Enhanced performance of bistable energy harvesters using auxiliary magnet oscillators

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**ENHANCED PERFORMANCE OF BISTABLE
ENERGY HARVESTERS USING AUXILIARY
MAGNET OSCILLATORS**

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ENERGY RESEARCH INSTITUTE @ NTU (ERI@N)**

2018

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ENERGY HARVESTERS USING AUXILIARY
MAGNET OSCILLATORS**

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A thesis submitted to the Nanyang Technological
University in partial fulfilment of the requirement for
the degree of
Doctor of Philosophy

2018

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research and has not been submitted for a higher degree to any other University or Institution.

9 – JAN – 2018

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Abstract

Recently, an emerging energy harvesting technology has been considered as a potential alternative to chemical batteries for self-powered electronic devices. Energy harvesting is a process to capture ambient energy sources, convert them to electrical energy, and provide power for small wireless micro-devices. Such an energy harvesting technology is an important part of ‘deploy-and-forget’ wireless sensor networks, which must be essential for Internet-of-Things paradigms in Industry 4.0.

Among various types of energy harvesters, a vibration energy harvester (VEH) is of great interest to researchers owing to the ubiquity of vibration energy sources, high energy density, and the ease of application. Recent research on a VEH system has been aimed particularly at broadening its operating frequency band, thereby enabling it to operate properly under an actual environment of ambient vibration sources with time-varying and/or multiple frequency components. Especially, during the past decade, a bistable energy harvester (BEH) has been one of the most promising designs of broadband VEH systems. A potential well escape phenomenon that occurs in a bistable system is well known to lead to a large-amplitude oscillation of a BEH (thus, produce high output power) in a broad frequency band. However, this mechanism has its intrinsic limitation that a potential energy barrier possibly prevents the broadband operation of a BEH under relatively weak excitation. As a research effort to overcome this drawback, new types of BEH systems with auxiliary magnet oscillators are proposed in this thesis and their nonlinear dynamic behaviors and the associated performance are theoretically and experimentally investigated and discussed.

This thesis is composed of two research subjects concerning bistable energy harvesters with i) horizontal and ii) vertical auxiliary magnet oscillators, which

are called BEH-HO and BEH-VO, respectively. Each of these BEHs is composed of a piezoelectric bimorph cantilever beam and two permanent magnets. One magnet is attached to the free end of the piezoelectric beam, and the other is an external magnet acting as an auxiliary magnet oscillator.

For the BEH-HO, an external magnet is designed to be movable in the horizontal direction. A mechanical rectifier is additionally inserted, such that the excessive oscillation of the external magnet is prohibited. The rectified half-sine motion of the auxiliary magnet oscillator tends to reduce the saddle barrier in the restoring potential energy function of the BEH-HO, leading to an easier occurrence of potential well escape phenomenon. Consequently, the operating frequency band of the BEH-HO is broadened in comparison to the conventional BEH under the same excitation condition.

For BEH-VO, an external magnet is attached to the free end of another cantilever beam in order to allow its vertical motion. In this case, it is found that the complicated nonlinear behaviors of the BEH-VO can be described with a novel concept of phase-dependent potential energy. In particular, there are two distinct dynamic regimes which depend on the relative motion of two magnets; i) the out-of-phase regime observed at the first primary resonance and ii) the in-phase regime at the second primary resonance. Moreover, the secondary resonances of the BEH-VO as well as the primary resonances can also be important routes to high-energy orbit motions. These multiple nonlinear resonances are properly arranged in series on the frequency domain, such that their frequency bands are gradually connected with the increase of excitation intensity and merged into a very broad single operating frequency band. Such a bridging effect notably enhances the broadband performance of the BEH-VO, when compared with its conventional counterpart.

The mathematical models for the abovementioned BEH-HO and BEH-VO systems are first derived based on the Euler-Bernoulli beam theory, linear piezoelectricity, and magnetic charge model. A series of nonlinear analyses are performed to investigate the nonlinear characteristics of the BEH-HO and BEH-VO systems in terms of energy harvesting performance. All the theoretical results are confirmed by comparing them with experimental ones. The results show a promising potential for the practical application of the proposed BEH-HO and BEH-VO systems for broadband vibration energy harvesting.

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Abbreviations

BEH	Bistable Energy Harvester
BEH-BO	Bistable Energy Harvester with Combined Oscillators
BEH-HO	Bistable Energy Harvester with a Horizontal auxiliary magnet Oscillator
BEH-VE	Bistable Energy Harvester with a Vertical auxiliary magnet Oscillator
BPH-EM	Bistable Piezoelectric Energy Harvester and an Elastic Magnifier
CBEH	Conventional Bistable Energy Harvester
DOF	Degree Of Freedom
HEOM	High-Energy Orbit Motion
LEOM	Low-Energy Orbit Motion
BEH-VO	Magnetically Coupled Bistable Energy Harvester
MDOF	Multiple Degree Of Freedom
MEMS	Micro Electro-Mechanical System
PEH	Piezoelectric Energy Harvester
PVDF	Polyvinylidene Fluoride
RMS	Roots Mean Square
VEH	Vibration Energy Harvester

List of publications

Main findings of this thesis have been published in:

International Journal Articles:

1. *Phase-dependent dynamics potential of magnetically coupled two-degree-of-freedom bistable energy harvester*, Pilkee Kim, Minh Sang Nguyen, Ojin Kwon, Young-Jin Kim, and Yong-Jin Yoon. Scientific Report, Nature Publishing Group, 2016 Sep 28; 6: 34411; Epub 2016 Dec 28. (Published)
2. *Lowering the potential barrier of a bistable energy harvester with the mechanically rectified motion of an auxiliary magnet oscillator*, Minh Sang Nguyen, Yong-Jin Yoon, Ojin Kwon and Pilkee Kim. Applied Physics Letter, 111, 253905 (2017). (Published)
3. *Enhanced broadband performance of a magnetically coupled 2-DOF bistable energy harvester with secondary intrawell resonances*, Minh Sang Nguyen, Yong-Jin Yoon and Pilkee Kim. International Journal of Precision Engineering and Manufacturing – Green Technology, 2017 August. (Accepted)
4. *Piezoelectric cylindrical design for harvesting energy in multi-directional vibration sources*, Minh Sang Nguyen, Ng Sum Huan Gary, Pilkee Kim and Yong-Jin Yoon. IOP Conference Series: Earth and Environmental Science, 2017 July. (Published)

Abstracts:

1. *Enhanced performance of a bistable energy harvester using a horizontal magnet oscillator*, Minh Sang Nguyen, Pilkee Kim and Yong-Jin Yoon. 5th Asia-Korea Conference Science & Technology, 16-18 November 2017, Singapore.

Chapter 1

Introduction

This chapter briefly introduces the motivation of the research on vibration energy harvester (VEH). In particular, this research addresses possible solutions to the limitation of a conventional bistable energy harvester (BEH). The focus of the research is on the challenges of designing broadband BEHs to keep its normal broadband operation under relatively weak excitations. In this chapter, the objectives and scope of the research are also provided, and lastly, the overview of the dissertation is mentioned.

1.1 Problem statement

Over the past decade, a vibration energy harvesting technology has been considered as one of the promising alternatives to conventional batteries in the low-powered electronic applications. In the early stage of development, most of the vibration energy harvesters (VEHs) were designed based on linear resonators; thus, their energy harvesting performances were guaranteed only in a narrow frequency band in which resonance occurs. However, real ambient vibrations in many applications were likely to be distributed over a wide frequency band with low intensity or occasionally have a time-varying frequency [1]. For these reasons, the linear VEHs were considered inappropriate for practical applications and, accordingly, many research works have been performed to overcome the drawback of the linear VEHs. Recently, a bistable energy harvester (BEH) has been proposed to enhance the frequency bandwidth of the VEH and its

outstanding broadband performance has drawn a significant attention from many researchers.

This study treats a bistable magnetic repulsion energy harvester based on piezoelectric energy conversion. This type of the BEH is typically composed of a piezoelectric bimorph cantilever beam and two permanent magnets. One magnet is attached to the tip of the beam and the other is located in external space. The post-buckled bimorph beam (i.e., statically bistable beam) can be formed by sufficiently imposing an axial force on the beam using the magnetic repulsion effect between two magnets.

Recently, many researchers have proposed various types of BEHs and showed that the BEHs could efficiently harvest vibration energy in a broader frequency range than linear VEHs. Actually, such broadband performance of BEHs attributes to their high-energy orbit motions which are caused by potential well escape phenomenon. In this regard, there has been an important design issue of BEHs –how to maintain high-energy orbit motions with regardless of the level of excitation intensity. In fact, the occurrence of high-energy orbit motions requires more than a certain level of external excitation, which is strong enough to overcome a potential energy barrier in the double-well potential of a BEH. Particularly when excitation strength is weak, most of BEHs can hardly initiate high-energy orbit motions and, accordingly, do not operate normally in a broad frequency band.

As a research effort to overcome such an intrinsic shortcoming of conventional BEHs, new types of BEHs using auxiliary magnet oscillators are proposed in this study, and their dynamic and energetic characteristics are investigated both theoretically and experimentally.

1.2 Objectives and Scope

The primary objective of the present study is to improve the energy harvesting performance of the BEH to retain its normal operation even under weak external excitations. To this end, this study uses a variety of external magnet oscillators instead of a fastened external magnet.

The conventional BEH (CBEH) uses an external magnet fixed on external space, and its potential energy function is determined by the location of the external magnet. On the other hand, the potential energy function of the BEH with the external magnet oscillator becomes more complicated and, specifically, it depends on the dynamic characteristics of the auxiliary oscillator such as the natural frequency and damping ratio. Accordingly, the external magnet oscillator can possibly be designed and optimized to find a better configuration of the potential energy function of the BEH (e.g., shallower and/or wider potential wells). Moreover, the BEH with an auxiliary oscillator has two degrees of freedom and thus it has multiple (linear and nonlinear) resonant frequency bands more than a single degree-of-freedom CBEH, which means additional operating frequency bands of the BEH. From this perspective, to enhance the performance of the BEH, this study considers the use of two different types of auxiliary oscillators, respectively, movable in the horizontal and vertical directions.

The main objectives and scope of the present study on the BEHs with the external magnet oscillators are to:

- Build the reasonable mathematical models of the proposed BEHs with auxiliary oscillators which describe their potential energy functions, nonlinear resonant behaviors and energy harvesting performances;
- Investigate the nonlinear characteristics of the systems such as

variations in dynamic potential energy, the resulting bifurcation behaviors, harmonic and subharmonic resonances, in order to disclose the underlying principles or mechanisms in the proposed systems;

- Develop design criteria or guidelines for the auxiliary oscillators and try to draw their optimal parameters to make the BEHs operate properly under relatively weak ambient vibrations;
- Verify, experimentally, the functionalities of the auxiliary oscillators in the proposed systems and their beneficial effects on broadband vibration energy harvesting.

1.3 Dissertation Overview

The thesis addresses the research on bistable energy harvesters using horizontal and vertical magnet oscillators. This thesis consists of six chapters as described below:

Chapter 1 provides a rationale for the research on broadband vibration energy harvester and outlines the goals and scope of the thesis.

Chapter 2 reviews the literature concerning the various types of energy harvesting technologies and provides a detailed history of the development of a vibration energy harvester, ranging from linear to non-linear designs. It also reports and evaluates the state-of-the-art of magnetic repulsion bistable energy harvesters along with the designs of multi-degree-of-freedom (MDOF) systems, and discusses the challenges faced in designing bistable energy harvesters.

Chapter 3 presents a novel bistable energy harvester with a horizontal auxiliary magnet oscillator (BEH-HO). The horizontal magnet oscillator with the mechanical rectifier is designed in a manner where its rectified half-sine motion tends to reduce the saddle barrier of restoring potential. This leads to an easier occurrence of the potential well escape phenomenon, and the resulting high-energy orbit motion can be initiated easily under relatively weak excitations. Such a beneficial aspect of the BEH-HO system is theoretically and experimentally verified by bifurcation analyses and frequency response analyses.

Chapter 4 concerns the theoretical and experimental studies of the bistable energy harvester with a vertical auxiliary magnet oscillator (BEH-VO), the nonlinear dynamic behaviors of which is very complicated and has not been understood completely yet. Particularly, this chapter introduces the novel concept of phase-dependent dynamic potential to explain the underlying principle of the complicated dynamics of the BEH-VO. The concept is based on two distinct dynamic regimes of out-of-phase and in-phase resonant modes. This approach to the dynamics of BEH-VO can be extended and applied to multi-degree-of-freedom (MDOF) systems, which sheds light on compact and efficient designs of magnetically coupled BEH chain structures.

Chapter 5, following the findings in *Chapter 4*, investigates possible routes to high-energy orbit motion (HEOM) in the BEH-VO: primary and secondary intrawell resonances. Generally, high electrical power generated by the BEH originates from HEOMs. Therefore, multiple routes to HEOMs can provide more chances for the BEH to harvest vibration energy. In this chapter, it is demonstrated that four intrawell resonant routes to HEOMs in the BEH-VO. Furthermore, with the increase of excitation intensity, the secondary intrawell resonances tend to bridge the gap between the frequency bands of the primary resonances, remarkably enhancing the operating frequency bandwidth. The enhanced broadband performance of the BEH-VO is theoretically and

experimentally evaluated and discussed in comparison with the CBEH.

Chapter 6 summarizes the works done in this thesis, provides the scientific contribution and novelty, and discusses the future works on BEH-HO and BEH-VO.

1.4 Findings and Outcomes/Originality

This research led to several novel outcomes by:

1. Proposing a novel design of a BEH with a horizontal auxiliary magnet oscillator (BEH-HO), of which the motion is mechanically rectified, particularly for lowering potential saddle barrier. For the proposed BEH-HO system, it is demonstrated theoretically and experimentally that the rectified half-sine motion of the auxiliary oscillator engages in broadening the operating frequency band of the BEH but does not distort its high-energy orbit motion, i.e., it does not deteriorate its energy harvesting efficiency.

The beneficial aspect of the auxiliary magnet oscillator in the BEH-HO system should be an important design consideration for a multimodal or multidirectional vibration energy harvester.

2. Introducing the novel concept of phase-dependent dynamic potential in a BEH with a vertical auxiliary magnet oscillator (BEH-VO). Particularly, through a series of nonlinear analyses, it is found that the BEH-VO possesses two distinct dynamic regimes: the out-of-phase mode in the first primary resonance and the in-phase mode in the second primary resonance. Under the out-of-phase mode regime, the frequency response of the BEH-VO exhibited the classical double-well dynamics of bistable systems. Under the in-phase mode regime, however, both intrawell and interwell motions belonged only to the single-well dynamics of monostable systems, even though the BEH-VO must be statically bistable.

This research, for the first time, presents the concept of the phase-dependent dynamic potential in the BEH-VO, providing a clear physical insight into its nonlinear resonant behavior – including the bifurcation structures and the associated potential well escape phenomena. This concept is not limited to the present BEH-VO, but can be extended and applied to all magnetically coupled multi-degree-of-freedom (MDOF) BEHs, of which the potential energies are functions of more than two displacement variables.

3. Demonstrating theoretically and experimentally that the secondary intrawell resonances (e.g., the one-half subharmonic resonance and the second superharmonic resonance), in addition to the first and second primary resonances, played important roles in generating the HEOM in the BEH-VO. Each of these four multiple routes of the BEH-VO to the HEOM forms an individual operating frequency band in the different frequency range. With the increase of the excitation intensity, the frequency bands of the HEOM, caused by the secondary intrawell resonances, tended to bridge the gap between frequency bands for the 1st and 2nd primary resonances, making a single combined operating frequency band.

Although only the BEH-VO (which is one of the simplest multi-DOF BEHs) was investigated in this study, the bridging effect of secondary intrawell resonances can possibly be applied to a variety of multi-DOF vibration energy harvesting systems in order to further enhance their broadband performance and/or in order to respond to more complicated ambient vibration sources. Hence, secondary intrawell resonances should be considered as another important factor in designing broadband multi-DOF BEHs.

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Chapter 2

Literature Review

This chapter provides a state-of-the-art review of the research of vibration energy harvester (VEH). Initially, energy harvesting technology is mentioned as an emerging alternative to the conventional chemical battery. Next, the various types of potential energy sources for energy harvesting technology are discussed and compared. The VEH is accordingly introduced as one of the most potential candidates to replace the battery in the small electronic applications. Afterward, the various stages of development in VEH – ranging from the linear resonant based designs to nonlinear designs – are listed in sequence. Along with that, the fundamental designs of bistable energy harvester (BEH) – from the simple Duffing model to multi-degree-of-freedom (MDOF) BEH – are explicitly reported. In addition, the remaining challenges on the development of BEH are discussed. Finally, the research gaps in the knowledge, that the thesis is addressing, are pointed out.

2.1 Overview of energy harvesting techniques

The rapid advance in MEMS (micro-electromechanical system) technology has led to the remarkable progress of micro-scale electronic devices which require low electric power. Consequently, a variety of wireless and portable autonomous devices has been developed and used for remote data collection, context recognition, et cetera in widespread applications such as the structural health monitoring, health-care, industrial automation, and civil and military applications [1-5]. The main advantages of the autonomous micro-devices over the conventional cable devices are their compactness in size, lightness in weight, flexibility in deployment, etc. [6]. Recently, the concept of smart environments has further encouraged the development of multifunctional autonomous micro-devices designed to use low electric power [7].

Conventionally, chemical batteries are used as a common power source in commercial autonomous devices. These days, however, it seems no longer suitable for evolving autonomous micro-devices. Most of the conventional batteries are hardly scalable and their bulky size makes it difficult to miniaturize wireless autonomous devices. For example, a typical wireless sensor node is largely composed of a micro-controller unit, a radio frequency transceiver and a battery (as shown in Fig. 2.1). All the parts, except for the battery, in this sensor node can be readily scaled down based on the advanced circuit integration technologies. Reducing the weight and size of the battery, however, remains one of the challenging issues these days. Furthermore, it was expected in a report [8] that the chemical batteries would not satisfy the power requirement of micro-scale devices, especially in term of energy density, in the immediate future. Fig. 2.2 demonstrates the slow improvement of the battery energy density in contrast with the rapid high-performance electronics development which is related to ubiquitous computing.

Figure 2.1 Architecture of a typical sensor node.

Figure 2.2 Mobile computing improvement [7].

The disadvantages of chemical batteries also include finite power storage, limited life span, hazardous chemicals, etc. The chemical battery needs to be recharged or replaced periodically due to its finite power storage. This battery maintenance would be more time-consuming and costly. Particularly, in the case when the electronic device is installed in the remote area or in the places with extreme conditions. The permanent discharge of the battery (due to its limited life span) and accompanying disposal process (due to hazardous chemicals) can also incur

periodic maintenances or repairs.

In view of these challenges, energy harvesting technologies attracted a great interest from industry and academia as a promising alternative to chemical batteries. The energy harvesting is a series of processes to extract energy from environmental energy sources and convert it into usable electric energy which is used to power the devices or stored in a capacitor. The concept of energy harvesting is not entirely new. In fact, windmills, hydroelectric generators, solar panels, etc., are a type of macro-scale energy harvesting systems. Recent research interests, however, have been focusing on the micro-scale energy harvesting technologies to power low-power electronic devices.

2.2 Vibration energy harvesting

Various ambient energy sources such as sunlight, wind, earth heat and mechanical vibration have been exploited as auxiliary energy resources together with fossil fuels for the last several decades. The energy sources existing in nature (including sunlight, wind and earth heat) are generally affected by environmental conditions, and the energy generation efficiency is more likely to alter significantly. Table 2.1 shows the comparison of energy density among a few energy sources with respect to environmental conditions. It should be noted that the outdoor solar energy source apparently has the highest power density, but the power density of indoor solar energy source is significantly lower than that of the other indoor energy sources. Solar energy source sensitive to environmental conditions may not be suitable for embedded applications without sufficient amount of sunlight [9].

Unlike the other ambient energy sources, the mechanical vibration tends to be more durable regardless of environmental conditions (e.g., light intensity, temperature and humidity). It can also provide enough electrical power to operate

low-power electronic devices. Many researchers have reported that the power requirements of recent micro-scale devices dropped to sub-milli-watt levels. For example, the electronic microchips for health monitoring had an average power consumption of approximately $50 \mu\text{W}$ [10, 11] or the wireless transponders could be efficiently operated with electrical power less than 1 mW [12, 13]. The mechanical vibration energy source can sufficiently cover these small-power requirements. Also, it may be a sustainable power source without undesirable aging effect occurring in every chemical battery (shown in Fig. 2.3).

Another advantage of the mechanical vibration energy source is that it can be easily found in our real-life environment. As listed in Table 2.2, a variety of vibration energy sources possibly exist everywhere; such as building structures, traffic constructions, industrial machines, etc. Most of these ambient vibrations generally have an oscillation frequency ranging from a few Hz to a few hundreds of Hz and their acceleration amplitude is at most a dozen of m/s^2 .

Table 2.1 Various energy sources and their power densities with respect to environmental conditions [8].

