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Arun, Kattukandy Rajan

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**ANALYSIS AND EXPLOITATION OF
DISPERSIVE WAVES FOR
SOURCE LOCALIZATION ON SOLIDS**

**KATTUKANDY RAJAN ARUN
SCHOOL OF ELECTRICAL AND ELECTRONIC
ENGINEERING
2018**

ANALYSIS AND EXPLOITATION OF DISPERSIVE WAVES FOR SOURCE LOCALIZATION ON SOLIDS

KATTUKANDY RAJAN ARUN

**SCHOOL OF ELECTRICAL AND ELECTRONIC
ENGINEERING**

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ABSTRACT

This thesis mainly addresses different classes of algorithms for impact source localization in solids with particular focus on developing a human-computer interface (HCI) through touch impact. The challenge of this research lies in the fact that transverse propagating waves on a thin elastic plate are dispersive in nature. As opposed to wave propagation in a non-dispersive medium such as air, a dispersive medium results in a wave propagation speed that is frequency dependent. This results in the spreading of impact waveform as it propagates. Although this research is conducted for touch/tap localization in human-interaction technology, results arising from this work can potentially be applied to the areas of identification of impact damage, loose part monitoring in industrial applications and crack analysis in non-destructive evaluation. Depending on material properties and geometry of the body, several modes of wave propagation in solids are possible and are characterized by their specific particle oscillatory patterns. Based on research derived from this thesis, amplitudes of these mode responses are shown to vary with source location. In view of this, algorithms derived in this thesis are based on time-frequency analysis of the signal.

This first class of algorithms is based on the location template matching (LTM) method which estimates the index of input data by comparing features of the received signal with features obtained from a pre-recorded set of signals. LTM can be extended to localization on an interactive surface by creating a library of signal features generated by exciting each point of interest with an impact. During operation, an impact is localized by determining the index of the template signal that best matches the pattern of the signal received under test. In this inter-disciplinary research, mechanical vibration theories that model wave propagation of the flexural modes of vibration generated by an impact on the elastic plate surface are studied and analyzed. By analyzing the mathematical model of flexural vibration derived, a Zak transform based time-frequency classifier is proposed for the localization of a finger-tap based on LTM.

The research is then extended to localization using time-difference-of-arrival (TDOA) between two spatially separated sensors. The TDOA-based technique is one of the most widely used methods in acoustic source localization and its main advantage compared to LTM-based methods is that it does not require training of known points. This implies that such algorithm can achieve a large number of distinguishable tap locations with minimal setup. The proposed triangulation based method employs phase transit TDOA between the received signals received by the vibration sensors. A mathematical model of the wave propagation of flexural vibration is used to explain the frequency spectrum diversity of vibration signals and background noise. An algorithm is subsequently developed using the Kullback-Leibler discrimination information to determine the phase transit time of the vibration signal.

The velocity for each frequency mode needs to be estimated for phase transit based algorithm. The next stage of this research focuses on deriving a source localization algorithm which does not require pre-calibration. The proposed method exploits the similarity of warped dispersive signal in the time-frequency domain. The wave propagation of flexural vibration due to an impact on a plate surface is analyzed and the change in dispersion between two signals is shown to be dependent on the range difference between the propagation path of the signals. Utilizing this, a source localization algorithm is developed for impact localization on solid surfaces. As the proposed source localization algorithm jointly estimates the model parameters and source location, this method does not require pre-calibration of the velocity parameter.

Although pre-calibration of velocity parameter is avoided in warping-based algorithm, the mechanical parameter of a solid has to be estimated before localization. The final part of this research uses the cross-spectrum that is independent of any pre-calibration steps or material parameter for source localization. The wave propagation model is used to show how the non-linear wavenumber function represents the dispersive nature of wave propagation. The non-linear wavenumber function is then approximated using the Taylor series to derive an expression for the phase derivative of the cross-spectrum. Moreover, it is noted that the phase derivative contains coupled terms pertaining to the range difference between sensor signals and material-dependent constant. A function is then proposed to map the estimated phase derivatives to a domain that facilitates in the estimation of the source location. This is achieved by employing the Radon transform and subsequently particle swarm optimization in the new domain. The validity of all the proposed algorithms is evaluated via experiments conducted on aluminum and glass plates, for both stylus and finger taps.

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List of Abbreviations

AP-LTM	All-pole model based LTM
CC-LTM	Cross-correlation based LTM
DFT	Discrete Fourier transform
DZT	Discrete Zak transform
GCC	Generalized cross-correlation
HCI	Human computer interface
KL	Kullback-Leibler
KLID	Kullback-Leibler discrimination localization algorithm
LM	Levenberg-Marquardt
LTM	Location template Matching
PHAT	Phase transform
RCS	Radon-based cross-spectrum localization algorithm
RMSE	Root-mean-square error
RSE	Root-square error
STFT	Short-time Fourier transform
TiF	Time-Frequency domain
TiF-HA	Time-Frequency domain-Hermitian angle localization algorithm
TDOA	Time-difference-of-arrival
TOA	Time-of-arrival
TFR	Time-frequency representation
TI	Touch-interface
WD	Wigner distribution
Z-LTM	Zak based LTM

List of Notation

x, y, z	Spatial variable axes
n	sample index of digital signal
m	index of tap
M	Total number of tap
t, θ	Time and frequency variable respectively in TFR
J	Unit imaginary number
(l_m^x, l_m^y)	the coordinate of a tap location
s^n, s^t	Tangential and shearing component of strain
α^n, α^t	Tangential and shearing component of stress
E	Modulus of elasticity
ν	Poisson's ratio
G	Shear modulus
$w(x, y, t)$	Vertical displacement of plate
L_x, L_y, L_z	Length, breadth and thickness of plate
$\mathfrak{M}_x, \mathfrak{M}_y$	Bending moments
$\mathfrak{M}_{xy}$	Twisting moments
Q_x, Q_y	Vertical shear forces
$\tilde{D}$	Flexural rigidity of the plate
$\mathcal{P}$	Uniformly distributed load per unit area
$\mathfrak{h}$	Mass density
∇^2, ∇^4	Laplacian, Biharmonic operator respectively
l, b	Mode numbers along x and y respectively
W_{lb}	vibration amplitude for mode l, b
ω_{lb}	Natural frequency of plate for mode l, b
$f_{lb}(t)$	Eigenfunction of $\mathcal{P}$ in time
s	Laplace domain variable
$\mu, \tilde{\mu}$	Damping coefficient and Reduced coefficient of absorption respectively
ω	Circular frequency
$\delta(x), \delta(y)$	Dirac delta functions with respect to spatial variables x and y respectively
$k(\omega)$	Wavenumber of propagation of flexural wave
$c_p(\omega)$	Velocity of propagation of flexural wave
V_l, V_t	Bulk longitudinal and shear velocities
$\dot{c}$	Plate velocity
ϱ	Shear adjustment coefficient
$g(), \mathbf{g}_m()$	Received signal sample and array of signal sample respectively
N_u	Length of received signal $u(n)$
n	Sample index

$m, \hat{m}$	Position index and estimated position index respectively
$h_m, \mathbf{h}_m$	Pre-recorded signal signature and array of signal signature respectively
N_h	Length of signal signature
$\zeta(m), \zeta(m)$	Correlation coefficient and array of correlation coefficient
μ_g, μ_h	mean of $\mathbf{g}(n)$ and $\mathbf{h}_m(n)$
$\hat{\sigma}_u$	Standard deviation
I_{AP}	Prediction order of AP-LTM
$a()$	Prediction filter coefficient
$e()$	error signal
$P(t, \theta)$	Time frequency representation
$\Phi()$	Kernel function
$U(t)$	Analysis window
$E()$	Energy density spectrum
Q, R	Total number of time and frequency indices respectively in DZT
$\mathcal{Z}_u()$	DZT representation
$\mathcal{D}$	Dissimilarity matrix
C	Classification accuracy
χ	Feature identity ratio
Γ	Ratio given by $\sqrt{Q/R}$
$\Theta()$	Prefilter of GCC
$\mathfrak{M}$	Non-zero feature elements in σ_u^2
K	Frame length
τ_{ij}	TDOA between i and j sensor
$\Lambda_g(\hat{n}, \omega)$	smoothed periodogram
$D(g, g', n)$	KL discrimination information or signal g and g' at time index n
$c_b(\omega)$	phase velocity
$c_g(\omega)$	group velocity
$\mathbf{W}(r, \omega)$	far field solution
$\underline{P}(\omega)$	far field solution of bending wave equation
$\xi(t)$	time-domain wrapping function
$\xi(\omega)$	frequency-domain wrapping function
$\tilde{\tilde{g}}(t)$	wrapped signal in time-domain
l	mode index
$ w_m(t) $	instantaneous amplitude of mode
κ	warping function parameter
κ^{opt}	optimum warping function parameter
$\xi_\kappa(t)$	time-domain wrapping function parametrized by κ

B	data frame length
$\mathbf{F}_B$	Fourier transform matrix
b	frequency bin index
$\mathbf{w}_i(\hat{n})$	data frame
$\widetilde{\mathbf{W}}_i(\hat{n})$	Fourier analysis of one frame (instant $\hat{n}$)
$\underline{\mathbf{W}}_i(b, \hat{n})$	STFT value for bin b , time frame $\hat{n}$
$\overline{\mathbf{W}}_i(b)$	STFT values corresponding to one bin b
$\underline{\mathbf{W}}_i^\kappa(b)$	normalized dB representation of STFT values corresponding to one bin b
$\widetilde{\mathbf{g}}_j^\kappa(n)$	κ parametrized wrapped signal in time-domain
f_s	Sampling frequency
$\underline{\mathbf{W}}_j^\kappa(b, \hat{n})$	STFT value for bin b , time frame $\hat{n}$ of wrapped signal parametrized by κ
$\overline{\mathbf{W}}_j^\kappa(b)$	STFT values corresponding to bin b of wrapped signal
$\underline{\mathbf{W}}_j^\kappa(b)$	normalized dB of STFT values corresponding to bin b of wrapped signal
$\hat{\mathbf{d}}(i)$	difference between source and sensor coordinate
$\hat{\kappa}_{a,i}^o$	estimated warping parameter between sensor a and i .
$\alpha_1, \alpha_2, \alpha_3$	Polynomial coefficients of order two.
$\hat{\alpha}_1, \hat{\alpha}_2$	estimated Polynomial coefficients of order two.
$\epsilon_{i,j}$	error function between sensor i and j .
$W_i(b, \hat{n})$	absolute value of STFT value for bin b , time frame $\hat{n}$
$\check{\theta}_{b,\hat{n}}$	Hermitian angle for bin b , time frame $\hat{n}$
$\sigma_i(\hat{n})$	standard deviation of Hermitian angle for time frame $\hat{n}$
$\eta_i(\hat{n})$	mean of Hermitian angle for time frame $\hat{n}$
σ_i^{thr}	threshold for standard deviation of Hermitian angle
$(l_{\text{src}}^x, l_{\text{src}}^y)$	coordinate of tap location.
(l_i^x, l_i^y)	coordinate of i sensor location.
r	distance measurement function
Δr_{ij}	range distance measurement function
$\check{\Gamma}_{ij}(\Delta r_{ij}, \omega)$	cross-spectrum of two sensor signals
$\Theta_{ij}(\omega)$	phase derivative of cross-spectrum
ϑ_ω	material dependent part of cross-spectrum
$o_\epsilon \left(\hat{l}_{\text{rz}}^x, \hat{l}_{\text{rz}}^y \right)$	Gaussian domain mapped representation of error function
$o \left(\hat{l}_{\text{rz}}^x, \hat{l}_{\text{rz}}^y \right)$	Mapping function of $o_\epsilon \left(\hat{l}_{\text{rz}}^x, \hat{l}_{\text{rz}}^y \right)$ with sparseness consideration
φ	empirical scaling factor
$\mathbf{o}_r(\rho, \theta)$	Radon transform representation

Chapter 1

Introduction

1.1 Motivation

The objective of this research is to develop source localization algorithms to estimate the location of an impact on solids using signals acquired by surface-mounted vibrational sensors. The research can be deployed for various applications including touch/tap localization in human-interaction technology in electronic devices [1] [2] [3] [4], identification of impact damage in machine parts [5], loose part monitoring in plants/factories and crack analysis in non-destructive evaluation. This research is focused on developing a human-computer interface (HCI) via impacts made by a stylus or a finger. Localization of the stylus or finger is achieved by analyzing elastic wave propagation through isotropic, homogeneous thin rectangular plate due to a sudden impact force caused by the tap.

Current approaches to HCI technology rely on the need for input devices such as the alphanumeric keyboard and the optical pointing device (mouse). These input devices have since become an important feature in personal as well as commercial computing. As new interactive digital media software applications continue to evolve over the years, one of the main drawbacks of such input devices is that they impede ease of software operation or data manipulation which require complex user input operations. As a

result, the use of keyboards or mouse limits, to a large extent, the scope, functionality as well as ease of use of the personal computer (PC).

To address these issues, research activities in HCI in recent years have been focused on the use of, for example, vision-based cameras for both eye and hand motion detection [6], gyro-based sensors to track head rotation [7], as well as localization of fingers on the surface of a waveguide medium via the frustrated total internal reflection principle of light-emitting diodes [8]. The common goal of these techniques is to enhance controllability of the PC in a way that is more natural for humans. One of the most commonly used HCI technologies is the touch screen, which has been widely deployed in portable devices such as smart-phones and laptops. These touch-sensitive screens often employ an embedded layer of capacitive sensors which detect changes in the electrostatic field caused by a finger tap. Localization of the finger tap is therefore achieved by detecting and locating such changes on the touch surface. Although this proven technology has gained much popularity, its high implementation cost often limits the size of the screens. As a result, capacitive touch screens are often limited to small portable devices.

One solution which has recently drawn attention in the research community is the development of an HCI platform that allows one to convert an ordinary surface such as table-top and window pane into a touch interface. This technology is based on the use of surface-mounted sensors as well as algorithms that have been developed for the localization of finger taps. In particular, the touch interface considered in [9], [3] describes impact localization methods based on well-known techniques such as time-differences-of-arrival (TDOA) and location template matching (LTM).

The TDOA algorithm utilizes an array of sensors and estimates the differences between the arrival times of the signals to these sensors. The TDOA between two sensors defines a hyperbolic function with the sensors corresponding to the foci of the hyperbola. Using several TDOA estimates, the intersection of these hyperbolae provides an estimate of the source location. In practice, the location of this intersection

can be estimated by solving a set of non-linear equations related to the assumed source and known sensor positions. Although the TDOA approach has been investigated for localizing finger taps on a surface [9], it assumes knowledge of the wave propagation speed. Determination of this speed is often a challenge particularly for solid surfaces due to the dispersive effects where propagating speeds vary with frequencies [2]. In addition, similar to acoustic propagation in air within an enclosed space, the presence of multipath within the material renders TDOA estimation a challenging task.

The LTM-based approach is another class of algorithms for source localization in solids. These LTM-based algorithms utilize only one sensor and localize finger taps by comparing the received signal with a set of pre-recorded signals from known locations. In order to achieve this comparison, LTM computes the similarity by employing the cross-correlation measure (CC-LTM) [1] as well as comparing the closeness of the autoregressive coefficients using the all-pole model (AP-LTM) [10], [4]. It is important to note that, unlike TDOA approaches, LTM-based approaches exploit the dissimilarity between the source-sensor paths for different source locations. Such dissimilarity arises, to a large extent, in the presence of multipath and dispersion and it is therefore expected that LTM-based algorithms achieve higher localization performance compared to TDOA-based approaches for anisotropic materials. The training required before the operation and the assumption that the library templates do not change with source environment condition make localization via LTM-based methods a challenging task.

The effectiveness of both methods has been verified experimentally for isotropic [11] and anisotropic [12] materials. Although progress has been made pertaining to the localization of finger taps on hard surfaces such as wood and glass, localization using signals received by the sensors mounted on such surfaces remains a challenge. These challenges include temperature variation, different types of wave propagation, multipath propagation of these waves through the material as well as wave distortion due to dispersive effects of the channel between the finger tap and the sensor. Due to the complexity of these challenges, which affects the accuracy of source localization to a

large extent, results presented thus far have been aimed on improving the accuracy of the source localization algorithms.

In order to mathematically analyze the signal and to have a better understanding of limits and properties of any algorithms developed, it is necessary to gain insights into the modes of wave propagation that is generated by an impact on the surface using mechanical vibration theories. Motivated by this, a mathematical model of elastic wave propagation through isotropic, homogeneous thin rectangular plate due to a sudden impact force caused by touch/tap is derived. The LTM-based algorithms utilize only one sensor and localize finger taps by comparing the received signal with a set of pre-recorded signals from known locations. The mathematical model derived motivates the study and exploitation of the dissimilarity between the source-sensor paths for different source locations. While TDOA based algorithms have the advantage that they do not require training of known locations, it is expected, in the presence of dispersion, that conventional TDOA estimation algorithms fail. This motives the utilization of mathematical model of wave propagation to compensate the error in estimation of time-of-arrival of the signal from impact location.

1.2 Scope and outline of the thesis

The objective of this thesis is to develop algorithms for impact source localization in solids with particular focus on developing HCI applications. In this thesis, several classes of algorithms which utilize the wave propagation model in solids to estimate source location are proposed. Depending on material properties and geometry of the body, several modes of wave propagation in solids which are characterized by their specific particle oscillatory patterns, are possible. Based on research arising from this thesis, the amplitudes of these mode responses vary with source location and therefore all algorithms derived in this thesis are based on the use of time-frequency analysis of the signal.

This thesis is organized as follows:

Chapter 2 highlights the importance of this inter-disciplinary research by gaining insights into the modes of wave propagation that is generated by an impact on the surface using mechanical vibration theories. The aim of this chapter is to model the wave propagation of flexural modes of vibration that is generated by an impact on a surface through the use of mechanical vibration theories. The initial part of the chapter is to investigate several modes of vibration that can propagate on the surface of solids. This research further models how the momentary force applied to the surface of the plate can be represented in terms of several fundamental kinematics motion elements such as stress and strains. By using the mechanical laws of equilibrium and assumption of Kirchoff's approximation for thin plates, a differential equation which encompasses the flexural behavior is derived. Finally, the mathematical model of flexural wave propagation due to touch is derived as a solution for this differential equation for the particular case of an impulsive input function being the input.

Chapter 3 presents an LTM-based approach for impact localization. This approach utilizes the Zak transform based time-frequency classifier to achieve source localization on solid surfaces using a single sensor. The mathematical model of wave propagation that is generated by an impact on the interactive surface is exploited for the proposed LTM method. This wave propagation model allows one to gain insights into the uniqueness of signal features obtained by an impact on each position on a given surface and motivates the use of time-frequency analysis for signal analysis. More importantly, it is shown that the amplitude variance across time for each frequency-bin is unique for each location. This chapter further shows that the Zak transform allows one to achieve faithful representation of these variations and that the variance in each frequency bin with respect to time is proposed as a unique time-frequency classifier for each location. Since mode frequencies generated by mechanical wave propagation have now been taken into account, the performance of the proposed Z-LTM source localization algorithm for various materials including the aluminum, glass as well as plywood is higher than

existing LTM methods.

Chapter 4 presents a technique for impact localization using TDOA between two spatially separated sensors – a commonly used method that has been applied for acoustic source localization. As described in Chapter 3, the main advantage of this method compared to that of LTM-based methods is that it does not require training of known locations. This implies that such algorithm is able to achieve a large range of distinguishable tap locations with minimal setup/calibration. In this chapter, a new method that employs the Kullback-Leibler (KL) discrimination information for localizing a single tap made by a metal stylus on various flat surfaces is proposed. This is achieved through the use of the phase transit time of the vibration signal. Similar to Chapter 3, we exploit properties of wave propagation in a solid medium which, in turn, allow us to understand the frequency spectrum of vibrating signals. As will be seen, this insight will allow us to employ the KL distance for source localization on solids. The performance of the proposed algorithm is compared and verified with existing TDOA approaches on real surfaces.

In the context of HCI, existing TDOA based methods require calibration to estimate the phase velocities. This, in turn, may cause inconvenience to potential users. Chapter 5 presents a new source localization algorithm based on the range difference between pairs of source-sensor in order to avoid the need of velocity estimation. The wave propagation model of flexural vibration due to an impact on a plate surface from Chapter 5 is analyzed and it is shown that the change in dispersion between two signals is dependent on the range difference between the propagation path of the signals. This implies that a difference in dispersion can be mapped directly to a difference in relative propagation distances if the signals are properly time-warped. The proposed algorithm, therefore, exploits similarity of the warped dispersive signal in the time-frequency domain. As the proposed source localization algorithm jointly estimates the model parameters and source location, this method does not require pre-calibration. For every pair of the sensor signals, one of the two sensor signals is warped over a range of warping parameter and

each warped signal will be checked for similarity with the other signals by analyzing them in the time-frequency domain. The value of the warping parameter giving the maximum similarity will then be taken as optimal warping parameter for that sensor pair. From the experimental results conducted on various plate surfaces, it is observed that a polynomial function of order two is sufficient to describe the relationship between the range differences and the optimal warping parameters. The parameters for this polynomial function, which depends on the material type, along with source position is jointly estimated using optimization algorithms. The performance of the proposed algorithm is validated via experiments conducted on aluminum and glass plates for both stylus and finger taps.

In Chapter 5, the relationship between the range differences and the optimal warping parameters is modeled as a second-order polynomial function, which is restrictive in nature. Chapter 6 presents another calibration-free method which extracts the range difference information embedded in phase derivatives of the cross-spectrum of flexural vibration signals. Unlike conventional cross-spectrum based source localization methods, research derived in this chapter shows that, when the sensor signals are caused by the flexural vibration on the solid surface, the phase derivative of the cross-spectrum will be a scaled version of the range difference between source-to-sensor locations. The scaling factor depends on the material properties of the medium and frequency of operation. The effect of dispersion on the frequency spectrum of the source signal is also studied and this is achieved by considering the Gaussian narrow-band pulse as an example source signal. This explains why conventional spectrum comparison methods fail to extract the time/range difference information when the medium of propagation is dispersive. To achieve effective source localization, the phase derivative of the cross-spectrum is mapped to a different domain using a newly proposed mapping function and applying the Radon transform on it. Particle swarm optimization is subsequently used for estimating the impact location. The algorithm is validated via experiments conducted on aluminum and glass plates, for both stylus and finger taps.

Conclusions arising from this research is presented in Chapter 7, with recommendations for future research.

1.3 Main contributions

The main contributions of this dissertation are summarized as follows:

(1) The analysis of elastic wave propagation through isotropic, homogeneous thin rectangular plate such as panels due to a sudden impact force caused by touch/tap for the localization of source position. A mathematical wave model for wave propagation of flexural modes of vibration is derived by applying mechanical vibration theories. This contribution is highlighted in Chapter 2.

(2) Development of an LTM-based algorithm which is robust to dispersion and multipath. By analyzing the mathematical model of flexural vibration, the Zak transform based time-frequency classifier for the localization of a finger-tap in the context of HCI is proposed in Chapter 3.

(3) Development of a TDOA-based source localization algorithm using phase transit TDOA between the received signals from sensors. A mathematical model of the wave propagation of flexural vibration due to an impact on a surface is analyzed in order to explain the frequency spectrum diversity of vibration signals and background noise. The Kullback-Leibler discrimination information is then used to determine the phase transit time of the vibration signal. This contribution is highlighted in Chapter 4.

(4) A source localization algorithm on solids which exploits the similarity of the warped dispersive signal in the time-frequency domain has been proposed. The wave propagation of flexural vibration due to an impact on a plate surface is analyzed and the change in dispersion between two signals is shown to be dependent on the range difference between the propagation path of the signals. As the proposed source localization algorithm jointly estimates the model parameters and source location, this method does not require pre-calibration. This contribution is highlighted in Chapter 5.

(5) Finally, a source localization algorithm which does not require any explicit calculation of material-dependent parameters has been proposed. This technique is based on the phase derivative of the cross-spectrum between two sensor outputs. The localization problem is mathematically formulated in terms of phase derivative of the cross-spectrum between two sensor outputs due to vibration of impact. To decouple the range difference between sensor signals and material-dependent scaling parameter a new function is proposed. The source localization is estimated from this new function using the Radon transform and subsequently applying the particle swarm optimization. This contribution is highlighted in Chapter 6.

Chapter 2

Wave Propagation Model In Thin Plates Due To An Impact

The mathematical model to represent the displacement of plate surface as a function of space and time is essential for deriving algorithms as well to infer several unique characteristics of time and frequency-domain representation of the collected data. For this purpose, this chapter presents an in-depth analysis of the wave propagation in solids caused by an impact. The primary causality behind the wave mechanics is the tendency for all matter to remain in a state of equilibrium with the surrounding environment. A sudden application or variation of an external force such as an impact on the material disturbs the equilibrium state of matter. The resultant internal electrostatic (elastic) restoring force within the matter along with particle inertia leads to oscillatory motions of particles, which propagate as waves. Examples of these elastic waves include waves generated in the Earth's crust during earthquakes as well as sound and ultrasonic waves in liquids and gases. Solid materials will resist any change in size (volume) and shape dissimilar to fluids which resist changes in just size. This results in the presence of several complex modes of wave propagation in solids compared to fluids. The aim of this chapter is to model the elastic wave propagation due to an impact force on a plate-like structure.

This chapter begins by studying several modes of vibration that can propagate on the surface of solids. The research is concentrated on the flexural modes of vibration under the assumption of thin plates. The momentary force that is applied to the surface of the plate can be represented in terms of several fundamental kinematics motion elements such as stress and strains. Using the mechanical laws of equilibrium and assumption of Kirchoff's approximation for thin plates, a differential equation which encompasses the flexural behavior is then derived. The mathematical model of flexural wave propagation due to the touch is finally obtained as the solution of this differential equation for the particular case of an impulsive input.

2.1 Modes of wave propagation in solids

Depending on material properties and the geometry of the body, several modes of wave propagation, which is characterized by their particular particle oscillatory patterns, are possible in solids. The aim of this section is to identify the mode of propagation in thin plates due to an impact.

Modes of wave propagation can broadly be classified as body and surface waves. Body waves propagate within the interior of a medium as longitudinal and transverse modes of vibration. In longitudinal waves (also known as irrotational, compressional, dilatational and primary or P wave), particles of the medium move back and forth (termed as compression and rarefaction movement) parallel to the direction of wave propagation. In the transverse wave (also known as shear, rotational and secondary or S wave), the particles oscillate perpendicular to the direction of propagation.

In an infinite homogeneous isotropic solid, only pure longitudinal and transverse waves exist. However, when the medium does not extend to infinity in all directions, other types of waves can be generated [13]. Surface waves propagate along the interface between differing media and are limited to the region of surfaces that bound the medium. When the medium is bounded by a free solid half-space, surface waves are called

Figure 2.1: Particle motion of Rayleigh wave [after [17]].

Rayleigh waves [14]. These waves can be conceived as a particular combination of longitudinal and transverse wave components with a definite phase relation to each other [15]. During the wave propagation, a particle traverses an elliptical path [16] whose major axis is normal to the surface of the solid as shown in Figure 2.1. As the depth of an atom from the surface increases the width of its elliptical motion decreases so that the amplitude of this wave motion decreases exponentially with depth [18]. This results in nearly zero displacement at a depth of two wavelengths from the surface [16]. The geometric spreading of Rayleigh waves is lower than that of body waves [19].

When the thickness of medium of propagation is much lesser than the order of wavelength, Rayleigh waves degenerate to Lamb waves (also known as guided, plate wave) [20] [21]. With Lamb waves, several modes of particle vibration are possible, but the two most common are symmetrical and asymmetrical which depend on the distribution of the displacements about the mid-plane of the plate [22] [23]. Symmetrical or extensional waves move in a symmetrical fashion about the median plane of the plate. This mode is commonly generated when the exciting force is parallel to the medium. On the contrary, during the asymmetrical or flexural Lamb wave mode majority of the wave

Figure 2.2: Particle motion in (a) Symmetrical Lamb wave and (b) Asymmetrical Lamb wave.

motion occurs in the normal direction of the propagation [24]. The particle motion for symmetrical and asymmetrical Lamb wave is shown in Figure 2.2.

It is assumed that the rigid planes used for this research is a thin plate which is homogeneous and isotropous. Lamb wave propagation occurs when the wavelength is much greater than the thickness of the plate. Several researchers recommend the wavelength be ten times higher than the thickness [25] and the remaining dimensions (length and width) of the plate must be greater than the wavelength. Both symmetrical and asymmetrical Lamb waves can propagate in an infinite number of modes. However, within the frequency range of measurement using conventional contact transducer (where the output signal is proportional to the displacement normal to the surface), only the lower mode of flexural vibration is significant [25] [26]. This motivates the use of classical plate theory for deriving the mathematical model for flexural vibration.

2.2 Mathematical modeling of wave propagation in thin plates

This research is focused on analyzing elastic wave propagation through an isotropic, homogeneous thin rectangular plate due to a sudden impact force caused by a tap to localize the source position. The foundation for wave propagation in solids lies in three conservation laws: conservation of energy, conservation of momentum and conservation of mass. When an external force is applied to a deformable plate, it gets deformed producing internal forces. The resultant flexural motion of the plate can be divided into the free vibration of a plate, which occurs in the absence of an applied load and the forced vibration, which is caused by the application of time-dependent loads. The free vibration mode deals with natural characteristics of the plates which depend only on the geometry and material of the plates. This state of static equilibrium should be analyzed to obtain fundamental differential equations which can be easily extended to analyze the dynamic motion due to time-dependent load. Natural or free vibrations is first investigated in this chapter by deriving the differential equation of motion of plates in the static case by taking the effect of inertia forces into account. This derivation involves obtaining a relationship between the applied forces and phenomenon related to physical deformations; stress and strains. The theoretical study of the relationship between components of stress and strain is thus conducted by considering an infinitesimal cubic inside plate. With the approximation using Kirchhoff hypotheses a differential equation governing free vibrations of thin, isotropic plates is derived based on the relationship between stress and strain. Finally, a mathematical model for wave propagation in thin plates is obtained as the solution of this differential equation by considering point force as input.

As mentioned at the beginning of this chapter, elastic wave propagation is due to the tendency of the body to resist the change in size and shape due to an external force. For a body in static equilibrium not to deform and rotate, the total force and moment

Figure 2.3: Components of stress [after [27]].

along any path should be zero. The theory of elasticity relates the applied forces to the resulting changes in size and shape. The relations between the applied forces and the deformations are expressed in terms of the physical phenomenon concepts of stress and strains. Since this research is interdisciplinary in nature, several foundational conceptual definitions of vibrational mechanics, based on [27] [28] [10] [29] [14] [15], are re-derived till (2.36) to provide the reader a deeper understanding of fundamental assumptions involved in the model.

2.2.1 Stress

When a force is applied to a body, the stress is defined as the ratio of the force to the area on which the force is exerted. Stress measures the distribution of internal forces. The value of stress at any point of a continuous and cohesive plate is found by considering an infinitesimally small element centered at the point and dividing the total force acting on this area by the magnitude of the area. Any force acting on the area of the element can be resolved into its perpendicular component, normal stress s^n and the tangential component, shearing stress s^t . The stresses acting on each of the six faces of a small element of volume can be resolved into a three-dimensional state of stress as depicted

in Figure 2.3.

