity mismatch and the sign of material dispersion, namely that the dark part is in the anomalous dispersion regime and the bright part is in normal dispersion regime, are required. On the other hand, an another form of bright-dark vector solitons formed by the coherently polarization coupling was also theoretically predicted [54,55]. This type of bright-dark vector solitons can exists in a weakly birefringent optical fiber [54,56]. In normal dispersion regime, vector dark solitons are also theoretically predicted and experimental observed [57,58].

A fiber laser cavity is essentially a nonlinear cavity. Hence, there is no surprise that nearly all the nonlinear dynamic effects predicted for nonlinear cavities, such as chaos [59–62], periodic doubling bifurcations [63–65], quasi-periodicity [62,66] and bi-stability [67,68], also occur in fiber lasers. An important property of an optical cavity is that light circulating in it must fulfil the cavity boundary condition. This is especially important in the case of fiber lasers, as the phase mismatch /cavity detuning can be caused either by the linear or the nonlinear phase delay, and the cavity detuning sensitively affects the laser operation. One of the effects as firstly shown in the thesis is that it can lead to the cavity induced modulation instability (CIMI) [69,70]. Unlike the traditional MI which can only occur in anomalous dispersion fibers, the CIMI can occur both in the normal and anomalous dispersion regimes. And its modulation frequency is related to the total cavity detuning. CIMI will cause the breaking up of a strong CW beam in fiber lasers into a periodic pulse train. Under suitable conditions, such a periodic pulse train could further be shaped into dissipative solitons [71,72]. Similar as the temporal cavity soliton

formation in the passive fiber ring cavities [73–75], this process leads to the temporal cavity soliton formation in fiber lasers.

1.2 Motivation and objectives

It's well known that pulse propagation in optical fibers can be described by the Nonlinear Schrödinger Equation (NLSE) and pulse propagation in fiber lasers is described by the Grinzburg-Laudau Equation (GLE) [46,76]. Compared with NLSE, there are additional terms such as the laser gain, gain bandwidth limitation (gain dispersion) and laser losses in the GLE. Moreover, as the light is circulating in the laser cavity, the cavity detuning in each cavity roundtrip also plays a role [72]. As discussed above, under the steady state operation and if the gain bandwidth limiting effect is ignorable, the solitons formed in a fiber laser are similar to those of solitons described by the NLSE. However, when the effects of gain bandwidth limitation and the cavity detuning become significant, the simplified NLSE modal must be replaced by the more complicated GLE model. To achieve a comprehensive understanding on the soliton formation in a fiber laser as well as the soliton pulse shaping mechanism, the actual effects of gain bandwidth limitation and cavity detuning need to be considered.

For the steady state laser operation, the light circulation in the fiber cavity should satisfy the cavity boundary condition. However, in practice many factors could change the phase delay of the light in the fiber cavity, e.g. a sudden change of the pump power or the cavity net birefringence, which drives the cavity detuning. What would be the consequence of such an event on the CW fiber laser operation? Previous study on the externally driven passive cavities have shown that cavity detuning could induce a kind of modulation instability [69,77], known as cavity induced modulation instability (CIMI), which can occur in both the anomalous and normal cavity dispersion regimes. Whether the CIMI could also appear in a fiber laser is worth for studying. On the other hand, in recent years there are reports on the temporal cavity solitons in externally driven passive fiber ring cavities [73,74]. These temporal cavity solitons could be stored in the cavity, generated or destroyed in a controlled

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manner. They could be potentially used in the all optical information storage [73,78]. However, it is difficult to form and maintain temporal cavity solitons in a passive fiber cavity. Nevertheless, formation of cavity solitons is an intrinsic feature in a strong nonlinear active fiber cavity.

A fiber laser cavity is a quasi-vector cavity if there is no polarization selective component inside. Such a fiber cavity supports two orthogonal polarization modes. As a result of the fiber nonlinearity light along the two orthogonal polarization direction would either coherently or incoherently coupled depending on the cavity birefringence. It is known that polarization domains could be formed in a fiber laser due to the incoherent coupling between two orthogonal CW polarization components [79–82]. Under strong coupling even the domain wall solitons could be formed [83,84]. Under coherent coupling coherent energy exchange between two polarization components could also occur [85]. Recently, there are reports about using the polarization domain walls as the topological bits for data transmission in optical fibers, revealing the potential of transferring the theory into real applications [86,87]. The research on domain wall in other nonlinear systems also attracts many focuses [88–90]. In the theory research, the previous researches had mainly focused on the polarization coupling between the CW-CW or soliton-soliton components. What would happen if a soliton is coupled to a CW beam? Such studies will provide comprehensive understanding on the properties of quasi-vector cavity fiber lasers. Besides, the coherently coupled optical black-white vector soliton is still undiscovered even it was first theoretical predicted in 1988 [54]. On other systems based on coupled NLSE such as the Bose-Einstein condensates, the coupled dark-bright vector solitons have been well studied both theoretically and experimentally [91,92]. Therefore, studying this novel vector soliton is meaningful for the nonlinear physics research, especially for the nonlinear optics.

In the soliton community, there is still debate on the observation and measurement of dark solitons in fiber lasers [37,93,94]. Obviously, the narrowest dark soliton experimentally observable is limited by the speed, sensitivity and noise level of the photodetector and the oscilloscope used to

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measure it. For example, the rise edge of a commercial 40 GHz photodetector is around 9 ps. Besides, considering the available sensitivity and noise level, it only can detect the dark solitons in tens of picoseconds pulse width range. For even narrow dark solitons it would become undetectable. However, experimentally based on the detected dark soliton spectrum, it seems much narrower dark solitons could also be detected. Are they authentic dark solitons? What has happened behind the phenomenon?

Based on the above discussion there are at least the following aspects about fiber lasers worthy a deeper study: 1) stability of strong CW oscillation under cavity detuning? Would the cavity induced modulation instability also occur in a CW fiber laser, could the temporal cavity solitons automatically form in fiber lasers? 2) Vector soliton formation and vector soliton properties in fiber lasers; 3) Dark soliton dynamics and evolution, as well as the black-white vector soliton formation in fiber lasers.

1.3 Main contributions of the research

First of all, similar as [69], we derive a theoretical model to study the cavity induced modulation instability effect in a fiber ring laser. The theoretical model shows that a fiber ring laser has similar properties as an externally driven passive fiber ring cavity. Under suitable conditions the cavity induced modulation instability effect could also occur in a fiber ring laser. We have experimentally set up an erbium-doped fiber ring laser in the net anomalous cavity dispersion regime, and also experimentally confirmed the CIMI effect. Our experiments show that under strong pumping, stable CIMI lasing could occur in a fiber ring laser. Under the CIMI a stable periodic pulse train is automatically formed in the laser cavity, moreover the pulse repetition rate varies with the pump power and the cavity detuning. The experimental observations are well in agreement of the theoretical predictions.

The CIMI leads the formation of a periodic pulse train in fiber laser. Experimentally it was observed that as the pump power is further increased, the pulses could be shaped into temporal cavity solitons. The experimental

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observation is also well supported by numerical simulations. Depending on the concrete cavity birefringence, in our experiments either the scalar or the vector temporal cavity solitons were obtained. Similar to the vector soliton formation in the mode locked fiber lasers with a near isotropic cavity, the two orthogonal polarization components of the vector temporal cavity solitons are either coherently or incoherently coupled. Induced solitons also exist. The vector temporal cavity soliton formation in the fiber laser is also theoretically studied.

On the other hand, in normal dispersion regime, vector dissipative solitons were also observed in a fiber laser. The formation of the dissipative solitons could be traced back to the combined effect of the CIMI and the gain bandwidth limitation [95]. To distinguish the dissipative solitons from those formed in a mode locked fiber laser, we have named them the gain-guided dissipative solitons, which are also the temporal cavity solitons formation in the normal cavity dispersion regime. Our experimental results have shown that the dissipative soliton formation is a generic property of fiber lasers under high power pumping. As our fiber laser has a quasi-vector cavity, the dissipative solitons formed in our fiber laser are phased-locked vector solitons. There is coherent energy exchange between the two orthogonal polarization components of the formed vector solitons. Moreover, vector noise like pulse emission has also been observed in the same fiber laser, which suggests that the occurrence of the noise like pulse emission in the mode locked fiber lasers may not due to the action of the saturable absorber used, as suggested by some researchers.

The various temporal vector cavity soliton formations and their properties in fiber lasers were experimentally investigated in details. In the experiments with fiber lasers of various cavity parameters and laser operation conditions, a kind of soliton-dark pulse pair formation was observed [96]. In a quasi-vector cavity fiber laser, a bright cavity soliton polarized could be formed along one principal cavity polarization axis, and a CW is formed along the orthogonal polarization direction. Due to the incoherent cross polarization coupling, the bright soliton could always automatically induce a wide dark pulse on the CW background. The formation mechanism of the dark pulse is similar to that of the polarization domain formation in the fiber lasers. Under coherent coupling, experimentally

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we found that induced dark solitons could also be formed on the CW background [97]. Numerical simulations based on the coupled complex GLEs have also well confirmed the experimental observations. The numerical simulations have further revealed that whether a dark or bright soliton is induced depends on if there is a phase jump in the dark pulse.

In the research of dark solitons, we find an interesting phenomenon of dark envelope emission of a fiber ring laser [98]. When a high frequency modulation and a dark-soliton-like dark pulse coexist in a fiber laser, they will be shaped into a high repetition rate dark pulse train and a dark soliton, respectively. Due to the superposition of their optical fields, the narrow dark soliton will form a wide dark envelope embedded in the dark pulse train background. The formed dark envelope makes the narrow dark soliton easily detectable. This finding brings a new understanding on the dark soliton formation in fiber lasers, especially on their experimental detection. Based on this understanding, the formation mechanism of the experimentally observed bright-dark vector solitons also becomes clear [99]. In a weak birefringence cavity fiber laser as a result of the strong coherent coupling between the two orthogonal polarization components, either the phase locked bright-bright vector cavity solitons or the black-white vector cavity solitons will be formed, which type of the vector cavity solitons will be formed depends on the phase relationship between the solitons. The coherent polarization coupling could lead to novel types of vector soliton formation.

1.4 Overview of the thesis

This thesis is organized as follows:

Chapter 1 serves as the introduction to the research, containing a brief introduction to ultrashort fiber lasers and pulse shaping mechanism in fiber lasers. Research motivations, objectives and contributions are also included in the part.

Chapter 2 presents a comprehensive and quantitative discussion on ultrashort pulse generation in fiber lasers. It starts on the pulse propagation in optical

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fibers. Fiber parameters that affect the pulse evolution, such as the GVD, fiber nonlinearity, and birefringence are discussed. Then pulse propagation in fiber laser cavity is further discussed. The effects of gain, loss and gain bandwidth limitation are emphasized. The equations of pulse propagation in optical fibers and fiber lasers are derived, both in scalar and vector cases. Meanwhile, the mathematic model of fiber lasers with cavity detuning is derived. Pulse shaping mechanism, such as soliton shaping, dissipative soliton and self-similar pulse evolution are introduced.

Chapter 3 discusses the cavity induced modulation instability (CIMI) lasing of fiber lasers. Firstly, a theoretical model of cavity induced modulation instability in a fiber ring laser is derived. Guided by the theoretical predictions, an erbium-doped fiber ring laser is then constructed whose CIMI effect is investigated. Ultra-stable GHz level repetition rate periodic pulse trains are always generated in the fiber laser. Experimental studies confirmed the relation between the pulse repetition rate with the cavity detuning and powering pump, as predicted by the CIMI theory. Under even stronger pump power, coexistence of the CIMI and the conventional MI in the same fiber laser was also experimentally observed. Besides, we also confirm the existence of CIMI in the normal dispersion regime.

Chapter 4 focuses on the temporal cavity soliton formation in fiber laser. We start with the temporal scalar cavity soliton formation. The scalar temporal cavity soliton formation is experimentally observed and confirmed by numerical simulations. We find that the temporal cavity soliton formation is related with CIMI and gain bandwidth limitation effect. Then the vector temporal cavity soliton formation in fiber laser is studied. We show the experimental results of vector cavity solitons in a net anomalous dispersion cavity fiber laser. The induced cavity solitons, formed either under coherent or the incoherent coupling between the two orthogonal polarization components of light in the cavity, have also been experimentally obtained. In addition, the numerical simulations support the experimental results well. At last, we show the vector gain-guided solitons in a normal dispersive fiber laser. The vector solitons are phased-locked and the coherent energy exchange between

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dissipative vector solitons is first experimentally revealed. Moreover, the vector noise like pulse emission has also been observed.

Chapter 5 addresses some special features of the cross polarization coupling between the two orthogonal polarization components of a quasi-vector cavity fiber laser. We experimentally show that under incoherent coupling, a soliton-dark pulse pair can be formed, which can be interpreted as an extreme case of polarization domain formation. On the other hand, we observe induced dark solitons in experiment. Through numerical simulation, we find that the formation of induced dark solitons is due to the coherent coupling between soliton and a dark pulse with phase jump. Without phase jump, a bright soliton will be induced instead.

Chapter 6 addresses on the dynamics and evolution of dark solitons formed in fiber lasers. Experimentally, dark pulse emission is reproduced in our fiber laser with a normal dispersive cavity. Based on the experimentally measured soliton spectrum, the dark solitons should have narrow pulse width which theoretically should not be detected by our detection system. Our main task now is to understand why the dark solitons could still be detected. To the purpose, we have conducted numerical simulations. We found that if a high repetition pulse train is simultaneously formed as a background of the dark solitons, a dark envelop with much broader pulse width will be formed at position of the dark soliton. Such a dark envelope pulse could be easily experimentally resolved.

Chapter 7 discusses the black-white vector soliton formation in fiber laser. Experimentally we have firstly obtained a stable soliton-dark pulse emission state under the coherent coupling between two polarization modes. To understand the result, we further numerically simulated the laser operation. The simulation results revealed that the observed dark pulse is actually the envelope dark pulse, which is the superposition of a dark soliton with a stable train of bright pulses. In this case the dark soliton is represented by the existence of a weak dark pulse on the side. The weak dark pulse appears at the same position as the bright soliton, and are coupled to it, have the same pulse width.

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Chapter 8 concludes the thesis, summarizes the achievements of the research, and gives recommendations for the future works.

2

Theory of pulse propagation in fiber lasers

An optical pulse propagating in a single mode fiber is deformed by the dispersion (variation of the group velocity as a function of the frequency) and nonlinearity (variation of the phase velocity as a function of the wave intensity) of the optical fiber. The dispersive property of fibers is decided by the fiber's GVD, while the nonlinear properties of a fiber are characterized by the optical Kerr effect (an effect of the increase in the refractive index n in proportion to the light intensity), whose manifestations in optical fiber are self-phase modulation (SPM) and cross phase modulation (XPM). In a fiber laser, the gain and gain dispersion introduced by the erbium-doped fiber amplifier will further complicate the pulse propagation. However, "single" mode fiber supports two orthogonal polarization modes while the term "single" mode refers only to the transverse profile. Therefore, the vector properties of optical field must be considered.

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The objective of Chapter 2 is to provide a theoretical description of pulse propagation and shaping in a vector fiber laser cavity. In Section 2.1, a theoretical model of pulse propagation in optical fibers which includes the group velocity dispersion and nonlinear effect is constructed. The nonlinear Schrödinger equation (NLSE) is derived and the soliton solutions of the equation are given. In Section 2.2, we theoretically discuss the operation of erbium doped fiber lasers. The pulse propagation in fiber laser is described by the Grinzburg-Laudau equation (GLE). A quantitative discussion on the dissipative solitons and self-similar pulses are also introduced. An extend GLE model with cavity detuning is derived, which is a more precise description on the light circulating in the fiber laser cavity. In Section 2.3, the vector model of NLSE and GLE are introduced. In Section 2.4, procedures of numerically solving the coupled GLEs are briefly demonstrated, which is the basics of numerical simulation of my research work.

2.1 Pulse propagation in optical fiber

2.1.1 Dispersion effects in optical fiber

When an optical pulse propagates in an optical fiber, if only consider linear effect, the pulse will be distorted as a result of the group velocity dispersion (GVD). Group velocity is the speed at which light travels along the fiber. For an optical pulse containing a finite number of frequency components, GVD will occur because different frequency components travel at different group velocities. Thus, GVD will lead to spreading of an optical pulse. A brief demonstration of the effect has shown in Fig. 1.4. The physics behind the GVD is the chromatic dispersion. When an electromagnetic wave, such as an optical wave, interacts with a dielectric, the medium response will depend on the optical frequency ω . This property is referred to as the chromatic dispersion, which manifests through the frequency dependence of the refractive index $n(\omega)$. On a fundamental level, the origin of dispersion is related to the characteristic resonance frequencies at which the medium absorbs the electromagnetic

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radiation through oscillations of bound electrons. The refractive index can be well approximated by the *Sellmeier Equation* [100],

$$n^2(\omega) = 1 + \sum_{j=1}^m \frac{B_j \omega_j^2}{\omega_j^2 - \omega^2} \quad (2.1.1)$$

where ω_j is the resonance frequency and B_j is the strength of j^{th} resonance. The sum in Eq. (2.1.1) extends over all material resonances that contribute to the frequency range. For silica optical fiber, a three-term *Sellmeier equation* is typically used, as shown in Eq. (2.1.2),

$$n^2(\omega) = 1 + \frac{B_1 \omega_1^2}{\omega_1^2 - \omega^2} + \frac{B_2 \omega_2^2}{\omega_2^2 - \omega^2} + \frac{B_3 \omega_3^2}{\omega_3^2 - \omega^2}. \quad (2.1.2)$$

Mathematically, the effects of fiber dispersion are accounted for by expanding the mode-propagation constant β in a Taylor series about the frequency ω_0 at which the pulse spectrum is centered:

$$\beta(\omega) = n(\omega) \frac{\omega}{c} = \beta_0 + \beta_1 (\omega - \omega_0) + \frac{1}{2} \beta_2 (\omega - \omega_0)^2 + \dots \quad (2.1.3)$$

$$\beta_m = \left(\frac{d^m \beta}{d\omega^m} \right)_{\omega=\omega_0} \quad (m=0,1,2,\dots) \quad (2.1.4)$$

The parameters β_1 and β_2 are related to the refractive index n and its derivatives through the relations

$$\beta_1 = \frac{1}{v_g} = \frac{n_g}{c} = \frac{1}{c} \left(n + \omega \frac{dn}{d\omega} \right), \quad (2.1.5)$$

$$\beta_2 = \frac{1}{c} \left(2 \frac{dn}{d\omega} + \omega \frac{d^2 n}{d\omega^2} \right), \quad (2.1.6)$$

where n_g is the group index and v_g is the group velocity. The whole envelop of the pulse moves at group velocity v_g and β_2 represents the dispersion of group velocity and is responsible for pulse broadening. β_2 is referred as the group velocity dispersion (GVD) parameters. In practice, the dispersion parameter D is also used and is defined as following,

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$$D = \frac{d\beta_1}{d\lambda} = -\frac{2\pi c}{\lambda^2} \beta_2 = -\frac{\lambda}{c} \frac{d^2 n}{d\lambda^2}. \quad (2.1.7)$$

We can see that D has opposite β_2 and the expression of D shows a more direct relationship between the refractive index and wavelength sign as Figure 2.1 shows the relationship between β_2/D and wavelength λ in fused silica [46].

Figure 2.1. Relation of dispersion parameter and GVD with wavelength for fused silica (d_{12} in the figure is the walk-off parameter). (After [46], © Elsevier)

In the figure, both β_2 and D become zero around 1.27 μm , which is referred as the zero-dispersion wavelength (λ_D). In practice, one can shift λ_D by changing the doping and core diameter in fiber design. For wavelength $\lambda < \lambda_D$, fiber is called normal dispersion as $\beta_2 > 0$ and $D < 0$. In normal dispersion regime, the higher frequency (blue shifted) components of an optical pulse travel slower than the lower frequency (red-shifted) components. On the contrary, fiber shows anomalous dispersion as $\beta_2 < 0$ and $D > 0$, when $\lambda > \lambda_D$. In this regime, the blue-shifted components of optical pulse travel faster than red-shifted components.

Here a brief mathematical description to how the dispersion effects affect the light propagation in optical fiber is given. As an electromagnetic wave, the propagation of optical field in optical fiber is also governed by Maxwell's equation. From Maxwell's equation, we can get the fundamental wave equation

$$\nabla \times \nabla \times \vec{E} = -\frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} - \mu_0 \frac{\partial^2 \vec{P}}{\partial t^2}, \quad (2.1.7)$$

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where $\vec{E}$ and $\vec{P}$ are electric field and the induced electric polarization. c is the speed of light in vacuum and has a relationship of $1/c^2 = \mu_0 \epsilon_0$, where μ_0 is the vacuum permeability and ϵ_0 is the vacuum permittivity.

Before solving the wave equation, two simplifications are made. First, by just considering linear polarized light propagation in the media, the vectors in Eq. (2.1.7) can be replaced by scalars. Second, by ignoring the nonlinear effects temporarily in this section. Eq. (2.1.7) is linear and can be easily converted to the frequency domain.

Using these two simplifications and we can simplify Eq. (2.1.7) as following,

$$\nabla \times \nabla \times \vec{E} = \nabla (\nabla \cdot \vec{E}) - \nabla^2 \vec{E}. \quad (2.1.8)$$

And we can get the wave propagation equation in frequency domain:

$$\nabla^2 E(z, \omega) + n^2(\omega) \frac{\omega^2}{c^2} E(z, \omega) = 0, \quad (2.1.9)$$

where $E(z, \omega)$ is the Fourier transform of $E(z, t)$ defined as:

$$E(z, \omega) = \int_{-\infty}^{\infty} E(z, t) \exp(i\omega t) dt. \quad (2.1.10)$$

Next, another function $F(z, \omega)$ is brought in to further simplify the wave equation. $F(z, \omega)$ is assumed to be a slowly varying function of z ,

$$F(z, \omega) = E(z, \omega) \exp[-i\beta(\omega)z]. \quad (2.1.11)$$

Therefore, the wave equation (2.1.9) can be further simplified as:

$$\frac{\partial F(z, \omega)}{\partial z} = -i\beta(\omega) F(z, \omega), \quad (2.1.12)$$

where $\beta(\omega) = n(\omega) \frac{\omega}{c}$ is the propagation constant varying with frequency ω .

When a pulse propagation in an optical fiber, its electric field can be generally written as:

$$f(z, t) = E(z, t) \exp\{i[\omega_0 t - \beta(\omega_0)z]\}. \quad (2.1.13)$$

$E(z, t)$ is the pulse envelop which contains both amplitude and phase. ω_0 is the central frequency of the pulse and $\beta(\omega_0)$ is the propagation constant at the frequency of ω_0 . Correspondingly, in frequency domain, an optical pulse can be described by the Fourier transform of $f(z, t)$ and written as:

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$$F(z, \omega) = E(z, \omega - \omega_0) \exp[i\beta(\omega_0)z]. \quad (2.1.14)$$

The function of $F(z, \omega)$ obeys the conventional transmission relations of Eq. (2.1.12). If the pulse is sufficiently wide, dispersion effect can be ignored and each spectral component of pulse at ω can be well approximated to propagate with the same phase constant $\beta(\omega_0)$. But when the width of an optical pulse t_p is so short that the spectral bandwidth $\delta\omega \propto \frac{1}{t_p}$ is a significant fraction of ω_0 , we must take account of the fact each spectral component of $F(z, \omega)$ has its own phase constant $\beta(\omega)$. In this case, by substituting Eq. (2.1.3) and Eq. (2.1.14) into Eq. (2.1.12), we can finally obtain an equation describing the pulse wave function $E(z, \omega - \omega_0)$:

$$\frac{\partial E(z, \omega - \omega_0)}{\partial z} = -i\delta\beta R - i\beta_1 E - i\frac{\beta_2}{2}(\omega - \omega_0)^2 E. \quad (2.1.15)$$

Since a perturbation induced by the amplitude usually exist (i.e. $n = n(\omega) + \Delta n$), the factor $\delta\beta$ is imported to denote the perturbation it induced on β (i.e. $\beta = \beta(\omega) + \delta\beta$).

At this point, we can go back to time domain by applying inverse Fourier transform to Eq. (2.1.15) and obtain the propagation equation of $E(z, t)$:

$$\frac{\partial E(z, t)}{\partial z} + \beta_1 \frac{\partial E(z, t)}{\partial t} = i\frac{\beta_2}{2} \frac{\partial^2 E(z, t)}{\partial t^2} - i\delta\beta E(z, t). \quad (2.1.16)$$

In practice, a synchronous coordinate system moving at group velocity v_g is always used to simplify Eq. (2.1.16). The simplification is realized by applying the following transformations:

$$\begin{cases} T = t - \frac{z}{v_g} \\ z' = z \end{cases} \quad (2.1.17)$$

Finally, the propagation equation of an optical pulse can be rewrite as:

$$\frac{\partial E(z', T)}{\partial z'} = i\frac{\beta_2}{2} \frac{\partial^2 E(z', T)}{\partial T^2} - i\delta\beta E(z', T). \quad (2.1.18)$$

Eq. (2.1.18) describes the propagation of a short pulse in dispersive media. It will be used to derive the pulse propagation in single mode fiber in following sections.