Type of energy sources	Characteristics	Power density
Solar	Outdoor	$100 \text{ mW} / \text{cm}^2$
	Indoor	$100 \mu\text{W} / \text{cm}^2$
Thermal	Human	$60 \mu\text{W} / \text{cm}^2$
	Industrial	$\sim 1\text{-}10 \text{ mW} / \text{cm}^2$
Vibration	Human	$\sim 4 \mu\text{W} / \text{cm}^3$
	Machines	$\sim 800 \mu\text{W} / \text{cm}^3$

Figure 2.3 Comparison of the output power density and the lifetime among battery, solar and vibration energy sources [10].

Table 2.2 Ambient vibration sources [10].

Vibration sources	Acceleration (m/s ²)	Peak frequency (Hz)
Car engine compartment	12	200
Base of 3-axis machine tool	10	70
Blender casing	6.4	121
Clothes dryer	3.5	121
Person nervously tapping their heel	3	1
Car instrument panel	3	13
Door frame just after door closes	3	125
Small microwave oven	2.5	121
HVAC vents in office building	0.2–1.5	60
Windows next to a busy road	0.7	100
CD on notebook computer	0.6	75
Second story floor of busy office	0.2	100

As mentioned above, the mechanical vibration energy source is considered as one of the most suitable micro-power sources for low-power micro-electronics because of its abundance, versatility and power density [14, 15]. To scavenge this

energy, various types of vibration energy harvester (VEH) have been proposed and developed. A VEH is an electromechanical device which can easily extract energy from mechanically oscillating subjects using several energy conversion mechanisms. For example, electrostatic, electromagnetic and piezoelectric energy conversions as shown in Fig. 2.4.

Figure 2.4 Three types of electromechanical transducers: (a) electrostatic, (b) electromagnetic, and (c) piezoelectric transducer [10].

The electrostatic energy conversion mechanism is based on the principle that the relative movement between two plates of capacitors generates a voltage across the output circuit. Many authors have proposed the designs and investigated the power generations of the electrostatic VEH at various scales.

Meninger et al. [17] proposed a variable capacitor to convert ambient vibration into electrical energy, and the resulting power density was about $3.8 \mu\text{W}/\text{cm}^3$. Roundy et al. [18] also designed an electrostatic VEH which has the power density of about $116 \mu\text{W}/\text{cm}^3$ when the input vibration was 2.25 m/s^2 in amplitude and 120 Hz in frequency. Tashiro et al. [19] proposed a honey-comb-type variable capacitor generator to harness energy from the motion of the living body. The design was capable to generate electric power up to $58 \mu\text{W}$ while supplying a load of $1 \text{ M}\Omega$. Although the electrostatic VEH is considered an easy integration into embedded electronic systems, its design usually requires an

external power source which decreases the energy efficiency.

The electromagnetic energy conversion mechanism has also been studied for many decades, but its energy density tends to be relatively lower than the other energy conversion mechanisms [16]. The principle of this electromagnetic design is based on the Faraday's law: when an oscillating coil cuts through a magnetic flux field formed by permanent magnets, a voltage will be induced into the coil.

To demonstrate this idea, Shearwood et al. [20] developed an electromagnetic micro-generator with the power density of about $2.4 \mu\text{W} / \text{cm}^3$. Glynne-Jones et al. [21] presented two prototypes of electromagnetic energy harvester using two and four permanent magnets. These magnets are arranged to couple with a moving coil. The electricity will be generated when the moving coil cuts across the magnetic flux formed by the permanent magnets. When tested on a car engine, the device was able to produce a power up to $157 \mu\text{W}$. For micro-scale applications, Liu et al. [22] introduced a MEMS electromagnetic energy harvester. This device consists of a circular suspension structure on an energy harvesting chip. After testing the device at three different vibration modes, the maximum power densities obtained are 0.157, 0.014 and $0.117 \mu\text{W}/\text{cm}^3$. Electromagnetic VEH demonstrated a great benefit with its relatively high energy output generation. Thus, further investigation on it is needed in the immediate future.

Recently, the piezoelectric energy conversion mechanism has also been proposed as a potential candidate, along with electrostatic and electromagnetic mechanism. The main idea of piezoelectric energy harvester is based on the properties of piezoelectric materials. This type of materials performs a direct piezoelectric effect, which is the ability to generate electric charges when the material is under mechanical stress or strain.

Most of the recent designs of piezoelectric energy harvester adopted the idea of a cantilever beam based structure due to its large amplitude oscillation. The piezoelectric patches are usually attached to the surface of the cantilever beam to generate the voltage in response to the beam's deflection. In fact, the piezoelectric energy conversion mechanism has been reported to be the most common choice because of its high- power density in practical applications [23-25]. Furthermore, this piezoelectric energy conversion is a type of reciprocal conversion which directly transduces structural strain energy to electrical energy without any external input. Therefore, the piezoelectric energy harvester (PEH) is likely to be designed as a simpler structure, thus facilitating its application to MEMS devices. Literature review for piezoelectric energy harvester will be provided in the subsequent subsections for more details. The merits and demerits of three energy conversion mechanisms are summarized in detail in Table 2.3.

Table 2.3 Comparisons of three types of vibration energy harvesters [16-25].

Types	Electrostatic	Electromagnetic	Piezoelectric
Advantages	Easily integrated into micro-systems small size; precise control of the mechanical resonance	No voltage source required; relatively high power output	No voltage source required; higher output voltage; small size; simple structure, high efficiency; precise mechanical control
Disadvantages	Require separate voltage source; need higher frequency than piezoelectric	Complex design; difficult to integrated into micro-system	Difficult to integrated into micro-system; need high frequency and stress; limited material selection

2.3 Linear resonant-type vibration energy harvesters toward broadband energy harvesting

Early designs of the VEHs were based on the principle of linear resonators which generated high electric power in the frequency bandwidth of resonance. The maximum power generation of this linear VEH can only be achieved if its natural frequency is synchronized with the excitation frequency. However, in spite of

high-energy generation, the linear VEH hardly responded to ambient vibrations with a wide spectrum of frequency components (refer to Table 2.2) [26], due to its very narrow bandwidth. As a result, the energy harvesting performances of the linear VEHs were limited to the resonance frequency bands. To circumvent this problem of the linear VEH, many alternative strategies have been proposed such as tunable energy harvesters, multimodal energy harvester and nonlinear energy harvesters.

The basic concept of tunable energy harvesters is to alter the natural frequency of the linear VEHs, actively or passively, to match the oscillation frequency of an ambient vibration. The tuning mechanisms can be categorized into three groups based on the mechanical, magnetic, and piezoelectric method.

As a mechanical tuning method, Leland and Wright [27] proposed an axial preload to modulate the stiffness (thus, the natural frequency) in their VEHs. Eichhorn et al. [28] presented a frequency-tunable cantilever energy harvester with additional wing-arm structures, applying pre-stress to the free ends (as shown in Fig. 2.5a). Instead of these mechanical tuning approaches, Challa et al. [29] used the combined magnetic interaction of magnetic repulsion and attraction effects caused by external magnets (placed on space) and tip magnets (attached to the free edge of a tunable cantilever VEH). Fig. 2.5b shows the schematic diagram of the magnetically tunable VEH. By adjusting the distance between the external and tip magnets, the natural frequency of this energy harvester was tunable over the range of 22-32 Hz with a power output of 240-280 μW when the base acceleration was fixed at 0.8 m/s^2 .

Fig 2.5c shows the schematic of a piezoelectric bimorph cantilever energy harvester in which the upper piezoelectric layer was connected to various capacitive loads for the frequency-tuning purpose [30]. The tunable bandwidth was 3 Hz: from 91.5 Hz to 94.5 Hz. Although the improved performance was

observed, this PEH had critical drawbacks in that it needed an external power supply and caused the system's structure to become more complicated. Furthermore, such tuning approaches could only respond to a small change of excitation frequency against a wide frequency range of ambient vibrations in actual environments [31].

Figure 2.5 Tunable energy harvesters using (a) mechanical method [28] (b) magnetic method [29] (c) piezoelectric method [30].

Another strategy to overcome the limitations of the linear VEHs is related to multimodal structures, e.g., a continuous beam with multiple bending modes or an array of different resonators. Roundy et al. [32] first proposed the multi-degree-of-freedom (MDOF) energy harvesting system that was basically a clamped-clamped beam with multiple proof masses loaded along the direction of its length (shown in Fig. 2.6a). This multi-DOF structure has been designed so

that several different natural modes could be well distributed in a wide but confined frequency range. The enhanced frequency bandwidth of the VEH could thus be achieved.

Unlike the multimodal single-beam structure above, Shahruz [33] developed an array of multiple cantilevers with different lengths and with tip masses respectively, attached to a common base (Fig. 2.6b). This design enabled to form a set of continuously increasing natural frequencies. Yang and Yang [34] designed a coupled bimorph energy harvester – of which the resonant frequencies could be tuned very close to each other. The aforementioned multi-modal VEHs required no external power supply and were more beneficial to simplify the structure when compared with the frequency-tunable VEHs. However, the multimodal VEHs remain several limitations in the non-uniform power generation in the frequency bandwidth, the low power density caused by large-volume design and the complicated interface circuit [35].

Figure 2.6 Multimodal energy harvesters using (a) multiple vibration modes [32] [(b) array of cantilever beams [33] (c) coupling of cantilever beams [34].

2.4 Broadband nonlinear vibration energy harvesters

One of the recent approaches to broaden the bandwidth of the PEH is to use the nonlinear properties of the system. The nonlinearity due to hardening- or softening-stiffness is well-known to extend the resonance bandwidth of the system and can be used to enhance the performance of VEHs under a wide range of excitation frequency [36-39]. Many research works have investigated the

inherent nonlinearities in geometry and materials of the VEHs, such as the nonlinear strain-deflection relationship due to large deformation [40] and the nonlinear constitutive relationships of piezoelectricity [41]. The effect of these inherent nonlinearities, however, were not easily controlled and were also proven to be less useful in broadening the bandwidth of the VEH. More recently, intentionally imposing a magnetic or mechanical load on the VEH structure has been preferred in order to strengthen the nonlinear effects as much as possible [42-45].

The most promising one among existing nonlinear VEHs must be a bistable energy harvester (BEH) using a potential well escape phenomenon for broadband energy harvesting. The remaining parts of this section will address the fundamental mechanisms of the BEH, various types of the BEHs reported in the literature and the associated challenging issues.

2.4.1 Fundamentals of bistable energy harvesters: Duffing oscillator

The simplest electromechanical oscillator model of the BEH systems can be expressed by

$$\begin{aligned}\ddot{x} &= -\frac{dU(x)}{dx} - \gamma\dot{x} + \kappa v - f(t) \\ C_p \dot{v} + \frac{v}{R} &= \kappa \dot{x}\end{aligned}\tag{2.1}$$

where x is the displacement of the oscillator; the over-dot denotes the differentiation with respect to time t ; $U(x)$ is the potential function; γ is the viscous damping constant; κ is the electromechanical coupling coefficient; v is the output voltage across the resistive load R ; C_p is the capacitance; and $f(t)$ is the inhomogeneous forcing term.

Consider a Duffing-type oscillator which is a good simplification to qualitatively investigate the dynamics of the bistable system. For the Duffing-type oscillator,

the potential energy function is given as a quadratic function of x , written by [46, 47],

$$U(x) = -\frac{1}{2}ax^2 + \frac{1}{4}bx^4 \quad (2.2)$$

where a and b are the coefficients characterizing the potential energy function. Accordingly, the nonlinear spring force of the Duffing-type oscillator is given as

$$F(x) = -ax + bx^3 \quad (2.3)$$

Substituting Eq. (2.3) or (2.2) into Eq. (2.1), the forced Duffing-type oscillator model can be written as [48-50],

$$\begin{aligned} \ddot{x} + \gamma\dot{x} - \kappa v - ax + bx^3 &= f(t) \\ C_p \dot{v} + \frac{v}{R} &= \kappa\dot{x} \end{aligned} \quad (2.4)$$

The system is given in Eq. (2.4) significantly depends on the parameters a and b . Three different realms of the system are listed below according to the parameter condition:

- I. For $a \neq 0$ and $b = 0$, the linear system,
- II. For $ab \leq 0$ and $b \neq 0$, the nonlinear monostable system with hardening stiffness ($b > 0$) or softening stiffness ($b < 0$),
- III. For $ab > 0$, the nonlinear bistable system with double-well potential shown in Fig. 2.7.

Figure 2.7 Different potential functions of Duffing oscillator.

The system model, given in Eq. (2.4), becomes a simple linear oscillator when $b = 0$; whereas, it becomes nonlinear when $b \neq 0$, and its dynamic characteristics can vary with the sign of parameters a and b . For $ab \leq 0$, the system always only has a single potential well monostable. For $ab > 0$, the system becomes bistable. The potential function of the system has two potential wells separated by a potential barrier as shown in Fig. 2.7.

Two stable equilibriums, which correspond to two minima of the potential function, $\pm\sqrt{a/b}$, define the separation between two potential wells. It must be noted that when b increases, the distance between stable equilibriums decreases, and the height of the potential barrier, $h_b = a^2/4b$, also decreases. This facilitates the motion from one potential well to the other, commonly known as inter-well motion. This type of motion will be discussed in the next parts.

The dynamics and the establishment of the Duffing-type bistable system can be explained using the pitchfork bifurcation diagram as seen in Fig. 2.8. The

potential function of the system can be re-expressed as $U(x) = -\frac{1}{2}rx^2 + \frac{1}{4}x^4$, where the equilibrium points of the system can be changed with the varying of the parameter r . This change is called a bifurcation.

As shown in the bifurcation diagram, the monostable Duffing-type is gradually changed into the bistable Duffing-type oscillator. While a monostable oscillator consists of one stable fixed point, a bistable oscillator consists of two stable fixed points and one unstable fixed point. It should be noted that the change in the parameters is mainly dependent on the design of the system. Hence, the design will determine the potential function shape as well as the configuration of the nonlinearity.

Figure 2.8 Supercritical pitchfork bifurcation diagram [51]

A bistable oscillator typically has a double-well restoring force potential as shown in Fig. 2.9. This configuration of potential energy function leads to three distinct regimes in the dynamics of the system (composed of intra-well, chaotic and inter-well motions) depending on the intensity of external excitation. When the excitation intensity is weak, the bistable oscillator tends to oscillate with small

amplitude within one of two potential wells. This small oscillation is called an intra-well motion or a low-energy orbit motion (denoted by the dotted lines in Fig. 2.9). As the excitation strength increases, the system possibly gains enough energy to overcome the potential barrier between two potential wells (i.e., a potential well escape phenomenon). The bistable oscillator then starts to oscillate with large amplitude across two potential wells, which is called an inter-well motion or a high-energy orbit motion (denoted by the continuous lines in Fig. 2.9).

Figure 2.9 Double-well restoring force potential of a bistable oscillator (a) intra-well motion (b) chaotic motion (c) inter-well motion.

Additionally, there is an intermediate state between the intra-well and inter-well motions called a chaotic motion (denoted by the dashed lines in Fig. 2.9). The chaotic motion is a mix of the intra-well and inter-well motions occurring when the excitation intensity is around a threshold value. Among these three motions, the inter-well motion is well-known to be desirable to realize the broadband energy harvesting performance of the BEH [35-52].

2.4.2 Post-buckled beam type bistable energy harvester

The post-buckled beam (or plate), one of the typical bistable structures, has been used as a basic structure of the BEH. As shown in Fig. 2.10, the post-buckled beam can be simply formed by forcing an axial load into a clamped-clamped beam. The post-buckled beam has two different stable equilibria. The beam snaps from one stable state to the other state under some perturbations; called the post-buckled snap-through. This snap-through mechanism was first employed to micro-systems by Casals-Terré et al. [53], but it was not for the purpose of energy harvesting. Baker et al. [54] proposed first a BEH system based on the post-buckled snap-through and experimentally showed the broadened bandwidth of the BEH. Since their first report, many research works have been performed on various post-buckled beam energy harvesters.

Figure 2.10 Typical post-buckled beam configuration [54].

Arrieta et al. [55] proposed a BEH using post-buckled composite plates. This BEH was considered as the one suitable for the MEMS application since it did not include any magnetic components and had two-dimensional geometry. Sneller et al. [56] took advantage of the snap-through mechanism of a post-buckled beam with a central proof mass, to develop a BEH, by adding piezoelectric patches on the top of the beam. For this BEH, the addition of the small mass into the system significantly broadened the frequency bandwidth. Cotton et al. [57] also

developed a magnetic-free BEH based on a buckled structure and examined the difference in energy harvesting performance between pre- and post-buckled states under exponentially correlated noise. In this study, the output voltage for the post-buckled case was enhanced up to an order of magnitude when compared with the pre-buckled one. Friswell et al. [58] proposed an inverted elastic beam-mass system as a basic structure of the BEH. The results demonstrated some advantages of the BEH under the low-frequency range of ambient vibrations.

The aforementioned post-buckled structure is the most fundamental one to induce the bi-stability of energy harvesting systems. In general, this type of the BEH can scavenge high electrical power if it is well designed. However, most of the mechanically post-buckled systems seem to be inefficient because it is difficult to trigger the inter-well motion.

2.4.3 Magnetic attraction bistable energy harvester

The magnetic attraction was first used to induce the bi-stability in a ferromagnetic beam structure proposed by Moon and Homes [59]. However, the first application of Moon's beam to energy harvesting systems was found in the recent literature reported by Erturk et al. [60]. In their study, a piezo-magneto-elastic structure (shown in Fig. 2.11) was composed of a ferromagnetic cantilevered beam with two patches of piezoelectric layers and two external permanent magnets. The experimental results showed that, under a harmonic base excitation, the magnetic attraction BEH resulted in a 200% increase in the output voltage in comparison with the conventional VEH. Ferrari et al. [61] proposed a magnetic attraction BEH using a single external magnet that was more appropriate for MEMS applications. Experimental results demonstrated an improvement, up to about 400 %, of the RMS output voltage compared to the linear VEH.

Figure 2.11 Schematic of the piezo-magneto-elastic power generator [59].

2.4.4 Magnetic repulsion bistable energy harvester

In recent researches on the BEHs, the magnetic repulsion effect has been intensively used to induce the bi-stability in the energy harvesting systems. The magnetic repulsion BEH (shown in Fig. 2.12) was first proposed in Ref [45] and it has been investigated by many researchers [46, 62].

The basic structure of the magnetic repulsion BEH comprises a piezoelectric cantilever beam, with a permanent magnet on its tip, and a fixed external magnet. As shown in Fig. 2.12, the two permanent magnets face each other with the same polarity, generating a magnetic repulsion effect. The magnetic forces, axially exerting on the cantilever beam, result in bending moment on the beam element. The beam eventually buckles and become bistable.

Figure 2.12 Bistable piezoelectric energy harvester using external magnet [45].

Zhou et al. [63] developed a BEH with rotatable magnets, which was able to match its resonant frequency band with the frequencies of ambient vibrations by regulating the angular orientation of the magnets. Tang and Yang [64] introduced a novel nonlinear BEH using a magnetic oscillator to realize a wider bandwidth than conventional BEH with a fixed magnet. Fan et al. [65] proposed a magnet-induced dual-cantilever beam PEH which generated power in a wide frequency range. Su et al. [66] presented a broadband PEH using magnetic coupling between two movable magnets. Gao et al. [67] investigated a bistable PEH with elastic support and external magnets. The results showed that the elastic support systems with an attached magnet led to a better power output performance than the rigid counterpart, especially when excited at low-intensity vibrations.

While magnetic interaction has been exploited to broaden the bandwidth of the energy harvester, many researchers have also utilized the magnetic repulsion to enhance the harvesting capacity of VEHs. Kang-Qi et al. [68] developed a bi-directional PEH using two piezoelectric cantilever beams magnetically coupled in two orthogonal directions. The results showed that the design not only improved the performance of PEH under random excitation but also helped to prevent the piezoelectric materials from the fatigue failure. Su and Zu [69] developed a tri-directional PEH that included the main cantilever with

piezoelectric patches, an auxiliary elastic beam, and a spring-mass system. This PEH system has a broadband energy harvesting performance in three orthogonal directions. By incorporating the magnetic nonlinearity into a linear 2-DOF system, Wu et al. [70] further broadened the operating bandwidth of their PEHs.

As mentioned above, various designs of PEHs using magnetic repulsion effects have been developed and most of them represented a significant enhancement in frequency bandwidth and output power, even in different harvesting directions.

2.5 Multi-degree-of-freedom (MDOF) magnetic repulsion vibration energy harvesters

Recently, the multi-degree-of-freedom (MDOF) structures have been integrated into vibration energy harvester for two main purposes: broadening the effective frequency bandwidth [65-57] and increasing the numbers of harvesting directions [68, 69]. The MDOF configuration can simply be constructed by adding proof masses into the system. As discussed above for the multimodal linear VEHs, the number of proof masses is possibly equal to the number of the resonant peaks in the frequency response. The frequency bandwidth of the VEH is consequently broadened due to the appearance of these extra resonant peaks. However, for the case of MDOF nonlinear energy harvester, the dynamic characteristics of the system are possibly more complicated and require further sophisticated studies.

It is well known that the nonlinearity can be added to the design of the VEH by using either the intrinsically nonlinear structure or magnetic coupling. Many recent research works focus on the magnetically coupled MDOF energy harvester due to the advantages in non-contact magnetic forces, which are used to form the desired dynamic characteristics in the system. This subsection reviews the most recent literature on the MDOF nonlinear energy harvesters using magnetic coupling technique.

