Broadband energy harvesting using nonlinear techniques

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BROADBAND ENERGY HARVESTING USING NONLINEAR TECHNIQUES

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2017

This thesis is dedicated to:

Knowledge,

And

My wife and daughter.

MY INSPIRATION

My journey so far has shown me the cycles of life in its own charming ways. I learned early on in my life that every success comes at a cost, and every happy moment is laced with sorrow on its circumference; life is a bosom of seasons and it teaches each one of us the lessons that we need to learn. When I was a child, during a monsoon season my family was stuck in a heavy cyclone and ensuing flood which lasted for more than a week. During that period, my dad had asked of me if we lost everything that very moment could we really survive in this world. This question frightened me initially but with age my search for an answer provided me with an understanding that *knowledge* is the very wealth that helps us survive and make a living in this world.

I have been blessed with numerous wise teachers and mentors who shaped me, as would a potter shape a mound of earth into a beautiful vase. This very vase is the vessel that can contain some elixir of knowledge, which ages gracefully with passing time. In my finite understanding, the true essence of *knowledge* is to pass it forward, and apply it for the benefit of the world.

I was often fascinated by mechanics and vibrations from an early age. As a kid, I would open parts of electrical and mechanical equipment just out of curiosity. This passion drove me to undertake engineering studies and finally doctoral research in the field of harnessing energy from vibrations. The process of a PhD has taught me some of the most valuable lessons in my life, first and the foremost being the ability to enjoy the process. I understood the beauty and the joy in performing simple everyday tasks like solving an equation or validating a hypothesis through experimentation. And slowly over the years, I found myself. I have often come to cross-roads with my own thoughts; the world is an unforgiving place, either one sways to its tunes or succumbs to its qualms. We all try to play the game and win yet the world always gets the better of us. In this chaos that the world throws at us, I

have always found my peace in pursuing *knowledge* for its own sake. For, it is a true liberator and a destination in itself.

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CONTENTS

CONTENTS	i
LIST OF TABLES	v
LIST OF FIGURES	vi
SUMMARY	xii
CHAPTER 1 INTRODUCTION	1
1.1. Motivation.....	1
1.2. Background	2
1.3. Research objectives and scope	5
1.4. Original contributions	6
1.5. Organization of the report.....	8
CHAPTER 2 VIBRATION BASED PIEZOELECTRIC ENERGY HARVESTING	10
2.1. Piezoelectricity and piezoelectric materials.....	10
2.2. Energy Harvesting using piezoelectric materials	13
2.3. Analytical modeling of piezoelectric energy harvesters.....	17
2.3.1 Basic single degree of freedom (S'DOF) model.....	18
2.3.2 Coupling of S'DOF model.....	19
2.3.3 Introduction of correction factors in S'DOF models.....	20
2.3.4 Distributed parameter model.....	22
2.3.5 Rayleigh-Ritz approach to solve complex coupled models.....	24
2.3.6 Modeling using Finite element methods	25
2.3.7 Analysis using equivalent circuit modeling (ECM).....	27
2.4. Efficiency enhancing techniques and Broadband energy harvesting	28
2.4.1 Advances in piezoelectric and structural configurations	29
2.4.2 Broadband Energy harvesting	31
2.4.3 Efficiency enhancement by optimizing the circuit interface	41
2.5. Longevity of piezoelectric energy harvesting systems	43
2.5.1 Fatigue in piezoelectric energy harvesting systems	43

2.6. Summary.....	46
CHAPTER 3 FORMULATION OF MAGNETOSTATIC INTERACTION EQUATIONS AND MULTIPLE EQUILIBRIUM STATES.	49
3.1. Introduction.....	49
3.2. Dipole-Dipole magnetostatic interaction formulation.....	49
3.3. Enhanced integral magnetostatic interaction formulation	50
3.3.1 Case: I – Magnetostatic interaction between two similar magnets.	51
3.3.2 Case: II – Magnetostatic interaction between two dissimilar magnets.	55
3.4. Magnetostatic interaction using finite element analysis (FEA).....	58
3.5. Different types of equilibrium conditions.....	67
3.6. Introduction to bistable and multiple stable states	68
3.7. Summary.....	74
CHAPTER 4 ENHANCEMENT OF BANDWIDTH USING MAGNETIC NONLINEARITY	75
4.1. Introduction.....	75
4.2. Effect of axial and transverse forces on a cantilever beam.....	75
4.2.1 Cantilever beam subjected to an axial force	76
4.2.2 Cantilever beam subject to a concentrated transverse force.....	83
4.2.3 Cantilever beam subject to both axial and transverse forces.....	86
4.3. Effect of magnetic nonlinearity on a PEH cantilever beam.....	99
4.3.1 Formulation for a linear PEH cantilever beam	100
4.3.2 Formulation for a nonlinear PEH cantilever beam.....	105
4.3.3 Experimental investigation of a nonlinear PEH cantilever beam	108
4.3.4 Results and discussion.....	110
4.4. Conclusion	118
4.5. Summary.....	119
CHAPTER 5 VARIATION OF THE BANDWIDTH AND EFFECTIVE STRAIN TRANSFER OF A PIEZOELECTRIC ENERGY HARVESTER WITH CHANGE IN STIFFNESS OF THE SYSTEM	120
5.1. Introduction.....	120
5.2. Correlation between strain transfer and bandwidth.....	122
5.2.1 Calculation of strain in a PEH beam	122

5.2.2	Calculation of equivalent strain and effective strain transfer	127
5.2.3	Effective strain transfer and variation of bandwidth in a nonlinear PEH beam	131
5.3.	Experimental investigation of variation of bandwidth in the presence of magnetic induced nonlinearity.....	136
5.3.1	Experimental setup	137
5.3.2	Results and discussion	139
5.4.	Conclusion	150
5.5.	Summary	152
CHAPTER 6 ENHANCEMENT OF BANDWIDTH BY COUPLING TWO CANTILEVER BEAMS UNDERGOING TRANSVERSE AND PARAMETRIC VIBRATIONS		154
6.1.	Introduction.....	154
6.2.	Transverse and parametric vibrations.....	155
6.2.1	Transverse vibrations.....	156
6.2.2	Parametric vibrations.....	160
6.3.	Potential energy distribution of a combined system	162
6.4.	Experimental investigation of two coupled cantilever PEH beams	166
6.4.1	Experimental setup	167
6.4.2	Results and discussion.....	169
6.5.	Conclusion	183
6.6.	Summary.....	184
CHAPTER 7 LONG TERM FATIGUE BEHAVIOR OF A CANTILEVER PIEZOELECTRIC ENERGY HARVESTER.....		186
7.1.	Introduction.....	186
7.2.	Linear piezoelectric energy harvesting beam.....	189
7.2.1	Analytical modeling	189
7.2.2	Finite element formulation.....	192
7.3.	Experimental investigation	195
7.3.1	Experimental setup	196
7.3.2	Measurement of voltage, resonant frequency and tip displacement	198
7.3.3	Electromechanical impedance readings.....	206
7.4.	Crack propagation	216

7.4.1	Crack behavior from experiments.....	217
7.4.2	Crack behavior from FEA.....	219
7.5.	Results and discussion.....	222
7.5.1	Output Voltage.....	222
7.5.2	Strain.....	226
7.5.3	Experimental data (Voltage, frequency, and tip displacement).....	228
7.5.4	Impedance values	229
7.5.5	Crack propagation	231
7.6.	Conclusions.....	232
7.7.	Summary.....	234
CHAPTER 8 CONCLUSIONS AND FUTURE WORK		235
8.1.	Conclusions.....	235
8.2.	Recommendations for future work.....	240
REFERENCES		242
AUTHOR’S PUBLICATIONS		255
APPENDIX – A		256

LIST OF TABLES

Table-1.1: Sources for ambient vibration (Roundy et al., 2003).....	3
Table-2.1: Analogy between the electrical and mechanical domain (Yang and Tang, 2009).....	28
Table-3.1: Properties of different types of magnets	53
Table-3.2: Percentage change in the forces derived using analytical methods and FEA modeling....	66
Table-3.3: Properties of the SDOF system	70
Table-4.1: Properties of the beam	82
Table-4.2: Equivalent system parameters for the cantilever beam.....	84
Table-4.3: Experimental resonant frequencies for a cantilever beam.....	93
Table-4.4: Material Properties	110
Table-4.5: Properties of the PEH beam	110
Table-5.1: Properties of the PEH beam	126
Table-5.2: Effective strain transfer and bandwidth for the PEH beam	129
Table-5.3: Effective strain transfer and bandwidth for the PEH beam at constant base excitation (0.3g) with varying Stiffness	134
Table-5.4: Material Properties	139
Table-5.5: Properties of the PEH beam	139
Table-5.6: Effective strain transfer values.....	140
Table-5.7: Variation of Bandwidth at 10 V (100 μ W)	145
Table-5.8: Properties of the PEH beam with aluminum substrate	146
Table-5.9: Effective strain transfer values at higher excitation levels	147
Table-6.1: Properties of the PEH beam	169
Table-6.2: Variation of Bandwidth at 100 μ W (10 V)	174
Table-6.3: Variation of Bandwidth at quarter power (Peak power/4)	174
Table-7.1: Properties of the PEH beam	192
Table-7.2: Material properties of constituents of PEH beam (ASM Inc., 2015; Steiger and Mokry, 2015).....	194

LIST OF FIGURES

Figure 2.1 Bending principles of a unimorph in d_{31} and d_{33} modes; E, P, and M represent the directions of the electric field, dielectric polarization and the bending moment respectively (Safari and Akdogan, 2008).....	11
Figure 2.2 Bending principles of a bimorph cantilever with electrical connections in parallel and in series (Safari and Akdogan, 2008).....	11
Figure 2.3 (a) Conventional PZT ceramics (PI Ceramic Co.) and (b) Macro fiber composite (MFC) (JEC Group).	13
Figure 2.4 Representaion of type of energy harvesting using (a) surface bonded piezo transducers, and (b) harvesting structure attached on the host structure (Liang and Liao, 2010).	14
Figure 2.5 Energy flow in a typical PEH device (Liang and Liao, 2012).....	15
Figure 2.6 A typical layout of PEH system with a rectifier and secondary regulator.....	16
Figure 2.7 A layout of a wireless sensing node which uses harvested energy (Arms <i>et al.</i> , 2005). ...	17
Figure 2.8 A wireless temperature and humidity sensor powered by a PEH device (Arms <i>et al.</i> , 2005).....	17
Figure 2.9 A schematic representation for a basic S'DOF model (Williams and Yates, 1996).	19
Figure 2.10 A schematic representation of coupled S'DOF model (duToit <i>et al.</i> , 2005).	20
Figure 2.11 Schematic representation of a typical beam with proof mass undergoing base excitation (Kim <i>et al.</i> , 2010).....	22
Figure 2.12 A schematic for the unimorph PEH cantilever beam (Erturk and Inman, 2011b).	22
Figure 2.13 (a) Shifting of the peaks of the Voltage frequency response function (FRF) with changing external resistive load, and (b) The peak of power amplitude FRF obtained with the distributed parameter model for changing resistance (Erturk and Inman, 2011b).	24
Figure 2.14 Equivalent circuit model of a PEH for different modes of the harvester (Yang and Tang, 2009).....	28
Figure 2.15 Distribution of strain along the length for beams with changing widths (Roundy <i>et al.</i> , 2005).....	30
Figure 2.16 The L-shaped beam to induce constant strain along the main beam (Xu <i>et al.</i> , 2010)....	30
Figure 2.17 Bidirectional frequency tuning mechanism (Eichhorn <i>et al.</i> , 2009).	33
Figure 2.18 Movable tip mass running on centrifugal force (Gu and Livermore, 2012).	33
Figure 2.19 Prototype to tune frequency using adjustable magnetic slider (Ahmadian <i>et al.</i> , 2009).	34
Figure 2.20 PEH sytem consisting of scavenging and tuning parts (Roundy and Zhang, 2005).....	35
Figure 2.21 PEH sytem for frequency-up conversion (Rastegar <i>et al.</i> , 2006).....	36
Figure 2.22 PEH sytem utilizing two different modes (Arafa <i>et al.</i> , 2011).	37
Figure 2.23 L-shaped PEH sytem utilizing two different modes (Erturk <i>et al.</i> , 2009b).	37
Figure 2.24 2DOF PEH sytem with rigid mass (Kim <i>et al.</i> , 2011).	38
Figure 2.25 Softening and Hardening behaviors of a monostable configuration (Stanton <i>et al.</i> , 2009).	40
Figure 2.26 Piezomagnetoelastic power generator (Erturk <i>et al.</i> , 2009a).....	41
Figure 2.27 Layout of a standard interface circuit connected to (a) Battery and (b) Resistive load. ...	42
Figure 2.28 Layout of impedance adaptation interface.	42
Figure 2.29 Layout of a SSHI circuit interface (a) Parallel and (b) Series.	43
Figure 2.30 Layout of SCE interface.....	43
Figure 2.31 Experimental prototype of a DuraAct transducer based energy harvester (Soma and De Pasquale, 2013).....	46
Figure 2.32 Experimental setup of a fluid flow nozzle based energy harvester (Sherrit <i>et al.</i> , 2015) 46	

Figure 3.1 Schematic representation of two cylindrical magnets with their magnetization vectors aligned.....	51
Figure 3.2 (a) Graphical representation of the variation of axial component of magnetostatic interaction force between two similar magnets of type-I, (b) Graphical representation of the variation of transverse component of magnetostatic interaction force between two similar magnets of type-I when $a = 0.01\text{m}$ and $0.005\text{m} \leq r \leq 0.005\text{m}$	55
Figure 3.3 (a) Graphical representation of the variation of axial component of Magnetostatic interaction force between two dissimilar magnets (b) Graphical representation of the variation of transverse component of magnetostatic interaction force between two dissimilar magnets (type-I and type-II) when $a = 0.01\text{m}$ and $0.005\text{m} \leq r \leq 0.005\text{m}$	57
Figure 3.4 (a) Meshed FEA model for two similar magnets facing each other, (b) Distribution of the magnetic flux density (T) within the control volume.	61
Figure 3.5 (a) Meshed FEA model for three similar magnets, (b) Distribution of the magnetic flux density (T) within the control volume.	62
Figure 3.6 (a) Graphical representation of the variation of axial component of magnetostatic interaction force between three similar magnets of type-I, (b) & (c) Graphical representation of the variation of axial and transverse components of magnetostatic interaction forces between three similar magnets of type-I when $a = 0.01\text{m}$ and $0.005\text{m} \leq r \leq 0.005\text{m}$	63
Figure 3.7 Schematic representation of a rotating cantilever beam (the deflection is exaggerated for the sake of clarity).	64
Figure 3.8 (a) & (b) Graphical representation of the variation of axial and transverse components of magnetostatic interaction forces between two similar magnets of type-I when $0.006\text{m} \leq a \leq 0.014\text{m}$. and $0.005\text{m} \leq r \leq 0.005\text{m}$	65
Figure 3.9 (a) & (b) Graphical representation of the variation of axial and transverse components of magnetostatic interaction forces between three similar magnets of type-I when $0.006\text{m} \leq a \leq 0.014\text{m}$. and $0.005\text{m} \leq r \leq 0.005\text{m}$	65
Figure 3.10 Schematic representation of different equilibrium positions.	68
Figure 3.11 Schematic representation of a cantilever beam with induced magnetic nonlinearity.	69
Figure 3.12 An equivalent SDOF lumped parameter system.	69
Figure 3.13 Representation of potential barrier and valley.	71
Figure 3.14 Potential energy variation of a SDOF system in the presence of one tip magnet and one end magnet, (a) Repulsive configuration, (b) Attractive configuration.	72
Figure 3.15 Schematic representation of a cantilever beam with one tip magnet and two end magnets.	72
Figure 3.16 Potential energy variation of a SDOF system in the presence of one tip magnet and two end magnets, (a) Repulsive configuration, (b) Attractive configuration.	73
Figure 4.1 Schematic representation of a cantilever beam with tip and end magnet in repulsive configuration.	76
Figure 4.2 Schematic representation of a cantilever beam with tip and end magnet in repulsive configuration.	77
Figure 4.3 Variation of fundamental frequency with changing axial force (tension to compression), (a) in the presence of tip mass, and (b) absence of tip mass.	83
Figure 4.4 Variation of tip velocity of the cantilever beam with frequency, (a) in the presence of tip mass, and (b) absence of tip mass.	85
Figure 4.5 Variation of tip displacement of the cantilever beam with frequency, (a) in the presence of tip mass, and (b) absence of tip mass.	86
Figure 4.6 Variation of tip velocity of the cantilever beam with frequency, (a) in the presence of tip mass, and (b) absence of tip mass.	90
Figure 4.7 Variation of tip displacement of the cantilever beam with frequency, (a) in the presence of tip mass, and (b) absence of tip mass.	91

Figure 4.8 Comparison of the variation of tip displacement with frequency for a cantilever beam subject to (a) repulsive configuration, and (b) attractive configuration at 8mm separation in the absence of tip mass, (c) repulsive configuration in the presence of a tip mass, and (d) repulsive configuration in the presence of a tip mass and undergoing a base excitation of 5g .92	92
Figure 4.9 Experimental setup of a cantilever beam with a tip magnet, (a) in linear configuration, and (b) vibrating in nonlinear configuration with tip magnet.....	94
Figure 4.10 (a) Variation of potential energy with change in spacing between the tip and end magnets, (b) variation of transverse component of magnetic force (F_y) and, (c) variation of axial component of magnetic force (F_z) with changing transverse displacement of the cantilever beam.....	96
Figure 4.11 Experimental setup of a cantilever beam with a tip magnet, (a) in linear configuration, and (b) vibrating in nonlinear configuration with tip magnet.	99
Figure 4.12 Typical representation of a linear PEH cantilever beam.	101
Figure 4.13 Schematic representation of the cross-section of a typical PEH beam.....	105
Figure 4.14 (a) Experimental setup for testing a PEH beam, and (b) Schematic representation of the experimental process.....	109
Figure 4.15 Variation of potential energy in the presence of one end magnet for (a) repulsive configuration, and (b) attractive configuration.	112
Figure 4.16 Variation of potential energy in the presence of two end magnets for (a) repulsive configuration, and (b) attractive configuration.	113
Figure 4.17 Variation of output voltage with frequency obtained from: (a) Experimental setup, and (b) analytical formulation, for a base vibration of 0.1g	115
Figure 4.18 Variation of output voltage with frequency obtained from: (a) Experimental setup, and (b) analytical formulation, for a base vibration of 0.2g	115
Figure 4.19 Variation of power output with frequency for: (a) & (b) 1 end magnet, and (c) & (d) 2 end magnets, for base vibrations of 0.1g and 0.2g respectively.....	117
Figure 4.20 Variation of bandwidth for output power greater than 100 μ W for PEH beam with (a) 1 end magnet, and (b) 2 end magnets.....	118
Figure 5.1 Experimental setup for strain testing of a PEH beam.	125
Figure 5.2 Results for validation of the PEH beam attached with a strain gauge – variation of voltage with frequency.....	125
Figure 5.3 Results for validation of the PEH beam attached with a strain gauge – variation of strain with frequency.	126
Figure 5.4 Effective strain transfer for a PEH beam with varying acceleration levels.	129
Figure 5.5 Variation of resonant frequencies of a PEH beam with varying stiffness.....	129
Figure 5.6 Variation of (a) effective strain transfer and (b) bandwidth with stiffness, and (c) shows the variation of both the effective strain transfer and bandwidth on the same plot.	131
Figure 5.7 Variation of (a) effective strain transfer and (b) bandwidth with stiffness, and (c) shows the variation of both the effective strain transfer and bandwidth on the same plot.	133
Figure 5.8 Variation of output voltage with frequency for (a) attractive and (b) repulsive configuration along with variation of K_{eq} values.	134
Figure 5.9 Variation of the effective strain transfer and bandwidth for (a) attractive and (b) repulsive configuration with changing K_{eq} values.	136
Figure 5.10 (a) Experimental setup for testing a PEH beam, and (b) Schematic representation of the experimental process.....	138
Figure 5.11 Variation of output voltage in the presence of 1 end magnet at an excitation amplitude of 0.1g for (a) PEH beam with fiberglass substrate, (b) PEH beam with aluminum substrate, and (c) V21BL bimorph PEH beam.....	142

Figure 5.12 Variation of output voltage in the presence of 1 end magnet at an excitaiton amplitude of 0.2g for (a) PEH beam with fiberglass substrate, and (b) PEH beam with aluminum substrate.	143
Figure 5.13 Variaiton of output voltage in the presence of 1 end magnet at an excitaiton amplitude of 0.2g for (a) PEH beam with fiberglass substrate, (b) PEH beam with aluminum substrate, and (c) V21BL bimorph PEH beam.	144
Figure 5.14 Variaiton of output voltage in the presence of 2 end magnets at an excitaiton amplitude of 0.2g for (a) PEH beam with fiberglass substrate, and (b) PEH beam with aluminum substrate.	144
Figure 5.15 Variaiton of output voltage in the presence of 2 end magnets (attractive configuration) at higher excitaiton amplitudes for (a) beam-1 subjected to 0.3g base excitation, (b) beam-1 subjected to 0.4g base excitation, and (c) beam-2 subjected to 0.3g base excitation.	149
Figure 5.16 Variaiton of output voltage in the presence of 2 end magnets (repulsive configuration) at higher excitaiton amplitudes for (a) beam-2 subjected to 0.3g base excitation, and (b) beam-3 subjected to 0.3g base excitation.	150
Figure 6.1 Layout of a cantilever beam subjected to (a) transverse vibrations and, (b) parametric vibrations.	156
Figure 6.2 Schematic representation of (a) beam undergoing transverse vibrations in the presence of nonlinear magnetic forces and, (b) cross-section of two cylindrical permanent magnets.	160
Figure 6.3 Scematic representation of the proposed system with (a) Case-I: Auxiliary beam is fixed, (b) Case-II: Main beam and auxiliary beam are vibrating in sync, (c) Case-III: Main beam and auxiliary beam are vibrating out of sync, (d) curves representing the potential energy distribution with the deflection for different cases, and (e) 3-dimensional representation of the potential energy curves for the main beam.	166
Figure 6.4 Representation of the proposed PEH system with (a) layout of the experimental setup, (b) linear configuration of the experimental setup, and (c) nonlinear configuration of the experimental setup.	169
Figure 6.5 Variation of output voltage for a linear configuration of the beams when (a) both PEH beams are made of aluminum substrate, and (b) both PEH beams are made of fiberglass substrate.	171
Figure 6.6 Variation of output power for nonlinear configuration ($a = 10$ mm) of the beams when (a) & (b) both PEH beams are made of aluminum substrate, (c) & (d) both PEH beams are made of fiberglass substrate, and (e) both the PEH beams are made of fiberglass substrate and the tip are placed 15 mm apart	173
Figure 6.7 Circuit configuration for the PEH system (a) Series connection, and (b) Parallel connection of the component PEH beams.	176
Figure 6.8 (a) Acceleration level for the random base vibration; Time history voltage response of (b) Linear configuration (unrectified) (c) Linear configuration – Series connection, (d) Linear configuraiton – Parallel connection, (e) Nonlinear configuration (unrectified voltage), (f) Nonlinear configuration – Series connection, (g) Nonlinear configuration – Parallel connection	179
Figure 6.9 (a) Acceleration level for the random base vibration; Time history voltage response of (b) Linear configuration (unrectified) (c) Linear configuration – Series connection, (d) Linear configuraiton – Parallel connection, (e) Nonlinear configuration (unrecified voltage), (f) Nonlinear configuration – Series connection, (g) Nonlinear configuration – Parallel connection	181
Figure 6.10 Energy stored across a 510 μ F capacitor for (a) Two PEH beams subjected to 0.1g, (b) Two PEH beams subjected to 0.2g, (c) One PEH beam subjected to 0.1g, and (d) One PEH beam subjected to 0.2g random base excitation.	182

Figure 7.1 (a) Schematic representation of the PEH beam and its constituent components, (b) Detailed representation of various layers of the PEH beam and MFC transducer.	189
Figure 7.2 Representation of (a) 3D FEA model of PEH beam and (b) 2D FEA model of PEH beam.	194
Figure 7.3 Representation of (a) experimental setup for fatigue test of PEH beam, (b) prepared samples for different excitation levels, (c) schematic representation of sensing and actuation modes of experimental procedure.	198
Figure 7.4 Variation of (a) Voltage Vs Frequency for the pilot PEH beam, and (b) voltage at a constant frequency for the pilot PEH beam.	199
Figure 7.5 Variation of frequency with increasing number of fatigue cycles for (a) 0.4g base excitation level, (b) 0.5g base excitation level, and (c) 0.6g base excitation level.	202
Figure 7.6 Variation of voltage ratio with frequency for the first 50000 cycles across different base excitation levels for (a) positive peak-peak values, and (b) negative peak-peak values.	203
Figure 7.7 Variation of (a)-(c) voltage ratio across increasing base excitation levels, (d)-(f) velocity of the tip of PEH beam, and (g)-(i) displacement of the tip of PEH beams with increasing fatigue cycles.	206
Figure 7.8 Impedance readings at various stages of the fatigue loading for different PEH beams. (a)&(b): Plots for real (Resistance) and imaginary (Reactance) parts of the impedance values for PEH beam undergoing 0.4g base excitation; (c)&(d): Plots for real and imaginary parts of the impedance values for PEH beam undergoing 0.5g base excitation; (e)&(f): Plots for real and imaginary parts of the impedance values for PEH beam undergoing 0.6g base excitation; (g)-(i): Plots for the first few fatigue cycles (fewer than 500k) showing a magnified range for real and imaginary parts of the impedance values for the PEH beams undergoing 0.4g, 0.5g, and 0.6g base excitation respectively.	214
Figure 7.9 Variation of RMSD with increasing fatigue cycles for (a)-(c) impedance values, and (d)-(f) voltage values across increasing base excitation levels.	216
Figure 7.10 Variation of rRMSD with increasing fatigue cycles for (a) 0.4g base excitation, (b) 0.5g base excitation, and (c) 0.6g base excitation levels.	216
Figure 7.11 Crack propagation with increasing number of fatigue cycles on subjecting the PEH beam to a base excitation level of (a) 0.4g, (b) 0.5g, and (c) 0.6g.	219
Figure 7.12 Recorded response of output voltage (a) initial damage, (b) final damage to the MFC transducer, and (c) at complete fatigue failure of the beam (x-axis: time, y-axis: magnitude of voltage).	219
Figure 7.13 Representation of (a) stress distribution in the PZT layer of an uncracked 2d FEA model, and (b) sequential induction of cracks in the MFC transducer layer of the 2d FEA model.	221
Figure 7.14 Modeling results for variation of (a) voltage, and (b) resonant frequency, with increasing number of cracks in the respective PEH beams.	222
Figure 7.15 (a-c) Plots showing the comparison of output voltages for analytical, FEA, and experimental formulations, (d) comparison of the variation of peak output voltages for the PEH beams across different formulations, (e) comparison of the frequency sweep for output voltage across the 3D and 2D FEA formulations for PEH beam undergoing 0.5g acceleration, and (f) comparison of the frequency sweep for 3D and 2D formulation using voltage ratio and frequency ratio.	226
Figure 7.16 Variation of Strain with frequency for different PEH beams.	228
Figure 7.17 Variation of Strain (peak strain) with the total number of fatigue cycles.	228

SUMMARY

In the quest for long-term service of various low power sensor systems and wireless sensor networks, there has been prominent research in the field of energy harvesting. Vibration based piezoelectric energy harvesting (PEH) has gained popularity since vibrations are ubiquitous and the piezoelectric materials provide a high-power density for conversion of vibrations to electrical energy. Most initial harvesters consisted of a linear PEH device (a cantilever bimorph beam) vibrating at resonance which provided very limited bandwidth of frequency for field operations. Due to such drawbacks of linear PEH devices a gradual shift towards broadband energy harvesters has been observed lately. The broadband energy harvesters can perform efficiently in a much broader range of frequency and such an enhanced design is implemented by the usage of various techniques, the foremost of them being the introduction of nonlinearity into the system.

The nonlinearity can be introduced into the system using magnets. The interaction between magnets is nonlinear in nature and in the presence of magnetic interaction forces the PEH system's response becomes nonlinear. This thesis discusses the importance of enhancing the bandwidth and the various parameters effecting it, along with introduction of innovative designs to enhance the bandwidth of a PEH system. Traditionally the interaction forces between magnets present in a PEH system were defined using a dipole-dipole formulation which assumes the magnets to be point dipoles. Though this formulation provided a decent understanding into the behavior of the PEH system, it's accuracy is limited when the magnets are placed close to each other, which is a commonplace in many nonlinear PEH systems. This discrepancy was overcome by using an enhanced magnetostatic interaction formulation. The usage of this enhanced integral formulation provides an added advantage in the analytical modeling and minimizes the discrepancies in the interaction forces when the magnets are placed close to each other wherein shape effects come into the picture.

Moreover, the conventional analytical models assumed the non-rotation of magnets, and neglected the effect due to magnetostatic forces acting in the axial direction. These fundamental limitations were addressed explicitly in this work and the limiting conditions where the assumptions from conventional models provide a decent degree of accuracy are also stated. Using the formulation developed, a PEH beam with one and two end magnets was analyzed and validated experimentally. The improvement in the bandwidth of operation for a nonlinear PEH system subjected to low level base excitation magnitudes has been dealt in detail.

The study on enhanced magnetostatic formulation was followed by a detailed investigation of the dependence of the operating bandwidth on the stiffness and effective strain transfer values of a nonlinear PEH system. An analytical approach was explored augmented with a comprehensive experimental investigation to assess the performance of nonlinear PEH systems using substrates of different materials, thus inducing the variation of stiffness into the system. It was concluded that the bandwidth is considerably higher for a flexible beam but the effective strain transfer rate is much lower, thereby minimizing the peak output response of the system.

In addition to the study of enhancing the bandwidth of operation in the presence of end magnets for a nonlinear PEH system. An innovative approach of utilizing a coupled PEH system consisting of two magnetically coupled cantilever PEH beams undergoing transverse and parametric vibrations was also explored. Even though the PEH system was subjected to lower levels of base excitation ($0.2g$, $g = 9.81 \text{ m/s}^2$), the response obtained was far superior to that of the linear configuration. The bandwidth at a level of $100 \text{ }\mu\text{W}$ was enhanced by 300%. This design provides a promising alternative to the traditional PEH systems.

Over the course of experimental investigation of the harvesters, damage due to prolonged exposure to external loading was observed. To address this, a unique fatigue based study to understand the damage progression and the corresponding usability of the PEH beam has been presented with detailed experimental, analytical and finite element based studies. A comprehensive FEA model for the piezoelectric macro fiber composite transducer has been introduced. This not only enhances the

accuracy in modeling PEH beams but also provides a detailed overview into the crack propagation of the damaged piezo transducers. The study ended with making provisions for a few limiting criterion and design recommendations of minimizing the number of times the maximum strain value exceeds $500 \mu\epsilon$ during the normal functioning of the PEH system.

CHAPTER 1 INTRODUCTION

1.1. Motivation

Over the past few decades, with new strides in technology man has been able to come up with new and innovative ways of sensing his environment around him and even within the human body itself. Such revolutionary innovations have in turn encouraged a lot of new findings in the fields of powering the sensor circuits. The traditional way to power these sensor systems consists of the usage of alkaline batteries. But, this possesses a major drawback since the alkaline batteries need to be changed frequently. A formidable solution to this standoff would be a green initiative to harness the energy from ambient environment itself.

Harvesting power from the environment has been a popular topic from the day man realized that the conventional sources of energy like coal, petroleum products and natural gases etc. are unlikely to last forever. Thus, harnessing energy from renewable sources like wind, ocean waves, geothermal sources etc. has been a topic of much importance for large scale power production. But coming to a much smaller scale where most sensors run on a few milliwatts and microwatts of power, the exploration of harnessing the ambient energy from the environment has been relatively new (Martinez-Val, 2013). The various modes of tapping the ambient energy will be discussed in detail in the subsequent sections.

Focusing on the application of these low power sensors in the field of civil engineering where they invariably play a very important role in data acquisition from different structures like bridges, tunnels, dams etc., a few examples of such sensors are, the accelerometers, wireless strain gauges, anemometers etc... Most of these sensors are either connected to a global hose wire which powers them or the usage of wireless sensors powered by batteries. Both the above-mentioned applications have respective inherent disadvantages, in the first case there is a need for a lot of wiring which is costly and the sensors can only be placed at locations where the wiring can be provided. In the second case, the major drawback of the usage of batteries to power the wireless sensors is the need for periodical replacement of batteries.

So, the search for an alternative to power small scale sensors has been the motivating factor for many researchers working in this field. The best and the safest alternative is the usage of ambient sources of energy like solar, vibrations, wind etc. to power these sensors at a small scale, moreover such small-scale energy harvesters can prove to be an economical alternative on a long run.

1.2. Background

The challenge to provide a long-term solution to power various low power sensor systems and wireless sensor network systems has thrown open a wide floor of opportunities. Over the last decade, there has been a lot of research to tap energy from the ambient environment to run these low power systems. There have been numerous approaches; the most popular ways of harnessing this energy consists of solar, vibrations, wind, temperature gradients etc. (Erturk and Inman, 2011b).

Solar energy harvesting is one of the most popular ways of harnessing the ambient energy; one can easily find that solar panels find their applications in calculators and watches also. The main drawback of harnessing the solar power is the fact that it becomes hard to run in dark places and such harvesters need to be attached to rechargeable batteries because the solar panels are of much use during the nights and cloudy days.

A few researchers have tried to harness energy from temperature gradients using thermoelectric or pyroelectric materials, but the sources for such energy harvesting (EH) and thus its application are very limited, because it is very unlikely that large temperature gradients would exist abundantly.

Vibration based energy harvesting where the harvesters utilize the vibrations from the ambient environment or caused by wind flow or water flow, have gained popularity owing to the simplicity of design and their flexibility of placement at various locations. From a civil or a mechanical engineering perspective, almost all the structures are under the effect of vibrations continuously, for example the rail and road bridges, the engines in automobiles etc., though a lot of research goes into finding ways of reducing and minimizing the structural vibrations, yet some stray

vibrations exist all the time. Thus, harnessing such stray vibrations to run a few low power sensors for monitoring systems or wireless sensors not only increases the applicability of the structures but also helps in keeping a continuous check on its health and robustness (duToit *et al.*, 2005). This study focuses on such a concept to harness the stray vibrations from civil and mechanical structures.

Table-1.1: Sources for ambient vibration (Roundy *et al.*, 2003)

Source of vibration	Acceleration (m/s^2)	Frequency (Hz)
Base of a 5 HP 3-axis machine tool	10	70
Kitchen blender casing	6.4	120
Clothes dryer	3.5	120
Door frame just after door closes	3	125
Small microwave oven	2.25	120
HVAC vents in office building	0.2 - 1.5	60
Wooden deck with people walking	1.3	385
Bread maker	1.03	120
Windows (0.6m x 1m) next to a busy street	0.7	100
Notebook computer while CD is being read	0.6	75
Washing machine	0.5	109
Second story floor of a wood frame office building	0.2	100
Refrigerator	0.1	240

In the past few years, researchers have worked on various kinds of harnessing methods for vibration based energy harvesting. These can be broadly classified as: electromagnetic, electrostatic, and piezoelectric. There has been a considerable stride to harvest energy using electroactive polymers and electromagnetic radiation also, but the amount of energy extracted remains relatively small. The electromagnetic harvesters run on the principle of Faraday's law of induction and can generate micro currents when magnets are attached to a vibrating cantilever beam and moved through conductors. The electrostatic harvesters employ the properties changing capacitance of capacitors which are sensitive to vibrations (Cook-Chennault *et al.*, 2008; Roundy *et al.*, 2003).

The piezoelectric energy harvesters work on the principle of direct and indirect piezoelectric effects of the piezoelectric materials. Energy harvesting using the piezoelectric materials gained much prominence over the years due to its high

power density in comparison to other modes of harvesting and its ease of handling and application (Erturk and Inman, 2011b).

Majority of the piezoelectric energy harvesters come in the form of cantilever beams attached to a structure which acts as a host and transfers vibrations to the cantilever. A simple piezoelectric energy harvester can be in the form of a metallic cantilever substrate beam attached with piezoelectric layers either on top and bottom faces or a single face. The harvester undergoes vibration whenever the host structure is under vibration, this is understood to be analogous to base vibration of a cantilever beam. Most real-life vibrations consist of low frequency vibrations and hence to tune the harvester to such frequencies a mass is attached to the tip of the beam. The vibration in the beam causes strain and in turn results as the potential difference in the electrodes connecting the piezoelectric elements because of the direct piezoelectric effect. The obtained voltage can be harmonic or chaotic depending on the type of vibrations induced onto the beam and the voltage is rectified and smoothed out using electrical circuitry to make it usable for low power sensor systems. There has also been pioneering research in the electrical circuitry part of the harvester circuits to improve the efficiency in tapping the harvested power (Anton and Sodano, 2007). Thus, a simple energy harvesting device brings in a combination of different fields of engineering.

A simple linear piezoelectric energy harvester is discussed in the above paragraph, but such a model of harvester poses a major drawback, it being the operating frequency of the harvester. If a frequency sweep were to be performed for such a harvester, a single peak at the harvester's resonant frequency would be observed. Many improved designs have been proposed over the past few years to overcome this drawback; '*multimodal harvesters*' was a method in which a harvester was designed to have two closely spaced modal frequencies thus resulting in a broader range of operating frequency or simply broader bandwidth, another method includes '*frequency broadening techniques*' for the harvester, then came the advent of the '*bistable harvesters*'. The bistability was introduced using the buckled beam concept, using the magnets etc... (Tang *et al.*, 2010). These techniques introduce non-linearity into the harvester's structural system, and the introduction of non-

linearity using magnets not only increases the bandwidth of the operating frequency but also is one of the simplest designs to explore the feasibility of practical applications. With this goal in mind, a sincere effort has been made during the conduct of this study to improve the usability of a piezoelectric energy harvester, and understand the factors effecting the efficient functioning to improve the operating bandwidth of the harvester.

1.3. Research objectives and scope

The present research focuses on improving the workability of piezoelectric energy harvesters. The handicap that most harvesters face is the bandwidth of frequency in which they are efficient enough to provide sufficient power for running the electronics connected to them. The ambient vibrations which can be harvested in field conditions are random in nature and constitute low frequency vibrations (less than 100Hz) with varying amplitude. Thus, to apply the energy harvesters for practical conditions, a comprehensive model needs to be developed which can help optimize the various parameters effecting the bandwidth.

- A. In the present work, the enhancement of bandwidth of operation frequency has been explored using the magnet induced nonlinearity into the PEH system. The foremost parameters effecting the bandwidth are the magnetostatic forces acting on the PEH system. The presence of magnetostatic forces induces modification to the potential energy of the whole PEH system. Conventional methods have inherent drawbacks in accurately analyzing the forces. Thus, a comprehensive integral formulation for the magnetic forces has been employed to provide an accurate investigation of the magnetostatic forces.
- B. After an accurate model for the magnetostatic forces was established, a thorough understanding of the effect of these forces on the PEH system and the increase in bandwidth of operation of the PEH system has been explored. In conventional methods, the effect from the transverse forces alone is considered. The effect of axial forces is often neglected, the limiting conditions for the presence of axial forces and their incorporation has been dealt in detail in the present work.

- C. The next major objective is to scrutinize the effect of beam stiffness on the effective transfer of strain and the bandwidth for the PEH system. Like prior investigations, detailed analytical studies were performed followed by experimental investigations. An array of experiments has been performed on varying substrate materials across various base excitation levels to establish the effect of stiffness on bandwidth and strain transfer values.
- D. To augment the understanding of bandwidth for a PEH beam, an innovative solution was proposed. A PEH system consisting of two magnetically coupled cantilever beams of different stiffness. The main cantilever PEH beam being subjected to transverse vibrations, and the auxiliary beam to parametric vibrations. The magnetic coupling causes changes in the potential energy sync of the system resulting in nonlinearity which enhances the bandwidth of the whole system. A rigorous experimental investigation was performed across different substrate materials to enhance the operational bandwidth.
- E. The constant vibration of PEH beams at varying excitation levels causes fatigue damage. Especially, the nonlinear PEH beams are subjected to higher strain levels in the beams to provide a larger bandwidth of operation. This inevitably causes deterioration of both the piezo transducer and the beam substrate. A detailed fatigue analysis at different base excitation levels was undertaken to study the damage characteristics and the limiting strain levels to harness energy efficiently using PEH systems. Analytical, finite element and experimental methods were utilized to provide a detailed picture of the fatigue damage and crack propagation.

