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**STUDY ON EXTREMELY ANISOTROPIC MEDIA
AND NOVEL CLOAKING DESIGNS**

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SCHOOL OF PHYSICAL AND MATHEMATICAL SCIENCES

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STUDY ON EXTREMELY ANISOTROPIC MEDIA AND NOVEL CLOAKING DESIGNS

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LIST OF ABBRIVIATIONS

SRR: Split-Ring-Resonator

TM: Transverse Magnetic

ENZ: Epsilon-Near-Zero

EMNZ: Epsilon-Mu-Near-Zero

0D: Zero-Dimensional

1D: One-Dimensional

2D: Two-Dimensional

3D: Three-Dimensional

PEC: Perfect Electric Conductors

PMC: Perfect Magnetic Conductors

ENI: Epsilon-Near-Infinity

ABSTRACT

Metamaterials and invisibility cloak designs have drawn massive research interests in the past two decades. This thesis studies extremely anisotropic metamaterial and invisibility cloak designs for electromagnetic waves and diffusive fluid. We study the diffraction-free properties of extremely anisotropic media. We realize an extremely anisotropic metamaterial at microwave frequencies with a metallic-plate-stacking structure. With the metamaterial, we demonstrate the reflectionless diffraction-free electromagnetic wave routing at a broad bandwidth. Combining the extremely anisotropic metamaterial design and epsilon-near-zero media, we realize a metamaterial with permittivity approaching zero and infinity in two directions, and permeability approaching zero in the third direction, which is key to achieve omnidirectional impedance match. With the special metamaterial, we demonstrate an omnidirectional invisibility cloak at microwave frequencies. At last, we extend the cloak concept and propose a cloaking design for diffusive fluid. We use the geometry change of a single homogeneous isotropic material in the third dimension to realize the experimental demonstration of the diffusive fluid cloak.

CHAPTER 1 INTRODUCTION

1.1 Metamaterials

Metamaterials are rationally designed macroscopic composites of cellular structures that exhibit unusual effective material properties [1-4]. In contrast to conventional natural materials whose properties are determined by structures and arrangements of their constituent atoms, the properties of metamaterials derive from the geometries, arrangements as well as the material composition of the constituent cellular structures (as shown in **Figure.1-1**). By engineering these constituent units, many intriguing effective material properties that are not available in nature can be achieved. The prefix “meta” comes from Greek, implying the meaning of properties beyond conventional natural materials.

Figure.1-1 Schematics of conventional natural material and metamaterial compositions. The properties of conventional natural materials derive from their constituent atoms. The properties of metamaterials are determined by the geometries, arrangements and material compositions of their constituent cellular structures.

The early study of metamaterials initiated from the field of electromagnetism, and can date back to the end of 19th century. In 1898, Indian scientist Jagadish Chunder Bose demonstrated artificial linear and circular birefringence with designed dielectric composites [5]. In 1920, German scientist Karl Lindman reported large electromagnetic resonance with metamaterial composed of metallic helices and cotton balls [6]. In the 1980s and 1990s, several researches have been done in designing radar absorbers with artificial chiral media [2, 3].

The recent prosperity of metamaterial researches initiated from the proposal of the split-ring-resonators (SRR) by John Pendry in 1999 [7]. Previously, negative permeability had never been found in natural materials. However, the special structure of the SRR supports a circulating and oscillating electric current when the incident electromagnetic waves hits it, that can generate a large magnetic response, leading to a negative effective permeability. Such a negative permeability opens possibility to realize a very interesting abnormal physical phenomenon – negative refraction that was first theoretically proposed by Victor Veselago in 1968 [8]. A negative refraction means that the refracted electromagnetic wave is on the same side of the normal with the incident wave, and the phase velocity is anti-parallel to the Poynting vector. In a negative refraction, the Snell's law, the Doppler shift as well as the Cherenkov radiation are all inversed [8-13]. Nevertheless, this abnormal physical phenomenon had not attracted much attention in the scientific community for over 30 years due to the non-existence of negative-index material in nature until the proposal of SRR metamaterial. In 2000, John Pendry proposed one of the most important applications of negative refraction – superlens that can transfer information carried by evanescent waves, which has inspired many studies on the realization of negative refraction [14]. In 2001, David Smith et al. first experimentally demonstrated the negative refraction phenomenon at microwave frequencies [15]. They combined the SRR structure

together with a thin wire in the metamaterial unit that produces negative permeability and negative permittivity simultaneously. Many studies about negative refraction in theory and experiment at both microwave and optical frequencies have been done, which blazes potential paths to achieve near field superlens, optical nanolithography and nanotechnology circuitry beyond diffraction limit etc. [1, 14].

Inspired by the periodic cellular designs of negative-index metamaterials, researchers find that the power of metamaterials is far beyond this. By engineering the cellular structures and material compositions of the metamaterials, various unusual material properties such as large anisotropy, inhomogeneity and singularities etc. can be realized, which greatly facilitates the material family for electromagnetic wave manipulation. However, how to make use of these metamaterials to manipulate electromagnetic waves arbitrarily was still ambiguous until the theory of transformation optics was firstly proposed by John Pendry et al. and Ulf Leonhardt in 2006 [16, 17]. The theory suggests that electromagnetic waves can be controlled at will with coordinate transformations, which has soon become a systematical tool to design many novel electromagnetic and optical devices including electromagnetic/optical invisibility cloaks [16-26], Luneburg lenses [27-29], field/polarization rotators [30-32], beam splitters [31, 33], reflectionless beam bends/expanders/squeezers [34-36], waveguide adaptors [37-39] etc. Moreover, the structures of metamaterials are no longer limited to metallic SRR structures but various geometries and material compositions like layered dielectrics/metals, periodic or non-periodic hole structures, metallic/dielectric pillars/ellipsoid and many other more complicated structures.

Besides three-dimensional (3D) bulk metamaterials, the two-dimensional (2D) version of metamaterial – metasurface, has also drawn many research interests due to its much easier practical implementation especially in the optical regime compared with 3D

metamaterials. Making use of the flexibility of metamaterial units to engineer the phase and amplitude of transmitted/reflected electromagnetic waves/lights at a subwavelength scale, metasurfaces with many novel functionalities have been experimentally demonstrated such as propagating direction manipulation, polarizations control and wave front reshaping [40-47] etc. Particularly, computer-generated-holograms have been realized with geometric metasurfaces that consist of arrays of plasmonic nano structures with spatially varying orientations. Many novel features over traditional holograms have been achieved such as broad bandwidth, high efficiency, multi-color and frequency/polarization selectivity [48-52] etc.

Moreover, the concept of metamaterial to engineer the effective material properties by adjusting the cellular units has extended to many other physical fields such as acoustics, thermal dynamics and mechanics etc. Utilizing metamaterials, many novel physical properties and functionalities such as super acoustic absorbing [53], extraordinary acoustic transmission [54, 55], negative acoustic/mechanical index [56-58], acoustic/thermal/mechanic cloaks [59-70], thermal rotators, concentrators and camouflages [71-75] etc. have been demonstrated.

All in all, the concept of metamaterial has greatly enriched the material family, enabling various novel material properties, phenomena and devices in not only electromagnetism and optics but also acoustics, thermal dynamics and mechanics etc.

1.2 Singular media

Besides negative index media that enable abnormal phenomenon of negative refraction, singular media that refer to media with extreme constitutive parameters have also drawn many research interests due to their unusual physical features.

A typical example of singular media is epsilon-near-zero (ENZ) media. As firstly theoretically proposed by Nader Engheta et al. in 2006, ENZ media enable a supercoupling phenomenon, as shown in **Figure.1-2(a)**, which means that electromagnetic waves can tunnel through a very narrow waveguide channel filled with ENZ media regardless of the shape, bending or twisting of the channel [76, 77]. The physical origin behind the supercoupling phenomenon is that the electromagnetic fields in ENZ media are near static with near infinitely long wavelengths. As a result, any geometrical details of the narrow waveguide channel are “felt” as subwavelength-sized, which generates negligible scatterings, resulting in the reflectionless tunneling of the electromagnetic waves. The consequence of the supercoupling phenomenon is two-fold: firstly, the energy density of the electromagnetic waves in the narrow channel is greatly enhanced due to the reflectionless squeezing of the electromagnetic waves when they tunnel through the narrow channel; secondly, the electromagnetic waves do not experience phase evolution in the whole ENZ-filled channel. The experimental demonstrations of the supercoupling phenomenon were first carried out later in 2008 by Nader Engheta et al. and David Smith et al with two different approaches [78, 79], respectively. The former group made use of the structural dispersion of the rectangular waveguide that leads to an effective near-zero permittivity near the cutoff frequency of the waveguide, as shown in **Figure.1-2(b)**. The latter group took advantage of the large electric response of compensatory SRRs to realize the near-zero permittivity, as shown in **Figure.1-2 (c)**. The experimental demonstrations in optical regime were

implemented in 2013 with metal-dielectric metamaterials [80-82] (two examples are shown in **Figure.1-2**(d) and (e)). The unique features of ENZ media lead to many applications of them in nonlinear effect boosting [83-85], photon density enhancing [80, 86] and defect sensing [87] etc.

Figure.1-2 Epsilon-Near-Zero (ENZ) Media. (a) Schematics of supercoupling phenomenon of ENZ media. (b) Experimental demonstration of supercoupling phenomenon of ENZ media using waveguide structural dispersion at microwave frequencies[78]. (c) Experimental demonstration of ENZ media using complimentary SRR at microwave frequencies[79]. (d) All dielectric ENZ metamaterial made of Si and SiO₂ at infrared frequencies[81]. (e) ENZ metamaterial made of silver Ag and SiN multilayer structure at visible wavelengths[82].

Following ENZ media, mu-near-zero (MNZ) media, as well as epsilon-mu-near-zero (EMNZ) media have also drawn many research interests. Since the impedance ($\eta = \sqrt{\mu/\epsilon}$) of ENZ media is actually near infinity, it requires the ENZ-filled channel to be very narrow so that a reflectionless transmission can be achieved. This feature sometimes limits the application of ENZ media. Researchers then started studying media with permittivity and permeability both approaching zero, namely EMNZ media. Compared with ENZ media, EMNZ media also enable supercoupling phenomenon, and they further liberate the narrow thickness limitation of the tunneling channel, which means that the reflectionless electromagnetic wave tunneling can occur for arbitrary-shaped waveguide filled with EMNZ media as can be illustrated in Figure.1-3(a) and (b) [88-91]. Such property of EMNZ media can greatly facilitate the manipulation of electromagnetic waves and lights. It has been proposed and demonstrated by C. T. Chan et al. that dielectric photonic crystals can be designed to exhibit Dirac cone dispersion at the center of the Brillouin zone to achieve EMNZ media, as shown in **Figure.1-3(c)** [88]. Furthermore, very recent work from Nader Engheta et al. shows that by randomly doping dielectric or perfect conducting dopants to ENZ media, as shown in Figure.1-3(d), the magnetic response of ENZ media can be dramatically changed, which can also be used to realize EMNZ media [91]. However, we should point out that the reflectionless electromagnetic wave tunneling in isotropic zero-index media including ENZ media, MNZ media as well as EMNZ media, all require the electromagnetic wave to be normal incidence, otherwise reflections will be generated.

On the other hand, another type of singular media besides zero-index media is near-infinity medium. An example of near-infinity media is a perfect electric conductor (PEC) whose conductivity is near infinite, resulting in a near-infinite permittivity. Common metals at microwave, terahertz and far infrared frequencies can all be regarded as PEC. No propagating electromagnetic waves are supported in PEC, thus all the electromagnetic waves

illuminated to a PEC block will be totally reflected. However, if only one or two diagonal components of the permittivity tensor is infinite and the others are finite, namely extremely anisotropic media, the media support propagating wave, but possess unique wave-manipulating properties. As an example, extremely anisotropic media with one permittivity component approaching infinity (single-ENI media) have been studied to realize efficient light transmission in deep-subwavelength aperture [92], subwavelength imaging [93-98], as well as reflectionless diffraction-free TM polarized subwavelength electromagnetic wave routing [99] etc.

Figure.1-3 Epsilon-Mu-Near-Zero (EMNZ) Media. (a) (b) EM wave tunneling through an arbitrary-shaped waveguide channel filled with EMNZ media[90]. (c) EMNZ metamaterial realized by dielectric photonic crystals at microwave frequencies[88]. (d) EMNZ media realized by randomly

doping dielectric rod to ENZ media at microwave frequencies. The ENZ media are achieved by waveguide structural dispersion method[91].

1.3 Transformation optics and invisibility cloak designs

1.3.1 Transformation optics

As introduced in **Chapter 1.1**, the developments of metamaterials in the past two decades have greatly enriched the material family. However, how to apply these various metamaterials to achieve arbitrary manipulation of electromagnetic waves or lights was still elusive until the theory of transformation optics was proposed by John Pendry et al. and Ulf Leonhardt in 2006 [16, 17]. A coordinate transformation can be regarded as a distortion of the space, and the electromagnetic waves or lights propagating in the space are all distorted following the space distortion (as shown in **Figure.1-4**).

Figure.1-4 Transformation optics. Electromagnetic waves are distorted following the coordinate transformation [16].

Therefore, with coordinate transformations, electromagnetic waves including both their phases and power flows can be manipulated at will. More importantly, the coordinate

transformations do not change the form of Maxwell's equations, but only scale the constitutive parameters in the following manner [16, 100]

$$\bar{\bar{\epsilon}}' = \frac{\bar{J}\bar{\bar{\epsilon}}\bar{J}^T}{\det(\bar{\bar{\epsilon}})}, \quad \bar{\bar{\mu}}' = \frac{\bar{J}\bar{\bar{\mu}}\bar{J}^T}{\det(\bar{\bar{\mu}})} \quad (1.1)$$

where $\bar{\bar{\epsilon}}'$ and $\bar{\bar{\mu}}'$ are the constitutive parameters in the new coordinate, $\bar{\bar{\epsilon}}$ and $\bar{\bar{\mu}}$ are the constitutive parameters of the original coordinate, $\bar{J}$ is the Jacobian matrix that relates two coordinates

$$\bar{J} = \begin{bmatrix} \frac{\partial x'}{\partial x} & \frac{\partial x'}{\partial y} & \frac{\partial x'}{\partial z} \\ \frac{\partial y'}{\partial x} & \frac{\partial y'}{\partial y} & \frac{\partial y'}{\partial z} \\ \frac{\partial z'}{\partial x} & \frac{\partial z'}{\partial y} & \frac{\partial z'}{\partial z} \end{bmatrix}.$$

Thus, arbitrary manipulations of electromagnetic waves can be achieved by designing the permittivity and permeability tensors according to Eq. (1.1)

More powerful than Snell's law that tells how different refractive indexes redirect and focus the light rays, transformation optics tells how the phases and power flows of electromagnetic waves can be manipulated with designed permittivity and permeability tensors in an intuitive inverse manner. Such manipulations also work for subwavelength electromagnetic waves found in plasmonic systems and even the static electric [101] and magnetic [102] regimes. Many novel electromagnetic and optical devices have been studied based on transformation optics. Certainly, one of the most fascinating application of transformation optics is the invisibility cloak design.