Recently, Wang et al. [71] combined the idea of multi-modal structure and nonlinear bistability in a two-degree-of-freedom (2-DOF) bistable energy harvester (BEH) as seen in Fig. 2.13. The operating bandwidth is broadened owing to the presence of the second resonant peaks. It is worth noting that the second resonant peaks may exhibit a strong nonlinear response due to the nonlinear magnetic forces. Thus, when compared with the linear VEH or the one-degree-of-freedom (1-DOF) BEH, this system performed a broader operating bandwidth.

Figure 2.13 A 2-DOF bistable energy harvester with magnetic coupling [71].

Another similar idea was proposed by Yang et al. [72]. In that paper, the authors used a system of two cantilever beams: one vertical cantilever beam and one horizontal cantilever beam, with tip magnets at the end of each beam. As shown in Fig. 2.14, the piezoelectric layer was attached to the surface of the horizontal beam. The design is modeled as a 2-DOF system. The authors indicated that the internal resonances resulting from the 2-DOF system enhanced the operating bandwidth of the energy harvester.

Figure 2.14 A 2-DOF bistable energy harvester with internal resonances [72].

Wu et al. [73] presented a compact 2-DOF piezoelectric energy harvester in Fig. 2.15. The design is composed of two beams with a tip mass at the free end of each beam. One beam is cut out from the other beam and both are covered by piezoelectric materials. The system is modelled as a 2 DOF and therefore two resonant peaks are observed in the frequency responses curves. The authors concluded that the advantage of the design is based on its large bandwidth, compactness, and material-saving design.

Figure 2.15 A compact 2-DOF piezoelectric energy harvester and its associated frequency response curves [73].

Zou et al. [74] proposed a compressive-mode piezoelectric energy with magnetic stoppers. In this research, the authors were using the flextensional transducers.

These transducers have been experimentally proven to achieve a higher electromechanical coupling coefficient compared to the conventional bend-mode transducers. However, the flextensional transducer tends to have low efficiency under weak excitation conditions. Therefore, to take advantage of the flextensional transducer, Zou et al. proposed the design of four oscillating permanent magnets as seen in Fig. 2.16. The results showed a good performance of the bistable energy harvester with flextensional transducers at weak excitation condition.

Figure 2.16 A compressive-mode piezoelectric energy harvester with flextensional transducers (a) 2 magnets and (b) 4 magnets [74].

Wang et al. [75] combined a bistable piezoelectric energy harvester and an elastic magnifier (BPH-EM) within one design. The elastic magnifier is expected to maintain the large amplitude oscillation of the bistable system. In comparison to the conventional BEH without an elastic magnifier, the design exhibited some advantages under certain excitation intensities and frequencies. Particularly, the dynamic responses of the proposed design were most probably attracted by the interwell motions under weak excitation conditions. This eventually led to a higher energy harvesting performance compared to the conventional design. Furthermore, the authors also found that the mass and stiffness ratio could significantly affect the frequency responses. Therefore, a great care must be taken when choosing these parameters. The experimental setup and the prototype of the BPH-EM are shown in Fig. 2.17.

Figure 2.17 Prototype and experimental setup of the BPH-EM and (a) 2 magnets and (b) 4 magnets [75].

In a different approach, Su et al. [69] used an MDOF design to trigger the operation of the piezoelectric energy harvester (PEH) in two directions. To do that, spring-mass systems were added as external oscillators in order to excite the main harvesting beam in additional directions. As seen in Fig. 2.18, the primary beam and the supporting spring-mass systems are connected through the magnetic forces formed by permanent magnets. There are particularly two designs proposed: beam-spring bi-directional PEH and beam-beam bi-directional PEH. The results demonstrated that additional structures efficiently improve the performance of the PEH in the auxiliary direction.

Figure 2.18 The schematic diagram of (a) the beam-spring bi-directional PEH and (b) the beam-beam bi-directional PEH [69].

Another compact bi-directional PEH is proposed by Fan et al. as seen in Fig. 2.19 [68]. The design consists of two cantilever beam in the perpendicular directions of the deflection. The two beams are coupled through a pair of permanent magnets attached to the tip of each beam. In comparison to the linear counterpart, the design has a higher output voltage and a lower resonant frequency.

Figure 2.19 The schematic diagram of the bi-directional PEH proposed by Fan et al. [68].

In Fig. 2.20, Gao et al. [67] presented a BEH with an elastic support external magnet. The elastic support magnet is connected to a cantilever beam and oscillates under the external excitation. Under low-intensity random excitation, the effect of the elastic support on the system through the magnetic coupling forces is theoretically and experimentally demonstrated to enhance the performance of the BEH. However, the authors also pointed out that the conventional BEH with rigid support tended to exhibit a better stability in harvesting performance at high-intensity random excitation.

Figure 2.20 The schematic diagram of the BEH with an elastic support external magnet [67].

Different designs of BEH using MDOF structure have been proposed recently. Most of these research works were trying to use the permanent magnets as the tip masses to integrate the nonlinearity into the system. The results showed that the MDOF structure tends to significantly broaden the bandwidth of BEHs and also increase the power output in many aspects. However, the sophisticated models and deep understanding of the nonlinear dynamic behaviours of these systems still remain as challenging tasks. Further research works are required to explore different benefit aspects of the structures as well as investigate the physical insights of individual characteristic behaviours.

2.6 Challenging issues on bistable energy harvesting systems

Although bistable energy harvesters have been proven to outperform the linear VEHs in past research works, it is worth noting that there are still important issues to further improve and advance the energy harvesting performance of the BEHs. This section covers several challenging issues and the associated potential solutions.

2.6.1 Dependency of the energy harvesting performance on the external excitation intensity

It should be noted that the maximum power generation of the BEH can only be achieved when it operates under the inter-well motion regime. However, from existing theoretical analyses and experimental investigations, it has been reported that the inter-well motion needs a sufficient intensity of external excitation over a certain threshold value. Furthermore, it has been revealed in Refs. [76-78] that the nonlinear BEHs can exhibit a wider bandwidth than linear counterpart only if its oscillation represents a high-energy orbit motion. Therefore, one of the main challenges is to maintain the high-energy orbit or inter-well motion even under low excitation intensity.

The large-amplitude inter-well motion usually depends on the excitation amplitude, the excitation frequency and the properties of the nonlinear design. For example, when the excitation strength is too small to activate the inter-well oscillations, the system's oscillation remains confined to one potential well, i. e., the BEH's dynamics becomes similar to a monostable one. Cotton et al. [45] concluded that the power generation performance of the BEH is improved as the separation between two stable states increases or as the potential barrier height decreases. Daqaq [77] also stated that there exists an optimal design in potential

shape that maximizes the output power for a certain intensity and frequency of an external excitation.

Hence, it is very important to investigate and uncover the effects of the excitation strength and the potential configuration of the dynamic behaviors of the BEH in order to optimize the energy harvesting performance.

2.6.2 Response to stochastic excitation

Generally, ambient vibrations in real environments are composed of multi-frequency harmonics with low-pass filtered noise. Although it has been shown that the BEHs have a great advantage over the linear counterparts, this conclusion would be valid only when the BEH exhibits the high-energy orbits. It has been reported by Zhao and Erturk [76] that a linear monostable VEH can outperform its bistable counterpart for the very low intensity of excitation or highly-stochastic excitation. Daqaq [77] has also shown that both the linear VEH and nonlinear BEH generated the same level of output power under white Gaussian excitation. Thus, the performance of the BEH needs to be investigated and examined thoroughly under various types of ambient excitations.

Many researchers [79, 80] have proposed the stochastic resonance to enhance the energy harvesting performance of a BEH. Stochastic resonance is normally used to amplify the weak signal by adding noises. In this case, the noises can be considered as a stochastic forcing excitation and the harvesting signal is needed to be amplified. Under the forcing excitation, the potential energy barrier oscillates. If the forcing rate matches with the mean time between transitions from one potential well to another potential well – i.e. the inverse Kramer's rate [81], the stochastic resonance will be triggered.

2.6.3 Performance metrics

From the literature review, it can be seen that the performance of the BEH is directly compared to the linear VEHs in most of the research works. Such comparisons may be appropriate to simply demonstrate the outstanding performance of the BEH but it may be not suitable to provide physical insight in detail on the characteristics of the BEHs [22]. Thus, a standing consensus is preferred in the literature to evaluate the performance of energy harvesting [82-84]. However, this is a challenging work because the testing conditions such as excitation vibration or frequency bandwidth usually vary among different research works.

2.7 Ph.D. in context of literature

The literature review showed that the benefits of bistable energy harvester are sustained by the active occurrence of the inter-well motion. The inter-well motion or high-energy orbital motion occurs when the dynamic of the system surmounts the potential saddle barrier and moves back and forth between two potential wells.

At the same excitation intensity, it is obviously that the higher the potential saddle barrier is, the harder the system triggers interwell motions. In other words, the existence of the potential saddle barrier indirectly heightens the threshold excitation intensity to trigger the interwell motions. Therefore, a design to alleviate the potential saddle barrier is required to maintain the advantage of the bistable energy harvester.

One of such potential designs to resolve these issues is presented in Chapter 3. In Chapter 3, we propose a bistable energy harvester with a horizontal auxiliary magnet oscillator (BEH-HO), of which the motion is mechanically rectified using a mechanical rectifier. The design of the proposed BEH-HO system is based on a local modification of the double-well potential. The auxiliary magnet oscillator with the mechanical rectifier is designed so that its rectified half-sine motion

tends to reduce the saddle barrier of restoring potential, leading to a possibility of an easier escape from the potential well, while keeping other parts of the potential, thereby maintaining high-energy orbital motion.

Another research direction to enhance the performance of the bistable energy harvester is by using the MDOF structure, as reviewed in Section 2.5. A bistable energy harvester with a vertical auxiliary magnet oscillator (BEH-VO) – recently considered as a potential design – is presented in Chapter 4. This design features two cantilever beams with a tip magnet at the free end of each beam. The magnetic nonlinearity, which initiated from the interaction between two tip oscillating magnets, endows the BEH-VO with the ability to perform an enhancement in terms of frequency bandwidth and energy harvesting power. However, the lack of sophisticated studies, which can provide a sufficiently deep understanding of the dynamic characteristics, hinders the straightforward design of the BEH-VO. For instance, the complicated nonlinear magnetic interaction with various types of resonant characteristics has yet to be completely understood. Therefore, Chapter 4 covers the study on the nonlinear resonant characteristics of this system.

Of particular interest is the concept of phase-dependent dynamic potential which explains the underlying principle of the complicated dynamics of the BEH-VO. Through theoretical simulations and analyses, two distinct dynamic regimes, called the out-of-phase and in-phase mode regimes in this report, are found to exist in the frequency regions of the first and second primary intrawell resonances.

For the out-of-phase mode regime, the frequency displacement (and output power) responses of the BEH-VO system exhibit typical double-well dynamics, whereas for the in-phase mode regime, only single-well dynamics is observed though the system is statically bistable. These dynamic regimes are also revealed to be caused by the difference in the dynamic potential energy trajectories propagating

on a high-dimensional potential energy surface. The present approach to the dynamics of the BEH-VO system can be extended and applied to higher-DOF systems, which sheds light on compact and efficient designs of magnetically coupled

Following Chapter 4, Chapter 5 expounds on the merits of the BEH-VO, in terms of energy harvesting performance. This chapter investigates possible routes to high-energy orbit motion (HEOM), from which high electrical power can be generated. By examining the frequency responses of the BEH-VO, four main routes to HEOM are found to originate from two primary and two secondary intrawell resonances. The enhanced broadband performance of the BEH-VO is theoretically and experimentally evaluated and discussed in comparison with the CBEH.

In conclusion, this Ph. D. thesis covers the challenges faced related to the design and implementation of the BEH to enhance its performance in terms of frequency bandwidth and energy harvesting performance.

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Chapter 3*

Bistable energy harvester with a horizontal auxiliary magnet oscillator: Enhanced performance by lowering the potential energy barrier

In this chapter, a bistable energy harvester with a horizontal auxiliary magnet oscillator (BEH-HO) is proposed. The design of the BEH-HO system is based on a local modification of the double-well potential. The auxiliary oscillator with the mechanical rectifier is designed in such a manner that its rectified half-sine motion tends to reduce the saddle barrier of restoring potential, leading to a possibility of an easier escape from the potential well. Meanwhile, this local modification does not disturb the other parts of the potential function, thereby maintaining high-energy orbital motion. Consequently, a broader frequency band of the BEH-HO is observed when compared with the CBEH's frequency band. This beneficial effect of the BEH-HO system is theoretically and experimentally supported by bifurcation analyses and frequency response analyses.

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3.1 Introduction

The bistable electromechanical oscillator is considered as a promising structure of a vibration energy harvester for broadband energy harvesting [1-3]. As discussed in Section 2.4, the main characteristic of a bistable oscillator is the double-well configuration of the potential energy function. The two potential wells of the bistable oscillator are separated by a potential saddle barrier. This potential energy function produces two distinct dynamic motions of a bistable energy harvester (BEH) – intrawell and interwell motions – depending on the excitation intensity [1, 2]. When the ambient excitation is weak, the oscillation of a BEH exhibits small-amplitude intrawell motion or low-energy orbit motion, confined to one of the two potential wells by the potential saddle barrier.

On the other hand, when the excitation is sufficiently strong for inducing potential well escape phenomenon [1, 2], the BEH achieves large-amplitude interwell motion or high-energy orbit motion, oscillating across the two potential wells in a broad frequency band. From this interwell motion, the BEHs extract electrical energy, using energy conversion mechanisms such as piezoelectric, electrostatic, and electromagnetic conversions [2, 3], which outperform linear and nonlinear monostable energy harvesters [3-9] in terms of broadband energy harvesting.

As the performance of BEHs is contingent on the occurrence of high-energy orbital motion, the majority of theoretical and experimental research studies were focused mainly on examining and characterizing the dependence of the frequency bandwidth of the interwell motion on the type and intensity of ambient vibration sources (e.g., swept sine [7, 10], impulse [11], and Gaussian white noise [12, 13]) and design parameters affecting the potential well configuration (e.g., magnetic field or structural stiffness [14, 15]). Recent research efforts were devoted to either further enhancing the operating frequency bandwidth of a BEH or reducing the threshold excitation intensity required for triggering interwell motion.

Multistable (tri- or quad-stable) energy harvesters [16-20] were proposed to lower potential saddle barriers and were shown to be favorable for energy harvesting, particularly from low-intensity ambient vibration. Multimodal or multi-degree-of-freedom (MDOF) systems of BEHs [14, 21] were reported to be suitable for multidirectional vibration sources, and their high-order resonances provided additional operating frequency bands for energy harvesting.

However, most of the energy harvesting systems mentioned above [14-21] inevitably produce distortion of high-energy orbital motion and/or undesirable changes of its frequency band, which adversely affect the efficiency of energy harvesting and/or make evaluating the performance of the energy harvesting system more difficult.

This chapter proposes a BEH system with a horizontal auxiliary magnet oscillator (BEH-HO) to achieve energy harvesting performance in a broader frequency band compared to conventional BEHs (CBEHs). In particular, the motion of the auxiliary oscillator in the proposed BEH-HO system is mechanically rectified to prevent any severe distortion of high-energy orbital motion or large changes in its frequency band.

3.2 Design and parameters of BEH-HO

The BEH-HO, shown in Fig. 3.1, is composed of a bimorph cantilever beam oscillator, with a permanent magnet at its free end, and an external magnet oscillator. The two identical permanent magnets face each other with the same polarities, and thus, a repulsive magnetic interaction acts as an axial force on the beam and external oscillators, imposing bistability on the system. Unlike CBEHs [14-20], the external magnet of the BEH-HO is not fixed in space but movable. As the tip magnet approaches its trivial position (at $w = 0$ in Fig. 3.1(a)), it drives the external magnet away from the equilibrium state to the right by magnetic repulsion. This motion of the external magnet oscillator results in weak magnetic

repulsion around the trivial position of the bimorph beam oscillator (when compared with the CBEH with a fixed external magnet), reducing the potential saddle barrier, thus enabling easier occurrence of high-energy orbital motion of the bimorph oscillator in a broader frequency band.

Figure 3.1 (a) Schematic of the BEH-HO system and (b) its experimental prototype. In this figure, the two identical magnets are located at their static equilibrium states, and a mechanical rectifier prevents the external magnet from moving past the equilibrium to the left.

Conversely, as the tip magnet moves far away from the trivial position after the potential well escape phenomenon occurs, the external magnet tends to return to the equilibrium position. A mechanical rectifier prevents the external magnet from moving past the equilibrium state to the left; therefore, the distortion of high-energy orbital motion can be minimized. Thus, the design of the BEH-HO

provides a broader operating frequency band in comparison to CBEHs, without deteriorating the efficiency of energy harvesting.

In this chapter, the geometric and material properties of the BEH-HO were chosen arbitrarily, within realistic ranges, to demonstrate the operation mechanism of the proposed BEH-HO system. The detailed parameter values are listed in Table 3.1.

Table 3. 1 Geometric and material properties of the BEH-HO system.

Parameter	Value
Substrates	
Length	70 mm
Width	10 mm
Thickness	0.3 mm
Density	8000 kg/m ³
Young's modulus	190 GPa
Piezo-polymer layers	
Length	70 mm
Width	10 mm
Thickness	0.052 mm
Density	1780 kg/m ³
Young's modulus	3 GPa
Piezo-strain constant	−23 pm/V
Permittivity	110 pF/m
Magnet	
Length	6 mm
Height	10 mm
Thickness	2 mm
Magnetization	7.2×10 ⁵ A/m
Distance d between two magnets	12.4 mm

3.3 Mathematical modelling of BEH-HO

A mathematical model of the bimorph cantilever beam is established on the basis of Euler–Bernoulli beam theory and linear piezoelectricity [11]. The external magnet oscillator is modeled as a simple mass-damper-spring system. The magnetic charge model [22] is employed to describe the two permanent magnets, and the resulting magnetic coupling forces are considered as external forces exerted at the tip of the beam and the center of mass of the external oscillator in the model. The governing equation of the beam is discretized using the Galerkin projection method [23]. The detailed derivations are provided separately in the following subsections.

3.3.1 Modelling the bimorph piezoelectric cantilever beam

Compared to its unimorph counterpart, a bimorph piezoelectric cantilever beam structure with a tip mass, owing to its high energy conversion efficiency, has been used widely to harvest vibration energy [11]. The working mechanism is simple as the two piezoelectric layers laminated on both sides of the substrate beam will simultaneously generate electricity under the beam's deflections.

In this research, a bimorph piezoelectric cantilever beam with a tip magnet, as schematically illustrated in Fig. 3.2(a), will be employed as the main energy conversion part. The two piezoelectric layers are electrically connected in serial mode to maximize the output power as shown in Fig. 3.2(b). The bending motion of the bimorph piezoelectric cantilever beam's modelling part is the first required.

Figure 3.2 (a) Schematic diagram of a bimorph piezoelectric cantilever beam with a tip mass and (b) the cross-section of the bimorph beam with the serial mode electrical connection.

There are several methods to obtain the mathematical model of the bimorph piezoelectric cantilever beam with a tip magnet such as the energy approach [24] or moment balance approach [25]. In this research, an energy approach based on extended Hamilton's Principle is used. The derivation steps in this chapter follow the energy approach which is reported in Refs [26, 27]. Accordingly, the following assumptions are applied in the modelling process:

- The bimorph cantilever beam is modeled after the linear Euler-Bernoulli beam theory which states that the cross-section along the beam's length will remain planar and normal to the deformed axis during the beam's oscillation. Consequently, the shear deformation and the rotary inertia of the deformed beam are negligible.

- The small rotation assumption is applied for the beam's deflection; thus, the concerning displacement is in the transverse direction only.
- The damping factors include the internal electrical and mechanical damping factor. These damping factors are separately added to the governing equations by using the appropriate damping coefficients.
- In this structure, the piezoelectric surfaces are assumed to be perfectly coated with electrode layers that are negligibly thick. Hence, the effect of electrode layers is not mentioned in the modelling part.

The generalized form of Hamilton's Principle for an electromechanical system is given by,

$$\int_{t_1}^{t_2} (\delta\Pi - \delta\Psi + \delta W_{ie} + \delta W_{nc}) dt = 0, \quad (3.1)$$

where $\delta\Pi$, $\delta\Psi$ and δW_{ie} are the first variations with respect to time t of the total kinetic energy, the total potential energy, and the internal electrical energy; and δW_{nc} is the virtual work of the non-conservative mechanical force and electrical charge components.

Since the mechanical damping effect is to be introduced later, the non-conservative virtual work is only a result of the electric charge output $Q(t)$. Thus, the non-conservative virtual work can be expressed as,

$$\delta W_{nc} = \delta W_{nce} = Q(t) \delta V(t), \quad (3.2)$$

where $V(t)$ is the voltage across the electrodes of the piezoelectric layers.