1.4. Original contributions

This research primarily aims at analyzing and enhancing the bandwidth of the operation for a PEH system subjected to magnetic nonlinearity. The original contributions from this research can be stated as:

- a) Most of the work reported in the literature for the usage of magnetic induced nonlinearity utilized magnetostatic interaction equations of a very basic nature and lacked a complete rigor to define the nonlinearity introduced into the PEH

system. This research brings forth a comprehensive model to define the magnetostatic interaction equations to accurately define the forces coming on to the harvester system in the presence of magnets. Moreover, these equations can be combined with the harvester models to obtain a mathematical solution for the nonlinear harvester system.

- b) Secondly, the limiting criteria for the application of magnetic forces was seldom reported in the literature. The present work explores the effects of different components of the magnetostatic forces and defines the criteria for efficient modeling of the PEH systems. Apart from this, the effect of stiffness on the bandwidth and effective strain transfer has been investigated. A PEH beam designed to certain combination of parameters may not be utilized efficiently, the stiffness plays a vital role in deciding the maximum bandwidth of operation and the efficiency of the beam to harness vibration energy.
- c) Moreover, this research pioneers in the application of closely spaced magnetostatic interactions inducing multi-stable nonlinear configurations to develop a PEH system which can be utilized over a broader frequency range. In addition, the techniques to enhance the mono-stability of the nonlinear PEH system to improve the utilization ratio of the potential energy sync have been presented.
- d) An innovative design to explore the coupling between transverse and parametric vibrations of PEH beams in the presence of magnetic induced nonlinearity has been presented. The magnetostatic nonlinearity distorts the potential energy of the whole system resulting in an enhanced bandwidth of operation frequency for the whole PEH system. The effect of stiffness in enhancing the bandwidth at 100 μ W and quarter-power levels have been thoroughly examined. Additionally, the behavior of the whole PEH system when subjected to random vibrations has also been studied.
- e) A unique fatigue based study to understand the damage progression and the corresponding usability of the PEH beam has been presented with detailed experimental, analytical and finite element based studies. A comprehensive FEA model for the piezoelectric macro fiber composite transducer has been introduced. This enhances the accuracy in modeling PEH beams. The overall

fatigue study provided a limiting criterion of minimizing the number of times maximum strain in the PEH beam exceeds $500\ \mu\epsilon$, to prolong the life of the harvester beam.

1.5. Organization of the report

The present thesis has been organized into eight chapters. The first chapter gives a brief introduction to the area of energy harvesting, the various research objectives considered and the original contributions through this work. The second chapter presents a detailed chronological development of the field of vibration based piezoelectric energy harvesting, it also presents the work done by various researchers which aid this present study and the limitations and drawbacks faced by them which become the stepping stones for this work. Chapter three focuses on the enhancement of the magnetostatic interaction formulation and its derivation. A detailed evaluation of various magnetostatic formulations and their applicability is presented. The fourth chapter illustrates the effect of magnetic forces on a piezoelectric energy harvesting system. A detailed derivation to show the importance of the effect of magnetostatic forces has been presented. In addition, the enhancement in bandwidth of the operating frequency in the presence of one or more than one end magnet is explored in details with the help of experimental investigations. The fifth chapter presents the importance of the beam stiffness and the strain transfer with respect to the operating bandwidth of the PEH system. PEH beams with varying substrate configurations were investigated in the presence of magnetic induced nonlinearities to understand the relationship between the three parameters. An innovative design to harness the vibration energy with the help of two coupled PEH beams is presented in the sixth chapter. The beams are magnetically coupled and are excited in transverse and axial directions respectively. The response obtained for the system provides two closely spaced resonant frequencies and an enhanced bandwidth of operation for the whole system. This chapter is followed by a thorough investigation of the effect of fatigue in the PEH beams. The progress of fatigue related damage has been explored till the ultimate failure of the substrate. Recommendations for safe usage of the harvesters are also presented in the seventh chapter. The eighth chapter summarizes the work presented

in this study and a few recommendations for future work are also presented. It is followed by references and appendix.

CHAPTER 2 VIBRATION BASED PIEZOELECTRIC ENERGY HARVESTING

A brief introduction to the concepts of energy harvesting and various types of energy harvesting mechanisms has been covered in the previous chapter. This chapter deals in detail about the recent advances in piezoelectric energy harvesters, which convert the vibration energy to electrical potential difference with the help of piezoelectric transducers.

2.1. Piezoelectricity and piezoelectric materials

Piezoelectricity is the phenomenon of developing an electrical potential difference across the material in the presence of a mechanical deformation and the development of a mechanical strain when an electrical field is applied to the same material. These are named as the direct and converse effects of piezoelectricity, and the physicists Jacques and Pierre Curie discovered these effects in the year 1880.

Many materials both naturally occurring and synthetically manufactured exhibit this property in varying degrees. Some naturally occurring materials are: Berlinite, sucrose, quartz, Rochelle salt, topaz, tourmaline group minerals, bone, tendon, silk, enamel, DNA, bacteriophage proteins etc. some synthetic materials are: Barium titanate, lead titanate, lead zirconate titanate (PZT), lithium niobate, sodium tungstate, zinc oxide etc... Of all the synthetic materials, PZT is the most commonly used piezo material because of its higher piezoelectric constant.

The commercially available materials undergo a poling process to have the piezo properties activated. The process includes heating the material to its curie temperature and applying a strong electric field of the order of 1 kV/mm (Crawley and Anderson, 1990) to orient the ions in the poling direction. Most commercially available piezo materials have their poling coefficients as d_{33} , d_{31} , and d_{15} which correspond to the longitudinal, transverse and shear deformations when an electrical field is applied along or perpendicular to the poling direction. Moreover, for thin films, the contribution from shear deformations becomes negligible. The effective configurations of the unimorph and bimorph beams with piezo materials attached to

them in both d_{33} mode and d_{31} mode are as shown in the Figures 2.1 and 2.2 (Safari and Akdogan, 2008). Most piezo materials used in energy harvesting constitute of the d_{31} mode.

Figure 2.1 Bending principles of a unimorph in d_{31} and d_{33} modes; E, P, and M represent the directions of the electric field, dielectric polarization and the bending moment respectively (Safari and Akdogan, 2008).

Figure 2.2 Bending principles of a bimorph cantilever with electrical connections in parallel and in series (Safari and Akdogan, 2008).

The fundamentals of all the mathematical modeling of the piezoelectric energy harvesting (PEH) beams initiates from the basic constitutive equations of the piezo materials. The linear forms of these constitutive equations which are used in the modeling are shown below.

$$d - \text{form} \quad \begin{cases} S = s^E T + d E \\ D = d T + \varepsilon^T E \end{cases} \quad (2.1)$$

$$e - \text{form} \quad \begin{cases} T = c^E S - eE \\ D = eT + \varepsilon^S E \end{cases} \quad (2.2)$$

$$h - \text{form} \quad \begin{cases} T = c^D S - hD \\ E = -hS + \beta^S D \end{cases} \quad (2.3)$$

$$g - \text{form} \quad \begin{cases} S = s^D T + gD \\ E = -gT + \beta^T D \end{cases} \quad (2.4)$$

Where, D (C/m²) is the electric displacement tensor; S is the strain tensor; E (V/m) is the applied electric field tensor; T (N/m²) is the stress tensor; ε (F/m) is the dielectric constant tensor; $\beta^T \text{ or } S$ (m/F) is the dielectric impermeability tensor; d (m/V), e (C/m²), h (V/m) and g (m²/C) are four forms of piezoelectric coefficient tensor; c (N/m²) is the elastic constant tensor; s (m²/N) is the elastic compliance tensor; and the superscripts of ‘ T ’, ‘ S ’, ‘ D ’, and ‘ E ’, indicate the constants being measured at constant stress, constant strain constant electric displacement and a constant electric field, respectively (IEEEStandard, 1988; Ikeda, 1990). Most piezo manufacturers provide the coefficients for the d – form and e – form and these are widely used for mathematical formulations. In the literature terms like forward electromechanical coupling and the backward electromechanical coupling are used to refer to the coupling caused by the direct piezoelectric effect and the converse piezoelectric effect, respectively (Erturk and Inman, 2011b).

The most popular commercially available piezo material is the lead zirconate titanate (PZT) ceramics. It exhibits high modulus of elasticity and low tensile strength and is extremely brittle, which results in difficulty of handling and bonding on curved surfaces (Sirohi and Chopra, 2000; Sodano *et al.*, 2004b). Over the years many composites have been developed, one of them being polyvinylidene fluoride or simply PVDF. Though the piezoelectric coefficient of PVDF is small, its enhanced flexibility enables its usage for fluid-induced or wind-induced vibrations (Akaydin *et al.*, 2010; Allen and Smits, 2001; Li *et al.*, 2011; Ovejas and Cuadras, 2011). Other varieties of piezo composites are the active fiber composites (AFC) and Macro fiber composites (MFC), these consist of layers of active piezoceramic fibers embedded in a polymeric matrix phase giving the composites increased

flexibility and ability to be bonded on curved surfaces (Bent and Hagood, 1997; Wilkie *et al.*, 2000). Many other materials like PMN-PT single crystals, AlN and ZnO nanowires have been reported in the literature for usage at the micro electromechanical systems (MEMS) level or the nano-scale levels of energy harvesting (Lefevvre *et al.*, 2009; Ren *et al.*, 2006). The latest of the developments is the discovery of engineered graphene, though its applications have not been reported in the literature so far, yet it holds a promising market in the immediate future (Ong and Reed, 2012).

Figure 2.3 (a) Conventional PZT ceramics (PI Ceramic Co.) and (b) Macro fiber composite (MFC) (JEC Group).

2.2. Energy Harvesting using piezoelectric materials

Out of the various methods illustrated in the literature, a broad classification can be given based on the type of installation of the patches (Liang and Liao, 2010). One of these methods include attaching the piezo transducers on the structure directly,

this method has a major advantage that the energy extracted from the structure does not have any dampening effects on the host structure as the coupling of the piezo elements and the structure would barely hamper any properties of the host structure. But, such a method involves direct extraction of energy from the structure, intuitively the harvested energy depends on the magnitude of the vibrations in the structure, thus it will always be very low. On the other hand, the second method involves attaching a smaller harvesting structure on the main structure. The piezo transducers are in turn attached to the harvesting structure which is usually a cantilever beam designed to vibrate at tuned resonant frequencies. Thus, accommodating the vibrations coming onto the main host structure. These have been schematically depicted in the Figure 2.4. As the harvesting structure is tunable, it provides an enormous opportunity to tap a higher amount of vibration energy.

Figure 2.4 Representaion of type of energy harvesting using (a) surface bonded piezo transducers, and (b) harvesting structure attached on the host structure (Liang and Liao, 2010).

The complete energy harvesting (EH) system consists of the harvesting structure as shown in Figure 2.4 and the EH circuit. This whole system can be used to supply

energy for low power electronic devices. Fig 2.5 shows a schematic representation of the energy flow in an EH system. This gives a better understanding into the behavior of an EH system when it is subjected to external mechanical vibrations (Liang, 2010). The backward coupling can be caused at both the mechanical and the electrical levels. The mechanical backward coupling can cause an effect on the host structure and the electrical backward coupling can cause an effect on the harvester as discussed above. Recently researchers reported that the impedance properties of the harvesting circuitry cause an effect on the vibration of the harvesting mechanical components (Szarka *et al.*, 2012).

Figure 2.5 Energy flow in a typical PEH device (Liang and Liao, 2012).

As the energy gets harvested, the mechanical vibration energy is converted to the electrical energy with the help of the piezo transducers; this cannot be utilized in its raw form as many low power electronic circuits have stringent requirements. Thus, the EH circuitry comes into the picture, in a usual circuit the raw energy which is in the form of alternating current (AC) is rectified using a bridge converter (AC-DC converter) to convert it into a direct current (DC) supply. It is further smoothened using a DC-DC converter so that the harvested energy can be stored or utilized by electronic circuits. This whole process is shown in Figure 2.6.

Figure 2.6 A typical layout of PEH system with a rectifier and secondary regulator.

The following figures Figure 2.7 and 2.8 show an example of the application of the harvested energy. Arms *et al.* (2005) came out with a temperature and humidity sensing node which utilized the energy that was harvested from mechanical vibrations of a cantilever beam. This application was one of the foremost works in the utilization of harvested energy for wireless sensors. Though there are many major drawbacks, yet it paved a way for many researchers to consider the problem and propose numerous solutions for the same. Many researchers have forayed into the field of applications of the harvested energy but two major drawbacks remain. Namely, the utilizable bandwidth of frequency needed to induce the mechanical vibrations in the PEH device, and the longevity of the harvester system, which have handicapped all the efforts. Thus, over the years there has been tremendous research to broaden the bandwidth of operation frequency for the mechanical components and prolonging the usage of the overall PEH system. There by, optimizing its operation and enhancing the response.

Figure 2.7 A layout of a wireless sensing node which uses harvested energy (Arms *et al.*, 2005).

Figure 2.8 A wireless temperature and humidity sensor powered by a PEH device (Arms *et al.*, 2005).

2.3. Analytical modeling of piezoelectric energy harvesters

The modeling of the PEH system broadly consists of modeling the mechanical components and the electrical circuitry. The present work is more focused on the mechanical domain of the PEH system, thus a detailed review of all the important methods used in analytical modeling of the mechanical components i.e. the vibrating cantilever beam and its coupling with the piezoelectric materials is given precedence and dealt in detail. A brief overview of different types of electrical analogies for modeling is also presented towards the end of this chapter.

The analytical modeling of the PEH system (henceforth, PEH system mostly refers to the mechanical parts) was initially developed as a simple lumped parameter model which was based on the study of a combination of mass, spring and damper systems (duToit *et al.*, 2005; Sodano *et al.*, 2004a; Williams and Yates, 1996). This was evolved into a fully-fledged study of the system using the distributed parameter models where the beam is analyzed based on its modal parameters. This was further simplified using Rayleigh-Ritz approach (Erturk and Inman, 2011b). In the recent past, models have been developed to analyze the PEH system using finite

element analysis (FEA) also, but they are limited to the harmonic vibration field owing to the formulation difficulties for random vibration analysis in most commercially available softwares. Apart from these, the PEH system has also been simulated as an equivalent circuit to aid easier integration and understanding when the mechanical parts are connected to the electrical circuits (Yang and Tang, 2009).

2.3.1 Basic single degree of freedom (S'DOF) model

In this model, the PEH system is modeled as a combination of mass spring and damper system subjected to a base excitation. This was applied to model an electromagnetic generator (Williams and Yates, 1996) and the coupling effect was considered to be similar to viscous damping effect, though this assumption does not hold good for a PEH system as the coupling consists of both forward and backward effects. Using this basic model, the behavior of a system can be understood though the accuracy is limited.

$$m\ddot{z}(t) + c\dot{z}(t) + kz(t) = -m\ddot{y}(t) \quad (2.5)$$

Where, m is the seismic mass, c is the damping in the system, k is the stiffness of the system, $z(t)$ is the relative displacement of the seismic mass and $y(t)$ is the base displacement of the system. The Figure 2.9 shows a schematic representation for this model.

Figure 2.9 A schematic representation for a basic S'DOF model (Williams and Yates, 1996).

2.3.2 Coupling of S'DOF model

As discussed above and shown in the energy flow diagram, the piezoelectric coupling effect on the structure has a backward coupling phenomenon which is due to the constitutive equations of the piezo materials. This behavior was included in the mathematical modeling by duToit *et al.* (2005), he established a model of S'DOF system which considered the coupling effect caused by a d_{33} - piezo transducer attached to the harvester substrate. The governing equations for such a system are shown below.

$$\ddot{w}(t) + 2\zeta_m \omega_n \dot{w}(t) + \omega_n^2 w(t) - \omega_n^2 d_{33} v(t) = -\ddot{w}_b(t) \quad (2.6)$$

$$R_{eq} C_P \dot{v}(t) + v(t) + m_{eff} R_{eq} d_{33} \omega_n^2 \dot{w}(t) = 0 \quad (2.7)$$

Where, $w_b(t)$ is the base displacement, $w(t)$ is the relative displacement of the proof mass as shown in Figure 2.10, $v(t)$ is the output voltage from the PEH beam, m_{eff} is the effective mass, ζ_m is the mechanical damping ratio, ω_n is the natural frequency of the system, C_P is the capacitance of the piezo transducer and R_{eq} is the equivalent resistance of the circuit.

Figure 2.10 A schematic representation of coupled S'DOF model (duToit *et al.*, 2005).

2.3.3 Introduction of correction factors in S'DOF models

The S'DOF models discussed above had several drawbacks in the modeling, they could not represent the system behavior accurately at resonance and more over the effect of the tip mass was not considered. To include these effects, the distributed parameter models were introduced and the correction factors for the S'DOF models were proposed to obtain a more accurate estimation of the system dynamics using the S'DOF models (Erturk and Inman, 2011b). Using these correction factors the equations for a S'DOF system are modified as shown below.

$$m\ddot{z}(t) + c\dot{z}(t) + kz(t) = -\mu_l m\ddot{y}(t) \quad (2.8)$$

$$m\ddot{z}(t) + c\dot{z}(t) + kz(t) = -\kappa_l m\ddot{y}(t) \quad (2.9)$$

Here, μ_l and κ_l are the correction factors for the transverse and the longitudinal vibrations respectively, and these are obtained from curve fitting expressions as follows:

$$\mu_1 = \frac{\left(\frac{M_t}{mL}\right)^2 + 0.0603\left(\frac{M_t}{mL}\right) + 0.08955}{\left(\frac{M_t}{mL}\right)^2 + 0.4637\left(\frac{M_t}{mL}\right) + 0.05718} \quad (2.10)$$

$$\kappa_1 = \frac{\left(\frac{M_t}{mL}\right)^2 + 0.7664\left(\frac{M_t}{mL}\right) + 0.2049}{\left(\frac{M_t}{mL}\right)^2 + 0.6005\left(\frac{M_t}{mL}\right) + 0.161} \quad (2.11)$$

Where, M_t is the proof mass or the tip mass of a cantilever harvester and m is the mass per unit length of the beam, L being the total length of the beam. These correction factors have been used widely in the literature and have been proven to yield a better estimate of the system characteristics.

The correction factors presented were limited to a system with unimorph and bimorph configurations of the substrates. One major drawback of the study was the omission of effects due to the presence of a considerably larger proof mass or tip mass. Kim *et al.* (2010) provided substantial evidence in their work that there is considerable effect when a larger proof mass or tip mass is attached to the PEH beam. They introduced the corrections by including static moment S_0 , mass of the overhang M_0 and changes to moment of inertia I_0 in the presence of proof mass. These were incorporated into the conventional analytical models for PEH systems.

$$M_0 = m_0 L_0 + mL_0 \quad (2.12)$$

$$S_0 = M_0 \frac{L_0}{2} \quad (2.13)$$

$$I_0 = \frac{m_0 L_0}{3} (L_0^2 + h_0^2) + \frac{m L_0}{3} (L_0^2 + h^2) \quad (2.14)$$

Where, mass, length and height are denoted by m , L , and h respectively and the subscript ‘0’ denotes the corresponding parameter for the proof or tip mass. A typical representation of the PEH beam with tip mass is shown as below.

Figure 2.11 Schematic representation of a typical beam with proof mass undergoing base excitation (Kim *et al.*, 2010).

2.3.4 Distributed parameter model

One of the major drawbacks for the S'DOF model was the limitation to model the system dynamics only at the fundamental mode. Thus Erturk and Inman (2011b) proposed a distributed parameter model which represented the PEH cantilever beam to a higher degree of accuracy. Figure 2.12 shows a schematic diagram of the cantilever unimorph beam without any tip or proof mass.

Figure 2.12 A schematic for the unimorph PEH cantilever beam (Erturk and Inman, 2011b).

The governing equations of motion for such a system is shown below in Equation 2.15.

$$\begin{aligned}
 & \frac{\partial^2}{\partial x^2} \left[YI \frac{\partial^2 w_{rel}(x,t)}{\partial x^2} \right] + c_s I \frac{\partial^5 w_{rel}(x,t)}{\partial x^4 \partial t} + c_a \frac{\partial w_{rel}(x,t)}{\partial x} + m \frac{\partial^2 w_{rel}(x,t)}{\partial x^2} \\
 & + \Theta V(t) \left[\frac{d\delta(x)}{dx} - \frac{d\delta(x-L)}{dx} \right] = -m \frac{\partial^2 w_b(x,t)}{\partial x^2} - c_a \frac{\partial w_b(x,t)}{\partial x}
 \end{aligned} \tag{2.15}$$

where YI is the average bending stiffness, I is the equivalent moment of inertia of the cross section of the beam, c_s is the strain rate damping, c_a is the damping due to air resistance, L is the total length of the beam, m is the mass distribution of the beam, Θ is the electromechanical coupling coefficient, $V(t)$ is the output voltage across the resistor, $\delta(x)$ is the dirac delta function, $w_b(x,t)$ and $w_{rel}(x,t)$ are the base motion and the motion relative to the base at a point x , and time t along the length of the beam.

This equation can further be simplified using the modal expansion for a cantilever beam under harmonic vibration, and the governing electromechanical coupled equations for the PEH beam can be written as:

$$w_{rel}(x,t) = \sum_{r=1}^{\infty} \phi_r(x) \eta_r(t) \quad (2.16)$$

$$\ddot{\eta}_r(t) + 2\zeta_r \omega_r \dot{\eta}_r(t) + \omega_r^2 \eta_r(t) + \chi_r v(t) = - \frac{\partial^2 w_b(x,t)}{\partial t^2} \left(m \int_0^L \phi_r(L) dx \right) \quad (2.17)$$

$$\frac{v(t)}{R_l} + C_P \dot{v}(t) - \sum_{r=1}^{\infty} \phi_r \dot{\eta}_r(t) \quad (2.18)$$

In the above equations, the system dynamics are represented for an r^{th} mode of vibration and, χ_r and ϕ_r represent the electromechanical coupling coefficients.

The equations were validated with the help of experimental investigation and the response obtained from the analytical model provided a good insight into the resonance shifting phenomenon which is caused by the backward coupling effect as shown in Figure 2.13. The prediction of the maximum power output for a PEH system when it is connected to a variable resistive load is as shown in Figure 2.13.

(a)

(b)

Figure 2.13 (a) Shifting of the peaks of the Voltage frequency response function (FRF) with changing external resistive load, and (b) The peak of power amplitude FRF obtained with the distributed parameter model for changing resistance (Erturk and Inman, 2011b).

2.3.5 Rayleigh-Ritz approach to solve complex coupled models

Solving for the shape function of the mode shape can be a tedious task as the behavior of the structure gets more complex. In order to overcome this problem, the Rayleigh-Ritz approach was utilized to find an approximate solution for the dynamics of the PEH (Elvin and Elvin, 2009a; Elvin and Elvin, 2009b; Sodano *et al.*, 2004a).

$$M\ddot{r} + C\dot{r} + Kr - \Theta v = -\ddot{w}_b \left(\int_0^L m\varphi(x)dx + M_t\varphi(L) \right) \quad (2.19)$$

$$\Theta^T \dot{r} + C_P v = I \quad (2.20)$$

where M , C , K , Θ and C_P are the lumped parameters for mass, damping, stiffness, piezoelectric coupling coefficient matrix and Capacitance of the piezo transducer respectively, r is the displacement vector, m is the distribution of mass along the length of the beam, M_t is the proof mass, w_b is the base excitation, φ is the vector of the assumed mode shape function, I is the current vector and v is the voltage vector.

In the Rayleigh-Ritz approach, the deflection curve for the PEH beam is assumed to be a combination of various sinusoidal and hyperbolic functions. The accuracy of this approach depends on the accuracy in approximating the mode shape function. The main advantage of this method is that it is computationally easier to solve. There are a few researchers who solved for the mode shape functions of the complex dynamic problems, though these methods provide a better accuracy, they are computationally very taxing and thus one need to draw a line between accuracy and computational effort (Abdelkefi *et al.*, 2013).

2.3.6 Modeling using Finite element methods

The developments in many powerful finite element analysis (FEA) commercial softwares like ANSYS, ABAQUS, COMSOL etc. and high power computing, have enabled many researchers to rely on finite element modeling for solving complex problems (De Marqui Junior *et al.*, 2009). An electromechanical modeling of the piezoelectric energy harvester plates was accomplished by De Marqui Junior *et al.* (2009) and it was found to be in good confirmation with experimental observations.

Zhu *et al.* (2009) developed a finite element coupled piezoelectric-circuit model to assess the power output of PEH system. They analyzed the response of a harvester to study the effect of a load resistance using ANSYS. In a later study, the geometric parameters were varied to optimize the power output of the system. Yang and Tang (2009) presented an equivalent circuit model and resorted to the finite element

techniques to identify the system parameters for the respective modes of vibration. Arafa *et al.* (2011) explored the analysis of a cantilever PEH beam attached with a dynamic magnifier, a close agreement between the responses obtained from FEA and experimental investigations were reported. More recently, Abdelkefi *et al.* (2014) performed a detailed study on modeling and analysis of piezoelectric energy harvesters designed for low frequency; they used FEA to validate the experimental and analytical models. The FEA model was validated against experimental results with a very high level of accuracy. Wu *et al.* (2013) demonstrated a detailed investigation into the enhancement of the power output and bandwidth with the help of a secondary beam attached to a primary cantilever PEH beam. The results obtained from the investigation showed a similar trend in both the experiment and FEA. Upadrashta and Yang (2015) provided an innovative solution to the problem on the modeling of magnetic nonlinearity using FEA. They modeled the magnetic interaction in nonlinear piezoelectric energy harvesters with the help of COMBIN39 nonlinear spring element in ANSYS. Though the applicability of the procedure was limited, it provided an initial step to solving the tedious problem of nonlinear magnetic interactions.

Furthermore, the commercially available MFCs constitute piezoelectric fibers enclosed in a composite of epoxy, copper electrodes and kapton layers (High *et al.*, 2003; Smart Material Corp., 2015; Wilkie *et al.*, 2002; Williams *et al.*, 2004a). Numerous finite element analysis studies have been performed to understand the behavior of the MFC as a composite, and many equivalent models of the composite MFC are available in the literature (Collet *et al.*, 2010; Deraemaeker and Nasser, 2010; Novakova and Mokry, 2011; Steiger and Mokry, 2015). Among these, the recently published equivalent model by Katerina and Pavel (2015) presents a very accurate representation of the layered MFC. In the work done by Meiling *et al.* (2009; 2010) the model was represented by a uniform piezoelectric layer attached on a brass substrate, and FEA was used to perform a parametric study for arriving at an optimum configuration of the system. The major drawback of most of the above mentioned FEA models is that they presented a simple uniform PZT layer attached on the vibrating beam, while in reality the MFC is a composite and requires more

detailed modeling as a composite to obtain a more realistic behavior of the transducer (Steiger and Mokry, 2015).

Though the FEA modeling holds a greener prospect for future research in analysis of energy harvesting systems, the main drawback lies in the fact that most commercial softwares can solve the problem accurately in the mechanical domain only. The integration of the mechanical and electrical domains of a PEH system remains at a very elemental level. Moreover, the integration of fluid-structure interactions, magnetic interactions, the electromechanical-structure interactions in piezo aero elastic energy harvesters, piezo fluidic multi-physics analysis etc. render the FEA formulation too tedious and computationally expensive. Considering all the drawbacks in the present FEA models, an FEA model representing the composite structure of MFC is worked out using the finite element multi-physics package of COMSOL in this work.

2.3.7 Analysis using equivalent circuit modeling (ECM)

Owing to the drawbacks in most analytical modeling methods which can simulate the mechanical parameters efficiently but fail to analyze the whole system by combining the electrical circuitry and the mechanical domain; an innovative method was developed in which the mechanical parts were modeled as equivalent components of an electrical circuit (Elvin and Elvin, 2009b; Yang and Tang, 2009).

The parameters in the mechanical domain were matched with the electrical domain as shown in the Table 2.1 (Yang and Tang, 2009). The concept that drives such matching is the structural and electrical impedance of both the mechanical and electrical domains respectively. Once the parameters of the corresponding mechanical domain are obtained in terms of the equivalent parameters in the electrical domain, the circuit simulation softwares can be utilized to obtain the response of the equivalent system. Yang and Tang (2009) stated that there was a good agreement between the mathematical models, ECM and experiments results. Thus, cementing the fact that ECM can be used as an alternative. Moreover, ECM accommodates the multimodal simulation of the PEH system. The analysis of many modes for complex structural configurations can become a tedious and expensive

computational procedure, thus the distributed parameter modeling is more advantageous in such situations.

Table-2.1: Analogy between the electrical and mechanical domain (Yang and Tang, 2009)

Electrical circuitry parameters for an r^{th} mode of vibration	Corresponding mechanical parameters
Charge $q_r(t)$	Modal coordinate $\eta_r(t)$
Current $i_r(t)$	Modal velocity $d\eta_r(t)/dt$
Inductance L_r	I
Resistance R_r	$2\zeta_r\omega_r$
Capacitance C_r	$1/\omega_r^2$
Voltage source $v_r(t)$	$-f_r\ddot{u}_g(t)$
Ideal transformer ratio N_r	Electromechanical coupling χ_r

Figure 2.14 Equivalent circuit model of a PEH for different modes of the harvester (Yang and Tang, 2009).

2.4. Efficiency enhancing techniques and Broadband energy harvesting

The field of wireless electronics and sensor technologies is constantly evolving and strives to make the technology more advanced and efficient. To match this rate of constant evolution many researchers in energy harvesting have come up with various methods to enhance the efficiency of the energy harvesters. These can broadly be classified into: (1) Advances in piezoelectric and structural

configurations; (2) Broadband energy harvesting techniques; (3) Advances in circuit interface to utilize the harvested energy.

2.4.1 Advances in piezoelectric and structural configurations

The effect of proof mass was discussed earlier in the modeling section, to take this further a few researchers (Anderson and Sexton, 2006; Kim *et al.*, 2011) studied the effect of proof mass on the performance of the beam. Their studies also included variation of other geometrical parameters like length and width of the beam. It was concluded that the proof mass plays a vital role in enhancing the harvesting properties of the beam but the major drawback is that the beam under vibration will be more susceptible to fatigue failure with increasing proof mass (Shafer *et al.*, 2012).

From mechanics, it is obvious that the strain distribution in a cantilever beam is not constant but varies along the length. Thus, the strain distribution varies along the length of the piezo layer; to maintain a constant strain in the beam along its length and enhance the efficiency of the harvester, studies were conducted on different shapes of the beam. Roundy *et al.* (2005) proposed that a trapezoidal beam sloping along the width was a much better option compared to the conventional rectangular shape as shown in Figure 2.15. The trapezoidal beam could supply twice the energy per unit volume of PZT, but the major hurdle lied in its manufacturing and analytical modeling for further optimization as a varying cross-section would only add to the computational woes.

Another enhancement from a strain distribution point of view was proposed by Xu *et al.* (2010). The proposed design consisted of an auxiliary beam which was attached at the end of the main beam as shown in Figure 2.16. This resulted in an L-shaped beam which works on the concept of the torque produced by the auxiliary beam inducing a constant strain on the main beam. Even this design generated twice the power output in comparison to the conventional rectangular cantilever beam.

Figure 2.15 Distribution of strain along the length for beams with changing widths (Roundy et al., 2005).

Figure 2.16 The L-shaped beam to induce constant strain along the main beam (Xu et al., 2010).

A few other methods to improve the power output include the multilayer piezoelectric stack configurations (Platt et al., 2005); an initial curved configuration of the PZT unimorph which gives an enhanced charge output (Danak et al., 2003); and the usage of meandering shape or a rolled up layer ring type configuration to harvest energy at MEMS scale (Massaro et al., 2011). Most of these proposed

fundamental designs aim for an enhanced output response, but the bandwidth of operation frequency was overlooked. The next section lists a few advances for enhancing the bandwidth of PEH systems.

2.4.2 Broadband Energy harvesting

The initial designs of harvesters found in the literature were mostly based on the concept of linear resonators where the harvester structure (cantilever beam) vibrates with maximum amplitude with a single peak at the structure's resonant frequency. This behavior caused a great limitation on the performance of the harvester over a broad range of frequency, as most real life field scenarios consist of a broad range of frequencies and a random range of amplitudes. Thus, to overcome this limitation many techniques were developed over the years.

Frequency tuning methods

The frequency tuning methods primarily consist of tuning the harvester frequency to make it more functional over a broader range of frequency. These can be further classified as *active* and *passive tuning methods* depending on the mode of tuning. In active methods, a constant supply of external power is required to maintain the vibration of the harvester; and in passive methods, an intermittent power input is required to disturb the harvester (Roundy and Zhang, 2005). This can be achieved using a mechanical tuning mechanism, magnetic methods and piezoelectric methods.

The mechanical tuning methods consist of prestressing the beam either in tension or compression to change the resonant frequency (Eichhorn *et al.*, 2009; Hu *et al.*, 2007; Leland and Wright, 2006). The models presented by the three researchers needed to be tuned manually. Hu *et al.* (2007) came out with an analytical model for the axially prestressed cantilever beam and inferred that using a tensile and compressive load of 50 N, the frequency can be increased from 129.3 Hz to 169.4 Hz and from 129.3 Hz to 58.1 Hz respectively. The analytical model was comprehensive but it lacked experimental validation. Eichhorn *et al.* (2009) proposed a bidirectional frequency tuning mechanism as shown in Figure 2.17 and

concluded from their experiments that a compressive load could shift the frequency by 22% and the tensile load shifted the frequency by about 4%. Another interesting mechanical tuning mechanism was presented by Wu *et al.* (2006) and Gu and Livermore (2012), the researchers investigated the option of varying the center of gravity of the proof mass. Wu *et al.* (2006) investigated the option of a movable tip mass controlled by an adjustable screw and the Gu and Livermore (2012) used the option of centrifugal force to control the movement of the tip mass as shown in Figure 2.18.

Figure 2.17 Bidirectional frequency tuning mechanism (Eichhorn *et al.*, 2009).

(a)

(b)

Figure 2.18 Movable tip mass running on centrifugal force (Gu and Livermore, 2012).

The usage of magnets to tune the vibration of the harvesters was employed by various researchers, Challa *et al.* (2008) proposed a tunable cantilever harvester with magnets fixed on the top and bottom of the tip mass. The proposed model could be tuned over a separation of 3cm in a frequency range of 22-32 Hz operating at an acceleration of 0.8 m/s^2 . Improvement over this conceptual model were carried by many researchers, Ahmadian *et al.* (2009) demonstrated a prototype where the

magnet was placed in vertical slider at the free end of the cantilever as shown in Figure 2.19; the model could achieve a workable range of almost 11.38 Hz. A similar improvement over this was achieved by using a smart controller to tune the distance between the tip magnet and the movable magnet Zhu *et al.* (2008).

(a)

Figure 2.19 Prototype to tune frequency using adjustable magnetic slider (Ahmadian *et al.*, 2009).

The piezoelectric methods are based on the utilization of piezoelectric properties of the piezo materials. The piezo transducers are used to vary the stiffness of the cantilever beam using a shunt electrical load (Peters *et al.*, 2009; Wu *et al.*, 2006). This is a major research area known as the piezoelectric shunt damping, but when the similar procedures are applied to the energy harvesting, they seldom resulted eventful response of the system. The major drawback lies in the fact that the power required to sustain the cyclic shunt loading was far more than the power harvested in the process. This was clearly visible in the work by Peters *et al.* (2009) though a 30% tunable frequency was achieved from an initial frequency of 78 Hz, the power consumption for the piezo actuator circuit was 150mW and the power harvester was only 1.4mW.

Figure 2.20 PEH sytem consisting of scavenging and tuning parts (Roundy and Zhang, 2005).

A few other techniques for frequency-up conversion were also proposed in the literature, these techniques were mostly proposed to utilize different mechanisms to pump up the low frequencies available in nature to cause vibration in energy harvesting systems. Rastegar *et al.* (2006) came up with a system of frequency transfer mechanism as shown in Figure 2.21. The system consisted of impacting teeth which would cause vibration in the piezo cantilevers. A further enhancement for this design was performed by replacing the teeth by magnets at the end and placing a magnet in the tip mass (Wickenheiser and Garcia, 2010). Whenever the low frequency excitation is passed to this mechanism it would result in excitation of the PEH cantilever system. Though these designs resulted in an enhanced output for the PEH system, they were limited by their vibration amplitudes necessary to sustain the output. Most of the proposed designs required higher magnitudes of base vibration and at a higher frequency. This is a major limitation for practical application of the proposed designs, as most ambient vibrations have their energy concentrated at frequencies lower than 100 Hz under and at low magnitude. Moreover, higher amplitudes give rise to excessive strain levels in the PEH beams resulting in an early fatigue damage to the PEH system.

Figure 2.21 PEH sytem for frequency-up conversion (Rastegar *et al.*, 2006).

Multimodal techniques

Multimodal energy harvesting refers to the process of exploiting different modes of vibration of a PEH system to harvest energy efficiently over a wider range of frequencies. There are two major methods of applying these techniques, exploitation of the different modes of a single beam and using an array of cantilevers to harvest energy (Tang *et al.*, 2010).

A beam using two vibration modes was proposed by Roundy *et al.* (2005). Though higher modes could be exploited, in most cases the higher modes are widely spaced from the lower modes. Arafa *et al.* (2011) reported a PEH system in which one proof mass served as a dynamic magnifier as shown in Figure 2.22, though it could help in magnifying the power, two closely spaced resonant frequencies could not be obtained. Erturk *et al.* (2009b) presented an L-shaped cantilever PEH system which could utilize the resonance response from two different modes. The harvester was mounted on a UAV for testing and showed considerable improvement over the previous models, the resonant frequencies were achieved around 16 Hz and 49 Hz respectively and the peaks of voltage FRF obtained were close to each other as shown in Figure 2.23. More recently, Wu *et al.* (2013) proposed a new model where

the resonant frequencies could be closely spaced to achieve a maximum bandwidth. One major drawback observed in most of these works was the charge cancellation and deep valleys between the two peaks, and a probable solution for this were to use discontinuous electrodes.

Figure 2.22 PEH sytem utilizing two different modes (Arafa *et al.*, 2011).

Figure 2.23 L-shaped PEH sytem utilizing two different modes (Erturk *et al.*, 2009b).

When an array of cantilever beams is utilized to harvest energy, a wider bandwidth can be achieved by selecting the geometrical properties of the cantilevers

appropriately. Yang and Yang (2009) conducted an analytical study on the performance of a harvester in which two cantilever bimorphs were connected by springs at the tip of their proof masses and concluded that a widened bandwidth can be obtained using this method. Kim *et al.* (2011) proposed a 2DOF PEH system with two piezoelectric cantilevers connected by a rigid mass as shown in Figure 2.24. This system could utilize the translational and rotational modes of vibration of the rigid mass and it exhibited a 280% increase in bandwidth at a voltage level of 55 V/g. Many other prototypes mentioned in the literature consisted of decoupled cantilevers arranged in an array. For efficient functioning of these PEH systems the whole array needs to be connected with an enhanced circuitry to minimize the losses due to cancellations (Xue *et al.*, 2008).

Figure 2.24 2DOF PEH system with rigid mass (Kim *et al.*, 2011).

Nonlinear techniques

The usage of magnets and axial forces to tune the frequency was explained in a previous section. In addition to that, it is interesting to note that the presence of magnets induces nonlinear behavior into the system as it affects the potential energy of the system. Such nonlinear behavior can be used to enhance the bandwidth of a PEH system. From the literature, there are two modes of utilization of the nonlinearity due to magnets: the monostable configuration and the bistable

configuration (Ahmadian *et al.*, 2009; Tang *et al.*, 2012). The bistability in a system can also be induced using the axial force (bucking the beam) as indicated earlier.