1.3.2 Invisibility cloak

An invisibility cloak is a device that can steer lights around a certain region, and then make the lights propagate along their original trajectories as if they propagate in free space. The fictional concept has been a fantasy of people for decades until the proposal of transformation optics. To explain the approach, we take a cylindrical cloak as an example.

Figure.1-5 Coordinate transformation and field pattern of a cylindrical invisibility cloak. (a) Free space coordinate in the virtual space. The black circle denotes the transforming region. Red solid line denotes a light ray. (b) Coordinate of the cylindrical cloak in the physical space. (c) Field pattern of a TM polarized plane wave propagating through the cylindrical cloak. The inner boundary of the cloak is PEC boundary. (d) First experimental demonstration of the cylindrical cloak with SRR metamaterials[18].

As can be seen from **Figure.1-5(a)**, we select an infinitely small point (marked by green dot) in the center of a circular region (marked by blue solid line) in a 2D virtual free space. Then we transform the point into a hole (marked by green solid line), and meanwhile the all the space in the circular region is squeezed into the ring in the physical space. The ring has an inner radius of R_1 and a outer radius of R_2 . As a result, all the lights or electromagnetic waves (as shown by the red solid line in **Figure.1-5(b)**) will follow the coordinate distortion, going around the hole and then keep propagating in their original trajectories as if propagating in the virtual free space. Anything in the hole region will be invisible from outer detection. Thus the ring is serving as an invisibility cloak.

The transformation optics provides a perfect theoretical solution to achieve an invisibility cloak. However, the media of the original cylindrical and spherical cloaks [16] are required to be inhomogeneous, anisotropic and with singularities, which are very challenging to realize in practice even with metamaterials. The first experimental demonstration of invisibility cloak was carried out at microwave frequencies in 2006 soon after transformation optics was proposed. They use 10 discrete rings consist of SRR to realize the inhomogeneity and anisotropy required by the cloak. However, instead of realizing the exact parameters required for a cylindrical cloak, they made a so called eikonal approximation that only preserves the refractive index ($n = \sqrt{\epsilon\mu}$) but sacrifices the impedance ($\eta = \sqrt{\mu/\epsilon}$) match between the cloak and the surrounding environment. Therefore, extreme anisotropy and singularities are dodged with such an approximation, but reflections and reduced transmissions are caused when electromagnetic waves propagate through the cloak.

Researchers then started searching for approaches to simplify the cloaking design so that it can be easier and more practical to realize. In 2008, Jensen Li and John Pendry proposed a new strategy of cloaking to conceal objects under a carpet cloak that appears as a line of PEC or PMC on the ground [103]. The new cloaking strategy has two advantages. Firstly, it only requires isotropic media rather than previous complex anisotropic media because the coordinate transformation used is quasi-conformal. Hence, the cloak can work for full polarizations at a broad bandwidth. Secondly, singular parameters are no longer involved since the coordinate transformation starts from a one-dimensional (1D) line rather than the previous zero-dimensional (0D) point. This greatly simplify the practical implementation of the cloak. Thank to these two advantages, carpet cloaks based on quasi-conformal mapping have been successfully demonstrated in several previous works at microwave frequencies [19, 23] and optical frequencies [20-22]. However, the quasi-conformal mapping method introduces a lateral shift that can make cloaking bump detectable [104].

Figure.1-6 Carpet cloak based on quasi-conformal mapping. (a) Schematics of carpet cloak. (b) Experimental realization at microwave frequencies using non-resonant SRR metamaterials[19]. (c)

Experimental realization at optical frequencies with hole-drilling dielectric metamaterials[21]. (d)
Three dimensional experimental demonstration at optical frequencies[22].

An alternative design, that can be regarded as another type of carpet cloak, is to use a bi-linear coordinate transformation rather than quasi-conformal method to transform a 1D straight PEC or PMC line in the virtual free space to a polygonal line in the physical space. Different from carpet cloak based on quasi-conformal mapping, the carpet cloak based on the bi-linear coordinate transformation does not suffer from lateral shift, which has been experimentally demonstrated with SRR and dielectric at microwave frequencies [26, 105] and natural calcite at visible light frequencies [24, 25], respectively. Yet the polarization is still limited, and the eikonal approximation is still required in optical regime due to the lack of magnetic materials.

Figure.1-7 Invisibility cloak demonstration based on bi-linear coordinate transformation. (a)(b) Experimental realizations with natural calcite at visible light frequency range[24, 25]. (b)

Experimental realization of a full-parameter cloak based on bi-linear coordinate transformation with SRR metamaterials at microwave frequencies[26]. (d) Experimental realization of a dielectric temperature-controlled cloak based on bi-linear coordinate transformation at microwave frequencies[105].

Recently, An ultrathin invisibility skin cloak for visible light using metasurface has been demonstrated by XingJie Ni et al [106]. Instead of using the transformation method, they borrow the concept of the carpet cloak but achieve the cloaking performance by reshaping the reflected light at a subwavelength scale with an ultrathin metasurface. Similar idea at microwave frequencies have been demonstrated by Yihao Yang et al in 2016 [107]. The ultrathin design with metasurface is closer to the concept of a perfect invisibility cloak, and is more practical to be achieved with the current developing state of the metamaterials. Nevertheless, the metasurface skin cloak still has problems, e.g. the cloak is actually limited to certain incident angles and a pre-known electromagnetic wave sources e.g. plane waves in Ref. [106, 107], which is due to the limitation of metasurfaces that can hardly reshape the waves incident from all angles to the required reflecting directions.

Figure.1-8 Metasurface invisibility cloak. (a) Schematic of the metasurface cloak[106]. (b) Field patterns when EM waves are normally and obliquely incident on the metasurface cloak[106]. (c)

Experimental realization of the metasurface invisibility cloak working at optical frequencies[106]. (d)

Experimental realization of the metasurface invisibility cloak at microwave frequencies[107].

Besides all of limitations, an unavoidable problem of the carpet cloak is that it is not free-standing which always requires a “ground plane” to camouflage the virtual PEC line, otherwise it can only work for unidirection in free space [26]. In one word, there is still a long way to go to achieve a perfect invisibility cloak as described in the Sci-fi novels in practice.

1.4 Laplace-equation-based invisibility cloaks

The transformation optics and the concept of invisibility cloaks have also inspired cloaking designs in other physical fields such as acoustics, thermal dynamics etc. Similar to Maxwell's equations whose form is invariant to coordinate transformation, the governing equations for other physical fields such as acoustics, thermal dynamics, diffusion, fluid dynamics and mechanics etc. also possess or approximately possess the similar feature. Thus, several acoustic cloaks [59, 61, 62] and thermal cloaks [63, 64, 108] based on coordinate transformation have been proposed and demonstrated. However, the cloak designs based on coordinate transformation always introduce either homogeneity or anisotropy to the required materials that are often challenging for practical implementation.

Figure.1-9 Acoustic, static electric and thermal cloaks based on coordinate transformation. (a) Cylindrical acoustic cloak demonstration[59]. (b) Broadband 3D ground plane acoustic cloak demonstration[62]. (c) Cloak demonstration for static electric field[101]. (d) Cloak demonstration for heat flux[71].

Recently, alternative approaches have been proposed to realize cloaks for static magnetic/electric fields, heat, diffusion and mechanics thank to the less complexity of Laplace equation. This method was firstly proposed and demonstrated in designing static magnetic cloaks [102, 109]. The core idea of this method is to use materials of the core and shell with different permeability to cancel the field distortions generated by each other, as shown in **Figure.1-10**, so that the external field maintains un-disturbed. Mathematically, by solving the Laplace-equation for a core-shell structure under the condition that the external uniform field is un-disturbed, the relation between the permeability of the core and shell can be obtained. The detailed calculation can be found in Ref. [109]. Figure.1-8 shows the two experimental demonstration of the magnetic cloaks [102, 109]. The shells are both made of magnetic materials (with permeability larger than one), and the cores are both superconducting material with a permeability of zero.

Figure.1-10 Principle of Laplace-equation based cloak and DC magnetic cloak demonstrations. (a) Magnetic field lines are attracted to a cylindrical magnetic shell with a permeability of 3.54[109]. (b) Magnetic field lines are repelled by a superconducting core with a permeability of zero[109]. (c) The

external magnetic field lines are unchanged when the superconducting core and the magnetic shell are combined due to the distortion cancellation[109]. (d)(e) Experimental demonstrations of the DC magnetic cloaks[102, 109].

This method extends other physical fields that follow or approximately follow Laplace equation, including thermal dynamics, photon diffusion and mechanical displacements etc., because it only requires two isotropic materials that greatly facilitates the practical implementations compared with previous cloaking approach that requires either inhomogeneity or anisotropy or singularities. **Figure.1-11** shows experimental demonstrations of cloaks for heat flux[65, 66], diffusive photons[110] and mechanical displacements[69], respectively.

Figure.1-11 Laplace-equation-based cloak demonstrations. (a) 3D ultrathin spherical thermal cloak demonstration[65]. The shell layer is copper. The core is air with near zero thermal conductivity. The background medium is stainless steel. (b) 2D cylindrical thermal cloak demonstration[66]. The core

is Alloy. The shell is Polystyrene. The background medium is sealant. (c) Cylindrical and spherical cloak demonstrations for diffusive photons[110]. The core is stainless steel with zero photon diffusivity. The shell is polydimethylsiloxane (PDMS) doped with melamine-resin micro-particles. (d) Cylindrical cloak demonstration for mechanical displacement[69]. The core region has zero bulk modulus and the red region serving as the shell whose bulk modulus can be tuned by the unit geometry of the pentamode-metamaterial.

1.5 Organization of the thesis

The thesis mainly contains three parts that study extremely anisotropic media and novel cloak designs.

The first part studies the properties of extremely anisotropic media and their applications in reflectionless diffraction-free electromagnetic wave routing which includes bending, splitting, compressing/expanding in 2D scenario, and reflectionless bending and subwavelength image transmission in 3D scenario.

The second part studies the design and implementation of an omnidirectional invisibility cloak based on a linear coordinate transformation. A metamaterial with constitutive parameters $\epsilon_x \rightarrow \infty$, $\epsilon_y \rightarrow 0$, and $\mu_z \rightarrow 0$ is implemented with metal-ENZ-stacking structure. The omnidirectional cloak is demonstrated by both simulation and experiment in a plane waveguide at microwave frequencies.

The third part studies the design and demonstration of a 2D cloak composed of a single isotropic homogenous medium that works for diffusive fluid. The height of the cloak in the third dimension can be applied to tune the effective mass density of the fluid. The cloaking performance is demonstrated by both simulation and experiment.

CHAPTER 2 EXTREMELY ANISOTROPIC MEDIA

Manipulating electromagnetic waves without reflection or diffraction is fundamentally difficult to realize due to the wave nature of electromagnetic waves. Previously, reflectionless manipulation of electromagnetic waves can be achieved by transformation optics [16, 17] techniques which map a virtual space into the physical space so that electromagnetic waves can be controlled at will without reflection. A typical example of reflectionless electromagnetic wave manipulation with transformation optics is the invisibility cloak which guides electromagnetic waves around a certain cloaking region and makes the electromagnetic waves preserve their original trajectory and phase after they propagate through the cloak [16]. However, the invisibility cloak designs suffer from the complex parameters that require to be implemented with complicated metamaterials [18, 19, 23, 26] or photonic structures [20-22]. Besides invisibility cloaks that are often based on continuous coordinate transformations, finite embedded coordinate transformations [35], which means the transformation-optical medium and the surrounding media are not continuous at the boundary, have been applied to design many photonic devices, such as beam bends, splitters and expanders [31, 111-113] etc. Nevertheless, these approaches cannot overcome the diffraction of EM waves since the virtual space that the transformation starts from is not free of diffraction. Thus, deep-subwavelength information or subwavelength images is not able to be delivered.

Another approach to realize reflectionless electromagnetic wave manipulation is to use ENZ media. It has been proposed and demonstrated that electromagnetic waves can tunnel through a narrow ENZ-media-filled waveguide channel filled regardless of the length and geometrical shape of the channel [76, 78, 79]. With an additional “sampling” process of inserting stacked thin metallic plates, sub-diffraction-limited images can be delivered

through a narrow right-angled bend filled with ENZ media [114]. However, the realizations of ENZ media fundamentally suffer from narrow bandwidth [78-80, 82, 114]. Besides, subwavelength information cannot be preserved in ENZ media if no anisotropy is introduced.

Recently, a new type of metamaterials have been proposed, which possess one permittivity component approaching infinity (we hereafter refer this type of metamaterials to “single-ENI” media) [92], and have been studied to realize efficient light transmission in deep-subwavelength aperture [92], subwavelength imaging [93-98], as well as reflectionless diffraction-free TM polarized subwavelength electromagnetic wave routing on a two-dimensional (2D) platform [99] etc. However, no bending or sharp corner is involved in the subwavelength imaging demonstration with wire media [94-96, 98]. While the single-ENI designs in Ref.[92, 99] is based on plasmonic resonances among metal/dielectric multilayers, thus being limited to narrow bandwidth. Besides, the reflectionless diffraction-free EM wave routing phenomenon has never been demonstrated in reality.

In this chapter, we will introduce the properties of extremely anisotropic media (one or two permittivity components approaching infinity) and the realization of an extremely anisotropic metamaterial using metallic-plate-stacking structure. The reflectionless diffraction-free EM wave routing phenomena are demonstrated at a broad bandwidth at microwave frequencies.

