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**NANYANG
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SINGAPORE

**TRACKING OF REAL AND COMPLEX
SINUSOIDS USING PILOTED ADAPTIVE
NOTCH FILTER**

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School of Electrical and Electronic Engineering

2018

TRACKING OF REAL AND COMPLEX SINUSOIDS USING PILOTED ADAPTIVE NOTCH FILTER

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Dedicated to my beloved parents,

Mr. Dwarka Prasad Chobey and Ms. Kiran Devi Chobey

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List of Abbreviations

AC	Alternating current
ANF	Adaptive notch filter
ALE	Adaptive line enhancer
CANF	Complex adaptive notch filter
CPANF	Complex piloted adaptive notch filter
DC	Direct current
FIR	Finite impulse response
GAL	Gradient adaptive lattice
GSM	Global system for mobile communications, originally Groupe Spécial Mobile
HDD	Hard disk drive
IIR	Infinite impulse response
ISR	Interference-to-noise ratio
LMS	Least mean square
MPPANF	Multiple pilot-pairs adaptive notch filter

MSE	Mean square error
ODE	Ordinary differential equation
PANF	Piloted adaptive notch filter
PSD	Power spectral density
QPSK	Quadrature phase shift keying
SAR	Synthetic aperture radio
SNR	Signal-to-noise ratio
VS-LMS	Variable step-size least mean square
VP-MPPANF	Variable pole radius multiple pilot-pairs adaptive notch filter
VP-PANF	Variable pole radius piloted adaptive notch filter

List of Notations

P	Probability of event X
$\Re\{.\}$	Real part of a complex number
$\Im\{.\}$	Imaginary part of a complex number
$(.)^*$	Conjugate of a complex number
$\mathbf{x}$	Vector
$\mathbf{X}$	Matrix
$\mathbf{x}^T$	Transpose operation
$\exp(.)$	Exponential function
$f(x)$	Function of x
$\mathbb{E}$	Expectation operator
∇	Gradient operator
$\bar{x}$	Mean of $\mathbf{x}$
$ \cdot $	Modulus operator

Summary

In digital signal processing, frequency estimation and tracking of sinusoids in noise using adaptive notch filter is an important area of research to solve problems in radar, communications, biomedical and other related areas. This filter is popularly implemented using the least mean square (LMS) algorithm in which the notch frequency is updated iteratively using either a fixed or an adaptive step-size to eventually converge to the frequency of the input signal. A large step-size will increase the rate of convergence but will result in larger misadjustment, while a small step-size will decrease the rate of convergence but will yield smaller misadjustment. So, in view of this trade-off, a variable step-size LMS algorithm is preferred over a fixed step-size LMS algorithm. Conventional variable step-size algorithms determine the step-size based on time-domain averaging of the gradient estimate at each sampling instance. The piloted adaptive notch filter (PANF) is a new concept in adaptive filtering which determines the step-size value based on the estimated distance between the main notch frequency and the input frequency with the help of pilot notches. A steady-state performance comparison of the PANF with conventional adaptive notch filters is conducted to verify the excellent performance exhibited by the PANF over other algorithms. Hence, this thesis aims at investigating the concept of PANF, originally proposed to estimate and track real sinusoid, to further enhance its performance and to extend the concept of pilot notches for signals in complex domain.

A new steering direction determination mechanism, using a time-domain averaging based gradient analysis of the piloted notch cost function at several frequency points at the same sampling instant, is proposed to determine the direction of the

main notch with respect to the input sinusoid frequency. This frequency domain information is then combined with time domain information to develop an algorithm for the determination of variable step-sizes for improved speed of convergence with significant reduction in steady-state mean square error (MSE).

An improved PANF with a variable pole radius mechanism is introduced. This scheme significantly reduces the transient effect and improves the steering direction determination mechanism of the notch filter. Computer simulations demonstrate the excellent performance of the variable pole radius PANF (VP-PANF) to significantly outperform the PANF with respect to the speed of convergence and steady-state MSE.

Aforementioned work leads to the development of a generalized formulation for multiple pilot-pairs adaptive notch filter (MPPANF) structure with the introduction of more number of pilots, variable pole radius mechanism, and improved steering direction determination. The transition from a very large step-size value to a very small step-size value is achieved in one or more intermediate step-size values. This reduces the probability that the main notch overshoots the optimum with large step-size producing large output error when it is close to the input frequency. Simulation results are presented to verify the excellent performance exhibited by the MPPANF and variable pole radius MPPANF (VP-MPPANF) over single-pair PANF with respect to the speed of convergence and steady-state MSE. A detailed theoretical analysis of the proposed adaptive notch filter and mathematical formulation for the determination of the probability of obtaining a correct steering direction along with the effect of input SNR, notch bandwidth and notch frequency on probability of obtaining a correct steering direction is presented.

We formulate a complex piloted adaptive notch filter (CPANF) to estimate and track the frequency of complex sinusoidal signal. A novel complex coefficient filter structure with a main notch and pilot notches to track the frequency of the input complex sinusoid with a variable step-size least mean squares (LMS) based

algorithm is presented. Simulation results verify the excellent performance exhibited by the CPANF over conventional complex adaptive notch filter (CANF) with respect to the speed of convergence and steady-state MSE. Theoretical analysis is also presented which closely follows the simulation results. Finally, the filter is implemented to suppress complex sinusoidal interference in a QPSK spread spectrum communication systems which shows improvement over conventional CANF in overall bit error rate (BER). This research leads to a detailed understanding and development of piloted adaptive notch filter to estimate and track both real and complex sinusoid

Chapter 1

Introduction

The estimation and tracking of signals is a fundamental research problem in digital signal processing. This has been achieved using digital filters which has seen significant research for its advantage over conventional analog filters in terms of stability, accuracy, flexibility and cost. Digital notch filter is an important class of these filters, used in systems which are required to eliminate or retrieve a specific frequency or multiple frequencies embedded in a broadband signal. A fixed notch filter is used when the frequency is time invariant. However, when the frequency of the sinusoid is unknown or time-variant, an adaptive notch filter (ANF) is required which can adapt the notch frequency to the frequency of the sinusoid.

ANF being capable of working in unknown environments is widely used in radar, communications, biomedical and other related areas [1]. They are used in biomedical system to remove power line interferences in biomedical data, particularly for ECG signals [2–9]. They are used in radar and communication systems for various applications which includes a recent work [10] where the authors have used modified ANF to autofocus complex moving structures in synthetic aperture radar (SAR) data. They have also been reported to be used in missile control [11], in global navigation satellite systems [12], in GSM mobile telephone [13], and in wireless multimedia systems [14]. They are used in power systems for tracking

and estimation of the magnitude, phase angle, and frequency of three-phase AC systems [15–19]. They are also used in hard disk drive (HDD) industry to mitigate mechanical resonant modes [20, 21]. Interestingly, they are also used in effective bass enhancement in hand-held consumer devices [22] and in shaking table system for earthquake simulators to compensate for reaction force [23]. Thus, with the huge scope of diverse applications, design and implementation of ANF is always a promising research area.

ANF is implemented both as finite-impulse response (FIR) filter and infinite-impulse response (IIR) filter. Extensive research has been carried out in designing and analysis of both types of adaptive notch filters to estimate and track the frequency of the input sinusoid corrupted with noise. The FIR based adaptive line enhancer (ALE) has relatively slow rate of convergence and is highly responsive to the autocorrelation of the input signal. The speed of convergence can be increased, but the filter might fall short in tracking. It requires a relatively larger number of filter coefficients as compared to the IIR based ALE [24–26] for the same narrow bandwidth notch to reject the interfering noise. As reported in [27], an IIR filter can be used to exactly model a pole-zero system with higher accuracy as compare to FIR filter. Thus, with the inherent potential for reduction in computational complexity and improvement in performance, there have been significant efforts to develop adaptive IIR notch filters. The IIR digital notch filters have been developed in the literature using various methods [24, 28–31] which popularly include analog filter transformation, implementation using all pass filters and pole-zero placement techniques. The simplest and the most effective design technique for IIR notch filter is by pole-zero placements [31], although it has the limitation of asymmetric

structure with uncontrollable gain.

The key feature of an adaptive IIR notch filter is the adaptive algorithm which helps to adapt the filter parameters to converge and track the target frequency. They are popularly implemented using the least mean square (LMS) algorithm [32–36] in which the notch frequency is updated iteratively to eventually converge to the frequency of the input signal. This algorithm is widely used in various applications for its computational simplicity and robustness. The filter parameters are adapted by minimizing the mean square of the output error sequence which is basically the difference between the desired output signal and the output of the filter. The LMS algorithm employs an adaptation step-size parameter to update the filter weights. This adaptation step-size plays a vital role in determining the rate of convergence and the misadjustment at steady-state [33]. A large step-size will increase the rate of convergence but will result in larger misadjustment, while a small step-size will decrease the rate of convergence but will yield smaller misadjustment. So, in view of this trade-off, a variable step-size LMS algorithm is preferred over a fixed step-size LMS algorithm as it appropriately generates a larger step-size when the notch is away from the input frequency for faster convergence and a smaller step-size when the notch frequency is close to the input frequency for smoother tracking. There has also been contributions for theoretical analysis of rate of convergence and complexity of these adaptive algorithms [37].

In literature, various step-size update techniques [38–50] have been proposed which performs time-domain averaging on the output error to obtain suitable step-sizes. In [38], the sign change between consecutive gradient estimates has been used to update the step-size. In this approach, the notch frequency is assumed to

be far away from the frequency of the input sinusoid, if the signs of consecutive gradient estimate do not change; and consequently, the step-size is incremented. The step-size is decreased if the signs of consecutive gradient estimate alternates. While in [44], the authors used the product of consecutive gradient estimates to update the step-size, the authors in [41], updated the step-size as a function of the low pass filtered values of the gradient estimates. In [45], the author replaced the linear update recursions of step-sizes by multiplicative update recursions. All these techniques were based on time domain information but the authors in [51] proposed a novel method of estimating the distance between the notch frequency and the frequency of the input sinusoid with a filter architecture incorporating the pilot notches. The signs of the gradient estimate at the pilot notches are used to determine whether the notch is far away from the input frequency. This frequency domain information is then used to choose an appropriate step-size value. The authors have shown that the algorithm has higher speed of convergence compared to the time-averaging based variable step-size algorithms.

Motivation for the research

The piloted adaptive notch filter (PANF) is a new concept, in adaptive filtering, of assisting the main notch with pilot notches in choosing an appropriate step-size value for convergence. The concept brings in important knowledge of the position of the main notch frequency with respect to the frequency of the input sinusoid. This is an interesting algorithm significantly different from other previously reported in literature with the authors claiming an improved performance.

A study of this novel algorithm to evaluate its steady state performance with

roles of variables like notch bandwidth and signal-to-noise ratio (SNR) will be crucial to evaluate its performance with other algorithms. Moreover, it will be interesting to study the introduction of more number of pilots which can be used to have a detailed frequency domain profile. It is also noted that the PANF has been developed for frequency estimation and tracking of real signal. However, it will add to the merits of the algorithm, if the concept can be further extended to track complex signals, that is when the signal consists of in-phase and quadrature-phase components.

With the prospects of enormous applications using adaptive notch filter, a research project in this area is motivational to solve complex problems in diverse areas. The development of the PANF concept can be seen as a new approach for other adaptive systems which fundamentally search for an optimal solution in an unknown environment with possible applications in defence, space and biomedical systems.

Contribution and organisation of the thesis

This research leads to a detailed understanding and further development of piloted adaptive notch filter to estimate and track both real and complex sinusoids with improved performance. The contributions has been organised as follows:

Chapter 1 presents the background and motivation of the current research work, major contribution and thesis organization.

Chapter 2 introduces a detailed literature review of adaptive filtering and the

mathematical details of the LMS algorithm and various VS-LMS algorithms reported in literature. The subsequent sections in the chapter introduces conventional adaptive notch filter structures and their mathematical formulations, and the concept and mathematical formulations of PANF.

Chapter 3 presents a simulation study of steady-state performance comparison of PANF with conventional fixed step-size LMS algorithm based second-order adaptive IIR lattice notch filter and the adaptive IIR notch filter with constrained poles and zeros. This study verifies the excellent performance exhibited by the PANF over other algorithms with respect to the speed of convergence and MSE.

Chapter 4 presents a new steering direction determination mechanism and step-size update algorithm for PANF. A time-domain averaging based gradient analysis of the piloted notch cost function has been utilized to determine the relative position of the main notch with respect to the input sinusoid frequency. This frequency domain information is then combined with time domain information to develop an algorithm for the determination of variable step-sizes for improved speed of convergence with significant reduction in mean square error (MSE). The simulation results verifies the excellent performance exhibited by our new steering mechanism and the step-size update algorithm over original PANF with respect to the speed of convergence and MSE.

Chapter 5 presents a detailed analysis of pole position approximation for pilot notches to understand its overall effect on steering direction determination and convergence of the PANF. An improved PANF with a variable pole radius mechanism (VP-PANF) is presented. This scheme significantly reduces the transient effect and improves the steering direction determination mechanism of the notch filter.

Computer simulations demonstrate the excellent performance of the variable-pole radius PANF to significantly outperform the original PANF with respect to the speed of convergence and steady-state MSE. A simulation based comparison of VP-PANF with Farhang's algorithm is also presented in this chapter.

Chapter 6 presents the generalized formulation and filter structure of a multiple pilot-pairs adaptive notch filter (MPPANF) with more than one pair of pilot notches. The transition from a very large step-size to a very small step-size is achieved in one or more intermediate step-sizes. This reduces the probability that the main notch overshoots the optimum with large output error when it is close to the input frequency. The new time-averaging based process is used to determine the steering direction of the main notch. The variable pole radius MPPANF (VP-MPPANF) with reduced transient response and improved probability of obtaining a correct steering direction is presented. Simulation results show that the performance of the MPPANF and VP-MPPANF is superior to that of the PANF. A detailed theoretical analysis of the proposed adaptive notch filter is also presented. Mathematical formulation for the determination of the probability of obtaining a correct steering direction along with the effect of input SNR, notch bandwidth and notch frequency on the probability of obtaining a correct steering direction are discussed.

Chapter 7 presents a complex piloted adaptive notch filter (CPANF) to estimate and track the frequency of complex sinusoidal signal. Simulation are presented to verify the excellent performance exhibited by the CPANF over conventional complex adaptive notch filter (CANF) with respect to the speed of convergence and steady-state MSE. Theoretical analysis is also presented which closely follows the

simulation results. Mathematical formulation for the determination of the probability of obtaining a correct steering direction along with the effect of input SNR, notch bandwidth and notch frequency on probability of obtaining a correct steering direction is presented. The filter is also implemented to suppress complex sinusoidal interference in a QPSK communication systems which shows improvement over conventional CANF in overall bit error rate (BER).

Finally, the conclusions and scope of future work is drawn in Chapter 8.

Chapter 2

Literature Review

In this chapter, we review the literature which led to the development of the adaptive notch filters. We first briefly present the history of adaptive filter theory and then introduce the LMS algorithm. The VS-LMS algorithms are discussed to demonstrate the significance of PANF. We then discuss, in detail, the adaptive IIR filter which is used as ANF in this thesis. The CANF which is used for complex signal is also presented.

2.1 Historical Background

It is known from the historical notes that people have always been highly interested in interpretation of observations to make estimations and predictions. The earliest evidence of the development of linear estimation theory can be found in astronomical studies, with telescopic measurement data, to study the movement of stars, planets and other celestial bodies. A rudimentary form of Fourier series was used by the Babylonians, as noted by Neugebauer [52], for these studies. However, the starting point for the development of the “Theory” of estimation is dated back to 1632 [53], when Galileo Galilei made various attempts to minimize the functions of errors. His works were followed by series of studies being made by prominent people such as Euler, Lagrange, Laplace, Bernoulli and many others. However, the

birth of linear estimation theory is attributed to Gauss [54] who, in 1795, invented the method of least squares. It initially remained controversial during the 19th century for the original inventor as it was first published by Legendre in 1805 [55] who was working independently to study the motion of celestial objects. This was followed by extensive research to study various aspects of the least squares method which is reported by Harter [56] and also in survey by Sorenson [57].

We will focus on least squares estimation in stochastic process which was first studied by Kolmogorov [58, 59] and Krein [60, 61] and Wiener (was working independently) [62] during late 1930s and 1940s. The work of Wold [59] on discrete-time stationary processes motivated Kolmogorov to develop a generic method to solve the linear prediction problem for discrete-time stationary processes. Krein used bilinear transformation to further extend Kolmogorov's results to continuous time stationary process by noting the relationship between works of Kolmogorov's and Szegö [63, 64]. However, Wiener, who was working independently, developed the optimum formula for the "optimum predictor" to solve the continuous-time linear prediction problem in order to find the solution for some anti-aircraft fire-control problem. His formulation was based on the solutions of an integral equation called the Wiener-Hopf equation [65]. He also mentioned in his autobiography [66] about the "filtering" problem. It was the problem of estimation of a process being corrupted by a "noise" process. His attempts to examine the various aspects of least squares and the subsequent results had great influences in the engineering fraternity.

However, the filtering problem in discrete time was later formulated by Levinson in 1947. The Wiener-Hopf equation which was originally formulated as an integral

equation, takes on matrix form as shown in (2.1)

$$\mathbf{R}\mathbf{w}_{opt} = \mathbf{p} \quad (2.1)$$

where, $\mathbf{R}$ is the correlation matrix of tap inputs and $\mathbf{p}$ is the cross-correlation of the tap inputs and the desired response. $\mathbf{w}_{opt}$ is the tap-weight vector of the optimum Wiener filter structured in the form of a transversal filter. It can be interestingly noted that the correlation matrix $\mathbf{R}$ assumes a special form, popularly known as Toeplitz, and using the properties of Toeplitz matrix, Levinson developed a recursive method to solve the matrix form of the Wiener-Hopf equation [67]. This method was latter revisited by Durbin who considered a special case of (2.1) in which vector $\mathbf{p}$ and correlation matrix $\mathbf{R}$ have same elements to develop a method to recursively fit autoregressive models to scalar time-series data. This work was latter extended by Whittle in 1963 to derive a multivariate generalization of the Levinson-Durbin recursion.

The main limitation of the works of Kolmogorov and Wiener is that they considered stochastic processes which are stationary and needed infinite amount of data. It motivated many authors to generalize the results for finite observations and estimation for non-stationary processes. However, they appeared to be rather complicated and difficult to track when there is an increase in the time between observations and, it was also difficult to modify the vector form. These limitations were evident, particularly in determining the orbits of satellites, as this included accumulation of data in some form of vectors of position and velocity of satellite passing over a tracking station. It was Swerling's work [68] which first proposed a recursive algorithm for this limitation. However, Kalman's work [69] with more restricted algorithm for discrete-time processes particularly become useful for the

space application and thus gained more prominence. He latter collaborated with Bucy to derive the formulation of the continuous time linear filtering, popularly known as Kalman-Bucy filter [70]. The latter works on linear filtering were attributed to Kailath [71–74] who used the Innovations approach. It was considered that each sample of the process provides entirely new information as it is statistically independent of all past samples. This work was documented in the book *Signal processing the model based approach* by Candy [75] in 1986.

Stochastic gradient algorithms and recursive least square algorithms are popular categories of linear adaptive filter. In this thesis, we will discuss the stochastic gradient algorithms.

Stochastic Gradient Algorithms: The development of adaptive filters can be related to various researcher’s works during 1950s. The most prominent being the LMS algorithm developed for operation on transversal filters devised by Widrow and Hoff in 1959 [76,77]. It was developed during their study on pattern-recognition scheme called as “Adaptive Linear (threshold Logic) element”. Gradient adaptive lattice (GAL) algorithm [78,79] is another popular stochastic gradient algorithm which is used for lattice based filters. In 1996, Hassibi *et al.* [80] theoretically proved that the LMS algorithm is robust and optimal under the H^∞ criterion (introduced by Zames [81] in 1982). Details of the LMS algorithm are discussed in the next section.

2.2 The LMS Algorithm

Consider an M -tap transversal filter, as shown in figure 2.1. Its input and desired output at time n are $u(n)$ and $d(n)$, respectively. The k^{th} filter weights is $w_k(n)$.

Figure 2.1: An M -tap transversal filter.

The output of the filter is expressed as

$$y(n) = \sum_{k=0}^{M-1} w_k(n) u(n-k). \quad (2.2)$$

It is assumed that $u(n)$, $y(n)$, $d(n)$ and $w_k(n)$ are real-valued. The filter adjusts the tap weights $w_0(n)$, $w_1(n)$, $w_2(n)$, $\dots$, $w_{M-1}(n)$, such that the mean value of $|e(n)|^2$, shown in (2.3), is minimized.

$$|e(n)| = |d(n) - y(n)| \quad (2.3)$$

It is important to note that the LMS algorithm is an iterative process which, at every time instant n , adapts the filter weights $w_k(n)$ such that the output error is minimized in the mean-square sense. As discussed above, if the input $u(n)$ and the desired output $d(n)$ are jointly stationary, then the algorithm, on average, converges the filter weights to the solution of Wiener-Hopf equation. Thus, it is a practical scheme of solving the Wiener-Hopf Equation to adapt the filter weights of Wiener filters. The LMS algorithm is a member of the family of stochastic gradient algorithms implemented with method of the steepest descent. It works on the principle of estimating gradient vector $\nabla J(n)$ at each iteration, where $J(n) = E[e^2(n)]$. The negative of the gradient vector points in the direction of the greatest decrease in the squared error, with respect to the change in weight values. It simply

replaces the cost function for the method of steepest decent $J(n) = E[e^2(n)]$ by its instantaneous value, that is, $\hat{J}(n) = e^2(n)$. The filter weight update equation is expressed as

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \mu \nabla e^2(n) \quad (2.4)$$

where, $\mathbf{w}(n) = [w_0(n)w_1(n)w_2(n)\dots w_{M-1}(n)]^T$, μ is the adaptation step-size parameter, and ∇ is the gradient operator defined as

$$\nabla = \left[\frac{\partial}{\partial w_0} \frac{\partial}{\partial w_1} \dots \frac{\partial}{\partial w_{M-1}} \right]^T \quad (2.5)$$

Using (2.5), (2.3), and (2.2)

$$\nabla e^2(n) = \begin{bmatrix} \frac{\partial e^2(n)}{\partial w_0} \\ \vdots \\ \frac{\partial e^2(n)}{\partial w_M} \end{bmatrix} = 2e(n) \begin{bmatrix} \frac{\partial e(n)}{\partial w_0} \\ \vdots \\ \frac{\partial e(n)}{\partial w_M} \end{bmatrix} = -2e(n)\mathbf{u}(n) \quad (2.6)$$

$$\nabla e^2(n) = -2e(n)\mathbf{u}(n) \quad (2.7)$$

where, $\mathbf{u}(n) = [u(n)u(n-1)\dots u(n-M+1)]^T$. Substituting (2.7) into (2.4), the LMS algorithm based weight update is shown in (2.8),

$$\mathbf{w}(n+1) = \mathbf{w}(n) + 2\mu e(n)\mathbf{u}(n) \quad (2.8)$$

Eq. (2.8) is a simple recursive procedure which adapts the filter weights, at every iteration, for the desired output. Thus, filter output (2.2), estimation error (2.3) and update formula (2.8) are used for the LMS algorithm based filter. The LMS algorithm is inherently simple to implement as it requires $2M+1$ multiplications and $2M$ additions per iteration.

