Soundfield control using loudspeaker array

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SOUNDFIELD CONTROL USING LOUDSPEAKER ARRAY

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**SCHOOL OF ELECTRICAL AND ELECTRONIC ENGINEERING
2018**

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Abstract

Control of loudspeaker arrays have been studied in the past few years for a wide variety of applications. One of such application is to control a pre-defined listening environment to achieve audible and inaudible zones so as to gain a personalized and immersive sound experience. One existing feasible approach to achieve soundfield control is to compute time- or frequency-domain coefficients for each loudspeaker such that the generated sound signals at the pre-defined listening locations can achieve the desired sound energy level for the superimposed sound signals. Several methodologies have recently been proposed to achieve a desired soundfield within a listening zone of interest. These methods include maximizing the energy ratio between energies of audible and inaudible zones, maximizing energy difference between two zones and maximizing the energy ratio while maintaining a desired sound energy level in the zone of interest. Although the above methods provide directional sound to the zone of interest, they cannot guarantee precise control of the soundfield within the audible/inaudible regions. As a result, undesired fluctuations of the soundfield or even sound distortion may occur. To address the issue, least squares (LS) methods along with regularisation techniques such as ordinary cross validation (OCV), generalized cross validation (GCV), the L-curve and the truncation method have also been proposed to achieve a desired sound energy level in a pre-defined zone of interest. Although these LS methods can improve the performance of soundfield control system, direct application of these methods will however introduce regularisation error (RE).

In this thesis an iterative LS based algorithm is proposed. This algorithm utilizes the round-off error (ROE) as a regularisation parameter for the ill-conditioned system matrix over the zone of interest. Unlike existing LS methods, the proposed approach estimates the system matrix inverse iteratively and identifies the best estimate of the matrix inverse. In addition, the proposed algorithm does not accumulate significant ROE when the system matrix is ill-conditioned. Instead, the accumulated ROE is taken as a regularisation parameter which is evaluated iteratively to achieve a good trade-off between matrix inverse accuracy and the amount of RE. To achieve the above, a matrix sparseness criterion is proposed to evaluate the accuracy of the inverse estimate. This measure is subsequently used to termi-

nate the iteration process. Furthermore, incorporating this criterion to the proposed method of estimating the loudspeaker coefficients, one can take into account the trade-off between the estimation error (during the convergence of the matrix inverse estimate) and the ROE (caused by mathematical computation when the system is ill-conditioned). The proposed algorithm is further extended to the scenario when the misplacement error (MPE) is present in the sound control system. It is found that the proposed algorithm can compensate and minimize the effect of MPE on the generated soundfield by exploiting the characteristics of the MPE when computing the regularisation parameter. Simulation results show that the proposed algorithm can outperform other LS-based regularisation methods in terms of normalized output error (NOE) in the presence or absence of MPE.

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List of Abbreviations

ADC	Analog-Digital Converter
BSI	Blind Source Identification
GCV	Generalized Cross Validation
LNN	Linear Neural Network
LS	Least Squares
MLN	Multi-layer Network
MPE	Mis-Placement Error
MSE	Mean Square Error
NCZ	Near Common-Zeros
NOE	Normalized Output Error
OD	Over-determined
OE	Output Error
OCV	Ordinary Cross Validation
RE	Regularisation Error
RIR	Room Impulse Response
ROE	Round-Off Error
SVD	Singular Value Decomposition
WFS	Wave Field Synthesis
UD	Under-determined

Chapter 1

Introduction

1.1 Background of Soundfield Control

There has been considerable research interest in active soundfield control [1] using loudspeaker array and modern signal processing techniques. These methods have been studied recently since they can be employed to realize controlled personal listening zones using loudspeaker arrays. One application of the loudspeaker array is to create immersive acoustic environment where listeners perceive a realistic but virtual three-dimensional soundfield within a spatial region [2]. The existing problem of soundfield control using loudspeaker array has been investigated to address issues related to spatial loudspeaker placement and calculation of the weight coefficients for accurate soundfield generation. The ambisonics method based on circular harmonics representation of soundfield and wave field synthesis (WFS) method based on Kirchhoff-Helmholtz integral have been proposed to design a user-defined soundfield [3]. These methods require a large number of loudspeakers and space to reproduce a soundfield which is as close as that compared to the actual sound experience.

Another application of the loudspeaker array is the control of the soundfield so as to provide, for example, audible and inaudible zones for listeners in an environment [4]. In this application, the audible zone refers to the listening zone with high sound energy whereas

the inaudible zone has low sound energy. Such application requires an accurate control of soundfield in the audible zone, and the sound energy outside this zone is not considered [5]. Therefore, this approach requires fewer loudspeakers and less complicated spatial location requirements and is therefore practically feasible. The research in this thesis focuses on soundfield control using a linear loudspeaker array in the frequency domain.

1.2 Existing Algorithms for Soundfield Control

One approach to achieve the control of the soundfield is to compute time- or frequency-domain coefficients for each loudspeaker. In frequency-domain approaches, a scalar weighting is applied while a vector of coefficients is applied in time-domain approaches for each loudspeaker. Several algorithms have been proposed to achieve desired soundfield within a required listening environment. Choi and Kim [6] proposed a method to maximize the energy ratio between the sound energy for the audible zone and that for the inaudible zone. Their method maximizes the acoustic energy in audible zone with a given input sound signal using the loudspeaker array. Although this ratio maximization method can effectively control the soundfield, it cannot guarantee the desired amount of energy in the audible and inaudible zones. By modifying Choi and Kim's method, Shin *et al.* [7] proposed to maximize the difference between energies of the two acoustic zones. They exploited a cost function that maximizes the energy difference between sound energies of the two acoustic zones, instead of their ratio. It is noted that with a given input source, maximization of the energy difference can be achieved by maximizing the acoustic energy in audible region while minimizing the acoustic energy in the inaudible region. It was reported that maximizing such energy difference improves the efficiency of acoustic radiation in the audible region while maintaining the sound energy difference between audible and inaudible regions [7]. Although this approach achieves higher soundfield control performance than the method of energy ratio maximization presented in [6], it cannot guarantee that desirable energy is

achieved in the audible zone when energy contrast ratio is maximized. In order to address the above disadvantages, a multi-objective optimization that maximizes the directivity to within a desired audible energy level has been proposed [4]. This optimization is achieved by finding the best value of a constrained directivity parameter. This constrained directivity parameter controls the energy ratio between the audible region and inaudible region while maintaining the acoustic energy in the audible region at a desired level. Although the above methods provide, to some extent, directional sound to a pre-defined audible region, they are unable to control the soundfield *within* the audible/inaudible regions. As a result, undesired fluctuations of the soundfield may occur [8].

Least squares (LS) optimization methods have also been proposed to produce a desired soundfield [1,9,10]. Given a set of acoustic transfer functions between the loudspeakers and some pre-defined locations which are of interest to listeners in an environment, the method of LS computes the inverse of the system matrix that is constructed from these transfer functions. The desired soundfield is generated at specific locations by computing weights of the loudspeaker array using a transfer function matrix. It is therefore important to note that the performance of such technique depends largely on the conditioning of the system matrix [11]. In addition, soundfield control can be achieved by generating a desired soundfield within the zone of interest with predefined listening points using a well-controlled loudspeaker array [1] [9]. The loudspeaker array is controlled by weights such that the array is equivalent to a virtual sound source.

In practice, direct application of the LS method will not always achieve the desired sound energy distribution for audible and inaudible zones. This is due to the inaccurate inversion of an ill-conditioned matrix [1,6,12]. The performance of the LS method degrades even further for matrices with large dimensions [13]. In addition, the presence of various kinds of errors, such as misplacement of loudspeakers, and inaccuracies in estimation of the system matrix error, makes the ill-conditioning problem even worse. To address these challenges, a variety of techniques that employ regularisation have been proposed in [1, 9,

14]. These regularisation techniques can broadly be classified into Tikhonov regularisation and the truncation method [11]. The performance of these methods are largely dependent on the appropriate choice of the regularisation parameter. Popular algorithms for estimating such parameter include the ordinary cross validation (OCV), generalized cross validation (GCV), and the L-curve methods [12, 13].

Existing regularisation techniques are generally used to control the computed loudspeaker weights to suppress the misplacement error (MPE) and/or the round-off error (ROE) [1, 9, 14, 15]. The MPE is caused by the erroneous location estimates of the listening points and the loudspeakers while the ROE is caused by the inaccurate estimation of an ill-conditioned system matrix inverse. In addition, these regularisation techniques introduce regularisation error (RE) compared to conventional LS method when the system is moderately well-conditioned.

Research in this thesis shows that while MPE and ROE can be suppressed by existing regularisation methods, any ill-conditioning of the system matrix can also amplify the effect of MPE in the generated output soundfield. Therefore, an improved regularisation-based LS optimization algorithm is proposed to achieve soundfield control over a given area to overcome the ill-conditioning issue of system matrix. The proposed algorithm utilizes the fact that ROE accumulates during iteration process and that this dynamic changing accumulation is modeled as a regularisation parameter. A matrix sparseness measure is also proposed to monitor the amount of regularisation resulted from ROE such that a more accurate matrix inverse estimate is achieved. Therefore, as opposed to existing techniques that compute the inverse of the ill-conditioned system matrix directly, the proposed method prevents a large amount of ROE by estimating the matrix inverse iteratively via the proposed matrix sparseness measure. As a result, the trade-off between the reduction of estimation error (during the convergence of the matrix inverse estimate) and accumulation of ROE (caused by ill-conditioned inverse matrix) is taken into account. Furthermore, the MPE is taken into consideration as well and consequently the proposed regularisation method can reduce the

amplification effect of MPE in the generated soundfield. As a result, the soundfield control system is more robust against MPE [16].

1.3 Organization of the Thesis

This thesis presents a new adaptive LS-based soundfield control algorithm for a linear loudspeaker array by modeling the dynamically accumulated ROE as regularisation parameter when the system is ill-conditioned. The organization of this thesis is as follows:

The thesis begins, in Chapter 2, by formulating the problem of the soundfield control using the loudspeaker array with knowing that the existing methodologies control the soundfield by computing a set of coefficients for loudspeakers. The performance of these methodologies are however dependent on the conditioning of the system, especially when the system is ill-conditioned [6]. To solve this ill-conditioning problem, Chapter 3 reviews several LS-based inverse techniques which can reduce the effect of the generated soundfield due to the ill-conditioned system. It is important to note that these LS based techniques such as the OCV, GCV, L-curve and truncation methods suffer from RE when the conditioning of the system matrix is improved. To better understand the effect of ill-conditioning on the generated acoustic sound level, the round-off error (ROE) is discussed in detail in the first part of Chapter 4. It is found that existing LS methods can reduce the ROE caused by ill-conditioning at the cost of introducing RE in the second part of Chapter 4. On top of the ROE discussion and analysis, this thesis also discusses another error source in soundfield control system — MPE. The MPE problem as discussed in Chapter 5 is found always present in a practical soundfield system and that its effect on generated soundfield has been shown in the second part of Chapter 5. To gain a better insight of the effect of MPE on the generated soundfield, it is shown that the normalized output error of the soundfield is highly related to the conditioning of the system matrix in the third part of Chapter 5. To overcome the effects of the ROE and MPE on the generated soundfield, an improved LS-

based algorithm with iteratively estimating the system matrix inverse for computing the loudspeaker array weight coefficients over the zone of interest has been proposed in Chapter 6. The proposed algorithm makes use of a linear network that was originally proposed for the diagonalization of the correlation matrix constructed using a pair of stereophonic input vectors [17] and the estimation of the inverse of a joint-input correlation matrix of input vectors [18] for stereophonic acoustic echo cancellation application. The single-layer linear network is then generalized to a multi-layer network for a better accuracy of matrix inverse estimation. Instead of eliminating accumulating ROE, the proposed algorithm models the dynamic ROE accumulation as a regularisation parameter which effectively improves the conditioning of the system matrix, which results in better accuracy of the generated soundfield. In addition, a new criterion called *matrix sparseness* is introduced to terminate the iteration at the optimal point where the trade-off between accumulating of large amount ROE and introducing of RE is achieved. It is also found that the proposed algorithm can reduce the effect of MPE by taking the characteristics of the MPE as the regularisation parameter when the MPE is present in the soundfield control system. As will be shown in Chapter 7, both simulation and experimental results show that the proposed algorithm outperforms other LS based algorithms. Finally Chapter 8 concludes this thesis and discusses potential future research directions.

1.4 Statement of the Originality and Main Contributions

This thesis details contributions made by the candidate in the development of an improved LS based optimization algorithm for matrix inverse estimation in Chapter 6. Chapter 6 describes the proposed algorithm which employs a linear multi-layer network incorporating the use of matrix sparseness. Control of the soundfield is achieved by using the proposed adaptive matrix inverse estimator. The starting point of the algorithmic development is the estimation of the inverse of a joint-input correlation matrix of input vectors [18]

which avoids the use of regularisation and therefore does not introduce any RE. As will be described in the first part of Chapter 6, it is found that direct application of the adaptive network suffers from the problem of slow convergence. The single-layer network is generalized to a multi-layer network and the matrix sparseness measurement is incorporated to avoid divergence of the estimation process. Unlike existing LS methods, the proposed approach models the accumulated ROE as regularisation parameter iteratively and utilizes a matrix sparseness criterion to stop the iteration process. As a result, the proposed algorithm neither accumulates significant ROE when the system matrix is ill-conditioned nor introduces large RE when system matrix is well-conditioned. Therefore, as opposed to existing techniques that compute the inverse of a system matrix directly, the matrix inverse is estimated iteratively and the dynamic changing regularisation parameter is checked accordingly. This, in turn, allows the proposed algorithm to evaluate the quality of the matrix inverse estimate at each iteration. The matrix sparseness criterion is proposed to evaluate the accuracy of the inverse estimate and is subsequently used to generate the desired sound-field over the pre-define zone of interest. Furthermore, by incorporating this criterion to the proposed method, one can take into account the trade-off between the reduction of estimation error (during the convergence of the matrix inverse estimate) and the ROE (caused by mathematical computation when the system matrix is ill-conditioned). Performance of the proposed method is compared with that of the conventional LS method and regularisation based LS methods. Simulation and experimental results show that the proposed method outperforms existing methods especially when the condition number of the system matrix is high.

Chapter 2

Problem Formulation

In this chapter, the soundfield control system is introduced and presented.

2.1 Soundfield Control System Mathematic Modeling

To achieve the control of a desired soundfield, a set of M loudspeakers is employed as shown in Fig. 2.1. Similar to that of [9], the objective is to control the soundfield generated by the loudspeaker array for a set of user-defined listening points within a listening zone. These zones are stated in this thesis as audible listening zones and inaudible listening zones. The audible listening zones refer to user-defined listening zones where the sound energy is high while in contrast, the inaudible listening zones refer to the user-defined listening zones having low or negligible sound energy. Both audible listening zone and inaudible listening zone constitute the zone of interest corresponding to the listening environment that will be discussed and analyzed throughout this thesis.

To formulate the problem, it is first assumed that input signal $X(f)$, for each frequency f , is multiplied with coefficients $w_j(f)$, $j = 1, \dots, M$, for the loudspeaker array. The

Figure 2.1: Schematic of a soundfield control system.

output signal $Y_j(f)$ corresponding to the j th loudspeaker can therefore be expressed as

$$Y_j(f) = X(f)w_j(f), \quad (2.1)$$

where $w_j(f)$ is the weight coefficient applied to the j th loudspeaker. Defining N as number of listening locations for which the control of sound field is desired, the measured signal at the i th listening location is given by

$$\hat{Y}_i(f) = \sum_{j=1}^M X(f)w_j(f)G_{ij}(f), \quad i = 1, \dots, N, \quad (2.2)$$

where

$$G_{ij}(f) = \frac{1}{4\pi} \frac{e^{-j2\pi f \|\mathbf{s}_j - \mathbf{o}_i\|_2 / c}}{\|\mathbf{s}_j - \mathbf{o}_i\|_2} \quad (2.3)$$

is the Green's function [9], $\mathbf{s}_j$ and $\mathbf{o}_i$ are the position vectors containing (x, y, z) positions of the j th loudspeaker and the i th listening point, respectively, c is the speed of sound and $\|\cdot\|_2$ denotes the l_2 -norm of a vector. For clarity of presentation, the frequency notation f is removed from the remainder of this chapter. In addition, since $X(f)$ for each frequency

can be represented by a complex scalar α , (2.2) can be expressed as

$$\hat{Y}_i = \sum_{j=1}^M \alpha w_j G_{ij}, \quad i = 1, \dots, N. \quad (2.4)$$

For mathematical compactness, the coefficients w_j across all channels are concatenated giving $\mathbf{w} = [w_1, w_2, \dots, w_M]^T$. Noting that the magnitude response for the acoustic signal X for *each* frequency f is a scalar, the input matrix $\mathbf{X}$ is formulated as a diagonal matrix with diagonal entries corresponding to the input channels respectively. As a result, the vector of acoustic signal $\hat{\mathbf{Y}} = [\hat{Y}_1, \hat{Y}_2, \dots, \hat{Y}_N]^T$ in the listening zone is given by

$$\hat{\mathbf{Y}} = \mathbf{G}\mathbf{X}_M\mathbf{w}, \quad (2.5)$$

where

$$\mathbf{G} = \begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1M} \\ G_{21} & G_{22} & \cdots & G_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ G_{N1} & G_{N2} & \cdots & G_{NM} \end{bmatrix} \quad (2.6)$$

is an $N \times M$ matrix of acoustic transfer function between the loudspeakers and listening points in the free space.

As stated at the beginning of this chapter, the objective of the soundfield control is to achieve a desired soundfield in terms of sound energy distribution across N listening points by estimating $\mathbf{w}$ for each frequency f . This desired soundfield in the zone of interest can be expressed as [1]

$$\mathbf{Y} = \mathbf{X}_N\mathbf{D} = X_N\mathbf{D}, \quad (2.7)$$

where $\mathbf{X}_N = X\mathbf{I}_{N \times N}$ and $\mathbf{D} = [D_1, D_2, \dots, D_N]^T$ is the user-defined sound energy distribution vector. This vector refers to the contrast ratio between the sound energy for each listening point in the zone of interest and that for original input signal. We therefore have

$\alpha \mathbf{ID} = \mathbf{G} \alpha \mathbf{I} \mathbf{w}$. Therefore $\mathbf{D}$ describes the desired spatial filtering function of the loudspeaker array [1].