2.1 Properties of extremely anisotropic media

We firstly discuss the properties of extremely anisotropic media. We start from extremely anisotropic media with one permittivity approaching infinity (single-ENI media). We consider a single-ENI medium with permittivity and permeability as:

$$\bar{\epsilon} = \epsilon_0 \begin{bmatrix} \infty & 0 & 0 \\ 0 & \epsilon_{\perp} & 0 \\ 0 & 0 & \epsilon_{\perp} \end{bmatrix}, \quad \bar{\mu} = \mu_0 \quad (2.1)$$

The dispersion relations are

$$\text{Ordinary waves:} \quad k_o^2 = \epsilon_{\perp} k_0^2 \quad (2.2)$$

$$\text{Extraordinary waves:} \quad \frac{k_{ex}^2}{\epsilon_{\perp}} = k_0^2 \quad (2.3)$$

where k_o is the wave number of ordinary waves, k_{ex} is the x component of the wave vector of the extraordinary waves. We can find that the isofrequency surface of the ordinary waves is a spherical surface with a radius of $\sqrt{\epsilon_{\perp}} k_0$, while that of the extraordinary waves is a pair of parallel plane. The isofrequency surface of the extraordinary waves supports infinitely large real transverse wave vectors, and meanwhile guarantees the same phase accumulation as the extraordinary waves propagate. Thus the extraordinary waves in single-ENI media are free of diffraction.

We show the comparison between wave propagation in free space and single-ENI media in a 2D scenario in **Figure.2-1**. **Figure.2-1(a)** and **(d)** show the 2D isofrequency contours of free space and a single-ENI medium, respectively. As can be seen that the isofrequency contour of free space is a circle which supports power flow in arbitrary directions with a uniform wave number of k_0 . Note in 2D scenario, the extraordinary waves are Transverse-Magnetic (TM) polarized (the magnetic field is out of plane). When a TM polarized Gaussian beam (with a waist size of 0.6 wavelength), which is an integral of plane

waves with different wave vectors, propagates in free space, the beam soon diffracts as shown in **Figure.2-1(b)**. When the Gaussian beam hits a perfect conducting obstacle with a random shape and a size of approximate one wavelength, the beam gets scattered into the whole space due to diffraction. The geometrical details of the obstacle can hardly be retrieved from the scattered waves.

Figure.2-1 Comparison between wave propagations in isotropic media and extremely anisotropic media. (a)(d) Isifrequency contours of isotropic media and extremely anisotropic media. The thin arrows denote the wave vectors supported and the wide arrows denote the Poynting vectors supported. (b)(e) TM polarized Gaussian beams propagation in an isotropic medium and an extremely anisotropic medium. The waist size is 0.6 wavelength. (c)(f) Waves scattering when TM polarized Gaussian beams hit a perfect conducting obstacle with a random shape and a size of approximate one wavelength. The waist size is 0.6 wavelength.

However, the isofrequency contour of a single-ENI media in 2D scenario is a pair of parallel straight lines that supports wave vectors in arbitrary directions but power flow only in the

directions perpendicular to the isofrequency lines. When a TM polarized Gaussian beam with the same feature of that in **Figure.2-1(b)** propagates in a single-ENI media, as shown in **Figure.2-1(e)**, the Gaussian profile of the beam is maintained, which implies the diffraction-free property of single-ENI media. Even when the TM polarized Gaussian beam hits a perfect conducting obstacle (features the same with that in **Figure.2-1(c)**), no wave can bypass the obstacle to propagate in the shadow region, as shown in **Figure.2-1(f)**, and the width of the shadow is exactly the same with the vertical dimension (perpendicular to the incident direction of the beam) of the obstacle. Moreover, it can be found that the geometrical details of the obstacle shined by the beam can be retrieved from the reflected waves directly, showing a unique imaging property of single-ENI media.

Now, we talk about the extremely anisotropic media with two permittivity components approaching infinity (double ENI media). We assume the permittivity tensor and the permeability are

$$\bar{\bar{\epsilon}} = \epsilon_0 \begin{bmatrix} \infty & 0 & 0 \\ 0 & \epsilon_{\parallel} & 0 \\ 0 & 0 & \infty \end{bmatrix}, \quad \bar{\bar{\mu}} = \mu_0 \quad (2.4)$$

The dispersion relations in a double-ENI medium are:

$$\text{Ordinary waves:} \quad k_o^2 = \infty k_0^2 \quad (2.5)$$

$$\text{Extraordinary waves:} \quad \frac{k_{ex}^2}{\epsilon_{\parallel}} + \frac{k_{ez}^2}{\epsilon_{\parallel}} = k_0^2 \quad (2.6)$$

where k_o is the wave number of ordinary waves, k_{ex} and k_{ez} are the x and z components of the wave vector for extraordinary waves, respectively. We can find that the isofrequency surface of the ordinary waves is a spherical surface with a near infinite radius. Thus, all ordinary waves are suppressed. The isofrequency surface of the extraordinary waves is a cylindrical surface oriented along the direction of finite component and with a radius of

$\sqrt{\varepsilon_{\parallel}}k_0$. Different from single-ENI media, the extraordinary waves in double-ENI media are only free from diffraction in the direction of the optical axis. Thus the subwavelength-imaging ability of double ENI media is limited to the direction of the optical axis. However, an advantage of double ENI media over single-ENI media is that double-ENI media are not bothered by birefringent effect since the ordinary waves are suppressed in double ENI media. We shall be discussing about the details of the transmission between two extremely anisotropic media in the **Chapter 2.2**.

2.2 Birefringent effect in single-ENI media

We firstly discuss the transmission between two single-ENI media. We consider two single-ENI media are connected along y - z plane. The optical axes of two media lie in x - y plane. As shown in **Figure 2-2**, the isofrequency surface of ordinary waves is a spherical surface, while that of extraordinary waves is a pair of parallel planes. We set that the finite components $\varepsilon_{\perp}$ of the two single-ENI media 1 and 2 are ε_1 and ε_2 . The optical axes of the two media (perpendicular to the parallel isofrequency planes of extraordinary wave in each medium) lie in the x - y plane making angles of θ_1 and θ_2 to the interface plane (y - z plane), respectively.

Figure.2-2 Isofrequency surfaces of extraordinary waves incident at an interface between two single-ENI media. The optical axes of two media are perpendicular to the two parallel planes of isofrequency surfaces in each medium, making angles of θ_1 and θ_2 with the interface (y - z plane). $\bar{k}_{ie}$, $\bar{k}_{re}$ and $\bar{k}_{te}$ are wave vectors of incident, reflected and transmitted extraordinary waves. $\bar{k}_{ro}$, $\bar{k}_{to}$ are wave vectors of reflected and transmitted ordinary waves. $\bar{S}_{ie}$, $\bar{S}_{re}$ and $\bar{S}_{te}$ are Poynting vectors of

incident, reflected and transmitted extraordinary waves. $\bar{\mathbf{S}}_{ro}, \bar{\mathbf{S}}_{to}$ are Poynting vectors of reflected and transmitted ordinary waves. The black dotted line in y-z plane shows the projections of wave vectors, satisfying phase matching condition.

Specifically, the expressions of two isofrequency surfaces of extraordinary waves in two single-ENI media are:

$$(k_x \sin(\theta_1) + k_y \cos(\theta_1))^2 = \varepsilon_1 k_0^2, \quad (2.7)$$

$$(k_x \sin(\theta_2) - k_y \cos(\theta_2))^2 = \varepsilon_2 k_0^2. \quad (2.8)$$

The expressions of two isofrequency surfaces of ordinary waves are:

$$k_x^2 + k_y^2 + k_z^2 = \varepsilon_1 k_0^2, \quad (2.9)$$

$$k_x^2 + k_y^2 + k_z^2 = \varepsilon_2 k_0^2. \quad (2.10)$$

Considering a plane electromagnetic wave incident from medium 1 to medium 2 with an extraordinary wave vector $\bar{\mathbf{k}}_{ie}$ which has arbitrary y component k_y and z component k_z

($\sqrt{|\bar{\mathbf{k}}_{ie}|^2 - k_y^2 - k_z^2} > 0$), it can be obtained according to phase matching conditions that:

$$k_{rey} = k_{roy} = k_{tey} = k_{toy} = k_y \quad (2.11)$$

$$k_{rez} = k_{roz} = k_{tez} = k_{toz} = k_z \quad (2.12)$$

where k_{rey}, k_{roy} (k_{rez}, k_{roz}) are the y (z) components of the extraordinary and ordinary wave vectors of the reflected waves, respectively, and k_{tey}, k_{toy} (k_{tez}, k_{toz}) are the y (z) components of the extraordinary and ordinary wave vectors of the transmitted waves, respectively. Substituting Eq. (2.11, 2.12) into Eq. (2.7-2.10), we can obtain the extraordinary incident, reflected and transmitted wave vectors $\bar{\mathbf{k}}_{ie}, \bar{\mathbf{k}}_{re}, \bar{\mathbf{k}}_{te}$ and the ordinary reflected and transmitted wave vectors $\bar{\mathbf{k}}_{ro}, \bar{\mathbf{k}}_{to}$. The permittivity tensors of two media are:

$$\bar{\bar{\boldsymbol{\varepsilon}}}_1 = \bar{\bar{\mathbf{R}}}_z(\theta_1) \bar{\bar{\boldsymbol{\varepsilon}}}'_1 \bar{\bar{\mathbf{R}}}_z^{-1}(\theta_1) \quad (2.13)$$

$$\bar{\bar{\epsilon}}_2 = \bar{\bar{R}}_z(-\theta_2)\bar{\bar{\epsilon}}_2'\bar{\bar{R}}_z^{-1}(-\theta_2) \quad (2.14)$$

where $\bar{\bar{\epsilon}}_1' = \epsilon_0 \begin{bmatrix} \infty & 0 & 0 \\ 0 & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_1 \end{bmatrix}$, $\bar{\bar{\epsilon}}_2' = \epsilon_0 \begin{bmatrix} \infty & 0 & 0 \\ 0 & \epsilon_2 & 0 \\ 0 & 0 & \epsilon_2 \end{bmatrix}$ and $\bar{\bar{R}}_z(\theta)$ is the rotation matrix

rotating a vector by θ angle about $\hat{z}$ axis $\bar{\bar{R}}_z(\theta) = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

To express the fields in anisotropic media, we define five groups of base unit vectors $\{\hat{i}_{e1}, \hat{i}_{e2}, \hat{i}_{e3}\}$, $\{\hat{r}_{e1}, \hat{r}_{e2}, \hat{r}_{e3}\}$, $\{\hat{r}_{o1}, \hat{r}_{o2}, \hat{r}_{o3}\}$, $\{\hat{t}_{e1}, \hat{t}_{e2}, \hat{t}_{e3}\}$, $\{\hat{t}_{o1}, \hat{t}_{o2}, \hat{t}_{o3}\}$ for incident extraordinary wave, reflected extraordinary wave, reflected ordinary wave, transmitted extraordinary wave and transmitted ordinary wave, respectively, each of which is defined in a manner as:

$$\hat{u}_3 = \frac{\bar{k}}{|\bar{k}|}, \quad \hat{u}_1 = \frac{\bar{k} \times \hat{O}}{|\bar{k} \times \hat{O}|}, \quad \hat{u}_2 = \hat{u}_3 \times \hat{u}_1 \quad (2.15)$$

where $\bar{k}$ is the wave vector, $\hat{O}$ is the unit direction vector of the optical axis, $\{\hat{u}_1, \hat{u}_2, \hat{u}_3\}$ is the corresponding base unit vectors group.

According to the properties of anisotropic medium, we know that the magnetic fields of extraordinary waves are along $\hat{u}_1$ direction, and the magnetic fields of ordinary waves are along $\hat{u}_2$ direction. We set that the magnetic field amplitude of the incident wave is H_0 , the reflection and transmission coefficients of the magnetic fields of extraordinary and ordinary waves are r_{He} , r_{Ho} , t_{He} , t_{Ho} , respectively, then we can write the magnetic fields of all waves as:

$$\begin{aligned} \bar{H}_{ei} &= \hat{i}_{e2} H_0 e^{i\bar{k}_{ie}\bar{r}} \\ \bar{H}_{re} &= \hat{r}_{e1} r_{He} H_0 e^{i\bar{k}_{re}\bar{r}} \\ \bar{H}_{ro} &= \hat{r}_{o2} r_{Ho} H_0 e^{i\bar{k}_{ro}\bar{r}} \\ \bar{H}_{te} &= \hat{t}_{e1} t_{He} H_0 e^{i\bar{k}_{te}\bar{r}} \end{aligned} \quad (2.16)$$

$$\bar{\mathbf{H}}_{\mathbf{to}} = \hat{\mathbf{t}}_{o2} t_{Ho} H_0 e^{i\bar{\mathbf{k}}_{\mathbf{to}} \bar{\mathbf{r}}}$$

The corresponding electric fields can be calculated with Ampere's law of Maxwell's equations as:

$$\begin{aligned}\bar{\mathbf{E}}_{te} &= -\frac{1}{\omega} \bar{\bar{\epsilon}}_1^{-1} (\bar{\mathbf{k}}_{te} \times \bar{\mathbf{H}}_{te}) \\ \bar{\mathbf{E}}_{re} &= -\frac{1}{\omega} \bar{\bar{\epsilon}}_1^{-1} (\bar{\mathbf{k}}_{re} \times \bar{\mathbf{H}}_{re}) \\ \bar{\mathbf{E}}_{ro} &= -\frac{1}{\omega} \bar{\bar{\epsilon}}_1^{-1} (\bar{\mathbf{k}}_{ro} \times \bar{\mathbf{H}}_{ro}) \\ \bar{\mathbf{E}}_{te} &= -\frac{1}{\omega} \bar{\bar{\epsilon}}_2^{-1} (\bar{\mathbf{k}}_{te} \times \bar{\mathbf{H}}_{te}) \\ \bar{\mathbf{E}}_{to} &= -\frac{1}{\omega} \bar{\bar{\epsilon}}_2^{-1} (\bar{\mathbf{k}}_{to} \times \bar{\mathbf{H}}_{to})\end{aligned}\tag{2.17}$$