The description of the energy is essential to derive the mathematical model of the system. The total potential energy, total kinetic energy, and internal electric energy of a bimorph cantilever beam with a tip magnet are expressed as follows,

$$\Psi = \frac{1}{2} \left(\int_{\mathcal{G}_s} S T_s d\mathcal{G}_s + 2 \int_{\mathcal{G}_p} S T_p d\mathcal{G}_p \right), \quad (3.3)$$

$$\Pi = \frac{1}{2} \left(\int_{\mathcal{G}_s} \rho_s \dot{u}^2 d\mathcal{G}_s + 2 \int_{\mathcal{G}_p} \rho_p \dot{u}^2 d\mathcal{G}_p + m_t \dot{u}_L^2 + I_t \omega_L^2 \right), \quad (3.4)$$

$$W_{ie} = \int_{\mathcal{G}_p} E D d\mathcal{G}_p. \quad (3.5)$$

where the dot over the ' u ' symbol represents the differentiation with respect to time t ; u is the absolute displacement of the beam structure; ρ is the mass density; $\mathcal{G}$ is the volume; the subscripts s and p denote the values corresponding to the substrate and piezoelectric layers, respectively; the subscript L denote the values at the length L of the beam; m_t , I_t and ω are the mass, the moment of inertia and the angular velocity of the tip magnet, respectively; E is the electric field component; D is the electric displacement component; S is the applied strain; and T is the stress.

The relationship between mechanical stress T_p and the electric field, which are generated within one piezoelectric layer, can be obtained from the constitutive laws of the linear piezoelectric material [28]. As for the piezo-elastic behaviors of a linear Euler-Bernoulli beam, the piezoelectric constitutive equations can be written as,

$$\begin{aligned} T_p &= c_{11}^E S - e_{31} E, \\ D &= e_{31} S + \epsilon_{33}^S E. \end{aligned} \quad (3.6)$$

where c_{11}^E is the compliance at a constant electric field, e_{31} is the piezoelectric constant, and ε_{33}^S is the permittivity at a constant stress. As shown in Fig. 3.3, the subscript 3 denotes the direction along the length of the beam (x-direction) and subscript 1 denotes the direction along the thickness of the beam (z-direction). In this research, the polarized direction of the piezoelectric material is in the z-direction.

Figure 3.3 Polarization of the piezoelectric layer.

In the bending mode, only the transverse displacement is considered by assuming that the axial displacement is negligible (small rotation assumption). When the beam deflects, the transverse displacement at a certain point on the surface of the beam is expressed as,

$$u(x, t) = w(x, t) + w_b(x, t), \quad (3.7)$$

where $w(x, t)$ is the relative transverse displacement with respect to the moving base and $w_b(x, t)$ denotes the base displacement.

For most of the beam-type energy harvester, the bending motion of the beam under a low frequency excitation condition is of main interest due to its high deflection's amplitude. Therefore, the fundamental mode approximation can be used to simply derive the model. With the fundamental mode approximation, the transverse displacement is expressed as a multiplication of the mode-shape

function and an unknown generalized modal coordinate. This transverse displacement is given by,

$$w(x, t) = \phi_r(x) r(t), \quad (3.8)$$

Here, $\phi_r(x)$ is the first eigen-function for the undamped cantilever beam with the shorted electrode and a tip mass at the end of the beam, and $r(t)$ is the associated modal coordinate.

The electromechanical Lagrange Equations based on Extended Hamilton Principle [30] are used to derive the governing equations of the system. In the Lagrange Equations, the generalized coordinates are the tip displacement and the output voltage. Thus, the equations are written as,

$$\frac{d}{dt} \left(\frac{\partial \Pi}{\partial \dot{w}} \right) - \frac{\partial \Pi}{\partial w} + \frac{\partial \Psi}{\partial w} - \frac{\partial W_{ie}}{\partial w} = 0, \quad (3.9)$$

$$\frac{d}{dt} \left(\frac{\partial \Pi}{\partial \dot{V}} \right) - \frac{\partial \Pi}{\partial V} + \frac{\partial \Psi}{\partial V} - \frac{\partial W_{ie}}{\partial V} = Q, \quad (3.10)$$

where Q is the total electric charge of the piezoelectric layers. The dissipating energy due to mechanical dissipation effect is introduced in the form of a proportional damping at a later stage.

After a trivial derivation process, we can obtain the form of a bimorph cantilever beam with a tip mass's governing equations under the harmonic excitation condition as,

$$\ddot{w} + 2\zeta_b \omega_b \dot{w} + \omega_b^2 w - \kappa V = -f \sin \Omega t, \quad (3.11)$$

$$\dot{V} + 2R^{-1}C^{-1}V + 2C^{-1}\kappa \dot{w} = 0, \quad (3.12)$$

where w is the tip deflection of the beam; ζ and ω are, respectively, the damping ratio and the natural frequency, and their subscripts b denote the quantities for the beam; V is the output voltage across the electrical resistance R ; κ and C are, respectively, the electromechanical coupling constant and the equivalent capacitance; and f and Ω are the amplitude and frequency of the base acceleration, respectively.

Next, we need to find the first mode-shape function $\phi_r(x)$ of an undamped cantilever beam with a tip mass.

3.3.2 Modal analysis of a cantilever beam with a tip mass

For a cantilever beam oscillating at the first bending mode, the linear Euler-Bernoulli beam theory is sufficient to give an accurate approximation. Therefore, the linear Euler-Bernoulli beam theory will be applied to derive the first mode-shape function of the bi-morph cantilever beam with a tip mass.

Firstly, an infinitesimal beam segment undergoing the bending motion is considered, as shown in Fig. 3.4. The unknown fields are the transverse displacement $w(x, t)$, the rotation $\theta(x, t)$, the curvature $\kappa(x, t)$, bending moment $\Gamma(x, t)$ and transverse shear $Y(x, t)$.

Figure 3.4 An infinitesimal beam segment undergoing the bending motion.

Using the Newtonian approach, we can derive the governing equations of motion for the undamped free vibration of a linear Euler-Bernoulli beam as follow,

$$\text{Balancing equation: } Y(x, t) = -\frac{\partial \Gamma(x, t)}{\partial x}, \quad (3.13)$$

$$\text{Newton's Law: } \frac{\partial^2 \Gamma(x, t)}{\partial x^2} = -m \frac{\partial^2 w(x, t)}{\partial t^2}, \quad (3.14)$$

$$\text{Kinematic equations: } \theta(x, t) = \frac{\partial w(x, t)}{\partial x}, \quad \kappa(x, t) = \frac{\partial^2 w(x, t)}{\partial x^2}, \quad (3.15)$$

$$\text{Constitutive equation: } \Gamma(x, t) = Y_b I_b \kappa(x, t). \quad (3.16)$$

where m is the mass per length of the cantilever beam, Y_b is the elastic modulus, and I_b is the area moment of inertia.

From the above equations, the equation motion of a cantilever beam with a tip mass is given as,

$$Y_b I_b \frac{\partial^4 w(x, t)}{\partial x^4} = -m \frac{\partial^2 w(x, t)}{\partial t^2}. \quad (3.17)$$

The beam is with clamped-free boundary conditions at $x = 0$ and with a tip mass attachment at $x = L$. Therefore, at the clamped-end of the cantilever beam, both the bending mode-shape and its first derivative with respect to the transverse direction are equal to zero:

$$w(0, t) = 0, \quad \left. \frac{\partial w(x, t)}{\partial x} \right|_{x=0} = 0. \quad (3.18)$$

At the free end of the cantilever beam ($x = L$), the shear force and the bending moment vanish. The mass and the moment of inertia with respect to the y-axis are denoted as m_t and I_t , respectively. Thus, we can obtain,

$$\left[Y_b I_b \frac{\partial^2 w(x, t)}{\partial x^2} + I_t \frac{\partial^3 w(x, t)}{\partial t^2 \partial x} \right]_{x=L} = 0, \quad \left[Y_b I_b \frac{\partial^3 w(x, t)}{\partial x^3} - m_t \frac{\partial^2 w(x, t)}{\partial t^2} \right]_{x=L} = 0. \quad (3.19)$$

To find the mode-shape function of the system, the method of separation of variables is utilized to separate the mode-shape function (position-dependent function) and the modal coordinate function (time-dependent function). The total solution when the beam oscillates in a pure mode is expressed as,

$$w(x, t) = \phi(x) r(t). \quad (3.20)$$

where $\phi(x)$ denotes the mode-shape function and $r(t)$ denotes the associated modal coordinate. Substituting the solution form (3.20) into the equation (3.17),

$$\frac{Y_b I_b}{m} \frac{1}{\phi(x)} \frac{d^4 \phi(x)}{dx^4} = -\frac{1}{r(t)} \frac{d^2 r(t)}{dt^2}. \quad (3.21)$$

The solution forms of the equation (3.21) are,

$$\begin{aligned} \phi(x) &= A \cos\left(\frac{\lambda}{L} x\right) + B \cosh\left(\frac{\lambda}{L} x\right) + C \sin\left(\frac{\lambda}{L} x\right) + D \sinh\left(\frac{\lambda}{L} x\right), \\ r(t) &= E \cos \omega t + F \sin \omega t. \end{aligned} \quad (3.22)$$

where A, B, C, D, E, and F are unknown constants and $\lambda^4 = \omega^2 \frac{mL^4}{Y_b I_b}$.

Boundary conditions in (3.18) and (3.19) can be rewritten as,

$$\phi(0) = 0, \quad \left. \frac{d\phi(x)}{dx} \right|_{x=0} = 0. \quad (3.23)$$

$$\left[Y_b I_b \frac{d^2 \phi(x)}{dx^2} - \omega^2 I_t \frac{d\phi(x)}{dx} \right]_{x=L} = 0, \quad \left[Y_b I_b \frac{\partial^3 \phi(x)}{\partial x^3} + m_t \omega^2 \phi(x) \right]_{x=L} = 0. \quad (3.24)$$

By substituting the solution forms into the first two boundary conditions in (3.23), we can obtain,

$$A + B = 0 \quad \text{and} \quad C + D = 0. \quad (3.25)$$

Thus, the mode-shape function can be rewritten as,

$$\phi(x) = A \left[\cos\left(\frac{\lambda}{L}x\right) - \cosh\left(\frac{\lambda}{L}x\right) \right] + C \left[\sin\left(\frac{\lambda}{L}x\right) - \sinh\left(\frac{\lambda}{L}x\right) \right]. \quad (3.26)$$

By substituting the solution form from (3.26) into the two remaining boundary conditions, we get the relations in matrix form. In order to obtain the non-trivial solution of A and C, the determinant of the matrix has to be singular. Thus, we have the characteristic equation as,

$$\begin{aligned} & 1 + \cos \lambda \cosh \lambda + \lambda \frac{m_t}{mL} (\cos \lambda \sinh \lambda - \sin \lambda \cosh \lambda) \\ & - \frac{\lambda^3 I_t}{mL^3} (\cosh \lambda \sin \lambda - \sinh \lambda \cos \lambda) + \frac{\lambda^4 m_t I_t}{m^2 L^4} (1 - \cos \lambda \cosh \lambda) = 0. \end{aligned} \quad (3.27)$$

For given system parameters, the solution of the above characteristic equation is the eigenvalue of the system. Since the boundary-value problem is a positive

definite, the system has an infinite number of positive eigenvalues for the infinite number of natural modes of vibration. The eigenvalue of the r^{th} vibration mode is denoted here as λ_r , and it is associated with the r^{th} eigen-function denoted by,

$$\phi_r(x) = A_r \left[\left[\cos\left(\frac{\lambda_r}{L}x\right) - \cosh\left(\frac{\lambda_r}{L}x\right) \right] + \chi_r \left[\sin\left(\frac{\lambda_r}{L}x\right) - \sinh\left(\frac{\lambda_r}{L}x\right) \right] \right], \quad (3.28)$$

$$\text{where } \chi_r = \frac{\sin \lambda_r - \sinh \lambda_r + \frac{\lambda_r m_t}{mL} (\cos \lambda_r - \cosh \lambda_r)}{-\cos \lambda_r - \cosh \lambda_r + \frac{\lambda_r m_t}{mL} (\sin \lambda_r - \sinh \lambda_r)}. \quad (3.29)$$

3.3.3 Magnetic forces between two oscillating magnets

The magnetic charge model [22] is employed for the permanent magnets with the coordinate system given in Fig. 3.5. For further simplification, the magnetic poles of each magnet are approximated to the equivalent point charges. Accordingly, for the present system, one magnetic dipole (which corresponds to Magnet 1 in Fig. 3.5) can be considered to be subjected to an external magnetic field applied by another magnetic dipole (Magnet 2 in Fig. 3.5).

Thus, when the magnets oscillate, the magnetic potential energy that the system acquires is given by,

$$U_m = -\int_V \mathbf{M}_1 \cdot \mathbf{B}_{\text{ext}} dv \approx -\int_l (M_s A_p \mathbf{e}_1) \cdot \left(\frac{\mu_0}{4\pi} \sum_{j=1,2} Q'_j \frac{\mathbf{x} - \mathbf{x}'_j}{|\mathbf{x} - \mathbf{x}'_j|^3} \right) dl. \quad (3.30)$$

where μ_0 is the permeability of free space; $\mathbf{M}_1 (= M_s \mathbf{e}_1)$ is the magnetization vector of Magnet 1; A_p is the cross-sectional area of the magnet or its pole face area; $\mathbf{B}_{\text{ext}}$ is the magnetic field generated by Magnet 2; Q'_j denotes the

equivalent magnetic charge of Magnet 2 given by $(-1)^j M_s A_p$ at the negative and positive poles ($j = 1$ and 2 , respectively); and $\mathbf{x}'_j$ is the position vector of the charge Q'_j .

Figure 3.5 Schematic diagram of a magnetic model.

By manipulating Eq. (3.30) with the geometric configuration of the system presented in Fig. 3.5, the magnetic potential energy can be obtained in the following form:

$$U_m = \frac{\mu_0 Q Q'}{4\pi} \sum_{j,k=1,2} \frac{1}{\sqrt{(x+d+\beta^{(j,k)})^2 + \alpha^{(k)2} w^2}}. \quad (3.31)$$

where $\alpha^{(k)} = 1 + (l_t + (-1)^k l_m) \eta$; $\beta^{(j,k)} = (-1)^j l_m - (-1)^k l_m$; μ is the permeability of free space; Q and Q' are the magnetic surface charges of the tip magnet and external magnet, respectively; l_m is the half-length of the magnet; l_t is the

distance from the tip of the beam to the mass center of the tip magnet; and η is the constant coefficient.

The magnetic forces are derived from the magnetic potential energy by taking derivative with respect to the corresponding general coefficients. The magnetic coupling forces are given in the forms:

$$F_b = \frac{\mu QQ'}{4\pi} \sum_{j,k=1,2} \frac{(-1)^{j+k} \alpha^{(k)2} w}{((x+d+\beta^{(j,k)})^2 + \alpha^{(k)2} w^2)^{3/2}}, \quad (3.32a)$$

$$F_e = \frac{\mu QQ'}{4\pi} \sum_{j,k=1,2} \frac{(-1)^{j+k} (x+d+\beta^{(j,k)})}{((x+d+\beta^{(j,k)})^2 + \alpha^{(k)2} w^2)^{3/2}}. \quad (3.32b)$$

where F_b and F_e represent the magnetic coupling forces exerted on the tip magnet and the external magnet, respectively.

3.3.4 Modelling the external magnet oscillator

Figure 3.6 (a) Schematic diagram of the external magnet oscillator with mechanical rectifier and (b) the corresponding spring-mass-damping model.

The oscillation of the external magnet with mechanical rectifier is analyzed in this section by using a typical spring-mass-damping system, which vibrates horizontally with a displacement x under the magnetic force $\vec{F}_e$. A mechanical rectifier prevents the external magnet from moving past the equilibrium state to the left as schematically drawn in Fig. 3.6(a) and is represented by a stopper as in Fig. 3.6(b). The equation of motion can be simply given as,

$$m_e \ddot{x} + c_e \dot{x} + k_e x + F_e + F_c = 0 . \quad (3.33)$$

where m_e, c_e, k_e are subsequently the equivalent mass, damping and stiffness of the spring-mass-damping system given in Fig. 3.6(b) (note that m_e is equal to m_t which is the tip mass).

And F_c accounts for the operation of the mechanical rectifier during the oscillation of the system. The force F_c is simply presented as a piecewise function that includes contact restoring and damping forces as:

$$F_c = \gamma_e^{-1} \left(2\zeta_c \omega_c \dot{x} + \omega_c^2 (x - \bar{x}) \right) H(-x + \bar{x}) . \quad (3.34)$$

where ζ_c and ω_c are, respectively, the damping ratio and natural frequency involved during the contact of the stopper and the external magnet; $\bar{x}$ is the equilibrium position of the external magnet; and H denotes the Heaviside function.

The typical form of Eq. (3.33) can be written as,

$$\ddot{x} + 2\zeta_e \omega_e \dot{x} + \omega_e^2 x + \gamma_e (F_e + F_c) = 0 . \quad (3.35)$$

where $\omega_e = \sqrt{k_e / m_e}$ is the natural frequency, $\zeta_e = c_e / 2\sqrt{k_e m_e}$ is the damping ratio and γ_e is the force coefficient.

3.3.5 Governing equations

After some derivations, the magneto-electro-mechanical oscillator model of the BEH-HO can be expressed as,

$$\ddot{w} + 2\zeta_b \omega_b \dot{w} + \omega_b^2 w - \kappa V - \gamma_b F_b = -f \sin \Omega t, \quad (3.36a)$$

$$\dot{V} + 2R^{-1}C^{-1}V + 2C^{-1}\kappa\dot{w} = 0, \quad (3.36b)$$

$$\ddot{x} + 2\zeta_e \omega_e \dot{x} + \omega_e^2 x + \gamma_e (F_e + F_c) = 0. \quad (3.36c)$$

where w and x are, respectively, the tip deflection of the beam and the displacement of the external oscillator; ζ and ω are, respectively, the damping ratio and the natural frequency, and their subscripts b and e respectively denote the quantities for the beam and external magnet oscillators; V is the output voltage across the electrical resistance R ; κ and C are, respectively, the electromechanical coupling constant and the equivalent capacitance; F_b and F_e are the magnetic coupling forces; and f and Ω are the amplitude and frequency of the base acceleration, respectively.

The mathematical model for a CBEH to be compared with the present BEH-HO can be obtained as a limiting case of Eqs. (3.36a) – (3.36c), when the displacement of the external magnet oscillator is fixed at its equilibrium position, i.e., $x \rightarrow \bar{x}$.

3.4 Numerical method and experimental setup of BEH-HO

Numerical dynamic simulations:

The equations of motion of the BEH-HO derived by Eqs. (3.36a) – (3.36c) has been rewritten, for numerical simulations, in a more convenient matrix-vector form as:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t) = \begin{bmatrix} -2\zeta_b \omega_b x_1 - \omega_b^2 x_2 + \kappa x_3 + \gamma_b F_b - f \sin \Omega t \\ x_1 \\ -2R^{-1}C^{-1}x_3 - 2C^{-1}\kappa x_1 \\ -2\zeta_e \omega_e x_4 - \omega_e^2 x_5 - \gamma_e (F_e + F_c) \\ x_4 \end{bmatrix}. \quad (3.37)$$

where $\mathbf{x}$ denotes the state variable vector defined as $\mathbf{x} = [\dot{w}, w, V, \dot{x}, x]^T$. By taking the direct numerical integration of Eq. (3.37), the stationary forced response of the BEH-HO was obtained and its stability is determined, if necessary, by the Floquet theory [21].

For frequency response analyses, a swept sine excitation was exerted on the base of the BEH-HO with the following excitation frequency: $\Omega = \Omega_0 + 0.5\Omega_r t$ where Ω_0 and Ω_r are the initial excitation frequency and linear frequency sweep rate, respectively.

The geometric dimensions and material properties used in this numerical study are listed in Table 3.1. For the numerical simulation, the values of ζ_e and ω_e are set to be 0.15 and 103 rad/s, respectively.

Experimental measurement of frequency voltage responses:

The prototype of the BEH-HO shown in Fig. 3.1(b) was prepared first. For the bimorph beam oscillator, piezoelectric polyvinylidene fluoride layers with a

thickness of $52\ \mu\text{m}$ were entirely laminated on the upper and lower surfaces of a stainless steel substrate ($70 \times 10 \times 0.3\ \text{mm}$). The two identical neodymium magnets ($2 \times 10 \times 6\ \text{mm}$) were used for the tip and external magnets.

The prototype of the BEH-HO was placed on a vibration exciter (Type 4808, B&K) connected with a closed-loop vibration controller (VR8500, Vibration Research). During forward/backward swept excitation, the base acceleration was held constant at a given magnitude using the control system. The voltage generated across a resistor of $1\ \text{M}\Omega$ (as a simple energy harvesting circuit in this study) was simultaneously measured using a signal acquisition system (CDAQ-9171, National Instruments). The schematic diagram of this experimental setup is explicitly illustrated in Fig. 3.7.

Figure 3.7 Schematic diagram of experiment setup for the frequency voltage response.