When the medium is in static equilibrium, the forces acting on area element must be balanced. This implies that the stress acting on one face must be equal and opposite to the corresponding stresses in the opposite face. Although in the general stress state, there are three normal stress components and six shear stress components, under moment equilibrium conditions it can be shown that the shearing stress acting opposite to the co-ordinate axis is identical [27],

$$s_{xy}^t = s_{yx}^t, \quad s_{xz}^t = s_{zx}^t, \quad s_{yz}^t = s_{zy}^t. \quad (2.1)$$

2.2.2 Strain

The strain is used to quantify deformation of an area element that is subject to force or a set of forces as depicted in Figure 2.4. This section derives the mathematical expression of each component of strain, which is used later for formulating differential equations representing static equilibrium of plate. Consider an infinitesimal area element inside plate whose initial undeformed coordinates are x , y and z . Let dx , dy and dz be the length of its face along x , y and z axis, respectively. Due to external force, the element is deformed [30] and displaced into $x + u$, $y + v$ and $z + w$, respectively, where u , v and w are respective projections of displacement vector on the coordinate axes x , y and z as shown in Figure 2.5. In addition to the element moving in space, it also elongates or contracts by $\Delta(dx)$, $\Delta(dy)$ and $\Delta(dz)$ relative to its dimensions. The first three deformations shown in Figures 2.4 (a), (b) and (c) define the elongation of edges of the parallelepiped in the direction of the coordinate axis. It can be shown that the normal or linear strains in the x , y and z directions can be expressed respectively as [27]

$$\alpha_x^n = \frac{\Delta(dx)}{dx}, \quad \alpha_y^n = \frac{\Delta(dy)}{dy}, \quad \alpha_z^n = \frac{\Delta(dz)}{dz}. \quad (2.2)$$

Figure 2.4: Components of strain [after [27]].

In (2.2) the increment $\Delta(dx)$ can be expressed by the second term in the Taylor series as

$$\Delta(dx) = \frac{\partial u}{\partial x} dx,$$

to obtain

$$\alpha_x^n = \frac{\partial u}{\partial x}, \quad \alpha_y^n = \frac{\partial v}{\partial y}, \quad \alpha_z^n = \frac{\partial w}{\partial z}. \quad (2.3)$$

The other three deformations shown in Figures 2.4 (d), (e) and (f) are referred to as shear strains because they describe distortions of an initially right angle between the edges of the parallelepiped. They are denoted by α_{xy}^t , α_{xz}^t , α_{yz}^t , where the subscripts indicate

Figure 2.5: Components of strain [after [27]].

the coordinate planes on which the shear strain occur. To determine, for example, the shear strain on the xy coordinate plane, consider the projection of the parallelepiped, shown in Figure 2.4 (d) on this coordinate plane. Figure 2.5 shows this projection in the form of the rectangle before deformation $ABCD$ and after deformation $(A'B'C'D')$. The angle BAD in Figure 2.5 deforms to the angle $B'A'D'$, with the deformation angle being $\alpha^t + \hat{\alpha}^t$. The shear strain is therefore given by

$$\alpha_{xy}^t = \alpha^t + \hat{\alpha}^t,$$

which can be determined, under small angle approximation, in terms of the in-plane displacement u and v as

$$\begin{aligned}
 \alpha_{xy}^t &= \tan^{-1}\left(\frac{\frac{\partial v}{\partial x}dx}{dx + \frac{\partial u}{\partial x}dx}\right) + \tan^{-1}\left(\frac{\frac{\partial u}{\partial y}dy}{dy + \frac{\partial v}{\partial y}dy}\right) \\
 \alpha_{xy}^t &\approx \frac{\frac{\partial v}{\partial x}dx}{dx + \frac{\partial u}{\partial x}dx} + \frac{\frac{\partial u}{\partial y}dy}{dy + \frac{\partial v}{\partial y}dy} \\
 &= \frac{\frac{\partial v}{\partial x}}{1 + \frac{\partial u}{\partial x}} + \frac{\frac{\partial u}{\partial y}}{1 + \frac{\partial v}{\partial y}}.
 \end{aligned} \tag{2.4}$$

Due to small deformations, the quantities $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y}$ in (2.4) is negligibly small compared with unity and thus omitted. Similar approximation can also be applied to the shear strains α_{xz}^t and α_{yz}^t . The approximated shear strains components can therefore be obtained as

$$\begin{aligned}
 \alpha_{xy}^t &= \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}, \\
 \alpha_{xz}^t &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \\
 \alpha_{yz}^t &= \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}.
 \end{aligned} \tag{2.5}$$

It is evident that the strain tensor is also symmetric since $\alpha_{xy}^t = \alpha_{yx}^t$, $\alpha_{xz}^t = \alpha_{zx}^t$ and $\alpha_{yz}^t = \alpha_{zy}^t$.

Having derived equations for various components of stress and strain in terms of displacement, Hooke's law [28] can further be used to map the relationship between each of these quantities.

2.2.3 Equations of stress and strain components during equilibrium state

This section is used to consolidate all the stress and strain components into a set of equations based on their mutual relationships. These equations can further be used to

formulate the differential equation of motion of the plate surface during equilibrium.

Hooke's law denotes the linear relationship between stress and strain components during the linear elastic range of operation. For the case of a three-dimensional isotropic body, the generalized Hooke's law can be used to relate the stress components to the strain components as given by [27]

$$\begin{aligned}
 \alpha_x^n &= \frac{1}{E}[\varsigma_x^n - \nu(\varsigma_y^n + \varsigma_z^n)], \\
 \alpha_y^n &= \frac{1}{E}[\varsigma_y^n - \nu(\varsigma_x^n + \varsigma_z^n)], \\
 \alpha_z^n &= \frac{1}{E}[\varsigma_z^n - \nu(\varsigma_y^n + \varsigma_x^n)], \\
 \alpha_{xy}^t &= \frac{1}{G}\varsigma_{xy}^t, \\
 \alpha_{xz}^t &= \frac{1}{G}\varsigma_{xz}^t, \\
 \alpha_{yz}^t &= \frac{1}{G}\varsigma_{yz}^t,
 \end{aligned} \tag{2.6}$$

where E , ν , G are the modulus of elasticity, Poisson's ratio and shear modulus, respectively. For (2.5) the six strain components are shown to be related to the three displacement components. Eliminating the displacement components by successive differentiation, the following equations are obtained

$$\begin{aligned}
 \frac{\partial^2 \alpha_x^n}{\partial y^2} + \frac{\partial^2 \alpha_y^n}{\partial x^2} &= \frac{\partial^2 \alpha_{xy}^t}{\partial x \partial y}, \\
 \frac{\partial^2 \alpha_y^n}{\partial z^2} + \frac{\partial^2 \alpha_z^n}{\partial y^2} &= \frac{\partial^2 \alpha_{yz}^t}{\partial y \partial z}, \\
 \frac{\partial^2 \alpha_z^n}{\partial x^2} + \frac{\partial^2 \alpha_x^n}{\partial z^2} &= \frac{\partial^2 \alpha_{xz}^t}{\partial x \partial z}, \\
 \frac{\partial}{\partial z} \left[\frac{\alpha_{xy}^t}{\partial x} + \frac{\alpha_{yz}^t}{\partial y} - \frac{\alpha_{xz}^t}{\partial z} \right] &= 2 \frac{\partial^2 \alpha_z^n}{\partial x \partial y}, \\
 \frac{\partial}{\partial x} \left[\frac{\alpha_{xz}^t}{\partial y} + \frac{\alpha_{xy}^t}{\partial z} - \frac{\alpha_{yz}^t}{\partial x} \right] &= 2 \frac{\partial^2 \alpha_x^n}{\partial y \partial z}, \\
 \frac{\partial}{\partial y} \left[\frac{\alpha_{xy}^t}{\partial z} + \frac{\alpha_{yz}^t}{\partial x} - \frac{\alpha_{xz}^t}{\partial y} \right] &= 2 \frac{\partial^2 \alpha_y^n}{\partial x \partial z}.
 \end{aligned} \tag{2.7}$$

Instead of the exact three-dimensional differential equation related to the motion of surface as mentioned in (2.7), the closed-form solution of the surface motion can be derived by simplifying (2.3), (2.5) and (2.6) using the classical theory of isotropic plates.

2.2.4 Classical plate theory of thin plates

To derive the differential equation governing deflection of the plate surface, the components of stress and strain need to be substituted in terms of moments and shear forces. In analyzing the behavior of thin plates, the fundamental assumptions of the classical theory for isotropic, homogeneous, elastic thin plates are employed, which simplify several constraints of (2.3), (2.5) and (2.6). These assumptions are known as the Kirchhoff hypotheses.

As mentioned in Section 2.2.2, consider the components of displacement at a point on a thin plate that occur in the x , y and z directions as denoted by u , v and w , respectively. Consider an arbitrary point, A , on the mid plane of the thin plate along xz plane after lateral loading as shown in Figure 2.6. The Kirchhoff hypotheses [27] state that:

{AP.1} The deflection of the mid-surface w is small compared with the thickness of the plate L_z . The slope of the deflected surface is therefore very small, and the square of the slope is a negligible quantity compared with unity.

{AP.2} The mid-plane remains unstrained subsequent to bending.

{AP.3} Plane sections initially normal to the mid-surface remain plane and normal to that surface after bending, which results in negligible vertical shear strains, α_{xz}^t and α_{yz}^t . Hence, deflection of the plate is associated principally with bending strains, due to which the normal strain α_z^n resulting from transverse loading is also negligible.

Figure 2.6: Deflection of A on thin plate along xz plane before and after loading.

{AP.4} The stress normal to the mid-plane s_z^n is small compared with the other stress components and may be neglected. This supposition becomes unreliable in the vicinity of highly concentrated transverse loads.

Taking {AP.3} into account (2.3) and (2.5) can be reduced to

$$\begin{aligned}
 \alpha_z^n &= \frac{\partial w}{\partial z} = 0, \\
 \alpha_{xz}^t &= \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} = 0, \\
 \alpha_{xy}^t &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 0, \\
 \alpha_{yz}^t &= \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} = 0.
 \end{aligned} \tag{2.8}$$

Integrating (2.8) and utilizing {AP.2} such that the initial values of $u(u_0(x, y))$ and $v(v_0(x, y))$ are zero at mid plane, we obtain [28]

$$\begin{aligned}
 u &= -z \frac{\partial w}{\partial x} + u_0(x, y) = -z \frac{\partial w}{\partial x} \\
 v &= -z \frac{\partial w}{\partial y} + v_0(x, y) = -z \frac{\partial w}{\partial y}.
 \end{aligned} \tag{2.9}$$

Substituting (2.9) into α_x^n and α_y^n of (2.3) and α_{xy}^t of (2.5), we obtain the following equations

$$\begin{aligned}\alpha_x^n &= -z \frac{\partial^2 w}{\partial x^2}, \\ \alpha_y^n &= -z \frac{\partial^2 w}{\partial y^2}, \\ \alpha_{xy}^t &= -2z \frac{\partial^2 w}{\partial x \partial y}.\end{aligned}\tag{2.10}$$

Having obtained a relation between strain and deflection (or curvature), one can involve stress in it and derive expressions for the stress resultants. Based on {AP.3}

$$\alpha_z^n = \alpha_{yz}^t = \alpha_{xz}^t = 0$$

can be substituted into (2.6) to obtain

$$s_x^n = \frac{E}{1-\nu^2}(\alpha_x^n + \nu \alpha_y^n),\tag{2.11}$$

$$s_y^n = \frac{E}{1-\nu^2}(\alpha_y^n + \nu \alpha_x^n),\tag{2.12}$$

$$s_{xy}^t = G \alpha_{xy}^t.\tag{2.13}$$

Substituting (2.10) in (2.11), (2.12) and (2.13), the following equations are obtained:

$$\begin{aligned}s_x^n &= -\frac{Ez}{1-\nu^2} \left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right), \\ s_y^n &= -\frac{Ez}{1-\nu^2} \left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} \right), \\ s_{xy}^t &= -2Gz \frac{\partial^2 w}{\partial x \partial y} = -\frac{Ez}{1+\nu} \left(\frac{\partial^2 w}{\partial x \partial y} \right).\end{aligned}\tag{2.14}$$

It is important to note that the stress vanishes at the mid surface and varies linearly over the thickness of the plate. These stress components distributed over the plate thickness produce bending moments ($\mathfrak{M}_x, \mathfrak{M}_y$), twisting moments ($\mathfrak{M}_{xy}$) and vertical shear forces (Q_x, Q_y). All these moments have units corresponding to forces per unit distance. With

Figure 2.7: (a) Stresses in the bottom half and stress normal to the midplane of a plate element and (b) variation of the stresses through the thickness of the plate.

reference to Figure 2.7 (b),

$$\begin{aligned} \int_{-\frac{L_z}{2}}^{\frac{L_z}{2}} z s_x^n dy dz &= dy \int_{-\frac{L_z}{2}}^{\frac{L_z}{2}} z s_x^n dz \\ &= \mathfrak{M}_x dy. \end{aligned} \quad (2.15)$$

Similarly,

$$\begin{Bmatrix} \mathfrak{M}_x \\ \mathfrak{M}_y \\ \mathfrak{M}_{xy} \end{Bmatrix} = \int_{-\frac{L_z}{2}}^{\frac{L_z}{2}} \begin{Bmatrix} s_x^n \\ s_y^n \\ s_{xy}^t \end{Bmatrix} z dz \quad (2.16)$$

and

$$\begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = \int_{-\frac{L_z}{2}}^{\frac{L_z}{2}} \begin{Bmatrix} s_{xz}^t \\ s_{yz}^t \end{Bmatrix} dz. \quad (2.17)$$

Substituting (2.14) into (2.16), the following equations for the bending and twisting

Figure 2.8: Positive stress resultants and load on a plate element.

moments in terms of the curvatures and the deflection can be derived as

$$\begin{aligned} \mathfrak{M}_x &= -\dot{D} \left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right), \\ \mathfrak{M}_y &= -\dot{D} \left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} \right), \\ \mathfrak{M}_{xy} &= -\dot{D}(1 + \nu) \left(\frac{\partial^2 w}{\partial x \partial y} \right), \end{aligned} \quad (2.18)$$

where

$$\dot{D} = \frac{EL_z^3}{12(1 - \nu^2)}$$

is the flexural rigidity of the plate.

The components of stress (and thus the stress resultants) generally vary from point to point in a loaded plate. These variations are governed by the conditions of equilibrium of statics. Figure 2.8 illustrates how the stress resultants vary from face to face. Depicted on this figure is an element $dx dy$ of the plate subject to an uniformly distributed load per unit area $\mathcal{P}$. It is assumed that inclusion of the plate weight (a small quantity) in $\mathcal{P}$ cannot affect the accuracy of the result. For simplicity, since the element is infinitesimally small, it can be noted that, the force and moment components may be

considered to be distributed uniformly over each face. Therefore they are shown in Figure 2.8 by a single vector, representing the mean values applied at the center of each face. Note that the variation with position is expressed by a truncated the Taylor's expansion.

The condition that the sum of the z directed forces equal to zero leads to

$$\begin{aligned} \frac{\partial Q_x}{\partial x} dxdy + \frac{\partial Q_y}{\partial y} dxdy + \mathcal{P} dxdy &= 0, \\ \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + \mathcal{P} &= 0. \end{aligned} \quad (2.19)$$

Likewise, the equilibrium of moments about the x and y axes are governed respectively by

$$\begin{aligned} \frac{\partial \mathfrak{M}_{xy}}{\partial x} + \frac{\partial \mathfrak{M}_y}{\partial y} - Q_y &= 0, \\ \frac{\partial \mathfrak{M}_{xy}}{\partial y} + \frac{\partial \mathfrak{M}_x}{\partial x} - Q_x &= 0. \end{aligned} \quad (2.20)$$

It can be noted that products of infinitesimal terms, such as the moment of p and the moment due to the change in Q_x and Q_y , have been omitted. Combining these two latter expressions into (2.19) finally leads to the differential equation of equilibrium for bending of thin plates

$$\frac{\partial^2 \mathfrak{M}_x}{\partial x^2} + 2 \frac{\partial^2 \mathfrak{M}_{xy}}{\partial x \partial y} + \frac{\partial^2 \mathfrak{M}_y}{\partial y^2} = -\mathcal{P}. \quad (2.21)$$

Expressions for vertical shear forces Q_x and Q_y may now be written in terms of deflection w . These expressions can be derived from (2.20) together with (2.18) giving

$$\begin{aligned} Q_x &= -\dot{D} \frac{\partial}{\partial x} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) = -\dot{D} \frac{\partial}{\partial x} \left(\nabla^2 w \right), \\ Q_y &= -\dot{D} \frac{\partial}{\partial y} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) = -\dot{D} \frac{\partial}{\partial y} \left(\nabla^2 w \right). \end{aligned} \quad (2.22)$$

Finally, combining (2.19) and (2.22) to obtain the governing differential equation for deflection of thin isotropic plates, we obtain

$$\begin{aligned} \frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} &= \frac{\mathcal{P}}{\dot{D}} \\ \nabla^4 w &= \frac{\mathcal{P}}{\dot{D}}, \end{aligned} \quad (2.23)$$

where

$$\begin{aligned} \nabla^4 &= \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4} \\ &= \nabla^2 \nabla^2. \end{aligned} \quad (2.24)$$

The dynamic motion of the plate can be divided into free vibration of plate, which occurs in the absence of an applied load, and due to forced vibration caused by the application of time-dependent loads. The free vibration mode deals with natural characteristics of the plates, and these natural vibrations occur at discrete frequencies, depending only on the geometry and material of the plates. To study the dynamic motion due to time-dependent load, all the time-dependent effective forces need to be considered. Inertia force, caused by the acceleration of the mass of plate's lateral motion, is subtracted using the D'Alembert's principle. In addition, other time-dependent forces such as the damping force caused by internal friction or by surrounding media have to be considered.

Natural or free vibrations are first investigated by extending the differential equation of motion of plates in the static case by taking the effect of inertia forces into account. Structural dynamics due to damping is ignored since it has an insignificant effect on the natural frequencies of the plate. In view of this, (2.23) can be expressed by

$$\dot{D} \nabla^4 w(x, y, t) = \mathcal{P}(x, y, t) - \mathfrak{h} L_z \frac{\partial^2 w(x, y, t)}{\partial t^2},$$

where $\mathfrak{h}$ is defined as the mass density. Since the natural vibrations are independent of

load $\mathcal{P}(x, y, t)$ is set equal to zero and the above equation becomes

$$\dot{D}\nabla^4 w(x, y, t) + \mathfrak{h}L_z \frac{\partial^2 w(x, y, t)}{\partial t^2} = 0. \quad (2.25)$$

This differential equation can be solved (see Appendix A.1) according to the boundary condition of simply supported plate to give the natural shape function

$$W(x, y) = \sum_{l=1}^{\infty} \sum_{b=1}^{\infty} W_{lb} \sin\left(\frac{l\pi x}{L_x}\right) \sin\left(\frac{b\pi y}{L_y}\right), \quad (2.26)$$

where L_x and L_y are the plate dimensions in x and y directions, respectively, and W_{lb} is the vibration amplitude for each value of l and b with natural frequency given as

$$\omega_{lb} = \sqrt{\frac{\dot{D}}{\mathfrak{h}L_z} \left(\frac{l^2\pi^2}{L_x^2} + \frac{b^2\pi^2}{L_y^2} \right)}. \quad (2.27)$$

Thus far, the considered plate is assumed to be in equilibrium. For an isotropic wave medium, (2.23) describes the equilibrium between transversal stresses on an infinitesimal plate element with coordinates (x, y) . However, when considering the dynamics of the plate, one should add additional terms to the differential equation of (2.25) giving

$$\dot{D}\nabla^4 w(x, y, t) + \mu \frac{dw(x, y, t)}{dt} + \mathfrak{h}L_z \frac{d^2 w(x, y, t)}{dt^2} = \mathcal{P}(x, y, t), \quad (2.28)$$

where the third term accounts for the inertia and is necessary when analyzing the vibrations in or waves through the plate. It can be observed that these terms represent stresses, i.e., forces per unit area. The second term accounts for the damping with μ being the damping coefficient. This term is included when the wave propagation appears to be dispersive, i.e., the phase speed and damping are significantly dependent on frequency.

To derive the displacement caused in the z direction $w(x, y, t)$ due to an external

load, $\mathcal{P}(x, y, t)$ is expressed in the form of series extended in natural mode shape (eigenfunctions) of the plate given by

$$\mathcal{P}(x, y, t) = \sum_{l=1}^{\infty} \sum_{b=1}^{\infty} f_{lb}(t) W_{lb}(x, y). \quad (2.29)$$

To obtain the closed-form solution of (2.29), we exploit (2.28) giving

$$w(x, y, t) = \sum_{l=1}^{\infty} \sum_{b=1}^{\infty} \mathfrak{F}_{lb}(t) W_{lb}(x, y), \quad (2.30)$$

where $\mathfrak{F}_{lb}(t)$ is an arbitrary function of time which needs to be determined in terms of t and depending on $\mathcal{P}(x, y, t)$ as explained in Appendix A. Substituting w and $\mathcal{P}$ in (2.28), we obtain

$$\dot{D} \mathfrak{F}_{lb}(t) \nabla^4 W_{lb}(x, y) + \mu F'_{lb}(t) W_{lb}(x, y) + \mathfrak{h} L_z F''_{lb}(t) W_{lb}(x, y) = f_{lb}(t) W_{lb}(x, y) \quad (2.31)$$

from which dividing both sides by $\mathfrak{F}_{lb}(t) W_{lb}(x, y)$,

$$\frac{\dot{D}}{\mathfrak{h} L_z} \frac{\nabla^4 W_{lb}(x, y)}{W_{lb}(x, y)} = \frac{1}{\mathfrak{h} L_z} \frac{f_{lb}(t)}{\mathfrak{F}_{lb}(t)} - \frac{\mu}{\mathfrak{h} L_z} \frac{F'_{lb}(t)}{\mathfrak{F}_{lb}(t)} - \frac{F''_{lb}(t)}{\mathfrak{F}_{lb}(t)}. \quad (2.32)$$

From (A.2), the left hand side of (2.32) can be written as

$$\frac{\dot{D}}{\mathfrak{h} L_z} \frac{\nabla^4 W_{lb}(x, y)}{W_{lb}(x, y)} = \omega_{lb}^2 \quad (2.33)$$

while the right hand side becomes

$$F''_{lb}(t) + \tilde{\mu} F'_{lb}(t) + \omega_{lb}^2 \mathfrak{F}_{lb}(t) = \frac{1}{\mathfrak{h} L_z} f_{lb}(t), \quad (2.34)$$

where $\tilde{\mu} = \frac{\mu}{\mathfrak{h} L_z}$ is the reduced coefficient of absorption. Since it can be assumed that the HCI will be supported on edges, the condition of simple-supported plate can be applied

to obtain solution of (2.34) in the Laplace transform domain as given by

$$\mathbf{F}_{lb}(s) = \frac{1}{\mathfrak{h}L_z} \frac{\mathbf{f}_{lb}(s)}{s^2 + \tilde{\mu}s + \omega_{lb}^2}. \quad (2.35)$$

In this case the mathematical model of a damped structure becomes [27] [10]

$$\mathbf{W}(x, y, s) = \sum_{l=1}^{\infty} \sum_{b=1}^{\infty} \frac{1}{\mathfrak{h}L_z} \frac{W_{lb}}{s^2 + \tilde{\mu}s + \omega_{lb}^2} W_{lb}(x, y), \quad (2.36)$$

where l and b denote the mode number of frequency of propagation along x and y directions, respectively. When the plate is subjected to a point force $\delta(t)$ applied to coordinate x_0 and y_0 (such as the case where a finger makes an impact to the rigid surface), (2.29) can be represented as

$$\mathcal{P}(x, y, t) = \delta(t)\delta(x - x_0)\delta(y - y_0). \quad (2.37)$$

Extending the applied force into a series, $\mathcal{P}(x, y, t)$ is given by

$$\begin{aligned} \mathcal{P}(x, y, t) &= \delta(t)\delta(x - x_0)\delta(y - y_0) \\ &= \delta(t) \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{lb} \sin(\tilde{\beta}x) \sin(\beta y), \end{aligned} \quad (2.38)$$

where $\tilde{\beta} = \frac{\pi l}{L_x}$ and $\beta = \frac{\pi b}{L_y}$, such that, W_{lb} is derived as shown in Appendix A.2,

$$\begin{aligned} W_{lb} &= \frac{4}{L_x L_y} \int_0^{L_x} \int_0^{L_y} \delta(x - x_0)\delta(y - y_0) \sin(\tilde{\beta}x) \sin(\beta y) dx dy \\ &= \frac{4}{L_x L_y} \sin(\tilde{\beta}x_0) \sin(\beta y_0). \end{aligned} \quad (2.39)$$

Substituting (2.39) into (2.36), the transfer function between the impact and sensor is

given by

$$\begin{aligned} \mathbf{W}(x, y, s) &= \frac{4}{L_x L_y \hbar L_z} \sum_{l=1}^{\infty} \sum_{b=1}^{\infty} \frac{1}{s^2 + \tilde{\mu}s + \omega_{lb}^2} W_{lb}(x, y) \sin(\tilde{\beta}x_0) \sin(\beta y_0) \quad (2.40) \\ &= \frac{4}{L_x L_y \hbar L_z} \sum_{l=1}^{\infty} \sum_{b=1}^{\infty} \frac{W_{lb}(x, y) \sin(\tilde{\beta}x_0) \sin(\beta y_0)}{\left(s + \frac{\tilde{\mu}}{2}\right)^2 + \left(\omega_{lb}^2 - \left(\frac{\tilde{\mu}}{2}\right)^2\right)}. \end{aligned}$$

The solution (Laplace transform) of (2.40) using standard Laplace transform pair equation (Laplace transform pair of $\frac{1}{\left(s + \frac{\tilde{\mu}}{2}\right)^2 + \left(\omega_{lb}^2 - \left(\frac{\tilde{\mu}}{2}\right)^2\right)}$ and $\frac{e^{-\frac{\tilde{\mu}}{2}t} \sin\left(t\sqrt{\omega_{lb}^2 - \left(\frac{\tilde{\mu}}{2}\right)^2}\right)}{\sqrt{\omega_{lb}^2 - \left(\frac{\tilde{\mu}}{2}\right)^2}}$) is given by

$$w(x, y, t) = \frac{4}{L_x L_y \hbar L_z} \sum_{l=1}^{\infty} \sum_{b=1}^{\infty} W_{lb}(x, y) \sin(\tilde{\beta}x_0) \sin(\beta y_0) \frac{e^{-\frac{\tilde{\mu}}{2}t} \sin\left(t\sqrt{\omega_{lb}^2 - \left(\frac{\tilde{\mu}}{2}\right)^2}\right)}{\sqrt{\omega_{lb}^2 - \left(\frac{\tilde{\mu}}{2}\right)^2}} \quad (2.41)$$

where $W_{lb}(x, y) = \sin(\tilde{\beta}x) \sin(\beta y)$. This is an important expression in the context of HCI, since (2.41) estimates the vertical displacement at any point (x, y) due to a tap made at (x_0, y_0) .

2.2.5 Wavenumber and phase velocity of propagation

To derive the velocity of wave propagation, conditions under which harmonic plane waves may propagate need to be investigated. Assume a plane wave propagating at circular frequency of ω in the x-direction of the form $w(x, t) = A e^{jk(\omega)(x - c_p(\omega)t)}$, where A is the wave amplitude, $k(\omega)$ is the wavenumber and $c_p(\omega)$ is the phase velocity of propagation. Utilizing $\nabla w(x, t) = jk(\omega)w(x, t)$ and $\nabla^2 w(x, t) = -k^2(\omega)w(x, t)$, substituting $A e^{jk(\omega)(x - c_p(\omega)t)}$ to (2.25),

$$c_p(\omega) = \sqrt{\frac{\tilde{D}}{\hbar L_z}} k(\omega). \quad (2.42)$$

Since $k(\omega) = \frac{\omega}{c_p(\omega)}$, we obtain

$$c_p(\omega) = \sqrt[4]{\frac{\dot{D}}{\hbar L_z}} \sqrt{\omega} \quad (2.43)$$

which corresponds to that described in [31]. This equation implies that the phase velocity of propagation is dependent on the signal frequency. In a multitone signal each frequency component will therefore travel with its own velocity which will result in the modification of source signal spectrum such that each frequency has different phase shift. This phenomenon makes the propagation of $w(x, y, t)$ dispersive. For example, a pulse propagating through a dispersive media becomes a sweep signal [32].

The wavenumber function $k(\omega)$ denotes the spatial frequency of the wave and can be derived from (2.43) as

$$\begin{aligned} k(\omega) &= \frac{\omega}{c_p(\omega)} \\ &= \sqrt[4]{\frac{\hbar L_z}{\dot{D}}} \sqrt{\omega}. \end{aligned} \quad (2.44)$$

This research employs the above expression $k(\omega)$ in Chapters 4, 5 and 6 for the derivation of spatial range difference based algorithms.

2.3 Reliability of modeling using classical plate theory

In order to accurately predict the propagation of waves in plates at high frequencies both Lord Rayleigh and H. Lamb in 1899 independently obtained the frequency equation for a plate using exact equations of elasticity. Their work, now known as the Rayleigh-Lamb theory, pertains to the propagation of continuous, straight crested waves in a plate, which is infinite in extent and has traction-free surfaces. Since mathematical derivation is beyond the scope of this work, the Rayleigh-Lamb frequency equation for flexural

waves in plate is stated below as [28] [33] [14]

$$\frac{\tan(\beta b)}{\tan(\alpha b)} = -\frac{((k(\omega))^2 - \beta^2)^2}{4\alpha\beta(k(\omega))^2}, \quad (2.45)$$

where

$$\begin{aligned} \alpha^2 &= \frac{\omega^2}{V_l^2} - (k(\omega))^2, \\ \beta^2 &= \frac{\omega^2}{V_t^2} - (k(\omega))^2, \end{aligned}$$

and V_l and V_t are the bulk longitudinal and shear velocities. Therefore, given ω , $k(\omega)$ is determined and (2.45) has to be satisfied. Using the expression $\omega/k(\omega)$, the velocity of propagation of the wave can be calculated.

R. Mindlin developed an approximate theory for flexural wave propagation in plates which included corrections for both rotary inertia and shear effects [29]. The dispersion equation from Mindlin's theory is given by

$$\left(\frac{(c_p(\omega))^2}{V_t^2} \right)^2 - \left(\varrho^2 + \frac{\dot{c}^2}{V_t^2} - \frac{12\varrho^2}{L_z^2(k(\omega))^2} \right) \left(\frac{(c_p(\omega))^2}{V_t^2} \right) + \varrho^2 \left(\frac{\dot{c}^2}{V_t^2} \right)^2 = 0, \quad (2.46)$$

where $\dot{c} = \sqrt{\frac{E}{\rho(1-\nu^2)}}$ is the plate velocity and L_z is the plate thickness. As in the Rayleigh-Lamb theory, values for the wavenumber are assumed and the equation is solved for the velocity $c_p(\omega)$. To find the shear adjustment coefficient ϱ , the behaviour of $\varrho = c_r/V_t$ for $k(\omega) \rightarrow \infty$ (high frequency) is used, where c_r is the Rayleigh wave velocity.

Based on studies performed in [14] and [28], for long wavelength, low-frequency conditions, classical plate theory (such as described in (2.41)) predicts the correct response. However, for short wavelength, high-frequency conditions, unbounded wave velocities (as opposed to model given in (2.46)) are predicted. This is the result of imperfections in the model due to rotary inertia and shear effects. Since our experiment lies within a frequency range of less than 10 kHz, mathematical model derived using

classical plate theory should give a reasonable approximation of the vertical deflection due to an impact.