We match the boundary condition in the tangential directions of the boundary (y-z plane):

$$\text{y component of } \bar{\mathbf{H}}: \hat{\mathbf{i}}_{e1} \cdot \hat{\mathbf{y}} + r_{He} \hat{\mathbf{r}}_{e1} \cdot \hat{\mathbf{y}} + r_{Ho} \hat{\mathbf{r}}_{o2} \cdot \hat{\mathbf{y}} = t_{He} \hat{\mathbf{t}}_{e1} \cdot \hat{\mathbf{y}} + t_{Ho} \hat{\mathbf{t}}_{o2} \cdot \hat{\mathbf{y}} \tag{2.18}$$

$$\text{z component of } \bar{\mathbf{H}}: \hat{\mathbf{i}}_{e1} \cdot \hat{\mathbf{z}} + r_{He} \hat{\mathbf{r}}_{e1} \cdot \hat{\mathbf{z}} + r_{Ho} \hat{\mathbf{r}}_{o2} \cdot \hat{\mathbf{z}} = t_{He} \hat{\mathbf{t}}_{e1} \cdot \hat{\mathbf{z}} + t_{Ho} \hat{\mathbf{t}}_{o2} \cdot \hat{\mathbf{z}} \tag{2.19}$$

$$\begin{aligned}\text{y component of } \bar{\mathbf{E}}: & \bar{\bar{\epsilon}}_1^{-1} (\bar{\mathbf{k}}_{te} \times \hat{\mathbf{i}}_{e1}) \cdot \hat{\mathbf{y}} + r_{He} \bar{\bar{\epsilon}}_1^{-1} (\bar{\mathbf{k}}_{re} \times \hat{\mathbf{r}}_{e1}) \cdot \hat{\mathbf{y}} \\ & + r_{Ho} \bar{\bar{\epsilon}}_1^{-1} (\bar{\mathbf{k}}_{ro} \times \hat{\mathbf{r}}_{o2}) \cdot \hat{\mathbf{y}} \\ & = t_{He} \bar{\bar{\epsilon}}_2^{-1} (\bar{\mathbf{k}}_{te} \times \hat{\mathbf{t}}_{e1}) \cdot \hat{\mathbf{y}} + t_{Ho} \bar{\bar{\epsilon}}_2^{-1} (\bar{\mathbf{k}}_{ro} \times \hat{\mathbf{t}}_{o2}) \cdot \hat{\mathbf{y}}\end{aligned}\tag{2.20}$$

$$\begin{aligned}\text{z component of } \bar{\mathbf{E}}: & \bar{\bar{\epsilon}}_1^{-1} (\bar{\mathbf{k}}_{te} \times \hat{\mathbf{i}}_{e1}) \cdot \hat{\mathbf{z}} + r_{He} \bar{\bar{\epsilon}}_1^{-1} (\bar{\mathbf{k}}_{re} \times \hat{\mathbf{r}}_{e1}) \cdot \hat{\mathbf{z}} + r_{Ho} \bar{\bar{\epsilon}}_1^{-1} (\bar{\mathbf{k}}_{ro} \times \hat{\mathbf{r}}_{o2}) \\ & \cdot \hat{\mathbf{z}} = t_{He} \bar{\bar{\epsilon}}_2^{-1} (\bar{\mathbf{k}}_{te} \times \hat{\mathbf{t}}_{e1}) \cdot \hat{\mathbf{z}} + t_{Ho} \bar{\bar{\epsilon}}_2^{-1} (\bar{\mathbf{k}}_{ro} \times \hat{\mathbf{t}}_{o2}) \cdot \hat{\mathbf{z}}\end{aligned}\tag{2.21}$$

By matching the boundary conditions, we obtain the reflectance and transmittance of the extraordinary waves and the ordinary waves as shown in **Figure 2-3**.

Figure.2-3 Transmittance and reflectance of the extraordinary wave and ordinary wave when an extraordinary wave is incident. The angles between the optical axes of two single-ENI media and the interface are $\theta_1 = \theta_2 = 45^\circ$. The finite permittivity components of two single-ENI media are equal to ε_0 . (a) Transmittance of the extraordinary wave. (b) Transmittance of the ordinary wave. (c) Reflectance of the extraordinary wave. (d) Reflectance of the ordinary wave.

As can be seen from **Figure.2-3**, the transmittance of the extraordinary waves is always one when the out-of-plane wave vector component k_z is zero, referring to TM polarized waves in a 2D scenario. However, when the out-of-plane wave vector component k_z is not zero, some portion of the incident extraordinary waves will diffract to the ordinary

waves in Medium 2 which are not free of diffraction and some portion of the wave will be reflected back as extraordinary wave and ordinary waves in Medium 1, showing the birefringent effect. Thus, when the out-of-plane wave vector component is not zero, the transmission from single ENI Medium 1 to single ENI Medium 2 is not reflectionless or diffraction-free.

We show the birefringent effect in single ENI media in **Figure.2-4**. As shown in **Figure.2-4(a)**, a 2D Gaussian beam (waist size $3\lambda_0$; λ_0 is the free-space wavelength), which is polarized with in-plane electric field, is incident on the interface between two single ENI media characterized by the same permittivity tensor of $\text{diag}\{\varepsilon_{\parallel}, \varepsilon_{\perp}, \varepsilon_{\perp}\}$ (the subscripts “ $\parallel$ ” and “ $\perp$ ” denote the directions parallel and perpendicular to the optical axis of the medium, respectively), where $\varepsilon_{\parallel} = \infty$ and $\varepsilon_{\perp} = \varepsilon_0$ (permittivity of free space). The optical axes (along the direction of $\varepsilon_{\parallel}$) of the two single ENI media form angles of 45° with the interface. This is because the Gaussian beam propagates as extraordinary waves in both single ENI media and the extreme anisotropy for extraordinary waves in the single ENI media fully suppresses wave diffraction. However, when an out-of-plane wave vector is involved, this Gaussian beam will contain not only extraordinary waves, but also ordinary waves, the latter of which are subject to intrinsic diffraction. Even if only extraordinary waves are considered as incident waves, the transmission will be seriously deteriorated. As shown in **Figure.2-4(b)**, an incident Gaussian beam of extraordinary waves with a finite out-of-plane wave vector component k_z (for illustration purposes, $k_z = 0.5 k_0$ in **Figure.2-4(b)**; k_0 is the wave number in free space) will have a significant portion of energy refracting into ordinary waves when entering the other single-ENI medium. Therefore, the reflectionless diffraction-free wave routing in **Figure.2-4(a)** is limited to 2D beam propagation with zero out-of-plane wave vector.

Figure.2-4 Birefringent effect in single ENI media. (a) Reflectionless diffraction-free wave bending in single ENI media in 2D scenario. (b) Birefringence in single-ENI media when the out-of-plane wave vector is not zero. $\vec{S}_{ie}$, $\vec{S}_{te}$, $\vec{S}_{re}$, are Poynting vectors of incident, transmitted and reflected extraordinary waves, respectively. $\vec{S}_{to}$ and $\vec{S}_{ro}$ are Poynting vectors of transmitted and reflected ordinary waves, respectively.

2.3 Reflectionless transmission condition

Now we discuss the transmission in double-ENI media. We set that the finite components of two double-ENI medium 1 and medium 2 are ε_1 and ε_2 , and the optical axes of two media, as shown in **Figure.2-5**, are O_1 and O_2 , lying in x - y plane making angles of θ_1 and θ_2 to the interface plane (y - z plane), respectively. Then, the isofrequency surfaces of extraordinary waves in medium 1 and medium 2 are two cylindrical surfaces oriented along O_1 and O_2 as shown in **Figure.2-5**.

Figure.2-5 Isofrequency surfaces of extraordinary waves incident at an interface between two double-ENI media. The optical axes O_1 and O_2 (white dashed lines) of the two media are in the x - y plane forming angles of θ_1 and θ_2 with the interface (y - z plane). $\bar{k}_{ie}$, $\bar{k}_{re}$ and $\bar{k}_{te}$ ($\bar{S}_{ie}$, $\bar{S}_{re}$ and $\bar{S}_{te}$) are wave (Poynting) vectors of incident, reflected and transmitted extraordinary waves, respectively.

The expressions of these two isofrequency surfaces are:

$$(k_x \cos(\theta_1) + k_x \sin(\theta_1))^2 + k_z^2 = \varepsilon_1 k_0^2 \quad (2.22)$$

$$(k_x \cos(\theta_2) - k_x \sin(\theta_2))^2 + k_z^2 = \varepsilon_2 k_0^2 \quad (2.23)$$

Considering a plane electromagnetic wave is incident from medium 1 to medium 2 with an extraordinary wave vector $\bar{\mathbf{k}}_{ie}$ which has arbitrary y component k_y and z component k_z

($\sqrt{|\bar{\mathbf{k}}_{ie}|^2 - k_y^2 - k_z^2} > 0$), it can be obtained according to phase matching conditions:

$$k_{rey} = k_{roy} = k_{tey} = k_{toy} = k_y \quad (2.24)$$

$$k_{rez} = k_{roz} = k_{tez} = k_{toz} = k_z \quad (2.25)$$

where k_{rey}, k_{roy} (k_{rez}, k_{roz}) are the y (z) components of the extraordinary and ordinary wave vector of the reflected waves, respectively and k_{tey}, k_{toy} (k_{tez}, k_{toz}) are the y (z) components of the extraordinary and ordinary wave vectors of the transmitted waves, respectively. Substituting Eq. (s17, s18) into Eq. (s13, s15, s16), we can write the extraordinary wave vectors:

$$\bar{\mathbf{k}}_{ie} = \hat{\mathbf{x}} \left(-k_y \tan(\theta_1) + \frac{\sqrt{k_1^2 - k_z^2}}{\cos(\theta_1)} \right) + \hat{\mathbf{y}} k_y + \hat{\mathbf{z}} k_z \quad (2.26)$$

$$\bar{\mathbf{k}}_{re} = -\hat{\mathbf{x}} \left(k_y \tan(\theta_1) + \frac{\sqrt{k_1^2 - k_z^2}}{\cos(\theta_1)} \right) + \hat{\mathbf{y}} k_y + \hat{\mathbf{z}} k_z \quad (2.27)$$

$$\bar{\mathbf{k}}_{te} = \hat{\mathbf{x}} \left(k_y \tan(\theta_2) + \frac{\sqrt{k_2^2 - k_z^2}}{\cos(\theta_2)} \right) + \hat{\mathbf{y}} k_y + \hat{\mathbf{z}} k_z \quad (2.28)$$

and the ordinary wave vectors:

$$\bar{\mathbf{k}}_{ro} = -\hat{\mathbf{x}} \sqrt{\infty} k_0 + \hat{\mathbf{y}} k_y + \hat{\mathbf{z}} k_z \approx -\hat{\mathbf{x}} \sqrt{\infty} k_0 \quad (2.29)$$

$$\bar{\mathbf{k}}_{to} = \hat{\mathbf{x}} \sqrt{\infty} k_0 + \hat{\mathbf{y}} k_y + \hat{\mathbf{z}} k_z \approx \hat{\mathbf{x}} \sqrt{\infty} k_0 \quad (2.30)$$

Following Eq. (s10, s11), we can write the magnetic fields. The corresponding electric fields can be obtained with Ampere's law. Then by matching the boundary conditions, we obtain the reflection and transmission coefficients of magnetic fields are:

$$r_{He} = \frac{\sqrt{\varepsilon_1 k_0^2 - k_z^2} \cos(\theta_2) - \sqrt{\varepsilon_2 k_0^2 - k_z^2} \cos(\theta_1)}{\sqrt{\varepsilon_1 k_0^2 - k_z^2} \cos(\theta_2) + \sqrt{\varepsilon_2 k_0^2 - k_z^2} \cos(\theta_1)} \quad (2.31)$$

$$t_{He} = \frac{2\sqrt{\varepsilon_1 k_0^2 - k_z^2} \cos(\theta_1)}{\sqrt{\varepsilon_1 k_0^2 - k_z^2} \cos(\theta_2) + \sqrt{\varepsilon_2 k_0^2 - k_z^2} \cos(\theta_1)} \frac{\sqrt{\varepsilon_1}}{\sqrt{\varepsilon_2}} \quad (2.32)$$

Then, the reflectance and transmittance can be readily obtained as:

$$R = \left(\frac{\sqrt{\varepsilon_1 k_0^2 - k_z^2} \cos(\theta_2) - \sqrt{\varepsilon_2 k_0^2 - k_z^2} \cos(\theta_1)}{\sqrt{\varepsilon_1 k_0^2 - k_z^2} \cos(\theta_2) + \sqrt{\varepsilon_2 k_0^2 - k_z^2} \cos(\theta_1)} \right)^2 \quad (2.33)$$

$$T = \frac{4\sqrt{\varepsilon_1 k_0^2 - k_z^2} \sqrt{\varepsilon_2 k_0^2 - k_z^2} \cos(\theta_1) \cos(\theta_2)}{\left(\sqrt{\varepsilon_1 k_0^2 - k_z^2} \cos(\theta_2) + \sqrt{\varepsilon_2 k_0^2 - k_z^2} \cos(\theta_1) \right)^2} \quad (2.34)$$

Letting $R = 0$, we can obtain the general reflectionless transmission condition

$$\sqrt{\varepsilon_1 \frac{\omega^2}{c^2} - k_z^2} \cos(\theta_2) = \sqrt{\varepsilon_2 \frac{\omega^2}{c^2} - k_z^2} \cos(\theta_1) \quad (2.35)$$

where c is the light speed in free space. The perfect transmission condition in Eq. (1) involves: (i) the finite permittivity components ε_1 and ε_2 of the two media, (ii) angles θ_1 and θ_2 between each optical axis of the two media and the interface, (iii) the frequency ω , and (iv) the out-of-plane wave vector component k_z . We consider two cases in the following.

Case 1) $k_z = 0$. In this 2D case, the electric fields are all in plane (i.e. $E_z = 0$). The perfect transmission condition in Eq. (2.35) becomes

$$\sqrt{\varepsilon_1} \cos(\theta_2) = \sqrt{\varepsilon_2} \cos(\theta_1) \quad (2.36)$$

which means the reflectionless diffraction-free transmission is irrelevant to either frequency or the incident angle $\alpha_i = \arcsin\left(\sqrt{k_y^2}/|\bar{\mathbf{k}}_{ie}|^2\right)$. This 2D scenario of perfect transmission in double-ENI media is similar to that of single-ENI media [99], and can be applied to bend, split, compress and expand EM waves. We shall be discussing about this case in **Chapter 2.4**.

Case 2) $k_z \neq 0$. The wave propagation in this case is beyond 2D. When and only when $\varepsilon_1 = \varepsilon_2$ and $\theta_1 = \theta_2$ are satisfied in Eq. (2.35) can the perfect transmission be independent on the frequency, the incident angle ($\alpha_i = \arcsin\left(\sqrt{k_y^2 + k_z^2}/|\bar{\mathbf{k}}_{ie}|^2\right)$), and the out-of-plane wave vector component k_z . Otherwise, the perfect transmission occurs only at a single frequency. In the experimental demonstration, we adopt the condition of $\varepsilon_1 = \varepsilon_2$ and $\theta_1 = \theta_2$ for a broad working bandwidth. The details of this case will be introduced in **Chapter 2.5**.