Soundfield control is therefore achieved by minimizing the cost function $J = E\{e^2\}$ where

$$e = \|\hat{\mathbf{Y}} - \mathbf{Y}\| = \|\mathbf{G} \mathbf{X}_M \mathbf{w} - \mathbf{X}_N \mathbf{D}\| \quad (2.8)$$

is the output error. Minimizing J for each frequency leads to different forms of Wiener solution given by [19]

$$\mathbf{w}_d^{\text{opt}} = \mathbf{A}^{-1} \mathbf{D}, \quad (2.9a)$$

$$\mathbf{w}_o^{\text{opt}} = \mathbf{A}^{-1} \mathbf{G}^H \mathbf{D}, \quad (2.9b)$$

$$\mathbf{w}_u^{\text{opt}} = \mathbf{G}^H \mathbf{A}^{-1} \mathbf{D}, \quad (2.9c)$$

where $\mathbf{w}_d^{\text{opt}}$, $\mathbf{w}_o^{\text{opt}}$, $\mathbf{w}_u^{\text{opt}}$ are the optimal ideal weights for the well-determined, over-determined and under-determined case, respectively, and the system matrix $\mathbf{A}$ is defined as

$$\mathbf{A} = \begin{cases} \mathbf{G}, & \text{well-determined case } (M = N); \\ \mathbf{G}^H \mathbf{G}, & \text{over-determined case } (M < N); \\ \mathbf{G} \mathbf{G}^H, & \text{under-determined case } (M > N), \end{cases} \quad (2.10)$$

where the system matrix is defined as $\mathbf{G}$ for the well-determined ($M = N$) case, $\mathbf{G}^H \mathbf{G}$ for the over-determined ($M < N$) case, and $\mathbf{G} \mathbf{G}^H$ for the under-determined ($M > N$) case, respectively.

As can be seen from (2.9a)-(2.9c), $\mathbf{w}^{\text{opt}}$ is related to the transfer function $\mathbf{G}$ and the desired sound distribution $\mathbf{D}$. Since the desired ratio vector $\mathbf{D}$ is predefined and controlled by users, conditioning of the transfer function $\mathbf{G}$ dominates the accuracy of the optimal solution $\mathbf{w}^{\text{opt}}$. In practice, $\mathbf{w}^{\text{opt}}$ is highly dependent on the conditioning of the system matrix in (2.9a)-(2.9c). This conditioning is often quantified using the l_2 -norm condition

number which can be expressed, for a matrix $\mathbf{A}$, by [20]

$$\begin{aligned}\chi_2(\mathbf{A}) &= \|\mathbf{A}\|_2 \|\mathbf{A}^{-1}\|_2 \\ &= \frac{\sigma_{\max}}{\sigma_{\min}},\end{aligned}\tag{2.11}$$

where $\sigma_{\max}$ and $\sigma_{\min}$ are the maximum and minimum eigenvalues of $\mathbf{A}$.

Therefore, when the system matrix $\mathbf{A}$ is ill-conditioned, $\chi_2(\mathbf{A})$ is high. As a result, the estimate of $\mathbf{w}^{\text{opt}}$ may not be reliable since, as will be explained in Chapter 4, ROE will be introduced during matrix inversion.

2.2 Chapter Summary

This chapter presents a signal model for a soundfield control system. Taking the soundfield control system for a loudspeaker array as a black box, the input is the original sound signal while the output is the desired sound energy level at the zone of interest. The key to an effective soundfield control system is the loudspeaker weight coefficients which can be computed by inverting the system matrix. However, computation of the loudspeaker weight coefficients is usually unreliable and inaccurate when the system matrix is ill-conditioned. In the following chapter, existing methods for solving the ill-conditioned problem are reviewed and discussed. These algorithms serve as a baseline for the proposed algorithm in performance comparison.

Chapter 3

Review of Existing Methods

To address the above ill-posed problem, regularisation methods have been proposed. These methods include two well-known techniques - Tikhonov regularisation method and the truncation method. These techniques are described and reviewed in the following sections.

3.1 The Tikhonov Regularisation Method

The Tikhonov regularisation method [14] [21] was proposed to improve the conditioning of the system matrix by adding a diagonal matrix $\lambda \mathbf{I}$ to the system matrix, where $\mathbf{I}$ is an identity matrix and λ is the regularisation parameter. Its effect on the conditioning of the system matrix $\mathbf{A}$ is illustrated by defining

$$\mathbf{A} = \begin{cases} \mathbf{G}, & \text{well-determined case;} \\ \mathbf{G}^H \mathbf{G}, & \text{over-determined case;} \\ \mathbf{G} \mathbf{G}^H, & \text{under-determined case,} \end{cases} \quad (3.1)$$

such that the regularized system matrix is given by

$$\tilde{\mathbf{A}} = \mathbf{A} + \lambda \mathbf{I}. \quad (3.2)$$

The estimated weights can be expressed, after regularisation, as

$$\hat{\mathbf{w}}_d = (\mathbf{A} + \lambda \mathbf{I})^{-1} \mathbf{D}, \quad (3.3a)$$

$$\hat{\mathbf{w}}_o = (\mathbf{A} + \lambda \mathbf{I})^{-1} \mathbf{G}^H \mathbf{D}, \quad (3.3b)$$

$$\hat{\mathbf{w}}_u = \mathbf{G}^H (\mathbf{A} + \lambda \mathbf{I})^{-1} \mathbf{D}. \quad (3.3c)$$

Applying singular value decomposition (SVD), we have $\mathbf{A} = \mathbf{U}_A \mathbf{\Lambda}_A \mathbf{V}_A^H$, where $\mathbf{U}_A$, $\mathbf{V}_A$ are unitary matrices and $\mathbf{\Lambda}_A = \text{diag}\{\sigma_1, \sigma_2, \dots, \sigma_N\}$ is a diagonal matrix with $\sigma_i > \sigma_{i+1}$. It can be shown that the eigenvalue σ_i of $\mathbf{A}$ is changed to $\tilde{\sigma}_i = (\sigma_i^2 + \lambda)/\sigma_i$ after regularisation [22]. Since $\sigma_i > 0$ and $\lambda > 0$, we have $\tilde{\sigma}_i > \sigma_i$ and therefore, the condition number after regularisation is given by

$$\chi_2(\tilde{\mathbf{A}}) = \frac{\tilde{\sigma}_{\max}}{\tilde{\sigma}_{\min}} = \frac{\sigma_{\max}}{\sigma_{\min}} \left(\frac{1 + \lambda/\sigma_{\max}^2}{1 + \lambda/\sigma_{\min}^2} \right) < \frac{\sigma_{\max}}{\sigma_{\min}} = \chi_2(\mathbf{A}). \quad (3.4)$$

Therefore the effective eigenvalues of $\mathbf{A}^{-1}$ and $\tilde{\mathbf{A}}^{-1}$ will be $1/\sigma_i$ and $1/\tilde{\sigma}_i = \sigma_i/(\sigma_i^2 + \lambda)$, respectively. As a result, the conditioning of $\mathbf{A}$ is improved. To find the best value of λ , the truncation method and the OCV, GCV, L -curve methods have been proposed. These methods will be described in next section.

3.2 The Truncation Method

Another form of regularisation is to truncate the SVD expansion of the solution by disregarding SVD components corresponding to the small singular values [13] [23]. Taking the well-determined case as example, this is achieved by defining a $N \times N$ filtration matrix [23]

$$\mathbf{T} = \text{diag}\{t_1, \dots, t_N\}, \quad (3.5)$$

where, for each listening point $i = 1, \dots, N$, $t_i = 1$ for $\sigma_i \geq \lambda$, and $t_i = 0$ for $\sigma_i < \lambda$. Applying $\mathbf{T}$ to the SVD of $\mathbf{A}^{-1} = \mathbf{V}_A \mathbf{\Lambda}_A^{-1} \mathbf{U}_A^H$, we obtain

$$\check{\mathbf{A}}^{-1} = \mathbf{V}_A \mathbf{\Lambda}_A^{-1} \mathbf{T} \mathbf{U}_A^H. \quad (3.6)$$

The above process eliminates the small eigenvalues from $\mathbf{A}$ and as a result, the conditioning of $\mathbf{A}$ is improved. Therefore, the estimated weights are given by

$$\hat{\mathbf{w}}_d = \check{\mathbf{A}}^{-1} \mathbf{D}, \quad (3.7a)$$

$$\hat{\mathbf{w}}_o = \check{\mathbf{A}}^{-1} \mathbf{G}^H \mathbf{D}, \quad (3.7b)$$

$$\hat{\mathbf{w}}_u = \mathbf{G}^H \check{\mathbf{A}}^{-1} \mathbf{D}. \quad (3.7c)$$

It has been shown in [24] that the accuracy of estimating the matrix inverse using the truncation method is similar to that of the Tikhonov method. However, the performance of the truncation method is highly dependent on the selection of the truncation parameter λ since λ determines how many eigenvalues in $\mathbf{V}_A$ will be ignored.

3.3 Determination of Regularisation Parameter λ

It is foreseeable that the value of λ will determine the performance of the regularisation methods. The OCV, GCV and L-curve methods [12] [13] have been proposed to determine the optimal values for λ .

3.3.1 The OCV Method

The OCV method omits the q th measured data and uses the remaining $N - 1$ data to predict the omitted data, given N measurement data and $q \in [1, N]$ [13]. This method finds the

best regularisation parameter λ by minimizing the cost function

$$J_O(\lambda, q) = \sum_{\substack{i=1 \\ i \neq q}}^N (D_i - \mathbf{g}_i \hat{\mathbf{w}}(\lambda)) + \lambda \hat{\mathbf{w}}^H(\lambda) \hat{\mathbf{w}}(\lambda), \quad (3.8)$$

where $\mathbf{g}_i = [G_{i1} \cdots G_{iM}]$. Therefore, the cross-validation function

$$\Gamma_O(\lambda) = \frac{1}{N} \left\| \frac{(\mathbf{I} - \mathbf{C}(\lambda)) \mathbf{G} \hat{\mathbf{w}}(\lambda)}{\mathbf{I} - \text{diag}\{\mathbf{C}(\lambda)\}} \right\|_2^2 \quad (3.9)$$

is obtained by minimizing (3.8) for $q = 1, \dots, N$ [13]. The variable $\mathbf{C}(\lambda) = \mathbf{G}^H(\mathbf{G}\mathbf{G}^H + \lambda\mathbf{I})^{-1}\mathbf{G}$ is defined for the under-determined case, $\mathbf{C}(\lambda) = \mathbf{G}(\mathbf{G}^H\mathbf{G} + \lambda\mathbf{I})^{-1}\mathbf{G}^H$ for the over-determined case and $\mathbf{C}(\lambda) = \mathbf{G}(\mathbf{G} + \lambda\mathbf{I})^{-1}$ for the well-determined case. Therefore, (3.9) is the solution for minimizing the cost function (3.8). The OCV method is a search-based approach aiming to determine the value of λ that minimizes both (3.8) and (3.9). It has been shown that the OCV is effective when the condition number of the system matrix is high [21] [25].

In extreme cases where entries of the system matrix $\mathbf{A}$ are zero except for the j th entries ($j = 1, 2, \dots, N$), i.e., $\mathbf{A}$ being a diagonal matrix, it can be shown [13] that (3.9) is simplified to

$$\Gamma_O(\lambda) = \frac{1}{N} \|\tilde{\mathbf{Y}}\|_2^2. \quad (3.10)$$

As a result, the OCV function is independent of λ and hence this method fails to compute the optimal λ in cases where $\mathbf{A}$ is a near-diagonal or diagonal matrix.

3.3.2 The GCV Method

To address the above issue, a modification to the OCV method is made to find a λ that is invariant to any structure of the system matrix [13]. It does so by first defining

$$\mathbf{e}(\lambda) = \mathbf{D} - \mathbf{G}\hat{\mathbf{w}}_u(\lambda) \quad (3.11)$$

and $\mathbf{\Omega}$ as a unitary rotation matrix where its (i, j) th element is given by $e^{j2\pi(ij/N)}/\sqrt{N}$, $i, j = 1, \dots, N$. Decomposing $\mathbf{G}$ as $\mathbf{U}_G \mathbf{\Lambda}_G \mathbf{V}_G^H$, pre-multiplying (3.11) by $\mathbf{\Omega} \mathbf{U}_G^H$ and applying the OCV model to the result, the corresponding cross-validation function is given by [13]

$$\Gamma_G(\lambda) = \frac{(1/N) \|(\mathbf{I} - \mathbf{C}(\lambda))\hat{\mathbf{w}}_u(\lambda)\|_2^2}{[(1/N) \text{tr}(\mathbf{I} - \mathbf{C}(\lambda))]^2}, \quad (3.12)$$

where $\text{tr}(\cdot)$ is the trace operator and $\mathbf{C}(\lambda) = \mathbf{G}^H(\mathbf{G}\mathbf{G}^H + \lambda\mathbf{I})^{-1}\mathbf{G}$. Similar to OCV, an optimal λ is found by minimizing $\Gamma_G(\lambda)$ in (3.12).

3.3.3 The L-curve Method

The L-curve method is another effective method to determine the appropriate value for λ . It has been shown that L-curve is more effective than GCV when the problem is relatively well-conditioned whereas GCV provides a more reasonable amount of regularisation for ill-conditioned problems where the noise levels are relatively low [25].

The L-curve is essentially a plot of $\|\mathbf{D} - \mathbf{G}\hat{\mathbf{w}}_u(\lambda)\|_2$ against $\|\hat{\mathbf{w}}_u(\lambda)\|_2$, where $\hat{\mathbf{w}}_u(\lambda)$ is computed using different λ values, as shown in Fig. 3.1. This curve is approximately L-shape so that it is approximately vertical for small λ and approximately horizontal for large λ with its turning point being the optimal λ . The reason for the L-shaped is due to the fact that when λ is too large, the regularisation error increases and that when λ is too small, $(\mathbf{A} + \lambda\mathbf{I})$ becomes very ill-conditioned and as a result, the output error increases. The resulting curve is approximately L-shaped and the turning point where it first converges

Figure 3.1: An illustrative plot of L-curve for l_2 -norm of error $\|\mathbf{G}\mathbf{w} - \mathbf{D}\|_2$ against the l_2 -norm of loudspeaker weight $\|\mathbf{w}\|_2$.

will correspond to the optimal λ .

Figure. 3.1 shows the l_2 -norm error between the desired sound signal $\mathbf{D}$ and the generated sound signal $\mathbf{G}\hat{\mathbf{w}}_u(\lambda)$ against the l_2 norm of the loudspeaker weight vector $\hat{\mathbf{w}}_u(\lambda)$ for $0 \leq \lambda \leq 0.25$. As can be seen, the resulting curve is approximately L-shaped and the maximum curvature point (L-curve corner point) will correspond to the optimal λ which establishes the trade-off between the error and the norm of the weight vector [12].

In order to determine the turning point, we note that this point has a maximum curvature, which corresponds to the optimal λ . Therefore, the curvature κ can be quantified using [12]

$$\kappa(\lambda) = \frac{\zeta' \phi'' - \zeta'' \phi'}{[(\zeta')^2 + \phi^2]^{3/2}}, \quad (3.13)$$

where $\phi = \log\|\mathbf{G}^H(\mathbf{G}\mathbf{G}^H + \lambda\mathbf{I})^{-1}\mathbf{D}\|_2$, $\zeta = \log\|\lambda(\mathbf{G}\mathbf{G}^H + \lambda\mathbf{I})^{-1}\mathbf{D}\|_2$ while ϕ' and ϕ'' are the first and second derivatives of ϕ with respect to λ . Similarly, ζ' and ζ'' are the first and second derivatives of ζ with respect to λ . Therefore the maximum curvature can be found by evaluating $\kappa(\lambda)$ over the entire L-curve from which λ is obtained. The maximum curvature can be found by (3.13) at the cost of computational complexity due the depreciative

operation involved in the computation.

3.4 Summary

In this chapter, the regularisation methodologies such as the Tikhonov method, the truncation method, OCV, GCV and L-curve have been presented and reviewed. Although regularisation methods improve matrix inversion accuracy by suppressing the ROE, they introduce RE to the solution when λ is over-estimated. On the other hand, ROE is accumulated and becomes significant when λ is under-estimated. In addition, mitigating MPE depends on the accuracy of estimating the value of λ and it cannot avoid the effect of MPE thoroughly. This is a consequence of inverting $\check{\mathbf{A}}$ instead of the true system matrix $\mathbf{A}$. Since RE increases with λ , there is often a trade-off between estimation accuracy of $\mathbf{A}^{-1}$ and reducing the RE. Last but not least, the success of regularisation depends on the appropriate choice of the regularisation parameter [26], which is often very challenging to derive an optimal regularisation parameter when MPE is considerable.

Chapter 4

The ROE and Its Effect on Soundfield Control

Although previous methods aim at improving the estimation accuracy, further simulation results show that they still have a limited accuracy in soundfield control which limits their deployment in actual scenarios. This chapter presents insights into the conventional and LS-based regularisation method by investigating the different types of errors introduced and their effects on soundfield control.

4.1 The Round-off Error (ROE)

In computer-based computation, each arithmetic operation is usually affected by the round-off error (ROE). The cause of this error is the limited precision for storing and representing real numbers [27], [28], [29]. To illustrate the ROE, the variable a_{ij} is first defined as the (i, j) th element of $\mathbf{A}$. A stored version of $\hat{a}_{ij}$ can then be represented as

$$\hat{a}_{ij} = a_{ij}(1 + \epsilon_{ij}), \quad |\epsilon_{ij}| \leq u, \quad (4.1)$$

where ϵ_{ij} is a round error relative to a_{ij} and u is the unit round-off corresponding to the largest ϵ_{ij} that may occur during a round operation [30]. The value of u is therefore defined as half of the gap between 1 and its next largest floating number in a floating-number computing system. It has been shown that in a system with a mantissa of s decimal digits, the value of u will be $0.5 \times 10^{1-s}$ [30]. This unit round-off determines the precision of the system and its value is system dependent. Defining $\mathbf{P} = \mathbf{X}\mathbf{Y}$ for ease of notation, the ROE associated with $\mathbf{P}$ is therefore given by [30, 31]

$$\|\mathbf{P} - \hat{\mathbf{P}}\|_F \leq 1.01Lu\|\mathbf{X}\|_F\|\mathbf{Y}\|_F, \quad (4.2)$$

where $\|\cdot\|_F$ represents the operation of Frobenius norm and L is the dimension of squared matrices $\mathbf{X}, \mathbf{Y}, \mathbf{P}$. An iterative process for the computation of $\mathbf{P}(n)$ is given by [32]

$$\mathbf{P}(n) = (1 + \epsilon_n)[\mathbf{P}(n-1) + \Delta\mathbf{P}(n-1)], \quad (4.3)$$

where ϵ_n is an independent round error associated with iteration n and $\Delta\mathbf{P}(n-1)$ is the update term. Therefore, employing (4.1) and (4.2), we define the iterative process for $p_{ij}(n)$ as

$$\begin{cases} \hat{p}_{ij}(n) = \hat{p}_{ij}(n-1) + \Delta(n-1) + \epsilon_n \hat{p}_{ij}(n) & \text{for } n > 0; \\ \hat{p}_{ij}(0) = p_{ij}(0) + \epsilon_0 p_{ij}(0) & \text{for } n = 0, \end{cases} \quad (4.4)$$

where ϵ_n is the relative round error at the iteration n for p_{ij} . Employing (4.2), (4.4) can be rewritten as [33]

$$\hat{p}_{ij}(n) = p_{ij}(n-1) + \sum_{m=1}^{n-1} \epsilon_m p_{ij}(m). \quad (4.5)$$

It can be seen from (4.5) that ROE accumulates with iteration n and associates with the value in previous iteration for a recursive computation process. In addition, it has been

shown that the ROE associated with $a = \mathbf{x}^T \mathbf{y}$ is given by [30] [31]

$$|a - \hat{a}| \leq 1.01Lu|\mathbf{x}|^T|\mathbf{y}|, \quad (4.6)$$

where L is the length of vectors $\mathbf{x}$ and $\mathbf{y}$. As a result, for a matrix $\mathbf{A} = \mathbf{X}\mathbf{Y}$, the ROE bound is

$$\|\mathbf{A} - \hat{\mathbf{A}}\|_F \leq \gamma \|\mathbf{X}\|_F \|\mathbf{Y}\|_F, \quad (4.7)$$

where $\gamma = Lu/(1 - Lu)$ [31] and L is the dimension of $\mathbf{A}$.