To support the validity of the mathematical model, a simple comparison of numerical and experimental results is also given as Fig. 3.8. Figure 3.8(a) presents the output voltage responses of the BEH-HO obtained numerically. In this figure, the peak points of output voltage are drawn for (quasi-)periodic responses but the stroboscopic points for chaotic responses. Note that the

scattered dots (in Fig. 3.8(a)) denote the stroboscopic points which represent the intersections of closed orbits with the Poincaré section at every forcing period. The random distribution of these stroboscopic points indicates the chaotic behaviors of the responses. Such stroboscopic points can also be obtained from experimental data after somewhat complicated data processing. But, in the experimental results (in Fig. 3.8(b)), the peak points of output voltage are drawn for both periodic and chaotic responses for convenience. The random fluctuation of peak points can be seen in Fig. 3.8(b), indicating chaotic responses.

Figure 3.8 Comparison of (a) numerical and (b) experimental results for swept sine responses of the output voltages of the BEH-HO.

The forward and backward sweep responses are denoted by the blue and red colors, respectively. The saddle-node bifurcations of intrawell and interwell motions are designated by points SN1 and SN2, respectively. Numerical and experimental results agree well in terms of frequency bandwidth. In both numerical and experimental results, the jump-up and jump-down phenomena occurred at around 10 Hz and 15 Hz, respectively. This preliminary result validates the above-described mathematical model.

3.5 Results and discussions

3.5.1 Potential barrier-lowering effect

First the potential energy diagram will be investigated and compared for both systems: BEH-HO and CBEH. For the BEH-HO, the tip magnet travels in the varying magnetic field generated by the movable external magnet. Thus, the repulsive magnetic forces are functions of the relative motion between the two magnets, and accordingly the potential well configuration of the BEH-HO cannot be easily evaluated.

The potential energy function of the BEH-HO, $U(w, x)$, forms a three-dimensional potential energy surface, on which a dynamic trajectory determines the configuration of the actual potential well experienced by the BEH-HO during its oscillation. In this chapter, to evaluate such potential well configurations, the steady-state response of the BEH-HO is first obtained using numerical integration of Eqs. (3.36a) – (3.36c), and then the dynamic path on potential energy surface is calculated using the following equation:

$$U = \int F_{R1}(w, x)dw + F_{R2}(w, x)dx . \quad (3.38)$$

where F_{R1} and F_{R2} are the restoring forces acting on the beam oscillator and the external magnet oscillator, respectively.

Figure 3.9 shows (a) the interwell motion of the BEH-HO, (b) the associated restoring force response, and (c) the dynamic potential well configuration, which is obtained numerically to demonstrate the operation of the auxiliary oscillator with the mechanical rectifier in the BEH-HO. In Figs. 3.9(a)–(c), the solid and dotted blue lines denote two portions of positive and negative velocities, respectively, for each response of the BEH-HO.

Figure 3.9 (a) Stationary interwell motion of the BEH-HO obtained numerically with a base acceleration of 0.6 g and frequency of 11 Hz, (b) the associated restoring force responses, and (c) the resulting potential well configurations with respect to the deflection of the beam.

The results of the CBEH (red lines) are also provided for comparison. The values of ω_e , ω_c , ζ_e , and ζ_c used in this simulation are 130 rad/s, 110 rad/s, 0.15, and 0.45, respectively. Particularly for ω_c , ζ_e , and ζ_c , except ω_e , the above values were consistently used for all of the simulations conducted in Figs. 3.9–3.12.

As the mechanical rectifier is fixed at the equilibrium position of the external magnet, the motion of the external magnet takes a sort of a rectified half-sine oscillation rather than a simple harmonic oscillation, as shown in Fig. 3.9(a). The rectified motion of the external magnet [the green line in Fig. 3.9(a)] becomes magnified with relatively strong magnetic repulsion when the tip magnet [the blue line in Fig. 3.9(a)] passes through the region of small displacement around the trivial position (i.e., when two magnets are close to each other). However, it becomes smaller with weaker magnetic repulsion when the tip magnet passes into the region of large displacement (i.e., when two magnets are far away from each other).

Accordingly, the resistance of the restoring force that the BEH-HO experiences while crossing the potential saddle [areas A and B in Fig. 3.9(b)] is reduced without significant distortion of interwell motion [as shown in Fig. 3.9(a) by the blue and red lines for the BEH-HO and CBEH, respectively]. In Fig. 3.9(c), the resulting asymmetric potential well configuration of the BEH-HO exhibits a lower potential saddle barrier when compared with the CBEH, which is beneficial to triggering interwell motion in a broader frequency band.

3.5.2 Nonlinear frequency responses and bifurcations

Figure 3.10 shows the swept sine responses for the tip deflections of (a) BEH-HO and (b) CBEH (refer to Fig. 3.8 in the previous subsection for the output voltage responses associated with Fig. 3.10). A negative static equilibrium state was used as an initial condition for the numerical simulation. In this simulation, the value of ω_e was set as 103 rad/s.

Figure 3.10 Swept sine responses for the tip deflections of (a) BEH-HO and (b) CBEH obtained with a base acceleration of 0.52 g.

In Figs. 3.10(a) and (b), the maximum displacements of the periodic responses and the stroboscopic points of the chaotic motions are represented. The forward and backward sweep responses are denoted by the blue and red colors, respectively. The saddle-node bifurcations of intrawell and interwell motions are designated as points SN1 and SN2, respectively. In Fig. 3.10(b), the dashed line indicates the interwell motions of the CBEH that do not appear in this swept sine response but coexist with the chaotic or intrawell motion.

For the forward sweep result of the BEH-HO [denote by blue color in Fig. 3.10(a)], the intrawell motion jumps up to the large-amplitude interwell motion through its saddle-node bifurcation [point SN1 in Fig. 3.10(a)]. The SN1

bifurcation occurs at a frequency where a chaotic attractor does not exist, and thereafter, the interwell motion is unique in a certain frequency band. In the case of the forward sweep, this frequency band is the only route to high-energy orbital motion without any external perturbation. The initiated interwell motion increases with the excitation frequency, and reduces with the chaotic motion to the intrawell motion after its saddle-node bifurcation [point SN2 in Fig. 3.10(a)]. It is worth noting that the interwell motion of bistable systems typically belongs to a hardening-type frequency response [1-3, 17-24].

The branch of the interwell motion may not be continuously developed if the system loses its consistent hardening characteristics, implying that it will fail to harvest vibrational energy in a broad frequency band. This situation could occur if the mechanical rectifier was not used in the BEH-HO, leading to a large distortion of interwell motion. It was observed that with the mechanical rectifier in use, the interwell motion branches of the BEH-HO and CBEH are nearly the same, as shown in Fig. 3.10, and their SN2 bifurcations are found at nearly the same frequency as well.

However, for the backward sweep of the BEH-HO [denoted by the red color in Fig. 3.10(a)], the intrawell motion changes to chaotic motion first, through a cascade of periodic doubling bifurcations, and the chaotic motion is sustained before being terminated at its boundary crisis, where the interwell motion is initiated. The bifurcation structure for the frequency response of the BEH-HO discussed above is in accordance with that of the CBEH [Fig. 3.10(b)]; however, there is a clear difference in between. In fact, the potential barrier-lowering effect of the BEH-HO indicates that a stiffness–softening effect strengthens around the potential saddle barrier, making the two potential wells shallower. Accordingly, the intrawell resonance curve of the BEH-HO and its local bifurcation points are shifted towards a lower frequency band.

As shown in Figs. 3.10(a) and (b), the SN1 bifurcation of the BEH-HO occurs at a lower frequency than that of the CBEH. It is obvious that the BEH-HO possesses a broader frequency band of interwell motion, bounded by two bifurcation points SN1 and SN2, where intrawell motion does not coexist. Such potential barrier-lowering effect is also beneficial for harvesting relatively weak vibration energy, as observed in Fig. 3.11.

The benefits of lowering the potential energy barrier in BEHs generally include the reduction of threshold excitation intensity required for triggering interwell motion as well as the increase in frequency bandwidth. This is because the threshold excitation intensity is directly related to the potential energy barrier. It is not easy to quantitatively evaluate the threshold excitation intensity of a BEH owing to the lack of standard metrics. However, as shown in Fig. 3.11 below, we can observe that the interwell motion of the BEH-HO tends to be initiated at weaker excitation compared with that of the CBEH.

Particularly, the forward sweep responses for the tip deflections of the BEH-HO and CBEH are obtained with three different base accelerations of 0.4 g, 0.49 g, and 0.57 g. At the base acceleration of 0.4 g, both systems exhibited the intrawell motion. However, when the base acceleration is increased up to 0.49 g, the interwell motion was triggered for the BEH-HO. Meanwhile, for the CBEH, only intrawell motions was observed. As the base acceleration at 0.57 g, both systems exhibited interwell motion. However, it is obvious that the BEH-HO performed a broader bandwidth compared to the CBEH. This results reaffirmed the merits of the BEH-HO for the sake of harvesting energy at weak excitation condition.

Figure 3.11 Forward sweep responses for the tip deflections of the BEH-HO (first column) and CBEH (second column), obtained with three different base accelerations of 0.4 g (first row), 0.49 g (second row), and 0.57 g (third row). The values of all other parameters used in this simulation were same as those used in Fig. 3.10.

3.5.3 Broadening effect of operating frequency band

According to a previous report [29], points SN1 and SN2 are known to be the lower and upper bounds of the operating frequency band, respectively, and provide more accurate predictions than Melnikov's criteria. In this sense, the BEH-HO system is better-adapted to energy harvesting from low frequency vibrations than CBEHs. Figure 3.12 shows variations in (a) the lower (SN1) and upper (SN2) bounds of operating frequency band and (b) the resulting frequency bandwidth with respect to base acceleration.

Figure 3.12 Variations in (a) the lower and upper bounds of operating frequency band and (b) the frequency bandwidth with respect to base acceleration.

The results for the BEH-HO obtained with several different values of ω_e are compared with the results obtained for the CBEH. In Fig. 3.12(a), the upper bounds where SN2 bifurcations occur are denoted by the lines without points, and the lower bounds where SN1 bifurcations occur are denoted by the lines with points. In this figure, the frequency at every SN1 bound for both the BEH-HO and CBEH tends to decrease with increasing excitation intensity. However, owing to the potential barrier-lowering effect, each SN1 bound for the BEH-HO is placed, to some degree, at a lower frequency than the corresponding value for the CBEH.

This frequency shift in the SN1 bound of the BEH-HO against the CBEH depends on the natural frequency of the external magnet oscillator; as seen in Fig. 4(a), the smaller the value of ω_e , the greater the frequency shift. A higher compliance of the external oscillator must enhance its half-sine motion and further reduce the potential saddle barrier experienced by the beam oscillator; therefore the SN1 bound is shifted further towards lower frequencies with a corresponding extension of the total operating frequency band. However, an excessive motion of the external oscillator may also produce a significant frequency shift of the SN2 bound, causing a significant reduction of the operating frequency band [as shown in Fig. 3.12(a) in the case when ω_e is 70 rad/s]. Therefore, the external magnet oscillator should be well-designed, allowing the frequency shift of the lower bound but preventing the frequency shift of the upper bound, such that the total operating frequency band of the BEH-HO becomes broader than for the CBEH. The results presented in Fig. 3.12(b) show that the bandwidth of the proposed BEH-HO increased by 3.43–4.47 Hz (or by 114–545%) compared to that of the CBEH, depending on the intensity of base excitation.

To validate the enhanced frequency band of the BEH-HO, the voltage responses generated by the BEH-HO and CBEH were experimentally obtained with several base accelerations, and the results are compared, as shown in Fig. 3.13. The output voltage responses obtained with a base acceleration of 0.52 g [Figs. 3.13(a) and 3.13(b)] clearly reveal that the interwell motion of the BEH-HO tends to occur more easily than that of the CBEH, in agreement with the theoretical predictions [Figs. 3.10(a) and 3.10(b)]. Thus, it can be converted to a high electrical voltage more efficiently.

Figure 3.13 Experimental comparison of the generated output voltages from the BEH-HO (first column) and the CBEH (second column), obtained when the base acceleration is given at 0.52 g (first row), 0.6 g (second row), and 0.7 g (third row).

With increasing base acceleration, the lower bound frequency (at point SN1 in Fig. 3.13) of the operating frequency band decreases in the experimental measurement: 9.6 Hz, 9.2 Hz, and 8.3 Hz for the BEH-HO [in Figs. 3.13(a), (c),

and (e), respectively], and 13.3 Hz, 12.7 Hz, and 12.0 Hz for the CBEH [in Figs. 3.13(b), (d), and (f), respectively]. The lower bound frequencies of the BEH-HO and CBEH are very similar in their rate of decrease but deviate from each other by an approximately constant value of 3.6 Hz.

As the upper bound frequencies (at point SN2 in Fig. 3.13) are indistinguishably similar to within an error of $O(10^{-2})$, this constant frequency deviation in the lower bound directly contributes to a broader operating frequency band of the proposed BEH-HO, which is well-matched by the theoretical predictions displayed in Fig. 3.12.

Another jump-up phenomenon can be seen at around 7.14 Hz in the results of the CBEH. This phenomenon is related to the secondary resonance – specifically, to the super-harmonic resonance of order 2. This type of secondary resonance can also trigger the large-amplitude motion of the CBEH, but its bandwidth is known to be narrow, when compared to that of the primary resonance. Such a weak effect of the secondary resonance was not discussed because it was out of the main scope of the research on the BEH-HO.

It should be noted that the enhanced broadband performance of the BEH-HO (discussed above with Figs. 3.9 – 3.13) is not guaranteed if x-directional ambient excitation exists, because it could terminate the normal operation of the BEH-HO. Further studies on the nonlinear dynamics of the BEH-HO under multidirectional excitation are needed for general applications.

3.6 Conclusion

In this chapter, a horizontal auxiliary magnet oscillator with a mechanical rectifier was introduced into the BEH system, resulting in the broadening of the frequency band for high-energy orbital motion without significant changes to the

global dynamics of the system (e.g., an amplitude reduction of interwell motion or an overall shift of operating frequency band). Mathematical modeling and simulation provided clear and understandable insights into the beneficial effects of the mechanically rectified motion of the auxiliary oscillator in the BEH-HO.

It was also supported theoretically and experimentally that the half-sine oscillation of the external magnet helped the bifurcation SN1 of intrawell motion to occur at a lower frequency, while keeping the global bifurcation SN2 of interwell motion nearly unchanged. Accordingly, the voltage output response of the BEH-HO was directly comparable to that of the CBEH, which clearly demonstrated the broader operating frequency band of the BEH-HO; specifically, broader by 114–545%, depending on excitation intensity, in this chapter.

In addition, the results and discussion presented in this chapter indicate that the local modification of the potential well configuration could be a potential design approach for enhancing energy harvesting performance of the BEH, possibly leading to a branch of research in this area.

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Chapter 4**

Bistable energy harvester with a vertical auxiliary magnet oscillator: Phase-dependent dynamic potential energy

Following Chapter 3, Chapter 4 covers the research on the BEH with a vertical auxiliary magnet oscillator (BEH-VO). Unlike the BEH-HO, the external magnet of the BEH-VO is connected to a cantilever beam structure. And thus, the system is considered as magnetically coupled oscillators. The BEH-VO has been recently considered as a promising compact structure to integrate multiple bistable energy harvesters (BEHs). However, its design is somewhat complicated owing to its varying potential energy pattern, which has yet to be fully investigated. In view of this challenge, this chapter introduces and thoroughly examines the concept of phase-dependent dynamic potential in the BEH-VO to explain the underlying principle of the complicated dynamics of the system.

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4.1 Introduction

As discussed in Chapter 2, bistable energy harvester (BEH) is known to have a double-well potential energy function, and its broadband performance can be achieved by high-energy orbit motion or interwell motion. However, as reviewed in Section 2.5.1, this potential well escape phenomenon occurs only when the kinetic energy of the system is large enough to surmount the potential barrier between the two wells. For this reason, many theoretical and experimental studies have been devoted to the performance evaluation of BEHs under various types of ambient vibration sources such as swept sine excitations [1, 2] and random noises [3, 4]. Concurrently, a tristable or quadstable energy harvester [5, 6] has been proposed as an extended version of a BEH to reduce the potential barriers (thus, the threshold excitation intensity), and several theoretical and experimental studies [5-10] showed their beneficial effect on the performance improvement over BEHs, especially under low-intensity ambient excitation.

Recently, multi-degree-of-freedom (MDOF) BEHs [11-13] have also drawn attention from researchers that attempted to further enhance the energy harvesting performance in a broader operating frequency bandwidth or under multidirectional vibration sources. Particularly, the bistable energy harvester with a vertical auxiliary magnet oscillator (BEH-VO) should be a potential candidate for compact arrangement of BEH chains, as both bistable structures and noncontact couplings between multiple oscillators can be formed by the proper use of magnetic forces.

However, the design of the BEH-VO is not straightforward owing to its complicated dynamics. The potential merits of BEH-VO were presented only in a few reports [11-13]. But deep physical insights into their nonlinear dynamics, which arise from the varying potential energy pattern with the relative motion of two magnet oscillators, are not provided.

This chapter presents, for the first time, a comprehensible approach to the nonlinear dynamics of the BEH-VO at the first and second primary resonances, introducing the concept of the phase-dependent dynamic potential energy.

4.2 Design and mathematical modelling of BEH-VO

The BEH-VO is composed of two bimorph cantilever beam oscillators with permanent magnets at the free end of each beam, as shown in Fig. 4.1. The two tip magnets are arranged to face each other with same polarities. When the separation distance between two tip magnets decreases, the magnetic repulsive force becomes large. Accordingly, the two beam oscillators become mutually bistable.

Figure 4.1 Schematics of the bistable energy harvester with a vertical auxiliary magnet oscillator (BEH-VO).

In this chapter, it is assumed for simplicity that the two oscillators have the same geometric dimensions and material properties, except for the lengths of the beams. Because two different lengths are used for the two beams (as illustrated in Fig. 4.1), the BEH-VO system possesses two distinct frequency bands for the first and second primary resonances. Thus, this geometric dimension of the BEH-VO is in accordance with the main design objective of multi-modal VEHs to harvest vibration energy in multiple frequency bands.

Table 4. 1 Geometric and material properties of the present system

Parameter	Value
Substrates	
Length	70 mm (47 mm)*
Width	10 mm
Thickness	0.3 mm
Density	7850 kg/m ³
Young's modulus	200 GPa
Piezo-polymer layers	
Length	70 mm (47 mm)*
Width	10 mm
Thickness	0.052 mm
Density	1780 kg/m ³
Young's modulus	3 GPa
Piezo-strain constant	−23 pm/V
Permittivity	110 pF/m
Magnet	
Length	2 mm
Height	10 mm
Thickness	6 mm
Magnetization	9×10 ⁵ A/m

* The parameter value in parentheses is the length of the external bimorph beam.

The details of geometric dimension and material properties in the design are summarized in Table 4.1. The beam lengths are 70 mm and 47 mm for the left (main beam) for the right (external beam) beams, respectively. The piezoelectric layers, with a very thin thickness of 0.052 mm, are laminated on both sides of the beams. These piezoelectric layers play the role of the energy conversion. The two tip magnets have the same mass of 0.876 g and are identical with the size of

$2 \times 10 \times 9 \text{ mm}^3$. The separation distance is chosen arbitrarily within the realistic range. There was no particular reason to use the distance d of 11.45 mm but merely to illustrate the mechanism of the proposed BEHs in the best way.

A mathematical model of the BEH-VO is built based on the energy approach presented in Chapter 3. The governing field equations and boundary conditions for two bimorph cantilever beams are derived on the basis of the Euler–Bernoulli beam theory and linear piezoelectricity [14]. Then, the fundamental mode approximation is applied to the field differential equations in order to obtain the lumped parameter model. Such a derivation process is somewhat complicated but straightforward, and its detailed steps can be found in Chapter 3. Noted that the effect of the lateral oscillation is ignored in the theoretical analysis by assuming the oscillations of the beams are only in the vertical direction.

The general form of the magneto-electro-mechanical oscillator model of the BEH-VO can be expressed as,

$$\ddot{w}_i + 2\zeta_i \omega_i \dot{w}_i + \omega_i^2 w_i - \kappa_{bi} V_{ei} + \gamma_i F_i = -\alpha_i \ddot{z}_b, \quad i = 1, 2, \quad (4.1a)$$

$$\dot{V}_{ei} + \rho_i V_{ei} + \kappa_{ei} \dot{w}_i = 0, \quad i = 1, 2, \quad (4.1b)$$

where index i is used to denoting the quantities of the two beam oscillators on the left ($i = 1$) and right ($i = 2$) in Fig. 4.1; w_i and V_{ei} are the tip deflection and the output voltage across the electrical resistance, respectively; ζ_i and ω_i are the damping ratio and the natural frequency, respectively; ρ_i is the cut-off frequency; κ_{bi} and κ_{ei} are the electromechanical coupling coefficients; $\ddot{z}_b$ is the base acceleration; and F_i is the magnetic repulsive force.

The magnetic force is derived by taking the first derivative with respect to time of the interaction energy between two permanent magnets as,

$$F_i = \frac{\partial U_m}{\partial w_i} \quad (4.2)$$

where U_m is the magnetic interaction energy which can be obtained by using the magnetic charge model [5].