2.4 Bending, splitting, squeezing/expanding of electromagnetic waves in 2D scenario

In this Chapter, we will introduce bending, splitting, squeezing and expanding of electromagnetic waves with extremely anisotropic media applying the **Case 1** of the reflectionless transmission condition as shown in Eq. (2.36) in a 2D scenario.

Figure.2-6 Reflectionless diffraction-free electromagnetic wave bending with extremely anisotropic media. (a)(c)(e) Power density distribution of TM polarized Gaussian beams bent by 60, 90 and 120 degrees, respectively. The waist size is 0.2 wavelengths. The dashed lines denote the boundaries. (b)(d)(f) Magnetic field distribution and wave vectors (black arrows).

Figure.2-6 shows simulations results of obtuse-angled (120 degree), right-angled and acute-angled (60 degree) reflectionless diffraction-free electromagnetic wave bending cases applying the reflectionless transmission condition. The results are obtained from commercial simulation software COMSOL Multiphysics. As shown in **Figure.2-6** (a)(c)(e), three TM polarized Gaussian waves (with wave size of 0.2 wavelength) propagate in regions formed by extremely anisotropic media (with an infinite permittivity and a finite permittivity of ϵ_0) oriented in different directions. The infinite permittivity components of the central regions in Figure.2-6 (a)(c)(e) form 30, 0 and -30 degree angles with the vertical direction, respectively. The boundaries between the adjacent two media are along the angle bisector line of the angles formed by the optical axes of two media. We can find that the propagation of electromagnetic waves are both reflectionless and diffraction-free regardless of the bending angles. **Figure.2-6** (b)(d)(f) show the magnetic field distribution and the wave vectors. These imply a near free circuit-like manipulations of electromagnetic waves.

Besides bending, if we re-design the boundary between two media in a way as shown in **Figure.2-7**, we can readily realize reflectionless diffraction-free splitting of electromagnetic waves.

Figure.2-7 Reflectionless diffraction-free electromagnetic wave splitting with extremely anisotropic media. (a) Splitting of a TM polarized Gaussian beam. (b) Splitting of waves passing through two subwavelength-distanced slits. The slits have size of $1/10$ wavelength and the center-to-center distance between two slits is $1/10$ wavelength. The dashed lines denote the boundaries.

As can be seen in **Figure.2-7(a)**, when a TM polarized Gaussian beam (with a waist size of 0.2 wavelength) propagates through the boundary of two extremely anisotropic media (the reflectionless condition is satisfied), the wave is split into two waves right along the central line of the Gaussian beam, and each of the waves maintains half of the Gaussian profile after the splitting. If we put a PEC mask with two slits (the center to center distance between two slits is $1/10$ wavelength, and the width of two slits is $1/10$ wavelength) in the path of the wave propagation, as shown in **Figure.2-7(b)**, two deep subwavelength ray-like waves are generated by the two slits and propagate without interfering with each other. When two ray-like waves go through the re-designed boundary, they are split downward and upward respectively without reflection, then re-directed to propagate in $+\hat{x}$ direction. Note that the waves go through two deep-subwavelength-distanced slits will soon interfere with each other in free space after a very short distance of propagation, making them impossible to distinguish or split. However, in extremely anisotropic media, the information of two slits can not only be maintained, but also split to a much larger distance so that the waves can still be distinguished even they are coupled to free space.

At last, we discuss electromagnetic wave compressing and expanding with extremely anisotropic media. When we take different values for the finite permittivity components of two extremely anisotropic media in Eq. (2.35), we can realize reflectionless diffraction-free compressing and expanding of electromagnetic waves. We explain the electromagnetic wave compressing and expanding in extremely anisotropic media here first.

Under the reflectionless transmission condition in Eq.(2-36), if the magnetic field of the incident wave is $\bar{H}_i = \hat{z}e^{i\bar{k}_i\bar{r}}$, the electric field of the incident wave can be calculated as $\bar{E}_i = \hat{o}_1\sqrt{\varepsilon_1}e^{i\bar{k}_i\bar{r}}$, and the magnetic field and electric field of the transmitted wave can be written as $\bar{H}_t = \hat{z}e^{i\bar{k}_t\bar{r}}$ and $\bar{E}_t = \hat{o}_2\sqrt{\varepsilon_2}e^{i\bar{k}_t\bar{r}}$, respectively, where $\bar{k}_i$ ($\bar{k}_t$) is the wave vector of the incident (transmitted) wave, $\hat{o}_1$ and $\hat{o}_2$ are the unit vector in the directions of the optical axes of two media. We can readily find that $|\bar{H}_i| = |\bar{H}_t|$, $\sqrt{\varepsilon_2}|\bar{E}_i| = \sqrt{\varepsilon_1}|\bar{E}_t|$ and $\sqrt{\varepsilon_2}|\bar{S}_i| = \sqrt{\varepsilon_1}|\bar{S}_t|$, which means that the magnitude of the magnetic fields of the electromagnetic waves in extremely anisotropic media are unchanged, while the magnitude of the electric fields and Poynting vectors (power flow) are related by the values of the finite components of the permittivity tensors of two media. In other word, by adjusting the ratio of the finite components of the permittivity tensors of two media, we can control the compressing or expanding ratio of the electromagnetic waves at will.

Now we proceed to demonstrate the compressing and expanding of electromagnetic waves with extremely anisotropic media with numerical simulations obtained by COMSOL Multiphysics. As can be seen from **Figure.2-8(a)**, the permittivity tensors of two media used are $\{\infty, \varepsilon_1\}$ and $\{\varepsilon_2, \infty\}$, respectively, where $\varepsilon_1 = 9\varepsilon_2$. The angles formed by the optical axes (the axes where the finite permittivity components lie in) of two infinitely anisotropic media and boundaries are θ_1 and θ_2 , where $\theta_2 = 3\theta_1 = 71.57^\circ$ ($\theta_1 + \theta_2 = 90^\circ$), satisfying the reflectionless transmission condition in Eq. (2.36). A TM polarized Gaussian beam (with a waist size of 0.2 wavelength) is incident from the left side of the region propagating in +x direction. When the TM polarized Gaussian beam propagates through the first boundary, it is compressed to 1/3 of its original width and the wave energy density is tripled. Specifically, the amplitude of the electric field is tripled, while the amplitude of the magnetic field is unchanged, as shown in **Figure.2-8(b)**. **Figure.2-8(c)** shows the normalized energy density along the cross lines Line1 and Line2 (white solid lines in **Figure.2-8(a)**). During the whole

bending and compressing process, the propagation of the electromagnetic wave is free from both diffraction and reflection.

Figure.2-8 Reflectionless diffraction-free electromagnetic wave compressing and expanding with extremely anisotropic media. (a)(b) Power density and magnetic field distributions of TM polarized Gaussian beam compressing and bending. (c) Normalized power density along the cross lines Line1 and Line2 (white solid lines) in (a). (d)(e) Power density and magnetic field distributions of TM polarized Gaussian beam compressed in its original direction. (f) Normalized power density along the cross lines Line3 and Line4 (white solid lines) in (d). (g)(h) Power density and magnetic field distributions of TM polarized Gaussian beam expanded in its original direction. (i) Normalized power density along the cross lines Line5 and Line6 (white solid lines) in (g).

We can also make the compressed wave propagate in the same direction of the incident wave. As can be seen in **Figure.2-8(d)**, the permittivity tensors for three media are $\{\infty, \varepsilon_1\}$, $\{\varepsilon_2, \infty\}$ and $\{\infty, \varepsilon_2\}$, where $\varepsilon_1 = \varepsilon_3 = 9\varepsilon_2$. The angles formed by the optical axes

of the media and the boundaries are $\theta_2 = 3\theta_1 = 71.57^\circ$ and $\theta_3 = 45^\circ$, satisfying the reflectionless transmission condition at both boundaries. When a TM polarized Gaussian beam propagates through the first boundary, we can see that the wave is firstly compressed to 1/3 of its original width and bent downward after propagating through the first boundary as in **Figure.2-8(a)**. Then the beam gets bent back to its original propagating direction after propagating through the second boundary. **Figure.2-8(e)** shows the magnetic field distribution and **Figure.2-8(f)** shows the normalized power density along the cross lines Line3 and Line4 in **Figure.2-8(d)**.

If we inverse the whole structure, as shown in **Figure.2-8(g)** and (i), electromagnetic wave can be expanded to triple of its original width and the energy density of the EM wave is diluted to 1/3. The magnitude of the magnetic field is unchanged as shown in **Figure.2-8(h)**. By changing the ratio of the finite components oriented angles of extremely anisotropic media, we can in theory realize arbitrary compressing and expanding of electromagnetic waves without reflection or diffraction. Particularly, when $\varepsilon_2 \rightarrow 0$ (means $\varepsilon_1/\varepsilon_2 \rightarrow \infty$), the electromagnetic wave can be compressed to an infinitely small width with infinitely large energy density, which perfectly matches the “supercoupling” phenomenon of ENZ media[76] .

2.5 Experimental demonstration of broadband reflectionless diffraction-free electromagnetic wave routing in 3D scenario

In this chapter, we introduce the experimental demonstration of the broadband reflectionless diffraction-free wave routing phenomenon of double-ENI media with metallic-plate-stacking metamaterials. We firstly propose a double-ENI metamaterial design at microwave frequencies with metallic-plate-stacking structure, as shown in **Figure.2-9**.

Figure.2-8 Double-ENI metamaterial with metal-dielectric-stacking structure at microwave frequencies. The stacking period is p that is much smaller than the wavelength. The thickness of the metallic plate δ is much smaller than the stacking period p . The effective permittivity of the metamaterial is finite in the stacking direction (y direction), and near infinity in the other two directions (x and z directions).

An explicit calculation of effective permittivity components of the metal-dielectric-stacking metamaterial can be conducted from effective medium theory. For a multilayer structure consisting of two constituents with permittivity of ϵ_1 and ϵ_2 , filling factors of f and $1-f$ and

a stacking period of p that is at a subwavelength scale, the effective permittivity components can be obtained according to the following standard formulae [115, 116]:

$$\varepsilon_{\text{eff}\perp} = \varepsilon_1 f + \varepsilon_2 (1 - f) \quad (2.37)$$

$$\varepsilon_{\text{eff}\parallel} = \frac{\varepsilon_1 \varepsilon_2}{(1 - f)\varepsilon_1 + f\varepsilon_2} \quad (2.38)$$

It is well-known that in the frequency spectrum ranging from microwave to far infrared, common metals can be regarded as PECs with infinitely large permittivity [117, 118]. Therefore, for the structure in **Figure.2-9**, $\varepsilon_1 = \infty$, $\varepsilon_2 = \varepsilon_d$, and $f = \delta/p \ll 1$, which result in $\varepsilon_{\text{eff}\perp} = \infty$ (in x and z directions) and $\varepsilon_{\text{eff}\parallel} = \varepsilon_d$ (in y direction). Physically, we can understand the effective permittivity of the structure in the following manner. When the electric fields of electromagnetic waves are perpendicular to the thin metallic plates, the waves in fact do not “feel” the existence of these metallic plates. Therefore, in the stacking direction, the effective permittivity component ($\varepsilon_{\text{eff}\parallel}$) is just the permittivity of the filling dielectric(ε_d). On the other hand, electric fields parallel to the thin metallic plates are required to be zero at the surface of each metallic plate. As the metallic plates are stacked at the deep-subwavelength scale, in the bulk metallic-plate-stacking structure, the average electric field in the two directions parallel to the metallic plates is almost zero. Therefore, the effective permittivity components ($\varepsilon_{\text{eff}\perp}$) in these two directions are both infinite. Similar metamaterial structures have been adopted previously in designs of unconventional lenses with subwavelength resolution [119, 120].

The values of these effective permittivity components are further verified and compared with single-ENI media with numerical simulations obtained by COMSOL Multiphysics, as shown in **Figure.2-10**. As shown in **Figure.2-10(a)**, when a TE-polarized Gaussian beam with a waist size of 3 wavelengths are incident at a 30 degree angle to a slab

of metallic-plate-stacking metamaterial (stacking period is $\lambda_0/10$, and thickness of the metallic plates is $\lambda_0/100$), the wave is totally reflected, which is exactly same with the double-ENI slab as shown in **Figure.2-10(c)**. However, for a single-ENI slab, the TE-polarized wave propagates as if it propagates in free space, as shown in **Figure.2-10(e)**. This is because TE-polarized waves follow the dispersion of the ordinary waves, and the ordinary waves in double-ENI media are all suppressed, while the isofrequency contour of the ordinary waves in single-ENI media is circle that is exactly the same with that of free space. For TM polarized waves, the wave responses of the metallic-plate-stacking metamaterial slab and the double-ENI slab are also the same, as shown in **Figure.2-10(b)** and (d). Note that the wave response of single-ENI slab is also the same with double-ENI slab as shown in **Figure.2-10(f)**, since in 2D scenario, the isofrequency contours of two media are the same. **Figure.2-10(a-d)** verifies that the metallic-plate-stacking metamaterial is an effective double-ENI medium.

Figure.2-10 Double-ENI performance of metallic-plate-stacking metamaterials. (a) (c) (e) Field patterns when TE polarized Gaussian waves (waist size $3\lambda_0$) are incident with an incident angle of

30° to slabs of metallic-plate-stacking metamaterial, double-ENI medium, and single-ENI medium. (b) (d) (f) Field patterns when TM polarized Gaussian waves (waist size $3\lambda_0$) are incident with an incident angle of 30° to slabs of metallic-plate-stacking metamaterial, double ENI-medium, and single ENI-medium. The stacking period of the metamaterial in (a) and (b) is $1/20 \lambda_0$.

Now we proceed to introduce the experimental implementation. A self-designed rectangular waveguide with two right-angle sharp corners is shown in **Figure.2-11**. Thin metallic plates (thickness $\delta = 0.4$ mm) stacked periodically (period $p = 3.4$ mm) are inserted into the waveguide as double-ENI media.

Figure.2-11 Verification of reflectionless transmission in double-ENI media. (a) Schematic of the homemade rectangular waveguide with two right-angle sharp corners filled with metallic-plate-

stacking metamaterials. The port dimensions of the waveguide are $w = 72$ mm and $h = 34$ mm. The white dashed lines show two interfaces. The blue dashed lines show the directions of the optical axes. (b) Illustration of the equivalent media distribution and the wave propagation of TE₁₀ mode in the geometry of the waveguide. (c) Simulated magnetic field distribution at 3.2GHz. (d) Measured and simulated transmission through the bent waveguide with/without double-ENI metamaterial structures.

