Randomized modulation schemes for digital modulators of switched-mode DC-DC converters

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2018

Cui, K. (2018). Randomized modulation schemes for digital modulators of switched-mode DC-DC converters. Doctoral thesis, Nanyang Technological University, Singapore.

<http://hdl.handle.net/10356/75926>

<https://doi.org/10.32657/10356/75926>

**NANYANG
TECHNOLOGICAL
UNIVERSITY**

SINGAPORE

**RANDOMIZED MODULATION SCHEMES FOR
DIGITAL MODULATORS OF
SWITCHED-MODE DC-DC CONVERTERS**

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2018

Randomized Modulation Schemes for Digital Modulators of Switched-Mode DC-DC Converters

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A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirements for the degree of
Doctor of Philosophy

2018

ACKNOWLEDGEMENTS

My doctoral study at Nanyang Technological University would not have been so memorable without the guidance and help from many people in my life. With this opportunity, I would like to express my sincere gratitude to them.

I wish to express my sincere appreciations to my supervisor, Professor Gwee Bah-Hwee, for guiding me in the field of research and providing me the valuable study opportunity. I am also thankful to him for allowing a great deal of freedom to do my research. I am grateful that he has always been encouraging and supportive throughout my candidature. He has been always willing to offer his suggestions whenever I encounter problems. He has helped me to develop positive attitude toward difficulties.

I wish to thank my co-supervisor, Dr. Victor Adrian, who has mentored me throughout my study with his invaluable wealth of advices and in-depth technical expertise. He has taught me many things, amongst them, the rigorousness in writing and communication, how to think critically. I owe him my deepest gratitude for all the time and effort he spent working with me on my reports, papers and thesis.

I would like to thank all the friends as well as former and current students and colleagues with whom I have worked in the Integrated System Research Laboratory for their kind help and support during my doctoral studies. In particular, I would like to thank Mr. Sun Yin for his invaluable guidance and

suggestions in fabricating a dc-dc converter output stage IC, and his insights and advices on all the technical problems I have discussed with him; Ms. Salma Nashit for her generous help in my research project; Mr. Ne Kyaw Zwa Lwin for the help and support in the process of fabricating the output stage; Dr. Ho Weng Geng for his assistance and suggestions.

Last but not least, I would like to express my gratitude to my parents and my family members for their consistent support and encouragement. Thank you all for your patience and accompany throughout my doctoral study.

ABSTRACT

Conventional switched-mode dc-dc converters usually operate based on the Pulse Width Modulation scheme (PWM). Nonetheless, the frequent switching-activities of the PWM result in strong harmonics at multiples of the switching-frequency in both the input current and output voltage spectra. An effective way to suppress the switching-frequency harmonics is to replace the PWM with randomized modulation schemes. The randomized modulation schemes can spread the harmonic power over the frequency spectrum to reduce the peak spectral power. Nonetheless, due to the randomization, the noise floor in the input current and output voltage spectra are usually increased. This noise floor will translate to ripple noise current at the input and ripple noise voltage at the output.

This thesis pertains to the proposal, analysis, and verification of randomized modulation schemes that feature low-harmonic and low-noise for single- and multi-phase switched-mode dc-dc converters. The intended applications of the converters are portable devices where the circuit area is scarce, low level of Electromagnetic Interference is mandatory, and power source is limited.

There are three proposed randomized modulation schemes in this thesis. The first proposed scheme is a noise-shaped randomized modulation scheme for applications in single-phase switched-mode dc-dc converters with low-harmonics and low input noise current requirements. An analytical expression for the input current spectrum of the proposed scheme is derived to analyze the

harmonics and low-frequency noise therein. A novel pulse generator structure is proposed in order to realize a dc-dc converter that embodying the proposed scheme. Experimental measurements of the input current spectrum, the output voltage spectrum, the transient-response, and the operating range are carried out on the realized converter. On the basis of the measurements, the proposed scheme features very low peak spectral power in the input current spectrum (18.1 dB lower than the PWM at 3.3 V input voltage, 0.5 duty cycle, and 100 kHz average switching-frequency). The input noise current of the proposed scheme, obtained at ~73mA rms (integrated over a 200 kHz bandwidth without an input filter), and is comparable to that of the PWM.

The second proposed randomized modulation scheme is the Randomized Wrapped-Around Pulse Position Modulation scheme (RWAPPM) with adjustable limits for applications in single-phase switched-mode dc-dc converters that require relatively low load current, low harmonics, and low output ripple noise voltage. This scheme can trade-off the output ripple noise voltage and the peak spectral power by varying/adjusting the limits on the randomized pulse position of the RWAPPM. An analytical expression of the output voltage spectrum is derived to provide an analytical means of calculating the trade-offs between the peak spectral power and the output ripple noise voltage. SPICE simulations and measurements on a dc-dc converter realized with discrete electronic components are conducted to verify the derived analytical expression. The results show that when the minimum (or maximum) limit is increased (or decreased) by 1% of the switching-period, the peak spectral power increases by 1 dB, whereas the ripple noise decreases by 0.6 mV. The ripple noise is

improved by 9.1 mV and 10.7 mV respectively for the minimum limit value at 15% of the switching-period and the maximum limit value at 85% of the switching-period, as compared to that for the RWAPPM without adjustable limits.

The third proposed randomized modulation scheme is denoted as RWAPPM scheme with Wrapped-Around Phase-Shift (RWAPPM+WAPS). It is proposed for applications in multi-phase dc-dc converters that require relatively high load current, low harmonics, and low output ripple noise voltage. Unlike other randomized modulation schemes, it does not negate the ripple cancellation effect of the multi-phase converters. A general N -phase analytical expression for the input current spectrum of the RWAPPM+WAPS is derived to analyze the harmonics therein. The derived analytical expression is verified by means of SPICE simulation on two- and three-phase dc-dc converters. The results (using a three-phase converter with 3.3 V input, 0.75 duty cycle, and 100 kHz switching-frequency) show that the RWAPPM+WAPS has the attribute of low peak spectral power in the input current spectrum (-29.9 dBFS and lower by ~ 19 dB compared to that of the PWM). It also features a low output ripple noise voltage at 4.9 mV, and is comparable to that of the PWM.

TABLE OF CONTENTS

Acknowledgements	i
Abstract	iii
List of Figures.....	x
List of Tables	xv
Nomenclature and Abbreviations.....	xvi
Chapter 1 Introduction.....	1
1.1 Motivation	1
1.2 Objectives	7
1.3 Contributions	9
1.4 Organization of the Thesis	12
Chapter 2 Literature Review	15
2.1 Switched-Mode DC-DC Converter	15
2.1.1 Single-phase Switched-Mode DC-DC Converter	15
2.1.2 Multi-phase Switched-Mode DC-DC Converter	17
2.2 The Pulse Width Modulation Scheme (PWM).....	20
2.3 Randomized Modulation Schemes	23
2.3.1 Randomized Pulse Position Modulation Scheme (RPPM)	24
2.3.2 Randomized Pulse Width Modulation Scheme (RPWM)	27
2.3.3 Randomized Carrier Frequency Modulation with Fixed Duty Cycle (RCFMFD).....	30

2.3.4	Randomized Carrier Frequency Modulation with Variable Duty Cycle (RCFMVD).....	32
2.3.5	Randomized Wrapped-Around Pulse Position Modulation scheme (RWAPPM).....	34
2.3.6	Other Randomized Modulation Schemes	39
2.3.7	Summary	40
2.4	Computational Complexity	41
2.5	Conclusions	43

Chapter 3 A Noise-Shaped Randomized Modulation for Switched-Mode

	DC-DC Converters	45
3.1	Introduction	45
3.2	The Proposed Noise-Shaped Randomized Modulation Scheme	49
3.3	The Proposed Pulse Generator	53
3.4	Analysis of the Input Current Spectra	56
3.4.1	Input Current Spectrum of the RWAPPM	56
3.4.2	Input Current Spectrum of the Proposed Scheme	59
3.4.3	Input Filter Design	62
3.4.4	Output Filter Design	65
3.5	Measurement Results	67
3.5.1	Input Current Spectrum (Steady-State Operation).....	67
3.5.2	Output Voltage Spectrum	70
3.5.3	Transient Response of the Converter Operating Based on the Proposed Scheme	72

3.5.4	Operating Range	75
3.6	Conclusions	78
Chapter 4	RWAPPM with Adjustable Limits	79
4.1	Introduction	79
4.2	Output Voltage Spectrum of the RWAPPM with the Adjustable Limits	81
4.3	Output Voltage Spectra and Ripple Noise with Various $\varepsilon_{\min}$ and $\varepsilon_{\max}$..	86
4.4	Simulation and Experimental Results	91
4.5	Conclusions	93
Chapter 5	A Low-Harmonic Low-Noise Randomized Modulation Scheme for Multi-Phase DC-DC Converters	95
5.1	Introduction	95
5.2	The RWAPPM with Wrapped-Around Phase-Shift (RWAPPM+WAPS)	98
5.3	Input Current Spectrum of the RWAPPM+WAPS	101
5.3.1	Derivation of an N-phase Converter Input Current Spectrum	101
5.3.2	Analytical Input Current Spectra	105
5.4	Simulation Results	107
5.4.1	Verification of the Analytical Input Current Spectra	107
5.4.2	Performance of the RWAPPM+WAPS	109
5.5	Conclusions	113

Chapter 6	Conclusions and Recommendations for Future Work	115
6.1	Conclusions	115
6.2	Recommendations for Future Work	121
Appendix A	Derivation of the Input Current Spectrum of the	
	RWAPPM.....	124
Appendix B	Derivation of the Input Current Spectra of the PWM, the	
	RCFMFD, and the PCFM	128
Author's Publications		132
Bibliography		133

LIST OF FIGURES

Figure 1.1: General structure of a switched-mode dc-dc converter.....	3
Figure 1.2: (a) Case 1 and (b) Case 2 of the RWAPPM.....	6
Figure 1.3: A typical multi-phase dc-dc buck converter.....	7
Figure 2.1: Three basic dc-dc converter configurations: (a) buck, (b) boost and (c) buck-boost converter	16
Figure 2.2: A typical implementation of buck converter.....	17
Figure 2.3: Switched-mode dc-dc buck converter with an N -phase output stage	18
Figure 2.4: Cancellation of the ripple currents at i_C for a two-phase PWM	20
Figure 2.5: Basic working principle of generating PWM pulses.....	21
Figure 2.6: Analytical output voltage spectrum of PWM at $H = 3.3$ V, $D = 0.5$ and $f_{sw} = 100$ kHz.....	23
Figure 2.7: Analytical output voltage spectrum of RPPM at $H = 3.3$ V, $D = 0.5$ and $f_{sw} = 100$ kHz.....	27
Figure 2.8: Analytical output voltage spectrum of RPWM at $H = 3.3$ V, $D = 0.5$ and $f_{sw} = 100$ kHz.....	29
Figure 2.9: Analytical output voltage spectrum of RCFMFD at $H = 3.3$ V, $D =$ 0.5 , $f_{sw} = 100$ kHz, and peak frequency deviation of 15%.....	31
Figure 2.10: Analytical output voltage spectrum of RCFMVD at $H = 3.3$ V, $D =$ 0.5 , $f_{sw} = 100$ kHz, and peak frequency deviation of 15%.....	34
Figure 2.11: Two cases for RWAPPM: (a) $0 \leq \varepsilon \leq (\tau - \alpha)$ and (b) $(\tau - \alpha) < \varepsilon$ $\leq \tau$	35

Figure 2.12: Analytical output voltage spectrum of RWAPPM at $H = 3.3$ V, $D = 0.5$, and $f_{sw} = 100$ kHz	38
Figure 2.13: Parameters in randomized modulation schemes	41
Figure 3.1: A typical switched-mode dc-dc buck converter	46
Figure 3.2: An example of using binary sequences to represent pulses of: (a) $PWM[n]$, (b) $RWAPPM[n]$, and (c) $E[n] = PWM[n] - RWAPPM[n]$	50
Figure 3.3: (a) Time-domain error-feedback noise-shaper to shape $E[n]$, and (b) Linear z -domain model of the noise-shaper.....	52
Figure 3.4: Amplitude response of the noise-shaper with $NTF(z) = (z^{-2} - 1.87z^{-1} + 1)$	53
Figure 3.5: Proposed pulse generator of the noise-shaped randomized modulation scheme	54
Figure 3.6: Analytical input current spectra of the RWAPPM and the PWM at $V_{in} = 3.3$ V, $V_{out} = 1.65$ V, $R_{load} = 4 \Omega$, and average $f_{sw} = 100$ kHz.....	59
Figure 3.7: Analytical input current spectra at $V_{in} = 3.3$ V, $V_{out} = 1.65$ V, $R_{load} = 4 \Omega$, and average $f_{sw} = 100$ kHz. (a) The proposed scheme and PWM, (b) RWAPPM (without the noise-shaper), (c) RCFMFD (at peak frequency deviation of 15%), and (d) PCFM (with a sawtooth signal at peak frequency deviation of 15% & modulation index of 16).....	61
Figure 3.8: Input filter design	63
Figure 3.9: Output impedance of the filter (Z_{filter}) and input impedance of the dc-dc buck converter (Z_{in}).....	64