Monostable configurations consist of a single stable equilibrium. The system can be represented analytically using a Duffing's equation for dynamic systems given by Equation 2.21 (Mann and Sims, 2009; Ramlan *et al.*, 2010).

$$\ddot{x} + c\dot{x} - ax + bx^3 = f(t) \quad (2.21)$$

Here, the stiffness which originates from the potential energy of the system is modified due the presence of nonlinearity of the magnetic interaction. For $a \leq 0$, the system remains monostable. $b > 0$ represents a hardening system response and $b < 0$ a softening response.

Ramlan *et al.* (2010) reported the study of the hardening mechanism and through numerical and analytical studies it was stated that the maximum power harvested by a system remained constant, however the frequency of its occurrence depended on the degree of nonlinearity. Stanton *et al.* (2009) performed experiments on the monostable configurations and stated that the monostable configurations can be bidirectional and can lead to an enhanced bandwidth of operating frequency when a frequency sweep is performed as shown in Figure 2.25. In addition, Daqaq (2010) demonstrated that under white Gaussian excitation or colored Gaussian excitations, the performance of the hardening type non-linearity caused diminished performance of the harvester, thus suggesting its applicability for frequency sweep excitations only.

(a)

Figure 2.25 Softening and Hardening behaviors of a monostable configuration
(Stanton *et al.*, 2009).

The bistability comes into existence when $a > 0$ in Equation 2.21, this means that the system has two stable equilibriums. Many researchers came up with different designs using different mechanisms, but the most popular of the bistable mechanisms remain the usage of magnets to induce bistability. Erturk *et al.* (2009b) designed a broadband piezo-magneto elastic generator as shown in Figure 2.26. The beam consisted of ferromagnetic material and was subject to oscillation in the presence of two magnets placed at the end. Though the results obtained showed an enhance root mean square (RMS) voltage, but they needed an initial disturbance to excite the beam. Cottone *et al.* (2009) stated a similar bistable mechanism by placing a magnet at the tip of a beam facing another magnet with an opposite polarity. The maximum power obtained exceeded the linear configuration by 4-6 times. To enhance the performance of a PEH in the presence of bistability some researchers proposed the utilization of stochastic resonance, which occurs if the base vibrations are strong enough to oscillate the potential barrier itself and matching this with the mean time between transitions (Formosa *et al.*, 2009; McInnes *et al.*, 2008). The main drawbacks observed in the literature were: most the system needed external perturbation to get excited to higher energy levels, and larger RMS acceleration (magnitudes) were required to maintain the higher energy states. Thus, the main challenge posed for a practical application of such harvesters are to find a solution which would enhance the probability of transition between the potential wells under lower magnitudes of base vibrations.

Figure 2.26 Piezomagnetoelastic power generator (Erturk *et al.*, 2009a).

2.4.3 Efficiency enhancement by optimizing the circuit interface

The previous sections, the outlook of the mechanical domain of the PEH system was dealt in detail. This section presents a brief overview of the electrical domain. Many researchers with a background in electrical circuitry have been coming up with innovative advances to enhance the power output from a PEH circuitry. In practice, the energy harvesting process involves the rectification of the output from the harvester followed by energy storage or connecting it to an electrical load. This is referred to as a standard interface and is shown in Figure 2.27 (Shu and Lien, 2006; Wickenheiser and Garcia, 2010). To enhance the efficiency of the electrical interface many impedance adaptation interface circuits have been proposed (Kong *et al.*, 2010; Ottman *et al.*, 2002; Ottman *et al.*, 2003). Ottman *et al.* (2002) proposed an adaptive circuit using a buck DC-DC converter and an adaptive control algorithm to continuously implement optimal power transfer. It was reported that the usage of the adaptive buck converter could increase the power transferred by over 400% in comparison with charging a battery directly. However, the converter could only work when the input voltage was higher than the output voltage for the circuit and this is a major hindrance at lower excitation levels.

Figure 2.27 Layout of a standard interface circuit connected to (a) Battery and (b) Resistive load.

Figure 2.28 Layout of impedance adaptation interface.

The other major advances in Electrical circuitry involve the *synchronized charge extraction* (SCE) interface and the *synchronized switch harvesting on inductor* (SSHI), which utilize various switching techniques to enhance the power output (Lefeuvre *et al.*, 2010; Liang and Liao, 2010).

Figure 2.29 Layout of a SSHI circuit interface (a) Parallel and (b) Series.

Figure 2.30 Layout of SCE interface.

2.5. Longevity of piezoelectric energy harvesting systems

Enhancing the bandwidth of operation and improving the longevity of a harvester are two major challenges which have inspired research in the field of energy harvesting. Previous sections described recent developments in enhancing the operational bandwidth of PEH systems, this section provides an insight into the research efforts undertaken to study the overall life of a harvester system.

2.5.1 Fatigue in piezoelectric energy harvesting systems

Research in the field of piezoelectric energy harvesting has grown by leaps and bounds in the past few years. Of the various forms of conversion mechanisms available, namely electrostatic, electromagnetic, piezoelectric etc., the PEH retains an added advantage of ease of application and a higher output power density (Erturk and Inman, 2011b). As discussed earlier the first use of PEH was presented by Sodano *et al* (2004b) where a lead zirconate titanate (PZT) patch was attached at the root of a cantilever beam. Over the due course of time, advances in piezoelectric transducers led to the influx of a large variety of transducers like PZT ceramics of

different shapes and sizes, Polyvinylidene fluoride (PVDF), Zinc Oxide (ZnO), diphenylalanine peptide nanotubes (PNTs), active fibre composites and macro fiber composites (MFCs) (Anton and Sodano, 2007). Of all these piezoelectric transducers, MFC is one of the most popular in both sensing and actuation owing to its simplistic design and flexibility (Erturk and Inman, 2011b; Schönecker *et al.*, 2006). Two types of MFC transducers have been widely used for a variety of applications, P1 and P2 respectively. The type P1 transducers having d_{33} configuration are poled lengthwise and are used primarily as actuators in vibration and noise control, piezoelectric morphing, lamb wave propagation etc. The type P2 transducers consisting d_{31} configuration are used for sensing applications like energy harvesting, piezoelectric impedance based structural health monitoring etc...(Schönecker *et al.*, 2006). Energy harvesting from ambient vibrations is achieved by attaching P2 type MFC transducers to a substrate and vibrating it in sync with the substrate. Many researchers have experimented with the piezoelectric materials and different design configurations for vibration energy harvesting, of which the cantilever PEH beam stands out due to its high strain levels at the root of the beam and its simplicity in design (Erturk and Inman, 2011b). It was also observed that cantilever PEH beam forms a base configuration for various enhancements proposed in the earlier sections to increase the bandwidth and applicability of the system, including the use of tip magnets to induce magnetic nonlinearity into the harvesting system, utilization of a systematic array of beams having different natural frequencies and various enhancements in power management circuits (Davidson and Mo, 2014; Pellegrini *et al.*, 2013).

In addition to bandwidth enhancement, there have been considerable efforts in optimizing the design of energy harvesters on the basis of strain and material strength considerations (Kim *et al.*, 2015; Shafer *et al.*, 2012; Upadrashta and Yang, 2015). In order to improve the performance and the operation bandwidth of the PEH beams, higher base excitation levels were often considered (Erturk and Inman, 2011b). Higher base excitation levels result in higher stress-strain at the root of the cantilever-type harvester, but higher levels of such reversal of stresses eventually resulted in fatigue failure of the PEH beam. Wilkie *et al* (2002), Williams *et al* (2004b), and Daue and Kunzmann (2008) addressed the issue of reliability and

strength of a P1 MFC transducer when it was subjected to static mechanical loading and actuated with varying cyclic electrical loads. They reported that under static conditions, the behavior of an MFC can be categorized as linear till a strain level of about 1000-2000 $\mu\epsilon$, but their testing was limited to the actuation mode of MFC. This was further supplemented for energy harvesting applications in the study performed by Stanton *et al* (2012), where an acceleration level of 0.4g-0.5g ($g = 9.81 \text{ m/s}^2$) was deemed enough to limit the behavior of the PEH beam within the linear limits. Soma and Pasquale (2011; 2013) explored the feasibility of attaching a DuraAct piezo transducer (manufactured by PI Ceramic, Germany) on a railway bogie for self-reliant damage monitoring by performing experiments on scaled laboratory models. They stated that at a base excitation level of 1g the DuraAct transducers failed in fatigue after 106 cycles. Sherrit *et al* (2015) conducted a study on the performance of a fluid flow nozzle based energy harvester using commercially available Vulture energy harvester beam from Mide Technology Corporation, United States. They observed that the harvester beams failed within minutes under a flow rate of 15 L/min (resonant frequencies: 120 and 450 Hz). Subsequently, a steel shim reinforced harvester beam endured a flow rate of 16.5 L/min for about 40 min, and no damage was observed under a flow rate of 9.1 L/min after 9 h of testing (approx. 3.9×10^6 cycles). A few other authors explored the fatigue and material properties of MFC transducers and PZT transducers under varying temperature conditions, but they were primarily limited to the actuation modes of the transducers (Isaac *et al.*, 2012; Okayasu *et al.*, 2009a; Okayasu *et al.*, 2009b). The studies concluded that the material strength and the electromechanical coupling parameters of the transducers are greatly affected by cyclic electrical and mechanical loading and varying temperatures. Apart from these, there are few studies in the literature addressing the fatigue issues of PEH beams utilizing the P2 type MFC transducers. To fill this gap, a comprehensive experimental study is performed in this work to investigate the fatigue behavior of a PEH beam subjected to varying levels of base excitation from 0.4g to 0.6g.

Figure 2.31 Experimental prototype of a DuraAct transducer based energy harvester
(Soma and De Pasquale, 2013)

Figure 2.32 Experimental setup of a fluid flow nozzle based energy harvester
(Sherrit et al., 2015)

2.6. Summary

In this chapter, an attempt has been made to present a complete picture into the functioning of a vibration based piezoelectric energy harvester. Firstly, the fundamentals on the working of piezoelectric materials were presented followed by ways to harvest energy using the piezo materials. Then, the analytical modeling procedures of the PEH systems were detailed along with advances in that domain. Subsequently, various fundamental designs for efficiency enhancing techniques were listed which involved the advances in piezoelectric and structural

configurations, then the techniques to enhance the bandwidth were discussed in detail focusing mainly on nonlinear techniques and an overview of advances in energy harvesting circuitry was also presented. Lastly, the importance of prolonging the life of an energy harvesting system is explained.

Apart from the advances various existing limitations were also stated in this chapter. Most of the bandwidth enhancing techniques listed in this chapter have an inherent drawback of a complex mechanism to enhance the bandwidth. For example, the frequency tuning mechanisms require an external assembly to enhance the bandwidth, and the multimodal techniques can result in delicate designs where the parameters need to be controlled efficiently to enhance the bandwidth. On the other hand, nonlinear PEH systems functioning in the presence of magnets follows the simplest design comprising of a vibrating cantilever beam with magnets at the tip. From a practical perspective and field desirability, a simpler design will pave way for a maintenance free harvester system. Thus, an effort has been made to study the nonlinear PEH systems in detail in this work.

The primary focus of this work is to enhance the bandwidth of the PEH systems and to understand the factors effecting the bandwidth and the longevity of the system. As observed from literature the maximum bandwidth obtained using the nonlinearity in the presence of magnets was limited to 5 Hz at a considerable RMS voltage level (40-50% of peak RMS voltage). Moreover, most of the methods used by researchers required initial disturbance of the PEH cantilevers to induce the harmonic vibration. Thus, to overcome these drawbacks new and enhanced designs are proposed in the coming chapters. Additionally, it was observed that most researchers used the simplified formulations to define parameters for the magnetic nonlinearity, the usage of this formulation possesses certain drawbacks, this is elaborated in the next chapter and an enhanced formulation for a greater accuracy is introduced. Furthermore, the literature states that the efficiency of the PEH system is optimum for a transition zone between the monostable and the bistable states, but a clear explanation could not be given for such an observation. An attempt to explain this behavior and a detailed investigation into such an occurrence has been

performed over the course of this study. Lastly, the fatigue issues arising due to continuous usage of the harvester system are addressed in detail.

CHAPTER 3 FORMULATION OF MAGNETOSTATIC INTERACTION EQUATIONS AND MULTIPLE EQUILIBRIUM STATES.

A review of the piezoelectric energy harvesting techniques and recent advances has been presented in the previous chapter. This chapter deals with the formulation of magnetostatic interaction equations and the various equilibrium states that come into existence in the PEH system when it is subject to magnetostatic interactions.

3.1. Introduction

The usage of magnets to enhance the performance of the PEH cantilever beam has been introduced in the previous chapter. The analytical formulation of a system consisting of the magnetic force was represented as dynamic equations for a duffing oscillator as shown in Equation 2.21. In the literature, the nonlinearity induced by the presence of the magnets was defined using simplified magnetostatic interaction equations in the form of dipole-dipole formulation and empirical equations derived from experimental values (Ahmadian *et al.*, 2009; Andò *et al.*, 2012; Cottone *et al.*, 2009; Daqaq, 2010; Erturk *et al.*, 2009a; Erturk and Inman, 2011a; Formosa *et al.*, 2009; Lin *et al.*, 2010; Mann and Sims, 2009; Mansour *et al.*, 2010; McInnes *et al.*, 2008; Pellegrini *et al.*, 2013; Ramlan *et al.*, 2010; Stanton *et al.*, 2009; Tang and Yang, 2012a; Tang *et al.*, 2012; Tang *et al.*, 2011; Vocca *et al.*, 2012; Wickenheiser and Garcia, 2010; Yang *et al.*, 2011). This chapter aims at redefining this formulation to define the magnetostatic interactions in a more accurate formulation. The primary purpose behind this enhanced formulation is to overcome the limitations that most dipole-dipole formulations possess, which is elaborated in the forthcoming sections.

3.2. Dipole-Dipole magnetostatic interaction formulation

Magnetostatic interaction can be defined as the time invariant interaction of two magnetic fields distributed in space. The magnetic fields are produced by two or more than two magnetic materials like permanent magnets, solenoid coils etc. The presence of these magnetic interactions in a PEH system distorts the linear potential energy distribution and induces nonlinearity. Thus, the system behaves nonlinearly.

The influence of this nonlinear potential energy has been represented mathematically using the dipole-dipole formulation equations as stated in Equation 3.1. Though the dipole-dipole formulation has been widely used to represent the magnetostatic interaction between two magnets, the accuracy of the formulation is uncertain when these equations are used to express interaction between magnets placed at distances in the order of the size of the magnets (Vokoun *et al.*, 2009).

$$U_m(x, D) = \frac{\mu_0 m_1 m_2}{2\pi d} (d^2 x^2 + D^2)^{-\frac{3}{2}} \quad (3.1)$$

where, $U_m(x)$ is the potential energy distribution, μ_0 is the permeability constant, m_1 and m_2 are the magnetic moments of two magnets, d is a geometrical parameter which depends on the point of measurement of the variable x , and D is the axial distance between the two magnets.

The drawback in accuracy of modeling the magnets using dipole expressions at close spaces is attributed to the assumptions that are considered for modeling magnets using these expressions. The important assumptions to be taken into consideration are: (1) The magnets are replaced by point dipoles, (2) The magnets are axially aligned always and (3) The magnets are separated with considerable distance from each other. The drawbacks resulting from these simplified assumptions are discussed with the help of mathematical derivations in the next section. To overcome these drawbacks, the usage of an enhanced formulation for the magnetostatic interaction forces is also proposed in this chapter.

3.3. Enhanced integral magnetostatic interaction formulation

Most PEH cantilever systems demonstrate nonlinear behavior when the magnets attached at the tip mass and those facing it are placed at close distances (in the order of the size of the magnets). The distances being in the order of less than 15mm. It is interesting to note that, when the distance between the magnets is close enough the size effects need to be considered. This was proven experimentally by Vokoun *et al.* (2009), and the analytical formulation for two cylindrical magnets was derived in their study. The major assumptions involved in the enhanced magnetostatic

interaction formulation are: (1) The magnets are uniformly magnetized, (2) The magnetization vectors of the magnets are along the axis of symmetry of the cylindrical magnets and (3) The magnetization vector rotation is restricted. Within the constraints of the assumptions for a Euler-Bernoulli beam these assumptions can be applied to model the magnetostatic interaction with a much better accuracy. In this section the derivations for the analytical formulation to simulate the interaction behavior between cylindrical magnets have been presented.

3.3.1 Case: I – Magnetostatic interaction between two similar magnets.

Figure 3.1 Schematic representation of two cylindrical magnets with their magnetization vectors aligned.

Considering a system of two magnets which are placed as shown in Figure 3.1. The magnets are placed with their magnetization vectors pointing in the same direction, thus the magnets will be attracting each other. Now, for a system with magnets having same radii, the equations for energy are as given below in Equation 3.2.

$$E_{i,l-2}(r, \varsigma) = 8\pi K_d R^3 \int_0^\infty J_0 \left(\frac{rq}{R} \right) \frac{J_1^2(q)}{q^2} \sinh(q\tau_1) \sinh(q\tau_2) e^{-q\varsigma} dq \quad (3.2)$$

$$\text{where } K_d = \frac{\mu_0 M_1 M_2}{2}, \tau_i = \frac{t_i}{2R}, \varsigma = \frac{Z}{R}, \text{ and } Z = \frac{\sum t_i}{2} + a$$

here $E_{i,l-2}$ is the interaction energy between the magnets 1 and 2, μ_0 is the permeability of vacuum, M_1 and M_2 are the magnetizations of magnets 1 and 2

respectively, R is the radius of the cylindrical magnets, J_0 and J_1 are the modified Bessel functions of the first type of order 0 and 1 respectively, r is the transverse separation between the magnetic axes along Y-direction, a is the axial separation between the ends of the magnets along Z-direction, the Bessel functions in q are used to define the shape of the magnets and t_i is the height of the cylindrical magnets. From elementary mechanics, the force vector can be obtained by deriving the gradient of the interaction energy.

$$\bar{F} = -grad(E_{i,l-2}(r, \varsigma)) \quad (3.3)$$

$$\text{where } \bar{F} = F_X \hat{i} + F_Y \hat{j} + F_Z \hat{k}$$

F_X , F_Y , and F_Z are the components of force in X, Y, and Z directions respectively.

$$\bar{F} = -\left(\frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k}\right) E_{i,l-2}(r, \varsigma) \quad (3.4)$$

$$F_Z = 8\pi K_d R^2 \int_0^\infty J_0\left(\frac{rq}{R}\right) \frac{J_1^2(q)}{q} \sinh(q\tau_1) \sinh(q\tau_2) e^{-q\varsigma} dq \quad (3.5)$$

Which on simplification using $\tau_1 = \tau_2 = \tau$, i.e. using the same thickness for the magnets, yields,

$$F_Z = 2\pi K_d R^2 \int_0^\infty J_0\left(\frac{rq}{R}\right) \frac{J_1^2(q)}{q} \left(e^{-q\left(\frac{a}{R}\right)} - 2e^{-q\left(\frac{a}{R}+2\tau\right)} + e^{-q\left(\frac{a}{R}+4\tau\right)}\right) dq \quad (3.6)$$

$$F_Y = 2\pi K_d R^2 \int_0^\infty J_1\left(\frac{rq}{R}\right) \frac{J_1^2(q)}{q} \left(e^{-q\left(\frac{a}{R}\right)} - 2e^{-q\left(\frac{a}{R}+2\tau\right)} + e^{-q\left(\frac{a}{R}+4\tau\right)}\right) dq \quad (3.7)$$

Considering a special case when the movement in the transverse direction is restricted, i.e. $r = 0$. Substituting this into Equation 3.6 and Equation 3.7 gives,

$$F_Z = 2\pi K_d R^2 \int_0^\infty \frac{J_1^2(q)}{q} \left(e^{-q\left(\frac{a}{R}\right)} - 2e^{-q\left(\frac{a}{R}+2\tau\right)} + e^{-q\left(\frac{a}{R}+4\tau\right)}\right) dq \quad (3.8)$$

$$\text{and, } F_Y = 0 \quad (3.9)$$

Note: $J_0(0) = 1$ and $J_1(0) = 0$.

When the magnets are placed far from each other, the Bessel functions can be expanded around $q = 0$ and F_z can be approximated to:

$$F_z \approx \frac{I}{2} \pi K_d R^4 \left[\frac{I}{a^2} + \frac{I}{(a+2t)^2} - \frac{2}{(a+2)^2} \right] \quad (3.10)$$

When the magnets are point dipoles and $t \ll a$, Equation 3.8 can be further simplified as:

$$F_z \approx 3\pi K_d R^4 t^2 \frac{I}{a^4} \quad (3.11)$$

Differentiating the Equation 3.1 and simplifying the terms gives Equation 3.11

$$F_z = -\frac{\partial U_m(x, D)}{\partial D} \approx \frac{3\mu_0 m_1 m_2 D}{2\pi} (x^2 + D^2)^{-\frac{5}{2}} \quad (3.12)$$

$$F_y = -\frac{\partial U_m(x, D)}{\partial x} \approx \frac{3\mu_0 m_1 m_2 x}{2\pi} (x^2 + D^2)^{-\frac{5}{2}} \quad (3.13)$$

Equation 3.11 is the most simplified form of a dipole-dipole approximation. Now, to understand the effect of magnet size which plays an important role in determining the interaction force, the Equations 3.8, 3.10 and 3.11 have been analyzed numerically using data for two magnets of type-1 (Table 3.1), the forces obtained are shown graphically in Figure 3.2.

Table-3.1: Properties of different types of magnets

Magnet	Radius 'm'	Height 'm'
Type-1 (NdFeB)	0.004	0.015
Type-2 (SmCo5)	0.0025	0.004
Type-3 (NdFeB)	0.003	0.002
Material: NdFeB – Neodymium-Iron-Boron, SmCo5 – Samarium-Cobalt * The coercivity and remanence for the magnets are provided in the manufacturer's data sheet. (Appendix A)		

It was observed that for a system of two magnets of type-1, as the proximity between the magnets is reduced the dipole expression produced force values which

deviated largely from the force values produced by the Integral expression (Equation 3.8) and the simplified expression of the integral expression (Equation 3.10). Similar studies were proved experimentally to establish the accuracy of the integral expressions over the simplified expressions by Vokoun *et al.* (2009). The dipole expressions can be used to approximate the magnetostatic interaction forces when the distance between the magnets is large enough. This distance usually depends on the size of the magnets being used and the magnetization of the material of the magnet. It is also important to note that at such large distances the effect of the nonlinearity induced by magnetostatic interactions is negligible.

(a)

(b)

Figure 3.2 (a) Graphical representation of the variation of axial component of magnetostatic interaction force between two similar magnets of type-I, (b) Graphical representation of the variation of transverse component of magnetostatic interaction force between two similar magnets of type-I when $a = 0.01\text{m}$ and $0.005\text{m} \leq r \leq 0.005\text{m}$.

3.3.2 Case: II – Magnetostatic interaction between two dissimilar magnets.

The previous case dealt with two similar magnets. In most practical scenarios, it will be very unlikely that two identical magnets would be used. Thus, an improved formulation was necessary to model the interaction between magnets accurately. Such expressions were not available in the literature thus they were derived separately as shown in this section.

$$E_{i,l-2}(a, r, q) = \varepsilon\mu_0 M_I M_2 \pi R_I R_2^2 \int_0^\infty J_0\left(\frac{rq}{R_2}\right) \frac{J_1\left(\frac{R_I}{R_2}q\right)}{q^2} J_1(q) U(a, q) dq$$

$$\text{Where, } U(a, q) = \left[e^{\left(-2d_2 - d\frac{q}{R_2}\right)} + e^{\left(-2d_1 - d\frac{q}{R_2}\right)} - e^{\left(-2d_1 + 2d_2 + d\frac{q}{R_2}\right)} - e^{\left(-d\frac{q}{R_2}\right)} \right] \quad (3.14)$$

The interaction energy expression shown above was derived by modifying the equations for interaction between two concentrically sliding cylindrical magnetic particles used in magnetic guns (Vokoun *et al.*, 2012). The force equations are obtained by differentiating these expressions in their respective directions.

$$F_Z(a, r, q) = -\frac{\partial E(a, r, q)}{\partial z},$$

$$F_Y(a, r, q) = -\frac{\partial E(a, r, q)}{\partial r} \quad (3.15)$$

$$F_Z(a, r, q) = \varepsilon\mu_0 M_I M_2 \pi R_I R_2^2 \int_0^\infty J_0\left(\frac{rq}{R_2}\right) \frac{J_1\left(\frac{R_I}{R_2}q\right)}{q} J_1(q) U(a, q) dq \quad (3.16)$$

$$F_Y(a, r, q) = -\varepsilon\mu_0 M_I M_2 \pi R_I R_2^2 \int_0^\infty J_1\left(\frac{rq}{R_2}\right) \frac{J_1\left(\frac{R_I}{R_2}q\right)}{q} J_1(q) U(a, q) dq \quad (3.17)$$

The Equation 3.16 and Equation 3.17 give the force distribution in a 2-dimensional plane, as these equations have no analytical solutions they were solved numerically using numerical solvers in MATLAB. When dipole approximations are applied to these equations, these are simplified to the dipole-dipole force equations as given by Equations 3.18 and 3.19 respectively.

$$F_z \approx \varepsilon \frac{3\mu_0 m_l m_z a}{2\pi} (r^2 + a^2)^{-\frac{5}{2}} \quad (3.18)$$

$$F_y \approx -\varepsilon \frac{3\mu_0 m_l m_z r}{2\pi} (r^2 + a^2)^{-\frac{5}{2}} \quad (3.19)$$

$$\text{Where, } m_i = M_i (\pi R_i^2 t_i)$$

(a)

Figure 3.3 (a) Graphical representation of the variation of axial component of Magnetostatic interaction force between two dissimilar magnets (b) Graphical representation of the variation of transverse component of magnetostatic interaction force between two dissimilar magnets (type-I and type-II) when $a = 0.01\text{m}$ and $0.005\text{m} \leq r \leq 0.005\text{m}$.

The interaction between a system of more than two magnets can be expressed as a linear summation of the magnetic potential energy. Vokoun *et al.* (2011) proved this experimentally and concluded that the linear summation can be used with a great degree of accuracy to model a symmetric system having more than two magnets. Hence the potential energy of magnetostatic interaction can be expressed using Equation 3.20. Where, $E_{i,Total}$ is the total interaction potential energy and $E_{i,l-m}$ is the interaction energy between l^{th} and m^{th} magnet. The corresponding magnetostatic interaction forces acting on the magnets can be derived using Equations 3.20 and 3.21.

$$E_{i,Total} = \sum_l^n \sum_{n+l}^m E_{i,l-m} \quad (3.20)$$

$$\begin{aligned} F_z(a, r, q) &= -\frac{\partial E(a, r, q)}{\partial z}, \\ F_y(a, r, q) &= -\frac{\partial E(a, r, q)}{\partial r} \end{aligned} \quad (3.21)$$

3.4. Magnetostatic interaction using finite element analysis (FEA)

The analytical formulation for the magnetostatic interaction between magnets with axial and transverse separation has been discussed in the previous section. Lately, finite element analysis has been developed to provide solutions for engineering problems to a great degree of accuracy. Many commercial software packages like ANSYS Multiphysics, COMSOL, ABAQUS, ATILA FEA etc., can deconstruct complex engineering problems involving coupling between various domains of physics. In the area of piezoelectric energy harvesting, various authors utilized the rigor of FEA to understand the behavior of the coupled piezoelectric, mechanical and electrical systems (Abdelkefi *et al.*, 2014; Arafa *et al.*, 2011; Upadrashta and Yang, 2015; Wu *et al.*, 2013; Yang and Tang, 2009; Yang *et al.*, 2015; Zhu *et al.*, 2009; Zhu *et al.*, 2010). The linear PEH systems can be modeled easily using the FEA packages, but nonlinear PEH systems especially in the presence of magnetic induced nonlinear forces can be very tedious and time consuming.

FEA has been widely used for modeling and analysis of magnetic fields in permanent magnet machines for understanding flux switching and magnetic induction (Ozkaya and Beyaz, 2015; Tomczuk *et al.*, 2007; Troster *et al.*, 2006; Zhu *et al.*, 2005). A FEA model of two magnets to determine the variation of magnetostatic force was utilized by Upadrashta and Yang (2015) to supplement the analytical formulation for a nonlinear PEH system. Most of the FEA models found in the literature are limited to a two-dimensional analysis or simplified configurations were used for the ease of modeling. Most two-dimensional models fail to provide an accurate analysis of magnets having cylindrical or spherical shapes. Moreover, the shape effects which play a major role in defining the magnetostatic interactions at close distances cannot be modelled with a good degree of accuracy. In this study, an exhaustive three-dimensional FEA analysis was performed to understand the variation of magnetostatic forces in the presence of various combinations of magnets.

A classic example was considered to validate the FEA model with the analytical formulation derived in the previous sections. Two cylindrical magnets of type-1

were modeled facing each other in the COMSOL Multiphysics package. A ‘*magnetic fields, with no currents (mfnc)*’ module was utilized to perform the FEA analysis. The air surrounding the magnets was also considered to facilitate the permittivity of the magnetic field as shown in Figure 3.4. The air was modeled to be confined inside a cuboid of size 0.5 m X 0.5 m X 0.5m, two layers of cuboids were modeled to facilitate a gradual meshing pattern and minimize any irregularities. Tetrahedral elements were used to mesh the whole assembly; the magnets were modelled with the finest size of the mesh. The whole assembly consisted of 3156262 elements, resulting in a system consisting of more than 4 million degrees of freedom. The whole system was analyzed using a computer with core i7 processor with a 16 GB ram; the time taken to carry out the analysis over 18 data points was about thirty minutes. The results obtained from the FEA analysis are plotted along with the values obtained from the analytical formulation in Figure 3.2.

In the presence of a system of more than two magnets, the magnetostatic interaction forces was calculated analytically using Equation 3.21. Figure 3.5 shows the meshed FEA model considered to analyze the three-magnet system for finding the resulting forces on the magnet placed at the top. The results obtained from the analytical and the FEA modeling are presented graphically in Figure 3.6. The forces obtained from the analytical modeling of the magnetostatic equations are in good agreement with the forces obtained from FEA modeling.

(a)

(b)

Figure 3.4 (a) Meshed FEA model for two similar magnets facing each other, (b) Distribution of the magnetic flux density (T) within the control volume.

(a)

(b)

Figure 3.5 (a) Meshed FEA model for three similar magnets, (b) Distribution of the magnetic flux density (T) within the control volume.

(a)

(b)

(c)

Figure 3.6 (a) Graphical representation of the variation of axial component of magnetostatic interaction force between three similar magnets of type-I, (b) & (c)

Graphical representation of the variation of axial and transverse components of magnetostatic interaction forces between three similar magnets of type-I when $a = 0.01\text{m}$ and $0.005\text{m} \leq r \leq 0.005\text{m}$.

A further investigation of the magnetostatic forces was performed by introducing the rotation of the magnets with respect to one another. This investigation was performed to understand the error in the analytical formulation for magnetostatic forces arising due to the assumption of non-rotation of the magnetization vectors as stated in Section 3.3. To incorporate this rotation, it was assumed that one magnet was connected to the tip of a cantilever beam of length ' L ' as shown in Figure 3.7. Based on the assumptions for small deflections as per the Euler-Bernoulli beam theory, the rotation of the beam for a tip deflection of ' r ' is assumed as ' $\sin^{-1}r/L$ '. The corresponding changes in the orientation of the magnets were included into the FEA modeling, and the resulting variation of the magnetostatic forces in comparison to the analytical formulation are shown in Figures 3.8 and 3.9.

Figure 3.7 Schematic representation of a rotating cantilever beam (the deflection is exaggerated for the sake of clarity).

(a)

(b)

Figure 3.8 (a) & (b) Graphical representation of the variation of axial and transverse components of magnetostatic interaction forces between two similar magnets of type-I when $0.006\text{m} \leq a \leq 0.014\text{m}$. and $0.005\text{m} \leq r \leq 0.005\text{m}$.

(a)

(b)

Figure 3.9 (a) & (b) Graphical representation of the variation of axial and transverse components of magnetostatic interaction forces between three similar magnets of type-I when $0.006\text{m} \leq a \leq 0.014\text{m}$. and $0.005\text{m} \leq r \leq 0.005\text{m}$.

From Figures 3.8 and 3.9, it is evident that the assumption of non-rotation of the magnetization vectors results in a higher estimate of the corresponding transverse force. The effect of rotation on the axial force is negligible as shown in Table – 3.2.

In addition to the graphical representation in Figures 3.8 and 3.9, Table – 3.2 provides the percentage change in the peak values obtained using the enhanced analytical formulation and the FEA modeling. The enhanced analytical formulation over-estimates the force by about 10% when the magnitude of the magnetic force is high (magnets are very close to each other), and the deflection from the mean position is large (more than 5mm). Therefore, it is necessary to investigate the maximum tip deflection of the PEH cantilever beam, and consider the effect of the change in magnetostatic interaction forces due to the assumption of non-rotation of the magnets accordingly.

Table-3.2: Percentage change in the forces derived using analytical methods and FEA modeling

		0.006m	0.008m	0.01m	0.012m	0.014m
Two Magnets	F_Y ‘N’	8.23	6.85	5.23	7.34	7.02
	F_Z ‘N’	0.58	0.33	0.41	0.02	0.87
Three Magnets	F_Y ‘N’	10.79	10.68	11.30	9.41	9.55
	F_Z ‘N’	0.53	0.44	0.35	0.45	0.19

The magnetostatic forces between two magnets can be efficiently modeled using FEA packages as discussed above, but the inclusion of a PEH system gives rise to coupling between magnetic, piezoelectric, mechanical and electrical systems. Moreover, the air surrounding the nonlinear PEH system needs to be modeled to provide a medium for interaction between the magnets. This leads to a highly complicated and intricate system where the meshing is required to be updated at each time step, leading to a tedious and computationally expensive procedure. To overcome this drawback, the magnets are modeled separately and the forces obtained can be applied to the PEH system directly.

The analytical modeling of interaction between two magnets assumes non-rotation of the magnetization vectors. This assumption can lead to a considerable error when the maximum tip rotation of a cantilever PEH system is relatively higher than 5 degrees. Moreover, the assumption of dipole formulation for the magnetostatic interaction leads to erroneous results as the distance between the magnets is decreased. A detailed analysis to overcome this error and model a nonlinear PEH system with an increased accuracy has been dealt in the next chapter.

3.5. Different types of equilibrium conditions

The effect on the magnetostatic interaction forces arising in the presence of different configuration of magnets is presented in the previous sections. The forthcoming sections provide an insight into the effect of the magnetostatic interaction forces on cantilever systems, and a broader picture into the equilibrium states of the PEH cantilever system. ‘*Equilibrium*’ at a point in space and time can be defined as the state where the sum total of external forces acting on a body is zero (Walker *et al.*, 2011). It is obtained as the point where the slope of the potential energy distribution vanishes. From elementary mechanics, there are three types of equilibrium positions as shown in Figure 3.10, these are named as the *stable equilibrium*, the *unstable equilibrium* and the *neutral or indifferent equilibrium*. Much of the literature available in the field of utilizing nonlinear energy harvesting, revolves around harnessing either of these equilibrium states to enhance the bandwidth of operation of the PEH system.

The stable equilibrium is the state where the system tends to restore back to its equilibrium in the presence of a small perturbation; the stable equilibrium position is a local minimum of the potential energy distribution and can be obtained when the second derivative is less than zero.

The unstable equilibrium is the state where the system cannot restore back to its equilibrium in the presence of a small perturbation; the unstable equilibrium position is a local maximum of the potential energy distribution and can be obtained when the second derivative is greater than zero.

The neutral equilibrium is the state where the system neither restores nor moves away from its equilibrium in the presence of a small perturbation, it reaches a new state of quasi equilibrium. The neutral equilibrium position can be obtained when the second derivative is equal to zero or does not exist.

Figure 3.10 Schematic representation of different equilibrium positions.

3.6. Introduction to bistable and multiple stable states

Many researchers worked on the magnetic field induced bistability (Erturk *et al.*, 2009a; Harne and Wang, 2013; Stanton *et al.*, 2010; Tang *et al.*, 2012; Wang and Liao, 2016; Zhou *et al.*, 2015; Zhou *et al.*, 2013). One of the ways in which the bistability is induced in a cantilever PEH system is with the help of magnets. In a typical bistable system, one magnet is placed at the tip of the cantilever PEH beam and a second magnet is placed directly opposite to the end of the beam as shown in Figure 3.4. Depending on the distance between the magnets and their magnetization, the system exhibits monostable or bistable states. The bistable state can be described as the state where a system exhibits two stable equilibrium positions. Similarly, various configurations of magnets can be used to introduce multiple stable states where the system exhibits more than two stable equilibrium positions.

Figure 3.11 Schematic representation of a cantilever beam with induced magnetic nonlinearity.

Figure 3.12 An equivalent SDOF lumped parameter system.

This has been further elaborated with the help of a cantilever beam simplified as a single degree of freedom (SDOF) system. In analytical modeling terms, it is also called as the lumped parameter system, where the mass, stiffness and damping of the cantilever system are represented by a lumped mass, spring element and a viscous damper respectively as shown in Figure 3.12. The mass is represented by M_{eq} , stiffness by K_{eq} and the damping by C_{eq} . The total energy of the system is represented as the summation of potential energy, kinetic energy, damping energy, and the external work done as shown in Equation 3.22 (Meirovitch, 2010). The potential energy due to the internal strain energy is represented in terms of the stiffness of the SDOF system given by Equation 3.23. As discussed in the previous section, the presence of magnets induces a change in the potential energy of the system. Hence the total potential energy of an SDOF system in the presence of magnets is given by Equation 3.24.

$$E_{Total} = E_{Potential} + E_{Kinetic} + E_{Damping} + E_{External} \quad (3.22)$$

$$E_{Potential,SDOF} = \frac{1}{2} K_{eq} y^2(t) \quad (3.23)$$

$$E_{Potential,M} = E_{Potential,SDOF} + E_{i,Total} \quad (3.24)$$

where E_{Total} is the total energy of the SDOF system, $E_{Potential}$ is the total potential energy of the SDOF system, and $E_{Potential,M}$ is the total potential energy of the SDOF system in the presence of magnets. $E_{Kinetic}$ is the kinetic energy from the acceleration of the mass, $E_{Damping}$ consists of the damping energy, $E_{External}$ consists of the work done by the external force, $E_{Potential,SDOF}$ is the potential energy due to stiffness of the spring element or equivalently stiffness of the beam, $y(t)$ is the displacement produced at the tip of the SDOF system due to the external excitation and $E_{i,Total}$ is the total magnetic interaction potential energy between the magnets at the tip of the beam and the end.

Figure 3.11 shows the configuration of cantilever system when the tip magnet faces a single end magnet. There are two possible combinations of this configuration depending upon the orientation of the magnetization vectors. One combination consists of the magnets attracting each other and the other consists of the magnets repelling each other. Thus, using Equations 3.23 and 3.24 the total potential energy of a SDOF system can be determined. A simple SDOF system with the properties given in Table-3.3 is considered for the calculation of the potential energy.

Table-3.3: Properties of the SDOF system

Property	Value
Equivalent Mass (g)	4.5
Equivalent Damping (Ns/m)	0.04025
Equivalent Stiffness (N/m)	100
Damping ratio	0.03
Natural frequency (Hz)	23.73
Tip magnet	Type – 2
End magnet	Type – 1

Figures 3.14 and 3.16 represent the variation of the potential energy with transverse displacement of a typical SDOF system described above. In the presence of one tip magnet and one end magnet (2 magnets), the nonlinearity induced by the magnetostatic interaction results in the formation of stable, unstable and neutral regions of the potential energy curves respectively. The linear potential energy curve is represented by the black line, the nonlinearity results in the formation of potential barriers (crests) and potential wells (troughs) as shown in Figure 3.13.

Figure 3.13 Representation of potential barrier and valley.

Figure 3.14 Potential energy variation of a SDOF system in the presence of one tip magnet and one end magnet, (a) Repulsive configuration, (b) Attractive configuration.

Figure 3.15 Schematic representation of a cantilever beam with one tip magnet and two end magnets.

Figure 3.16 Potential energy variation of a SDOF system in the presence of one tip magnet and two end magnets, (a) Repulsive configuration, (b) Attractive configuration.

With increasing separation distance between the tip magnets and the end magnets, the potential barrier for the repulsive cases diminishes. When magnets are placed in attraction, a significant potential well is formed. The potential energy for an attractive case is represented as negative only to provide an intuitive representation of the nonlinear system. When the number of the end magnets is increased the number of potential barriers and wells are also increased as shown in Figure 3.16.