The adaptation step-size is an important parameter which controls the rate of convergence of the filter weights to the optimum filter weights, and the misadjustment at steady state. In [33], the magnitude of step-size inversely affect the

magnitude of misadjustment, i.e, a smaller value of step-size results in reduced steady-state error but with slower speed of convergence and vice versa. It is important to have bounds on step-size for the convergence as discussed in [33] and the step-size should follow the bounds shown in (2.9)

$$0 < \mu < \frac{1}{\lambda_{max}} \quad (2.9)$$

where, λ_{max} is the maximum eigenvalue of the autocorrelation matrix ($\mathbf{R}$) of the input signal $u(n)$. It is important to note that the convergence behavior of the LMS algorithm is affected by the eigenvalue spread of $\mathbf{R}$. Moreover, as discussed in [1], the use of stochastic gradient in the LMS algorithm makes it more sensitive to the step-size value, as a result, the upper bound of step-size parameter needs to be much lower. The authors in [82] suggested a stricter upper bound as in (2.10)

$$0 < \mu < \frac{1}{3\text{trace}[\mathbf{R}]} \quad (2.10)$$

2.3 Variable Step-size LMS Algorithms

As discussed in the previous section, the step-size values plays a key role in deciding the speed of convergence and steady state misadjustment. If the algorithm uses a large step-size then it will result in larger misadjustment but will show a higher speed of convergence, whereas if the algorithm uses a smaller step-size, then it will yield smaller misadjustment but will show reduced speed of convergence. Thus, to address this trade-off, the LMS algorithm with variable step-size mechanism or VS-LMS is preferred over a fixed step-size LMS algorithm. There has been significant research in the development of variable step-size mechanisms for the LMS algorithm [38–50]. These schemes determine the step-size value based on the

error estimated at several time instants. We will discuss, in detail, a few of these schemes to appreciate the development of VS-LMS algorithm.

The tap weight vector of the filter can be updated using a common step-size as shown in the weight update equation in (2.11)

$$\mathbf{w}(n+1) = \mathbf{w}(n) + 2\mu e(n)\mathbf{u}(n) \quad (2.11)$$

or, individual step-size for each tap weight as expressed in (2.12)

$$\mathbf{w}(n+1) = \mathbf{w}(n) + 2\boldsymbol{\mu}(n)e(n)\mathbf{u}(n) \quad (2.12)$$

where, $\boldsymbol{\mu}(n)$ denotes the diagonal matrix of step-sizes $\mu_k(n)$ at each iteration n .

VS-LMS algorithm based on sign-change of gradient estimates

In [38], Harris *et al.* proposed a scheme for determining the adaptation step-size by using the sign changes between consecutive gradient estimates $\nabla_{k,n}$ defined in (2.13).

$$\nabla_{k,n} = -2e(n)u(n-k) \quad (2.13)$$

The scheme was designed to have an individual step-size for each tap weight, and each individual step-size value $\mu_k(n)$ is adapted using a predetermined factor α as shown in (2.14)

$$\mu_k(n+1) = \begin{cases} \frac{\mu_k(n)}{\alpha}, & \text{if } \nabla_{k,n} \text{ alternates sign for } m_0 \text{ samples} \\ \alpha\mu_k(n), & \text{if } \nabla_{k,n} \text{ has the same signs for } m_1 \text{ samples} \\ \mu_k, & \text{otherwise} \end{cases} \quad (2.14)$$

The maximum step-size μ_{max} , as proposed by the authors of [38] should be bounded as in (2.9) and the minimum value of step-size, μ_{min} , is chosen as per the

desired steady state misadjustment. The authors reported four useful implementation cases as follows:

1. $m_0 = 1, m_1 = \infty$, This will cause the value of the step-size to decrease at every sign change of $\nabla_{k,n}$
2. $m_0 > 1, m_1 = \infty$, This will cause the value of the step-size to decrease, if m_0 consecutive sign changes occur.
3. $m_0 > m_1 > 0$, This will be the most desirable case for stationary input signals as it will in general decrease the step-size value.
4. $m_0 = m_1 \geq 0$, This will be the most desirable case for non-stationary input signal as it will symmetrically increase or decrease the step-size value.

This scheme of adapting the step-size value was designed by taking the hardware cost into consideration. Though, the authors claimed that this VS-LMS algorithm “provide faster convergence and less misadjustment than constant μ or LMS algorithm” with the assumption that this scheme will add 15% to the hardware cost compared to LMS algorithm [32]. The algorithm used parameters which would have to be tuned as per the application.

This algorithm was latter compared by Evans *et al.* in [43], where the authors proposed an additive rather than a multiplicative factor to adapt the step-size at each iterations, as shown below

$$\mu_k(n+1) = \begin{cases} \mu_k(n) - \alpha, & \text{if } \nabla_{k,n} \text{ alternates sign for } r_0 \text{ samples} \\ \mu_k(n) + \alpha, & \text{if } \nabla_{k,n} \text{ has the same signs for } r_1 \text{ samples} \\ \mu_k(n), & \text{otherwise} \end{cases} \quad (2.15)$$

The authors claimed that this algorithm performs slightly better than [38] but will be slightly difficult to implement as it can have a range of values for α .

VS-LMS algorithm based on function of gradient estimates

The next category of VS-LMS algorithm determines the value of step-size as a function of the gradient estimate. This is based on the fact that the expected value of the gradient estimate will approach zero as the algorithm converges. The authors in [40, 41] proposed low pass filtering of the gradient estimate at every sampling instant to obtain the expected value of the gradient estimate. Then, the value of the step-size is determined using this filtered gradient estimate.

In [40], the filtered gradient estimates at each time instant n , denoted as $\rho(n)$, is derived using a forgetting factor γ , as shown in (2.16)

$$\rho(n) = \lambda\rho(n-1) + (1-\lambda)\bar{u}(n)e(n) \quad (2.16)$$

where, $\bar{u}(n)$ is the mean value of the input, expressed in (2.17)

$$\bar{u}(n) = \frac{1}{M} \sum_{k=0}^{M-1} u(n-k) \quad (2.17)$$

The step-size update at each time instant n , calculated as

$$\mu(n+1) = \frac{\alpha|\rho(n)|}{\mathbf{u}^T(n)\mathbf{u}(n)} \quad (2.18)$$

where, α is the scaling factor. Though, λ is chosen in the range of 0.9 to 1, the value of α is determined experimentally.

The algorithm was later modified to be known as correlation LMS algorithm with bounds on step-size ($\mu < \mu_{max}$). The new filtered gradient estimate, denoted as $\rho'(n)$, is derived using instantaneous input value as follows

$$\rho'(n) = \lambda\rho'(n-1) + (1-\lambda)u(n)e(n) \quad (2.19)$$

and the step-size is updated as

$$\mu(n+1) = \frac{\alpha |\rho'(n)|}{\mathbf{u}^T(n)\mathbf{u}(n)} \quad (2.20)$$

The author in [40] claimed that the algorithm is robust to disturbances and sudden changes in SNR with excellent tracking capabilities.

Karni *et al.* in [41] also proposed step-size update formulation based on the similar principle using a damping factor α shown in (2.21)

$$\mu(n) = \mu_{max}(n)(1 - \exp(-\alpha \|u(n)e(n)\|^2)) \quad (2.21)$$

The authors suggested that α should be greater than 1 and observed that as α approaches ∞ , the scheme functions as conventional LMS algorithm with fixed step-size value μ_{max} . The authors claimed that the algorithm performs better in terms of convergence and misadjustment especially under non-stationary conditions. However, it is a single parameter algorithm compared to [40] which needs two parameters.

VS-LMS algorithm based on the product of consecutive gradient estimates

The next category of algorithms for the step-size adaptation is implemented by using product of consecutive gradient estimates. This approach was adopted in [44] and [45]. This was developed from the observations that, after convergence, the weights oscillate around the optimal solution leading to change of signs. Thus, when the product of two consecutive gradient estimates is negative, it would imply that the step-size needs to be decreased. However, the step-size needs to be bounded.

Mathews *et al.* in [44] proposed the step-size update formula as shown in (2.22)

$$\mu_k(n) = \mu_k(n-1) + \rho e(n)e(n-1)u(n-k)u(n-k-1) \quad (2.22)$$

where, ρ is a positive constant which controls the convergence of the algorithm. The algorithm needs to initialize the step-size with some small value and the choice may be crucial for initial speed of convergence. The authors claimed “close to best possible performance” in a non-stationary environmental conditions.

VS-LMS algorithm based on the energy of error signal

The variable step-size algorithm has also been developed by using the energy of the output error signal [42]. This works on the fact that a large step-size can be used when the error is large and a small step-size when the solution is near the optimal solution. Kwong *et al.* in [42] formulated the step-size update as shown in (2.23) and (2.24)

$$\hat{\mu}(n+1) = \alpha\mu(n) + \gamma e^2(n); \quad (2.23)$$

$$\mu(n+1) = \begin{cases} \mu_{max}, & \text{if } \hat{\mu}(n+1) > \mu_{max} \\ \mu_{min}, & \text{if } \hat{\mu}(n+1) < \mu_{min} \\ \hat{\mu}(n+1), & \text{otherwise} \end{cases} \quad (2.24)$$

where, α and γ are controlling parameter. α is chosen in the range 0 to 1, and the authors in [42] used the value of α as 0.97. γ is small and controls the convergence rate and steady-state MSE. The authors claimed that the filter gave comparable results as in [38].

However, the above algorithm may show unpredictable behaviour in the presence of measurement noise as observed by Aboulnasr *et al.* [47]. They proposed a scheme based on the estimation of correlation between $e(n)$ and $e(n-1)$. A filtered version of this correlation, denoted as $\hat{p}(n)$, using an exponential weighting

Figure 2.2: Adaptive line enhancer.

parameter β , is calculated, as shown in (2.25)

$$\hat{p}(n) = \beta \hat{p}(n-1) + (1 - \beta)e(n)e(n-1); \quad (2.25)$$

This estimate $\hat{p}(n)$ is used to update the step-size as in (2.26)

$$\hat{\mu}(n+1) = \alpha \mu(n) + \gamma \hat{p}(n); \quad (2.26)$$

The parameters α and γ is the same as proposed in [42]. The value of β influences the quality of the estimation and has values in the range $0 < \beta < 1$. The step-size follows the same bounds as in (2.24). The algorithm enhances the performance in measurement noise but at the cost of an additional parameter, β .

VS-LMS algorithm based on controlled bandwidth

In [83], the authors proposed new mechanism of controlling the bandwidth of the ALE (shown in Fig. 2.2) based on its frequency response peak. The transfer function $H(z)$ in [83] is derived by setting Δ equal to 1 as shown in (2.27),

$$H(z) = \frac{(1-r)(W - z^{-1})}{1 - (1+r)Wz^{-1} + rz^{-1}} \quad (2.27)$$

Algorithm 2.1: Farhang's algorithm [83] when r and W are adjusted simultaneously

Initialization;

$$y(n) =$$

$$(1 + r(n))W(n)y(n-1) - r(n)y(n-2) + (1 - r(n))(W(n)u(n) - u(n-1));$$

$$e(n) = u(n+1) - y(n);$$

$$\alpha(n) =$$

$$(1 + r(n))W(n)\alpha(n-1) - r(n)\alpha(n-2) + (1 + r(n))y(n-1) + (1 - r(n))u(n);$$

$$W(n+1) = W(n) + \mu_W(n)e(n)\alpha(n);$$

$$\beta(n) = (1 + r(n))W(n)\beta(n-1) - r(n)\beta(n-2) - (W(n)e(n-1) - e(n-2));$$

$$r(n+1) = r(n) + \mu_r \frac{1}{(1-r(n))^2} y(n)^2 + \frac{(1+r(n))}{(1-r(n))} y(n)\beta(n);$$

$$\mu_W(n+1) = 0.5(1 - r(n+1))^3$$

where, r is the parameter which controls the bandwidth of the IIR ALE and takes value in the range 0 to 1. The variables $\alpha(n)$ and $\beta(n)$ are the variables used to update the $W(n)$ and $r(n)$, respectively. $\mu_W(n)$ and μ_r are the algorithm step-sizes. It can be seen from Algorithm 2.1 that step-size for updating the filter weights is chosen based on the value of $r(n)$. The authors claim that this mechanism of varying the bandwidth guarantees fast convergence.

It is clear that there has been extensive research in determining an appropriate value of step-size. Most of this scheme fall in the categories described above or use some form of time-domain information to update the step-size. However, the PANF proposed in [51] is a new mechanism where the information about the position of main notch with respect to the input sinusoid frequency is used to determine an appropriate step-size. It is a novel technique significantly different from the schemes reported in the literature. This is discussed in detail in subsequent chapters.

2.4 Adaptive IIR Notch Filter

Notch filters are widely used in systems which are required to eliminate a specific frequency or multiple frequencies interfering with the useful signal. They are used in frequency estimation and tracking of sinusoids in noise. It is an important area of research to solve problems in radar, communications, bio-medical and other related areas [1]. The details of its applications in diverse areas are already discussed in the Introduction. It is also discussed that although it can be implemented both as FIR and IIR type filters, we will focus our work on adaptive IIR notch filter because of its inherent potential for reduction in complexity and improvement in performance.

The IIR digital notch filters have been developed in the literature using various methods which popularly include analog filter transformation [28], implementation using all-pass filters [29,30], and by pole-zero placement techniques [24]. The analog filter transformation [28] is used to transform an analog notch filter into a digital notch filter using bilinear transform, impulse invariance and step invariance. This filter is then characterized by parameters with minimum number of multipliers for optimized realizations. Similarly, digital notch filters are also implemented, by using all-pass filter sections [29, 30]. These all-pass filter sections possess mirror-image symmetry in their transfer function which can be effectively utilized to realize notch filters with computationally efficient lattice structure and having very low sensitivity. The simplest and the most effective technique for IIR notch filter design is by pole-zero placements [31], although it has the limitations of asymmetric structure with uncontrollable gain.

Adaptive notch filters are designed for scenarios when the frequency, to estimate

and track, is variable. The filters are adapted using adaptive algorithms. They are popularly implemented using the LMS algorithm [32–36] in which the notch frequency is updated iteratively to eventually converge to the frequency of the input signal. This algorithm is widely used in various applications for its computational simplicity and robustness. The filter parameters are adapted by minimizing the mean square of output error sequence which is basically the difference between the desired output signal and the output of the filter. This has been discussed in detail in section 2.2.

The bandwidth of these adaptive filters also influences the speed of convergence. The bandwidth of the notch filter which determines the sharpness of the notch filter is calculated from the frequencies for which the magnitude of the transfer function is reduced to -3 dB. The notch filter attenuates the frequencies between its lower and upper -3dB points and rejects any frequency between these frequencies. The effect of bandwidth for infinite impulse response (IIR) adaptive line enhancer (ALE)) has been studied in [83], in which the author proposed a mechanism for controlling the bandwidth of the ALE based on its frequency response peak. In [8, 9, 84, 85], the authors proposed adaptive notch filters with time-varying bandwidth. The authors in [8] and [9], studied the effect of bandwidth on the duration of transient response for adaptive notch filter and proposed mechanism for varying the bandwidth.

Thus, there has been significant research in the design of adaptive notch filter structure as well as on various adaptive mechanism for desired performance. We shall now discuss the filter structure used in this thesis.

Figure 2.3: Pole-zero plot of the second-order adaptive IIR notch filter.

Figure 2.4: Structure of an IIR adaptive notch filter.

2.4.1 Traditional Adaptive Notch Filters

Conventional fixed step-size LMS-based (direct form) notch filter

In this thesis, we consider the conventional LMS-based second-order IIR adaptive notch filter for performance comparison. A second order IIR notch filter can be developed by constraining the zeros of the notch filter to the unit circle [24]. It is implemented by pole-zero placement technique where a zero is placed on the unit circle and a pole is placed at a location between the zero and the origin as shown in Fig. 2.3. The direct form implementation of a second order IIR adaptive notch filter is shown in Fig. 2.4. The z-transform transfer function of the time-varying notch filter is stated in (2.28).

Figure 2.5: Structure of the lattice notch filter.

$$H(z, n) = \frac{1 + W(n)z^{-1} + z^{-2}}{1 + rW(n)z^{-1} + r^2(n)z^{-2}} \quad (2.28)$$

where, r is the pole radius which determines the bandwidth of the notch filter and the range of r is $0 \ll r < 1$. The bandwidth of the notch can be approximated as, $BW \approx (1 - r)f_s/\pi$ in hertz [9], where f_s is the sampling frequency. A narrow bandwidth notch is obtained when the value of r is close to 1. $W(n)$ is the adjustable filter weight updated adaptively using the LMS algorithm. The Widrow's technique [32] is used to update $W(n)$ at sampling instant n as shown in (2.29) (refer to Appendix A for derivation)

$$W(n + 1) = W(n) - 2\mu_c x(n - 1)e(n) \quad (2.29)$$

where, μ_c is the fixed step-size for the conventional notch. It will inherit the limitations of fixed step-size LMS as discussed earlier. $x(n)$ is an intermediate filter state and $e(n)$ is the output of the filter. The notch frequency at any time n can be calculated using (2.30)

$$\theta(n) = \cos^{-1}(-W(n)/2) \quad (2.30)$$

LMS-based adaptive lattice notch filter

The structure of the LMS-based adaptive lattice filter is shown in Fig. 2.5. The transfer function for lattice notch filter as mentioned in [46] can be written as

$$\frac{H(z)}{E(z)} = \frac{1 + W(n)z^{-1} + z^{-2}}{1 + (1 + r^2)\frac{W(n)}{2}z^{-1} + r^2z^{-2}} \quad (2.31)$$

where, $E(z)$ is the z-transform of the error signal $e(n)$ and r is the pole radius. The notation of filter weight in (2.29) is modified as $W(n)/2$ to have a common platform of comparison with other adaptive algorithms. The weight update equation for lattice algorithm is shown in (2.32).

$$W(n+1)/2 = W(n)/2 - 2\mu_{lat}x(n-1)e(n) \quad (2.32)$$

where, μ_{lat} is the fixed step-size for the lattice notch.

2.5 The Piloted Adaptive Notch Filter

The piloted adaptive notch filtering is a new concept in adaptive filtering proposed in [51]. This is an IIR notch filter which can be realized with any adaptive algorithm reported in literature. The implementation using the LMS algorithm is considered in this thesis for illustration as it is currently one of the most popular adaptive algorithm. The notch frequency of the filter is updated iteratively using an adaptive step-size to steer the zero of the notch towards the input sinusoid based on the gradient information.

As discussed in the Section 2.3, variable step-size algorithms are preferred over fixed step-size algorithms. The conventional variable step-size algorithms determine the step-size based on the time averaging of the gradient estimate at several sampling instances. This gradient estimate can only provide the direction in which

the notch should be steered without any information on the distance between the notch frequency and the frequency of the input sinusoid.

The PANF having a main notch and the pilot notches, being incorporated into the filter structure at minimum cost, helps to estimate the relative position of the notch frequency with respect to the frequency of the input sinusoid. This information is then used to determine the appropriate step-sizes for adapting the filter coefficient. The information about the steering direction of the notch is estimated by checking the signs of the gradient estimate, at each sampling instance, of the main notch and the pilot notches at several frequency points. If the notch is found to be far away from the input sinusoid, a larger step-size is chosen, whereas, a smaller step-size is used if the notch is close to the input frequency. The following section introduces the PANF with its filter structure and mathematical formulations.

2.5.1 Mathematical formulations

As proposed in [51], the PANF consists of a main notch and two pilot notches. These are second order IIR filters with transfer function as stated in (2.33), which have a pole followed by a zero and the zero of the main notch being sandwiched between that of the pilot notches as in Fig. 2.6.

$$H(z, n) = \frac{1 + W(n)z^{-1} + z^{-2}}{1 + rW(n)z^{-1} + r^2z^{-2}} \quad (2.33)$$

where, r is the pole radius which determines the bandwidth of the notch filter. $W(n)$ is the adjustable filter weight updated adaptively using the LMS algorithm. The Widrow's technique [32] has been used to update $W(n)$ as shown in (2.34)

Figure 2.6: Pole-zero plot of the piloted notch filter.

(refer to Appendix A for derivation)

$$W(n+1) = W(n) - 2\mu(n)x(n-1)e(n) \quad (2.34)$$

where, $\mu(n)$ is the adaptation step-size which influences the rate of convergence and the misadjustment. $x(n)$ is an intermediate filter state and $e(n)$ is the output of the filter. The input $y(n)$ is assumed to be a single sinusoid as shown in (2.35)

$$y(n) = a \cos(\omega_0 n + \varphi) \quad (2.35)$$

where, ω_0 is the frequency of the input sinusoid, φ is a arbitrary phase constant, and a is the magnitude of the input sinusoid.

The notch frequency at any time n can be calculated using (2.36)

$$\theta(n) = \cos^{-1}(-W(n)/2) \quad (2.36)$$

The notch frequency of one of the pilot notches is kept higher by ϕ_{high} and the other lower by ϕ_{low} as shown in Fig. 2.6. The complexity of the structure is reduced by integrating the pilot notches with that of the main notch filter (as shown in Fig. 2.7) by keeping the same denominator for all the notches. The transfer

Figure 2.7: Structure of the piloted notch filter.

functions of the pilot notches are formulated as in (2.37) and (2.38), denoted by $H_{high}(z, n)$ and $H_{low}(z, n)$.

$$H_{high}(z, n) = \frac{1 + W_{high}(n)z^{-1} + z^{-2}}{1 + rW(n)z^{-1} + r^2z^{-2}} \quad (2.37)$$

$$H_{low}(z, n) = \frac{1 + W_{low}(n)z^{-1} + z^{-2}}{1 + rW(n)z^{-1} + r^2z^{-2}} \quad (2.38)$$

where, W_{high} and W_{low} are obtained as

$$W_{high}(n) = W(n) + \alpha_{high} \quad (2.39)$$

$$W_{low}(n) = W(n) - \alpha_{low} \quad (2.40)$$

The values of α_{high} and α_{low} correspond to ϕ_{high} and ϕ_{low} , respectively, and can be calculated in advance for a particular application. The precision of ϕ_{high} and ϕ_{low} need not to be high as the pilot notches are used only to get the steering direction. Substituting the value of $W_{high}(n)$ and $W_{low}(n)$ into (2.37) and (2.38) would lead to

$$H_{high}(z, n) = H(z, n) + \frac{\alpha_{high}z^{-1}}{1 + rW(n)z^{-1} + r^2z^{-2}} \quad (2.41)$$

$$H_{low}(z, n) = H(z, n) - \frac{\alpha_{low}z^{-1}}{1 + rW(n)z^{-1} + r^2z^{-2}} \quad (2.42)$$

The output of the pilot notches being denoted as $e_{high}(n)$ and $e_{low}(n)$ can be

calculated as

$$e_{high}(n) = e(n) + \alpha_{high}x(n-1) \quad (2.43)$$

$$e_{low}(n) = e(n) - \alpha_{low}x(n-1) \quad (2.44)$$

2.5.2 Determination of steering direction

The concept of PANF was formulated with the objective of estimating the difference between the frequency of the input sinusoid and the notch frequency. The steering direction of each pilot notch and the main notch is interpreted from the signs of the gradient estimate of the main notch and pilot notches at several frequency points. The notch is said to be close to the input sinusoid if the frequency the input sinusoid is sandwiched between the frequencies of the pilot notches $H_{high}(z, n)$ and $H_{low}(z, n)$. And, the notch is assumed to be far away from the input sinusoid if the input sinusoidal frequency is higher than the frequency of $H_{high}(z, n)$ or lower than the frequency of $H_{low}(z, n)$.