4.2 The Effect of ROE On Soundfield Control

One of the contributions of this thesis is to illustrate the effect of ROE in the context of soundfield control. This is achieved by noting that $\mathbf{A} = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^H$ by virtue of SVD. The computed system matrix $\hat{\mathbf{A}}$ can therefore be represented by

$$\hat{\mathbf{A}} = \hat{\mathbf{U}}\hat{\mathbf{\Lambda}}\hat{\mathbf{V}}^H, \quad (4.8)$$

where $\hat{\mathbf{U}} = \mathbf{U} + \mathbf{E}_u$, $\hat{\mathbf{\Lambda}} = \mathbf{\Lambda} + \mathbf{E}_\Lambda$, $\hat{\mathbf{V}} = \mathbf{V} + \mathbf{E}_v$ given that $\mathbf{E}_u$, $\mathbf{E}_\Lambda$ and $\mathbf{E}_v$ are the ROEs associated with $\mathbf{U}$, $\mathbf{\Lambda}$ and $\mathbf{V}$, respectively. Therefore, when the condition number of $\mathbf{A}$ is high, the eigenvalue spread is governed by the smallest element of $\mathbf{\Lambda}$. Hence, it can be shown, using matrix inversion lemma [34], that the ROE is given by

$$\mathbf{\Lambda}^{-1} - \hat{\mathbf{\Lambda}}^{-1} = (\mathbf{I} + \mathbf{E}_\Lambda^{-1}\mathbf{\Lambda})^{-1} \mathbf{\Lambda}^{-1}. \quad (4.9)$$

Since this difference can be significant in the presence of small values in $\mathbf{\Lambda}$, ROE is significant when $\mathbf{A}$ is ill-conditioned. Therefore $\hat{\mathbf{\Lambda}}^{-1} = (\mathbf{\Lambda} + \mathbf{E}_s)^{-1} = \mathbf{\Lambda}^{-1} + \hat{\mathbf{E}}_s$, where $\hat{\mathbf{E}}_s$ is an error matrix with considerably high diagonal elements. As a result, $\hat{\mathbf{E}}_s$ causes $\hat{\mathbf{\Lambda}}^{-1}$ to devi-

ate from $\mathbf{\Lambda}^{-1}$ and consequently $\widehat{\mathbf{V}}\widehat{\mathbf{\Lambda}}^{-1}\widehat{\mathbf{U}}^H$ completely differs from $(\mathbf{G}\mathbf{G}^H)^{-1} = \mathbf{V}\mathbf{\Lambda}^{-1}\mathbf{U}^H$. Therefore when $\mathbf{G}\mathbf{G}^H$ is ill-conditioned, ROE is significant in the generated sound field in terms of sound energies distributions in the zone of interest.

In the Tikhonov method, λ is added to avoid the small eigenvalues since $(\mathbf{A} + \lambda\mathbf{I})^{-1} = \widehat{\mathbf{V}}_T\widehat{\mathbf{\Lambda}}_T^{-1}\widehat{\mathbf{U}}_T^H$, $\widehat{\mathbf{\Lambda}}_T^{-1}$ contains diagonal elements $\sigma_i/(\sigma_i^2 + \lambda)$ [22]. It is however important to note that, as described in Section 3.4, introduction of λ introduces RE. This improvement in condition number effectively reduces the ROE in (4.9) and consequently the output error e defined in (2.8) is reduced. While this effect may not be significant for small λ such as when $\mathbf{A}$ is relatively well-conditioned, output error e may increase significantly when a high value of λ is used such as occur when $\mathbf{A}$ is severely ill-conditioned. Therefore, there is often a need to reduce ROE without introducing significant amount of RE so as to achieve a low output error, particularly when $\mathbf{A}$ is severely ill-conditioned.

In the Tikhonov method, the high contribution of ROE in estimating the matrix inverse is reduced significantly and, as a result, the output error (OE) reduces. However in such cases, while a high λ reduces the ROE, a higher RE is introduced. Therefore, a combination of RE and ROE and their trade-off limits the performance of these methods for the soundfield control.

4.3 Summary

In this Chapter, the intrinsic characteristic of the computer-based computation of ROE is discussed and reviewed. It is found that the ROE accumulates and cannot be avoided in an iterative computation process as seen from (4.5). Further study of the ROE shows that it is associated with the size and conditioning of the matrix for a matrix inverse computation, derived from (4.9). It is therefore noted that the ROE will affect the performance of soundfield control and this is of particular importance since the system matrix inverse computation introduces the ROE accumulation during iterative process. As a result, the

regularisation method is employed to suppress the large amount of ROE while regularisation method introduces RE at the same time. Therefore, the trade-off between the RE and ROE needs to be achieved such that the overall OE is minimized, which, in turn, optimizes the soundfield control performance.

Chapter 5

The MPE and Its Effect On Soundfield Control

This chapter begins by modeling the MPE mathematically from soundfield control system point of view. Section 5.1 describes how the MPE is introduced in the soundfield control system as previously described in Section 2.1 while Section 5.2 illustrates how the MPE affects the acoustic transfer function between loudspeaker array and the listening points, which, in turn, degrades the soundfield control performance. Section 5.3 analyzes how the MPE effect can be minimized for improving the soundfield control system performance.

5.1 Introduction of MPE

It is noted that the measurement noise and measurement error play an important role in the accuracy of soundfield control system. However, for soundfield control application in this thesis, and similar to that of [35–38], feedback signals from microphones are not available, i.e., only the locations of listening points are provided. This implies that there is no feedback from listening points to train the weights of the loudspeakers. As a result, unlike a microphone-based soundfield control system, no measurement noise is involved in

the soundfield control system in this thesis. Since the locations of the loudspeakers are to be known a priori as desired positions, any measurement error is contributed mainly by the misplacement error (MPE) - an error caused by inaccurate measurement of the loudspeaker positions. In a real application of soundfield control, the physical loudspeakers must be placed at the required positions when the desired audible zone is given for soundfield control purpose. Therefore, the measurement error in terms of loudspeaker position deviation will always be present when placing the loudspeaker in the desired position, which is the focus of this thesis. This loudspeaker position placement (misplacement) error can further propagate and is reflected in the acoustic transfer function between the loudspeaker arrays and the desired listening points, which, as a result, degrades the soundfield control accuracy. To improve the accuracy of soundfield control system, the proposed algorithm should also take MPE into consideration to obtain the best optimal loudspeaker weight coefficients.

For clarity of presentation, the case of M loudspeakers and one listening point is illustrated as example. This discussion can directly be extended to a multi-listening point scenario. Since the aim of this chapter is to analyze the effect of MPE on soundfield control, the following analysis assumes that ROE is not present in the system. The distance between the i th loudspeaker and the listening point is denoted by R_i and to describe the uncertainty of the loudspeaker placement on a 2-dimensional plane, a random perturbation represented by a radius r_i and angle θ_i is given as shown in Fig. 5.1. Employing the cosine law, we have

$$\widehat{R}_i^2 = R_i^2 + r_i^2 - 2R_i r_i \cos \theta_i. \quad (5.1)$$

The theoretical acoustic transfer function from the i th loudspeaker to the listening point at a specific frequency f is then given by [10]

$$G(R_i) = \frac{1}{4\pi R_i} e^{jkR_i}, i = 1, \dots, M, \quad (5.2)$$

Figure 5.1: Illustration of the MPE which described by a radius r and angle θ .

where $j = \sqrt{-1}$, $k = 2\pi f/c$ is the wave number and c is the speed of the sound. In (5.2) and the remainder of this thesis, the variable f has been left out for ease of notation. To describe the MPE, it is assumed that the theoretical position of the i th loudspeaker in a two-dimensional plane is (x_i, y_i) m while its actual location is $(x_i + r_i \cos \theta_i, y_i + r_i \sin \theta_i)$ m. It is therefore assumed that the perturbation is described by r_i drawn from a Gaussian distribution with $E\{r_i\} = r$ for $i = 1, \dots, M$ and θ_i that is uniformly distributed at the range $(0, 2\pi]$. Substituting (5.1) to (5.2), the new transfer function can therefore be expressed as

$$\begin{aligned} G(\hat{R}_i) &= \frac{1}{4\pi \hat{R}_i} e^{jk\hat{R}_i} \\ &= \frac{e^{jk\sqrt{R_i^2 + r_i^2 - 2R_i r_i \cos \theta_i}}}{4\pi \sqrt{R_i^2 + r_i^2 - 2R_i r_i \cos \theta_i}}. \end{aligned} \quad (5.3)$$

It is noted that $G(R_i)$ and $G(\hat{R}_i)$ are uncorrelated with respect to θ_i .

As θ_i is assumed uniformly distributed from 0 to 2π , $\cos \theta$ would be also uniformly distributed from -1 to $+1$ [39]. The expectation of the transfer function $g(\theta_i)$ over the parameter θ is then given by

$$E_{\theta}\{g(\theta_i)\} \approx \frac{1}{2} \int_{-1}^1 g(\theta_i) d \cos \theta_i. \quad (5.4)$$

It is noted that both the displacement parameter r_i and the angle θ_i are random variables

in a measurement system for each process step [40]. Therefore, the overall displacement is evaluated by integrating all angles uniformly since the displacement angle θ_i has negligible effect on the the point source (loudspeaker) compared with the displacement parameter r_i . Consequently, employing (5.4), we have

$$\begin{aligned}
E_\theta \left\{ \frac{G(\hat{R}_i)}{G(R_i)} \right\} &= E_\theta \left\{ \frac{G(\hat{R}_i)G^*(R_i)}{G(R_i)G^*(R_i)} \right\} \\
&= \oint_0^{2\pi} \frac{\frac{1}{16\pi^2 \hat{R}_i R_i} e^{jk(\hat{R}_i - R_i)}}{\frac{1}{16\pi^2 R_i^2}} d\theta \\
&= R_i \oint_0^{2\pi} \frac{e^{jk(\sqrt{R_i^2 + r_i^2 - 2R_i r_i \cos \theta} - R_i)}}{\sqrt{R_i^2 + r_i^2 - 2R_i r_i \cos \theta}} d\theta \\
&= R_i \oint_0^{2\pi} \frac{e^{jk(\sqrt{R_i^2 + r_i^2 - 2R_i r_i \cos \theta} - R_i)}}{\sqrt{R_i^2 + r_i^2 - 2R_i r_i \cos \theta}} d\theta \\
&\approx 0.5 R_i \int_{-1}^1 \frac{e^{jk(\sqrt{R_i^2 + r_i^2 - 2R_i r_i \cos \theta} - R_i)}}{\sqrt{R_i^2 + r_i^2 - 2R_i r_i \cos \theta}} d \cos \theta \\
&= 0.5 R_i \int_{R_i + r_i}^{R_i - r_i} \frac{e^{jk(\hat{R}_i - R_i)}}{-R_i r_i} d\hat{R}_i \\
&= \frac{-e^{-jkR_i}}{2r_i} \frac{e^{jk\hat{R}_i}}{jk} \Big|_{R_i + r_i}^{R_i - r_i} \\
&= \frac{\sin kr_i}{kr_i}, \tag{5.5}
\end{aligned}$$

where $k = 2\pi f/c$ is the wave number for input frequency f and sound speed c . To make the approximation in (5.4) valid, $kr \leq \pi$ is needed. Since $G(R_i)$ is independent of θ , (5.5) can be rewritten as

$$E_\theta \{G(\hat{R}_i)\} = \frac{\sin kr_i}{kr_i} G(R_i). \tag{5.6}$$

The ratio term $\frac{\sin kr_i}{kr_i}$ in (5.6) can be taken as unnormalized sinc function $\text{sinc}(kr_i)$. As we can see from (5.5), the perturbed transfer function $G(\hat{R}_i)$ caused by MPE is dependent on input signal frequency f , the displacement r and theoretical distance R_i between the i th loudspeaker and listening point.

Figure 5.2: Illustration of estimated error for MPE versus displacement r in terms of λ

Figure 5.2 illustrates how the estimated new transfer function $G(\hat{R}_i)$ varies with the displacement r in terms of wavelength λ . The plot was generated using a setup where $R = 1$ m, $f = 1000$ Hz, $c = 345$ m/s and $\theta = (0, 2\pi]$. The estimated error is computed as

$$e_G = \frac{\left| \frac{\sin kr}{kr} G(R) - G_{\text{ave}}(\hat{R}) \right|}{|G_{\text{ave}}(\hat{R})|}, \quad (5.7)$$

where

$$G_{\text{ave}}(\hat{R}) = \frac{1}{360} \sum_{\theta=\pi/180}^{2\pi} G(\sqrt{R_i^2 + r_i^2 - 2R_i r_i \cos \theta}). \quad (5.8)$$

As can be seen from Fig. 5.2, the estimated error increased with the increase the displacement r . We note that when $r = 0.5n\pi$, e_G becomes very large since $\sin kr = 0$ and this results in the estimated $G(\hat{R})$ being zero. This estimation error comes from (5.4) as integration over θ does not equal to integration over $\cos \theta$.

5.2 Effect of the MPE On Soundfield Control

To quantify the effect of the MPE on generated sound signal, the mean-square error (MSE) e_{mse} between the desired soundfield $D = \sum_{i=1}^M D_i = \sum_{i=1}^M w_i G(R_i)$ and generated soundfield $Y = \sum_{i=1}^M Y_i = \sum_{i=1}^M w_i G(\hat{R}_i)$ due to the misplacement of r_i and moving direction θ_i is given by

$$\begin{aligned} e_{mse} &= E_{\theta} \left\{ \|D - Y\|^2 \right\} \\ &= E_{\theta} \left\{ \left\| \sum_{i=1}^M [w_i G(\hat{R}_i) - w_i G(R_i)] \right\|^2 \right\}, \end{aligned} \quad (5.9)$$

where E_{θ} represents the expectation operation over variable θ and w_i is the weight for the i th loudspeaker. Expanding (5.9) result in

$$\begin{aligned} e_{mse} &= E_{\theta} \left\{ \left\| \sum_{i=1}^M \left[\left(\frac{G(\hat{R}_i)}{G(R_i)} - 1 \right) (w_i G(R_i)) \right] \right\|^2 \right\} \\ &= E_{\theta} \left\{ \sum_{i=1}^M \left(\frac{G(\hat{R}_i)}{G(R_i)} - 1 \right) D_i \sum_{j=1}^M \left(\frac{G^*(\hat{R}_j)}{G^*(R_j)} - 1 \right) D_j^* \right\} \\ &= E_{\theta} \left\{ \sum_{i=1}^M \sum_{j=1}^M \left[\left(\frac{G(\hat{R}_i)}{G(R_i)} - 1 \right) \left(\frac{G^*(\hat{R}_j)}{G^*(R_j)} - 1 \right) D_i D_j^* \right] \right\} \\ &= E_{\theta} \left\{ \sum_{i=1}^M \sum_{j=1}^M \left[D_i D_j^* \frac{G(\hat{R}_i)}{G(R_i)} \frac{G^*(\hat{R}_j)}{G^*(R_j)} - D_i D_j^* \frac{G(\hat{R}_i)}{G(R_i)} - D_i D_j^* \frac{G^*(\hat{R}_j)}{G^*(R_j)} + D_i D_j^* \right] \right\} \\ &= E_{\theta} \left\{ \sum_{i=1}^M \sum_{j=1}^M \left[D_i D_j^* \frac{G(\hat{R}_i) G^*(\hat{R}_j)}{G(R_i) G^*(R_j)} - D_i D_j^* \frac{G(\hat{R}_i) G^*(R_i)}{G(R_i) G^*(R_i)} \right. \right. \\ &\quad \left. \left. - D_i D_j^* \frac{G^*(\hat{R}_j) G(R_j)}{G^*(R_j) G(R_j)} + D_i D_j^* \right] \right\}, \end{aligned} \quad (5.10)$$

where the superscript $[\cdot]^*$ denotes the complex conjugation operator. To calculate each sum item in (5.10), the Taylor series the 0th order of approximation technique [39] is applied. Taking a function $g(x_1, x_2, \dots, x_n)$ as an example, $g(x_1, x_2, \dots, x_n)$ is a function of random variables $x_1, x_2, \dots, x_n$ where $E\{x_j\} = \bar{x}_j, j = 1, \dots, n$. After applying Taylor

series to $g(x_1, x_2, \dots, x_n)$, we have [39]

$$E_\theta\{g(x_1, \dots, x_n)\} = g(\bar{x}_1, \dots, \bar{x}_n) + \sum_{j=1}^n g'(\bar{x}_j)(x_j - \bar{x}_j) + \mathcal{O}(x_1, \dots, x_n), \quad (5.11)$$

where $g'(\bar{x}_j)$ is the 1st order derivative of $g(x_j)$ and $\mathcal{O}(x_1, \dots, x_n)$ corresponds to the higher-order items of the Taylor series expansion. Therefore, taking the 0th order approximation to (5.11), we obtain

$$E_x\{g(x_1, \dots, x_n)\} \approx g(\bar{x}_1, \dots, \bar{x}_n). \quad (5.12)$$