Figure 3.10: Control-to-output magnitude and phase transfer functions of the converter with and without the input filter	64
Figure 3.11: The schematic of dc-dc buck converter used in the measurements	68
Figure 3.12: (a) Measured output pulses at the node V_x and input current time-domain waveforms. (b) Measured and analytical input current spectra of the proposed scheme at $V_{in} = 3.3$ V, $V_{out} = 1.65$ V, $R_{load} = 4$ Ω , and average $f_{sw} = 100$ kHz	69
Figure 3.13: Measured output voltage spectra of (a) the proposed scheme and (b) the RWAPPM at $V_{in} = 3.3$ V, $V_{out} = 1.65$ V, $R_{load} = 4$ Ω , and average $f_{sw} = 100$ kHz	71
Figure 3.14: Load regulation of the converter with the proposed scheme and the converter with the PWM scheme for (a) a load current step from 130 mA to 900 mA, and (b) a load current step from 900 mA to 130 mA	73
Figure 3.15: Line regulation of the converter with the proposed scheme and the converter with the PWM scheme for (a) a supply voltage step from 2.4 V to 5.5 V, and (b) a supply voltage step from 5.5 V to 2.4 V	74
Figure 3.16: (a) Analytical and measured input noise current vs. duty cycle; (b) Analytical and measured normalized noise current vs. duty cycle ..	76
Figure 3.17: Analytical and measured peak spectral power vs. duty cycle	77
Figure 4.1: Output voltage waveforms with the adjustable limits in (a) Case 1, and (b) Case 2	82

Figure 4.2: Analytical output voltage spectrum of the RWAPPM at (a) $\varepsilon_k \in [\varepsilon_{\min}, \tau]$, where $\varepsilon_{\min}$ is swept from 0 to 15% τ , and (b) $\varepsilon_k \in [0, \varepsilon_{\max}]$, where $\varepsilon_{\max}$ is swept from 85% τ to τ	89
Figure 4.3: Analytical output ripple noise when (a) $\varepsilon_{\min}$ is swept from 0 to 15% τ and (b) $\varepsilon_{\max}$ is swept from 85% τ to τ	90
Figure 4.4: The schematic of the dc-dc buck converter used in simulations and experimental measurements.....	91
Figure 4.5: Simulated and measured output voltage spectra of the RWAPPM at (a) $\varepsilon_k \in [0, \tau]$, (b) $\varepsilon_k \in [15\% \tau, \tau]$ and (c) $\varepsilon_k \in [0, 85\% \tau]$	93
Figure 5.1: Switched-mode dc-dc buck converter with an N -phase output stage	98
Figure 5.2: Negation of the ripple current cancellation for a two-phase RWAPPM	100
Figure 5.3: Cancellation of the ripple currents at i_C for a two-phase RWAPPM+WAPS.....	101
Figure 5.4: The possible two cases in each switching-period.....	102
Figure 5.5: Analytical input current spectra of the (a) two-phase and (b) three-phase RWAPPM+WAPS at $V_{in} = 3.3$ V, $V_{out} = 2.5$ V, $R_{load} = 4$ Ω , and average $f_{sw} = 100$ kHz	106
Figure 5.6: Analytical and simulated input current spectra of the (a) two-phase and (b) three-phase RWAPPM+WAPS at $V_{in} = 3.3$ V, $V_{out} = 2.5$ V, $R_{load} = 4$ Ω , and average $f_{sw} = 100$ kHz.....	108
Figure 5.7: Input current spectra of (a) multi-phase PWM, and (b) multi-phase RWAPPM+WAPS in three-phase converters	110

Figure 5.8: i_C waveforms of single-, two-, and three-phase converters using PWM, multi-phase RWAPPM, and multi-phase RWAPPM+WAPS.....	111
Figure A.1: Input current (i_{in}) waveforms with the corresponding V_x waveforms for (a) Case 1, and (b) Case 2 of the RWAPPM.....	125

LIST OF TABLES

Table 2.1: Characteristics of various randomized modulation schemes.....	41
Table 2.2: Summary of computational complexity of various randomized modulation scheme.....	43
Table 3.1: Analytical input current peak spectral power and noise current of different modulation schemes	62
Table 3.2: Performance of the converter operating based on the proposed scheme	74
Table 4.1: Analytical peak spectral power and ripple noise in the output voltage and the input current of the RWAPPM at different limit values for $V_{in} = 3.3 \text{ V}$, $V_{out} = 2.5 \text{ V}$, and average $f_{sw} = 100 \text{ kHz}$	88
Table 4.2: Measured ripple noise in the output voltage of the RWAPPM at different limit values	92
Table 5.1: Peak spectral power at the input current for single-, two-, and three- phase converters using different modulation schemes.....	112
Table 5.2: Output ripple noise voltages and i_C for single-, two-, and three-phase converters using different modulation schemes.....	113

NOMENCLATURE AND ABBREVIATIONS

α	constant pulse width
A_k	pulse width of the k -th switching period
ADC	analog-to-digital converter
χ_{nk}	random variable for the occurrence of Case 2 in Phase- n , k -th switching-period of the RWAPPM+WAPS
C_{out}	output capacitor
CFM	Chaotic Frequency Modulation scheme
Δ	random delay in Case 2 of the RWAPPM
D	duty cycle
D_0	desired duty ratio
DSM	Delta-Sigma Modulation scheme
d_k	duty cycle ratio of the k -th switching period ($d_k = A_k / T_k$)
dB	decibel
dBFS	decibel Full-Scale (dB with respect to the power of a full-scale sine-wave)
ε_k	delay time of the pulse in the k -th switching period
E_D	accumulated error
EMI	Electromagnetic Interference
$E[n]$	sampled binary error sequence
ϕ_{nk}	delay time from the beginning of the k -th switching-period to the starting position of a pulse in Case 1 or the second pulse in Case 2 of the RWAPPM+WAPS
FFT	Fast Fourier Transform
FPGA	Field Programmable Gate Array
f_c	carrier frequency or input sampling frequency
f_{in}	input frequency
f_{sw}	constant switching frequency
H	high-state level of output pulses
$H_{fb}(z)$	transfer function of feedback filter
I_{load}	nominal load current

I_o	nominal output current
IC	integrated circuit
i_C	output capacitor current
i_{in}	input current
i_{ph}, i_{phn}	phase current
k	k -th switching period
LFSR	Linear-Feedback Shift Register
MOSFET	Metal-Oxide Semiconductor Field-Effect Transistor
$NSRWAPPM(f)$	input current spectrum of the RWAPPM with noise-shaper
$NSRWAPPM[n]$	output sequence of noise-shaped RWAPPM pulses
$NTF(z)$	noise transfer function
P	power of discrete harmonics
P_{dc}	dc power
PCB	Printed Circuit Board
PCFM	Periodic Carrier Frequency Modulation scheme
PDF	Probability Density Function
PFM	Pulse Frequency Modulation scheme
PMF	Probability Mass Function
PWM	Pulse Width Modulation scheme
$PWM[n]$	sampled binary sequence of PWM pulses
R_{load}	load resistance
RBW	Resolution Bandwidth
RCFMFD	Randomized Carrier Frequency Modulation with Fixed Duty Cycle
RCFMVD	Randomized Carrier Frequency Modulation with Fixed Duty Cycle
RPPM	Randomized Pulse Position Modulation scheme
RPWM	Randomized Pulse Width Modulation scheme
RWAPPM	Randomized Wrapped-Around Pulse Position Modulation scheme
$RWAPPM(f)$	input current spectrum of the RWAPPM
$RWAPPM[n]$	sampled binary sequence of RWAPPM pulses
rms	root-mean-square
$S(f)$	output voltage spectrum

SPICE	Simulation Program with Integrated Circuit Emphasis
τ	constant switching period
T	nominal switching period
T_k	period of the k -th switching period
V_G	gate voltage
V_{in}	input voltage
V_{out}	output voltage
V_x	switching node voltage of a converter output stage
$W(f)$	power spectral density of a continuous noise-spectrum
WAPS	Wrapped-Around Phase-Shift
ψ	random delay in Case 1 of the RWAPPM

CHAPTER 1

INTRODUCTION

1.1 Motivation

Portable electronic devices, such as cell-phones and tablets, employ a multitude of circuits and systems that each has its own voltage level requirements different from the battery level. Moreover, the voltage of the battery usually declines as its energy is consumed by various circuits and systems. Therefore, dc-dc converters are extensively employed in these portable devices to convert the battery voltage level to the required voltage levels, and to perform regulations including compensating the slowly declined battery voltage.

Linear dc-dc converters or linear regulators usually use resistive voltage drop to create the desired output voltage. They are usually low cost and consist of few external components. They have the advantages of low output ripple and noise, and fast transient response [1]. Therefore, they are a great choice for low power devices, radio frequency and precise analog applications. Nonetheless, the power dissipation of a linear regulator largely depends on the difference between its input and output voltages. Therefore, the linear regulators have the disadvantage of a low power efficiency, especially when the input voltage is much larger than the output voltage [1]. Apart from that, the linear regulators can only perform voltage down conversions.

On the other hand, switched-mode dc-dc converters are converters with electronic switches, such as power MOSFETs, to convert the input dc voltage to the desired output voltage level. They are highly efficient regardless of the difference between its input and output voltages [2], and the output voltage can be either larger or smaller than the input voltage. Considering its advantages over the linear regulators and the requirements for high-performance and high-integration in portable devices, the switched-mode dc-dc converters have been employed prevalently in the power management of these devices.

Figure 1.1 depicts the general structure of a switched-mode dc-dc converter [3]. It consists of a modulator, an output stage, and a low-pass filter. The output stage comprises power MOSFETs and gate drivers for the switches. The switching activities of the power switches are controlled by the modulator. Through the power switches, the modulator modulates the input dc voltage into pulses that are subsequently demodulated by the low-pass filter into the required dc output voltage. The output voltage is usually sampled and fed back to the modulator so that it can be regulated and maintained near to the desired voltage level.

Amongst the three main parts of a switched-mode dc-dc converter, the modulator plays an important role in obtaining the desired output voltage level. The pulse density or widths of the output pulses from the modulator are modulated according to the modulation scheme employed in the modulator. The commonly

employed modulation scheme is the Pulse Width Modulation scheme (PWM).

The PWM features a simple algorithm and is easy to be realized in hardware.

Figure 1.1: General structure of a switched-mode dc-dc converter

By employing the PWM, the power switches usually switch on and off at a high constant switching-frequency. These frequent switching-activities result in strong harmonics at multiples of the switching-frequency [4]–[7] and the harmonic power can be comparable to the desired dc power. The high harmonic power at the input current of the switched-mode dc-dc converter conducts through the common power line and causes interference to other circuits and function units, thereby translating to conducted Electromagnetic Interference (EMI) [8]–[10]. On the other hand, the harmonics caused by the switching activities at the output of the switched-mode dc-dc converter radiates through the output traces and wires, thereby causing radiated EMI and interfering circuits that are near to the switched-mode dc-dc converter [11].

According to reported works, there are a few approaches that can be adopted to resolve the EMI caused by the switching-frequency harmonics. A simple and straightforward method is to employ passive EMI filters to filter out the conducted EMI [12]–[15], but this increases the cost and weight of the converter [16], [17]. As weight and size of the portable devices are an important consideration, this method becomes somewhat undesirable. Some other reported methods slow down the transitions of the power switches by using soft-switching techniques [18]–[24] so that the high-frequency harmonics are reduced. However, the circuit layout largely affects the effectiveness of this approach [20], [23]. Other works propose to employ a dedicated spread-spectrum clock generator [25]–[30], but the circuit complexity and the power consumption are largely increased as a trade-off [31].

An effective and prevalent method to mitigate the harmonics is to employ randomized modulation schemes instead of the conventional PWM [32]–[40]. The randomized modulation schemes can spread the harmonic power over the whole frequency spectrum so that the strong power at multiple of the switching-frequency is largely reduced [41], [42]. Some randomized modulation schemes achieve the spread-spectrum effect by dithering/randomizing one of the modulation parameters, such as the switching-frequency/switching-period [43]–[48], the pulse position [44], [49]–[52], the duty cycle/pulse width [44], [53]–[56], or a combination of these parameters [44], [54], [57]. Some other randomized modulation schemes spread the harmonic power by modulating the switching-frequency with periodic [48], [58]–[63] or chaotic [46], [64]–[69] signals.

Compared to the other methods, the randomized modulation schemes are easier to realize. Nonetheless, some reported works have shown that most of the randomized modulation schemes are ineffective in reducing the peak spectral power of the harmonics [8], [70]. Furthermore, due to the dithering/randomization, the noise floor is generally increased as compared to that of the PWM [70]–[72]. The increased noise floor will translate to higher ripple noise that can affect sensitive circuits. Therefore, larger and more expensive filters are required to filter the noise current and noise voltage [71]. To resolve the incurred cost on the filters and at the same time, maintain low cost and complexity on the modulator, it is imperative to have randomized modulation schemes that can address the switching-frequency harmonics and the ripple noise in the input current and output voltage.