The analytical expression of U_m is acquired with the coordinate system given in Fig. 4.2. For further simplification, the magnetic poles of each magnet are approximated to the equivalent point charges. The magnetic field at a point in space can be evaluated by taking the summation of the magnetic fields generated by other point charges. In this figure, any distortion of the beams in the x-y plane or the x-axis did not occur.

Figure 4.2 The geometric configuration of two tip magnets with the coordinate system.

Accordingly, for the present system, one magnetic dipole (which corresponds to Magnet 1 in Fig. 4.2) can be considered to be subjected to an external magnetic field applied by another magnetic dipole (Magnet 2 in Fig. 4.2). Thus, when the magnets oscillate, the magnetic potential energy that the system acquires is given by,

$$U_m = -\int_V \mathbf{M}_1 \cdot \mathbf{B}_{\text{ext}} dv \approx -\int_l (M_s A_p \mathbf{e}_1) \cdot \left(\frac{\mu_0}{4\pi} \sum_{j=1,2} Q'_j \frac{\mathbf{x} - \mathbf{x}'_j}{|\mathbf{x} - \mathbf{x}'_j|^3} \right) dl, \quad (4.3)$$

where μ_0 is the permeability of free space; $\mathbf{M}_1 (= M_s \mathbf{e}_1)$ is the magnetization vector of Magnet 1; A_p is the cross-sectional area of the magnet or its pole face

area; $\mathbf{B}_{\text{ext}}$ is the magnetic field generated by Magnet 2; Q'_j denotes the equivalent magnetic charge of Magnet 2 given by $(-1)^j M_s A_p$ at the negative and positive poles ($j = 1$ and 2 , respectively); and $\mathbf{x}'_j$ is the position vector of the charge Q'_j .

By manipulating Eq. (4.3) with the geometric configuration of the system presented in Fig. 4.2, the magnetic potential energy can be obtained in the following form:

$$U_m = \frac{\mu_0}{4\pi} \sum_{j,k=1,2} \frac{Q_k Q'_j}{\sqrt{G_{jk}^2 + H_{jk}^2}} \quad (4.4)$$

$$\text{with } \begin{cases} G_{jk} = d_s + h_{mk} \theta_1 + h_{mj} \theta_2 \\ H_{jk} = w_1 - w_2 + l_m \theta_1 - l_m \theta_2 + h_{mk} - h_{mj} \end{cases}$$

where $Q_k = (-1)^k M_s A_p$; $h_{mj} = (-1)^j h_m$; $h_{mk} = (-1)^k h_m$; l_m and h_m are the half-length and half-height of the magnets, respectively; and θ_i ($i=1,2$) is the rotation angle of the beam at the free end.

The magnetic repulsive forces are then calculated and the analytical expression of these forces are provided as follows,

$$F_i = -\frac{dU_m}{dw_i} = -\frac{\mu_0}{4\pi} \sum_{j,k=1,2} Q_k Q'_j \left(\frac{G_{jk} (h_{mk} \Phi_i) + H_{jk} (1 + l_m \Phi_i)}{(G_{jk}^2 + H_{jk}^2)^{3/2}} \right), i=1,2, \quad (4.5)$$

$$\text{with } \begin{cases} G_{jk} = d_s + h_{mk} \theta_1 + h_{mj} \theta_2 \\ H_{jk} = w_1 - w_2 + l_m \theta_1 - l_m \theta_2 + h_{mk} - h_{mj} \end{cases}$$

and where Φ_i , $i = 1, 2$ are the constant coefficients.

The total potential energy of the system model is evaluated by the summation of the strain energy and magnetic potential energy, as follows:

$$U_{tot} = \sum_{i=1,2} \frac{1}{2} k_i w_i^2 + U_m, \quad (4.6)$$

where $k_i, i = 1, 2$ are the equivalent spring constants of the beams.

The potential energy function of the BEH-VO is a two variable function of the relative position of the two oscillators, forming a high-dimensional potential energy surface. This potential energy function is a state function by definition and thus should not be misunderstood as a path-dependent function. In fact, only an apparent potential energy path that the BEH-VO experiences during oscillation varies with the excitation intensity and frequency on the prescribed potential energy surface.

In this chapter, such apparent potential energy paths of the BEH-VO system are theoretically and experimentally examined along with its nonlinear resonant behavior in order to investigate the underlying principles of its complicated dynamics.

4.3 Numerical simulation and frequency responses experiment of BEH-VO

4.3.1 Numerical dynamic simulations

For numerical simulations, second-order differential equations are transformed

into an autonomous system of first-order differential equations. Eqs. (4.1a) and (4.1b) can be written by using state vector $\mathbf{x} = [\dot{w}_1, w_1, V_{e1}, \dot{w}_2, w_2, V_{e2}, \Theta]^T$ in a more convenient matrix-vector form:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t) = \begin{bmatrix} -2\zeta_1\omega_1x_1 - \omega_1^2x_2 + \kappa_{b1}x_5 - \gamma_1 \frac{\partial U_m}{\partial x_2} - \alpha_1 A \cos(\Theta) \\ x_1 \\ -\rho_1x_3 - \kappa_{e1}x_1 \\ -2\zeta_2\omega_2x_4 - \omega_2^2x_5 + \kappa_{b2}x_6 - \gamma_2 \frac{\partial U_m}{\partial x_5} - \alpha_2 A \cos(\Theta) \\ x_4 \\ -\rho_2x_6 - \kappa_{e2}x_4 \\ \Omega \end{bmatrix}, \quad (4.7)$$

where Θ and Ω are the cyclic variable and the excitation angular frequency, respectively. The dynamic simulations are performed using the direct numerical integration of Eq. (4.7) and the stationary periodic solutions are obtained after a large number of forcing cycles (more than 10^3 cycles), such that the steady-state condition is guaranteed. For all the numerical simulations, the following parameter values are used: $\zeta_i = (0.01, 0.01)$ for $i = 1$ and 2 , respectively, $\omega_i = (1.73 \times 10^2, 3.30 \times 10^2)$, $\kappa_{bi} = (-1.89 \times 10^{-3}, -3.13 \times 10^{-3})$, $\rho_i = (1.32 \times 10^2, 1.96 \times 10^2)$, $\kappa_{ei} = (-3.34 \times 10^3, -7.49 \times 10^3)$, $\alpha_i = (1.17, 1.11)$, and $\gamma_i = (7.47 \times 10^2, 8.21 \times 10^2)$. All of these parameter values are determined during the process of deriving the nonlinear oscillator model (specifically, the process of discretizing the governing field equations for the two bimorph beams). The geometric dimensions and material properties used to calculate the parameter values are listed in Table 4.1. For the purpose of demonstrating the phase-dependent dynamic potential, the geometric dimensions of the BEH-VO system were chosen arbitrarily, within realistic ranges, based on the geometric configuration shown earlier in Fig. 4.1. The material properties of a stainless steel and a

piezoelectric polyvinylidene fluoride (PVDF) are used for the metal substrates and piezo-layers of the bimorph beams. A resistance of $10 \text{ M}\Omega$ is used for external electrical loads R_1 and R_2 .

The stability analysis for the stationary oscillation is performed based on the Floquet theory [15]. To this end, following monodromy matrix $\mathbf{M}$ of stationary oscillation $\mathbf{x}_s(t)$ is constructed as,

$$\mathbf{M} = \prod_{i=N}^1 \exp \left[\int_{t_{i-1}}^{t_i} \mathbf{J}(t) dt \right] \quad \text{with} \quad \mathbf{J} = \left[\frac{\partial \mathbf{f}(\mathbf{x})}{\partial \mathbf{x}} \right]_{\mathbf{x}=\mathbf{x}_s}, \quad (4.8)$$

where $\mathbf{J}$ is the Jacobian matrix and N is the number of equally spaced subintervals (i.e., $t_i - t_{i-1}$) during one period. The Floquet multiplier μ can be obtained by solving the eigenvalue problem of $\mathbf{M}$. The destabilization of $\mathbf{x}_s$ occurs when the largest magnitude of μ becomes 1, with certain bifurcation phenomena: saddle-node or symmetry-breaking bifurcation for $\mu = +1$, period-doubling bifurcation for $\mu = -1$, and Neimark–Sacker bifurcation for $|\mu| = 1$ and $\text{Im}[\mu] \neq 0$. In this chapter, each type of bifurcations is identified in the frequency response results by examining the variations in the Floquet multipliers with the change in the excitation frequency.

4.3.2 Frequency responses experiment of BEH-VO

The prototype of the BEH-VO is presented in Fig. 4.3(a). The two bimorph cantilever beams on the left and right have the same geometric dimensions, except for their lengths. The detailed dimensions of these beams are given in Table 4.1. For each bimorph beam, piezo-polymer layers are fully laminated on the upper and lower surfaces of a metal substrate. These piezo-polymer layers are connected to

an external electrical load. The two beams are attached to two manual stages as shown in Fig. 4.3(a). These two stages with side-mounted micrometer heads allow for accurate adjustments of the distance between the main and the external beams. The experimental set up is designed to minimize the lateral oscillation of the magnets.

Two identical neodymium magnets are used as the tip magnets. As shown in Fig. 4.3(b), the base frame of this prototype is fastened to a vibration exciter (Type 4808, B&K) connected to a power amplifier (Type 2719, B&K). A vibration controller (VR 8500, Vibration Research) is used to provide a harmonic voltage signal for the power amplifier. The beam deflections and the base displacement are simultaneously measured by a pair of laser displacement sensors (optoNCDT 1610, Micro-Epsilon). All output signals are collected by a data acquisition system (NI 9239, National Instruments). A LabVIEW platform is built to collect and pre-process the data collected from the NI-DAQ. All the experimental procedures are similar to the case with the BEH-HO in Chapter 3.

Figure 4.3 (a) Prototype of the BEH-VO and (b) experimental set-up for frequency response experiment.

4.4 Results and discussions

4.4.1 Static bistability of BEH-VO

A stationary bifurcation analysis has been performed for the BEH-VO. The separation distance between two magnets is chosen to be the bifurcation parameter. The bifurcation diagram was obtained by solving the static-state equations, $\omega_i^2 x_i + \gamma_i f_M = 0$, $i = 1, 2$ – which are deduced from governing equations 4.1(a), (b) and (c) by eliminating all the time derivative terms. Figure 4.4(a) shows the bifurcation diagram of the equilibrium states for main beam and external beam with respect to the separation distance d . The separation distance is chosen as the bifurcation parameter. When the separation distance decreases down to a critical value (at point PF in Fig. 4.4(a)), the symmetrical bistability (with two stable equilibrium branch symmetrically separated by an unstable

equilibrium branch) appears. This phenomenon is called the supercritical pitchfork (PF) bifurcation.

Through this bifurcation point, the stable trivial solution (ST) loses its stability and becomes unstable (UT), and concurrently, the two pairs of stable nontrivial solutions, $(\bar{w}_1^{(+)}, \bar{w}_2^{(-)})$ and $(\bar{w}_1^{(-)}, \bar{w}_2^{(+)})$, are bifurcated. In details, the bifurcation diagram can be explained by the effect of the interaction forces between two tip magnets. In particular, a magnetic repulsive force is exerted mutually on the two permanent magnets, and it becomes larger as the separation distance decreases. When the magnetic force reaches the certain intensity level (at point PF in Fig. 4.4(a)), structural instability occurs in the equilibrium state(s) of the oscillators. The two beams buckle at the same time and these bucklings accompany with a pitchfork bifurcation phenomenon.

The supercritical pitchfork bifurcation leads to the destabilization of the trivial equilibrium (from ST to UT in Fig. 4.4(a)) and the generation of the two stable nontrivial equilibria (nonzero branches in Fig. 4.4(a)), which indicates the transition of the static stability from mono to bistability. The dynamic characteristic of the system will obviously change according to the change of the static stability. More particularly, the phases and amplitudes of the oscillations are shifted depending on the new stability's state. Each stable nontrivial equilibrium comprises two components with opposite signs, owing to the magnetic repulsion effect, e.g., a pair of positive $\bar{w}_1$ and negative $\bar{w}_2$ (indicated by $\bar{w}_1^{(+)}$ and $\bar{w}_2^{(-)}$, respectively, in Fig. 4.4(a)) or vice versa ($\bar{w}_1^{(-)}$ and $\bar{w}_2^{(+)}$ in Fig. 4.4(a)). The magnitude of static deflection $\bar{w}_1$ is always larger than that of $\bar{w}_2$ because of a higher compliance of the main beam oscillator.

To clearly demonstrate such static bistability, the total potential energy was evaluated as a function of the deflections of the two beam oscillators when $d = 11.45$ mm. In Fig. 4.4(b), the contour plot of the total potential energy is used for

illustration. As seen in this figure, the points S and C denote the trivial saddle and nontrivial center points, respectively. It can be evidently observed that one stationary potential saddle exists at the trivial midpoint between the two nontrivial potential centers; thereby two skew-symmetric potential wells are formed in three-dimensional space.

Figure 4.4 (a) Bifurcation diagram of the static equilibrium state with respect to separation distance d and (b) the contour plot of the potential energy as a function of deflections w_1 and w_2 , obtained when $d = 11.45$ mm.

As discussed above, the BEH-VO obviously possesses the double-well potential energy function after the supercritical pitchfork bifurcation occurs. The fundamental mechanism of this static bistability seems to be very similar to those of existing conventional bistable oscillators [2, 3]. However, the actual trajectory of the potential energy that the BEH-VO experiences during its oscillation does not follow a certain fixed path on the three-dimensional potential energy surface in Fig. 4.4(b), but unpredictably varies with the relative motion of the two magnets, which makes it difficult to design the BEH-VO. To address this problem, we must study the nonlinear resonant behavior of the BEH-VO system along with the potential energy trajectory.

4.4.2 Out-of-phase and in-phase intrawell resonances

As discussed in Section 2.5.1, all bistable systems exhibit two distinct dynamic motions depending on the intensity of external excitation: small-amplitude intrawell motion or low energy orbit motion (under weak excitation) and large-amplitude interwell motion or high energy orbit motion (under relatively strong excitation). In general, the transition from intrawell to interwell motion (i.e., the potential well escape phenomenon) is likely to occur through intrawell resonances owing to high vibration transmissibility. For this reason, intrawell resonances were considered as the main route to high-energy orbit motion in designing BEHs. For the present BEH-VO, the two primary intrawell resonances exist and the associated vibration modes make differences in the resonant behaviors and potential energy trajectories.

Figure 4.5 presents the first and second intrawell resonant behaviors of deflections w_1 and w_2 obtained when the base acceleration is small ($|\ddot{z}_b| = 2 \text{ m/s}^2$). Without loss of generality, only the intrawell motion within the right potential well is presented in Fig. 4.5 (also, all the result figures hereafter). As observed in Figs. 4.5(a) and (b), the vibration modes at the two resonances appear with the opposite trends of the amplitude ratios. The first resonant mode tends to be of large w_1 and small w_2 (Fig. 4.5(a)), whereas the second resonant mode tends to be of small w_1 and large w_2 (Fig. 4.5(b)). This observation indicates that each of the two oscillators would possess its own frequency range in which it operates with a larger amplitude.

Figure 4.5 Out-of-phase and in-phase intrawell resonant motions obtained with a base acceleration of 2 m/s^2 . (a), (b) The frequency marching responses obtained in the frequency regions for the first and second primary resonances. (c), (d) The stationary time responses of out of phase and in phase, respectively, evaluated at 17.5 Hz and 49 Hz.

The most remarkable feature is the phase difference between the first and second resonant modes that make significant differences in the potential energy trajectories. From a series of simulations, deflections w_1 and w_2 are found to always oscillate out of phase in the frequency region of the first resonance (Fig. 4.5(c)), but almost in phase in the region of the second resonance (Fig. 4.5(d)). These phase differences are evaluated based on the dominating frequency components of the responses in order to roughly see the basic dissimilarities of the two deflections in term of phase.

For the out-of-phase mode, a strong magnetic repulsive force tends to form the potential energy barrier between the two magnets approaching each other, which prohibits the crossing-over of the oscillator motions. Thus, the BEH-VO experiences a strong stiffness softening effect at the first intrawell resonance, and its frequency response tends to follow a typical hysteretic resonant curve (Fig. 4.5(a)).

For the in-phase mode, the two magnets tend to oscillate simultaneously in the same direction. Accordingly, typical bistable characteristics such as the potential barrier and the softening effect are weakened significantly. The resulting resonant curve at the second resonance (Fig. 4.5(b)) is slightly bended without visible hysteresis and covers only a narrow frequency band.

The above-mentioned behaviors of the out-of-phase and in-phase resonant modes can be easily identified by tracing the associated potential energy trajectories. As illustrated in Fig. 4.6, the out-of-phase path – illustrated by the blue color – intersects the contour lines of the potential energy along the direction of the skew-symmetric axis. This path is obstructed by the potential saddle where stiffness softening is maximized. Meanwhile, the in-phase path – illustrated by the red line – is almost in the perpendicular direction to the skew-symmetric axis. Evidently, it does not suffer from the potential barrier on its own path. These observations partially explain the dissimilarities of the dynamic behaviors of the out-of-phase and in-phase resonant modes, which will be deeply presented in the next section.

Figure 4.6 The paths intersecting the potential contour lines which are associated with the out-of-phase and in-phase intrawell resonant motions in Fig. 4.5.

4.4.3 Potential well escape phenomena under the out-of-phase and in-phase mode regimes

When external excitation is strong, both the first and second intrawell resonances can lead to the transition into the interwell motion. This is called the potential well escape phenomenon. However, the processes are quite different for each type of resonance. Fig. 4.7 shows numerical simulation results for the interwell motions triggered by the out-of-phase and in-phase modes (the first and second columns, respectively) when the base acceleration is 6 m/s^2 . To demonstrate the difference in the transition processes, the forward and backward frequency-sweep responses (the first and second rows, respectively) are presented in this figure with several bifurcation points of periodic oscillations that can be identified based on the Floquet theory [15]. In Fig. 4.7(a) and (b), points SN1–SN3 denote the saddle-node bifurcations where the jump-up or jump-down phenomenon of periodic oscillation can be observed. In Fig. 4.7(c), point PD means the period-doubling bifurcation, which leads to chaotic motion through a subsequent period-doubling cascade. Point NS is the Neimark–Sacker

bifurcation, at which the periodic interwell motion is destabilized. In Fig. 4.7(d), points SB1 and SB2 indicate the subcritical and supercritical symmetry-breaking bifurcations, respectively. All of these bifurcation point types are classified by solving monodromy matrix to get the corresponding Floquet multipliers – which are presented in section 4.3.1.

Figure 4.7 Interwell motions triggered by (the first column: (a) forward sweep and (c) backward sweep) the out-of-phase and (the second column: (b) forward sweep and (d) backward sweep) in-phase interwell resonances, when the base acceleration is 6 m/s^2 .

The complicated bifurcation structure of the deflection response is observed particularly in the frequency region of the first resonance. For the forward frequency-sweep response in Fig. 4.7(a), periodic interwell motion is initiated with a sudden jump-up phenomenon by the saddle-node bifurcation of the intrawell motion (designated by point SN1), then continues to grow with its amplitude (from SN1 to SN2), and finally jumps down through the chaotic region into the intrawell motion after passing the saddle-node bifurcation of the interwell

motion (SN2).

As for the backward frequency-sweep response in Fig. 4.7(c), the intrawell motion first experiences a period-doubling bifurcation (PD) and a subsequent period-doubling cascade that leads to chaotic motion. In this case, periodic interwell motion emerges after the chaotic motion disappears, and it persists until it becomes destabilized at the Neimark–Sacker bifurcation point (NS in Fig. 4.7(c)).

In fact, such a bifurcation structure in the first resonance is in accordance with those of existing CBEHs [16]. This is because the out-of-phase trajectories of both chaotic and interwell motions always pass through the potential saddle in order to escape from one potential well into another, as seen in Fig. 4.8(a). Thus, fundamental bistable natures are preserved without significant deterioration.

Figure 4.8 (a) Out-of-phase and (b) in-phase trajectories on the contour plot of the potential energy. In (a), the interwell, chaotic, and intrawell paths correspond to the stationary responses obtained at 14, 17, and 20 Hz, respectively, from Fig. 4.7(c). In (b), one interwell and three intrawell paths are obtained at 47.3, 48, 49, and 51 Hz, respectively, from Fig. 4.7(d).

The bifurcation structure of the in-phase oscillation in the second resonance is simpler than that of the out-of-phase oscillation, as shown in Figs. 4.7(b) and (d),

and it comprises one saddle-node and two symmetry-breaking bifurcations.

For the forward frequency-sweep response in Fig. 4.7(b), interwell motion (if exists) would be abruptly activated by the saddle-node bifurcation of the intrawell motion (SN3). However, for the backward case in Fig. 4.7(d), the transition process from the intrawell to interwell motion is smooth and continuous. The asymmetric intrawell motion continuously gains its amplitude with the decrease in the excitation frequency, until it grows into symmetric interwell motion through a supercritical symmetry-breaking bifurcation (SB2). Then, the smoothly initiated interwell motion is terminated at its subcritical symmetry-breaking point (SB1 in Fig. 4.7(d)).