As a result, applying approximation technique obtained in (5.12) to (5.10), (5.10) becomes

$$\begin{aligned} e_{\text{mse}} \approx & D^2 + \sum_{i=1}^M \sum_{j=1}^M D_i D_j^* E_\theta \left\{ \frac{G(\hat{R}_i) G^*(\hat{R}_j)}{G(R_i) G^*(R_j)} \right\} - \sum_{i=1}^M \sum_{j=1}^M D_i D_j^* E_\theta \left\{ \frac{G(\hat{R}_i) G^*(R_i)}{G(R_i) G^*(R_i)} \right\} \\ & - \sum_{i=1}^M \sum_{j=1}^M D_i D_j^* E_\theta \left\{ \frac{G^*(\hat{R}_j) G(R_i)}{G^*(R_j) G(R_i)} \right\}. \end{aligned} \quad (5.13)$$

We note that $E_\theta\{G(R_i)\} = G(R_i)$, $E_\theta\{G(R_j)\} = G(R_j)$, $E_\theta\{G(R_i)G^*(R_i)\} = G(R_i)G^*(R_i)$ and $E_\theta\{G(\hat{R}_i)G^*(R_i)\} = G^*(R_i)E_\theta\{G(\hat{R}_i)\}$ due to the fact that $G(R_i)$, $G(R_j)$ and $G^*(R_i)$ are all independent of variable θ . Since R_i is independent of r_i and θ_i , it is therefore each sum item in (5.13) can be further simplified as

$$E\left\{ \frac{G(\hat{R}_i) G^*(\hat{R}_j)}{G(R_i) G^*(R_j)} \right\} = \frac{E\{G(\hat{R}_i) G^*(\hat{R}_j)\}}{E\{G(R_i) G^*(R_j)\}}, \quad (5.14a)$$

$$E\left\{ \frac{G(\hat{R}_i) G^*(R_i)}{G(R_i) G^*(R_i)} \right\} = \frac{E\{G(\hat{R}_i) G^*(R_i)\}}{E\{G(R_i) G^*(R_i)\}}, \quad (5.14b)$$

$$E\left\{ \frac{G^*(\hat{R}_j) G(R_i)}{G^*(R_j) G(R_i)} \right\} = \frac{E\{G^*(\hat{R}_j) G(R_i)\}}{E\{G^*(R_j) G(R_i)\}}. \quad (5.14c)$$

Substituting (5.14a)-(5.14c) to (5.13), (5.13) can be rewritten as

$$\begin{aligned}
e_{\text{mse}} \approx & D^2 + \sum_{i=1}^M \sum_{j=1}^M D_i D_j^* \frac{E_{\theta}\{G(\hat{R}_i)G^*(\hat{R}_j)\}}{E_{\theta}\{G(R_i)G^*(R_j)\}} - \sum_{i=1}^M \sum_{j=1}^M D_i D_j^* \frac{E_{\theta}\{G(\hat{R}_i)G^*(R_i)\}}{E_{\theta}\{G(R_i)G^*(R_i)\}} \\
& - \sum_{i=1}^M \sum_{j=1}^M D_i D_j^* \frac{E_{\theta}\{G^*(\hat{R}_j)G(R_i)\}}{E_{\theta}\{G^*(R_j)G(R_i)\}}. \tag{5.15}
\end{aligned}$$

Applying (5.2), (5.3) and (5.5), the Expectation operation item in (5.15) can be further rewritten as

$$E_{\theta}\{G(\hat{R}_i)G^*(\hat{R}_j)\} = \frac{1}{16\pi^2} \frac{e^{jk(R_i-R_j)}}{R_i R_j} \frac{\sin(kr_i)}{kr_i} \frac{\sin(kr_j)}{kr_j}, \tag{5.16a}$$

$$E_{\theta}\{G(R_i)G^*(R_j)\} = \frac{1}{16\pi^2} \frac{e^{jk(R_i-R_j)}}{R_i R_j}, \tag{5.16b}$$

$$E_{\theta}\{G(\hat{R}_i)G^*(R_i)\} = \frac{1}{16\pi^2 R_i^2} \frac{\sin(kr_i)}{kr_i}, \tag{5.16c}$$

$$E_{\theta}\{G^*(\hat{R}_i)G(R_i)\} = \frac{1}{16\pi^2 R_i^2} \frac{\sin(kr_i)}{kr_i}, \tag{5.16d}$$

$$E_{\theta}\{G(R_i)G^*(R_i)\} = \frac{1}{16\pi^2 R_i^2}, \tag{5.16e}$$

$$E_{\theta}\left\{\frac{G(\hat{R}_i)}{G(R_i)}\right\} = \frac{E_{\theta}\{G(\hat{R}_i)G^*(R_i)\}}{E_{\theta}\{G(R_i)G^*(R_i)\}} = \frac{\sin kr_i}{kr_i}. \tag{5.16f}$$

Substituting (5.16a)-(5.16f) to (5.15), (5.15) can be expressed as

$$\begin{aligned}
e_{\text{mse}} \approx & D^2 + \sum_{i=1}^M \sum_{j=1}^M D_i D_j^* \frac{E\{G(\hat{R}_i)G^*(\hat{R}_j)\}}{E\{G(R_i)G^*(R_j)\}} - \sum_{i=1}^M \sum_{j=1}^M D_i D_j^* \frac{E\{G(\hat{R}_i)G^*(R_i)\}}{E\{G(R_i)G^*(R_i)\}} \\
& - \sum_{i=1}^M \sum_{j=1}^M D_i D_j^* \frac{E\{G^*(\hat{R}_j)G(R_i)\}}{E\{G^*(R_j)G(R_i)\}} \\
= & D^2 + \sum_{i=1}^M \sum_{j=1}^M D_i D_j^* \frac{\sin kr_i}{kr_i} \frac{\sin kr_j}{kr_j} - \sum_{i=1}^M \sum_{j=1}^M D_i D_j^* \frac{\sin kr_i}{kr_i} - \sum_{i=1}^M \sum_{j=1}^M D_i D_j^* \frac{\sin kr_j}{kr_j} \\
= & D^2 + \sum_{i=1}^M D_i \frac{\sin kr_i}{kr_i} \sum_{j=1}^M D_j^* \frac{\sin kr_j}{kr_j} - \sum_{i=1}^M D_i \frac{\sin kr_i}{kr_i} \sum_{j=1}^M D_j^* - \sum_{i=1}^M D_i \sum_{j=1}^M D_j^* \frac{\sin kr_j}{kr_j}. \tag{5.17}
\end{aligned}$$

With the assumption that $E_\theta\{r_i\} = E_\theta\{r_j\} = r$ for the MPE of each loudspeaker since θ and r are exclusively independent to each other, (5.17) can be written as

$$\begin{aligned} e_{\text{mse}} &= D^2 + \left(\frac{\sin kr}{kr}\right)^2 D^2 - \frac{\sin kr}{kr} D^2 - \frac{\sin kr}{kr} D^2 \\ &= \left(1 - \frac{\sin kr}{kr}\right)^2 D^2. \end{aligned} \quad (5.18)$$

As can be seen from (5.18), to minimize the mean-square error (MSE) of the generated output soundfield, the displacement parameter r must be minimized given that the input signal is known with k being a constant. When r is approaching zero, we have

$$\lim_{r \rightarrow 0} \frac{\sin kr}{kr} = 1. \quad (5.19)$$

Substituting (5.19) into (5.18), we achieve

$$\begin{aligned} \lim_{r \rightarrow 0} e_{\text{mse}} &= \lim_{r \rightarrow 0} \left(1 - \frac{\sin kr}{kr}\right)^2 D^2 \\ &= 0. \end{aligned} \quad (5.20)$$

Therefore, in an ideal situation where MPE is not present, we have $r = 0$ and the MSE caused by the MPE is 0. When the MPE is present in the system, the MSE is related to the displacement parameter r only. In the following section, it will be shown how the MPE propagates in the acoustic transfer function which results in the normalized output error being a key contributor to the soundfield performance.

5.3 Minimizing the Effect of the MPE on Soundfield Control

It is shown that the MPE arises from loudspeaker location measurement error in the soundfield control system and will affect the acoustic transfer function G as discussed in Section 5.1. In this section the effect of the MPE on the soundfield control system especially when the system matrix is ill-conditioned is discussed. The user-defined soundfield signal $\mathbf{D} = [D_1, D_2, \dots, D_N]^T$ can, in theory, be achieved by controlling the weights of loudspeakers where N is the total number of listening points, so that

$$\mathbf{G}\mathbf{w} = \mathbf{D}. \quad (5.21)$$

Assuming that MPE exists, the transfer function $\mathbf{G}$ becomes

$$\hat{\mathbf{G}} = \mathbf{G} + \Delta\mathbf{G}, \quad (5.22)$$

where $\Delta\mathbf{G}$ is the deviation in transfer function caused by MPE and $\hat{\mathbf{G}}$ can be expressed as

$$\hat{\mathbf{G}} = \begin{bmatrix} G(\hat{R}_{11}) & G(\hat{R}_{12}) & \cdots & G(\hat{R}_{1M}) \\ G(\hat{R}_{21}) & G(\hat{R}_{22}) & \cdots & G(\hat{R}_{2M}) \\ \vdots & \vdots & \ddots & \vdots \\ G(\hat{R}_{N1}) & G(\hat{R}_{N2}) & \cdots & G(\hat{R}_{NM}) \end{bmatrix}, \quad (5.23)$$

where $\hat{R}_{ij}$ is the actual distance between the j th listening point to the i th loudspeaker and M is the number of loudspeakers. Therefore, the (i, j) th element in $\Delta\mathbf{G}$ can be expressed

as

$$\begin{aligned}\Delta \mathbf{G}(i, j) &= G(\hat{R}_{ij}) - G(\hat{R}_{ij}) \\ &= \frac{1}{4\pi} \left(\frac{e^{-jk\hat{R}_{ij}}}{\hat{R}_{ij}} - \frac{e^{-jkR_{ij}}}{R_{ij}} \right).\end{aligned}\quad (5.24)$$

For simplicity of presentation, we assume all the loudspeakers have the same misplacement r and the misplacement direction is uniformly distributed in direction $0 \leq \theta < 2\pi$. Consequently the loudspeaker weights deviate from the exact values $\mathbf{w}$ by an amount of $\Delta \mathbf{w}$. Therefore, (5.21) is modified to

$$(\mathbf{G} + \Delta \mathbf{G})(\mathbf{w} + \Delta \mathbf{w}) = \mathbf{D}. \quad (5.25)$$

It is noted that when $r = 0$, $\Delta \mathbf{w} = \mathbf{0}$ and $\Delta \mathbf{G} = \mathbf{0}$. On the other hand, assuming that $\mathbf{w} = \mathbf{G}^{-1}\mathbf{D}$ is known and that MPE exists, the generated soundfield can be expressed as

$$\mathbf{Y} = (\mathbf{G} + \Delta \mathbf{G})\mathbf{w}. \quad (5.26)$$

As a result, the generated soundfield error $\Delta \mathbf{D}$ is obtained as

$$\begin{aligned}\Delta \mathbf{D} &= \mathbf{Y} - \mathbf{D} \\ &= (\mathbf{G} + \Delta \mathbf{G})\mathbf{w} - \mathbf{G}\mathbf{w} \\ &= \Delta \mathbf{G}\mathbf{w}.\end{aligned}\quad (5.27)$$

Expansion of (5.25) results in

$$\mathbf{G}\mathbf{w} + \mathbf{G}\Delta \mathbf{w} + \Delta \mathbf{G}\mathbf{w} + \Delta \mathbf{G}\Delta \mathbf{w} = \mathbf{D}. \quad (5.28)$$

Since $\Delta \mathbf{G}$ and $\Delta \mathbf{w}$ are negligibly small compared with $\mathbf{G}$ and $\mathbf{w}$, we can neglect $\Delta \mathbf{G}\Delta \mathbf{w}$,

and (5.28) can then be expressed as

$$\mathbf{G}\Delta\mathbf{w} + \Delta\mathbf{G}\mathbf{w} = \mathbf{0}. \quad (5.29)$$

Substituting $\mathbf{G}\mathbf{w} = \mathbf{D}$ into (5.29), (5.29) can be simplified as

$$\mathbf{G}\Delta\mathbf{w} = -\Delta\mathbf{G}\mathbf{w}. \quad (5.30)$$

For clarity of presentation and ease of notation in the following section, the well-determined case is taken as an example for analysis in which $N = M$. Therefore $\mathbf{G}$ is a squared matrix. This well-determined case can be further extended to under- or over-determined case easily. Multiplying $\mathbf{G}^{-1}$ at the both sides of (5.30), we obtain

$$\Delta\mathbf{w} = -\mathbf{G}^{-1}\Delta\mathbf{G}\mathbf{w}. \quad (5.31)$$

Taking the l_2 norm at both sides of (5.31),

$$\begin{aligned} \|\Delta\mathbf{w}\| &= \|\mathbf{G}^{-1}\Delta\mathbf{G}\mathbf{w}\| \\ &\leq \|\mathbf{G}^{-1}\| \|\Delta\mathbf{G}\| \|\mathbf{w}\|. \end{aligned} \quad (5.32)$$

Moving the scalar item $\|\mathbf{w}\|$ from right to the left, we have

$$\begin{aligned} \frac{\|\Delta\mathbf{w}\|}{\|\mathbf{w}\|} &\leq \|\mathbf{G}^{-1}\| \|\mathbf{G}\| \left\| \frac{\Delta\mathbf{G}}{\mathbf{G}} \right\| \\ &= \chi_2(\mathbf{G}) \left\| \frac{\Delta\mathbf{G}}{\mathbf{G}} \right\|, \end{aligned} \quad (5.33)$$

where $\chi(\mathbf{G}) = \|\mathbf{G}^{-1}\| \|\mathbf{G}\|$ is the condition number of $\mathbf{G}$ [30]. Rewriting (5.33), we have

$$\frac{\|\mathbf{G}\Delta\mathbf{w}\|}{\|\mathbf{G}\mathbf{w}\|} \leq \chi(\mathbf{G}) \left\| \frac{\Delta\mathbf{G}}{\mathbf{G}} \right\|. \quad (5.34)$$

Consequently, the relationship between the normalized output error and the system matrix condition number is given by

$$\frac{\|\Delta \mathbf{D}\|}{\|\mathbf{D}\|} \leq \chi(\mathbf{G}) \left\| \frac{\Delta \mathbf{G}}{\mathbf{G}} \right\|. \quad (5.35)$$

Taking expectation of (5.23) and substituting (5.5) to (5.23), (5.23) is modified as

$$E\{\widehat{\mathbf{G}}\} = \begin{bmatrix} \frac{\sin kr_1}{kr_1} G(R_{11}) & \frac{\sin kr_2}{kr_2} G(R_{12}) & \cdots & \frac{\sin kr_N}{kr_N} G(R_{1N}) \\ \frac{\sin kr_1}{kr_1} G(R_{21}) & \frac{\sin kr_2}{kr_2} G(R_{21}) & \cdots & \frac{\sin kr_N}{kr_N} G(R_{2N}) \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\sin kr_1}{kr_1} G(R_{N1}) & \frac{\sin kr_2}{kr_2} G(R_{N2}) & \cdots & \frac{\sin kr_N}{kr_N} G(R_{NN}) \\ \frac{\sin kr_1}{kr_1} G(R_{11}) & \frac{\sin kr_2}{kr_2} G(R_{12}) & \cdots & \frac{\sin kr_N}{kr_N} G(R_{1N}) \\ \frac{\sin kr_1}{kr_1} G(R_{21}) & \frac{\sin kr_2}{kr_2} G(R_{21}) & \cdots & \frac{\sin kr_N}{kr_N} G(R_{2N}) \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\sin kr_1}{kr_1} G(R_{N1}) & \frac{\sin kr_2}{kr_2} G(R_{N2}) & \cdots & \frac{\sin kr_N}{kr_N} G(R_{NN}) \end{bmatrix}, \quad (5.36)$$

where R_{ij} is the distance between the j th loudspeaker and i . Assuming $E\{r_j\} = r$, (5.36)

is simplified to

$$E\{\widehat{\mathbf{G}}\} = \frac{\sin kr}{kr} \mathbf{G}. \quad (5.37)$$

Therefore, $E\{\Delta \mathbf{G}\}$ is obtained and is given by

$$E\{\Delta \mathbf{G}\} = E\{\widehat{\mathbf{G}}\} - \mathbf{G} = \left(\frac{\sin kr}{kr} - 1 \right) \mathbf{G}. \quad (5.38)$$

Substituting (5.38) to (5.35), we obtain

$$E\left\{ \frac{\|\Delta \mathbf{D}\|}{\|\mathbf{D}\|} \right\}_{\text{wd}} \leq \chi(\mathbf{G}) \left| \frac{\sin kr}{kr} - 1 \right|, \quad (5.39)$$

Equation (5.39) implies that the effect of MPE on the generated soundfield performance,

are determined by the conditioning of the system, input signal and misplacement. In addition, the term $E\{\|\frac{\Delta \mathbf{D}}{\mathbf{D}}\|\}$ will be amplified by a factor of $\chi(\mathbf{G})$ for well-determined case. This observation can also be extended to under- and over-determined cases. Given that the system matrix for over- and under- determined cases are $\mathbf{G}^H \mathbf{G}$ and $\mathbf{G} \mathbf{G}^H$ respectively, applying (2.9a)-(2.9c) defined in Chapter 2 to (5.30), the MPE effect on relative output error for under-determined case can be expressed as

$$\begin{aligned} E\{\mathbf{G} \Delta \mathbf{w}\}_{\text{ud}} &= E\{\Delta \mathbf{G}\} \mathbf{G}^H (\mathbf{G} \mathbf{G}^H)^{-1} \mathbf{D} \\ &= \left(\frac{\sin kr}{kr} - 1\right) \mathbf{G} \mathbf{G}^H (\mathbf{G} \mathbf{G}^H)^{-1} \mathbf{D}. \end{aligned} \quad (5.40)$$

Taking the norm at both sides of (5.40), we obtain

$$\|E\{\Delta \mathbf{D}\}\|_{\text{ud}} \leq \left\| \frac{\sin kr}{kr} - 1 \right\| \|(\mathbf{G} \mathbf{G}^H)\| \|(\mathbf{G} \mathbf{G}^H)^{-1}\| \|\mathbf{D}\|. \quad (5.41)$$

Equation (5.41) can be rewritten as

$$E\left\{\frac{\|\Delta \mathbf{D}\|}{\|\mathbf{D}\|}\right\}_{\text{ud}} \leq \chi(\mathbf{G} \mathbf{G}^H) \left| \frac{\sin kr}{kr} - 1 \right|, \quad (5.42)$$

where $\chi(\mathbf{G} \mathbf{G}^H)$ is the condition number of the under-determined system matrix $\mathbf{G} \mathbf{G}^H$ [30]. Similarly, the relationship between the expected relative OE and the conditioning of system for over-determined case can be expressed as

$$\begin{aligned} E\{\mathbf{G} \Delta \mathbf{w}\}_{\text{od}} &= E\{\Delta \mathbf{G}\} (\mathbf{G}^H \mathbf{G})^{-1} \mathbf{G}^H \mathbf{D} \\ &= \left(\frac{\sin kr}{kr} - 1\right) \mathbf{G} (\mathbf{G}^H \mathbf{G})^{-1} \mathbf{G}^H \mathbf{D}. \end{aligned} \quad (5.43)$$

Taking the norm at both sides of (5.43), we obtain

$$\begin{aligned}\|E\{\Delta\mathbf{D}\}\|_{\text{od}} &\leq \left\|\frac{\sin kr}{kr} - 1\right\| \|\mathbf{G}\| \|(\mathbf{G}^H \mathbf{G})^{-1}\| \|\mathbf{G}^H\| \|\mathbf{D}\| \\ &= \left\|\frac{\sin kr}{kr} - 1\right\| \|(\mathbf{G}^H \mathbf{G})\| \|(\mathbf{G}^H \mathbf{G})^{-1}\| \|\mathbf{D}\|. \quad (5.44)\end{aligned}$$

Equation (5.44) can be rewritten as

$$E\left\{\frac{\|\Delta\mathbf{D}\|}{\|\mathbf{D}\|}\right\}_{\text{od}} \leq \chi(\mathbf{G}^H \mathbf{G}) \left|\frac{\sin kr}{kr} - 1\right|. \quad (5.45)$$

Substituting the system matrix in (5.39), (5.42) and (5.45) with (2.9a)-(2.9c) defined in Chapter 2, the effect of MPE on the normalized output error can be further generalized as

$$E\left\{\frac{\|\Delta\mathbf{D}\|}{\|\mathbf{D}\|}\right\} \leq \chi(\mathbf{A}) \left|\frac{\sin kr}{kr} - 1\right|. \quad (5.46)$$

It therefore can be seen from (5.46) that, the conditioning of the system matrix $\mathbf{A}$ is important as it can amplify the effect of MPE on the performance of the soundfield control during the inversion process. It is noted that when $\mathbf{A}$ is ill-conditioned, χ is significantly large and therefore the effect of MPE could be amplified considerably in the normalized OE $\|\Delta\mathbf{D}\|/\|\mathbf{D}\|$. This indicates the severity of ill-conditioned problem in the system matrix inversion process, for well-, under- and over- determined cases of soundfield control system. Therefore, minimizing MPE effect on soundfield control and solving the ill-conditioning problem of the system matrix both can minimize the effect of MPE of the normalized OE. As a result, in addition to the MPE, the ill-conditioned problem should be addressed in soundfield control application. This supports our original objective of investigating the effect of ROE for different LS-based methods under the assumption that MPE is not present.