In Figure 3.14 (a) it is evident that the system displays bistable potential system with two stable equilibrium states on either side of the potential barrier. The external force used to excite the SDOF system will need to be sufficient enough to instigate an inter-well transition to utilize the bistability induced into the system. Similarly, for an attractive configuration the system needs sufficient excitation to escape the deep potential well.

In Figure 3.16 (a), the repulsive configuration results in a tristable potential system at close axial separation. This quickly converts to a bistable and monostable system with increasing axial distance. The monostable system is a special case of neutral equilibrium state where the system exhibits minimum potential energy at

consecutive transverse spacing. The response of various cantilever PEH beams when subjected to nonlinearities resulting from magnetostatic interactions are discussed in detail in the coming chapters.

3.7. Summary

This chapter provides a brief outlook into the enhanced formulation for the magnetostatic interaction equations. The first section gives a brief introduction of the available formulations followed by the dipole-dipole formulation and its limitations for usage in analytical modeling. An elaborate description of different cases of the enhanced integral formulation for the magnetostatic interactions is presented in the third section. The fourth section and the fifth sections give an overview of the application of these formulations and a comparison between the dipole formulation and the integral formulations, and the existence of different stable states is also presented in this chapter.

CHAPTER 4 ENHANCEMENT OF BANDWIDTH USING MAGNETIC NONLINEARITY

A comparison of various formulations for determining the magnetostatic interaction forces, and the accuracy of the enhanced integral formulation has been presented in the previous chapter. This chapter gives an account of the effect of nonlinearity in a cantilever PEH system. The nonlinearity being introduced due to the presence of magnetostatic forces from the tip, and the end magnets.

4.1. Introduction

A brief overview of various techniques to induce nonlinearity into a linear PEH has been reviewed in chapter two. Cottone *et al.* (2009) and Erturk *et al.* (2009a) introduced the usage of magnets for inducing nonlinearity to enhance the effective bandwidth of a PEH system. Over the years, various researchers utilized the concept to introduce many designs for the application of magnetic based nonlinearity in PEH systems. Most of the researchers utilized the dipole-dipole formulation to induce the magnetostatic interaction into a PEH system. The drawbacks arising due to the usage of this formulation have been dealt at length in the previous chapter. Moreover, the advantages of using an enhanced integral formulation for the magnetostatic interaction has also been discussed. The effects of the error in the calculation of magnetostatic force on the cantilever PEH system has seldom been addressed in the literature. This chapter focusses on this issue and provides a comprehensive account of the limiting criteria to model the nonlinear PEH system.

4.2. Effect of axial and transverse forces on a cantilever beam

The presence of magnets at the end of a cantilever PEH beam results in a nonlinear response of the system. This nonlinearity is caused due to the magnetostatic interaction between the magnets. The magnetostatic interaction between two magnets can be expressed as point forces acting from the concerned magnetic materials. For axisymmetric magnets, the forces can be broadly classified as acting in axial and transverse directions as shown in Figure 4.1. The axial force and the transverse force are represented by F_Z and F_Y respectively. The effect of the action

of axial and transverse forces on a plane cantilever beam have been analyzed in this section. The equations derived in this section have been modified at a later stage to incorporate the presence of piezoelectric effect in a cantilever PEH system.

Figure 4.1 Schematic representation of a cantilever beam with tip and end magnet in repulsive configuration.

4.2.1 Cantilever beam subjected to an axial force

The presence of magnets inadvertently results in an axial force F_Z which acts at the tip of the beam. The effect of the axial force was seldom examined in the literature where the magnets were used to enhance the bandwidth. It is only pragmatic to understand the limiting conditions of the variation of an axial force on a cantilever beam before assessing the effect of the whole magnetic force.

Various researchers investigated the effect of an axial force on a cantilever beam system (Bokaian, 1988; Bokaian, 1990; Li *et al.*, 2013; Pratiher and Dwivedy, 2007; Rao, 2007; Shaker, 1975). The effect of axial force on the natural frequency of vibration of a cantilever beam has been long established. Most of these studies focus on the variation of natural frequency over a broad spectrum of axial forces. The present work delves into the effect on the natural frequency of the vibrating cantilever system with the variation of the corresponding magnetic forces.

Figure 4.2 Schematic representation of a cantilever beam with tip and end magnet in repulsive configuration.

A cantilever beam subjected to an axial force F_Z and undergoing transverse vibration is considered as shown in Figure 4.2 (a). Figure 4.2 (b) shows the forces acting on an infinitesimal segment of the cantilever beam of length dz . The change in length for the infinitesimal segment can be approximated as given in Equations 4.1 and 4.2.

$$ds - dz = \left[(dz)^2 + \left(\frac{\partial w}{\partial z} dz \right)^2 \right]^{\frac{1}{2}} - dz \quad (4.1)$$

$$ds - dz \approx \frac{1}{2} \left(\frac{\partial w}{\partial z} \right)^2 dz \quad (4.2)$$

In the above equations, ds is the length of the infinitesimal segment and dz is its length along Z-axis. The work done by the axial component of the magnetic force

W_F , and the transverse force W_f acting on the cantilever beam are given by Equations 4.3 and 4.4. Where, F_Z is the axial component of the magnetic force and $f(z,t)$ is the external transverse force acting on the cantilever beam. Equation 4.5 represents the total work done by the external forces W acting on the cantilever beam. Equations 4.6 and 4.7 depict the total potential energy U and the kinetic energy T_E of the cantilever beam system. In the above equations, E is the Young's modulus of beam having a second moment of area I , and m being the mass per unit length of the beam. The partial differentials $\frac{\partial^2 w}{\partial z^2}$ and $\frac{\partial w}{\partial z}$ are the curvature and the slope of the cantilever beam respectively, along the length of the beam.

$$W_F = -\frac{I}{2} \int_0^L F_Z \left(\frac{\partial w}{\partial z} \right)^2 dz \quad (4.3)$$

$$W_f = \int_0^L f(z,t) w dz \quad (4.4)$$

$$W = W_F + W_f \quad (4.5)$$

$$U = \frac{I}{2} \int_0^L EI(z) \left(\frac{\partial^2 w}{\partial z^2} \right)^2 dz \quad (4.6)$$

$$T_E = \frac{I}{2} \int_0^L m \left(\frac{\partial w}{\partial t} \right)^2 dz \quad (4.7)$$

Using the Hamilton's principle, the total energy of the cantilever beam system can be represented using Equation 4.8. On substitution of the values of T_E , U , W from the above equations into Equation 4.8 and simplification yields the differential equation of motion given by Equation 4.9.

$$\delta \int_{t_1}^{t_2} (T_E - U + W) dt = 0 \quad (4.8)$$

$$EI \frac{\partial^4 w}{\partial z^4} + m \frac{\partial^2 w}{\partial t^2} + F_Z \frac{\partial^2 w}{\partial z^2} = f(z,t) \quad (4.9)$$

The free vibration case of a cantilever beam having a uniform flexural rigidity is represented using Equation 4.10. Thus, EI is used in-lieu of $EI(z)$. Utilizing the

method of separation of variables, the response to Equation 4.10 is given as a convergent series of two variable $\phi_r(z)$ and $\eta_r(t)$. Where, $\phi_r(z)$ and $\eta_r(t)$ represent the mode shape and the modal coordinates for the r^{th} mode of vibration for the cantilever beam respectively.

$$EI \frac{\partial^4 w}{\partial z^4} + m \frac{\partial^2 w}{\partial t^2} + F_z \frac{\partial^2 w}{\partial z^2} = 0 \quad (4.10)$$

$$w(z, t) = \sum_{r=1}^{\infty} \phi_r(z) \eta_r(t) \quad (4.11)$$

By substituting Equation 4.11 in Equation 4.10, and rearranging the terms yields Equation 4.12. Where, the mode shape and the modal coordinate functions are separated and equated to the square of a constant ω_r which represents the natural frequency of the r^{th} mode of vibration.

$$\left[\frac{EI}{m} \frac{\partial^4 \phi_r(z)}{\partial z^4} + \frac{F_z}{m} \frac{\partial^2 \phi_r(z)}{\partial z^2} \right] \frac{1}{\phi_r(z)} = \frac{-1}{\eta_r(t)} \frac{\partial^2 \eta_r(t)}{\partial t^2} = \omega_r^2 \quad (4.12)$$

$$\frac{\partial^4 \phi_r(z)}{\partial z^4} + \frac{F_z}{EI} \frac{\partial^2 \phi_r(z)}{\partial z^2} - \frac{m}{EI} \omega_r^2 \phi_r(z) = 0 \quad (4.13)$$

$$\frac{\partial^2 \eta_r(t)}{\partial t^2} + \omega_r^2 \eta_r(t) = 0 \quad (4.14)$$

Utilizing the Rayleigh-Ritz approach, the solution to Equations 4.13 and 4.14 is represented as shown in Equations 4.15 and 4.16. A, B, C, D, E , and F are unknown constants which are obtained by using the boundary and the initial conditions for the cantilever beam. The terms λ_{1r} and λ_{2r} are related to the axial component of the force, mass per unit length of the beam, natural frequency, and the flexural stiffness are shown in Equation 4.17.

$$\phi_r(z) = A \cos \lambda_{1r} z + B \sin \lambda_{1r} z + C \cosh \lambda_{2r} z + D \sinh \lambda_{2r} z \quad (4.15)$$

$$\eta_r(t) = E \cos \omega_r t + F \sin \omega_r t \quad (4.16)$$

$$\lambda_{1r}^2, \lambda_{2r}^2 = \pm \frac{F_z}{2EI} + \sqrt{\frac{F_z^2}{4E^2 I^2} + \frac{m\omega_r^2}{2EI}} \quad (4.17)$$

$$\begin{aligned}
\phi_r(0) &= 0, \\
\frac{d\phi_r}{dz}(0) &= 0, \\
EI \frac{d^2\phi_r}{dz^2}(L) - \omega_r^2 I_t \frac{d\phi_r}{dz}(L) &= 0, \\
EI \frac{d^3\phi_r}{dz^3}(L) + \omega_r^2 M_t \phi_r(L) - F_z \frac{d\phi_r}{dz}(L) &= 0,
\end{aligned} \tag{4.18}$$

The boundary conditions for a cantilever beam in terms of the modal coordinates are shown in Equation 4.18. Where, the first two parts of the Equation 4.18 represent the fixed end (deflection and slope are zero), and the next two parts represent the free end of the cantilever beam (bending moment and the shear force are zero). The effect of the tip mass is incorporated with the help of the boundary conditions, M_t and I_t are the mass and second moment of area for the tip mass respectively. The substitution of Equation 4.15 in Equation 4.18 yields an eigenvalue problem. The solution of which yields the undamped natural frequency of r^{th} mode for the cantilever beam. The general solution for the displacement of the system can be obtained by utilizing the initial conditions for displacement ($w(z,0)$) and velocity ($\partial w(z,0)/\partial t$) respectively.

The cantilever beam has infinite number of mode shapes, modeling several mode shapes is cumbersome and unnecessary. The response of a uniform cantilever beam can be approximated to a good degree of accuracy using its first mode of vibration (Erturk and Inman, 2011b; Meirovitch, 2010). Thus, the distributed parameter formulation of a cantilever as shown above is modeled as a simple mass, spring and damper system by lumping the equivalent properties of the cantilever beam. This is achieved by using the concepts of orthogonality of mode shapes and mass normalization techniques for a cantilever PEH beam as demonstrated by Erturk and Inman (2011b). The equivalent mass (M_{eq}), stiffness (K_{eq}), and damping (C_{eq}) for the cantilever beam are represented using Equations 4.19, 4.20, and 4.21 respectively; and the equation of motion for the lumped parameter model of the cantilever beam subjected to free vibration is given by Equation 4.22.

$$M_{eq} = \int_0^L m\phi(z)^2 dz + M_t\phi(L)^2 \quad (4.19)$$

$$K_{eq} = \int_0^L EI \left(\frac{d^2\phi(z)}{dz^2} \right)^2 dz$$

$$\text{or } K_{eq} = M_{eq}\omega^2 + F_z \left(\frac{d\phi(L)}{dz} \right)^2 \quad (4.20)$$

$$C_{eq} = \alpha M_{eq} + \beta K_{eq} \quad (4.21)$$

$$M_{eq} \frac{d^2\eta(t)}{dt^2} + C_{eq} \frac{d\eta(t)}{dt} + K_{eq}\eta(t) = 0 \quad (4.22)$$

On being subjected to an external excitation, the forcing function of $f(z,t)$ in Equation 4.9 comes into existence. The response of the system depends on this forcing function. Thus, the forcing function for a cantilever beam subjected to a base excitation of $\frac{d^2\eta_0(t)}{dt^2}$ is as represented in Equation 4.23. Equation 4.24 is the equation of motion of a lumped parameter system subjected to an external force. α and β in Equation 4.21 are equivalent viscous damping parameters for the cantilever beam system. For a simple cantilever beam, the values of α and β can be approximated as 0 and $\xi/\pi f_0$ respectively (Avvari *et al.*, 2016). Where, f_0 is the frequency at which the damping ratio ξ is measured.

$$F(t) = - \left(M_t\phi(L) + \int_0^L m\phi(z) dz \right) \frac{d^2\eta_0(t)}{dt^2} \quad (4.23)$$

$$M_{eq} \frac{d^2\eta(t)}{dt^2} + C_{eq} \frac{d\eta(t)}{dt} + K_{eq}\eta(t) = F(t) \quad (4.24)$$

The effect of the axial force acting on a cantilever beam was modeled analytically using the beam properties specified in Table 4.1. The graphs shown in Figure 4.3 (a) and (b) represent the variation of the fundamental frequency of the beam in the presence and the absence of the tip mass respectively. It was observed that the fundamental frequency of the cantilever beam drops drastically as the compressive axial load approaches the buckling load of the beam. Moreover, the buckling load of the beam is independent of the tip mass. Thus, the effect of the axial component of

the magnetic force must be thoroughly scrutinized to understand the effect of the nonlinearity introduced into the system.

Table-4.1: Properties of the beam

Property	Value
Thickness (mm)	0.7
Width (mm)	10
Length (mm)	90
Tip mass (g)	10.28
Young's modulus (N/m ²)	71.7×10^9
Density (kg/m ³)	2810
Euler buckling load (N)	6.605

(b)

Figure 4.3 Variation of fundamental frequency with changing axial force (tension to compression), (a) in the presence of tip mass, and (b) absence of tip mass.

4.2.2 Cantilever beam subject to a concentrated transverse force

The effect of an axial force on a cantilever beam is discussed in the previous section. In the presence of magnets, the vertical component of the magnetic force F_Y acts at the surface of the magnet in the transverse direction as shown in Figure 4.1. The variation of the axial and transverse forces acting in the presence of magnets placed at varying distances (axial and transverse) has been dealt at length in the previous chapter. This section presents the effect of transverse forces acting at the tip of a vibrating cantilever beam.

$$F(t) = - \left(M_t \phi(L) + \int_0^L m \phi(z) dz \right) \frac{d^2 \eta_0(t)}{dt^2} - F_Y \quad (4.25)$$

$$\phi_r(0) = 0,$$

$$\frac{d\phi_r}{dz}(0) = 0,$$

$$EI \frac{d^2 \phi_r}{dz^2}(L) - \omega_r^2 I_t \frac{d\phi_r}{dz}(L) = 0, \quad (4.26)$$

$$EI \frac{d^3 \phi_r}{dz^3}(L) + \omega_r^2 M_t \phi_r(L) - F_Y \phi_r(L) = 0,$$

The analytical formulation of the cantilever beam discussed in the previous section can be modified to incorporate the transverse forces accordingly. There are two major ways in which the transverse forces can be induced into a cantilever beam system. Method-I: The transverse force can be modeled as a point force acting along with the forcing term as shown in Equation 4.25. Method-II: The transverse force can be induced as a boundary condition for the corresponding shear force as shown in Equation 4.26. The magnetic force varies with the distance between the magnets, thus the force component F_Y is a time varying function. Method-I gives rise to a straight forward procedure of finding a solution for the dynamic equation. On the other hand, method-II requires the calculation of mode shape function and

the equivalent system parameters at every time step. This renders the method-II computationally intensive and inefficient for analytical formulation.

Table-4.2: Equivalent system parameters for the cantilever beam

Property	Tip mass present	Tip mass absent
Equivalent mass (g)	10.68	0.43
Equivalent stiffness (N/m)	91.78	94.55
Equivalent damping (Ns/m)	5.5×10^{-3}	5.7×10^{-3}

A simple cantilever beam discussed in the previous section is used to demonstrate the effect of the transverse force. The lumped system parameters for the cantilever beam are calculated and listed in Table-4.2. The cantilever beam is assumed to be subjected to the action of magnetostatic forces arising due to the presence of a tip magnet and an end magnet of type-3 and type-2 respectively. The magnetization values are determined based on the values for coercivity and remanence provided in the manufacturer's data sheet. The magnetostatic interaction force F_Y is obtained from the enhanced integral equations discussed in the previous chapter. The cantilever beam is subjected to a base vibration of 0.1g ($g = 9.81 \text{ m/s}^2$). The analytical solution for a steady state response for displacement and velocity at various frequencies and varying magnetic distances are obtained. The variation of velocity, and displacement with frequency is shown in Figures 4.4 and 4.5.

(a)

(b)

Figure 4.4 Variation of tip velocity of the cantilever beam with frequency, (a) in the presence of tip mass, and (b) absence of tip mass.

(a)

(b)

Figure 4.5 Variation of tip displacement of the cantilever beam with frequency, (a) in the presence of tip mass, and (b) absence of tip mass.

Figures 4.4 and 4.5 exhibit the variation of the magnitude for both the velocity and tip displacements with varying frequency, and increasing distance between the magnets. It was noticed that in the presence of a tip mass, the peaks of tip displacement and tip velocity decrease in the presence of the magnets. They deviate from the linear configuration with considerable offset on changing the distance between the tip and the end magnets. In the absence of a tip mass, the peak magnitude of velocity remains unchanged for different configurations, but the displacement of the tip of the cantilever beam is further increased as the magnets are brought closer in a repulsive configuration. This analytical model bolsters the intuitive understanding of the effect of magnets on a cantilever beam and provides a quantification for the magnetic effect on a vibrating cantilever beam. Moreover, the presence of a tip mass decreases the natural frequency of a vibrating cantilever beam. At lower frequencies, both the tip displacement and tip velocity display a similar variation with frequency.

4.2.3 Cantilever beam subject to both axial and transverse forces

The effect of an axial force, and transverse force on a cantilever beam have been discussed in the previous sections. It has been a widely accepted practice of modeling the magnetic nonlinearity induced into a vibrating cantilever system with the help of a transverse magnetic force (Ahmadian *et al.*, 2009; Erturk *et al.*, 2009a; Erturk and Inman, 2011a; Ramlan *et al.*, 2010; Tang *et al.*, 2012; Upadrashta and Yang, 2015; Vocca *et al.*, 2012; Wickenheiser and Garcia, 2010). Though this procedure provides a decent solution to understand the behavior of a vibrating cantilever beam subjected to a magnetic force. The absence of axial force does induce an error into the solution.

The effect of the action of both axial (F_z) and transverse (F_y) magnetostatic forces on a vibrating cantilever beam has been dealt in detail in this section. The corresponding boundary conditions and the equations of motion to incorporate the

respective forces are as shown in Equations 4.27 – 4.29. These equations are obtained by introducing suitable modifications to the equations stated in sections 4.2.1 and 4.2.2. The solution to the equation of motion is obtained using the state space formulation given in Equations 4.30 and 4.31.

$$\begin{aligned}\phi_r(0) &= 0, \\ \frac{d\phi_r}{dz}(0) &= 0, \\ EI \frac{d^2\phi_r}{dz^2}(L) - \omega_r^2 I_t \frac{d\phi_r}{dz}(L) &= 0,\end{aligned}\tag{4.27}$$

$$\begin{aligned}EI \frac{d^3\phi_r}{dz^3}(L) + \omega_r^2 M_t \phi_r(L) - F_z \frac{d\phi_r}{dz}(L) &= 0, \\ F(t) &= -\left(M_t \phi(L) + \int_0^L m \phi(z) dz\right) \frac{d^2\eta_0(t)}{dt^2} - F_Y(t)\end{aligned}\tag{4.28}$$

$$M_{eq} \frac{d^2\eta(t)}{dt^2} + C_{eq} \frac{d\eta(t)}{dt} + K_{eq} \eta(t) = F(t)\tag{4.29}$$

$$k_1(t) = \eta(t), \quad k_2(t) = \frac{d\eta(t)}{dt}\tag{4.30}$$

$$\begin{aligned}\dot{k}_1(t) &= k_2(t) \\ \dot{k}_2(t) &= -\frac{C_{eq}}{M_{eq}} k_2(t) - \frac{K_{eq}}{M_{eq}} k_1(t) + \frac{F(t)}{M_{eq}}\end{aligned}\tag{4.31}$$

The state variables assumed are the displacement and the velocity components as shown in Equation 4.30. Using these variables, the equation of motion is reorganized as shown in Equation 4.31. The state space equation was solved using an ordinary differential equation solver ODE45 in MATLAB. Furthermore, the transverse component of magnetic force (F_Y) was evaluated at every time step corresponding to the change in tip displacement and the distance between the magnets. The change in distance between the magnets was calculated based on the small deflection assumption as discussed in chapter 3. Moreover, for a small deflection assumption, the change in axial component of the magnetic force is negligible. Thus, a constant axial force is assumed when the cantilever beam undergoes transverse vibration, and the corresponding axial magnetic force (F_Z) is incorporated into the boundary conditions as shown in Equation 4.27.

The variation of tip velocity and tip displacement have been graphically presented in Figures 4.6 and 4.7. The observed trend of the plots shows a similar variation of both the parameters with changing frequency. In the absence of a tip mass, the tip displacement gradually increases with closer spacing of the magnets in a repulsive configuration. The proximity of the magnets, causing a strong repulsive force between them results in such a behavior. Moreover, with an increase in the magnitude of the magnetostatic forces at closer spacing of the magnets, a considerable variation in the response obtained from analytical formulation using the transverse force (F_Y), and combined transverse (F_Y) and axial (F_Z) forces was observed. This variation for a cantilever system with 8 mm separation between the tip and end magnets has been explicitly shown in Figure 4.8. Thus, it is evident that the presence of the axial component of the magnetic force decreases the stiffness of the cantilever beam system.

To further illustrate the need for including axial force in addition to transverse force, three different cases are presented. Case I studies a simple cantilever beam (tip mass absent) subjected to a base vibration of 0.1g in the presence of magnetostatic forces. This hypothetical scenario was explored to exhibit the sensitivity of the cantilever beam to axial loads. Here, $F_Y = 0.487$ N ($a = 0.008$ m) and $F_{cr} = 6.605$ N. Though the applied axial force is only 7.37% of the critical buckling load, the effect on the response of the vibrating beam is quite significant as shown in Figure 4.8 (a). For this case, the non-consideration of axial load leads to an error of nearly 1 Hz in the determination of the resonant frequency. Case II is the same as Case I except a tip mass being present. From Figure 4.8 (b), it is evident that the exclusion of axial force in the analytical formulation marginally under predicts the system response. Case III is the same as Case II but the spacing between the tip and end magnets was reduced to 5 mm, producing $F_Y = 1.491$ N (22.57 % of F_{cr}). At such close spacing, the system requires considerable amount of energy to overcome the potential barrier and traverse across all the stable states, thus a base excitation level of 5g was used in the analytical formulation. Figure 4.8 (c) represents the response obtained for this loading condition. The exclusion of axial force leads to a considerable error for such a scenario. The three hypothetical cases clearly emphasize the necessity to employ both the axial and transverse

components of the magnetostatic forces for arriving at an accurate analytical formulation of the vibrating cantilever system in the presence of tip and end magnets. Moreover, the response of the system is influenced by the parameters like tip mass, spacing between the magnets, and magnitude of base excitation. Therefore, it is of prime importance to infuse the effects due to both the axial and transverse components to arrive at a more accurate analytical model for a vibrating cantilever beam subjected to magnetostatic forces induced nonlinearity.

Figure 4.6 Variation of tip velocity of the cantilever beam with frequency, (a) in the presence of tip mass, and (b) absence of tip mass.

Figure 4.7 Variation of tip displacement of the cantilever beam with frequency, (a) in the presence of tip mass, and (b) absence of tip mass.

Figure 4.8 Comparison of the variation of tip displacement with frequency for a cantilever beam subject to (a) repulsive configuration, and (b) attractive configuration at 8mm separation in the the absence of tip mass, (c) repulsive configuration in the presence of a tip mass, and (d) repulsive configuration in the presence of a tip mass and undergoing a base excitation of 5g .

Additionally, a further experimental investigation was performed to understand investigate the effect of combined action of magnetostatic forces on a cantilever beam system. The experimental setup consisting of a vibrating cantilever beam attached with a tip and end magnets is as shown in Figure 4.9. The experimental setup consisted of a simple aluminum cantilever beam attached with a tip mass having an embedded magnet. The cantilever beam was attached firmly to a frame and bolted to the shaker platform. An accelerometer was attached to the shaker platform to track the base acceleration. The displacement and velocity of the tip of the cantilever beam were monitored using a laser vibrometer. The beam was subjected to increasing base excitation levels of 0.02g, 0.05g, and 0.1g respectively. A movable end magnet arrangement was attached to the frame to induce the magnetic effect onto the vibrating cantilever beam.

The initial readings for resonant frequencies obtained from a linear configuration (absence of end magnet) were observed. Thereafter, readings obtained in the presence of an end magnet in repulsive configuration were also monitored; and the velocity and displacement data at the resonant frequencies were collected. The

repulsive configuration was considered to examine the behavior of the cantilever system in the presence of more than one stable states. The experimental data for the corresponding base excitations is shown in Table – 4.3. The beam properties are as shown in Table – 4.1. The shift in potential energy curves of the system with changes in spacing of the magnets is as shown in Figure 4.10 (a). As discussed in the previous chapter, Figures 4.10 (b) and (c) portray the comparison between the forces obtained by using the integral formulation and the FEA formulation which incorporates the rotation of magnets. In addition to the investigation for non-consideration of both the components of magnetostatic forces, this investigation was performed to determine the error resulting from ignoring the rotation of magnets in the integral formulation for obtaining magnetic forces. Thus, the variation of both axial and transverse components of magnetic forces for a close spacing of 8 mm is presented. From the plots, it is evident that, there is hardly any error induced into the cantilever system due to non-consideration of the rotation of magnets.

Table-4.3: Experimental resonant frequencies for a cantilever beam

Beam configuration	Resonant Frequencies		
	0.02g	0.05g	0.1g
Linear	14.75 Hz	14.75 Hz	14.75 Hz
Nonlinear, $a = 0.008$ m	5.25 Hz	5.75 Hz	7.8 Hz
Nonlinear, $a = 0.01$ m	11.1 Hz	11.25 Hz	11.65 Hz
Nonlinear, $a = 0.015$ m	13.5 Hz	13.4 Hz	14.6 Hz

(a)

(b)

Figure 4.9 Experimental setup of a cantilever beam with a tip magnet, (a) in linear configuration, and (b) vibrating in nonlinear configuration with tip magnet.

(a)

(b)

(c)

Figure 4.10 (a) Variation of potential energy with change in spacing between the tip and end magnets, (b) variation of transverse component of magnetic force (F_y) and, (c) variation of axial component of magnetic force (F_z) with changing transverse displacement of the cantilever beam.

The comparison of the experimental observations presented in Table – 4.3, and the results from the analytical modeling of the vibrating cantilever system presented in Figures 4.6 and 4.7 reveal the accuracy of the analytical modeling. Figure 4.11 displays various plots obtained from the experimental and the analytical investigations for the cantilever beam system. The steady state velocities obtained at the resonant frequencies for both the cases shows a good match. Furthermore, the phase plots between the velocity and displacement for nonlinear cases having magnet spacing of 15 mm, 10 mm and 8 mm, presented in Figures 4.11 (c) – (h) provide very identical variations. This validates the accuracy of the analytical formulation developed over the course of this study. The phase plots provide a deeper insight into the nonlinearity induced into the cantilever beam system. As the system traverses from a linear to nonlinear, a single stable equilibrium bifurcates to two stable equilibria resulting in an inter-well vibration as shown experimentally in Figure 4.11 (g).

(b)

(c)

(d)

(e)

(f)

(g)

Figure 4.11 Experimental setup of a cantilever beam with a tip magnet, (a) in linear configuration, and (b) vibrating in nonlinear configuration with tip magnet.

The use of both axial (F_z) and transverse (F_y) components of the magnetostatic forces to model the response of a cantilever beam has been demonstrated. The fundamental understanding of the response of a cantilever beam system subjected to nonlinear forces due to magnetostatic interaction between the magnets developed so far will be extended to cantilever piezoelectric energy harvester systems in the forthcoming sections.

4.3. Effect of magnetic nonlinearity on a PEH cantilever beam

Chapter 3 provides a detailed outlook into the force and potential energy formulations for the magnetostatic interactions for a system of magnets. The components of the resultant magnetic forces act in the axial and transverse directions. The orientation of these forces is governed by the axes of the magnetization vectors. The presence of magnets induces nonlinearity in a vibrating cantilever beam system, resulting in shifting of resonant frequency and change in the magnitudes of the system response. These have been analyzed with the help of analytical and experimental results in section 4.2. This section is dedicated to understanding the effect of magnets on a vibrating PEH cantilever beam.

4.3.1 Formulation for a linear PEH cantilever beam

Piezoelectric materials exhibit a peculiar property of converting mechanical stress acting on the material to an electrical field and vice versa. A variety of engineering applications have been developed based on these properties. One such application is harvesting energy from external vibrations. Out of the wide variety of energy harvesting systems, the PEH cantilever beams have been widely used due to their simplicity of design and applicability. The analytical formulation to model a cantilever energy harvesting system is based on the following assumptions: (a) The Euler-Bernoulli beam assumptions are valid, (b) the effect of external excitation from air damping is negligible, (c) the strain rate damping and viscous air damping are assumed to be proportional to the bending stiffness and mass per unit length of the beam, and (d) a uniform electric field is assumed across the piezo layer (Elvin and Elvin, 2009b; Erturk and Inman, 2011b; Kim *et al.*, 2010; Tang and Yang, 2012b). With these assumptions in place, and the constitutive relations for a piezoelectric material given in Equation 4.32 (IEEE Standard, 1988) in its d-form, a simple PEH cantilever beam can be modeled as a linear PEH beam.

$$\begin{bmatrix} S \\ D \end{bmatrix} = \begin{bmatrix} s^E & d^c \\ d^d & \varepsilon^T \end{bmatrix} \begin{bmatrix} T \\ E^f \end{bmatrix} \quad (4.32)$$

Figure 4.12 Typical representation of a linear PEH cantilever beam.

A single PEH beam subjected to transverse vibrations, commonly referred as the linear PEH beam is as shown in Figure 4.12. A typical linear PEH beam consists of a piezo transducer attached at the root of the substrate of the beam, and connected to an external resistor. The whole assembly is subjected to a base vibration. A proof mass is attached to the tip of the beam to reduce the resonant frequency, and enhance the strain transfer between the substrate and the transducer. The governing equations for a linear PEH beam can be derived using the Hamilton principle (Elvin and Elvin, 2012; Erturk and Inman, 2011b; Sodano *et al.*, 2004b; Upadrashta *et al.*, 2014) and approximated for a single mode. This single mode approximation is also known as the lumped parameter system. The lumped parameter modeling for a simple cantilever beam has been discussed in the previous sections. The presence of a piezoelectric transducer introduces extra terms in the energy equations as shown in Equations 4.33 – 4.35. In other words, the presence of a piezoelectric transducer produces a coupled electro-mechanical system.

$$W = \int_0^L f(z, t) w dz - \int_0^L \phi(z, t) q dz \quad (4.33)$$

$$U = \frac{1}{2} \int_0^L \frac{M^2(z, t)}{EI(z)} dz \quad (4.34)$$

$$M(z, t) = EI(z) \frac{\partial^2 w}{\partial z^2} + \mathcal{G}v(t) [H(z) - H(z - L)] \quad (4.35)$$

$$T_E = \frac{1}{2} \int_0^L m(z) \left(\frac{\partial w}{\partial t} \right)^2 dz \quad (4.36)$$

$$W_e = \frac{1}{2} \int_{V_p} E^t D dV_p \quad (4.37)$$

$$W_e = \frac{1}{2} \int_{V_p} E^t e S dV_p + \frac{1}{2} \int_{V_p} E^t \varepsilon^s E dV_p \quad (4.38)$$

$$\int_{t_1}^{t_2} [\delta(T - U + W_e) + \delta W] dt = 0 \quad (4.39)$$

In addition to the terms described earlier, a few more parameters are introduced in Equations 4.33 – 4.39. The movement of a charge q in a scalar electrical potential field $\varphi(z,t)$ contributes to the external work done. The macroscopic electromechanical coupling coefficient, $\vartheta = -e_{31}b(t_s+t_p)$ contributes to the bending moment $M(z,t)$ of the PEH beam. $H(\cdot)$ is the Heaviside step function. $v(t)$ is the voltage across the piezoelectric macro fiber composite (MFC) layer having a piezoelectric coefficient e_{31} and a thickness of t_p , and attached to the substrate of thickness t_s . The electrical energy W_e is the integral of the electric displacement vector D in an applied electrical field E (superscript t indicates the transpose of the matrix) across the whole volume V_P of the piezoelectric MFC layer. Following a similar approach discussed in section 4.2.1 and the formulation for a PEH system illustrated in the literature (duToit *et al.*, 2005; Erturk and Inman, 2011b; Kim *et al.*, 2010), the equations of motion for a base excited cantilever PEH beam system were derived. The PEH beam system is further simplified by lumping the parameters. Thus, Equations 4.40 and 4.41 represent the equations of motion for a lumped PEH cantilever beam system.

$$M_{eq} \frac{d^2\eta(t)}{dt^2} + C_{eq} \frac{d\eta(t)}{dt} + K_{eq}\eta(t) + \Theta V(t) = F(t) \quad (4.40)$$

$$\frac{V(t)}{R_L} + C^P \frac{dV(t)}{dt} - \Theta \frac{d\eta(t)}{dt} = 0 \quad (4.41)$$

Where

$$M_{eq} = \int_0^{L_c} m_C(z) \phi_C(z)^2 dz + \int_{L_c}^L m_S(z) \phi_S(z)^2 dx + M_T \phi(L)^2 \quad (4.42)$$

$$K_{eq} = \int_0^{L_c} (EI)_C(z) \left(\frac{d^2\phi_C(z)}{dz^2} \right)^2 dz + \int_{L_c}^L (EI)_S(z) \left(\frac{d^2\phi_S(z)}{dz^2} \right)^2 dz \quad (4.43)$$

$$\Theta = - \int_{V_P} y \left(\frac{d^3\phi_C(z)}{dz^3} \right) e_{31} \psi(y) dV_P \quad (4.44)$$

$$F(t) = - \left(M_T \phi(L) + \int_0^{L_c} m_C(z) \phi_S(z) dz + \int_{L_c}^L m_S(z) \phi_S(z) dz \right) \frac{d^2\eta_0(t)}{dt^2} \quad (4.45)$$

$$C_{eq} = \alpha M_{eq} + \beta K_{eq} \quad (4.46)$$

Most of the parameters have been described in section 4.2. The subscripts S , C , and P signify the substrate, the composite, and the piezoelectric part of the PEH beam, respectively. The composite part is comparable to the one reported in the literature, where the MFC is considered as a single composite layer attached to the substrate; z is measured along the length of the beam varying from zero to L ; and y is the distance from the neutral axis. e_{31} is the piezoelectric stress constant and $\psi(y)$ represents the constant electric field across the volume of the piezoelectric material V_P (Sodano *et al.*, 2004b), and C^P is the overall capacitance of the piezoelectric transducer.

In the PEH applications, it is widely accepted that the fundamental mode of vibration approximately represents the system behavior when subjected to an external excitation with a frequency close to the system's first natural frequency. Thus, the properties of the system are lumped together, and a solution for a single mode approximation can be obtained. In the above equations, M_{eq} , K_{eq} , C_{eq} and Θ are the equivalent mass, stiffness, damping, and electromechanical coupling parameters, respectively. The equations are solved for voltage $V(t)$ across the resistor R_L and time component of the displacement $\eta(t)$. For a single mode approximation problem, the total displacement is given by $w(z,t) = \eta(t)\phi(x)$, where $\phi(x)$ is the fundamental mode shape of PEH beam which is mass normalized to unity corresponding to the fundamental natural frequency ω_n .

$$\phi(z) = \begin{cases} \phi_C(z) & 0 \leq z \leq L_C \\ \phi_S(z) & L_C < z \leq L \end{cases} \quad (4.47)$$

The mode shapes are represented as a combination of trigonometric and hyperbolic functions

$$\begin{aligned} \phi_C(z) &= A_1 \cos \lambda_C z + B_1 \sin \lambda_C z + C_1 \cosh \lambda_C z + D_1 \sinh \lambda_C z \\ \phi_S(z) &= A_2 \cos \lambda_S z + B_2 \sin \lambda_S z + C_2 \cosh \lambda_S z + D_2 \sinh \lambda_S z \end{aligned} \quad (4.48)$$

The non-trivial mode shape values of ϕ_C and ϕ_S , are determined by substituting the values of λ_C and λ_S which are, in turn, obtained using the geometric and natural boundary conditions given below

$$\begin{aligned}
 \phi_C(0) &= 0; \frac{d\phi_C}{dz}(0) = 0; \\
 \phi_C(L_C) &= \phi_S(L_C); \frac{d\phi_C}{dz}(L_C) = \frac{d\phi_S}{dz}(L_C); \\
 (EI)_C \frac{d^2\phi_C}{dz^2}(L_C) - (EI)_S \frac{d^2\phi_S}{dz^2}(L_C) &= 0, \\
 (EI)_C \frac{d^3\phi_C}{dz^3}(L_C) - (EI)_S \frac{d^3\phi_S}{dz^3}(L_C) &= 0, \\
 (EI)_S \frac{d^2\phi_S}{dz^2}(L) - \omega_n^2 I_t \frac{d\phi_S}{dz}(L) &= 0, \\
 (EI)_S \frac{d^3\phi_S}{dz^3}(L) + \omega_n^2 M_t \phi_S(L) &= 0
 \end{aligned} \tag{4.49}$$

In the above equations, the value of EI varies along the length of the beam depending on the cross section. The height of the neutral axis and the corresponding EI for a composite section are shown in Equations 4.50 – 4.51. Equation 4.52 gives the magnitude of strain at a time t (a detailed derivation is provided in section 5.2). These equations were derived considering a composite beam consisting of the piezoelectric layer (subscript P) attached to the substrate (subscript S) with an epoxy-glue layer (subscript G). The bending stress for any cross-section at its neutral axis is zero, and the expression for the depth of the neutral axis was obtained using this fundamental property.

$$h = \frac{t_S}{2} \left(\frac{I - \frac{E_P b_P}{E_S b_S} \left(\frac{t_P}{t_S} \right)^2 - \frac{E_G b_G}{E_S b_S} \left(\frac{t_G}{t_S} \right)^2 - 2 \frac{E_P b_P}{E_S b_S} \left(\frac{t_P t_G}{t_S^2} \right)}{I + \frac{E_P b_P t_P}{E_S b_S t_S} + \frac{E_G b_G t_G}{E_S b_S t_S}} \right) \tag{4.50}$$

$$\begin{aligned}
 (EI)_C &= \frac{1}{3} E_S b_S (t_S - h)^3 + \frac{1}{3} E_S b_S h^3 + \frac{1}{3} E_G b_G (t_G + h)^3 - \frac{1}{3} E_G b_G h^3 \\
 &+ \frac{1}{3} E_P b_P (t_P + t_G + h)^3 - \frac{1}{3} E_P b_P (t_G + h)^3
 \end{aligned} \tag{4.51}$$

$$\varepsilon_{max}(t) = \frac{-K_{eq} \eta(t) \phi(L) (h + t_P) L}{(EI)_C} - \frac{e_{31} V(t)}{E_P t_P} \tag{4.52}$$

In the above equations, h is the depth of the neutral axis from the top surface of the substrate; E is the modulus of elasticity; and b and t are the width and thickness of the corresponding layers, respectively. Figure 4.13 shows a schematic diagram of the cross-section of a typical PEH beam with the corresponding distances for h , y , b , t , etc...