Let us take the noiseless input signal $y(n)$ as shown in (2.35). The intermediate state $x(n)$ which is also a sinusoidal signal is expressed as

$$x(n) = A \cos(\omega_0 n + \beta) \quad (2.45)$$

The error output $e(n)$ is expressed as

$$e(n) = A \cos(\omega_0 n + \beta) + W(n)A \cos(\omega_0(n-1) + \beta) + A \cos(\omega_0(n-2) + \beta) \quad (2.46)$$

The product term $x(n-1)e(n)$ is obtained as

$$\begin{aligned}
& x(n-1)e(n) \\
&= [A \cos(\omega_0(n-1) + \beta)][A \cos(\omega_0 n + \beta) \\
&+ W(n)A \cos(\omega_0(n-1) + \beta) + A \cos(\omega_0(n-2) + \beta)] \\
&= A^2 \cos^2(\omega_0(n-1) + \beta)[W(n) + 2 \cos \omega_0] \\
&= 2A^2 \cos^2(\omega_0(n-1) + \beta)[\cos \omega_0 - \cos(\theta(n))] \tag{2.47}
\end{aligned}$$

From (2.47), it can be inferred that if $x(n-1)e(n)$ is positive at a particular instant n , then it indicates $\omega_0 < \theta(n)$. Conversely, if $x(n-1)e(n)$ is negative at a particular instant n , then it indicates $\omega_0 > \theta(n)$. This analysis also follows for the product terms of the pilot notches, $x(n-1)e_{high}(n)$ and $x(n-1)e_{low}(n)$. Thus, the relative positions of ω_0 with respect to the main notch frequency $\theta(n)$ can be obtained from the signs of $x(n-1)e_{high}(n)$, $x(n-1)e(n)$ and $x(n-1)e_{low}(n)$. However, the steering direction determination is affected if the input sinusoid $y(n)$ is corrupted with noise as shown in (2.48)

$$y(n) = a \cos(\omega_0 n + \beta) + s(n) \tag{2.48}$$

where, $s(n)$ is a zero mean white Gaussian noise. The input SNR is defined in this thesis as

$$SNR(dB) = 10 \log_{10} \left(\frac{a^2}{2\sigma_s^2} \right) \tag{2.49}$$

where, σ_s^2 is the variance of noise. The steering direction determination mechanism is explained further in Section 2.5.3.

2.5.3 Variable step-size mechanism

The PANF uses two step-sizes, denoted as μ_{high} and μ_{low} , which are chosen according to the position of the main notch w.r.t. the input sinusoid. The information of the steering direction obtained from the signs of $x(n-1)e_{high}(n)$, $x(n-1)e(n)$ and $x(n-1)e_{low}(n)$ is used to choose an appropriate step-size. Given that the input $y(n)$ is a noiseless sinusoid, the signs of $x(n-1)e_{high}(n)$, $x(n-1)e(n)$ and $x(n-1)e_{low}(n)$ indicate whether the frequency of the input sinusoid is higher or lower than the frequency of the respective notch. If the sign of $x(n-1)e_{low}(n)$ is positive, it implies that the frequency of the input sinusoid is lower than the notch frequency of $H_{low}(z, n)$. Since, $H_{low}(z, n)$ is the lower pilot notch, the sign $x(n-1)e_{high}(n)$ and $x(n-1)e(n)$ will also be positive in this case. Similarly, if the sign of $x(n-1)e_{high}(n)$ is negative, it implies that the frequency of input sinusoid is higher than the notch frequency of $H_{high}(z, n)$. And thus, the sign $x(n-1)e(n)$ and $x(n-1)e_{low}(n)$ will also be negative. So, if the sign of $x(n-1)e_{high}(n)$, $x(n-1)e(n)$ and $x(n-1)e_{low}(n)$ are the same, the input frequency is either higher than the notch frequency of $H_{high}(z, n)$ or lower than that of $H_{low}(z, n)$. This is considered to be the case when the notch frequency of the main notch is assumed to be far away from the frequency of the input sinusoid. Thus, a larger step-size i.e., μ_{high} can be used to increase the speed of convergence.

In the case, when the frequency of the input sinusoid is in the close vicinity of the main notch, it will be sandwiched between the notch frequency of $H_{high}(z, n)$ and $H_{low}(z, n)$. While, the sign of $x(n-1)e_{high}(n)$ will be positive and the sign of $x(n-1)e_{low}(n)$ will be negative, the sign of $x(n-1)e(n)$ will be dependent on the position of the input frequency w.r.t. the main notch. This is the condition,

when a smaller step-size i.e., μ_{low} , can be used to reduce the mis-adjustment error at steady-state.

2.6 Complex Adaptive IIR Notch Filter

When the signal consist of in-phase and quadrature-phase components leading to complex signals, the filter structure need to be modified and implemented using complex coefficients. Thus, the CANF was developed and studied in literature [86–91]. They can be used in many applications as discussed in [87], especially in communication systems using QPSK modulation [86], and radar signal processing. They are implemented both as FIR and IIR based filters, but IIR filters are computationally more efficient as compared to FIR filters as discussed earlier.

In [87], the authors proposed complex adaptive IIR notch filter by implementing it as a first order IIR filter with constrained poles and zeros. They modified the complex Gauss-Newton type algorithm [92] to update its complex coefficients. Although, their filter has shown improved speed of convergence, the algorithm is inherently biased and also requires stability check. In [88], the authors proposed complex filter structure based on first order complex notch filter and used stochastic gradient algorithm to update the filter co-efficient but the filter showed degradation in the speed of convergence when the notch approached the input signal frequency.

The authors in [90] proposed the CANF using lattice all-pass filter and claimed that the convergence is unbiased and has higher speed of convergence compared to the one proposed in [87]. However, the filter showed degradation in performance with increase in pole radius or narrowing the bandwidth. The authors in [93] also proposed an all-pass filter sections based CANF which can track multiple complex

Figure 2.8: Structure of an complex adaptive IIR notch filter.

sinusoids, which are close in frequency, based on an output learning algorithm.

The filters proposed in [87,88,90,93] have been developed as an extension to the filters capable of processing real signals. Moreover, they have used a fixed step-size parameter to update the filter weights. The value of step-size influences the speed of convergence and misadjustments at steady-state as discussed earlier. A variable step-size algorithm for CANF will provide better performance matrix compared to a fixed step-size algorithm. However, reports on variable step-size CANF is rare in the published literature.

2.6.1 First order complex adaptive IIR notch filter

We have used first order complex adaptive IIR Notch filter in this thesis. The direct form of first order complex adaptive IIR notch filter is shown in Fig. 2.8 and the pole-zero plot of first order complex adaptive IIR notch filter is shown in Fig. 2.9. Thus, the transfer function of the time-varying notch filter is as stated in (2.50).

$$H(z, n) = \frac{1 + W(n)z^{-1}}{1 + rW(n)z^{-1}} \quad (2.50)$$

where, r is the pole radius which indicates the bandwidth of the notch filter and the value of r is always less than unity for stability. $W(n)$ is the adjustable complex filter weight and is given by

$$W(n) = -e^{j\theta(n)} = -(\cos \theta(n) + j \sin \theta(n)) \quad (2.51)$$

Figure 2.9: Pole-zero plot of first order complex adaptive IIR notch filter.

where, $\theta(n)$ is the notch frequency. It can be updated using any adaptive algorithms as discussed earlier.

2.7 Chapter Summary

In this chapter, we have discussed the historical background of adaptive filter theory to appreciate the development of adaptive algorithms, and particularly the LMS algorithm. The VS-LMS algorithms have been discussed to exhibit the significance of PANF as a new mechanism to determine the step-size. The adaptive IIR notch filters both in real and complex form have been presented which will be used later in this thesis.

Chapter 3

Steady State Performance Evaluation of Piloted Adaptive Notch Filter

3.1 Introduction

As discussed in the literature survey, frequency estimation and tracking of signals is an important area of research for its applications in various domains. There has been significant research in the development of ANFs to improve the speed of convergence and the steady-state MSE. As these schemes of frequency estimation are becoming important in various applications, a considerable amount of work has been done for the performance analysis of these filters. There are works for both theoretical as well as simulation based performance evaluation, with deeper insight into the functioning of these adaptive filters.

The performance analysis of IIR notch filter is more involved as compared to the FIR notch filter due to their non-linear nature. However, there has been several efforts to analyse the performance of the adaptive algorithms for the IIR filter [94–100]. In [94,95], the authors obtained the analytical results for the performance of the adaptive IIR notch filter with constrained poles and zeros using recursive algorithms [24, 101]. The authors in [96] and [97] analysed the performance of

adaptive lattice notch filter using the ordinary differential equation (ODE) and MSE techniques. In [98], the authors presented the stochastic convergence analysis and the stability bounds for a second-order adaptive IIR notch filter. In [99], the authors proposed a method for the performance analysis of stochastic gradient based notch filters similar to the analysis of adaptive lattice notch filter. Although the authors in [100] derived a closed form expression for bias and MSE of ANF with constrained poles and zeros but it remains valid only for small step-sizes leading to slower convergence. The authors in [51] presented the theoretical analysis of the performance of the steering direction mechanism of PANF, however, the detailed performance comparison of the PANF at steady state is not yet available.

In this chapter, we present a detailed performance comparison of the PANF with the adaptive lattice notch filter and the adaptive notch filter with constrained poles and zeros. With the objective of determination of superior algorithms for practical applications, we compare these filters for various performance parameters. Extensive computer simulations are done to test these filters under various conditions.

3.2 Performance Comparison

The adaptive IIR notch filters discussed in Sections 2.4.1 and 2.5 employs LMS algorithm for adapting $W(n)$. The frequency of the notch is updated at each sampling instance in order to converge to the frequency of the input sinusoid. We have done computer simulations to compare the performance of the three approaches being discussed in 2.4.1 and 2.5. The bandwidth of all the notches is kept the same at $r = 0.95$. The parameters for piloted notch filter, α_{high} and α_{low} are kept

Figure 3.1: MSE comparison at the same speed of convergence for conventional notch, lattice notch and PANF.

symmetrical and equal to 0.625. The frequency of the input sinusoid is one eighth of the sampling frequency with input SNR equal to 10 dB.

3.2.1 Comparison of mean square error (MSE)

In this section, we estimate the MSE through simulation for the same speed of convergence by the three schemes. The step-sizes are chosen such that they all converge approximately around the same number of iteration. The results are shown in Fig. 3.1. The step-sizes being used in the simulation are as follows: $\mu_c = 1.4 \times 10^{-3}$ (see (2.29)), $\mu_{lat} = 7.0 \times 10^{-4}$ (see (2.32)), $\mu_{high} = 3 \times 10^{-3}$ and $\mu_{low} = (1/3) \times 10^{-3}$.

We can clearly visualize from the results in Fig. 3.1 that the MSE of PANF for the same convergence speed is fairly low as compared to the other two schemes. This is because the PANF uses a smaller step-size after convergence as compared to the other schemes which continue to use the same step-size, thus yielding lower

Figure 3.2: Comparison of speed of convergence for the same MSE for conventional notch, lattice notch and PANF.

misadjustment.

3.2.2 Comparison of speed of convergence

In this section, we will review the performance of the PANF for the comparison of speed of convergence. The step-sizes for the three schemes have been chosen such that they have approximately the same MSE at steady state. The step-sizes being used for this analysis are as follows: $\mu_c = 3.3 \times 10^{-3}$, $\mu_{lat} = 1.7 \times 10^{-4}$, $\mu_{high} = 3 \times 10^{-2}$ and $\mu_{low} = (1/3) \times 10^{-2}$. The results in Fig. 3.2 show that PANF shows an improved speed of convergence for the same MSE in comparison to the other two schemes. This is because of the fact that the PANF uses a higher initial step-size to increase the speed of convergence and then adapt to lower step-size to maintain the required steady-state MSE.

Figure 3.3: Standard deviation of frequency estimate versus input SNR for conventional notch, lattice notch and PANF.

3.2.3 Deviation in estimated frequency after convergence

We know that the performance of adaptive IIR notch improves with the increase in input SNR. Fig. 3.3 shows the performance of the three schemes for frequency tracking with variation in the noise conditions, where the frequency deviation from the input frequency is plotted versus the input signal's SNR. The parameters being used are the same as those of Section 3.2.1 and the frequency deviation is calculated by averaging the last 3000 values of the estimated weights with an input data length of 10000 samples. The result shows that even in the worst case scenarios, the PANF performs better than the other two schemes with a standard deviation less than 0.0001π under various noise conditions.

Figure 3.4: Steady-state MSE versus input SNR for conventional notch, lattice notch and PANF.

3.2.4 Steady-state MSE comparison

In this section, we estimate the steady-state MSE of the three approaches using simulations. A large input data length of 10000 is chosen and the last 3000 output data samples of the filter in each run are time averaged to produce the estimated values of MSE. We consider the same filter parameters as in Section 3.2.1.

For different input SNR

The estimated MSE are plotted versus the input signal's SNR as shown in Fig. 3.4. The result shows that the PANF has comparatively smaller MSE at steady state with respect to the other two algorithm over a range of SNR values. This shows that the steering direction determination is helping the notch to choose an appropriate step-size under noisy conditions. However, it will be interesting to examine the performance degradation under low SNR conditions which is covered in Chapter 4.

Figure 3.5: Steady-state MSE versus pole radius for conventional notch, lattice notch and PANF.

For different pole radius

The pole radius r plays an important role in the estimation of the frequency. Simulations are done for different values of pole radius r . The results are shown in Fig. 3.5 which clearly show that the pole radius affects the performance of the filters as the pole radius approaches unity. Detailed analysis of the effect of the pole is done in Chapter 5.

For different input frequency

We estimate the steady-state MSE for different input frequency through simulation. The estimated values are plotted as shown in Fig. 3.6. The results show that all the schemes have high MSE, when the frequency of the input sinusoid is either close to zero or half-sampling rate frequency with PANF performing the best among the three schemes.

Figure 3.6: Steady-state MSE for different input frequency for conventional notch, lattice notch and PANF.

3.3 Chapter Summary

In this chapter, we have presented a computer simulation based performance comparison of the three schemes of frequency estimation and tracking using adaptive second order IIR notch filters. These filters have been simulated under various noise conditions with different filter parameters for proper evaluation of the algorithms. The steady-state analysis in terms of MSE has been observed to determine the best possible solution for a given scenario. It is concluded through various performance parameters that the PANF which uses a steering mechanism is superior to the other two widely used filter structures in frequency estimation and tracking.

Chapter 4

A Piloted Notch Time-Frequency based Variable Step-Size (VS) Algorithm

4.1 Introduction

In literature, variable step-size determination algorithms for LMS based adaptive filtering are extensively reported with the objective to develop efficient filters with improved rate of convergence and low misadjustment error at steady-state. Various step-size update techniques proposed in [38–50] performs time-domain averaging on the output error to obtain suitable step-sizes. In [38], the sign changes between consequent gradient estimates was used to update the step-size. While in [44], the authors used the product of consecutive gradient estimates to update the step-size, the authors in [41] updated the step-size as a function of the low pass filtered values of gradient estimates. In [45], the authors replaced the linear update recursions of step-sizes by multiplicative update recursions. All these techniques are based on time domain information but the PANF uses a novel method of estimating the distance between the notch frequency and the frequency of the input sinusoid with a filter architecture incorporating the pilot notches. This frequency domain information is used to choose an appropriate step-size. Thus, it depends on the

probability of obtaining a correct steering direction. Hence, there is room for improvement particularly at low SNR [102].

In this chapter, we present a more robust time-domain averaging based steering direction determination mechanism for the PANF which has higher probability of obtaining a correct steering direction. A weighted averaging of the gradient analysis of the piloted notch cost function at several frequency points at the same sampling instant is performed to estimate the distance between the notch frequency and the frequency of the input sinusoid. This frequency-domain information from the steering direction is combined with time domain information to develop a step-size update algorithm which improves the speed of convergence with very low MSE. Extensive computer simulations are done to test the proposed method under various conditions.

4.2 Proposed Steering Direction Determination Mechanism for PANF

In this section, a time-domain averaging based steering direction determination mechanism is proposed for the PANF architecture. In [51], the information about the steering position is determined using the signs of $x(n-1)e(n)$, $x(n-1)e_{high}(n)$ and $x(n-1)e_{low}(n)$ as discussed in Section 2.5 of Chapter 2. This scheme determines the instantaneous steering direction at time n . However, the instantaneous steering direction from $x(n-1)e(n)$ is contaminated with noise and hence the probability of obtaining a correct steering direction decreases with decrease in input SNR. Here, we develop a more robust steering mechanism to correctly steer the notch in the desired direction.

The input signal $y(n)$ is as shown in (2.48). The intermediate state $x(n)$ which is also a sinusoidal signal, is expressed as

$$x(n) = A \cos(\omega_0 n + \beta) + \xi(n) \quad (4.1)$$

where, $\xi(n)$ is the filtered version of $s(n)$ through the all-pole section of the notch filter.

The error output $e(n)$ from (2.46) can be expressed as (4.2) using (2.36),

$$\begin{aligned} e(n) &= A \cos(\omega_0 n + \beta) + W(n)A \cos(\omega_0(n-1) + \beta) \\ &\quad + A \cos(\omega_0(n-2) + \beta) + \eta(n) \\ &= A \cos(\omega_0(n-1) + \beta) \tau(\theta, n) + \eta(n) \end{aligned} \quad (4.2)$$

where,

$$\tau(\theta, n) = 2(\cos \omega_0 - \cos(\theta(n))) \quad (4.3)$$

$$\eta(n) = \xi(n) + W(n)\xi(n-1) + \xi(n-2) \quad (4.4)$$

The product term $x(n-1)e(n)$ can be expressed as

$$\begin{aligned} &x(n-1)e(n) \\ &= [A \cos(\omega_0(n-1) + \beta) + \xi(n-1)][A \cos(\omega_0 n + \beta) \\ &\quad + W(n)A \cos(\omega_0(n-1) + \beta) + A \cos(\omega_0(n-2) + \beta) + \eta(n)] \\ &= A^2 \cos^2(\omega_0(n-1) + \beta)[W(n) + 2 \cos \omega_0] + \nu(n) \\ &= 2A^2 \cos^2(\omega_0(n-1) + \beta)[\cos \omega_0 - \cos(\theta(n))] + \nu(n) \\ &= A^2 \cos^2(\omega_0(n-1) + \beta) \tau(\theta, n) + \nu(n) \end{aligned} \quad (4.5)$$

$$= \frac{A^2}{2} \tau(\theta, n) + \frac{A^2}{2} \tau(\theta, n) \cos(2(\omega_0(n-1) + \beta)) + \nu(n) \quad (4.6)$$

Table 4.1: Power at dc and peak power frequency

Notch	Power at dc (dB)	Peak power (dB)	Freq. of peak Power
Higher	15.171	9.332	$2\omega_0$
Main	11.662	8.358	$2\omega_0$
Lower	5.683	7.266	$2\omega_0$

where, $\nu(n)$ is the noise term given by,

$$\begin{aligned} \nu(n) = & A \cos(\omega_0(n-1) + \beta)\eta(n) \\ & + \xi(n-1)\{A \cos(\omega_0(n-1) + \beta)\tau(\theta, n) + \eta(n)\} \end{aligned} \quad (4.7)$$

In the original PANF [51], the authors used (4.5) for the determination of the steering direction based on the sign of $\tau(\theta, n)$. It can be seen from (4.5) that when $|\cos^2(\omega_0(n-1))|$ is zero, the steering direction determination is solely decided by the noise component of product term, $\nu(n)$. Hence, it may lead to wrong choice of step-size and will affect the performance of the filter.

The power spectral density (PSD) estimates of $x(n-1)e_{high}(n)$, $x(n-1)e(n)$ and $x(n-1)e_{low}(n)$ for PANF are shown in Fig. 4.1 and the power at dc and peak power at $2\omega_0$ for the three notches is shown in Table 4.1 for the same simulation. From (4.6) and (4.3), it can be observed that the dc component of $A^2 \cos^2(\omega_0(n-1) + \beta)\tau(\theta, n)$ gives the correct information of the steering direction. Thus, we introduce a low-pass filtering of the product term $x(n-1)e(n)$ to extract the dc component of the $A^2 \cos^2(\omega_0(n-1) + \beta)\tau(\theta, n)$. The low-pass filtering process, denoted $\Psi(n)$, using a first-order filter is shown in (4.8),

$$\Psi(n) = \gamma\Psi(n-1) + x(n-1)e(n) \quad (4.8)$$

Figure 4.1: Power spectral density (PSD) estimates (in dB per rad) of the the product terms (a) $x(n-1)e_{high}(n)$, (b) $x(n-1)e(n)$ and (c) $x(n-1)e_{low}(n)$ with input frequency of 0.25π . The notch frequency is fixed at 0.75π and the input SNR is 10 dB. 0 dB is defined when the PSD is 1.

Figure 4.2: Graphical analysis of steering direction determination mechanism for PANF. (a) Comparison of the actual weights and the estimated weights for proposed and original steering mechanism, (b) and (c) Time instants of wrong prediction of steering direction by proposed ($\gamma = 0.3$) and original ($\gamma = 0$) steering mechanism respectively.

where, γ is the weighting parameter and is chosen in the range 0 to 1. The introduction of γ effectively takes weighted average of the present and past estimates with reduction in noise. The choice of γ is essential as it introduces a delay. If $\gamma = 0$ then it is the case where the instantaneous steering direction is obtained from $x(n-1)e(n)$ and if the value of γ is taken very close to unity, the delay of the filter will be too long for appreciable performance.

Similarly, the processes $\Psi_{high}(n)$ and $\Psi_{low}(n)$ for the pilot notches is obtained

as

$$\Psi_{high}(n) = \gamma\Psi_{high}(n-1) + x(n-1)e_{high}(n) \quad (4.9)$$

$$\Psi_{low}(n) = \gamma\Psi_{low}(n-1) + x(n-1)e_{low}(n) \quad (4.10)$$

From (4.6) and (4.8) and assuming that the filtering process of (4.8) causes negligible delay, it can be inferred that if $\Psi(n)$ is positive at a particular instant n , then it indicates $\omega_0 < \theta(n)$. Inversely, if $\Psi(n)$ is negative at a particular instant n , then it indicates $\omega_0 > \theta(n)$. Thus, the relative positions of ω_0 can be obtained from the signs of $\Psi_{high}(n)$, $\Psi(n)$ and $\Psi_{low}(n)$. If the signs of $\Psi_{high}(n)$, $\Psi(n)$ and $\Psi_{low}(n)$ are the same, then it can be concluded that the input frequency is away from the main notch. And, if they have different signs, then it will indicate that the input frequency is sandwiched in between the pilot notch frequencies, i.e., the input frequency is closer to the main notch. Simulations are done to compare the proposed mechanism to the one used in [51]. Fig. 4.2 shows the relative improvement of the steering directions where the vertical dark coloured lines indicates the time-instants of wrong prediction of steering direction by the respective algorithms.

4.3 Proposed Time-Frequency Information Based Step-size Update Algorithm

In this section, we present a time-frequency information based step-size update adaptive algorithm for PANF. The information of the steering direction is interpreted as the notch frequency is located in either of the two regions, denoted as L and S , in the frequency domain. The notch is said to be in

- region L , if it is far away from the input frequency.

- region S , if it close to the input frequency.

The position of the notch in a particular region in the frequency domain helps to define a suitable initial step-size. As discussed in Chapter 2, a larger step-size, μ_L , should be used when the distance between the zero of the main notch and the frequency of the input sinusoid is large. And a smaller step-size, μ_S , should be used when the zero of the notch is in the close vicinity of the frequency of the input sinusoid.