5.4 Summary

This chapter introduces the MPE and describes its presence in the soundfield control system. Followed by the fact that the MPE is mainly present in the loudspeaker location measurement, this misplacement for the loudspeaker can be described by the misplacement parameter r and the angle θ . The effect of the MPE is therefore analyzed and quantified in the soundfield control system. It is found that the MPE can be amplified by the condition number of the system matrix $\chi(\mathbf{A})$ in the normalized output error (NOE) which indicates the performance of the soundfield control system. Given a known MPE prior where r is constant, the l -2 norm output error is proportional to the l -2 norm of desired soundfield output $\mathbf{D}$ with the amplifying factor of $\chi(\mathbf{A})$. This finding motivates the need to solve the ill-conditioning problem of the system matrix for the soundfield control system. Knowing the effects of the MPE, the discussion and analysis of how to minimize such detrimental effect on the soundfield control system will be discussed in the next chapter.

Chapter 6

The Proposed Method

In this chapter, the new LS-based algorithm utilizing the linear network is proposed to estimate the system matrix inverse iteratively, even when the system matrix is ill-conditioned. Instead of avoiding largely accumulated ROE, the proposed algorithm takes the dynamic changing ROE accumulation as a regularisation parameter to improve the conditioning of system matrix. As a result, the accuracy of the system matrix inverse is improved, leading better soundfield control performance. The trade-off between reducing accumulation of residual ROE and introducing of RE is achieved with the help of matrix sparseness measure to terminate the iteration process at an optimal point.

6.1 The Proposed LS-based Iterative Matrix Inverse Algorithm

As opposed to the direct inversion of $\mathbf{A}$ which can introduce large amount of ROE, the proposed approach estimates $\mathbf{A}^{-1}$ iteratively so as to reduce the accumulated ROE. As will be shown in Section 6.3, such iterative approach allows the algorithm to evaluate the accuracy of $\mathbf{A}^{-1}$ estimate at every iteration so as to limit the effect of ROE. Instead of ignoring the effect of ROE, the proposed method utilizes ROE as a form of regularisation

during the inverse estimation. More importantly, it achieves a dynamic balance between the reduction of ROE and introduction of RE using the matrix sparseness measure.

The starting point of the proposed algorithm is the use of a linear network that was originally proposed for the diagonalization of the correlation matrix constructed using a pair of stereophonic input vectors [17] and the estimation of the inverse of a joint-input correlation matrix of input vectors [18] for stereophonic acoustic echo cancellation application. The techniques proposed in [17] and [18] were originally developed for reducing the inter-channel coherence and partial decorrelation of stereophonic signals. In [17] these signals are employed for reducing the eigenvalue spread of input signals for improving the convergence performance of multichannel adaptive filters. In [18] these signals are employed for obtaining a coarse estimation of inverse of joint-input correlation matrix to be used in LMS-Newton adaptive filter in stereophonic acoustic echo cancellation. For both applications, the adaptive linear network is utilized for de-correlating the tap-input vectors. A linear network is proposed for estimating the inverse of a system matrix incorporated with matrix sparseness control in an adaptive manner. It is however important to note that direct application of the above for the soundfield control application will not achieve the desired result due to the ill-conditioning of the system matrix. It is therefore proposed to estimate the inverse of a system matrix iteratively while taking into account the conditioning of this matrix.

As will be discussed in the next section of this chapter, a single-layer linear network is employed for estimating the inverse of the system matrix $\mathbf{A}$. To improve convergence performance further, the generalization of the algorithm to a multi-layer network architecture is then illustrated. As the proposed algorithm operates on $\mathbf{A}$, an initial estimate of $\mathbf{A}^{-1}$ is given by an identity matrix. It is also important to note that during the iterative process, the estimate approaches toward $\mathbf{A}^{-1}$ and hence, if $\mathbf{A}^{-1}$ is ill-conditioned, the algorithm will misconverge after some iterations. This misconvergence is caused by the accumulation of ROE. To this end, a sparseness measure is then proposed to quantify how close the iterative estimate is to $\mathbf{A}^{-1}$. This measure is then employed to determine the best estimate of

$\mathbf{A}^{-1}$ during the iterative process. When the MPE is present, a regularisation based on the characteristics of MPE is proposed to suppress the effect of MPE on NOE.

During adaptive process, in an ill-conditioned system, the matrix inverse estimate approaches the actual $\mathbf{A}^{-1}$ and as a result, its conditioning worsens gradually. If $\mathbf{A}$ is ill-conditioned, it is expected, from adaptive theory point of view, that estimating $\mathbf{A}^{-1}$ is challenging. This reduces the accuracy of estimation when $\mathbf{A}$ is extremely ill-conditioned. Therefore there will be a trade-off between the estimation error and numerical error. The best estimation is then extracted using a measure for accuracy of estimation, which will be explained in later in this chapter. In cases where $\mathbf{A}$ is well-conditioned, the adaptive network can estimate it with higher accuracy. The proposed method includes the adaptive linear network and a sparseness control.

6.2 Single-Layer Linear Network for Matrix Inverse Computation

The structure of the proposed linear network employed to estimate $\mathbf{A}^{-1}$ defined in (2.10) is illustrated in Fig. 6.1. As can be seen, the output of the network is given by the matrix

$$\mathbf{Z}(n) = \begin{cases} \mathbf{P}(n)\mathbf{G}, & M > N; \\ \mathbf{P}(n)\mathbf{G}^H, & M \leq N, \end{cases} \quad (6.1)$$

where $\mathbf{P}(n)$ is the $N \times N$ matrix for under-determined case and $M \times M$ matrix for over- or well-determined case. In addition, $\mathbf{Z}(n)$ is employed in the network to train the weights such that the network weight update is given by

$$\mathbf{P}(n+1) = \mathbf{P}(n) + \eta[\mathbf{I} - \mathbf{Z}(n)\mathbf{Z}^H(n)], \quad (6.2)$$

Figure 6.1: The schematic structure for the proposed linear network for the transfer function $\mathbf{G}$.

where η is the step-size that determines the convergence rate of the network and $\mathbf{I}$ is an identity matrix with a size of $N \times N$ for under-determined case and $M \times M$ for over- or well-determined case. We assume that no *a priori* information of $\mathbf{A}^{-1}$ is available and hence we consider $\mathbf{P}(0) = \mathbf{I}$ initially.

To understand how the algorithm can estimate $\mathbf{A}^{-1}$, it is noted that after convergence, $\mathbf{P}(n+1) = \mathbf{P}(n)$ in (6.2) which implies that

$$\mathbf{Z}\mathbf{Z}^H = \mathbf{I}, \quad (6.3)$$

where $\mathbf{Z}$ is defined as the steady-state value of $\mathbf{Z}(n)$. Substituting steady-state values of (6.1) into (6.3), we have

$$\mathbf{P}\mathbf{A}\mathbf{P}^H = \mathbf{I}, \quad (6.4)$$

where $\mathbf{P}$ is the steady-state value of $\mathbf{P}(n)$. Since $\mathbf{P}$ is a square matrix and assuming $\mathbf{A}$ is nonsingular, (6.4) becomes

$$\mathbf{P}^H\mathbf{P} = \mathbf{A}^{-1}. \quad (6.5)$$

Equation (6.5) always holds as long as $\mathbf{A}$ is non-singular. It can be seen from (6.5) that the computation of $\mathbf{A}^{-1}$ can be realized by calculating $\mathbf{P}^H \mathbf{P}$ instead which can avoid introducing large amount of ROE when the $\mathbf{A}$ is ill-conditioned. However, small amount of ROE is still involved in the computation of $\mathbf{P}^H \mathbf{P}$ due to the fact that number of iterations for the adaptive process would not be infinitely large. Therefore, the computed values of $\hat{\mathbf{P}}^H \hat{\mathbf{P}}$ can be expressed as

$$\hat{\mathbf{P}}^H \hat{\mathbf{P}} \approx \mathbf{A}^{-1}. \quad (6.6)$$

Therefore, using (6.1), (6.2) and (6.6), we can estimate $\mathbf{A}^{-1}$ iteratively. As shown in Appendix 8.2.3, convergence is ensured when

$$0 < \eta < \frac{2}{\sqrt{\lambda_{\max}}(1 + \sqrt{\lambda_{\max}(n)})}, \quad (6.7)$$

where $\lambda_{\max}$ and $\lambda_{\max}(n)$ are the maximum eigenvalue of $\mathbf{A}$ and that of $E\{\mathbf{Z}(n)\mathbf{Z}^H(n)\}$, respectively. In practice, after the algorithm achieves its steady state, $\mathbf{P}(n+1) \approx \mathbf{P}(n)$ and therefore $\mathbf{P}^H \mathbf{P} \approx \mathbf{A}^{-1}$.

6.3 Generalization to Multi-layer Linear Network

In order to achieve higher rate of convergence and better estimate, the single-layer network is generalized to a multi-layer network (MLN). In such network, each layer is cascaded such that the output of the previous layer is used as the input of the next layer. To illustrate the above, we define K as the user-defined total number of layers in the network, n as the global iteration index, and $\mathcal{N}_k$ as the user-defined network layer boundary iteration index, such that for the k th layer, $k = 1, \dots, K$, the network iterates from iteration index $n = \mathcal{N}_{k-1} + 1$ to $\mathcal{N}_k$ and for the first layer, $\mathcal{N}_0 = 0$. In addition, update matrix for the k th layer at $n = \mathcal{N}_{k-1} + 1$ is initialized with an identity matrix. Therefore, the weight update matrix for each layer is

summarized as

$$\mathbf{P}_k(n+1) = \begin{cases} \mathbf{P}_k(n) + \eta_k(n) [\mathbf{I} - \mathbf{Z}_k(n) \mathbf{Z}_k^H(n)], & n > \mathcal{N}_{k-1}; \\ \mathbf{I}, & n \leq \mathcal{N}_{k-1}, \end{cases} \quad (6.8)$$

where

$$\eta_k(n) = \begin{cases} \tilde{\eta}_k, & \mathcal{N}_{k-1} < n \leq \mathcal{N}_k; \\ 0, & \text{otherwise} \end{cases} \quad (6.9)$$

and $\tilde{\eta}_k$ is the step-size for the k th layer. In addition, weight update matrix $\mathbf{P}_k(n)$ is of size $N \times N$ for under-determined case and $M \times M$ for well- or over-determined case. Consequently, the output of the k th layer is

$$\mathbf{Z}_k(n) = \mathbf{P}_k(n) \mathbf{Z}_{k-1}(n), \quad (6.10)$$

where

$$\mathbf{Z}_0(n) = \begin{cases} \mathbf{G}, & M > N; \\ \mathbf{G}^H, & M \leq N. \end{cases} \quad (6.11)$$

Substituting (6.8) into (6.10) for steady state, the final output can be expressed as

$$\mathbf{Z}(n) = \begin{cases} \mathbf{P}_K(n) \mathbf{P}_{K-1}(n) \cdots \mathbf{P}_1(n) \mathbf{G}, & M > N; \\ \mathbf{P}_K(n) \mathbf{P}_{K-1}(n) \cdots \mathbf{P}_1(n) \mathbf{G}^H, & M \leq N \end{cases} \quad (6.12)$$

from which, after convergence of (6.8), we achieve

$$\mathbf{P}_1 \mathbf{P}_2 \cdots \mathbf{P}_K \mathbf{A} \mathbf{P}_K^H \cdots \mathbf{P}_2^H \mathbf{P}_1^H = \mathbf{I}, \quad (6.13)$$

where $\mathbf{P}_k$ is steady-state value of $\mathbf{P}_k(n)$. As a result, $\mathbf{A}^{-1}$ can be estimated for all cases, after convergence, using

$$\mathbf{\Psi} = \mathbf{P}_1^H \mathbf{P}_2^H \dots \mathbf{P}_K^H \mathbf{P}_K \dots \mathbf{P}_2 \mathbf{P}_1. \quad (6.14)$$

With reference to (2.9a) to (2.9c), the estimated loudspeaker weight can then be computed as

$$\hat{\mathbf{w}}_w = \mathbf{\Psi} \mathbf{D}, \quad (6.15a)$$

$$\hat{\mathbf{w}}_o = \mathbf{\Psi} \mathbf{G}^H \mathbf{D}, \quad (6.15b)$$

$$\hat{\mathbf{w}}_u = \mathbf{G}^H \mathbf{\Psi} \mathbf{D}. \quad (6.15c)$$

As shown in Appendix 8.2.3, convergence of MLN is ensured when

$$0 < \tilde{\eta}_k < \frac{2}{\sqrt{\lambda_{k-1,\max}}(1 + \sqrt{\lambda_{k,\max}(n)})}, \quad (6.16)$$

where $\lambda_{k-1,\max}$ and $\lambda_{k,\max}(n)$ are the maximum eigenvalues of $\mathbf{Z}_{k-1}(\mathcal{N}_{k-1}+1)\mathbf{Z}_{k-1}^H(\mathcal{N}_{k-1}+1)$ and $E\{\mathbf{Z}_k(n)\mathbf{Z}_k^H(n)\}$, respectively.

It is found that K affects the accuracy of estimating $\mathbf{A}^{-1}$ in (6.14). During convergence, the ROE is accumulated but negligible until the convergence process makes the matrix inverse estimate very close to the actual matrix inverse. In such cases, some elements of the estimated matrix achieves significantly high values, especially when $\mathbf{A}$ is considerably ill-conditioned. As a result, the accumulated ROE increases exponentially, which, in turn, increases the output error. Simulation results show that when K is larger, the best estimate of $\mathbf{A}$ can be obtained in fewer number of iterations. It is found after K achieves a critical value, the estimation accuracy of $\mathbf{A}^{-1}$ cannot increase further. Therefore, the optimal balance point should be accomplished between the possible maximum accuracy of $\mathbf{A}^{-1}$ es-

Figure 6.2: Schematic setup diagram for soundfield control system with loudspeaker array and planar listening point array.

time and the convergence rate. Simulation results exhibit that this balance is compromised when $2 \leq K \leq 4$.

It will be shown, by way of simulation, that the performance of the MLN is better than that of single-layer network. To illustrate the performance of the algorithm at each iteration, an estimate of (6.14) for the k th layer is first defined as

$$\Psi_k(n) = \tilde{\mathbf{P}}_k^H(n) \tilde{\mathbf{P}}_k(n), \quad (6.17)$$

where $\tilde{\mathbf{P}}_k(n) = \prod_{l=1}^k \mathbf{P}_{k-l+1}(n)$. In this illustrative example, elements $G_{ij}(f)$ were generated synthetically using $f = 500$ Hz, $c = 340$ m/s with uniform linear array of $M = 20$ loudspeakers and uniform square listening point planar array with $N = 25$ listening points. The above setup corresponds to an over-determined case. An inter-loudspeaker spacing of $r_s = 0.08$ m is used while the listening points were uniformly separated at a distance $r_o = 0.05$ m. The center of the listening points is positioned 2.5 m away from the centroid of the loudspeaker array. The distribution ratio $\mathbf{D}$ which has been defined in (2.7) is set in a

way such that the magnitude of its elements in the audible as one and that in the inaudible listening zone as zero. This corresponds to an ideal situation where the contrast is high. The audible and inaudible zones are set so that inner nine listening points are located at the audible zone and the remaining sixteen listening points are located at the outer side corresponding to the inaudible zone. In the same manner as [41] [42], the magnitude of sound signal in the audible listening zone is set to a maximum level of one while the sound signal magnitude in the inaudible listening zone is set to zero. In addition, similar to [42], the average delay that sound signal propagates from loudspeaker array center to each listening point is added to the corresponding element of $\mathbf{D}$ to ensure causality. Figure 6.2 shows the system setup. A three-layer network is employed with $\tilde{\eta}_1 = 0.8$, $\tilde{\eta}_2 = 0.4$ and $\tilde{\eta}_3 = 0.08$ while network layer boundary iteration indices $\mathcal{N}_1 = \mathcal{N}_2 = \mathcal{N}_3 = 3000$ are used. To quantify the performance of the proposed algorithm, we compare the normalized output error (NOE) defined as

$$\tilde{e}(n) = \|\mathbf{X}\mathbf{D} - \mathbf{G}\mathbf{X}\hat{\mathbf{w}}_o(n)\|/\|\mathbf{X}\mathbf{D}\|, \quad (6.18)$$

where $\hat{\mathbf{w}}_o(n) = \mathbf{\Psi}_k(n)\mathbf{G}^H\mathbf{D}$. As can be seen from Fig. 6.3, $\tilde{e}(n)$ first reduces and then increases significantly beyond 6200 iterations. For simplicity of comparison, we define another form of $\tilde{e}(n)$ in dB as

$$\tilde{e}_{\text{dB}}(n) = 20 \times \log_{10}(\tilde{e}(n)). \quad (6.19)$$

The NOE in dB in (6.19) has been illustrated in Fig. 6.3 for better resolution when NOE is varying from a small value to a significant large value with number of iterations for MLN system. This simulation therefore allows us to gain an insight into this misconvergence problem for MLN during the iteration process and allows one to propose a new method to address it in following sections.