Figure 4.13 Schematic representation of the cross-section of a typical PEH beam.

4.3.2 Formulation for a nonlinear PEH cantilever beam

Various techniques to enhance the bandwidth of operation of a PEH system have been explored in the literature. Introducing nonlinearity into the system is one such technique. The different sources of nonlinearity have been discussed in detail in chapter 2. The present section concentrates on the behavior of a PEH cantilever beam system subjected to a magnetic induced nonlinearity. In the presence of a magnet, the response of a PEH system undergoes a change depending on the strength and orientation of the magnetostatic forces. The effect of magnetostatic forces on a simple cantilever beam have been thoroughly investigated in section 4.2. With the presence of a piezoelectric MFC, an extra response in the form of voltage is obtained from the system. The axial and the transverse forces due to presence of

magnet induced nonlinearity are incorporated into the equations of motion as shown in the following equations.

$$\begin{aligned}
 \phi_C(0) &= 0; \frac{d\phi_C}{dz}(0) = 0; \\
 \phi_C(L_C) &= \phi_S(L_C); \frac{d\phi_C}{dz}(L_C) = \frac{d\phi_S}{dz}(L_C); \\
 (EI)_C \frac{d^2\phi_C}{dz^2}(L_C) - (EI)_S \frac{d^2\phi_S}{dz^2}(L_C) &= 0, \\
 (EI)_C \frac{d^3\phi_C}{dz^3}(L_C) - (EI)_S \frac{d^3\phi_S}{dz^3}(L_C) &= 0, \\
 (EI)_S \frac{d^2\phi_S}{dz^2}(L) - \omega_n^2 I_t \frac{d\phi_S}{dz}(L) &= 0, \\
 (EI)_S \frac{d^3\phi_S}{dz^3}(L) + \omega_n^2 M_t \phi_S(L) - F_Z \frac{d\phi_S}{dz}(L) &= 0
 \end{aligned} \tag{4.53}$$

$$F(t) = - \left(M_t \phi(L) + \int_0^{L_C} m_C(z) \phi_S(z) dz + \int_{L_C}^L m_S(z) \phi_S(z) dz \right) \frac{d^2 \eta_0(t)}{dt^2} - F_Y(t) \tag{4.54}$$

$$F_Z(a, r, q) = \varepsilon \mu_0 M_I M_2 \pi R_I R_2 \int_0^\infty J_0 \left(\frac{rq}{R_2} \right) \frac{J_1 \left(\frac{R_I}{R_2} q \right)}{q} J_1(q) U(a, q) dq \tag{4.55}$$

$$F_Y(a, r, q) = -\varepsilon \mu_0 M_I M_2 \pi R_I R_2 \int_0^\infty J_1 \left(\frac{rq}{R_2} \right) \frac{J_1 \left(\frac{R_I}{R_2} q \right)}{q} J_1(q) U(a, q) dq \tag{4.56}$$

Where,

$$U(a, q) = \left[e^{\left(+2d_2 - a \left| \frac{q}{R_2} \right| \right)} + e^{\left(-2d_1 - a \left| \frac{q}{R_2} \right| \right)} - e^{\left(-2d_1 + 2d_2 + a \left| \frac{q}{R_2} \right| \right)} - e^{\left(+a \left| \frac{q}{R_2} \right| \right)} \right] \tag{4.57}$$

Equations 4.53 represent the boundary conditions for a vibrating PEH cantilever beam with a magnet attached at the tip mass. The axial component of the magnetostatic force (F_Z) is introduced as the point force acting at the tip of the beam. For small deflection, the variation of F_Z due to the vibration of the beam has been neglected. This has been discussed at length in the previous sections. While the maximum rotation of the tip of the beam is within 5° , or the deflection of tip of the beam is less than 10mm; the effect of the rotation of magnets can also be neglected. The transverse component (F_Y) presented in Equation 4.54, is directly

dependent on the tip deflection of the PEH beam, and induced as an external forcing term. Thus, the PEH cantilever beam system can be represented using equivalent lumped parameters as shown in Equations 4.58 and 4.59. The equations are solved numerically using the state space formulation. In addition to the displacement and velocity, the output voltage is also assumed to be a state variable. The response of the system obtained using this formulation provides a reasonable estimate to predict the system behavior.

$$M_{eq} \frac{d^2\eta(t)}{dt^2} + C_{eq} \frac{d\eta(t)}{dt} + K_{eq}\eta(t) + \Theta V(t) = F(t) \quad (4.58)$$

$$\frac{V(t)}{R_L} + C^P \frac{dV(t)}{dt} - \Theta \frac{d\eta(t)}{dt} = 0 \quad (4.59)$$

$$k_1(t) = \eta(t), \quad k_2(t) = \frac{d\eta(t)}{dt}, \quad k_3(t) = V(t) \quad (4.60)$$

$$\begin{aligned} \dot{k}_1(t) &= k_2(t) \\ \dot{k}_2(t) &= -\frac{C_{eq}}{M_{eq}} k_2(t) - \frac{K_{eq}}{M_{eq}} k_1(t) - \frac{\Theta}{M_{eq}} k_3(t) + \frac{F(t)}{M_{eq}} \\ \dot{k}_3(t) &= \frac{\Theta}{C^P} k_2(t) - \frac{1}{R_L C^P} k_3(t) \end{aligned} \quad (4.61)$$

$$P(t) = \frac{V^2(t)}{R_L} \quad (4.62)$$

The variation of output voltage and power with frequency has been widely used to demonstrate the improvement in bandwidth of a vibrating PEH beam. The bandwidth of a PEH system, or the effective usability across a varied frequency range has been a widely-used metric to determine the versatility of a proposed PEH design (Cottone *et al.*, 2009; Erturk and Inman, 2011b; Kim *et al.*, 2015; Tang *et al.*, 2010; Wickenheiser and Garcia, 2010). To this end, a plot displaying the statistical root mean square (RMS) average of the output voltage or power under steady state conditions is used. The instantaneous power across a resistive load can be obtained using Equation 4.62. When RMS voltage is used in lieu of instantaneous voltage, the equation provides RMS power. A detailed experimental study is conducted to support the analytical formulation discussed in this section. A

PEH cantilever beam is subjected to a base excitation and the resultant behavior of the system is discussed in the next section.

4.3.3 Experimental investigation of a nonlinear PEH cantilever beam

The experimental setup consisted of a cantilever PEH beam attached to a frame which was fixed to the arm of a seismic shaker. The seismic shaker (APS 455) was attached with an accelerometer and connected to a shaker controller (VR 9500) which provided a controlled feedback system to maintain constant root mean square (RMS) acceleration. The feedback system was controlled from a computer using the Vibration View software which helped maintain the constant acceleration levels. A schematic representation of the experimental process is shown in Figure 4.14. The cantilever PEH beam was prepared by attaching a d_{31} MFC transducer (M2807P2; Smart Material Corp. (2015)) to an aluminum substrate. A high strength epoxy was used to bond the transducer on the substrate. The MFC transducer has a total thickness of about 300 μm (Smart Material Corp., 2015). The material properties for MFC, substrate and epoxy layers for the PEH beam are shown in Table 4.4. The physical properties of the PEH cantilever beam are displayed in Table 4.5. Electrical wires were soldered to the electrodes of the transducer and the whole system was connected to a national instruments data acquisition system (NI-DAQ). The output voltage was measured using the NI-9229 Voltage DAQ card with an internal impedance of 1 $\text{M}\Omega$. The system response was observed at two different base excitation levels of 0.1g and 0.2g respectively. Figure 4.14 (a) shows a typical representation of the experimental setup.

The PEH beam was attached with a tip mass consisting of an embedded magnet. The nonlinearity was introduced with the help of end magnets, which were encased into a movable acrylic holder as shown in Figure 4.14. Initial readings for a linear configuration were obtained at different base excitation levels. Thereafter, the end magnets were introduced and the output voltage was recorded. Few studies consider the usage of optimal resistance for achieving optimal power output for linear systems, the optimal resistance is dependent on the natural frequency, capacitance, and excitation force on the PEH system (Cammarano *et al.*, 2014; Guyomar *et al.*,

2005; Renno *et al.*, 2009; Szarka *et al.*, 2012) . Taking into consideration the number of experimental variables (attractive and repulsive configurations, varying base excitation levels, etc...) which would create complexities in determining the optimal resistance for each configuration, a constant external resistance was utilized to obtain the output power across all the configurations. Thus, all the voltage and power readings were obtained at a constant resistance of 1 M Ω .

(a)

(b)

Figure 4.14 (a) Experimental setup for testing a PEH beam, and (b) Schematic representation of the experimental process.

During the experimental testing, the PEH beam was subjected to increasing base excitation levels. The PEH beam was subjected to base excitation levels of 0.1g and 0.2g. The corresponding steady state RMS voltage was obtained for every

frequency increment. Thus, the power output was also calculated using Equation 4.62. The results obtained from the analytical model and the experiments for various configurations are shown in Figures 4.17, 4.18, and 4.19, and discussed in detail in the next section. There is a great correlation between them, which signifies the advantages of using the enhanced integral formulation for modeling nonlinear PEH cantilever systems.

Table-4.4: Material Properties

Material	Young's modulus (Pa)	Poisson's ratio	Density (kg/m ³)
Aluminum substrate	72×10^9	0.35	2700
Epoxy	2×10^7	0.3	1100
Piezoelectric MFC	30.34×10^9	0.34	5440

Table-4.5: Properties of the PEH beam

Property	Values
Beam size (mm x mm)	$105^a \times 10^a$
Total thickness (mm)	1.38^b
Tip mass (g)	8.162
Length of glue (mm)	38
Damping coefficient	0.0266
Equivalent mass (g)	8.581
Equivalent stiffness (N/m)	122.88
Equivalent damping (Ns/m)	0.0542
Coupling coefficient	-1.905×10^{-4}
Capacitance (F)	12.4×10^{-9}
Resistance (Ω)	1×10^6
Tip magnet	Type – 2
End magnet	Type – 1
^a Due to the manual preparation of the sample, there can be an error of ± 1 mm	
^b An average of five measurement points was used to obtain the total thickness	
* The properties of the magnets are provided in the Appendix A	

4.3.4 Results and discussion

The analytical modeling, and the experimental procedure to validate the analytical modeling have been discussed at length in the preceding sections. A detailed discussion of all the results obtained and their importance for understanding the magnetic induced nonlinear behavior of a PEH beam is presented in this section.

The potential energy distribution of a PEH system provides a preliminary glimpse into its behavior. The experiments were performed for low amplitude base

vibrations in the order of 0.1g and 0.2g. The major outcome of this study is to validate the analytical formulation and understand the behavior of a nonlinear PEH system. It has been an established fact that, monostable states provide a better frequency response under low amplitude base vibrations (Lin *et al.*, 2010; Pellegrini *et al.*, 2013; Tang *et al.*, 2012; Upadrashta and Yang, 2015). Thus, monostable states with axial separation values, $a = 0.008$ m and $a = 0.01$ m were considered for the experimental investigation.

The potential energy of a nonlinear PEH system is the sum of the potential energy due to the stiffness of the beam and the potential energy due to the presence of magnetostatic interaction. Figures 4.15 and 4.16 present the distribution of potential energy for the PEH beam with a single end magnet and two end magnets respectively. It was observed that, a single end magnet results in a system with two potential wells, and two end magnets give rise to three potential wells (in the present case, only one potential well is prominent). The transition between the bistable and the monostable states gives rise to neutral equilibrium states where the beam traverses the potential energy curve with much ease. This in turn paves the way for an enhanced bandwidth of operation under such conditions. The enhancement of bandwidth and the system response under monostable conditions has been discussed with the help of output voltage and power.

Figure 4.15 Variation of potential energy in the presence of one end magnet for (a) repulsive configuration, and (b) attractive configuration.

Figure 4.16 Variation of potential energy in the presence of two end magnets for (a) repulsive configuration, and (b) attractive configuration.

Voltage and power

The tip displacement and velocity of a simple cantilever beam have been used to assess its behavior as shown in section 4.2. Similarly, with the presence of a piezoelectric component, the output voltage across a resistor plays a vital role in evaluating the behavior of a PEH cantilever beam. Moreover, unlike displacement and velocity which require expensive equipment like a laser vibrometer, voltage can be readily examined even with a simple multimeter. Thus, voltage has been widely used to demonstrate the behavior of various PEH systems. In the present study, the output voltage was obtained using a NI DAQ system and it was directly recorded into the computer. The steady state voltage was recorded at various frequencies to provide a response as shown in Figures 4.17 and 4.18.

The analytical formulation for the corresponding PEH system is also shown in Figures 4.17 and 4.18. It is evident that the analytical formulation using the enhanced magnetostatic force equations provides a very close match with the experimental values. The presence of magnets, introduces stiffening or softening nonlinearities into the PEH system. When the tip and the end magnets were placed repelling each other, the resonant frequency shifted to a lower value than the linear

configuration giving rise to a softening nonlinearity. Similarly, when the magnets were placed attracting each other, the resonant peaks shifted to a higher value. The presence of these nonlinearities becomes more prominent with increase in the base excitation levels. In other words, the system could traverse a wider part of the potential energy curves with increasing base excitation. This in turn, gave rise to an enhanced bandwidth of operation.

Figure 4.17 Variation of output voltage with frequency obtained from: (a) Experimental setup, and (b) analytical formulation, for a base vibration of 0.1g.

Figure 4.18 Variation of output voltage with frequency obtained from: (a) Experimental setup, and (b) analytical formulation, for a base vibration of 0.2g.

Recent advances in sensor technologies gave rise to sensor circuits which can be run with power as low as $100 \mu\text{W}$ (Tang *et al.*, 2010). Thus, various designs have been proposed over the years to improve the bandwidth of operation of the PEH systems. Out of the wide variety of designs, the use of nonlinearities introduced due to magnets provides a PEH system which is simple and practicable. For a linear PEH cantilever beam, the output power is limited to a narrow bandwidth, which is evident from Figure 4.19. In the presence of magnets, the bandwidth increases. The variation of bandwidth for various configurations is presented in Figure 4.20. The observed bandwidth was higher than the linear PEH beam for a configuration with 1

end magnet. Moreover, the repulsive configuration provided a higher bandwidth than the attractive configuration. The bandwidth was also dependent on the base acceleration; this behavior is attributed to the ability of the beam to traverse a greater part of the potential energy curve with an increased input energy in the form of base excitation. When two end magnets were placed facing the tip magnet, the bandwidth of the PEH beam fell, this configuration might require higher base acceleration before the nonlinear configuration displays an enhanced bandwidth. This has been addressed in detail in the next chapter.

(c)

(d)

Figure 4.19 Variation of power output with frequency for: (a) & (b) 1 end magnet, and (c) & (d) 2 end magnets, for base vibrations of 0.1g and 0.2g respectively.

(a)

(b)

*Sequence (Left to Right): Nonlinear-repulsive-8mm; Nonlinear-repulsive-10mm; Linear; Nonlinear-attractive-10mm; Nonlinear-attractive-8mm.

Figure 4.20 Variation of bandwidth for output power greater than 100 μ W for PEH beam with (a) 1 end magnet, and (b) 2 end magnets.

4.4. Conclusion

A candid investigation into the effects of magnetostatic forces acting on a cantilever beam have been presented in this chapter. Analytical formulation to obtain the response of a simple cantilever beam in the presence of magnets has been derived. The primary contribution from this formulation is the inclusion of the enhanced magnetostatic force terms into the lumped parameter formulation. In addition to the use of transverse force, the necessity for the inclusion of axial force is presented with the help of analytical and experimental investigations. The non-inclusion of axial magnetostatic force can lead to a considerable error especially in the repulsive configuration. Thereafter, the necessity for understanding the influence of the rotation of tip magnet on the behavior of the cantilever beam system has been addressed. It was concluded that the error is not significant while the maximum rotation is within 5° .

This formulation for a simple cantilever beam was extended to a PEH cantilever beam system. In the presence of the piezoelectric element, an extra response in the form of voltage is obtained. The output voltage obtained from a PEH system can reveal comprehensive information about the system. The formulation for the PEH system was validated with the help of an experimental investigation. The PEH

cantilever beam was subjected to base excitation levels of 0.1g and 0.2g, and various configurations for both 1 end magnet and 2 end magnets were explored. The output voltage obtained from the experimental PEH system matched well with the values obtained from the analytical formulation. The output voltage was converted to power to provide a better understanding of the bandwidth of operation of the PEH system.

Moreover, the improvement in the system bandwidth in the presence of magnetic induced nonlinearity has also been investigated. An improvement of 30% in the bandwidth for output power greater than 100 μ W was obtained for a configuration undergoing a base excitation of 0.2g. It was inferred that with an increase in the base excitation the nonlinear PEH system could perform better than the linear PEH system. The bandwidth depended on the base excitation and the nonlinear configuration, a further investigation with higher base excitation levels and varying stiffness levels is required to completely understand the enhancement in bandwidth. This will be dealt in detail in the next chapter.

4.5. Summary

A detailed perspective of the functioning of a PEH cantilever beam has been provided in the present chapter. The first section lists out Various ways in which the magnetostatic interaction in a PEH system has been dealt in the literature, and the inherent shortcomings present, are listed out in the first section. It has been followed up with a detailed derivation of the effects of both the axial and transverse forces, and their combination on a simple cantilever beam. A simple experimental investigation to prove the derived formulation, has also been illustrated in the second section. The third section has been dedicated to the derivation of the response of a PEH cantilever beam in the presence of magnetostatic interaction. An experimental approach to validate the analytical formulation, and the concept of bandwidth enhancement due to the presence of magnets has also been discussed. It is followed by a concluding portion of the present chapter.

CHAPTER 5 VARIATION OF THE BANDWIDTH AND EFFECTIVE STRAIN TRANSFER OF A PIEZOELECTRIC ENERGY HARVESTER WITH CHANGE IN STIFFNESS OF THE SYSTEM

The analytical and experimental formulations to comprehend the behavior of a PEH cantilever beam system subjected to magnetic nonlinearity has been presented in detail in the previous chapter. The concepts of bandwidth and the necessity of enhancing the bandwidth of operation for a PEH beam were explained with experimental findings. The previous chapter ended with a brief discussion on the variation of bandwidth with increasing magnetic nonlinearity induced into the system. In addition to the variation in magnetic nonlinearity, the stiffness of the PEH system is also susceptible to change with practical applications. Thus, an effort has been made in this chapter to reinforce the understanding of the variation of bandwidth and effective strain transfer with change in stiffness of the system. Both analytical and experimental methods have been explored to demonstrate the same.

5.1. Introduction

The bandwidth of an energy harvester is defined as the range of frequency within which the harvester performs efficiently. A simple cantilever shaped linear energy harvester has a very narrow bandwidth of operation. Various techniques were investigated over the years to enhance the bandwidth of the harvesters (Daqaq *et al.*, 2014; Harne and Wang, 2013; Pellegrini *et al.*, 2013; Tang *et al.*, 2010). Introducing magnetic induced nonlinearity is one of the popular techniques employed to enhance bandwidth. The usage of magnets to enhance the performance of a vibration based energy harvester was introduced by Cottone *et al.* (2009), Erturk *et al.* (2009a) and Xing *et al.* (2009). In their work, Cottone *et al.* (2009) presented the behavior of a piezoelectric inverted pendulum model when it was subjected a base vibration in the presence of a tip magnet. They concluded that at a critical separation distance of 11.6 mm, the system produced the maximum power under a monostable configuration. Erturk *et al.* (2009a) investigated the response of a piezo magneto elastic power generator in the presence of end magnets. The power

generator displayed broadband characteristic under a bistable configuration at a root mean square (RMS) base acceleration of 0.35g ($g = 9.81 \text{ m/s}^2$). Xing *et al.* (2009) demonstrated the broadening of bandwidth of a magnetostatic energy harvester undergoing a base vibration of 0.57g. They concluded that the presence of end magnets and a highly permeable magnetic beam induce an alternating magnetic flux which enhances the system output to 18.5% of the operating frequency. Review papers by Tang *et al.* (2010), Pellegrini *et al.* (2013), Harne and Wang (2013), and Daqaq *et al.* (2014) enumerate the various attempts to achieve enhanced broadband performance of the energy harvesters in the presence of magnet induced nonlinearities. More recently, De Paula *et al.* (2015) reported an enhanced design for piezo magneto elastic energy harvesting cantilever beam in a bistable and monostable configuration under random base excitation. Wang *et al.* (2015) explored the possibility of a tristable configuration of a piezoelectric cantilever beam for human motion. It was concluded that under large amplitude vibrations which induce inter-well traversing of the cantilever beam, the nonlinear harvester outperforms the linear harvester.

Substantial efforts have been made in understanding the broadband behavior of a PEH system having magnet induced nonlinearity (Ahmadian *et al.*, 2009; Andò *et al.*, 2012; Cottone *et al.*, 2009; Daqaq, 2010; Erturk *et al.*, 2009a; Erturk and Inman, 2011a; Formosa *et al.*, 2009; Lin *et al.*, 2010; Mann and Sims, 2009; Mansour *et al.*, 2010; McInnes *et al.*, 2008; Pellegrini *et al.*, 2013; Ramlan *et al.*, 2010; Stanton *et al.*, 2009; Tang and Yang, 2012a; Tang *et al.*, 2012; Tang *et al.*, 2011; Vocca *et al.*, 2012; Wickenheiser and Garcia, 2010; Yang *et al.*, 2011). The analytical formulation to understand the effect of magnet induced nonlinearity has been discussed in the previous chapter. Furthermore, the broadband characteristics of a nonlinear PEH system are also investigated with the help of experimental studies.

Many studies were conducted to enhance the bandwidth of a PEH system by introducing various designs. Most of the experimental research in the literature was conducted on a PEH system having a constant stiffness for the beam. The variation of bandwidth with varying stiffness of the PEH system was seldom investigated in the literature. It is only rational to understand the most efficient configuration for a

design. This opens a whole new arena with a compelling need to be explored and understood. This chapter addresses this critical aspect with analytical and experimental studies on various cantilever PEH systems having different beam stiffness. This was achieved experimentally by changing the material used for the substrate of the PEH beam.

5.2. Correlation between strain transfer and bandwidth

The potency of a cantilever PEH system depends on its ability to transfer the strain efficiently to the piezoelectric component from the substrate, and its affinity to perform under a broad range of vibration frequencies. The enhancement of the bandwidth of operation of a PEH system in the presence of magnetic nonlinearity has been introduced in the previous chapter. For a given set of parameters of the PEH system, the bandwidth and the strain transfer may not be mutually efficient to produce an optimum output. This critical concern has been examined in the following sections.

5.2.1 Calculation of strain in a PEH beam

The longitudinal strain in a vibrating PEH cantilever beam can be derived from the first principles of solid mechanics. Assuming, that the beam undergoes small vibrations and the piezoelectric MFC transducers are subjected to a small electric field, the constitutive relations for a piezoelectric material in the d-form are given in Equation 5.1 (IEEEStandard, 1988).

$$\begin{bmatrix} S \\ D \end{bmatrix} = \begin{bmatrix} s^E & d^c \\ d^d & \varepsilon^T \end{bmatrix} \begin{bmatrix} T \\ E^f \end{bmatrix} \quad (5.1)$$

In the above equation, the strain S and the electric displacement D are dependent on the stress T and the electric field E^f . The elastic compliance vector s^E (elastic compliance under a constant electric field), the piezoelectric constants d^c and d^d (superscripts c and d represent the converse and the direct piezoelectric effects respectively) and the dielectric permittivity ε^T (dielectric permittivity under a

constant stress field) are the constants which influence the strain and electric displacements.

A perfect bonding layer between the substrate, the epoxy layer and the MFC transducer are assumed and the corresponding axial strain due to bending of the cantilever PEH beam can be represented as shown in Equation 5.2. The axial strain can be expressed as a combination of the strain resulting from direct bending of various layers, and the resultant strain due to the electric field across the piezoelectric layer.

The first part of the Equation 5.2 was obtained from the Euler-Bernoulli beam theory and presented in Equation 5.3 (Erturk and Inman, 2011b). M_x represents the bending moment across a cross-section of a cantilever beam having an area moment of inertia I and situated at a distance z from the fixed end. y being the distance of the farthest layer from the neutral axis, and E_z being the elastic modulus in the direction of length of the beam. The bending moment across a cross-section can be approximated using the equivalent stiffness of the PEH beam, and tip displacement as shown in the Equation 5.4.

$$S_z = s^E T_z + d_{31} E^f \quad (5.2)$$

$$T_z = -\frac{M_x y}{I} \text{ and } s^E = \frac{I}{E_z} \quad (5.3)$$

$$M_x = K_{eq} w(L, t)(L - z) \quad (5.4)$$

$$d_{31} = \frac{e_{31}}{E_p} \text{ and } E^f = \frac{-V(t)}{T_p} \quad (5.5)$$

The second part of Equation 5.2 was obtained from the fundamental properties of the piezoelectric materials. To harvest energy using a piezoelectric material, the constituent material is poled along its thickness giving rise to a d_{31} configuration. The piezoelectric constant d_{31} was obtained from e_{31} and the elastic modulus of the piezoelectric material E_p which are provided in the manufacturer's datasheet. A uniform electric field E^f was assumed across the piezoelectric material which can be

represented as the ratio of voltage $V(t)$ and the thickness of the piezoelectric transducer T_P as shown in Equation 5.5 (Bhalla *et al.*, 2012; Sodano *et al.*, 2004b).

The maximum strain ε_{max} in a PEH beam occurs at the farthest most layer from the neutral axis, and near the root of beam (i.e. $z = 0$). The following equations for strain were derived considering a composite beam consisting of the piezoelectric layer (subscript P) attached to the substrate (subscript S) with a glue layer (subscript G). The bending stress for any cross-section at its neutral axis is zero and the expression for the depth of the neutral axis was obtained using this fundamental property.

$$\varepsilon_{max}(t) = \frac{-K_{eq}\eta(t)\phi(L)(h+t_P)L}{(EI)_C} - \frac{e_{31}V(t)}{E_P t_P} \quad (5.6)$$

Where,

$$h = \frac{t_S}{2} \left(\frac{I - \frac{E_P b_P}{E_S b_S} \left(\frac{t_P}{t_S} \right)^2 - \frac{E_G b_G}{E_S b_S} \left(\frac{t_G}{t_S} \right)^2 - 2 \frac{E_P b_P}{E_S b_S} \left(\frac{t_P t_G}{t_S^2} \right)}{I + \frac{E_P b_P t_P}{E_S b_S t_S} + \frac{E_G b_G t_G}{E_S b_S t_S}} \right) \quad (5.7)$$

$$(EI)_C = \frac{1}{3} E_S b_S (t_S - h)^3 + \frac{1}{3} E_S b_S h^3 + \frac{1}{3} E_G b_G (t_G + h)^3 - \frac{1}{3} E_G b_G h^3 \\ + \frac{1}{3} E_P b_P (t_P + t_G + h)^3 - \frac{1}{3} E_P b_P (t_G + h)^3 \quad (5.8)$$

In the above equations, h is the depth of the neutral axis from the top surface of the substrate; E is the modulus of elasticity, and B and T are the width and thickness of the corresponding layers. Figure 4.13 shows a schematic diagram of the cross-section of a typical PEH beam with the corresponding distances for h , y , b , t , etc... From the above equations, it can be deduced that the maximum strain in a PEH beam is a function of its stiffness and the voltage obtained.

An analytical representation of a PEH beam with the help of a lumped spring-mass-damper system with equivalent values of stiffness, mass, damping and electromechanical coupling is illustrated in the previous chapter. On subjecting a PEH beam to base vibration, the output voltage $V(t)$ and the corresponding

displacement $w(z,t)$ can be recorded from both experimental observations and analytical formulation.

Figure 5.1 Experimental setup for strain testing of a PEH beam.

Figure 5.2 Results for validation of the PEH beam attached with a strain gauge – variation of voltage with frequency

Figure 5.3 Results for validation of the PEH beam attached with a strain gauge – variation of strain with frequency.

To validate the formulation for strain given in Equation 5.6; an experiment was conducted on a PEH beam attached with a strain gauge as shown in Figure 5.1. The corresponding beam parameters are provided in Table 5.1. The PEH beam was subjected to incremental base vibrations from 0.1g to 0.3g in a frequency range of 20Hz - 35Hz, the resonant frequency of the beam was observed to be in a range of 26.8Hz for 0.1g and 26.5Hz for 0.3g. The output voltage and the strain gauge readings for the PEH beam were obtained for all the observation points.

The maximum strain was obtained using Equation 5.6; the magnitude of the output voltage $V(t)$ and the tip displacement $w(L,t) = u(t)\Phi(L)$, and other corresponding parameters for the respective beam were obtained from experimental observations and the analytical formulation respectively.

Table-5.1: Properties of the PEH beam

Property	Values
Beam size (mm x mm)	90 ^a X 10 ^a
Total thickness (mm)	1.05 ^b
Tip mass (g)	4.344
Length of glue (mm)	35
Equivalent mass (g)	4.672
Equivalent stiffness (N/m)	128.144
Coupling coefficient	-1.943 X 10 ⁻⁴
Capacitance (F)	12.4 X 10 ⁻⁹
Resistance (Ω)	1 X 10 ⁶
^a Due to the manual preparation of the sample, there can be an error of ± 1 mm	
^b An average of five measurement points was used to obtain the total thickness	

Figure 5.2 shows the variation of voltage with changing frequency across base acceleration levels of 0.1g, 0.2g and 0.3g. The output voltage obtained from experiments matches well with the output voltage obtained from analytical formulation. The variation of strain with changing frequency and base excitation levels is presented in Figure 5.3. The strain represented by ‘-Experimental’ was obtained directly from the strain gauge readings. The strain readings represented by ‘-analytical’ were obtained from the analytical formulation directly. It was observed

that, the analytical formulation for strain provided a good approximation for the curves obtained experimentally. Though there was a slight mismatch at lower frequencies, this was due to the low amplitude vibrations resulting in a considerable lower magnitude of strain, displacement and voltage values. Thereby driving the data acquisition equipment to the limits of their sensitivity.

Based on the above discussion, the analytical formulation for maximum strain in a PEH beam given by the Equation 5.6 can be utilized efficiently to obtain the variation of strain with frequency in a PEH system.

5.2.2 Calculation of equivalent strain and effective strain transfer

A PEH cantilever system comprises a multitude of layers, which include the substrate, epoxy, piezoelectric MFC consisting of a matrix of kapton, piezoelectric fibers, electrodes and epoxy. In the presence of a composite cross-section for a PEH cantilever beam, the strain obtained at the outer most layer is dependent on the material properties of the constituent layers. To understand the rate at which the strain is transferred to the outermost layers, it is imperative to obtain the strain for a cross-section with homogenous material. In PEH cantilever beams, the equivalent strain across a homogeneous cross-section of the substrate material (aluminum) provides a good yardstick to obtain the effective strain transfer. Thus, an equivalent beam with a homogeneous cross-section and having the same equivalent system parameters is considered.

For a cantilever beam with a uniform cross-section, the stiffness can be obtained using Equation 5.9. h_{eq} represents the thickness of a beam with homogeneous cross-section. Based on the same principles used in the previous section, strain for the equivalent beam is derived and represented using Equation 5.12. It is assumed that the equivalent beam with a uniform cross-section has a linear variation of strain till the outermost fibers. The effective strain transfer ϵ_r is represented as the percentage ratio of maximum strain obtained for a PEH beam ϵ_{max} and the maximum strain obtained for an equivalent beam with a uniform cross-section ϵ_{eq-max} . It is represented as shown in Equation 5.13 and the effective strain transfer values for the PEH beam are tabulated in Table 5.2.

$$K_{eq} = \frac{3EI}{L^3} = \frac{Eb h_{eq}^3}{4L^3} \quad (5.9)$$

Where,

$$I = \frac{b h_{eq}^3}{12} \quad (5.10)$$

$$h_{eq} = \left(\frac{4K_{eq}L^3}{Eb} \right)^{\frac{1}{3}} \quad (5.11)$$

$$\varepsilon_{eq-max}(t) = \frac{-K_{eq} \eta(t) \phi(L) h_{eq} L}{2EI} \quad (5.12)$$

$$\varepsilon_r = \frac{\varepsilon_{max}}{\varepsilon_{eq-max}} \times 100 \quad (5.13)$$

The composite section has constituent layers with Young's modulus less than that of the substrate. Thus, for a given thickness of the PEH beam the equivalent section will have an effective thickness less than that of the composite section. Therefore, the effective strain transfer values obtained from Equation 5.13 will be greater than 100. In other words, there is an increased rate of strain transfer in a PEH beam compared to a simple cantilever beam with uniform cross section having similar lumped parameters. The variation of the effective strain transfer values for the PEH beam discussed in the previous section is as shown in Figure 5.4. From the figure, it is evident that the variation of effective strain transfer values for a beam of constant stiffness and varying base excitation levels is negligible.

Figure 5.4 Effective strain transfer for a PEH beam with varying acceleration levels.

Figure 5.5 Variation of resonant frequencies of a PEH beam with varying stiffness.

Table-5.2: Effective strain transfer and bandwidth for the PEH beam

$K_{eq} = 128.144 \text{ N/m}$		
Base excitation		Effective strain transfer
0.1g		111.14
0.2g		111.10
0.3g		110.94
Constant base excitation (0.3g) with varying Stiffness		
Stiffness x K_{eq}	Effective strain transfer	Bandwidth 'Hz'
0.5	108.01	5.2
0.75	109.85	4.4
1	110.94	4
1.25	111.67	3.6
1.5	112.19	3.3

The stiffness plays a very important role in determining the behavior of a PEH system. Thus, an efficient setup for a PEH beam pivots around the determination of optimum stiffness which would concede maximum strain transfer and maximum output bandwidth. Therefore, an analytical investigation of the effect of variation of stiffness on the effective strain transfer and bandwidth was undertaken by varying the stiffness K_{eq} for a PEH beam from $0.5 \times K_{eq}$ to $1.5 \times K_{eq}$ ($K_{eq} = 128.144 \text{ N/m}$). All the other lumped parameters (M_{eq} , Θ) were assumed to be constant. This variation in stiffness resulted in a shift of the resonant frequency of the PEH system as shown in Figure 5.5. The bandwidth was measured at a level of 10 V (Power = $100 \mu\text{W}$). The corresponding values are shown in Table 5.2. It was observed that

with the increase in stiffness the response of the system (voltage) decreased, this is due to the increase in resonant frequency and consequently the damping term C_{eq} . Figure 5.6 represents the variation of effective strain transfer and bandwidth with changing stiffness. In an ideal case, a PEH system with maximum bandwidth and maximum strain transfer (indicating efficient utilization of the cross-section) would make for an optimum configuration. Since these values vary reciprocally with respect to stiffness as shown in Figure 5.6, the optimum stiffness is about $0.8 \times K_{eq}$ for the present configuration.

(a)

(b)

Figure 5.6 Variation of (a) effective strain transfer and (b) bandwidth with stiffness, and (c) shows the variation of both the effective strain transfer and bandwidth on the same plot.

5.2.3 Effective strain transfer and variation of bandwidth in a nonlinear PEH beam

The increase in bandwidth of a PEH system in the presence of magnetic nonlinearity has been discussed in the previous chapter. Moreover, the variation of bandwidth with varying magnetic distance has also been introduced in the previous chapter. This section outlays the importance of PEH beam stiffness on the bandwidth of a nonlinear PEH system. To understand the effect of change in stiffness, the analytical model derived in the previous chapter has been used to obtain the system response (Voltage, power, strain etc.). The linear beam discussed in the previous section is used as the base configuration and magnetic forces resulting from the interaction between two similar cylindrical magnets (Radius = 2.5 mm, height = 4 m and Magnetization = 2000 kA/m) have been considered. The smaller size of the magnets was selected to ensure the deviation in the stiffness is minimal.

Figure 5.7 Variation of (a) effective strain transfer and (b) bandwidth with stiffness, and (c) shows the variation of both the effective strain transfer and bandwidth on the same plot.

The K_{eq} values were varied in a range of $0.5 \times K_{eq}$ to $1.5 \times K_{eq}$. The magnets were assumed to be placed 8 mm apart and the whole system was subjected to a base vibration of 0.3g. The potential energy distribution for a system with $0.5 \times K_{eq}$, K_{eq} , and $1.5 \times K_{eq}$ are shown in Figure 5.7. The potential energy was obtained using the equations derived in the previous chapter. As shown in the graphs, the potential energy for a repulsive configuration varies from a bistable state at $0.5 \times K_{eq}$ to a monostable state at $1 \times K_{eq}$, and an almost linear state at $1.5 \times K_{eq}$. This variation presents a holistic approach to explore the behavior of a nonlinear PEH system. Figure 5.8 shows the variation of output voltage for PEH systems with different K_{eq} values. The variation of both the effective strain transfer and bandwidth (calculated at 100 μ W) is presented in Figure 5.9.

Figure 5.8 Variation of output voltage with frequency for (a) attractive and (b) repulsive configuration along with variation of K_{eq} values.

The output parameters were obtained using the lumped parameter formulation for a nonlinear PEH system provided in Section 4.3.2. The variation of output voltage with frequency provides a good understanding into the effect of the nonlinear magnetic force acting on the PEH beam. The PEH system depicted a hardening behavior in the presence of repulsive magnetic force and softening behavior in the presence of attractive magnetic force. Almost all the curves for the output voltage portrayed a similar behavior with varying degree of hardening or softening apart from the curve obtained for a PEH system with $0.5 \times K_{eq}$ under a repulsive configuration. The bistable behavior of this case restricted the PEH system to a single potential well. Moreover, the output base excitation was unlikely enough to push the system over the potential barrier. This was overcome with an increase in stiffness ($0.75 \times K_{eq}$), thereby resulting in a configuration which provided a much higher bandwidth. The effective strain transfer and the bandwidth values are provided in Table 5.3.

Table-5.3: Effective strain transfer and bandwidth for the PEH beam at constant base excitation (0.3g) with varying Stiffness

Attractive configuration: $K_{eq} = 128.216 \text{ N/m}$		
Stiffness $\times K_{eq}$	Effective strain transfer	Bandwidth 'Hz'
0.5	106.49	3.6

0.75	109.23	3.5
1	110.65	3
1.25	111.52	2.9
1.5	112.11	2.6
Repulsive configuration: $K_{eq} = 128.118$ N/m		
Stiffness x K_{eq}	Effective strain transfer	Bandwidth 'Hz'
0.5	115.25	6.3
0.75	111.65	8.2
1	111.81	8
1.25	112.12	5.9
1.5	112.44	4.6

A simple comparison of the values for strain transfer and bandwidth provided in Tables 5.2 and 5.3 gives a broad idea into the effect of nonlinearity on the variation of stiffness in a PEH system. These values have been shown graphically in Figure 5.9. On comparison with the linear PEH system, the repulsive configuration enhances the bandwidth of the system along with an increase in effective strain transfer. On the other hand, both the effective strain transfer and bandwidth are diminished for an attractive configuration. Moreover, the variation of effective strain transfer in a repulsive configuration is far less compared to that of the linear or attractive configurations. Almost 100% increase in the bandwidth was observed for a repulsive configuration especially at lower stiffness levels. This is conclusive of the fact that; a repulsive configuration provides a better utilization of the magnetic induced nonlinearity in a PEH system. The next section bolsters this concept with exhaustive experimental investigation of PEH systems subjected to magnetic induced nonlinearity.

(a)

(b)

Figure 5.9 Variation of the effective strain transfer and bandwidth for (a) attractive and (b) repulsive configuration with changing K_{eq} values.

5.3. Experimental investigation of variation of bandwidth in the presence of magnetic induced nonlinearity.

The previous section explains how a nonlinear repulsive configuration out-performs the linear PEH system in the presence of a single end magnet. Following the similar experimental setup explained in the previous chapter, PEH cantilever beams with an aluminum and fiberglass substrate along with a commercially available bimorph beam were tested.