2.4 Chapter Summary

This chapter is focused on analyzing elastic wave propagation through an isotropic, homogeneous thin rectangular plate due to a sudden impact force caused by touch/tap. When an external force is applied to a deformable plate, internal forces are produced. The resulting flexural motion of the plate can be divided into free vibration of plate which occurs in the absence of an applied load, and due to forced vibration caused by the application of time-dependent loads. The free vibration mode deals with natural characteristics of the plates which depend only on the geometry and material of the plates. The effect of natural or free vibrations is first investigated in this chapter by deriving the differential equation of plate motion in the static case and taking the effect of inertia forces into account. This derivation requires one to obtain the relationship between the applied forces and phenomenon related to physical deformations; stress and strains. The theoretical study of relationship between components of stress and strain is thus conducted by considering an infinitesimal cubic inside the plate. Using approximation and Kirchhoff hypotheses, a differential equation governing free vibrations of thin isotropic plates is derived based on the relationship between stress and strain. The mathematical model for wave propagation in thin plates is finally obtained as the solution of this differential equation by considering point force as input. This solution, given by (2.41), paves the way for further source localization algorithm solutions pertaining to HCI in subsequent chapters. The time-frequency variation of free vibrations as given in (2.36) will be presented in detail in Chapter 3. The remaining chapters deal with mitigating the effects of dispersion in (2.28) and (2.41) to estimate TDOA or range difference to achieve localization of an impact made on a rigid surface.

Chapter 3

A Location Template Matching Technique for HCI

3.1 Introduction

Location template matching (LTM) methods estimate the index of input data by comparing features of the received signal with features obtained from a pre-recorded set of signals. LTM can be extended to localization on an interactive surface by creating a library of signal features for each location during the training phase. During the testing phase, an impact is localized by finding the index of the template signal that best matches the pattern of the signal received under test. Two LTM methods which compute the similarity of features for localizing an impact made on a rigid surface have been proposed. The first employs the cross-correlation measure (CC-LTM) [1] while the second compares the closeness of the auto-regressive coefficients using the all-pole model (AP-LTM) [10], [4].

As briefly described in Chapter 1, time-difference-of-arrival (TDOA) based algorithms employ differences between signals arriving at each sensor pair to achieve source localization. Although the TDOA approach has been investigated for localizing finger taps on a surface [9], it assumes knowledge of the wave-propagation speed. Determina-

tion of this speed is often a challenge particularly for solid surfaces due to the dispersive effects where frequencies propagate at different speeds [2]. In addition, similar to acoustic propagation in air within an enclosed space, the presence of multipath within the material render TDOA estimation a challenging task. It is important to note that, unlike TDOA approaches, LTM-based approaches exploit the dissimilarity between the source-sensor paths for different source locations. Such dissimilarity arises, to a large extent, in the presence of multipath and dispersion and it is therefore expected that LTM-based algorithms can achieve higher localization accuracy compared to TDOA-based approaches for anisotropic materials.

In this chapter the mathematical model of wave propagation that is generated by an impact on the interactive surface is exploited for the proposed LTM method. This interdisciplinary study gains insights into the uniqueness of signal features on each position of a given surface and the need to utilize time-frequency analysis. This chapter further illustrates the use of Zak transform for faithful time-frequency domain representation of a tap signal. More importantly, this chapter shows that the amplitude variance across time for each frequency-bin is unique for each location. This property is exploited as a unique time-frequency classifier for the localization of the finger-tap in the context of HCI. Since mode frequencies generated by mechanical wave propagation are now taken into account, the performance of the proposed Zak based LTM (Z-LTM) source localization algorithm for various materials including the aluminum, glass as well as plywood is expected to outperform that of CC-LTM [1] and AP-LTM [4].

3.2 Review of existing location template matching techniques

Consider a single-input, single-output system shown in Figure 3.1. Assume that a vibration caused by an impact generates a received signal $\mathbf{g}(n) = [g(n), \dots, g(n + N_g - 1)]^T$, where N_g is the length of $\mathbf{g}(n)$. The aim of LTM is therefore to estimate the

Figure 3.1: Diagram of TI with a matrix of predefined points.

source location using $\mathbf{g}(n)$. To achieve this, LTM requires prior knowledge of the source signature at a known location. This signature can be obtained during the calibration phase, where

$$\mathbf{h}_m(n) = [h_m(n), \dots, h_m(n + N_h - 1)]^T \quad (3.1)$$

is defined as the pre-recorded signal corresponding to a known source location at position index m . It assumes that the length of $\mathbf{h}_m(n)$ is equivalent to $\mathbf{g}(n)$ and that there is one such pre-recorded signal for each of the M pre-defined locations. The goal of LTM-based algorithms is therefore to compute a similarity measure between $\mathbf{g}(n)$ with that of $\mathbf{h}_m(n)$ for $m = 1, \dots, M$. The estimated location is then given by the location corresponding to the pre-recorded signal that is most similar to the received signal.

3.2.1 Cross-correlation based LTM (CC-LTM)

To compute the similarity, the use of the cross-correlation between $\mathbf{g}(n)$ and $\mathbf{h}_m(n)$, for $m = 1, \dots, M$, has been proposed [1], [34]. This approach exploits the fact that the received signal is unique with respect to each of the source location due to variations in scattering and reflections of the wave along the source-sensor path. The $M \times 1$ vector $\boldsymbol{\zeta}(m) = [\zeta(1), \dots, \zeta(M)]^T$ is defined as a vector containing zero-lag cross-correlation-

coefficients between $\mathbf{g}(n)$ and $\mathbf{h}_m(n)$ such that each element is given by

$$\zeta(m) = \frac{1}{N_g \hat{\sigma}_g \hat{\sigma}_h} \sum_{i=1}^{N_g} (g(i) - \mu_g)(h_m(i) - \mu_h), \quad (3.2)$$

where μ_g and μ_h denote the mean of $\mathbf{g}(n)$ and $\mathbf{h}_m(n)$ while $\hat{\sigma}_g$ and $\hat{\sigma}_h$ denote the standard deviation of $\mathbf{g}(n)$ and $\mathbf{h}_m(n)$, respectively. The cross-correlation based LTM (CC-LTM) estimates the location of the impact $\hat{m}$ corresponding to the maximum of $\zeta(m)$ over all position index $m = 1, \dots, M$, i.e.,

$$\hat{m} = \arg \max_m (\zeta(m)). \quad (3.3)$$

It has been shown in [1] that location estimation using CC-LTM achieves a localization accuracy of ± 1.5 cm. The time-reversal approach proposed in [35] have the same computational complexity and accuracy as CC-LTM.

3.2.2 The all-pole filter model approach to LTM (AP-LTM)

To increase the localization accuracy, the all-pole model for LTM (AP-LTM) algorithm has been proposed [4]. This algorithm exploits the dissimilarity between spectral peaks of the received signal at each tap location. Differences between spectral peaks of $\mathbf{g}(n)$ and $\mathbf{h}_m(n)$ can be made by comparing the locations of these dominant frequencies in the unit circle using an all-pole filter. Given a received signal $\mathbf{g}(n)$, the error between the actual and predicted signal is given by

$$e(n) = g(n) - \sum_{i=1}^{I_{AP}} a^g(i)g(n-i), \quad (3.4)$$

where I_{AP} is the prediction order and $a^g(i)$ is the i th prediction filter coefficient. Defining $a_m^h(i)$ similar to that of (3.4) for the pre-recorded signal $\mathbf{h}_m(n)$ at location index m , the

sum of the squared differences of the prediction coefficients is given by

$$S_m = \sum_{i=1}^{I_{AP}} (a^g(i) - a_m^h(i))^2. \quad (3.5)$$

As opposed to CC-LTM, this approach estimates the location of the source corresponding to the minimum distance between the prediction coefficients, i.e.,

$$\hat{m} = \arg \min_m S_m. \quad (3.6)$$

As shown in [4], AP-LTM achieves an improvement in localization accuracy with lower computational costs compared to CC-LTM. However to achieve high accuracy, there is a need to pre-process $\mathbf{g}(n)$ and $\mathbf{h}_m(n)$ such that their bandwidths match that of the sensor as well as the frequency response of the material.

3.3 The proposed vibrational model based time-frequency analysis and classification algorithm

Unlike CC-LTM and AP-LTM, the LTM-based localization problem on a surface is formulated by considering the problem using mechanical vibration theories as described in Chapter 2. The model is based on vibration waves on the surface of a thin rectangular plate - a common structure covering a wide range of objects including tables, walls, and glass windows. Using the mathematical model, the mode frequencies of vibrational signal propagating on a rigid surface is analyzed. This insight showed that one can represent the signal in the time-frequency domain using the Zak transform. This, in turn, is used to localize the finger tap using a time-frequency classifier that is unique to the position of the finger tap.

3.3.1 Mathematical model for flexural vibration

With reference to Figure 3.1, the coordinate of a tap location is defined as (l_m^x, l_m^y) where $m = 1, \dots, M$ is the location index while M denotes the total number of possible tap locations. The well-known Kirchoff's hypotheses is employed as described in Section 2.2.4 for linear, elastic small-deflection theory of thin plates. With reference to (2.28) and defining $\mathcal{P}_m(x, y, t)$ as the applied force at location (x, y) at time t due to a finger impact made at location (l_m^x, l_m^y) , the vertical displacement $w_m(x, y, t)$ of the plate at a particular location (x, y) must satisfy the motion equation [27]

$$\mathcal{P}_m(x, y, t) = \dot{D} \nabla^4 w_m(x, y, t) + \mu \frac{dw_m(x, y, t)}{dt} + \mathfrak{h} L_z \frac{d^2 w_m(x, y, t)}{dt^2}, \quad (3.7)$$

where μ is the absorption coefficient, $\mathfrak{h}$ the density, L_z the thickness of the plate, $\dot{D} = EL_z^3/12(1 - \nu^2)$ the stiffness of the plate, E the Young's modulus, ν Poisson's ratio and the biharmonic operator $\nabla^4 = \frac{\partial^4}{\partial x^4} + 2 \frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}$.

In order to solve (3.7), $\mathcal{P}_m(x, y, t)$ due to impact at (l_m^x, l_m^y) is expressed as

$$\mathcal{P}_m(x, y, t) = \delta(t) \cdot \delta(x - l_m^x) \cdot \delta(y - l_m^y), \quad (3.8)$$

where $\delta(t)$, $\delta(x)$, and $\delta(y)$ are the Dirac delta functions with respect to variables t , x and y , respectively. This condition is identical to the derivation in Section 2.2.4. From (2.41), an important expression is therefore obtained for the vertical displacement at (x, y) due to a tap made at (l_m^x, l_m^y) and is given by

$$w_m(x, y, t) = \frac{4}{\mathfrak{h} L_x L_y L_z} \sum_{u=1}^{\infty} \sum_{v=1}^{\infty} W_{uv}(x, y) \left(\frac{e^{-0.5\tilde{\mu}t} \sin(\xi_{uv}t)}{\xi_{uv}} \right), \quad (3.9)$$

 Figure 3.2: Shaping function $W_{5,5}(x, y)$ for mode $u = 5, v = 5$.

where $\xi_{uv} = \sqrt{\omega_{uv}^2 - (\frac{\tilde{\mu}}{2})^2}$, and

$$\begin{aligned} W_{uv}(x, y) &= \sin(\tilde{\beta}x) \sin(\beta y) \sin(\tilde{\beta}l_m^x) \sin(\beta l_m^y), \\ \omega_{uv} &= (\tilde{\beta}^2 + \beta^2) \sqrt{\frac{\hbar \dot{D}}{\hbar L_z}}, \end{aligned} \quad (3.10)$$

such that $\tilde{\beta} = \pi u / L_x$, $\beta = \pi v / L_y$, ω_{uv} is the mode frequency while $\tilde{\mu} = \mu / (\hbar L_z)$ is the reduced coefficient of absorption and that $u, v \in \mathbb{Z}^+$ are the mode index of wave propagation.

As shown in (3.9), $w_m(x, y, t)$ is represented as the summation of mode response corresponding to each mode frequency ω_{uv} . For a given tap location and material, each mode response can be represented as the product of the shaping function $W_{uv}(x, y)$ and the time-dependent function $e^{-0.5\tilde{\mu}t} \sin(\xi_{uv}t) / \xi_{uv}$. It can be seen that the time-dependent function is inversely proportional to ξ_{uv} . This implies that, as ω_{uv} increases with u and v , the contribution of that mode to $w_m(x, y, t)$ reduces with respect to mode $u = 1, v = 1$. In addition, as shown in Figure 3.2 for a tap made at location $(l_m^x, l_m^y) = (0.1, 0.1)$ m, the shaping function $W_{uv}(x, y)$ is dependent on (x, y) . More importantly, as shown in (3.10), $W_{uv}(x, y)$ is a function of (l_m^x, l_m^y) . This implies that the time-dependent function is weighted by a location-dependent shaping function $W_{uv}(x, y)$ and as a result, the vertical

Figure 3.3: Variation of mode response, $W_{uv}(x, y)(\frac{e^{-0.5\tilde{\mu}t} \sin(\xi_{uv}t)}{\xi_{uv}})$, for tap positions (a) $(l_m^x, l_m^y) = (0.01, 0.01)$ m, (b) $(l_m^x, l_m^y) = (0.1, 0.1)$ m.

displacement $w_m(x, y, t)$ will vary with tap position due to its amplitude variation of mode frequencies with respect to time. To illustrate the above, Figure 3.3 (a) and (b) show the variation of mode response $W_{uv}(x, y)(\frac{e^{-0.5\tilde{\mu}t} \sin(\xi_{uv}t)}{\xi_{uv}})$ for a tap made at location $(l_m^x, l_m^y) = (0.01, 0.01)$ m and $(l_m^x, l_m^y) = (0.1, 0.1)$ m, respectively. For each of these locations, mode responses corresponding to $[u = 4, v = 3]$ and $[u = 4, v = 9]$ are illustrated. It is noted that the amplitudes of these mode responses vary with tap location (l_m^x, l_m^y) . In addition, the amplitude reduces with increasing u, v , as expected. Therefore these amplitude variations are exploited for source localization through the use of time-frequency analysis.

It is also important to note that although the sensor's transfer function has not been taken into account for the development of LTM-based approaches, the performance of such algorithms is not affected by this function. This is because the transfer function of the sensor has the same effect on $w_m(x, y, t)$ and consequently the amplitude variation of mode frequencies due to the sensor are consistent across all positions on the surface. As a result, the uniqueness of $w_m(x, y, t)$ is still preserved for each tap location.

3.3.2 Time-frequency analysis

For classification purpose, it is important to represent signals in a domain that is unique for each tap location. Utilizing the mechanical model and as shown in Figure 3.3, the

amplitudes of the mode frequencies vary with respect to the tap location and hence a time-frequency representation (TFR) for the analysis of these mode frequencies in $\mathbf{g}(n)$ and $\mathbf{h}_m(n)$ defined in (3.1) is proposed. Although it is noted that for LTM classification, TFR needs not be an ideal representation of the signal, this chapter proposes to represent the signal using a TFR that faithfully represents the mode frequencies [36]. Unlike existing ad-hoc LTM approaches, this accurate representation allows one to formulate the LTM problem in the context of wave propagation which, in turn, allows the algorithm to accurately track any amplitude variation of such mode frequencies with time for localization. It is therefore important to track the instantaneous change in time and frequency content [37] such that summing the energy distribution of the TFR across all frequencies at a particular time instant gives the instantaneous energy while summing the TFR across time at a particular frequency gives the energy density spectrum. These desirable properties of a TFR are referred to as the time- and frequency-marginal properties [38] and is represented, respectively, as

$$\int P(t, \omega) d\omega = |q(t)|^2, \quad (3.11)$$

$$\int P(t, \omega) dt = |q(\omega)|^2, \quad (3.12)$$

where $P(t, \omega)$ is the TFR of $q(t)$ with time variable t and frequency variable ω .

It is also useful to note that there are different types of TFR and the choice of TFR is important to accurately track the amplitude variation of the mode frequencies. It is therefore important to verify whether these TFRs satisfy the marginal properties. This chapter utilizes the properties of a subclass of TFR falling under the category of Cohen's class of bilinear TFR which can be expressed as [38]

$$P(t, \omega) = \frac{1}{2\pi} \iiint_{-\infty}^{\infty} q\left(p + \frac{\alpha}{2}\right) q\left(p - \frac{\alpha}{2}\right) \Phi(\varphi, \alpha) \cdot e^{-j(\varphi t + \omega \alpha - \varphi p)} dp d\alpha d\varphi, \quad (3.13)$$

where $j = \sqrt{-1}$ and $\Phi(\varphi, \alpha)$ is the kernel function which is dependent on the TFR that

one wishes to represent, while α and φ denote the frequency-shift and time-lag variables, respectively. Based on Cohen's class, the time- and frequency-marginal conditions are satisfied when $\Phi(\varphi, 0) = 1$ and $\Phi(0, \alpha) = 1$, respectively.

3.3.2.1 Short-Time Fourier Transform (STFT)

The short-time Fourier transform (STFT) is one of the most commonly used TFR which divides a time-domain signal into overlapping/non-overlapping frames. Each frame is first windowed before being Fourier transformed giving

$$P_{\text{STFT}}(t, \omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} w_m(x, y, \tau) U(\tau - t) e^{-j\omega\tau} d\tau, \quad (3.14)$$

where $U(t)$ is the analysis window. One of the most well-known forms of STFT is the spectrogram which is defined by the energy density spectrum of STFT $E_{\text{STFT}}(t, \omega) = |P_{\text{STFT}}(t, \omega)|^2$. As noted in (3.14), the STFT of a given signal is dependent on the choice of the analysis window and the Gaussian-shaped window is often used since it satisfies the Heisenberg uncertainty equality [39] giving one of the best representations of the signal both in time and frequency. This Gaussian window is described by

$$U(t) = (\hat{\gamma}/\pi)^{\frac{1}{4}} e^{0.5\hat{\gamma}t^2}, \quad (3.15)$$

where $\hat{\gamma}$ is a parameter which controls the spread of the window in the time- and frequency-domain.

To verify if the STFT will achieve good representation of our wave propagation

model when operated on $w_m(x, y, t)$, (3.9) and (3.15) are substituted into (3.14) giving

$$\begin{aligned}
 P_{\text{STFT}}(t, \omega) = & \frac{1}{2J(\hat{\gamma}\pi)^{1/4}} \sum_{u=1}^{\infty} \sum_{v=1}^{\infty} W_{uv} \\
 & \cdot \left[\exp \left[\frac{(0.5\tilde{\mu})^2 - \tilde{\mu}\hat{\gamma}t - (\xi_{uv} - \omega)^2}{2\hat{\gamma}} \right] \cdot \exp \left[\frac{J(-0.5\tilde{\mu} + \hat{\gamma}t)(\xi_{uv} - \omega)}{\hat{\gamma}} \right] \right. \\
 & \left. + \exp \left[\frac{(0.5\tilde{\mu})^2 - \tilde{\mu}\hat{\gamma}t - (\xi_{uv} + \omega)^2}{2\hat{\gamma}} \right] \cdot \exp \left[\frac{-J(-0.5\tilde{\mu} + \hat{\gamma}t)(\xi_{uv} + \omega)}{\hat{\gamma}} \right] \right].
 \end{aligned} \tag{3.16}$$

It is noted that, unlike $w_m(x, y, t)$, the STFT contains terms which are not only a function of ξ_{uv} but also undesirable terms which depend on $\hat{\gamma}$ that are derived from $U(t)$ and not $w_m(x, y, t)$ alone. These imply that the STFT will not give a true representation of $w_m(x, y, t)$ and consequently will not be suitable for our algorithm to accurately track any amplitude variations of mode frequencies with time for localization.

To verify that $\Phi(\varphi, \alpha)$, as defined after (3.13), does not satisfy marginal conditions, it is noted that the spectrogram is a member of Cohen's class whose kernel function for real signals is given by

$$\begin{aligned}
 \Phi(\varphi, \alpha) &= \int_{-\infty}^{\infty} U(p + \frac{\alpha}{2})U(p - \frac{\alpha}{2})e^{-J\varphi p} dp \\
 &= \sqrt{\frac{\hat{\gamma}}{\pi}},
 \end{aligned} \tag{3.17}$$

where the definition of $U(t)$ in (3.15) is utilized. Equation (3.17) implies that our model under the STFT does not satisfy the time- and frequency-marginal conditions. This result is as expected since the spectrogram is a function of both the energy distribution of $w_m(x, y, t)$ as well as that of the analysis window $U(t)$. The presence of undesirable frequency components can also be observed in the derived STFT expression given by (3.16).

3.3.2.2 Wigner Distribution (WD)

The Wigner distribution (WD) is another TFR computed by correlating $w_m(x, y, t)$ with a time and frequency translated version of itself [38], i.e.,

$$P_{\text{WD}}(t, \omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} w_m(x, y, t + \frac{\tau}{2}) w_m(x, y, t - \frac{\tau}{2}) e^{-j\omega\tau} d\tau. \quad (3.18)$$

As shown in [38], the WD is also a member of Cohen's class whose kernel function is $\Phi(\varphi, \alpha) = 1$. While this implies that the WD satisfies the time- and frequency-marginal conditions for any input signal, the main drawback of the WD is the presence of cross-terms, similar to that of the STFT. This can be verified by substituting (3.9) into (3.18) giving

$$\begin{aligned} P_{\text{WD}}(t, \omega) = & -\frac{e^{-j\omega t}}{4} \left[\Pi_1 \delta(\omega - \xi_{l_1 b_1}) + \Pi_1 \delta(\omega + \xi_{l_1 b_1}) \right. \\ & + \Pi_2 \cos(\xi^- t) \delta\left(\omega - \frac{\xi^+}{2}\right) + \Pi_2 \cos(\xi^- t) \delta\left(\omega + \frac{\xi^+}{2}\right) \\ & \left. + 2 \left(\Pi_1 \delta(\omega) \cos(\xi_{l_1 b_1} t) + \Pi_2 \cos(\xi^+ t) \delta\left(\omega - \frac{\xi^+}{2}\right) \right) \right], \end{aligned} \quad (3.19)$$

where

$$\begin{aligned} \Pi_1 &= \sum_{l_1=1}^{\infty} \sum_{b_1=1}^{\infty} W_{l_1 b_1}^2, \\ \Pi_2 &= 2 \sum_{l_1=1}^{\infty} \sum_{b_1=1}^{\infty} \sum_{l_2=l_1+1}^{\infty} \sum_{b_2=b_1+1}^{\infty} W_{l_1 b_1} W_{l_2 b_2}, \end{aligned}$$

while $\xi^+ = \xi_{l_1, b_1} + \xi_{l_2, b_2}$ and $\xi^- = \xi_{l_1, b_1} - \xi_{l_2, b_2}$. It is noted that an impulse term at ξ_{uv} is our desired term containing the mode frequencies while all other cross-terms will cause non-zero values in the TFR even though the mode frequencies are not present in $w_m(x, y, t)$. As such, the WD is not suitable for us to compute the amplitude variance of each mode frequency.

3.3.2.3 Zak Transform

The validity of representing our signal using the Zak transform [40] [41] is now presented. Similar to the STFT and WD, the Zak transform maps a signal into the time and frequency-domains and is defined by

$$P_{\text{Zak}}(t, \omega) = \sum_{\tau=-\infty}^{\infty} w_m(x, y, t + \tau) e^{-j\omega\tau}. \quad (3.20)$$

To analyze the model using the Zak transform, (3.9) is substituted into (3.20) giving

$$|P_{\text{Zak}}(t, \omega)| = \sum_{u=1}^{\infty} \sum_{v=1}^{\infty} W_{uv} e^{-0.5\tilde{\mu}t} \left[\frac{e^{j\xi_{uv}t}}{\sqrt{4(\omega - \xi_{uv})^2 + \tilde{\mu}^2}} - \frac{e^{-j\xi_{uv}t}}{\sqrt{4(\omega + \xi_{uv})^2 + \tilde{\mu}^2}} \right]. \quad (3.21)$$

As shown in [42], time- and frequency-marginal conditions are satisfied for the Zak transform. This implies that the Zak transform can be utilized to track any instantaneous change in time and frequency which the mode frequencies are dependent on. In addition, as can be seen from (3.21), $|P_{\text{Zak}}(t, \omega)|$ consists of only a linear combination of functions relating to ξ_{uv} . Unlike STFT and WD, the absence of undesirable frequencies as well as cross-terms in the Zak transform provide a reliable estimate of the mode frequencies which, when combined with the marginal conditions, can be used for tracking the amplitude variations of the mode frequencies faithfully.

For numerical implementation, the discrete Zak transform (DZT) is used where it is operated on a signal $\mathbf{g}(n)$ of length N_g resulting in a two-dimensional time-frequency representation. Defining $\acute{n}_1, \dots, \acute{n}_R$ and $\flat_1, \dots, \flat_Q$ as the DZT time and frequency indices respectively, such that $R \times Q = N_g$, the DZT is defined as [41]

$$\mathcal{Z}_g(\acute{n}, \flat) = \sum_{i=0}^{Q-1} g(\acute{n} + iR) e^{(-j2\pi i \flat)/Q}. \quad (3.22)$$

Figure 3.4: (a) Time domain and (b) Zak transform representation of a tap on an aluminum when tapped at location $(0.1, 0.1)$ m with $Q = 256$ and $R = 16$.

The variable $\mathcal{Z}_g(\acute{n}, b)$ is computed by first defining

$$\mathcal{G}(n) = \begin{pmatrix} g(0) & g(1) & \cdots & g(R-1) \\ g(R) & g(R+1) & \cdots & g(2R-1) \\ \vdots & \vdots & \ddots & \vdots \\ g((R-1)Q) & g((R-1)Q+1) & \cdots & g(N_g-1) \end{pmatrix}, \quad (3.23)$$

such that each row is constructed from R consecutive samples in $\mathbf{g}(n)$. The DZT of $\mathbf{g}(n)$ is then given by taking the column-wise discrete Fourier transform (DFT) of $\mathcal{G}(n)$, i.e.,

$$\begin{aligned} \mathcal{Z}_g &= |\mathbf{F}\mathcal{G}(n)| \\ &= \begin{pmatrix} |\mathcal{Z}_g(\acute{n}_1, b_1)| & |\mathcal{Z}_g(\acute{n}_2, b_1)| & \cdots & |\mathcal{Z}_g(\acute{n}_R, b_1)| \\ |\mathcal{Z}_g(\acute{n}_1, b_2)| & |\mathcal{Z}_g(\acute{n}_2, b_2)| & \cdots & |\mathcal{Z}_g(\acute{n}_R, b_2)| \\ \vdots & \vdots & \ddots & \vdots \\ |\mathcal{Z}_g(\acute{n}_1, b_Q)| & |\mathcal{Z}_g(\acute{n}_2, b_Q)| & \cdots & |\mathcal{Z}_g(\acute{n}_R, b_Q)| \end{pmatrix}, \end{aligned} \quad (3.24)$$

where $\mathbf{F}$ denotes the $Q \times Q$ discrete Fourier matrix.

Figure 3.4 (a) shows an illustrative plot of $\mathbf{g}(n)$ for the case of a tap made on an aluminum surface at location $(l_m^x, l_m^y) = (0.1, 0.1)$ m while Figure 3.4 (b) shows its

corresponding plot of $\mathcal{Z}_g$ with $Q = 256$ and $R = 16$. It is noted from Figure 3.4 (b) that $\mathcal{Z}_g$ is sparse since its energy is concentrated within the lower frequencies. This observation is consistent with that of (3.9) and Figure 3.3, where it is noted that the amplitudes of mode frequencies reduce with increasing mode indices u, v . More importantly, as noted in Figure 3.4 (b), the amplitudes of these dominant frequencies vary with time and therefore, this chapter proposes to employ the time variation of these amplitudes as a classifier for the localization.

3.3.3 Time-frequency classification

In order to determine the location of the finger tap using LTM and similar to that of CC-LTM and AP-LTM, the similarity between $\mathbf{g}(n)$ and $\mathbf{h}_m(n)$ is compared. The choice of a classifier is, therefore, important since this classifier must be dependent on the position of the tap location.

As noted in Figure 3.3, the amplitude of each mode frequency varies with position. One therefore expects that the variance across time between two signals for each frequency will be minimum if these two signals are generated from the same location. To verify that this variance is indeed dependent on the location,

$$\vartheta_{uv}(t) = W_{uv} \left(\frac{e^{-0.5\tilde{\mu}t} \sin(\xi_{uv}t)}{\xi_{uv}} \right) \quad (3.25)$$

is defined. Taking the variance across time for each mode frequency u and v ,

$$\begin{aligned} \hat{\sigma}_{uv}^2 &= E(t^2) - E^2(t) \\ &= \int_0^\infty t^2 \vartheta_{uv}(t) dt - \left[\int_0^\infty t \vartheta_{uv}(t) dt \right]^2 \\ &= \frac{W_{uv}}{\xi_{uv}} \left(\frac{\tilde{\mu}^2 \xi_{uv}^2 (3 - \xi_{uv} - \tilde{\mu}) - \xi_{uv}^5 + 0.375 \tilde{\mu}^4}{\omega_{uv}^2} \right) \end{aligned} \quad (3.26)$$

is obtained, where $E(\cdot)$ is the expectation operator. Based on the mechanical model, it can be, therefore, noted that the variance across each mode frequency u, v is dependent

on $W_{uv}(x, y)$ and hence dependent on tap location (l_m^x, l_m^y) as given by (3.10).

Based on the above discussion, the Zak transform given by (3.24) is proposed to accurately represent a received signal $\mathbf{g}(n)$ in the TF domain. Since the variance across time for each mode frequency is dependent on the tap location, the variance across each row of $\mathcal{Z}_g$, i.e.,

$$\hat{\sigma}^2(k) = \frac{1}{R} \sum_{i=1}^R \left[|\mathcal{Z}_g(\hat{n}_i, b_k)| - \frac{1}{R} \sum_{j=1}^R |\mathcal{Z}_g(\hat{n}_j, b_k)| \right]^2 \quad (3.27)$$

is computed. To estimate the location of the tap, a $(Q/2) \times 1$ vector

$$\boldsymbol{\sigma}_u^2 = [\hat{\sigma}_g^2(1), \dots, \hat{\sigma}_g^2(Q/2)]^T \quad (3.28)$$

is defined, where $\hat{\sigma}_g^2(q)$ is computed up to frequency index $q = Q/2$ due to the symmetric property of the FFT magnitude spectrum of real signals. As seen from Figure 3.4 (b), the frequency changes significantly with time and in order to track changes in the amplitude for each mode frequency, $Q/2 - \mathfrak{M}$ smallest elements in $\boldsymbol{\sigma}_u^2$ is nulled out such that there are only $\mathfrak{M}$ non-zero elements in $\boldsymbol{\sigma}_u^2$. Using this sparse feature vector, the similarity between $\mathbf{g}(n)$ and $\mathbf{h}_m(n)$ is given, for the proposed Z-LTM algorithm, by

$$S_m = \sum_{i=1}^{Q/2} \left(\hat{\sigma}_g^2(i) - \hat{\sigma}_{h_m}^2 \right)^2. \quad (3.29)$$

The estimated location is then given by

$$\hat{m} = \arg \min_m S_m. \quad (3.30)$$

3.3.4 Computational and memory requirement

The computational complexity of the algorithms is now evaluated by computing the number of real multiplications each algorithm requires. As the proposed Z-LTM al-

Table 3.1: Computation complexity.

Algorithm	No. of real multiplications in detection phase	Illustration of real multiplications in detection phase
CC-LTM	$M(3N_g + 10)$	307 450
AP-LTM	$I_{AP}^2 + 2I_{AP} + I_{AP}N_g - (I_{AP} - 1)I_{AP}/2 + M \cdot I_{AP}$	12 375
Z-LTM	$2N_g \cdot \log_2 N_g + \frac{0.5(1 + M)}{\sqrt{N_g}} \cdot$	99 136

gorithm is to be implemented for a real-time HCI, only the computational complexity during the detection phase (as opposed to the pre-recording phase) will be of concern. The computational complexity is shown in Table 3.1 for an example where there are $M = 25$ tap positions, $N_g = 4096$ samples while $I_{AP} = 3$ for AP-LTM and $M = 5$ for Z-LTM. It is noted that CC-LTM, AP-LTM and Z-LTM require approximately 307 450, 12 375 and 99 136 real multiplications, respectively. This implies that AP-LTM requires 4% while Z-LTM requires 32.24% that of CC-LTM in terms of real multiplications. Although the proposed Z-LTM algorithm requires higher complexity compared to AP-LTM, it will be shown in Section 3.4 that Z-LTM achieves higher performance compared to both CC-LTM and AP-LTM.