It is observed through simulations than the notch iterates for more than one iteration in a particular region in the frequency domain. This brings in a motivation to update the step-size for that region. For example, if it is observed that the notch remains in region L for consecutive iterations, then the step-size $\mu(n)$ must be incremented by δ_L for faster convergence. The step-size $\mu(n)$ which is initialized by an experimentally obtained suitable value corresponding to a region in the frequency domain is updated at each sampling instance based on the process $\Phi(n)$, obtained by low pass filtering the product term $x(n-1)e(n)$, as in (4.6), to reduce the inherent noise. The $\Phi(n)$ can be expressed as

$$\Phi(n) = \alpha\Phi(n-1) + x(n-1)e(n) \quad (4.11)$$

where, α is the weighting parameter. The value of α is taken close to unity which significantly reduces the noise content. The delay will not significantly effect the performance of the filter as this process is used when the notch iterates in a particular region (L or S) and the step-size is updated with a very small value. The change in step-size for each region is formulated as

$$\delta_L = \rho_L | \Phi(n) | \quad (4.12)$$

$$\delta_S = \rho_S | \Phi(n) | \quad (4.13)$$

where, ρ_L and ρ_S are the scaling factors for regions L and S , respectively. The value of ρ_L and ρ_S are different and defines the change in the step-sizes in a particular region. While, ρ_L affects the speed of convergence, the value of ρ_S controls the variance in the estimated weights $W(n)$, once the zero of the notch is in the vicinity of the input frequency. The step-size is updated using

$$\mu(n) = \mu(n-1) + \delta_L \quad (4.14)$$

$$\mu(n) = \mu(n-1) - \delta_S \quad (4.15)$$

Equations (4.14) and (4.15) are chosen according to the frequency domain information.

When the notch is observed to be in region L , multiple large step-sizes are used till the notch is in the vicinity of the input sinusoidal frequency. Once the notch is close to the input frequency, i.e., in the region S , a smaller step-size helps in convergence which is updated at each sampling instance based on the same process $\Phi(n)$. Thus, we are using multiple step-sizes instead of only two step-sizes for faster and smooth tracking of the input frequency. Figure 4.3 shows the algorithm to be followed to update the step-size. The variables $flag_L$ and $flag_S$ are taken to check the position of the notch if it has continued to stay in the regions L and S for consecutive iteration, respectively. The variable SS is the absolute sum of the signs of $\Psi_{high}(n)$, $\Psi(n)$ and $\Psi_{low}(n)$ to determine the steering direction for an iteration.

Figure 4.3: Flow diagram for step-size update algorithm.

Figure 4.4: Probability of obtaining a correct steering direction using proposed steering direction determination mechanism for different input SNR.

Figure 4.5: Comparison of the steady-state MSE versus input SNR for original PANF and proposed PANF.

Figure 4.6: Comparison of the actual weight and the estimated weight for original PANF and proposed PANF.

4.4 Simulation Results and Discussions

We have done computer simulations to demonstrate the excellent performance of the proposed PANF with new steering direction detection mechanism and step-size update algorithm over the original PANF [51]. The bandwidth of all the notches is kept the same at $r = 0.9$. The parameters for PANF, α_{high} and α_{low} are kept symmetrical and equal to 0.4. The step-size μ_L and μ_S are taken as 3×10^{-2} and $(1/3) \times 10^{-2}$ respectively. The value of γ is taken to be 0.6 while the value of α is 0.95. The frequency of the input sinusoid is 0.3π . The simulation have been done over a range of SNR values to compare the robustness of the proposed method. Fig. 4.4 shows that the proposed steering mechanism have higher probability of obtaining a correct steering direction as compared to the original steering mechanism of PANF in [51]. Fig. 4.5 shows that the steady-state MSE of the proposed PANF is highly reduced as compared to the original PANF with the use of a variable step-size update algorithm. Fig. 4.6 shows the improvement in the speed of convergence with smoother tracking when the input SNR is 10 dB and the input sinusoidal frequency is abruptly changed from 0.3π to 0.4π at 300^{th} iteration. A detailed theoretical analysis of the new steering direction determination mechanism is presented in Chapter 6.

4.5 Chapter Summary

In this chapter, a new steering direction determination mechanism and step-size update algorithm have been proposed for the PANF. The new steering direction determination mechanism based on time-averaging of the gradient analysis of the piloted notch cost function has proved to be superior with higher probability of

providing a correct steering direction. This highly accurate steering direction information has been utilized to derive the frequency domain information which is used for the step-size update algorithm. A new step-size update algorithm has been introduced based on time-frequency domain information which has improved the speed of convergence with a comparatively huge reduction in steady-state MSE.

Chapter 5

Effect of Pole Radius in a Piloted Adaptive Notch Filter

5.1 Introduction

The bandwidth of the adaptive notch filter influences the speed of convergence and the steady-state performance. The effect of the bandwidth for IIR adaptive line enhancer (ALE) was studied in [83] in which the author proposed a mechanism for controlling the bandwidth of the ALE based on its frequency response peak. In [8, 9, 84, 85], the authors proposed ANFs with time-varying bandwidth. The authors in [8] and [9], studied the effect of bandwidth on the duration of transient response for ANF and proposed a mechanism for varying the bandwidth. Thus, there has been efforts for the design of variable pole-radius mechanism for improved performance of ANF.

Chapters 3 and 4 have given an insight into the PANF. However, the effect the pole position approximation for pilot notch, i.e., the use of the same pole of the main notch for the pilot notches has not been investigated. The effect of pole radius on the duration of transient response and its influence on steering direction determination needs to be studied for robust design of PANF.

In this chapter, we study the effect of the pole position for a PANF and present

a variable pole radius PANF (VP-PANF) to significantly improve its performance under low SNR conditions. The use of variable pole radius not only helps in transient suppression but also significantly improves the probability of obtaining a correct steering direction. This improved steering direction determination increases the rate of convergence and also help to reduce the MSE at steady-state. Simulations are presented to show the improved steering direction determination, speed of convergence and steady-state MSE.

5.2 Effect of Pole Radius in a PANF

In Section 2.5 of Chapter 2, we have shown the transfer functions of the pilot notches in (2.37) and (2.38) denoted by $H_{high}(z, n)$ and $H_{low}(z, n)$. In order to simplify the theoretical analysis, we assume that $W(n)$, $W_{high}(n)$ and $W_{low}(n)$ can be approximated by time-invariant W , W_{high} and W_{low} respectively within a given short duration of interest. The frequency response gain for pilot notches are obtained as shown in (5.1) and (5.2),

$$|H_{high}(e^{jw})|^2 = \frac{(2\cos(\omega) + W_{high})}{(1 - r^2)^2 + r(2\cos(\omega) + rW)(2r\cos(\omega) + W)} \quad (5.1)$$

$$|H_{low}(e^{jw})|^2 = \frac{(2\cos(\omega) + W_{low})}{(1 - r^2)^2 + r(2\cos(\omega) + rW)(2r\cos(\omega) + W)} \quad (5.2)$$

The above response is shown in Fig. 5.1(d). The effect of pole can be studied in the following terms.

Noise Amplification at the position of the pole

Since, the pole of the main notch is used as the pole of the pilots, the zero, pole and origin of the pilot notches are not in a straight line, i.e., there is a pole-zero

Figure 5.1: (a), (b) and (c) are the pole-zero plots of the main notch, higher pilot notch and lower pilot notch respectively. (d) is the frequency response of the main and pilot notches.

misalignment. The poles and zeros for $H_{high}(z, n)$ and $H_{low}(z, n)$ are plotted in Figs. 5.1(b) and 5.1(c) respectively. The pole-zero misalignment will result in the amplification of noise near the pole of the pilot's output. This amplified noise may lead to wrong determination of steering direction as will be explained further in Section 5.3. It can also be observed that the steering direction information from the pilot notch towards the input frequency will be most affected as its pole will amplify the noise in the wrong side of the pilot notch frequency. A larger pole radius will cause larger amplification of the noise but a smaller pole radius will result in degraded steady-state performance.

Transient State of the notch filter

The effect of pole radius r on the transient response of a conventional ANF is studied in [8] and [9]. The transient response is significantly important for PANF

Figure 5.2: Analysis of transient state for PANF at different notch frequencies for an input frequency of 0.25π

as it will affect the steering direction determination and the step-size being used for updating the filter weights. Fig. 5.2 shows the number of transient iterations required to give the correct steering direction increases with increase in pole radius r . Hence, the speed of convergence of PANF is affected by a higher value pole radius. This effect is greatly reduced by the introduction of variable pole radius mechanism as discussed in Section 5.3.

5.3 Variable Pole Radius Mechanism

The determination of the correct steering direction is key to decide the step-size for updating the filter coefficients. In [51], the signs of $x(n-1)e(n)$, $x(n-1)e_{high}(n)$ and $x(n-1)e_{low}(n)$ were used to determine the position of the main notch with respect to the input frequency. The input SNR and subsequent amplification of the noise terms are critical for the probability of obtaining a correct steering direction. As discussed in Section 2.5 of Chapter 2, the product term $x(n-1)e(n)$ can be

expressed as

$$x(n-1)e(n) = A^2 \cos^2(\omega_0(n-1) + \beta)\tau(\theta, n) + \nu(n) \quad (5.3)$$

where, $\nu(n)$ is the noise term. It is clear from (5.3) that the steering direction is correct if $|\nu(n)| < |A^2 \cos^2(\omega_0(n-1) + \beta)\tau(\theta, n)|$.

PANF may predict the wrong steering direction when the noise term $\nu(n)$ does not satisfy the criterion $|\nu(n)| < |A^2 \cos^2(\omega_0(n-1) + \beta)\tau(\theta, n)|$. As a consequence the pilots may give wrong estimate of the distance between the notch frequency and frequency of the input sinusoid. This effect is particularly serious when the pole-zero misalignment causes the wrong side of the noise being amplified. The noise components being amplified may swamp the input sinusoid, resulting in noise term dominating the determination of the steering direction.

In this chapter, we propose a variable pole radius PANF (VP-PANF) to reduce this pole-zero misalignment effect by varying the pole radius according to the distance of the notch from the input frequency. The modified transfer function of the main notch filter is as shown in (5.4).

$$H(z, n) = \frac{1 + W(n)z^{-1} + z^{-2}}{1 + r(n)W(n)z^{-1} + r^2(n)z^{-2}} \quad (5.4)$$

where, $r(n)$ is the pole radius at any particular time instant n .

We use two values of pole radius, denoted as r_L and r_S , which are chosen based on the information whether the notch is close or far away from the input frequency. The relative distance of the notch from the input frequency is estimated from the signs of product term $x(n-1)e(n)$, $x(n-1)e_{high}(n)$ and $x(n-1)e_{low}(n)$ as discussed in Section 2.5 of Chapter 2. It is important to note that the steering direction determination mechanism proposed in Chapter 4 is not used as the objective is

Figure 5.3: Probability of obtaining a correct steering direction versus input SNR for PANF and VP-PANF. The input frequency is 0.25π and the notch frequency is fixed at 0.75π .

to investigate only the effect of the pole. However, the detailed comparison are performed in Chapter 6.

If the signs of $x(n-1)e(n)$, $x(n-1)e_{high}(n)$ and $x(n-1)e_{low}(n)$ are the same, then it can be inferred that the input frequency is far away from the main notch and a comparatively smaller value of pole radius, that is, r_S is used. A smaller value of pole radius will ensure that the noise is not highly amplified at the position of the pole and will also have a lower transient time as discussed in Section 5.2. This will also increase the speed of convergence of the notch. If the signs of $x(n-1)e(n)$, $x(n-1)e_{high}(n)$ and $x(n-1)e_{low}(n)$ are different, then it will indicate that the input frequency has been sandwiched in between the pilot notch frequencies, that is, the input frequency is close to the main notch. A larger value of pole radius, r_L is used to improve the output SNR with higher quality factor.

Simulations are done to compare the proposed mechanism to a fixed pole-radius notch filter as used in [51]. Fig. 5.3 shows the relative improvement of the steering

direction determination of the proposed filter. The result clearly shows that the variation of pole radius play an important role for the pilot notch towards the input frequency as the probability of obtaining a correct steering is improved from less than 0.4 to 0.5 under low SNR conditions which significantly improves the estimation and tracking as steering direction determination is crucial to the overall notch performance.

5.4 Simulation Results and Discussions

In this section, we present computer based simulation results to show the improved performance of the VP-PANF over PANF. The value of r_S and r_L for VP-PANF are 0.6 and 0.9 respectively. The pole radius of PANF is kept fixed at $r = 0.9$. The parameters α_{high} and α_{low} are kept symmetrical and equal to 0.2. The step-size μ_L and μ_S are taken as 3×10^{-3} and $(1/3) \times 10^{-3}$ respectively. The frequency of the input sinusoid is 0.25π . The simulations are done over a range of SNR values for performance evaluation.

5.4.1 Comparison of speed of convergence

The speed of convergence is an important criterion to look at the performance of the filter to justify the introduction of variable pole position mechanism. The step-sizes for the three structures are chosen such that they have approximately the same MSE at steady-state. The results in Fig. 5.4 show that VP-PANF has a smooth and higher speed of convergence for the same MSE. This is due to the fact that with the use of a smaller pole radius, when the notch is far away from the input frequency, not only reduces the transient time to give correct steering

Figure 5.4: Comparison of speed of convergence for the same MSE for PANF and VP-PANF.

direction but also increases the probability of obtaining a correct steering direction due to a reduced effect of the pole.

5.4.2 Steady-state performance versus input SNR

In this section, we estimate the steady-state MSE of the PANF and VP-PANF. A large input data length of 10000 samples has been chosen and the last 3000 output data samples of the filter in each run are time averaged to produce the estimated values of MSE. Simulations for various noise conditions are performed. The estimated MSE is plotted versus the input signal's SNR as shown in Fig. 5.5. The results clearly show that VP-PANF has comparatively smaller MSE at steady state. This is due to the fact that a higher value of pole radius can be used when the notch is closer to the input frequency.

Figure 5.5: Steady-state MSE versus input SNR for PANF and VP-PANF.

5.4.3 Comparison with Farhang's Algorithm

In this section, we have presented a detailed comparison of the VP-PANF with the Farhang's algorithm proposed in [83]. The algorithm is discussed in Chapter 2 as Algorithm 2.1. The parameters $r(n)$ for the Algorithm 2.1 [83] is initialized to 0.25 and is allowed to vary in the range 0.25 to 0.9 with step-size parameter μ_r equal to 0.001 as used in [83]. The step-size μ_L and μ_S for VP-PANF are taken as 11×10^{-3} and $(1/5) \times 10^{-3}$, respectively.

It can be clearly seen from Fig. 5.6 and Fig. 5.7 that VP-PANF converges faster for the same steady-state MSE. The results in Fig. 5.7 clearly shows the VP-PANF maintains lower MSE throughout. It can be observed from Algorithm 2.1 [83] in Chapter 2 requires 17 multiplications and 18 additions to compute the value of pole radius and step-size in each iteration, whereas VP-PANF requires 8 multiplications and 8 additions to select the appropriate value of pole-radius and step-size for each iteration. Hence, it can be concluded that the Algorithm 2.1 [83] requires

Figure 5.6: Comparison of speed of convergence for VP-PANF and Farhang's algorithm.

Figure 5.7: Comparison of MSE for VP-PANF and Farhang's algorithm.

higher computation in comparison to VP-PANF to choose a particular value r and subsequent derivation of step-size parameter.

5.5 Chapter Summary

In this chapter, we have presented the effect of pole position in a piloted adaptive notch filter especially in the context of steering direction determination. It is important to consider the effect of transient response and the pole-zero misalignment in designing the notch filter for a particular application. The introduction of variable pole radius in a piloted notch filter has shown significant improvement in the probability of obtaining a correct steering direction and the speed of convergence. The adaptive nature of the pole radius converges to a sharper notch to give improved steady-state results. The results can be further improved by the use of multiple pilots for multiple values of pole radius for smoother transitions.

Chapter 6

Multiple Pilot Pairs Adaptive Notch Filter

6.1 Introduction

The PANF [51] has only two step-sizes which are chosen based on the relative position of the notch with respect to the input sinusoidal frequency. It uses a larger step-size for higher speed of convergence when the notch is far away from the input sinusoid and then chooses a smaller step-size when the notch is close to the input frequency. This transition from a large step-size to a small step-size is abrupt which increases the MSE and also overshooting the input frequency if the difference between the step-sizes are very large. This is because a single pair of pilot notches provides limited frequency domain information, i.e., close or far away from the input sinusoidal frequency.

In this chapter, we present the generalized formulation and filter structure of a multiple pilot-pairs adaptive notch filter (MPPANF) with more than one pair of pilot notches. The transition from a very large step-size to a very small step-size is achieved in one or more intermediate step-sizes. The estimation of the steering direction mechanism is improved by using the time-averaging process introduced in

Chapter 4. The variable pole radius technique, discussed in Chapter 5, may be employed to further improve the performance of MPPANF, particularly at low SNR. This variable pole radius MPPANF method is called VP-MPPANF in this thesis. Mathematical formulation for the determination of the probability of obtaining a correct steering direction along with the effect of input SNR, notch bandwidth and notch frequency on the probability of obtaining a correct steering direction are discussed. Extensive simulations are done to show the superiority of MPPANF and VP-MPPANF over the original PANF [51].

6.2 The Multiple Pilot-Pairs Adaptive Notch Filter

As discussed in Section 2.5 of Chapter 2, the PANF consists of two pilot notches in addition to the main notch for steering direction determination. In this chapter, we introduce the structure of a generalized MPPANF to detect an input $y(n)$, assumed to be a single sinusoid with additive white Gaussian noise, as shown in (6.1)

$$y(n) = a \cos(\omega_0 n + \varphi) + s(n) \quad (6.1)$$

where, ω_0 is the frequency of the input sinusoid, φ is an arbitrary phase constant, a is the magnitude of the sinusoid, and $s(n)$ is a zero mean white Gaussian noise. This filter consists of a traditional main notch assisted by multiple pairs of pilot notches.

Figure 6.1: Structure of an IIR adaptive notch filter with time varying pole radius.

6.2.1 Main notch

The main notch consists of a zero on the unit circle and a pole placed at a location between the zero and the origin. The direct form implementation of a second order IIR adaptive notch filter is shown in Fig. 6.1. The transfer function, at time n , of the time-varying notch filter is as stated in (6.2)

$$H(z, n) = \frac{1 + W(n)z^{-1} + z^{-2}}{1 + r(n)W(n)z^{-1} + r^2(n)z^{-2}} \quad (6.2)$$

where, $r(n)$ is the pole radius of the notch which determines the bandwidth of the notch filter. The value of $r(n)$ is always less than unity for stability. $W(n)$ is the adjustable filter weight and can be updated using any adaptive technique reported in the literature. However, we choose the LMS algorithm [32] to illustrate our concept since it is currently one of the most popular algorithms. The update equation is shown in (6.3)

$$W(n+1) = W(n) - 2\mu(n)x(n-1)e(n) \quad (6.3)$$

where, $\mu(n)$ is the adaptation step-size which influences the rate of convergence and the misadjustment. $x(n)$ is an intermediate filter state and $e(n)$ is the output of the filter.

Figure 6.2: Pole-zero plot of the multiple pilot pairs notch filter.

The notch frequency at time n can be calculated using (6.4)

$$\theta(n) = \cos^{-1}(-W(n)/2) \quad (6.4)$$

6.2.2 Pilot notches

The pilot notches are incorporated in the system for determining the steering direction of the notch in such a way that the zero of the main notch is being sandwiched between that of the zeros of pilot notches as shown in Fig. 6.2. In this chapter, we present a generalized MPPANF with M pairs of pilot notches, where $M \geq 1$. The case of conventional notch (as shown in Fig. 6.1) corresponds to $M = 0$ and the original PANF [51] corresponds to $M = 1$. The m^{th} pilot-pair ($m = 1, 2, \dots, M$) consists of 2 notches, one of which has a notch frequency of $\phi_{h,m}$ higher than that of the main notch and the other has a notch frequency $\phi_{l,m}$ lower than that of the main notch. In order to reduce the complexity of the structure, the pilot notches and the main notch filter share the same pole as shown in Fig. 6.2. The filter structure of the MPPANF is shown in Fig. 6.3. The transfer function of

Figure 6.3: Structure of the multiple pilot pairs notch filter.

the pilot notches, denoted as $H_{h,m}(z, n)$ and $H_{l,m}(z, n)$, respectively, are shown in (6.5) and (6.6)

$$H_{h,m}(z, n) = \frac{1 + W_{h,m}(n)z^{-1} + z^{-2}}{1 + r(n)W(n)z^{-1} + r^2(n)z^{-2}} \quad (6.5)$$

$$H_{l,m}(z, n) = \frac{1 + W_{l,m}(n)z^{-1} + z^{-2}}{1 + r(n)W(n)z^{-1} + r^2(n)z^{-2}} \quad (6.6)$$

where, $W_{h,m}(n)$ and $W_{l,m}(n)$ are defined as

$$W_{h,m}(n) = W(n) + \alpha_{h,m} \quad (6.7)$$

$$W_{l,m}(n) = W(n) - \alpha_{l,m} \quad (6.8)$$

The value of $\alpha_{h,m}$ and $\alpha_{l,m}$ corresponds to $\phi_{h,m}$ and $\phi_{l,m}$ respectively and can be calculated in advance for a particular application. The precision of $\alpha_{h,m}$ and $\alpha_{l,m}$ need not to be high as the pilots are used only to get the steering direction. Substituting the value of $W_{h,m}(n)$ and $W_{l,m}(n)$ into (6.5) and (6.6), we have

$$H_{h,m}(z, n) = H(z, n) + \frac{\alpha_{h,m}z^{-1}}{1 + r(n)W(n)z^{-1} + r^2(n)z^{-2}} \quad (6.9)$$

$$H_{l,m}(z, n) = H(z, n) - \frac{\alpha_{l,m}z^{-1}}{1 + r(n)W(n)z^{-1} + r^2(n)z^{-2}} \quad (6.10)$$

The output of the pilot notches being denoted as $e_{h,m}(n)$ and $e_{l,m}(n)$ can be calculated as

$$e_{h,m}(n) = e(n) + \alpha_{h,m}x(n-1) \quad (6.11)$$

$$e_{l,m}(n) = e(n) - \alpha_{l,m}x(n-1) \quad (6.12)$$

6.3 Variable Step-size Mechanism

The adaptation step-size plays a vital role in determining the performance of the notch filter. As discussed in Chapter 2, the choice of a particular step-size $\mu(n)$ can lead to various performance outcomes in terms of the rate of convergence and the misadjustment error at steady-state. We consider a system (MPPANF having M pairs of pilot notches) with $M+1$ step-sizes, denoted by μ_k where $k = 0, 1, 2, \dots, M$ and $\mu_k > \mu_{k-1}$. The step-size $\mu(n)$ can be chosen based on the information of the relative position of the notch frequency to the frequency of the input sinusoid, i.e., the steering direction information. As discussed in Chapter 4, the PANF uses the product term $x(n-1)e(n)$ to determine the instantaneous steer direction at time n . However, the instantaneous steer direction from $x(n-1)e(n)$ is contaminated with noise when the SNR is very low. In this chapter, we use the new steering direction determination mechanism discussed in Chapter 4. The time-averaging process $\Psi(n)$ on the product term $x(n-1)e(n)$ for the main notch (shown in Fig. 6.3) obtained using a first-order filter as expressed in (6.13)

$$\Psi(n) = \gamma\Psi(n-1) + x(n-1)e(n) \quad (6.13)$$

where, γ is the weighting parameter and is chosen in the range 0 to 1. Similarly, the processes $\Psi_{h,m}(n)$ and $\Psi_{l,m}(n)$ for the pilot notches can also be obtained as

$$\Psi_{h,m}(n) = \gamma\Psi_{h,m}(n-1) + x(n-1)e_{h,m}(n) \quad (6.14)$$

$$\Psi_{l,m}(n) = \gamma\Psi_{l,m}(n-1) + x(n-1)e_{l,m}(n) \quad (6.15)$$

The relative positions of ω_0 can be obtained from the signs of $\Psi(n)$, $\Psi_{h,m}(n)$ and $\Psi_{l,m}(n)$.