Among all the randomized modulation schemes, the reported Randomized Wrapped-Around Pulse Position Modulation scheme (RWAPPM) [70] outperforms others because of its salient features of simple algorithm and strong attenuation of the harmonic power in its output voltage. Figure 1.2 depicts the pulses that are generated by employing the RWAPPM. The RWAPPM achieves the attenuation by means of adding a random delay before the pulse in each switching-period. The minimum and maximum limits of the random delay are fixed at 0 to 100% of the switching-period respectively. When the random delay is small, the pulse ends within the current switching-period, as depicted by Case 1 in Figure 1.2(a). When the random delay is large, part of the pulse runs out of

the current switching-period and it is shifted to the beginning of the switching-period. This is depicted by Case 2 in Figure 1.3(b).

Figure 1.2: (a) Case 1 and (b) Case 2 of the RWAPPM

Although strong attenuation on the output voltage switching-harmonics attenuation is achieved, the effects of the RWAPPM on the input current remain unknown. Considering the desirable feature of high harmonic-attenuation, it is worthwhile to investigate the performance of the RWAPPM on the input current in the process of designing randomized modulation schemes for switched-mode dc-dc converters in this thesis. Further, the RWAPPM was proposed without optimizing the output ripple noise voltage [70]. It would be desirable to devise a method that can reduce the switching harmonics, and at the same time, can optimize the trade-off between the peak spectral power and the ripple noise.

To improve the output ripple noise voltage, the multi-phase dc-dc converter configuration can be employed [73]–[79]. In this configuration, a parallel connection of identical output stages and inductors are switched at the same frequency but with staggered phase shifts [80]–[82]. A typical multi-phase dc-dc converter is depicted in Figure 1.3. This configuration allows the output

ripple to be cancelled between the different phases so that smaller output ripple is achieved [83]. Nonetheless, the multi-phase configuration is inherently non-compatible with the randomized modulation schemes. This is because the dithering/randomization process will negate the ripple cancellation effect in multi-phase converters [84]. Hence, it is desirable that there is a randomized modulation scheme that can be employed for multi-phase switched-mode dc-dc converters without the inherent drawback of the loss of the ripple cancellation effect.

Figure 1.3: A typical multi-phase dc-dc buck converter

1.2 Objectives

The overall objectives of this thesis pertain to the design, analysis, and verification of randomized modulation schemes for low-harmonics low-noise single- and multi-phase switched-mode dc-dc converters. The converters are intended for applications in portable devices where the circuit area is scarce, low level of EMI is mandatory, and power source is limited.

For the application to single-phase switched-mode dc-dc converters, the specific objectives are:

- (a) To design randomized modulation schemes that incorporate the RWAPPM to mitigate the harmonics, and concurrently, reduce the ripple noise in the input current and the output voltage;
- (b) To derive analytical expressions for the input current spectrum or the output voltage spectrum of the randomized modulation schemes designed in (a), thereby depicting the non-linearities and parameters that affect the switching-harmonics and the ripple noise;
- (c) To realize switched-mode dc-dc converters embodying the scheme proposed in (a), verify the derived expression in (b) by means of experimental measurements on the realized converters, and benchmark the performance of the proposed schemes in (a) against other modulation schemes.

For the application to multi-phase switched-mode dc-dc converters, the specific objectives are:

- (a) To design a randomized modulation scheme that can be employed in multi-phase switched-mode dc-dc converters without negating the output ripple cancellation effect;
- (b) To derive an analytical expression for the input current spectrum of the modulation scheme proposed in (a), thereby depicting the mechanisms of mitigating the harmonics in multi-phase dc-dc converters;

- (c) To verify the expressions derived in (b) by SPICE simulations, benchmark the performance of the modulation scheme proposed in (a) against the performance of the multi-phase PWM.

1.3 Contributions

This section summaries the contributions of the work described in this thesis. The contributions pertaining to the objectives (a) – (c) for the application to single-phase switched-mode dc-dc converters are as follows:

- (i) A noise-shaped randomized modulation scheme is proposed. The proposed scheme is a hybrid of the RWAPPM and a 2nd-order noise-shaper. The RWAPPM mitigates the switching-frequency harmonics and the noise-shaper mitigates the low-frequency noise current therein;
- (ii) The analytical expression for the input current spectrum of the noise-shaped randomized modulation scheme in (i) is derived, and the input current spectrum is benchmarked against other randomized modulation schemes. The peak spectral power in the input current spectrum of the proposed scheme is 18.1 dB lower than the PWM under the same condition (3.3 V input voltage, 1.65 V output voltage, 4 Ω load and 100 kHz average switching-frequency). Other randomized modulation schemes, in general, have undesirably higher input noise current than that of the PWM. The input noise current of the proposed scheme, obtained at ~73mA rms (integrated over a 200 kHz bandwidth without an input filter), is

comparable to that of the PWM, and is lower by ~18% (~16 mA rms) compared to that of the RWAPPM without the noise-shaper;

- (iii) A novel pulse generator structure that embodies the noise-shaped randomized modulation scheme in (i) is proposed. A closed-loop dc-dc converter employing the pulse generator is constructed, and experimental measurements are done on the converter to verify the derived expression and the characteristics of the proposed scheme. The output voltage spectrum, the transient-response, and the operating range of the converter are also measured to verify the closed-loop performance. The measurement results have shown that the PWM can be readily replaced by the proposed hybrid scheme.
- (iv) The RWAPPM with adjustable limits on the range of the random delay is proposed. The proposed randomized modulation scheme is able to vary the minimum and maximum limit values of the random delay so as to trade-off the peak spectral power with the output ripple noise voltage;
- (v) An analytical expression for the output voltage spectrum of the RWAPPM with adjustable limits in (iv) is derived. This expression provides an analytical means of calculating the trade-offs between the peak spectral power and the output ripple noise voltage;
- (vi) By means of the derived expression in (v), the output voltage spectra with different limit values on the random delay are obtained, and the trade-off

between the peak spectral power and the output ripple noise voltage can be analyzed. The trade-off analysis shows that when the minimum (or maximum) limit is increased (or decreased) by 1% of the switching-period, the peak spectral power increases by 1 dB, whereas the ripple noise decreases by 0.6 mV. This trade-off and the analytical output voltage spectra are verified by means of SPICE simulations and experimental measurements at 3.3 V input voltage, 0.75 duty cycle, and 100 kHz switching-frequency. The minimum limit of the random delay is varied from 0 to 15% of the switching-period, and the maximum limit of the random delay is varied from 85% to 100% of the switching-period. The results show that when the extreme limit values are used for the RWAPPM, the ripple noise is the highest at 56.8 mV. When either the minimum limit value increases or the maximum limit value decreases, the ripple noise decreases. Compared to the RWAPPM without adjustable limits, the ripple noise is improved by 16% (9.1 mV) for the minimum limit value at 15% of the switching-period, and by ~19% (10.7 mV) for the maximum limit value at 85% of the switching-period.

The contributions pertaining to the objectives (a) – (c) for the application to multi-phase switched-mode dc-dc converters are as follows:

- (i) A scheme denoted as RWAPPM with Wrapped-Around Phase-Shift (WAPS) method, or RWAPPM+WAPS is proposed. The WAPS is a novel method to render the RWAPPM compatible with the multi-phase configuration of switched-mode dc-dc converters. The RWAPPM+WAPS

successfully restores the ripple cancellation effect of the multi-phase configuration;

- (ii) An analytical expression for the input current spectrum of the multi-phase RWAPPM+WAPS is derived. Analytical simulations based on the derived expression are performed using two-phase and three-phase converters;
- (iii) The RWAPPM+WAPS is simulated using SPICE for two- and three-phase converters. The performance of the RWAPPM+WAPS is benchmarked against the multi-phase PWM and the multi-phase RWAPPM. Simulation results (using a three-phase converter with 3.3 V input, 100 kHz switching-frequency, and 0.75 duty cycle) show that the RWAPPM+WAPS features a low output ripple noise voltage at 4.9 mV, and is comparable to that of the PWM (4.4 mV). The RWAPPM+WAPS also features very low peak spectral power of the input current harmonics at -29.9 dB Full-Scale (dBFS) and this is lower by ~ 19 dB compared to that of the PWM when simulated at the same conditions.

1.4 Organization of the Thesis

The remainder of this thesis is organized in the following manner.

Chapter 2 provides background and a literature review of switched-mode dc-dc converters and their modulation schemes. It commences with a brief

introduction on different configurations of single-phase switched-mode dc-dc converters. This is followed by a review of multi-phase converters, including the ripple cancellation effect and its features and advantages over single-phase converters. Finally, Chapter 2 reviews well-known modulation schemes for switched-mode dc-dc converters, and provides the analytical output voltage spectra for some of the reviewed modulation schemes.

Chapter 3 presents a proposed noise-shaped randomized modulation scheme for switched-mode dc-dc converters and a proposed pulse generator structure that embodies the noise-shaped randomized modulation scheme. Chapter 3 subsequently presents the derivation of an analytical expression for the input current spectrum of the proposed scheme. Thereafter, it presents the benchmarking results of the proposed scheme against the conventional PWM, the RWAPPM without the noise-shaper, and some other well-known randomized modulation schemes. Finally, a dc-dc converter employing the proposed pulse generator is realized, and measurement results of the output voltage spectrum, the transient-response, and the operating range of the converter are presented to verify the derived expression and the characteristics of the proposed scheme.

Chapter 4 describes the proposed RWAPPM with adjustable limits. It subsequently presents the derivation for the output voltage spectrum with the adjustable limits. Thereafter, it presents the analytical output voltage spectra and ripple noise when the range of the random delay is varied. Subsequently, the simulation and experimental measurement results are presented to verify the derived expression.

Chapter 5 describes the proposed RWAPPM+WAPS for multi-phase converters. Subsequently, it presents the derivation for the input current spectrum of a general N -phase dc-dc converter. Thereafter, it provides the simulation results to verify the analytical input current spectrum and compare the performance of the RWAPPM+WAPS against the multi-phase PWM and the multi-phase RWAPPM.

Chapter 6 concludes this thesis by summarizing the major contributions described in the previous chapters, and offers suggestions for future work.

CHAPTER 2

LITERATURE REVIEW

This chapter provides a review on switched-mode dc-dc converter and some well-known randomized modulation schemes. The review of the modulation schemes commences with the non-randomized PWM scheme since many randomized modulation schemes are based on the PWM scheme.

2.1 Switched-Mode DC-DC Converter

2.1.1 Single-phase Switched-Mode DC-DC Converter

Switched-mode dc-dc converters are routinely employed in power supplies of various portable electronic systems to convert a supply voltage from one voltage level to another. The switched-mode dc-dc converter employs a modulator, either digital or analog, to modulate the output pulses. The modulator often modulates the pulses based on certain modulation schemes, such as non-randomized PWM scheme, which is the most commonly employed modulation scheme.

There are three basic configurations of switched-mode dc-dc converters generally [18]. The three configurations are depicted in Figure 2.1. The first configuration in Figure 2.1(a) is the buck converter, whose output voltage is lower than its input

voltage ($V_{out}/V_{in} = D$ where D is the duty cycle). Figure 2.1(b) shows the boost converter that steps the voltage up with $V_{out}/V_{in} = 1/(1-D)$. Figure 2.1(c) depicts the buck-boost converter that can generate an inverted polarity output voltage of any magnitude with $V_{out}/V_{in} = -D/(1-D)$. In the aforementioned three configurations, the desired output voltage can be achieved by adjusting the value of duty cycle D . The final dc output voltage can be obtained by passing the output pulses through a low-pass filter.

Figure 2.1: Three basic dc-dc converter configurations:

(a) buck, (b) boost and (c) buck-boost converter

The buck converter is the most commonly used configuration when used to verify various modulation schemes for its simple relation between output voltage and input voltage. Figure 2.2 depicts a typical implementation of the buck converter. The transistors T_1 and T_2 function switches and they are controlled by the pulses generated from the modulator. This configuration will be used throughout the rest of the thesis. The switched-mode dc-dc converters can be either in closed-loop configuration or open-loop configuration [85], while in Chapters 4 and 5 of this thesis, the converter is employed in open-loop configuration. This is because it is easy to benchmark the effectiveness of various modulation schemes against the non-randomized PWM scheme.