3.4 Experiment results

Experiment results are now presented to verify the performance of the proposed algorithm in the context of a touch interface. For these LTM-based approaches, comparisons between time-domain signals or features extracted from these time-domain signals are required. As such, time alignment is necessary before computing the cross-correlation coefficients for the CC-LTM, extracting the auto-regressive coefficients for AP-LTM or constructing the feature vector σ_u^2 for the proposed Z-LTM algorithm. The signals $\mathbf{g}(n)$

and $\mathbf{h}_m(n)$ are therefore time-aligned such that the maximum of each signal corresponds to a sample index of 512, i.e., the maximum amplitudes of $\mathbf{g}(n)$ and $\mathbf{h}_m(n)$ correspond to $g(n + 511)$ and $h_m(n + 511)$, respectively. In addition, an input length of $N_g = 4096$ is used with a sampling rate of 96 kHz. There is no windowing operation is involved in calculation of ZAK transform [43].

To quantify the performance of the algorithms, the notion of classification accuracy which defines the ability to correctly identify the observations belonging to the tap location (l_m^x, l_m^y) is adopted. Defining $\mathcal{M}_c$ as the number of correct classifications and $\mathcal{M}_t$ as the total number of observations under test, the classification accuracy is defined as

$$C = \frac{\mathcal{M}_c}{\mathcal{M}_t}. \quad (3.31)$$

To further verify the robustness of the algorithms the notion of a dissimilarity matrix

$$\mathcal{D} = \begin{pmatrix} \mathcal{D}_{1,1} & \mathcal{D}_{1,2} & \dots & \mathcal{D}_{1,M} \\ \mathcal{D}_{2,1} & \mathcal{D}_{2,2} & \dots & \mathcal{D}_{2,M} \\ \vdots & \vdots & \vdots & \vdots \\ \mathcal{D}_{M,1} & \mathcal{D}_{M,2} & \dots & \mathcal{D}_{M,M} \end{pmatrix} \quad (3.32)$$

is exploited, where the i th and j th element $\mathcal{D}_{i,j} = \sqrt{(\mathcal{F}_i - \mathcal{F}_j)^T (\mathcal{F}_i - \mathcal{F}_j)}$ denotes the distance measure between two instances of feature vectors for taps made at position indices i and j . Therefore, for CC-LTM and AP-LTM, $\mathcal{F}_i = \mathbf{g}(n)$ and $\mathcal{F}_i = \mathbf{a}(n)$, respectively, where $\mathbf{a}(n) = [a(1), \dots, a(I_{AP})]^T$ is a vector of prediction coefficients corresponding to the i th position index. Similarly for Z-LTM, $\mathbf{F}_i = \sigma_u^2$ corresponds to the i th position index.

Figures 3.5 (a) and (b) illustrate a plot of $\mathcal{D}$ for CC-LTM and Z-LTM respectively, where five taps are made for each of the twenty-five tap locations giving $M = 125$. For this illustrative example, the first set of five taps corresponds to the first location while the second set of five taps corresponds to the next location. It is noted from

Figure 3.5: Dissimilarity matrix $\mathcal{D}$ for (a) CC-LTM and (b) Z-LTM for an experimental data set of aluminum.

these plots that $\mathcal{D}$ exhibits greater disparity between the off-diagonal elements and the diagonal elements for Z-LTM compared to CC-LTM. This large disparity between the diagonal/off-diagonal elements for Z-LTM implies that different instances of a received signal corresponding to the same tap location will exhibit a low distance when σ_u^2 is used as a feature vector compared to when $\mathbf{g}(n)$ is used. It is therefore expected that the Z-LTM approach achieves a higher source localization accuracy than CC-LTM.

To quantify the difference in disparity between diagonal and off-diagonal elements in $\mathcal{D}$, the feature identity ratio

$$\chi = \frac{\sum_i \mathcal{D}_{i,i}}{\sum_{i,j} \mathcal{D}_{i,j}}, \quad i \neq j \quad (3.33)$$

is defined. This measure therefore allows us to quantify, for the same tap location, the robustness of the algorithm to any inconsistencies in the feature vectors brought about by external factors such as background noise. It is therefore expected that a low value of χ denotes lower dissimilarity between features corresponding to the same tap locations which in turn implies better localization performance compared to one with a higher χ .

Figure 3.6: Experimental setup for collection of data for aluminum.

3.4.1 Experiment results using real materials

To verify the performance of the proposed algorithm, an experiment was conducted on aluminum, glass and plywood with dimensions $(L_x, L_y, L_z) = (0.6, 0.6, 0.005)$ m, $(0.6, 0.6, 0.004)$ m and $(0.75, 0.6, 0.005)$ m, respectively. A Murata PKS1-4A1 shock sensor is used to measure the flexural vibrations caused by the taps. Twenty-five tap locations are arranged in a grid of 5×5 array as shown in Figure 3.1 where the distance between the second, third, fourth and fifth row of tap positions with respect to the first row is 1, 3, 6 and 10 cm, respectively. The tap locations are also separated column-wise in the same manner. The first tap location is positioned at $(l_1^x, l_1^y) = (10, 10)$ cm with respect to sensor position. As seen in Figure 3.1, the sensor is denoted by with a ‘o’ and each of the tap locations is denoted by a ‘•’. The tap locations are sequentially tapped and received signals are digitized using a PreSonus FireStudio at 96 kHz with 16-bit resolution. Each position is tapped five times to evaluate the feature identity ratio χ and to test the ability of the classifier to identify taps of the same tap location. Results of classification accuracy and feature identity ratio from these experiments are presented in Figures 3.7 - 3.9 for the aluminum, glass and plywood, respectively. Although a human finger tap will not deviate the results significantly, a

Figure 3.7: (a) Classification accuracy C and (b) feature identity ratio χ for aluminum.

mechanized tapper is employed for our 125 taps for consistency. This setup is shown in Figure 3.6.

For each of these plots, the performance of AP-LTM is studied by varying the prediction order I_{AP} . The number of non-zero elements $\mathfrak{M}$ in $\hat{\sigma}_u^2$ for Z-LTM is also varied. In addition, the performance of Z-LTM was studied for various values of Q and R . Since $Q \times R = N_g$, increasing the value of Q will increase the spectral resolution at the expense of time resolution. It is therefore expected that the performance of Z-LTM will depend on the ratio between time and frequency resolution samples $\Gamma = \sqrt{Q/R}$. The upper limit on N_g is restricted due to short duration of vibration signal. Therefore, with $\Gamma = 4$, the number of frequency-bins is 16 times that of time samples.

The performance of the algorithms in terms of C for an aluminum plate with various non-zero feature elements $\mathfrak{M}$ in σ_u^2 and prediction order I_{AP} is shown in Figure 3.7 (a). The CC-LTM algorithm achieves an accuracy of $C = 97.6\%$ and is independent of $\mathfrak{M}$ and I_{AP} while the accuracy of AP-LTM varies with the prediction order I_{AP} such that the highest accuracy of $C = 85\%$ for $I_{AP} = 3$. In addition, the proposed Z-LTM algorithm achieves nearly 100% accuracy for various Γ . For each Γ , the accuracy increases with the number of non-zero features $\mathfrak{M}$ since a larger $\mathfrak{M}$ implies that more features are taken into account during classification. For the case of $\Gamma = 16$, the lowest value of $\mathfrak{M}$ which gives the highest accuracy of 100% is $\mathfrak{M} = 9$ for the Z-LTM algorithm. Figure 3.7 (b)

Figure 3.8: (a) Classification accuracy C and (b) feature identity ratio χ for glass.

shows the robustness of the algorithms in terms of the feature identity index χ . It is noted that the proposed Z-LTM algorithm achieves a low value of χ for all cases of $\Gamma = 4, 16, 32$ compared to that of AP-LTM. This implies that there is a greater disparity between the diagonal and off-diagonal elements of $\mathcal{D}$ which translates to Z-LTM being more robust to any inconsistencies in the feature vectors for the same tap location. As before, it is noted that AP-LTM achieves good localization performance when $I_{AP} = 3$.

The classification accuracy of the algorithms for a glass plate is shown in Figure 3.8 (a). As can be seen, AP-LTM achieves the highest $C = 76.8\%$ when $I_{AP} = 3$, similar to the case of aluminum. For clarity of presentation, the performance of Z-LTM for $\Gamma = 4, 16, 32$ are shown and it is noted that, for each Γ , C increases with $\mathfrak{M}$. It is interesting to note that the performance of Z-LTM reduces with increasing Γ and this implies that for this glass material, a good time resolution is desirable. The performance of Z-LTM achieves that of CC-LTM at $C = 99.2\%$ when $\Gamma = 4$ and $\mathfrak{M} = 5$. However, as shown in Table 3.1, this comparable performance is achieved with a lower complexity for Z-LTM compared to CC-LTM. Figure 3.8 (b) illustrates the variation of χ with $\mathfrak{M}$ and I_{AP} for the glass plate. It is noted, as before, that AP-LTM is robust to inconsistencies in the feature vectors for the same tap location with $I_{AP} = 3$. As seen from the lower values of χ , the proposed Z-LTM algorithm achieves higher robustness for the cases of $\Gamma = 4$ and 16.

Figure 3.9: (a) Classification accuracy C and (b) feature identity ratio χ for plywood.

The same experiment was conducted on a piece of plywood, an anisotropic material. The performance in terms of C and χ are illustrated in Figures 3.9 (a) and (b), respectively. As can be seen in Figure 3.9 (a), CC-LTM achieves an accuracy of $C = 73.6\%$ while AP-LTM achieves an accuracy of $C = 39.2\%$ when $I_{AP} = 3$. The accuracy of Z-LTM is consistently higher, for the cases of $\Gamma = 16$ and 32 , reaching $C = 94\%$, compared to CC-LTM and AP-LTM. For the case where $\Gamma = 4$, C reduces to 67% implying that a high frequency resolution is required for plywood. Figure 3.9 (b) shows the variation of χ with $\mathfrak{M}$ and I_{AP} for Z-LTM and CC-LTM, respectively. This result is consistent with the fact that AP-LTM achieves the highest robustness to any inconsistencies in the feature vectors for the same tap location when $I_{AP} = 3$. Similar to the case of glass and aluminum, the proposed Z-LTM achieves the highest robustness for the cases of $\Gamma = 4, 16$ and 32 .

3.4.2 Variation of accuracy with Γ and $\mathfrak{M}$ for Z-LTM

The variation of C with Γ and $\mathfrak{M}$ is now illustrated for Z-LTM. Figures 3.10 (a) - (c) show results for the aluminum, glass and plywood, respectively. As described in Section 3.4.1, a higher value of Γ implies higher frequency resolution. Comparing these figures, it is noted that, for each $\mathfrak{M}$, the accuracy increases initially with Γ and thereafter reduces

Figure 3.10: Variation of C with M and Γ for (a) aluminum, (b) glass and (c) plywood with increasing Γ for aluminum and glass. This reduction in C beyond a certain value of Γ implies that a high frequency resolution is no longer able to ensure the uniqueness of the frequency-bins since the time-resolution is lost for high Γ values. For the case of plywood shown in Figure 3.10 (c), the modest reduction in C for increasing Γ implies that it is important to have high frequency resolution to achieve high localization accuracy. For all the cases shown, a higher M achieves a high localization accuracy since more features are used in the feature vector σ_u^2 . The selection range of value of M depends on N_g and Γ . The upper limit of M depends on the processing and storage limit of the processing system. To select the lower limit, users have to analyze a few signals from the materials which they operate and select based on their desired C or χ .

Further insights into the performance of Z-LTM for these materials can be gained by illustrating $|Z_g|$ for aluminum, glass, and plywood as shown in Figures 3.11 (a), (b) and (c), respectively. For these illustrative examples, $Q = 256$ and $R = 16$ is chosen giving $\Gamma = 4$. For all these cases, a peak is observed at approximately 2.5 kHz corresponding to the resonant frequency brought about by the transfer function of the Murata sensor. Comparing Figure 3.11 (a) with that of Figures 3.11 (b) and (c), it can

Figure 3.11: Zak transform based time-frequency representation, $|Z_g|$, of $g(n)$ using $Q = 256$, $R = 16$ for (a) aluminum, (b) glass and (c) plywood when tapped at point $(0.14, 0.1)$.

be noted that the TFR for aluminum decays less significantly with time compared to that of glass and plywood while the TFR for plywood decays most significantly. This implies that for plywood, a higher frequency resolution is desirable and this observation is consistent with a high value of Γ as shown in Figure 3.10 (c).

3.4.3 Illustration of touch interface

To further illustrate the localization performance of the algorithms, a tap location where Z-LTM performs the worst compared to other test locations on a piece of plywood is chosen. Five taps were made using mechanical tapper at the chosen location denoted by ‘●’ shown in Figure 3.12 are used for localization. The estimated tap locations for the CC-LTM, AP-LTM and Z-LTM are denoted by ‘▽’, ‘○’ and ‘*’, respectively. As can be seen from Figure 3.12, none of the estimated positions by CC-LTM or AP-LTM correspond to the true position. The closest position estimated by CC-LTM and AP-LTM is 1 cm (two instances) and 3 cm (one instance) away. The Z-LTM achieves consistently good localization accuracy with three out of five taps being localized accurately. For

Figure 3.12: Illustration of estimated positions by CC-LTM, AP-LTM and Z-LTM which are marked using ‘▽’, ‘o’ and ‘*’, respectively. The true source location is marked as ‘•’. The sensor is located at (0, 0).

this illustrative example, CC-LTM and AP-LTM achieves 0% classification accuracy while Z-LTM is able to achieve a 60% classification accuracy. Since plywood is a highly damped surface, the low signal-to-noise ratio is the reason for the low performance of CC-LTM and AP-LTM, where as the Z-LTM looks for the features in the time-frequency domain, and is less immune to signal amplitude.

3.5 Chapter Summary

A time-frequency classification algorithm for source localization on solid surfaces using a single sensor has been presented in this chapter. The mathematical model of wave propagation that is generated by an impact on the interactive surface is exploited for the proposed LTM method. This inter-disciplinary study gains insights into the uniqueness of signal features obtained by impact on each position on a given surface. The time-frequency analysis is exploited to track amplitude variations of mode frequencies with respect to time. This chapter further illustrates the use of Zak transform for faithful

time-frequency domain representation of a tap signal. More importantly, the amplitude variance across time for each frequency-bin is shown to be unique for each location. This in turn allows us to exploit this variance as a unique time-frequency classifier for the localization of the finger-tap in the context of HCI. Since Z-LTM depends on the time-frequency variation of mode frequencies generated by mechanical wave propagation, the algorithm as such is less sensitive to additive noise and low signal-to-noise ratio signals. This is reflected in an improvement of 2.4% and 20.4% for the case aluminum and plywood, respectively by Z-LTM when compared with CC-LTM. The significance of performance is relevant since the computational complexity of Z-LTM is only 32.24% of CC-LTM. Although the proposed Z-LTM algorithm requires higher complexity compared to AP-LTM, it achieves high performance compared to both CC-LTM and AP-LTM. Although LTM-based algorithms require only a single sensor to work, these methods are impractical for detection of large number of tap locations due to the training of known locations. To address this challenge, algorithms proposed in Chapter 4, 5 and 6 exploit multi-sensor architecture for source localization of a large range of distinguishable tap locations with minimal setup/calibration. These chapters utilizes inter-relationship between time-frequency spectrum of each channel to perform source localization in dispersive environment. Here preserving the time-frequency nature of original modes is no longer necessary since same transform is operated on each pair of signals.

Chapter 4

A Time-Difference-Of-Arrival

Algorithm for HCI

4.1 Introduction

This chapter presents a technique for impact localisation using time-difference-of-arrival (TDOA) between two spatially separated sensors - a commonly used method that has been applied for acoustic source localization. The main advantage of this method is that, compared to LTM-based methods such as described in Chapter 3, it does not require training of known locations. This implies that such algorithm is able to achieve a large range of distinguishable tap locations with minimal setup/calibration. The generalized cross-correlation (GCC) algorithm has been one of the most popular algorithms to estimate the TDOAs [44]. In the current context of touch interface, this technology can be applied by employing an array of low-cost surface-mounted sensors as well as developing source localization algorithms to estimate the position of each tap. However, the performance of existing TDOA methods is poor since it does not take into account effects of wave dispersion as described in Chapter 1. In the presence of dispersion, which occurs, e.g., in a solid medium where the propagation speed varies across frequencies, it is expected that conventional TDOA estimation algorithms fail. This degradation in

performance is expected since the received signals are distorted - a direct effect of wave dispersion [45].

Since the input signals for this research are impulsive in nature, the TDOA between each sensor pair can be derived from the estimated value of time-of-arrival (TOA) corresponding to each sensor. The TOA can be identified as the instant from which a sudden change in the statistical nature of signal happens. Several techniques have been proposed in various engineering fields to detect the change-point or onset time of a signal. In the seismic field similar approaches to detect seismic events have been proposed in [46], [47] and [48], while other techniques have been proposed in the biomedical field that detects onset of muscle activity [49], distinguishing heart sound from lung sound [50], and the use of singular spectrum analyses for change point detection as described in [51]. Another class of change-point detection using subspace identification have been proposed in [52]. In addition, damage detection for structural health monitoring using acoustic emission has been proposed in [53], [54] and [55]. Several fundamental statistical methods have been described in [56] and probabilistic modeling [57], log-transform [58], sub-Gaussian fitting [59], support-vectors [60] as well as for the application of climate time series [61] also deal with similar problem statement. Most of these methods depend on parametric modeling of signal generation, while others propose the use of a hard threshold to achieve onset detection.

In this chapter a new method that employs the Kullback-Leibler (KL) discrimination information for localizing a single tap made by a metal stylus on various flat surfaces is proposed. This is achieved through the use of the phase transit time of the vibration signal. Similar to Chapter 3, properties of wave propagation in a solid medium is exploited which, in turn, allows us to understand the frequency spectrum of vibrating signals. As will be seen, this insight will allow us to employ the KL distance effectively for source localization on solids.

Figure 4.1: Variation in width of Gaussian envelope $f(0, t)$ with of propagation distance r_i .

4.2 Review of flexural vibration of plate

One of the phenomena that deteriorate elastic waves in solids during propagation is dispersion. Dispersion occurs when the velocity of wave propagation is wavelength dependent. The detrimental effect of dispersion on multitone signals is the modification of signal spectrum and, in particular, its phase. As a result of this dependence, each frequency has a different phase shift and therefore a pulse propagating through a dispersive media will result in a sweep [32]. As mentioned in (2.43), the velocity of flexural wave propagation in plate is a non-linear function of ω . Each frequency component will therefore travel with its own velocity, making the propagation of input signals dispersive. To analyze the change in a pulse width due to signal propagation, a Gaussian narrow-band pulse signal centered at ω_0 with an Gaussian envelope given by

$$f(0, t) = e^{-\frac{t^2}{2\tau_0}} \quad (4.1)$$

will be employed, where τ_0 is the time-domain width of the pulse. Figure 4.1 illustrates that the pulse width increases with distance of propagation r_i [62] which results in the spreading of time-domain pulse signal.

In common terminology, wave speed, phase velocity and group velocity define the speed of wave propagation [25]. Wave speed is defined as the ratio of distance travelled

Figure 4.2: Diagram representing difference between transit time, phase transit time and centroid transit time.

by the wave to the time required during propagation. On the other hand, the transit time is defined by the time from which the signal amplitude exceeds a pre-defined amplitude threshold as shown in Figure 4.2 for a typical signal generated by a metal stylus on a piece of aluminum. Since signal amplitude is used, determination of the transit time is particularly challenging in the presence of noise or background vibration. Although cross-correlation based TDOA estimation is, to some extent, robust to additive noise, its performance reduces significantly in the presence of wave distortion caused by dispersion.

As opposed to wave speed, phase and group velocities are functions of the phase point transit time of individual peaks (frequencies) within a signal and the centroid transit time of the signal as shown in Figure 4.2 [25]. Phase velocity is then calculated by dividing the inter-sensor distance with the change in phase point transit time. The group velocity, on the other hand, is computed by dividing the inter-sensor distance with the change in centroid transit time. For non-dispersive wave propagation, phase velocity and group velocity are equivalent. However, this is not necessarily true for dispersive wave propagation.

For dispersive wave propagation, the individual peaks within the wave packet will vary relative to the centroid as the wave propagates through a medium. Direct application of an amplitude threshold may not be effective for estimating the transit time of

Figure 4.3: Diagram of a touch interface with $\bullet$ representing a predefined point.

a distorted waveform particularly when the source-to-sensor distance increases. Interference from edge reflections with the wave packet may also lead an incorrect estimate of the wave packet centroid and therefore pose challenges in defining the trailing edge of the wave packet. Estimations of phase velocities, on the other hand, are often more accurate since phase points are easier to locate or estimate.

With reference to Figure 4.3, the coordinate of a tap location is defined as (l_m^x, l_m^y) , where $m = 1, \dots, M$ is the tap index while M denotes the total number of possible tap locations. Similar to that described in Chapter 3, the sensor is an accelerometer where its output is dependent on the surface's transfer function $\mathbf{W}_m(x, y, s)$ and that of the accelerometer [2] [63]. Consider the following parameters for the plate: the absorption coefficient μ , the density $\mathfrak{h}$, the thickness of the plate L_z , the stiffness of the plate $\tilde{D} = EL_z^3/12(1 - \nu^2)$, the Young's modulus E and the Poisson's ratio ν . Using the well-known Kirchoff's hypothesis for linear, elastic small-deflection theory of thin supported plates and by considering a Dirac delta forcing function as per (2.40), the surface transfer function can be expressed as

$$\mathbf{W}_m(x, y, s) = \sum_{u=1}^{\infty} \sum_{v=1}^{\infty} \frac{4}{\mathfrak{h}L_xL_yL_z} \frac{W_{uv}(x, y)}{s^2 + \tilde{\mu}s + \omega_{uv}^2}, \quad (4.2)$$

where s is the Laplacian variable and

$$\begin{aligned} W_{uv}(x, y) &= \sin(\tilde{\beta}x) \sin(\beta y) \sin(\tilde{\beta}l_m^x) \sin(\beta l_m^y), \\ \omega_{uv} &= (\tilde{\beta}^2 + \beta^2) \sqrt{\frac{\hbar D}{\hbar L_z}}, \end{aligned}$$

such that $\tilde{\beta} = \pi u / L_x$, $\beta = \pi v / L_y$, ω_{uv} is the mode frequency while $\tilde{\mu} = \mu / (\hbar L_z)$ is the reduced coefficient of absorption and that $u, v \in \mathbb{Z}^+$ are the modes of wave propagation.

Equation (4.2) implies that each frequency ω_{uv} increases with u, v , and that the contribution of that mode to $|W_m(x, y, s)|$ reduces with respect to mode $u = 1, v = 1$. This, in turn, limits the frequency distribution of the sensor output to a specific bandwidth with certain frequencies having higher contributions. This frequency distribution will be different from that of background noise, which is assumed to have normal distribution in nature. This chapter, therefore, exploits such difference in the distribution of frequency spectrum between noise and signal for detecting the phase transit time. Similar to Chapter 3, the sensor's transfer function need not be taken into account since it affects the signal output due to a tap as well as noise equally.

4.3 Review of the GCC source localization method

The generalized cross-correlation (GCC) is widely used in acoustic TDOA estimation. It unifies various correlation-based algorithms into a single framework and it improves TDOA estimation through the use of prefilters [44]. In GCC, the TDOA, $\hat{\tau}_{ij}$, between sensors i and j is estimated based on

$$\hat{m}_{ij} = \arg \max_m (\Psi_{ij}[m]), \quad (4.3)$$

where m_{ij} denotes time lag index and

$$\Psi_{ij}[m] = \sum_{b=0}^{\mathcal{B}-1} \Theta[b] \Gamma_{ij}[b] e^{j2\pi mb/\mathcal{B}} \quad (4.4)$$

is the generalized cross-correlation function. Defining b as the frequency-bin index, the variable $\Gamma_{ij}[b] = \underline{g}_i[b] \underline{g}_j^*[b]$, $b = 0, \dots, \mathcal{B} - 1$, defines the cross-spectrum estimated between Sensor i and Sensor j , where $\underline{g}_i[b]$ is the discrete Fourier Transform (DFT) of $g[n]$ and $\mathcal{B}$ is the length of the DFT. The variable $\Theta[b]$ is the weighing function which is often referred to as a prefilter. There are different member algorithms in the GCC family depending on which weighting function is chosen. A widely used prefilter for GCC is the phase transform (PHAT) where

$$\Theta_{\text{PHAT}}[b] = 1 / |\Gamma_{ij}[b]|.$$

It has been shown in [64] that the PHAT prefilter achieves good performance in the presence of low noise and multipath environment.

4.4 Review of change-point analysis using Kullback-Leibler discrimination information

A method for detecting abrupt changes to the time-varying power spectrum of a time series has been proposed [65], where it assumes that the time series is locally stationary in the neighborhood of the change point. Consider a sequence of observations $\mathbf{g}(n) = [g(1), \dots, g(N_g)]^T$ where N_g is the total length of acquired signal. If the initial section of the observations $[g(1), g(2), \dots, g(\check{n})]$ is derived from some process while the remaining $[g(\check{n} + 1), g(\check{n} + 2), \dots, g(N_g)]$ is derived from another process, a change point can therefore be defined between $g(\check{n})$ and $g(\check{n} + 1)$. Let $\mathbf{g}(n')$ be a length $\mathfrak{N}$ stationary time sequence frame within $\mathbf{g}(n)$. Assuming suitable regularity conditions [65], the

periodogram at frequency $\omega \in [-\pi, \pi]$ is given by

$$\Xi_g(\hat{n}, \omega) = \frac{1}{2\mathfrak{N}\pi} \left| \sum_{n=\hat{n}}^{\hat{n}+\mathfrak{N}-1} g(n) e^{-jn\omega} \right|^2. \quad (4.5)$$

One of the main issues with the use of periodogram $\Xi_g(\hat{n}, \omega)$ is leakage, where the variance at the frequency of interest leaks into other frequencies in the estimated spectrum. The net effect is the presence of erroneous peaks in the estimated spectrum. The objective of tapering is therefore to reduce such leakage and it consists of altering the ends of the mean-adjusted time series so that their magnitudes reduce gradually to zero. A frequently used taper function is the split cosine bell given by [66],

$$u_{\mathfrak{d}}(n) = \begin{cases} \frac{1}{2}(1 - \cos(\frac{\pi(n-0.5)}{\hat{n}})), & 1 \leq n \leq \hat{n}; \\ 1, & \hat{n} + 1 \leq n \leq \mathfrak{N} - \hat{n}; \\ \frac{1}{2}(1 - \cos(\frac{\pi(\mathfrak{N}-n+0.5)}{\hat{n}})) & \mathfrak{N} - \hat{n} + 1 \leq n \leq \mathfrak{N}, \end{cases} \quad (4.6)$$

where $\mathfrak{d}$ is the proportion of data desired to be tapered, n is the time index, $\mathfrak{N}$ is the total length of signal frame and $\hat{n} = \mathfrak{N}\mathfrak{d}/2$. A value of $\mathfrak{d} = 0.10$ is suggested [66], which implies that 5% of the signal is tapered on each end.

The periodogram is a wildly fluctuating estimate of the spectrum with high variance. For a stable estimate, periodogram is smoothed over an odd number r of equally spaced frequencies to obtain spectral estimates

$$\hat{\Lambda}_g(\hat{n}, \omega) = \sum_{\hat{\omega}=-(r-1)/2}^{(r+1)/2} \frac{1}{\bar{u}_r(\hat{\omega})} \Xi_g(\hat{n}, \omega + \hat{\omega}), \quad (4.7)$$

where $\bar{u}_r(\hat{\omega})$ is the weight for respective bins. A modified Daniell window which also reduces leakage due to the influence of variance at non-Fourier frequencies on Fourier frequencies has been proposed [66] [67]. The weighting function of the modified

Daniell window of odd length r is given by

$$\bar{u}_r(\omega) = \begin{cases} \frac{1}{2(r-1)} & \text{for } \omega = 1 \text{ or } \omega = r; \\ \frac{1}{(r-1)} & \text{for } \omega \text{ otherwise.} \end{cases} \quad (4.8)$$

Instead of the power spectrum, for a non-stationary series, the power spectrum is computed using a window centered at every time-frequency point giving $\Lambda_g(n, \omega)$ which varies in both time and frequency.

The KL discrimination information is defined as the expected log likelihood ratio of any two distributions. Therefore, such discrimination information is a measure of how far apart the two distributions are. The authors of [68] showed that the KL discrimination information for two spectra $\Lambda_g(n, \omega)$ and $\Lambda_{g'}(n, \omega)$ can be computed using

$$I(g, g', n) = \sum_{\omega} \left(\frac{\Lambda_g(n, \omega)}{\Lambda_{g'}(n, \omega)} - \log\left(\frac{\Lambda_{g'}(n, \omega)}{\Lambda_g(n, \omega)}\right) - 1 \right). \quad (4.9)$$

Based on (4.9), the authors of [65] derived a test metric for quantifying the symmetrized Kullback-Leibler discrimination information distance between two spectra given by

$$D(g, g', n) = \frac{1}{\mathfrak{N}} \sum_{\omega} \left(\frac{\Lambda_g(n, \omega)}{\Lambda_{g'}(n, \omega)} + \frac{\Lambda_{g'}(n, \omega)}{\Lambda_g(n, \omega)} \right), \quad (4.10)$$

where $\mathfrak{N}$ is the total length of frame.

4.5 Proposed phase transit time estimation

From Section 4.2, it is expected that the frequency distribution of signal due to flexural vibration and that of background noise will be significantly different in the spectral domain. Utilizing this frequency disparity, the main contribution of this chapter is in estimating the change point of time series based on Kullback-Leibler discrimination information of each received sensor signal. The statistical change point of time-series

can be identified as an estimate of time of arrival, and thus the localization problem is formulated in terms of time of arrival.

Suppose for every $n, \mathfrak{N} \leq n \leq (N_g - \mathfrak{N})$, the periodogram $\Lambda_-(n, \omega)$ is estimated from the signal frame $\mathbf{g}_-(n) = [g(n - \mathfrak{N} + 1), \dots, g(n)]^T$ and $\Lambda_+(n, \omega)$ using $\mathbf{g}_+(n) = [g(n + 1), \dots, g(n + \mathfrak{N})]^T$. These signal buffers $\mathbf{g}_-(n)$ and $\mathbf{g}_+(n)$ are referred as left and right buffers respectively in this chapter. The KL distance $D(n)$ between the two distributions corresponding to $\Lambda_-(n, \omega)$ and $\Lambda_+(n, \omega)$ is then estimated. The sample time n at which maximum $D(n)$ is obtained will show maximum frequency discrepancy between $\mathbf{g}_-(n)$ and $\mathbf{g}_+(n)$ frames. It can therefore be assumed that the point corresponding to the maximum frequency disparity is the phase transit point $\hat{n}_p$ since, by definition, this point corresponds to the maximum change in frequency distribution.

To estimate $\hat{D}(n)$ the signal buffers $\mathbf{g}_-(n)$ and $\mathbf{g}_+(n)$, where $\mathfrak{N} \leq n \leq (N_g - \mathfrak{N})$, are normalized and the left and right buffers are de-trended to achieve zero mean. The de-trending process allows to remove the effect of dc-component from calculation of spectral estimates, since this algorithm utilizes Kullback-Leibler discrimination information of only non-zero mode frequencies. Each of the buffers is then multiplied with the taper window using (4.6). The spectral estimates from periodograms $\Lambda_-(n, \omega)$ and $\Lambda_+(n, \omega)$, which corresponds to left and right buffer, are then estimated using (4.5). Each periodogram is further smoothed using the modified Daniel window of odd size r using weights from (4.8) to obtain $\hat{\Lambda}_-(n, \omega)$ and $\hat{\Lambda}_+(n, \omega)$ respectively.