If the input $y(n)$ is a noiseless sinusoid and the sign of $\Psi_{l,M}(n)$ is positive and assuming that the filtering process of (6.13), (6.11) and (6.15) causes negligible delay, it will imply that the frequency of the input sinusoid is lower than the notch frequency of $H_{l,M}(z, n)$. This also implies that the frequency of the input sinusoid will also be lower than the notch frequencies of $H_{h,M}(z, n)$ and $H(z, n)$ and hence the sign of $\Psi_{h,M}(n)$ and $\Psi(n)$ must also be positive. Similarly, if the sign of $\Psi(n)$, $\Psi_{h,M}(n)$ and $\Psi_{l,M}(n)$ are all negative, the frequency of the input sinusoid is higher than the notch frequency of $H_{h,M}(z, n)$. Hence, if the sign of $\Psi(n)$, $\Psi_{h,M}(n)$ and $\Psi_{l,M}(n)$ are the same, this implies that frequency of the input sinusoid is either higher than the notch frequency of $H_{h,M}(z, n)$ or lower than the notch frequency of $H_{l,M}(z, n)$. This is the case when the input sinusoid frequency is far away from the main notch frequency, a larger step-size μ_M can be used to increase the speed of convergence.

If the frequency of the input sinusoid is sandwiched between the notch frequencies of $H_{h,M}(z, n)$ and $H_{l,M}(z, n)$, the sign of $\Psi_{h,M}(n)$ will be positive and $\Psi_{l,M}(n)$ will be negative. In this case, the signs from the inner pilot notches are to be checked. Suppose that the signs of $\Psi_{h,i}(n)$ and $\Psi_{h,i+1}(n)$ where $i = 1, 2, \dots, M-1$ (or $\Psi_{l,i}(n)$ and $\Psi_{l,i+1}(n)$) are different. This means that the frequency of the input

sinusoid is sandwiched between the notch frequencies of $H_{h,i}(z, n)$ and $H_{h,i+1}(z, n)$ (or $H_{l,i}(z, n)$ and $H_{l,i+1}(z, n)$). In this case, an appropriate step-size μ_i is used. The step-size μ_0 will be used when the frequency of the input sinusoid is sandwiched between the notch frequencies of $H_{h,1}(z, n)$ and $H(z, n)$ (or $H_{l,1}(z, n)$ and $H(z, n)$).

6.4 Variable Pole Radius

The pole radius affects the transient response, speed of convergence and steering direction determination mechanism of PANF [51] as discussed in Chapter 5. A larger pole radius will lead to longer duration of transient response and consequently reduced speed of convergence. A smaller pole radius will lead to larger bandwidth and degrade the steady-state performance of the notch filter. In this chapter, we use the variable pole radius mechanism discussed in chapter 5 for MPPANF. The information of the steering direction from the main notch and the pilot notches is used to determine the pole radius of the notch. This is computationally simpler compared to the algorithms reported in literature [8, 9] which uses mathematical formulation to define the pole radius for each iteration of the adaptive algorithm. The information of steering direction obtained from signs of $\Psi(n)$, $\Psi_{h,m}(n)$ and $\Psi_{l,m}(n)$; ($m = 1, 2, \dots, M$), are checked at each iteration to select an appropriate value of pole radius for that iteration. Our system employs $M + 1$ values of pole radius, denoted by r_k , where $k = 0, 1, 2, \dots, M$ and $r_k < r_{k-1}$. Each value of the pole radius r_k is used according to the position of the input sinusoidal frequency with respect to the notches. This is similar to the scheme discussed in Section 6.3. Thus, the pole radius r_k is closely linked with the step-size μ_k in such a way that if μ_k is selected, r_k is also selected.

Table 6.1: Filter parameter: $\alpha_{h,m}$ and $\alpha_{l,m}$

No. of pilots	$\alpha_{h,m}$		$\alpha_{l,m}$	
	$\alpha_{h,1}$	$\alpha_{h,2}$	$\alpha_{l,1}$	$\alpha_{l,2}$
2 ($M = 1$)	0.3	-	0.3	-
4 ($M = 2$)	0.125	0.3	0.125	0.3

Table 6.2: Filter parameter: r_k

No. of pilots	r_k		
	r_0	r_1	r_2
2 ($M = 1$)	0.9	0.7	-
4 ($M = 2$)	0.9	0.8	0.6

6.5 Simulation Results and Discussions

As discussed in Section 6.2, the MPPANF employs LMS algorithm for adapting the filter weight $W(n)$ in order to converge the notch frequency to the frequency of the input sinusoid using a step-size $\mu(n)$. We have done computer simulations to compare the performance of the original PANF [51] with the proposed MPPANF and VP-MPPANF for different M values. We have performed simulations for $M = 1$ and 2, with the parameters $\alpha_{h,m}$ and $\alpha_{l,m}$ as shown in Table 6.1. The pole radius for ANF with fixed pole radius is $r_0 = 0.9$. For variable pole radius, the values of r_k are shown in Table 6.2. The value of γ for (6.13), (6.14) and (6.15) is 0.3. The frequency of the input sinusoid is 0.25π . The results are shown in Fig. 6.4, Fig. 6.5 and Fig. 6.6 and are discussed in Sections 6.5.1, 6.5.2 and 6.5.3 respectively.

6.5.1 Comparison of speed of convergence

In this comparison, the step-sizes for all the schemes are chosen such that they produce approximately the same MSE at steady-state. A data length of 5000 samples is considered and the results are obtained by averaging 100 independent

Figure 6.4: Comparison of speed of convergence for PANF, MPPANF and VP-MPPANF for the same MSE when the input SNR is 10 dB.

simulation runs which are shown in Fig. 6.4. The step-sizes being used for this analysis are as follows: For PANF with two pilots, $\mu_0 = (1/4) \times 10^{-2}$ and $\mu_1 = 1 \times 10^{-2}$; For MPPANF with $M = 1$, $\mu_0 = (1/4) \times 10^{-2}$ and $\mu_1 = 1.5 \times 10^{-2}$, and $M = 2$, $\mu_0 = (1/4) \times 10^{-2}$, $\mu_1 = (1/2) \times 10^{-2}$ and $\mu_2 = 2.2 \times 10^{-2}$; For VP-MPPANF with $M = 1$, $\mu_0 = (1/4) \times 10^{-2}$ and $\mu_1 = 2 \times 10^{-2}$, and $M = 2$, $\mu_0 = (1/4) \times 10^{-2}$, $\mu_1 = 1 \times 10^{-2}$ and $\mu_2 = 2.5 \times 10^{-2}$. The pole radius being used for VP-MPPANF with $M = 1$, $r_0 = 0.9$ and $r_1 = 0.7$, and $M = 2$, $r_0 = 0.9$, $r_1 = 0.8$ and $r_2 = 0.6$.

The results in Fig. 6.4 show that the proposed MPPANF has a higher speed of convergence for the same MSE over PANF. This is because, a much larger step-size is chosen when the notch is far away from the input frequency. The use of four pilots increases the speed of convergence since, with the introduction of an intermediate step-size, an even larger step-size can be chosen when the input frequency is far

Figure 6.5: MSE comparison at same speed of convergence for PANF and VP-MPPANF by varying number of pilots when the input SNR is 10 dB.

away from the notch frequency. VP-MPPANF shows the best performance as it employs the variable pole radius and suppresses transient response to facilitate a higher speed of convergence.

6.5.2 Comparison of mean square error (MSE)

In this comparison, the mean square error is estimated for the same speed of convergence. The same data length and simulation runs as in Section 6.5.1 are used and the averaged results are shown in Fig. 6.5. For better observation, here we present the MSE comparison results for PANF and VP-MPPANF. Later, in Section 6.5.3 we present the steady-state MSE comparison plots for all the schemes. The step-sizes are chosen such that all the schemes converge approximately around the same number of iteration. The step-sizes being used in the simulation are as follows: For PANF, $\mu_0 = (1/2) \times 10^{-3}$ and $\mu_1 = 2 \times 10^{-2}$; For VP-MPPANF with $M = 1$, $\mu_0 = (1/4) \times 10^{-2}$ and $\mu_1 = 1.8 \times 10^{-2}$, and $M = 2$, $\mu_0 = (1/10) \times 10^{-2}$,

$\mu_1 = (1/3) \times 10^{-2}$ and $\mu_2 = 1.8 \times 10^{-2}$. The pole radius being used for VP-MPPANF with $M = 1$, $r_0 = 0.9$ and $r_1 = 0.7$, and $M = 2$, $r_0 = 0.9$, $r_1 = 0.8$ and $r_2 = 0.6$.

The results in Fig. 6.5 show that the MSE of VP-MPPANF is lower than that of PANF. The use of four pilots in VP-MPPANF allows the use of very small step-size when the input frequency is very close to the notch frequency.

6.5.3 Steady-state MSE comparison versus input SNR

We have also estimated the steady-state MSE for the PANF, MPPANF and VP-MPPANF under different noisy conditions. The filter parameters are chosen such that all the schemes have the same speed of convergence as in Section 6.5.2. A large input data length of 10000 samples is chosen and the last 3000 output data samples of the filter in each run are time averaged to produce the estimated values of MSE. Simulations are done over a range of SNR values for performance evaluation. The estimated steady-state MSE are plotted versus the input signal's SNR as shown in Fig. 6.6.

The results show that when input SNR is good, the steady-state MSE decreases with the increasing number of step-sizes and pole radius available to choose. It can be clearly seen that MPPANF and VP-MPPANF with four pilots performs better than the PANF with two pilots. When the input SNR is low, the performance is mainly decided by the ability of estimating correct steering direction. Thus, VP-MPPANF performs better in low SNR conditions.

Figure 6.6: Steady-state MSE versus input SNR for PANF, MPPANF and VP-MPPANF by varying number of pilots.

6.6 Theoretical Analysis of Multiple Pilot pairs Adaptive Notch Filter

6.6.1 Filter states

In order to simplify the theoretical analysis, we assume that $W(n)$ and $r(n)$ can be approximated by a pair of time-invariant parameters W and r , respectively, within a given short duration of interest. For input $y(n)$, as defined in (6.1), the intermediate state $x(n)$ (shown in Fig. 6.3) is also a sinusoidal signal expressed as

$$x(n) = A \cos(\omega_0 n + \beta) + \xi(n) \quad (6.16)$$

where,

$$A = \frac{a}{\sqrt[2]{(1 - r^2)^2 + r(2\cos(\omega_0) + rW)(2r\cos(\omega_0) + W)}} \quad (6.17)$$

$$\beta = \varphi + \tan^{-1} \left(\frac{rW \sin(\omega_0) + r^2 \sin(2\omega_0)}{1 + rW \cos(\omega_0) + r^2 \cos(2\omega_0)} \right) \quad (6.18)$$

and $\xi(n)$ is the filtered version of $s(n)$ through the all-pole section of the notch filter. It is assumed that $\xi(n)$ is Gaussian distributed with zero mean. The auto-correlation function of $\xi(n)$, denoted as $R_{\xi\xi}(k)$, is given by

$$R_{\xi\xi}(k) = \frac{1}{2\pi j} \oint |H_D(z)|^2 \Phi_{ss}(z) z^{k-1} dz \quad (6.19)$$

where, $H_D(z)$ is given by

$$H_D(z) = \frac{1}{1 + rWz^{-1} + r^2z^{-2}} \quad (6.20)$$

$H_D(z)$ is the transfer function of the all-pole section of the notch filter and $\Phi_{ss}(z)$ is the power spectrum of input Gaussian noise, which is equal to σ_s^2 . $R_{\xi\xi}(0)$, $R_{\xi\xi}(1)$ and $R_{\xi\xi}(2)$ can be calculated by applying Residue theorem to (6.19),

$$\begin{aligned} \sigma_\xi^2 &= R_{\xi\xi}(0) = \frac{1}{2\pi j} \oint H_D(z) H_D^*(z^{-1}) \sigma_s^2 \frac{dz}{z} \\ &= \frac{\sigma_s^2}{2\pi j} \oint \frac{z}{(z^2 - rWz + r^2)(1 - rWz + r^2z^2)} dz \\ &= \frac{(r^2 + 1)\sigma_s^2}{(1 - r^2)(r^4 + 2r^2 + 1 - r^2W^2)} \end{aligned} \quad (6.21)$$

$$\begin{aligned} R_{\xi\xi}(1) &= \frac{1}{2\pi j} \oint H_D(z) H_D^*(z^{-1}) \sigma_s^2 dz \\ &= \frac{rW\sigma_s^2}{(1 - r^2)(r^4 + 2r^2 + 1 - r^2W^2)} \\ &= -\frac{rW}{(1 + r^2)} \sigma_\xi^2 \end{aligned} \quad (6.22)$$

and

$$\begin{aligned} R_{\xi\xi}(2) &= \frac{r^2(W^2 - r^2 - 1)\sigma_s^2}{(1 - r^2)(r^4 + 2r^2 + 1 - r^2W^2)} \\ &= \frac{r^2(W^2 - r^2 - 1)}{(1 + r^2)} \sigma_\xi^2 \end{aligned} \quad (6.23)$$

The error output $e(n)$ from (2.46) can be expressed as (4.2) using (2.36),

$$\begin{aligned}
 e(n) &= A \cos(\omega_0 n + \beta) + W A \cos(\omega_0(n-1) + \beta) \\
 &\quad + A \cos(\omega_0(n-2) + \beta) + \eta(n) \\
 &= A \cos(\omega_0(n-1) + \beta) \tau(\theta) + \eta(n)
 \end{aligned} \tag{6.24}$$

where,

$$\tau(\theta) = 2(\cos \omega_0 - \cos(\theta)) \tag{6.25}$$

$$\eta(n) = \xi(n) + W\xi(n-1) + \xi(n-2) \tag{6.26}$$

$\eta(n)$ is the filtered noise through the all-zero section of the notch filter. Thus, it can also be assumed to be Gaussian distributed with zero mean and variance σ_η^2 given by

$$\sigma_\eta^2 = E[\eta(n)\eta^*(n)] \tag{6.27}$$

Substituting (6.26) into (6.27) and considering the results of (6.22) and (6.23), we have

$$\begin{aligned}
 \sigma_\eta^2 &= (2 + W)\sigma_\xi^2 + 4WR_{\xi\xi}(1) + 2R_{\xi\xi}(2) \\
 &= \left[2 + W^2 + \frac{2r^2(W^2 - r^2 - 1)}{1 + r^2} - \frac{4rW^2}{1 + r^2} \right] \sigma_\xi^2
 \end{aligned} \tag{6.28}$$

6.6.2 The product term $x(n-1)e(n)$

As stated earlier, the steering direction can be obtained using the product term $x(n-1)e(n)$ expressed as

$$\begin{aligned}
 & x(n-1)e(n) \\
 &= [A \cos(\omega_0(n-1) + \beta) + \xi(n-1)][A \cos(\omega_0(n-1) \\
 &+ \beta)\tau(\theta) + \eta(n)] \\
 &= A^2 \cos^2(\omega_0(n-1) + \beta)\tau(\theta) + \nu(n) \tag{6.29}
 \end{aligned}$$

$$= \frac{A^2}{2}\tau(\theta) + \frac{A^2}{2}\tau(\theta) \cos(2(\omega_0(n-1) + \beta)) + \nu(n) \tag{6.30}$$

where, $\nu(n)$ is the noise component of the product term $x(n-1)e(n)$ given by

$$\begin{aligned}
 \nu(n) &= A \cos(\omega_0(n-1) + \beta)\eta(n) \\
 &+ \xi(n-1)\{A \cos(\omega_0(n-1) + \beta)\tau(\theta) + \eta(n)\} \tag{6.31}
 \end{aligned}$$

The mean value of the product term $x(n-1)e(n)$ can be obtained as

$$\begin{aligned}
 & E[x(n-1)e(n)] \\
 &= \frac{1}{2\pi} \int_{-\pi}^{\pi} H_D^*(e^{-j\omega})H(e^{j\omega})S_{yy}(\omega)e^{j\omega}d\omega \tag{6.32}
 \end{aligned}$$

where, $S_{yy}(\omega)$ is the power spectral density of the input signal (6.1) expressed as

$$S_{yy}(\omega) = \frac{a^2}{2}\delta(\omega - \omega_0) + \sigma_s^2 \tag{6.33}$$

Solving (6.32) using Residue Theorem,

$$\begin{aligned}
 & E[x(n-1)e(n)] \\
 &= \frac{a^2}{2} \frac{\tau(\theta)}{(1-r^2)^2 + r(2\cos(\omega_0) + rW)(2r\cos(\omega_0) + W)} + \frac{(r-1)^2W}{(1+r^2)}\sigma_\xi^2 \tag{6.34}
 \end{aligned}$$

Substituting (6.17) into (6.34), we get

$$E[x(n-1)e(n)] = \frac{A^2\tau(\theta)}{2} + \frac{(r-1)^2W}{(1+r^2)}\sigma_\xi^2 \quad (6.35)$$

Thus, the mean value of the noise component $\nu(n)$, denoted by E_ν , can be obtained from (6.35) as

$$E_\nu = \frac{(r-1)^2W}{(1+r^2)}\sigma_\xi^2. \quad (6.36)$$

The variance of $\nu(n)$, denoted as σ_ν^2 , can be obtained as

$$\sigma_\nu^2 = E[(\nu(n) - E_\nu)^2] \quad (6.37)$$

For small E_ν , and using (6.31),

$$\begin{aligned} \sigma_\nu^2 &= E[(\nu(n))^2] \\ &= E[(A \cos(\omega_0(n-1) + \beta)\eta(n) + \xi(n-1)\{A \cos(\omega_0(n-1) + \beta)\tau(\theta) + \eta(n)\})^2] \\ &= \frac{A^2\sigma_\eta^2}{2} + \sigma_\xi^2 \left[\frac{A^2\tau^2(\theta)}{2} + \sigma_\eta^2 \right] + A^2\tau(\theta)E[\eta(n)\xi(n-1)] \\ &\quad + 2E[A \cos(\omega_0(n-1))(\eta(n) + \tau(\theta)\xi(n-1))(\eta(n)\xi(n-1))] \end{aligned} \quad (6.38)$$

Multiplying (6.26) on both side by $\xi(n-1)$ and taking expectation, we have,

$$\begin{aligned} E[\eta(n)\xi(n-1)] &= E[\xi(n)\xi(n-1) + W\xi(n-1)\xi(n-1) + \xi(n-1)\xi(n-2)] \\ &= 2R_{\xi\xi}(1) + WR_{\xi\xi}(0) \end{aligned} \quad (6.39)$$

Substituting the results from (6.21) and (6.22) into (6.39), we get

$$E[\eta(n)\xi(n-1)] = \frac{(r-1)^2}{(1+r^2)}\sigma_\xi^2 \quad (6.40)$$

From (6.40), it can be inferred that $E[\eta(n)\xi(n-1)]$ approaches zero when the pole radius r approaches unity. As the mean values of $\eta(n)$ and $\xi(n-1)$ are also equal

to zero, the variance of $\nu(n)$ is obtained from (6.38) and (6.40), as shown in (6.41)

$$\sigma_\nu^2 = \frac{A^2\sigma_\eta^2}{2} + \sigma_\xi^2 \left[\frac{A^2\tau^2(\theta)}{2} + \sigma_\eta^2 \right] \quad (6.41)$$

The variance of the product term $x(n-1)e(n)$, denoted as σ_{pt}^2 , can be obtained as,

$$\begin{aligned} \sigma_{pt}^2 &= E[(x(n-1)e(n) - E[x(n-1)e(n)])^2] \\ &= E[(x(n-1)e(n))^2] - E[x(n-1)e(n)]^2 \end{aligned} \quad (6.42)$$

Substituting (6.35) and the square of (6.30) into (6.42) and using (6.41), we obtained

$$\sigma_{pt}^2 = \frac{A^2\sigma_\eta^2}{2} + \sigma_\xi^2 \left[\frac{A^2\tau^2(\theta)}{2} + \sigma_\eta^2 \right] + \frac{A^4\tau^2(\theta)}{8} \quad (6.43)$$

Neglecting the fourth order terms with respect to A , we have

$$\sigma_{pt}^2 = \frac{A^2\sigma_\eta^2}{2} + \sigma_\xi^2 \left[\frac{A^2\tau^2(\theta)}{2} + \sigma_\eta^2 \right] \quad (6.44)$$

6.6.3 The process $\Psi(n)$

From (6.30) and (6.13) and assuming that the filtering process of (6.13) causes negligible delay, if $\Psi(n)$ is positive then it indicates $\omega_0 < \theta$. Conversely, if $\Psi(n)$ is negative then it indicates $\omega_0 > \theta$. Hence, the relative positions of ω_0 can be obtained from the signs of $\Psi(n)$, $\Psi_{h,m}(n)$ and $\Psi_{l,m}(n)$.

In order to simplify the theoretical analysis, for a slow change in $\Psi(n)$, it is assumed that the mean of the process $\Psi(n)$ at any particular time instant n , denoted as E_Ψ , can be obtained from (6.13) by taking expectation,

$$E_\Psi = \frac{1}{(1-\gamma)} E[x(n-1)e(n)] \quad (6.45)$$

Substituting for $E[x(n-1)e(n)]$ using the results of (6.35), we have

$$E_\Psi = \frac{A^2\tau(\theta)}{2(1-\gamma)} + \frac{(r-1)^2W}{(1+r^2)(1-\gamma)} \sigma_\xi^2 \quad (6.46)$$

Figure 6.7: Variance of the $\Psi(n)$ for different input SNR with input signal power=1.

The variance of the process $\Psi(n)$, denoted as σ^2_Ψ , can be obtained as

$$\begin{aligned}
 \sigma^2_\Psi &= E[(\Psi(n) - E_\Psi)^2] \\
 &= E[(\Psi(n))^2 - 2\Psi(n)E_\Psi + E_\Psi^2] \\
 &= E[(\Psi(n))^2] - E_\Psi^2
 \end{aligned} \tag{6.47}$$

$E[(\Psi(n))^2]$ can be obtained by squaring (6.13) on both sides and taking expectation, assuming $\Psi(n-1)$ and $x(n-1)e(n)$ are uncorrelated,

$$E[(\Psi(n))^2] = \frac{1}{(1-\gamma^2)} E[(x(n-1)e(n))^2] \tag{6.48}$$

Substituting (6.48) and the square of (6.45) into (6.47), and rearranging we have,

$$\begin{aligned}
 \sigma^2_\Psi &= \frac{1}{(1-\gamma^2)} E[(x(n-1)e(n))^2] - E[x(n-1)e(n)]^2 \\
 &\quad - \frac{2\gamma}{(1+\gamma)(1-\gamma)^2} E[x(n-1)e(n)]^2 \\
 &= \frac{1}{(1-\gamma^2)} \sigma_{pt}^2 - \frac{2\gamma}{(1+\gamma)} E_\Psi^2
 \end{aligned} \tag{6.49}$$

It can be assumed that the second term in (6.49) is smaller compared to the first term for small γ and can be neglected. Thus, substituting (6.44) into (6.49), σ_Ψ^2 is expressed as

$$\sigma_\Psi^2 = \frac{A^2\sigma_\eta^2 + \sigma_\xi^2[A^2\tau^2(\theta) + 2\sigma_\eta^2]}{2(1 - \gamma^2)} \quad (6.50)$$

Fig. 6.7 shows the theoretical and simulation results of σ_Ψ^2 for different input SNR.