2.1.2 Nonlinear effects in optical fibers

The refractive index of optical media is not only frequency dependence, but also optical intensity dependent. When the real power of a pulse is very high, self-phase modulation (SPM) will arise from the Kerr nonlinearity. SPM will lead to the spectral broadening of an optical pulse [46].

On a fundamental level, the origin of nonlinearity is related with the nonlinear response of the electric dipolar polarization $\vec{P}$ induced by the applied electric field $\vec{E}$:

$$\vec{P} = \epsilon_0 \left(\chi^{(1)} \cdot \vec{E} + \chi^{(2)} : \vec{E}\vec{E} + \chi^{(3)} : \vec{E}\vec{E}\vec{E} + \dots \right), \quad (2.1.19)$$

where ϵ_0 is the vacuum permittivity and $\chi^{(j)}$ is the j th order susceptibility. The linear susceptibility $\chi^{(1)}$ represents the dominant contribution to P , whose effects are included through the refractive index n . The second order susceptibility $\chi^{(2)}$ is caused by the asymmetry in molecular structure, which is responsible for the nonlinear effects such as second harmonic generation (SHG) and sum frequency generation. However, as SiO_2 has a symmetric molecular structure, no $\chi^{(2)}$ effect in a silica optical fiber. So the nonlinear effects in silica optical fiber mainly origin from $\chi^{(3)}$. The third order nonlinear effects include the third harmonic generation and four wave mixing (FWM), which need strict phase matching condition. There are other nonlinear effects such as stimulated Raman and Brillouin scattering, which are not concerned in my research work. But it's important to point out that all these nonlinear effects are light intensity related which can automatically occur after the light intensity reaches certain thresholds. In a simplest form, the nonlinear refractive index can be written as:

$$n(\omega, |E|^2) = n(\omega) + n_2 |E|^2, \quad (2.1.20)$$

where $n(\omega)$ is the linear part given by Eq. (2.1.1), $|E|^2$ is the optical intensity and n_2 is the nonlinear index coefficient determined by $\chi^{(3)}$. Normally n_2 of silica is very weak which is in the range of $2.2 \sim 3.4 \times 10^{-20} \text{ m}^2 \cdot \text{W}^{-1}$ [46]. However, as the core area of SMF is very small, 9 μm for the commonly used SMF-28 fiber, the light intensity in core area is very high. On the other hand,

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the light loss in fiber is very low and light-matter interaction distance can be very long, especially when the light circulating in fiber laser, it makes the efficiency of nonlinear optical process very high in fiber laser cavity. The two most important nonlinear effects in optical fiber are self-phase modulation (SPM) and cross phase modulation (XPM). Self-phase modulation refers to the self-induced phase shift on the spectral components of an optical pulse when propagating in optical fiber. The phase of an optical field changes by:

$$\phi = nk_0L = (n + n_2|E|^2)k_0L, \quad (2.1.21)$$

where $k_0 = \frac{2\pi}{\lambda}$ and L is the fiber length. In Eq. (2.1.21), the intensity-dependent nonlinear phase shift due to SPM is $\phi_{NL} = n_2|E|^2k_0L$. SPM is responsible for the spectral broadening of ultrashort pulses and the formation of optical solitons [46].

Cross phase modulation refers to the nonlinear phase shift of an optical field induced by another field with different wavelength, or polarization states. Its origin can be explained from the expression of total electric field E :

$$E = \frac{1}{2} \hat{x} [E_1 \exp(-i\omega_1 t) + E_2 \exp(-i\omega_2 t) + c.c.] , \quad (2.1.22)$$

where two optical fields at frequencies ω_1 and ω_2 , polarized along the same axis, propagate simultaneously inside a fiber. The nonlinear phase shift introduced by field E_2 to field E_1 is $\phi_{nl} = 2n_2k_0L|E_2|^2$. Therefore the total nonlinear phase shift to field E_1 is

$$\phi_{nl} = n_2k_0L \left(|E_1|^2 + 2|E_2|^2 \right). \quad (2.1.23)$$

The two terms on ϕ_{nl} are due to SPM and XPM. XPM also results in spectral broadening and paired vector soliton formation in optical fibers.

2.1.3 Nonlinear Schrödinger equation (NLSE) and soliton solutions

By including the nonlinear part of refractive index into $\delta\beta$ of Eq. (2.1.18) using

$$\beta(\omega) = n(\omega) \frac{\omega}{c}, \text{ we can get:}$$

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$$i \frac{\partial E}{\partial z} = \frac{\beta_2}{2} \frac{\partial^2 E}{\partial T^2} - \gamma |E|^2 E, \quad (2.1.24)$$

where $\gamma = \pi n_2 / \lambda$ is the nonlinear parameter. Eq. (2.1.24) is in a form called Nonlinear Schrödinger equation (NLSE). Here we should notice that the frame is in the reference moving with pulse at the group velocity v_g , so no β_1 term exists. The two terms on the right-hand side of Eq. (2.1.24) stand for the dispersion and nonlinearity, respectively. Depending on the peak power P_0 and width T_0 of the incident pulse, either dispersion or nonlinear effects would dominate along the fiber. There are two important concepts here: dispersion length L_D and nonlinear length L_{NL} . They are expressed as: $L_D = \frac{T_0^2}{|\beta_2|}$, $L_{NL} = \frac{1}{\gamma P}$.

We can normalize Eq. (2.1.24) by introducing the normalized time as $\tau = \frac{T}{T_0}$ and normalized amplitude U as $E(z, \tau) = \sqrt{P_0} U(z, \tau)$. So Eq. (2.1.24) can be rewrite as:

$$i \frac{\partial U}{\partial z} = \frac{\text{sgn}(\beta_2)}{2L_D} \frac{\partial^2 U}{\partial \tau^2} - \frac{1}{L_{NL}} |U|^2 U, \quad (2.1.25)$$

where $\text{sgn}(\beta_2) = \pm 1$ depending on the sign of β_2 . Depending on the relative magnitudes of fiber length L , L_D and L_{NL} , the pulse propagation behavior can be classified into four different types:

1. When $L \ll L_{NL}$ and $L \ll L_D$, neither dispersion nor nonlinearity plays a significant role during pulse propagation. Two terms in Eq. (2.1.25) can be neglected.
2. When $L \ll L_{NL}$ and $L \sim L_D$, dispersion dominates and nonlinearity effect is negligible.
3. When $L \ll L_D$ and $L \sim L_{NL}$, Nonlinearity dominates and dispersion effect is negligible.
4. When L is longer and comparable to both L_D and L_{NL} , dispersion and nonlinearity act together. In anomalous dispersion regime ($\beta_2 < 0$), the fiber supports solitons; in normal dispersion regime ($\beta_2 > 0$), the GVD and SPM effect can be used for pulse compression.

We can further normalize Eq. (2.1.24) using $\xi = \frac{z}{L_D}$:

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$$i \frac{\partial U}{\partial \xi} = \text{sgn}(\beta_2) \frac{1}{2} \frac{\partial^2 U}{\partial \tau^2} - N^2 |U|^2 U. \quad (2.1.26)$$

Here we introduce a new parameter N as:

$$N^2 = \frac{L_D}{L_{NL}} = \frac{\gamma P_0 T_0^2}{|\beta_2|}. \quad (2.1.27)$$

Parameter N can be eliminated from Eq. (2.1.26) by introducing

$$u = NU = \sqrt{\gamma L_D} E, \quad (2.1.28)$$

and Eq. (2.1.26) is in a standard form of NLSE:

$$i \frac{\partial u}{\partial \xi} + \frac{1}{2} \frac{\partial^2 u}{\partial \tau^2} + |u|^2 u = 0. \quad (2.1.29)$$

Here we assume the fiber is in anomalous dispersion and $\text{sgn}(\beta_2) = -1$.

For Eq. (2.1.29), it has a soliton solution as [46]:

$$u(\xi, \tau) = \text{sech}(\tau) \exp(i\xi/2). \quad (2.1.30)$$

When $N=1$, we have $U(\xi, \tau) = \text{sech}(\tau) \exp(i\xi/2)$ which is called the fundamental soliton. Eq. (2.1.30) describes a sech-shape pulse whose pulse shape is irrelevant with the propagation distance. It means that this optical pulse propagation in optical fiber will always keep its pulse shape and pulse width unchanged. This feature makes the fundamental solitons attractive for optical fiber communication systems [24]. We can also get the peak pulse power P_0 required to support the fundamental soliton:

$$P_0 = \frac{|\beta_2|}{\gamma T_0^2} \approx \frac{3.11 |\beta_2|}{\gamma T_{FWHM}^2}, \quad (2.1.31)$$

where the FWHM (Full width half maximum) of soliton is defined using $T_{FWHM} \approx 1.76 T_0$ for a sech-shape pulse. Besides the fundamental soliton, high order solitons also exist where N equals to an integral other than one [46], but they will not be discussed in this thesis.

On the other hand, in normal dispersion regime with $\text{sgn}(\beta_2) = 1$, Eq. (2.1.26) is written as:

$$i \frac{\partial u}{\partial \xi} - \frac{1}{2} \frac{\partial^2 u}{\partial \tau^2} + |u|^2 u = 0. \quad (2.1.32)$$

Eq. (2.1.32) has the dark soliton solution as [29]:

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$$u(\xi, \tau) = \eta \left[B \tanh(\xi) - i\sqrt{1-B^2} \right] \exp(i\eta^2 \xi), \quad (2.1.33)$$

where $\xi = \eta B (\tau - \tau_s - \eta B \sqrt{1-B^2})$. The parameters η and τ_s represent the background amplitude and dip location of the dark soliton respectively. In contrast to the previous bright solitons, dark solitons have a new parameter B , standing for the depth of dark soliton. For $|B|=1$, the dip intensity falls to zero and we call these dark solitons as “black solitons”; for $|B|<1$, the dip intensity doesn’t reach zero and we call them “gray solitons”. Fig. 2.2 shows the intensity and phase profiles of dark solitons with different values of B [46].

Figure 2.2. Intensity and phase profile of dark solitons for several values of B .
(After Ref. [46], ©Elsevier)

In Eq. (2.1.33), when $B = \eta = 1$, the solution becomes:

$$u(\xi, \tau) = \tanh(\xi) \exp(i\xi). \quad (2.1.34)$$

There is a phase jump of π at the bottom of the dark soliton. Correspondingly, we also can call it the fundamental dark soliton. It represents a pulse with tanh-shape amplitude, exhibiting an intensity dip at the center, propagate unchanged in a normal dispersive optical fiber. The dark solitons also have potential applications in optical fiber communication [39,101].

2.2 Pulse propagation in fiber lasers

2.2.1 Gain profile of rare-earth ion doped gain fibers

Rare-earth doped gain fibers are the single mode fibers with the active rare-earth ions doped in their fiber cores. Various rare-earth ions could be doped in order to achieve a wide range of wavelength emission of fiber lasers, as shown in Table 1.1. Among the various rare-earth doped fibers, erbium-doped fibers (EDF) have attracted special attentions as they have a broadband gain in the 1.55 μm optical fiber communication window [15]. Erbium-doped fiber amplifiers (EDFA) have been a necessary component for the long range optical fiber communication systems [102]. Inserting a piece of EDF into a fiber ring, one can construct a typical erbium doped fiber ring laser, which is the testbed of my research work in this thesis.

Here we briefly introduce the operation principles of an erbium doped fiber ring laser: By pumping the Erbium-ions from their ground state to the excited state, population inversion could be achieved between the two energy levels. Then the stimulated emission happens and amplifies the light incident to the fiber whose frequency is within the gain profile. The optical spectrum of a fiber amplifier is mainly determined by the property of dopants. This is the basic principle for any laser system. For erbium ions, they could be schematically described as a 3-level system, as shown in Fig. 2.3(a) [103]. In practice, the pumping light wavelength can be either 980 nm or 1480 nm.

Figure 2.3. (a) Energy levels of erbium ions in silica fibers. (b) Absorption and gain spectra of an erbium-doped fiber. (After Ref. [103], © IEEE.)

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Generally, the EDF is characterized by its small signal gain, gain bandwidth, gain saturation power and the noise figure. For a homogeneously broaden gain medium as Er^{3+} ions, the gain coefficient of EDF can be written as [76]:

$$g(\omega) = \frac{g_0}{1 + (\omega - \omega_0)^2 T_0^2 + P/P_s}, \quad (2.2.1)$$

where g_0 is the small signal gain, ω is the frequency of incident light, ω_0 is the resonant frequency of the doped Er^{3+} ions, T_2 is the dipole relaxation time of the doped Er^{3+} ions, P is the power of pumping light and P_s is the saturation power of gain medium. The saturation power P_s depends on the stimulated emission cross section and the upper energy level of doped Er^{3+} ions. For a 3-level pumping scheme as Er^{3+} , it's also related with the pumping power. The gain spectrum of EDF is shown in Fig 2.3(b). The gain bandwidth of EDF can be nearly 40 nm, which supports as short to 60 fs pulse. But in practice, not all the gain spectrum can be efficiently used due to some results such as the non-uniform gain profile, fiber birefringence and the cavity effect in fiber lasers.

2.2.2 Ginzburg-Laudau Equation

When an optical pulse propagates in a doped gain fiber, the gain effects must be considered. Generally, the interaction of doped ions with light is governed by the Maxwell-Bloch equation [76]. However, when the incident light pulse width is much larger comparing with the dipole relaxation time (< 0.1 fs) of Er^{3+} ions, the rate equation approximation can be made and the gain term can be directly added into the NLSE Eq. (2.1.24).

Due to the gain dispersion, the frequency dependence of the fiber gain coefficient can be approximated by:

$$g(\omega) = g(\omega_0) + \left. \frac{\partial g}{\partial \omega} \right|_{\omega=\omega_0} (\omega - \omega_0) + \frac{1}{2} \left. \frac{\partial^2 g}{\partial \omega^2} \right|_{\omega=\omega_0} (\omega - \omega_0)^2 + \dots \quad (2.2.2)$$

Assuming that the pulse carrier frequency of the light just coincides with the gain peak, the NLSE Eq. (2.1.24) with gain term becomes:

$$\frac{\partial E}{\partial z} = -i \frac{\beta_2}{2} \frac{\partial^2 E}{\partial T^2} + i \gamma |E|^2 E + \frac{g}{2} E + \frac{g}{2\Omega_g^2} \frac{\partial^2 E}{\partial T^2}, \quad (2.2.3)$$

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where g is the peak gain coefficient and Ω_g is the gain bandwidth. Eq. (2.2.3) is in the form called Grinzburg-Laudau equation (GLE). The time dependence of the peak gain coefficient varies with time due to the gain saturation and is determined by [76]:

$$\frac{\partial g}{\partial t} = \frac{g_0 - g}{T_1} - \frac{g|E|^2}{E_s}, \quad (2.2.4)$$

where g_0 is the small signal gain, T_1 is the population decay time and E_s is the saturation energy. For Er^{3+} ions, T_1 is in the time scale of 10 ms, if the pulse width is far narrower than this, such is normal for ultrashort pulse, T_1 term is negligible during the pulse amplification, so $g(t)$ can be rewritten as:

$$g(t) = g_0 \exp \left[-\frac{1}{E_s} \int_{-\infty}^t |E|^2 dt \right]. \quad (2.2.5)$$

The typical value of E_s for EDF is about 10 uJ. As the pulse energy is usually much smaller than E_s , so gain saturation is negligible over the duration of a single pulse. However, in an ultrashort pulse fiber laser, the pulse will circulate in the cavity, so the average light power may still saturate the gain and determines the saturated gain value.

In practice, to apply Eq. (2.2.3) to the real fiber ring laser, we must consider the laser cavity losses. These include the propagation loss in the fiber and the cavity output loss. So Eq. (2.2.3) is extended as:

$$\frac{\partial E}{\partial z} = -i \frac{\beta_2}{2} \frac{\partial^2 E}{\partial T^2} + i \gamma |E|^2 E + \frac{g - \alpha}{2} E + \frac{g}{2\Omega_g^2} \frac{\partial^2 E}{\partial T^2}, \quad (2.2.6)$$

Here the loss is averaged into the whole cavity length and represented by α . Then Eq. (2.2.6) can be normalized to:

$$i \frac{\partial U}{\partial \xi} + \frac{1}{2} (1 - id) \frac{\partial^2 U}{\partial \tau^2} + N^2 |U|^2 U = \frac{i}{2} \mu U, \quad (2.2.7)$$

using the normalization as Eq. (2.1.29) and

$$\begin{cases} d = \frac{g T_2^2}{n_0 |\beta_2|} \\ \mu = \left(\frac{g}{n_0} - \alpha \right) L_D \end{cases}. \quad (2.2.8)$$

2.2.3 Dissipative solitons and self-similar pulses/similaritons

For the GLE, there is no exact soliton solution as the NLSE. However, when $N=1$, using a trial-and-error method, it still can find a soliton solution with some parameters [76]:

$$U(\xi, \tau) = \text{sech}(p\tau) \exp[iq \ln \cosh(p\tau) + i\Gamma \xi], \quad (2.2.9)$$

where p, q, Γ are parameters, which are related with the gain and loss in fiber laser. Due to the existence of gain and loss, fiber laser is a dissipative system and the soliton described by Eq. (2.2.9) is called the dissipative soliton, also named parameter soliton. In physics, GLE is widely used in described different kinds of nonlinear dissipative solitons [104]. The dissipative solitons also have a sech-shape pulse profile. However, different from the NLSE solitons, the dissipative solitons are highly chirped. More importantly, dissipative solitons can be formed in both normal and anomalous dissipative regime.

In recent years, there is another kind of wave-breaking free pulse formed in normal dispersion regime attracted many researchers' attention. The pulse is called the self-similar pulse or similariton. Starting from the NLSE, when the dispersion is normal, NLSE Eq. (2.1.4) has a solution taking the form [34]:

$$E(z, T) = \begin{cases} E_0 \sqrt{1 - (T/\tau(z))} e^{ib(z)T^2} & , \text{if } |t| \leq \tau(z) \\ 0 & , \text{if } |t| \geq \tau(z) \end{cases}, \quad (2.2.10)$$

which has parabolic pulse shape profile. In propagation, the pulse will maintain its shape, but is always a scaled version of itself, as shown in Fig. 1.6, which is called self-similarity. Tamura and Nakazawa first reported that the parabolic pulse can reduce wave breaking in a short fiber amplifier [105]. Kruglov *et al.* systematically investigated a new class of solutions and showed theoretically that the parabolic pulses propagate with self-similarity in an amplifier [106–108]. This kind of self-similar parabolic pulse is also named as “similariton” to highlight its solitary wave nature. There are many experimental reports about the self-similar pulses in EDFA and fiber lasers [42,44,109–114]. Among them, the work by Ilday *et al.* reported a soliton-similariton fiber laser [44]. In a mode locked erbium doped fiber ring laser, the pulse evolves into the similariton in

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the gain fiber with normal dispersion, and then is transformed into a regular soliton in the rest of the cavity. However, in the theoretical discussion of the self-similar pulse, the gain bandwidth in amplifier is neglected. The unlimited bandwidth is impossible for a real fiber laser cavity, either due to the bandpass width of fiber components or the gain spectrum. The gain bandwidth limitation can be neglected only when the pulse spectrum is narrower than the effective gain bandwidth.

2.2.4 Theoretical modelling of light circulation in fiber lasers

Here we consider the light circulation in a unidirectional fiber ring laser. Starting from the GLE Eq. (2.2.3), the light propagation in the n th cavity roundtrip is described as:

$$i \frac{\partial E^n}{\partial z_n} - \frac{\beta_2}{2} \frac{\partial^2 E^n}{\partial T^2} + \gamma |E^n|^2 E^n - i \frac{g}{2} E^n - i \frac{g}{2\Omega_g} \frac{\partial^2 E^n}{\partial T^2} = 0, \quad (2.2.11)$$

where $E^n = E^n(z_n, T)$ are the complex envelopes of the optical field, n denotes the n th passage through the fiber laser cavity, β_2 is the group velocity dispersion (GVD) coefficient, γ is the nonlinear coefficient, g is the gain coefficient and Ω_g is the effective gain bandwidth of the fiber laser. z_n is the light propagation distance parameter and T is the retarded time in a reference frame traveling with the group velocity of light. When light is circulating in the cavity, it should also fulfill the cavity boundary condition:

$$E^{n+1}(z_n = 0, T) = \rho \exp(-i\phi) E^n(z_n = L, T), \quad (2.2.12)$$

where L is the cavity length, ρ and θ are the amplitude reflection and transmission coefficient between each cavity cycle, $\rho^2 = 1 - \theta^2$. ϕ is the linear phase delay generated when light propagating one round trip in the cavity.

For a good cavity operating near the resonance and having a cavity length (L) far shorter than the dispersion length (L_d) in the cavity, we have the following approximation:

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$$\begin{cases} \rho \approx 1 - \theta^2/2 \equiv 1 - \varepsilon\theta^2/2 \\ (2m\pi - \phi)/\pi = \delta/\pi \approx \varepsilon \\ L/L_d \approx \varepsilon \end{cases} \quad (2.2.13)$$

Where ε is an arbitrary small parameter ($\varepsilon \ll 1$), m is the order of the closest cavity resonance and δ is the cavity detuning parameter, therefore the boundary condition Eq. (2.2.12) can be simplified as

$$E^{n+1}(z_n = 0, T) = (1 - i\varepsilon\pi - \varepsilon\theta^2/2)E^n(z_n = L, T) + O(\varepsilon^2) \quad (2.2.14)$$

and Eq. (2.2.12) can be rewritten in one cavity cycle as

$$\begin{aligned} & E^n(z_n = \varepsilon L_d, T) - E^n(z_n = 0, T) \\ &= -i\varepsilon L_d \frac{\beta_2}{2} \frac{\partial^2 E^n}{\partial T^2} + i\varepsilon L_d \gamma |E^n|^2 E^n + \frac{\varepsilon L_d g}{2} E^n + \frac{\varepsilon L_d g}{2\Omega_g} \frac{\partial^2 E^n}{\partial T^2}. \end{aligned} \quad (2.2.15)$$

Combining Eq. (2.2.14) and (2.2.15), we can get

$$\begin{aligned} E^{n+1}(z_n = 0, T) &= E^n(z_n = 0, T) - i\varepsilon L_d \frac{\beta_2}{2} \frac{\partial^2 E^n}{\partial T^2} + i\varepsilon L_d \gamma |E^n|^2 E^n \\ &+ \frac{\varepsilon L_d g}{2} E^n + \frac{\varepsilon L_d g}{2\Omega_g} \frac{\partial^2 E^n}{\partial T^2} - (i\varepsilon\pi E^n + \varepsilon\theta^2/2)E^n(z_n = 0, T) + O(\varepsilon^2). \end{aligned} \quad (2.2.16)$$

Under good cavity condition, the variation of the optical field over one cavity roundtrip is very small, so we can approximate the spatial differential as

$$\left. \frac{\partial E(z, t)}{\partial z} \right|_{z=n\varepsilon L_d} = \frac{E(z = (n+1)\varepsilon L_d, T) - E(z = n\varepsilon L_d, T)}{\varepsilon L_d} \quad (2.2.17)$$

Substituting Eq. (2.2.16) into Eq. (2.2.17) we get

$$i \frac{\partial E}{\partial z} - \frac{\beta_2}{2} \frac{\partial^2 E}{\partial T^2} + \gamma |E|^2 E + i \frac{\theta^2}{2L} E - i \frac{g}{2} E - i \frac{g}{2\Omega_g} \frac{\partial^2 E}{\partial T^2} - \frac{\delta}{L} E = 0. \quad (2.2.18)$$

Normalize Eq. (2.2.18) by $L_{NL} = 1/\gamma P_0$, $L_d = T_0^2/|\beta_2|$, $u\sqrt{P_0} = E$, $\eta = \text{sgn}(\beta_2)$,

$\xi = z/L_d$, $N^2 = L_d/L_{NL} = 1$, where P_0 and T_0 are the peak power and pulse width

of a reference pulse. For simplicity, the normalized cavity detuning is set as

$\Delta = L_d \delta/L$ and $a = \theta^2 L_d/L$ is the normalized loss coefficient of the cavity. $G = gL_d$ is

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the normalized gain coefficient and $\Omega = \Omega_g T_0$ is the normalized gain bandwidth, then Eq. (2.2.18) can be rewritten as

$$i \frac{\partial u}{\partial \xi} - \frac{\eta}{2} \frac{\partial^2 u}{\partial \tau^2} + |u|^2 u - i \frac{(G-a)}{2} u - i \frac{G}{2\Omega^2} \frac{\partial^2 u}{\partial \tau^2} - \Delta u = 0. \quad (2.2.19)$$

Compared with the traditional GLE, Eq. (2.2.19) considers the cavity property of fiber lasers. The last term on the left-hand side of Eq. (2.2.19) stands for the cavity detuning. When the fiber laser is in a steady state operation, $\delta = \Delta = 0$. So the detuning term vanished and Eq. (2.2.19) comes to the traditional GLE form. However, when the cavity is detuned, by either tuning the intracavity polarization controller (PC) or changing the pumping power in experiment, the cavity detuning will occur and the fiber laser comes into a nonlinear unstable state until it come back to a new steady state operation. In this unstable period, the cavity detuning will lead to some nonlinear phenomena such as the cavity induced modulation instability (CIMI).