Figure 2.2: A typical implementation of buck converter

2.1.2 Multi-phase Switched-Mode DC-DC Converter

Multi-phase (interleaving) configuration in switched-mode dc-dc converters is a parallel connection of identical power stages that are switched at the same frequency but with staggered phase shifts [80]. Figure 2.3 depicts a dc-dc buck converter with an N -phase power stage, where each phase comprises a pair of

power transistors and an inductor. The modulator generates the gate voltage signals, V_G 's, for the power transistors in the different phases. The V_G 's are staggered sequentially by a constant and equal phase-shift. The multi-phase configuration allows for a high current carrying capability, and is prevalent configuration for the power stage of dc-dc converters in applications that demand high load currents, such as microprocessors and communications systems. Additional salient features of the multi-phase configuration are as follows [86]–[89]:

Figure 2.3: Switched-mode dc-dc buck converter with an N -phase output stage

- The interleaving of the power stages achieves a cancellation effect that results in a smaller combined ripple current i_C at the output capacitor, C_{out} ; Figure 2.4 illustrates this using a two-phase converter operating based on the PWM. The V_{GP} 's are the gate voltages to the M_P 's power transistors in Figure 2.3; V_{GN} 's are not shown, but they are similar to V_{GP} 's. The rising part of the ripple current in a Phase-1 overlaps with the falling part

of the ripple current in Phase-2 such that there is a ripple cancellation (subtractive) effect at i_C . The starting position of each PWM pulse is always at the beginning of its switching cycle—no random delay, and this ensures that the multi-phase PWM always achieves the ripple cancellation (ideally). However, this is not the case for the randomized modulation schemes, as will be discussed later in Chapter 5. The smaller ripple at i_C (compared to i_{ph} 's) allows a smaller C_{out} , and leads to a smaller output ripple voltage at V_{out} ;

- The transient response at V_{out} is improved due to the smaller ripple current as compared to that in single-phase converters; and
- The current i_{in} in Figure 2.3 is shared among the different phases (as i_{ph1} , ..., i_{phN}) to reduce the current stress on the power transistors.

Figure 2.4: Cancellation of the ripple currents at i_C for a two-phase PWM

2.2 The Pulse Width Modulation Scheme (PWM)

The PWM scheme is a type of modulation scheme that used to encode the input signal into pulsing signals. The basic working principle of the PWM scheme is illustrated in Figure 2.5. The amplitude of input signal is compared against the amplitude of a carrier signal with frequency f_c , where f_c is much higher than the input signal frequency f_{in} . When the input signal is digital, f_c is the input sampling frequency. In the case shown in Figure 2.5, f_c is also the PWM pulse

switching frequency. When the amplitude of the input signal is higher than that of the carrier signal, the resultant pulse will be at high state ('1'), and the pulse will be at low state ('0') otherwise. The resultant normalized value of the pulse width, or duty cycle in the case of switched-mode dc-dc converters, is denoted as D . After the output stage, the PWM pulses are filtered by a low-pass filter to recover the encoded information.

Figure 2.5: Basic working principle of generating PWM pulses

The spectrum analyses of the modulation schemes are usually done for the switching-point output voltage (V_x in Figure 2.2) by means of a Fast Fourier Transform (FFT) expression that takes f_c , f_{in} and all relevant parameters into consideration. The analyses become much simpler in the cases of switched-mode dc-dc converters since the input signal frequency f_{in} is 0. In such case, the output spectrum of the non-randomized PWM scheme is the FFT of a square

waveform with switching frequency f_{sw} and duty cycle D . Specifically, in expressions, the output spectrum can be found as [70]:

$$S_{\text{PWM}}(f) = P_{\text{PWM}} + P_{\text{dcPWM}}, \quad (2.1)$$

where P_{PWM} is the power of the discrete harmonics and is given by

$$P_{\text{PWM}} = \frac{H^2}{\tau^2} \left(\alpha \text{sinc}(\pi f D \tau) \right)^2 \sum_{\substack{q=-\infty \\ q \neq 0}}^{q=\infty} \delta\left(f - \frac{q}{\tau}\right), \quad (2.1a)$$

P_{dcPWM} is the dc power and is given by

$$P_{\text{dcPWM}} = D^2 H^2, \quad (2.1b)$$

α is the constant pulse width, H is high-state level of output pulses, and τ is the constant switching period ($1/f_{sw}$). By means of Equation (2.1), the analytical plot of the PWM output voltage spectrum can be obtained and it is depicted in Figure 2.6 [70]. It can be observed that strong discrete harmonics appear in the output voltage spectrum (obtained after the output stage and before the output filter) and their power is comparable to that of the desired dc voltage.

Figure 2.6: Analytical output voltage spectrum of PWM

at $H = 3.3$ V, $D = 0.5$ and $f_{sw} = 100$ kHz

2.3 Randomized Modulation Schemes

As indicated by the term P_{PWM} in equation (2.1) and the plot shown in Figure 2.6, the power spectrum of non-randomized PWM scheme contains strong discrete harmonics at multiples of the switching frequency. These harmonics can often be observed both in the PWM input current and output voltage power spectra, and they may generate undesired noise for other working units. Specifically, the harmonics at the input current is the major cause of conducted EMI [9] and they may affect other circuits through power bus or the PCB traces. On the other hand, the harmonics at the output may radiate to other circuits through output traces and wires, hence generating radiated EMI [9]. These harmonics are the

source of EMI and are highly undesirable as they may degrade the quality of power supply and the performance of other sensitive circuits.

To attenuate the discrete harmonics in the spectrum of non-randomized PWM scheme, randomization was proposed in many literatures [43], [49], [53], [71], [90]–[92]. These randomized modulation schemes attenuate the power of high frequency harmonics by means of spreading it over the frequency spectrum as a continuous noise-spectrum.

In general, the randomized modulation schemes introduce a random variable with probability distribution function P to randomize the switching parameters such as the switching-frequency, the pulse width, the pulse position. A randomness level R that is in the range $[0, 1]$ can be defined for each randomized modulation scheme. The effectiveness on spreading the harmonics power is determined by the R and the P . Considering the computational complexity, the uniform distribution function is usually chosen as the P of the random variable. For a randomized modulation scheme, a larger R will generally result in lower discrete harmonics power and higher continuous noise-power. In other words, a better spreading effect can be obtained by increasing the randomness level of the randomized modulation scheme.

2.3.1 Randomized Pulse Position Modulation Scheme (RPPM)

The RPPM was proposed based on the PWM. It is different from the PWM by adding a random delay ε_k at the beginning of each switching period so that the

pulse's starting position is randomly distributed in each switching period. In this modulation scheme, the switching period is kept constant as $\tau = 1/f_{sw}$, therefore the pulse width $\alpha = D\tau$ is a constant in the case of dc-dc converter.

As the random variable ε_k is introduced to the RPPM, the randomness level can be defined as

$$R_{\text{RPPM}} = \frac{\varepsilon_b - \varepsilon_a}{\tau}, \quad (2.2)$$

where ε_a and ε_b are the minimum and maximum limits of the pulse position in each cycle.

The output voltage spectrum of the RPPM can be obtained by calculating the autocorrelation of the output voltage equation and taking its Fourier Transform [54]. A general randomized modulation scheme power spectrum equation has been derived in [91] as follows

$$S(f) = \frac{1}{\tau} \left[E \{ |V(f)|^2 \} - |E \{ V(f) \}|^2 + \frac{1}{\tau} |E \{ V(f) \}|^2 \cdot \sum_{q=-\infty}^{\infty} \delta(f - \frac{q}{\tau}) \right], \quad (2.3)$$

where $E\{.\}$ is the expectation (statistical average) operator, $V(f)$ is the Fourier transform of a cycle of the voltage time-domain waveform expression $v(t)$ with randomness level R_{RPPWM} , and is given by

$$V(f) = \frac{j \cdot H}{2\pi f} \cdot (e^{-j2\pi f D\tau} - 1) \cdot e^{-j2\pi f \varepsilon_k}. \quad (2.4)$$

The expectations in Equation (2.3) can be determined as follows

$$E \{ V(f) \} = \int_{\varepsilon_a}^{\varepsilon_b} P(\varepsilon_k) V(f) d\varepsilon_k, \quad (2.5)$$

$$E\{|V(f)|^2\} = \int_{\varepsilon_a}^{\varepsilon_b} P(\varepsilon_k) V(f) V^*(f) d\varepsilon_k, \quad (2.6)$$

where $V^*(f)$ is the complex conjugate of $V(f)$ and $P(\varepsilon_k)$ is the probability distribution function of ε_k . With ε_k uniformly distributed in $[\varepsilon_a, \varepsilon_b]$, $P(\varepsilon_k)$ can be obtained as

$$P(\varepsilon_k) = \frac{1}{\varepsilon_b - \varepsilon_a} = \frac{1}{R_{\text{RPPM}} \cdot \tau}. \quad (2.7)$$

By substituting Equations (2.4) – (2.7) into (2.3), the output voltage spectrum of the RPPM can be obtained as the following expressions [53], [54], [91]:

$$S_{\text{RPPM}}(f) = W_{\text{RPPM}}(f) + P_{\text{RPPM}} + P_{\text{dcRPPM}}, \quad (2.8)$$

where $W_{\text{RPPM}}(f)$ is the continuous noise-spectrum and is given by

$$W_{\text{RPPM}}(f) = \frac{H^2}{\tau} G_{\text{RPPM}_1} (1 - G_{\text{RPPM}_2}), \quad (2.8a)$$

P_{RPPM} represents the discrete harmonics and is given by

$$P_{\text{RPPM}} = \frac{H^2}{\tau^2} G_{\text{RPPM}_1} G_{\text{RPPM}_2} \sum_{\substack{q=-\infty \\ q \neq 0}}^{q=\infty} \delta\left(f - \frac{q}{\tau}\right), \quad (2.8b)$$

P_{dcRPPM} is the dc power and is given by

$$P_{\text{dcRPPM}} = D^2 H^2, \quad (2.8c)$$

The symbols in Equation (2.8) are given by

$$G_{\text{RPPM}_1} = (\alpha \text{sinc}(\pi f D \tau))^2, \quad (2.9)$$

$$G_{\text{RPPM}_2} = \frac{2}{(\omega \cdot R_{\text{RPPM}} \cdot \tau)^2} [1 - \cos(\omega \cdot R_{\text{RPPM}} \cdot \tau)], \quad (2.10)$$

the constraints for ε_a and ε_b are: $\{\varepsilon_a, \varepsilon_b\} \in [0, \tau - D\tau]$ and $\varepsilon_a < \varepsilon_b$.

By means of Equation (2.8), the analytical plot of the RPPM output voltage spectrum can be obtained and it is depicted in Figure 2.7 [70]. Based on Equation (2.8) and Figure 2.7, the output spectrum of the RPPM exhibits a continuous noise-spectrum and attenuated discrete harmonics.

Figure 2.7: Analytical output voltage spectrum of RPPM

at $H = 3.3$ V, $D = 0.5$ and $f_{sw} = 100$ kHz

2.3.2 Randomized Pulse Width Modulation Scheme (RPWM)

As indicated by its name, the RPWM randomizes the pulse width A_k in each switching cycle. In this modulation scheme, the switching period τ is still kept constant while the pulse width is allowed to vary in each switching cycle, but the average pulse width is equal to the required duty cycle. For an A_k varies in $[A_a, A_b]$, the randomness level of the RPWM, R_{RPWM} , is

$$R_{\text{RPWM}} = \frac{A_b - A_a}{\tau} = D_b - D_a. \quad (2.11)$$

Similar to the RPPM output voltage spectrum derivation, for A_k uniformly distributed in $[A_a, A_b]$, the output voltage spectrum of the RPWM can be derived as [53], [54], [91]:

$$S_{\text{RPWM}}(f) = W_{\text{RPWM}}(f) + P_{\text{RPWM}} + P_{\text{dcRPWM}}, \quad (2.12)$$

where

$$W_{\text{RPWM}}(f) = \left[\frac{H^2}{\tau} \left\{ \frac{2}{\omega^2} \left(1 - \frac{1}{\omega R_{\text{RPWM}} \cdot \tau} \right) (\sin(\omega A_b) - \sin(\omega A_a)) \right\} - G_{\text{RPWM}} \right]_{f \neq 0}, \quad (2.12a)$$

$$P_{\text{RPWM}} = \frac{H^2}{\tau^2} G_{\text{RPWM}} \sum_{\substack{q=-\infty \\ q \neq 0}}^{q=\infty} \delta\left(f - \frac{q}{\tau}\right), \quad (2.12b)$$

$$P_{\text{dcRPWM}} = \left(\frac{A_a + A_b}{2\tau} \right)^2 H^2, \quad (2.12c)$$

$$G_{\text{RPWM}} = \left\{ \frac{2}{\omega^4 (R_{\text{RPWM}} \cdot \tau)^2} (1 - \cos(\omega \cdot R_{\text{RPWM}} \cdot \tau)) - \omega \cdot R_{\text{RPWM}} \cdot \tau (\sin(\omega A_b) - \sin(\omega A_a)) + \frac{\omega^2 (R_{\text{RPWM}} \cdot \tau)^2}{2} \right\}, \quad (2.13)$$

the constraints for A_k is $\{A_a, A_b\} \in [(D - \sigma)\tau, (D + \sigma)\tau]$ in order to obtain the desired average duty cycle D ; the maximum value of σ is the smaller between D and $1 - D$.