The equivalent geometry of the waveguide filled with double-ENI media is illustrated in **Figure.2-11(b)**. In the same x - y - z coordinate system, the double-ENI media 1 and 3 have a permittivity tensor of $\text{diag}\{\infty, \varepsilon_0, \infty\}$, and the double-ENI medium 2 has a permittivity tensor of $\text{diag}\{\varepsilon_0, \infty, \infty\}$. Since the optical axes (blue dashed lines in **Figure.2-11(a)**) of double-ENI media filled in the waveguide form the same angle of 45° with the interface (white dashed lines in **Figure.2-11(a)** or red planes in Figure.2-10(b)) between each pair of adjacent double-ENI media, the general perfect transmission condition in Eq.(2-35) is satisfied. We adopt the fundamental waveguide mode (TE₁₀ mode) for demonstration. Note that the waves of TE₁₀ mode follow a zig-zag path in the waveguide, as illustrated in **Figure.2-11(b)**, exhibiting a finite out-of-plane wave vector component k_z . The transverse standing wave along the z direction can be treated as the superposition of two plane waves with opposite k_z values. Because these two plane waves are fully mirror-symmetric with respect to the x - y plane, the transmission of TE₁₀ mode through the waveguide is equivalent to the transmission of a single plane wave with the finite k_z through the corresponding metallic-plate-stacking metamaterial structure that is infinite along the z direction. Therefore, the setup of the bent waveguide can demonstrate the unique advantage of double-ENI media.

A full-wave simulation for the corresponding metallic-plate-stacking metamaterial structure that is infinite along the z direction is provided in **Figure.2-12**.

Figure. 2-12 Reflectionless diffraction-free wave routing in a 3D structure composed of metallic-plate-stacking metamaterials. (a) Prospective view of the field pattern. The white dashed lines denote two interfaces. The inset shows the input field pattern. The electric field of the incident wave is in y direction. (b) Top view of the field pattern. (c) Front view of the field pattern and zoom-in view of the output.

The 3D structure is composed of three regions constructed by metallic-plate-stacking metamaterials, as shown in **Figure.2-12(a)**. The stacking direction of Region 1 and 3 is along the y direction, and the stacking direction of Region 2 is along x direction. The stacking period is $3/80 \lambda_0$. At two interfaces (white dashed lines), the perfect transmission condition is satisfied (the optical axes of the metallic-plate-stacking metamaterials make angles of 45 degrees with the interfaces). Electromagnetic waves with Gaussian profile in x - z plane (waist size is $2.65 \lambda_0$) are separated by two subwavelength slits, as shown in **Figure.2-12(c)** and the inset of **Figure.2-12(a)**. The thickness of each slit is $3/40 \lambda_0$, the edge-to-edge distance

between two slits are $3/40 \lambda_0$. The waves with subwavelength information of the two slits are obliquely incident from Region 1 (the incident angle is 20 degree, the corresponding out-of-plane wave vector $k_z = 0.342 k_0$). As shown in **Figure.2-12(a)** and (b), the waves propagate through two interfaces almost without reflection, and the subwavelength information in y direction are perfectly transmitted to the output end of the structure, as shown in **Figure.2-12(c)**.

Getting back to our waveguide demonstration, we choose the working frequency range from 2.2 GHz to 4.1 GHz, in which only the waves of TE₁₀ mode can propagate in the waveguide. The magnetic field distribution at 3.2 GHz simulated with CST Microwave Studio is shown in **Figure.2-13(c)**. Almost perfect transmission through the bent waveguide can be observed. The measured transmission over the broad frequency range from 2.2 GHz to 4.1 GHz in **Figure.2-13(d)** shows that, the waveguide filled with double-ENI media can reach almost unity transmission with only slight transmission reduction compared to the simulation results. The slight transmission reduction is mainly due to the manufacturing imperfection and the coupling loss from the homemade waveguide to the vector network analyzer. In contrast, the empty bent waveguide without double-ENI media shows resonant transmission spectral and the transmission is significantly reduced. The resonant spectral shape is a result of the structural resonance formed between the two right-angle sharp corners. Note that there is also a slight reduction in transmission through the empty waveguide compared to the simulation results. This further confirms that the slight transmission reduction is from the measurement setup, rather than from the double-ENI media.

We then demonstrate that the double-ENI media can deliver deep-subwavelength information through sharp corners. An aluminous mask with two subwavelength slits (width 3.4 mm) separated by a center-center distance of $d = 6.8$ mm, as shown in the inset of **Figure.2-13(a)**, is attached to the input of the double-ENI metamaterial structure. We

illuminate the aluminous mask by launching waves from the left port of the waveguide. The field distribution at the output of the double-ENI metamaterial structure is measured with a near-field probe as shown in Figure.2-13(b).

Figure.2-13 Demonstration of diffraction-free subwavelength information transmission. (a) Simulated electric field $|\vec{E}|$. A metallic mask with subwavelength slits (width 3.4 mm; center-to-center distance $d = 6.8$ mm) is attached on the right side of the double-ENI metamaterial structure. The field at the output of the double-ENI metamaterial structure diffracts into free space. (b) Measured source images at different frequencies. (c) Measured transmitted images at different frequencies.

As the simulated electric field distribution in **Figure.2-13(a)** shows, the waves emitted from the two subwavelength slits are routed through the bent waveguide. Note that at the output the waves will diffract into free space. This will cause reflection and form standing waves in the two subwavelength wave channels. We plot the measured source image (field distribution immediately behind the mask) in **Figure.2-13(b)**, and the measured transmitted image (field distribution immediately behind the double-ENI metamaterial structure) in

Figure.2-13(c). It is evident that the image with deep-subwavelength information can be transmitted through the bent waveguide with two sharp corners in a broad bandwidth.

In the end, we show the experimental setup and real experimental sample in **Figure.2-14**. The experimental setup for measurement of the transmission of the metamaterial-filled waveguide with two right-angle corners is shown in **Figure.2-14(a)**. Two ports of the waveguide are connected to the vector network analyzer by waveguide-to-coaxial adaptors. **Figure.2-14(b)** shows the experimental setup for the measurement of the subwavelength images through the bent waveguide. The electric fields are measured by a near-field probe with a 2 mm copper tip oriented along the vertical direction, as shown in the inset of **Figure.2-14(b)**. **Figure.2-14(c)** shows the prospective photo of the waveguide sample. The inner height of the rectangular waveguide is 36 mm. The inner width of the waveguide is 72 mm. The inset shows the cross section on the output of the metamaterial structure. The side photo of the waveguide sample is shown in **Figure.2-14(d)**. The waveguide sample has a total length of 416 mm and a total height of 160 mm. The distance from the ports to the

metallic-plate-stacking metamaterial is 111 mm. The right port can be mechanically disassembled for the convenience of image transmission measurement.

Figure.2-14 Experimental setups and waveguide samples. (a) Experimental setup of measuring the transmission of the bent waveguide filled by metallic-plate-stacked metamaterials. (b) Experimental setup of measuring subwavelength images. The inset shows mapping the electric field with a near-field prob. (c) Photo of the waveguide sample filled by metallic-plate-stacked metamaterials. The inner height of the rectangular waveguide is 36 mm. The inner width of the waveguide is 72 mm. The inset shows cross section of the waveguide on the output of the metamaterial structure. (d) Side photo of the waveguide sample. The waveguide is made of aluminum. The total length of the waveguide sample is 416 mm. The total height is 160 mm. The distance between the port and the metallic-plate-stacking metamaterial is 111 mm. The right port can be disassembled for the convenience of image mapping.

2.6 Conclusion

In this chapter, we study the properties of extremely anisotropic media. The properties of single-ENI media and double-ENI media are compared. The reflectionless transmission condition between two double-ENI media are introduced. We verify the bending, splitting, squeezing and expanding of electromagnetic waves in 2D scenarios with double-ENI media by numerical simulations. We propose a double-ENI metamaterial design with metal-dielectric-stacking structure. We experimentally demonstrate the reflectionless diffraction-free transmission electromagnetic wave routing phenomenon over a broad bandwidth. An image with deep-subwavelength information is transported through two sharp corners in a bent waveguide over a broad bandwidth. The above results further the study of singular media, and may find novel application in subwavelength electromagnetic wave manipulation and high-efficiency deep-subwavelength information interaction in photonic systems.

CHAPTER 3 OMNIDIRECTIONAL INVISIBILITY CLOAK

Realizing an invisibility cloak has captured wide scientific research interests since the proposal of transformation optics [16, 17] in 2006. A perfect invisibility cloak should be able to guide electromagnetic waves around a certain region and maintain their original trajectories and phases after the electromagnetic waves interact with the cloak as if the waves propagate in an isotropic free space. To achieve a perfect invisibility cloak, it requires the coordinate transformation to initiate from a 0D point in the virtual space. However, such coordinate transformations will introduce extreme anisotropy and singularities to the constitutive parameters in the physical space. Thus, the first cloak demonstration was carried out using the so called eikonal approximation that only preserves the refractive index but sacrifices the impedance match between the cloak and the surrounding environment [18]. As a result, reflections are generated when electromagnetic waves hit the cloak and the transmission through the cloak are reduced.

The carpet cloak has been a promising strategy to realize an invisibility cloak in practice, since it does not require singularities and extreme anisotropy. As introduced in **Chapter 1.3**, several demonstrations at microwave frequencies and optical frequencies have been successfully implemented based on quasi-conformal mapping, bi-linear coordinate transformation and recent redirection of the reflected waves with ultrathin metasurfaces. However, the carpet cloaks are not free-standing that always require a “ground plane” to camouflage the virtual perfect conducting line, otherwise it can only work for unidirection in free space [26, 105].

In this chapter, we propose and demonstrate a full-parameter omnidirectional invisibility cloak without eikonal approximation at microwave frequencies. The cloak is

based on a linear coordinate transformation that starts from a 0D point in the virtual space, which requires extreme anisotropy with both zero and infinity. We extend the recent photonic doping approach to achieve the required media with permittivity approaching zero and infinity in two orthogonal directions and permeability approaching zero in the third direction. We implement the cloak design with periodically-stacking metallic metamaterials in a plane waveguide. This work could further the study of extremely anisotropic media and push the research of realizing a perfect invisibility cloak in practice one step forward.

3.1 Coordinate transformation for the omnidirectional invisibility cloak

We introduce the coordinate transformation for the omnidirectional invisibility cloak here. We firstly select an infinitely large slab region with thickness of A in the virtual space for the coordinate transformation, as shown by the dashed blue lines in **Figure.3-1(a)**. An infinitely small perfect conducting point (solid green point) with a size of $\delta \ll \lambda_0$ lies in the center of the slab. Note that no scattering will be generated by the point if EM waves propagate in the virtual space due to the infinitely small size of the point.

Figure.3-1 Coordinate transformation for the omnidirection invisibility cloak. (a) Coordinate mesh of a free virtual space. A PEC point (green solid point) with an infinitely small size of $\delta \ll \lambda_0$ lies in the center of the space. The yellow line denotes a light ray that is not scattered by the infinitely small point. (b) The thin slit (marked by black dashed lines) together with the PEC point inside is stretched to a slab with a thickness of B . The PEC point then becomes a PEC line (marked by solid green line). Meanwhile, two regions on two sides of the slit are squeezed to a thickness of $(A - B)/2$ so that the space in the transforming region is still intact. (c) A bi-linear coordinate transformation is applied to bend the middle region and the straight PEC line so that a rhombic cloaking region (marked by green solid line) is generated.

We then divide the slab into three regions: a thin slab (with a thickness of δ , marked by dashed black lines) containing the perfect conducting point and the two regions on two sides. Firstly, as shown in **Figure.3-1(b)**, we stretch the space in the thin slab region along x direction to a thickness of B , meanwhile the two regions on two sides are compressed to $(A-B)/2$ to make the whole space intact. As a result, the perfect conducting point is stretched to a perfect conducting line (marked by solid green line) with a length of B . Then we do a bi-linear coordinate transformation to bend the straight perfect conducting line to two polygonal lines (marked by solid green line) so that a rhombic cloaking region is generated, as shown in **Figure.3-1(c)**. Note that the coordinate transformation starts from an infinitely small point, thus the cloak should be working for omnidirection.

Figure.3-2 Simulated field patterns comparison between free space and cloak for normal incidence and 30-degree oblique incidence. (a) Magnetic field pattern when a Gaussian wave

propagates in $+x$ direction in free space. (b) Magnetic field pattern when a Gaussian wave is incident normally to the invisibility cloak. (c) Magnetic field pattern when a Gaussian wave propagates obliquely with a 30-degree incident angle in free space. (d) Magnetic field pattern when a Gaussian wave is obliquely incident to the cloak with a 30-degree incident angle. The waist size of the Gaussian waves is 4 wavelengths. The Gaussian waves are TM polarized.

Figure.3-2 shows the simulated field pattern comparison between free space and cloak for normal incidence and 30-degree oblique incidence. The field patterns are obtained from COMSOL Multiphysics. We can see that the cloak works perfectly for both normal and oblique incidences.

The mathematical relations between virtual space (x, y) and the physical space (x', y') are

$$x'_{\text{in}} = \frac{B}{\delta} x; \quad y'_{\text{in}} = y + \frac{H}{B} \left(B - \frac{B}{\delta} x \right), \quad (3.1)$$

$$x'_{\text{ex}} = \frac{B}{2} + \frac{A-B}{B} x; \quad y'_{\text{ex}} = y, \quad (3.2)$$

where the subscripts “in” and “ex” denote the inner and external regions of the cloak, respectively. The constitutive parameters for the cloak can be calculated according to the standard formulae as shown in Eq. (1-1)

$$\bar{\epsilon}'_{\text{in}} = \bar{\mu}'_{\text{in}} = \frac{\bar{J}_{\text{in}} \cdot \bar{J}_{\text{in}}^T}{|\bar{J}_{\text{in}}|} = \begin{pmatrix} \frac{B}{\delta} & \pm \frac{H}{\delta} & 0 \\ \pm \frac{H}{\delta} & \frac{H^2}{\delta B} + \frac{\delta}{B} & 0 \\ 0 & 0 & \frac{\delta}{B} \end{pmatrix} \quad (3.3)$$

$$\bar{\epsilon}'_{\text{ex}} = \bar{\mu}'_{\text{ex}} = \frac{\bar{\mathbf{J}}_{\text{ex}} \cdot \bar{\mathbf{J}}_{\text{ex}}^T}{|\bar{\mathbf{J}}_{\text{ex}}|} = \begin{pmatrix} \frac{A-B}{A} & 0 & 0 \\ 0 & \frac{A}{A-B} & 0 \\ 0 & 0 & \frac{A}{A-B} \end{pmatrix}, \quad (3.4)$$

where the $\bar{\mathbf{J}}_{\text{in}}$ and $\bar{\mathbf{J}}_{\text{ex}}$ are the Jacobian matrices for the inner and external regions, respectively. Eq. (3) can be diagonalized as

$$\bar{\epsilon}''_{\text{in}} = \bar{\mu}''_{\text{in}} = \begin{pmatrix} \infty & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (3.5)$$

where ∞ is in the direction parallel to the polygonal perfect conducting lines and 0 is in the direction perpendicular to them.