6.7 Probability of Correct Steering Direction Determination

The use of pilot notches is an excellent technique to obtain the steering direction and choose a suitable step-size for convergence. In Section 6.3, we have shown that the signs of $\Psi(n)$, $\Psi_{h,m}(n)$ and $\Psi_{l,m}(n)$ can provide information on whether the notch is far away or close to the input frequency. However, the steering direction may not be correct when the input signal is corrupted by noise as discussed in Chapter 4. Thus, it is important to analyse the probability of obtaining a correct steering direction from the signs of $\Psi(n)$, $\Psi_{h,m}(n)$ and $\Psi_{l,m}(n)$. In this section, we will discuss in detail the probability of obtaining a correct steering direction from the sign of $\Psi(n)$ at any time instant n in noisy conditions. The results of other terms can be obtained in a similar way. Denote $\Psi(n)$ at any time instant n as the steering term, $st(n)$, given by

$$st(n) = \Psi(n) \quad (6.51)$$

It is assumed that the input noise is Gaussian with zero mean and variance σ_s^2 . Since, $\xi(n)$ is also Gaussian distributed with zero mean as discussed in Section 6.6, the output noise $\eta(n)$ is also Gaussian distributed as it is the sum of two Gaussians

with variance calculated in (6.28). From (6.40), it is clear that $E[\eta(n)\xi(n-1)]$ approaches zero as the pole radius r approaches unity. From (6.13), it can be observed that $\Psi(n)$ is a sum of many random variables. Hence, using the central limit theorem it can be assumed that $\Psi(n)$ is Gaussian distributed. From (6.51), $st(n)$ is Gaussian distributed with mean, denoted by E_{st} , as calculated in (6.46),

$$E_{st} = \frac{A^2\tau(\theta)}{2(1-\gamma)} + \frac{(r-1)^2W}{(1+r^2)(1-\gamma)}\sigma_\xi^2 \quad (6.52)$$

The variance of $st(n)$, denoted by σ_{st}^2 , is the variance of $\Psi(n)$ calculated in (6.50). Thus,

$$\sigma_{st}^2 = \frac{A^2\sigma_\eta^2 + \sigma_\xi^2[A^2\tau^2(\theta) + 2\sigma_\eta^2]}{2(1-\gamma^2)} \quad (6.53)$$

Although $st(n)$ may not be always Gaussian, but in order to simplify the theoretical analysis, we assume it to be Gaussian. Thus, the probability density function (pdf) of the steering term $st(n)$, is expressed as

$$f_{st}(st(n)) = \frac{1}{\sqrt{2\pi\sigma_{st}^2}} \exp\left(-\frac{(st(n) - E_{st})^2}{2\sigma_{st}^2}\right) \quad (6.54)$$

The sign of $st(n)$ is key to the success of determining the position of the input sinusoid on the z-plane. The probability that the sign of $st(n)$ is positive, denoted by P_+ , is given by

$$\begin{aligned} P_+ &= \int_0^\infty f_{st}(st(n))d(st(n)) \\ &= \int_0^\infty \frac{1}{\sqrt{2\pi\sigma_{st}^2}} \exp\left(-\frac{(st(n) - E_{st})^2}{2\sigma_{st}^2}\right) d(st(n)) \end{aligned} \quad (6.55)$$

It is important to know the factors which affect the probability of obtaining a correct steering direction. Substituting (6.21) into (6.28), we obtained

$$\sigma_\eta^2 = \frac{2(1+r)(r^2+1) - W^2(3r-1)}{(1+r)(r^4+2r^2+1-r^2W^2)}\sigma_s^2 \quad (6.56)$$

Considering (6.56) and (6.4), we expressed σ_η^2 as,

$$\sigma_\eta^2 = f_1(r, \theta) \sigma_s^2 \quad (6.57)$$

where,

$$f_1(r, \theta) = \frac{2(1+r)(r^2+1) - W^2(3r-1)}{(1+r)(r^4+2r^2+1-r^2W^2)} \quad (6.58)$$

Considering (6.21) and (6.4), we expressed σ_ξ^2 as

$$\sigma_\xi^2 = f_2(r, \theta) \sigma_s^2 \quad (6.59)$$

where,

$$f_2(r, \theta) = \frac{(r^2+1)}{(1-r^2)(r^4+2r^2+1-r^2W^2)} \quad (6.60)$$

Substituting (6.57) and (6.59) into (6.53), we obtained

$$\sigma_{st}^2 = \frac{A^2 f_1(r, \theta) + f_2(r, \theta) [A^2 \tau^2(\theta) + 2f_1(r, \theta) \sigma_s^2]}{2(1-\gamma^2)} \sigma_s^2 \quad (6.61)$$

Considering (6.61), (6.25) and (6.17), we expressed σ_{st}^2 as

$$\sigma_{st}^2 = f_3(a, \omega_0, \sigma_s^2, r, \theta, \gamma) \quad (6.62)$$

substituting (6.59) into (6.52), we have

$$E_{st} = \frac{A^2 \tau(\theta)}{2(1-\gamma)} + \frac{(r-1)^2 W f_2(r, \theta)}{(1+r^2)(1-\gamma)} \sigma_s^2 \quad (6.63)$$

Considering (6.63), (6.25) and (6.17), we expressed E_{st} as

$$E_{st} = f_4(a, \omega_0, \sigma_s^2, r, \theta, \gamma) \quad (6.64)$$

substituting (6.62) and (6.64) into (6.55), we obtained (6.65),

$$P_+ = \int_0^\infty \frac{1}{\sqrt{2\pi f_3(a, \omega_0, \sigma_s^2, r, \theta, \gamma)}} \exp\left(-\frac{(st(n) - f_4(a, \omega_0, \sigma_s^2, r, \theta, \gamma))^2}{2f_3(a, \omega_0, \sigma_s^2, r, \theta, \gamma)}\right) d(st(n)) \quad (6.65)$$

Figure 6.8: Probability of obtaining a correct steering direction from the sign of $\Psi(n)$ versus input SNR for an input frequency of 0.4π . The notch frequency is fixed at 0.75π and the pole radius r is 0.9.

From (6.65), it can be derived that the probability of obtaining a correct steering direction is a function of i) the frequency of the input sinusoid ω_0 , ii) the input SNR, iii) the notch frequency θ and iv) the notch bandwidth i.e, pole radius r , v) the value of γ . The choice of γ is application specific, as it depends on SNR conditions and the acceptable lag in the tracking of the input frequency. The following Sections 6.8-6.10 present the detailed analysis of the effect of the variables (i)-(iii) on the probability of obtaining a correct steering direction.

6.8 Effect of Input SNR on Steering Direction Determination

The effect of input SNR on the steering direction determination can be obtained from (6.54) and (6.55). As there are multiple factors affecting the steering direction determination, the effect of input SNR can be obtained by fixing the input

sinusoid's frequency ω_0 , the notch frequency θ and the notch bandwidth i.e, pole radius r . The probability of obtaining a correct steering direction as a function of input SNR can be obtained from (6.55) using the results from (6.21), (6.25), (6.28), (6.30), (6.52), (6.53) and (6.54) as done in Section 6.7. The obtained results are shown in Fig. 6.8 together with simulation results. The array of symbols corresponds to the results obtained through computer simulations and the solid line corresponds to the results predicted by (6.55). The results represented by each symbol are obtained by averaging 100 independent simulation runs. The notch frequency θ of the main notch is fixed at 0.75π , the input frequency ω_0 at 0.4π and the pole radius is set to be $r = 0.9$. The steering direction is correct if the sign of $\Psi(n)$ is positive for $\theta > \omega_0$. It can be seen from Fig. 6.8 that with decreasing SNR of input signal, the probability of obtaining a correct steering direction decreases.

6.9 Effect of Notch Position on Steering Direction Determination

The effects of the position of the notch on the z-plane w.r.t., the input frequency on the probability of correct steering direction determination are shown in Figs. 6.9 and 6.10. The steering direction is accurate if the sign of $\Psi(n)$ is positive when $\theta > \omega_0$ and negative when $\theta < \omega_0$. The input frequency ω_0 is fixed at 0.4π and 0.6π for Figs. 6.9 and 6.10, respectively, with input SNR at 10 dB and pole radius $r = 0.9$. It can be clearly seen from the figures that the theoretical values obtained from (6.55) closely follows the simulation results. It is observed from the results that there is degradation in performance of the steering direction determination when the notch is far away from the input frequency. This is because when the

Figure 6.9: Probability of obtaining a correct steering direction from the sign of $\Psi(n)$ versus notch frequency θ for an input frequency of 0.4π . The input SNR is 10 dB and the pole radius r is 0.9.

notch is far away from the input sinusoidal frequency the pole amplifies the noise more than the input sinusoid resulting in degradation of the SNR. Although the results show degradation at some notch positions, the probability of obtaining a correct steering direction from the sign of $\Psi(n)$ is always greater than 0.5. Thus, it will always be better than a random guess. Each symbol in Figs. 6.9 and 6.10 are obtained by averaging the results obtained from 100 independent simulation runs.

6.10 Effect of Pole Radius r on Steering Direction Determination

It will be interesting to understand the effect of notch bandwidth or pole radius r on the probability of obtaining a correct steering direction for a given input frequency ω_0 , notch position θ , and input SNR. In Fig. 6.11, the results corresponding to $\theta = 0.75\pi$, input frequency ω_0 and input SNR of 10 dB are shown.

Figure 6.10: Probability of obtaining a correct steering direction from the sign of $\Psi(n)$ versus notch frequency θ for an input frequency of 0.6π . The input SNR is 10 dB and the pole radius r is 0.9.

Figure 6.11: Probability of obtaining a correct steering direction from the sign of $\Psi(n)$ versus pole radius r for an input frequency of 0.4π . The notch frequency is fixed at 0.75π and the input SNR is 10 dB.

It can be inferred from Fig. 6.11 that when the notch has not converged to the frequency of the input sinusoid, i.e., $\theta \neq \omega_0$ the accuracy of steering direction determination decreases with increasing pole radius. This is due to the amplification of the noise component by the pole which increases with the increase in pole radius r . Thus, a smaller pole radius will always be preferred for steering direction determination when the algorithm has not converged, particularly, at low SNR. However, with the increase in the bandwidth of the notch, the steady-state performance will degrade. Thus, a variable pole radius scheme will have the following advantages. It will use a smaller pole radius when the notch frequency is far away from the frequency of the input sinusoid for faster speed of convergence, and a larger pole radius when the notch frequency is close to the input frequency for better steady state results. The theoretical and simulation results are consistent as shown in Fig. 6.11. The simulation results shown in Fig. 6.11 are obtained by averaging the results over 100 independent simulation runs.

6.11 Chapter Summary

In this chapter, we have presented a generalized formulation for piloted adaptive notch filter structure to include multiple pilots (MPPANF). The introduction of more number of pilots to the filter structure has been useful in increasing the number of step-sizes which helps in smoothing the transition from a very large step-size to a very small step-size. The new steering direction determination mechanism using a time-averaging based process has significantly improved the determination of the steering direction of the main notch and pilot notches when the input SNR

is very low. The introduction of variable pole radius in MPPANF has shown significant improvement in the probability of obtaining a correct steering direction and the speed of convergence. The theoretical analysis of the filter has been presented which helped in the detailed understanding of the mechanisms proposed. The choice of the filter structure and adaptive mechanisms is application specific for a robust performance.

Chapter 7

A Complex Piloted Adaptive Notch Filter

7.1 Introduction

Adaptive notch filter (ANF) has been widely used in the literature to estimate and track the frequency of time-varying sinusoidal signals in noise as reported in [1]. There has been significant research in the development of these filters for real signals [24–26, 29, 30, 51, 96, 101, 103–109] by using various methods which includes implementation using all pass-filter sections [29] [30] and by pole-zero placement techniques [24]. They employ adaptive algorithms like the least mean squares (LMS) algorithm [32] [46] in which the notch frequency is updated iteratively to eventually converge to the frequency of the input signal. However, when the signal consists of in-phase and quadrature-phase components leading to complex signals, the filter structure need to be modified and implemented using complex coefficients. Thus, the complex adaptive notch filter was developed and studied in literature [86–91, 93]. They can be used in many applications as discussed in [87], especially in communication systems using QPSK modulation [86], and radar signal processing. They are implemented both as FIR and IIR based filters, but IIR filters are computationally more efficient as compared to FIR filters.

In Chapter 2, we have discussed the complex adaptive notch filters reported in the literature. These filters have been developed as an extension to the filters capable of processing real signals. They used a fixed step-size to update the filter weights. However, a variable step-size algorithm will provide better performance matrix compared to a fixed step-size algorithm. There has been significant research in the development of the techniques to determine the variable step-sizes for tracking real signal [38, 41, 43–45] as discussed in Chapter 2. However, all these techniques are based on past history of time-domain data without any instantaneous frequency-domain information. The PANF was originally developed for tracking real signals. They incorporated pilot notches where the signs of the gradient estimates of the main notch and pilot notches are used to estimate the distance between the notch frequency and the frequency of the input sinusoid. This estimate of the steering direction is used to determine the appropriate step-size. In Chapters 4-6, we have presented mechanisms to improve the performance of PANF particularly at low SNR and showed the effectiveness of their technique . It will be interesting to incorporate the concept of using pilot notches in tracking and estimating the frequency of complex signals.

In this chapter, we formulate the complex piloted adaptive notch filter (CPANF) which is used to track a complex sinusoid. The information from the pilot notches are used to determine the steering direction of the notch and to choose an appropriate step-size. The use of variable step-size not only increases the speed of convergence but also helps to reduce the mean square error (MSE) at steady-state. Simulation are presented to verify the excellent performance exhibited by the CPANF over conventional complex adaptive notch filter (CANF) with respect

to the speed of convergence and steady-state MSE. Theoretical analysis is also presented which closely follows the simulation results. Mathematical formulation for the determination of probability of obtaining a correct steering direction along with the effect of input SNR, notch bandwidth and notch frequency on probability of obtaining a correct steering direction is presented. The filter is also implemented to suppress complex sinusoidal interference in a QPSK communication systems which shows improvement over conventional CANF in overall bit error rate (BER).

7.2 The Complex Piloted Adaptive Notch Filter

The structure of PANF for frequency estimation and tracking was first introduced in [51]. It was the first time when an adaptive update algorithm was guided by a steering mechanism to choose a suitable step-size for a given relative position of the main notch with respect to the input frequency. This system consists of pilot notches in addition to the main notch for steering direction determination mechanism. However, it was developed for real signals. In this section, we will introduce the structure of the piloted adaptive notch filter system to detect an input $y(n)$ which is assumed to be a signal consisting of in-phase and quadrature-phase components, thus a complex signal, as shown in (7.1)

$$y(n) = ae^{j(\omega_0 n + \beta)} \quad (7.1)$$

where, ω_0 is the frequency of the input signal, β is an arbitrary phase constant, a is the magnitude of the input signal.

The system consists of a traditional main notch assisted by two pilot notches which share the same pole with the main notch.

Figure 7.1: Structure of an IIR complex adaptive notch filter (CANF).

7.2.1 Main notch

The main notch of the complex piloted adaptive notch filter system is developed by constraining the zeros of the notch filter to the unit circle [24]. It is implemented by pole-zero placement technique where a pole is followed by a zero to have an overall improved output SNR. The direct form implementation of a first order IIR complex adaptive notch filter is shown in Fig. 7.1. The transfer function of the time-varying notch filter is stated in (7.2).

$$H(z, n) = \frac{1 + W(n)z^{-1}}{1 + rW(n)z^{-1}} \quad (7.2)$$

where, r is the pole radius of the notch which determines the bandwidth of the notch filter. The value of r is always less than unity for stability. $W(n)$ is the adjustable complex filter weight and is given by

$$W(n) = -e^{j\theta(n)} = -(\cos \theta(n) + j \sin \theta(n)) \quad (7.3)$$

where, $\theta(n)$ is the notch frequency. It can be updated using any algorithm reported in the literature. However, we choose the LMS algorithm [32] to illustrate our concept since it is one of the most popular algorithms.

7.2.2 Pilot notches

The pilot notches are incorporated in the system for determining the steer direction of the notch in such a way that the zero of the main notch is sandwiched between

Figure 7.2: Pole-zero plot of the complex piloted adaptive notch filter (CPANF).

that of the zeros of pilot notches as shown in Fig. 7.2. The notch frequency of one of the pilot notch is kept higher by ϕ_{high} and the other pilot notches is kept lower by ϕ_{low} with respect to the main notch. The complexity of the structure is reduced by using the same pole for the pilot notches and the main notch filter (shown in Fig. 7.3). The transfer functions of the pilot notches are formulated as in (7.4) and (7.5) denoted by $H_{high}(z, n)$ and $H_{low}(z, n)$.

$$H_{high}(z, n) = \frac{1 + W_{high}(n)z^{-1}}{1 + rW(n)z^{-1}} \quad (7.4)$$

$$H_{low}(z, n) = \frac{1 + W_{low}(n)z^{-1}}{1 + rW(n)z^{-1}} \quad (7.5)$$

where, $W_{high}(n)$ and $W_{low}(n)$ are defined as

$$W_{high}(n) = -e^{j(\theta(n) + \phi_{high})} = \alpha_{high}W(n) \quad (7.6)$$

$$W_{low}(n) = -e^{j(\theta(n) - \phi_{low})} = \alpha_{low}W(n) \quad (7.7)$$

where, α_{high} and α_{low} are defined as

$$\alpha_{high} = e^{j\phi_{high}} = \cos \phi_{high} + j \sin \phi_{high} \quad (7.8)$$

Figure 7.3: Structure of the complex piloted adaptive notch filter (CPANF).

$$\alpha_{low} = e^{-j\phi_{low}} = \cos \phi_{low} - j \sin \phi_{low} \quad (7.9)$$

The values of α_{high} and α_{low} correspond to ϕ_{high} and ϕ_{low} respectively and can be calculated in advance for a particular application. The precision of ϕ_{high} and ϕ_{low} need not be high as the pilot notches are used only to get the steering direction. Furthermore, if we arbitrarily choose

$$\phi_{high} = \phi_{low} \quad (7.10)$$

then,

$$\alpha_{high} = \alpha \quad \text{and} \quad \alpha_{low} = \alpha^* \quad (7.11)$$

The output of the pilot notches being denoted as $e_{high}(n)$ and $e_{low}(n)$ can be calculated as

$$e_{high}(n) = x(n) + \alpha_{high}W(n)x(n-1) \quad (7.12)$$

$$e_{low}(n) = x(n) + \alpha_{low}W(n)x(n-1) \quad (7.13)$$

7.3 Adaptive Algorithm

The last four decades of development of adaptive update algorithms have seen numerous techniques being proposed to achieve the best performing adaptive filters. The theoretical analysis, performance results and computational complexity depends upon the algorithm being used. However among all, the LMS algorithm

is most widely used because of its simplicity and robustness. In this chapter, we will use this as reference updating algorithm though other techniques can also be used for updating the filter weights.

As the input $y(n)$ is complex, the filter states are complex numbers. The notch frequency $\theta(n)$ which is real valued can be updated using the gradient information from either the real part or imaginary part (or both parts) of the function of filter states. The two forms of $\theta(n)$ updates are as shown in (7.14) and (7.15)

Real Part Update

$$\theta(n+1) = \theta(n) - 2\mu(n)\Re\{x^*(n-1)e(n)\} \quad (7.14)$$

Imaginary Part Update

$$\theta(n+1) = \theta(n) + 2\mu(n)\Im\{x^*(n)e(n)\} \quad (7.15)$$

where, $(.)^*$, $\Re\{.\}$ and $\Im\{.\}$ denote the conjugate, real and imaginary parts of a complex number respectively. $\mu(n)$ is the adaptation step-size which influences the rate of convergence and the misadjustment. $x(n)$ is an intermediate filter state and $e(n)$ is the output of the filter (as shown in Fig. 7.3).

As stated earlier, either (7.14) or (7.15) can be used to update the filter weights to converge to the frequency of the input sinusoid. They can also be used simultaneously or alternately. The use of (7.14) leads to faster converging notch whereas using (7.15) shows better steady-state performance. This is explained theoretically in Section 7.6. So, in view of this trade-off, a mixed update LMS will always be preferred as it will appropriately use (7.14) when the notch is away from the input frequency for faster convergence and (7.15) to be used when the notch is close to the input frequency for better misadjustment after convergence.

Equations (7.14) and (7.15) may be substituted into (7.3) to obtain the update equation for $W(n)$. However, doing so will involve computation of $\cos(\cdot)$ and $\sin(\cdot)$ at each iteration, which have high computational complexity. Hence, we update $W(n)$ using the approximation (7.18) and (7.19) by assuming that the change in $\theta(n)$, denoted as $\delta\theta(n)$, is small. From (7.3), we express

$$\begin{aligned} W(n+1) &= -e^{j\theta(n+1)} \\ &= -e^{j(\theta(n)+\delta\theta(n))} \end{aligned} \quad (7.16)$$

For small $\delta\theta(n)$, we obtained

$$\begin{aligned} W(n+1) &= -e^{j\theta(n)}[1 + j\delta\theta(n)] \\ &= W(n)[1 + j\delta\theta(n)] \end{aligned} \quad (7.17)$$

Substituting the value of $\delta\theta(n)$ from (7.14) and (7.15) into (7.17), we have

Real Part Update

$$W(n+1) = W(n)[1 - j2\mu(n)\Re\{x^*(n-1)e(n)\}] \quad (7.18)$$

Imaginary Part Update

$$W(n+1) = W(n)[1 + j2\mu(n)\Im\{x^*(n)e(n)\}] \quad (7.19)$$

However, the approximation (7.18) and (7.19) will lead to shifting of the zero of the notch filter away from the unit circle by $\frac{(\delta\theta(n))^2}{2}$ in each iteration as shown in (7.20). Using binomial series expansion and neglecting the higher order terms, we have

$$\begin{aligned} |W(n+1)| &= \sqrt[2]{1 + (\delta\theta(n))^2} \\ &= 1 + \frac{(\delta\theta(n))^2}{2} - \frac{(\delta\theta(n))^4}{8} + \dots \\ &\approx 1 + \frac{(\delta\theta(n))^2}{2} \end{aligned} \quad (7.20)$$

Although the shift is very small, for each iteration, the cumulative shift will be significant after a large number of iterations. Thus, the zero of the notch filter have to be restored back to the unit circle at regular intervals. This can be done by restoring $|W(n)|$ to unity after a finite number of iterations which depends upon the input SNR and the step-size being used for that particular application of the notch filter.

7.4 Variable Step-size Mechanism

The adaptation step-size plays a vital role in determining the performance of the notch filter. As discussed in Section 7.1, the choice of a step-size can lead to various performance outcomes in terms of the rate of convergence and the misadjustment error at steady-state. For a variable step-size system, the step-size can be chosen based on the information of the relative position of the notch frequency to the frequency of the input sinusoid. Thus, the pilot notches are introduced to estimate whether the notch frequency is close or far away from the input sinusoid. This is discussed in Section 7.7. The information of steering direction obtained from the signs of the real part of product terms $x^*(n-1)e(n)$, $x^*(n-1)e_{high}(n)$, and $x^*(n-1)e_{low}(n)$ or the imaginary part of product terms $x^*(n)e(n)$, $x^*(n)e_{high}(n)$, and $x^*(n)e_{low}(n)$ are used to choose an appropriate step-size.