By means of Equation (2.12), the analytical plot of the RPWM output voltage spectrum can be obtained and it is depicted in Figure 2.8 [70]. Similar to the spectrum of the RPPM, the output spectrum of the RPWM exhibits a continuous noise-spectrum and attenuated discrete harmonics, which can be observed from Equation (2.12) and Figure 2.8.

Figure 2.8: Analytical output voltage spectrum of RPWM

at $H = 3.3$ V, $D = 0.5$ and $f_{sw} = 100$ kHz

2.3.3 Randomized Carrier Frequency Modulation with Fixed Duty Cycle (RCFMFD)

In contrast to the previous two modulation schemes, the RCFMFD randomizes the switching period (T_k) while keeping the duty cycle d_k at a fixed value D in each switching cycle. As $A_k = DT_k$, the resultant pulse width is randomized. For T_k varies in $[T_a, T_b]$, the randomness level of the RCFMFD, R_{RCFMFD} , can be defined as

$$R_{\text{RCFMFD}} = \frac{T_b - T_a}{\tau}. \quad (2.14)$$

For T_k uniformly distributed in $[T_a, T_b]$, the expression for the output voltage spectrum of the RCFMFD is given by [54], [91]:

$$S_{\text{RCFMFD}}(f) = W_{\text{RCFMFD}}(f) + P_{\text{dcRCFMFD}}, \quad (2.15)$$

where

$$W_{\text{RCFMFD}}(f) = \frac{2H^2}{T_a + T_b} \left[G_{\text{FD1}} + 2\text{Re} \left(\frac{G_{\text{FD2}} G_{\text{FD3}}}{1 - G_{\text{FD4}}} \right) \right], \quad (2.15a)$$

$$P_{\text{dcRCFMFD}} = D^2 H^2, \quad (2.15b)$$

The symbols in Equation (2.15) are given by

$$G_{\text{FD1}} = \frac{2}{\omega^3 \cdot R_{\text{RCFMFD}} \cdot \tau} \left[\omega \cdot R_{\text{RCFMFD}} \cdot \tau - \frac{1}{D} (\sin(\omega D T_b) - \sin(\omega D T_a)) \right], \quad (2.16)$$

$$G_{\text{FD2}} = \frac{1}{\omega^2 \cdot R_{\text{RCFMFD}} \cdot \tau} \left[\frac{1}{(1-D)} (e^{j\omega(1-D)T_b} - e^{j\omega(1-D)T_a}) - (e^{j\omega T_b} - e^{j\omega T_a}) \right], \quad (2.17)$$

$$G_{\text{FD3}} = \frac{1}{\omega^2 \cdot R_{\text{RCFMFD}} \cdot \tau} \left[\frac{1}{D} \left(e^{j\omega DT_a} - e^{j\omega DT_b} \right) + j\omega \cdot R_{\text{RCFMFD}} \cdot \tau \right], \quad (2.18)$$

$$G_{\text{FD4}} = \frac{j}{\omega \cdot R_{\text{RCFMFD}} \cdot \tau} \left(e^{j\omega T_a} - e^{j\omega T_b} \right), \quad (2.19)$$

the constraint for T_a is $0 \leq T_a < T_b$.

By means of Equation (2.15), the analytical plot of the RCFMFD output voltage spectrum can be obtained and it is depicted in Figure 2.9 [70]. As compared to the spectrum of the RPPM and RPWM, the spectrum of the RCFMFD exhibits only the continuous noise-spectrum.

Figure 2.9: Analytical output voltage spectrum of RCFMFD at $H = 3.3$ V, $D = 0.5$, $f_{\text{sw}} = 100$ kHz, and peak frequency deviation of 15%

2.3.4 Randomized Carrier Frequency Modulation with Variable Duty Cycle (RCFMVD)

In the RCFMVD scheme, the pulse width is kept constant at α , whilst the switching period T_k is randomized, consequently the resultant duty cycle d_k is still randomized in each switching cycle. Nevertheless, the average duty cycle is equal to the required nominal value. For T_k varies in $[T_a, T_b]$, the randomness level of the RCFMVD, R_{RCFMVD} , can be defined as

$$R_{\text{RCFMVD}} = \frac{T_b - T_a}{\tau}. \quad (2.20)$$

For T_k uniformly distributed in $[T_a, T_b]$, the expression for the output voltage spectrum of the RCFMVD scheme is given by [54], [91]:

$$S_{\text{RCFMVD}}(f) = W_{\text{RCFMVD}}(f) + P_{\text{dcRCFMVD}}, \quad (2.21)$$

where

$$W_{\text{RCFMVD}}(f) = \frac{2H^2}{T_a + T_b} \left[G_{\text{VD1}} + 2\text{Re} \left(\frac{G_{\text{VD2}} G_{\text{VD3}}}{1 - G_{\text{VD4}}} \right) \right], \quad (2.21a)$$

$$P_{\text{dcRCFMVD}} = D^2 H^2. \quad (2.21b)$$

The symbols in Equation (2.21) are given by

$$G_{\text{VD1}} = \frac{2}{\omega^2} [1 - \cos(\omega\alpha)], \quad (2.22)$$

$$G_{\text{VD2}} = \frac{1}{\omega^2 \cdot R_{\text{RCFMVD}} \cdot \tau} (e^{j\omega T_b} - e^{j\omega T_a}) (e^{-j\omega\alpha} - 1), \quad (2.23)$$

$$G_{\text{VD3}} = \frac{j}{\omega} (1 - e^{j\omega\alpha}), \quad (2.24)$$

$$G_{\text{VD4}} = \frac{j}{\omega \cdot R_{\text{RCFMVD}} \cdot \tau} \left(e^{j\omega T_a} - e^{j\omega T_b} \right), \quad (2.25)$$

the constraint for T_a is $0 \leq T_a < T_b$.

Note that if T_a is set to α , the value of α that will yield the average duty cycle to be equal to the desired duty cycle D is

$$\alpha = \frac{DT_b}{2-D}. \quad (2.26)$$

To obtain an average switching period of τ when T_a is set to α , T_b has to be set to $2\tau - \alpha$ (so that $\frac{T_a + T_b}{2} = \tau$). Consequently, from Equation (2.26), α and hence, T_a

have to be set to $D\tau$.

By means of Equation (2.21), the analytical plot of the RCFMVD output voltage spectrum can be obtained and it is depicted in Figure 2.10 [70]. From Equation (2.15), Equation (2.21), Figure 2.9, and Figure 2.10, it can be clearly observed that the output spectrum of the randomized modulation schemes with a randomized switching period exhibit a continuous noise-spectrum without any discrete harmonics.

Figure 2.10: Analytical output voltage spectrum of RCFMVD at $H = 3.3$ V, $D = 0.5$, $f_{sw} = 100$ kHz, and peak frequency deviation of 15%

2.3.5 Randomized Wrapped-Around Pulse Position Modulation scheme (RWAPPM)

A randomized modulation scheme has been proposed in an earlier work, known as RWAPPM [70]. The RWAPPM is a bit similar to the RPPM in the way that both of them feature a constant switching frequency and they delay the starting time of the pulse by a random delay ε in each switching cycle with a period of $T = 1/f_{sw} = \tau$. However, they impose different limits on the maximum value of ε . In the RPPM, the maximum value of ε is limited to $(1 - D)\tau$. This is to prevent the pulse from overrunning the current switching cycle. As opposed to RPPM, the maximum value of ε in RWAPPM is set to a full switching period τ instead,

i.e. a full randomness level ($R_{RWAPPM} = 1$). As a result, two cases will occur in RWAPPM depending on the value of ε . The first case occurs when $0 \leq \varepsilon \leq (\tau - \alpha)$ as depicted in Figure 2.11(a). This is exactly the same as that of RPPM scheme where the pulse starting point is delayed by ε and the whole pulse still ends within the current switching cycle. The random delay ε is uniformly distributed in the interval $[0, (\tau - \alpha)]$, therefore the probability that this case occurs is $(\tau - \alpha)/\tau = 1 - D$. With a smaller value of D , the probability will be higher, which means this case will happen more frequently when the pulse widths are shorter.

Figure 2.11: Two cases for RWAPPM:

(a) $0 \leq \varepsilon \leq (\tau - \alpha)$ and (b) $(\tau - \alpha) < \varepsilon \leq \tau$

The second case is illustrated in Figure 2.11(b) and it occurs when $(\tau - \alpha) < \varepsilon \leq \tau$, where the pulse S is delayed by the random value ε . It can be observed from Figure 2.11(b) that the pulse runs out of the current switching cycle because of the large random delay value. In order to prevent the pulse from overlapping with the pulse in the next switching cycle, the overrun portion of the pulse $S2$ is cut off at the edge of current switching cycle and shifted to the beginning of this cycle to form pulse $S2^*$ and the portion $S1$ remains at the end of the switching

cycle. The duty cycle remains to be D since the pulse width is unchanged. The probability for this case to occur is $(\tau - (\tau - \alpha))/\tau = D$. As compared to the first case, a higher D value will cause this case to occur more frequently since a pulse with longer pulse width is easier to be pushed out of the current switching cycle.

The expression for the output voltage spectrum of the RWAPPM has been derived and given by [70]:

$$S_{\text{RWAPPM}}(f) = W_{\text{RWAPPM}}(f) + P_{\text{RWAPPM}}(f) + P_{\text{dcRWAPPM}}, \quad (2.27)$$

where $W_{\text{RWAPPM}}(f)$ is the continuous noise-spectrum and is given by

$$W_{\text{RWAPPM}}(f) = \left[\frac{H^2}{\tau} (G_{1A}(G_{1B} - G_{1C}) + G_{2A}(G_{2B} - G_{2C}) + G_{4A}(G_{4B} - G_{4C})) \right]_{f \neq 0}, \quad (2.27a)$$

P_{RWAPPM} is the PWR of the discrete harmonics and is given by

$$P_{\text{RWAPPM}} = \frac{H^2}{\tau^2} (G_{1A}G_{1C} + G_{2A}G_{2C} + G_{3A}G_{3C} + G_{4A}G_{4C}) \sum_{\substack{q=-\infty \\ q \neq 0}}^{q=\infty} \delta\left(f - \frac{q}{\tau}\right), \quad (2.27b)$$

P_{dcRWAPPM} is the dc power given by

$$P_{\text{dcRWAPPM}} = D^2 H^2, \quad (2.27c)$$

The symbols in Equation (2.27) are given by

$$G_{1A} = (\alpha \operatorname{sinc}(\pi f \alpha))^2, \quad (2.28)$$

$$G_{1B} = 1 - D, \quad (2.29)$$

$$G_{1C} = (D^2 - 2D + 1) \operatorname{sinc}^2(\pi f(\tau - \alpha)), \quad (2.30)$$

$$G_{2A} = \left((\alpha - \tau) \operatorname{sinc}(\pi f(\alpha - \tau)) \right)^2, \quad (2.31)$$

$$G_{2B} = G_{3C} = G_{4B} = D, \quad (2.32)$$

$$G_{2C} = \left(D \operatorname{sinc}(\pi f \alpha) \right)^2, \quad (2.33)$$

$$\begin{aligned} G_{3A} = & 2 \operatorname{Re} \left[A_V(f) A_Z(-f) E\{e^{i\omega\psi}\} \right] \\ & + 2 \operatorname{Re} \left[A_W(f) A_Z(-f) E\{e^{i\omega\psi}\} E\{e^{-i\omega\Delta}\} \right], \end{aligned} \quad (2.34)$$

$$\begin{aligned} G_{4A} = & \left((\tau \operatorname{sinc}(\pi f \tau)) \right)^2 + 2 \operatorname{Re} \left[A_V(f) A_W(-f) E\{e^{i\omega\Delta}\} \right] \\ & - 2 \operatorname{Re} \left[A_V(f) A_Z(-f) E\{e^{i\omega\psi}\} \right] \\ & - 2 \operatorname{Re} \left[A_W(f) A_Z(-f) E\{e^{i\omega\psi}\} E\{e^{-i\omega\Delta}\} \right], \end{aligned} \quad (2.35)$$

$$G_{4C} = D^2, \quad (2.36)$$

$$E\{e^{i\omega\psi}\} = \frac{1}{\omega(\tau - \alpha)} \left[\sin(\omega(\tau - \alpha)) - i(\cos(\omega(\tau - \alpha)) - 1) \right], \quad (2.37)$$

$$\begin{aligned} E\{e^{i\omega\Delta}\} = & \frac{1}{\alpha\omega} \left[(\sin(\omega\tau) - \sin(\omega(\tau - \alpha))) \right. \\ & \left. - i(\cos(\omega\tau) - \cos(\omega(\tau - \alpha))) \right], \end{aligned} \quad (2.38)$$

$$\begin{aligned} E\{e^{-i\omega\Delta}\} = & \frac{1}{\alpha\omega} \left[(\sin(\omega\tau) - \sin(\omega(\tau - \alpha))) \right. \\ & \left. + i(\cos(\omega\tau) - \cos(\omega(\tau - \alpha))) \right], \end{aligned} \quad (2.39)$$

$$A_Z(f) = \frac{1 - e^{-i\omega\alpha}}{i\omega}, \quad (2.40)$$

$$A_V(f) = \frac{1 - e^{-i\omega\tau}}{i\omega}, \text{ and} \quad (2.41)$$

$$A_W(f) = \frac{1 - e^{-i\omega(\alpha - \tau)}}{i\omega}, \quad (2.42)$$

where ψ is the random delay in Case 1, Δ is the random delay in Case 2.