Finally, the KL discrimination information is computed using the distance metric

$$\hat{D}(n) = \frac{1}{\mathfrak{N}} \sum_{\omega} \left(\frac{\hat{\Lambda}_-(n, \omega)}{\hat{\Lambda}_+(n, \omega)} + \frac{\hat{\Lambda}_+(n, \omega)}{\hat{\Lambda}_-(n, \omega)} \right). \quad (4.11)$$

The phase transit point is finally estimated as

$$\hat{n}_p = \arg \max \hat{D}(n), \quad \mathfrak{N} \leq n \leq (N_g - \mathfrak{N}).$$

To estimate the TDOA, the variable $\hat{n}_{p_{ij}} = \hat{n}_{p_j} - \hat{n}_{p_i}$ is defined as the transit time

Figure 4.4: Spectral estimation calculation of received sensor signal where (a) denotes sensor signal $\mathbf{g}(n)$ which is divided into (b) $\mathbf{g}_-(n)$ and (c) $\mathbf{g}_+(n)$ to estimate (d) $\Lambda_-(n, \omega)$ and (e) $\Lambda_+(n, \omega)$ respectively.

difference (in samples) between Sensor j and Sensor i . This variable corresponds to the time delay $\hat{\tau}_{p_{ij}}$ between the two sensors. The sum of square errors between $\hat{\tau}_{p_{ij}}$ and that obtained using the initial guess positions $(l_{\text{src}_0}^x, l_{\text{src}_0}^y)$ can then be represented by

$$\sum_{i,j} \left(\frac{\sqrt{\hat{\mathbf{d}}^T(i)\hat{\mathbf{d}}(i)} - \sqrt{\hat{\mathbf{d}}^T(j)\hat{\mathbf{d}}(j)}}{c_v} - \hat{\tau}_{p_{ij}} \right)^2, \quad (4.12)$$

where $\hat{\mathbf{d}}(i) = [\hat{l}_{\text{src}}^x, \hat{l}_{\text{src}}^y]^T - [l_i^x, l_i^y]^T$ denotes the distance between coordinates of estimated tap position $(\hat{l}_{\text{src}}^x, \hat{l}_{\text{src}}^y)$ and the i th known sensor position (l_i^x, l_i^y) . The tap location is estimated by minimizing the cost function defined by (4.12) iteratively using the Levenberg-Marquardt (LM) algorithm [69]. The LM algorithm converges to a minimum in the case of correct estimate of TDOA. The empirical value for c_v is computed using the calibration method described in [2] based on data from known source positions.

4.6 Experiment results

To quantify the performance of the algorithm in terms of accuracy, the root-square error (RSE) and the root-mean-square error (RMSE) is adopted given, for the m^{th} tap, by

$$\text{RSE}(m) = \sqrt{(l_m^x - \hat{l}_m^x)^2 + (l_m^y - \hat{l}_m^y)^2}, \quad (4.13)$$

$$\text{RMSE} = \sqrt{\frac{1}{M} \sum_{m=1}^M ((l_m^x - \hat{l}_m^x)^2 + (l_m^y - \hat{l}_m^y)^2)}, \quad (4.14)$$

where $1 \leq m \leq M$, M is the total number of taps used in the experiment, and $[l_m^x, l_m^y]$ is actual tap position while $[\hat{l}_m^x, \hat{l}_m^y]$ is estimated tap position at m^{th} index.

To verify the performance of the proposed method, experiments were conducted on aluminum and glass with dimensions $(L_x, L_y, L_z) = (0.6, 0.6, 0.005)$ m and $(0.6, 0.6, 0.004)$ m, respectively. Similar to that of Chapter 3, the Murata PKS1-4A1 shock sensor is used to detect the flexural vibrations caused by the taps. A frame length

Figure 4.5: Diagram of a touch interface where the sensor and test tap coordinates are denoted by ‘•’ and ‘×’ respectively.

Figure 4.6: Spectrogram of a sensor output due to a metal stylus tap on an aluminum plate surface.

of $\mathfrak{N} = 256$ and smoothing factor $r = 13$ are used to estimate $\hat{\Lambda}_-(n, \omega)$ and $\hat{\Lambda}_+(n, \omega)$. Figure 4.6 shows the spectrogram of a sensor output due to a stylus tap on an aluminum plate. It is noted that beyond frequency bin $b > 30$ (corresponding to a frequency of 11 kHz), the energy of the signal is negligibly small. This is consistent with the frequency response of the Murata PKS1-4A10 [70] used in our experiment. As per sensor specifications frequency-bin indices 14 to 20 are selected for the computation of $\hat{D}(n)$. The values of $c_v = 1540$ m/s and $c_v = 1440$ m/s for aluminum and glass, respectively, are determined during calibration where taps were made at known location and solving (4.12).

Figure 4.7: (a) Received signal at Sensor S_1 during first tap at tap position T_1 and (b) estimated KL discrimination information distance $\hat{D}(n)$ of aluminum.

Nine tap locations are arranged in a grid of 3×3 array as shown in Figure 4.5, where the separation between the points along each row and column is 10 cm. The first tap location is positioned at (20, 20) cm. The sensor is denoted by ‘•’ while each of the tap location is denoted by a ‘×’. The tap locations are sequentially tapped using a metal stylus and received signals are digitized at 96 kHz with 16-bit resolution. Each position is tapped five times (which implies $M = 9 \times 5 = 45$) to evaluate the mean deviation of estimated tap location from actual tap location. Figure 4.7 (a) shows an example of the received signal received by Sensor S_1 when a tap is made at location T_1 . The estimated KL discrimination information distance $\hat{D}(n)$ computed using (4.11) is plotted as shown in Figure 4.7 (b). This result shows that the KL discrimination information distance is able to discriminate the noise from a tap which can then be used to detect the phase transit point $\check{n}_p$.

To verify the localization performance of the proposed algorithm, the RSEs for the forty-five taps are presented in Figures 4.8 (a) and 4.8 (b) for the aluminum and glass, respectively. For comparison, performance of the proposed algorithm as well as that of localization using the well-known GCC-PHAT method is plotted. For clarity

Figure 4.8: Illustration of root square error of each tap index $T_1 - T_9$ of (a) aluminum and (b) glass (neglecting outliers for GCC-PHAT).

of presentation and for, the case of GCC-PHAT, RSE value ≥ 0.2 m are considered as outliers whose values are omitted from the computation of RSE and only RMSE values is considered. Results presented show that the proposed KL-based localization technique consistently outperforms that of GCC-PHAT. The RMSE across the taps were 0.96 cm for the proposed algorithm compared to 8.17 cm for the GCC algorithm in the case of the aluminum. For the case of glass, the RMSE was 2.13 cm for the proposed algorithm compared to 7.86 cm for the GCC algorithm. Poor localization performance is expected for GCC-PHAT due to non-linear distortion of propagating vibration signals brought about by dispersion and multipath reflections from the edges. Although Figures 4.8 (a) and (b) may imply that GCC-PHAT outperforms the proposed KL based algorithm for midpoint T_5 , further analysis show that this result is insufficient to prove the usability of GCC-PHAT for points away from edges. Tapping at the mid-point of the plate results in signal propagating through equal distances on isotropic medium to reach every sensors. This results in similar dispersion for every sensor signal, giving rise to zero delay for GCC-PHAT. As a result, such taps results in lower RSE compared to taps located off-center

Feasibility of employing the proposed algorithm for human-computer touch interface is further illustrated. Figures 4.9 (a) and (b) illustrate the performance of the proposed

Figure 4.9: Source localization using our proposed algorithm on (a) aluminum and (b) glass where the sensor coordinates, actual tap locations and estimated tap locations are denoted by ‘■’, ‘•’ and ‘×’, respectively.

localization algorithm employing the KL discrimination distance on aluminum and glass, respectively. In these experiments, nine tap locations were chosen and each location were tapped five times using a human finger. The sensor coordinates, actual and estimated tap locations are denoted by ‘■’, ‘•’ and ‘×’, respectively. As can be seen from these results, the estimated positions are very close to the true position.

4.7 Chapter Summary

A source localization algorithm is proposed using the TDOA between two spatially separated sensors - a commonly used method that has been applied for acoustic source localization. The main advantage of this method is that, compared to LTM-based methods such as described in Chapter 3, it does not require training of known locations. This implies that such algorithm is able to achieve a large range of distinguishable tap locations with minimal setup/calibration. In this chapter a new method that employs the Kullback-Leibler (KL) discrimination information for localizing a single tap made by a metal stylus on various flat surfaces is proposed. This is achieved through the use of the phase transit time of the vibration signal. The wave propagation of flexural vibration due

to an impact on a surface has been analyzed to explain the frequency spectrum diversity of vibration signals and background noise. Change-point detection algorithm is then devised for detecting abrupt changes to the time-varying power spectrum of a time series when the signal is transiting from background noise to that caused by an impact. The KL discrimination information is used to determine phase transit time of vibration signal. A few frequency bins are considered in order to mitigate the effect of dispersion during KL calculation. The performance of the proposed algorithm is compared and verified with existing TDOA approaches on real surfaces. With an RMSE of 0.96 cm for aluminum and 2.13 cm for glass, this algorithm is feasible for large-scale touch interfaces. The algorithms proposed in Chapter 5 and 6 further exploits multi-sensor architecture to avoid the need of velocity estimation.

Chapter 5

Source Localization on Solids Utilizing Time-Frequency Analysis of Parameterized Warped Signals

5.1 Introduction

In the context of HCI, existing TDOA based source localization methods, including that proposed in Chapter 4, require calibration for the estimation of phase velocities. This, in turn, may cause inconvenience to potential users. This chapter presents a source localization algorithm on solids that is based on the range difference between pairs of source-sensor in order to avoid the need of velocity estimation.

Passive source localization techniques such as the time-frequency Hermitian angle (TiF-HA) approach [71] and Kullback-Leibler discrimination information (KLID) [72] have shown to achieve promising localization accuracy without prior knowledge of material properties. These algorithms achieve source localization via time-of-arrival (TOA) estimation by exploiting the spectral diversity of a signal before and after the onset point. The KLID algorithm as described in Chapter 4 estimates the phase transit time by maximizing the Kullback-Leibler distance between the periodogram of neighboring

signal frames. In the case of TiF-HA, an analysis vector is formed by concatenating a complex element (taken from a uniformly distributed random variable) with a signal element in the time-frequency domain. The TOA is then estimated by finding the crossover point where the standard deviation of the Hermitian angles between the analyzing vector and a randomly selected reference vector (calculated across frequency bins) exceeds a pre-determined threshold. Since TOA-based methods estimate the source location by assuming a single onset point within each received signal, localization accuracy is sensitive to additive noise. It is important to note that, unlike conventional TDOA based methods such as generalized cross-correlation (GCC) [44] which exploit the coherence between the received signals, the above TDOA-based source localization methods are not robust against uncorrelated sensor and/or thermal noise. In addition, existing methods such as those presented in [71] [72] require calibration for the estimation of phase velocities. This, in turn, may cause inconvenience to potential users.

Unlike in [71] and [72], the range difference between source-sensor pairs is estimated in order to avoid the need of velocity estimation. To achieve this, wave propagation is analyzed through solids and it will be shown that any dispersion in solids can be represented by a non-linear change in phase function. Therefore, a difference in dispersion can be mapped directly to a difference in relative propagation distances if the signals are properly time-warped. This property is then exploited for source localization [73] by quantifying the similarity of the warped signal in the time-frequency domain. As the proposed source localization algorithm jointly estimates warping-based polynomial parameters and source location, it does not require pre-calibration such as required in algorithms proposed in [74], [75], [73], [76].

Warping methods have been extensively employed for matching various signals. These applications include warping for generating matching pursuit dictionary [77], modeling acoustic propagation in shallow water [78] [79], matching audio and auditory criteria [80] [81], modeling dispersion characteristics of guided waves [82], analysis of sound of marine animals [83] [84], designing a unitary signal transformation that

is covariant to scale changes [85], [86], [87], [88], calculation of warping using the Laguerre transform [89].

5.2 Review of the TiF-HA method

One of the recent source localization algorithms for impacts made on solid surfaces is TiF-HA [71]. This algorithm detects the TOA of the signal by exploiting the variation of Hermitian angle between random noise signal and impulsive impact signal. To describe the algorithm, short-time Fourier transform (STFT) of the received signal $g_i(n)$ from Sensor i is performed to obtain time-frequency coefficient $\underline{g}_i(b, \hat{n})$ corresponding to frequency bin index b and time-frame index $\hat{n}$. A uniformly distributed random element $\underline{r}_i(b, \hat{n})$ is next generated in the TiF domain. The Hermitian angle $\check{\theta}$ between $[1 + j, 0]$ and $[\underline{g}_i(b, \hat{n}), \underline{r}_i(b, \hat{n})]$ is computed for each coefficient using

$$\cos \check{\theta}_{b, \hat{n}} = \frac{|\underline{g}_i(b, \hat{n})|}{\sqrt{|\underline{g}_i(b, \hat{n})| + |\underline{r}_i(b, \hat{n})|}}. \quad (5.1)$$

Variation of Hermitian angles is then analyzed across b for each $\hat{n}$ by defining the standard deviation

$$\sigma_i(\hat{n}) = \sqrt{\sum_{b=b_{\min}}^{b_{\max}} (\check{\theta}_{b, \hat{n}} - \eta_i(\hat{n}))^2 / (\Delta b + 1)}, \quad (5.2)$$

where

$$\eta_i(\hat{n}) = \sum_{b=b_{\min}}^{b_{\max}} \check{\theta}_{b, \hat{n}} / (\Delta b + 1), \quad (5.3)$$

$\Delta b = b_{\max} - b_{\min}$, such that $b_{\min}$ and $b_{\max}$ correspond to minimum and maximum bin index of the analyzed frequency range, respectively. The point where $\sigma_i(\hat{n})$ crosses a pre-determined threshold σ_i^{thr} corresponds to the onset of the signal. Differences between these onset times will result in TDOAs of the signals.

As shown in [71], TiF-HA achieves an overall improvement in localization accuracy compared to methods based on the Kullback-Leibler discrimination information [72]

Figure 5.1: Diagram of a touch interface with $\bullet$ representing a predefined point.

and wavelet [90]. Generally an onset detection algorithm for source localization assumes a single change-point which may easily be corrupted by impulsive noise. This method also requires a velocity calibration phase which has to be repeated for any change in material properties or dimensions.

5.3 Proposed Algorithm

The proposed algorithm takes into account waves propagating on thin rectangular plate surfaces and we gain insights into the dispersive behavior of the signal propagation (due to non-linear phase function) from the solution of the wave equation as shown in (2.41). These non-linearities can be linearized by an appropriate time-frequency resampling warping function, which is dependent on material properties. To avoid material-based calibration, the warping is generalized by approximating the solution using a parametrized warping function.

5.3.1 Flexural Vibration of Thin Plate

With reference to the Fig 5.1, the coordinate of the source location is defined as $(l_{\text{src}}^x, l_{\text{src}}^y)$. According to the wave propagation model as described in Chapter 2, at time t , the vertical displacement $w_i(l_i^x, l_i^y, t)$ of the plate surface at the i th sensor location (l_i^x, l_i^y) due to a

force function $\mathcal{P}(l_{\text{src}}^x, l_{\text{src}}^y, t)$ at location $(l_{\text{src}}^x, l_{\text{src}}^y)$ is defined as

$$\mathcal{P}(l_{\text{src}}^x, l_{\text{src}}^y, t) = D\nabla_i^4 w_i(l_i^x, l_i^y, t) + \mu \frac{dw_i(l_i^x, l_i^y, t)}{dt} + \mathfrak{h}L_z \frac{d^2 w_i(l_i^x, l_i^y, t)}{dt^2}, \quad (5.4)$$

where μ is the absorption coefficient of the plate material, $\mathfrak{h}$ is the density, L_z is the thickness, D is the stiffness and ∇_i^4 represents the biharmonic operator evaluated at (l_i^x, l_i^y) such that $\nabla^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}$. To solve (5.4), $\mathcal{P}(l_{\text{src}}^x, l_{\text{src}}^y, t)$ is expressed as $\mathcal{P}(l_{\text{src}}^x, l_{\text{src}}^y, t) = p(t)\delta(x - l_{\text{src}}^x)\delta(y - l_{\text{src}}^y)$, where $p(t)$ is any arbitrary function in time and $\delta(x)$, $\delta(y)$ are the Dirac delta functions with respect to spatial variables x and y , respectively. The approximate solution for (5.4) in the space-frequency domain for angular frequency ω in terms of wavenumber $k(\omega)$ is given by [91]

$$\underline{w}_i(r_i, \omega) = \underline{P}(\omega)e^{-jk(\omega)r_i}, \quad (5.5)$$

where $\underline{P}(\omega)$ is the Fourier transform of $\mathcal{P}(l_{\text{src}}^x, l_{\text{src}}^y, t)$ and $r_i = \sqrt{(l_i^x - l_{\text{src}}^x)^2 + (l_i^y - l_{\text{src}}^y)^2}$ is the distance between the point of impact and the i th sensor. As described in Section 2.2.5, the wavenumber $k(\omega)$ can be obtained from homogeneous solution of (5.4) as

$$k(\omega) = \sqrt[4]{\frac{\mathfrak{h}L_z}{D}}\sqrt{\omega}. \quad (5.6)$$

The non-linear nature of (5.6) with respect to ω results in non-linear phase variation in (5.5). This results in frequencies propagating at different velocities. The phase velocity, which defines the rate of particle propagation corresponding to a single angular frequency ω is then given by

$$\begin{aligned} c_p(\omega) &= \frac{\omega}{k(\omega)} \\ &= \sqrt[4]{\frac{D}{\mathfrak{h}L_z}}\sqrt{\omega} \end{aligned} \quad (5.7)$$

while the group velocity

$$\begin{aligned} c_g(\omega) &= \frac{d\omega}{dk(\omega)} \\ &= 2c_p(\omega) \end{aligned} \quad (5.8)$$

defines the rate of propagation of the signal energy. The difference between $c_p(\omega)$ and $c_g(\omega)$ results in (inter-modal) dispersion of the wave. This dispersive behavior of (5.6) results in a non-linear variation in phase of (5.5) with respect to ω .

As mentioned in Chapter 4, this variation in velocity across different frequencies causes constitutes dispersion or spreading of waves during propagation. The effect of dispersion happening to the signals received by the sensor depends on the range or distance traveled by the wave from source-to-sensor. Therefore, a difference in the effect of dispersion between two received sensor signals can be mapped directly to a difference in relative propagation distances. Any dispersive linear time-varying systems can be characterized by the non-linear phase function of the signal with respect to time [92] [84]. These non-linear functions are specific to the nature of the channel through which the signal propagates. Such transformations can be modeled using warping-based time-frequency resampling of the original signals [93]. Therefore, a difference in dispersion can be mapped directly to a difference in relative propagation distances if the signals are properly time-warped. This property is then exploited for source localization by quantifying the similarity of the warped signal in the time-frequency domain. As the proposed source localization algorithm jointly estimates warping-based polynomial parameters and source location, it does not require pre-calibration such as that required in [94].

5.3.2 Time-frequency Warping

The warping transform is a time-frequency resampling transformation which transforms the time-domain variable $w_i(t)$ into a warped version $\tilde{w}_i(t)$ using a unitary mapping

Figure 5.2: Schematic diagram illustrating the mapping between time-frequency samples of original signal and warped signal.

function [95]. This mapping can either be performed in the time-domain using $\xi(t)$, or in the frequency-domain using $\underline{\xi}(\omega)$, where $\xi(t)$ is termed as time-domain mapping function. Therefore the warped version of $w_i(t)$ is given by

$$\tilde{w}_i(t) = \sqrt{\left| \frac{d\xi(t)}{dt} \right|} w_i[\xi(t)], \quad (5.9)$$

where the operation $w_i[\xi(t)]$ implies sampling $w_i(t)$ according to the warping function $\xi(t)$. The functional inverse of $\xi(t)$ can then be defined as $\xi^{-1}(t)$ such that $\xi^{-1}[\xi(t)] = t$. The corresponding mapping function $\underline{\xi}(\omega)$ which maps the change in sampling of frequency axis due to $\xi(t)$ for a given value of t is given by

$$\underline{\xi}(\omega) = \omega \sqrt{\left| \frac{d\xi^{-1}(t)}{d\omega} \right|}. \quad (5.10)$$

Similarly, the functional inverse of $\underline{\xi}(\omega)$ can be defined as $\underline{\xi}^{-1}(\omega)$ such that $\underline{\xi}^{-1}[\underline{\xi}(\omega)] = \omega$. Noting that $\tilde{w}_i(t)$ and $w_i(t)$ have the same energy, the warping function satisfies unitary equivalence.

Figure 5.2 shows the resampling of time-frequency response of $w(t)$ and $\underline{w}(t, \omega)$ due to warping function $\xi(t)$ or $\underline{\xi}(\omega)$. This allows a warping interpretation in term of unitary equivalence.

Consider the case where $\underline{\xi}^{-1}(\omega) = k(\omega)$. Applying the warping operator to (5.5) yields signal $\widetilde{w}_i(t)$ in the space-frequency domain given by

$$\underline{\widetilde{w}}_i(r_i, \omega) = \underline{P}[\underline{\xi}(\omega)] \sqrt{\left| \frac{d\underline{\xi}^{-1}(\omega)}{d\omega} \right|} e^{-j\omega r_i} \quad (5.11)$$

in which the non-linear change in phase of $\underline{w}_i(r_i, \omega)$ is transformed to a linear function in warped domain $\underline{\widetilde{w}}_i(r_i, \omega)$. Now, for a signal that has been propagated by a distance of $r_i + \Delta r$,

$$\underline{\widetilde{w}}_i(r_i + \Delta r, \omega) = \underline{\widetilde{w}}_i(r_i, \omega) e^{-j\omega \Delta r}. \quad (5.12)$$

It is therefore noted that proper selection of $\underline{\xi}(t)$ will translate the change in dispersion due to $k(\omega)$ to a linear function of range difference. The accuracy of this characterization is highly dependent on the priori information of $k(\omega)$. As described in [96] [97], $k(\omega)$ may be available or can be derived based on physical dimensions and mechanical properties of the medium. Such pre-calibration imposes strong assumptions on the material properties and any variation in $k(\omega)$ due to, for example, caused by a change in environment [34], will degrade the localization performance.

5.3.3 Parametrized Time Warping

The above problem is addressed by jointly estimating the warping-related polynomial parameters and source location. With reference to Section 5.3.2, (5.12) also indicates that the range difference between two propagation paths can be estimated from the change in dispersion between the two signals. This chapter, therefore, proposes to utilize time-frequency representations of the parametrized warped signals.

Unlike (5.11) which requires prior information pertaining to material properties, wave propagation in plates is approximated with that of normal mode propagation in an

Figure 5.3: Effect of warping the signal in time-domain (after time aligning both signals), where $\kappa = 300f_s^{-1}$.

ideal waveguide. In an ideal waveguide, propagation of particles in the time-domain, for mode l with an angular frequency of ω_l , is given by [93] [98]

$$w_i^l(t) = |w_i^l(t)|e^{J\omega_l\sqrt{t^2-\kappa^2}}, \quad (5.13)$$

where $|w_i^l(t)|$ is the instantaneous amplitude of mode l and κ is the unknown propagation delay in a non-dispersive environment. Therefore the time-domain warping function parametrized by κ to satisfy phase linearity is given by [99]

$$\xi_\kappa(t) = \sqrt{t^2 + \kappa^2}, \quad (5.14)$$

and in an ideal waveguide, the time-warped signal will be

$$\tilde{q}_i^l(t) = \sqrt{\left|\frac{d\xi_\kappa(t)}{dt}\right|} |w_i^l[\xi_\kappa(t)]| e^{J\omega_l t}. \quad (5.15)$$

From (5.15), for each warped mode, the instantaneous phase $\omega_l t$ is now linear in time. Here κ is used to parametrize the dispersion or spreading of the signal as illustrated in Figure 5.3, for the case of $\kappa = 300f_s^{-1}$, where f_s is the sampling frequency. In the context of our application, assuming $g_i(t)$ and $g_j(t)$ as the two sensor outputs induced by the vertical displacement $w_i(t)$ and $w_j(t)$ respectively, the objective is to find an optimal

Figure 5.4: Spectrogram of a sensor output due to a metal stylus tap on an aluminum plate surface.

warping parameter κ^{opt} such that

$$g_i(t) \approx \sqrt{\left| \frac{d\xi_{\kappa}^{\text{opt}}(t)}{dt} \right|} |g_j[\xi_{\kappa}^{\text{opt}}(t)]| e^{j\omega_j t}, \quad (5.16)$$

where

$$\xi_{\kappa}^{\text{opt}}(t) = \sqrt{t^2 + (\kappa^{\text{opt}})^2}. \quad (5.17)$$

To estimate κ^{opt} , one of the two sensor signals is warped over a range of κ and each warped signal will be checked for similarity with the other signal by analyzing them in the time-frequency domain. The value of κ giving the maximum similarity will then be taken as κ^{opt} .

To describe the above, $\tilde{g}_j^{\kappa}(n)$ is defined as the time-warped version of $g_j(n)$ computed via (5.14) and (5.9) using a value of κ . Both $g_i(n)$ and $\tilde{g}_j^{\kappa}(n)$ are next transformed into the time-frequency domain using short-time Fourier transform (STFT) so that $\underline{g}_i(b, \hat{n})$ and $\underline{\tilde{g}}_j^{\kappa}(b, \hat{n})$ are the STFT coefficients of $g_i(n)$ and $\tilde{g}_j^{\kappa}(n)$, respectively, where b is the frequency bin index and $\hat{n}$ is the time frame index. Since the algorithm is insensitive to spectral leakage across frequency bins, in order to save computational cost and to avoid amplitude smoothing around the onset point, the rectangular window is used for the STFT analysis. To preserve maximum time resolution, the hop size is fixed as one sample.

Due to the trade-off between high-frequency resolution and computational complex-

ity, the maximum number of frequency bins has been chosen as 256. As mentioned in Chapter 4, Figure 5.4 shows the spectrogram of a sensor output due to a stylus tap on an aluminum plate. It is noted that beyond frequency bin $b > 30$ (corresponding to a frequency of 11 kHz), the energy of the signal is negligibly small. This is consistent with the frequency response of the Murata PKS1-4A10 [70] used in our experiment. Hence, in order to reduce the computational cost and influence of noise, analysis of the frequency bins is limited to $10 \leq b \leq 20$.

Here the warped signals $\tilde{g}_j^\kappa(n)$ are estimated by resampling the samples of $g_j(n)$ at $\xi_\kappa(t)$ using linear interpolation. Interpolation is required since most of the samples are lying in the non-integer multiple of the sampling frequency $f_s = 96$ kHz. The signal similarity analysis is iterated over $0 \leq \kappa \leq 100 \times 2f_s^{-1}$.

The estimate of the optimal warping operator κ^{opt} is then obtained as

$$\hat{\kappa}^{\text{opt}} = \arg \min_{\kappa} \mathcal{J}(\kappa), \quad (5.18)$$

where $\mathcal{J}(\kappa)$ quantifies the similarity between a received signal and the warped signal of the other channel given by

$$\mathcal{J}(\kappa) = \sum_{b=b_{\min}}^{b_{\max}} \frac{\left(\underline{\mathbf{g}}_i(b) - \underline{\tilde{\mathbf{g}}}_j^\kappa(b) \right) \left(\underline{\mathbf{g}}_i(b) - \underline{\tilde{\mathbf{g}}}_j^\kappa(b) \right)^T}{\mathfrak{N}(b_{\max} - b_{\min} + 1)}, \quad (5.19)$$

given that $\underline{\mathbf{g}}_i(b)$ and $\underline{\tilde{\mathbf{g}}}_j^\kappa(b)$ are row vectors representing the frequency bin-wise normalized version of $\underline{g}_i(b, \mathfrak{n})$ and $\underline{\tilde{g}}_j^\kappa(b, \mathfrak{n})$, respectively, in decibel (dB) across $\mathfrak{n} = 0, 1, \dots, \mathfrak{N} - 1$, $\mathfrak{N}$ is the total number of time frames, and $b_{\min}$ and $b_{\max}$ are, respectively, the lower and upper range of the frequency bins.

Since $g_i(n)$ and $\tilde{g}_j^\kappa(n)$ are defined for $t \geq 0$, (5.14) is valid only for $t > \kappa$. This implies that the sensor nearest to the point of impact is chosen as the reference sensor to avoid negative range difference. This reference sensor can be identified using one of the onset detection algorithms such as presented in [71] [72].

Figure 5.5: Variation of optimal warping parameter $\kappa_{\gamma,i}$ with range difference $\Delta r_{\gamma,i}$.

5.3.4 Source Position Estimation

To estimate the source location, a polynomial relation model between differences in the source-sensor range and the optimal warping parameter is proposed. Utilizing this model, the source position is estimated by minimization of a non-linear cost function.

Establishing the relationship between the range differences and the optimal warping parameters can be achieved by tapping at known locations on an aluminum plate surface using a metal stylus. Since the tap/source locations are known, the range differences are also known and these points are plotted against the corresponding optimal warping parameters as shown in Fig. 5.5 as dotted points. In this plot, $\hat{\kappa}_{\gamma,i}^{\text{opt}}$ denotes the estimated optimal warping parameter corresponding to the range difference $\Delta r_{\gamma,i}$ between the nearest sensor γ and the i th sensor. A polynomial curve is then fitted on the data as shown in Fig. 5.5. From experimental results conducted on different plate surfaces it is observed that a polynomial function of order two is sufficient to describe the relationship between the range differences and the optimal warping parameters. Hence, a second-order polynomial function is proposed to relate $\Delta r_{\gamma,i}$ and $\hat{\kappa}_{\gamma,i}^{\text{opt}}$, which is given by

$$\widehat{\Delta r}_{\gamma,i} = \alpha_1(\hat{\kappa}_{\gamma,i}^{\text{opt}})^2 + \alpha_2(\hat{\kappa}_{\gamma,i}^{\text{opt}}) + \alpha_3, \quad (5.20)$$

where α_1 , α_2 and α_3 are the unknown polynomial parameters which depend on the medium of wave propagation. The relative range difference $\hat{r}_{i,j}$ between Sensors i and

j can now be expressed in terms of (5.20) as

$$\begin{aligned}\widehat{\Delta r}_{i,j} &= \widehat{\Delta r}_{\gamma,i} - \widehat{\Delta r}_{\gamma,j} \\ &= \widehat{\alpha}_1((\widehat{\kappa}_{\gamma,i}^{\text{opt}})^2 - (\widehat{\kappa}_{\gamma,j}^{\text{opt}})^2) + \widehat{\alpha}_2(\widehat{\kappa}_{\gamma,i}^{\text{opt}} - \widehat{\kappa}_{\gamma,j}^{\text{opt}}),\end{aligned}\quad (5.21)$$

where $\widehat{\alpha}_1$ and $\widehat{\alpha}_2$ are estimates of α_1 and α_2 . These estimates are not required apriori since they are jointly estimated along with source location $(\widehat{l}_{\text{src}}^x, \widehat{l}_{\text{src}}^y)$ using optimization algorithms as will be described below. Assuming known sensor positions and defining r_i as the distance between the source and the i th sensor, the unknown source position $(l_{\text{src}}^x, l_{\text{src}}^y)$ can be estimated by minimizing the error function

$$(\widehat{l}_{\text{src}}^x, \widehat{l}_{\text{src}}^y) = \arg \min_{l_{\text{src}}^x, l_{\text{src}}^y, \alpha_1, \alpha_2} \sum_{i,j} \left(r_i - r_j - \widehat{\Delta r}_{i,j} \right), \quad (5.22)$$

where $\widehat{\Delta r}_{i,j}$ is calculated from (5.21) and $r_i = \sqrt{(l_{\text{src}}^x - l_i^x)^2 + (l_{\text{src}}^y - l_i^y)^2}$. Similar to that presented in earlier chapters, such minimization can be performed using, for example, the iterative Levenberg-Marquardt optimization algorithm [69].