If the input $y(n)$ is a noiseless sinusoid, the signs of $\Re\{x^*(n-1)e(n)\}$, $\Re\{x^*(n-1)e_{high}(n)\}$ or $\Re\{x^*(n-1)e_{low}(n)\}$ indicate whether the input sinusoidal frequency is higher or lower than the respective notch frequency. If the sign of $\Re\{x^*(n-1)e_{low}(n)\}$ is positive, it implies that the frequency of the input sinusoid is lower than the notch frequency of $H_{low}(z, n)$. Since $H_{low}(z, n)$ is the lower pilot notch,

the signs of $\Re\{x^*(n-1)e_{high}(n)\}$ and $\Re\{x^*(n-1)e(n)\}$ will also be positive in this case. Similarly, if the sign of $\Re\{x^*(n-1)e_{high}(n)\}$ is negative, it implies that the frequency of the input sinusoid is higher than the notch frequency of $H_{high}(z, n)$. And thus, the signs of $\Re\{x^*(n-1)e_{low}(n)\}$ and $\Re\{x^*(n-1)e(n)\}$ will also be negative. So, if the signs of $\Re\{x^*(n-1)e(n)\}$, $\Re\{x^*(n-1)e_{high}(n)\}$ and $\Re\{x^*(n-1)e_{low}(n)\}$ are the same, the input frequency is either higher than the notch frequency of $H_{high}(z, n)$ or lower than that of $H_{low}(z, n)$. This is the case when the notch frequency of the main notch is assumed to be far away from the frequency of the input sinusoid. Thus, a larger step-size i.e., μ_{large} can be used to increase the speed of convergence.

When the frequency of the input sinusoid is in the close vicinity of the main notch, it will be sandwiched between the notch frequency of $H_{high}(z, n)$ and $H_{low}(z, n)$. In this case, the sign of $\Re\{x^*(n-1)e_{high}(n)\}$ will be positive and the sign of $\Re\{x^*(n-1)e_{low}(n)\}$ will be negative. The sign of $\Re\{x^*(n-1)e(n)\}$ will be dependent on the position of the input frequency w.r.t the main notch. This is the condition when a smaller step-size i.e., μ_{small} can be used to reduce the misadjustment error at steady-state. This same analysis follows if we use the signs of $\Im\{x^*(n)e(n)\}$, $\Im\{x^*(n)e_{high}(n)\}$, and $\Im\{x^*(n)e_{low}(n)\}$ except that they are positive when the frequency of input sinusoid is higher than that of the notch frequency and vice-versa.

7.5 Simulation Results and Discussions

As discussed in Section 7.3, the CPANF employs LMS algorithm for adapting the filter weight $W(n)$ in order to converge to the frequency of the input sinusoid

using a step-size $\mu(n)$. Fig. 7.4 shows the convergence of the notch frequency to the actual frequency which is abruptly changing at every 1000th iteration. The proposed CPANF uses real part update from (7.14) to update the notch frequency.

We have done computer simulations to compare the performance of the proposed CPANF with CANF. While the CPANF uses two step-sizes μ_{large} and μ_{small} , the CANF uses a fixed step-size, denoted as μ_{fixed} . In Figs. 7.5-7.7 the results for CPANF (real update) corresponds to CPANF using real part update from (7.14), CPANF (imag update) corresponds to CPANF using imaginary part update from (7.15) and CPANF (mixed update) corresponds to alternative use of (7.14) and (7.15) as discussed in Section 7.3. When the notch is far away from the input frequency CPANF (mixed update) uses (7.14) and step-size μ_{large} , and when the notch is close to the input frequency it uses (7.15) and step-size μ_{small} . The parameters ϕ_{high} and ϕ_{low} are taken as 0.056π to keep the pilot notches symmetrical to that of the main notch. The bandwidth of all the notches is kept unchanged at $r = 0.9$. The frequency of the input sinusoid is 0.25π . The results are shown in Fig. 7.5, Fig. 7.6 and Fig. 7.7 which are discussed in subsections 7.5.1, 7.5.2, 7.5.3 respectively.

7.5.1 Comparison of speed of convergence

In this comparison, the step-sizes for the schemes are chosen such that they have approximately the same MSE at steady-state. A data length of 5000 samples is considered and the results are obtained by averaging 100 independent simulation runs which are shown in Fig. 7.5. The step-sizes being used for this analysis are as follows: For CANF, $\mu_{fixed} = (1/6) \times 10^{-3}$; For CPANF (real update) and CPANF

Figure 7.4: Convergence of the notch frequency to the actual frequency when input SNR is 10 dB using step-size $\mu_{large} = 3 \times 10^{-3}$ and $\mu_{small} = (1/3) \times 10^{-3}$.

Figure 7.5: Comparison of speed of convergence for the same MSE for CANF, CPANF (real update), CPANF (imag update) and CPANF (mixed update) when the input SNR is 5 dB.

Figure 7.6: MSE comparison at the same speed of convergence for CANF, CPANF (real update) and CPANF (mixed update) when the input SNR is 5 dB.

(imag update), $\mu_{large} = (1/4) \times 10^{-3}$ and $\mu_{small} = (1/6) \times 10^{-3}$; For CPANF (mixed update), $\mu_{large} = (1/2) \times 10^{-3}$ and $\mu_{small} = (1/6) \times 10^{-3}$. The μ_{large} step-size for CPANF (mixed update) is chosen larger to have the same MSE as compared to the other schemes.

The results in Fig. 7.5 show that the CPANF has a higher speed of convergence for the same MSE over CANF. This is because, a much larger step-size is chosen when the notch is far away from the input frequency. CPANF (mixed update) shows the fastest convergence it can use an even larger step-size when the input frequency is far away from the notch frequency.

7.5.2 Comparison of mean square error (MSE)

In this comparison, the step-size are selected such that the algorithms have roughly the same speed of convergence. The same data length and simulation runs as in Section 7.5.1 are used and the averaged results are shown in Fig. 7.6. For better

Figure 7.7: Steady-state MSE versus input SNR for CANF, CPANF (real update), CPANF (imag update) and CPANF (mixed update).

observation, here we present the MSE comparison results for CANF, CPANF (real update) and CPANF (mixed update). Later, in Section 7.5.3 we present the steady-state MSE comparison plots for all the schemes. The step-sizes being used in the simulation are as follows: for CANF, $\mu_{fixed} = 1 \times 10^{-3}$; For CPANF (real update), $\mu_{large} = 1.1 \times 10^{-3}$ and $\mu_{small} = (1/4) \times 10^{-3}$; For CPANF (mixed update), $\mu_{large} = 1.1 \times 10^{-3}$ and $\mu_{small} = (1/6) \times 10^{-3}$.

The results in Fig. 7.6 show that the MSE for CPANF is smaller as compared to CANF. The variable step-size algorithm in CPANF allows to the use of a smaller step-size after convergence. CPANF (mixed update) shows the lowest MSE as it can choose an even smaller step-size.

7.5.3 Steady-state MSE comparison versus input SNR

In this comparison, we estimate the steady-state MSE of the complex notch filters under different noise conditions. The filter parameters are chosen such that all the

schemes have the same speed of convergence as in Section 7.5.2. A large input data length of 10000 is chosen and the last 3000 output data samples of the filter in each run are time averaged to produce the estimated values of MSE. Simulations are done over a range of SNR values for performance evaluation. The estimated MSE are plotted versus the input signal's SNR as shown in Fig. 7.7. The results show that the CPANF has comparatively smaller MSE at steady state. CPANF (mixed update) performs better overall as it uses appropriate update and step-size to attain the lowest steady-state MSE.

7.6 Theoretical Analysis of Complex Piloted Adaptive Notch Filter

7.6.1 Filter states

The information from the three notches gives an estimate of the relative position of the notch frequency with respect to the frequency of the input complex sinusoid. This estimate is used to determine the steering direction of the main notch. The input $y(n)$, as shown in Fig. 7.1 and Fig. 7.3, is assumed to be a signal consisting of in-phase and quadrature-phase components with additive complex Gaussian noise and thus, a complex noisy signal as shown in (7.21)

$$y(n) = ae^{j(\omega_0 n + \beta)} + s(n) \quad (7.21)$$

where, ω_0 is the frequency of the input sinusoid, β is an arbitrary phase constant, a is the magnitude of the input sinusoid. $s(n)$ is the complex Gaussian noise with zero mean and variance σ_s^2 . In order to simplify the theoretical analysis, we have assumed that $\theta(n)$ can be approximated by a time invariant constant θ within a given short duration of interest.

The intermediate state $x(n)$, which is also a complex signal, is expressed as

$$x(n) = Ae^{j(\omega_0 n + \gamma)} + \eta(n) \quad (7.22)$$

where,

$$A = \frac{a^2}{\sqrt[2]{1 - 2r \cos(\theta - \omega_0) + r^2}} \quad (7.23)$$

$$\gamma = \beta + \tan^{-1} \left(\frac{r \sin(\theta - \omega_0)}{1 - r \cos(\theta - \omega_0)} \right) \quad (7.24)$$

and $\eta(n)$ is the filtered version of $s(n)$ through the all-pole section of the notch filter. It is assumed that $\eta(n)$ is also Gaussian distributed with zero mean. The autocorrelation function of $\eta(n)$, denoted as $R_{\eta\eta}(k)$, is given by

$$R_{\eta\eta}(k) = \frac{1}{2\pi j} \oint |H_D(z)|^2 \Phi_{ss}(z) z^{k-1} dz \quad (7.25)$$

where, $H_D(z)$ is given by

$$H_D(z) = \frac{1}{1 + rWz^{-1}} \quad (7.26)$$

$H_D(z)$ is the transfer function of the all-pole section of the notch filter and $\Phi_{ss}(z)$ is the power spectrum of the input Gaussian noise, which is equal to σ_s^2 . $R_{\eta\eta}(0)$ and $R_{\eta\eta}(1)$ is calculated by applying Residue theorem to (7.25).

$$\begin{aligned} R_{\eta\eta}(0) &= \frac{1}{2\pi j} \oint H_D(z) H_D^*(z^{-1}) \sigma_s^2 \frac{dz}{z} \\ &= \frac{\sigma_s^2}{2\pi j} \oint \frac{1}{z - re^{j(\theta)}} \cdot \frac{1}{1 - re^{-j(\theta)}z} dz \\ &= \frac{\sigma_s^2}{1 - r^2} \end{aligned} \quad (7.27)$$

$$\begin{aligned} R_{\eta\eta}(1) &= \frac{1}{2\pi j} \oint H_D(z) H_D^*(z^{-1}) \sigma_s^2 dz \\ &= \frac{\sigma_s^2}{2\pi j} \oint \frac{1}{z - re^{j(\theta)}} \cdot \frac{1}{1 - re^{-j(\theta)}z} z dz \\ &= \frac{re^{j(\theta)}}{1 - r^2} \sigma_s^2 \end{aligned} \quad (7.28)$$

The error output $e(n)$ is expressed as

$$\begin{aligned} e(n) &= Ae^{j(\omega_0 n + \gamma)} - e^{j\theta} Ae^{j(\omega_0(n-1) + \gamma)} + \nu(n) \\ &= Ae^{j(\omega_0(n-1) + \gamma)} \tau_r(\theta) + \nu(n) \end{aligned} \quad (7.29)$$

$$= Ae^{j(\omega_0(n) + \gamma)} \tau_i(\theta) + \nu(n) \quad (7.30)$$

where,

$$\tau_r(\theta) = e^{j\omega_0} - e^{j\theta} \quad (7.31)$$

$$\tau_i(\theta) = 1 - e^{j(\theta - \omega_0)} \quad (7.32)$$

and

$$\nu(n) = \eta(n) - e^{j\theta} \eta(n-1) \quad (7.33)$$

$\nu(n)$ is the filtered noise through the all-zero section of the notch filter. Thus, it can also be assumed to be Gaussian distributed with zero mean and variance σ_ν^2 given by

$$\begin{aligned} \sigma_\nu^2 &= E[\nu(n)\nu^*(n)] = \frac{2\sigma_s^2}{1-r^2}(1-r) \\ &= \frac{2\sigma_s^2}{1+r} \end{aligned} \quad (7.34)$$

7.6.2 Steer direction determining terms

The steering direction is obtained from $\Re\{x^*(n-1)e(n)\}$ or $\Im\{x^*(n)e(n)\}$. Hence, we shall investigate the product term $x^*(n-1)e(n)$ and $x^*(n)e(n)$.

1. $\Re\{x^*(n-1)e(n)\}$: The product term $x^*(n-1)e(n)$ is obtained using (7.22)

and (7.29) and considering the real part, we get

$$\begin{aligned}
 & \Re\{x^*(n-1)e(n)\} \\
 &= \Re\{(Ae^{-j(\omega_0(n-1)+\gamma)} + \eta^*(n-1))(Ae^{j(\omega_0(n-1)+\gamma)}\tau_r(\theta) + \nu(n))\} \\
 &= \Re\{A^2\tau_r(\theta) + \xi_r(n)\} \tag{7.35}
 \end{aligned}$$

where,

$$\xi_r(n) = Ae^{-j(\omega_0(n-1)+\gamma)}\nu(n) + \eta^*(n-1)(Ae^{j(\omega_0(n-1)+\gamma)}\tau_r(\theta) + \nu(n)) \tag{7.36}$$

Substituting (7.31) into (7.35), we obtained

$$\begin{aligned}
 & \Re\{x^*(n-1)e(n)\} \\
 &= \Re\{A^2(\cos\omega_0 - \cos(\theta) - j(\sin\omega_0 - \sin(\theta)))\} + \Re\{\xi_r(n)\} \tag{7.37}
 \end{aligned}$$

It can be seen from (7.35), (7.36) and (7.37) that $\Re\{A^2\tau_r(\theta)\}$ is the steering component and $\Re\{\xi_r(n)\}$ is the noise component of $\Re\{x^*(n-1)e(n)\}$.

The mean value of $\Re\{x^*(n-1)e(n)\}$ is obtained as

$$\Re\{E[x^*(n-1)e(n)]\} = \Re\left\{\frac{1}{2\pi} \int_{-\pi}^{\pi} H_D^*(e^{-j\omega})H(e^{j\omega})S_{yy}(\omega)e^{j\omega}d\omega\right\} \tag{7.38}$$

where, $S_{yy}(\omega)$ is the power spectral density of the input signal (7.21),

$$S_{yy}(\omega) = a^2\delta(\omega - \omega_0) + \sigma_s^2 \tag{7.39}$$

Solving (7.38) using Residue Theorem,

$$\Re\{E[x^*(n-1)e(n)]\} = \Re\left\{\frac{a^2\tau_r(\theta)}{1 - 2r\cos(\theta - \omega_0) + r^2} - \frac{e^{j(\theta)}}{1+r}\sigma_s^2\right\} \tag{7.40}$$

The mean value of $\xi_r(n)$, denoted by E_{ξ_r} , can be obtained from (7.40) as

$$E_{\xi_r} = -\frac{e^{j(\theta)}}{1+r}\sigma_s^2 \tag{7.41}$$

Figure 7.8: Comparison of mean of noise component $\Re\{E_{\xi_r}\}$ and steering component $\Re\{E[A^2\tau_r(\theta)]\}$ of $\Re\{E[x^*(n-1)e(n)]\}$.

It can be seen from Fig. 7.8 that $|\Re\{E_{\xi_r}\}|$ is always less than or equal to $|\Re\{E[A^2\tau_r(\theta)]\}|$, the real part of the mean of the steering component. $\Re\{E_{\xi_r}\}$ and $\Re\{E[A^2\tau_r(\theta)]\}$ carries the same sign.

The variance of $\xi_r(n)$, denoted by $\sigma_{\xi_r}^2$, can be obtained as

$$\sigma_{\xi_r}^2 = E[(\xi_r(n) - E_{\xi_r})(\xi_r(n) - E_{\xi_r})^*] \quad (7.42)$$

For small E_{ξ_r} ,

$$\begin{aligned} \sigma_{\xi_r}^2 &= E[|\xi_r(n)|^2] \\ &= E[|Ae^{-j(\omega_0(n-1)+\gamma)}\nu(n) \\ &\quad + \eta^*(n-1)(Ae^{j(\omega_0(n-1)+\gamma)}\tau_r(\theta) + \nu(n))|^2] \end{aligned} \quad (7.43)$$

As the mean values of $\eta(n)$, $\nu(n)$ and $e^{j\theta}$ are each equal to zero, and that

Figure 7.9: Variance of $\Re\{x^*(n-1)e(n)\}$ for different SNR conditions for input signal power of 0.5.

$\eta(n)$ and $\nu(n)$ are uncorrelated,

$$\begin{aligned}
 \sigma_{\xi_r}^2 &= A^2 E[\nu(n)\nu^*(n)] \\
 &\quad + E[\eta^*(n-1)\eta(n-1)](A^2|\tau_r(\theta)|^2 + E[\nu(n)\nu^*(n)]) \\
 &= A^2\sigma_\nu^2 + R_{\eta\eta}(0)[A^2|\tau_r(\theta)|^2 + \sigma_\nu^2]
 \end{aligned} \tag{7.44}$$

The variance of $\Re\{x^*(n-1)e(n)\}$ is the variance of $\Re\{\xi_r(n)\}$, denoted by

$\sigma_{\xi_{real}}^2$, and is given by

$$\sigma_{\xi_{real}}^2 = \frac{A^2}{2}\sigma_\nu^2 + \frac{R_{\eta\eta}(0)}{2}[A^2|\tau_r(\theta)|^2 + \sigma_\nu^2] \tag{7.45}$$

Fig. 7.9 shows the computer simulated results and the theoretical results obtained from (7.45) for different SNR conditions.

2. $\Im\{x^*(n)e(n)\}$: The product term $x^*(n)e(n)$ is obtained using (7.22) and (7.30)

and considering the imaginary part, we get

$$\begin{aligned}
 & \Im\{x^*(n)e(n)\} \\
 &= \Im\{(Ae^{-j(\omega_0(n)+\gamma)} + \eta^*(n))(Ae^{j(\omega_0(n)+\gamma)}\tau_i(\theta) + \nu(n))\} \\
 &= \Im\{A^2\tau_i(\theta) + \xi_i(n)\}
 \end{aligned} \tag{7.46}$$

where,

$$\xi_i(n) = Ae^{-j(\omega_0(n)+\gamma)}\nu(n) + \eta^*(n)(Ae^{j(\omega_0(n)+\gamma)}\tau_i(\theta) + \nu(n)) \tag{7.47}$$

Substituting (7.32) into (7.46), we obtained

$$\begin{aligned}
 & \Im\{x^*(n)e(n)\} \\
 &= \Im\{A^2(1 - \cos(\theta - \omega_0) - j \sin(\theta - \omega_0))\} + \Im\{\xi_i(n)\}
 \end{aligned} \tag{7.48}$$

It can be seen from (7.46), (7.47) and (7.48) that $\Re\{A^2\tau_i(\theta)\}$ is the steering component and $\Re\{\xi_i(n)\}$ is the noise component of $\Im\{x^*(n)e(n)\}$.

The mean value of $\Im\{x^*(n)e(n)\}$ is obtained as

$$\Im\{E[x^*(n)e(n)]\} = \Im\left\{\frac{1}{2\pi} \int_{-\pi}^{\pi} H_D^*(e^{-j\omega})H(e^{j\omega})S_{yy}(\omega)d\omega\right\} \tag{7.49}$$

Solving (7.49) using the Residue Theorem,

$$\Im\{E[x^*(n)e(n)]\} = \Im\left\{\frac{a^2\tau_i(\theta)}{1 - 2r \cos(\theta - \omega_0) + r^2} + \frac{1}{1+r}\sigma_s^2\right\} \tag{7.50}$$

The mean value of $\xi_i(n)$, denoted by E_{ξ_i} , can be obtained from (7.50) as

$$E_{\xi_i} = \frac{1}{1+r}\sigma_s^2 \tag{7.51}$$

It can be seen from (7.51) that E_{ξ_i} is real and has no imaginary part. However, when E_{ξ_i} is obtained using a finite number of samples, the imaginary part may be non-zero although its magnitude is small.

The variance of $\xi_i(n)$, denoted by $\sigma_{\xi_i}^2$, can be obtained as

$$\sigma_{\xi_i}^2 = E[(\xi_i(n) - E_{\xi_i})(\xi_i(n) - E_{\xi_i})^*] \quad (7.52)$$

For small E_{ξ_i} ,

$$\begin{aligned} \sigma_{\xi_i}^2 &= E[|\xi_i(n)|^2] \\ &= E[|Ae^{-j(\omega_0(n)+\gamma)}\nu(n) + \eta^*(n)(Ae^{j(\omega_0(n)+\gamma)}\tau_i(\theta) + \nu(n))|^2] \end{aligned} \quad (7.53)$$

As the mean values of $\eta(n)$, $\nu(n)$ and $e^{j\theta}$ are each equal to zero, and that $\eta(n)$ and $\nu(n)$ are uncorrelated,

$$\begin{aligned} \sigma_{\xi_i}^2 &= A^2 E[\nu(n)\nu^*(n)] \\ &\quad + E[\eta^*(n)\eta(n)](A^2|\tau_i(\theta)|^2 + E[\nu(n)\nu^*(n)]) \\ &= A^2\sigma_\nu^2 + R_{\eta\eta}(0)[A^2|\tau_i(\theta)|^2 + \sigma_\nu^2] \end{aligned} \quad (7.54)$$

The variance of $\Im\{E[x^*(n)e(n)]\}$ is the variance of $\Im\{\xi_i(n)\}$, denoted by $\sigma_{\xi_{imag}}^2$, and is given by

$$\sigma_{\xi_{imag}}^2 = \frac{A^2}{2}\sigma_\nu^2 + \frac{R_{\eta\eta}(0)}{2}[A^2|\tau_i(\theta)|^2 + \sigma_\nu^2] \quad (7.55)$$

7.7 Steering Direction Determination Mechanism for Complex Piloted Adaptive Notch Filter

The determination of the steering direction of the notch is fundamental to the functioning of the CPANF. The steering direction of a notch is obtained from either using $\Re\{x^*(n-1)e(n)\}$ or $\Im\{x^*(n-1)e(n)\}$. Consider the term $\Re\{x^*(n-1)e(n)\}$ from (7.37), it can be inferred that if $\Re\{x^*(n-1)e(n)\}$ is positive at a particular instant n , then it indicates $\omega_0 < \theta$. Conversely, if $\Re\{x^*(n-1)e(n)\}$ is negative

Figure 7.10: Comparison of mean of $|\Re\{E[A^2\tau_r(\theta)]\}|$ (steering component of $\Re\{x^*(n-1)e(n)\}$) and $|\Im\{E[A^2\tau_i(\theta)]\}|$ (steering component of $\Im\{x^*(n)e(n)\}$).

at a particular instant n , then it indicates $\omega_0 > \theta$. Thus, the relative positions of ω_0 can be obtained from the signs of $\Re\{x^*(n-1)e_{high}(n)\}$, $\Re\{x^*(n-1)e(n)\}$ and $\Re\{x^*(n-1)e_{low}(n)\}$. If the signs of $\Re\{x^*(n-1)e_{high}(n)\}$, $\Re\{x^*(n-1)e(n)\}$ and $\Re\{x^*(n-1)e_{low}(n)\}$ are the same, then it can be concluded that the input frequency is far away from the main notch. If they have different signs, then it will indicate that the input frequency is sandwiched in between the frequencies of the pilot notches, that is, the input frequency is close to the main notch. This steering direction information can also be obtained using $\Im\{x^*(n)e(n)\}$ from (7.48). For high SNR, it is unimportant whether $\Re\{x^*(n-1)e(n)\}$ or $\Im\{x^*(n)e(n)\}$ is used. For low SNR, the choice becomes important as can be seen from (7.37) and (7.48); the sign of $\Re\{x^*(n-1)e(n)\}$ or $\Im\{x^*(n)e(n)\}$ is significantly influenced by ξ_r or ξ_i respectively, when $|\Re\{\xi_r\}| > |\Re\{A^2\tau_r(\theta)\}|$ or $|\Im\{\xi_i\}| > |\Im\{A^2\tau_i(\theta)\}|$.