The RWAPPM features a simple algorithm and a constant switching frequency. By means of Equation (2.27), the analytical plot of the RWAPPM output voltage spectrum can be obtained and it is depicted in Figure 2.12. The output voltage spectrum has shown that the output voltage has a spectrum of completely eliminated (or highly attenuated) discrete harmonics, low peak spectral power and low noise.

Figure 2.12: Analytical output voltage spectrum of RWAPPM

at $H = 3.3$ V, $D = 0.5$, and $f_{sw} = 100$ kHz

2.3.6 Other Randomized Modulation Schemes

Delta-Sigma Modulation Scheme (DSM)

Delta-sigma modulation scheme is one of the randomized modulation schemes which can effectively achieve a higher signal-to-noise ratio in a limited frequency band. It is usually employed in data converters [93], [94] to modulate input signal into 1-bit pulse signals. In view of its varying switching frequency that can spread discrete harmonics over the entire frequency spectrum, it is suitable to be adopted in dc-dc converters to suppress EMI. However, the 1st-order DSM dc-dc converter has been shown to exhibit a periodic cycle [90], [95] that appears as discrete harmonics in the frequency spectrum. Reducing the discrete harmonics requires more complex hardware and further dithering which will introduce more low-frequency noise. The 2nd-order DSM dc-dc converter exhibits similar periodic cycle problem too [90], [96]. Moreover, the required higher sampling frequency for 2nd-order DSM dc-dc converter will translate to higher power consumption [90].

Periodic Carrier Frequency Modulation Scheme (PCFM)

In the PCFM, the switching frequency is modulated with a periodic signal such as a periodic sinusoidal, triangular or exponential signal to achieve randomization. Instead of spreading the harmonics into continuous noise-spectrum, the PCFM distributes the discrete harmonics into smaller sidebands around the peaks [46], [60].

Chaotic Frequency Modulation Scheme (CFM)

In contrast to the PCFM, the CFM modulates the switching frequency with a chaotic modulating signal to achieve randomization. The CFM spreads each discrete harmonic into continuous noise-spectrum. However, normally requires a complex design of the power transistors that leads to undesired power dissipation [45], [97], [98].

2.3.7 Summary

This sub-section summaries the properties of the randomized modulation schemes reviewed in Section 2.3. The properties of the reviewed modulation schemes are tabulated in Table 2.1 below with the help of Figure 2.13. The parameters are as follows: ε_k is the delay time of the pulse in the k -th switching cycle, A_k is the pulse width in the k th switching cycle, T_k is the period of the k -th switching cycle, H is the voltage level of the logic-high pulses, and d_k is the duty cycle of the k th switching cycle.

Table 2.1: Characteristics of various randomized modulation schemes

Modulation Scheme	T_k	A_k	ε_k	$d_k = A_k/T_k$
PWM	Fixed	Fixed	Zero	Fixed
RPPM	Fixed	Fixed	Randomized	Fixed
RPWM	Fixed	Randomized	Zero	Randomized
RCFMVD	Randomized	Fixed	Zero	Randomized
RCFMFD	Randomized	Randomized	Zero	Fixed
RWAPPM	Fixed	Fixed	Randomized	Fixed
PCFM	Periodic	Periodic	Zero	Fixed
CFM	Chaotic	Chaotic	Zero	Fixed

Figure 2.13: Parameters in randomized modulation schemes

2.4 Computational Complexity

The computational complexity of the randomized modulation schemes reviewed in the previous Sections 2.3 will now be discussed. The area and the power dissipation of each scheme are commensurable with the complexity and the computation speed/frequency. The mathematical computations to determine the output pulses can be implemented using digital or analog circuits; nonetheless, they are easier to realize digitally because digital approaches offer some advantages [58], [88], [99] including lower sensitivity to process and parameter variations, easy integration with other digital systems, programmability,

reusability, and potentially lower power consumption. In this discussion, digital computations are assumed.

The RPPM, RPWM, RCFMFD, RCFMVD and RWAPPM require random numbers to randomize their modulation parameters. The random numbers are generated using a pseudorandom number generator that is usually implemented using the Linear Feedback Shift Register (LFSR) method [100] that features simple hardware comprising only M shift registers and a few XOR gates. The random number output is subsequently constrained (scaled/multiplied) to the required randomness level by each modulation scheme. This multiplication computation is not required for the RWAPPM since it requires a full randomness level [70]. On the other hand, different from the aforesaid schemes, the PCFM requires a periodic signal, such as a triangular or sawtooth signal, to modulate f_{sw} [58], and this signal can be implemented using a digital counter. The CFM requires a pseudochaotic signal generator, comprising the same M shift registers with more hardware overhead [101], and is typically more complex as compared to the LFSR pseudorandom number generator [99].

Table 2.2 provides a summary of the computational complexity of the randomized modulation schemes described in Section 2.3. The power dissipation of each randomized modulation scheme is commensurable with the switching-frequency and the computational complexity. It can be observed from Table 2.2 that the RWAPPM features the lowest computational complexity and low non-linearities.

Table 2.2: Summary of computational complexity of various randomized modulation schemes

Randomized Modulation Scheme	Computational Complexity			Random Number Type
	+ , −	x	/	
RPPM	1	1	0	Pseudo random
RPWM	2	2	0	Pseudo random
RCFMFD	2	1	0	Pseudo random
RCFMVD	4	4	0	Pseudo random
RWAPPM	2	0	0	Pseudo random
PCFM	2	3	1	Periodic
CFM	3	3	1	Pseudo chaotic

2.5 Conclusions

In this chapter, the switched-mode dc-dc converters and various randomized and non-randomized modulation schemes have been reviewed. The buck converter has been shown to be the best configuration for verifying modulation schemes because of the simple relation between the input and output voltages. The multi-phase converter has been shown to outperform the single-phase counterpart in reducing the output ripple noise voltage. The output voltage spectra of the modulation schemes have been studied and evaluated. It has been shown that strong discrete harmonics appear in the output voltage spectrum of the PWM. The randomized modulation schemes can spread the harmonic over the frequency spectrum as continuous noise. The frequency modulation schemes can even totally eliminate the discrete harmonics, whereas the continuous noise floor is largely increased. Amongst all the randomized modulation schemes, the RWAPPM outperform others by achieving low peak spectral power and relatively low continuous noise floor with simple algorithm and hardware

complexity. The analyses provide insights into the high harmonics and high ripple noise drawbacks of current-art randomized modulation schemes, and they subsequently provide impetus for the proposal of the low-harmonics low-noise randomized modulation schemes described in the thesis.

CHAPTER 3

A NOISE-SHAPED RANDOMIZED MODULATION FOR SWITCHED-MODE DC-DC CONVERTERS

3.1 Introduction

A large portion of this chapter is extracted from a paper published in the IEEE Transactions on Circuits and Systems I: Regular Papers [102] co-authored by the author of this thesis.

As discussed in Chapter 1, the switched-mode dc-dc converters are usually modulated using the conventional PWM. When a typical switched-mode dc-dc converter as depicted in Figure 3.1 operates using the PWM, the power transistors M_H and M_L switch on and off at a high switching frequency, thereby resulting in fast transitions at the input current drawn from V_{in} by the converter and at the node voltage V_x . At steady-state operation, the fast transients render strong harmonics at multiples of the switching-frequency in the input current and the output voltage spectra. The harmonics at the input current can cause conducted EMI noise [9], [10] that travels along power lines or PCB traces, and causes interference to other electronic circuits near the converter. It is imperative to suppress the conducted EMI that is generated by the dc-dc converter, particularly when noise-sensitive analog circuits are connected to the same power line.

Figure 3.1: A typical switched-mode dc-dc buck converter

It was also discussed in Chapter 1 that the prevalent approach to mitigate the conducted EMI is to suppress it at the origin by employing randomized modulation schemes instead of the PWM. These schemes are relatively easy to realize; nonetheless, most of them have been shown to be rather ineffective in reducing the peak spectral power of the harmonics [8], [70]. Further, when compared to the conventional PWM, the randomized schemes generally increase the low-frequency noise [70], [71] due to the dithering of the modulation parameters. The larger noise current, consequently, requires larger and more expensive input filters [71]. Besides the randomized schemes, the delta-sigma modulation scheme—widely employed in data converters [93], [94]—has also been used for mitigating the switching-frequency harmonics. Nonetheless, when the duty cycle is constant, this scheme exhibits a periodic cycle that appears as harmonics in the frequency spectrum [90], [95], [96]. Put simply, it is desirable that a randomized modulation scheme has the attributes of low switching-

frequency harmonics and attenuated low-frequency noise.

In this chapter, a novel hybrid scheme that combines the randomized modulation scheme with a noise-shaper (a critical building block of the delta-sigma modulation scheme) is proposed. The randomized modulation scheme mitigates the switching-frequency harmonics, while the noise-shaper shapes/attenuates the low-frequency noise in the frequency spectrum. The dithering of the modulation parameter in the randomized modulation scheme is modeled as noise, and a novel method to obtain this noise is proposed so that the noise can be subsequently shaped.

As discussed in Chapter 2, the RWAPPM is highly effective in reducing the peak spectral power of the harmonics in the output voltage [70]. Therefore, the RWAPPM is employed as the randomized scheme. Note that the analyses of the RWAPPM in [70] and Chapter 2 focus on the output voltage, whereas the focus of this chapter is on the input current. An analytical expression for the input current spectrum of the noise-shaped randomized modulation scheme is derived. This expression is useful to elucidate the harmonics and the noise that are present in the input current, and to identify their parameters.

The proposed scheme is benchmarked against the PWM, the RWAPPM, and other well-known randomized and spread-spectrum modulation schemes discussed in Chapter 2. At a duty cycle of 0.5 and an average switching frequency of 100 kHz, the peak spectral power in the input current spectrum of the proposed scheme is lower by 18.1 dB compared to the PWM. The input

noise current of the proposed scheme is lower by ~ 16 mA rms (obtained over a 200 kHz bandwidth and without an input filter) compared to the noise current of the RWAPPM without the noise-shaper, and is relatively low among other schemes.

In this chapter, a novel pulse generator structure that translates the duty cycle input into output pulses of the noise-shaped randomized modulation scheme is also proposed. A dc-dc converter employing the pulse generator is realized. The converter is measured in the steady-state operation to verify the derived expression and the characteristics of the proposed scheme. The output voltage spectrum is measured as well, and interestingly, the low-frequency noise is also shaped, resulting in ~ 9.6 mV rms lower output ripple noise voltage compared to that of the RWAPPM. The transient-response of the converter is measured and the results are benchmarked against the same converter but operating based on the PWM scheme. The benchmarking results show that the proposed scheme introduces minimal effects on the dynamic performance of the converter. Finally, the operating range, or the duty cycle range, of the converter operating based on the proposed scheme is measured.

This chapter is organized in the following manner. In Section 3.2, the proposed noise-shaped randomized modulation scheme is described. Section 3.3 describes the proposed pulse generator structure and its working principle. Section 3.4 presents the derivation of the expression for the input current spectrum of the proposed scheme. In Section 3.5, the measurement results of the converter are presented. Finally, the conclusions are drawn in Section 3.6.

3.2 The Proposed Noise-Shaped Randomized Modulation Scheme

In this section, the proposed noise-shaped randomized modulation scheme is described and its output pulse is modeled mathematically. The PWM will be used to facilitate the modeling of the output pulse. A PWM pulse with a period of τ can be represented as a sequence of N consecutive binary samples with each sample having a period of τ/N . As an example, Figure 3.2(a) depicts a binary sequence $PWM[n]$ forming a PWM pulse with a 50% duty cycle, where n is the n -th binary value in every period, and '1' and '0' are the binary samples representing the high and the low of the pulse respectively. Similarly, the output pulses of the RWAPPM can also be represented as a binary sequence $RWAPPM[n]$. The $RWAPPM[n]$ now can be regarded as $PWM[n]$ that has been dithered by a binary error sequence $E[n]$, such that some of the binary values in $PWM[n]$ flip values, as illustrated in Figure 3.2(b) for Case 1 of the RWAPPM. The $E[n]$ exists because of the random delay ε_k that randomizes the pulse position. To obtain $E[n]$, the $RWAPPM[n]$ is subtracted from $PWM[n]$, as shown in Figure 3.2(c).