5.4 Experiment Results

To quantify the performance of the algorithm in terms of accuracy, the root-mean-square error

$$\text{RMSE} = \sqrt{\frac{1}{M} \sum_{m=1}^M \left(l_{\text{src}}^x(m) - \widehat{l}_{\text{src}}^x(m) \right)^2 + \left(l_{\text{src}}^y(m) - \widehat{l}_{\text{src}}^y(m) \right)^2} \quad (5.23)$$

where tap index $1 \leq m \leq M$, M is the total number of taps, and $[l_{\text{src}}^x(m), l_{\text{src}}^y(m)]^T$ is the actual tap position while $[\widehat{l}_{\text{src}}^x(m), \widehat{l}_{\text{src}}^y(m)]^T$ is the estimated position of the m th tap.

To verify the performance of the proposed method, experiments were conducted on aluminum and glass surfaces having dimension of $0.6 \text{ m} \times 0.6 \text{ m} \times 2.5 \text{ mm}$ and $0.6 \text{ m} \times 0.6 \text{ m} \times 5.0 \text{ mm}$, respectively. Eight Murata PKS1-4A1 shock sensors were

Figure 5.6: Variation of $\mathcal{J}$ with respect to κ (scaled by $2f_s^{-1}$) for various values of range difference for aluminum plate surface.

placed 10 cm apart along the inner square perimeter of the test surfaces. Nine tap locations are arranged in a uniform grid of 3×3 array inside the area enclosed by the sensor array where the separation between the nearest row and column tap positions is 10 cm as described in [71]. The tap locations were sequentially tapped using a metal stylus and a finger. Each position is tapped five times giving $M = 45$. The received signals were digitized at $f_s = 96$ kHz sampling frequency with 24-bit resolution. The value of $B = 1024$ is used for TiF-HA and for the proposed algorithm, $B = 256$ was found to be suitable in terms of achieving a reasonable trade-off between computational complexity and frequency resolution. Figure 5.6 illustrates the variation of $\mathcal{J}$ with respect to κ using data collected on aluminum surface, tapped with the metal stylus. It is interesting to see that $\mathcal{J}$ is a convex function with respect to κ and this allows one to easily determine its minimum. Hence, for the cases shown in Fig. 5.6, $\hat{\kappa}_{\gamma,1}^{\text{opt}} = 0$, $\hat{\kappa}_{\gamma,2}^{\text{opt}} = 49$ and $\hat{\kappa}_{\gamma,3}^{\text{opt}} = 64$. As expected, value of κ^{opt} increases with range difference.

As shown in [11], TiF-HA achieves an overall improvement in localization accuracy compared to methods based on the Kullback-Leibler discrimination information [72] and wavelet [90]. Hence, in this work performance of the proposed algorithm is compared with that of TiF-HA only.

Figure 5.7 compares the performance of the proposed algorithm with TiF-HA in terms of RMSE and standard deviation in RMSE for source localization. The results

Figure 5.7: Performance comparison of the proposed algorithm with TiF-HA algorithm (a) RMSE in source localization and (b) standard deviation of RMSE in source localization.

show that, for taps made by the stylus, the proposed algorithm can achieve reduction in RMSE of 7 mm and 2 mm compared to TiF-HA, for aluminum and glass surfaces, respectively. The standard deviations of proposed algorithm is also lower compared to TiF-HA. For impacts made by the finger, the RMSE and standard deviation are only modestly higher than that of TiF-HA. The increase in RMSE for aluminum is about 1 mm whereas for glass it is 0.1 mm. Similarly, the standard deviation in RMSE is also modestly higher; approximately 3 mm for aluminum and 2 mm for glass, compared to that of TiF-HA. For a large surface considered in this experiment, such degradation is insignificant and in addition, unlike TiF-HA, the proposed algorithm does not require pre-calibration.

5.5 Chapter Summary

In the context of HCI, existing TDOA based methods, like KLID in Chapter 4, require calibration for the estimation of phase velocities. This, in turn, may cause inconvenience to potential users. The proposed source localization algorithm on solids is based on the range difference between pairs of source-sensor in order to avoid the need of velocity estimation. The wave propagation model of flexural vibration due to an impact on a plate surface from Chapter 5 is analyzed and it was shown that the change in dispersion between two signals is dependent on the range difference between the propagation path of the signals. Therefore, a difference in dispersion can be mapped directly to a difference in relative propagation distances if the signals are properly time-warped. This algorithm thus exploits the similarity of the warped dispersive signal in the time-frequency domain. As the proposed source localization algorithm jointly estimates the model parameters and source location, this method does not require pre-calibration. For every pair of sensor signal, one of the two sensor signals is warped over a range of warping parameter and each warped signal will be checked for similarity with the other signal by analyzing them in the time-frequency domain. The value of warping parameter giving the maximum similarity will then be taken as optimal warping parameter for that sensor pair. From the experimental results conducted on different plate surfaces it is observed that a polynomial function of order two is sufficient to describe the relationship between the range differences and the optimal warping parameters. The parameter for this polynomial function, which depends on the material type, along with source positions are jointly estimated using optimization algorithms. The algorithm is validated via experiments conducted on aluminum and glass plates, for both stylus and finger taps. Results presented in this chapter show that, for taps made by the stylus, the proposed algorithm can achieve significant reduction in RMSE compared to TiF-HA. However, RMSE for finger is modestly higher than TiF-HA by approximately 1 mm for aluminum and 0.1 mm for glass. This slight reduction in accuracy is due to model

mismatch from the second-order polynomial function for signals generated using finger. The algorithm proposed in Chapter 6 is a model-independent method which extracts range difference information embedded in the derivative of phase to achieve higher localization accuracy.

Chapter 6

Source Localization on Solids Utilizing Radon Transform of Mapped Phase Derivative Information

6.1 Introduction

In the context of HCI, warping-based source localization methods, proposed in Chapter 5, is one of the calibration-free algorithm to estimate source localization of impact. However due to model mismatch, the algorithm may achieve less accurate results than the TiF-HA algorithm. A novel calibration-free algorithm, which extracts range difference information embed in the derivative of phase spectrum, is proposed in this chapter.

Unlike in [71] and [100], the phase derivative based localization approach is employed by formulating the problem in terms of cross-spectrum across the signals at the sensor outputs. The proposed cross-spectrum model is derived based on the wave propagation model derived from mechanical vibration theories as discussed in Chapter 2. Unlike conventional cross-spectrum based source localization methods, it will be shown in this chapter that, when the sensor signals are caused by flexural vibration on

the solid surface, the phase derivative of the cross-spectrum will be a scaled version of the range difference between source-to-sensor locations and that this scaling factor depends on the material properties of the medium and frequency of operation. One of the main advantages of this proposed method is that it does not require prior knowledge of the material properties. As supplementary information, insights into the effect of dispersion on the frequency spectrum of the source signal is also derived by considering a Gaussian narrow-band pulse as an example source signal. This helps to explain why conventional spectrum comparison methods such as GCC [44] fail to extract the time/range difference information when the medium of propagation is dispersive. It will also be shown that the phase derivatives of cross-spectrum depend on the range difference and material based physical constants. This explains the significance of the proposed method which does not require knowledge of the material properties or pre-calibration unlike methods such as described in [101], [102], [103]. To achieve this, the phase derivative of the cross-spectrum is initially mapped to a Radon domain by applying the Radon transform on a newly proposed mapping function of cross-spectrum values. Particle swarm optimization is subsequently used on the Radon domain for estimating the impact location.

This chapter is organized as follows: In Section 6.2 mathematical formulation of the localization problem in terms of phase derivative of the cross-spectrum between two sensor outputs due to vibration of an impact is shown. A new mapping function is then proposed to decouple the range difference and material-dependent scaling parameters. The location of impact is subsequently estimated from this new function using the Radon transform followed by particle swarm optimization. In Section 6.3, localization performance of the algorithm with varying parameters is evaluated using real experiment data. Finally, Section 6.4 concludes the chapter.

Figure 6.1: Block diagram of the proposed source localization algorithm.

6.2 The Proposed Radon-based Cross-Spectrum (RCS) Localization Algorithm

For the development of the proposed Radon-based cross-spectrum (RCS) localization algorithm, the propagation of flexural vibration signals on glass and aluminum is considered. Based on wave propagation model derived using mechanical vibration theories on flexural vibrations described in Chapter 2, the non-linear wavenumber function is shown to represent the dispersive nature of wave propagation. The non-linear wavenumber function is then approximated using the Taylor series and an expression for phase derivative of the cross-spectrum is derived. More importantly, it will be shown that the phase derivative contains coupled terms comprising of range difference between sensor signals and material-dependent constants. A method that maps the estimated phase derivatives to the Radon domain is proposed. This method achieves location estimation without any explicit computation of the material-dependent parameters. This is achieved via the Radon transform and subsequently particle swarm optimization in the transformed domain. Figure 6.1 shows the block diagram of the proposed algorithm.

Assuming a rigid flat surface on which vibration sensors are mounted at different

locations, an impact is applied at location $(l_{\text{src}}^x, l_{\text{src}}^y)$ by tapping finger or a metal stylus which generates vibrations on the surface. These vibration signals are picked up by the vibration sensors mounted on the surface. The aim of this work is to estimate the impact location $(l_{\text{src}}^x, l_{\text{src}}^y)$ relative to the sensor positions by either estimating difference in distance between the source-to-sensor locations (i.e., the range difference) or the difference in time-of-arrival (TOA) of the signals arriving at the sensors. The dispersive nature of flexural waves propagation on the plate surface is studied using the signal propagation model based on mechanical vibration theories.

6.2.1 Dispersion during flexural wave propagation

An approximate solution for the vertical/flexural displacement $w(l_i^x, l_i^y, t)$ of the plate surface at the i th sensor location (l_i^x, l_i^y) due to a force function $P(l_{\text{src}}^x, l_{\text{src}}^y, t)$ at location $(l_{\text{src}}^x, l_{\text{src}}^y)$ has been derived in Section 5.3.1. This solution can be represented in the space-frequency domain for angular frequency ω as [91]

$$\underline{w}(r_i, \omega) = \underline{P}(\omega)e^{-jk(\omega)r_i}, \quad (6.1)$$

where $\underline{P}(\omega)$ is the Fourier transform of $P(l_{\text{src}}^x, l_{\text{src}}^y, t)$ and $r_i = \sqrt{(l_i^x - l_{\text{src}}^x)^2 + (l_i^y - l_{\text{src}}^y)^2}$ is the distance between the point of impact and the i th sensor. The wavenumber $k(\omega)$ for each propagating frequency ω is derived and described in Section 2.2.5 as

$$k(\omega) = \sqrt[4]{\frac{\rho L_z}{D}} \sqrt{\omega}. \quad (6.2)$$

The non-linear nature of (6.2) with respect to ω leads to non-linear variation in phase of (6.1) resulting in different frequencies propagating at different velocities (since velocity can be calculated as $\omega/k(\omega)$). This variation in velocity across frequencies constitutes velocity dispersion or spreading of waves during its propagation. To quantify the dispersion in terms of range difference, an equation for cross-spectrum between a pair

of sensor outputs based on generic wave propagation model [28] is derived in the following section.

6.2.2 Cross-spectrum of flexural vibration signals

In source localization, it is common to estimate the source location via range difference. In this section, an expression for the cross-spectrum between two spatially separated sensor outputs is derived. The range difference information embedded in the cross-spectrum will then be used for the estimation of the source location. However, due to the detrimental effect of dispersion, the range difference will be a non-linear function of the signal frequency which makes the source localization a challenging problem. To extract range difference information embedded in the cross-spectrum, information is mapped onto the Radon domain where the maxima will provide a good estimate of the source location.

Assuming $w(r_i, t)$ and $w(r_j, t)$ are the vertical displacement of the plate surfaces at a distance of r_i and r_j respectively from the source location, while $g(r_i, t)$ and $g(r_j, t)$ are the corresponding received sensor signals. The cross-spectrum between two sensor signals at an angular frequency ω is given by

$$\check{\Gamma}(\Delta r_{ij}, \omega) = \underline{g}(r_i, \omega) \underline{g}^*(r_j, \omega), \quad (6.3)$$

where $\Delta r_{ij} = r_i - r_j$ is the range difference and $\underline{g}(r, \omega)$ denotes the sensor signal in the frequency-domain. Here it is assumed that the cross-spectrum is a function of Δ_{ij} and ω . Note that in this chapter, notations with underline represent the frequency-domain version of the corresponding time-domain signals. To derive a closed-form solution for the frequency-domain signal $\underline{g}(r_i, \omega)$, a monochromatic plane wave which has propagated by a distance r_i in a homogeneous medium of wave-number function $k(\omega)$ can be considered. The wave propagation can then be modeled by a linear differential equation with constant coefficients, whose solution is given by $e^{J(k(\omega)x - \omega t)}$ [28]. Therefore, the

Fourier transform of the signal received at Sensor i can be approximated as [91]

$$\underline{g}(r_i, \omega) \approx e^{jk(\omega)r_i} \underline{g}(0, \omega), \quad (6.4)$$

where $\underline{g}(0, \omega)$ is the Fourier transform of the source signal at a given tap location. Taking the inverse Fourier transform of (6.4), the time-domain sensor signal can be obtained as

$$g(r_i, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{jk(\omega)r_i} \underline{g}(0, \omega) d\omega. \quad (6.5)$$

Using the Taylor's series, assuming a narrow-band source signal with central frequency ω_0 , the non-linear function $k(\omega)$ can be approximated as

$$k(\omega) = k_0 + (\omega - \omega_0)k'_0 + \frac{(\omega - \omega_0)^2}{2}k''_0, \quad (6.6)$$

where $k_0 = k(\omega)$, $k'_0 = \frac{dk(\omega)}{d\omega}$ and $k''_0 = \frac{d^2k(\omega)}{d^2\omega}$ evaluated at $\omega = 0$. Substituting (6.6) and (6.4) to (6.3), the cross-spectrum is given by

$$\check{\Gamma}(\Delta r_{ij}, \omega) = e^{-j(k_0 + k'_0\omega + k''_0\frac{\omega^2}{2})\Delta r_{ij}} |\underline{g}(0, \omega)|^2. \quad (6.7)$$

It can therefore be observed, from (6.7), that the phase of $\check{\Gamma}(\Delta r_{ij}, \omega)$ is a non-linear function which makes the source localization problem challenging in a dispersive medium.

To study the effect of dispersion on the characteristics of the source signal spectrum and hence for the derivation of an equation that describes the range difference, the source signal is assumed to be a Gaussian narrow-band pulse [28] centered at ω_0 with a Gaussian envelope given by

$$f(0, t) = e^{-\frac{t^2}{2\tau_0}}, \quad (6.8)$$

where τ_0 is termed as the time-domain width of the pulse. Utilizing the modulation property of the Fourier transform, the relationship between $\underline{g}(0, \omega)$ and $\underline{f}(0, \omega)$ can be

Figure 6.2: Variation in width of gaussian envelope $f(r_i, t)$ with of propagation distance r_i .

written as $\underline{g}(0, \omega) = \underline{f}(0, \omega - \omega_0)$ which corresponds to

$$g(r_i, t) = e^{j\omega_0 t - k_0 r_i} f(r_i, t), \quad (6.9)$$

where, from (6.5), the source signal is given by

$$f(r_i, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{j(\omega - \omega_0)t - (k(\omega) - k_0)r_i} \underline{f}(0, \omega - \omega_0) d\omega \quad (6.10)$$

after propagating a distance of r_i . Using (6.6), $f(r_i, t)$ in the frequency-domain can be written as

$$\underline{f}(r_i, \omega) = e^{-jk'_0 r_i} e^{-j \frac{k''_0 r_i \omega^2}{2}} \underline{f}(0, \omega). \quad (6.11)$$

As the Fourier transform of the envelope in (6.8) is given by $\underline{f}(0, \omega) = \sqrt{2\pi\tau_0^2} e^{\frac{\tau_0^2 \omega^2}{2}}$, (6.11) can be re-written as

$$\underline{f}(r_i, \omega) = \sqrt{2\pi\tau_0^2} e^{-jk'_0 r_i} e^{-\frac{jk''_0 r_i + \tau_0^2}{2} \omega^2}. \quad (6.12)$$

The inverse Fourier transform of (6.12) corresponds to the time-domain signal at a

distance r_i from the source location and is given by

$$f(r_i, t) = \sqrt{\frac{\tau_0^2}{\tau_0^2 + jk_0''r_i}} e^{-\frac{(t-k_0'r_i)^2}{2(\tau_0^2 + jk_0''r_i)}}. \quad (6.13)$$

Assuming that k_0' and k_0'' are real [62],

$$|f(r_i, t)| = \left(\frac{\tau_0^4}{\tau_0^4 + k_0''r_i} \right)^{\frac{1}{4}} e^{-\frac{(t-k_0'r_i)^2\tau_0^2}{2(\tau_0^4 + (k_0''r_i)^2)}} \quad (6.14)$$

whose effective width is $\frac{\tau_0^4 + (k_0''r_i)^2}{\tau_0^2}$ which is a higher value while comparing with width τ_0 of source signal. Figure 6.2 illustrates this increase in pulse width when a source signal of time-domain width $\tau_0 = 0.0001$ propagates a distance of $r_i = 0.2$ m through dispersive medium characterized by $k(\omega) = 0.0005\omega^2$.

Equation (6.13) corresponds to the Fourier transform of a sinusoidal chirp signal modulated by a Gaussian envelope whose frequency varies linearly due to the linear term $\frac{k_0''r_i}{\tau_0^4 + (k_0''r_i)^2}$ [62], which, in turn, is a function of the propagation distance. Since the above term is a function of the instantaneous frequency k_0 , the center frequency of narrow-band Gaussian pulse shifts according to the propagation distance. This implies that conventional frequency-domain comparison techniques such as the GCC can only be applied for signals traveling in dispersive media only if the effect of dispersion due to the range difference is compensated as that described in [104].

To achieve a linear approximation for range difference, the phase derivative of $\check{\Gamma}(\Delta r_{ij}, \omega)$ given by $\dot{\Theta}_{ij}(\omega) = (k_0' + k_0''\omega)\Delta r_{ij}$ can be employed. This phase derivative $\dot{\Theta}_{ij}(\omega)$ can therefore be viewed as a scaled version of the range difference

$$\Delta r_{ij} = \vartheta_\omega \dot{\Theta}_{ij}(\omega), \quad (6.15)$$

where $\vartheta_\omega = \frac{1}{(k_0' + k_0''\omega)}$ is constant for a given material at a particular frequency ω . The problem of source localization is now translated to the problem of estimation of the

phase derivatives using all the sensor pairs.

6.2.3 Source Position Estimation

In order to extract source location from the range difference equation in (6.15), an error function which quantifies the error between actual range difference and estimated range difference in terms of tap location on the plate surface is proposed. Deviation of the proposed error function from its true value is then studied against different values of the material properties ϑ_ω . The extrema conditions identified are then exploited along with the Radon transform and particle swarm optimization technique to extract the source location irrespective of the actual value of ϑ_ω .

To estimate the source location an error function

$$\epsilon(\hat{l}_{tz}^x, \hat{l}_{tz}^y) = \sum_i \sum_j \epsilon_{ij}(\hat{l}_{tz}^x, \hat{l}_{tz}^y) \quad (6.16)$$

is defined, where $(\hat{l}_{tz}^x, \hat{l}_{tz}^y)$ is the rasterized value of position on touch surface and

$$\epsilon_{ij}(\hat{l}_{tz}^x, \hat{l}_{tz}^y) = r_i(\hat{l}_{tz}^x, \hat{l}_{tz}^y) - r_j(\hat{l}_{tz}^x, \hat{l}_{tz}^y) - \Delta r_{ij}(l_{src}^x, l_{src}^y), \quad (6.17)$$

where $r_i(\hat{l}_{tz}^x, \hat{l}_{tz}^y)$ is the range between Sensor i and rasterized position and $\Delta r_{ij}(l_{src}^x, l_{src}^y)$ is the range difference between Sensor i and Sensor j with respect to (l_{src}^x, l_{src}^y) . Generalizing (6.17) for multiple sensor pairs,

$$\epsilon(\hat{l}_{tz}^x, \hat{l}_{tz}^y) = \sum_i \sum_j \left(\left(r_i(\hat{l}_{tz}^x, \hat{l}_{tz}^y) - r_i(l_{src}^x, l_{src}^y) \right) - \left(r_j(\hat{l}_{tz}^x, \hat{l}_{tz}^y) - r_j(l_{src}^x, l_{src}^y) \right) \right) \quad (6.18)$$

where $\left(r_i(\hat{l}_{tz}^x, \hat{l}_{tz}^y) - r_i(l_{src}^x, l_{src}^y) \right) = \sqrt{(l_i^x - \hat{l}_{tz}^x)^2 + (l_i^y - \hat{l}_{tz}^y)^2} - \sqrt{(l_i^x - l_{src}^x)^2 + (l_i^y - l_{src}^y)^2}$. The non-linear function (6.17) can be approximated by using the Taylor series, by representing $(\hat{l}_{tz}^x, \hat{l}_{tz}^y)$ in terms of Δx and Δy , the estimation error in the x and y coordinate respectively as $(\hat{l}_{tz}^x, \hat{l}_{tz}^y) = (l_{src}^x + \Delta x, l_{src}^y + \Delta y)$. This approximation results in

linearizing $\left(r_i(\hat{l}_{TZ}^x, \hat{l}_{TZ}^y) - r_i(l_{src}^x, l_{src}^y)\right)$ as given by

$$\left(r_i(\hat{l}_{TZ}^x, \hat{l}_{TZ}^y) - r_i(l_{src}^x, l_{src}^y)\right) \approx \frac{(l_{src}^x - l_i^x)}{r_i(l_{src}^x, l_{src}^y)} \Delta x + \frac{(l_{src}^y - l_i^y)}{r_i(l_{src}^x, l_{src}^y)} \Delta y. \quad (6.19)$$

Substituting (6.19) in (6.17)

$$\epsilon_{ij}(\Delta x, \Delta y) \approx \left(\frac{(l_{src}^x - l_i^x)}{r_i} - \frac{(l_{src}^x - l_j^x)}{r_j}\right) \Delta x + \left(\frac{(l_{src}^y - l_i^y)}{r_i} - \frac{(l_{src}^y - l_j^y)}{r_j}\right) \Delta y. \quad (6.20)$$

This result is inline with the common notion that two sensors are insufficient to achieve triangulation based source localization using range differences. This can be seen from (6.20) where $\epsilon(\Delta x, \Delta y) = 0$ is satisfied by all pairs of $\Delta x, \Delta y$ which are linearly dependent. Therefore, provided that the true value of ϑ_ω is known, the source location can be estimated using three or more sensors by a common point of intersection of minimum lines corresponding to each sensor pair.

To reduce the effect of noise in (6.16) and for easier analytical treatment, $\epsilon(\hat{l}_{TZ}^x, \hat{l}_{TZ}^y)$ is mapped to Gaussian function $o_\epsilon(\hat{l}_{TZ}^x, \hat{l}_{TZ}^y)$ using

$$o_\epsilon(\hat{l}_{TZ}^x, \hat{l}_{TZ}^y) = e^{-\varphi \epsilon^2(\hat{l}_{TZ}^x, \hat{l}_{TZ}^y)}, \quad (6.21)$$

where φ is an empirical scaling factor with no units. The variable $o_\epsilon(\hat{l}_{TZ}^x, \hat{l}_{TZ}^y)$ can be approximated as Gaussian function since $\epsilon_{ij}(\hat{l}_{TZ}^x, \hat{l}_{TZ}^y)$ is a linear function based on (6.20) and therefore (6.21) describes an exponential of sum of square of a linear function which intersects at one common minimum. It can be noted that source location at minima of $\epsilon(\Delta x, \Delta y)$ will be mapped to maxima of $o_\epsilon(\hat{l}_{TZ}^x, \hat{l}_{TZ}^y)$, which corresponds to the center of a Gaussian function.

To formulate a method to extract the maxima of $o_\epsilon(\hat{l}_{TZ}^x, \hat{l}_{TZ}^y)$ independent of ϑ_ω , variation of $o_\epsilon(\hat{l}_{TZ}^x, \hat{l}_{TZ}^y)$ due to deviation of ϑ from its true value ϑ_ω is investigated. This is achieved by way of simulation using sensors being positioned as illustrated in Fig. 6.7. The simulation is conducted using (6.15), (6.16) and (6.21) with $\vartheta_\omega = 0.1$

and a tap location being simulated at position $(l_{\text{src}}^x, l_{\text{src}}^y) = (20, 20)$ cm. The simulation is conducted to compute $o_\epsilon(\hat{l}_{\text{tz}}^x, \hat{l}_{\text{tz}}^y)$ for $\vartheta = -0.05, 0.00, 0.05, 0.10$ and 0.15 and the results are respectively shown in Figures 6.3 (a), (b), (c), (d) and (e). The global maxima of $o_\epsilon(\hat{l}_{\text{tz}}^x, \hat{l}_{\text{tz}}^y)$ should ideally correspond to the true location $(l_{\text{src}}^x, l_{\text{src}}^y)$. This only occurs when the material parameter value matches the true value, i.e., when $\vartheta = \vartheta_\omega = 0.1$ as shown in Fig. 6.3 (d). To further distinguish values of $o_\epsilon(\hat{l}_{\text{tz}}^x, \hat{l}_{\text{tz}}^y)$ for various ϑ , the sparseness metric defined by

$$\left(\frac{\arg \max_{x,y} o_\epsilon(\hat{l}_{\text{tz}}^x, \hat{l}_{\text{tz}}^y)}{\sum_{x,y} o_\epsilon(\hat{l}_{\text{tz}}^x, \hat{l}_{\text{tz}}^y)} \right) \quad (6.22)$$

is proposed in this thesis. This measure quantifies the spread of $o_\epsilon(\hat{l}_{\text{tz}}^x, \hat{l}_{\text{tz}}^y)$ around the global maxima for a given ϑ . The sparseness factor can be used to quantify the spread/concentration of $o_\epsilon(\hat{l}_{\text{tz}}^x, \hat{l}_{\text{tz}}^y)$ such that a high sparseness factor implies that majority of relative-high values of $o_\epsilon(\hat{l}_{\text{tz}}^x, \hat{l}_{\text{tz}}^y)$ is concentrated in the neighborhood region of the global maxima of $o_\epsilon(\hat{l}_{\text{tz}}^x, \hat{l}_{\text{tz}}^y)$. In other words a high sparseness factor implies the spread of $o_\epsilon(\hat{l}_{\text{tz}}^x, \hat{l}_{\text{tz}}^y)$ is similar to impulsive or Super-Gaussian function. As observed from Fig. 6.3 (d), the sparseness factor of $o_\epsilon(\hat{l}_{\text{tz}}^x, \hat{l}_{\text{tz}}^y)$ is maximum when $\vartheta = \vartheta_\omega$. This is because all the sensor pairs will point to a unique minimum for this condition. On the contrary, as shown in Fig.6.3 (a), due to the deviation of $\vartheta = 0.05$ from $\vartheta_\omega = 0.1$, the global maxima of $o_\epsilon(\hat{l}_{\text{tz}}^x, \hat{l}_{\text{tz}}^y)$ is away from its true source location value of $(20, 20)$ cm to $(35, 35)$ cm. It is also important to note that, for this case, the global maxima value of $o_\epsilon(\hat{l}_{\text{tz}}^x, \hat{l}_{\text{tz}}^y)$ is lesser when compared to that when $\vartheta = \vartheta_\omega$. At the same time the sparseness factor of $o_\epsilon(\hat{l}_{\text{tz}}^x, \hat{l}_{\text{tz}}^y)$ distribution is also lower for $\vartheta = 0.05$. By further analyzing the simulation results it can be observed that as the value of ϑ approaches to that of ϑ_ω , the global maxima of $o_\epsilon(\hat{l}_{\text{tz}}^x, \hat{l}_{\text{tz}}^y)$ shifts towards the actual source position $(20, 20)$ cm as illustrated in Fig. 6.3(b), (c) and (e). From this simulation it can therefore be inferred that the largest value is observed for the global maxima of $o_\epsilon(\hat{l}_{\text{tz}}^x, \hat{l}_{\text{tz}}^y)$ and the sparseness factor when ϑ approximates near to ϑ_ω .

Figure 6.3: Simulated values $o_\epsilon(\hat{l}_{rz}^x, \hat{l}_{rz}^y)$ using (a) $\vartheta = -0.05$, (b) $\vartheta = 0.0$, (c) $\vartheta = +0.05$, (d) $\vartheta = +0.10$ and (e) $\vartheta = +0.15$ where uniform noise of $\pm 1e-3$ seconds added to $\dot{\Theta}_{ij}(\omega)$ and the true value $\vartheta_\omega = 0.1$.

The sparseness metric defined at (6.22) quantifies the behavior of sparseness factor and global maxima of $o_\epsilon(\hat{l}_{tz}^x, \hat{l}_{tz}^y)$ such that the scaling amplifies the values when ϑ is near to ϑ_ω . This dynamic scaling factor can be exploited such that the summation of all scaled versions of $o_\epsilon(\hat{l}_{tz}^x, \hat{l}_{tz}^y)$, corresponding to each ϑ , will still preserve the global maxima of final summation at true location (l_{src}^x, l_{src}^y) , without the knowledge of ϑ_ω . Therefore, defining $o(\hat{l}_{tz}^x, \hat{l}_{tz}^y)$ as the sum of dynamic scaled version of $o_\epsilon(\hat{l}_{tz}^x, \hat{l}_{tz}^y)$ given by

$$o(\hat{l}_{tz}^x, \hat{l}_{tz}^y) = \sum_{\vartheta} \left(\frac{\arg \max_{\hat{l}_{tz}^x, \hat{l}_{tz}^y} o_\epsilon(\hat{l}_{tz}^x, \hat{l}_{tz}^y)}{\sum_{\hat{l}_{tz}^x, \hat{l}_{tz}^y} o_\epsilon(\hat{l}_{tz}^x, \hat{l}_{tz}^y)} \right) o_\epsilon(\hat{l}_{tz}^x, \hat{l}_{tz}^y). \quad (6.23)$$

The dynamic weighting factor of $o(\hat{l}_{tz}^x, \hat{l}_{tz}^y)$ for $\left(\frac{\arg \max_{\hat{l}_{tz}^x, \hat{l}_{tz}^y} o_\epsilon(\hat{l}_{tz}^x, \hat{l}_{tz}^y)}{\sum_{\hat{l}_{tz}^x, \hat{l}_{tz}^y} o_\epsilon(\hat{l}_{tz}^x, \hat{l}_{tz}^y)} \right)$ will help to retain the contribution based on nearest value of ϑ to ϑ_ω , which in turn makes the algorithm independent of knowledge of any material parameters. The source localization problem can therefore be formulated as detecting the peak of the a two-dimensional array $\mathbf{O}$ consisting of $o(\hat{l}_{tz}^x, \hat{l}_{tz}^y)$ for rasterized values of dimensions within touch location boundaries in x and y dimensions.

Figure 6.4 shows an illustrative example of $\mathbf{O}$ calculated using signals collected by tapping on an aluminum plate surface with a metal stylus. As observed from Fig 6.4, due to the presence of noise when estimating $\dot{\Theta}_{ij}(\omega)$, the extrema variation of $\mathbf{O}$ will be noisy, which, in turn, will result in an inaccurate location estimation. The Gaussian behavior of $o(\hat{l}_{tz}^x, \hat{l}_{tz}^y)$ can be utilized in terms of the Radon transform which will solve the problem of global maxima detection using parametric optimization. This algorithm therefore exhibits higher immunity to noise affecting global maxima $\mathbf{O}$ in case of a real world signal.