Figure 6.3: The amount of NOE $\tilde{e}(n)$ in dB with the iteration in MLN when $K = 3$.

6.4 The Misconvergence Problem for MLN within Iteration Process

It is noted, from the above illustrative example, that the proposed algorithm cannot be directly applied for reducing the NOE since the algorithm may misconverge. This misconvergence is due to the intrinsic problem of estimating $\mathbf{A}^{-1}$ particularly when $\mathbf{A}$ is ill-conditioned. To understand the root cause of this misconvergence, Fig. 6.4 shows how the condition number of $\Psi_k(n)$ varies with the number of iterations for the same simulation setup as described in Fig. 6.2. Since the initial value of $\Psi_1(n)$ is an identity matrix, it has a condition number equivalent to one which allows $\Psi_k(n)$ to converge toward $\mathbf{A}^{-1}$. In cases where $\mathbf{A}$ is ill-conditioned, the condition of $\mathbf{A}^{-1}$ is correspondingly poor. Therefore, as the algorithm converges to $\mathbf{A}^{-1}$, $\chi_2(\Psi_k(n))$ increases accordingly, as shown in Fig. 6.4. As a result, this poor conditioning will lead to high ROE accumulating as described in Sec-

Figure 6.4: The changing magnitude of $\chi_2(\Psi_k(n))$ during iteration within an MLN.

tion 4.2.

To gain insights into the above, Figs. 6.5 (a) and (b) show the magnitude of the elements of $\mathbf{P}_3(n)$ at the 6200th and 9000th iteration, respectively. It can be seen that the magnitude of elements in $\mathbf{P}_3(n)$ is comparatively smaller at the 6200th iteration than that at the 9000th iteration. As a result of (4.1) and (4.9), ROE that arises from the computation of $\Psi_3(n)$ accumulates and increases significantly which, in turn, results in a high $\tilde{e}_{\text{dB}}(n)$ for $n > 6200$ as illustrated in Fig. 6.3 at the third layer. The effect of ROE can be further illustrated by showing how the magnitudes of elements in $\Psi_3(n)\mathbf{G}^H\mathbf{G}$ varies with the number of iterations. Figures 6.6 (a) and (b) show a 3D plot of the magnitudes of elements in $\Psi_3(n)\mathbf{G}^H\mathbf{G}$ at the 6200th and 9000th iteration, respectively. As can be seen, $\Psi_3(n)\mathbf{G}^H\mathbf{G}$ is closer to an identity matrix that is more diagonal-dominated at $n = 6200$ compared to when $n = 9000$. It is therefore foreseeable that a close-identity matrix $\Psi_3(n)\mathbf{G}^H\mathbf{G}$ is desirable at $n = 6200$. As a result, the performance of the algorithm degrades with increasing

Figure 6.5: The 3D plot of magnitude for each element in matrix $P_3(n)$ at (a) 6200th and (b) 9000th iteration.

Figure 6.6: The 3D plot of magnitude for each element in matrix $\Psi_3(n) \mathbf{G}^H \mathbf{G}$ at (a) 6200th and (b) 9000th iteration.

number of iterations beyond the initial convergence at the 6200th iteration. With reference to (6.14) where $(\mathbf{G}^H \mathbf{G})^{-1}$ is estimated by Ψ , the misconvergence may be quantified by how close $\Psi_3(n) \mathbf{G}^H \mathbf{G}$ is to an identity matrix and as such, a sparseness control mechanism is proposed to avoid the misconvergence problem during iteration in MLN system.

6.5 Misconvergence Analysis and Corresponding Improvement for the Proposed Method

In practical situation, the ROE is always present in the network update process [32]. Based on (4.1), the stored version of $\mathbf{P}(n+1)$ can be expressed as

$$\bar{\mathbf{P}}(n+1) = \mathbf{P}(n) + \sum_{i=1}^n \mathbf{E}(i), \quad (6.20)$$

where $\mathbf{E}(i) = \Upsilon_i \mathbf{P}(i)$ is the ROE for i th iteration and Υ_i is the relative round error matrix. Substituting (6.20) into (6.8), (6.8) can be expanded as

$$\bar{\mathbf{P}}(n+1) = \mathbf{P}(n) + \eta[\mathbf{I} - \mathbf{P}(n) \mathbf{A} \mathbf{P}^H(n)] + \Upsilon_n \mathbf{P}(n+1). \quad (6.21)$$

Substituting (6.20) into (6.21) and we obtain

$$\eta[\mathbf{I} - \mathbf{P}(n) \mathbf{A} \mathbf{P}^H(n)] = \Phi(n), \quad (6.22)$$

where $\Phi(n) = \sum_{i=1}^n \mathbf{E}(i) - \Upsilon_n \mathbf{P}(n+1)$ is the accumulating ROE matrix at iteration n . At steady state where n is adequately large enough, $\Phi(n) \approx \sum_{i=1}^{n-1} \mathbf{E}(i)$ when $\mathbf{E}(n) \approx \mathbf{E}(n+1)$.

Under ill-conditioned situations, the ROE increases significantly with accumulation and therefore $\mathbf{E}(n)$ cannot be ignored when the update process exceeds a specific number of

iterations denoted by the critical point $\mathcal{N}_c$. After the process reaches $n = \mathcal{N}_c$, ROE begins to increase significantly with accumulation. As shown in Fig. 6.5, the magnitudes of elements in $\mathbf{P}(n = 9000)$ are much larger than those in $\mathbf{P}(n = 6200)$. As a result, with reference to (4.1), $\Phi(n = 9000)$ has significant value while $\Phi(n = 6200)$ is comparatively negligible. Consequently, there is no steady state for the update equation (6.21) as $\Phi(n > \mathcal{N}_c)$ cannot be considered as $\mathbf{0}$.

Substituting $\hat{\mathbf{P}}(\mathcal{N}_c + 1)$ into (4.1) in Section 4.1 where the ROE is introduced, it is concluded that

$$\hat{\mathbf{P}}(\mathcal{N}_c + 1) = \mathbf{P}(\mathcal{N}_c + 1) + \epsilon_{\mathcal{N}_c+1} \mathbf{P}(\mathcal{N}_c + 1). \quad (6.23)$$

Therefore, substituting (6.23) into (6.22), we obtain

$$\Phi = \eta [\mathbf{P} \mathbf{A} \mathbf{P}^H - \mathbf{I}] \quad (6.24)$$

with $\mathbf{P} = \mathbf{P}(\mathcal{N}_c) \approx \mathbf{P}(\mathcal{N}_c + 1)$ at steady state and $\Phi = \Phi_c(\mathcal{N}_c) - \epsilon_{\mathcal{N}_c+1} \mathbf{P}(\mathcal{N}_c + 1) \approx \Phi_c(\mathcal{N}_c - 1)$.

To further investigate how the proposed method can mitigate the effect of ROE during iteration process, (6.22) is rewritten as

$$\mathbf{I} = \mathbf{P}(n) \mathbf{A} \mathbf{P}^H(n) + \frac{1}{\eta} \Phi(n). \quad (6.25)$$

On the other hand, (6.25) can be rewritten as

$$\mathbf{I} = \mathbf{P}(n) (\mathbf{A} + \mathcal{R}(n)) \mathbf{P}^H(n), \quad (6.26)$$

where

$$\mathcal{R}(n) = (\eta \mathbf{P}^H(n) \Phi^{-1}(n) \mathbf{P}(n))^{-1}. \quad (6.27)$$

As a result, we have

$$\mathbf{P}^H(n)\mathbf{P}(n) = (\mathbf{A} + \mathcal{R}(n))^{-1}. \quad (6.28)$$

Substituting $n = \mathcal{N}_c$ into (6.28), we have

$$\mathbf{P}^H\mathbf{P} = (\mathbf{A} + \mathcal{R})^{-1}, \quad (6.29)$$

where $\mathbf{P} = \mathbf{P}(\mathcal{N}_c)$ and $\mathcal{R} = \mathcal{R}(\mathcal{N}_c)$.

Substitute (6.28) into (6.15a) - (6.15c), the equivalent form of estimated loudspeaker weight coefficients for well-, over- and under-determined cases can be expressed as

$$\hat{\mathbf{w}}_w = (\mathbf{G} + \mathcal{R})^{-1}\mathbf{D}, \quad (6.30a)$$

$$\hat{\mathbf{w}}_o = (\mathbf{G}^H\mathbf{G} + \mathcal{R})^{-1}\mathbf{G}^H\mathbf{D}, \quad (6.30b)$$

$$\hat{\mathbf{w}}_u = \mathbf{G}^H(\mathbf{G}\mathbf{G}^H + \mathcal{R})^{-1}\mathbf{D}. \quad (6.30c)$$

From (6.30a) and (6.30c), it can be seen that $\mathcal{R}$ represents an equivalent regularisation matrix that is a function of the accumulated ROE and network weight matrix $\mathbf{P}$. Consequently, the new form of Weiner solution is equivalent to the LS optimization solution for the following cost function

$$J = \|\mathbf{Y} - \mathbf{D}\|^2 + \|\mathcal{A}\mathbf{w}\|^2, \quad (6.31)$$

where $\mathcal{A}$ is a $M \times M$ matrix which controls the gain of the loudspeaker array and $\mathcal{A} = \mathbf{U}\mathcal{R}^{1/2}$ where $\mathbf{U}$ is a unitary matrix. In this case, $\mathbf{P}$ can be further represented as

$$\mathbf{P} = (\mathbf{A} + \mathcal{R})^{-1/2}, \quad (6.32)$$

where $(\mathbf{A} + \mathcal{R})^{-1/2}$ is the symmetric square-root factor of the inverse $\mathbf{A} + \mathcal{R}$. Substituting

(6.32) into (6.27), we have

$$\begin{aligned}
\mathcal{R}(n) &\approx [\eta(\mathbf{A} + \mathcal{R}^H(n))]^{-1/2} \Phi(n)^{-1} [\mathbf{A} + \mathcal{R}(n)]^{-1/2}]^{-1} \\
&\approx [\eta \mathbf{A}^{-1/2} \Phi^{-1}(n) \mathbf{A}^{-1/2}]^{-1} \\
&= \eta^{-1}(\mathbf{A})^{1/2} \Phi(n) \mathbf{A}^{1/2}.
\end{aligned} \tag{6.33}$$

It is therefore noted from (6.29) and (6.33) that, in the absence of ROE, $\mathcal{R}(n) = \mathbf{0}$ and we achieve exact matrix inverse using (6.28) while in the presence of ROE, $\mathcal{R}(n)$ is a non-zero matrix which avoids a large deviation of the computed inverse matrix from the actual inverse matrix by direct inverse method. This explains why NOE reduces first and subsequently increases in Fig. 6.3. As $\mathcal{R}$ is extracted at the critical point $n = \mathcal{N}_c$ to give the minimum NOE, before the algorithm reaches steady-state ($n = \mathcal{N}_c$), the RE is greater than the ROE while after steady-state, the converse is true as shown in Fig. 6.3. After iteration reaches $\mathcal{N}_c$, the ROE dominates the NOE and therefore, NOE increases with iterations. Therefore, it is critical and necessary to find a solution which stops the iteration at $n = \mathcal{N}_c$ to prevent accumulation of large amount of ROE.

6.6 Matrix Sparseness Measure for Preventing Misconvergence Issue

To identify the critical point $\mathcal{N}_c$ so as to prevent misconvergence, we introduce a *matrix sparseness* measure to stop the iterative process described by (6.8) when NOE achieves its minimum. The proposed matrix sparseness is derived from vector sparseness defined by [43]

$$\xi(\mathbf{v}) = \frac{L}{L - \sqrt{L}} \left(1 - \frac{\|\mathbf{v}\|_1}{\sqrt{L}\|\mathbf{v}\|} \right), \tag{6.34}$$

where $\mathbf{v}$ is a $L \times 1$ vector and $\|\cdot\|_1$ denotes the l_1 -norm of the vector. It has been shown that $0 \leq \xi(\mathbf{v}) \leq 1$ and that for a perfectly sparse vector where $\mathbf{v}$ is a Kronecker delta function, $\xi(\mathbf{v}) = 1$. On the other hand, for a perfectly dispersive vector where elements of $\mathbf{v}$ are of the same magnitude, $\xi(\mathbf{v}) = 0$. The notion of sparseness has been applied for sparse adaptive filtering to improve convergence rate [44] for sparse/dispersive impulse responses.

We extend the notion of vector sparseness to *matrix sparseness* for a $L \times L$ square matrix $\mathbf{V}$ given by

$$\begin{aligned} \xi(\mathbf{V}) = & \frac{L}{(2L+1)(L-\sqrt{L})} \left[2L - \frac{1}{\sqrt{L}} \sum_{i=1}^L \left(\frac{\|\mathbf{v}_r(i)\|_1}{\|\mathbf{v}_r(i)\|} + \frac{\|\mathbf{v}_c(i)\|_1}{\|\mathbf{v}_c(i)\|} \right) \right] \\ & + \frac{1}{2L+1} \left[1 - \frac{L}{L-\sqrt{L}} \left(1 - \frac{\|\mathbf{v}_d\|_1}{\sqrt{L}\|\mathbf{v}_d\|} \right) \right], \end{aligned} \quad (6.35)$$

where $\mathbf{v}_r(i)$, $\mathbf{v}_c(i)$ and $\mathbf{v}_d$ are the i th row, i th column, and diagonal vector of $\mathbf{V}$, respectively. The matrix sparseness $\xi(\mathbf{V})$ is therefore computed by summing $\xi(\mathbf{v}_c(i))$, $\xi(\mathbf{v}_r(i))$, for $i = 1, \dots, L$, and $\xi(\mathbf{v}_d)$ after which the result is normalized by $2L+1$ such that $0 < \xi(\mathbf{V}) \leq 1$. We therefore note that $\xi(\mathbf{V}) \approx 0$ when $\mathbf{V} = \alpha \mathbf{\check{U}}$ where α is a scalar and $\mathbf{\check{U}}$ is an $L \times L$ unit matrix of ones. On the other hand, $\xi(\mathbf{V}) = 1$ when $\mathbf{V} = \mathbf{I}$.

To visualize the implication of matrix sparseness, it is found that the matrix sparseness will increase with the matrix being diagonally dominant - the matrix sparseness $\xi(\mathbf{V})$ can effectively approximate the $\chi_2(\mathbf{V})$ in an iterative process if $\mathbf{V}$ varies with time.

We show how $\xi(\mathbf{V})$ varies with the matrix condition number $\chi_2(\mathbf{V})$. In this illustrative example, 100 independent matrices $\mathbf{V}$ with size of 20×20 are generated. After the initialization, $\mathbf{V}$ is updated using

$$\mathbf{V}(n+1) = \beta \mathbf{I} + (1-\beta)\mathbf{V}(n), \quad (6.36)$$

where $\mathbf{V}(0)$ is a 20×20 random matrix generated using Gaussian distribution process with

Figure 6.7: The averaged matrix sparseness $\xi(\mathbf{V})$ across 100 trials varies with β .

a standard deviation of one and n is the number of iterations. Different $\mathbf{V}(n)$ are generated in a single trial by varying β from 0 to 1 with a step-size of 0.01. The variables $\xi(\mathbf{V}(n))$ and $\chi_2(\mathbf{V}(n))$ are determined for each $\mathbf{V}(n)$. The above process is repeated for 100 different values of $\mathbf{V}(n)$ and the average $\xi(\mathbf{V}(n))$ and $\chi_2(\mathbf{V}(n))$ are computed across these 100 trials. It can be seen from (6.36) that the diagonal elements of $\mathbf{V}(n)$ are increased by a factor of β . This implies that $\chi_2(\mathbf{V}(n))$ decreases with increasing β . This observation is consistent with the Tikhonov regularisation method [26]. Figure 6.7 shows that $\xi(\mathbf{V})$ increases monotonically with β , which further indicates that $\xi(\mathbf{V})$ increases monotonically with $\chi_2(\mathbf{V})$. Therefore, the exact value of condition number is *not* required for the sound-field control system, which further implies that $\xi(\mathbf{V})$ is a good reflection of $\chi_2(\mathbf{V})$ in an efficient manner.

After exploiting the characteristic of the matrix sparseness to prevent misconvergence, we incorporate the matrix sparseness measure to the proposed algorithm so that the point

Figure 6.8: Variation of matrix sparseness $\xi(\Psi_k(n)\mathbf{G}^H\mathbf{G})$ for over-determined soundfield control system when $k = 1, 2, 3$ with iterations.

of misconvergence can be identified and the iterative process described by (6.8) can be terminated. To illustrate the above, Fig. 6.8 shows the variation of $\xi(\Psi_k(n)\mathbf{G}^H\mathbf{G})$ for an over-determined case with iterations for the same setup as described in Section 6.4. It can be seen that the matrix sparseness achieves its maximum at approximately 6200th iteration just before the starting point of misconvergence as shown in Fig. 6.3. Therefore, as $\Psi_k(n)\mathbf{G}^H\mathbf{G}$ approaches an identity matrix during convergence, $\xi(\Psi_k(n)\mathbf{G}^H\mathbf{G})$ allows the algorithm to quantify how close $\Psi_k(n)\mathbf{G}^H\mathbf{G}$ is to an identity matrix and as a result, how well $\Psi_k(n)$ can estimate $(\mathbf{G}^H\mathbf{G})^{-1}$ which is equivalent to the minimum NOE as shown in Fig. 6.3. It is noted that finding $\mathcal{N}_c$ by way of searching for minimum NOE is not possible in practice since the parameter $\mathcal{R}(n)$ is unknown. As a result we employ matrix sparseness $\xi(\Psi\mathbf{G}^H\mathbf{G})$ to estimate the critical point. It is found that a more accurate $\Psi_k(n)$ results in $\Psi_k(n)\mathbf{G}^H\mathbf{G}$ being closer to $\mathbf{I}$. And hence, sparseness $\xi(\Psi_k(n)\mathbf{G}^H\mathbf{G})$ is closer to one which corresponds to the sparseness of the identity matrix. In other words, $\Psi_k(n)$ is closer the actual $(\mathbf{G}^H\mathbf{G})^{-1}$.