2.3 Vector pulse propagation in fiber lasers

In all the discussion above, we didn't involve the polarization characters of light propagation in the optical fibers. In practice, the SMF is not actually completely single mode because it supports two degenerate modes that polarize along two orthogonal directions. All fibers exhibit modal birefringence because of the deviation from cylindrical symmetry due to the random variations in core shape and/or mechanical induced anisotropy along the fiber. The strength of modal birefringence is defined as:

$$B = \Delta\beta / k_0 = |n_x - n_y|, \quad (2.3.1)$$

where $\Delta\beta = |\beta_x - \beta_y|$ is the difference in the propagation constant between two polarization modes, n_x and n_y are the respective modal refractive index of two polarization modes. For a given value of B , after the incident light propagating at a length of L , the phase difference between two polarization modes is:

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$$\Delta\phi = \frac{2\pi(n_x - n_y)L}{\lambda} = \frac{2\pi BL}{\lambda} = 2\pi \cdot \frac{L}{L_B}, \quad (2.3.2)$$

where $L_B = 2\pi/|\beta_x - \beta_y| = \lambda/B$ is called the beat length. From Eq. (2.3.2) we can see that the polarization state of incident light varies periodically along the fiber. At every propagation distance of L_B , a phase difference of 2π will be generated between the two polarization modes, which means the light returns to its initial polarization state. Beat length indicates the strength of fiber birefringence. Weak birefringence means long beat length. For a standard SMF, L_B is in the range of several meters; for the polarization-maintaining (PM) fiber, as the strong built-in birefringence, L_B can be just several millimeters. In practice, the fiber has intrinsic birefringence because of the manufacture; on the other hand, the fiber birefringence is also can be externally changed by mechanical twisting or bending. In experiment the polarization controller (PC) is usually used for tuning the fiber birefringence. Take the three-ring PC for example, the fiber was coiled inside the rings, once the angle of rings is changed, the fiber inside is twisted and fiber birefringence is also changed.

When the nonlinear effects in optical fiber become significant, the sufficiently intense optical field can induce nonlinear birefringence whose magnitude is light intensity dependent. The total refractive index for each polarization modes can be described as:

$$n_j = n_j^L + \Delta n_j \quad (j = x, y), \quad (2.3.3)$$

where n_j^L is the nonlinear part of refractive index of one polarization mode. The nonlinear contribution Δn_x and Δn_y are given by:

$$\Delta n_x = n_2 \left(|E_x|^2 + \frac{2}{3} |E_y|^2 \right), \Delta n_y = n_2 \left(|E_y|^2 + \frac{2}{3} |E_x|^2 \right), \quad (2.3.4)$$

The value of n_2 is usually treated as the same for two polarization modes. In Eq. (2.3.4), the first term on the right-hand side is responsible for SPM and the second term stands for XPM. In physics, the presence of XPM term induces a nonlinear coupling between the two polarization modes of E_x and E_y . Therefore, we can extend the NLSE to a coupled vector form:

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$$\begin{cases} \frac{\partial E_x}{\partial z} + \beta_{1x} \frac{\partial E_x}{\partial t} + \frac{i\beta_2}{2} \frac{\partial^2 E_x}{\partial t^2} = i\gamma \left(|E_x|^2 + \frac{2}{3} |E_y|^2 \right) + \frac{i\gamma}{3} E_x^* E_y^2 \exp(-2i\Delta\beta z) \\ \frac{\partial E_y}{\partial z} + \beta_{1y} \frac{\partial E_y}{\partial t} + \frac{i\beta_2}{2} \frac{\partial^2 E_y}{\partial t^2} = i\gamma \left(|E_y|^2 + \frac{2}{3} |E_x|^2 \right) + \frac{i\gamma}{3} E_y^* E_x^2 \exp(2i\Delta\beta z) \end{cases}, \quad (2.3.5)$$

here $\Delta\beta = \beta_x - \beta_y = (2\pi/\lambda)B = \frac{2\pi}{L_B}$ is related with the linear birefringence of the

fiber. The modal birefringence leads to different group velocities for the two polarization modes because $\beta_x \neq \beta_y$ in general. The last term in Eq. (2.3.5) stands for the coherent coupling between two polarization modes and leads to degenerate four-wave mixing (FWM). In practice, the significance of FWM term is related to the birefringence. When the birefringence is strong and the value of L_B is very small compared with fiber length L , the FWM term will frequently change the sign and its contribution averages out to zero. The FWM term in a highly birefringent fiber can be neglected. So the coherently coupling only can happen in the weakly birefringent fibers. Similarly, the GLE (2.2.3) can also be extended to the coupled vector form, to describe a birefringent vector fiber laser cavity:

$$\begin{cases} \frac{\partial E_x}{\partial z} + \beta_{1x} \frac{\partial E_x}{\partial T} + \frac{i\beta_2}{2} \frac{\partial^2 E_x}{\partial T^2} = i\gamma \left(|E_x|^2 + \frac{2}{3} |E_y|^2 \right) + \frac{i\gamma}{3} E_x^* E_y^2 \exp(-2i\Delta\beta z) + \frac{g}{2} E_x + \frac{g}{2\Omega_g^2} \frac{\partial^2 E_x}{\partial T^2} \\ \frac{\partial E_y}{\partial z} + \beta_{1y} \frac{\partial E_y}{\partial T} + \frac{i\beta_2}{2} \frac{\partial^2 E_y}{\partial T^2} = i\gamma \left(|E_y|^2 + \frac{2}{3} |E_x|^2 \right) + \frac{i\gamma}{3} E_y^* E_x^2 \exp(2i\Delta\beta z) + \frac{g}{2} E_y + \frac{g}{2\Omega_g^2} \frac{\partial^2 E_y}{\partial T^2} \end{cases}. \quad (2.3.6)$$

By numerically solving Eq. (2.3.6) and choosing suitable parameters, we can simulate the operation of a specific fiber laser. The basic numerical simulation steps are presented in the next section.

2.4 Procedures for numerically solving the coupled Grinzburg-Laudau equations

The Grinzburg-Laudau equation can be numerically solved using the split-step Fourier method [46]. Here we briefly introduce the basics about split-step Fourier method. We can rewrite GLE in the following form:

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$$\frac{\partial E}{\partial z} = (\hat{D} + \hat{N})E, \quad (2.4.1)$$

where $\hat{D}$ is a differential operator accounting for dispersion and $\hat{N}$ is the nonlinear operator that governs the effect of nonlinearity. Physically, dispersion and nonlinearity act simultaneously during the pulse propagation. The split-step Fourier method takes an approximate solution by assuming that in a small propagation distance of h , the dispersion and nonlinearity effects act separately. The pulse propagation from Z to $Z+h$ is divided into two steps. In the first step, nonlinearity acts alone while ignoring the dispersion. In the second step, dispersion acts alone but no linearity. Mathematically, Eq. (2.4.1) can be transformed into:

$$E(z+h, T) \approx \exp(h\hat{D})\exp(h\hat{N})E(z, T). \quad (2.4.2)$$

By separating the dispersion and nonlinearity effects, the propagation under the influence of dispersion alone can be solved in frequency domain and the Fast Fourier Transform (FFT) algorithm can be employed to make the calculation very fast. In order to improve the accuracy of split-step Fourier method, an improved symmetrized split-step Fourier method can be applied. Eq. (2.4.2) is replaced by:

$$E(z+h, T) \approx \exp\left(\frac{h}{2}\hat{D}\right)\exp(h\hat{N})\exp\left(\frac{h}{2}\hat{D}\right)E(z, T), \quad (2.4.3)$$

The main difference is that the effect of nonlinearity is included in the middle of the segment h rather than at the segment boundary. A schematic illustration of the symmetrized split-step Fourier method is shown in Fig. 2.4 [46].

Figure 2.4. Schematic illustration of the symmetrized split-step Fourier method used for numerical simulations. Fiber length is divided into many

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segment of width h . Within each segment, nonlinearity effect is included at the midplane shown by a dashed line. (After Ref. [46], ©Elsevier)

Here we show the basic steps of solving the GLE using the symmetric split-step Fourier method. In Eq. (2.1.17), we have defined $T = t - z/v_g$, which stands for the retarded frame where the Eq. (2.3.6) is in a coordinate system travelling at the group velocity v_g . Here we use the average group velocities of two polarization components, because due to the birefringence, v_{gx} varies from v_{gy} .

And we introduce a new parameter $\delta = (\beta_{1x} - \beta_{1y})/2$ to simplify Eq. (2.3.6):

$$\begin{cases} \frac{\partial E_x}{\partial z} + \delta \frac{\partial E_x}{\partial T} + \frac{i\beta_2}{2} \frac{\partial^2 E_x}{\partial T^2} = i\gamma \left(|E_x|^2 + \frac{2}{3} |E_y|^2 \right) + \frac{i\gamma}{3} E_x^* E_y^2 \exp(-2i\Delta\beta z) + \frac{g}{2} E_x + \frac{g}{2\Omega_g^2} \frac{\partial^2 E_x}{\partial T^2} \\ \frac{\partial E_y}{\partial z} - \delta \frac{\partial E_y}{\partial T} + \frac{i\beta_2}{2} \frac{\partial^2 E_y}{\partial T^2} = i\gamma \left(|E_y|^2 + \frac{2}{3} |E_x|^2 \right) + \frac{i\gamma}{3} E_y^* E_x^2 \exp(2i\Delta\beta z) + \frac{g}{2} E_y + \frac{g}{2\Omega_g^2} \frac{\partial^2 E_y}{\partial T^2} \end{cases} \quad (2.4.4)$$

Then we can eliminate the exponential components in the FWM term by replace E_x and E_y with $E_x = F_h \exp(-i\Delta\beta z/2)$ and $E_y = F_v \exp(i\Delta\beta z/2)$, Eq. (2.4.1) changes to:

$$\begin{cases} \frac{\partial F_h}{\partial z} - i\frac{\Delta\beta}{2} F_h + \delta \frac{\partial F_h}{\partial T} + \frac{i\beta_2}{2} \frac{\partial^2 F_h}{\partial T^2} = i\gamma \left(|F_h|^2 + \frac{2}{3} |F_v|^2 \right) F_h + \frac{i\gamma}{3} F_h^* F_v^2 + \frac{g}{2} F_h + \frac{g}{2\Omega_g^2} \frac{\partial^2 F_h}{\partial T^2} \\ \frac{\partial F_v}{\partial z} + i\frac{\Delta\beta}{2} F_v + \delta \frac{\partial F_v}{\partial T} + \frac{i\beta_2}{2} \frac{\partial^2 F_v}{\partial T^2} = i\gamma \left(|F_v|^2 + \frac{2}{3} |F_h|^2 \right) F_v + \frac{i\gamma}{3} F_v^* F_h^2 + \frac{g}{2} F_v + \frac{g}{2\Omega_g^2} \frac{\partial^2 F_v}{\partial T^2} \end{cases} \quad (2.4.5)$$

Here the gain is the saturated gain, which is defined as:

$$g(t) = g_0 \exp \left[-\frac{1}{E_s} \int_{-\infty}^t \left(|F_h|^2 + |F_v|^2 \right) dt \right]. \quad (2.4.6)$$

Now we can apply the symmetric split-step Fourier method to solve the modified coupled-GLEs Eq. (2.4.5).

1. Ignore the nonlinear terms. The coupled GLEs then becomes:

$$\begin{cases} \frac{\partial F_h}{\partial z} = i\frac{\Delta\beta}{2} F_h - \delta \frac{\partial F_h}{\partial T} - \frac{i\beta_2}{2} \frac{\partial^2 F_h}{\partial T^2} + \frac{g}{2\Omega_g^2} \frac{\partial^2 F_h}{\partial T^2} \\ \frac{\partial F_v}{\partial z} = -i\frac{\Delta\beta}{2} F_v + \delta \frac{\partial F_v}{\partial T} - \frac{i\beta_2}{2} \frac{\partial^2 F_v}{\partial T^2} + \frac{g}{2\Omega_g^2} \frac{\partial^2 F_v}{\partial T^2} \end{cases} \quad (2.4.7)$$

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Since Fourier Transform can be defined as $\tilde{F}(\omega) = \int_{-\infty}^{+\infty} F(T) \exp(-i\omega T) dT$, the Fourier Transform of $\frac{\partial F(T)}{\partial T}$ is $i\omega \tilde{F}(\omega)$.

Perform Fourier Transform on Eq. (2.4.7), we obtain:

$$\begin{cases} \frac{\partial \tilde{F}_h}{\partial z} = i \frac{\Delta\beta}{2} \tilde{F}_h - i\omega\delta \tilde{F}_h + \frac{i\beta_2}{2} \omega^2 \tilde{F}_h^2 - \frac{g}{2\Omega_g^2} \omega^2 \tilde{F}_h^2 \\ \frac{\partial \tilde{F}_v}{\partial z} = -i \frac{\Delta\beta}{2} \tilde{F}_v + i\omega\delta \tilde{F}_v + \frac{i\beta_2}{2} \omega^2 \tilde{F}_v^2 - \frac{g}{2\Omega_g^2} \omega^2 \tilde{F}_v^2 \end{cases} \quad (2.4.8)$$

For an increment step of $\frac{\partial z}{2}$, $\tilde{F}_h\left(z + \frac{\partial z}{2}, T\right)$ and $\tilde{F}_v\left(z + \frac{\partial z}{2}, T\right)$ are given below:

$$\begin{cases} \tilde{F}_h\left(z + \frac{\partial z}{2}, \omega\right) = \tilde{F}_h(z, \omega) \times \exp\left[\left(-\frac{g_p}{2\Omega_g^2} \omega^2 + i\left(\frac{\beta_2}{2} \omega^2 + \frac{\Delta\beta}{2} - \omega\delta\right)\right) \frac{\partial z}{2}\right] \\ \tilde{F}_v\left(z + \frac{\partial z}{2}, \omega\right) = \tilde{F}_v(z, \omega) \times \exp\left[\left(-\frac{g_p}{2\Omega_g^2} \omega^2 + i\left(\frac{\beta_2}{2} \omega^2 - \frac{\Delta\beta}{2} + \omega\delta\right)\right) \frac{\partial z}{2}\right] \end{cases} \quad (2.4.9)$$

Then the $\tilde{F}_h\left(z + \frac{\partial z}{2}, T\right)$ and $\tilde{F}_v\left(z + \frac{\partial z}{2}, T\right)$ can be obtained through performing the inverse Fourier transform to Eq. (2.4.9).

2. Ignore the dispersion terms. The coupled GLEs then becomes:

$$\begin{cases} \frac{\partial F_h}{\partial z} = i\gamma \left(|F_h|^2 + \frac{2}{3}|F_v|^2\right) F_h + \frac{i\gamma}{3} F_h^* F_v^2 + \frac{g}{2} F_h \\ \frac{\partial F_v}{\partial z} = i\gamma \left(|F_v|^2 + \frac{2}{3}|F_h|^2\right) F_v + \frac{i\gamma}{3} F_v^* F_h^2 + \frac{g}{2} F_v \end{cases} \quad (2.4.10)$$

In order to eliminate the FWM terms $\frac{i\gamma}{3} F_h^* F_v^2$ and $\frac{i\gamma}{3} F_v^* F_h^2$ in the coupled GLEs,

a so-called Circular Transformation is employed. We defined $F_r = \frac{F_h + iF_v}{\sqrt{2}}$ and

$F_l = \frac{F_h - iF_v}{\sqrt{2}i}$, from which we can obtain $F_h = \frac{F_r + F_l}{\sqrt{2}}$ and $F_v = \frac{F_r - F_l}{\sqrt{2}i}$. Substitute

them into Eq. (2.4.10), after some algebra transformation, Eq. (2.4.10) can be simplified as:

$$\begin{cases} \frac{\partial F_r}{\partial z} = \frac{g}{2} F_r + i \frac{2\gamma}{3} \left(|F_r|^2 + 2|F_l|^2\right) F_r \\ \frac{\partial F_l}{\partial z} = \frac{g}{2} F_l + i \frac{2\gamma}{3} \left(|F_l|^2 + 2|F_r|^2\right) F_l \end{cases} \quad (2.4.11)$$

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Hence for an increment step of ∂z , $F_r(z + \partial z, T)$ and $F_l(z + \partial z, T)$ are given as:

$$\begin{cases} F_r(z + \partial z, T) = F_r(z, T) \times \exp\left[\left(\frac{g}{2} + i\frac{2\gamma}{3}(|F_r|^2 + 2|F_l|^2)\right)\partial z\right] \\ F_l(z + \partial z, T) = F_l(z, T) \times \exp\left[\left(\frac{g}{2} + i\frac{2\gamma}{3}(|F_l|^2 + 2|F_r|^2)\right)\partial z\right] \end{cases} \quad (2.4.11)$$

$F_h(z + \partial z, T)$ and $F_v(z + \partial z, T)$ can be obtained by performing the inverse Circular Transformation to Eq. (2.4.11)

In the simulation, all the parameters are based on the experimental conditions. For the parameters related to the cavity birefringence:

$$\Delta\beta = \beta_x - \beta_y = (2\pi/\lambda)B = \frac{2\pi}{L_B}, \quad L_B \text{ is the beat length;}$$

$$\delta = \frac{\beta_{lx} - \beta_{ly}}{2} = \frac{1}{2c} \left(n_x(\omega_0) + \omega_0 \left. \frac{dn_x}{d\omega} \right|_{\omega=\omega_0} - n_y(\omega_0) - \omega_0 \left. \frac{dn_y}{d\omega} \right|_{\omega=\omega_0} \right), \text{ where } c \text{ is the speed of}$$

light in vacuum. We can assume $\left. \frac{dn_x}{d\omega} \right|_{\omega=\omega_0} = \left. \frac{dn_y}{d\omega} \right|_{\omega=\omega_0}$ in general, so

$$\delta = \frac{\beta_{lx} - \beta_{ly}}{2} = \frac{n_x - n_y}{2c} = \frac{\lambda_0}{2cL_B}, \quad \text{where } \lambda_0 \text{ is the central wavelength of the}$$

propagating light. If we want to simulate the fiber laser with very weak birefringence and achieve coherent coupling between two polarization modes, a relatively long L_B (in hundreds of meters) should be applied. On the other hand, a fiber ring laser may contain different kinds of fibers, such as EDF, serving as gain fiber; dispersion compensation fiber (DCF) which is usually used to shift the average cavity dispersion and SMF. In the simulation, the fiber parameters vary in different fiber parts. In the lasing wavelength of erbium-doped fiber laser, dispersion parameter varies between these fibers, for example: $D(\text{SMF}) = 18 \text{ ps/nm*km}$, $D(\text{EDF}) = -48 \text{ ps/nm*km}$, $D(\text{DCF}) = -4 \text{ ps/nm*km}$. The gain bandwidth plays a very important role in the pulse shaping in fiber lasers [72]. Although the EDF has a very wide gain spectrum, in practice, due to the fiber components' bandpass width, cavity birefringent and the ring cavity effect, the effective gain bandwidth may not be as wide as the theoretical value.

3

Cavity induced modulation instability lasing of fiber ring lasers

Modulation instability (MI) is an important nonlinear phenomenon in fiber optics, referring to a process of continuous wave (CW) light breaking up into periodic pulse trains. The occurrence of modulation instability is the result of interplay between nonlinearity and dispersion effect. As the nature of its nonlinear properties, MI only can occur in the anomalous dispersion regime [46]. The first experimental observation of modulation instability in optical fiber is reported by K. Tai *et al.* in [115]. The modulation frequency of MI depends on the light intensity and group velocity dispersion (GVD), which is usually tens to hundreds of GHz. Therefore, the modulation instability fiber

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lasers could be a potential source to provide high repetition rate pulse trains for optical communication system [116–118].

Moreover, M. Haelterman, S. Trillo and S. Wabnitz proposed a theoretical model of a new kind of modulation instability based on a synchronously pumped fiber ring cavity [77,119]. Different from the conventional MI, this new MI is induced by the cavity boundary condition and can occur either in normal or anomalous dispersion regime. Later, S. Coen *et al* experimentally confirm the existence of this new modulation instability [69]. For its occurrence mechanism, the new MI is called as the cavity induced modulation instability (CIMI). The modulation frequency of CIMI is related with the cavity phase detuning, which is determined by the total phase shift after light circulating in one cavity round-trip. If we consider this phenomenon in a more general case, CIMI should exist in every nonlinear optical cavity with cavity phase detuning. The fiber ring laser is a favorite system for the research of nonlinear optics phenomena. For its high intra cavity light intensity, the nonlinearity in fiber ring laser cavity could be very strong. Similar to the synchronously pumped passive fiber ring cavity, the fiber ring lasers also have a nonlinear ring cavity, so we anticipate that the CIMI could also occur in a fiber laser. Actually, Tang *et al.* has reported the experimental observation of CIMI effect in a fiber ring laser [70]. In this chapter, we focus on the theory and experiments of the cavity induced modulation instability lasing. Based on CIMI, stable periodic pulse train emission with tunable repetition rate can be achieved. The cavity induced modulation instability theory of the active fiber ring laser model will be briefly reviewed in Section 1. The experimental results of stable cavity induced modulation instability lasing in both normal and anomalous dispersive fiber lasers will be shown in Section 2. Some discussions on the CIMI lasing will be included in Section 3.

3.1 Theory of CIMI in fiber ring lasers

Here we briefly review the theory of cavity induced modulation instability in a fiber ring laser as firstly derived in [70]. The description of light propagation in

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a fiber ring laser can be simplified as the extended nonlinear Schrodinger equation (NLSE), which is in the form

$$\frac{\partial E^n}{\partial z} = -i \frac{\beta_2}{2} \frac{\partial^2 E^n}{\partial T^2} + i \gamma |E^n|^2 E^n + g E^n, \quad (3.1.1)$$

where E^n is the amplitude of the optical field in the n^{th} fiber ring cavity round-trip, β_2 is the second-order GVD coefficient, γ is the fiber's nonlinear coefficient, g is gain coefficient of the fiber laser, z is the light propagation distance and T is the retarded time in a reference frame moving at the speed of light. For convenience, we ignore the gain bandwidth limitation in Eq. (3.1.1). Similar as the light in the synchronously pumped fiber ring cavity, the light circulating in an active fiber ring laser also should fulfill the cavity boundary condition

$$E^{n+1}(0, t) = \sqrt{\rho} E^n(L, t) \exp(i\psi_0), \quad (3.1.2)$$

where L is the length of the fiber ring and ρ is the light intensity transmission rate of the coupler in fiber ring laser. ψ_0 is linear phase delay of the light generated after propagating one cavity roundtrip, which is related with the optical path length of one cavity roundtrip.

To simplify Eq. (3.1.1), we normalize it with $\xi = z/L$, $U^n = \sqrt{\gamma L} E^n$, $\tau = t / \sqrt{|\beta_2| L}$, $G = gL$ and Eq. (3.1.1) becomes

$$\frac{\partial U^n}{\partial \xi} = -i \frac{\eta}{2} \frac{\partial^2 U^n}{\partial \tau^2} + i |U^n|^2 U^n + G U^n, \quad (3.1.3)$$

where η is the sign of average cavity dispersion, $\eta = \text{sgn}(\beta_2)$. Eq. (3.1.3) has the following CW solution

$$U^n = U_0 e^{G\xi} \exp\left(i \frac{U_0^2}{2G} \exp(2G\xi)\right), \quad (3.1.4)$$

where U_0 is the light intensity at the beginning of the n^{th} cavity round-trip. Following the same procedures in [69], we analyze the stability of the CW solution against small sideband perturbations. We add perturbation terms v to the CW solution Eq. (3.1.4),

$$U^n = (u_0 e^{G\xi} + v) \exp\left(i \frac{u_0^2}{2G} \exp(2G\xi)\right), \quad (3.1.5)$$

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where

$$\begin{aligned} v = v_1^n(\xi) \exp(i\Omega\tau) + v_{-1}^n(\xi) \exp(-i\Omega\tau) = \exp(\mu\xi) \left[a_n \exp(i\Omega\tau) + b_n \exp(-i\Omega\tau) \right] \\ + \exp(-\mu\xi) \left[c_n \exp(i\Omega\tau) + d_n \exp(-i\Omega\tau) \right], \end{aligned} \quad (3.1.6)$$

Take Eq. (3.1.5) back into Eq. (3.1.3) and solve v , the eigenvalue is

$$\mu = iK = \frac{1}{2} i\Omega \sqrt{\eta^2 \Omega^2 + 4\eta U_0^2} = \Omega \sqrt{-\frac{\Omega^2}{4} - \eta U_0^2}. \quad (3.1.7)$$

The perturbation term also should obey the cavity boundary condition,

$$v_{\pm 1}^{n+1}(\xi=0) = \sqrt{\rho} \exp \left[i \left(\psi_0 + |U_0^2| \right) \right] v_{\pm 1}^n(\xi=1). \quad (3.1.8)$$

Express these relations in the terms of components $a_n, c_n, a_{n+1}, c_{n+1}$, we get

$$\begin{pmatrix} a_{n+1} \\ c_{n+1} \end{pmatrix} = \sqrt{\rho} \begin{pmatrix} \exp(iK)(\cos(\phi) + i\beta \sin(\phi)) & i \exp(-iK)(1 + \beta) \sin(\phi) \\ i \exp(iK)(\beta - 1)(-2i \sin(\phi)) & \exp(-iK)(\cos(\phi) - i\beta \sin(\phi)) \end{pmatrix} \begin{pmatrix} a_n \\ c_n \end{pmatrix}, \quad (3.1.9)$$

where $\beta = \frac{\eta \Omega^2 + 2U_0^2}{-2i\mu}$ and $\phi = \psi_0 + |U_0^2|$. ϕ is the total phase delay of light propagating in a cavity round trip, combining with the linear phase delay ψ_0 and the nonlinear phase delay $|U_0^2|$.