5.3.1 Experimental setup

The experimental setup consisted of a cantilever PEH beam attached to a frame which was fixed to the arm of a seismic shaker similar to the experimental setup shown in Figure 4.14. The seismic shaker (APS 455) was attached with an accelerometer and connected to a shaker controller (VR 9500) which provided a controlled feedback system to maintain constant root mean square (RMS) acceleration. The feedback system was controlled from a computer using the Vibration View software which helped maintain the constant acceleration levels. The same schematic representation of the experimental process is shown again in Figure 5.10 for the sake of comprehensibility of the section. The cantilever PEH beam was prepared by attaching a d_{31} MFC transducer (M2807P2/M2814P2; Smart Material Corp. (2015)) to the respective substrate. A high strength epoxy was used to bond the transducer on the substrate. The MFC transducer has a total thickness of about 300 μm (Smart Material Corp., 2015). The material properties for MFC, substrate and epoxy layers for the PEH beam are shown in Table 5.4 and the physical properties of the PEH cantilever beam are displayed in Table 5.5. Electrical wires were soldered to the electrodes of the transducer and the whole system was connected to a national instruments data acquisition system (NI-DAQ). For a commercially available bimorph harvester (Model: V21BL – Midé Technology Corporation), the harvester is available in a pre-manufactured state and the electrodes are connected to the data acquisition system directly. The output voltage was measured using the NI-9229 Voltage DAQ card with an internal impedance of 1 $\text{M}\Omega$. The system response was observed at both lower excitation levels of 0.1g and 0.2g, and higher excitation levels of 0.4g and above respectively. Figure 5.10 (a) shows a typical representation of the experimental setup for an aluminum substrate.

The PEH beam was attached with a tip mass consisting of an embedded magnet. The nonlinearity was introduced with the help of end magnets, which were encased into a movable acrylic holder as shown in Figure 5.10. Initial readings for a linear configuration were obtained at different base excitation levels. Thereafter, the end magnets were introduced and the output voltage was recorded. All the voltage and

power readings were obtained at a constant resistance of $1\text{ M}\Omega$ for all the substrate materials.

(a)

(b)

Figure 5.10 (a) Experimental setup for testing a PEH beam, and (b) Schematic representation of the experimental process.

During the experimental testing, the PEH beam was subjected to increasing base excitation levels. The corresponding steady state RMS voltage was obtained for every frequency increment. Thus, the power output was also calculated using Equation 4.62. The validation for the analytical model has been provided in the

previous chapter. This chapter is more concentrated on the experimental investigation of the effect of stiffness on the bandwidth enhancement in the presence of magnetic induced nonlinearity.

Table-5.4: Material Properties

Material	Young's modulus (Pa)	Poisson's ratio	Density (kg/m ³)
Aluminum substrate	72×10^9	0.35	2700
Fiberglass substrate	18.62×10^9	0.2	1820
Epoxy	2×10^7	0.3	1100
Piezoelectric MFC	30.34×10^9	0.34	5440

Table-5.5: Properties of the PEH beam

Property	Aluminum	Fiberglass	Bimorph
Beam size (mm x mm)	$105^a \times 10^a$	$100^a \times 10^a$	$60.96^c \times 16.76^c$
Total thickness (mm)	1.38^b	1.42^b	0.79^c
Tip mass (g)	8.162	10.775	10.775
Length of glue (mm)	38	40	-
Resistance (Ω)	1×10^6	1×10^6	1×10^6
MFC	M2807-P2	M2807-P2	-
Tip magnet	Type – 2	Type – 2	Type – 2
End magnet	Type – 1	Type – 1	Type – 1
Equivalent mass (g)	8.581	11.021	11.576
Equivalent stiffness (N/m)	122.88	58.13	292.55
Equivalent damping (Ns/m)	0.0542	0.1671	0.0985
Coupling coefficient	-1.905×10^{-4}	-9.895×10^{-5}	-4.656×10^{-4}
Capacitance (F)	12.4×10^{-9}	12.4×10^{-9}	13×10^{-9}
^a Due to the manual preparation of the sample, there can be an error of ± 1 mm			
^b An average of five measurement points was used to obtain the total thickness			
^c Vulture Data Sheet – Mide Technology, US			

5.3.2 Results and discussion

The dependence of bandwidth on the stiffness of a PEH beam has been shown analytically in the previous sections. A controlled variation of the stiffness is practically unattainable; thus, the variation of bandwidth across PEH beams with different substrate beams was explored. PEH beams with aluminum, fiberglass and the commercially available V21BL bimorph beams were initially subjected to excitation levels of 0.1g and 0.2g. The lower level of excitation was used to minimize the fatigue damage which results when the strain levels exceed $500\mu\epsilon$. The fatigue damage resulting in PEH beams due to varying strain levels has been

thoroughly examined in chapter 7. Based on the results obtained from the lower excitation levels, aluminum beams proved to be the most congenial option for higher excitation levels.

Lower excitation levels

The equivalent mass of all the three beams was maintained as close as possible, but a considerable variation in the equivalent beam stiffness was observed as shown in Table 5.5. The fiberglass PEH beam had the least equivalent stiffness and the V21BL bimorph had the maximum equivalent stiffness. The corresponding equivalent stiffness and strain transfer values for linear configurations are shown in Table 5.6.

Table-5.6: Effective strain transfer values

Material	Equivalent Stiffness (N/m)	Excitation level	RMS Strain ($\mu\epsilon$)	Effective strain transfer (%)
Fiberglass	58.13	0.1g	69.65	48.234
		0.2g	139.31	48.335
Aluminum	122.88	0.1g	134.41	110.94
		0.2g	247.36	110.94
V21BL Bimorph	292.6	0.1g	304.71	390.11
		0.2g	-	-

During the experimental investigation, the V21BL bimorph beam underwent damage at an excitation level of 0.2g and the voltage output readings could not be obtained. The higher stiffness of the beam resulted in very high effective strain values even at lower excitation level of 0.1g. Thus, at 0.2g base excitation level the absolute strain levels were close to the maximum recommended strain level $800\mu\epsilon$ (Midé Technology Corporation – V21BL datasheet), which culminated in failure of the V21BL bimorph PEH beam after a few excitation cycles.

All the beams were subjected to both linear and nonlinear (attractive and repulsive) configurations in the presence of 1 and 2 end magnets separated from the tip magnets at 8 and 10 mm respectively. The corresponding variation of output voltage is presented in Figures 5.11-5.14.

(a)

(b)

(c)

Figure 5.11 Variation of output voltage in the presence of 1 end magnet at an excitaiton amplitude of 0.1g for (a) PEH beam with fiberglass substrate, (b) PEH beam with aluminum substrate, and (c) V21BL bimorph PEH beam.

(a)

(b)

Figure 5.12 Variation of output voltage in the presence of 1 end magnet at an excitaiton amplitude of 0.2g for (a) PEH beam with fiberglass substrate, and (b) PEH beam with aluminum substrate.

Figure 5.13 Variation of output voltage in the presence of 1 end magnet at an excitaiton amplitude of 0.2g for (a) PEH beam with fiberglass substrate, (b) PEH beam with aluminum substrate, and (c) V21BL bimorph PEH beam.

(a)

(b)

Figure 5.14 Variation of output voltage in the presence of 2 end magnets at an excitaiton amplitude of 0.2g for (a) PEH beam with fiberglass substrate, and (b) PEH beam with aluminum substrate.

A simple look at the peak RMS voltage of linear configurations indicates the importance of effective strain transfer as an indicative parameter. Effective strain transfer is a direct indication of the rate at which the mechanical base excitation is converted to voltage. A higher effective strain transfer indicates a higher output voltage as observed in the V21BL bimorph PEH beam. The fiberglass PEH beam which had an effective strain transfer of less than 100% resulted in a low peak voltage of less than 10 V even at a base excitation level of 0.2g.

Though the peak values are lower for a fiberglass PEH beam, the bandwidth enhancement (considering bandwidth at half the peak voltage of a linear configuration) is higher in comparison to the other two PEH beams. This property is enhanced with increase in base excitation and increase in the number of end magnets, wherein the nonlinearity induced into the system is enhanced. For example, in Figure 5.13 the bandwidth for a repulsive configuration at the half of peak voltage for a fiberglass PEH beam is enhanced to nearly 200% of the values obtained from a linear configuration. The primary reason for such an enhancement is the ability of the fiberglass PEH beam to traverse across all the potential wells owing to its lower stiffness levels. This behavior was discussed with analytical examples in the previous section.

Although the fiberglass PEH beam provides a greater enhancement in the operation bandwidth in the presence of magnetic nonlinearity, the practical usability of the fiberglass PEH beam is limited owing to its lower effective strain transfer values. Considering a power level of $100 \mu\text{W}$ as a benchmark for practical usability of the PEH system. The bandwidth at a voltage level of 10 V ($100 \mu\text{W}$) provides a more appropriate measure for determining the best configuration. Table 5.7 enumerates the variation of bandwidth for various configurations of the aluminum and the V21BL bimorph PEH beams respectively.

Table-5.7: Variation of Bandwidth at 10 V ($100 \mu\text{W}$)

Material	Equivalent Stiffness (N/m)	Configuration	0.1g		0.2g	
			Att.*	Rep.*	Att.*	Rep.*
Aluminum	122.88	Linear	0.9		2.4	
		Nonlinear, 8 mm	0.7	1.05	1.7	3
		Nonlinear, 10 mm	0.95	1	2	2.7
V21BL Bimorph	292.6	Linear	1.25		-	
		Nonlinear, 8 mm	1.05	1.7	-	-
		Nonlinear, 10 mm	0.45	1.05	-	-

A comparison of the values shown in Table 5.7 indicates that the nonlinear repulsive configuration provides a congenial environment for an enhanced bandwidth. For a, given level of base excitation, though the peak values obtained for a V21BL bimorph PEH beam are higher, the aluminum PEH beam provides a higher increase in bandwidth for a nonlinear repulsive configuration. This is further

enhanced with increase in the base excitation values. The increase in base excitation pumps enough energy into the system for the beam to traverse a larger distance, and a closer spacing of the magnets provides an enhanced interaction between the tip and end magnets respectively. Taking these points into consideration, a few more PEH beams consisting of aluminum substrate were experimentally tested at higher excitation levels.

Higher excitation levels

Higher excitation levels of 0.3g and 0.4g provide enough energy for the PEH system to traverse higher deflections. Thus, three PEH beams were prepared using an aluminum substrate. The PEH beams were prepared by varying the glue thickness and length to provide a variation in the equivalent stiffness of the PEH beam system. As stated earlier, a major drawback of subjecting the PEH beams to higher excitation levels is the increase in the maximum strain values at the root of the PEH cantilever beam. This was evident during the experimental investigation and resulted in early damage of the PEH cantilever beams. Thus, limited data was obtained for each PEH beam at these higher excitation levels. The beam properties and the corresponding equivalent parameters are shown in Table 5.8.

Table-5.8: Properties of the PEH beam with aluminum substrate

Property	Beam-1	Beam-2	Beam-3
Beam size (mm x mm)	120 ^a X 15 ^a	120 ^a X 15 ^a	120 ^a X 15 ^a
Total thickness (mm)	1.12 ^b	1.24 ^b	1.26 ^b
Tip mass (g)	10.775	8.12	10.775
Length of glue (mm)	40	45	50
Resistance (Ω)	1 X 10 ⁶	1 X 10 ⁶	1 X 10 ⁶
MFC	M2807-P2	M2814-P2	M2814-P2
Tip magnet	Type – 2	Type – 2	Type – 2
End magnet	Type – 1	Type – 1	Type – 1
Equivalent mass (g)	11.367	8.668	11.309
Equivalent stiffness (N/m)	60.41	80.492	96.223
Equivalent damping (Ns/m)	0.093	0.0790	0.1076
Coupling coefficient	-1.337 X 10 ⁻⁴	-2.0 X 10 ⁻⁴	-2.491 X 10 ⁻⁴
Capacitance (F)	12.4 X 10 ⁻⁹	24.8 X 10 ⁻⁹	24.8 X 10 ⁻⁹
^a Due to the manual preparation of the sample, there can be an error of ± 1 mm			
^b An average of five measurement points was used to obtain the total thickness			

A longer substrate beam was used in these PEH beams to reduce the equivalent stiffness of the system. Similar experimental setup as shown in Figure 5.10 was used for the investigation of these PEH beams. The base acceleration was gradually increased to 0.3g from 0.1g and the necessary data was recorded. Damage was observed in two of the three PEH beams after recording the data for few cycles of excitation at 0.3g base excitation. Only the undamaged beam was further subjected to a base excitation level of 0.4g.

Table-5.9: Effective strain transfer values at higher excitation levels

Material	Equivalent Stiffness (N/m)	Excitation level	RMS Strain ($\mu\epsilon$)	Effective strain transfer (%)
Beam-1	60.41	0.3g	417.21	122.26
Beam-2	80.49	0.3g	367.91	98.255
Beam-3	96.22	0.3g	342.33	104.02

The output voltage data has been categorized depending on the configuration of the PEH system. Moreover, the effective strain values at an excitation level of 0.3g for all the PEH beams are shown in Table 5.9. It was observed that the usage of M2807-P2 MFC gave rise to a higher RMS strain value at the base of the PEH beam. Though a similar substrate configuration was used for all the PEH beams, there was a change in the piezoelectric transducers attached to the substrate in beam-2 and beam-3, the primary purpose of this change was to reduce the RMS strain levels. Although the strain levels were reduced by nearly 20% in beam-2 and beam-3, they proved to be more susceptible to failure at higher excitation levels. In all the PEH beams, failure was determined based on the initial damage to the PEH beam which occurred in the MFC transducers as presented in Chapter 7. Akin to the lower base excitation levels, the magnets were placed at 8 mm and 10 mm axial separation wherever feasible. In addition to that, data was recorded for an axial separation of 5 mm between the magnets wherever available. The primary purpose being the examination of the extent to which the tip displaces under strong nonlinear magnetic forces at 5 mm separation.

Attractive configuration

At lower excitation levels, the improvement in the bandwidth due to introduction of nonlinearity was limited when the tip and end magnets were placed in attractive configuration. But at higher excitation levels, there is considerable enhancement for both the attractive and the repulsive configurations. Figure 5.15 displays the output voltage recorded for beam-1 and beam-2 in the presence of magnet induced nonlinearity. Two major inferences from this configuration are: the validation of the dependence of bandwidth on the stiffness of the PEH beam. The plots obtained for beam-1 bolster the fact that a stiffer beam does provide a diminished bandwidth even at higher base excitation levels. Secondly, the effect of the MFC transducer on the output parameters. In addition to the effect of stiffness, a bigger MFC transducer has a larger capacitance, thereby inducing a higher damping into the system, and reducing the bandwidth of the output voltage.

(c)

Figure 5.15 Variation of output voltage in the presence of 2 end magnets (attractive configuration) at higher excitaiton amplitudes for (a) beam-1 subjected to 0.3g base excitation, (b) beam-1 subjected to 0.4g base excitation, and (c) beam-2 subjected to 0.3g base excitation.

Repulsive configuration

The placement of the tip magnets and the end magnets in repulsive configuration gave rise to an improvement of bandwidth only at larger separation of the magnets. When the magnets were closely spaced (8mm and 5mm), it was observed that the inter-well transition of the PEH beams was limited. This primarily indicated that a much higher excitation level is necessary to induce the inter-well behavior, whereby the PEH beam has enough energy to traverse through different potential wells. The study inferred that for a repulsive configuration the bandwidth enhancement is prominent only in the presence of inter-well transition of the tip of the PEH beam. Moreover, inter-well transition of a PEH system requires higher base excitation levels to be induced into the system.

Figure 5.16 Variation of output voltage in the presence of 2 end magnets (repulsive configuration) at higher excitation amplitudes for (a) beam-2 subjected to 0.3g base excitation, and (b) beam-3 subjected to 0.3g base excitation.

5.4. Conclusion

Thus, a tradeoff between the stiffness, effective strain transfer, bandwidth and excitation level should be thoroughly studied to arrive at an efficient configuration for a PEH system subjected to given design parameters.

The objective of this chapter was to understand the relationship between stiffness of a cantilever PEH beam system and its bandwidth of operation. Various techniques have been explored in the literature to enhance the bandwidth of a PEH system, the usage of magnetic forces to induce nonlinearity into the PEH system is one of the popular techniques. Of the numerous models developed to study a nonlinear PEH

system, the representation of magnetic forces using an enhanced integral formulation provides an accurate account of the magnetic forces. The magnetic forces are broadly classified as axial and transverse forces which act at the tip of a vibrating PEH beams. The expressions for the determination of strain in a vibrating cantilever PEH beam provide a detailed understanding of a very important parameter. Strain provides a comprehensive understanding of the usability of the PEH beam cross-section. Experimental investigation validating the analytical model for a linear PEH system subjected to increasing base excitation levels of 0.1g, 0.2g and 0.3g is presented. The output voltage and strain match well with the analytical model. To further illustrate the utilization of the cross-section, a parameter termed 'effective strain transfer' was calculated. Effective strain transfer facilitates the comparison of various composite cross sections by providing a ratio of the strain across a composite cross-section with respect to a uniform aluminum cross-section. A detailed analytical study was performed to illustrate the relationship between stiffness, effective strain transfer, and bandwidth of operation for both linear and nonlinear PEH beams (bandwidth was calculated at a level of $100 \mu\epsilon$). It was inferred that, when the PEH beam stiffness was varied in a range of $0.5 \times K_{eq}$ to $1.5 \times K_{eq}$ ($K_{eq} = 128.144 \text{ N/m}$), for base excitation levels of 0.3g the nonlinear repulsive configuration provided a maximum bandwidth of operation with an increase in a range of 10% to 40%. Moreover, for a given base excitation, an increase in the stiffness of a cross-section contributes to an increase in the effective strain transfer values and decrease in the bandwidth of operation. The effective strain transfer is inversely proportional to the bandwidth, therefore for a given set of conditions the PEH beam should be designed for an optimum value of stiffness so that there is maximum utilization of the cross-section and the PEH system provides an improved bandwidth.

An experimental investigation was performed to reinforce the analytical investigation. PEH beams constituting of fiberglass, aluminum substrates along with commercially available V21BL harvester were experimentally tested and the corresponding output parameters were recorded. All the beams were initially subjected to base excitation levels of 0.1g and 0.2g for both linear and nonlinear configurations. It was evident that the PEH beam with aluminum substrate out

performs other two PEH beams. There was better utilization of the cross-section which was evident from the effective strain transfer values being more than 100, and an improved bandwidth of operation for a nonlinear repulsive configuration. Thus, PEH beams with aluminum substrate were further examined for higher level base excitation of 0.3g and 0.4g. At higher base excitation levels, the PEH beams were very susceptible to damage; and the nonlinear configuration outperformed the linear configuration in terms of the bandwidth. In conclusion, it was confirmed that a PEH beam having a lower beam stiffness provided an enhanced bandwidth of operating frequency for both linear and nonlinear configurations. Moreover, the effective strain transfer under such conditions is less which states that the cross-section is not optimally utilized. Thus, a tradeoff between the stiffness, effective strain transfer, bandwidth and excitation level should be thoroughly investigated to arrive at an efficient configuration for a PEH system subjected to given design parameters.

5.5. Summary

The present chapter illustrates the relationship between the equivalent stiffness of the PEH cantilever beam, effective strain transfer and its bandwidth of operation. An introduction to the concept of bandwidth and advancement in the PEH systems using various techniques for improving the bandwidth has been stated in the first section. Moreover, a few gaps in the literature have also been stated which have been addressed in this chapter. The second section enumerates a detailed analytical account of the correlation between cantilever PEH beam stiffness, strain transfer between the piezo transducer and substrate, and the bandwidth of operation of the PEH system. The derivation for strain was validated with the help of experimental investigation and a new term '*effective strain transfer*' was proposed to provide a guideline for comparison of the strain transfer between PEH beams having varying stiffness. The analytical modeling was followed by a detailed experimental investigation in the third section to study the behavior of PEH beams manufactured from various substrate materials. Various PEH beams were tested under different base excitation conditions in the presence and absence of magnet induced

nonlinearities. The final section lists out the highlights of the present chapter along with a few concluding remarks and inferences.

CHAPTER 6 ENHANCEMENT OF BANDWIDTH BY COUPLING TWO CANTILEVER BEAMS UNDERGOING TRANSVERSE AND PARAMETRIC VIBRATIONS

The concept of bandwidth and its improvement in the presence of magnetic induced nonlinearity in a PEH system have been presented in the previous chapters. Moreover, the dependence of the strain and bandwidth on the stiffness of the cantilever PEH beam has been investigated in the previous chapter. In most of the investigations presented so far, fixed end magnets were primarily used to induce nonlinearity into the system. This chapter explores the possibility of a translating end magnet. This was achieved by introducing a secondary cantilever beam into the PEH system which undergoes parametric vibration in the presence of external base excitation. The improvement in bandwidth of operation due to the presence of a secondary beam has been illustrated with the help of experimental investigations.

6.1. Introduction

Over the last decade, there has been substantial development in the realm of PEH; various designs were reported in the literature stating the added benefits of each design (Abdelkefi *et al.*, 2012; Erturk and Inman, 2011b; Szarka *et al.*, 2012; Tang *et al.*, 2010; Vokoun *et al.*, 2012). The initial harvester designs consisted of a transversely vibrating cantilever beam with a piezoelectric element attached at the root of the beam (Erturk and Inman, 2011b). This was the simplest designs and had a major drawback owing to a narrow bandwidth of operational frequency. This was overcome by the introduction of magnets at the tip of the beam, L-shaped beam designs etc... (Erturk and Inman, 2011b; Erturk *et al.*, 2009b; Tang *et al.*, 2010; Xu *et al.*, 2010). Researchers have even investigated the application of the basic cantilever designs for axial mode of vibrations, this results in a parametric vibration of the cantilever beam (Abdelkefi *et al.*, 2012; Daqaq *et al.*, 2009; HaQuang *et al.*, 1987; Jia *et al.*, 2013). Thus, to further contribute to the effort of numerous researchers working on widening the bandwidth of operation of energy harvesters, a contemporary design utilizing both the transverse and the parametric vibrations to enhance the bandwidth of a PEH system has been presented in this chapter.

In most of the designs stated in the literature, the magnetic forces were induced into the PEH system by using tip magnets and end magnets (Ahmadian *et al.*, 2009; Andò *et al.*, 2012; Cottone *et al.*, 2009; Daqaq, 2010; Erturk *et al.*, 2009a; Erturk and Inman, 2011a; Formosa *et al.*, 2009; Lin *et al.*, 2010; Mann and Sims, 2009; Mansour *et al.*, 2010; McInnes *et al.*, 2008; Pellegrini *et al.*, 2013; Ramlan *et al.*, 2010; Stanton *et al.*, 2009; Tang and Yang, 2012a; Tang *et al.*, 2012; Tang *et al.*, 2011; Vocca *et al.*, 2012; Wickenheiser and Garcia, 2010; Yang *et al.*, 2011). Often the end magnet is fixed so that the vibrating tip magnet induces nonlinearity into the system. One major drawback of this design is that the potential energy curves are constant, and the effect of a dynamic potential energy curve is rather unexplored. Thus, to induce dynamic variation of the magnetic nonlinearities in the PEH system a new design is proposed. This design consists of a simple cantilever flexural member (main beam) undergoing transverse vibrations and a secondary cantilever flexural member (auxiliary beam) undergoing parametric vibration, the auxiliary beam subjects the system to an off-resonance peak thereby inducing a dynamic variation of the potential energy curves. This modification of the potential wells and barriers is explored with the help of an experimental investigation across two substrate materials. Both the sets of beams were subjected to base excitation levels of 0.1g and 0.2g, and an enhanced bandwidth of operation was observed for both the substrate materials.

6.2. Transverse and parametric vibrations

A brief overview of transverse and parametric vibrations has been discussed in this section. When an external excitation is applied out of the plane of bending for a flexural member, the member is subjected to a transverse vibration. Similarly, when an external excitation is applied along the axial direction of a flexural member, the member undergoes transverse instability at a frequency twice its modal frequencies and this is known as the principle parametric resonance; this was first reported by Faraday in 1831 and investigated by many researchers after that (Daqaq *et al.*, 2009; Erturk *et al.*, 2009b; Jia and Seshia, 2014). Figure 6.1 shows the transverse and the parametric vibrations of a simple cantilever beam.

Figure 6.1 Layout of a cantilever beam subjected to (a) transverse vibrations and, (b) parametric vibrations.

6.2.1 Transverse vibrations

Transverse vibrations of a PEH beam have been studied in detail over the years. A detailed analytical formulation for the behavior of a PEH beam subjected to transverse vibrations in the presence of magnetic induced nonlinearity has been presented in Chapter 4. Magnetic induced nonlinearity generates a force which is represented as point force acting at the tip of the PEH beam. The magnetic force comprises an axial (F_Z) and a transverse component (F_Y), both the forces contribute to the nonlinearity of the response depending on the properties of the PEH beam. A

simplified lumped parameter formulation for the transverse vibration of a PEH beam subjected to a harmonic base excitation is expressed using Equations 6.1 – 6.7. The transverse component of the magnetic force F_Y acts with the external forcing term and the axial component is incorporated in the boundary conditions.

$$M_{eq} \frac{d^2 \eta(t)}{dt^2} + C_{eq} \frac{d\eta(t)}{dt} + K_{eq} \eta(t) + \Theta V(t) = F(t) \quad (6.1)$$

$$\frac{V(t)}{R_L} + C^P \frac{dV(t)}{dt} - \Theta \frac{d\eta(t)}{dt} = 0 \quad (6.2)$$

Where

$$M_{eq} = \int_0^{L_C} m_C(z) \phi_C(z)^2 dz + \int_{L_C}^L m_S(z) \phi_S(z)^2 dz + M_T \phi(L)^2 \quad (6.3)$$

$$K_{eq} = \int_0^{L_C} (EI)_C(z) \left(\frac{d^2 \phi_C(z)}{dz^2} \right)^2 dz + \int_{L_C}^L (EI)_S(z) \left(\frac{d^2 \phi_S(z)}{dz^2} \right)^2 dz \quad (6.4)$$

$$\Theta = - \int_{V_P} y \left(\frac{d^3 \phi_C(z)}{dz^3} \right) e_{31} \psi(y) dV_P \quad (6.5)$$

$$C_{eq} = \alpha M_{eq} + \beta K_{eq} \quad (6.6)$$

$$F(t) = - \left(M_T \phi(L) + \int_0^{L_C} m_C(z) \phi_S(z) dz + \int_{L_C}^L m_S(z) \phi_S(z) dz \right) \frac{d^2 \eta_0(t)}{dt^2} - F_Y(t) \quad (6.7)$$

The subscripts S , C , and P signify the substrate, the composite, and the piezoelectric part of the PEH beam, respectively. The composite part is comparable to the one reported in the literature, where the MFC is considered as a single composite layer attached to the substrate; z is measured along the length of the beam varying from zero to L ; and y is the distance from the neutral axis. e_{31} is the piezoelectric stress constant and $\psi(y)$ represents the constant electric field across the volume of the piezoelectric material V_P (Sodano *et al.*, 2004b), and C^P is the overall capacitance of the piezoelectric transducer. M_T is the tip mass attached at the end of the PEH beam. The forcing function $F(t)$ is dependent on the external excitation and the transverse component of the magnetic force. α and β are the equivalent viscous damping parameters; in this case, the parameters can be approximated as $\alpha = 0$ and $\beta = \zeta/\pi f_0$. Furthermore, I_T is the moment of inertia of the tip mass; ζ is the damping ratio

measured at frequency f_0 in the immediate neighborhood of the natural frequency ($f_0 \leq \pm 0.05\omega_n$); and ω_n is the natural frequency of the beam.

In the above equations, M_{eq} , K_{eq} , C_{eq} and Θ are the equivalent mass, stiffness, damping, and electromechanical coupling parameters, respectively. The equations are solved for voltage $V(t)$ across the resistor R_L and time component of the displacement $\eta(t)$. For a single mode approximation problem, the total displacement is given by $w(z,t) = \eta(t)\phi(x)$, where $\phi(x)$ is the fundamental mode shape of PEH beam which is mass normalized to unity corresponding to the fundamental natural frequency ω_n .

$$\phi(z) = \begin{cases} \phi_C(z) & 0 \leq z \leq L_C \\ \phi_S(z) & L_C < z \leq L \end{cases} \quad (6.8)$$

The mode shapes are represented as a combination of trigonometric and hyperbolic functions

$$\begin{aligned} \phi_C(z) &= A_1 \cos \lambda_C z + B_1 \sin \lambda_C z + C_1 \cosh \lambda_C z + D_1 \sinh \lambda_C z \\ \phi_S(z) &= A_2 \cos \lambda_S z + B_2 \sin \lambda_S z + C_2 \cosh \lambda_S z + D_2 \sinh \lambda_S z \end{aligned} \quad (6.9)$$

The non-trivial mode shape values of ϕ_C and ϕ_S , are determined by substituting the values of λ_C and λ_S which are, in turn, obtained using the geometric and natural boundary conditions given below

$$\begin{aligned} \phi_C(0) &= 0; \frac{d\phi_C}{dz}(0) = 0; \\ \phi_C(L_C) &= \phi_S(L_C); \frac{d\phi_C}{dz}(L_C) = \frac{d\phi_S}{dz}(L_C); \\ (EI)_C \frac{d^2\phi_C}{dz^2}(L_C) - (EI)_S \frac{d^2\phi_S}{dz^2}(L_C) &= 0, \\ (EI)_C \frac{d^3\phi_C}{dz^3}(L_C) - (EI)_S \frac{d^3\phi_S}{dz^3}(L_C) &= 0, \\ (EI)_S \frac{d^2\phi_S}{dz^2}(L) - \omega_n^2 I_t \frac{d\phi_S}{dz}(L) &= 0, \\ (EI)_S \frac{d^3\phi_S}{dz^3}(L) + \omega_n^2 M_t \phi_S(L) - F_z \frac{d\phi_S}{dz}(L) &= 0 \end{aligned} \quad (6.10)$$

In the above equations, the value of EI varies along the length of the beam depending on the cross section. The height of the neutral axis and the corresponding EI for a composite section have been represented in Equations 5.7 – 5.8. These equations were derived considering a composite beam consisting of the piezoelectric layer (subscript P) attached to the substrate (subscript S) with an epoxy-glue layer (subscript G). The bending stress for any cross-section at its neutral axis is zero, and the expression for the depth of the neutral axis was obtained using this fundamental property.

Figure 6.2 Schematic representation of (a) beam undergoing transverse vibrations in the presence of nonlinear magnetic forces and, (b) cross-section of two cylindrical permanent magnets.

$$F_Z(a, r, q) = \varepsilon \mu_0 M_1 M_2 \pi R_1 R_2 \int_0^\infty J_0\left(\frac{rq}{R_2}\right) \frac{J_1\left(\frac{R_1}{R_2} q\right)}{q} J_1(q) U(a, q) dq \quad (6.11)$$

$$F_Y(a, r, q) = -\varepsilon \mu_0 M_1 M_2 \pi R_1 R_2 \int_0^\infty J_1\left(\frac{rq}{R_2}\right) \frac{J_1\left(\frac{R_1}{R_2} q\right)}{q} J_1(q) U(a, q) dq \quad (6.12)$$

Where,

$$U(a, q) = \left[e^{\left(-2d_2 - d_1 \frac{q}{R_2}\right)} + e^{\left(-2d_1 - d_1 \frac{q}{R_2}\right)} - e^{\left(-2d_1 + 2d_2 + d_1 \frac{q}{R_2}\right)} - e^{\left(-d_1 \frac{q}{R_2}\right)} \right] \quad (6.13)$$

In the above equations, F_Z and F_Y are the axial and transverse forces between magnets 1 and 2 as shown in Figure 6.2, μ_0 is the permeability of vacuum, M_1 and M_2 are the magnetizations of magnets 1 and 2 respectively, R_i is the radius of the cylindrical magnets, J_0 and J_1 are the modified Bessel functions of the first type of order 0 and 1 respectively, r is the transverse separation between the magnetic axes along Y-direction, a is the axial separation between the ends of the magnets along Z-direction as shown in Figure 6.2, the Bessel functions in q are used to define the shape of the magnets, t_i is the height of the cylindrical magnets and $d_i = t_i/2$.

In the absence of magnetic nonlinearity, the system behaves as a linear PEH beam. The behavior of a PEH system is governed by the potential energy of the system. The equivalent beam stiffness plays a vital role in determining the potential energy of the configuration and the corresponding potential wells and barriers.

6.2.2 Parametric vibrations

Various researchers explored the feasibility of the usage of parametric vibrations for PEH (Abdelkefi *et al.*, 2012; Daqaq *et al.*, 2009; Jia and Seshia, 2014; Jia *et al.*, 2013). The usage of parametric vibrations for PEH is not as widely popular as transverse vibrations, because of the narrow bandwidth of operation. The parametric

vibrations are a special case of nonlinear vibrations where the system response is limited to the neighborhood of natural harmonics and fall to the trivial zero response state at other frequencies. The analysis of parametric systems is usually performed using multiple scale or harmonic balance methods which provide the steady state solutions for the system (Nayfeh, 2008; Nayfeh and Pai, 2008). Abdelkefi *et al.* (2012) developed a reduced order model for a PEH beam derived based on the energy formulation and Galerkin procedure. Daqaq *et al.* (2009) utilized the lumped parameter and second order multiple scale analysis for deriving the response of a PEH beam. Based on these the lumped parameter formulation, the equations of motion for a PEH beam subjected to parametric excitation are shown in Equations 6.14 and 6.15.

$$M_{eq} \frac{d^2\eta(t)}{dt^2} + 2M_{eq}\mu_1 \frac{d\eta(t)}{dt} + K_{eq}\eta(t) + M_{eq}\mu_2 \left| \frac{d\eta(t)}{dt} \right| \frac{d\eta(t)}{dt} + M_{eq}\alpha_b\eta(t)^3 + 2M_{eq}\beta_b \left(\eta(t)^2 \frac{d^2\eta(t)}{dt^2} + \eta(t) \left(\frac{d\eta(t)}{dt} \right)^2 \right) + \Theta V(t) = \eta(t)F(t) \quad (6.14)$$

$$\frac{V(t)}{R_L} + C^P \frac{dV(t)}{dt} - \Theta \frac{d\eta(t)}{dt} = 0 \quad (6.15)$$

In the above equations, M_{eq} , K_{eq} , C_{eq} and Θ are the equivalent mass, stiffness, damping and electromechanical coupling parameters, respectively. The equivalent parameters are same as shown in the previous section. μ_1 and μ_2 represent the viscous and quadratic damping terms respectively. The terms consisting of α_b and β_b depict the nonlinearities resulting due to the geometry and inertia respectively. Even in the absence of these higher degree nonlinearities, the presence of $\eta(t)$ along with the forcing function results in parametric instability of the system (Daqaq *et al.*, 2009). The steady state solutions to the above equations are obtained by employing the multiple scale analysis or harmonic balance method. Consequently, the corresponding steady state output voltage at a given frequency of base excitation can be obtained.

6.3. Potential energy distribution of a combined system

The potential energy of a system of magnets can be expressed using many different formulations, the merits and demerits of various formulations have been discussed in detail in Chapter 3. For the present PEH system, the enhanced integral formulation was used. The potential energy of the system is represented using Equations 6.16 – 6.18.

$$E_{i,l-2}(a, r, q) = \varepsilon \mu_0 M_l M_2 \pi R_l R_2^2 \int_0^\infty J_0\left(\frac{rq}{R_2}\right) \frac{J_1\left(\frac{R_l}{R_2} q\right)}{q^2} J_1(q) U(a, q) dq$$

$$\text{Where, } U(a, q) = \left[e^{\left(+2d_2 - d_l \frac{q}{R_2}\right)} + e^{\left(-2d_l - d_l \frac{q}{R_2}\right)} - e^{\left(-2d_l + 2d_2 + d_l \frac{q}{R_2}\right)} - e^{\left(+d_l \frac{q}{R_2}\right)} \right]$$
(6.16)

$$E_{i,Total} = \sum_l^n \sum_{n+l}^m E_{i,l-m}$$
(6.17)

$$F_z(a, r, q) = - \frac{\partial E(a, r, q)}{\partial z},$$

$$F_y(a, r, q) = - \frac{\partial E(a, r, q)}{\partial r}$$
(6.18)

Here $E_{i,l-2}$ is the interaction energy between the magnets 1 and 2, μ_0 is the permeability of vacuum, M_l and M_2 are the magnetizations of magnets 1 and 2 respectively, R_i is the radius of the cylindrical magnets, J_0 and J_1 are the modified Bessel functions of the first type of order 0 and 1 respectively. r is the transverse separation between the magnetic axes along Y-direction, a is the axial separation between the ends of the magnets along Z-direction as shown in Figure 6.2. The Bessel functions in q are used to define the shape of the magnets and t_i ($t_i = 2d_i$) is the height of the cylindrical magnets. From elementary mechanics, the force vector can be obtained by deriving the gradient of the interaction energy.

As discussed in earlier chapters, the interaction between a system of more than two magnets can be expressed as a linear summation of the magnetic potential energy. Thus, the potential energy of magnetostatic interaction is expressed using Equation 6.17. Where, $E_{i,Total}$ is the total interaction potential energy and $E_{i,l-m}$ is the interaction energy between l^{th} and m^{th} magnet.

The total energy of the PEH system is represented as the summation of potential energy, kinetic energy, damping energy, and the external work done as shown in Equation 6.19 (Meirovitch, 2010). The potential energy due to the internal strain energy is represented in terms of the equivalent stiffness of the beam given by Equation 6.20. The presence of magnets induces a nonlinearity into the system changing the potential energy. Hence the total potential energy of a cantilever PEH system in the presence of magnets is given by Equation 6.21.

$$E_{Total} = E_{Potential} + E_{Kinetic} + E_{Damping} + E_{External} \quad (6.19)$$

$$E_{Potential,PEH} = \frac{1}{2} K_{eq} y^2(t) \quad (6.20)$$

$$E_{Potential,M} = E_{Potential,PEH} + E_{i,Total} \quad (6.21)$$

In the above equations, E_{Total} is the total energy of the PEH system, $E_{Potential}$ is the total potential energy of the PEH system, and $E_{Potential,M}$ is the total potential energy of the PEH system in the presence of magnets. $E_{Kinetic}$ is the kinetic energy resulting from the inertial excitation of the system mass, $E_{Damping}$ consists of the damping energy, $E_{External}$ consists of the work done by the external forces, $E_{Potential,PEH}$ is the potential energy due to stiffness of the spring element or equivalently stiffness of the beam, $y(t)$ is the displacement produced at the tip of the PEH system due to the external excitation and $E_{i,Total}$ is the total magnetic interaction potential energy due to the presence of the magnets.

The PEH system presented in this chapter consists of two cantilever PEH beams with tip masses consisting of embedded magnets. The interaction between the magnets contributes to the potential energy of the whole system. To understand the effect of a translating end magnet, the PEH system is classified into three simple configurations as shown in Figure 6.3. Case-I exhibits a situation when the magnet on the auxiliary beam is static and the main beam vibrates under the external base excitation, this case is analogous to the vibration of a PEH beam discussed in Chapters 4 and 5. Case-II represents an extreme scenario where both the beams vibrate in sync with each other. Finally, case-III depicts the other extreme scenario where the vibration of both the beams is completely out of sync. These three cases

illustrate the extreme envelopes for the distribution of potential energy of the system. Thus, any other configuration is likely to fall within these enveloping curves for potential energy. These potential energy curves were obtained for the beams with aluminum substrate having equivalent stiffness values of 104.7 N/m and 10.42 N/m respectively, and the magnets were assumed to be placed 10 mm apart.

(a)

(b)

(c)

(d)

(e)

Figure 6.3 Schematic representation of the proposed system with (a) Case-I: Auxiliary beam is fixed, (b) Case-II: Main beam and auxiliary beam are vibrating in sync, (c) Case-III: Main beam and auxiliary beam are vibrating out of sync, (d) curves representing the potential energy distribution with the deflection for different cases, and (e) 3-dimensional representation of the potential energy curves for the main beam.

The above discussion and the potential energy curves shown in Figure 6.3 provide an evidence that the presence of parametrically vibrating auxiliary beam causes a change in the potential energy of the main beam and vice versa. Thus, the magnetic nonlinearity is induced into the system. Moreover, the variation of potential energy results in widening of the operation bandwidth of the output voltage and power. This has been investigated experimentally as discussed in the following section.