Moreover, the choice of the steering terms also becomes significant when considering the relative position of the notch frequency with respect to the input sinusoidal frequency. Fig. 7.10 shows the plots of $|\Re\{E[A^2\tau_r(\theta)]\}|$ and $|\Im\{E[A^2\tau_i(\theta)]\}|$ from (7.40) and (7.50) when the input sinusoidal frequency is 0.75π and the input SNR is 0 dB for different notch frequency. It can be observed that $|\Re\{E[A^2\tau_r(\theta)]\}|$ has larger magnitude compared to $|\Im\{E[A^2\tau_i(\theta)]\}|$ when the notch is far away from the input frequency. Hence, the steering information from $\Re\{x^*(n-1)e(n)\}$ will give better results as it is less affected by the noise term. Conversely, when the notch is closer to the input frequency, $|\Im\{E[A^2\tau_i(\theta)]\}|$ has larger magnitude compared to $|\Re\{E[A^2\tau_r(\theta)]\}|$, and hence, $\Im\{x^*(n)e(n)\}$ will give more accurate steering information under noisy conditions. Thus, a mixed steering mechanism will be preferred where the signs of $\Re\{x^*(n-1)e_{high}(n)\}$, $\Re\{x^*(n-1)e(n)\}$ and $\Re\{x^*(n-1)e_{low}(n)\}$ are used when the notch is far away from the input frequency and the signs of $\Im\{x^*(n)e_{high}(n)\}$, $\Im\{x^*(n)e(n)\}$ and $\Im\{x^*(n)e_{low}(n)\}$ are used when the notch is closer to the frequency of the input complex sinusoid.

The steering direction information can be improved by increasing the number of pilots which will provide more information about the position of the notch. Simulations are done with input signal corrupted by complex Gaussian noise. It is shown that the probability of obtaining a correct steering direction by the proposed mechanism is always greater than 0.5 even for very high noise conditions.

7.8 Probability of Correct Steering Direction Determination

The use of pilot notches is an excellent technique to obtain the steering direction and choose a suitable step-size for convergence. In Section 7.7, we have shown that the signs of the product terms can provide information on whether the notch is far away or closer to the input frequency. However, the steering direction may not be correct when the input signal is corrupted by noise as discussed in Section 7.6. Thus, it is important to analyse the probability of obtaining a correct steering direction from the signs of $\Re\{x^*(n-1)e_{high}(n)\}$, $\Re\{x^*(n-1)e(n)\}$ and $\Re\{x^*(n-1)e_{low}(n)\}$ or $\Im\{x^*(n)e_{high}(n)\}$, $\Im\{x^*(n)e(n)\}$ and $\Im\{x^*(n)e_{low}(n)\}$. In this section, we will discuss in detail the probability of obtaining a correct steering direction from the sign of $\Re\{x^*(n-1)e(n)\}$ at any time instant n in noisy conditions. The results of other terms can be obtained in the similar way. Denote $\Re\{x^*(n-1)e(n)\}$ at any time instant n as the steering term, $st(n)$, given by

$$st(n) = \Re\{x^*(n-1)e(n)\} \quad (7.56)$$

It is assumed that the input noise is complex Gaussian with zero mean and variance σ_s^2 . Since, $\eta(n)$ is also Gaussian distributed with zero mean as discussed in section 7.6, the output noise $\nu(n)$ is also Gaussian distributed as it is the sum of two Gaussians with variance calculated in (7.45). Also from (7.37), (7.36), (7.27)

and (7.28), we have

$$\begin{aligned}
 E[\eta^*(n-1)\nu(n)] &= E[\eta^*(n-1)\eta(n) - e^{j\theta}\eta^*(n-1)\eta(n-1)] \\
 &= R_{\eta\eta}(1) - e^{j\theta}R_{\eta\eta}(0) \\
 &= -\frac{1}{(1+r)}e^{j\theta}\sigma_s^2
 \end{aligned} \tag{7.57}$$

As discussed in Section 7.6, the correlation of $\nu(n)$ and $\eta(n-1)$ from (7.57) is very small compared to the other terms in the product term $\Re\{x^*(n-1)e(n)\}$. Thus, from (7.37), (7.36), (7.56) and (7.57), it can be concluded that at any time instant n , $st(n)$ is Gaussian distributed with mean as calculated in (7.40), denoted by E_{st} .

$$E_{st} = \Re\left\{\frac{a^2\tau_r(\theta)}{1 - 2r\cos(\theta - \omega_0) + r^2} - \frac{e^{j(\theta)}}{1+r}\sigma_s^2\right\} \tag{7.58}$$

The variance of the steering term, denoted by σ_{st}^2 , is the variance of the real part of $\xi_r(n)$ calculated in (7.45). Thus,

$$\sigma_{st}^2 = \frac{A^2}{2}\sigma_\nu^2 + \frac{R_{\eta\eta}(0)}{2}[A^2|\tau_r(\theta)|^2 + \sigma_\nu^2] \tag{7.59}$$

Although $st(n)$ may not be always Gaussian, but in order to simplify the theoretical analysis, we assume it to be Gaussian. The pdf of the steering term $st(n)$, can be expressed as

$$f_{st}(st(n)) = \frac{1}{\sqrt{2\pi\sigma_{st}^2}}\exp\left(-\frac{(st(n) - E_{st})^2}{2\sigma_{st}^2}\right) \tag{7.60}$$

The sign of $st(n)$ is key to deciding the notch position on the z-plane. The probability that the sign of $st(n)$ is positive, denoted by P_+ , is given by

$$\begin{aligned}
 P_+ &= \int_0^\infty f_{st}(st(n))d(st(n)) \\
 &= \int_0^\infty \frac{1}{\sqrt{2\pi\sigma_{st}^2}}\exp\left(-\frac{(st(n) - E_{st})^2}{2\sigma_{st}^2}\right)d(st(n))
 \end{aligned} \tag{7.61}$$

It is important to know the factors which affects the probability of obtaining a correct steering direction. Substituting (7.27) and (7.34) into (7.59), we have

$$\sigma_{st}^2 = \left[\frac{A^2(1+r)((2(1-r) + |\tau_r(\theta)|^2) + 2\sigma_s^2)}{2(1-r^2)(1+r)} \right] \sigma_s^2 \quad (7.62)$$

Considering (7.23) and (7.31), we expressed (7.62) as

$$\sigma_{st}^2 = F_1(a, \omega_0, \sigma_s^2, r, \theta) \quad (7.63)$$

From (7.58), we expressed E_{st} as

$$E_{st} = F_2(a, \omega_0, \sigma_s^2, r, \theta) \quad (7.64)$$

Substituting (7.63) and (7.64) into (7.61), we obtained (7.65)

$$\begin{aligned} P_+ &= \int_0^\infty f_{st}(st(n)) d(st(n)) \\ &= \int_0^\infty \frac{1}{\sqrt{2\pi F_1(a, \omega_0, \sigma_s^2, r, \theta)}} \exp\left(-\frac{(st(n) - F_2(a, \omega_0, \sigma_s^2, r, \theta))^2}{2F_1(a, \omega_0, \sigma_s^2, r, \theta)}\right) d(st(n)) \end{aligned} \quad (7.65)$$

It can be derived from (7.65) that the probability of obtaining a correct steering direction is a function of i) the frequency of the input complex sinusoid ω_0 , ii) the input SNR, iii) the notch frequency θ and iv) the notch bandwidth i.e, pole radius r . The following Sections 7.9-7.11 present detailed analysis of the effect of these variables on the probability of obtaining a correct steering direction.

7.9 Effect of Input SNR on Steering Direction Determination

The effect of input SNR on the steering direction determination can be obtained from (7.60) and (7.61). As there are multiple factors affecting the steering direction

Figure 7.11: Probability of obtaining a correct steering direction from the sign of $\Re\{x^*(n-1)e(n)\}$ versus input SNR for an input frequency of 0.4π . The notch frequency is fixed at 0.75π and the pole radius r is 0.9.

determination, the effect of input SNR can be obtained by fixing the input sinusoid's frequency ω_0 , the notch frequency θ and the notch bandwidth i.e, fixed pole radius r , the probability of obtaining a correct steering direction as a function of input SNR can be obtained from (7.61) using the results from (7.27), (7.31), (7.34), (7.37), (7.58), (7.59) and (7.60) as done in Section 7.8. The obtained results are shown in Fig. 7.11 together with simulation results. The array of symbols corresponds to the results obtained through computer simulations and the solid line corresponds to the results predicted by (7.61). The results represented by each symbol are obtained by averaging 100 independent simulation runs. The notch frequency θ of the main notch is fixed at 0.75π , the input frequency ω_0 at 0.4π and the pole radius is set to be $r = 0.9$. The steering direction is correct if the sign of $\Re\{x^*(n-1)e(n)\}$ is positive for $\theta > \omega_0$. It can be seen from Fig. 7.11 that the probability of obtaining a correct steering direction decreases with decreasing

Figure 7.12: Probability of obtaining a correct steering direction from the sign of $\Re\{x^*(n-1)e(n)\}$ versus pole radius r for an input frequency of 0.4π . The notch frequency is fixed at 0.75π and the input SNR is 0 dB.

SNR of the input signal.

7.10 Effect of Pole Radius r on Steering Direction Determination

It will be interesting to understand the effect of notch bandwidth or pole radius r on the probability of obtaining a correct steering direction for a given input frequency ω_0 , notch position θ , and input SNR. In Fig. 7.12, the results corresponding to $\theta = 0.75\pi$, input frequency ω_0 and input SNR of 0 dB are shown.

It can be inferred from Fig. 7.12 that when the notch has not converged to the frequency of the input sinusoid, i.e., $\theta \neq \omega_0$ the accuracy of steering direction determination decreases with increasing pole radius. This is due to the amplification of the noise component by the pole which increases with the increase in pole radius r . Thus, a smaller pole radius will always be preferred for steering direction

Figure 7.13: Probability of obtaining a correct steering direction from the sign of $\Re\{x^*(n-1)e(n)\}$ versus notch frequency θ for an input frequency of 0.4π . The input SNR is 0 dB and the pole radius r is 0.9.

determination when the algorithm has not converged, particularly, at low SNR. However, with the increase in the bandwidth of the notch, the steady-state performance will degrade. Thus, a variable pole radius scheme will have the following advantages. It will use a smaller pole radius r when the notch frequency is far away from the frequency of the input sinusoid for faster speed of convergence, and a larger pole radius r when the notch frequency is close to the input frequency for better steady state results. The theoretical and simulation results are consistent as shown in Fig. 7.12. The simulation results shown in Fig. 7.12 are obtained by averaging the results over 100 independent simulation runs.

It is important to note that the effect of noise become more prominent for pilot notches as the zero, pole and the origin are not in the straight line leading to unsymmetrical gain at the vicinity of the notch frequency.

Figure 7.14: Probability of obtaining a correct steering direction from the sign of $\Re\{x^*(n-1)e(n)\}$ versus notch frequency θ for an input frequency of 0.6π . The input SNR is 0 dB and the pole radius r is 0.9.

7.11 Effect of Notch Position on Steering Direction Determination

The position of the notch w.r.t., the input frequency affects the probability of obtaining a correct steering direction as shown in Figs. 7.13 and 7.14. The steering direction is accurate if the sign of $\Re\{x^*(n-1)e(n)\}$ is positive when $\theta > \omega_0$ and negative when $\theta < \omega_0$. The input frequency ω_0 is fixed at 0.4π and 0.6π for Figs. 7.13 and 7.14 with input SNR at 0 dB and pole radius $r = 0.9$. It can be seen from the figures that the theoretical values obtained from (7.61) closely follows the simulation results. It is observed from the results that there is degradation in performance of the steering direction determination when the notch is far away from the input frequency. This is because when the notch is far away from the input complex sinusoidal frequency, the noise is being amplified by the pole more than the input sinusoid. This results in a degradation of SNR. Although the results

show degradation at some notch positions, the probability of obtaining a correct steering direction from the sign of $\Re\{x^*(n-1)e(n)\}$ is always greater than 0.5. Thus, it will always be better than a random guess. Each symbol in Figs. 7.13 and 7.14 are obtained by averaging the results obtained from 100 independent simulation runs.

7.12 Application in QPSK Communication Systems

We choose a QPSK system where the received signal is corrupted with sinusoidal interference and additive white complex Gaussian noise as an example to illustrate our method. The system is shown in Fig. 7.15. The input of the complex adaptive notch filter, at time n , is $u(n)$ and is expressed as

$$u(n) = d(n) + Ae^{j(\omega_0 n + \beta)} + s(n) \quad (7.66)$$

and

$$d(n) = d_1(n) + jd_2(n) \quad (7.67)$$

where, $d(n)$ is the complex binary signal generated from the PN signals. In (7.66), A and ω_0 are unknown amplitude and frequency of the interfering tone, β is an arbitrary phase and $s(n)$ is the complex Gaussian noise with zero mean and variance σ_s^2 .

Simulation results for different ISR (interference-to-signal ratio) and SNR conditions with an interfering tone of frequency 0.3π are shown in Fig. 7.16-7.20. Fig. 7.16 and 7.18 show the results obtained when the input QPSK signal is only corrupted by an interfering sinusoid with ISR of 5 dB. Fig. 7.17 and 7.19 show

Figure 7.15: Noise model for a QPSK receiver.

Figure 7.16: Signal constellation of received QPSK signal (notch input) with ISR 5 dB.

the simulation results when the input QPSK signal is corrupted by an interfering tone with ISR of 5 dB and complex Gaussian noise with SNR of 10 dB. Fig. 7.20 shows the bit error rate (BER) improvement in the received signal with CPANF over conventional CANF [107] for varying SNR conditions. Fig. 7.21 shows stable BER performance of CPANF in improving the received signal over CANF for ISR ranging from 0 dB to 15 dB.

Figure 7.17: Signal constellation of received QPSK signal (notch input) with ISR 5 dB and SNR 10 dB.

Figure 7.18: Signal constellation of notch output for input QPSK signal with ISR 5 dB.

Figure 7.19: Signal constellation of notch output for input QPSK signal with ISR 5 dB and SNR 10 dB.

Figure 7.20: Comparison of BER for CANF and CPANF (real update) versus SNR. The frequency of the sinusoid interference ω_0 is 0.3π with ISR of 5 dB.

Figure 7.21: Comparison of BER for CANF and CPANF (real update) versus ISR. The frequency of the sinusoid interference ω_0 is 0.3π with SNR of 10 dB.

7.13 Chapter Summary

In this chapter, we have presented the filter structure and the mathematical formulation of CPANF for estimation and tracking of complex sinusoidal signal. We have shown that with the higher probability of obtaining a correct steering direction, the CPANF gives better estimate of the relative position of the main notch frequency with respect to the frequency of the input sinusoid. This is useful for choosing an appropriate step-size for high speed of convergence and lower MSE at steady state. The simulation results verify the excellent performance of the notch under various conditions. The performance of the filter can be further improved by introducing multiple pilots and multiple step-sizes.

We have also simulated the application of the filter in a QPSK communication systems for suppressing complex sinusoidal interference. Simulation results showed significant improvement in overall BER over conventional CANF. The performance

of the filter remained stable over a range of ISR values.

Chapter 8

Conclusion and Future Work

8.1 Conclusion

In this thesis, we have evaluated, enhanced and extended the mechanism of PANF to estimate and track real and complex sinusoid. The PANF is a novel technique of using pilot notches with the conventional notch filter to derive frequency domain information. The information is obtained by analyzing the signs of the gradient terms at each of these pilot notches. This frequency domain information, which was previously unavailable, is used to choose an appropriate step-size for the LMS filter coefficient update algorithm. A detailed computer simulation based performance evaluation of this filter with other conventional notch filters under various noise conditions has been discussed in Chapter 3. The results of the steady-state analysis clearly justify the use of pilots to determine the steering direction of the notch with significant improvement in the speed of convergence and the steady-state MSE. The steady-state analysis motivated us to further improve the steering direction determination with the objective of improvement in the performance of PANF under low SNR conditions.

The theoretical study of the product term used for deriving the instantaneous steering direction information gave an insight into its limitations under low SNR conditions. This led to the development of a new steering direction determination mechanism based on low pass filtering of the product term in Chapter 4. The

results show significant improvement in the probability of having a correct steering direction. Moreover, periodic instances when the sinusoids have zero crossing and the steering direction being derived only from the noise term is highly reduced. As the original PANF uses only two step-sizes, a new step-size update algorithm has also been proposed. This new step-size update algorithm which is based on the time-frequency domain information has improved the speed of convergence with a comparatively huge reduction in the steady-state MSE.

The pole radius plays an important role in determining the performance of the notch filter as discussed in Chapter 5. Though in the literature the effect of pole radius on the transient response has been studied but its effect in a PANF is very unique. As the PANF uses the same pole of the main notch for pilot notches, it led to the amplification of the noise in the wrong side of the pilot notch frequency. Hence, in addition to the effect of transient response on the steering direction, the amplified noise also degrades the steering direction determination. This detailed analysis of the effect of pole radius in a PANF led to the development of a computationally efficient variable pole-radius mechanism in Chapter 5. The proposed technique significantly reduced the transient response and also improved the steering direction determination of the PANF, resulting in higher speed of convergence of the notch filter. The steady-state MSE is also improved in comparison to the PANF with fixed pole-radius.

The PANF [51] uses two step-sizes and a single pair of pilot notches which provides limited frequency domain information, i.e., the notch is close or far away from the input sinusoidal frequency. The transition from a larger step-size value to a small step-size value is abrupt and may lead to huge output error when the

notch is close to the input frequency. Thus, we proposed a generalized mathematical formulation and filter structure of PANF with multiple pilots pairs, MPPANF, to derive more frequency domain information and having an option for a range of step-sizes. The transition from a very large step-size to a very small step-size is achieved in one or more intermediate step-sizes. The variable pole radius MP-PANF (VP-MPPANF) with reduced transient response and improved probability of obtaining a correct steering direction has also been presented. The MPPANF and VP-MPPANF have shown enhanced performance over original PANF. The detailed theoretical analysis of the proposed adaptive notch filter has been presented to substantiate the simulation claims. Mathematical formulation for the determination of the probability of obtaining a correct steering direction has been presented for theoretical investigation of the effect of input SNR, notch bandwidth and notch frequency on steering direction determination.

The original PANF [51] was proposed for tracking only real signals and hence, the extension of this mechanism of adaptive notch filtering to track sinusoid with both real and imaginary components was essential. Moreover, reports on variable step-size adaptive notch filter tracking complex signals is rare in the published literature. We have proposed a novel filter structure and the mathematical formulation of CPANF to estimate and track the frequency of a complex sinusoidal signal. The proposed variable step-size algorithm based on the steering direction information of the notch is a new technique in adaptive filtering for complex signals. The analysis of the product terms used for coefficient update and steering direction determination is helpful to develop the algorithm for a particular application. We have presented simulations results to show the excellent performance by

CPANF over conventional CANF with respect to the speed of convergence and the steady-state MSE. We have also presented the theoretical analysis of the CPANF with the mathematical formulation for determining the probability of obtaining a correct steering direction. The theoretical results for the effect of input SNR, notch bandwidth and notch frequency on the probability of obtaining a correct steering direction is presented which closely follows the simulation results. In order to showcase the effectiveness of the proposed filter, we have implemented it in a QPSK communication system to suppress complex sinusoidal interference. The results clearly show the improvement over conventional CANF in overall BER and the robust performance of the CPANF over a range of ISR values.

Thus, in conclusion, we have presented the research work on piloted adaptive notch filter to estimate and track both real and complex sinusoids.

8.2 Future Work

The PANF is new in adaptive filtering and brings in tremendous opportunity to be implemented in advanced tracking and noise cancellation systems. This system with a higher probability of correct steering direction makes frequency tracking possible at higher rate of convergence with low MSE. In continuation to the works mentioned above, the following are the areas which needs to be focused,

In Chapter 6, we have presented the MPPANF with more than one pair of pilot pairs. Though we have simulated a symmetric piloted notch structure, it will be interesting to study the unsymmetrical placement of pilot notches and the distribution of pilots around the main notch. As the number of pilots increases, the placement of pilots becomes crucial to reduce the overall complexity. This can

be studied through a heuristic study of the co-efficient update algorithm.

In Chapter 7 of this thesis, we have presented the CPANF with two pilot notches. Due to time constraint, some aspects of the filter have not been covered and further investigation for the effect of parameters as we have done for the PANF tracking real sinusoid will be of significant contribution for the development of CPANF.

The PANF has been developed to track single sinusoid. This fundamental filter section can be utilized to form filter structures for simultaneous detection and tracking of multiple sinusoids. Moreover, all these filter sections will have pilots which can be used co-operatively to map the frequency domain. This would reduce the overall complexity of introducing the pilots for the variable step-size algorithm.

Author's Publications

1. D. K Chobey, Y. C. Lim, and A. G. Constantinides, "Steady-state comparative performance evaluation of piloted adaptive notch filter", in 19th International Conference on Digital Signal Processing (DSP), Aug 2014, pp. 316-320.
2. D. K Chobey and Y. C. Lim, "A piloted notch time-frequency information based variable step-size algorithm", in IEEE International Symposium on Circuits and Systems (ISCAS), May 2015, pp. 2201-2204.
3. D. K Chobey and Y. C. Lim, "A multi-piloted notch filter for frequency estimation and tracking", in IEEE International Conference on in Digital Signal Processing (DSP), July 2015, pp. 848-852.
4. D. K Chobey and Y. C. Lim, "Effect of pole-radius in a piloted adaptive notch filter", in IEEE International Conference on in Digital Signal Processing (DSP), Oct 2016, pp. 618-622.
5. D. K Chobey and Y. C. Lim, "Multiple pilot pairs adaptive notch filter", to be submitted.
6. D. K Chobey and Y. C. Lim, "A complex piloted adaptive notch filter", is under review in IEEE Transactions on Signal Processing.

Appendix A

Derivation of Weight Update Equation

Let us consider a general system model of an adaptive noise cancellation as shown in Fig. A.1, where $x(n)$ is the signal with input sinusoid and wide-band noise and $e(n)$ notch output. Let us now consider a second order adaptive filter for noise cancellation as shown in the Fig. A.2.

The overall transfer function of the system shown in Fig. A.2 can be defined as

$$\xi(z) = \frac{E(z)}{X(z)} \quad (\text{A.1})$$

Thus, from Fig. A.2, we derive $\xi(z)$ as shown in (A.2)

$$\xi(z) = 1 + W_1(n)z^{-1} + W_2(n)z^{-2} \quad (\text{A.2})$$

The zeroes of (A.2) can be represented as in Fig. A.3.

For zeroes of the system to be on the unit circle, we take

$$W_2(n) = 1; \quad (\text{A.3})$$

Figure A.1: Adaptive noise cancellation

Figure A.2: Second order adaptive noise cancellation

Figure A.3: Zeros of the adaptive filter

Thus, using the derivation of LMS algorithm in Chapter 2, from (2.8), we can derive the update equation as

$$W_1(n+1) = W_1(n) - 2\mu e(n)x(n-1) \quad (\text{A.4})$$

It is important to note that the (-ve) sign has been introduced as $e(n)$ is obtained from $d(n) + \hat{y}(n)$.

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