The eigenvalue of the matrix is

$$q_{\pm} = \sqrt{\rho} \left(p \pm \sqrt{p^2 - 1} \right), \quad (3.1.10)$$

where $p = \cos(\phi) \cosh(\mu) - \frac{\eta \Omega^2 / 2 + U_0^2}{\mu} \sin(\phi) \sinh(\mu)$.

CIMI will occur if the modulus of the eigenvalues is larger than unity. So the gain coefficient of CIMI can be defined by $G_{MI} = |q_{\pm}| - 1$ and the gain spectrum can be drawn under different parameters. Under given parameters of η and ρ , the gain spectrum is determined by the total phase delay ϕ . We can define the cavity phase detuning as $\delta = \phi - 2m\pi$, where m is an integral. When the total phase delay equals with the integral multiple times of 2π , the cavity phase detuning $\delta = 0$, it becomes the situation of conventional modulation instability [46]. For the conventional modulation instability, its modulation frequency is purely determined by the dispersion and light intensity. In other

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cases, whenever the total phase delay is unequal to $2m\pi$, the modulation instability will exist and its modulation frequency is also related to the cavity phase detuning. This is regarded as the cavity induced modulation instability (CIMI). In Fig. 3.1, we compare two gain spectra under different linear phase delay. However, it's important to keep in mind that not only by changing the linear phase delay, but also by changing the nonlinear phase delay can lead to the occurrence of CIMI. It means the changing of light intensity also can induce CIMI.

Figure 3.1. The gain spectrum of CIMI under two different linear phase delay.

$$(a): \psi_0 = 0.1\pi, |U_0^2| = 1, \eta = -1 \text{ and } \rho = 0.95$$

$$(b): \psi_0 = 0.3\pi, |U_0^2| = 1, \eta = -1 \text{ and } \rho = 0.95.$$

Here we have some more discussions about the cavity boundary conditions and CIMI of an active fiber ring laser. Considering the fact that under the steady state laser operation, where the cavity gain always balances with cavity losses, the light intensity returns back to its original value after each cavity roundtrip. Therefore, the net gain is zero, so we set $G=0$ and $\sqrt{\rho} = 1$. Hence, the cavity boundary condition only introduces a phase change. When the fiber laser is under steady state operation, the total phase delay must be $2m\pi$. In this case if the intracavity polarization controller (PC) is altered, the linear phase delay will be changed. This will make the total phase delay deviates from $2m\pi$ and lead to the occurrence of CIMI. For the same reason, if we changed the intracavity

light intensity, the nonlinear phase delay will also change and it will trigger the occurrence of CIMI.

3.2 Experimental results of CIMI lasing

In this section, we will present some experimental results of CIMI lasing based on an erbium doped fiber ring laser with a net anomalous cavity dispersion. The setup of fiber laser is shown in Fig. 3.2. The fiber ring has a total cavity length of 13.5 m, consisting of a piece of 3 m Erbium doped fiber (EDF) with a normal dispersion parameter of -48 ps/nm/km and 9.5 m single mode fiber (SMF-28) with an anomalous dispersion parameter of 18 ps/nm/km. The fiber ring cavity is estimated to have an average net anomalous dispersion parameter of 2 ps/nm/km. The fiber laser is pumped by a 1480 nm single mode Raman fiber laser whose maximum output power is 5 W. A polarization independent isolator is inserted in the cavity to force the unidirectional circulation of light in the cavity. Besides, an intra-cavity polarization controller (PC) is used to fine-tune the linear cavity birefringence. A wavelength division multiplexer (WDM) is used to couple the pumping light into the cavity, and a 10% fiber output coupler is used to output the light. The pumping coupling efficiency from the pump source to the cavity is about 60%. All the components used in our experiment have very low polarization dependent loss (PDL) (WDM: 0.01 dB, Isolator: 0.04 dB, Coupler: 0.01 dB). Therefore, the effect of PDL induced mode locking can be excluded in our laser system [120]. The output signal is monitored with a high-speed electronic detection system consisting of a 40 GHz photo-detector (Newport Model 1014) and a 33 GHz bandwidth real-time oscilloscope (Agilent DSO-X 92804A). The output light spectrum is simultaneously measured by an optical spectrum analyzer (Yokogawa AQ6375).

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Figure 3.2. A schematic of the Erbium-doped fiber ring laser. EDF: Erbium-doped fiber. SMF: Single mode fiber. DCF: Dispersion compensation fiber. WDM: Wavelength division multiplexer. PC: Polarization controller. ISO: Isolator. OC: Output coupler. OSA: Optical spectrum analyzer.

The CW emission pumping power threshold of the fiber laser is about 30 mW. When increasing the pumping power to about 200 mW, by continually tuning the orientation of the intracavity PC, different states of stable periodic pulse train emission with different pulse repetition rates can be achieved, as shown in Fig. 3.3.

Figure 3.3. Periodic pulse train emission of fiber laser with different repetition rate under the same pumping power but different cavity detuning. Repetition rate: (a) 107 MHz, (b) 412 MHz, (c) 1.33 GHz, (d) 2.68 GHz.

The different pulse repetition rate represents different modulation instability frequency. We can see that the modulation instability frequency can be tuned

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from hundreds of MHz to several GHz. Under the same cavity parameters, just by tuning the intracavity PC, which stands for changing the linear cavity phase delay. The result clearly shows that the formation of the periodic pulse trains is due to the cavity induced modulation instability effect in the fiber laser. We note that the optical spectra of these emission states are just a broad envelop with no sign of modulation on it. That's because the limitation of the minimum resolution (0.05 nm) of our optical spectrum analyzer, such low frequency modulation can't be observed.

We noticed that there is always a weak CW background on these pulse trains. It means that the pulses haven't been fully shaped into the soliton yet, and no spectral sidebands appear on their spectrum either. Owing to the existence of CW background, it makes the pulse train very stable against the weak perturbations. In this case, the pulse is in coherent with the CW background and they propagate together in the cavity. The stabilization of pulse train is due to the limitation of the effective gain bandwidth [72,121]. However, if there is negligible gain bandwidth limitation, the stable pulse trains will be replaced by the breather emission, which are also in coherent with the CW background and will have energy exchange between them [122–124]. The gain bandwidth will restrain the energy exchange between pulse and CW background, thus makes the pulse train stable. The coherence between the pulse train and CW background also remind us the previously reported temporal cavity solitons, which are very similar to our results. Furthermore, if we keep increasing the pumping power and tuning the intra-cavity PC to reduce the repetition rate, single pulse energy of the pulses could be significantly increased. The pulses can be finally shaped into dissipative solitons and move freely in the cavity, just as we previously reported in [72]. The CW background will significantly decrease and the solitons are no longer in coherence with the CW background. The dissipative soliton formation triggered by the CIMI is a standard case of the temporal cavity soliton formation in fiber lasers. It is worth pointing out that the conventional MI didn't occur in these situations. We believe it can be attributed to the low light intensity in the cavity. This means the CIMI has lower power threshold than that of the conventional MI [70].

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According to the CIMI theory, the modulation frequency depends not only on the cavity detuning, but also on the light intensity. In Fig. 3.4, we represent the experimental results of periodic pulse train emission of different repetition rate, just by changing the pumping power and keeping other cavity parameters unchanged. Table 3.1 shows the relationship between the repetition rate, pumping power and intracavity light intensity of our experimental results. In our experiment, we notice that when we changing the pumping power, the repetition rate of the pulse train certainly is altered but after a few seconds it becomes stable again. It clearly shows that CIMI is a nonlinear process.

Figure 3.4. Periodic pulse train emission of fiber laser with different repetition rate under the different pumping power. Repetition rate: (a) 4 GHz, (b) 4.76 GHz, (c) 6.47 GHz, (d) 8.5 GHz.

Table 3.1. Relation between pumping power, intracavity light intensity and repetition rate

Figure	Pumping power (dBm)	Intracavity light intensity (mW)	Repetition rate (GHz)
(a)	28	133	4
(b)	30	173	4.76
(c)	31	290	6.47
(d)	32	375	8.5

From the above discussion, one can summarize the following characteristics of the CIMI in a fiber laser: When the pulse energy increases, the pulses can be

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shaped into dissipative solitons; While when the pulse energy is small, the pulse train is more like a kind of sinusoidal modulation of the CW beam. In experiment, when keeping other parameters unchanged and gradually increasing the pumping power, we can clearly observe the modulation on the CW beam evolved from the initial “sinusoidal wave” like modulation to a pulse train of tens of GHz pulse repetition. It provides a solid evidence of the CIMI effect in fiber lasers.

If the pump power is sufficiently high, the coexistence of the conventional MI and the CIMI can be obtained. In our experiment, when the pump power is increased to about 2 W, a pulse train with very high repetition rate can be observed. Fig. 3.5(a) shows the autocorrelation trace of the pulse train. From the autocorrelation trace, we can measure the repetition rate is about 250 GHz and the FWHM of the pulses is about 1.3 ps if a sech^2 shape pulse is assumed. Fig. 3.4(b) is the corresponding optical spectrum. From the spectrum we can observe some strong spectral peaks. The interval of those strong spectral peaks is about 1.34 nm and stands for a modulation frequency of 250 GHz. This is just the same with the repetition rate of the pulse train. Meanwhile, there are also some weak spikes on the spectrum. The interval of those weak spikes is about 0.1 nm, standing for a 12 GHz modulation. This results means the conventional MI and CIMI coexist in the cavity. The higher 250 GHz modulation is induced by the conventional MI and the lower 12 GHz modulation is caused by the CIMI. The conventional MI with much higher modulation frequency dominates and leads to the breaking up of the CW in fiber laser. It is expected that by tuning the intracavity PC, the frequency of CIMI can be changed. However, the effect of CIMI is hard to be observed on the autocorrelation trace and spectrum, for its low modulation frequency. The result also confirms CIMI has lower power threshold than the conventional MI.

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Figure 3.5. Pulse train emission with 250 GHz repetition rate. (a) Autocorrelation trace of pulse train, (b) The corresponding spectrum in linear scale

The experiment system we discussed above is a net anomalous dispersion fiber ring laser, where the conventional MI can occur. Previously Song *et al* reported a 280 GHz dark pulse emission in a positive dispersion fiber ring laser [39]. We notice that the author didn't give an exact explanation on how the 280 GHz modulation appeared. We built a new fiber laser with all normal dispersion. The new cavity has a piece of 4 m EDF and 9 m of DCF, which has an average normal dispersion parameter of -17.5 ps/nm/km. Under a relatively high pumping power of about 2 W, by appropriately tuning the intracavity PC, a high frequency modulation instability emission state was achieved. Fig. 3.6 shows the corresponding optical spectrum and the autocorrelation trace. From the spectrum, we can calculate the modulation frequency is about 240 GHz, which just coincides with the repetition rate of the pulse train shows in the autocorrelation trace. As the cavity is in the normal dispersion regime and there is no mode-locking effect, the pulse train should also be a dark pulse train. Under the same pumping power, by carefully tuning the PC, the modulation frequency can be altered. At last, we can tune the modulation frequency up to about 345 GHz, which is much higher than the 280 GHz reported in [39]. Fig. 3.7 shows the corresponding optical spectrum and autocorrelation trace. The experimental results of cavity detuning-dependent modulation frequency clearly show that the modulation should be originated from the CIMI. In the experiment with the dark pulses, it's hard to decrease the modulation frequency to a very low value. We believe it's due to the instability of the dark pulses.

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When the repetition rate of dark pulses is high, the dark pulse train becomes more stable, which is also opposite to the bright pulse case.

Figure 3.6. Dark pulse train emission of 240 GHz repetition rate. (a) The corresponding spectrum, (b) Autocorrelation trace of pulse train.

Figure 3.7. Dark pulse train emission of maximum 345 GHz repetition rate. (a) The corresponding spectrum, (b) Autocorrelation trace of pulse train.

3.3 Discussion and conclusion

In section 3.1 we have derived the theoretical foundation of the CIMI in fiber lasers and found that the total phase delay of light propagating in one cavity roundtrip, $\phi = \psi_0 + |U_0^2|$, determines the modulation instability gain spectrum. It's a common sense that when a fiber laser is under steady state operation, ϕ should be an integral multiple of 2π . In this case the CIMI can no longer occur but only the conventional MI. Would this mean that there is no CIMI in a fiber laser? To better understand the situation, we note that in the practice of a fiber laser operation, one can always change the laser operation e.g. by changing the

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intracavity PC orientation, in this case the linear cavity phase delay ψ_0 will be altered, consequently $\phi = \psi_0 + |U_0^2|$ is no longer equal to the multiples of 2π , or by changing the pump power, so the intracavity light intensity will be changed, and the nonlinear phase delay is altered. In all these cases, the CIMI would still appear. Once it occurs, it breaks up the CW laser emission into a pulse train. It is pointed out that a state where $\phi \neq 2m\pi$ is not a steady state operation of a laser, but rather a transient. Each time the light passing through the output coupler, it will interfere with the light in the previous round-trip and lead to the light intensity fluctuation. The light intensity fluctuation will cause the changing of nonlinear phase shift $|U_0^2|$ and so does the total phase shift. After several cavity roundtrips, ϕ will eventually become equal to $2m\pi$ and the fiber laser come back to a steady state operation again. However, in the new steady state laser operation the laser emission is in the pulse train emission rather than a CW emission. Therefore, the periodic pulse train emission states are very stable in our fiber laser. Based on its formation mechanism and features, we believe that the CIMI should be a general effect of different kinds of lasers with strong cavity nonlinearity, no matter if it is a fiber laser or a solid state laser.

It is well-known that when a strong CW beam propagates in a single mode optical fiber, due to the MI its intensity will temporally become periodically modulated, and such a periodical intensity modulation will evolve into a breather structure [123]. In the case of fiber lasers, under the effect of the gain bandwidth limitation, no breather structure will appear. The modulated CW beam will be directly shaped into a stable periodical pulse train. We emphasize that the gain bandwidth limitation is a key factor to achieve stable periodic pulse train laser emission. Besides, if one keeps increasing the pumping power and/or tuning the cavity detuning to make the modulation frequency becoming lower, so that the single pulse energy becomes large, the pulse can finally be shaped into a dissipative soliton. The CIMI lasing provides a new method to generate periodical pulse train in a fiber laser other than mode-locking. Moreover, the repetition rate of the pulse train can be easily and continuously changed by just tuning the intracavity PC, even under the same pumping power,

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and the tuning range is broad. In our experiments it varies from MHz to tens of GHz. This range of repetition frequency is very important because the pulse train can be resolved by the commercial photodetector. This makes the CIMI lasing a promise technique for the future optical communication system.

4

Temporal cavity solitons in fiber laser cavity

In chapter 3, we have discussed the origin of cavity induced modulation instability in a fiber laser cavity and the CIMI resulted pulse train formation. Under high power pumping, the pulses in the pulse train can be further shaped into solitons. This spontaneous formation of dissipative solitons as a result of the destabilization of a CW beam in a nonlinear cavity reminds us another kind of solitons: cavity solitons. First theoretically proposed by Lugiato and Lefever, cavity solitons stand for spontaneously formed transverse patterns in driven passive nonlinear optical cavities [125]. The formation of this spatial cavity solitons is due to the balance of competition between the transverse cavity diffraction and nonlinearity. Diffraction of light will trigger the instability on the external driven light field and a dissipative transverse pattern thus cavity solitons will spontaneously form. Experimentally, spontaneous formation of spatial cavity solitons has been observed in different optical systems, such as in a single mirror feedback system using sodium vapor as the nonlinear

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medium [126], semiconductor micro-resonator [127,128], and photorefractive crystal cavities [129]. Not only the spatial cavity solitons but also the temporal cavity solitons could be formed in a nonlinear optical cavity. Leo *et al.* have experimentally demonstrated the existence of temporal cavity solitons in an externally driven passive fiber cavity [73]. A temporal cavity soliton is formed due to the balance between dispersion and nonlinearity in an optical cavity. Just similar with spatial cavity solitons, temporal cavity solitons can be selectively excited, destroyed, or stored in a cavity [73,130].

In this chapter, we focus on the formation of temporal cavity solitons in fiber laser cavity. We show both theoretically and experimentally that temporal cavity solitons can be spontaneously formed in a fiber laser. Especially, the formation mechanism and dynamics of the temporal cavity solitons have close similarity to those of the spatial cavity solitons. In Section 1, the experimental observation of scalar temporal cavity soliton formation in an anomalous dispersive fiber laser cavity is presented. And the numerical simulation studies are also added to reveal the formation dynamics of temporal cavity soliton in fiber laser cavity. In Section 2, we will focus on the vector temporal cavity solitons in an anomalous dispersive fiber laser cavity. The induced temporal cavity solitons, formed either under coherent or the incoherent coupling between the two orthogonal polarization components of light in the cavity, have also been experimentally obtained. In addition, the numerical simulations support the experimental results well. In Section 3, the experimental results of vector temporal cavity solitons in normal dispersive fiber laser cavity are represented.

4.1 Scalar temporal cavity soliton formation in fiber laser cavity with net anomalous dispersion

4.1.1 Experimental setup and results

We set up a fiber ring laser with a cavity configuration as shown in Fig. 3.2. The fiber laser consists a piece of 3 m EDF and a piece of 10 m SMF. A

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polarization independent isolator was employed in the cavity to force the unidirectional operation of the cavity, and an intra-cavity polarization controller (PC) was used to tune the linear cavity birefringence. A wavelength division multiplexer (WDM) was used to couple the pump beam into the cavity. The laser was pumped by a 1480 nm high power Raman fiber laser source whose maximum output pump can be as high as 5 W. A fiber output coupler with 10% output coupling was used to output the laser emission. The fiber laser has a very simple cavity configuration. Different from the conventional soliton fiber lasers which need either an active or passive mode locker inserted in the cavity to initiate pulse operation, there is no mode locker in our experiment. Therefore, no mode locking will occur. Under low power pumping the laser is always in a CW emission state. Depending on the linear cavity birefringence, which is controlled by changing the orientations of the paddles of the intra-cavity PC, the CW laser emission either consists of two different wavelengths and the emission at each wavelength is linearly polarized along one of the two orthogonal polarization directions of the cavity, or has a single wavelength with an elliptical (or linear) polarization.

Figure 4.1. A schematic of the Erbium-doped fiber laser. EDF: Erbium-doped fiber. SMF: Single mode fiber. WDM: Wavelength division multiplexer. PC: Polarization controller

We first investigated the case that the laser cavity has very weak linear birefringence. It was found that under strong pumping power (pump power >500mW), linearly polarized CW laser emission could always be

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obtained. We attributed the linear polarized laser operation to the polarization instability of light propagation in weakly birefringent fibers [131]. Operating in such a linearly polarized state, as the pump power is further increased it is experimentally observed that the CW laser emission could be suddenly changed to a train of optical pulses, as shown in Fig. 4.2. The repetition rate of the pulse train is tunable by carefully changing the orientation of the intra-cavity PC, which means it is initiated by the cavity induced MI (CIMI) [69,70]. The pulses are very stable in the cavity. Carefully checking the pulse train, we can find that the pulse train is on a CW background, as shown in Fig. 4.2(a).

Figure 4.2. A typical case of the suddenly formed periodic pulse train emission of the fiber laser. (a) Oscilloscope trace of the laser emission. (b) The corresponding optical spectrum.

Once the periodic pulse train with CW background is obtained, further increasing the pump power, the pulse intensity will increase and the CW background level will decrease. Eventually the pulses start to move freely in the cavity, and the pulse distribution in the cavity becomes random, as shown in Fig. 4.3(a). Fig 4.3(b) is the corresponding optical spectrum. We can see that the optical spectrum of the laser emission becomes significantly broader. In particular, there are some obvious Kelly sidebands on the spectrum, which is the symbol of the formation of solitons in the fiber ring laser. Once solitons are formed in the cavity, it was found that the dynamics of the solitons are exactly the same as those observed on the conventional mode-locked soliton fiber lasers [132].

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Figure 4.3. A typical case of the suddenly formed dissipative soliton emission state of the fiber laser. (a) A snapshot of the oscilloscope trace. (b) The corresponding optical spectrum.

Fig. 4.4 and Fig. 4.5 further show the operation of the fiber laser under relatively large linear cavity birefringence. In this case the laser emissions along the two orthogonal cavity birefringence axes have different wavelength. Hence, the two orthogonally polarized laser beams are incoherently coupled. It was shown that the incoherent coupling results in the formation of the polarization domains, displaying that the laser emission alternates between the two coupled orthogonal polarization modes, as shown in Fig. 4.4.

Figure 4.4. Dissipative soliton formation in each polarization domain. The two oscilloscope traces show the laser emission along each of the two orthogonal polarizations of the cavity. Zoom-in the trace the dissipative soliton distribution looks like that shown in Fig. 4.2(a).

Under relatively low pumping intensity, the laser emission in each domain is CW. However, as the intensity of the laser beam in a domain becomes sufficiently strong, a periodic pulse train could automatically form in the

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domain. The formation of the periodic pulse train is attributed to the effect of the CIMI. Initially the pulses are uniform and static in a domain. As the beam intensity is increased, the pulse distribution then becomes non-uniform, as shown in Fig. 4.4; in the meantime, the Kelly spectral sidebands also appear on the optical spectrum. Note that some pulses in Fig. 4.4 have high intensity, this can be explained by the fact that two pulses were too close to each other.

Normally, the formed pulses are confined within a domain, like the case shown in Fig. 4.4. However, under strong beam intensity it was also experimentally observed that the pulses could even constantly move out of a domain. Fig. 4.5 shows a case where the intensities along the two polarizations are asymmetric. Along one polarization direction, as the intensity is weak, no MI occurred. The emission of the laser is in CW, while in the other polarization direction pulses are formed and the pulses have very high repetition rate. Because of the high pulse density, some of the pulses “diffused” out of the domain. On the real-time oscilloscope one could clearly monitor the pulse movement. The situation is similar to the so-called “soliton rain” observed in the mode-locked fiber lasers [133]. Again, the experimental result suggests strongly that the formed pulses are dissipative solitons.

Figure 4.5. Dissipative soliton formation in one of the asymmetrical polarization domains. The two oscilloscope traces show the laser emission along each of the two orthogonal polarizations of the cavity.

4.1.2 Numerical simulations

To find out the physical mechanism of soliton formation in experiment, we numerically simulate the operation of our fiber laser. The numerical method is

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based on the Split-Step Fourier method which is briefly introduced in Section 2.4. The simulation parameters are the same as in experiment. We use the fiber nonlinear coefficient $\gamma = 3 \text{ km}^{-1}\text{W}^{-1}$, cavity effective gain bandwidth $\Omega_g = 20\text{nm}$ and 10% cavity output loss. Considering the CW fiber laser operation destabilized by the cavity-induced modulation instability (CIMI), we artificially introduced a weak field $U(0, \tau) = U_0 \cos(2\pi f_c \tau)$ as an initial periodic modulation on the CW laser field, where f_c is the modulation frequency. Fig. 4.6 shows the evolution of the laser emission under existence of the effective gain bandwidth limiting. When the gain bandwidth is appropriately selected, a stable periodic pulse train is always formed initiated by the CIMI effect.

Figure 4.6. (a) Formation of stable periodic pulse pattern in a fiber laser under the effect of modulation instability and effective gain bandwidth limiting. $E_s = 0.3\text{pJ}$, $\Omega_g = 20\text{nm}$ and $f_c = 20\text{GHz}$. (b) A steady laser emission state.

Without the effective gain bandwidth limitation, the laser emission would display the Akhmediev breathers, as shown in Fig. 4.7. This simulation result is the same as reported in Ref. [123], which verifies that the light propagating in fiber laser cavity without gain bandwidth limitation has the same properties as the propagation in optical fiber described by NLSE.

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Figure 4.7. Formation of Akhmediev Breather structure in fiber laser cavity where ignoring the gain bandwidth limitation as the condition of Fig. 4.6.

Carefully checking the periodic pulse train in Fig. 4.6, it is identified that there is a CW background on the pulse train. However, as the saturation energy is gradually increased, the pulse width narrows down and the CW level decreases. Eventually all the pulses in the pulse train effectively become dissipative solitons and the soliton is highly localized with each other, as shown in Fig. 4.8.