Figure 3.2: An example of using binary sequences to represent pulses of:

- (a) $PWM[n]$,
- (b) $RWAPPM[n]$, and
- (c) $E[n] = PWM[n] - RWAPPM[n]$

In the delta-sigma modulation scheme used in data converters [93], a noise-shaper is employed to shape or high-pass the quantization noise so that the low-frequency noise inside the bandwidth is reduced. For the proposed noise-shaped randomized modulation scheme, the noise-shaper is employed to shape $E[n]$ that presents at the RWAPPM output. The noise-shaper is implemented using the error-feedback structure [93] depicted in Figure 3.3(a). This structure is chosen because of its simplicity in obtaining $E[n]$, and also because of its low-complexity feedback filter (H_{fb}). The output sequence is a noise-shaped $RWAPPM[n]$, and is denoted as $NSRWAPPM[n]$. The ε_k to $E[n]$ transformation, in Figure 3.3(a), is used for modeling purposes, and in practice, ε_k is directly used by the PWM to RWAPPM converter to generate $NSRWAPPM[n]$ by flipping the binary values.

The linear model of the noise-shaper in the z -domain can be obtained similar to the linear model of the noise-shaper for data converters [93]. This model is depicted in Figure 3.3(b), and is given by

$$NSRWAPPM(z) = PWM(z) - NTF(z) \cdot E(z), \quad (3.1)$$

where $NTF(z)$ is the noise transfer function, $(1+H_{fb}(z))$, that shapes $E(z)$;

$H_{fb}(z)$ is the transfer function of the feedback filter;

$PWM(z)$ is the input current spectrum of the PWM, and is given by Equation (B.3) in Appendix B;

$E(z)$ is the spectrum obtained by subtracting the spectrum of the RWAPPM (without the noise-shaper) from $PWM(z)$, i.e. $E(z) = PWM(z) - RWAPPM(z)$. The spectrum of the RWAPPM is given by Equation (3.3) which will be derived in Section 3.4.1 later;

$NSRWAPPM(z)$ is the input current spectrum of the noise-shaped randomized modulation scheme. It is given by Equation (3.16) which will be derived in Section 3.4.2 later.

Figure 3.3: (a) Time-domain error-feedback noise-shaper to shape $E[n]$, and
(b) Linear z -domain model of the noise-shaper

For the error-feedback noise-shaper, the $NTF(z)$ or $(1 + H_{fb}(z))$ is set to $(z^{-2} - 1.87z^{-1} + 1)$ [93]. This is a 2nd-order high-pass filter with optimized zeros, and its amplitude response is depicted in Figure 3.4. With a higher order $NTF(z)$, better noise-shaping can be realized but with reduced system stability and increased hardware complexity trade-offs. It will be shown later in Section 3.4.2 that $NTF(z)$ shapes the low-frequency components of the input current spectrum of the RWAPPM.

Figure 3.4: Amplitude response of the noise-shaper with $NTF(z) = (z^{-2} - 1.87z^{-1} + 1)$

3.3 The Proposed Pulse Generator

Figure 3.5 depicts the proposed pulse generator to realize the noise-shaped randomized modulation scheme described in Section 3.2. This pulse generator can be realized digitally, and converts the M -bit duty cycle data, D_0 , with a sampling period of τ into an output pulse with the same period. Each output pulse, in turn, comprises $N = 2^M$ discrete binary values with a period of τ/N .

Figure 3.5: Proposed pulse generator of the noise-shaped randomized modulation scheme

For the noise-shaper in Figure 3.3, the error $E[n]$ can be obtained directly from the difference between $PWM[n]$ and $NSRWAPPM[n]$. Nonetheless, the computation of H_{fb} in Figure 3.3 will result in multi-bit data in its output, and in turn, this will increase the number of bit in each sample of $NSRWAPPM[n]$ to more than one bit. A multi-bit $NSRWAPPM[n]$ would render the proposed scheme unfeasible for practical implementation since $NSRWAPPM[n]$ must be a pulse, and therefore, constrained to only one bit. To solve this problem, the pulse generator is constructed using two main building blocks (both are shaded in gray in Figure 3.5). The first building block performs as a noise-shaper with M -bit data, receives D_0 , and generates M -bit compensated duty cycle data, D . The second building block generates the 1-bit $PWM[n]$ from D , and thereafter, generates $NSRWAPPM[n]$ (the final output pulse).

The more detailed operation of the pulse generator is described as follows. The integrator, H_{int} , in Figure 3.5 integrates/accumulates N binary samples of $E[n]$ into E_D . The H_{int} , therefore, performs down-sampling or decimation of $E[n]$ from a period of τ/N to τ . Because of this decimation, E_D has the same period as D_0 . The output E_D of H_{int} is obtained as follows

$$E_D = \frac{1}{N} \sum_{i=1}^N E[i]. \quad (3.2)$$

Subsequently, the H_{fb} is moved after H_{int} so that instead of processing the error $E[n]$ in Figure 3.3, H_{fb} in Figure 3.5 now processes the accumulated error E_D . Both E_D and D_0 are M -bit, and after subtracting E_D from D_0 , the M -bit D that is converted into $PWM[n]$ by the PWM generator can be obtained, and

subsequently, it can be converted into the final output pulse $NSRWAPPM[n]$ in Figure 3.5. Note that H_{fb} now operates at a period of τ , but the overall amplitude response of $NTF(z)$ of the pulse generator remains the same as in Figure 3.4. Assuming the out-of-band components of H_{int} are negligible so that the aliasing error from the down-sampling process can be neglected, Equation (3.1) remains valid for this pulse generator structure. To compensate for the phase shift of N samples due to H_{int} , the input D_0 is delayed by one τ .

3.4 Analysis of the Input Current Spectra

3.4.1 Input Current Spectrum of the RWAPPM

The detailed derivation of the expression for the input current spectrum of the RWAPPM is provided in Appendix A. From (A.14), the input current spectrum of RWAPPM is obtained as

$$RWAPPM(f) = P_{RWAPPM_DC} + W_{RWAPPM}(f) + P_{RWAPPM}(f), \quad (3.3)$$

where P_{RWAPPM_DC} is the dc power and is given by

$$P_{RWAPPM_DC} = \left(\frac{\alpha I_o}{\tau} \right)^2. \quad (3.3a)$$

$W_{RWAPPM}(f)$ is a (continuous) noise-spectrum and is given by

$$W_{RWAPPM}(f) = \frac{1}{\tau} \cdot (M_{1A}M_{1B} - M_{2A}M_{2B} + M_{3A}M_{3B} - M_{4A}M_{4B} + M_{5A}M_{5B} - M_{6A}M_{6B})_{f \neq 0}, \quad (3.3b)$$

$P_{RWAPPM}(f)$ is the power of the (discrete) switching-frequency harmonics and is given by

$$P_{RWAPPM}(f) = \left(\frac{1}{\tau}\right)^2 \cdot (M_{2A}M_{2B} + M_{4A}M_{4B} + M_{6A}M_{6B}) \cdot \sum_{\substack{q=-\infty, \\ q \neq 0}}^{q=\infty} \delta(f - \frac{q}{\tau}), \quad (3.3c)$$

The expressions for the symbols in Equations (3.3a) – (3.3c) are defined as follows

$$M_{1A} = E\{\chi^2\} - 2E\{\chi\} + 1, \quad (3.4)$$

$$M_{1B} = E\{|I_A|^2\}, \quad (3.5)$$

$$M_{2A} = (E\{\chi\})^2 - 2E\{\chi\} + 1, \quad (3.6)$$

$$M_{2B} = |E\{I_A\}|^2, \quad (3.7)$$

$$M_{3A} = E\{\chi\} - E\{\chi^2\}, \quad (3.8)$$

$$M_{3B} = E\{I_A I_B^*\} + E\{I_B I_A^*\} + E\{I_A I_C^*\} + E\{I_C I_A^*\}, \quad (3.9)$$

$$M_{4A} = E\{\chi\} - (E\{\chi\})^2, \quad (3.10)$$

$$\begin{aligned} M_{4B} = & E\{I_A\} E\{I_B^*\} + E\{I_B\} E\{I_A^*\} \\ & + E\{I_A\} E\{I_C^*\} + E\{I_C\} E\{I_A^*\}, \end{aligned} \quad (3.11)$$

$$M_{5A} = E\{\chi^2\}, \quad (3.12)$$

$$M_{5B} = E\{|I_B|^2\} + E\{I_B I_C^*\} + E\{I_C I_B^*\} + E\{|I_C|^2\}, \quad (3.13)$$

$$M_{6A} = (E\{\chi\})^2, \quad (3.14)$$

$$\begin{aligned} M_{6B} = & |E\{I_B\}|^2 + E\{I_B\} E\{I_C^*\} \\ & + E\{I_B\} E\{I_C^*\} + E\{I_C\} E\{I_B^*\} + |E\{I_C\}|^2, \end{aligned} \quad (3.15)$$

where I_A , I_B , and I_C are the Fourier Transforms of i_A , i_B , and i_C shown in Figure A.1 in Appendix A respectively. The operator * denotes a conjugate operation

and $E\{\cdot\}$ is the expectation (statistical average) operation.

Figure 3.6 depicts the plot of the input current spectrum of the RWAPPM obtained analytically using the expression in (3.3). To appreciate its salient low-peak feature, the RWAPPM spectrum is superimposed with the analytical input current spectrum of the conventional PWM, whose expression is derived in Appendix B. The plot is obtained at $V_{in} = 3.3$ V, $V_{out} = 1.65$ V (duty cycle = 0.5), $R_{load} = 4$ Ω , average switching-frequency (f_{sw}) = 100 kHz, Resolution Bandwidth (RBW) = 200 Hz, and without any input filtering applied to clearly show the spectra across the frequencies. The peak spectral power is due to the $P_{RWAPPM}(f)$ term in (3.3). It is -23.86 dBFS at 100 kHz, and is lower by 24.39 dB compared to that of the PWM. On the other hand, as can be observed in Figure 3.6, the RWAPPM spectrum contains low-frequency components (the $W_{RWAPPM}(f)$ term in (3.3)) that are nonexistent in the PWM spectrum. These undesirable components translate to 89 mA rms noise current (integrated over a 200 kHz bandwidth) compared to 64.5 mA rms of the PWM. The noise-shaper of the proposed scheme will attenuate them as will be shown in Section 3.4.2.

Figure 3.6: Analytical input current spectra of the RWAPPM and the PWM

at $V_{in} = 3.3$ V, $V_{out} = 1.65$ V, $R_{load} = 4$ Ω , and average $f_{sw} = 100$ kHz

3.4.2 Input Current Spectrum of the Proposed Scheme

Having derived the input current spectrum of the RWAPPM, the expression for the input current spectrum of the overall noise-shaped randomized modulation scheme can now be obtained by substituting (3.3), (B.3), and $z = e^{j2\pi f}$ into (3.1). The input current spectrum of the proposed scheme is given by Equation (3.16) shown below

$$\begin{aligned}
 NSRWAPPM(f) = & P_{RWAPPM_DC} - (e^{-j4\pi f} - 1.87e^{-j2\pi f}) \cdot P_{PWM}(f) + \\
 & (e^{-j4\pi f} - 1.87e^{-j2\pi f} + 1) \cdot (W_{RWAPPM}(f) + P_{RWAPPM}(f)).
 \end{aligned}
 \tag{3.16}$$

The expression in (3.16) provides a means to analyze the input current spectrum

of the RWAPPM. It identifies the components that present in the input current as well as the parameters that affect them. With this expression, the peak power in the input current spectrum can be obtained, which is important in conducted EMI tests for compliance to regulations [103]. The input noise current can also be obtained from the low-frequency components of the spectrum.

Using (3.16), the analytical spectrum of the proposed scheme is plotted and shown in Figure 3.7(a). This spectrum is benchmarked with the spectra of the PWM (superimposed in Figure 3.7(a)), the RWAPPM (without the noise-shaper) in Figure 3.7(b), the RCFMFD in Figure 3.7(c), and the PCFM in Figure 3.7(d). As discussed in Chapter 2, the RCFMFD is a well-known modulation scheme that randomizes f_{sw} while keeping the desired output duty cycle constant [46]. The PCFM is a well-known spread-spectrum scheme, and is similar to the RCFMFD but instead of randomizing f_{sw} , it modulates f_{sw} with a periodic signal [58].

The spectra are obtained using the same test conditions used in Figure 3.6. The peak spectral power in the input current spectrum and the input noise current of different modulation schemes are summarized in Table 3.1. The peak spectral power in the input current spectrum of the proposed scheme is higher by 6.3 dB compared to the RWAPPM, but lower than the other schemes, as much as 18.1 dB lower compared to the PWM. The input noise current of the proposed scheme is lower (improved) by ~ 16 mA rms (17.98%) compared to the noise current of the RWAPPM without the noise-shaper, and is relatively low among other

schemes. In summary, the proposed scheme offers a low peak spectral power and a low input noise current.