6.2.3.1 Source position estimation using the Radon transform

In this chapter, the Radon transform defines the mapping of input function $o(\hat{l}_{tz}^x, \hat{l}_{tz}^y)$ defined in the (x, y) domain to the Radon domain (ρ, θ) such that each transformed

Figure 6.4: Estimated $\mathbf{O}$ for location (20.0, 20.0) cm of aluminum tapped with metal stylus.

value $o_r(\rho, \theta)$ is calculated by the integral of the input function along a line. This line is defined by $\rho = \hat{l}_{rz}^x \cos(\theta) + \hat{l}_{rz}^y \sin(\theta)$ which is at a distance ρ from the origin and at angle θ about the x axis [105] [106], i.e.,

$$o_r(\rho, \theta) = \sum_{\hat{l}_{src}^x} \sum_{\hat{l}_{src}^y} o(\hat{l}_{rz}^x, \hat{l}_{rz}^y)(\hat{l}_{rz}^x \cos(\theta) + \hat{l}_{rz}^y \sin(\theta) - \rho). \quad (6.24)$$

Equation (6.24) represents the mathematical formulation of the Radon transform and is illustrated in Fig 6.5, where the values of $o_r(\rho, \theta)$ is the line integral along the ray which is perpendicular to line from origin where both passes through common point $(\rho \cos(\theta), \rho \sin(\theta))$.

Since $o(\hat{l}_{rz}^x, \hat{l}_{rz}^y)$ is formed by summing Gaussian functions given by (6.23) of which the maximum value at location (l_{src}^x, l_{src}^y) is preserved due to the dynamic scaling of each summation by the sparseness factor, the resultant $\mathbf{O}$ can also be modeled as Gaussian function, centered at (l_{src}^x, l_{src}^y) given by

$$o(\hat{l}_{rz}^x, \hat{l}_{rz}^y) = \frac{1}{2\pi\sigma_g^2} e^{-\frac{(\hat{l}_{rz}^x - l_{src}^x)^2}{2\sigma_g^2} - \frac{(\hat{l}_{rz}^y - l_{src}^y)^2}{2\sigma_g^2}}, \quad (6.25)$$

Figure 6.5: Illustration of the Radon transform, $o_r(\rho, \theta)$, as line integral.

where σ_g denotes arbitrary standard deviation. Using (6.25), the Radon transform of $o(\hat{l}_{src}^x, \hat{l}_{src}^y)$ can now be written as [107]

$$o_r(\rho, \theta) = \frac{1}{\sqrt{2\pi\sigma_g^2}} e^{-\left(\frac{\rho - (l_{src}^x \cos(\theta) + l_{src}^y \sin(\theta))}{2\sigma_g^2}\right)}. \quad (6.26)$$

To estimate the value of (l_{src}^x, l_{src}^y) , the mathematical property of exponential can be utilized such that, when evaluated for any angle θ , $0 \leq \theta \leq 90$, the maximum value of $o_r(\rho, \theta)$ occurs for

$$\rho_{max}(\theta) = l_{src}^x \cos(\theta) + l_{src}^y \sin(\theta). \quad (6.27)$$

Each set of $(\theta, \rho_{max}(\theta))$ should jointly satisfy the condition (6.27). Figures 6.6 (a) and (b) show the variation of $o_r(\rho, \theta)$ for signals collected from an aluminum plate when tapped at $(l_{src}^x, l_{src}^y) = (.20, .30)$ m for specific case of $\theta = 30$ and $\theta = 60$ respectively. Estimation of (l_{src}^x, l_{src}^y) , which jointly satisfy the condition in (6.27) for all value sets $(\theta, \rho_{max}(\theta))$, represents an over-determined parametric optimization problem. In this chapter, the particle swarm optimization technique is used to solve this optimization problem for estimating the source location.

Figure 6.6: The Radon transform of O calculated for location $(.20, .30)$ m on aluminum plate for (a) $\theta = 30$ and (b) $\theta = 60$.

6.2.3.2 Particle swarm optimization

Particle swarm optimization [108] [109] [110] is an optimization technique where the problem space is divided into potential solutions called particles. Each optimization problem is characterized by the size of particles, or variables to optimize, and a fitness function to estimate quantitatively the distance of particle from optimum solution. The particles are randomly distributed over the entire solution range initially. This optimization technique tracks two states in each iteration - the best fitness solution p_{best} encountered so far for each particle and best fitness solution g_{best} among all the particles. The particle swarm optimization concept consists of changing the velocity (accelerating) and position of each particle towards its p_{best} and g_{best} locations at each time step according to $v_p = v_p + c_1 r_1 (p_{best} - \delta p) + c_2 r_2 (g_{best} - \delta p)$ and $\delta p = \delta p + v_p$, where empirical constants c_1 is known as the social constant and c_2 is the cognitive constant, r_1 and r_2 denotes uniformly distributed random numbers within $[0, 1]$.

In this research, (6.27) can be optimized among a set of values of $(\theta, \rho_{max}(\theta))$ to

find the best fit source location $(\hat{l}_{\text{src}}^x, \hat{l}_{\text{src}}^y)$. For this a particle is constructed using three optimization variables $\hat{l}_{\text{src}}^x, \hat{l}_{\text{src}}^y$ and θ_n , where θ_n represents any statistical noise in the angle θ . The fitting function is used as the minimization of mean-square-error of $\rho_{\text{max}}(\theta) = \hat{l}_{\text{src}}^x \cos(\theta - \theta_n) + \hat{l}_{\text{src}}^y \sin(\theta - \theta_n)$. This optimization technique will enable one to find the optimal solution for the source location $(\hat{l}_{\text{src}}^x, \hat{l}_{\text{src}}^y)$ from the noisy estimate of O .

6.3 Simulation and Experiment Results

6.3.1 Simulation Results

Simulations are performed to verify the performance of the proposed algorithm in the context of a human computer touch interface. The sensor arrangements for the experiments are shown in Fig. 6.7 where the sensors are denoted by ‘•’ while the tap locations are denoted by a ‘×’. The range difference Δr_{ij} is calculated with reference to the sensor setup given in Fig. 6.7 for the tap location $(l_{\text{src}}^x, l_{\text{src}}^y) = (20, 20)$ cm. By considering an arbitrary ground truth value $\vartheta_\omega = 0.1$, $\Theta_{ij}(\omega)$ is calculated for each value of Δr_{ij} using (6.15). Uniform distributed random noise is added to $\Theta_{ij}(\omega)$ with its varying maximum magnitude as specified in each experiment.

In the first simulation the effect of empirical scaling factor φ on $o(\hat{l}_{\text{rz}}^x, \hat{l}_{\text{rz}}^y)$ as given by (6.23) is studied. The simulation scenarios considered are no-solution case of two sensors, a three-sensor setup corresponding to the minimal number of sensors to obtain a solution and an eight-sensor setup. The two-sensor simulation setup is conducted using Sensor S_1 at $(l_1^x, l_1^y) = (10, 10)$ cm and S_5 at $(l_5^x, l_5^y) = (50, 50)$ cm as shown in Fig. 6.7. In case of three-sensor simulation, S_3 is added to previous setup at $(l_3^x, l_3^y) = (50, 10)$ cm as a third sensor. Finally all eight sensors as shown in Fig. 6.7 are used for the eight-sensor simulation. Figure 6.8 (a) shows the result for $\varphi = 1$ for the two-sensor case, where the Gaussian function equivalence of $o(x, y)$ can be observed for a single summation case. When three sensors are used for $\varphi = 1$ as shown in Fig. 6.8 (b) the maxima of

Figure 6.7: Simulated sensor configuration where the tap location is at position $(l_{\text{src}}^x, l_{\text{src}}^y) = (20, 20)$ cm and the sensor coordinates are denoted by ‘•’.

$o(\Delta x, \Delta y)$ is obtained correctly at $(\Delta x, \Delta y) = (0, 0)$. However, for this case, the width of the peak of $o(\Delta x, \Delta y)$ is large and this results in the maxima being sensitive to noise. Figure 6.8 (c) shows the result for $\varphi = 1$ for the eight-sensor case with a distinct peak of $o(\Delta x, \Delta y)$ whose value is comparatively higher than that of the three-sensor case. Figures 6.9 (a), (b) and (c) show the result for $\varphi = 1000$ for the two-sensor, three-sensor and eight-sensor scenarios where it can be observed that width of peak reduces when compared with $\varphi = 1$. This predominance of the main peak enhances peak-detection, since in a later stage Gaussian nature of $o(\Delta x, \Delta y)$ is exploited by using Gaussian fitting function for maxima estimation. The simulation is repeated for $\varphi = 1 \times 10^6$. As can be observed from Figs. 6.10 (a), (b) and (c) the behavior of $o(\Delta x, \Delta y)$ deviates from Gaussian function to that of an impulsive function. The Gaussian function variation of $o(\Delta x, \Delta y)$ near to peak is essential since this behavior is exploited while exploiting the Radon transform for peak detection. This is to overcome non-linear distortions that may occur in case of real data signal as shown in Fig. 6.4. In all remaining experiments of

Figure 6.8: $o(\Delta x, \Delta y)$ calculated by $\varphi = 1$ using (a) two sensors, (b) three sensors and (c) eight sensors for simulated data.

this chapter, $\varphi = 1000$ is chosen empirically since it suppresses the side lobes as well ensures a Gaussian function that is peaky.

6.3.2 Experiments using real data

To quantify the performance of the proposed algorithm in terms of accuracy, the root-mean-square error (RMSE) given by

$$\text{RMSE} = \sqrt{\frac{1}{M} \sum_{m=1}^M (l_{\text{src}}^x(m) - \widehat{l}_{\text{src}}^x(m))^2 + (l_{\text{src}}^y(m) - \widehat{l}_{\text{src}}^y(m))^2} \quad (6.28)$$

is used as the performance measure, where $1 \leq m \leq M$, M is the total number of taps, and $(l_{\text{src}}^x(m), l_{\text{src}}^y(m))$ is the actual tap location while $(\widehat{l}_{\text{src}}^x(m), \widehat{l}_{\text{src}}^y(m))$ is the estimated location of the m th tap.

To evaluate the performance of the proposed method, experiments were conducted

Figure 6.9: $o(\Delta x, \Delta y)$ calculated by $\varphi = 1000$ using (a) two sensors, (b) three sensors and (c) eight sensors for simulated data.

on aluminum and glass surfaces having dimensions $0.6 \text{ m} \times 0.6 \text{ m} \times 2.5 \text{ mm}$ and $0.6 \text{ m} \times 0.6 \text{ m} \times 5.0 \text{ mm}$, respectively. Similar to previous chapters, eight Murata PKS1-4A1 shock sensors were placed 10 cm apart along the inner square perimeter of the test surfaces.

This experiment is conducted on nine uniformly spaced tap locations on a 3×3 grid with an inter-location spacing of 10 cm, as shown in Fig. 6.11. The impact signals are then generated by tapping at the locations marked "x" using a metal stylus and finger. The received signals are then digitized at 96 kHz with 16-bit resolution. To minimize the human error in tapping at the exact location, five impacts are made at each location, resulting in $M = 45$.

Figure 6.10: $o(\Delta x, \Delta y)$ calculated by $\varphi = 1e6$ using (a) two sensors, (b) three sensors and (c) eight sensors for simulated data.

Figure 6.12 (a) shows a time-domain data frame of $g_i(n)$ and $g_j(n)$ received at two sensors placed at locations (0.1, 0.1) m and (0.5, 0.1) m respectively due to an impact at location (0.2, 0.2) m on an aluminum plate surface using a metal stylus. The normalized frequency spectra of these signals are shown in Fig. 6.12 (b), where it can be seen that the energy content remains simultaneously higher in frequency bins corresponding to 3000 and 6000 Hz. The energy spectrum of other sensor outputs also exhibit similar characteristics for these bins. Hence, in our experiments, these two frequency bins are chosen for the computation of phase derivatives. The cross-spectrum is computed using (6.3) for each time-sampling point of a sensor pair. The phase angle is subsequently

Figure 6.11: Diagram of a touch interface where the sensor and test tap coordinates are denoted by '•' and '×' respectively.

Figure 6.12: Signals corresponding to Sensor 1 and Sensor 3 (a) time-domain signal and (b) normalized frequency spectrum of signal.

Figure 6.13: Estimation of phase derivative, $\dot{\Theta}_{ij}(\omega)$, from phase angle.

determined from the cross-spectrum and thereafter, the phase derivative is calculated for frequency bins 2 and 3 from the unwrapped phase angle. For an ideal scenario, the phase derivative is expected to remain constant for a given pair, irrespective of the time-sampling point of frame in time-domain. For real-world signals however, the constant nature of phase derivative is only valid for a small duration due to the multipath effect. This problem is significant in solids since the propagation speed is high. The phase derivative calculated using data segment that is near to the TOA is less susceptible to the detrimental effect of multipath. Therefore, the phase derivative is calculated by averaging phase derivative values near to TOA of sensor signal. This is shown in Fig. 6.13 where the minimum of TOA values from the KLID algorithm is estimated to be at 530 sample index, and thus the phase derivative is derived using the average of phase derivative values around 530 sample index. Figure 6.13 also shows that the phase derivative approximately remains constant around TOA values. The phase jump of more than 2π usually implies that there is an onset point between 495 and 530 such that the phase of noise is having sudden transition to phase of signal. Thus it is sufficient to use the less computational intensive KLID algorithm [72] with a periodogram size of 32 and a frame size of 300 samples.

To estimate the $o(\hat{l}_{TZ}^x, \hat{l}_{TZ}^y)$ using (6.23), the touch area is rasterized with a step size of

Figure 6.14: Radon transform of $\mathbf{O}$ calculated for location (0.20, 0.20) m on aluminum plate for $\theta = 0$.

0.005 m for possible values of $(\hat{l}_{TZ}^x, \hat{l}_{TZ}^y)$. The variable ϑ is also iterated for values within -0.2 to 0.2 in steps of 0.07 to estimate $o(\hat{l}_{TZ}^x, \hat{l}_{TZ}^y)$ as given in (6.23). In addition, $\varphi = 1000$ is used to compute $o(\hat{l}_{TZ}^x, \hat{l}_{TZ}^y)$. Figure 6.4 shows a specific example of $\mathbf{O}$ calculated from the first tap on the location (0.20, 0.20) m of aluminum tapped with metal stylus for the setup shown in Fig. 6.11. The Radon transformed variable $o_r(\rho, \theta)$ is estimated on $\mathbf{O}$ for values $0 \leq \theta \leq 90^\circ$. Figure 6.14 shows a specific example of $\theta = 0^\circ$. The estimated data points corresponding $o_r(\rho, \theta)$ in Fig. 6.14 shows the validity of assumption of Gaussian nature of $\mathbf{O}$. To compensate resolution loss due to rasterization in x and y dimensions, the output of each the Radon transform is interpolated by polynomial fitting using a one-dimensional Gaussian fitting curve before determining $\rho_{max}(\theta)$. The variation of $\rho_{max}(\theta)$ with θ for location (0.20, 0.20) m on the aluminum plate is shown in Fig. 6.15. The variation of $\rho_{max}(\theta)$ with respect to θ in Fig. 6.15 satisfies the function $0.2 \cos(\theta) + 0.2 \sin(\theta)$. The particle swarm optimization is used to estimate the source location (0.20, 0.20) m from a set of values of $\rho_{max}(\theta)$. The particle swarm optimization is performed using 64 particles, with $c_1 = c_2 = 2$. The optimization is

Figure 6.15: Variation of $\rho_{\max}(\theta)$ for location (0.20, 0.20) m on aluminum plate.

performed for three variables, l_{src}^x , l_{src}^y and θ_n to find the minimization of the fitting function $\rho_{\max}(\theta) = l_{\text{src}}^x \cos(\theta - \theta_n) + l_{\text{src}}^y \sin(\theta - \theta_n)$ where θ_n is the phase shift error for the angle. The experiment is conducted on aluminum and glass plates using both metal and finger taps. The performance of the proposed algorithm is compared with that of TiF-HA [71] and warping-based algorithm [100].

The TiF-HA algorithm is performed on a data sample length of 1024 with 256 samples of leading zeros. The FFT size of 1024 is used, on a data frame length of 256 with proportionality constant of 0.0625, which determines the power of the random signal. For the case of parametrized warping algorithm, a signal length of 256 samples, with a FFT size of 128 is used. The reference sensor (sensor which is having the minimum onset time of arrival) is identified using the weighted average of transition onset time-of-arrival that is determined from the KLID [72] and the TiF-HA transition detection algorithms. Parameters chosen for TiF-HA is the same as that described in Section 5.4. For the KLID algorithm, a periodogram size of 512 is used and that bins are selected in the range of 14 to 20 for distance measurement. A cosine window of size 13 is also used to smooth the periodogram. The κ is iterated from 0 to 90 to obtain $\hat{\kappa}^{\text{opt}}$ which satisfies the optimization criteria.

Figure 6.16 compares the performance of the proposed RCS algorithm with TiF-HA and the warping-based algorithm in terms of RMSE and standard deviation. The

experiment is performed on aluminum and glass where each tap location is sequentially tapped using a metal stylus and a finger. These results show that, for taps made by the stylus on aluminum, the RCS algorithm can achieve reduction in RMSE of 11.5 mm and 3.9 mm compared to TiF-HA and the warping-based algorithm, respectively. The standard deviations of proposed algorithm are also lower by 5.2 mm and 2.5 mm compared to TiF-HA and the warping-based algorithm, respectively. When the aluminum surface is tapped using finger, the proposed RCS algorithm achieves a reduction of 13.0 mm and 14.0 mm (in terms of RMSE) and 5.7 mm and 8.4 mm (in terms of standard deviation) compared to TiF-HA and warping-based algorithm, respectively. The reduction in accuracy for finger tap when compare to stylus impact for aluminum is also reported for TiF-HA and warping-based algorithms. This is mainly due to the weak amplitude of signals received by far sensors (from source) during finger impact. For the case of impact localization on glass using stylus, the RCS algorithm outperforms TiF-HA and Warping algorithm by achieving reduction in RMSE of 11.8 mm and 9.6 mm as well as the reduction in standard deviation of 6 mm and 5.1 mm, respectively. Similar to finger impact on aluminum, the finger impact on glass also performs slightly less accuracy due to weaker signals when compare to its stylus based impact counterpart. The RCS algorithm achieves reduction in RMSE of 9.8 mm and 9.8 mm as well as reduction in standard deviation by 3.6 mm and 5.8 mm respectively when compared with TiF-HA and Warping algorithm. For all the experiment cases considered in this section, the performance of the proposed algorithm is higher than existing algorithms. For a large surface considered in this experiment, the RMS error of majority of tap locations is less than 1 cm with very few outliers. This validates the performance of RCS algorithm is less dependent on material when compared to TiF-HA and Warping algorithm. The improvement in localization accuracy with its material-independent nature makes the proposed RCS algorithm useful for touch applications.

Figure 6.16: Performance comparison of the proposed RCS algorithm with TiF-HA algorithm and Warping algorithm (a) RMSE in source localization and (b) standard deviation of RMSE in source localization.

6.4 Chapter Summary

In this interdisciplinary research, which employs the mechanical model of wave propagation in solids and signal processing techniques, an algorithm for localization of taps on solids via range difference based optimization approach is proposed. This algorithm extracts the range difference information embedded in phase derivatives of the cross-spectrum of flexural vibration signals. The effect of dispersion on the frequency spectrum of the source signal is also studied in this chapter by considering Gaussian narrow-band pulse as an exemplar source signal. This allows one to understand why conventional spectrum comparison methods such as the GCC fail to extract the time/range difference information when the medium of propagation is dispersive. Unlike conventional cross-spectrum based source localization methods, it is shown in this chapter that when the sensor signals are due to the flexural vibration on the solid surface, the phase derivative of the cross-spectrum will be a scaled version of the range difference between source-to-sensor locations, where the scaling factor depends on the material properties of the medium and frequency of operation. The phase derivatives of the cross-spectrum is shown to depend on the range difference and material based physical constants. This

explains the significance of the proposed method which does not require knowledge of the material properties or the need for pre-calibration. To achieve this, the phase derivative of the cross-spectrum is mapped to a different domain using a newly proposed mapping function and applying the Radon transform on it. Particle swarm optimization is subsequently used for estimating the impact location. The algorithm is validated via experiments conducted on aluminum and glass plates, for both stylus and figure taps.

Chapter 7

Conclusion and Recommendations

The objective of this research is to develop source localization algorithms to estimate the location of an impact on solids using signals acquired by surface-mounted vibrational sensors. In this thesis, several classes of algorithms which utilize the time-frequency analysis of the vibrational signal due to impact on solids to estimate source location are proposed.

The mathematical model derived in Chapter 2 is utilized in Chapter 3 to derive an LTM-based approach for impact localization using a single sensor. This wave propagation model allows one to gain insights into the uniqueness of signal features obtained by an impact on each position on a given surface. Moreover, the model highlights the need for time-frequency analysis to the study of these signals since it is shown that the amplitude variance across time for each frequency-bin is unique for each location. This chapter further shows that the Zak transform allows one to achieve faithful representation of these variations and that the variance in each frequency bin with respect to time is proposed as a unique time-frequency classifier for each location. Since mode frequencies generated by mechanical wave propagation have now been taken into account, the performance of the proposed Z-LTM source localization algorithm for various materials including the aluminum, glass as well as plywood is higher than existing LTM methods. Moreover, Z-LTM depends on the time-frequency variation of

mode frequencies generated by mechanical wave propagation, the algorithm as such is immune to additive noise to signals.

Chapter 4 presents a technique for impact localization using time-difference-of-arrival (TDOA) between two spatially separated sensors – a commonly used method that has been applied for acoustic source localization. The main advantage of this method compared to that of LTM-based methods is, it does not require training of known locations. This implies that such algorithm is able to achieve a large range of distinguishable tap locations with minimal setup/calibration. The change point detection algorithm is devised for detecting abrupt changes in the time-varying power spectrum of a time series while transition from background noise to vibration signal. In this chapter, a new method that employs the Kullback-Leibler (KL) discrimination information for localizing a single tap made by a metal stylus on various flat surfaces is proposed. This is achieved through the use of the phase transit time of the vibration signal.

In the context of HCI, existing TDOA based methods, like KLID in Chapter 4, require calibration for the estimation of phase velocities. This, in turn, may cause inconvenience to potential users. So a source localization algorithm on solids is proposed in Chapter 5 based on the range difference between pairs of source-sensor to avoid the need for velocity estimation. The wave propagation model of flexural vibration due to an impact on a plate surface from Chapter 5 is analyzed and show that the change in dispersion between two signals is dependent on the range difference between the propagation path of the signals. Therefore, a difference in the dispersion can be mapped directly to a difference in relative propagation distances if the signals are properly time-warped. This algorithm thus exploits the similarity of the warped dispersive signal in the time-frequency domain. As the proposed source localization algorithm jointly estimates the model parameters and source location, this method does not require pre-calibration. For every pair of the sensor signal, one of the two sensor signals is warped over a range of warping parameter and each warped signal will be checked for similarity with the other signal by analyzing them in the time-frequency domain. From the experimental

results conducted on different plate surfaces, it is observed that a polynomial function of order two is sufficient to describe the relationship between the range differences and the optimal warping parameters. Since the parameter for this polynomial function, which depends on the material type, along with source position are jointly estimated using optimization algorithms, this algorithm is calibration free.

In Chapter 5, the relationship between the range differences and the optimal warping parameters is modeled as a second-order polynomial function, which is restrictive in nature. Chapter 6 presents another calibration-free method which extracts the range difference information embed in phase derivatives of the cross-spectrum of flexural vibration signals. Unlike conventional cross-spectrum based source localization methods, this chapter shows that when the sensor signals are caused by the flexural vibration on the solid surface, the phase derivative of the cross-spectrum will be a scaled version of the range difference between source-to-sensor locations. The scaling factor depends on the material properties of the medium and frequency of operation. The effect of dispersion on the frequency spectrum of the source signal is also studied and this is achieved by considering the Gaussian narrow-band pulse as an example source signal. This explains why conventional spectrum comparison methods fail to extract the time/range difference information when the medium of propagation is dispersive. To achieve effective source localization, the phase derivative of the cross-spectrum is mapped to a different domain using a newly proposed mapping function and applying the Radon transform on it. Particle swarm optimization is subsequently used for estimating the impact location. The algorithm is validated via experiments conducted on aluminum and glass plates, for both stylus and figure taps and is found more accurate than all other proposed algorithms in this thesis.

The Z-LTM method is a single channel classification algorithm which is suitable for applications where the number of points to detect is finite and limited. The computational cost for classifying tap location increases with number of training points. But advantage is that the resolution for less than 1 cm is achieved during experiment

setup. The KLID, warping-based and RCS method is multi sensor approach which does not involves training and thus is more suitable for wider applications. The KLID method involves calibration of velocity of propagation, unlike warping-based and RCS methods. Due to this the accuracy achieved by KLID method is inferior when compare to warping-based and RCS based methods. The warping method assumes a model for its estimation parameters, and occasionally if there is a model mismatch the accuracy of warping-based is less than RCS based methods. The RCS algorithm achieves highest accuracy among the existing multi sensor source localization algorithm in solids which in terms of RMSE reduction of 11.5 mm and 3.9 mm compared to TiF-HA and the warping-based algorithm, respectively. Among all proposed multi sensor source localization algorithms in solids the computational complexity is lowest for KLID followed by warping method and Tif-HA whereas the RCS have highest computationally intensive. Accuracy wise RCS have the highest accuracy among the multi-channel algorithms followed by warping method, Tif-HA and least for KLID.

7.0.1 Recommendations for further research

The algorithms proposed in this thesis are to develop source locations corresponding to a single impact on the surface of solids. The LTM-based algorithms proposed in Chapter 3 utilize only one sensor and localize finger taps by comparing the received signal with a set of pre-recorded signals from known locations. On the contrary, the algorithms proposed in Chapters 4, 5 and 6 exploit multi-sensor architecture for source localization of a large range of distinguishable tap locations.

The TDOA based approach proposed in Chapter 4 is affected by noise, especially at the change point area. This algorithm can be further improved by operating the algorithm on significant mode signal separated from impact signal. Several mode separation methods such as empirical-mode-decomposition or blind-source-separation can be used to separate the signal into modes. Further research needs to be conducted on the selection of a significant mode from the separated signals for which mathematical

model of wave propagation can be exploited.

Two main research directions for algorithms proposed in Chapter 5 and 6 are identified. One is to extend the single tap algorithm to track sequential detection of several taps which mimics the tracking of drag operation over an interactive surface. This problem can be formulated as state-space tracking of tap location over time. The feasibility of using existing state-space tracking techniques like Kalman filter and grid-based filter or approximate suboptimal techniques like particle filters for this current problem has to be investigated [111]. The other direction is to design and develop algorithms suitable for localizing simultaneous multiple impacts. This study will be challenging due to the overlapping of the elastic waves and the reflection of those waves from the edge in a plate. The existing algorithms [112] for multiple impact localization in plates does not allow complete overlap of impact signal in time-domain and also uses material-dependent parameter for localization. The possible solution is to extend the research proposed in Chapter 6 with source separation techniques such as blind source separation [113] which exploits information from multiple channels to extract individual impacts from mixture.

Appendix A

A.1 Natural frequencies of plate vibration

We want to solve

$$\ddot{D}\nabla^4(w(x, y, t)) + \rho L_z \frac{\partial^2 w(x, y, t)}{\partial t^2} = 0 \quad (\text{A.1})$$

for plate in which $0 \leq x \leq L_x$ and $0 \leq y \leq L_y$ with simply supported plate boundary condition

$w(0, y, t) = 0$ and $w(L_x, y, t) = 0$ for every possible value of y and t

$w(x, 0, t) = 0$ and $w(x, L_y, t) = 0$ for every possible value of x and t

we would like to have solution

$$w(x, y, t) = W(x, y)T(t),$$

where

$$W(x, y) = X(x)Y(y).$$

Substitute in (A.1), we obtain

$$\ddot{D}T(t)\nabla^4(X(x)Y(y)) + \rho L_z X(x)Y(y)T''(t) = 0.$$

Dividing by $X(x)Y(y)T(t)$ on both sides

$$\begin{aligned} \dot{D} \frac{\nabla^4 (X(x)Y(y))}{X(x)Y(y)} + \rho L_z \frac{T''(t)}{T(t)} &= 0 \\ \frac{\dot{D}}{\rho L_z} \frac{\nabla^4 (X(x)Y(y))}{X(x)Y(y)} &= -\frac{T''(t)}{T(t)} = \omega^2 \\ \frac{\dot{D}}{\rho L_z} \frac{\nabla^4 (X(x)Y(y))}{X(x)Y(y)} = \varpi^4 \quad \Bigg| \quad \text{where } \varpi^4 &= \frac{\rho L_z}{\dot{D}} \omega^2 \end{aligned} \quad (\text{A.2})$$

$$\begin{aligned} \nabla^4 (X(x)Y(y)) &= \varpi^4 X(x)Y(y) \\ \nabla^4 (X(x)Y(y)) - \varpi^4 X(x)Y(y) &= 0 \\ \left(\nabla^2 + \varpi^2 \right) \left(\nabla^2 - \varpi^2 \right) X(x)Y(y) &= 0. \end{aligned}$$

Two possible solutions are

$$\left(\nabla^2 + \varpi^2 \right) X(x)Y(y) = 0 \quad \& \quad \left(\nabla^2 - \varpi^2 \right) X(x)Y(y) = 0. \quad (\text{A.3})$$

Consider the first case

$$\begin{aligned} \left(\nabla^2 + \varpi^2 \right) X(x)Y(y) &= 0 \\ \nabla^2 (X(x)Y(y)) + \varpi^2 X(x)Y(y) &= 0 \\ X(x)Y''(y) + X''(x)Y(y) + \varpi^2 X(x)Y(y) &= 0 \\ \frac{X''(x)}{X(x)} + \frac{Y''(y)}{Y(y)} &= -\varpi^2 \\ \frac{X''(x)}{X(x)} = -\left(\frac{Y''(y)}{Y(y)} + \varpi^2 \right) &= -\hat{\varpi} \\ X''(x) + \hat{\varpi} X(x) = 0, Y''(y) + \hat{\varpi} Y(y) &= 0 \\ \frac{X''(x)}{X(x)} = -\hat{\varpi}, \quad \frac{Y''(y)}{Y(y)} = -(-\varpi^2 + \hat{\varpi}) = -\varpi \quad \Bigg| \quad \text{where } \varpi^2 = \bar{\varpi} + \varpi & \quad (\text{A.4}) \\ X''(x) + \bar{\varpi} X(x) = 0 \quad \Bigg| \quad Y''(y) + \varpi Y(y) = 0. \end{aligned}$$

Consider $X''(x) + \bar{\omega}X(x) = 0$ with simply supported plate boundary condition. The variable $w(0, y, t) = 0$ and $w(L_x, y, t) = 0$ for every possible value of y and t , giving

$$\begin{aligned} w(0, y, t) &= 0 \\ X(0)Y(y)T(t) &= 0 \\ \Rightarrow X(0) &= 0. \end{aligned}$$

Since $Y(y) = 0$ and $T(t) = 0$ are trivial solutions.