Figure 4.8. (a) Formation of stable dissipative soliton pulse train in a fiber laser under the effect of modulation instability and effective gain bandwidth limiting. $E_s = 0.5 \text{ pJ}$, $\Omega_g = 20 \text{ nm}$ and $f_c = 20 \text{ GHz}$. (b) A steady state of the laser emission.

To confirm that each pulse in the periodic pulse train is eventually shaped into a dissipative soliton, whose parameters are determined by the laser

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operation condition, we have further compared the steady-state pulse amplitude and phase profiles with those of the theoretically predicted fundamental dissipative soliton [76]. A dissipative soliton has a chirped-Sech profile in the form of

$$u(\tau) = N_s \operatorname{sech} h(p\tau) \exp[-iq \ln(\cosh p\tau)], \quad (4.1)$$

where N_s , p , q are constants related to the pulse height, width, and frequency chirp, respectively. Taking N_s , p , q as the artificial fitting parameters, we found that the numerically calculated pulse profile and phase profile well match those of the dissipative solitons, as shown in Fig. 4.9.

Figure 4.9. Comparison of (Left) amplitude and (Right) phase profile of the steady state pulse with those of the analytic fundamental dissipative soliton.

The steady-state pulses under different effective laser gain bandwidths have been calculated and the pulse width varied with the effective gain bandwidth. However, no matter how the pulse width varies, the steady-state pulse profiles always have the characteristics of a fundamental dissipative soliton. When keeping increasing the saturation energy, like the operation of the conventional dissipative soliton fiber lasers, the solitons will break up and form a new pulse train with non-uniform soliton separation, as shown in Fig. 4.9.

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Figure 4.10. (Left) Dissipative soliton breaking when the beam intensity is strong. $E_s = 2\text{pJ}$, $\Omega_g = 20\text{nm}$ and $f_c = 20\text{GHz}$. (Right) A state of the laser emission.

In the simulations the laser emission under various modulation frequencies, gain bandwidth, and saturation energy are also checked. The above features were found independent on the concrete laser parameter selections, suggesting that they are general properties of the fiber lasers. We note that mathematically the state shown in Fig. 4.6 can be considered as a stable periodic solution of Eq. (2.2.19). Periodic solutions known as cnoidal waves were found for the NLSE [134]. It is known that as the periods approach infinity, the limiting case of the cnoidal waves is a soliton. Since Eq. (2.2.19) could be simplified to the NLSE, it is expected that a similar feature would also occur on the periodic solutions of Eq. (2.2.19). Increasing the pumping power, the intensity of the periodic waves could also increase. Therefore, it would be plausible that under sufficient high energy, a train of dissipative solitons could be formed as a result of the nonlinear shaping of its periodic waves.

In summary, our numerical simulations have shown that for a fiber ring laser with anomalous cavity dispersion, as far as the laser power is strong, the MI always destabilizes its CW operation. If there is no effective gain bandwidth limitation, the system is described by NLSE. MI or CIMI will only lead to the formation of the Akhmediev breathers or the formation of Peregrine soliton as the limiting case [122,135,136]. However, under existence of the effective gain

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bandwidth limiting, the system is described by the extend-GLE. The CIMI will lead to the formation of a stable train of periodic pulses or temporal cavity solitons (or dissipative solitons). Nevertheless, the periodic soliton trains could be easily destroyed by the system noises, especially in the case of low pulse repetition rate. Consequently, multiple mutually independent dissipative solitons would be formed in the laser cavity.

4.1.3 Discussion

Optical soliton formation in mode-locked fiber lasers is extensively studied. A characteristic of the mode-locked fiber lasers is that there is a saturable absorber in the cavity. Under effect of the saturable absorber, self-started mode locking could be achieved. Through mode locking high peak power mode-locked pulses can be formed in the fiber laser cavity. The mode-locked pulses are then shaped into dissipative solitons. In a passively mode-locked soliton fiber laser the saturable absorber plays an important role in the soliton formation. The features of the saturable absorber also affect the soliton dynamics. However, in our experiment, there is no mode-locking element in the cavity. Therefore, no mode locking is achievable. As shown both numerically and experimentally above, the soliton formation in our fiber laser follows a completely different mechanics from that of a mode-locked fiber laser. In our experiment it is the cavity-induced MI that generated an initial periodic optical field modulation, which under strong cavity nonlinearity is either shaped to the Akhmediev breathers or the stable periodic pulse train, depending on the effective gain bandwidth. As the MI is an intrinsic feature of strong CW light propagation in anomalous dispersion fibers, it is anticipated that the observed phenomenon is general to all anomalous dispersion cavity fiber lasers under high power operation.

Our experimental results have clearly shown that due to the effective gain bandwidth limitation in the fiber laser cavity, the stable periodic pulse train can be shaped in to dissipative solitons at last. Moreover, we found that the formed dissipative solitons have exactly the same features as those of the solitons formed in the conventional mode-locked soliton fiber lasers. A clear distinction between the stable periodic pulses and the dissipative solitons is that the

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periodic pulses are static in the cavity, while the dissipative solitons are freely movable in the cavity. Our experimental result clearly shows that as the pulse peak intensity increases, the pulses gradually change from static to freely movable in the cavity.

Finally, we point out that the periodic pulse train formation in our fiber laser is very similar to the spatial pattern formation described by Lugiato and Lefever in Ref. [125]. In the latter case it is the instability caused by the interaction between the diffraction and nonlinearity that destabilizes the initial uniform spatial field, leading to the formation of the stationary spatial pattern. In our fiber laser it is the MI that destabilizes the CW operation of the laser, leading to the formation of a stationary periodic pulse train. Therefore, such a periodic pulse train could be considered as a stationary temporal pattern. Under sufficiently strong pulse intensity and effective gain bandwidth limitation, the stationary periodic pulse train could even be converted to temporal cavity solitons.

4.2 Vector temporal cavity soliton formation in fiber lasers with net anomalous dispersion

4.1.1 Experimental setup and results

The experimental setup is almost the same as that shown in Section 4.2, as shown in Fig. 4.10, except the length of SMF change from 10 m to 9 m, so the net cavity dispersion becomes 1.5ps/nm/km. To experimentally resolve the two orthogonal polarization components of the laser emission, an external cavity polarization beam splitter (PBS) are used to separate the two orthogonal polarization components of the light field. A polarization controller is inserted before the external polarization beam splitter to balance the linear polarization change caused by the lead-fibers. The two orthogonal polarization components are simultaneously monitored with a high-speed detection system.

Figure 4.11. A schematic of the Erbium-doped fiber laser. EDF: Erbium-doped fiber. SMF: Single mode fiber. WDM: Wavelength division multiplexer. PC: Polarization controller.

Same as in Section 4.1 there is no saturation absorber (SA) or polarization dependent components in the laser cavity, so no mode locking could happen. Experimentally, as the pump power was increased beyond the lasing threshold, CW emission was always established in the fiber laser. Depending on the net cavity birefringence, it was found that the CW laser emission could have different operation states: When the net cavity birefringence was weak, the laser emissions along the two orthogonal polarization directions of the cavity would have almost the same frequency. In this case when the pump power became sufficiently strong, coherent coupling between the two polarization-components occurred; if the net cavity birefringence was strong, the laser emissions along the two orthogonal polarization directions of the cavity had obviously different frequencies. In this case the coupling between the two polarization components is incoherent.

We started from the laser operation under the coherent polarization coupling. In this situation the CW emission of the laser was elliptically polarized, and depending on the concrete laser operation condition, the ellipticity of the beam varied. Fig. 4.11 shows a case of the laser operation where the two polarization components had comparable intensity. It was experimentally observed that as the pump power was increased to about 600 mW, the CW emission along both polarizations simultaneously changed to periodic pulsation. The pulses along

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the two polarization directions were synchronized and had the same repetition rate. Carefully tuning the intra-cavity PC, the repetition rate of the periodic pulses could be continuously altered, which also suggests that the periodic pulsing was caused by the cavity induced modulation instability (CIMI). However, different from the experimental results in previous section, which is a scalar pulse train, we had now a vector periodic pulse train. With fixed laser operation condition, the periodic vector pulse train state was very stable. Contrary to the periodic pulse trains caused by modulation instability effect in the light propagation case, no breathing of the pulses was observed.

Figure 4.12. Laser emission state of the vector periodic pulse

Keeping increasing the pump power or decreasing the pulse repetition rate by tuning the intra-cavity PC, the peak power of each individual pulses increased. Eventually, the pulse distribution in the cavity became aperiodic. Especially the pulses started to move independently in the cavity, as shown in Fig. 4.13(a). Fig. 4.13(b) shows the corresponding polarization-resolved spectra. From the spectra one can clearly identify the appearance of two spectral sidebands, which are symmetric with respect to the central wavelength. The appearance of the spectral sidebands indicates that the pulses are solitons [137]. According to the theory of temporal cavity solitons, these solitons are dissipative solitons. We note that the sideband locations and the soliton center wavelengths of the two polarization components are almost the same, which suggests that the solitons along each of the two orthogonal polarizations had the

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same parameters and properties. Despite of the fact that there was a strong CW field along the two polarization directions, the solitons along each of the polarizations moved synchronously in the cavity. Once the vector solitons were formed, slightly changing the linear cavity birefringence couldn't destroy them. Carefully reducing the pump power, first the CW beam power was reduced. Eventually, on the optical spectra no CW components were visible, associated with the pump power decreasing the number of soliton in the cavity could also be reduced one by one. The similar features have also been observed on the solitons formed in the mode locked fiber lasers. Fig. 4.12(c) further shows the measured autocorrelation trace of the solitons. It has a FWHM width of about 1.37 ps, suggesting that the width of the solitons at the laser output is about 880fs if a Sech²-shape pulse profile is assumed.

Figure 4.13. A state of the vector cavity soliton emission of the laser. (a) Oscilloscope traces of the two orthogonal polarization components; (b) Polarization-resolved spectra; (c) the autocorrelation trace

Depending on the laser operation conditions, the relative strength of the two coherently coupled polarization components could be very different. Fig. 4.14 shows a case where the laser was almost linearly polarized along the horizontal polarization direction. Based on the intensity trace of the laser emission, the

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soliton pulses along the vertical polarization direction were hardly to be measured. However, from the polarization-resolved spectra it is to see that there were also solitons formed along the vertical polarization direction as suggested by the broad spectrum with some clear spectral sidebands. Carefully comparing the spectral sidebands with those of the solitons polarized along the orthogonal polarization direction, they located at the same spectral positions and exhibited the peak-dip relation, suggesting that they were formed due to the coherent coupling between the two orthogonal polarization components [85]. We note that there is also a big spectral dip near the center part of the spectrum of the vertical polarization component. We postulate that it was also formed due to the coherent coupling of the two polarization components. The spectral intensity difference of the two orthogonal polarization components is larger than 25 dB. Obviously, the beam intensity along the vertical polarization direction alone couldn't support the spontaneous soliton formation. Therefore, the formed soliton should be induced by the cavity solitons formed in the horizontal polarization direction. Zhang *et al.* reported a similar phenomenon of induced solitons in a mode locked fiber laser previously [52]. Our experimental result once again confirmed the formation of the type of induced solitons between the coherently coupled light beams.

Figure 4.14. Polarization-resolved spectra of the laser emission that shows the induced cavity soliton formation under weak net cavity birefringence.

In our experiment induced temporal cavity solitons were also observed even when the net linear cavity birefringence was relatively large, as shown in Fig. 4.15. Different from the case shown in Fig. 4.14, the induced solitons have

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obviously different central wavelength from that of the inducing solitons. Moreover, the Kelly sidebands of the induced solitons also locate at different positions from those of the inducing solitons. We note that similar phenomenon was also reported in [52], which was explained as a result of the incoherent coupling between the two polarization components. Our experimental result supports the explanation again, as shown in Fig. 4.13, on the polarization-resolved spectra of the solitons no energy exchange sidebands are visible.

Figure 4.15. Polarization-resolved spectra of the laser emission showing the induced soliton formation under relatively large net cavity birefringence

In addition to the results shown in Fig. 4.14 and Fig. 4.15, another situation of the induced cavity soliton formation as shown in Fig. 4.16 and Fig. 4.17 was also experimentally observed. Fig. 4.16 shows the case of the incoherent coupling between the two orthogonal polarization components, while Fig. 4.17 shows that of the coherent coupling. Different from the results shown in Fig. 4.14 and Fig. 4.15 where due to the cavity induced MI the strong CW beam itself was first destabilized, and then the temporal cavity solitons were formed from the destabilized CW beam, Fig. 4.16 and Fig. 4.17 show that a strong CW beam along one polarization direction could induce the formation of periodic pulsation along the orthogonal polarization direction where initially either a weak CW or no light exists, and further lead to the formation of the temporal cavity solitons along that polarization direction. Once the cavity solitons are formed, they further induce soliton formation on the CW side. However, in the case of incoherent coupling the induced solitons are too weak to be seen on the

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oscilloscope trace, while in the case of coherent coupling clear induced solitons are visible on the strong CW background, as shown in Fig. 4.16(a). Associated with the induced solitons broad intensity depths were also formed on the CW beam, whose formation is a manifestation of the polarization domains as reported previously [52].

The results shown in Fig. 4.16 and Fig. 4.17 can be attributed to the modulational polarization instability (MPI) effect [138,139]. As first theoretically predicted by S. Wabnis *et al.*, through the cross polarization coupling a strong CW beam in a linear birefringence single mode fiber polarized along one eigen-polarization direction could induce the MI of light polarized along the orthogonal polarization direction. Different research groups have previously experimentally observed the MPI effect [140–142]. However, to the best of my knowledge there is no experimental observation of the effect occurred in a laser cavity. Comparing to the previous experimental observations of the phenomenon, obviously in an active cavity the threshold of the MPI could be significantly reduced. Bearing in mind that the MPI could destabilize the light polarized along the orthogonal polarization, under the same mechanism of the temporal cavity soliton formation shown in Fig. 4.14 and Fig. 4.15, the appearance of the temporal cavity solitons on the weak CW side as shown in Fig. 4.16 and Fig. 4.17 could be easily understood.

Figure 4.16. Temporal cavity soliton formation caused by the MPI effect in the laser under large net cavity birefringence. (a) The polarization-resolved oscilloscope traces, (b) the corresponding optical spectra.

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Figure 4.17. Temporal cavity soliton formation caused by the MPI effect in the laser under small net cavity birefringence. (a) The polarization-resolved oscilloscope traces, (b) the corresponding optical spectra.

4.2.2 Numerical study

To verify the temporal vector cavity soliton formation in fiber lasers, the numerical simulations about the operation of the fiber laser were further conducted. The emphasis here is to show that in a quasi-vector cavity fiber laser, because of the combined effect of the cavity induced modulation instability, the coherent coupling between the orthogonal polarization components and the effective gain bandwidth limitation, a stable periodic vector pulse train could be automatically formed. As a fiber laser is intrinsically a dissipative system, when the pump of the laser is increased, and/or the repetition rate of the pulses is decreased, the intensity of the formed pulses would increase. Eventually the pulses could be shaped into the temporal vector cavity solitons, just like the case of the scalar temporal cavity soliton formation discussed previously. Moreover, once a temporal vector cavity soliton is formed, it is stable in the cavity.

Our simulations are based on the coupled Ginzburg-Landau equations (GLEs), which describe the light propagation in a weakly birefringent gain fiber,

$$\begin{aligned} \frac{\partial u}{\partial z} &= i\beta u - \delta \frac{\partial u}{\partial t} - \frac{ik''}{2} \frac{\partial^2 u}{\partial t^2} + \frac{ik'''}{6} \frac{\partial^3 u}{\partial t^3} + i\gamma \left(|u|^2 + \frac{2}{3}|v|^2 \right) u + \frac{i\gamma}{3} v^2 u^* + \frac{g}{2} u + \frac{g}{2\Omega_g^2} \frac{\partial^2 u}{\partial t^2} \\ \frac{\partial v}{\partial z} &= -i\beta v + \delta \frac{\partial v}{\partial t} - \frac{ik''}{2} \frac{\partial^2 v}{\partial t^2} - \frac{ik'''}{6} \frac{\partial^3 v}{\partial t^3} + i\gamma \left(|v|^2 + \frac{2}{3}|u|^2 \right) v + \frac{i\gamma}{3} u^2 v^* + \frac{g}{2} v + \frac{g}{2\Omega_g^2} \frac{\partial^2 v}{\partial t^2}, \end{aligned} \quad (4.1)$$

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where u and v describe the optical fields of the two orthogonal polarization components in the fiber, $2\beta = 2\pi\Delta n / \lambda = 2\pi / L_p$ is the wave number difference between these two polarization modes, $2\delta = 2\beta\lambda / 2\pi c$ is the group velocity difference, k'' is the second-order dispersion coefficient, k''' is the third-order dispersion coefficient, γ represents the nonlinearity of the fiber cavity, g is the gain coefficient of the gain fiber and Ω_g is the effective gain bandwidth. The gain saturation is defined as

$$g = g_0 \exp \left[-\frac{\int (|u|^2 + |v|^2) dt}{E_s} \right], \quad (4.3)$$

where g_0 is the small signal gain coefficient and E_s is the gain saturation energy, which is related to the pumping power in the experiment. In our simulations, we have possibly used the real experimental parameters such as the actual cavity length, the dispersion-managed cavity, and the fiber dispersion etc. At a certain CW laser operation state we start the simulation by adding a weak periodic modulation on the optical field, and then let the light circulate in the fiber cavity until a stable state is reached. We consider each such obtained stable state as a possible laser emission state experimentally observed.

We first considered the case of a fiber laser that has a weak cavity birefringence. To the end we have set the beat length L_B as 100 m in the simulation. It represents the averaged birefringence strength of the fiber cavity. When the lasing condition is fulfilled, CW emission is established in the cavity, whose intensity increases with the pump power. In the case the optical field along the two orthogonal polarization directions of the cavity is coherently coupled. To simulate the cavity induced modulation instability effect, we artificially introduced an initial field $u(0,t) = U_0 \cos(2\pi\Delta\nu t) \sin(\theta)$ and $v(0,t) = U_0 \cos(2\pi\Delta\nu t) \cos(\theta)$, where $\Delta\nu$ is the modulation frequency and θ is the polarization angle. Numerically it is found that if the effective gain bandwidth limitation term Ω_g in the simulation is ignored, the CW beam on each polarization components always turns into the Akhmediev breather structures, which is a typical modulation instability phenomenon supported by the NLSE. However, with an appropriate effective gain bandwidth selection,

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the modulated CW will become a periodic pulse train embedded on a CW background. Keeping increasing the saturation energy, the pulses will narrow down and the CW level will decrease. Eventually the periodic pulse train turns into cavity solitons, as shown in Fig. 4.17. The cavity soliton nature of the pulses is confirmed by their pulse profile, which is uniquely determined by the laser parameters such as the effective gain bandwidth and the balance between the gain and losses.

Figure. 4.18. Formation of vector cavity solitons in a fiber laser. The parameters used are: $E_s=25$ pJ, $\Omega g=10$ nm, $g_0=640/\text{km}$ and $\Delta\nu=15$ GHz.

We then numerically simulated the induced cavity soliton formation by the cross-phase modulation (XPM) effect in a weakly birefringent cavity. To simulate the experimental observation, the initial input state is set as $u(0,t)=U_0 \cos(2\pi\Delta\nu t)$ and $v(0,t)=V_0$, where $U_0 \gg V_0$. By appropriate choosing the effective gain bandwidth and saturation energy, we observe that not only a stable train of cavity soliton can be formed on the strong CW side, like the case of the scalar cavity soliton formation reported previously, but also a stable train of cavity soliton are formed on the weak CW side in the orthogonal polarization direction due to the XPM effect, as shown in Fig. 4.18. The induced solitons have very low peak power and are embedded in the weak CW beam. The solitons on both polarization components are synchronized and have nearly the same pulse width, which is the same as the experimental observations.

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Figure 4.19. The XPM induced temporal cavity solitons. The parameters used are: $E_s=75$ pJ, $\Omega g=10$ nm, $g_0=640$ /km, $L_b=100$ m and $\Delta\nu=15$ GHz.

At last we simulated the MPI induced temporal cavity soliton formation. To this end we have set the input as $u(0,t)=U_0$ and $v(0,t)=V_0 \cos(2\pi\Delta\nu t)$, where $U_0 \gg V_0$. The modulation in the vertical polarization direction is set as 15 GHz, which is induced by the cavity induced MPI effect. Again the beat length of the fiber cavity is set as 100 m to represent the weakly birefringence cavity. Under appropriate effective gain bandwidth and saturation energy, we can again get a stable pulse train in the vertical axis, as shown in Fig. 4.19. From the simulation result it is to see that the formed temporal cavity solitons further induce cavity solitons on the horizontal direction and the induced solitons are embedded in the strong CW beam.

Figure 4.20. The MPI induced temporal cavity soliton formation. Parameters used are $E_s=25$ pJ, $\Omega g=10$ nm, $g_0=640$ /km, $L_b=100$ m and $\Delta\nu=15$ GHz.

4.3 Temporal cavity solitons in normal dispersive fiber lasers

4.3.1 Experimental setup and results

Both our experimental and numerical studies have confirmed the temporal cavity solitons formation in anomalous dispersion cavity fiber lasers. Now it is time to move to the normal cavity dispersion regime. To this end we built up an erbium doped fiber ring laser with net normal cavity dispersion. The fiber laser is schematically shown in Fig. 4.20. It has a dispersion-managed fiber ring cavity that consists of a piece of 4 m Erbium doped fiber (EDF) with a dispersion parameter of -48 ps/nm/km, 10.7 m single mode fiber (SMF) with a dispersion parameter of 18 ps/nm/km and 15.6 m dispersion compensation fiber (DCF) with a dispersion parameter of -4 ps/nm/km. The total cavity length is ~ 30.3 m with a net normal average dispersion parameter of -2.04 ps/nm/km. A polarization independent isolator is inserted in the cavity to force the unidirectional circulation of light in the cavity. Besides, an intra-cavity polarization controller (PC) is used to fine-tune the linear cavity birefringence. The fiber laser is pumped by a 1480 nm high power Raman single-mode fiber laser source whose maximum output power can be as high as 5 W. A wavelength division multiplexer (WDM) is used to couple the pumping light into the cavity, and a 10% fiber output coupler is used to output the light. The pumping coupling efficiency from the pump source to the cavity is about 60%. An external cavity polarization beam splitter (PBS) is used to experimentally resolve the two orthogonal polarization components of the laser emission. And a polarization controller is inserted before the external polarization beam splitter to balance the linear polarization change caused by the lead-fibers. The two orthogonal polarization components are simultaneously monitored with a high-speed electronic detection system made of two 40 GHz photo-detectors and a 33 GHz bandwidth real-time oscilloscope. There is no saturation absorber (SA) or polarization dependent components in the laser cavity, so no mode locking could happen in the laser. Meanwhile, the components used in our experiment have very low polarization dependent loss (PDL) (WDM: 0.01 dB,

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Isolator: 0.04 dB, Coupler: 0.01 dB). Therefore, the PDL induced mode locking is also unlikely to occur in our experiment [120].

Figure 4.21. A schematic of the Erbium-doped fiber laser. EDF: Erbium-doped fiber. SMF: Single mode fiber. DCF: Dispersion compensation fiber. WDM: Wavelength division multiplexer. PC: Polarization controller.

The fiber laser started lasing when the pump power was ~ 100 mW. When the pump power was increased to about 1W, by carefully tuning the intra-cavity PC, a stable quasi-periodic vector pulse train emission state, as shown in Fig. 4.21(a), could be obtained. Fig. 4.21(b) is the corresponding polarization-resolved optical spectra of the laser emission, the sharp edges are typical for the dissipative solitons obtained in the normal dispersion cavity fiber lasers [143]. This confirms that the vector pulses are vector dissipative solitons. As there is no SA in the cavity, these dissipative solitons can be regarded as the temporal cavity solitons, which are in principal dissipative solitons. The 3-dB spectral bandwidth of the solitons is about 13.5 nm. Moreover, on the polarization resolved spectra spectral sidebands with clear peak-dip intensity relation are visible at 1593 nm. According to studies on the phase locked vector solitons formed in the anomalous dispersion cavity fiber lasers, such peak-dip spectral sidebands are formed as a result of the coherent coupling between the two orthogonal polarized soliton components. It is an indication of the energy exchange between them [85]. To the best of our knowledge, this is the first experimental observation of such energy exchange between the components of

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vector dissipative solitons formed in the normal dispersion cavity fiber lasers. We note that different from the coherent energy exchange sidebands observed on the solitons formed in the anomalous dispersion cavity fiber lasers, where they appear almost symmetrically with respect to the soliton central wavelength, here only one peak-dip structure is observed. We believe it could be due to the large frequency chirp of the dissipative solitons formed in the normal dispersion regime. Fig. 4.21(c) is the corresponding autocorrelation trace of the dissipative solitons. It has a FWHM width of about 18.6 ps, suggesting that the width of the solitons is about 12.1 ps if a Sech-shape pulse profile is assumed. The time-bandwidth product is about 19.5, indicating the solitons are strongly chirped. Here we measure the intra-cavity power is about 175 mW and the repetition rate is about 400 MHz, so we can calculate the single pulse energy is about 437.5 pJ.