Checking Eq. (3.4), we find that the external regions involve magnetic anisotropy which could possibly be implemented with SRR. However, the working bandwidth of the SRR design is very narrow due to the resonant nature, meanwhile the previous implementations of zero-index media all suffer from narrow bandwidth as well. Therefore, it can be expected that it would be very difficult to make both the inner and external regions work at the same frequency band. Besides, we can also find from **Figure.3-1(c)** and **Figure.3-2(d)** that the function of the external regions of the cloak is to complement the wave displacement generated by the inner region of the cloak. **Figure.3-3** shows the cases when the external magnetically anisotropic regions are removed. As can be seen in **Figure.3-3 (a)** and **(c)**, when a Gaussian wave with a waist size of 4 wavelengths is obliquely incident on the simplified cloak, the rhombic PEC cloaking region can still be cloaked, while a displacement in y direction is generated. These displacements of the waves only make the virtual image look closer to the detector in x direction, as shown in **Figure.3-3(b)** and **(d)**. The image shift is equal to the width of the cloak B . In our following demonstration we drop

the external magnetically anisotropic regions for the convenience of practical implementation.

Figure.3-3 Simulated field patterns of the simplified cloak. (a) Light ray trajectories when a plane wave is obliquely incident to the simplified cloak. (b) Light ray trajectories when the simplified cloak is illuminated by a point source (yellow round patch) on the left-hand side of it. The red round patch denotes the virtual image when a detector observes from the right-hand side of the simplified cloak. (c) Magnetic field pattern when a Gaussian wave is obliquely incident with a 30-degree incident angle to the simplified cloak corresponding to (a). (d) Magnetic field pattern when the simplified cloak is illuminated by a point source corresponding to (b). The electromagnetic waves in both (c) and (d) are both TM polarized.

3.2 Parameters implementation and experimental setup

Here we discuss the experimental implementation of the omnidirectional invisibility cloak. In consideration of the practical implementing difficulties, we only consider transverse-magnetic (TM) polarized waves. The required permittivity and permeability are

$$\bar{\epsilon}_{\text{in}}'' = \begin{pmatrix} \infty & 0 \\ 0 & 0 \end{pmatrix}, \quad \bar{\mu}_{\text{inz}}'' = 0, \quad (3.6)$$

We firstly consider how to implement the extreme anisotropy. We have introduced in **Chapter 2** that extremely anisotropic metamaterials can be implemented with metal-dielectric-stacking structure, which has been applied to demonstrate reflectionless diffraction-free EM wave routing phenomena [121]. For the stacking structure shown in **Figure.2-9**, the effective constitutive parameters realized by the metal-dielectric-stacking structure are

$$\bar{\epsilon}_{\text{eff}} = \begin{pmatrix} \infty & 0 & 0 \\ 0 & \epsilon_d & 0 \\ 0 & 0 & \infty \end{pmatrix}, \quad \bar{\epsilon}_{\text{eff}} = \begin{pmatrix} \mu_d & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \mu_d \end{pmatrix} \quad (3.7)$$

where ϵ_d and μ_d are the permittivity and permeability of the dielectric. Hence, to realize the constitutive parameters in Eq. (3.6), we need the constitutive parameters of the dielectric ϵ_d and μ_d to be both near zero. In other word, to achieve the parameters required by the cloak, we only need to insert EMNZ media into the metallic-plate-stacking structure.

As introduced in **Chapter 1.2**, EMNZ media can be achieved by dielectric photonic crystals exhibiting Dirac cone dispersion and photonic doping to ENZ media. Here we propose a much simpler approach to emulate the epsilon-mu-near-zero (EMNZ) media with ENZ media. To explain this, we firstly analyze the properties of EMNZ media and ENZ

media. **Figure.3-2(a)** shows the transmission when a plane wave in free space is incident to slabs of EMNZ (black line) and ENZ (red line) media as the thickness of the slabs d changes.

Figure.3-4 Emulating EMNZ media with ENZ media. (a) Transmissions through a slab of EMNZ medium and a slab of ENZ medium as the thickness of the slabs varies. (b) Field pattern when a plane wave propagates through a EMNZ slab. The thickness of the slab is 4 wavelengths. (c) Field pattern when a plane wave propagates through a “H”-shaped ENZ channel. The width of the “H”-shape ENZ region is 4 wavelengths.

We can see the transmission through the EMNZ slab is always one as the thickness changes, which is due to the fact that the impedance between free space and the EMNZ medium is always matched. While the transmission through a ENZ slab is one only when the thickness of the slab is at a deep subwavelength scale (near zero), and the transmission decreases rapidly as the thickness increases. For larger thickness, EM wave will be totally reflected. This is because the impedance of the ENZ media is near infinite. However, a unique property of ENZ medium as pointed out by Engheta et al. is that the wavelength in ENZ media is near infinity, thus the EM waves propagating in ENZ media cannot ‘feel’ the geometry changes of the ENZ media [76, 77]. Therefore, if we change the shape of the very thin ENZ slab to a “H”-shaped channel but maintain the area of the thin slab (the total width

of the channel is 4 wavelength), as shown in **Figure.3-4(c)**, the transmission remains one. The field pattern when a plane wave is incident to a 4-wavelength thick EMNZ slab is shown in **Figure.3-4(b)**. Comparing **Figure.3-4(b)** and (c), We can readily find that the EM wave responses are the same for a 4-wavelength-thick EMNZ slab and a 4-wavelength-wide “H”-shaped ENZ channel, which implies that we can actually use ENZ media to emulate EMNZ media.

ENZ media have been demonstrated in several previous works both in microwave and optical regime as introduced in **Chapter 1.3**. Here we adopt the waveguide structural dispersion method [78] to demonstrate our omnidirectional invisibility cloak at microwave frequencies. **Figure.3-5(a)** shows one unit of the cloak.

Figure.3-5 Unit of the cloak using waveguide structural dispersion. (a) Structure of the cloak unit. The background media are Teflon with a relative permittivity of 2.1 (white blocks). The two pieces of bent red blocks are PEC. Note that there is a thin air slit between two pieces of bent PEC blocks. The period of the unit is $p = 6$ mm. The width is $w = 35$ mm. The thickness of the thin slit is $d = 0.5$ mm. The total length of the bent channel in z direction is 342 mm. All the boundaries in x and y directions are PEC boundaries. (b) Field pattern when a TE₀₁ mode (electric field in y direction) EM wave is incident from the left port of the structure. The frequency is at 4.288 GHz that is near to the

cutoff frequency of the rectangular waveguide with a width of 35 mm and a height of 6 mm when it is filled with air.

The width of the waveguide is $w = 35$ mm. The thin channel is filled with air that has a cutoff frequency of 4.288 GHz, while the background regions are filled with Teflon. The effective wavelength in the waveguide is 67.7 mm. Thus, we set the period of the stacking is $p = 6$ mm that is at a subwavelength scale satisfying the effective media theory. The red bent blocks are PEC. There is a thin air channel between two pieces of red bent PEC blocks. The thickness of the thin channels is $\delta = 0.5$ mm. The width of the bent channel is 342 mm that is about five times of the effective wavelength. **Figure.3-5(b)** shows the field pattern when a TE₀₁ mode EM wave is incident from the left port of the unit. The frequency of the EM wave is 4.29 GHz that is near to the cutoff frequency of the rectangular waveguide with a width of 35 mm and a height of 6 mm when it is filled with air. The cloak can be constructed by stacking the unit structure in the vertical direction as shown in **Figure.3-6**.

Figure.3-6 Structure of the cloak. The background medium is Teflon ($\epsilon = 2.1$). The dark gray regions are PEC. The light gray regions are air. The stacking period $p = 6$ mm.

Figure.3-7 Schematics of the cloak and experimental setup. Construction of invisibility cloak experiment transmission measurement. (a) Schematic of the experimental setup. A horn antenna launches electromagnetic waves on one side of the parallel-plate waveguide. Another horn antenna measures transmission on the other side. Both antenna can sweep angles azimuthally. The height of the parallel plane waveguide is 35 mm (not illustrated). (b-d) Photos of the cloak details. $p = 6$ mm, $d_1 = 0.5$ mm, $d_2 = 0.5$ mm

Figure.3-7 shows the structure of the cloak design and the experimental setup. Note that we need the electric fields of the electromagnetic waves to be in x - y plane. Thus, different from previous works that usually make use of the fundamental mode (the electric field is in z direction), we need to excite the higher order mode rather than the fundamental mode of the plane waveguide. In our experiment we design a pair of lens antennas fed by cable-to-waveguide adaptors as the source and receiver. The lens antennas are filled with

Teflon as the background media and FR-4 convex lens. The FR-4 convex lens converts the curved wave front to a near flat one so that a near Gaussian wave can be achieved. The geometry of self-designed aluminous horn antenna and FR-4 lens are shown in **Figure.3-8(a)** and (b). Other space inside the horn antenna is filled by Teflon. All the geometrical parameters are optimized with parameter sweeping. The field pattern emitted by the self-designed aluminous horn antenna is shown in **Figure.3-8(c)**. As can be seen that the electromagnetic wave emitted from the self-designed antenna has a near Gaussian profile.

Figure.3-8 Horn antenna design. (a) Geometrical details of the aluminous horn antenna. The green dashed line shows the contour of the FR-4 lens. Other space inside the antenna is filled by Teflon. (b)

Geometrical details of the FR-4 lens. (c) Simulated field pattern of the antenna in Teflon background at 4.29 GHz.

For the convenience of measuring the omnidirectional performance of the cloak, we design the background Teflon to be two semicircular disks so that we can sweep the incident and receiving angles. The background Teflon is surrounded by triangular absorbing materials to guarantee no reflection is generated.

3.3 Simulation and Experimental results

Here we report our experimental results. We firstly confirm the working frequency of the proposed cloak. We measure the transmission spectrum (normal incidence & receiving) when there is bare Teflon background as the reference. The measuring setup is shown in **Figure.3-9(a)**. We then measure the transmission spectrum of the proposed cloak (normal incidence & receiving), as shown in **Figure.3-9(b)** and normalize it with the bare Teflon reference. The absorbing material and structure are not shown in the figure. The normalized transmission spectrum is shown in **Figure.3-10**. The transmission peak is at 4.27 GHz, which corresponds to the working frequency of the cloak. Note that the cutoff frequency of the parallel plane waveguide is 4.29 GHz. The little frequency shift is caused by fabrication imperfection.

Figure.3-9. Schematics of measuring setups of reference and cloak (normal incidence & receiving). The absorbing material and structure are not shown in the figures.

Figure.3-10 Measured normalized transmission spectrum of the cloak for normal incidence and normal receiving. The transmission peak is at 4.27 GHz.

Figure.3-11(a-c) show the simulated field patterns at 4.29 GHz for free Teflon background, a rhombic perfect conducting obstacle and cloak cases. As can be seen that the cloak can perfectly guild the normally incident wave around the rhombic cloaking region as if the wave propagates in free isotropic Teflon background while most of the waves are scattered away by the obstacle when there is no cloak. The inset of **Figure.3-11(c)** shows the field distributions of the subwavelength-thick ENZ channels. As can be seen that there is nearly no phase change as the EM wave propagates in the ENZ channels (a little phase evolvment is due to the thickness of the channels, which can be further decrease as we decrease the thickness of the channels). **Figure.3-11 (d-f)** show the measured angle-sweeping transmission of three cases in **Figure.3-11(a-c)** at 4.27 GHz, respectively. The

transmissions are normalized by the transmission spectrum of the case when there is only background Teflon.

Figure.3-11 Simulated field patterns and measured transmissions of the cloak. (a) Field pattern when the electromagnetic wave propagates in free Teflon background. (b) Field pattern when the EM wave illuminates on a rhombic PEC obstacle. (c) Field pattern when the EM wave propagates through the invisibility cloak. The inset shows field distributions in the subwavelength-thick channels. (d) Angle-sweeping normalized transmission when the EM wave propagates in the free Teflon background. (e) Angle-sweeping normalized transmission when the EM wave illuminates on a rhombic PEC obstacle. (f) Angle-sweeping normalized transmission when the EM wave propagates through the cloak. The frequency for the simulated results in (a-c) is at 4.29 GHz. The frequency for the measured results in (d-f) is at 4.27 GHz.

Then we examine the omnidirectional performance of the cloak. We take 10, 20 and 30 degrees as the incident angles respectively. For larger incident angles, it requires larger cloaking structure and setup, thus we only take angles smaller than 30 degrees for this

experimental demonstration. The simulation field patterns and the angle-sweeping transmission on the receiving side for different incident angles at 4.27 GHz are shown in the **Figure.3-12(a-c)** and (d-f), respectively. We can find that the transmission for different incident angles can all reach above 80%. For smaller incident angles (e.g. < 15 degree), the transmission can be above 95%. Above results demonstrate the omnidirectional performance of the cloak.

Figure.3-12 Demonstration of the omnidirectional cloaking performance. (a-c) Field patterns when the EM waves are incident on the cloak at 10, 20 and 30 degrees at 4.29 GHz, respectively. (d-f) Angle sweeping normalized transmission when the EM waves are incident on the cloak at 10, 20 and 30 degrees at 4.27 GHz, respectively.

For larger incident angles, we show two simulations obtained from COMSOL Multiphysics in **Figure.3-13**. As can be seen in **Figure.3-13(a)**, when we increase the incident angle to 45 degree, the transmission through the cloak is still very high. However when the incident angle increase to 60 degree, observable reflection can be seen, as shown in **Figure.3-**

13(b). This is due to the thickness of the thin channels are not thin enough in practical implementation. According to the photonic doping equation of ENZ media in Ref. [91], the effective permittivity of the channels is actually $1 - \frac{A_{\text{PEC}}}{A} = 0.083$, where A_{PEC} is the area of the PEC (in our work it is Aluminum) and A is the total area. As a result, the non-zero permittivity actually causes a little impedance mismatch between the background and the cloak, and the transmission decreases as the incident angle increases. This matches well with both our experimental result and simulations in Figure.3-12 and -13. One can further increase the transmission for larger incident angle by further decrease the thickness of the ENZ channels.