The sparseness measure is incorporated so that the critical point $\mathcal{N}_c$ is estimated as

$$\mathcal{N}_c = \underset{n}{\operatorname{argmax}} \xi \left(\Psi_k(n) \mathbf{G}^H \mathbf{G} \right). \quad (6.37)$$

Substituting $n = \mathcal{N}_c$ in (6.28) and employing the result in (6.30b), therefore the estimate of $(\mathbf{G}^H \mathbf{G})^{-1}$ is obtained using

$$\hat{\Psi} = \Psi_k(\mathcal{N}_c). \quad (6.38)$$

Finally $\hat{\mathbf{w}}_o$ for over-determined case is obtained using

$$\hat{\mathbf{w}}_o = \hat{\Psi} \mathbf{G}^H \mathbf{D}, \quad (6.39)$$

which is the estimate of $\hat{\mathbf{w}}_o^{\text{opt}}$ in (2.9b). With reference to (6.30b), the best estimate of system matrix inverse can also be expressed as

$$\hat{\Psi} = (\mathbf{G}^H \mathbf{G} + \mathcal{R})^{-1}. \quad (6.40)$$

We therefore note from (6.40) that a more accurate $\hat{\Psi}$ results in $\mathcal{R}$ being closer to $\mathbf{0}$. As $\mathcal{R}$ is related to ROE as shown in (6.27), it is always involved in the estimation of matrix inverse. As a result, as can be seen from (6.18) and (6.30b), if ROE approaches zero which results in $\mathcal{R}$ approaching $\mathbf{0}$, NOE will approach zero. On the other hand, it is evident that maximizing the matrix sparseness $\xi(\mathbf{A}(\mathbf{A} + \mathcal{R}(n))^{-1})$ is equivalent to minimizing elements in $\mathcal{R}(n)$ which can effectively reduce $\tilde{e}(n)$. Consequently, matrix sparseness is introduced as a measurement such that elements in $\mathcal{R}(n)$ are minimized for an accurate inverse estimate. Therefore, the minimum achievable NOE could be identified by finding the maximum matrix sparseness, as shown in Fig. 6.8. Table 6.1 summarizes the proposed iterative linear network algorithm.

6.7 Proposed Method to Mitigate the Effect of MPE

To evaluate the effect of MPE, and considering the over- or well- determined case as an example, we assume that $\Delta \mathbf{G}$ is a white Gaussian noise and uncorrelated with the $\mathbf{w}$ such that $E\{\Delta \mathbf{G}\} = \mathbf{0}$ and $E\{\Delta \mathbf{G}^H \Delta \mathbf{G}\} = \Theta$. When MPE is present in the system, the output error in (2.8) can be rewritten as

$$e_M = \|(\mathbf{G} + \Delta \mathbf{G})\mathbf{X}\mathbf{w} - \mathbf{X}\mathbf{D}\|_2. \quad (6.41)$$

As a result, the cost function $J_M = E\{e_M^2\}$ can be expanded as

$$J_M = X^2 [(\mathbf{G}\mathbf{w} - \mathbf{D})^H(\mathbf{G}\mathbf{w} - \mathbf{D}) + \mathbf{w}^H \Theta \mathbf{w}]. \quad (6.42)$$

The Wiener solution for (6.42) is obtained by

$$\tilde{\mathbf{w}}_o = (\mathbf{G}^H \mathbf{G} + \Theta)^{-1} \mathbf{G}^H \mathbf{D}. \quad (6.43)$$

As can be seen from (6.43), the solution obtained deviates from the optimal solution $\mathbf{w}_o^{\text{opt}}$ in (2.9b) which is free of MPE. The matrix Θ can be treated as a special regularisation matrix to the system matrix $\mathbf{A}$. Substituting $\tilde{\mathbf{G}} = \mathbf{G} + \Delta \mathbf{G}$ into (6.8), we achieve

$$\mathbf{P}(n+1) = \mathbf{P}(n) + \eta [I - \mathbf{P}(n)(\mathbf{G} + \Delta \mathbf{G})^H(\mathbf{G} + \Delta \mathbf{G})\mathbf{P}(n)]. \quad (6.44)$$

Taking the expectation at both sides of (6.44), with knowing that $E\{\mathbf{P}(n+1)\} = E\{\mathbf{P}(n)\} = \mathbf{P}$ for steady state, we have

$$\mathbf{I} = \mathbf{P} E\{(\mathbf{G} + \Delta \mathbf{G})^H(\mathbf{G} + \Delta \mathbf{G})\} \mathbf{P}^H. \quad (6.45)$$

With assumption that $E\{\Delta\mathbf{G}\} = \mathbf{0}$ and $E\{\Delta\mathbf{G}^H\Delta\mathbf{G}\} = \Theta$, (6.45) can be expanded as

$$\mathbf{I} = \mathbf{P}(\mathbf{G}^H\mathbf{G} + \Theta)\mathbf{P}^H. \quad (6.46)$$

Rearranging (6.46) to obtain the estimated new system inverse

$$(\mathbf{G}^H\mathbf{G} + \Theta)^{-1} = \mathbf{P}\mathbf{P}^H. \quad (6.47)$$

Substituting (6.47) into (6.43), the loudspeaker weight $\tilde{\mathbf{w}}_o$ therefore can be estimated using the proposed algorithm. The system matrix of the proposed algorithm can therefore be modified to

$$\hat{\mathbf{A}} = \mathbf{A} + \Theta, \quad (6.48)$$

where $\Theta = E\{\Delta\mathbf{G}^H\Delta\mathbf{G}\}$ for over-determined case, $\Theta = E\{\Delta\mathbf{G}\Delta\mathbf{G}^H\}$ for under-determined case and $\Theta = E\{\Delta\mathbf{G}\}$ for well-determined case. Hence for computing the corresponding system matrix inverse estimate, for the proposed algorithm can be changed as

$$\mathbf{P}_1\mathbf{P}_2\ldots\mathbf{P}_K\hat{\mathbf{A}}\mathbf{P}_K^H\ldots\mathbf{P}_2^H\mathbf{P}_1^H = \mathbf{I}. \quad (6.49)$$

Consequently the new system matrix inverse is estimated as

$$\mathbf{P}_1\mathbf{P}_2\ldots\mathbf{P}_K\mathbf{P}_K^H\ldots\mathbf{P}_2^H\mathbf{P}_1^H = (\mathbf{A} + \Theta)^{-1}. \quad (6.50)$$

As can be seen from (6.43) and (6.50), the proposed method can mitigate the effect of MPE on soundfield control by taking MPE characteristics Θ as the regularisation parameter.

6.8 Summary

This chapter introduces the proposed method which can iteratively estimate the system matrix inverse instead of direct computation when the matrix is ill-conditioned. The iterative matrix inverse estimation process can avoid accumulating large amount of ROE which can cause the computed value significantly deviating from its actual value. Inspired by the single-layer linear network utilized for de-correlating the acoustic tap-input vectors, the multi-layer network is proposed and generalized for the soundfield control system with taking the acoustic transfer function $\mathbf{G}$ as the network input while the network output is the corresponding identity matrix. Instead of attempting to eliminate the ROE from the computation process, the proposed MLN method takes the residual ROE as the regularisation parameter matrix, which effectively improves the conditioning of the system matrix. As a result, the matrix inverse estimation accuracy can be improved. However, it is also found that the MLN update process will misconverge, where the normalized output error $\tilde{e}(n)$ increases after sometime. This critical turning point, where the normalized output error starts increasing has been identified as critical process termination point denoted by $\mathcal{N}_c$. A new process measure - matrix sparseness is introduced and this measure is incorporated into the iterative estimation process for terminating the process at the $\mathcal{N}_c$ point. As a result, the soundfield control system performance is maximized in terms of suppressing the NOE to a minimize level by terminating the matrix inverse estimation process at the $\mathcal{N}_c$ point. In addition, it is also found that the MPE is always present in a real soundfield control system. This MPE affects the accuracy of the generated soundfield. The effect of the MPE is shown to be amplified by the conditioning of system matrix in Section 5.3. Therefore, improving the conditioning of the system matrix is equivalent to mitigating the effect of MPE on the soundfield control system. As discussed and analyzed in Section 6.5, the proposed MLN method utilizes the ROE as a regularization parameter, which implies that the proposed method effectively improves the conditioning of the system matrix. Therefore the proposed

method can mitigate the effect of MPE on soundfield control by taking MPE characteristics Θ as the regularisation parameter. Under the scenario that the residual ROE - the equivalent regularisation parameter, equals to the MPE characteristics Θ , the optimal balancing point between reducing the ROE and increasing of RE due to MPE is achieved. Consequently the optimal loudspeaker weight coefficients are obtained for the best performance setup of the soundfield control system.

Table 6.1: The proposed linear multi-layer network LS based matrix inverse algorithm.

User-defined parameters

K : Number of layers

n : Iteration index

η_k : Step-size for k th layer

$\xi(\mathbf{A})$: Matrix sparseness of matrix $\mathbf{A}$

$\mathcal{N}_c$: Critical iteration index where the process starts misconverge

$\mathcal{N}_k, 1 \leq k \leq K$: Maximum umber of iterations for the k th layer

Initialization

$$\mathbf{P}_k(n) = \begin{cases} \mathbf{0}_{N \times N}, & n \leq \mathcal{N}_{k-1} \\ \mathbf{I}_{N \times N}, & n = \mathcal{N}_{k-1} + 1 \end{cases}$$

For each iteration n

$$\mathbf{P}_k(n+1) = \mathbf{P}_k(n) + \eta_k(n) \{ \mathbf{I} - \mathbf{Z}_k(n) \mathbf{Z}_k^H(n) \}$$

$$\eta_k(n) = \begin{cases} \tilde{\eta}_k, & \mathcal{N}_{k-1} < n \leq \mathcal{N}_k \\ 0, & \text{otherwise} \end{cases}$$

$$\tilde{\mathbf{P}}_k(n) = \prod_{l=1}^k \mathbf{P}_{k-l+1}(n)$$

$$\mathbf{Z}_k(n) = \begin{cases} \tilde{\mathbf{P}}_k(n) \mathbf{G}_{N \times M}, & N \leq M \\ \tilde{\mathbf{P}}_k(n) \mathbf{G}_{N \times M}^H, & N > M \end{cases}$$

$$\Psi_k(n) = \tilde{\mathbf{P}}_k^H(n) \tilde{\mathbf{P}}_k(n)$$

Terminate Iteration Process

$$\mathbf{P}^H \mathbf{P} = (\mathbf{A} + \mathcal{R})^{-1}$$

$$\mathcal{R} = \eta^{-1} (\mathbf{A})^{1/2} \Phi(n) \mathbf{A}^{1/2})^{-1}$$

$$\mathcal{N}_c = \underset{n}{\operatorname{argmax}} \xi(\Psi_k(n) \mathbf{A})$$

$$\hat{\Psi} = \Psi_k(\mathcal{N}_c)$$

$$\hat{\mathbf{w}} = \begin{cases} \mathbf{G}^H \hat{\Psi} \mathbf{D}, & N \leq M \\ \hat{\Psi} \mathbf{G}^H \mathbf{D}, & N > M \end{cases}$$

Chapter 7

Simulation and Experiment Results

7.1 Simulation Results

We evaluate the performance of the proposed algorithm in the context of soundfield control for different frequencies f and loudspeaker/listening point configurations for both over-determined (OD) and under-determined (UD) cases. We first consider a similar setup in Fig. 6.4 with $M = 20, N = 25$ and $f = 500$ Hz for the OD case while $M = 30, N = 25$ and $f = 800$ Hz for the UD case where the audible zone is represented by a squared uniform planar listening point array. The centroid of loudspeaker array is 3 m away from that of the listening points. Similar with Fig. 6.2 setup, we have $r_s = 0.12$ m and $r_o = 0.06$ m. A three-layer network with step-sizes $\tilde{\eta}_1 = 0.8, \tilde{\eta}_2 = 0.4, \tilde{\eta}_3 = 0.05$ and iteration indices $\mathcal{N}_1 = \mathcal{N}_2 = \mathcal{N}_3 = 3000$ is employed for the proposed LNN method. The audible zone is such that 9 inner listening points are located in the audible zone and the remaining 16 listening points are located in the inaudible area. For the GCV and OCV methods, we search for an optimal λ between 0 and 0.6 in steps of 1×10^{-6} . For the truncation method, we have used $\lambda = 10^{-6}$. For performance evaluation, we employed the NOE defined in (6.19) for all LS methods.

Figure 7.1 compares the performance of various algorithms in terms of across iterations

for both OD and UD cases. For this simulation, $\tilde{e}_{\text{dB}}(n)$ varies with the number of iterations for the proposed algorithm while, on the other hand, $\tilde{e}_{\text{dB}}(n)$ does not depend on the number of iterations for the other methods. This is the key difference between the proposed iterative converging method and other search methods. It can be seen that $\tilde{e}_{\text{dB}}(n)$ is the highest for the conventional LS algorithm. This poor performance is due to the high ROE incurred without any form of regularisation. The regularisation-based methods (GCV, OCV and truncation) can achieve better performance due to the improved conditioning of $\hat{\mathbf{A}}$ with regularisation. Among these regularisation techniques, the OCV achieves the best performance since it searches for a λ that minimizes $\tilde{e}$. More importantly, we note that the proposed algorithm, with $\mathcal{N}_c \approx 8000$ for UD case and $\mathcal{N}_c \approx 7000$ for OD case detected using the sparseness control mechanism, achieves the best performance in these 2 scenarios. With $\tilde{e}_{\text{dB}}(n) = -2.6$ dB for $n \geq \mathcal{N}_c$ in UD case, the proposed algorithm achieves an improvement of 1, 1, 1 and 14.8 dB over that of the OCV, GCV, truncation and conventional LS algorithms, respectively. Meanwhile the proposed algorithm has similar performance in OD case compared with UD case, achieving $\tilde{e}_{\text{dB}}(n) = -2.7$ dB for $n \geq \mathcal{N}_c$, obtaining 2, 1.1, 0.4 and 24.5 dB improvement over that of the OCV, GCV, truncation and conventional LS methods. This improvement arises since the proposed algorithm introduces the regularisation in terms of taking ROE as the regularisation parameter via the sparseness control mechanism which described in Section 6.6.

Comparing Fig. 7.1(a) and (b), these results suggest that the performance of different algorithms do not vary with the loudspeaker/listening points setup change when input frequency is fixed. To further gain insight the effect of the input frequency on the performance of the LS based algorithms, the performance of the algorithms, in terms of $\tilde{e}_{\text{dB}}(n)$ across different input frequencies are compared in Figure 7.2 . We set the audible and inaudible areas as 1 and 0.01 in terms of ratio D . The inaudible zone with scale ratio of 0.01 is because it is more realistic and applicable in the real application instead of using 0. For this simulation, $\tilde{e}_{\text{dB}}(n)$ is evaluated for each algorithm with $1 \leq f \leq 20$ kHz in steps of 500 Hz. A three-

layer linear network with step-sizes $\tilde{\eta}_1 = 0.8$, $\tilde{\eta}_2 = 0.4$, $\tilde{\eta}_3 = 0.08$ and iteration indices $\mathcal{N}_1 = \mathcal{N}_2 = \mathcal{N}_3 = 5000$ for both OD and UD cases is employed. The linear loudspeaker array with $M = 75$ and square planner listening point array with $N = 125$ are configured for OD case setup while for UD case setup, $M = 130$ and $N = 125$. These results are shown in Fig. 7.2 for the proposed algorithm correspond to $\tilde{e}(n = \mathcal{N}_c)$. A regularisation parameter $\lambda = 10^{-6}$ is used for the truncation method in order to achieve the lowest averaged $\tilde{e}$ across frequencies while, similar to the above, an optimal value of $0 \leq \lambda \leq 0.6$ is searched in steps of 1×10^{-6} for the GCV, OCV methods.

As before, it can be seen that the conventional LS method achieves the worst average performance while the truncation method outperforms the OCV and GCV across most of the frequencies for OD case. Meanwhile the truncation method has the second worst performance, following by the OCV method, while the GCV and conventional method has similar better performance. The proposed LNN algorithm achieves the best performance with most of the input frequencies for both UD and OD cases. In addition, we note that the proposed method achieves the lowest $\tilde{e}_{\text{dB}}(n)$ with an average improvement of 6.1 dB over OCV, 2.2 dB over the method of truncation, 3 dB over GCV and 12.7 dB over conventional LS method in OD case while for UD case, the improvements are 11 dB, 12.7 dB, 5.4 dB and 26.3 dB over the OCV, truncation, GCV and conventional methods. For clarity of presentation, we did not include results for the conventional LS method since it fails to achieve comparable performance. We also note that the performance of each algorithm increases with frequency since $\chi_2(\mathbf{G}\mathbf{G}^H)$ has been found to reduce with increasing f [10]. It is also interesting to note that for high frequencies where $\mathbf{G}\mathbf{G}^H$ is modestly well-conditioned, the ROE is small and therefore, the conventional LS method and the proposed method outperform other regularisation-based methods since they do not introduce any RE.

We next evaluate the variation of $\tilde{e}_{\text{dB}}$ with respect to r_o for both OD and UD cases. We have fixed the listening point inter-spacing as $r_o = 0.08$ while the loudspeaker inter-spacing r_s varied between 0.05 m and 0.45 m with step-size of 0.01 m. A three-layer linear net-

work is employed with the same setup used for Fig. 7.1 and $f_0 = 2$ kHz. As before results shown in Fig. 10 for the proposed algorithm correspond to $\tilde{e}(n = \mathcal{N}_c)$. For the remaining regularisation-based methods, an optimal value of λ is obtained using same approach described in Fig. 7.1. Figure 7.3 compares the performance of the algorithms. As can be seen from Fig. 7.3, the proposed method achieves the best performance with the lowest $\tilde{e}_{\text{dB}}$ across r_s . Similar to Fig. 7.2, it is also interesting to note that the performance of the algorithms generally improve with increasing r_s since the conditioning of system matrix $\mathbf{A}$ improves with r_s [10]. It is also interesting to note that the regularisation-based methods cannot outperform the conventional LS method. For large values of r_s , the performance of the truncation method, L-curve and GCV degrade significantly due to the introduction of RE. The improvement in performance, averaged over $0.05 \leq r_o \leq 0.45$ m, of the proposed method is approximately 4.2 dB over OCV, 2.6 dB over truncation, 3.1 dB over GCV, and 9.7 dB over the conventional LS methods for OD case. Similarly, the improvement is approximately 7.8 dB over OCV, 3.5 dB over truncation, 5.2 dB over GCV, and 26.8 dB over the conventional LS methods for UD case. Again the conventional LS method is not included in the figures since it fails to achieve comparable performance with others. As discussed in Section 6.5, the proposed LNN regularizes the system matrix $\mathbf{A}$ by utilizing the varying ROE with iteration. The benefit of the the proposed LNN method is that the regularisation parameter relatively large when $\mathbf{A}$ is extremely ill-conditioned while it becomes negligible small when $\mathbf{A}$ is well-conditioned as ROE is very small.