Figure 4.22. A typical vector temporal cavity soliton emission state (a) Polarization resolved oscilloscope traces. (b) Polarization-resolved optical spectra. (c) The autocorrelation trace.

In the experiment once the near periodic vector temporal cavity soliton train is obtained, further changing the intra-cavity PC or the pumping power, the soliton pulse train will break up and the soliton distribution in the cavity will

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become inhomogeneous, as shown in Fig. 4.22(a). With less number of soliton in the cavity, the energy of each individual soliton also becomes larger. Consequently, the edges of the soliton spectrum stretch out and become no longer sharp. As the spectrum of the solitons extends out, different from the Fig. 4.21(b) with only one energy exchange structure, more energy exchange spectral sidebands appear on the two stretched edges of the polarization resolved spectra, as shown in Fig. 4.22(b).

Figure 4.23 Vector temporal cavity soliton operation of the fiber laser with random soliton distribution in cavity. (a) Polarization resolved oscilloscope traces. (b) Polarization-resolved optical spectra.

As there are no saturation absorber (SA) or polarization dependent components in the laser cavity, no self-started mode locking could occur in the laser, therefore, the formation of the vector dissipative solitons can't be attributed to the soliton shaping of the mode locked pulse. We believe it's induced by the cavity induced modulation instability (CIMI) which also exists in the normal dispersion regime. CIMI will break the CW emission and under effect of the effective gain bandwidth limitation, the CW changes into a periodic pulse train. In case that the formed pulse intensity is sufficiently strong the pulses are further shaped into dissipative solitons under the mutual interaction of the average normal cavity dispersion, the nonlinear Kerr effect, laser gain and loss [30]. We emphasize that the pulse formation in our laser is not through the mode locking process. In addition, the formed solitons are not subject the influence of the saturable absorber or a mode locker as in a mode locked laser case. To distinguish the dissipative solitons from those generated in the mode-locked fiber lasers and to emphasis the role played by the effective

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gain bandwidth limitation on the soliton formation, this dissipative soliton can be also regarded as the temporal cavity solitons.

From the state shown in Fig. 4.22 if the intra-cavity PC is further tuned so that the soliton spectrum further extends out, the solitons in the cavity could come together to form a giant energy pulse, as shown in Fig. 4.23(a). Zooming in the pulse shown in Fig. 4.13(a) with a larger time scale, random varying fine structures are visible within the pulse. Experimentally, the pulse width and energy increases with the pump power. At a pump power of ~ 2 W, where intra-cavity power is about 400 mW, so the energy of the whole “Big Pulse” can be calculated as 60 nJ. Such a pulse is known as the noise-like pulse [144–146]. The noise like pulse emission is one of the typical operation states of the soliton fiber laser in the normal dispersion cavity regime. Previous study of the laser emission was speculated that the noise like pulses could be caused by the saturable absorber in the cavity. However, there is no any mode-locker our fiber laser but the same state of laser emission could still be formed. Hence, the formation of noise-like pulse can't only be attributed to the effect of saturable absorber. In Fig. 4.23(b) we have compared the optical spectra of the above three laser operation states. In our experiment by simply tuning the intra-cavity PC the laser emission could move from one state to the other.

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Figure 4.24. (a) Polarization resolved oscilloscope traces of a vector noise-like pulse emission state of temporal cavity soliton. (b) Optical spectra of the laser emission in I: a near-periodic soliton train state; II: a state with random soliton distribution; III: a noise-like pulse state.

Occasionally, before the noise like pulse state is reached, experimentally another state of the laser operation as shown in Fig. 4.24 could also be observed. In the state the solitons don't come together to form a noise-like pulse, but form a so-called "soliton rain" state [30,133]. In the current soliton rain state the solitons continuously move away from the soliton condensate. Based on the experimental result we suspect that a noise like pulse could be actually an ensemble of many solitons that have come very close to each other in the cavity.

Figure 4.25. The vector soliton rain formed in the fiber laser cavity.

4.3.2 Discussion and conclusion

As shown above we have experimentally observed the spontaneous formation of near periodic vector dissipative soliton pulse trains in a dispersion-managed cavity fiber laser with net normal cavity dispersion. We explain that the vector soliton pulse train formation is a result of the soliton shaping of the periodic pulse trains caused by the cavity induced modulation instability and the gain bandwidth limitation. To the best of our knowledge, such a dissipative vector soliton formation mechanism has not been reported before. In our experiment, we have also first experimentally observed the coherent energy exchange between the two orthogonal polarization components of a dissipative vector soliton formed in the normal dispersion cavity fiber lasers, despite of the fact that the dissipative solitons are strongly chirped. Moreover, we have also experimentally observed the noise-like pulse emission and the vector cavity soliton rain effect, and investigated their relations. All these experimental results have not only shown that the dissipative soliton formation is a general feature of the high power fiber lasers, but also greatly improved our understanding on the dissipative solitons and their features.

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5

Soliton coupling between two eigen- polarization modes of a fiber laser

When two or more intense optical fields with different wavelengths propagate simultaneously in an optical fiber, they will couple with each other through the fiber nonlinearity. The cross-phase modulation (XPM) is the most common way of coupling. XPM occurs when the relative phase between the solitons varies much faster than the relaxation time of the nonlinear medium. It arises from the third order susceptibility $\chi^{(3)}$ and Eq. (2.1.23) gives the origins of XPM. XPM process is phase insensitive and only determined by the relative strength of light. There is no energy exchange between the two polarization modes and it's also called incoherent coupling. On the other hand, when the phase matching condition between two polarization modes is fulfilled, the four-wave-mixing

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(FWM) type of coupling occurs. This can be regarded as coherent coupling. To maximize the strength of coherent coupling, phase matching condition must be fulfilled. In optical fibers, it can only occur when the nonlinear medium is weakly anisotropic or weakly birefringent.

In this chapter, the soliton coupling effect between two polarization modes in a fiber laser will be discussed. In Section 1, we present the experimental observation of soliton-dark pulse pair formation in a fiber ring laser. It was experimentally observed in a quasi-vector cavity fiber laser that due to the incoherent cross polarization coupling, a bright cavity soliton polarized along one principal cavity polarization axis could always automatically induce a dark pulse on the CW background of the laser polarized along the orthogonal polarization axis. The appearance of the dark pulse is a result of the incoherent cross polarization coupling between the soliton and the CW beam, its mechanism is similar to that of the polarization domain formation in the fiber laser. In Section 2, based on the results in Section 1, we further show that under coherent polarization coupling, a bright soliton could induce either a weak bright or a dark solitary pulse on the bottom of a broad dark pulse. Numerical simulations have also well reproduced the experimental observations, and further shows whether a weak dark or bright solitary pulse is induced is determined by the presence or absence of a phase jump in the induced pulse.

5.1 Soliton-dark pulse pair formation in a fiber laser under the incoherent coupling

5.1.1 Experimental setup and results

The fiber ring laser we used has a cavity configuration as shown in Fig. 5.1. The fiber ring has a total length of 13.5 m, consisting of a piece of 3 m Erbium doped fiber (EDF) with a group velocity dispersion (GVD) parameter of -48 ps/nm/km, 9.7 m single mode fiber (SMF-28) with a GVD parameter of 18 ps/nm/km and 0.8 m of dispersion compensation fiber (DCF) with a GVD parameter of -4 ps/nm/km. The fiber laser is pumped by a 1480 nm single mode Raman fiber laser whose maximum output power is 5 W. A polarization

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independent isolator is inserted in the cavity to force the unidirectional circulation of light in the cavity. In addition, an intra-cavity polarization controller (PC) is used to fine-tune the linear cavity birefringence. A wavelength division multiplexer (WDM) is used to couple the pumping light into the cavity, and a 10% fiber output coupler is used to output the light. The components used in our experiment have very low polarization dependent loss (PDL) (WDM: 0.01 dB, Isolator: 0.04 dB, Coupler: 0.01 dB). The fiber ring cavity is estimated to have an average net anomalous GVD parameter of 2.03 ps/nm/km. An external cavity polarization beam splitter (PBS) is used to experimentally resolve the two orthogonal polarization components of the laser emission. To the end a polarization controller is inserted before the external polarization beam splitter to balance the linear polarization change caused by the lead-fibers. To appropriately set the PC orientation experimentally we first operate the laser in a stable polarization domain emission state. Based on the polarization switching between the domains we then carefully adjust the external PC position so that the two orthogonally polarized laser emissions can be well separated. The polarization resolved laser emissions are simultaneously monitored with a high-speed electronic detection system made of two 40 GHz photo-detectors and a 33 GHz bandwidth real-time oscilloscope.

Figure 5.1. A schematic of the Erbium-doped fiber laser. EDF: Erbium-doped fiber. SMF: Single mode fiber. DCF: Dispersion compensation fiber. WDM: Wavelength division multiplexer. PC: Polarization controller.

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Under low pump power the laser always emits continuous wave (CW). However, depending on the net cavity birefringence, the laser emission could exhibit different polarization features. In the special case of very weak net cavity birefringence, the laser could emit an elliptically polarized beam. However, the most frequent situation is that the laser emits simultaneously CW along the two orthogonal polarization directions of the cavity. Carefully tuning the orientation of the intra-cavity polarization controller, which alters the net linear cavity birefringence, the oscillation wavelength difference between the CWs could be tuned. The experimental result suggests that the laser emission wavelength difference is related to the linear cavity birefringence. In our experiment, we used the wavelength separation as an indication on the strength of the net cavity birefringence. Operating the laser under different pump strength and net cavity birefringence, we could experimentally observe various interesting features of the laser emission. At first, we operated the laser at very small net cavity birefringence and under a pump power of about 2 W, where the intra-cavity light intensity is about 630 mW. By carefully setting the orientation of the intra-cavity PC, it is experimentally observed that the CW emission along one polarization direction could suddenly break up into a periodic bright pulse train, and associated with each of the bright pulses there is a dark pulse formed on the CW background along the orthogonal polarization direction, as shown in Fig. 5.2(a).

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Figure 5.2. Experimental results of soliton-dark pulse pair formation in a fiber laser. (a) The polarization resolved oscilloscope traces of the laser emission. (b) Polarization-resolved spectra. (c) Autocorrelation trace of the bright solitons. (d) The zoom-in oscilloscope traces of a soliton-dark pulse pair.

The automatic formation of a periodic bright pulse train in the fiber laser could be understood as a result of the cavity induced modulation instability (CIMI). It has been shown numerically that such CIMI in a fiber laser could lead to the periodic pulse train or even bright soliton pulse train formation [72]. In our experiment, we have investigated the features of the formed bright and dark pulses. Fig. 5.2(b) shows the polarization-resolved spectra of the laser emission. The spectrum of the light polarized along the horizontal polarization is significantly broadened. Its 3-dB bandwidth is estimated ~ 8 nm. Moreover, clear Kelly spectral sidebands are formed on the spectrum. Formation of the Kelly sidebands is a typical characteristic of the soliton operation of a laser [137]. The appearance of Kelly sidebands on the pulse spectrum unambiguously shows that the formed bright pulses are solitons. The central wavelength of the solitons is at 1578.76 nm, which is different from that of the CW laser emission of the same polarization direction. So far we have not fully understood why the central soliton wavelength is far away from that of the CW. We suspect it could be due to that the soliton and the CW beam have different loss dispersion in the cavity. Due to the weak net cavity birefringence, the CW

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wavelengths of the laser along the two orthogonal polarization directions are almost the same, which is at ~ 1575.7 nm. The spectra of the CW lights are also slightly broadened. From the traces shown in Fig. 5.2(a), one can observe that the CW background is very noisy. This is due to the modulation instability of the beam, which occurs when the CW intensity is high and has a high modulation frequency [46]. This also explains why the spectra of the CW components are broad. Experimentally we measured the average output power of the laser emission along the two orthogonal polarizations. It has a ratio of about 1:6 (bright soliton side: dark pulse side).

Fig. 5.2 (c) shows the autocorrelation trace of the bright solitons. It has a FWHM width of about 0.558 ps, suggesting that the width of the solitons at the laser output is about 362 fs if a Sech-shape pulse profile is assumed. Thus the time-bandwidth product (TBP) of the pulses is 0.348, which means that the solitons are slightly chirped. Fig. 5.2(d) shows the zoom-in of a dark pulse. Different from the bright solitons, the dark pulses are broad intensity ‘holes’ embedded in the CW background. In our experiment if the total laser emission is measured, the oscilloscope trace then shows a dark-bright pulse pair on the CW background. A similar dark-bright pulse pair emission was also observed on a figure-of-eight fiber laser, and it was found that the dark and bright pulse belongs to different wavelength bands of the laser emission, respectively [147]. Experimentally we found that the dark pulse width decreased with the increase of the CW power and the coupling strength between the two polarizations. Under our experimentally accessible pump strength, dark pulses as narrow as several tens of picosecond could be obtained. However, the dark pulses are significantly broader than the bright pulses. Slightly tuning the intra-cavity PC or carefully reducing the pump power, the number of solitons in the cavity could be changed and the periodic pulse train could also become inhomogeneous. Nevertheless, no matter how the bright solitons are distributed in the cavity, corresponding to each bright soliton there is always a dark pulse appeared on the CW background. Here we note that if one observes the oscilloscope trace under a large time scale or using a low-speed detection

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system, this kind of soliton-dark pulse pair would look like a ‘bright-dark soliton pair’ [54,55].

We then operated the laser under relatively large net cavity birefringence. In this case the wavelength separation between the two orthogonally polarized CW emissions has a large value. Fig. 5.3 shows the case experimentally observed. Fig. 5.3(a) shows the oscilloscope traces of the laser emission along the two orthogonal polarizations. Fig. 5.3(b) is the polarization resolved optical spectra. Because of the incoherent cross coupling between the two orthogonally polarized CW components, polarization domains are formed, characterized as the intensity alternation of the CW laser emissions between the two orthogonal polarization directions. We have shown previously that the appearance of the polarization domains is an intrinsic feature of the fiber laser emission under the incoherent coupling of the two orthogonal polarization modes [80]. However, in the current state as a result of the strong pumping, simultaneously a periodic bright soliton train is also formed on one polarization direction. The bright soliton train fills up the whole cavity. Again the formed bright solitons have different central wavelength than the CW component. When the oscilloscope is triggered by the edge of the polarization domain shown in the upper trace, the bright solitons move with respect to the polarization domain. It is to point out that in the polarization domain where there is coupling between the bright solitons and a CW beam polarized along the orthogonal polarization direction, the bright solitons induce dark pulse formation on the CW background, as shown in Fig. 5.3(a). Zooming in the region of the coupled soliton-dark pulse pairs, again corresponding to each bright soliton there is a dark pulse. Moreover, the dark pulses have a much broader pulse width than the bright solitons.

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Figure 5.3. The soliton-dark pulse pairs formed in a polarization domain where there is incoherent coupling between solitons and a CW beam. (a) Laser emissions along the two orthogonal polarization directions. (b) The corresponding polarization resolved spectra.

The “soliton rain” is an interesting feature of the soliton fiber laser operation whose formation mechanism is not yet clearly explained [133]. Under the same pumping power, by appropriately setting the orientation of the intra-cavity PC, the soliton rain feature is also observed on the formed bright solitons in our fiber laser, as shown in Fig. 5.4. Because of the incoherent coupling of the orthogonally polarized CW components, polarization domains are formed. On one polarization bright solitons are formed. In particular, the formed bright solitons exhibit the soliton rain feature where the solitons move with accelerated speed away from the soliton condensate. Again, each bright soliton induces a dark pulse on the orthogonally polarized CW. Consequently the dark pulses also exhibit the similar pulse evolution.

Figure 5.4. The soliton rain effect of the soliton-dark pulse pairs. The two oscilloscope traces are the polarization resolved emissions of the fiber laser.

5.1.2 Discussion

Our experimental results clearly demonstrate that because of the cross-polarization coupling, a bright soliton could induce the formation of a dark pulse. To understand the dark pulse formation in our laser we note that the polarization domain formation is an intrinsic feature of the quasi-vector cavity fiber lasers under the cross-polarization coupling. Required by the system minimum energy condition, within one polarization domain the laser would only emit in one polarization state. A soliton-dark pulse pair could be considered as a special case of the polarization domains, where the appearance of a soliton in one polarization will demand a temporal no lasing, or an intensity hole, on the orthogonal polarization direction. Hence, a dark pulse will always be formed associated with a bright soliton. We note that D. N. Christodoulides had once theoretically predicted the formation of a kind of black-white soliton pair in weakly birefringent SMF under coherent polarization coupling [54]. Although the bright soliton and the dark pulse observed in our experiment are coupled, their coupling is incoherent. Moreover, the formed dark pulses have an asymmetric pulse form and a much broader pulse width than the bright solitons. It is unlikely that the observed dark pulses are dark solitons.

In our fiber laser as there is no any mode-locking element in the cavity, so no mode locking could occur. We describe that the bright soliton formation is initiated by the cavity induced modulation instability and the gain bandwidth limitation. This also explains why initially a stable soliton pulse train is always formed in our laser. It is also a unique feature of the fiber laser that differs from the soliton operation of the mode locked fiber laser. Finally, we point out that the observed bright soliton-dark pulse pair formation is independent on the concert laser cavity parameters. Actually under different cavity conditions (different cavity length, net anomalous dispersion) we have observed the phenomenon, which shows that it is a general effect of the fiber lasers.

5.2 Induced dark solitary pulse in fiber lasers under coherent coupling

5.2.1 Experimental setup and results

The experimental fiber laser setup is schematically shown in Fig. 5.5. The fiber ring has a total length of 22.6 m, consisting of a piece of 3 m Erbium doped fiber (EDF) with a group velocity dispersion (GVD) parameter of -48 ps/nm/km, 13.5 m single mode fiber (SMF-28) with a GVD parameter of 18 ps/nm/km and 6.1 m dispersion compensation fiber (DCF) with a GVD parameter of -4 ps/nm/km. The fiber ring cavity is estimated to have an average net anomalous GVD parameter of 3.3 ps/nm/km. The fiber laser is pumped by a 1480 nm single mode Raman fiber laser whose maximum output power is 5 W. A polarization independent isolator is inserted in the cavity to force the unidirectional circulation of light in the cavity. Besides, an intra-cavity polarization controller (PC) is used to fine-tune the linear cavity birefringence. A wavelength division multiplexer (WDM) is used to couple the pumping light into the cavity, and a 10% fiber output coupler is used to output the light. The pumping coupling efficiency from the pump source to the cavity is about 60%. All the components used in our experiment have very low polarization dependent loss (PDL) (WDM: 0.01 dB, Isolator: 0.04 dB, Coupler: 0.01 dB). Therefore, the PDL induced mode locking is unlikely to occur in our laser [120]. An external cavity polarization beam splitter (PBS) is used to experimentally resolve the two orthogonal polarization components of the laser emission. To this end a polarization controller is inserted before the external polarization beam splitter to balance the linear polarization change caused by the lead-fibers. The two orthogonal polarization components are simultaneously monitored with a high-speed electronic detection system made of two 40 GHz photo-detectors and a 33 GHz bandwidth real-time oscilloscope.

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Figure 5.5. A schematic of the Erbium-doped fiber ring laser. EDF: Erbium-doped fiber. SMF: Single mode fiber. DCF: Dispersion compensation fiber. WDM: Wavelength division multiplexer. PC: Polarization controller. ISO: Isolator. OC: Output coupler. OSA: Optical spectrum analyzer.

Under low pump power the laser always emits continuous wave (CW). However, depending on the net cavity birefringence, which can be altered experimentally by changing the orientation of the intracavity polarization controller, the laser emission could be in different polarization states. In the case of very weak net cavity birefringence, the laser could emit either an elliptically polarized beam where the two polarization modes have the same oscillation wavelength, or a linearly polarized beam which can be attributed to the effect of polarization instability [148]. However, the most common state is that the laser emits CW along the two orthogonal polarization modes simultaneously, and each CW has different oscillation wavelengths. Experimentally it is identified that the wavelength difference of them varies with the net cavity birefringence. Specifically, strong birefringence leads to big wavelength difference. Hence experimentally we used the wavelength difference as an indicator on the strength of the net cavity birefringence and operated the laser under different pump strength and net cavity birefringence.

Increasing the pumping power to about 1.0 W (30 dBm), by tuning the intracavity PC, a kind of soliton-dark pulse pair emission similar to that shown in Fig. 5.6(a) is obtained, as we have discussed in previous section. Briefly, because the bright solitons and the CW in the opposite polarization direction have different central wavelengths, there is incoherent polarization coupling between them. Consequently, an effect similar to the polarization domain formation takes place, leading to that corresponding to each bright soliton a

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broad dark pulse is generated on the CW background. We note that the formed dark pulse is much broader than the bright soliton, and the stronger the pump power the narrower the dark pulses. When we keep increasing the pumping power to about 3.2 W (35 dBm), in the meantime also slightly tuning the intra-cavity PC to make the net cavity birefringence very weak, a special phenomenon as shown in Fig. 5.6 is further observed. Fig. 5.6(a) shows the oscilloscope traces of the polarization resolved laser emission. It shows that the laser emits stable bright soliton-dark pulse pairs, as the phenomenon in previous section. However, different from that, associated with each of the bright solitons there is a narrow dark pulse formed on the bottom of each of the broad dark pulses. Fig. 5.6(b) shows the zoom-in of one bright-dark pulse pair. The broad dark pulse is about 250 ps wide. On the bottom of the dark pulse there is an additional narrower dark pulse of about 30 ps wide, which is the bandwidth limitation of our oscilloscope. The narrow dark pulse synchronizes with the bright soliton in the orthogonal polarization direction. Fig. 5.6(c) is the autocorrelation trace of the bright soliton, whose FWHM width is 1.46 ps, suggesting that the width of the bright soliton is about 944 fs if a Sech-shape pulse profile is assumed.

Figure 5.6. Experimental results of induced dark solitary pulse formation in a fiber laser. (a) The polarization resolved oscilloscope traces of the laser emission. (b) The zoom-in oscilloscope traces of a soliton-dark pulse pair. (c) Autocorrelation trace of the bright solitons. (d) The Polarization-resolved spectra.

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Figure 5.6(d) shows the polarization-resolved optical spectra of the laser emission. The central wavelength of CW light (in vertical axis) is 1575.2 nm. The central wavelength of the bright soliton (the broadened spectrum in horizontal axis) is about 1575.5 nm, which can be estimated by fitting the spectrum profile. A Kelly sideband locates at 1588.95 nm, which indicates that the bright pulses in the horizontal axis have fully evolved into solitons [137]. Besides, we can clearly see two peak-dip spectral structures locating at 1570.45 nm and 1580.6 nm, which stands for the energy exchange between the two polarization components and is the symbol of coherent coupling between the bright solitons and the dark pulses [85]. To confirm that the narrow dark pulse is induced by the coherent polarization coupling of the bright soliton, we have experimentally deliberately changed the cavity birefringence so that the two polarization components are in the incoherent coupling. In this case the broad dark pulse still exists, while the narrow dark pulse become unmeasurable. In the meantime, no coherent energy exchange spectral sidebands appear on the optical spectra. The experimental result clearly suggests that the weak dark pulse could be an induced dark soliton by the bright soliton through the coherent polarization coupling. We note that there is also some spectral broadening of the CW beam. It is due to the modulation instability effect. This is also reflected by the noise background of the oscilloscope trace of the CW beam.

In addition to the induced dark solitons, under the coherent polarization coupling, experimentally we have also observed the induced bright soliton formation inside the wide dark pulse, as shown in Fig. 5.7. Fig. 5.7 (a) shows again the polarization resolved oscilloscope traces of the laser emission. Note that in the cavity multiple bright solitons are formed. Associated with each of the bright solitons a weak bright soliton is induced in the orthogonal polarization. Fig. 5.7(b) is the polarization resolved optical spectra. Note that the soliton wavelength and the CW wavelength are not the same. Hence, there is incoherent polarization coupling between them. This explains why each bright soliton also induces a broad dark pulse on the CW beam. The polarization-resolved spectra further show that the induced bright solitons have

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the same central wavelength as that of the inducing bright solitons. This could be identified by drawing the spectral profiles of the pulses and comparing their Kelly sideband positions. The Kelly sidebands on both polarization directions are clearly visible and they have the same locations. Moreover, the energy exchange spectral sidebands are also visible on the optical spectra, suggesting that the weak bright solitons are coherently induced. We point out that formation of induced bright solitons in mode locked fiber lasers were reported before [52]. We found that the induced bright solitons have very similar features to those reported previously.

Figure 5.7. The induced bright soliton formation inside the wide dark pulse of bright-dark pulse emission state. (a) Laser emissions along the two orthogonal polarization directions. (b) The corresponding polarization resolved spectra.

For the formation mechanism of the narrow dark solitary pulse inside the dark pulse, we notice that D. N. Christodoulides theoretically proved the existence of bright-dark soliton pair by the coherent coupling between polarization modes in [54]. Therefore, we conjecture that in our experiment, the formation of the narrow dark solitary pulse in the wide dark pulse is induced by the bright soliton in its orthogonal polarization mode through coherent coupling. Besides, this is a special phenomenon because in SMF environment, only bright soliton can propagate steady in the anomalous dispersion regime and dark soliton in normal dispersion regime. In our experiment, we find the dark solitary wave also can exist in anomalous dispersion regime. We suggest it's due to the coherent coupling between two polarization modes and dissipative properties of fiber laser.