Such bifurcation scenarios in the second resonance are not completely similar to those in the first resonance, since the deflection responses remain almost in phase throughout the transition process. As depicted in Fig. 4.8(b), the intrawell oscillation of the in-phase mode does not seem to move across but gradually climbs during its growth. Then this intrawell motion disappears at the hilltop saddle, colliding with its skew-symmetric counterpart that has grown from the left potential well; instead, the interwell motion of the in-phase mode appears. This interwell motion does not reach deep inside either potential well, only staying around the hilltop saddle, which clearly indicates that the potential saddle does not act as a barrier.

As illustrated in Fig. 4.1(a), it was assumed in this study that the two cantilever beam oscillators of the BEH-VO system had the same geometric dimensions except for their beam lengths. Without any loss of generality, because of the symmetric nature of the system, we considered only the case of the main beam shown in Fig. 4.1(a) being longer than the external beam. Figure. 4.9 shows the out-of-phase/in-phase trajectories on the contour plot of the potential energy obtained for different lengths of the external beam, specifically, 57 mm, 52 mm,

and 42 mm for Cases I, II, and III, respectively. All other geometric dimensions were the same as those listed in Table 4.1. For each of the three cases, the equivalent parameter values for the nonlinear oscillator model (Eq. (4.7)) were calculated and are listed in Table 4.2. The results show that the dynamic behaviors of the system exhibited similar characteristics with different lengths of the external beam.

Figure 4.9 Out-of-phase (first column) and in-phase (second column) trajectories on the contour plot of the potential energy. The potential energy trajectories are obtained for different lengths of the external beam, specifically, (a)(b)(Cases I) 57 mm, (c)(d)(Cases II) 52 mm, and (e)(f) (Cases III) 42 mm. The distance d is set to be 12.1 mm, 11.9 mm, and 11.5 mm for Cases I–III, respectively.

Table 4.2 System parameters of the nonlinear oscillator model for the Cases I–III

Parameter	Case I		Case II		Case III	
	$i=1$	$i=2$	$i=1$	$i=2$	$i=1$	$i=2$
ζ_i	0.01	0.01	0.01	0.01	0.01	0.01
ω_i	1.73×10^2	2.42×10^2	1.73×10^2	2.81×10^2	1.73×10^2	3.95×10^2
κ_{bi}	-1.89×10^{-3}	-2.47×10^{-3}	-1.89×10^{-3}	-2.77×10^{-3}	-1.89×10^{-3}	-3.58×10^{-3}
ρ_i	1.32×10^2	1.62×10^2	1.32×10^2	1.77×10^2	1.32×10^2	2.20×10^2
κ_{ei}	-3.34×10^3	-5.06×10^3	-3.34×10^3	-6.10×10^3	-3.34×10^3	-9.40×10^3
α_i	1.17	1.14	1.17	1.13	1.17	1.09
γ_i	7.47×10^2	7.88×10^2	7.47×10^2	8.04×10^2	7.47×10^2	8.36×10^2

To verify the theoretical observations obtained in Fig. 4.7, the experimental results are collected and presented in Fig. 4.10. A small deviation in critical frequency regions (where the bifurcations occur) from numerical results in Fig. 4.7 and experimental results in Fig. 4.10 is encountered. However, the general tendency is preserved with a good agreement in both results. As in Fig. 4.10(a) and (c), the forward sweep and backward sweep frequency responses in the frequency region of the first resonance (out-of-phase resonance) showed typical hysteresis phenomena of the existing CBEHs. Of which the bistable features result from the existence of the potential saddle barrier and the potential well escape phenomenon. For the case of second primary resonance (in-phase resonance), as in Fig. 4.10(b) and (d), the interwell motion is smoothly established from intrawell resonances, which exhibits the characteristic of a monostable configuration.

Figure 4.10 Experimental results associated with the results in Fig. 4.7 for interwell motions triggered by the out-of-phase (first column) and in-phase (second column) interwell resonances when the base acceleration is 6 m/s^2 .

The bifurcation structures discussed in Figs. 4.7 – 4.10 can consistently apply to the electrical output powers of the BEH-VO. The numerical results and experimental results for electrical output power are provided in Fig. 4.11 and Fig. 4.12. Both results show a similar tendency in the frequency responses.

As shown in Fig. 4.11(a), the out-of-phase interwell motion produces an output power much higher than the chaotic or intrawell motion in a broad frequency band (between NS to SN2); whereas in Fig. 4.11(b), the interwell motion of the in-phase mode is bounded by a narrower band (between SB1 and SB2). However, the intrawell motion of the in-phase mode can also generate a high output power when it continuously grows towards point SB2. Thus, the operating frequency bandwidth of the BEH-VO in the second resonance should be evaluated with

such large-amplitude intrawell motion in addition to the interwell motion.

Figure 4.11 Frequency responses of the electrical output powers generated from the (a) main and (b) external bimorph beams, respectively, in the frequency regions of the (a) out-of-phase and (b) in-phase interwell motions.

These results also show that the external bimorph beam can actually generate a high output voltage in the in-phase mode. This can be explained by the fact that the interwell motion of the in-phase mode does not surmount the potential saddle to traverse from one potential well to another potential well. In other words, the potential saddle does not act as a barrier. Therefore, the kinetic energy is maintained during the oscillation. This boosts the velocity of the external beam oscillation and thus enhance the harvesting performance of the beam. Although the frequency bandwidth of these high energy output region is only limited at around 4 Hz (from 47 Hz to 51 Hz as in Fig. 11(b)), this should be an important consideration when designing the BEH-VO.

Figure 4.12 Experimental verification for frequency responses of the electrical output powers generated from the (a) main and (b) external bimorph beams, respectively, in the frequency regions of the (a) out-of-phase and (b) in-phase interwell motions.

4.4.4 Phase-dependent dynamic potential well

To clearly demonstrate the phase-dependent dynamic potential well of the out-of-phase and in-phase modes, Fig. 4.13 shows the potential well configurations that the motions of the out-of-phase and in-phase modes experience, obtained at six different excitation frequencies in Fig. 4.7. In this figure, for illustration, deflections w_1 and w_2 are chosen as the horizontal axes for the out-of-phase and in-phase modes, respectively. This result provides the most comprehensible demonstration of why the nonlinear dynamics of the BEH-VO is not the same in the first and second resonances.

Figure 4.13 Dynamic potential well configurations for (the first row) the out-of-phase and (the second row) in-phase modes.

The apparent dynamic potential wells illustrated in 4.13(a)–(f) are obtained at six different excitation frequencies used in Fig. 4.7. The solid and dashed lines denote the parts of positive and negative velocities in the stationary responses, respectively.

As shown in Figs. 4.13(a)–(c), the intrawell, chaotic, and interwell motions of the out-of-phase mode tend to form single-well (Fig. 4.13(a)), chaotic double-well (Fig. 4.13(b)), and symmetric double-well potential configurations (Fig. 4.13(c)), respectively. Thus, the underlying principle of the potential well escape phenomenon in the first resonance follows the typical double-well dynamics of CBEHs [26].

On the contrary, for the second resonance, the potential energy trajectories do not take any double-well configuration. As depicted in Figs. 4.13(d)–(f), only single-well potential configurations occur even for the interwell motion as well as the intrawell motion. Consequently, it can be concluded that the present BEH-VO

acts as a bistable system in the out-of-phase mode, but behaves as a monostable system in the in-phase mode.

To further support our conclusion concerning the phase-dependent dynamics of the BEH-VO system at primary resonances, additional simulation results (dynamic potential well configurations) were obtained for several different sets of geometric dimensions and are presented in Figs. 4.14. These results are associated with the potential energy trajectories obtained for different lengths of the external beam as shown in Fig. 4.9, specifically, (a)(b)(Cases I) 57 mm, (c)(d)(Cases II) 52 mm, and (e)(f) (Cases III) 42 mm. The distance d is also set to be 12.1 mm, 11.9 mm, and 11.5 mm for Cases I–III, respectively.

Figure 4.14 Dynamic potential well configurations for (the first column) the out-of-phase and (the second column) in-phase modes.

As Figs. 4.9 and 4.14 show, the phase-dependent dynamic potential of the present BEH-VO system leads to double-well dynamics for the out-of-phase mode, as in Fig. 4.14 (a)(c) and (e), at the first resonance but single-well dynamics for the in-phase mode, as in Fig. 4.14 (b)(d) and (f), at the second primary resonance. All of the above simulation results confirm that the conclusion remains unchanged

with the variation in the geometric parameters such as the length of the beam and the separation distance.

4.5 Conclusions

Using the vertical auxiliary magnet oscillator (VO) to further enhance the performance of the BEHs in term of harvesting energy and frequency bandwidth is a potential design. However, this design is not straightforward, owing to the complicated dynamics behaviors which result from the magnetic interaction between its movable magnets.

In this chapter, a series of theoretical analyses and experimental verification was performed to investigate the complicated resonant behaviors of the BEH-VO. The results revealed that the BEH-VO possesses two distinct dynamic regimes: the out-of-phase mode in the first resonance and the in-phase mode in the second resonance.

Under the out-of-phase mode regime, the frequency responses of the BEH-VO exhibited the classical double-well dynamics of bistable systems. Accordingly, the periodic out-of-phase interwell motion could generate a high output power in a broad frequency band bounded by the Neimark–Sacker and saddle-node bifurcations.

However, under the in-phase mode regime, although the BEH-VO is statically bistable, both intrawell and interwell motions belong only to the single-well dynamics of monostable systems. In this case, the interwell motion branch originated from the supercritical or subcritical symmetry-breaking bifurcation. Even though this branch is bounded by a narrow frequency band, the large-amplitude intrawell motion could additionally extend the operating frequency bandwidth to a high output power.

As summarized above, this chapter introduces the concept of the phase-dependent dynamic potential in the BEH-VO, for the first time, providing a clear physical insight into its nonlinear resonant behavior, including the bifurcation structures and the associated potential well escape phenomena. This concept is not limited to the present BEH-VO, but can be extended and applied to all magnetically coupled BEHs, of which the potential energies are functions of more than two displacement variables.

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Chapter 5***

Bistable energy harvester with a vertical auxiliary magnet oscillator: Enhanced broadband performance with secondary intrawell resonances

Following the study in Chapter 4, this chapter investigates another feature of the bistable energy harvester with a vertical auxiliary magnet oscillator (BEH-VO). In particular, this chapter focuses on possible routes to high-energy orbit motion (HEOM), from which high electrical power can be generated, in the BEH-VO. For conventional bistable energy harvesters (CBEHs), only primary resonance has been considered important in the design process, because secondary resonances can hardly initiate HEOM. However, for the BEH-VO, the secondary resonances also form separate frequency bands of the HEOM for energy harvesting. Furthermore, with the increase of the excitation intensity, these secondary resonances tend to bridge the gap between the frequency bands of the two primary resonances, significantly enhancing the operating frequency bandwidth (up to 200% with respect to the CBEH used in this chapter). The enhanced broadband performance of the BEH-VO is theoretically and experimentally evaluated and discussed in comparison with the CBEH.

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5.1 Introduction

As being presented in Chapter 2, the bistable energy harvester (BEH) is the earliest and simplest type of multistable energy harvester [1], and its potential-energy function is generally in a double-well configuration that consists of two potential wells and one saddle in between, as illustrated in Fig. 5.1.

As discussed in Section 2.5.1, this potential-energy function produces two distinct dynamic motions of a BEH—intrawell and interwell motions—depending on the excitation intensity [2, 3]. When the ambient excitation is weak, the oscillation of a BEH exhibits small-amplitude intrawell motion or low-energy orbit motion (LEOM) [Fig. 5.1(a)], which is confined to either of the two potential wells by the potential saddle barrier. On the other hand, when the excitation is sufficiently strong for inducing the potential well escape phenomenon [6, 7], the BEH achieves large-amplitude interwell motion or high-energy orbit motion (HEOM) [Fig. 5.1(b)], oscillating across the two potential wells in a broad frequency band. From this interwell motion, the BEHs extract electrical energy, using energy-conversion mechanisms such as piezoelectric, electrostatic, and electromagnetic conversion [4–6].

It was previously believed that the complexity in the dynamic motion of a bistable energy harvester with vertical auxiliary magnet oscillator (BEH-VO) originated from the variable potential-energy function [7, 8] or the varying pattern of the magnetic force [9]. In Chapter 4, it was theoretically proven that the dynamic behaviors of the BEH-VO at primary intrawell resonances can be well-described using the phase-dependent dynamic potential. However, the relationships between the complicated dynamics of the BEH-VO and the resulting output power are not yet completely understood, e.g., the effect of secondary intrawell resonances and/or a combination of intrawell resonances on the energy-harvesting performance.

Figure 5.1 Schematic of the double-well potential of a bistable energy harvester with (a) intrawell motion or LEOM and, (b) interwell motion or HEOM.

In order to fill in this gap, this chapter theoretically and experimentally examines the displacement and output voltage responses of the BEH-VO to investigate the effect of secondary intrawell resonances (in addition to primary intrawell resonances) on the energy-harvesting performance. In the case of relatively weak excitation, it is shown that there are four different intrawell resonances: two primary resonances and the associated two secondary resonances. Subsequently, with the increase of the excitation intensity, these four intrawell resonances are demonstrated to be important routes to the HEOM, which can be converted into a high electric voltage.

In particular, the bridging effect of the secondary resonances between the two primary resonances, which dramatically enhances the operating frequency band of the BEH-VO for energy harvesting, is investigated. The broadband performance of the BEH-VO is evaluated and discussed in comparison with its conventional counterpart.

5.2 Design and mathematical model of BEH-VO

As being described in Chapter 4, the BEH-VO is composed of two cantilever beam oscillators as shown in Fig. 5.2(a). However, to focus on the nonlinear dynamic effects of the system, in this chapter, the external beam oscillator without piezopolymer layers is used. The main beam remains a bimorph beam oscillator covered with two piezoelectric layers for transducing its mechanical vibration to electrical current flowing through the connected electrical circuit. Note that the dynamic behaviors of both BEH-VO systems (in Chapter 4 and in this chapter) are almost identical (if with the same separation distance) because the stiffness of the piezopolymer layers is relatively negligible.

Each of the two beam oscillators carries a permanent magnet at its free end (called a tip magnet in this chapter). The two tip magnets are arranged to face each other, with the same polarities, and accordingly, the resulting magnetic repulsive force acts as an axial force that is mutually exerted on the beams. As the tip magnets move closer to each other (i.e., as the separation distance d in Fig. 5.2 decreases), the magnetic axial force becomes stronger, even enough to lead to a beam-buckling phenomenon. After the buckling occurs, the two beam oscillators become mutually bistable.

Figure 5.2 Schematics of the (a) bistable energy harvester with vertical auxiliary magnet oscillator (BEH-VO) and (b) its conventional counterpart.

As mentioned earlier, the design of BEH-VO in this chapter is similar to the one in Chapter 4, except that the external beam is without piezo-polymer layers. Therefore, the governing equation of the BEH-VO in this chapter can directly be deduced from the Eq (4.1) by removing the corresponding coefficients and equation. The general form of the oscillator model of the BEH-VO given in Fig. 5.2(a) is expressed as,

$$\ddot{w}_1 + 2\zeta_1\omega_1\dot{w}_1 + \omega_1^2w_1 - \kappa_eV_e + \gamma_1f_1 = -\alpha_1\ddot{z}_b, \quad (5.1a)$$

$$\ddot{w}_2 + 2\zeta_2\omega_2\dot{w}_2 + \omega_2^2w_2 + \gamma_2f_2 = -\alpha_2\ddot{z}_b, \quad (5.1b)$$

$$C_p\dot{V}_e + 2R_L^{-1}V_e + 2\kappa_e\dot{w}_1 = 0, \quad (5.1c)$$

where the indices 1 and 2 in the subscript indicate physical quantities for the main and external beams, respectively; w , ζ , and ω are the tip deflection of the beam, the damping ratio, and natural frequency, respectively; γ and α is the constant coefficients; V_e is the voltage across the electrical resistance R_L ; C_p and κ are the equivalent capacitance and electromechanical coupling coefficient, respectively; and z_b is the displacement excitation of the base frame.

In Eqs. (5.1a) and (5.1b), the magnetic forces f_1 and f_2 are given by,

$$f_1 = \frac{\partial U_m}{\partial w_1} \text{ and } f_2 = \frac{\partial U_m}{\partial w_2}, \quad (5.2a)$$

where U_m is the magnetic potential energy, which the BEH-VO system acquires during oscillation, in the form as similar to the one obtained in Chapter 4:

$$U_m = \frac{\mu_0}{4\pi} \sum_{j,k=1,2} \frac{Q_k Q'_j}{\sqrt{G_{jk}^2 + H_{jk}^2}}, \quad (5.2b)$$

$$G_{jk} = d + (-1)^k h_m \theta_1 + (-1)^j h_m \theta_2, \quad (5.2c)$$

$$H_{jk} = w_1 - w_2 + l_m \theta_1 - l_m \theta_2 + (-1)^k h_m - (-1)^j h_m, \quad (5.2d)$$

Here, μ_0 is the permeability of free space; Q and Q' are the equivalent magnetic charges for the tip magnets attached to the left and right beams, respectively; θ_1 and θ_2 are the small rotation angles of the left and right beams, respectively, at their free ends; d is the separation distance between the two tip magnets; and l_m and h_m are the half-length and half-height of the tip magnets, respectively.

The mathematical model for the conventional BEH (CBEH) to be compared with

the BEH-VO can be obtained as a limiting case of Eqs. (5.1) and (5.2) with $w_2 \rightarrow 0$.

The governing equations for the CBEH in this chapter can therefore be obtained as,

$$\ddot{w}_1 + 2\zeta_1 \omega_1 \dot{w}_1 + \omega_1^2 w_1 - \kappa_e V_e + \gamma_1 \frac{dU_m^*}{dw_1} = -\alpha_1 \ddot{z}_b, \quad (5.3a)$$

$$C_p \dot{V}_e + 2R_L^{-1} V_e + 2\kappa_e \dot{w}_1 = 0, \quad (5.3b)$$

where U_m^* is the magnetic potential energy of the CBEH system acquires during oscillation,

$$U_m^* = \frac{\mu_0}{4\pi} \sum_{j,k=1,2} \frac{Q_k Q_j'}{\sqrt{\left(d + (-1)^k h_m \theta_1\right)^2 + \left(w_1 + l_m \theta_1 + (-1)^k h_m - (-1)^j h_m\right)^2}}, \quad (5.4)$$

where all the coefficients are the same as in Eq. (5.2).

In this chapter, the CBEH refers to a conventional bistable energy harvester shown in Fig. 5.2(b), which is buckled by magnetic repulsive force (not by mechanical axial force).

5.3 Numerical simulations and experiments

To this end, the differential equations given in Eqs. (5.1a)–(5.1c) were rewritten in the following state vector equation for numerical integration:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t) = \begin{bmatrix} -2\zeta_1\omega_1x_1 + \omega_1^2x_2 - \kappa_e x_5 + \gamma_1 f_1(\mathbf{x}) - \alpha_1 \ddot{z}_b(t) \\ x_1 \\ -2\zeta_2\omega_2x_3 - \omega_2^2x_4 - \gamma_2 f_2(\mathbf{x}) - \alpha_2 \ddot{z}_b(t) \\ x_4 \\ -2R_L^{-1}C_p^{-1}x_5 - 2\kappa_e C_p^{-1}x_1 \end{bmatrix}, \quad (5.5)$$

where the state vector $\mathbf{x} = [x_1, x_2, x_3, x_4, x_5]^T = [\dot{w}_1, w_1, \dot{w}_2, w_2, V_e]^T$ and the base excitation is given by $\ddot{z}_b = A \sin \Omega t$ with the amplitude A and frequency Ω . The solutions of the displacement and voltage responses were obtained by the direct numerical integration of the state equation given in Eq. (5.5). The geometric and material parameters used in the simulation are listed in Table 5.1.

Table 5.1 Geometric and material properties of the BEH-VO system.

Parameters	Value
Separation distance	11.7 mm
<i>Substrate</i>	<i>Bimorph beam</i> <i>External beam</i>
Length	68 mm 53 mm
Width	10 mm
Thickness	0.3 mm
Density	7,850 kg/m ³
Young's modulus	200 GPa
<i>Piezoelectric layers</i>	
Length	68 mm
Width	10 mm
Thickness	0.05 mm
Density	1,780 kg/m ³
Young's modulus	3 GPa
Piezo-strain constant	−23 pm/V
Permittivity	110 pF/m
<i>Magnets</i>	

Length	2 mm
Height	10 mm
Thickness	6 mm
Magnetization	8.8×10^5 A/m

In this chapter, numerical simulation results are compared with experimental measurements for validation. Fig. 5.3 shows the prototype of the BEH-VO with a bimorph cantilever and an external beam without the lamination of piezo-layers. The experimental setup is not different from the one in Chapter 4 and is previously shown in Fig. 4.3(b). Therefore, it is not presented here.

For the bimorph beam oscillator, a stainless steel substrate with dimensions of $68 \times 10 \times 0.3$ mm³ is entirely covered on its upper and lower surfaces with piezoelectric polyvinylidene fluoride 50 μ m thick. The two piezo-layers are connected to a load resistance of 1 M Ω . On the other hand, for the external beam oscillator, a stainless steel substrate with dimensions of $53 \times 10 \times 0.3$ mm³ is used without the laminating of piezo-polymer layers. Neodymium magnets with dimensions of $2 \times 10 \times 6$ mm³ are attached to the free ends of the beams.