Similarly, $w(L_x, y, t) = 0 \Rightarrow X(L_x) = 0$. The solution of $X''(x) + \bar{\omega}X(x) = 0$ with boundary conditions $X(0) = 0$ and $X(L_x) = 0$ can be assumed to be form of $e^{\hat{\lambda}x}$. Substituting in above equation, we obtain

$$\begin{aligned} (\hat{\lambda}^2 + \bar{\omega})e^{\hat{\lambda}x} &= 0 \\ \Rightarrow \hat{\lambda}^2 &= -\bar{\omega}. \end{aligned}$$

There can be three case for the solution where for the first case $\bar{\omega} = 0$. This is the case of real and equal roots

$$\begin{aligned} X(x) &= c_1 + c_2x \\ X(0) = 0 &\Rightarrow c_1 = 0 \\ X(a) = 0 &\Rightarrow c_2 = 0 \end{aligned}$$

so that $X(x) = 0$ which is a trivial solution.

For the second case $\bar{\omega} < 0$ and this is the case of real and distinct roots. So assume

$\bar{\omega} = -k^2$ by which $\hat{\lambda} = \pm k$

$$X(x) = c_1 e^{kx} + c_2 e^{-kx}$$

$$X(0) = 0 \Rightarrow c_1 + c_2 = 0 \Rightarrow c_1 = -c_2$$

$$X(L_x) = 0 \Rightarrow c_1 (e^{kL_x} - e^{-kL_x}) = 0 \Rightarrow c_1 = 0.$$

This also leads to trivial solution.

For the third case $\bar{\omega} > 0$. This is the case of imaginary roots. Assume $\bar{\omega} = k^2$ by which $\hat{\lambda} = \pm jk$

$$X(x) = e^{0x} (c_1 \cos(kx) + c_2 \sin(kx))$$

$$X(0) = 0 \Rightarrow c_1 = 0$$

$$X(L_x) = 0 \Rightarrow c_2 \sin(kL_x) = 0 \Rightarrow kL_x = l\pi \Rightarrow k = \frac{l\pi}{L_x}.$$

The solution of $X''(x) + \hat{\omega}X(x) = 0$ for given boundary condition is given by

$$X(x) = c_l \sin\left(\frac{l\pi x}{L_x}\right) \quad \text{with } \hat{\omega} = \frac{l^2\pi^2}{L_x^2}.$$

Similar derivation can be performed for $Y''(y) + \varpi Y(y) = 0$ with boundary condition leading to $Y(0) = 0$ and $Y(L_y) = 0$, in turn, leads to

$$Y(y) = c_b \sin\left(\frac{b\pi y}{L_y}\right) \quad \text{with } \varpi = \frac{b^2\pi^2}{L_y^2}.$$

These two solutions can be combined according to superposition principle to form the solution set of the natural shape function as

$$W(x, y) = \sum_{l=1}^{\infty} \sum_{b=1}^{\infty} W_{lb} \sin\left(\frac{l\pi x}{L_x}\right) \sin\left(\frac{b\pi y}{L_y}\right). \quad (\text{A.5})$$

where L_x and L_y are the plate dimensions and W_{lb} is the vibration amplitude for each

value of l and b .

The natural frequency of vibration is obtained from (A.2) and (A.4) as

$$\begin{aligned}\dot{\omega}^2 &= \hat{\omega} + \varpi \\ \dot{\omega}^4 &= \frac{\rho L_z}{\dot{D}} \omega^2 \\ \omega &= \sqrt{\frac{\dot{D}}{\rho L_z}} \dot{\omega}^2 \\ &= \sqrt{\frac{\dot{D}}{\rho L_z}} (\hat{\omega} + \varpi) \\ \omega_{lb} &= \sqrt{\frac{\dot{D}}{\rho L_z}} \left(\frac{l^2 \pi^2}{L_x^2} + \frac{b^2 \pi^2}{L_y^2} \right).\end{aligned}$$

A.2 Solving W_{lb} for particular value of input function

$$f(x, y)$$

$$f(x, y) = \sum_{l=1}^{\infty} \sum_{b=1}^{\infty} W_{lb} \sin\left(\frac{l\pi x}{L_x}\right) \sin\left(\frac{n\pi y}{L_y}\right).$$

Multiplying $\sin(\frac{\hat{p}\pi x}{L_x})$ on both sides (where $\hat{p}$ is any arbitrary value) and integrating with respect to x ,

$$\int_0^{L_x} f(x, y) \sin\left(\frac{\hat{p}\pi x}{L_x}\right) dx = \sum_{b=1}^{\infty} \sin\left(\frac{b\pi y}{L_y}\right) \sum_{l=1}^{\infty} W_{lb} \int_0^{L_x} \sin\left(\frac{l\pi x}{L_x}\right) \sin\left(\frac{\hat{p}\pi x}{L_x}\right) dx.$$

Applying the identity

$$\int_0^{L_x} \sin\left(\frac{l\pi x}{L_x}\right) \sin\left(\frac{\hat{p}\pi x}{L_x}\right) dx = \begin{cases} \frac{L_x}{2} & \text{if } l = \hat{p} \\ 0 & \text{else} \end{cases},$$

we get,

$$\int_0^{L_x} f(x, y) \sin\left(\frac{\hat{p}\pi x}{L_x}\right) dx = \frac{L_x}{2} \sum_{b=1}^{\infty} W_{lb} \sin\left(\frac{b\pi y}{L_y}\right)$$

Similarly multiplying $\sin(\frac{\hat{q}\pi y}{L_y})$ on both sides (where $\hat{q}$ is any arbitrary value) and integrating with respect to y ,

$$\int_0^{L_y} \int_0^{L_x} f(x, y) \sin(\frac{\hat{p}\pi x}{L_x}) \sin(\frac{\hat{q}\pi y}{L_y}) dx dy = \frac{L_x}{2} \sum_{b=1}^{\infty} \int_0^{L_y} W_{lb} \sin(\frac{b\pi y}{L_y}) \sin(\frac{\hat{q}\pi y}{L_y}) dy.$$

Applying the identity

$$\int_0^{L_y} W_{lb} \sin(\frac{b\pi y}{L_y}) \sin(\frac{\hat{q}\pi y}{L_y}) dy = \begin{cases} \frac{L_y}{2} & \text{if } b = \hat{q} \\ 0 & \text{else} \end{cases},$$

we get,

$$\int_0^{L_y} \int_0^{L_x} f(x, y) \sin(\frac{\hat{p}\pi x}{L_x}) \sin(\frac{\hat{q}\pi y}{L_y}) dx dy = \frac{L_x L_y}{4} W_{lb}.$$

Therefore,

$$W_{lb} = \frac{4}{L_x L_y} \int_0^{L_y} \int_0^{L_x} f(x, y) \sin(\frac{\hat{p}\pi x}{L_x}) \sin(\frac{\hat{q}\pi y}{L_y}) dx dy.$$

Publications

- [i] K. R. Arun, X. Yap, and A. W. H. Khong, “A touch interface exploiting time-frequency classification using Zak transform for source localization on solids,” *IEEE Trans. Multimedia*, vol. 13, pp. 487-497, 2011.
- [ii] K. R. Arun, E. Ong, and A. W. H. Khong, “Source localization on solids using Kullback-Leibler discrimination information,” in *Proc. IEEE Conf. Information, Communications, Signal Processing*, 2011.
- [iii] K. R. Arun, V. G. Reju, and A. W. H. Khong, Source localization on solids utilizing time-frequency analysis of parameterized warped signals, in *Proc. IEEE Int. Conf. Acoust. Speech and Signal Process.*, pp. 689–693, 2014.
- [iv] K. R. Arun, V. G. Reju, and A. W. H. Khong, “Source Localization on Solids utilizing Radon Transform of Phase Value Performance Matrix,” *IEEE Trans. Multimedia*, under preparation.

References

- [1] D. Pham, M. Al-Kutubi, Z. Ji, M. Yang, Z. Wang, and S. Catheline, “Tangible acoustic interface approaches,” in *Proc. IPROMS Virtual Conf.*, 2005, pp. 497–502.
- [2] A. Sulaiman, K. Poletkin, and A. W. H. Khong, “Source localization in the presence of dispersion for next generation touch interface,” in *Proc. Int. Conf. Cyberworlds*. IEEE Computer Society, 2010, pp. 82–86.
- [3] D. Pham, Z. Wang, Z. Ji, M. Yang, M. Al-Kutubi, and S. Catheline, “Acoustic pattern registration for a new type of human-computer interface,” in *Proc. IPROMS Virtual Conf.*, 2005, pp. 481–486.
- [4] X. Yap, A. W. H. Khong, and W.-S. Gan, “Localization of acoustic source on solids: A linear predictive coding based algorithm for location template matching,” in *Proc. IEEE Int. Conf. Acoust. Speech and Signal Process.* IEEE, 2010, pp. 2490–2493.
- [5] H. Jeong and Y. S. Jang, “Fracture source location in thin plates using the wavelet transform of dispersive waves,” *IEEE Trans. Ultrason., Ferroelect., Freq. Control*, vol. 47, no. 3, pp. 612–619, 2000.
- [6] G. Shin and J. Chun, “Vision-based multimodal human computer interface based on parallel tracking of eye and hand motion,” in *Proc. IEEE Int. Conf. Convergence Inform. Technol.* IEEE, 2007, pp. 2443–2448.

- [7] Y. W. Kim, "A novel development of head-set type computer mouse using gyro sensors for the handicapped," in *Proc. IEEE 2nd Annual Int. EMB Special Topic Conf. Microtechnol. in Medicine & Biology*. IEEE, 2002, pp. 356–359.
- [8] J. Kim, J. Park, H. Kim, and C. Lee, "HCI (human computer interaction) using multi-touch tabletop display," in *Proc. IEEE Pacific Rim Conf. Communication, Computers, and Signal Process.* IEEE, 2007, pp. 391–394.
- [9] W. Rolshofen, D. T. Pham, M. Yang, Z. B. Wang, Z. Ji, and M. Al-Kutubi, "New approaches in computer-human interaction with tangible acoustic interfaces," in *Proc. IPROMS Virtual Conf.*, 2005.
- [10] K. Poletkin, X. Yap, and A. W. H. Khong, "A touch interface exploiting the use of vibration theories and infinite impulse response filter modeling based localization algorithm," in *Proc. IEEE Int. Conf. Multimedia and Expo.* IEEE, 2010, pp. 286–291.
- [11] C. Bornand, A. Camurri, G. Castellano, S. Catheline, A. Crevoisier, E. B. Roesch, K. R. Scherer, and G. Volpe, "Usability evaluation and comparison of prototypes of tangible acoustic interfaces," in *Proc. 2nd Int. Conf. Enactive Interfaces*, 2005.
- [12] T. Kundu, S. Das, S. A. Martin, and K. V. Jata, "Locating point of impact in anisotropic fiber reinforced composite plates," *Ultrasonics*, vol. 48, pp. 193–201, 2008.
- [13] D. Chimenti, "Guided waves in plates and their use in materials characterization," *Appl. Mech. Rev.*, vol. 50, p. 247, 1997.
- [14] Y.-H. Pao, R. R. Gajewski, and A. N. Ceranoglu, "Acoustic emission and transient waves in an elastic plate," *J. Acoust. Soc. Am.*, vol. 65, no. 1, pp. 96–105, 1979.
- [15] I. Tolstoy and E. Usdin, "Wave propagation in elastic plates: low and high mode dispersion," *J. Acoust. Soc. Am.*, vol. 29, no. 1, pp. 37–42, 1957.

- [16] C. Dikilitas, "Location template matching-based study of acoustic emission localization using e-puck robots," Ph.D. dissertation, Northeastern University Boston, 2015.
- [17] L. W. Braile, "Seismic Waves and the Slinky: A Guide for Teachers," <http://web.ics.purdue.edu/~braile/edumod/slinky/slinky.htm>, February 2010.
- [18] V. Giurgiutiu, J. Bao, and W. Zhao, "Active sensor wave propagation health monitoring of beam and plate structures," in *Proc. SPIE*. International Society for Optics and Photonics, 2001, pp. 234–245.
- [19] D. Aggelis and T. Matikas, "Influence of propagation distance on the acoustic emission parameters in metal plates," in *Proc. SPIE*, vol. 8346. International Society for Optics and Photonics, 2012, pp. 83 460F–1–83 460F–6.
- [20] A. Maji, D. Satpathi, and T. Kratochvil, "Acoustic emission source location using lamb wave modes," *J. Eng. mechanics*, vol. 123, no. 2, pp. 154–161, 1997.
- [21] D. Rovetta, A. Sarti, G. De Sanctis, and M. Fabiani, "Modelling elastic wave propagation in thin plates," in *Proc. 14th Eur. Signal Process. Conf.*, 2006, pp. 1–5.
- [22] Z. Su, L. Ye, and Y. Lu, "Guided lamb waves for identification of damage in composite structures: A review," *J. Sound Vib.*, vol. 295, no. 3, pp. 753–780, 2006.
- [23] W. H. Prosser, "The propagation characteristics of the plate modes of acoustic emission waves in thin aluminum plates and thin graphite/epoxy composite plates and tubes," *J. Acoust. Soc. Am.*, vol. 92, no. 6, pp. 3441–3442, 1992.
- [24] H. Dunegan, "Modal analysis of acoustic emission signals," *J. Acoustic emission*, vol. 15, pp. 53–61, 1997.

- [25] B. J. Tucker, “Ultrasonic plate waves in wood-based composite panels,” Ph.D. dissertation, Department of Civil and Environmental Engineering, 2001.
- [26] P. F. Pai, H. Deng, and M. J. Sundareshan, “Time-frequency characterization of lamb waves for material evaluation and damage inspection of plates,” *Mech. Syst. Signal Process.*, vol. 62, pp. 183–206, 2015.
- [27] E. Ventsel and T. Krauthammer, *Thin plates and shells: theory, analysis, and applications*. Marcel Dekker, Inc., 2001.
- [28] K. F. Graff, *Wave Motion in Elastic Solids*. Dover Publications, 1991.
- [29] R. D. Mindlin, “The influence of rotatory inertia and shear on the flexural motions of isotropic elastic plates,” *J. Appl. Mech. Trans. ASME.*, vol. 18, pp. 31–38, 1951.
- [30] C. Chang and W. Sachse, “Analysis of elastic wave signals from an extended source in a plate,” *J. Acoust. Soc. Am.*, vol. 77, no. 4, pp. 1335–1341, 1985.
- [31] W. Prosser and M. Gorman, “Plate mode velocities in graphite/epoxy plates,” *J. Acoust. Soc. Am.*, vol. 96, no. 2, pp. 902–907, 1994.
- [32] J. Martens, “Analysis of bending wave patterns using array technology,” Master’s thesis, Laboratory of Acoustic Imaging and Sound Control, 1998.
- [33] Z. Su and L. Ye, *Identification of Damage Using Lamb Waves: From Fundamentals to Applications*. Springer Verlag, 2009.
- [34] G. Ribay, S. Catheline, D. Clorennec, R. K. Ing, N. Quieffin, and M. Fink, “Acoustic impact localization in plates: properties and stability to temperature variation,” *IEEE Trans. Ultrason., Ferroelect., Freq. Control*, vol. 54, pp. 378–385, 2007.
- [35] F. Ciampa and M. Meo, “Acoustic emission source localization in anisotropic structures with diffuse field conditions using a time reversal approach,” in *Proc.*

- SPIE*, vol. 7984. International Society for Optics and Photonics, 2011, pp. 798 416–1–798 416–7.
- [36] W. H. Prosser, M. D. Seale, and B. T. Smith, “Time-frequency analysis of the dispersion of lamb modes.” *J. Acoust. Soc. Am.*, vol. 105, no. 5, pp. 2669–76, May 1999.
- [37] T. Boczar, “Time-frequency analysis of acoustic emission pulses generated by partial discharges,” *Journal of Electrical Engineering*, vol. 54, no. 3–4, pp. 63–68, 2003.
- [38] L. Cohen, “Time-frequency distributions-a review,” *Proc. IEEE*, vol. 77, no. 7, 1989, pp. 941 – 981.
- [39] F. J. Harris, “On the use of windows for harmonic analysis with the discrete Fourier transform,” *Proc. IEEE*, vol. 66, 1978, pp. 51 – 83.
- [40] P. Angelidis and G. Sergiadis, “Time-frequency representation of damped sinusoids using the Zak transform,” *J. Magn. Resonance*, vol. 103, pp. 191–195, 1993.
- [41] J. R. O’Hair and B. W. Suter, “The Zak transform and decimated time-frequency distributions,” *IEEE Trans. Signal Process.*, vol. 44, pp. 1099–1110, 1996.
- [42] H. Bolcskei and F. Hlawatsch, “Discrete Zak transforms, polyphase transforms, and applications,” *IEEE Trans. Signal Process.*, vol. 45, pp. 851–866, 1997.
- [43] R. L. Allen and D. Mills, *Signal analysis: time, frequency, scale, and structure*. John Wiley & Sons, 2004.
- [44] C. Knapp and G. C. Carter, “The generalized correlation method for estimation of time delay,” *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 24, no. 4, pp. 320–327, 1976.

- [45] B. Castagnede, W. Sachse, and K. Y. Kim, "Location of pointlike acoustic emission sources in anisotropic plates," *J. Acoust. Soc. Am.*, vol. 86, no. 3, pp. 1161–1171, 1989.
- [46] V. Pisarenko, A. Kushnir, and I. Savin, "Statistical adaptive algorithms for estimation of onset moments of seismic phases," *Physics of the earth and planetary interiors*, vol. 47, pp. 4–10, 1987.
- [47] R. Roberts, A. Christoffersson, and F. Cassidy, "Real-time event detection, phase identification and source location estimation using single station three-component seismic data," *Geophysical Journal International*, vol. 97, no. 3, pp. 471–480, 1989.
- [48] I. V. Nikiforov and I. N. Tikhonov, "Application of change detection theory to seismic signal processing," in *Detection of Abrupt Changes in Signals and Dynamical Systems*. Springer, 1986, pp. 355–373.
- [49] L. Vaisman, J. Zariffa, and M. R. Popovic, "Application of singular spectrum-based change-point analysis to emg-onset detection," *J. Electromyography and Kinesiology*, vol. 20, no. 4, pp. 750–760, 2010.
- [50] F. Ghaderi, H. R. Mohseni, and S. Sanei, "Localizing heart sounds in respiratory signals using singular spectrum analysis," *IEEE Transactions Biomedical Eng.*, vol. 58, no. 12, pp. 3360–3367, 2011.
- [51] V. Moskvina and A. Zhigljavsky, "An algorithm based on singular spectrum analysis for change-point detection," *Communications in Statistics-Simulation and Computation*, vol. 32, no. 2, pp. 319–352, 2003.
- [52] Y. Kawahara, T. Yairi, and K. Machida, "Change-point detection in time-series data based on subspace identification," in *Proc. 8th IEEE Int. Conf. Data Mining*. IEEE, 2007, pp. 559–564.

- [53] H. Sohn and C. R. Farrar, "Damage diagnosis using time series analysis of vibration signals," *Smart materials and structures*, vol. 10, no. 3, p. 446, 2001.
- [54] A. Deng, L. Zhao, and Y. Zhao, "Study on algorithms of acoustic emission source localization in complicated environment," in *Proc. Int. Conf. Electronic Measurement & Instruments*. IEEE, 2009, pp. 462–466.
- [55] K. K. Nair, A. S. Kiremidjian, and K. H. Law, "Time series-based damage detection and localization algorithm with application to the ASCE benchmark structure," *J. Sound Vib.*, vol. 291, no. 1, pp. 349–368, 2006.
- [56] M. Basseville and I. V. Nikiforov, *Detection of Abrupt Changes: Theory and Application*. Prentice Hall Englewood Cliffs, 1993, vol. 104.
- [57] J.-i. Takeuchi and K. Yamanishi, "A unifying framework for detecting outliers and change points from time series," *IEEE transactions Knowledge and Data Eng.*, vol. 18, no. 4, pp. 482–492, 2006.
- [58] M. Last, "Change point estimation with independent observations and piecewise continuous variance function," *Communications in Statistics-Theory and Methods*, vol. 37, no. 5, pp. 722–736, 2008.
- [59] A. Hoover, A. Singh, S. Fishel-Brown, and E. Muth, "Real-time detection of workload changes using heart rate variability," *Biomed. Signal Process. Control*, vol. 7, no. 4, pp. 333–341, 2012.
- [60] F. Camci, "Change point detection in time series data using support vectors," *Int. J. Pattern Recognition and Artificial Intelligence*, vol. 24, no. 01, pp. 73–95, 2010.
- [61] M. E. Mann and J. M. Lees, "Robust estimation of background noise and signal detection in climatic time series," *Climatic change*, vol. 33, no. 3, pp. 409–445, 1996.

- [62] S. J. Orfanidis, *Electromagnetic waves and antennas*. Rutgers University New Brunswick, NJ, 2002.
- [63] K. R. Arun, X. Yap, and A. W. H. Khong, “A touch interface exploiting time-frequency classification using Zak transform for source localization on solids,” *IEEE Trans. Multimedia*, vol. 13, no. 3, pp. 487–497, 2011.
- [64] C. Zhang, D. Florencio, and Z. Zhang, “Why does PHAT work well in lownoise, reverberative environments?” in *Proc. IEEE Int. Conf. Acoust. Speech and Signal Process.*, 2008.
- [65] M. Last and R. Shumway, “Detecting abrupt changes in a piecewise locally stationary time series,” *J. Multivariate Analysis*, vol. 99, pp. 191–214, 2008.
- [66] P. Bloomfield, *Fourier Analysis of Time Series: An Introduction, Second Edition*. John Wiley & Sons, Inc., 2005.
- [67] J. D. Cryer and K. Chan, *Time Series Analysis With Applications in R*. Springer New York, 2008.
- [68] R. Dahlhaus, “On the Kullback-Leibler information divergence of locally stationary processes,” *Stoch. Proc. Appl.*, vol. 62, pp. 139–168, 1996.
- [69] D. W. Marquardt, “An algorithm for least-squares estimation of nonlinear parameters,” *J. Soc. Ind. Appl. Math.*, vol. 11, no. 2, pp. 431–441, 1963.
- [70] “muRata PKS1-4A10 datasheet,” 2016, available at <http://www.mouser.com/catalog/specsheets/Murata%20PKS1-4A.pdf>, (Date last accessed 25-December-2016).
- [71] V. G. Reju, A. W. H. Khong, and A. Sulaiman, “Localization of taps on solid surfaces for human-computer touch interfaces,” vol. 15, no. 6. IEEE, 2013, p. 1365.

- [72] K. R. Arun, E. Ong, and A. W. H. Khong, “Source localization on solids using Kullback-Leibler discrimination information,” in *Proc. IEEE Conf. Information, Communications, Signal Process.*, 2011, pp. 1–5.
- [73] J. Smith and J. Abel, “Closed-form least-squares source location estimation from range-difference measurements,” *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 35, no. 12, pp. 1661–1669, 1987.
- [74] K. Grabowski, M. Gawronski, H. Nakatani, P. Packo, I. Baran, W. Spsychalski, W. Staszewski, T. Uhl, and T. Kundu, “Distance-domain based localization techniques for acoustic emission sources: a comparative study,” in *Proc. SPIE*, vol. 9438. International Society for Optics and Photonics, 2015, pp. 94 381T–1–94 381T–9.
- [75] M. Gawronski, K. Grabowski, P. Russek, W. Staszewski, T. Uhl, and P. Packo, “Acoustic emission source localization based on distance domain signal representation,” in *Proc. SPIE*, vol. 9805. International Society for Optics and Photonics, 2016, pp. 98 050I–1–98 050I–7.
- [76] M. J. Eaton, R. Pullin, and K. M. Holford, “Acoustic emission source location in composite materials using Delta T mapping,” *Composites Part A: Applied Science and Manufacturing*, vol. 43, no. 6, pp. 856–863, 2012.
- [77] A. Papandreou-Suppappola and S. B. Suppappola, “Analysis and classification of time-varying signals with multiple time-frequency structures,” *IEEE Signal Process. Lett.*, vol. 9, no. 3, pp. 92–95, 2002.
- [78] J. Bonnel, G. Le Touze, B. Nicolas, and J. I. Mars, “Physics-based time-frequency representations for underwater acoustics: Power class utilization with waveguide-invariant approximation,” *IEEE Signal Process. Mag.*, vol. 30, no. 6, pp. 120–129, 2013.

- [79] Y. Jiang, H. Shen, and A. Papandreou-Suppappola, “Characterization of shallow water environments with waveform design for diversity,” in *Proc. Int. Waveform Diversity Design Conf.*, 2006.
- [80] M. Karjalainen, A. Harma, U. K. Laine, and J. Huopaniemi, “Warped filters and their audio applications,” in *Proc. IEEE Workshop on Applications of Signal Processing to Audio and Acoustics*. IEEE, 1997, pp. 4–pp.
- [81] A. Härmä, M. Karjalainen, L. Savioja, V. Välimäki, U. K. Laine, and J. Huopaniemi, “Frequency-warped signal processing for audio applications,” *J. Audio Eng. Soc.*, vol. 48, no. 11, pp. 1011–1031, 2000.
- [82] P. Wilcox, “A rapid signal processing technique to remove the effect of dispersion from guided wave signals,” *IEEE Trans. Ultrason., Ferroelect., Freq. Control*, vol. 50, no. 4, pp. 419–427, 2003.
- [83] C. Ioana and A. Quinquis, “On the use of time-frequency warping operators for analysis of marine-mammal signals,” in *Proc. IEEE Int. Conf. Acoust. Speech and Signal Process.*, vol. 2. IEEE, 2004.
- [84] C. Ioana, J. I. Mars, A. Serbanescu, and S. Stankovic, “Time-frequency-phase tracking approach: Application to underwater signals in a passive context,” in *Proc. IEEE Int. Conf. Acoust. Speech and Signal Process.* IEEE, 2010, pp. 5634–5637.
- [85] R. G. Baraniuk, “Signal transform covariant to scale changes,” *Electronics Letters*, vol. 29, no. 19, pp. 1675–1676, 1993.
- [86] R. G. Baraniuk, “Warped perspectives in time-frequency analysis,” in *Proc. IEEE-SP Int. Symp. Time-Frequency and Time-Scale Analysis*. IEEE, 1994, pp. 528–531.

- [87] R. G. Baraniuk and D. L. Jones, "Unitary equivalence: A new twist on signal processing," *IEEE Trans. Signal Process.*, vol. 43, no. 10, pp. 2269–2282, 1995.
- [88] F. Hlawatsch, A. Papandreou-Suppappola, and G. Boudreaux-Bartels, "The power classes-quadratic time-frequency representations with scale covariance and dispersive time-shift covariance," *IEEE Trans. Signal Process.*, vol. 47, no. 11, pp. 3067–3083, 1999.
- [89] G. Evangelista, "Real-time time-varying frequency warping via short-time Laguerre transform," in *Proc. COST G-6 Conf. Digital Audio Effects.*, 2000, pp. 7–12.
- [90] L. Gaul and S. Hurlebaus, "Identification of the impact location on a plate using wavelets," *Mech. Syst. Signal Process.*, vol. 12, no. 6, pp. 783 – 795, 1998.
- [91] A. Berkhout, D. de Vries, and M. C. Brink, "Array technology for bending wave field analysis in constructions," *J. Sound Vib.*, vol. 300, no. 1-2, pp. 25 – 42, 2007.
- [92] A. Papandreou-Suppappola, R. L. Murray, B. Iem, and G. F. Boudreaux-Bartels, "Group delay shift covariant quadratic time-frequency representations," *IEEE Trans. Signal Process.*, vol. 49, no. 11, pp. 2549–2564, 2001.
- [93] Y. Jiang and A. Papandreou-Suppappola, "Discrete time-frequency characterizations of dispersive linear time-varying systems," *IEEE Trans. Signal Process.*, vol. 55, no. 5, pp. 2066–2076, 2007.
- [94] J. Doyle, "An experimental method for determining the location and time of initiation of an unknown dispersing pulse," *Experimental mechanics*, vol. 27, no. 3, pp. 229–233, 1987.
- [95] S. Caporale, "Design and computation of warped time-frequency transforms," Ph.D. dissertation, Advanced Research Center on Electronic Systems, 2009.

- [96] I. Bartoli, A. Marzani, F. L. di Scalea, and E. Viola, “Modeling wave propagation in damped waveguides of arbitrary cross-section,” *J. Sound Vib.*, vol. 295, no. 3Ü5, pp. 685 – 707, 2006.
- [97] A. Roueff, J. Mars, J. Chanussot, and H. Pedersen, “Dispersion estimation from linear array data in the time-frequency plane,” *IEEE Trans. Signal Process.*, vol. 53, no. 10, pp. 3738–3748, 2005.
- [98] C. Ioana, A. Jarrot, C. Gervaise, Y. Stéphan, and A. Quinquis, “Localization in underwater dispersive channels using the time-frequency-phase continuity of signals,” *IEEE Trans. Signal Process.*, vol. 58, no. 8, pp. 4093–4107, 2010.
- [99] J. Bonnel, B. Nicolas, J. I. Mars, and S. C. Walker, “Estimation of modal group velocities with a single receiver for geoacoustic inversion in shallow water,” *J. Acoust. Soc. Am.*, vol. 128, no. 2, pp. 719–727, 2010.
- [100] K. R. Arun, V. G. Reju, and A. W. H. Khong, “Source localization on solids utilizing time-frequency analysis of parameterized warped signals,” in *Proc. IEEE Int. Conf. Acoust. Speech and Signal Process.* IEEE, 2014, pp. 689–693.
- [101] E. Lympertos and E. Dermatas, “Acoustic emission source location in dispersive media,” *Signal Processing*, vol. 87, no. 12, pp. 3218–3225, 2007.
- [102] J. Hensman, R. Mills, S. Pierce, K. Worden, and M. Eaton, “Locating acoustic emission sources in complex structures using Gaussian processes,” *Mech. Syst. Signal Process.*, vol. 24, no. 1, pp. 211–223, 2010.
- [103] M. G. Baxter, R. Pullin, K. M. Holford, and S. L. Evans, “Delta T source location for acoustic emission,” *Mech. Syst. Signal Process.*, vol. 21, no. 3, pp. 1512–1520, 2007.

- [104] G. C. McLaskey, S. D. Glaser, and C. U. Grosse, “Acoustic emission beamforming for enhanced damage detection,” in *Proc. SPIE*, vol. 6932. International Society for Optics and Photonics, 2008, pp. 693 239–1–693 239–9.
- [105] G. Beylkin, “Discrete Radon transform,” *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 35, no. 2, pp. 162–172, 1987.
- [106] P. A. Toft and J. A. Sørensen, “The Radon transform-theory and implementation,” Ph.D. dissertation, Dept. Mathematical Modeling, 1996.
- [107] S. R. Deans, “Radon and Abel transforms,” in *The Transforms and Applications Handbook, Second Edition*. CRC Press, 2000.
- [108] R. Poli, J. Kennedy, and T. Blackwell, “Particle swarm optimization,” *Swarm intelligence*, vol. 1, no. 1, pp. 33–57, 2007.
- [109] J. Kennedy and R. Eberhart, “Particle swarm optimization,” in *Proc. IEEE Int. Conf. Neural Networks*, vol. 4, 1995, pp. 1942–1948.
- [110] R. C. Eberhart and Y. Shi, “Particle swarm optimization: developments, applications and resources,” in *Proc. IEEE Int. Conf. Evolutionary Computation*, vol. 1. IEEE, 2001, pp. 81–86.
- [111] M. Arulampalam, S. Maskell, N. Gordon, and T. Clapp, “A tutorial on particle filters for online nonlinear/non-Gaussian bayesian tracking,” *IEEE Trans. Signal Process.*, vol. 50, no. 2, pp. 174–188, 2002.
- [112] J. Park, Y. Moon, and S. Lee, “Source localization of multiple impacts in a plate,” in *Proc. Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, vol. 226, no. 2. SAGE Publications, 2012, pp. 423–438.
- [113] V. G. Reju, S. N. Koh, and Y. Soon, “Underdetermined convolutive blind source separation via time–frequency masking,” *IEEE Trans. Audio, Speech, Language Process.*, vol. 18, no. 1, pp. 101–116, 2010.