5.2.3 Numerical simulations and discussion

We have investigated both experimentally and theoretically the polarization domain formation in quasi-vector cavity fiber laser and shown that it is an intrinsic feature of the incoherent polarization coupling in the fiber lasers [80,81]. The bright soliton-dark pulse pair formation of the laser under relatively low pump power or even high pump power but incoherent polarization coupling, could be considered as a special case of the polarization domain formation [14]. Here due to the influence of the strong nonlinear self-phase modulation the domains are represented in a different form. What is interesting here is that once the polarization coupling becomes coherent, in addition to the broad dark pulse formation, a weak dark or bright soliton is further induced. The formation of induced dark soliton under the polarization coupling was theoretically predicted before [53,149,150], and formation of induced bright solitons in a mode locked fiber laser was also experimentally reported [52]. However, to the best of our knowledge, no induced dark solitons in the anomalous dispersion regime are experimentally observed. In order to understand how the dark soliton is formed, we numerical simulated the operation of the fiber ring laser. Our simulation is based on the coupled complex Ginzburg-Landau equations (CGLEs), which describe the light propagation in a weakly birefringent fiber cavity,

$$\begin{aligned}\frac{\partial u}{\partial z} &= i\beta u - \delta \frac{\partial u}{\partial t} - \frac{ik''}{2} \frac{\partial^2 u}{\partial t^2} + i\gamma \left(|u|^2 + \frac{2}{3}|v|^2 \right) u + \frac{i\gamma}{3} v^2 u^* + \frac{g}{2} u + \frac{g}{2\Omega_g^2} \frac{\partial^2 u}{\partial t^2} \\ \frac{\partial v}{\partial z} &= -i\beta v + \delta \frac{\partial v}{\partial t} - \frac{ik''}{2} \frac{\partial^2 v}{\partial t^2} + i\gamma \left(|v|^2 + \frac{2}{3}|u|^2 \right) v + \frac{i\gamma}{3} u^2 v^* + \frac{g}{2} v + \frac{g}{2\Omega_g^2} \frac{\partial^2 v}{\partial t^2},\end{aligned}\quad (5.1)$$

where u and v describe the optical fields of the two orthogonal polarization components in the fiber cavity. $2\beta = 2\pi\Delta n/\lambda = 2\pi/L_b$ is the wave number difference between these two polarization modes. $2\delta = 2\beta\lambda/2\pi c$ is the group velocity difference, which are related with the value of beat length L_b and stand for the cavity birefringence. k'' is the second-order dispersion coefficient, γ represents the nonlinearity of the fiber cavity, g is the gain coefficient of the gain fiber and Ω_g is the effective gain bandwidth. The gain saturation is defined as

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$$g = g_0 \exp \left[-\frac{\int (|u|^2 + |v|^2) dt}{E_s} \right], \quad (5.2)$$

where g_0 is the small signal gain coefficient and E_s is the gain saturation energy, which is related to the pumping power in the experiment.

To model the induced dark or bright soliton formation in the laser, we have set the initial state as a soliton-dark pulse pair, which is similar to the experimental situation. The initial bright soliton has the form of $u = A \cdot \text{sech}(B \cdot t)$, while for the initial dark pulse we have either the form of $v = C \cdot \tanh(D \cdot t)$, where there is a phase jump in the center of the dark pulse; or $v = C \cdot \sqrt{1 - \text{sech}^2(D \cdot t)}$, where there is no phase jump. Here A and C stand for the bright soliton intensity and dark pulse depth, usually A is set much bigger than C because the bright soliton peak power is much higher than the CW power in our experiment. B and D stand for the width of bright soliton and the dark pulse. B is set much smaller than D , in accordance with the experimental results that the dark pulse is much wider than the bright solitons. In all our simulations, we have possibly used the actual laser cavity parameters if possible, e.g. the cavity length $L=22.6$ m and the average cavity dispersion is 3.3 ps/nm/km. In our experiment results, the coherent coupling between two orthogonal polarization components can only happen under very weak cavity birefringence. So we set $L_b = 50$ km, which means the cavity birefringence is very weak.

First, we numerically simulated the case of dark pulse with a phase jumping $v = C \cdot \tanh(D \cdot t)$. Under appropriate parameters selection, we can obtain a stable state laser emission as shown in Fig. 5.8. Like what observed in the experiment, a narrow pulse width bright soliton polarized along one polarization eigenmode of the fiber laser could induce a weak dark soliton on the bottom of a broad dark, and consequently form a coupled bright-dark soliton pair. Both the broad and narrow dark pulses are stable and they are trapped with the bright solitons and propagate together in the cavity. To confirm that the narrow dark soliton is induced by the coherent coupling, we also deliberately removed the four-wave-mixing (FWM) terms from the CGLEs and did the simulations with the same

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parameters. No induced dark soliton could be formed inside the broad dark pulse. Therefore, we conclude that the formation of the induced dark solitary pulse is a result of the coherent polarization coupling between the two polarization modes.

Figure 5.8. (a) Stable soliton-dark pulse emission of the laser when the input dark pulse has a phase jump. The saturation energy $E_s=0.1$ pJ and the small signal gain $g_0=640$ m⁻¹. (b) Zoom-in of the dark pulse. The induced dark soliton is on the bottom of the broad dark pulse.

We then repeated the numerical simulations for the case of no phase jump dark pulse, $v=C \cdot \sqrt{1-\text{sech}^2(D \cdot t)}$. We used exactly the same simulation parameters as the previous one. The results are shown in Fig. 5.9. In this case instead of that an induced dark solitary pulse is formed, a bright solitary pulse is formed inside the broad dark pulse. The numerically simulated results are well in qualitative agreement with the experimental observations.

Figure 5.9. (a) Stable soliton-dark pulse emission of the fiber laser when the input dark pulse has no phase jump. The saturation energy $E_s=0.1$ pJ and small signal gain $g_0=640$ m⁻¹. (b) Zoom-in of the dark pulse. The induced bright soliton is on the bottom of the broad dark pulse.

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Based on the numerical simulations it is to see that as a result of the coherent polarization coupling, a bright soliton in an anomalous dispersion cavity fiber laser could induce either a dark soliton or a bright soliton on the bottom of a broad dark pulse polarized along the orthogonal polarization, depending on if there is a phase jump in the broad dark pulse or not. Finally, we point out that strictly speaking all solitons formed in a fiber laser should be dissipative solitons as a laser is intrinsically a dissipative system due to the gain and losses balance [30]. In view of that a bright dissipative soliton could either be formed in the normal or anomalous cavity dispersion regimes, it should also be possible that a dissipative dark soliton could be formed either in the normal or anomalous dispersion regime.

5.3 Conclusion

In this chapter, the soliton coupling between two orthogonal polarization modes of a birefringent fiber laser cavity is investigated. Depending on the cavity birefringence, it is shown that either incoherent or coherent coupling between light along the two polarization modes can be achieved, and different coupling phenomena could be obtained. Due to the incoherent coupling between the two orthogonal polarization modes of a fiber cavity, a bright soliton pulse always induces a dark pulse on the CW background polarized along the orthogonal polarization. In particular, the formed soliton-dark pulse pairs are stable in the cavity. Under our experimental accessible conditions, it is found that the formed dark pulses always have a broader pulse width than the bright solitons. The mechanism of the dark pulse formation could be understood based on the polarization domain formation. On the other hand, under the coherent polarization coupling, a bright soliton polarized along one polarization eigenmode of the cavity could either induce the formation of a weak dark soliton or a bright soliton on the bottom of a broad dark pulse in the orthogonal polarization direction in a net anomalous dispersion cavity fiber laser. In particular, the induced solitons are stable in the cavity. Numerical simulations based on the coupled complex GLEs have well confirmed the experimental

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observations. The numerical simulations have further revealed that whether a dark or bright soliton is induced depends on if there is a phase jump in the broad dark pulse.

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6

Dark soliton envelop formation in a fiber laser

The NLSE supports not only bright solitons, but also dark solitons [46]. Contrast to bright solitons, a dark soliton is an intensity dip on a constant background with a phase jump across its intensity minimum. Dark solitons were first theoretically proposed by Hasegawa *et al.* in 1973 [151]. Since then, dark soliton research has also attracted considerable attention, especially in the fields of nonlinear optics [29], oceanology [152] and Bose–Einstein condensates [153]. Like the bright solitons, dark solitons also can be formed in fiber lasers. Moreover, as dark solitons are less sensitive to noise and fiber losses compared with bright solitons, they are more promising for the future optical communications or optical signal processing applications. However, there are not many reports about the dark soliton fiber lasers until recent years.

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Sylvestre *et al.* generated stable dark-soliton-like pulse trains in a fiber laser using the dissipative four-wave mixing method [121]. Zhang and Tang *et al.* reported the spontaneously generated dark solitons in an all-normal dispersion fiber ring laser [37,38,41]. Song *et al.* also reported a 280 GHz dark soliton train emission of a fiber laser [39], which may find potential applications in optical communications [101].

The main challenge for experimental study of dark solitons is the detection and measurement. Unlike the bright soliton, which is an intensity peak and can be easily detected even with a moderate speed detector, the dark soliton is an intensity dip. If the dark soliton is too weak or too narrow, it will be difficult to be resolved from the background noise. For a dark pulse train with high repetition rate, one could also use an autocorrelator to resolve it [39,121]. Although high speed detector and oscilloscope with several tens of GHz bandwidth are available now, for single dark soliton the measurable pulse width is still limited to few picoseconds. This brings a barrier for the experimental study of dark solitons. In this chapter, we will discuss an interesting phenomenon of dark envelope pulse emission of a fiber ring laser. We show that when a high repetition rate pulse train and a dark-soliton pulse coexist in a fiber laser, their superposition will lead to the formation of a broad dark envelope pulse embedded in the high repetition rate pulse background. Since the formed dark envelope pulse could have much broader pulse width than the original dark soliton, it would be relatively easily detected. We have both experimentally and numerically confirmed the existence of the phenomenon. It explains why in the experiments frequently a much narrow dark soliton than the bandwidth of the detection system could still be measured.

6.1 Experimental setup and results

To generate high repetition dark pulse trains and dark solitons, we have built up a fiber ring laser with average normal cavity dispersion, following the same setup in [39]. The fiber laser has a cavity configuration as shown in Fig. 6.1. The fiber ring has a total length of 15.6 m, consisting of a piece of 3 m Erbium

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doped fiber (EDF) with a group velocity dispersion (GVD) parameter of -48 ps/nm/km, 8.5 m single mode fiber (SMF-28) with a GVD parameter of 18 ps/nm/km and 4.1 m of dispersion compensation fiber (DCF) with a GVD parameter of -4 ps/nm/km. The fiber laser is pumped by a 1480 nm single mode Raman fiber laser whose maximum output power is 5 W. A polarization independent isolator is inserted in the cavity to force the unidirectional circulation of light in the cavity. In addition, an intra-cavity polarization controller (PC) is used to fine-tune the cavity. A wavelength division multiplexer (WDM) is used to couple the pumping light into the cavity, and a 10% fiber output coupler is used to output the light. The components used in our experiment have very low polarization dependent loss (PDL) (WDM: 0.01 dB, Isolator: 0.04 dB, Coupler: 0.01 dB), to avoid the unwanted PDL-induced mode locking [120]. The fiber ring cavity is estimated to have an average net normal GVD parameter of 0.5 ps/nm/km. Such a low average dispersion helps to lower threshold of modulation instability to generate the high repetition pulse trains [14]. The laser emission was monitored by an electronic detection system consisting of a 45 GHz photo-detector and a 33 GHz bandwidth real-time oscilloscope.

Figure 6.1. A schematic of the Erbium-doped fiber laser. EDF: Erbium-doped fiber. SMF: Single mode fiber. DCF: Dispersion compensation fiber. WDM: Wavelength division multiplexer. PC: Polarization controller.

By tuning the intra-cavity PC, we can minimize the net cavity birefringence and make the fiber laser operating in a single polarization state. The single polarization operation can be attributed to the polarization instability effect of

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the fiber laser [147]. After increasing the pumping power to about 2W, while the intracavity light intensity is about 500 mW, and tuning the intra-cavity PC at appropriate orientation, a stable dark pulse emission state can be achieved, as shown in Fig. 6.2(a). The dark pulses are randomly distributed in the cavity but repeat themselves with the cavity roundtrip time. Besides, the amplitude of dark pulses also randomly varies. These properties fulfill the characters of dark solitons [29]. Fig. 6.2(b) shows the zoom-in trace of the dark pulses. The zoom-in trace clearly tells us that these dark pulses are all well separated single pulses. The average dark pulse width is about 90 ps, which is within the resolution of our measurement system. On the other hand, the lowest amplitude of dark pulses doesn't reach zero, meaning they are "grey dark pulses". Fig. 6.2(c) is the corresponding spectrum. The spectrum shows a high frequency modulation, which is similar with those reported in [39]. By measuring the distance between adjacent spectral spikes, it shows that the modulation frequency is about 230 GHz. If just judged by the spectrum, the experimental result seems have no difference to those shown in [39]. However, the fiber laser has a stable dark pulse emission. Carefully analyzing the output spectrum, the central CW peak at 1582.7 nm has a FWHM bandwidth of about 0.2 nm. Considered as a transform-limited pulse, the width of dark pulses is 13 ps [37]. Theoretically, such a narrow dark pulse could not be clearly detected with our measurement system.

To get an insight on the formation mechanism of the measured dark pulses, we measured the autocorrelation trace of the laser output, and the result is shown in Fig. 6.2(d). The autocorrelation trace shows an obvious difference to the pure high repetition rate pulse trains. Although the pulse spikes on the autocorrelation trace means the existence of a stable pulse train with 220 GHz repetition rate which coincides with that measured from the optical spectrum, the amplitude of pulse train exhibits a waving shape. Such a waving shape fluctuation of pulse amplitude indicates there should be another structure among the periodical high repetition pulses. Considered the dark pulse emission trace and normal dispersion cavity, we conjecture that the emission state should

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be a combination of a high repetition dark pulse train and dark solitons embedded in it.

Figure 6.2. A dark pulse emission state of our fiber laser. (a) The oscilloscope trace of the dark pulse emission state. (b) Zoom-in trace of the dark pulse emission. (c) The corresponding optical spectrum of the laser output. (d) The autocorrelation trace of the laser output.

6.2 Numerical simulation and discussion

To understand the experimental phenomenon, we further numerically simulated the fiber laser operation under coexistence of a stable high repetition rate pulse train and a dark soliton. The light circulation in a unidirectional fiber ring laser cavity is governed by the following Ginzburg-Laudau equations (GLE):

$$i \frac{\partial E}{\partial z} - \frac{\beta_2}{2} \frac{\partial^2 E}{\partial T^2} + \gamma |E|^2 E - i \frac{g}{2} E - i \frac{g}{2\Omega_g} \frac{\partial^2 E}{\partial T^2} = 0, \quad (6.1)$$

where E are the complex envelopes of the optical field, n denotes the n^{th} passage through the fiber laser cavity, β_2 is the group velocity dispersion (GVD) coefficient, γ is the nonlinear coefficient, g is the gain coefficient and Ω_g is the effective gain bandwidth of fiber laser. z is the light propagation distance parameter and T is the retarded time in a reference frame traveling with the

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group velocity of light. To derive the equation, it has been assumed that the light field is in resonance with the cavity in each cavity roundtrip and the cavity length is far shorter than the dispersion and nonlinear length of the light in the cavity. g is the saturation gain and described by

$$g = \frac{g_0}{1 + \int |u|^2 dt / E_s}, \quad (6.2)$$

where g_0 is the small gain coefficient and E_s is the saturation energy of the gain fiber. The simulation is based on numerically solving Eq. (1), using the split-step Fourier method. Our simulation model is very similar with power-energy saturation model reported in [40,154], except the nonlinear saturable absorption/loss terms. The simulation parameters are set according to the experimental parameters, such as a 15.6 m fiber ring cavity with an average net normal GVD parameter of 0.5 ps/nm/km.

The initial state of the simulation is set as a modulated CW, which can be shaped into a high repetition rate dark pulse train in a normal dispersion fiber laser cavity [39]. In addition, we also add dark pulses on the CW background, to simulate the formation of dark solitons in the laser. To match our experimental results, the modulation frequency of the CW is set as 200 GHz and the superposition trace of the initial state is shown in Fig. 6.3. The width of the added dark pulse is set as about 4 ps, which is far beyond the observation limitation of the experimental measurement system. It is to note that we added two *tanh*-shaped dark pulses, which is used to makes the initial state fulfilling the periodical boundary condition of the calculation window.

Figure 6.3. Initial input state of the numerical simulations

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By properly setting other simulation parameters, we can get the stable results as shown in Fig. 6.4(a). From the figures we can see that the initially modulated CW background has been shaped into stable dark pulse trains. Besides, each dark pulse has split into two identical dark pulses. The two dark pulses moved to opposite directions and they might collide with other dark pulse. However, after collisions they still remain their pulse profiles, showing a soliton-like property.

Figure 6.4. Stable simulated dark pulse evolution in a modulated CW background.

In Fig. 6.4(b), the simulated fiber laser emission state is presented. The ‘dark pulses’ embed in the dark pulse train background evolved into two dark envelopes. Moreover, as shown by the simulations, the dark envelope shows soliton-like features. In Fig. 6.4(b) we have roughly drawn a fitting of the dark envelopes. The falling and rising sides of the fitted pulses are about 12 ps, which is much wider than the initial dark pulse width. This simulation result explains why in real experiment a narrow dark soliton pulse could be detected by a detection system with relatively smaller bandwidth. Limited by the bandwidth of a 45 GHz photo-detector, 200 GHz repetition rate dark pulse train would be displayed just as a noisy CW background, however, the dark envelopes due to their large rising and falling time are resolved as dark pulses. Moreover, as the dark envelope pulses are embedded among the dark pulse train background, the lowest amplitude of those resolved dark pulses can’t reach zero and they are observed as “grey dark pulses”. The simulation results fully reproduce our experimental observations. For comparison, we also

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simulated the same situation but without the dark pulse train background. The corresponding result is shown in Fig. 5. Without existence of the dark pulse train background, one initial dark pulse would be shaped into two stable counter-propagating dark solitons. The final stable dark soliton would have a pulse width of about 3 ps. Obviously such a narrow dark soliton can't be resolved in the real experiment. The existence of the dark pulse train background would have the effect of making the much narrow dark soliton 'visible'.

Figure 6.5. Simulation results of dark soliton formation of a modulation-free CW background.

Previous theoretical studies have shown that the formation of bright solitons needs to fulfill a certain pulse threshold condition, while that of dark solitons has no threshold requirement. This means that any weak dark pulse would be able to evolve into a dark soliton in a normal dispersion nonlinear fiber. Our numerical simulations shown above well confirm it. An intensity dip has evolved into two counter-propagating dark solitons. The soliton parameters of the stable dark soliton are determined by the laser operation conditions, like the CW intensity strength, the net cavity dispersion etc. As shown by the simulations the stable dark solitons would have a pulse width of about 3ps. Such a narrow dark soliton is impossible to be detected with our 45GHz bandwidth detection system. However, if the dark soliton would be embedded in a 200GHz stable repetition rate dark pulse train, a dark envelope pulse would be formed, and the dark envelope would have a pulse width of more than 12ps. Such a pulse width would be detectable by our detection system. We note that

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the numerical result uncovers a novel experimental technique of detecting narrow dark pulses.

Based on the numerical simulation results, we could explain now the experimental observations shown in section 1. As mentioned above, the experimentally measured autocorrelation traces showed that there is a high repetition rate pulse train coexisted with the dark pulses. We believe that the high repetition pulse train formation could be a result of the cavity induced modulation instability effect. Depending on the laser operation conditions, such a CIMI could generate a stable pulse train with tunable pulse repetition. However, due to the high repetition rate the dark pulse train could not be resolved by the detection system. It is displayed as a noisy CW background on the oscilloscope trace. However, the high repetition rate pulse train can be resolved by an autocorrelator. In the normal dispersion fiber lasers the formation of dark soliton is an intrinsic feature. So dark solitons are always automatically formed in the fiber laser. However, in order to detect the dark solitons, one has to carefully control the CW power and other laser parameters. While our simulations suggest that by superimpose a narrow dark soliton with a stable high repetition rate pulse train, one could effectively increase the resolution power of a detection system so that with exactly the same system a relatively much narrow dark pulse could be detected. Finally, we note that based on the numerical simulations, physically the dark envelope pulses are a different form of the dark solitons formed in the fiber laser. Although they have much broader pulse width, they still maintain the features of the dark soliton. Or in other words, the existence of the high repetition rate pulse train does not affect the dark soliton shaping process in the fiber laser.

6.3 Conclusion

In this chapter, we have both experimentally and numerically investigated the dark soliton emission of a fiber laser. We found that under certain experimental conditions the CW background could be a high repetition rate pulse train. However, due to the limited resolution of the detection system, the high

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repetition rate pulse train could not be resolved. Hence on the oscilloscope trace it is displayed as a noisy CW background. Despite of the fact that the CW is a high repetition rate pulse train, it does not affect the dark soliton formation in the laser. Since now the dark soliton is embedded in a stable high repetition pulse train, the dark solitons formed would have a broad dark pulse envelope. Because of the feature of the dark solitons a relative narrower dark pulse could be detected with the same bandwidth detection system. Based on the good agreement between the experimental observation and numerical simulations, we conclude that our experimentally measured dark pulses are in fact such dark envelope pulses. They are dark solitons in nature, but due to the superposition with a high repetition pulse train they have much broader envelope pulse width than the conventional dark solitons.

7

Black-white vector solitons in fiber laser

In chapter 2, we have discussed the pulse propagation in a birefringent optical fiber and a vector fiber laser cavity. In the aspect of optical solitons, in the birefringent fiber environment there are vector solitons. Two types of vector solitons were predicted for the coupled NLSEs: i) Vector solitons whose formation has close relation to their scalar, one component counterparts. Examples of the vector solitons include the group velocity locked bright vector solitons [155], the phase-locked bright-bright vector solitons [156,157], the polarization rotating bright vector solitons [158], and the dark-dark vector solitons [57]. These vector solitons have the characteristic that if there were no coupling, each polarization component of them can still exist as a scalar soliton. All the type of vector solitons have already been experimentally observed [17,48,159,160]. ii) Vector solitons that have no scalar, single component counterparts. Examples of the vector solitons are the polarization

domain wall solitons [161] and the black-white vector solitons [54]. Formation of the type of vector solitons is purely a result of the cross polarization coupling of light in the fibers. They constitute a fundamentally new type of optical solitons. S. Pitois *et al.* have previously reported the experimental observation of a polarization domain wall soliton by mixing two intense counter propagating laser beams in a spun fiber [162]. Recently, the experimental observation of polarization domain wall solitons in a single mode fiber laser has been reported [84]. However, to the best of our knowledge, there is so far no experimental observation of the black-white vector solitons in SMFs. In this chapter, combining with the results in chapter 6, we will discuss a new approach to the experimental study of black-white vector solitons.

7.1 Theoretical background

The existence of black-white vector solitons in weakly birefringent SMFs was first theoretically predicted by D. N. Christodoulides in 1988 [54]. M. Haelterman had theoretically analyzed the link between polarization modulation instability (PMI) and the black-white vector solitons [55]. It was shown that the existence of the vector soliton is intimately related to the dynamics of the PMI in birefringent Kerr materials. To have a better understanding on the black-white vector solitons, we highlight some of their key features here.

Following Christodoulides, light propagation in a weakly birefringent SMF is governed by the coupled NLSEs [54]:

$$\begin{aligned} i\frac{\partial u}{\partial z} - \frac{\beta_2}{2}\frac{\partial^2 u}{\partial \tau^2} + \gamma\left(|u|^2 + \frac{2}{3}|v|^2\right) + \frac{\gamma}{3}v^2u^* \exp(-4i\rho z) &= 0 \\ i\frac{\partial v}{\partial z} - \frac{\beta_2}{2}\frac{\partial^2 v}{\partial \tau^2} + \gamma\left(|v|^2 + \frac{2}{3}|u|^2\right) + \frac{\gamma}{3}u^2v^* \exp(-4i\rho z) &= 0 \end{aligned} \quad (7.1)$$

where u , v are the normalized slowly varying optical fields along the two orthogonal polarization eigen-modes of the fiber, β_2 is the group velocity dispersion coefficient, γ is the nonlinear coefficient of the fiber, and z is the propagation distance along the fiber and τ is the time in a coordinator system

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that moves at the average group velocity, $\rho = \frac{\pi}{\lambda_0}(n_x - n_y)$ is the birefringence of the fiber.