As described earlier in Chapter 4, the base frame of the BEH-VO prototype is fastened to a vibration exciter (Type 4808, B&K) connected to a power amplifier (Type 2719, B&K). A vibration controller (VR 8500, Vibration Research) is used to provide a harmonic voltage signal for the power amplifier. The beam deflections and the base displacement are simultaneously measured by two laser displacement sensors (optoNCDT 1610, Micro-Epsilon). All output signals are collected by a data acquisition system (NI 9239, National Instruments) and are analyzed using LabVIEW software.

Figure 5.3 Prototype of the BEH-VO.

5.4 Results and discussion

5.4.1 Primary and secondary intrawell resonances

Generally, the dynamic motion of the BEH-VO is more complicated than that of the CBEH, owing to the larger variation in the magnetic interaction during oscillation [7]. In Chapter 4, it was revealed that the BEH-VO have two different dynamic regimes at the first and second intrawell resonances; the out-of-phase and in-phase mode regimes, respectively. These regimes well describe the nonlinear motions of the BEH-VO and the associated energy-harvesting characteristics at the primary intrawell resonances, but they have not been considered in the study of secondary intrawell resonances. This chapter investigates the nonlinear dynamic behaviors of the BEH-VO system induced by secondary intrawell resonances in addition to the primary intrawell resonances, in order to evaluate the effect of the secondary resonances on energy-harvesting performance.

Fig. 5.4 shows the displacement responses of the (a) BEH-VO and (b) CBEH with respect to the excitation frequency, which were numerically obtained with the relatively weak harmonic excitation of the base frame, and (c, d) the associated experimental measurements. The location of the nontrivial static equilibrium of the bimorph oscillator is set to be identical for the BEH-VO and CBEH for comparison. In this chapter, the static equilibrium states of the systems were used as initial

conditions for dynamic simulations. The symbols ω_{n1} and ω_{n2} denote the first and second natural frequencies of the BEH-VO, obtained with the linearized system at nontrivial equilibrium, and the symbol $\bar{\omega}_n$ is the natural frequency of the CBEH. In Figs. 5.4(a) and (c), four different types of intrawell-resonant behaviors of the BEH-VO are observed: (R1) the first primary resonance at $\Omega = \omega_{n1}$, (R2) the associated subharmonic resonance of order 1/2 at $\Omega = 2\omega_{n1}$, (R3) the second primary resonance at $\Omega = \omega_{n2}$, and (R4) the associated superharmonic resonance of order 2 at $\Omega = 0.5\omega_{n2}$. The resonances (R3) and (R4) do not exist in the CBEH.

Figure 5.4 Amplitudes of the displacement responses for the (a) BEH-VO and (b) CBEH, respectively, numerically obtained with base accelerations of 0.18 and 0.05 g. The associated experimental measurements are given in (c) and (d). The symbols ω_{n1} and ω_{n2} denote the first and second natural frequencies of the BEH-VO, obtained with the linearized system at nontrivial equilibrium, and the symbol $\bar{\omega}_n$ is the natural frequency of the CBEH.

The four resonances of the BEH-VO can be sorted and characterized into out-of-phase and in-phase resonances [as shown in Fig. 5.5(a)] by investigating the relative motions of two oscillators. As presented in Chapter 4, the harmonic motions of the two magnets appear out-of-phase at the first primary resonance [Fig. 5.5(b)], but almost in-phase at the second primary resonance frequency [Fig. 5.5(c)]. For the out-of-phase resonance (R1), the magnetic repulsive force becomes stronger as the two magnets approach each other towards the zero positions; thus, the BEH-VO experiences a certain level of the potential saddle barrier. However, for the in-phase resonance (R3), the two magnets do not try to cross over each other because they are oscillating simultaneously in the same direction; thus the BEH-VO never experiences a potential saddle barrier, which means that the system loses the feature of typical double-well dynamics.

Figure 5.5 (a) Phase difference of the displacement responses for the BEH-VO, and (b)–(d) displacement responses in the time domain obtained at the frequencies of four different resonances (R1)–(R4).

On the other hand, for the secondary intrawell resonances (R2) and (R4), the magnet motions of the BEH-VO seem to be in the intermediate out-of-phase mode, although they are more complicated owing to the addition of a second- or half-harmonic component [as shown in Figs. 5.5(d) and (e)]. Note that the phase differences between w_1 and w_2 responses were evaluated with a dominating

frequency component in the case that they had multiple frequency components (as in Figs. 5.5(d) and (e)), in order to show their relationship somewhat roughly.

The resonance peak of the BEH-VO has been obviously developed at the one-half subharmonic resonance (R2) in addition to the primary resonances (R1) and (R3). This indicates that the effect of the excitation is at a similar level in the primary and secondary resonances. Thus, for the BEH-VO, the secondary resonances as well as the primary resonances should be considered as important routes to the HEOM. However, in Figs. 5.4(b) and (d), the resonance peak of the CBEH is only observed at the primary resonance; it remains invisibly small at the associated one-half subharmonic resonance.

5.4.2 Multiple routes of BEH-VO to HEOM

When the BEH-VO is subjected to a weak ambient excitation force, it exhibits intrawell motion or LEOM with a small but relatively larger amplitude (than the LEOM of CBEH) at intrawell resonances, being confined to one of two potential wells [as demonstrated in the previous section]. However, if the BEH-VO acquires a higher kinetic energy than the potential energy barrier from a stronger ambient excitation, the intrawell motion is likely to be changed into large-amplitude interwell motion or HEOM, oscillating across two potential wells and producing a higher output power. Such a potential well escape phenomenon occurs easily around intrawell resonances, where the transmissibility is maximized. Thus, all four intrawell resonances of the BEH-VO are important routes to the HEOM.

In most of previous studies on the CBEH [9-11], only the primary intrawell resonance was considered to be the route to the HEOM, as it is the unique primary resonance for the conventional system, and the effect of the excitation in the associated secondary resonances (e.g., one-half subharmonic resonance) is not significant enough to induce the potential well escape phenomenon. In this chapter, it is demonstrated that the secondary intrawell resonances of the BEH-VO, as well

as primary intrawell resonances, play important roles in broadband vibration energy harvesting as main routes to the HEOM.

Fig. 5.6 shows the displacement responses of the BEH-VO with respect to the excitation frequency, obtained with four different base accelerations: 0.3, 0.5, 0.7, and 1.0 g (in the first to the fourth row, respectively). In Fig. 5.6, the simulation results (first column) are directly compared to the experimental measurements (second column) for validation. When the base acceleration is 0.3 g [Figs. 5.6(a) and (b)], the HEOM is initiated first around the frequency band of the first primary intrawell resonance (at 17.36 Hz), while the LEOM is only observed in all the other frequency bands. This indicates that for the BEH-VO, the first primary intrawell resonance is still the most important route to the HEOM, triggering the interwell motion with the lowest excitation intensity.

Figure 5.6 Displacement responses of the BEH-VO obtained with relatively strong base accelerations of (a, b) 0.3 g, (c, d) 0.5 g, (e, f) 0.7 g and (g, h) 1.0 g. The numerical and experimental results are presented on the first and second columns, respectively.

However, with further increase of the excitation intensity, the other intrawell resonances (the second primary and two secondary resonances) also generate HEOMs. When the base acceleration is increased to 0.5g [Figs. 5.6(c) and (d)], the

HEOMs occur at the second primary resonance (around 40.04 Hz) and the one-half subharmonic resonance (around 34.72 Hz). Lastly, with a larger base acceleration of 0.7g [Figs. 5.6(e) and (f)], the second superharmonic resonance gives rise to the HEOM as well. Each of these multiple routes to the HEOM provides an individual and separate operating frequency band of the BEH-VO to harvest vibration energy. Notably, four operating frequency bands tend to be connected to each other, with the increase of the excitation intensity, and finally form a whole broad frequency band.

As shown in Figs. 5.6(e) and (f), the bridging of the frequency bands starts to happen between two nearby intrawell resonances (e.g., between the first primary resonance and second superharmonic resonance or between the second primary resonance and one-half subharmonic resonance). When the base acceleration is 1.0 g [Figs. 5.6(g) and (h)], the frequency bands of the HEOM for all four intrawell resonances are finally merged into a very broad frequency band ranging from 10 to 42.5 Hz. This implies that secondary intrawell resonances in addition to primary resonances are essentially considered as important factors in designing broadband vibration energy harvesters. All the simulation results for the displacement responses in Fig. 5.6 agree well with the corresponding experimental ones, which supports the above-mentioned observations and discussion.

Fig. 5.7 compares the displacement responses between the BEH-VO and CBEH, illustrated in the time domain (first column) and phase plane (second column). In this figure, the displacement responses are obtained with a base acceleration of 0.7 g, at three different excitation frequencies: (first row) 30 Hz, (second row) 35 Hz, and (third row) 40 Hz. The excitation frequencies of 30 and 35 Hz correspond to the one-half subharmonic intrawell resonance of the BEH-VO, while the frequencies of 40 Hz correspond to the second primary intrawell resonance. In Fig. 5.7, the BEH-VO exhibits HEOMs for all cases of the excitation frequencies, but the CBEH only exhibits the LEOMs. This result demonstrates additional routes of

the BEH-VO to the HEOM which is an advantage over the CBEH.

Figure 5.7 Comparisons of the displacement responses for BEH-VO and CBEH illustrated in the time domain. The results are obtained with a base acceleration of 0.7 g, at the frequencies of (a) 30 Hz, (b) 35 Hz and (c) 40 Hz.

The associated phase planes and the Fourier analyses of the results in Fig. 5.7 are shown in Fig. 5.8. The results clearly demonstrate the periodic solutions of the HEOM and LEOM in both cases. The dotted points in Fig. 5.8(a)(c) and (e) denote the stroboscopic projections obtained at every forcing period. For the HEOM which is triggered by the one-half subharmonic resonance at 30 and 35 Hz, periodic-three motions are observed, as depicted by the numbers of stroboscopic points. Meanwhile, a periodic-one motion has been observed for the HEOM which is triggered by the second primary resonance at 40 Hz. In addition, it is observed that HEOM in Fig. 5.8(c) exhibits a monostable configuration.

Figure 5.8 The associated phase plane representations, and the Fourier components of the responses for BEH-VO and CBEH obtained with a base acceleration of 0.7 g at the frequency of (a) 30 Hz, (b) 35 Hz, and (c) 40 Hz.

The Fourier analysis shows the harmonic components of the periodic solutions for both systems. The subharmonic components at 1/3 fundamental frequency are observed in the periodic solutions of BEH-VO at 30 and 35 Hz. These subharmonic components result from the subharmonic resonances which are the routes to trigger HEOMs in this frequency range. In all cases, regardless of the complexity of the coexisting periodic solutions in the BEH-VO, the solutions are

dominated by the HEOMs. On the other hand, for CBEH, only LEOMs are observed.

5.4.3 Enhanced broadband performance of BEH-VO

In the previous section, it was shown that the multiple routes to the HEOM tend to form a very broad operating frequency band of the BEH-VO with the increase of the excitation intensity. This broadband characteristic of the HEOM consistently applies to the electrical output responses.

Fig. 5.9 compares the output voltage responses between the BEH-VO and the CBEH (denoted by the solid and dashed lines, respectively), obtained with four different base accelerations of 0.3, 0.5, 0.7, and 1.0 g (presented in the first to the fourth row, respectively). When the base acceleration is 0.3 g [Figs. 5.8(a) and (b)], for both the BEH-VO and CBEH, the HEOM occurs around the first primary intrawell resonance, producing high electrical voltage outputs. In this case, the generated voltage of the BEH-VO around the first primary intrawell resonance is smaller than that of the CBEH. This can be described with the out-of-phase mode regime.

The BEH-VO suffers from the weakening of the magnetic repulsion effect with a larger rate than the CBEH, as the magnets move away from each other; thus, the amplitude of the HEOM and the associated output voltage decrease. However, the reduced amount of the output voltage of the BEH-VO against the CBEH gradually decreases with the increase of the base acceleration [from the first to the fourth row in Fig. 5.8] and becomes trivial at 1.0 g [Figs. 5.8(g) and (h)].

Figure 5.9 Voltage responses of the BEH-VO obtained with relatively strong base accelerations of (a, b) 0.3 g, (c, d) 0.5 g, (e, f) 0.7 g, and (g, h) 1.0 g. The numerical and experimental results are presented in the first and second columns, respectively.

At this small cost of output voltage around the first primary resonance, the BEH-VO can produce a higher voltage against the CBEH, at other frequency ranges for the second primary resonance or the secondary resonances. Such an advantage of the BEH-VO in its operating frequency band is clarified with the increase in the base acceleration. When the base acceleration is 1.0 g [Figs. 5.8(g) and (h)], the

frequency bands of the HEOMs caused by four different intrawell resonances are all connected together. The operating frequency bandwidth of the BEH-VO, where the high output voltage is produced, has been enhanced up to approximately 200 % against the CBEH.

5.5 Conclusions

In this chapter, it was demonstrated theoretically and experimentally that the secondary intrawell resonances (e.g., the one-half subharmonic resonance and the second superharmonic resonance) in addition to the first and second primary resonances played important roles in generating the HEOM of the BEH-VO and thus producing high electrical power. Each of these four multiple routes of the BEH-VO to the HEOM formed an individual operating frequency band in a different frequency range.

Remarkably, with the increase of the excitation intensity, the frequency bands of the HEOM caused by the secondary intrawell resonances tended to bridge the gap between the frequency bands for the first and second primary resonances, yielding a single combined operating frequency band. The bridging effect could be observed with relatively strong excitation but, therewith, the bandwidth of the BEH-VO for vibration energy harvesting was notably enhanced up to 200 % against the CBEH.

Although only the BEH-VO (which is the simplest one of multi-DOF BEHs) was investigated in this study, the bridging effect of secondary intrawell resonances can be possibly applied to various multi-DOF vibration energy harvesting systems in order to further enhance their broadband performance and/or respond to more complicated ambient vibration sources. Hence, secondary intrawell resonances should be considered as another important factor in designing broadband multi-DOF BEHs.

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Chapter 6

Discussion and Future Works

This chapter summarizes the research outcomes in this thesis and discusses the possible steps for future works.

6.1 General discussion

In this thesis, the BEHs with auxiliary magnet oscillators (BEH-HO and BEH-VO systems) were proposed for further enhancing energy harvesting performance against the CBEH. One of the main features of a CBEH is double-well potential energy function, in which potential energy barrier separates two symmetric potential wells. If the system gains enough kinetic energy to surpass the potential energy barrier, it can move across two potential wells, which means the appearance of HEOM. This HEOM directly contributes to the broadband performance of a CBEH. However, if the kinetic energy of the system is not enough, it fails to initiate HEOM owing to the potential energy barrier. In this case, a CBEH is not likely to be under normal operation and generates even lower energy than a linear VEH. Such an intrinsic disadvantage has been one of the main challenging issues in designing CBEHs.

On the other hand, the proposed BEH-HO and BEH-VO systems use movable external magnets, unlike CBEHs that uses a fixed external magnet. The BEH-HO and BEH-VO significantly gain complexity in their nonlinear dynamic behaviours, which have not yet been understood completely. However, it was shown in this thesis that the use of external magnet oscillators could lead to

beneficial effects, e.g., lowering potential energy barrier and triggering interwell motion by secondary resonances as well as primary resonances. The complicated but advantageous aspects in the use of auxiliary magnet oscillators were thoroughly investigated and presented in this thesis.

The research results included theoretical observations and experimental verifications on the two subjects: bistable energy harvester with a horizontal auxiliary magnet oscillator (BEH-HO) and bistable energy harvester with a vertical auxiliary magnet oscillator (BEH-VO).

The works can be summarized as follows:

- 1) *To propose and develop a novel bistable energy harvester with a horizontal auxiliary magnet oscillator (BEH-HO):*

The auxiliary magnet oscillator of the BEH-HO is designed to be movable in the horizontal direction and its motion is rectified by a mechanical rectifier. The rectified half-sine motion of the auxiliary magnet oscillator tends to lower the saddle barrier of the double-well potential, while maintaining the other parts of the potential configuration. Therefore, when compared to the conventional design, the operating frequency bandwidth of the BEH-HO is broadened with an extended lower bound and an unchanged upper bound. Thus, a better performance in operating frequency bandwidth is observed in the proposed BEH-HO, when compared to the conventional design, especially under weak excitation conditions.

To theoretically study the system, a mathematical model of bistable energy harvester with a horizontal auxiliary magnet oscillator is established on the basis of the linear Euler-Bernoulli beam theory, the

linear piezoelectricity, and the magnetic dipole model. A series of nonlinear analysis and frequency response analysis is conducted to unveil the mechanisms and the underlying principles behind the design concept. Experimental verifications which validate and support the theoretical observations are also provided.

This design can be applied to the applications of vibration energy harvester (VEH) under weak excitation environments.

- 2) *To investigate the underlying principles of a bistable energy harvester with a vertical auxiliary magnet oscillator (BEH-VO):*

This study, for the first time, reveals the concept of phase-dependent dynamic potential energy. The potential energy of the BEH-VO is observed as a path-dependent function which propagates through the high dimensional space and significantly depends on the phase difference between the two oscillators.

Through theoretical simulations and analyses, two distinct dynamic regimes, called the out-of-phase and in-phase mode regimes in this thesis, are found to exist in the frequency regions of the first and second primary intrawell resonances. It is found that the present BEH-VO acts as a bistable system in the out-of-phase mode, but behaves as a monostable system in the in-phase mode.

These findings unveil the complicated dynamics of the BEH-VO, shedding light on compact and efficient designs of magnetically coupled bistable energy harvester (BEH) chain structures.

- 3) *To investigate multiple routes to high-energy orbit motion (HEOM) in*

the bistable energy harvester with a vertical auxiliary magnet oscillator (BEH-VO):

Owing to their potential to trigger the high-energy orbit motion (HEOM), the secondary resonance, along with the primary resonance, should be considered important when designing BEH-VO. Under a relatively strong excitation, the secondary resonances tend to bridge the gap between the frequency bands of the BEH-VO's two primary resonances. Thus, remarkably enhancing the operating frequency bandwidth (up to 200% with respect to the CBEH in this study).

This effect of the secondary resonance and the bridging phenomena can be extended and applied to multi-degree-of-freedom (MDOF) designs of nonlinear vibration energy harvesters (VEHs).

6.2 Future works

Further works, which can be considered to complete this line of research, are proposed as follows:

- 1) *Nonlinear analysis of bistable energy harvesters (BEHs) with auxiliary magnet oscillators using Harmonic Balance Analysis (HBA) and room for improvement in simulation and experimental approaches:*

a) Simulation approach

Numerical methods may have some limitations, such as computational time, numerical errors, and the impossibility to exhibit underlying principle of the solutions (i.e., multiple coexisting solutions) of the BEHs with auxiliary magnet oscillators. Therefore, a semi-analytical method can possibly be employed for finding the approximate analytical form of

the steady-state responses of the BEHs under harmonic excitation. When compared to the numerical results, semi-analytical solutions can provide a deeper physical insight into the complicated nonlinear dynamics of the proposed system.

Harmonic Balance Analysis (HBA) would be used owing to its straightforwardness and simplicity. The HBA is a sort of a frequency domain method to obtain semi-analytical solutions of nonlinear differential equations in the form of Fourier series, and accordingly it can give us frequency response results more informative than numerical approaches.

b) Experimental approach

A series of experiments were carried out to conceptually verify numerical simulation results. While giving a good agreement to the numerical results, such an experimental approach still has a couple of limitations. Undesirable noise could be mixed in measured signals (although it was eliminated by using proper filters in this thesis). Besides, some results could not be experimentally measured, e.g., unstable solutions and exact bifurcation points in frequency responses, which can be detected only by theoretical approaches.

2) Bistable energy harvester with combined oscillators (BEH-CO):

Bistable energy harvester with combined oscillators (BEH-CO) is a combination of BEH with a horizontal and vertical auxiliary magnet oscillator. In particular, the auxiliary magnet of the BEH-CO can possibly be constructed to be movable in both horizontal and vertical directions simultaneously. The mathematical model and theoretical study may be

carried out based on the existing models of the BEH-HO and BEH-VO. The same analyses, which were conducted for the two individual cases, should be applied to the combined case.

The possible synergetic effect of the BEH-CO, obtained from the unique features of the auxiliary oscillators, should be thoroughly examined. The bistable energy harvester with combined oscillators (BEH-CO) is expected to broaden the frequency bandwidth. Thus, further enhancing the energy harvesting performance of the BEHs.

3) *Field testing of proposed bistable energy harvesters (BEHs):*

It is important to evaluate the applicability of the proposed bistable energy harvester (BEH) designs by performing theoretical and experimental studies under various types of excitation conditions: harmonic excitation, swept-sine excitation, and random excitation (Gaussian noises or pink noises). These excitation conditions can be extracted from real-life ambient vibrations such as automobiles, mass rapid transits, micro-ovens, refrigerators, et cetera.

Field testing on the proposed bistable energy harvesters provides a critical appraisal of the design of bistable energy harvester with auxiliary magnet oscillators. Thus, it may promote the commercialization of these proposed designs.