6.4. Experimental investigation of two coupled cantilever PEH beams

The conceptualized PEH system was subjected to harmonic base excitation and the output voltage and power were monitored. A similar experimental setup as

discussed in the previous chapters was maintained in the present study. The major difference being the addition of auxiliary beam to induce the vibration of end magnets.

6.4.1 Experimental setup

The experimental setup consisted of two beams placed perpendicular to each other as shown in Figure 6.4, the main beam was placed perpendicular to the direction of excitation and underwent transverse vibrations; the auxiliary beam was placed parallel to the direction of excitation thus it underwent parametric vibrations. The beams were prepared by attaching MFC patches to the root of the beams. A total of two different substrate materials (aluminum and fiberglass) were used in the experiments. The corresponding properties of the beams are listed in Table 6.1. The material properties are same those shown in the previous chapter. The output voltage for each beam was monitored using National Instruments (NI) Voltage DAQ system having an internal resistance of 1M Ω . The output power was calculated based on these voltage values. The magnets were attached or embedded with the tip masses, thus they served the dual purpose of tuning the natural frequencies of the beams and inducing magnetic nonlinearity. The beams were supported on a frame which was in turn fixed to the moving arm of the seismic shaker. The seismic shaker (APS 455) was attached with an accelerometer and connected to a shaker controller (VR 9500) which provided a controlled feedback system to maintain constant root mean square (RMS) acceleration. The feedback system was controlled from a computer using the Vibration View software which helped maintain the constant acceleration levels. The whole setup was subjected to base excitation levels of 0.1g and 0.2g in a frequency range of 5 – 40 Hz. The experimental setup for both linear and nonlinear cases are as shown in Figure 6.4.

(a)

(b)

(c)

Figure 6.4 Representation of the proposed PEH system with (a) layout of the experimental setup, (b) linear configuration of the experimental setup, and (c) nonlinear configuration of the experimental setup.

Table-6.1: Properties of the PEH beam

Property	Aluminum		Fiberglass	
	Main beam	Auxiliary beam	Main beam	Auxiliary beam
Beam size (mm x mm x mm)	90 ^a X 10 ^a X 0.7 ^b	90 ^a X 10 ^a X 0.3 ^b	90 ^a X 10 ^a X 0.8 ^b	90 ^a X 10 ^a X 0.8 ^b
Tip mass (g)	8.14	9.06	8.14	9.06
MFC	M2807-P2	M2807-P2	M2807-P2	M2807-P2
Tip magnet	NdFeB–N35, ϕ 6 mm X 2 mm	NdFeB–N42, ϕ 8 mm X 15 mm	NdFeB–N35, ϕ 6 mm X 2 mm	NdFeB–N42, ϕ 8 mm X 15 mm
Resonant frequency (Hz)	17.2	11.4	11.6	14.2
^a Due to the manual preparation of the sample, there can be an error of ± 1 mm				
^b An average of five measurement points was used to obtain the thickness				

6.4.2 Results and discussion

The importance of minimizing the strain levels in the beam has been stated in the previous chapter and a detailed analysis of the effect of strain on the PEH beams is provided in chapter 7. Thus, to minimize the fatigue damage in the PEH beams, the

excitation levels for the present PEH system were limited to 0.1g and 0.2g. Moreover, at lower frequencies of less than 10 Hz, an increased magnitude of excitation resulted in disproportionate transfer of the vibration energy to the experimental setup. This was due to the experimental frame undergoing resonance at lower frequencies thereby interfering with the response of the PEH beam. Thus, this precarious scenario was avoided by limiting the excitation levels and designing the PEH beams to resonate at frequencies greater than 10 Hz.

Harmonic base excitation

The proposed coupled two beam PEH system was investigated under increasing base excitation levels of 0.1g and 0.2g. The initial readings for the two PEH beams were recorded by placing them in a linear configuration as shown in Figure 6.5. The harvesters were simulated to have been connected in series. Thus, the output voltage from both the harvesters was added arithmetically. The variation of output voltage for both the harvesters in their linear configuration is shown in Figure 6.6. Thereafter, the experimental investigation for nonlinear configuration was conducted by placing the tip magnets at 10 mm and 15 mm apart. It was observed that the effect of nonlinearity was not so prominent when the tips of the PEH beams were placed 15 mm apart as depicted in Figure 6.6.

(a)

(b)

Figure 6.5 Variation of output voltage for a linear configuration of the beams when (a) both PEH beams are made of aluminum substrate, and (b) both PEH beams are made of fiberglass substrate.

The fundamental difference between the PEH beams with aluminum and fiberglass substrates was the occurrence of resonant frequency (peaks of output voltage shown in Figure 6.5). In the PEH beams with aluminum substrate, the auxiliary beam had a resonant frequency lower than that of the main beam. Furthermore, in the PEH system using fiberglass substrate the auxiliary beams had a resonant frequency greater than that of the main beam. This combination was achieved by varying the thickness of substrate material for the aluminum beams and maintaining a constant thickness for both the beams in the case of fiberglass substrate. Thus, not only are the beams investigated for varying stiffness levels but also the variation in occurrence of resonance frequencies. The output power for the PEH system is obtained using equation 6.22, where P_O is the output power, V_O is the output voltage, and R_L is the resistive load used in the experimental setup.

$$P_O = \frac{V_O^2}{R_L} \quad (6.22)$$

In addition to the voltage, power is also used to represent the response of the PEH system in this chapter. The internal impedance of the NI DAQ system (1 M Ω), was used as the resistance for all the experimental cases. The variation of power with frequency as shown in Figure 6.6 provided a better representation of the coupled

PEH system consisting of two beams. So far, the bandwidth of a PEH system was acquired at 100 μW level; as the fiberglass beams yield lower levels of output voltage or power, the quarter power bandwidth (bandwidth at the level of peak power/4) was also utilized to quantify the bandwidth of the PEH system. The bandwidth values obtained for the various cases are presented in Tables 6.2-6.3.

(a)

(b)

(c)

(d)

(e)

Figure 6.6 Variation of output power for nonlinear configuration ($a = 10$ mm) of the beams when (a) & (b) both PEH beams are made of aluminum substrate, (c) & (d) both PEH beams are made of fiberglass substrate, and (e) both the PEH beams are made of fiberglass substrate and the tip are placed 15 mm apart .

Figure 6.6 presents the variation of output power for the PEH system across both the substrates. It was observed that there was marginal increase in the peak output power for all the nonlinear cases. The repulsive configuration outperformed (bandwidth comparison) the attractive configuration for all the cases when an aluminum substrate was used. On the other hand, though the output for the power was marginally higher in the PEH beam with fiberglass substrate subjected to an attractive configuration, the repulsive configuration displayed a higher bandwidth. The combination of output power for the aluminum substrate demonstrated a system with two closely placed resonant frequencies which were brought much

closer by introducing nonlinearity into the system. On the other hand, the PEH system with fiberglass substrate stretched both the resonant frequencies away from each other. In agreement with the concept of effective strain transfer presented in the previous chapter, the aluminum PEH beams provided higher voltage output for the PEH system.

Table-6.2: Variation of Bandwidth at 100 μ W (10 V)

Material	Configuration	0.1g		0.2g	
		Att.*	Rep.*	Att.*	Rep.*
Aluminum	Linear	0.25		1.3	
	Nonlinear, 10 mm	-	1	-	5.5

Table-6.3: Variation of Bandwidth at quarter power (Peak power/4)

Material	Configuration	0.1g		0.2g	
		Att.*	Rep.*	Att.*	Rep.*
Aluminum	Linear	1.15		1.3	
	Nonlinear, 10 mm	-	2.65	-	5.5
Fiberglass	Linear	2.4		2.4	
	Nonlinear, 10 mm	2.4	2.7	2.7	2.7

From Tables 6.2 & 6.3 a maximum bandwidth enhancement of more than 300% was achieved at 0.2g base excitation level. One of the primary inference from the present design, is the ability of the proposed design involving vibration of both the tip and end magnets for a PEH beam to enhance the operational bandwidth considerably. This was achieved when the secondary resonance or the resonance of the auxiliary PEH beam was less than the resonance of the primary PEH beam. Though a valley in the curve for power variation was observed, it was not deep enough to hamper the bandwidth. Moreover, the attractive configurations seldom provided any improvement in the bandwidth. During the experimental investigation, the two PEH beams stuck to each other for the beams with aluminum substrate.

The fiberglass beams provided only a marginal increment in the bandwidth of operation for the PEH system at 0.2g base excitation level. Though the resonant peaks were stretched apart for the fiberglass beams, the system was inefficient in transferring enough energy to the piezoelectric transducers for conversion to an electric potential. This major flaw inhibits the usage of fiberglass as a conventional substrate for PEH systems.

Random base excitation

The ambient vibration sources available for field conditions are primarily random in nature, with the vibration energy distributed over a wide spectrum of frequency. The proposed PEH system was initially subjected to harmonic base excitation levels as discussed in the previous section; the PEH beams with aluminum substrate were further subjected to a random base excitation. Like the harmonic base excitation, the acceleration levels in this case were limited to RMS amplitudes of 0.1g and 0.2g. Both the linear and nonlinear configurations were comprehensively investigated. Additionally, the response of the main beam in the presence of a single end magnet placed 10mm away from the tip magnet was investigated under the random base excitation. This provided a comparative analysis for the proposed system.

In addition to the experimental setup shown in Section 6.4.1, the PEH system output was connected to a full wave rectifier and capacitor to obtain the energy stored over a period. The circuit configurations for both series and parallel setups are as shown in Figure 6.7. In the figure, the harvester is represented as a combination of battery and a capacitor (internal capacitance of piezo material) having capacitance values of $C_{P,M}$ and $C_{P,A}$ for the main beam and the auxiliary beam respectively. The harvesting unit is connected to a resistive load R_L and capacitor C_P through a rectifier. The energy stored across a capacitor was calculated using Equation 6.23. Where, E_O is the output energy stored across the capacitor having a capacitance of C_P and V_O is the voltage output through the circuit.

(a)

(b)

Figure 6.7 Circuit configuration for the PEH system (a) Series connection, and (b) Parallel connection of the component PEH beams.

$$E_o = \frac{I}{2} C_p V_o^2 \quad (6.23)$$

The output acceleration levels were recorded using an accelerometer attached to the frame of the harvester and the raw acceleration signal is as shown in Figure 6.8 (a). The target RMS acceleration level for the random excitation was maintained at 0.1

and 0.2g respectively using a vibration controller system. The voltage response of the PEH system for a linear and nonlinear configuration subjected to a base excitation of 0.2g is as shown in Figure 6.8 (b) – (g). The voltage response was recorded for 30 seconds and consequently converted to the frequency domain using fast Fourier transform (FFT) functions in MATLAB. The response was rectified and combined using parallel or series configurations.

(c)

(d)

(e)

(f)

(g)

Figure 6.8 (a) Acceleration level for the random base vibration; Time history voltage response of (b) Linear configuration (unrectified) (c) Linear configuration – Series connection, (d) Linear configuration – Parallel connection, (e) Nonlinear configuration (unrectified voltage), (f) Nonlinear configuration – Series connection, (g) Nonlinear configuration – Parallel connection

The response of all the above voltage plots in frequency domain were plotted as shown in Figure 6.9 (a) – (d), the plots obtained in the frequency domain provide distinctive peaks for the nonlinear cases. Moreover, the bandwidth for the nonlinear harvester is visibly enhanced. It is interesting to note that, the frequency domain response of the rectified values is obtained in a range of 20 – 40 Hz as against 10 – 20 Hz for the unrectified cases. This was because the FFT of a rectified signal provides amplitudes at even multiples of frequency components of the original signal along with DC offset values. The DC offset values for the experimental cases are 12.9 V, 15.37 V, 12.9 V and 13.11 V respectively for linear – parallel, linear – series, nonlinear – parallel, nonlinear – series configurations respectively.

(a)

(b)

(c)

(d)

Figure 6.9 (a) Acceleration level for the random base vibration; Time history voltage response of (b) Linear configuration (unrectified) (c) Linear configuration – Series connection, (d) Linear configuration – Parallel connection, (e) Nonlinear configuration (unrectified voltage), (f) Nonlinear configuration – Series connection, (g) Nonlinear configuration – Parallel connection

Further experiments were carried out to investigate the energy storage across a capacitor. All the configurations consisting of linear and nonlinear cases were subjected to base excitation levels of 0.1g and 0.2g, and the response signal was rectified and combined either in series or parallel before being connected to a capacitor of 510 μ F. The capacitor was charged for about 300s and the voltage was recorded continuously. This was performed three times to average out the response for calculating the energy across the capacitor.

(a)

(b)

(c)

(d)

Figure 6.10 Energy stored across a 510 μF capacitor for (a) Two PEH beams subjected to 0.1g, (b) Two PEH beams subjected to 0.2g, (c) One PEH beam subjected to 0.1g, and (d) One PEH beam subjected to 0.2g random base excitation.

The above plots represent the energy storage capacity of the PEH system proposed in this chapter. To perform a comparative analysis the main beam was tested

separately in the presence of an end magnet (10mm apart) in repulsive configuration. It is evident that for the random base excitation conditions, the contribution for energy storage from the auxiliary beam subjected to parametric vibrations is marginal. Though the bandwidth is visibly larger than the linear counterpart, the nonlinear PEH system doesn't outperform the linear PEH system. The likelihood of this behavior is due to the property of parametric vibrations, which contribute to the system performance in a very narrow range of frequency only. Moreover, the harvesting circuitry used in the experiments was elementary, a better harvesting circuitry might minimize the energy losses through the circuits enhancing the performance of the system. Furthermore, the attractive configuration outperformed the linear and repulsive cases for a single PEH beam under the experimental conditions. The attractive configuration could not be tested for the proposed 2 PEH beam configuration as it results in both the tip magnets attaching to each other straightaway.

6.5. Conclusion

The main objective of the current chapter was to demonstrate the ability of the proposed design to enhance the operational bandwidth of a PEH system. The proposed design consisted of two flexural members undergoing transverse and parametric vibrations respectively, and coupled together using magnets. This induced vibration of both tip and end magnets for the main cantilever PEH beam. Piezoelectric MFC transducers were attached to both the beams and the resonant frequencies of the beams complement each other with the added nonlinearity from the magnets to give an enhanced bandwidth of the output power distribution. A detailed experimental analysis was performed by testing two types of beam materials subjected to incremental levels of base excitation values. The proposed design resulted in an additional resonant peak for the response parameters due to the presence of the beam subjected to parametric resonance. These resonant peaks were observed to be on either side of the resonant peak for the main beam undergoing transverse vibration for PEH beams with aluminum and fiberglass substrates.

It was observed that the PEH beams with aluminum substrate produced higher levels of output voltage and output power due to their higher effective strain transfer values. Moreover, a maximum enhancement of operation bandwidth of over 300% was obtained for a repulsive configuration of the PEH beams with aluminum substrate. The attractive configuration was inefficient in providing any productive output. Furthermore, the aluminum beams out-performed the fiberglass beams for all the repulsive configurations. Additionally, the configuration with aluminum substrate was subjected to random base excitation. Though the bandwidth was visibly larger than the linear counterpart, the contribution to energy storage across a capacitor from the auxiliary beam proved out to be marginal.

It was inferred that the performance of a PEH system with two resonant frequencies is governed by the magnitude of base excitation, the strength of magnets used to induce nonlinearity and couple the system, the substrate material used for the PEH beams, and the deviation of the resonant frequencies of the PEH beams with respect to one another. Farther placement of the resonant frequencies induces deep valleys in between the peak responses, thereby, minimizing the bandwidth of operation of the system.

6.6. Summary

A contemporary design to enhance the bandwidth of operation of a PEH system consisting of two cantilever PEH beams has been presented in this chapter. The first section provides an outlook into the evolution of various designs for improving the bandwidth of PEH systems. An attempt has been made to overcome the inherent shortcomings and drawbacks of some designs in this chapter. The second section outlines the procedures that can be used to represent the proposed PEH system analytically. It is followed by the representation of distribution of potential energy for the current PEH system. A thorough understanding of the variation of the potential energy is required to optimize the proposed design. An experimental approach to demonstrate the functioning of the proposed design is presented in the fourth section. The conditions for enhancement of bandwidth of operation have also

been illustrated in this section. It is followed by a concluding portion for the present chapter.

CHAPTER 7 LONG TERM FATIGUE BEHAVIOR OF A CANTILEVER PIEZOELECTRIC ENERGY HARVESTER

The previous chapters concentrated on illustrating the functioning of a PEH system. Detailed analytical and experimental studies were undertaken to understand the functioning of linear and nonlinear PEH beams. In most of the work, nonlinearity was introduced with the help of magnets. The concepts of bandwidth and various designs to enhance the bandwidth of operation have been discussed in detail. Moreover, the relationship between bandwidth, effective strain transfer and stiffness has also been introduced in chapter 5. During some of the experimental investigations, the drastic effect of higher excitation levels was observed. It was concluded that higher excitation levels induce higher strain levels in the PEH beams thereby subjecting them to fatigue damage and reducing the usable life of the harvester. An attempt has been made in this chapter to explore the damage arising in PEH beams on being subjected to prolonged fatigue loading under harmonic base excitation. In addition, a detailed analysis of the damage and crack growth, necessary precautions to minimize the damage have also been examined in this chapter.

7.1. Introduction

Research in the field of piezoelectric energy harvesting (PEH) has grown by leaps and bounds in the past few years. Consequently, there has been an influx of commercial forms of piezoelectric energy harvesters. Although the usage of such harvesters is limited to a narrow bandwidth of operational frequency, these are, nevertheless, a first step in the realization of self-powered sensor systems. Most of these harvesters are based on a cantilever beam design, the importance and the popularity of cantilever based designs has been illustrated in the preceding chapters. In order to improve the performance and the operation bandwidth of the PEH beams, higher base excitation levels were often considered (Erturk and Inman, 2011b). Higher base excitation levels result in higher stress-strain at the root of the cantilever-type harvester, but higher levels of such reversal of stresses will also

result in potential fatigue failure of the PEH beam which affects the longevity of the harvester system.

To counter these drawbacks, considerable efforts in optimizing the design of energy harvesters on the basis of strain and material strength considerations were performed (Kim *et al.*, 2015; Shafer *et al.*, 2012; Upadrashta and Yang, 2015). The studies concluded that the material strength and the electromechanical coupling parameters of the transducers are greatly affected by cyclic electrical and mechanical loading and varying temperatures. Yet they do not address the fatigue issues directly. As discussed in Chapter 2, there are few studies which solely addressed the fatigue issues of PEH beams utilizing the P2 type MFC transducers. To fill this gap, a comprehensive experimental study was performed to investigate the fatigue behavior of a PEH beam subjected to varying levels of base excitation from 0.4g to 0.6g. A total of three beams were tested till complete fatigue failure of the substrate, and the variation of voltage and the damage to the P2-type MFC with increasing number of fatigue cycles have been presented in the following sections.

The initial parameters of the PEH beams have been investigated with the help of detailed FEA models. The detailed 3D model shown in Figure 7.1 was analyzed and the experimental conditions for different acceleration levels were matched with a good level of accuracy. An attempt was made to understand the fatigue damage due to crack growth by refining the FEA models. The analysis of a 3D model for piezoelectric crack is computationally too expensive due to increased complexity. To mitigate this, the 3D model was simplified to a 2D model, and this model was used to investigate the behavior of the PEH beam after the formation of cracks in the piezoelectric domain. An indirect approach was employed to understand the effect of cracks in a PEH beam. This was done by inducing cracks at the highly-stressed regions of the piezoelectric material in the 2D model and a frequency domain analysis was performed to obtain the response of the damaged PEH beam. A lot of work has been done in the area of dynamic behavior of cracked conventional piezo materials (Wang and Han, 2007; Wang and Mai, 2007; Wang and Huang, 2002). Most of these works presented the effect of cracks on the behavior of PZT-4 material in a d_{33} configuration when it is used as an actuator by

inducing artificial cracks in the material and investigating the behavior mathematically using the first principles of solid mechanics for orthotropic piezoelectric materials. This work focuses on the application part of a cracked piezo material (PZT-5A) in d_{31} configuration which is primarily used for energy harvesting purposes, thus the emphasis is laid on the study of the consequent behavior of a cracked piezo material without probing deeper into the details on the mechanics of cracks.

Figure 7.1 (a) Schematic representation of the PEH beam and its constituent components, (b) Detailed representation of various layers of the PEH beam and MFC transducer.

7.2. Linear piezoelectric energy harvesting beam

The primary objective of this chapter is to study the behavior of three PEH beams subjected to long-term cyclic loading of a constant magnitude. Hence, the mechanics behind the working of a PEH beam have been summarized.

7.2.1 Analytical modeling

Most of the popularly used PEH beams were designed to work within their linear elastic limits. Nonetheless, from previous experience, it was observed that over a period, the PEH beams may lose their stiffness or the MFC transducers may undergo some damage. Shafer *et al* (2012) explored the limiting conditions for the utilization of a harvester beam from the ultimate strength considerations of the constituent materials. Upadrashta *et al* (2014) examined the material strength considerations and suggested that a limiting strain of $1000 \mu\epsilon$ for the MFC transducer would restrict it within the linear range. Though these studies have provided useful guidelines, it is unlikely that the field conditions would be uniform and any harvester designed based on such conditions would lead to a very conservative design. One of the very important aspects of design is the longevity and applicability; thus, a fatigue study would augment the basic design criteria and help in realizing the bigger goal of a more optimized design. As fatigue is a long-term phenomenon and it is exhaustive to explore all the possible conditions, a few important loading cases for the linear cantilever energy harvesters are considered in this chapter. The term linear is broadly used to imply that the behavior of the PEH beam is considered to be within the linear elastic range which has been limited to about 0.4g-0.5g of acceleration and no external nonlinearities are introduced into the harvester design (Stanton *et al.*, 2012). Thus, under these set of assumptions, the governing equations for a unimorph PEH beam were derived using the Hamilton

principle in chapter 4. These equations of motion are simplified for a linear PEH beam using lumped parameter formulation and shown as below.

$$M_{eq} \frac{d^2\eta(t)}{dt^2} + C_{eq} \frac{d\eta(t)}{dt} + K_{eq}\eta(t) + \Theta V(t) = F(t) \quad (7.1)$$

$$\frac{V(t)}{R_L} + C^P \frac{dV(t)}{dt} - \Theta \frac{d\eta(t)}{dt} = 0 \quad (7.2)$$

Where

$$M_{eq} = \int_0^{L_c} m_c(z) \phi_c(z)^2 dz + \int_{L_c}^L m_s(z) \phi_s(z)^2 dz + M_T \phi(L)^2 \quad (7.3)$$

$$K_{eq} = \int_0^{L_c} (EI)_c(z) \left(\frac{d^2\phi_c(z)}{dz^2} \right)^2 dz + \int_{L_c}^L (EI)_s(z) \left(\frac{d^2\phi_s(z)}{dz^2} \right)^2 dz \quad (7.4)$$

$$\Theta = - \int_{V_P} y \left(\frac{d^3\phi_c(z)}{dz^3} \right) e_3 \mathcal{M}(y) dV_P \quad (7.5)$$

$$C_{eq} = \alpha M_{eq} + \beta K_{eq} \quad (7.6)$$

$$F(t) = - \left(M_T \phi(L) + \int_0^{L_c} m_c(z) \phi_s(z) dz + \int_{L_c}^L m_s(z) \phi_s(z) dz \right) \frac{d^2\eta_0(t)}{dt^2} \quad (7.7)$$

The parameters represented in the above equations have been listed out in detail in the earlier chapters.

A vibrating beam has infinite number of modes, and it is practically impossible and unnecessary to solve the governing equations for all the modes. In the PEH applications, it is accepted that the fundamental mode of vibration approximately represents the behavior of a linear elastic system when subjected to an external periodic excitation with a frequency close to the system's first natural frequency. Thus, the properties of the system are lumped together, and a solution for a single mode approximation can be obtained. In the above equations, M_{eq} , K_{eq} , C_{eq} and Θ are the equivalent mass, stiffness, damping, and electromechanical coupling parameters, respectively. As discussed in the earlier chapters, the equations are solved for voltage $V(t)$ across the resistor R_L and time component of the displacement $\eta(t)$ for a single model approximation.

$$\phi(z) = \begin{cases} \phi_C(z) & 0 \leq z \leq L_C \\ \phi_S(z) & L_C < z \leq L \end{cases} \quad (7.8)$$

The mode shapes are represented as a combination of trigonometric and hyperbolic functions

$$\begin{aligned} \phi_C(z) &= A_1 \cos \lambda_C z + B_1 \sin \lambda_C z + C_1 \cosh \lambda_C z + D_1 \sinh \lambda_C z \\ \phi_S(z) &= A_2 \cos \lambda_S z + B_2 \sin \lambda_S z + C_2 \cosh \lambda_S z + D_2 \sinh \lambda_S z \end{aligned} \quad (7.9)$$

The non-trivial mode shape values of ϕ_C and ϕ_S , are determined by substituting the values of λ_C and λ_S which are, in turn, obtained using the geometric and natural boundary conditions given below

$$\begin{aligned} \phi_C(0) &= 0; \frac{d\phi_C}{dz}(0) = 0; \\ \phi_C(L_C) &= \phi_S(L_C); \frac{d\phi_C}{dz}(L_C) = \frac{d\phi_S}{dz}(L_C); \\ (EI)_C \frac{d^2\phi_C}{dz^2}(L_C) &- (EI)_S \frac{d^2\phi_S}{dz^2}(L_C) = 0, \\ (EI)_C \frac{d^3\phi_C}{dz^3}(L_C) &- (EI)_S \frac{d^3\phi_S}{dz^3}(L_C) = 0, \\ (EI)_S \frac{d^2\phi_S}{dz^2}(L) &- \omega_n^2 I_t \frac{d\phi_S}{dz}(L) = 0, \\ (EI)_S \frac{d^3\phi_S}{dz^3}(L) &+ \omega_n^2 M_t \phi_S(L) = 0 \end{aligned} \quad (7.10)$$

A total of three sets of PEH beams were prepared and tested under increasing periodic external excitations of 0.4g, 0.5g and 0.6g. Using the above governing equations, the variation of voltage across the National Instruments data acquisition system (NI-DAQ) voltage card having an internal resistance of 1 M Ω was obtained. The parameters for different beams are given in Table 7.1. As the output power is a function of voltage, and to minimize the complexity of obtaining the optimum resistance for the three different beams with passing time, voltage was deemed to be a more amiable parameter for the experiments. The longitudinal strain in the vibrating PEH beam can be calculated based on the fundamental principles of piezoelectricity. The maximum strain ε_{max} in a PEH beam occurs near the support and can be approximated analytically by Equation 5.6 and the detailed derivation is provided in Chapter 5.

The maximum strain in a PEH beam depends on the stiffness of that beam and the voltage obtained. Thus, the strain can be calculated using the tip displacement and the output voltage, and it slowly reaches a steady state as the voltage, and tip displacements reach their respective steady states. Over a period, the stiffness of the system changes and the loss of stiffness results in the fall in natural frequency of the system. This will be addressed in the forthcoming sections with the help of experimental investigations and FEA of a cracked PEH beam.

Table-7.1: Properties of the PEH beam

Property	Beam-0.4g base excitation	Beam-0.5g base excitation	Beam-0.6g base excitation
Beam size (mm x mm)	90 ^a X 10 ^a	90 ^a X 10 ^a	90 ^a X 10 ^a
Total thickness (mm)	1.2 ^b	1.075 ^b	1.175 ^b
Tip mass (g)	4.42	4.42	4.42
Length of glue (mm)	35	37	37
Damping coefficient	0.02896	0.02680	0.02615
Equivalent mass (g)	4.747	4.745	4.753
Equivalent stiffness (N/m)	130.4	139.1	131.3
Equivalent damping (Ns/m)	0.04522	0.04357	0.04079
Coupling coefficient	-1.912 x 10 ⁻⁴	-2.093 x 10 ⁻⁴	-2.065 x 10 ⁻⁴
Capacitance (F)	12.4 x 10 ⁻⁹	12.4 x 10 ⁻⁹	12.4 x 10 ⁻⁹
Resistance (Ω)	1 x 10 ⁶	1 x 10 ⁶	1 x 10 ⁶
^a Due to the manual preparation of the sample, there can be an error of ±1 mm			
^b An average of five measurement points was used to obtain the total thickness			

7.2.2 Finite element formulation

Though the analytical formulation of a linear PEH beam provides a good approximation of the behavior of a vibrating harvester beam, the inherent assumptions involved bring out a few drawbacks. A few of these drawbacks have been addressed with the help of FEA (Arafa *et al.*, 2011; Upadrashta and Yang, 2015; Yang and Tang, 2009) and broadly discussed in the previous section. COMSOL Multiphysics provides a simple yet powerful and user-friendly interface for solving various problems; thus, a detailed FEA model was developed and analyzed using this software. The schematic representation of the PEH beam and the constituent layers is presented in Figure 7.1 (Similar schematic representations are shown in Figures 4.12 and 4.13, these have been repeated to provide comprehensibility and completeness to the section). The MFC was modeled in its

layered composition, with the central piezoelectric layer consisting of 17 rectangular PZT-5A fibers spread along the length and the poling direction being along the thickness. The outer epoxy layers constitute the interdigitated copper electrodes, the kapton layer forms the outer most protective layers, and the whole composite is attached to the aluminum substrate using a high strength epoxy. The analytical modeling of the whole composite is challenging since it consists of multiple layers with different sizes and orientations; thus, an FEA model is constituted. The whole composite is meshed using 27-node second order brick elements. The brick elements were preferred over the tetrahedral elements as the former gives more flexibility in determining the strain levels (and higher derivatives of displacement) to a greater accuracy. COMSOL provides the necessary interface to describe the materials and their properties. In this study, the predefined PZT-5A was used directly from the COMSOL materials library, and the material properties for the epoxy layers and the Kapton layers were defined based on the values given in Table 7.2. The entire composite beam is meshed into about 16,000 elements (e.g., for 0.5g base excitation – 3D elements: 16024; 2D elements: 3193) having about 450,000 degrees of freedom (DOFs) (e.g., for 0.5g base excitation – 3D DOFs: 433916; 2D DOFs: 66992). The meshed configurations in both 3D and 2D are presented in Figure 7.2.

(a)

Figure 7.2 Representation of (a) 3D FEA model of PEH beam and (b) 2D FEA model of PEH beam.

In FEA, the accuracy and the time taken for the solution depend directly on the total DOFs of the model. The two copper electrode layers are defined as electrical terminals. One layer is designated as ground, and the other is connected to an electrical circuit consisting of a resistor element corresponding to the internal resistance of the voltage DAQ system to complete the circuit. Damping was introduced into the PEH beam using the Rayleigh damping formulation, and the values for α and β were entered accordingly. The external excitation was defined as an equivalent periodic body load having amplitude corresponding to the experimental excitation levels (0.4g, 0.5g and 0.6g). The steady-state frequency-domain analysis was performed in a frequency range of 20–35 Hz. The whole system was solved using the direct solvers on a workstation having four core i7 processors with a maximum speed of 3.4 GHz and 16 GB of ram, and the resultant output voltage versus frequency was stored in the solution file.

Table-7.2: Material properties of constituents of PEH beam (ASM Inc., 2015; Steiger and Mokry, 2015)

Material	Young's Modulus 'Pa'	Poisson's ratio	Density 'kg/m ³ '
Aluminum Substrate	72x10 ⁹	0.35	2700
Epoxy	2x10 ⁷	0.3	1300
Kapton	2x10 ⁷	0.34	1420
Tip mass	160x10 ⁹	0.24	7500
Copper	120x10 ⁹	0.34	8960
	Compliance Matrix	Coupling Matrix	Relative

	‘Pa ⁻¹ ’		‘C/N’		permittivity	
PZT-5A	$s_{11}=s_{22}$	1.64×10^{-11}	$d_{15}=d_{24}$	5.84×10^{-10}	ϵ_{11}	1730
	s_{33}	1.88×10^{-11}	$d_{31}=d_{32}$	-1.71×10^{-10}	ϵ_{22}	1730
	$s_{13}=s_{23}$	-7.22×10^{-12}	d_{33}	3.74×10^{-10}	ϵ_{33}	1700
	s_{12}	-5.74×10^{-12}				
	$s_{44}=s_{55}$	4.75×10^{-11}				
	s_{66}	4.43×10^{-11}				

The 3D model closely depicts the experimental conditions but takes a long time to solve (about 24 h for 75 frequency points in frequency domain and about 60-70 h for 5 s with up to 20 steps per cycle in time domain). To reduce the computational cost, the 3D model was converted into 2D without sacrificing the accuracy of modeling as explained earlier. As shown in Figure 7.2, the 2D model represents the mid plane of the 3D PEH beam with a few inherent assumptions. One of the important assumptions is that the piezoelectric layer and epoxy layer spread uniformly across the width of the PEH beam. These assumptions induce rigidity into the system, and consequently, the frequency and voltage values deviate slightly from their original values. The key idea behind the FEA formulation is to initially provide a detailed model replicating the experimental conditions to the maximum possible extent and consequently to dwell deeper into the effect of cracks on the piezoelectric surface. This will be qualitatively addressed in the section on crack formulation where the effect of formation of cracks in the piezo layer will be investigated using a 2D FEA formulation. The 3D formulation serves the primary purpose and provides a way for exhaustive parametric studies, and the 2D formulation provides a good approximation for the PEH beam system when subjected to damage in the form of cracks in the piezo layer.

7.3. Experimental investigation

An analytical and FEA overview of the cantilever PEH beam system was presented in the previous section. An experimental investigation to understand the fatigue damage in a PEH beam has been explored in this section. Moreover, the MFC transducer was utilized as a damage monitoring sensor in addition to its primary purpose of harvesting energy.

7.3.1 Experimental setup

The experimental setup as shown in Figure 7.3 consisted of a cantilever PEH beam attached to the arm of a seismic shaker. The seismic shaker LDS-V455 was attached with an accelerometer and connected to a shaker controller (VR 9500) which provided a controlled feedback system to maintain constant root mean square (RMS) acceleration. The feedback system was controlled from a computer using the Vibration View software which helped maintain the constant acceleration levels. The cantilever PEH beam was prepared by attaching a d_{31} MFC transducer (M2807P2, Smart Material Corp) to an aircraft grade aluminum substrate (Al-7075 T6). A high strength epoxy was used to bond the transducer on the substrate. Electrical wires were soldered to the electrodes of the transducer and the whole system was connected to a national instruments data acquisition system (NI-DAQ). The output voltage was measured using the NI-9229 voltage DAQ card with an internal impedance of 1 M Ω . The tip displacement of the beam was monitored intermittently with the Polytec (Polytec PT GmbH Polymere Technologien, Germany) Laser Vibrometer (OFV-5250) and the data was logged using the NI-9215 voltage or acceleration DAQ card. As the experimental work targeted at evaluating the behavior of the harvester when subjected to a long-term cyclic fatigue loading, the output voltage (V_O) was used as the indicative parameter for the performance of harvester. Thus, no variable external resistance was used to find the optimal resistance to minimize the complexity of using power as the indicative parameter. Figure 7.3 shows a typical representation of the experimental setup. A detailed cross-section of the PEH beam is shown in Figure 7.1. The MFC transducer has an effective size of 28 mm in length, 7 mm in width and a total thickness of about 300 μm (Smart Material Corp., 2015). In the modeling of a representative volume element (RVE) of a MFC by Steiger and Mokry (2015), the PZT fiber width of 350 μm and thickness of 180 μm with a fiber fill percentage of 83% was considered, and a copper electrode – epoxy layer of 18 μm and a kapton layer of 40 μm were used in the derivation of an equivalent model. Similar values were used in the present study to model the MFC transducer. The material properties for MFC, substrate and epoxy layers for the PEH beam are shown in Table 7.2. A total of three identical beam specimens were prepared and the PEH beams were attached

with a tip mass of 4.4235 g to limit the resonant frequency within 50 Hz (50 Hz is considered as the limit for low frequency vibrations by Priya (2007) and Abdelkefi *et al* (2014)). To induce fatigue cycles, the PEH beams were excited at accelerations of 0.4g, 0.5g and 0.6g separately till failure occurred in the substrate of the beam. Figure 7.3 (c) shows a schematic representation of the whole experimental setup constituting the connections for the feedback mechanism and the data acquisition. To monitor the healthy state of the PEH beam, the shaker was paused periodically, and the admittance readings were acquired by connecting the MFC electrodes to a high precision impedance analyzer (B&K 6500B). The admittance signatures for the PEH beams were obtained in the frequency range of 40-200 kHz; the high frequency range was used in order to minimize the interference from ambient sources (Bhalla *et al.*, 2012; De Pasquale *et al.*, 2011; Giurgiutiu *et al.*, 1999; Sodano *et al.*, 2004b).

(a)

Figure 7.3 Representation of (a) experimental setup for fatigue test of PEH beam, (b) prepared samples for different excitation levels, (c) schematic representation of sensing and actuation modes of experimental procedure.

7.3.2 Measurement of voltage, resonant frequency and tip displacement

When the electrodes of the MFC were connected to the DAQ system and the shaker was turned on, it induces harmonic vibration at a pre-defined frequency. Once the resonant frequency of the corresponding PEH beam was determined experimentally, the PEH beam was subjected to a continued vibration at the resonant frequency. The output voltage of the PEH beam was logged continuously, and the beam was monitored for any shift in resonant frequency after every 2 million cycles. The process was continued till the PEH beam failed in fatigue. To inspect the required rate at which the shift in resonant frequency occurred, the PEH beam needed to be probed. A pilot experiment was first conducted where a PEH beam (Substrate: Al6061-T6, size: 90 mm X 10 mm X 1 mm, MFC: M2807P2, initial resonant frequency: 31.7 Hz) was subjected to 0.2g base acceleration at a constant frequency equal to the initial resonant frequency. It was observed that after a few million cycles, as shown in Figure 7.4 the voltage output from the beam dropped from its initial value. On physical inspection, it was deemed that no damage occurred in the MFC. Thus, the decrease voltage was associated with the shifting of the resonant frequency of the beam due to the softening caused by the loss of stiffness in the beam over a period (It can be observed from Figure 7.4 that a steep drop in the

output voltage occurred as we moved slightly away from the resonant conditions). Taking cue from the pilot experiment, a 2 million cycle interval was considered apt for the measurement of shift in the resonant frequency and the corresponding output voltage and tip displacements.

Figure 7.4 Variation of (a) Voltage Vs Frequency for the pilot PEH beam, and (b) voltage at a constant frequency for the pilot PEH beam.

Hence, the appropriate connections were made to complete the circuit, and the linear behavior of the PEH beams was documented before the beams were subjected to fatigue loading. Once the initial readings for voltage, frequency, and

displacement were chronicled, the beam was subjected to the desired excitation amplitudes. The variations of the voltage, frequency and tip displacement were recorded intermittently and plotted as shown in Figures 7.5 to 7.7. The variation of frequency with increasing number of cycles has been plotted in Figure 7.5. It was observed that at the final stage, the resonant frequency of all PEH beams dropped by about 2 Hz. The variation of voltage ratios for both positive and negative parts is portrayed in Figure 7.6. The damage induced by the increasing fatigue cycles affects the whole PEH beam system. The voltage, velocity, and displacement are a few of the directly observable parameters from the experiments. The representation of the variation of these curves in Figure 7.7 presents a preliminary understanding of the increase in velocity and displacement as the stiffness of the beam decreases. An overall diminishing trend in the voltage values indicates that with increasing fatigue cycles, the PEH beam underwent considerable damage especially in the MFC transducer.

As opposed to the popular use of power as a metric for harvester performance in most literature, voltage was used in the present work to monitor the damage in the PEH beam. The primary reason is that voltage is the simplest measure that can be continuously monitored, without the hassle of determining the optimal resistance from time to time to obtain the optimal power. The drawback of using voltage over power is the inability to monitor the change in internal capacitance of the MFC transducer with passing time. Given the nature of the experimental study, the extraction of voltage over power was preferred. Thus, the voltage was monitored along with the tip displacement and resonant frequency with increasing number of fatigue cycles.

(a)

(b)

(c)

Figure 7.5 Variation of frequency with increasing number of fatigue cycles for (a) 0.4g base excitation level, (b) 0.5g base excitation level, and (c) 0.6g base excitation level.

(a)

(b)

Figure 7.6 Variation of voltage ratio with frequency for the first 50000 cycles across different base excitation levels for (a) positive peak-peak values, and (b) negative peak-peak values.