The coupled NLSEs admit the formation of coupled black-white vector solitons of the form:

$$\begin{aligned} u(z, \tau) &= U_0 \operatorname{sech}(\tau/\tau_0) \exp[i(\sigma - \rho)z] \\ v(z, \tau) &= V_0 \tanh(\tau/\tau_0) \exp[i(\sigma + \rho)z] \end{aligned} \quad (7.2)$$

where

$$\begin{cases} \tau_0^2 = \frac{\beta_2}{4\rho} = \frac{\lambda_0 \beta_2}{4\pi(n_x - n_y)} \\ U_0^2 = \frac{\lambda_0}{\pi n_2}(\sigma - 3\rho) \\ V_0^2 = \frac{\lambda_0}{\pi n_2}(\sigma + \rho) \end{cases} \quad (7.3)$$

In Eq. (7.2) and (7.3) U_0 and V_0 are the amplitude of the white and black solitons, respectively, τ_0 is the temporal soliton pulse width, and σ is an arbitrary constant.

To illustrate, we have drawn in Fig. 7.1 the black-white soliton profiles at different σ values and in Fig. 7.2 the variation of the soliton pulse width τ_0 with the fiber birefringence under different fiber dispersions. The black-white vector solitons have the following characteristics: i) Their formation is a result of the coherent cross polarization coupling of light in SMFs; ii) The vector solitons can be formed either in the normal ($\beta_2 > 0$) or the anomalous ($\beta_2 < 0$) dispersion fibers. While in the anomalous dispersion regime the white soliton polarizes along the fast birefringence axis, in the normal dispersion regime it polarizes along the slow birefringence axis. Moreover, in the normal dispersion fibers, $V_0^2 > U_0^2$, which means that the black soliton dominates the vector soliton, while in the anomalous dispersion fibers, $U_0^2 > V_0^2$, the white soliton dominates the vector soliton; iii) the pulse widths of the black and white solitons are always the same. Under fixed fiber group velocity dispersion, it is uniquely determined by the birefringence of the fiber; The features of the black-white vector solitons are different from those of the scalar NLSE solitons as well as

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those of the vector solitons so far experimentally observed, which make them unique and interesting.

Figure 7.1. The black-white solitons formed in the anomalous dispersion regime at different σ . The solid-lines are the white solitons and the dashed-lines are the black solitons. Here $\beta_2 = -1.55 \times 10^{-27} \text{ s}^2/\text{m}$, $\delta n = 1 \times 10^{-9}$, $n_2 = 1.2 \times 10^{-22} (\text{m}/V)^2$, the effective cross-section area of the fiber is $S = 20 \times 10^{-12} \text{ m}^2$, and the wavelength $\lambda = 1580 \text{ nm}$. The full width at half maximum of the solitons is $\tau_{FWHM} = 770 \text{ fs}$.

Figure 7.2. The dependence of T_{FWHM} on the fiber birefringence δn at different β_2 . Here $n_2 = 1.2 \times 10^{-22} (\text{m}/V)^2$, $S = 20 \times 10^{-12} \text{ m}^2$, and $\lambda = 1580 \text{ nm}$. Inset in panel is the zoom-in of the low birefringence area.

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However, it is challenge to experimentally observe the black-white vector solitons. Consider using the standard SMF to generate the vector solitons at 1.55 μm . Based on the theory, to form a soliton with 1ps pulse width, the fiber birefringence must be as small as $\sim 3 \times 10^{-9}$, and it would also require a CW beam with an average power of $\sim 20\text{W}$ to support the black soliton. In practice it is difficult to find a fiber that maintains such a small birefringence over a long distance. One option is to use high power light so that the required fiber length could be reduced, like S. Pitois *et al.* did for observing the polarization domain wall solitons [162]. The drawback is that high power optical pulses must be applied, and the strong light could also cause the occurrence of other nonlinear effects. We found an innovative approach to solve the problem: instead of traversing light over a long fiber, we circulate light in an active fiber ring cavity. It can be theoretically shown that the average dynamics of light circulating in an active fiber cavity with suitably selected parameters is equivalent to light propagation along an endless fiber. In addition, the approach also has the following advantages: as the average dynamics of light circulating in the cavity is governed by the interplay between the averaged fiber properties and the Kerr nonlinearity over the cavity length, one could use the cavity dispersion and birefringence management technique to fine tune the effective cavity dispersion and birefringence. In our experiment through using fibers with different signs of dispersion and inserting a polarization controller in the cavity, it is possible to operate a cavity at near net zero dispersion and birefringence position. Thus, not only broader black-white solitons could be obtained, but also the beam power required for the occurrence of the vector solitons could be significantly decreased. Using an active cavity the power of the beams can be easily controlled. For the erbium- or ytterbium-doped fibers, as their gain spectra are very broad and gain recovery times are very long, as far as the temporal soliton formation is concerned, no gain related dynamics needs to be considered. Using similar active fiber cavities we had previously experimentally successfully demonstrated the phase locked and polarization rotation bright vector solitons, whose formation also requires the coherent coupling between the orthogonal polarization components of light in a weakly birefringent fiber [17,48,85].

However, there is so far no cogent experimental observation of the black-white vector solitons in SMFs.

7.2 *Experimental setup and results*

Our fiber ring laser setup is schematically shown in Fig. 7.3. The fiber ring has a total length of 29.3 m, consisting of a piece of 3 m Erbium doped fiber (EDF) with a normal dispersion parameter of -48 ps/nm/km, 13.5 m single mode fiber (SMF-28) with an anomalous dispersion parameter of 18 ps/nm/km and 12.9 m dispersion compensation fiber (DCF) with a normal dispersion parameter of -4 ps/nm/km. The fiber ring cavity is estimated to have an average net anomalous dispersion of -2.15 ps². The fiber laser is pumped by a 1480 nm single mode Raman fiber laser whose maximum output power is 5 W. A polarization independent isolator is inserted in the cavity to force the unidirectional circulation of light in the cavity. Besides, an intra-cavity polarization controller (PC) is used to fine-tune the linear cavity birefringence. A wavelength division multiplexer (WDM) is used to couple the pumping light into the cavity, and a 10% fiber output coupler is used to output the light. The pumping coupling efficiency from the pump source to the cavity is about 60%. All the components used in our experiment have very low polarization dependent loss PDL. An external cavity polarization beam splitter is used to experimentally resolve the two orthogonal polarization components of the laser emission. To this end a polarization controller is inserted before the external polarization beam splitter to balance the linear polarization change caused by the lead-fibers. The two orthogonal polarization components are simultaneously monitored with a high-speed electronic detection system made of two 45 GHz photo-detectors and a 33 GHz bandwidth real-time oscilloscope.

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Figure 7.3. A schematic of the Erbium-doped fiber ring laser. EDF: Erbium-doped fiber. SMF: Single mode fiber. DCF: Dispersion compensation fiber. WDM: Wavelength division multiplexer. PC: Polarization controller. ISO: Isolator. OC: Output coupler. OSA: Optical spectrum analyzer.

Under low pump power the laser is always in CW operation. However, depending on the net cavity birefringence, which can be altered experimentally by changing the orientation of the intra-cavity PC, the laser emission could be in different polarization states. In the case of very weak net cavity birefringence, the laser could emit either an elliptically polarized beam where the two polarization modes have the same oscillation wavelength, or a linearly polarized beam which can be attributed to the effect of polarization instability of the fibers [148]. However, the most common state is that the laser emits CW along the two orthogonal polarization modes simultaneously, and each CW has different oscillation wavelengths. Experimentally it is identified that the wavelength difference of them varies with the net cavity birefringence. Specifically, strong birefringence leads to big wavelength difference. Hence experimentally we used the wavelength difference as an indicator on the strength of the net cavity birefringence and operated the laser under different pump strength and net cavity birefringence.

Increasing the pumping power to a relatively high value, in our experiment to about 28 dBm for instance, the spectra of the CW beams will become broad and their traces on the oscilloscope become noisy. We explain the effect as a result of the cavity induced modulation instability, which is an intrinsic property of optical fiber when the light intensity is sufficiently high. Keeping increasing the pumping power to 35.5 dBm, while the intracavity power is about 1W, we can achieve a stable bright-dark solitary wave pair, as shown in

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Fig. 7.4(a). Fig. 7.4(b) is the corresponding polarization-resolved spectrum. We can see clearly that the spectrum of light in the horizontal axis has significantly broadened and there are two sidebands located at 1565.5 nm and 1589.1 nm, which are symmetric with respect to the central wavelength (~ 1577 nm) of the broadened spectral envelope. The existence of spectral sideband means the bright pulses in horizontal axis have been shaped into solitons [137]. The autocorrelation trace of the bright solitons is shown in Fig. 4(c). It has a FWHM width of about 551 fs, suggesting that the width of the solitons is about 357 fs if a Sech-shape pulse profile is assumed.

Figure 7.4. Experimental results of bright-dark solitary wave pair. (a) The oscilloscope trace. (b) The polarization-resolved spectrum. (c) The autocorrelation trace of bright solitons.

Zooming in the bright-dark solitary pulse pair in Fig. 7.4(a), we found that the width of dark pulse (~ 30 ps) is much broader than the width of the bright pulse. Nevertheless, the ~ 30 ps pulse width is limited by the resolution of our detection system. And as discussed in Chapter 7, the narrowest dark pulse our detection can resolve is in tens of picosecond. Carefully analyzing the polarization resolved spectra, we note that the central wavelength of light on vertical axis locates at about 1575.5 nm, which is very close to the central wavelength of the solitons polarized along the horizontal axis. Therefore, we

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believe that when the light intensity is sufficient high, coherent coupling will be achieved between the two polarization modes. Under coherent coupling, the black-white vector solitons should exist.

If the black-white vector solitons really exist in our fiber laser, so how to explain that the dark soliton is much wider than the bright soliton? We note that the CW background of the dark pulse side is very noisy. The dark pulse side is measured by autocorrelator and corresponding trace is shown in Fig. 7.5. The CW beam is proved to be a high repetition rate pulse train with a repetition rate of about 330 GHz. The experimental results remind us the formation of the dark envelope pulses discussed in Chapter 7. Therefore, we believe that the noisy CW background on the dark pulse side is in fact a high repetition rate periodical pulse train. However, limited by the bandwidth of our electronic detection system (45 GHz photo-detector and 33 GHz bandwidth real-time oscilloscope), this high frequency pulse train can't be resolved. Therefore, it displayed as a noisy background on the oscilloscope trace. Unlike the previously reported autocorrelation trace of high repetition rate pulse with a flatted background, such as in [39], the autocorrelation trace in our result has clearly background fluctuations. Combined with the conclusions in Chapter 7, it's supposed that the dark solitons are also embedded in the high repetition pulse train background.

Figure 7.5. Autocorrelation trace of the dark pulse component.

7.3 Numerical simulation and discussion

To confirm the conjecture to the explanation of experimental results, we numerically simulate the operation of the fiber laser. The vector light propagation in a weakly birefringent fiber laser cavity is described by the coupled complex Ginzburg-Landau equations (CGLEs),

$$\begin{aligned}\frac{\partial u}{\partial z} &= i\beta u - \delta \frac{\partial u}{\partial t} - \frac{i\beta_2}{2} \frac{\partial^2 u}{\partial t^2} + i\gamma \left(|u|^2 + \frac{2}{3}|v|^2 \right) u + \frac{i\gamma}{3} v^2 u^* + \frac{g}{2} u + \frac{g}{2\Omega_g^2} \frac{\partial^2 u}{\partial t^2} \\ \frac{\partial v}{\partial z} &= -i\beta v + \delta \frac{\partial v}{\partial t} - \frac{i\beta_2}{2} \frac{\partial^2 v}{\partial t^2} + i\gamma \left(|v|^2 + \frac{2}{3}|u|^2 \right) v + \frac{i\gamma}{3} u^2 v^* + \frac{g}{2} v + \frac{g}{2\Omega_g^2} \frac{\partial^2 v}{\partial t^2}\end{aligned}, \quad (7.4)$$

where u and v describe the optical fields of the two orthogonal polarization components in the fiber cavity. $2\beta = 2\pi\Delta n/\lambda = 2\pi/L_b$ is the wave number difference between these two polarization modes and $2\delta = 2\beta\lambda/2\pi c$ is the group velocity difference, which are related with the value of beat length L_b and stand for the cavity birefringence. Different from Eq. (7.1), in fiber laser we must consider the gain effect, so we have the additional two terms: g is the gain coefficient of the gain fiber and Ω_g is the effective gain bandwidth. The gain saturation is defined as

$$g = G \exp \left[- \frac{\int (|u|^2 + |v|^2) dt}{E_s} \right], \quad (7.5)$$

where G is the small signal gain coefficient and E_s is the gain saturation energy, which is related to the pumping power in the experiment. The numerical method is also based on the split-step Fourier method used in the previous chapters. We set the initial state as a bright-dark soliton pair: $u = A \cdot \text{sech}(B \cdot t)$ and $v = C \cdot \tanh(D \cdot t)$. Here A and C stand for the bright soliton intensity and dark pulse depth, A is set much bigger than C because the bright soliton peak power is much higher than depth of dark soliton for a black-white vector solitons in anomalous dispersion regime, according to conclusion in [54]. B and D stand for the width of bright soliton and the dark soliton and they have the same value. In simulation we used the actual laser cavity parameters if possible, e.g. the cavity length $L=29.3$ m and the average anomalous cavity dispersion of 1.62 ps/nm/km. In our experiment the wavelength difference of

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light on the two polarization modes is very small. So we have set $L_b = 5000$ m, which stands for a very weak net cavity birefringence. After several thousand roundtrips, the bright pulse component evolves into a stable single soliton pulse and the dark soliton component break up into a stable pulse train, as shown in Fig. 7.6(a). Fig. 7.6(b) is an emission state of the simulation result of the previous bright and dark components. Since the net cavity dispersion is anomalous, the bright pulse along the horizontal axis is stable. It has a FWHM of 3.5 ps. Meanwhile, the dark pulse along the vertical axis has broken up into a pulse train with a repetition rate of 80 GHz. This high repetition rate is already beyond our detection range, so in experiment, this pulse train can be only measured as a noisy CW background. However, from the figure we can clearly see that near $t=0$ ps of the pulse train, there is an obvious gap between the bright pulses. The gap is about 30 ps. As a result, it could be detected as a dark pulse on the oscilloscope with our 33 GHz bandwidth detection system. The dark pulse formation therefore has exactly the same mechanism as that of the ‘dark envelop pulse’ discussed in chapter 6. The numerical result also clearly explains why the observed dark pulses have a broader pulse width in our experiment. We have also calculated the autocorrelation trace of the pulse train along the vertical axis. It shows a similar fluctuated background as shown in Fig. 7.6.

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Figure 7.6. Numerical simulation results of black-white vector solitons. (a) Stable simulation state of fiber laser. (b) Emission state of black-white vector soliton emission state. (c) Zoom in of black-white vector soliton.

Furthermore, if zooming in the central area of the gap, a very weak dark pulse could be detected. Comparing it with the bright pulse polarized along the horizontal axis, as shown in Fig. 7.6(c), the dark and the bright pulses have also exactly the same pulse width. We note that the effect is very stable. We used different laser cavity parameters, e.g. different beat lengths, to repeat the simulations. The same result can be obtained. Besides, the width of formed soliton pair is inversely proportional to the value of beat length. Those properties are quite similar with the black-white vector solitons predicted by D.N. Chiristodulides in [54]. Differently, the fiber ring laser is a dissipative system based on Coupled-GLEs, but not the Coupled-NLSEs which describes the conservative system. However, as mentioned previously, in the case of a fiber laser, because the saturated gain always balances the cavity losses, if the effective gain bandwidth is much broader than the bandwidth of the formed solitons, the dynamics of the solitons in the system is effectively similar to those of the NLSE system. Therefore, it is no surprise that similar features as those predicted by the coupled NLSEs could be observed in the lasers.

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We noticed that S. Wabnitz *et al.* have numerically simulated the propagation stability of black-white soliton pair in either normal or anomalous dispersive SMF [163]. They reported the coupled solitary wave decay is induced by the modulational polarization instability of the dark component in anomalous dispersion regime. Even in normal dispersion regime, the stable propagation of bright-dark soliton pair can only last for distances in the order of one linear beat length. In the optical fiber, modulational polarization instability restrains the stabilization of black-white vector solitons. However, in a fiber ring laser, the gain bandwidth limitation can stabilize the modulation instability and shapes the modulated continuous wave (CW) beam into stable periodic pulse trains [72,121]. Therefore, we anticipate that this effect would have different consequence on the black-white vector solitons formed in fiber ring lasers.

The formation mechanism of the black-white vector solitons in the fiber laser can be interpreted as following: the dark-bright soliton pair can be formed in a birefringent fiber laser cavity under coherent coupling between two polarization modes; however, due to the modulational polarization instability (MPI) and the gain bandwidth limitation, the CW background of the dark soliton will break up into a stable pulse train of high repetition rate; meanwhile, the breaking up of the CW background will not affect the existence of the dark-bright soliton pairs. Therefore, the wide dark pulse we observed experimentally is not the dark component of the dark-bright soliton pair, but its superposition with the high repetition pulse train formed by the cavity induced MI effect. Based on the numerical simulation results, we can conclude that stable dark-bright vector solitons truly exist.

When the birefringence is near zero, from Fig. 7.2 we can see that the pulse width increases rapidly as the decreasing of the birefringence. If the cavity birefringence is too small, the pulse will become too wide to be stable. When the birefringence is large, ρ in Eq. (7.3) will become a big value and the amplitude also becomes high. It requires very high laser intensity for the formation. Therefore, the formation of black-white vector solitons needs a good combination of cavity birefringence and high light intensity in the cavity. The

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experimental observation through our electronics detection system can't directly reflect the true condition in the fiber laser cavity, limited by the detection speed. However, the results of numerical simulation match with experimental results very well and reveal the existence of black-white vector solitons in fiber laser cavity objectively.

7.3 Conclusion

In this chapter, we have focused on the black-white vector solitons in fiber laser cavity. In experiment, we observed a soliton-dark pulse emission state under the coherent coupling between two polarization modes. In numerical simulation, the simulation results revealed that the dark pulse part is similar with the previously discussed dark envelope emission state. Furthermore, on the bottom of dark envelope there is a weak dark pulse just corresponding to the bright soliton on its orthogonal polarization modes. They are coupled together and have the same pulse width, which have the same properties as the black-white vector solitons. To the best of my knowledge, this is the first report about black-white vector solitons in fiber laser.

8

Conclusions and future works

In the thesis, the nonlinear cavity properties of fiber ring lasers have been extensively investigated. Various novel phenomena of the fiber lasers, including the cavity induced modulation instability, GHz range periodic pulse train emission, temporal cavity solitons, as well as the bright soliton-dark pulse pair formation, induced dark solitons in the anomalous dispersion cavity, and bright-dark vector solitons etc., were experimentally revealed and numerically simulated. Some of these phenomena could find their counterparts in the conventional mode locked fiber lasers, some are not. They are unique to the specific nonlinear active cavity of the fiber lasers. In this chapter, the main research results will be summarized and some other interesting research works which due to the limited time of the PhD candidature couldn't be completed will be recommended for the future work.

8.1 Conclusions

Cavity induced modulation instability lasing

The non- $2m\pi$ total phase delay after cavity roundtrip will lead to a non-zero cavity detuning, which is the origin of cavity induced modulation instability (CIMI). CIMI can both occur in normal and anomalous dispersion regime. In fiber laser, the modulation frequency of CIMI can be either altered by tuning the cavity or changing the intracavity light intensity. Due to the gain bandwidth limitation, the CIMI modulated CW beam in fiber laser can be shaped into periodical pulse trains. The repetition rate range can be continuously tuned from hundreds of MHz to tens of GHz. The CIMI lasing makes fiber laser being a potential stable and tunable optical pulse source.

Temporal cavity solitons in fiber laser

At sufficient high light intensity, the CIMI modulated CW beam in fiber laser can be fully shaped into dissipative solitons. To differentiate the previously reported solitons in mode-locked fiber lasers, we name this kind of spontaneously formed solitons as temporal cavity solitons. In anomalous dispersive fiber laser, both the scalar and vector solitons are discussed. The induced vector cavity solitons, formed either under coherent or the incoherent coupling between the two orthogonal polarization components of light in the cavity, have also been experimentally obtained and numerically studied. These theoretical and experimental results suggest that the temporal cavity solitons have no difference with the solitons in mode-locked fiber lasers. Besides, the vector cavity solitons are also observed in normal dispersive fiber laser. The coherent energy exchange is first observed between vector dissipative solitons in normal dispersion regime.

Cross polarization coupling of cavity solitons

Due to the incoherent coupling between the two orthogonal polarization modes of a fiber cavity, a bright soliton pulse always induces a dark pulse on the CW background polarized along the orthogonal polarization. Under our

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experimental accessible conditions, it is found that the formed dark pulses always have a broader pulse width than the bright solitons. The mechanism of the dark pulse formation could be understood based on the polarization domain formation. On the other hand, under the coherent polarization coupling, a bright soliton polarized along one polarization eigenmode of the cavity could either induce the formation of a weak dark soliton or a bright soliton on the bottom of a broad dark pulse in the orthogonal polarization direction in a net anomalous dispersion cavity fiber laser. In particular, the induced solitons are stable in the cavity. Numerical simulations based on the coupled complex GLEs have well confirmed the experimental observations. The numerical simulations have further revealed that whether a dark or bright soliton is induced depends on if there is a phase jump in the broad dark pulse.

Dark envelop emission and black-white vector solitons

When a high frequency modulation and a dark-soliton-like dark pulse coexist in a fiber laser, they will be shaped into a high repetition rate dark pulse train and a dark soliton, respectively. Due to the superposition of their optical fields, the narrow dark soliton will form a wide dark envelope embedded in the dark pulse train background. The formed dark envelope makes the narrow dark soliton easily detectable. This finding brings a new understanding on the dark soliton formation in fiber lasers, especially on their experimental detection. Based on this understanding, the formation mechanism of the experimentally observed bright-dark vector solitons also becomes clear. Under coherent coupling and sufficient high light intensity, the black-white vector solitons can be formed. However, due to the cross-phase coupling, the CW background of dark soliton is always break up into high repetition pulse trains and the dark soliton is embedded inside. This makes the experiment results of a black-white vector solitons as a narrow bright soliton and a much wider dark pulse. The experiment and numerical simulation results confirm the existence of black-white vector solitons in our fiber laser, which to the best of our knowledge, it's the first experimental report on black-white vector solitons.

8.2 Recommendations for future works

8.2.1 Stable tunable CIMI fiber lasers

In Chapter 3 I have discussed the CIMI fiber laser both theoretically and experimentally. In experiment, we found that stable modulation frequency range of CIMI is from hundreds of MHz to tens of GHz. This frequency range is meaningful because it can be directly resolved by the commercial photodetectors. And the tens of GHz modulation frequency is in the operation range of commercial optical fiber communication systems. Besides, the modulation frequency of CIMI can be continuously tuned by changing the total phase delay, realized by either tuning the cavity or changing the pumping power. These properties make the stable tunable CIMI fiber laser a promising application. The key point is to stabilize the repetition rate of pulse train emission. There are two potential stabilization method:

1. Repetition rate lock loop - PZT stabilization: Inspired by the phase lock loop operation of mode locked fiber laser [164], we can use the signal of photodetector and feedback to the PZT which can slightly change the fiber cavity length, to lock the repetition rate to certain value.
2. Photonics crystal fiber (PCF) assisted stabilization: We notice that Pang *et al.* have reported stable GHz-rate mode-locking of fiber lasers by optoacoustic effects in photonic crystal fibers [165–167]. This also can be used in the CIMI fiber laser. However, the repetition rate only can be locked at certain values determined by the optoacoustic interactions.

8.2.2 High power pulsed fiber laser

With the development of modern industry, there is huge demand for the fine manufacture devices. Fiber lasers, owning the advantage of compact size, stable operation, high power and excellent beam quality, attract the focus in industry. The high power pulsed fiber laser can provide pulse with high single pulse energy, especially good for the ‘cold processing’. In my research, once we have achieved the stable CIMI fiber laser, we can use it as a seed and amplify the

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pulses to achieve high pulse energy output. There are two potential method of external pulse amplification methods [45]:

1. Parabolic pulse amplification: It has been shown in chapter 2 that by amplifying short pulse in the gain fiber with normal dispersion, it's possible to form self-similar pulse. For the self-similar pulse, it can be amplified without splitting. More important, the self-similar pulse is linearly chirped and can be compressed to achieve high power ultrashort pulse using appropriate dispersive elements such as a grating or prism.
2. Fiber based chirp pulse amplification (CPA): In CPA, the short pulses are first largely stretched in time using a highly dispersive element. Then these stretched pulses can be amplified to high energy but low peak power, avoiding excess SPM induced pulse splitting. At last, the amplified-stretched pulse can be compressed using the opposite dispersion element.

8.2.3 Study of dark soliton transmission

Compared with bright pulse/soliton, dark pulse/soliton process many advantages, such as more robust to noise and low formation power threshold, this makes the dark pulse promising for the application of optical fiber communication system. But there are still many fundamental studies to be done. Now we have already had a stable high repetition rate dark soliton source [39], we can achieve different repetition rate and dark pulse width by changing the net cavity average dispersion and intracavity light intensity. Then we can input the dark soliton train into a long normal dispersion fiber, the fiber dispersion also can be altered by applying different types of fiber, even fiber combinations. Through these experiments, we expect to study the dark pulse/soliton transmission under different conditions, to get a comprehensive understanding of fundamental physics of dark solitons.

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