Figure.3-13 Simulation of large incident angles. (a) Field patterns when a Gaussian EM beam is incident on the cloak at 45 degrees. (d-f) Field patterns when a Gaussian EM beam is incident on the cloak at 60 degrees.

3.4 Conclusion

In summary, we propose a cloak design based on a linear coordinate transformation that starts from an infinitely small point. The cloak requires homogeneous but extremely anisotropic media with both zero and infinity that has never been achieved before. We propose to combine the previous designs of extremely anisotropic metamaterial and ENZ metamaterial to realize the media required. We demonstrate the omnidirectional cloaking performance of the cloak with both simulation and experiment at microwave frequencies. This work could further the study of extremely anisotropic media, and push the realization of free-standing omnidirectional invisibility cloak one step forward.

CHAPTER 4 SINGLE-MEDIUM CLOAK FOR FLUID

The transformation optics and the concept of invisibility cloaks, on the other hand, have also inspired cloaking designs in other physical fields such as acoustics, thermal dynamics etc. Similar to Maxwell's equations whose form is invariant to coordinate transformation, the governing equations for other physical fields such as acoustics, thermal dynamics, diffusion, fluid dynamics and mechanics etc. also possess or approximately possess the similar feature. Thus, several acoustic cloaks [59, 61, 62] and thermal cloaks [63, 64, 108] based on coordinate transformation have been proposed and demonstrated. However, these cloak designs based on coordinate transformations always introduces either homogeneity or anisotropy to the required materials that are often challenging for practical implementation. To dodge the inhomogeneous or anisotropic parameters required by coordinate transformation, alternative approaches have been proposed recently to realize cloaks for static magnetic fields [102, 109], heat flux [65, 66], photon diffusion [110] as well as mechanical displacement [69] etc. thanks to the less complexity of Laplace equation. Instead of using coordinate transformation, they realize the cloaking effect using a core-shell structure by solving the governing Laplace equation. The materials used to implement the cloaks, as a result, were simplified from inhomogeneous and anisotropic media to homogeneous and isotropic media. Significant accomplishments have been achieved in thermal, mechanical and static electric/magnetic cloak designs [65, 66, 69, 73, 102, 109, 110]. However, the propagations of electromagnetic wave, acoustic wave, heat and static electric/magnetic fields are all in a form of field propagation. There has been no experimental demonstration of cloak for mass flow e.g. fluid so far. Besides, the current cloak designs at least require two different materials.

In this chapter, we propose a new cloak design approach for diffusive fluid. Different from previous cloak demonstrations for heat, static electric/magnetic fields and

diffusion that use at least two materials, we use geometry change of a single isotropic material in the third dimension to realize the first experimental demonstration of cloaking for diffusive fluid. The cloak design might find applications in solving fluid blocking in relevant industries.

4.1 Theoretical design of the diffusive fluid cloak

We start from a steady diffusion process that follows Laplace equation

$$D\nabla^2 u = 0 \quad (4.1)$$

where D is diffusivity, u is the density of the diffusing media. We consider a homogeneous diffusion process in a 2D scenario, as shown in **Figure.4-1(a)**.

Figure.4-1 Core-shell-structured cloak for diffusion. (a) Density distribution and streamlines of diffusion fluxes of a homogeneous diffusion process. The diffusivity of the background media is D_1 .

(b) Density distribution and streamlines of diffusion fluxes when there is a rigid obstacle (with zero diffusivity). The radius of the obstacle is R_1 . (c) Density distribution and streamlines of diffusion fluxes when there is higher-diffusivity shell ($D_2 > D_1$). The shell has an inner radius of R_1 and an outer radius of R_2 . (d) Density distribution and streamlines of diffusion fluxes when there is a core-shell-structured cloak. The core has a zero diffusivity and a radius of R_1 . The shell has an inner radius of R_1 , an outer radius of R_2 and a diffusivity of D_2 . D_1 , D_2 , R_1 and R_2 satisfy Eq. (4.2).

The diffusivity of the background media is D_1 . When there is a rigid round obstacle with a radius of R_1 and zero diffusivity, we can find that diffusion flux of the density is dispelled by the rigid obstacle, as shown in **Figure.4-1(b)**. On the other hand, if there is a ring with an inner radius of R_1 and an outer radius of R_2 and a diffusivity of D_2 ($D_2 > D_1$), the diffusion flux is attracted to the ring region, as shown in **Figure.4-1(c)**. We combine the two material to build a core-shell structure. When the geometry (inner and outer radius) and the diffusivity satisfy certain mathematical relation, the dispelling effect of the rigid zero-diffusivity core and the attraction of large diffusivity ring can compensate each other so that the diffusion flux outside the core-shell structure can be undisturbed. To solve the mathematical relation, we can set the density distribution outside the core-shell structure to be the same with the homogeneous diffusion process and the Laplace equation can be solved. The relation between the diffusivity of the shell and the background medium to achieve cloaking for diffusion can be obtained following the approach in Ref.[109, 110] as

$$D_2 = \frac{R_2^2 + R_1^2}{R_2^2 - R_1^2} D_1 \quad (4.2)$$

As shown in **Figure.4-1(d)**, we set the diffusivity of the shell to satisfy Eq. (4.2). We can find that the diffusion fluxes outside the core-shell structure are exactly the same with the homogeneous diffusion process in **Figure.4-1(a)**, which shows perfect cloaking performance for diffusion.

Now we consider fluid. We assume that the fluid flows slowly and has very large viscosity so that the fluid flows like diffusion. The continuity equation for fluid in steady case can be written as

$$\rho \nabla^2 \Phi = 0 \quad (4.3)$$

where ρ is the fluid density, Φ is the velocity potential. For a 2D core-shell cloaking structure, where the core is the cloaking region with a radius of R_1 , while the shell with an outer radius of R_2 . The density of the core, cloak and the background region are ρ_1 , ρ_2 and ρ_3 , respectively. Following the design approach for diffusion cloak, we expect the velocity potential flow in the background region to be undisturbed while the fluid flows through the cloak and we can obtain the density ρ_2 of the cloak as

$$\rho_2 = \frac{R_2^2 + R_1^2}{R_2^2 - R_1^2} \rho_1. \quad (4.4)$$

Hence by setting the density of the fluid in the shell region to satisfy Eq. (4.4), we can achieve cloaking performance for fluid similar to that shown in **Figure.4-1(d)**. However, different from diffusion or heat transfer or other diffusion-like phenomena that the required shell parameters (e.g. thermal conductivity for thermal cloak, or diffusivity for diffusion, electric resistance/conductance for DC electric field, permeability for DC magnetic field etc.) can directly be achieved by materials, the cloak shell for the diffusive fluid requires the local fluid density to be different from that of the background region. Here we propose an approach to emulate the different local density of the fluid by using the geometrical change of the third dimension. We will be introducing the details in the next session.

4.2 Experimental design and simulation results

Now we discuss the practical implementation of the fluid cloak in experiment. Previously, different homogenous isotropic materials are used to realize the parameters required for the cloak. For example, copper/air and alloy/Polystyrene are used to realize thermal cloaks, and superconductor/ferromagnetic material are used to realize the static magnetic cloak etc. Different from previous Laplace-equation-based cloak demonstrations, the cloak for fluid requires different local density of the fluid, which is physically difficult to realize. Here we propose to use geometry change in the third dimension to emulate different local fluid density in the two-dimensional (2D) case.

Figure.4-2 Schematic of the single-medium fluid cloak. The whole structure is rigid with zero diffusivity. The shell region is sunk with an extra depth to generate a different local fluid density.

The three-dimensional (3D) model of the fluid cloak is shown in **Figure.4-2**. The thickness of the cloak region is extended in z direction. When the fluid flows through this

region, more amount of the fluid is needed to fill up the space. Hence, in a 2D view, the local density of the fluid in the cloak region is no longer the same with the background region. Thus, the 2D local density of the fluid can be tuned by adjusting the extended height in the third dimension. One can readily get the approximate relation between the effective density and the height in the third dimension as $\rho_s = \rho_b h_s / h_b$. We choose the rigid obstacle radius as $R_1 = 8$ mm and the cloak shell outer radius as $R_2 = 14$ mm. According to Eq. (4.4), the local density of the shell is supposed to be $\rho_s = 1.97 \rho_b$, which means that the height of the shell should be 1.97 times that of the background region. Here we choose the background thickness as 5 mm. Thus, the height of the shell should be 9.85 mm. We further optimize the height of the shell by parameters sweeping method and set the shell to be 10 mm high.

The simulation results obtained by COMSOL Multiphysics can be seen in **Figure.4-3**. **Figure.4-3(a)** shows fluid flow and velocity field when the fluid flows in from the input port with a uniform normal velocity. When there is a rigid round obstacle with a radius of 8 mm, the streamlines of the fluid flow are dispelled by the obstacle, and the velocity fields are on two sides of the obstacle increase since the flow channel becomes thinner due to the presence of the obstacle. The velocity fields right at the back of the obstacle becomes smaller because of the block of the obstacle. When there is a shell with a radius of 14 mm and a height of 10 mm concentrically surrounding the obstacle, as shown in **Figure.4.3(c)** we can see that the streamlines outside the shell are not disturbed by the obstacle, and the velocity fields outside are also the same with the case when the fluid flows without the obstacle, which verify the performance of our proposed single-medium cloak. We shall introduce the experimental setup and results in the next session.

Figure.4-3 Simulation results of the single-medium fluid cloak. (a) Streamlines (blue solid lines) and velocity fields (blue arrows) when the fluid in from the input port with a uniform normal velocity. (b) Streamlines (blue solid lines) and velocity fields (blue arrows) when there is a rigid round obstacle in the center of the flowing path. The radius of the obstacle is 8 mm. (c) Streamlines (blue solid lines) and velocity fields (blue arrows) of the single-medium cloak case. The outer radius of the shell is 14 mm. The depth of the shell is 10 mm.

4.3 Experimental setup and results

In this chapter, we introduce the detailed experimental setup and report the experimental results. **Figure.4-4** shows the schematics of the experimental sample. We choose glycerin dyed with acrylic paints as the diffusive fluid. We use an electrically driven piston pump to slowly pump the glycerin fluid into the setup through a thick rubber tube. The fluid will first flow into the sink. When the height of the fluid in the sink exceeds that of the bottom of the flow path, the fluid will flow into the path with a near uniform input velocity. To clearly show the streamline of the fluid. We dye the input fluid with white acrylic paints and we input black acrylic paints through four evenly distributed thin rubber tubes. The black acrylic paints are input by hand with injection syringes.

Figure.4-4 Schematic of the experimental setup. The glycerine fluid dyed by white paints is pumped into the sink first through a wide rubber tube. The black paints are injected into the fluid by hands with injection syringes through the injection tubes.

We show the experimental results in **Figure.4-5**.

Figure.4-5 Experimental snapshots of reference, obstacle and cloak cases at different 0 second, 4 second and 10 second.

We can find that when there is bare rigid obstacle in the flowing path, the streamlines of the glycerin fluid are dispelled to two sides of the obstacle. However, for the cloak case, we can find that the streamlines are un-disturbed and almost the same with the reference case when there is nothing in the flowing path. These results meet well with the simulations as well as our theoretical prediction, which demonstrate for the first time a cloak for diffusive fluid.

4.4 Conclusion

In this chapter, we propose a new cloak design approach for diffusive fluid. Different from previous cloak demonstrations for heat, static electric/magnetic fields and diffusion that use at least two materials, we use geometry change of a single isotropic material in the third dimension to realize the first experimental demonstration of cloaking for diffusive fluid. The cloak design might find applications in solving fluid blocking in relevant industries. The designing method could also be extended to design cloaks and other novel devices in other physical fields that are governed or approximately governed by Laplace equation.

CHAPTER 5 SUMMERY AND OUTLOOK

In this thesis we study the properties and applications of extremely anisotropic media, novel cloak designs for electromagnetic waves and diffusive fluid.

Firstly, we study the properties of extremely anisotropic media. We compare the properties of single-ENI and double-ENI media. We numerically demonstrate their applications in reflectionless diffraction-free electromagnetic wave routing which includes bending, splitting, compressing/expanding in 2D scenario. We for the first time experimentally reflectionless bending and subwavelength image transmission in 3D scenario. The results further the study of singular media, and may find novel application in subwavelength electromagnetic wave manipulation and high-efficiency deep-subwavelength information interaction in photonic systems.

Secondly, we extend the photonic doping method to achieve a special metamaterial with permittivity approaching zero and infinity in two orthogonal directions and permeability approaching zero in the third direction. We realize the metamaterial by inserting large volume periodic metallic structure into ENZ media. The ENZ media are emulated by structural dispersion waveguide. With the special metamaterial, we propose an omnidirectional invisibility cloak design based on transformation optics. The omnidirectional invisibility cloak is demonstrated by both simulation and experiment at microwave frequencies. This work could further the study of extremely anisotropic media, and contribute a very important step to the realization of free-standing omnidirectional invisibility cloak.

Thirdly, we propose a new cloak design approach for diffusive fluid. we use geometry change of a single isotropic material in the third dimension to realize required different local density required. We demonstrate the cloak for diffusive fluid experimentally for the first

time. The cloak design might find applications in solving fluid blocking in relevant industries. The designing method that makes use of the geometrical change of the third dimension to achieve effective 2D parameters could also be extended to design novel devices in other physical fields.

At last, I suggest some potential ideas for future work.

Firstly, we only discuss media with permittivity approaching infinity in one or two directions, which can be applied to manipulate wave propagation with certain polarization and control the energy density of the electric field. If the permeability of one medium has one or two component approaching infinity as well, the medium can manipulate waves with any polarizations and the energy density of magnetic field can also be controlled. With such media, the propagation of electromagnetic waves can really be manipulated freely.

Secondly, the special media with permittivity approaching zero and infinity in two orthogonal directions and permeability approaching zero in the third direction, as introduced in **Chapter 3**, could have potential applications in reflectionless wave front reshaping, reflectionless lens designing as well as subwavelength imaging etc. thanks to its unique impedance matching and near-zero phase delay properties. Besides, the realization of such media with other approaches, and in other frequency regime can also be promising research topics.

Thirdly, the single-medium cloak approach introduced in **Chapter 4** can be extended to any other physical fields that are governed or approximately governed by Laplace equations such as heat flow, DC electric/magnetic field as well as mechanics etc. Moreover, for a single-medium cloak structure, it could possibly achieve cloaking effects simultaneously for multi-physics thanks to the unique single-medium advantage of the approach.

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