We extended the evaluation of LS based methods performance when MPE is present in the system. The simulation setup is the same with setup in Fig. 7.1 except $M = 40$, $N = 125$, $r_o = 0.06$ m, $r_s = 0.1$ m and $f_0 = 2$ kHz. The misplacement error is set as 10% of the loudspeaker inter-spacing r_s with Gaussian distribution probability as discussed in Section 6.7. A three-layer linear network is employed with the same setup used for Fig. 7.1 and $f = 2$ kHz. As before results shown in Fig. 10 for the proposed algorithm correspond to $\tilde{e}(n = \mathcal{N}_c)$. For the remaining regularisation-based LS methods, an optimal value of λ is

obtained using same approach described in Fig. 7.1 again.

It can be seen from Fig. 7.4 that the proposed algorithm outperforms existing LS methods where it achieves an average improvement of 1.8 dB, 1.1 dB, 1.1 dB, and 21.7 dB over OCV, truncation, GCV and conventional method respectively for OD case while for UD case, the improvement are 3.9 dB, 2.7 dB, 2.7 dB, and 21.5 dB respectively. It is also worthwhile to note that the proposed algorithm can achieve consistently good performance even when the MPE is present. The conventional LS method has the least tolerance of MPE when system matrix is extremely ill-conditioned. This also indicates that the regularisation method, no matter what algorithm is used, can somehow reduce the effect of MPE which is unavoidable in the real application.

7.2 Experiment Results

The performance of the algorithms is evaluated in an anechoic room as shown in Fig. 7.5. This anechoic room is located in Siemens Medical Instruments Pte. Ltd Singapore. The dimension of the room is $4.7\text{ m} \times 4.7\text{ m} \times 3.5\text{ m}$ and impulse responses were collected using nineteen AKG-C47 II microphones and a Genelec 1029A loudspeaker. The experimental setup is illustrated in Fig. 7.6. In total, nineteen positions were used within this rectangular listening zone. In this experiment, the loudspeaker played the sound source which is a maximum length signal (MLS) and the Analog-Digital Converter (ADC) converted the analog sound signals received from the microphones to digitized data saved as audio files in the harddisk of the laptop. The laptop served as a control station and the loudspeaker and microphones are assumed to be omni-directional. For each listening position point in the zone, the recording was conducted for approximately six seconds and the sampling rate for the ADC was set to 44.1 kHz. After that, the recording procedure was repeated for the next position until all the nineteen positions were recorded. The original intention for using measured room impulse response (RIR) in anechoic room for the experiment is its closeness to

Green's function derived in free space. The RIR in this anechoic room is employed to verify that the observations are consistent with the analysis presented in Chapter 2 and Chapter 6. For performance comparison, a three-layer network with the same step-sizes as previous simulations and iteration indices $\mathcal{N}_1 = 12000$, $\mathcal{N}_2 = 5000$ and $\mathcal{N}_3 = 4000$ is employed in this experiment. For the GCV and OCV methods, we search for an optimal regularized parameter λ from 0 to 0.6 in steps of 1×10^{-4} while the truncation method employs $\lambda = 0.1$. The normalized output error of the LS methods are evaluated over $100 \leq f \leq 1000$ Hz. Figure 7.7 shows the variations of $\tilde{e}$ for the algorithms in terms of f . Experiment results in Fig. 7.7 are generated using theoretical simulations based on measured RIR. As before, the curves are obtained by averaging over five independent trials where for each trial, elements in $\mathbf{D}$ are randomly generated using a white Gaussian process with a standard deviation of one.

It can be seen from Fig. 7.7 that the proposed algorithm outperforms existing LS methods where it achieves an average improvement of 3.4 dB, 3.6 dB and 9.8 dB over OCV, truncation and GCV, respectively. For clarity of presentation, we did not include results for the conventional LS method since it fails to achieve comparable performance. It is also worthwhile to note that the proposed algorithm can achieve consistently good performance across frequencies.

7.3 Summary

In the first part of this chapter, we compare the soundfield control performance for the proposed method and other LS-based methods in terms of $\tilde{e}_{\text{dB}}$ for both the OD and the UD cases. The simulation result suggests that the performance of different algorithms do not vary with the loudspeaker/listening points setup change when input frequency is fixed. To further gain insight for the effect of the input frequency on the performance of the LS based algorithms, the soundfield control performance is evaluated among all the methods with

$1 \leq f \leq 20$ kHz in steps of 500 Hz. It is found that the performance of each algorithm improves with frequency since the system matrix conditioning is better. It is also noted that when the system matrix becomes well-conditioned, there is less ROE accumulation than the introduction of RE for those regularisation methods. Following simulation is to investigate the effect of loudspeaker inter-spacing on the soundfield control performance by varying the inter-spacing from 0.05 m to 0.45 m for all the methods. The proposed algorithm always outperform other methods in the varying input frequency scenario and varying loudspeaker inter-spacing scenario. As MPE is one of the issues existed in the real application of soundfield control, better soundfield control performance using the proposed algorithm compared with other methods is also verified in a new simulation setup where the MPE is present. Besides these simulation setups, the proposed algorithm and other algorithms are also evaluated using a semi-synthesized acoustic transfer function in an anechoic room. The result is in line with previous simulation results that the proposed algorithm achieve better soundfield control quality in terms of less NOE.

Figure 7.1: Comparison of $\tilde{e}_{dB}(n)$ with iterations among different LS based methods for (a) Over- and (b) Under-determined case with fixed input signal frequency and set-up.

Figure 7.2: The NOE ($\tilde{e}_{dB}$) with frequencies among different methods (a) Over- (b) Under- determined cases.

Figure 7.3: The NOE $\tilde{e}_{\text{dB}}$ of different methods with respect to loudspeaker inter-spacing when $f_0 = 2$ kHz
(a) OD (b) UD cases.

Figure 7.4: The NOE $\tilde{e}_{dB}$ of different methods when MPE is present with $f = 2000$ Hz for (a) Over- (b) Under- determined cases.

Figure 7.5: Experiment setup using microphone array and single loudspeaker in an anechoic room.

Figure 7.6: The schematic diagram of the experiment setup in the anechoic room.

Figure 7.7: Variation of $\tilde{e}_{\text{dB}}$ with frequency in an anechoic room.

Chapter 8

Summary and Future Research

Direction

8.1 Summary

In this thesis, recent soundfield control techniques such as audio energy ratio maximization, audio energy difference maximization and multi-objective optimization including the directivity optimization with the constraint of given input energy are reviewed. It is found that the energy ratio maximization method has the disadvantage as it cannot guarantee the desired acoustic energy in audible region. At the same time, the audio energy difference maximization technique generates the desired acoustic energy in the audible region by maximizing the energy in audible region and minimizing the energy in inaudible region. However, the directivity of the sound beam cannot be guaranteed. Therefore, a multi-objective optimization technique is proposed to maximize the audio directivity for audible and inaudible regions while maintaining the acoustic energy in audible region at the desired level. We found that the multi-objective optimization technique is challenging and cannot always have a smooth control of all listening points in the audible regions. Several researchers proposed LS methods to precisely control of the soundfield using the loudspeaker array. Direct application of

such LS methods cannot always guarantee good results due to the ill-conditioning system matrix. In Chapter 2, the soundfield control system setup is reviewed and the whole system is formulated such that the problem can be presented and solved in a systematic manner. We next reviewed the LS based optimization methods such as OCV, GCV, L-curve and the truncation method which are applied to solve the matrix inverse problems when system matrix is ill-conditioned in Chapter 3. In the first part of Chapter 4, the ROE is studied and its effect on the soundfield control during the matrix inverse estimation is investigated first. The ROE occurs due to the limitation of the hardware and accumulates significantly in the matrix inverse estimation when the system matrix is ill-conditioned. It is also found that the MPE is always present in a practical soundfield control system and its effect on the soundfield performance in terms of normalized output error cannot be negligible when the measurement error of loudspeaker position is large. It is also proven in the last part of Chapter 5 that the effect of MPE on the soundfield control can be amplified by the condition number of system matrix. As a result, a new LS-based iterative method is proposed to overcome the ROE and MPE issues as mentioned in previous chapters. The proposed algorithm employs a linear single/multi-layer network that incorporates a sparseness control mechanism to estimate the coefficients for loudspeakers. Such iterative approach allows the algorithm to take into account the conditioning of the system matrix when estimating its inverse. Instead of eliminating the large accumulation of ROE, the proposed algorithm takes the dynamic ROE accumulation as a regularisation parameter therefore the conditioning of system matrix is improved. As a result, the computed loudspeaker weight coefficients achieves higher accuracy which leads to a better soundfield control performance in terms of smaller normalized output error. Both the simulation and experimental results show that the proposed method outperforms the conventional LS method, the truncation method and the Tikhonov regularisation methods including OCV, GCV and L-curve techniques.

8.2 Future Research Direction

Below are the possible directions for our future research:

8.2.1 Improvement of the current multi-layer network with matrix sparseness control

The current proposed method employs a three-layer network with three different network weights P_1 , P_2 and P_3 by making use of the matrix sparseness control. One possible way to reduce the computation complexity is to use only one network weight variable P through the different layers. In addition, the convergence rate can be higher if current time-consistent step-sizes are changed to a single time-varying step-size. In addition, the weighting parameter for the diagonal elements in the matrix sparseness could also be changed to time-varying variable instead of a constant value such that an identity matrix can be identified more precisely. As a result, the mismatch between the actual critical point and the matrix sparseness peak can be reduced. Therefore, the misconvergence can be further reduced.

On the other hand, the cost functions for the OCV and GCV regularisation methods indicate that we can also improve the accuracy of our purposed method by introducing the output error as one criterion which stops the update of the network weights. The output error criterion combined with the sparseness criterion will make the proposed algorithm more robust and accurate under both well-conditioned and ill-conditioned cases.

8.2.2 Implementation of the proposed algorithm in time domain

Since the proposed algorithm is implemented in the frequency domain, it is possible to extend current research in frequency-domain to that of the time domain. It will be challenging to implement the proposed algorithm in time domain since the size of the system matrix will be much larger than that in frequency domain. Thus, it is suggested to employ the real time

subband system to reduce the computation complexity in the time domain for system matrix inverse. In time domain, the system matrix $\mathbf{A}$ is time varying and thus for each movement of listening point, the coefficients for loudspeaker need to be computed again. This is very challenging and requires high computational complexity. The subband technique can be incorporated with the proposed algorithm to improve the overall performance.

8.2.3 Blind source identification and equalization

Most of the high-order-statistics based blind source identification (BSI) rely on the channel identifiability conditions [45] where there are no common zeros across all channels. When common zeros exist in the BSI algorithms, the system matrix will be ill-condition and thus the accuracy of algorithm is reduced. In addition, common-zeros problem can be interpreted as the global channel matrix is not full rank [45] while the near common-zeros (NCZ) problem can be taken as ill-conditioning problem of the global channel matrix. A common way to solve the NCZ problem is to increase the number of the microphones which is computationally expensive and practically limited. It is possible to find the relation between the ill-conditioning problem and the NCZ problem thus mitigate the NCZ problem by incorporating our proposed algorithm.

In addition, the current research is to minimize the difference between $\mathbf{D}$ and $\mathbf{Y}$, which is similar to the blind source equalization where the blind equalizer is the inverse matrix of the channel bank [46]. To some extent, what we are solving in this thesis is similar to that of what is to be achieved for equalization, where the desired acoustic signal is to be generated/estimated at the interest zone by minimizing the the output error. It is possible to employ our proposed algorithm to find the blind equalizer by taking the inverse of the channel bank in an adaptive way. It is found that the matrix inverse estimation is required in multiple-input multiple-output systems. Therefore, our proposed algorithm may improve the accuracy and efficiency of the blind equalizer estimation.

Appendix A: Stability Analysis of Matrix Inverse Estimator

We first analyze the stability for single-layer network before generalizing it to a multi-layer network. Employing eigenvalue decomposition,

$$\mathbf{A} = \tilde{\mathbf{U}}\tilde{\mathbf{\Lambda}}\tilde{\mathbf{U}}^H \quad (8.1)$$

where $\tilde{\mathbf{U}} = [\tilde{\mathbf{u}}_1, \dots, \tilde{\mathbf{u}}_N]_{N \times N}$ is a square matrix whose i th column, $\tilde{\mathbf{u}}_i$, is the i th eigenvector of $\mathbf{A}$ and $\tilde{\mathbf{\Lambda}} = \text{diag}\{\tilde{\lambda}_1, \dots, \tilde{\lambda}_N\}$ is a diagonal matrix whose diagonal elements are the corresponding eigenvalues of $\mathbf{A}$. Taking the expectation of (6.2), we have

$$E\{\mathbf{P}(n+1)\} = E\{\mathbf{P}(n)\} + \eta[\mathbf{I} - E\{\mathbf{Z}(n)\mathbf{Z}^H(n)\}]. \quad (8.2)$$

By pre- and post-multiplying (8.2) by $\tilde{\mathbf{U}}^H$ and $\tilde{\mathbf{U}}$ respectively, and using (28), we obtain

$$\tilde{\mathbf{U}}^H E\{\mathbf{P}(n+1)\} \tilde{\mathbf{U}} = \tilde{\mathbf{U}}^H E\{\mathbf{P}(n)\} \tilde{\mathbf{U}} + \eta[\mathbf{I} - \tilde{\mathbf{U}}^H E\{\mathbf{P}(n)\mathbf{A}\mathbf{P}^H(n)\} \tilde{\mathbf{U}}]. \quad (8.3)$$

For small η , fluctuations of the elements in $\mathbf{P}(n)$ are small such that $E\{p_{i,j}(n)p_{i',j'}(n)\} \approx E\{p_{i,j}(n)\}E\{p_{i',j'}(n)\}$, for any indices $i, j, i',$ and j' between 0 and $N-1$. This assumption can yield a reliable performance analysis for small step-size values [47]. We can therefore

rewrite (8.3) as

$$\mathbf{\Lambda}_p(n+1) = \mathbf{\Lambda}_p(n) + \eta[\mathbf{I} - \mathbf{\Lambda}_p(n)\tilde{\mathbf{\Lambda}}\mathbf{\Lambda}_p(n)] \quad (8.4)$$

where $\mathbf{\Lambda}_p(n) = \tilde{\mathbf{U}}^H E\{\mathbf{P}(n)\}\tilde{\mathbf{U}}$ is a diagonal matrix when we choose $\mathbf{P}(0) = \mathbf{I}$. As a result,

$$\lambda_i(n+1) = \lambda_i(n) + \eta[1 - \tilde{\lambda}_i\lambda_i^2(n)] \quad (8.5)$$

where $\lambda_i(n)$ is the i th eigenvalue of $E\{\mathbf{P}(n)\}$. Defining $\check{\lambda}_i(n) = \lambda_i(n) - 1/\sqrt{\tilde{\lambda}_i}$ as the estimation error for the i th eigenvalue of $E\{\mathbf{P}(n)\}$, (8.5) can be rewritten as

$$\begin{aligned} \check{\lambda}_i(n+1) &= \check{\lambda}_i(n) + \eta[1 - \tilde{\lambda}_i\lambda_i^2(n)] \\ &= \check{\lambda}_i(n) - \eta[\tilde{\lambda}_i\check{\lambda}_i^2(n) + 2\sqrt{\tilde{\lambda}_i}\check{\lambda}_i(n)] \\ &= [1 - \eta(\tilde{\lambda}_i\check{\lambda}_i(n) + 2\sqrt{\tilde{\lambda}_i})]\check{\lambda}_i(n) \\ &= [1 - \eta(\tilde{\lambda}_i\lambda_i(n) + \sqrt{\tilde{\lambda}_i})]\check{\lambda}_i(n) \\ &= [1 - \eta\sqrt{\tilde{\lambda}_i}(\sqrt{\tilde{\lambda}_i}\lambda_i(n) + 1)]\check{\lambda}_i(n), \end{aligned} \quad (8.6)$$

where $\tilde{\lambda}_i(n) = \tilde{\lambda}_i\lambda_i^2(n)$ is the i th eigenvalue of $E\{\mathbf{P}(n)\}\mathbf{G}\mathbf{G}^H E\{\mathbf{P}^H(n)\} \approx E\{\mathbf{Z}(n)\mathbf{Z}^H(n)\}$.

To guarantee the convergence of estimation error $\check{\lambda}_i(n)$ to zero, we require that $|\check{\lambda}_i(n+1)| < |\check{\lambda}_i(n)|$ for each iteration. Using (8.6), this constraint leads to the stability condition

$$\left| 1 - \eta\sqrt{\tilde{\lambda}_i}(1 + \sqrt{\tilde{\lambda}_i(n)}) \right| < 1. \quad (8.7)$$

Convergence of $\check{\lambda}_i(n)$ is thus guaranteed if

$$0 < \eta < \frac{2}{\sqrt{\tilde{\lambda}_i}(1 + \sqrt{\tilde{\lambda}_i(n)})}. \quad (8.8)$$

Since these bounds must hold for all $\tilde{\lambda}_i$ and $\tilde{\lambda}_i(n)$, the necessary and sufficient condition for

stability is

$$0 < \eta < \frac{2}{\sqrt{\lambda_{\max}}(1 + \sqrt{\lambda_{\max}(n)})} \quad (8.9)$$

where $\lambda_{\max}$ and $\lambda_{\max}(n)$ are the maximum values of $\tilde{\lambda}_i$ and $\tilde{\lambda}_i(n)$, respectively.

For the multi-layer case, we may follow the above analysis using (6.8) and (6.10) from which the stability is given by

$$0 < \tilde{\eta}_k < \frac{2}{\sqrt{\lambda_{k-1,\max}}(1 + \sqrt{\lambda_{k,\max}(n)})}, \quad \mathcal{N}_{k-1} < n \leq \mathcal{N}_k, \quad (8.10)$$

where $\lambda_{k-1,\max}$ and $\lambda_{k,\max}(n)$ are the maximum eigenvalues of $\mathbf{Z}_{k-1}(\mathcal{N}_{k-1})\mathbf{Z}_{k-1}^H(\mathcal{N}_{k-1})$ and $E\{\mathbf{Z}_k(n)\mathbf{Z}_k^H(n)\}$, respectively.

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