(a)

(b)

(c)

(d)

(h)

(i)

(j)

Figure 7.7 Variation of (a)-(c) voltage ratio across increasing base excitation levels, (d)-(f) velocity of the tip of PEH beam, and (g)-(i) displacement of the tip of PEH beams with increasing fatigue cycles.

7.3.3 Electromechanical impedance readings

In the PEH applications, the MFC transducer converts the mechanical strain into voltage (direct piezoelectric effect), that is, acting as a sensor. In the SHM applications using the electromechanical impedance (EMI) technique, it acts as both an actuator and a sensor. When a piezoelectric transducer (PZT or MFC) is attached to the surface of a material, it can be used to detect damage in its vicinity using the

EMI technique. The converse piezoelectric effect comes to the fore in such cases, where the externally induced potential difference causes mechanical vibrations in the transducer. These vibrations are, in turn, transferred to the host material, and the response of the host material results in a variation in the electrical signals because of the electromechanical coupling. This variation can be captured using an impedance analyzer or an inductance (L)-capacitance (C)-resistance (R) (LCR) meter (Annamdas and Radhika, 2013; Bhalla *et al.*, 2012; Giurgiutiu *et al.*, 1999). When the host material is damaged, the relative change in signature of the damaged and undamaged states provides a quantification of damage. Bhalla *et al.* (2012) utilized this technique to monitor the damage in bolted steel joints when the whole structure was subjected to fatigue. In the EMI technique, the admittance values obtained from the impedance analyzer were processed to assess the damage in the structure using certain statistical method such as root mean square deviation (RMSD). Sodano *et al.* (2004b) introduced the use of MFC as an impedance transducer. MFC gives relatively lower values of magnitude of the real part of impedance in comparison to the PZT due to MFC's higher capacitance, but the main parameter which determines the loss of stiffness with damage, that is, frequency shift, varies at par in both cases for damaged and undamaged states.

This work focuses on the detection of damage in the PEH beam; thus, the elementary technique of RMSD variation in the real part of the impedance readings was utilized to monitor the PEH beam. The admittance function represents the compound behavior of the MFC and the substrate. The impedance function represented by Equation 7.12, which is the inverse of admittance function, has been popularly used in many SHM related studies. The impedance for the whole composite of PEH beam was obtained as shown in Figure 7.8. The real part of the impedance readings is more sensitive to damage; thus, the RMSD variation for the real part was obtained using Equation 7.13. Equation 7.14 represents the RMSD variation of the output voltage (vRMSD); the vRMSD is primarily utilized to provide a broader correlation between the impedance readings and the corresponding output voltage. In many RMSD studies involving PZT transducers, the RMSD reaches a maximum value (i.e. 100%) and remains constant as a slight damage to the PZT transducer can render it unusable. But, in the case of MFC

transducers, there exists slight variation in the impedance values even after the initial occurrences of damage, as the transducer retains some usability till eventual failure. The primary reason behind this behavior can be associated with the presence of numerous PZT fibers inside the MFC transducer. In general, the number of failed fibers increases with the number of cycles. To account for such incremental damage in the MFC transducer, the relative RMSD (rRMSD) given by Equation 7.15 is employed. The RMSD index provides the quantification of accumulative damage at a given state, whereas the secondary indices rRMSD and vRMSD provide the damage quantification at a given state with respect to its preceding state. The plots shown in Figures 7.9 and 7.10 display the variation of RMSD, vRMSD, and rRMSD values for the corresponding beam, and they are discussed in detail in the forthcoming section.

$$\bar{Y} = G + iB \quad (7.11)$$

$$\bar{Z} = X + iY = \bar{Y}^{-1} = (G + iB)^{-1} \quad (7.12)$$

$$RMSD(\%) = \sqrt{\frac{\sum_{k=1}^N (X_k^l - X_k^0)^2}{\sum_{k=1}^N (X_k^0)^2}} \times 100 \quad (7.13)$$

$$vRMSD(\%) = \sqrt{\frac{(V^l - V^0)^2}{(V^0)^2}} \times 100 \quad (7.14)$$

$$rRMSD(\%) = \sqrt{\frac{\sum_{k=1}^N (X_k^l - X_k^{l-1})^2}{\sum_{k=1}^N (X_k^{l-1})^2}} \times 100 \quad (7.15)$$

In the above equations, $\bar{Y}$ is the admittance obtained from the PEH beam; G and B are its real and imaginary parts, namely, conductance and susceptance, respectively; $\bar{Z}$ is the impedance of the PEH beam; and X and Y are the real and imaginary parts, namely, resistance and reactance, respectively; X_k^l is the value at the k^{th} measurement point out of the N measurement points at the l^{th} damaged state having a corresponding output voltage V^l ; and the k^{th} measurement point for the undamaged state is represented by X_k^0 having an output voltage of V^0 . X_k^{l-1} symbolizes the $(l-1)^{th}$

damage state for the k^{th} measurement point; $rRMSD(\%)$ is the relative variation of RMSD of impedance values of a damaged state with respect to its previous damaged state.

(c)

(d)

(e)

(f)

(g)

(h)

(i)

(j)

(k)

(l)

Figure 7.8 Impedance readings at various stages of the fatigue loading for different PEH beams. (a)&(b): Plots for real (Resistance) and imaginary (Reactance) parts of the impedance values for PEH beam undergoing 0.4g base excitation; (c)&(d): Plots for real and imaginary parts of the impedance values for PEH beam undergoing 0.5g base excitation; (e)&(f): Plots for real and imaginary parts of the impedance values for PEH beam undergoing 0.6g base excitation; (g)-(l): Plots for the first few fatigue cycles (fewer than 500k) showing a magnified range for real and imaginary parts of the impedance values for the PEH beams undergoing 0.4g, 0.5g, and 0.6g base excitation respectively.

(a)

(b)

(c)

(d)

(e)

(f)

Figure 7.9 Variation of RMSD with increasing fatigue cycles for (a)-(c) impedance values, and (d)-(f) voltage values across increasing base excitation levels.

(a)

(b)

(c)

Figure 7.10 Variation of rRMSD with increasing fatigue cycles for (a) 0.4g base excitation, (b) 0.5g base excitation, and (c) 0.6g base excitation levels.

7.4. Crack propagation

The propagation of cracks as observed in the experiment and the results from the FEA modeling are discussed in this section.

7.4.1 Crack behavior from experiments

The state of the MFC transducers deteriorated with increasing number of fatigue cycles. A pictorial illustration of the propagation of cracks in MFC with increasing cycles for the three PEH beams is shown in Figure 7.11. The maximum strain occurred at the top most layer of the MFC near the root of the PEH beam, as it was assumed that the MFC was attached exactly near the root of the beam. However, in practice the outer kapton layer extended beyond the active layers for enhanced protection, leaving some offset from the root of the beam and the active MFC layer. Therefore, the maximum strain occurred slightly away from the tip of the top most layers. In all the three sets of PEH beams, the crack patterns validated this hypothesis as can be seen from the chronological order of the cracks in the figure. The initial crack developed slightly away from the tip, followed by subsequent cracks along the length of the MFC. The output voltage fell considerably after the first crack, and the output signal was no more a uniform sinusoidal, as shown in Figure 7.12 (a). Only the negative part of the voltage remained sinusoidal indicating the voltage from the undamaged portion; even this declined slowly as the MFC deteriorated, and finally, only noise was observed, as shown in Figures 7.12 (b) and 12 (c). The cracks were observed intermittently using a Nikon microscope, and the loss in frequency and increase in displacement were logged at each intermittent pause.

(c)

Figure 7.11 Crack propagation with increasing number of fatigue cycles on subjecting the PEH beam to a base excitation level of (a) 0.4g, (b) 0.5g, and (c) 0.6g.

(a)

(b)

(c)

Figure 7.12 Recorded response of output voltage (a) initial damage, (b) final damage to the MFC transducer, and (c) at complete fatigue failure of the beam (x-axis: time, y-axis: magnitude of voltage).

7.4.2 Crack behavior from FEA

Very few studies in the literature are available on the modeling of cracked MFC transducers. In this chapter, the primary concern of cracked transducer is the orientation of the crack and the effect on the interdigitated layers. In this section, an attempt is made to understand the behavior of a cracked PEH beam using FEA. As

discussed in Section 7.2, the experimental conditions were replicated to a good degree of accuracy in 3D FEA model and the 2D model was used to understand the behavior of a cracked PEH beam. This was pursued by introducing a thin crevice in the MFC layer which penetrated into the bottom kapton layer (Wang and Mai, 2007), and leaving the substrate layer intact. The cracks were introduced at the point of maximum strain on the MFC surface. To minimize high stress concentration in the modeling, the bottom tip of the crack was shaped using a second-degree curve instead of a sharp triangular tip. A frequency domain analysis was performed to monitor the shift in resonant frequency with each crack. A total of 8-10 cracks were modeled, and the drop in resonant frequency of the PEH beam is shown in Figure 7.13 (b). The drop-in output voltage is evident after the initial few cracks where the variation of voltage has a negative slope with increasing damage (number of cracks; Figure 7.14). Moreover, due to the presence of idealized boundary conditions in the FEA, especially the electrical terminal connections, the output voltage (time response) obtained from the cracked PEH beam followed a sinusoidal wave. This is unlike the experimental case where the output voltage was a distorted sinusoidal wave after the initial damage, as shown in Figure 7.12 (a) and (b). This is probably due to the interaction between the damaged electrode layers where they might touch each other during the compression phase of the vibration or the complete disintegration of the electrode layer in the experimental PEH beams. Thus, leaving room for slight misconnection between the damaged and the undamaged parts of the MFC transducer. Furthermore, in the FEA model, the electrode layer is considered as a uniform copper layer attached on the epoxy layer, but in practice, the layer is interdigitated which may have given rise to the discrepancy in the FEA model.

Figure 7.13 Representation of (a) stress distribution in the PZT layer of an uncracked 2d FEA model, and (b) sequential induction of cracks in the MFC trasducer layer of the 2d FEA model.

Figure 7.14 Modeling results for variation of (a) voltage, and (b) resonant frequency, with increasing number of cracks in the respective PEH beams.

7.5. Results and discussion

The basic idea of the analytical, FEA and experimental procedures were presented in the preceding sections. A detailed discussion of all the results obtained and their importance for understanding the fatigue behavior of the PEH beams is presented in this section.

7.5.1 Output Voltage

Output voltage from the analytical, FEA and experimental studies has been used as a base parameter for analyzing the behavior of the PEH beams. The variation of voltage for a 0.1g base acceleration for all the PEH beams is shown in Figure 7.15. The plots reveal how the analytical, FEA and experimental values of voltage complement each other. Compared to the analytical model, the FEA model matches well with the experimental values for all the beams. This is evident in the plot for the PEH beam with a targeted base excitation of 0.6g, where the analytical formulation resulted in a plot which is offset from the experimental values by about 0.5 Hz. It is to be noted that, due to ideal boundary conditions and loading conditions, the peaks obtained from the analytical and FEA formulations are higher

than the experimental values. Nevertheless, the modeling results agree well with the experimental results, and the error in matching the frequencies at peak voltage values is well within 0.5-1 Hz.

The plot showing the variation of peak voltages for all the cases of analytical, FEA, and experimental values is of prime importance in understanding the limitations of the analytical and FEA studies. The frequency of the peak voltage for experimental cases shifted leftwards in all the cases, as shown in Figure 7.15 (d), and a slight leftward deviation in the experimental values was also observed as the base acceleration increases to 0.6g. This leftward shift is attributed to the softening of the beam or loss of stiffness due to non-ideal boundary conditions. One of the major contributors for the deviation is the nonlinearity which arises due to materials and geometry. The geometric nonlinearity can be accounted in the FEA formulation, but modeling of the material nonlinearities requires an in-depth investigation into the material properties of the various layers, which falls beyond the scope of this study. This study revolves around the assumption of a linear behavior for the PEH beams, which is pretty much valid for the experimental conditions for the targeted base accelerations of 0.4g and 0.5g, and agrees well with the literature (Stanton *et al.*, 2012).

To investigate the propagation of cracks in MFC transducer of the PEH beam using FEA, a 2D analysis was used to minimize the computational effort that a 3D analysis requires. A comparison of the voltage obtained for the frequency sweep for a PEH beam with a targeted base excitation of 0.5g is shown in Figure 7.15 (e). Due to the inherent assumptions involving a 2D FEA as stated in the previous sections, the PEH beam shows a more rigid behavior. Although the peak voltage and frequency at the peak voltage vary by about 15% and 1Hz, respectively, the overall behavior in both cases is identical as shown in the succeeding plot displaying the variation of voltage ratio with frequency ratio (Voltage ratio is the ratio of the voltage obtained at an observation point to the maximum voltage, and frequency ratio is the ratio of the frequency at an observation point to the frequency observed at the maximum voltage). Thus, the crack propagation analysis performed using the

2D FEA formulation can be used to understand the behavior of a damaged PEH beam.

(c)

(d)

Figure 7.15 (a-c) Plots showing the comparison of output voltages for analytical, FEA, and experimental formulations, (d) comparison of the variation of peak output voltages for the PEH beams across different formulations, (e) comparison of the frequency sweep for output voltage across the 3D and 2D FEA formulations for PEH beam undergoing 0.5g acceleration, and (f) comparison of the frequency sweep for 3D and 2D formulation using voltage ratio and frequency ratio.

7.5.2 Strain

The maximum strain in the PEH beam provides a deeper insight into the behavior of the PEH beam; as stated in the literature, a maximum strain of $1000 \mu\epsilon$ would limit the PEH beam to a linear range (Upadrashta *et al.*, 2014; Williams *et al.*, 2004b). The strain at any point in the PEH beam can be obtained from the first principles of solid mechanics and piezoelectric governing equations, and is given by Equation 5.6. The strain obtained using the equation and the analytical formulation for different PEH beams is shown in Figure 7.16. The maximum strain obtained from the analytical formulation considered a point exactly near the root of the beam, but as stated before, the protective kapton layer resulted in the effective piezo layers being slightly offset from the root. Thus, the values obtained from the analytical formulation provided a conservative estimate of the strain occurring in the beam.

Experimental determination of strain values was avoided as the addition of strain gauges and extra wiring could hamper the behavior of the harvester. Furthermore, the strain gauges could be more vulnerable to damage under fatigue loading (Figure 7.17).

It is evident that the maximum strain occurred in the PEH beam undergoing 0.6g base excitation, and it was below $700\mu\epsilon$. Hence, the nonlinearity induced during the experimental test was less (Upadrashta *et al.*, 2014). These strain values (about $430\mu\epsilon$ for 0.4g base excitation, $550\mu\epsilon$ for 0.5g base excitation, and $670\mu\epsilon$ for 0.6g base excitation, respectively) for different base excitation magnitudes can be used as the initial values for predicting the number of fatigue cycles that a particular PEH beam can sustain before damage occurs (Rolfe and Barsom, 1977). The method of predicting damage in a PEH system is a detailed topic on its own and will be dealt in future works.

Figure 7.16 Variation of Strain with frequency for different PEH beams.

Figure 7.17 Variation of Strain (peak strain) with the total number of fatigue cycles.

7.5.3 Experimental data (Voltage, frequency, and tip displacement)

As stated in the previous section that the output voltage was monitored continuously using the DAQ system, and the other experimental parameters (frequency and tip displacement) were recorded at about every 2 million cycles. The voltage obtained from the experiments replicated the sinusoidal base excitation during the cycles before the occurrence of the initial damage. After the initial cracks, the magnitude of voltage fell considerably and the positive peaks diminished to spikes as shown in Figure 7.12 (a). Thus, the negative peak voltage represents a more reliable parameter for determining the effect of fatigue on the PEH beam. It can be observed from Figure 7.6 that the initial damage in the MFC transducer occurred much earlier than the final damage in the beam. The peak voltages (-ve) obtained from the PEH beams did not follow any distinct pattern of decay, but an overall diminishing trend with steeper decay in the case of the beam undergoing a base excitation level of

0.6g was observed. At some point during the experiments, the MFC transducer completely deteriorated, and did not produce any further response for output voltage. This is evident in plots shown in Figure 7.7 where the voltage suddenly dropped to zero. After that point, the MFC cannot be effectively utilized for generating any voltage. The final failure in the PEH beams occurred at about 88 million cycles for 0.4g base excitation, 20 million cycles for 0.5g, and 6 million cycles for 0.6g. Correlating this with strain, it can be stated that as the maximum initial strain increases, the number of fatigue cycles beyond which the PEH beam cannot effectively generate voltage also reduces (0.4g-beam: $\sim 92\%$, 0.5g-beam: $\sim 85\%$, and 0.6g-beam: $\sim 70\%$ of the total fatigue cycles).

Apart from voltage, frequency and tip displacement are also the conjugate parameters which can be monitored to understand the damage caused by fatigue. Although continuous monitoring of these parameters is tedious, it was observed that in all the three cases the beam underwent final failure as the resonant frequency dropped by about 2 Hz and the tip displacement reached about 10 mm ($>0.1L$), as shown in Figures 7.5 and 7.7. The drop in the resonant frequency was due to the loss of stiffness in the PEH beam as a result of fatigue, and the loss in stiffness resulted in excessive deformation and drop in voltage, and eventual cracking leading to a complete failure. For an uncoupled beam, the loss in stiffness can be directly related to the decrease in Young's modulus (Boroński, 2004; Rolfe and Barsom, 1977). But for a coupled PEH beam, a closed form solution for the loss in stiffness is hard to obtain as it is dependent on many factors, and primarily due to the variation of electromechanical coupling resulting from the formation of cracks.

7.5.4 Impedance values

The fundamental purpose of the extraction of impedance values has been stated in section "Electromechanical impedance readings." The significance of observed data is discussed in this section. The impedance values provide a deeper insight into the health of the whole system of PEH beam and transducer. The variation of the real (resistance) and imaginary (reactance) parts of the impedance and the RMSD variation shown in Figures 7.8 and 7.9 are indicators of the healthy state of the

transducer, and the variation of the peaks in Figure 7.8 is an indicator of the health of the structure (Bhalla *et al.*, 2012; Sodano *et al.*, 2004b). The general trend of the variation of the resistance and reactance are represented in Figure 7.8 (a) to (f). With increasing number of cycles, the magnitude changed, and at failure the curve was almost a smooth line in all the cases. The change in magnitude signifies the increasing damage within the MFC transducer resulting from the loss in its electromechanical properties and capacitance; this agrees well with the trends observed in the literature.

The resistance and reactance curves for the first few hundred thousand cycles (fewer than 500,000 cycles) for the PEH beams undergoing 0.4g, 0.5 g, and 0.6g base excitations respectively, are presented in Figure 7.8 (g) to (l). Along with the change in magnitude, the slight shifts in the peak frequencies were also observed. The leftward shift in peak frequencies symbolizes the change in resonant frequency of the whole PEH beam, indicating the softening or loss in stiffness of the PEH beam. Moreover, the damage in the transducer can be represented by the variation of the reactance curves (Bhalla and Moharana, 2013; Moharana and Bhalla, 2015; Park *et al.*, 2008). Park *et al* (2008) performed detailed experimental studies on the self-diagnostics of piezoelectric sensors, and suggested that the health of the sensor is directly related to the slope of the imaginary part. It was concluded that an upward shift in the slope was indicative of debonding between the sensor and the epoxy layers, and a downward shift indicated the damage of the sensors. In this chapter, the reactance curves in Figure 7.8 present a conclusive evidence of damage in the MFC transducer with increasing number of cycles. It is also clear from Figure 7.8 (g) to (l) that there is very little variation in the slope, for the first few hundred thousand cycles. Thus, the bonding layer was almost intact, and the gradual damage in the MFC transducer along with loss of stiffness resulted in a drop in the voltage values and the peak frequencies.

Additional quantification of the impedance observations was performed by obtaining the RMSD values and plotting them as shown in Figure 7.9. The RMSD values revealed an increasing trend with passing fatigue cycles; as the damage progressed in the transducer, the effectiveness of the transducer decreased, and it no

longer acted as an actuator or sensor resulting in smooth resistance curves at the failure of the transducer. This was further inferred from the vRMSD plots where the RMSD of the output voltage was plotted. With an increase in the number of cycles the initial damage in the MFC resulted in a jump which was prominently visible in all the RMSD plots. Though the MFC transducer was not completely rendered useless after the initial damage the output voltage decreased and further damage gave rise to irregular peaks. In the plots for rRMSD, the peaks observed revealed the major occurrences of damage to the transducer. When correlated with voltage, it was observed that the initial peak in the rRMSD occurred near the point of initial damage to the MFC, and every consecutive peak depicts further damage in the PEH beam. In the 0.6g base excitation case, the initial damage gave rise to a major peak (near 250,000 cycles) resulting in considerable damage in the MFC, and high RMSD and rRMSD values. In the other two cases, gradual deterioration of the MFC was observed post initial damage resulting in consecutive peaks in the RMSD plots.

7.5.5 Crack propagation

The intermittent experimental observations of the crack propagation are shown in Figure 7.11, and the FEA procedure for the determination of the crack propagation is discussed in section “Crack behavior from FEA.” It is experimentally impossible to continuously track the exact crack growth and propagation due to the dynamic nature of the experiment. Moreover, it is very hard to obtain the depth and width of cracks due to the multitude of layers in the MFC transducer. Thus, the closest imitation of the crack propagation was obtained with the help of an equivalent FEA. The results obtained from the FEA modeling of the cracks are plotted in Figure 7.14. It was observed that with an increase in the number of cracks, the PEH beam softened and the resonant frequency of the beam decreased. The plot shown in Figures 7.5 to 7.7 portrays the variation of frequency and voltage ratio with increasing number of cycles. The gradual drop in frequency with increasing number of cycles in Figure 7.5 is analogous to the downward trend observed with increasing number of cracks in Figure 7.14 (b). This is conclusive of the fact that the PEH beam loses its stiffness with increasing number of fatigue cycles. However, the drop-in voltage with increasing number of cycles (Figure 7.6) is much steeper with

increasing base excitation levels. Moreover, with the increase in the number of cracks in the FEA model, the voltage also displayed a downward trend and a steep drop at base excitation level of 0.6g, as shown in Figure 7.14 (a). From the above observations, it is evident that the increasing number of fatigue cycles causes deterioration of the MFC transducer, resulting in loss of stiffness and voltage of the PEH system, before the ultimate failure of the substrate in fatigue. The magnitude of voltage and frequency obtained from the FEA formulation were slightly higher than the experimental values, due to the assumptions involving idealized boundary conditions, and uniform piezoelectric and electrode layers. Yet, the 2D FEA model presented to study the effect of cracks in the transducer provides a satisfactory outlook into the incremental deterioration of the PEH beam. The FEA crack propagation methodology in this work presents an elementary model used to understand the trend for variation of voltage and frequency from the model. A comprehensive model can be taken up in future studies and compared with experimental investigation for cracks by testing the MFC transducers under controlled conditions.

7.6. Conclusions

The reason for the inability of cantilever PEH beams to provide adequate response at higher excitation levels despite improvement in the strain levels has been addressed in this chapter. A candid investigation into the damage due to long term fatigue loading at constant amplitude base excitation in PEH beams undergoing excitation levels of 0.4g, 0.5g, and 0.6g has been presented. Analytical formulation to obtain the output response of voltage and strain for the PEH beams, followed by the FEA modeling of the layered MFC transducer attached on the aluminum substrate has been explored. The analytical and FEA results agreed well with the experimental values. The 3D FEA model was further simplified into 2D model to minimize the computation effort required to model the crack propagation in the PEH beam. An exhaustive experimental study was performed to investigate the behavior of the PEH beams when subjected to constant base excitation levels. The output voltage obtained from the PEH beams was monitored constantly, and the resonant frequency, tip displacement, and tip velocity were monitored intermittently

after every 2 million cycles. It was observed that after the initial damage in the MFC transducer, there was a considerable drop in the output voltage of the PEH beam. The MFC transducer underwent gradual damage in all the cases and completely failed after 92% (base excitation: 0.4g), 85% (base excitation: 0.5g) and 70% (base excitation: 0.6g) of the total fatigue cycles for the respective base excitation levels. The total fatigue cycles were 88 million, 20 million, and 6 million for the PEH beams subjected to base excitation levels of 0.4g, 0.5g, and 0.6g, respectively. The PEH beams underwent gradual deterioration, initial failure started with degradation of the MFC transducer, and followed by final failure of the substrate at the root of the beam. To augment the experimental analysis, the EMI approach was embraced by recording the necessary data using an impedance analyzer from time to time. The RMSD, ν RMSD, and r RMSD values were obtained to understand the propagation and extent of damage at different stages. Unlike the PZT transducers, an initial damage does not render the MFC transducer unusable, and it can still be utilized to monitor damage to both the substrate and the transducer with increasing fatigue cycles. With the increase in fatigue cycles, the PEH beam lost its stiffness and softened, thus resulting in the fall of natural frequency and an increase in tip displacement. It was observed that for the present PEH beams, the ultimate failure in fatigue occurred when the frequency dropped by >2 Hz and the tip displacement increased to $>10\%$ of the length of the beam. The crack propagation in the MFC transducers was investigated with FEA modeling using an indirect approach. The results obtained from the FEA modeling portrayed a similar trend in the variation of output voltage and resonant frequency. In conclusion, the results obtained from this study can be utilized to incorporate the effect of fatigue into the design of the piezoelectric energy harvesting beams by minimizing the number of times that the strain levels exceed $500 \mu\epsilon$ for a given set of base excitation conditions. This chapter provides a foundation for fatigue-inspired design, and can be applied to design harvesters to harness vibration energy from various civil or mechanical structures.

7.7. Summary

The present chapter illustrates the need for a fatigue inspired design of PEH beams to minimize the damage and prolong the life of the harvesters. The first section introduces the concepts of fatigue and a detailed overview of various component of a PEH beam. The analytical modeling of a PEH beam followed by detailed 3D and 2D FEA models are presented in the second section. The third section illustrates the experimental procedure to examine the fatigue behavior of three PEH beams when subjected to higher acceleration levels over a prolonged period. Section “Crack propagation” deals with the FEA modeling of the surface cracks in the PEH beams and the variation of resonant frequency with increasing damage. The results are presented and discussed in the fifth section; and the section “Conclusion” lists the major findings from the present chapter.

CHAPTER 8 CONCLUSIONS AND FUTURE WORK

The research work presented in this thesis illustrates the importance of enhancing the bandwidth of operating frequency for a PEH system. Most of the work revolved around understanding the parameters effecting the bandwidth, and consequently enhancing the bandwidth with the help of magnetic induced nonlinearity. Comprehensive analysis of the nonlinear PEH systems has been performed with the help of mathematical, finite element and experimental investigations. A few of the major conclusions realized from the study are reported as follows.

8.1. Conclusions

Major conclusions from the research work presented in this thesis are summarized as follows.

A. A candid investigation into the effects of magnetostatic forces acting on a cantilever beam have been discussed. Analytical formulation to obtain the response of a simple cantilever beam in the presence of magnets has been derived. A few highlights from this formulation are:

- The primary contribution from this formulation is the inclusion of the *enhanced magnetostatic force* terms into the lumped parameter formulation. In addition to the use of transverse force, the necessity for the *inclusion of axial force* is presented with the help of analytical and experimental investigations.
- The necessity for understanding the influence of the *rotation of tip magnet* on the behavior of the cantilever beam system has been addressed. It was concluded that the error is not significant while the maximum rotation is within 5° . The PEH cantilever beam was subjected to base excitation levels of 0.1g and 0.2g, and various configurations for both 1 end magnet and 2 end magnets were explored, and the analytical results were validated. The output voltage obtained from the experimental PEH system matched well with the values obtained from the analytical formulation.

- The *improvement in the system bandwidth* in the presence of magnetic induced nonlinearity has also been investigated. An improvement of 30% in the bandwidth for output power greater than 100 μW was obtained for a configuration undergoing a base excitation of 0.2g. It was inferred that with an increase in the base excitation the nonlinear PEH system could perform better than the linear PEH system. The bandwidth depended on the base excitation and the nonlinear configuration.

B. The need for arriving at a tradeoff between the stiffness, effective strain transfer, bandwidth and excitation level to arrive at an efficient configuration for a PEH system subjected to given design parameters was thoroughly investigated. The magnetic forces are broadly classified as axial and transverse forces which act at the tip of a vibrating PEH beams. The expressions for the determination of strain in a vibrating cantilever PEH beam provide a detailed understanding of a very important parameter. Strain provides a comprehensive understanding of the usability of the PEH beam cross-section. Experimental investigation validating the analytical model for a linear PEH system subjected to increasing base excitation levels of 0.1g, 0.2g and 0.3g was presented. The output voltage and strain match well with the analytical model. A few highlights from this part are:

- To illustrate the utilization of the cross-section, a parameter termed '*effective strain transfer*' was calculated. Effective strain transfer facilitates the comparison of various composite cross sections by providing a ratio of the strain across a composite cross-section with respect to a uniform aluminum cross-section. A detailed analytical study was performed to illustrate the relationship between stiffness, effective strain transfer, and bandwidth of operation for both linear and nonlinear PEH beams (bandwidth was calculated at a level of 100 $\mu\epsilon$). It was inferred that, when the PEH beam stiffness was varied in a range of $0.5 \times K_{eq}$ to $1.5 \times K_{eq}$ ($K_{eq} = 128.144 \text{ N/m}$), for base excitation levels of 0.3g the nonlinear repulsive configuration

provided a maximum bandwidth of operation with an increase in a range of 10% to 40%.

- Moreover, for a given base excitation, *an increase in the stiffness of a cross-section contributes to an increase in the effective strain transfer values and decrease in the bandwidth of operation*. The effective strain transfer is inversely proportional to the bandwidth, therefore for a given set of conditions the PEH beam should be designed for an optimum value of stiffness so that there is maximum utilization of the cross-section and the PEH system provides an improved bandwidth. An experimental investigation was performed to reinforce the analytical investigation. PEH beams constituting of fiberglass, aluminum substrates along with commercially available V21BL harvester were experimentally tested and the corresponding output parameters were recorded. All the beams were initially subjected to base excitation levels of 0.1g and 0.2g for both linear and nonlinear configurations. It was evident that the PEH beam with aluminum substrate outperformed the other two PEH beams. There was better utilization of the cross-section which was evident from the effective strain transfer values being more than 100, and an improved bandwidth of operation for a nonlinear repulsive configuration. Thus, PEH beams with aluminum substrate were further examined for higher level base excitation of 0.3g and 0.4g. At higher base excitation levels, the PEH beams were very susceptible to damage. The nonlinear configuration outperformed the linear configuration in terms of the bandwidth. It was confirmed that a PEH beam having a lower beam stiffness provided an enhanced bandwidth of operating frequency for both linear and nonlinear configurations. Moreover, the effective strain transfer under such conditions is less which states that the cross-section is not optimally utilized. Thus, a tradeoff between the stiffness, effective strain transfer, bandwidth and excitation level should be thoroughly investigated to arrive at an efficient configuration for a PEH system subjected to given design parameters.

C. An innovative design to enhance the operational bandwidth of a PEH system was proposed.

- The proposed design consisted of *two flexural members undergoing transverse and parametric vibrations* respectively, and coupled together using magnets. Piezoelectric MFC transducers were attached to both the beams and the resonant frequencies of the beams complement each other with the added nonlinearity from the magnets to give an enhanced bandwidth of the output power distribution. *A detailed experimental analysis was performed by testing two types of beam materials subjected to incremental levels of base excitation values.* The proposed design resulted in an additional resonant peak for the response parameters due to the presence of the beam subjected to parametric resonance. These resonant peaks were observed to be on either side of the resonant peak for the main beam undergoing transverse vibration for PEH beams with aluminum and fiberglass substrates. It was observed that the PEH beams with aluminum substrate produced higher levels of output voltage and output power due to their higher effective strain transfer values.
- A maximum enhancement of operation bandwidth of over 300% was obtained for a repulsive configuration of the PEH beams with aluminum substrate under a base excitation level of 0.2g. Furthermore, the aluminum beams out-performed the fiberglass beams for all the repulsive configurations. It was inferred that the performance of a PEH system with two resonant frequencies is governed by the magnitude of base excitation, the strength of magnets used to induce nonlinearity and couple the system, the substrate material used for the PEH beams, and the deviation of the resonant frequencies of the PEH beams with respect to one another. Farther placement of the resonant frequencies induces deep valleys in between the peak responses, thereby, minimizing the bandwidth of operation of the system.

- D.** The reason for the inability of cantilever PEH beams to provide adequate response at higher excitation levels despite improvement in the strain levels has been addressed with the help of an *experimental fatigue study*.
- A candid investigation into the damage due to *long term fatigue loading* at constant amplitude base excitation in PEH beams undergoing excitation levels of 0.4g, 0.5g, and 0.6g was investigated. Analytical formulation to obtain the output response of voltage and strain for the PEH beams, followed by the *FEA modeling of the layered MFC transducer* attached on the aluminum substrate was also explored. The analytical and FEA results agreed well with the experimental values. The 3D FEA model was further simplified into 2D model to minimize the computation effort required to model the crack propagation in the PEH beam.
 - An exhaustive experimental study was performed to investigate the behavior of the PEH beams when subjected to constant base excitation levels. The output voltage obtained from the PEH beams was monitored constantly, and the resonant frequency, tip displacement, and tip velocity were monitored intermittently after every 2 million cycles. It was observed that after the initial damage in the MFC transducer, there was a considerable drop in the output voltage of the PEH beam. The MFC transducer underwent gradual damage in all the cases and completely failed after 92% (base excitation: 0.4g), 85% (base excitation: 0.5g) and 70% (base excitation: 0.6g) of the total fatigue cycles for the respective base excitation levels. The total fatigue cycles were 88 million, 20 million, and 6 million for the PEH beams subjected to base excitation levels of 0.4g, 0.5g, and 0.6g, respectively. The PEH beams underwent gradual deterioration, initial failure started with degradation of the MFC transducer, and followed by final failure of the substrate at the root of the beam.
 - To augment the experimental analysis, the EMI approach was embraced by recording the necessary data using an impedance analyzer from time to time. The RMSD, vRMSD, and rRMSD values were obtained to understand the propagation and extent of damage at different stages. With the increase in fatigue cycles, the PEH beam lost its stiffness and softened, thus resulting in

the fall of natural frequency and an increase in tip displacement. It was observed that for the present PEH beams, the ultimate failure in fatigue occurred when the frequency dropped by >2 Hz and the tip displacement increased to $>10\%$ of the length of the beam.

- The *crack propagation* in the MFC transducers was investigated with FEA modeling using an indirect approach. The results obtained from the FEA modeling portrayed a similar trend in the variation of output voltage and resonant frequency. In conclusion, the results obtained from this study can be utilized to incorporate the effect of fatigue into the design of the piezoelectric energy harvesting beams by minimizing the number of times that the strain levels exceed $500\ \mu\epsilon$ for a given set of base excitation conditions.

8.2. Recommendations for future work

Recent advances in electrical circuits and low power electronics have raised the need for efficient PEH system designs to supply the necessary energy levels. Numerous researchers have reported new and innovative ways to address these issues. The work presented in this thesis lays a strong foundation for the enhancement of design configurations for PEH systems and improving the bandwidth of operation especially at low amplitude harmonic vibrations. Yet, further contribution from researchers is necessary to realize the true potential of PEH systems and implement them for harnessing energy from ambient vibrations on a large scale. A few of the recommendations to further extend this work are as follows.

- a) An innovative design to enhance the bandwidth of operation of a coupled two beam PEH system is presented in this work, and a bandwidth improvement of near 300% at a base excitation level of 0.2g was achieved. This design utilizes the same base excitation to induce transverse and axial vibrations in the respective beams. An extension of this design would be to include PEH beams from multiple directions and couple them with the help of strong magnetic material to induce nonlinear effects in all the beams. Moreover, this might lead

to a challenge in collecting the response of the harvester. To overcome this enhanced energy harvesting circuits like SCE and SSHI need to be utilized in tandem with the enhancements in the PEH beams.

- b)** The phenomenon of piezoelectric energy harvesting has been undergoing advancements for nearly a decade. Many researchers have contributed to the enhancement of the efficiency of the PEH systems by suggesting various design guidelines. Even the work presented in this thesis contributes to the implementation of an enhanced bandwidth of operation and limiting criterion for minimizing fatigue. The work in this area can be extended to produce a definite set of design guidelines considering all the various parameters that effect the efficient functioning of a PEH system. This would give rise to transfer of technology from the laboratory prototypes to large scale implementation of the most efficient designs for practical usage.
- c)** The designs and work discussed in this thesis focusses mostly on harmonic base vibration responses for the concerned PEH system. Though the base excitation levels were limited to 50-100 Hz (low frequency vibrations), the vibrations coming from real civil and mechanical structures are unlikely to be harmonic. Thus, investigations to arrive at an efficient configuration for random set of base vibrations in the low frequency range need to be studied. Work is presently underway to harness the random vibrations and provide energy for a low power structural health monitoring system. The proposed system draws energy from vibrations and powers a smart damage monitoring system working on the principle of pulse-echo.

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AUTHOR'S PUBLICATIONS

Journal papers

- **Avvari PV**, Yang YW and Soh CK. (2016) "Long-term fatigue behavior of a cantilever piezoelectric energy harvester." *Journal of intelligent material systems and structures* 28: 1188-1210.
- **Avvari PV**, Yang Y and Soh CK. (2017) " Effect of magnetic nonlinearity on vibration energy harvesting." *Journal of intelligent material systems and structures* (under review).
- **Avvari PV**, Yang Y and Soh CK. (2017) " A study demonstrating the effect of system stiffness on the bandwidth and utilization of the cross-section of a piezoelectric energy harvester." *Smart Materials and Structures* (to be submitted).

Conference papers

- **Avvari PV**, Tang L, Yang Y, and Soh CK. (2013) "Enhancement of piezoelectric energy harvesting with multi-stable nonlinear vibrations." *Proc. SPIE 8688, Smart Structures and Materials+ Nondestructive Evaluation and Health Monitoring*.
- **Avvari PV**, Yang Y, Soh CK, and Zhao L. (2014) "Bandwidth enhancement of a piezoelectric energy harvester using parametrically induced vibrations." *Proc. ICAST 2014, International conference on adaptive structures and technologies*.
- **Avvari PV**, Yang Y, Liu P, and Soh CK. (2015) " Experimental investigation of fatigue in a cantilever energy harvesting beam." *Proc. SPIE 9493, Smart Structures and Materials+ Nondestructive Evaluation and Health Monitoring*.

APPENDIX – A

Mag. values DIN IEC 60404-8-	Conditions	Min	Typ	Max	Unit
Magnet Material		Samarium Cobalt			
Remanent Induction Br		900	925		mT
Revers. temp. coeff. of Br				-0,042	%/K
Coercivity HcB		680	710		kA/m
Intrinsic Coercivity HcJ		1.750	2.000		kA/m
Max. operating temperature				250	°C
Magnetic moment	inclusive of 5% tolerance	6,8	7,1	7,3	E-6 Vs*cm
Direction of Magnetization		axial			
Density			8,3		g/cu cm

General data	Conditions	Min	Typ	Max	Unit
Remarks		all details acc. to supplier informations.

Magnetic values acc. to DIN 17410		Min.	Typ.	Max.	unit
Energy product	(B x H) max.	250	295	-	kJ/m^3
Remanence	B_r	1180	1240	-	mT
revers. temp. coeff. of B_r		-	-0,1	-	%/K
Coercivity	H_{cB}	860	920	-	kA/m
Coercivity	H_{cJ}	1750	1900	-	kA/m
revers. temp. toeff. of H_{cJ}		-	-0,6	-	%/K
Density		-	7,6	-	g/cm^3
max. operating temperature		-	-	160	°C

Remark : all details correspond to manufacturers information

Magnetic Moment	M	8,33	8,96	9,59	X10^{-5} Vs·cm
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Drawings not to scale
All dimensions in mm (inches)

SPECIFICATION

Magnet material	Neodymium Iron Boron (NdFeB) Nickel plated
Distance (mm) at which magnets measure 1000 gauss	2.0
Surface flux measurement (gauss)	2500
Grade/Energy density	Grade N35H, max. energy density 35 MGOe
Operating temperature	120°C max.

NdFeb is a high energy material with exceptional resistance to de-magnetisation.