Figure 3.7: Analytical input current spectra at $V_{in} = 3.3$ V, $V_{out} = 1.65$ V, $R_{load} = 4$ Ω , and average $f_{sw} = 100$ kHz. (a) The proposed scheme and PWM, (b) RWAPPM (without the noise-shaper), (c) RCFMFD (at peak frequency deviation of 15%), and (d) PCFM (with a sawtooth signal at peak frequency deviation of 15% & modulation index of 16)

Table 3.1: Analytical input current peak spectral power and noise current of different modulation schemes

Modulation Scheme	Peak Spectral Power (dBFS)	Noise Current* (mA rms)
Proposed Scheme	-17.59	73.0
PWM	0.53	64.5
RWAPPM	-23.86	89.0
RCFMFD	-5.40	60.1
PCFM	-12.10	159.8

*Integrated over a 200 kHz bandwidth for the spectra in Figure 3.7.

3.4.3 Input Filter Design

Based on the input current spectrum of the proposed hybrid scheme depicted in Figure 3.7(a), the designed input filter should have strong attenuation at ~20 kHz, since the low-frequency noise from this frequency onwards becomes high.

Figure 3.8 depicts a two-stage input filter suitable for the converter operating based on the proposed scheme. It is designed based on the impedance inequalities criteria [18] that ensure stability and minimize any effects on the control-to-output (the input of the switching power transistors to the converter output) transfer function of the converter.

Figure 3.8: Input filter design

Figure 3.9 depicts the magnitude plots of the output impedance of the input filter and the input impedance of the converter. The resonant frequency of the input filter is at 5 kHz, above the resonant frequency of the converter at 2.9 kHz, thereby satisfying the impedance inequalities. The control-to-output magnitude and phase transfer functions of the converter with and without the input filter are plotted in Figure 3.10. The filter has a cutoff frequency of ~ 8.7 kHz, 15 dB attenuation at 20 kHz, and 76 dB attenuation at the 100 kHz switching-frequency. It can be observed that the effects of the input filter on the transfer functions are negligible. With the attenuation provided by the input filter, the ripple noise at the input current is obtained at 4.1 mA rms (calculated by integrating the noise from 10 Hz to 200 kHz).

Figure 3.9: Output impedance of the filter (Z_{filter}) and input impedance of the dc-dc buck converter (Z_{in})

Figure 3.10: Control-to-output magnitude and phase transfer functions of the converter with and without the input filter

3.4.4 Output Filter Design

The output filter of a typical buck dc-dc converter consists of an inductor and a capacitor. The inductor directly influences the amount of current ripple noise seen on the inductor current. When designing the output filter, the inductance can be calculated first based on the target current ripple noise and other circuit specifications [104]. Thereafter, the minimum capacitance can be determined.

According to the method described in [104], the inductor value can be calculated using the following equation:

$$L = \frac{V_{out}}{f_{sw} \times I_{ripple}} \cdot (1 - D), \quad (3.17)$$

where I_{ripple} is the target ripple current. The ripple current is usually chosen to be 10% to 40% of maximum current [104].

The output capacitor directly affects the output voltage, the response time of the feedback loop, and the amount of voltage overshoot that occurs during load transients. The ripple current from the inductor will result in ripple voltage noise on the dc output voltage. The ripple voltage can be reduced by larger capacitors, but with trade-off of longer response time during load transients. In order to meet the ripple voltage and voltage overshoot requirements, a minimum capacitance must be considered. The minimum capacitance can be calculated by the following equation [104]:

$$C_{out} = \frac{\Delta I_L}{8 \times f_{sw} \times V_{ripple}}, \quad (3.18)$$

where V_{ripple} is the allowable output ripple voltage and ΔI_L is the total output ripple current that can be calculated using the following equation:

$$\Delta I_L = \frac{V_{out} \cdot (1-D)}{f_{sw} \cdot L}. \quad (3.19)$$

The above equations only consider the effect of output ripple voltage and inductor ripple current on the output capacitance. The capacitor must be able to meet the transient conditions as well. When a load step is initiated from full load to no load, the transient voltage overshoot must be within the limitation. By equating the capacitive energy and the inductive energy, the following equation can be derived:

$$C = \frac{L \cdot (I_H^2 - I_L^2)}{V_{out2}^2 - V_{out1}^2}, \quad (3.20)$$

where I_H is full load output current, I_L is no load output current, V_{out2} is the allowable transient voltage rise and V_{out1} is the initial output voltage. In order to satisfy both conditions, the output capacitance must be no less than both values calculated using equation (3.18) and (3.20).

Using the same conditions used in Section 3.4.2 (i.e. $V_{in} = 3.3$ V, $V_{out} = 1.65$ V, $R_{load} = 4$ Ω , and $f_{sw} = 100$ kHz), the L and C values are calculated to be 103 μ H and 29 μ F respectively, at a ripple current of $20\% \times$ the load current, and a ripple noise voltage of $10\% \times$ the output voltage. The output voltage spectrum and waveform are shown later in Sections 3.5.2 and 3.5.3 to demonstrate the performance of the output filter.

3.5 Measurement Results

3.5.1 Input Current Spectrum (Steady-State Operation)

The measurement results are presented in this section to verify the analytical expressions derived in Section 3.4. The measurements are performed using a dc-dc buck converter depicted in Figure 3.11, and are obtained using the same test conditions used in Figures 3.6 and 3.7 (i.e. $V_{in} = 3.3$ V, $V_{out} = 1.65$ V, $R_{load} = 4$ Ω , and $f_{sw} = 100$ kHz). The analog-to-digital converter (ADC) samples and converts the output voltage at the same rate as the switching-frequency, or 100 kHz. The FPGA is employed to implement the digital building blocks in Figure 3.11. The digital subtractor obtains the error data, $e[n]$, between the ADC output and the desired duty cycle. The digital compensator compensates the duty cycle data, $d[n]$, based on $e[n]$. The compensator employs the following discrete-time control law (derived using the Proportional-Integral-Derivative algorithm) [105]:

$$\begin{aligned} d[n] = & 3.54 \times 10^{-3} \cdot e[n] + 5.78 \times 10^{-3} \cdot e[n-1] \\ & - 3.46 \times 10^{-3} \cdot e[n-2] + 1.98 \cdot d[n-1] - 0.98 \cdot d[n-2]. \end{aligned} \quad (3.17)$$

The digital pulse generator embodies the proposed structure described in Section 3.3, and translates $d[n]$ into a pulse with a 6-bit width. The clock frequency required to generate this pulse width is $2^6 \times 100$ kHz = 6.4 MHz. A digital pseudorandom number generator based on the LFSR [106] method generates $N = 64$ ($M = 6$) pseudorandomized ε_k for the pulse generator. A digital dead-time generator provides a dead-time delay to avoid a short-circuit current through the power transistors (Si3585DV N- and P- channel MOSFET [107]). The time-

domain waveform of the input current is obtained differentially across a $0.01 \, \Omega$ current-sensing resistor between the supply and the power transistors.

Figure 3.11: The schematic of dc-dc buck converter used in the measurements

Figures 3.12(a) and (b) depict the measured input current in the time-domain and in the frequency domain, respectively, obtained at steady-state operation. In Figure 3.12(b), the measured spectrum plot is superimposed with the analytical spectrum plot of the proposed scheme from Figure 3.7. It can be observed that the measured spectrum generally agrees with the analytical spectrum, thereby verifying the derived expressions. The deviation between the measured spectrum and the analytical spectrum at low frequencies is due to the quantization noise from using a 6-bit resolution pulse width. This noise does not exist in the

analytical spectrum and is not shaped by the noise-shaper in the measured spectrum, and can be mitigated by using a higher pulse width resolution.

(a)

(b)

Figure 3.12: (a) Measured output pulses at the node V_x and input current time-domain waveforms. (b) Measured and analytical input current spectra of the proposed scheme at $V_{in} = 3.3$ V, $V_{out} = 1.65$ V, $R_{load} = 4 \Omega$, and average $f_{sw} = 100$ kHz

3.5.2 Output Voltage Spectrum

For completeness, the output voltage spectrum of the proposed scheme is also measured using the same dc-dc buck converter and the same conditions used in Section 3.5.1. Figure 3.13(a) depicts the output voltage spectrum of the proposed scheme at node V_x before the output filter, and Figure 3.13(b) depicts the spectrum of the RWAPPM for comparison. It can be observed from Figure 3.13(a) that the noise-shaper also has the desirable effect of shaping the low-frequency components in the output voltage spectrum. The output noise voltage of the proposed scheme (before the output filter) is obtained by integrating the spectrum over a 200 kHz bandwidth. The noise voltage is 9.2 mV rms, and is lower by ~9.6 mV rms compared to that of the RWAPPM. The peak spectral power of the proposed scheme is obtained at -16 dBFS, and is approximately equal to that of the RWAPPM.

Figure 3.13: Measured output voltage spectra of (a) the proposed scheme and

(b) the RWAPPM at $V_{in} = 3.3$ V, $V_{out} = 1.65$ V, $R_{load} = 4 \Omega$, and average $f_{sw} =$

100 kHz

3.5.3 Transient Response of the Converter Operating Based on the Proposed Scheme

This section presents the transient response of the converter operating based on the proposed scheme in Figure 3.11, and benchmarks the results against the same converter but based on the PWM scheme. Unless otherwise noted, the test conditions are the same conditions used in Section 3.5.1.

Figures 3.14 and 3.15 depict the output voltage waveforms of the converter with the proposed scheme, and the converter with the PWM scheme during load and line regulation, respectively. Figures 3.14(a) and (b) show the load regulation for a load step from 130 mA ($R_{load} = 13 \Omega$) to 900 mA ($R_{load} = 1.8 \Omega$), and vice versa. Figures 3.15(a) and (b) show the line regulation for a supply voltage step from 2.4 V to 5.5 V, and vice versa. From all the waveforms, it can be observed that both converters work properly and have nearly the same response/settling time, 210 μ s, to reach the steady-state for the load regulation in Figure 3.14, and 300 μ s for the line regulation in Figure 3.15. The undershoot/overshoot voltages for both converters are about 0.35 V for the load regulation in Figure 3.14, and 0.45 V for the line regulation in Figure 3.15. From the transient response results in Figures 3.14 and 3.15, it can be observed that the proposed scheme introduces minimal effects on the dynamic performance of the converter when benchmarked to that of the PWM scheme. On this basis, the PWM scheme in converters can be replaced with the proposed scheme to take advantage of its lower peak spectral power in the input current spectrum, particularly when the operation has reached

the steady-state. Table 3.2 summarizes the performance of the converter operating based on the proposed scheme.

Figure 3.14: Load regulation of the converter with the proposed scheme and the converter with the PWM scheme for

(a) a load current step from 130 mA to 900 mA, and

(b) a load current step from 900 mA to 130 mA

Figure 3.15: Line regulation of the converter with the proposed scheme and the converter with the PWM scheme for

(a) a supply voltage step from 2.4 V to 5.5 V, and

(b) a supply voltage step from 5.5 V to 2.4 V

Table 3.2: Performance of the converter operating based on the proposed scheme

Input voltage	2.4 ~ 5.5 V
Output voltage	0.7 ~ 3.85 V
(min, nominal, max) load	(130, 400, 900) mA
Line regulation	$\pm 8.8\%/V$
Load regulation	$\pm 0.35 V$
Output ripple noise (after output filter)	23.9 mV

3.5.4 Operating Range

Figure 3.16(a) depicts the analytical and measured input noise currents when the desired D is swept from 0 to 1, while Figure 3.16(b) depicts normalized input noise currents (input noise current / input dc current) against the desired D . It can be observed that as D is swept from 0 to 0.5, the analytical noise current as well as the analytical noise current percentage increases; when D is increase from 0.5 to 1, both the analytical noise current and the analytical noise current percentage decreases. However, at extreme high and low duty cycles, the measured noise current and its percentage deviate from the analytical ones.

Figure 3.17 depicts the results for the peak spectral power. Both Figures 3.16 and 3.17 show that the noise and the peak spectral power are dependent on D as predicted by (3.16), where D appears in all terms. From Figure 3.16, it can be observed that within the $0.3 \leq D \leq 0.8$ operating range, the measured input noise currents (and also the peak spectral power) agree well with the analytical results and remain relatively low. It is recommended to operate the converter within this range.

(a)

(b)

Figure 3.16: (a) Analytical and measured input noise current vs. duty cycle;

(b) Analytical and measured normalized noise current vs. duty cycle

Figure 3.17: Analytical and measured peak spectral power vs. duty cycle

On the other hand, outside the operating range, the measured noise is high due to the following reason. During operation, because of the feedback, the compensated D values — that the pulse generator translate into output pulses — fluctuate around the desired D value. At the extreme low and high of the desired D values (outside the recommended operating range), the compensated D values can be smaller than 0 or larger than 1, and are saturated to 0 and 1 in our digital pulse generator implementation. The ensuing error from the difference between both D values translates to higher components in the frequency spectrum, and therefore, higher measured input noise current and peak spectral power. The performance of the converter can be improved by using a higher resolution in the digital pulse generator implementation so that the error magnitude can be finer/smaller. Nonetheless, the higher resolution will come at the expense of a higher clock frequency requirement to generate the pulses.

3.6 Conclusions

A proposed noise-shaped randomized modulation scheme for switched-mode dc-dc converters has been presented in this chapter. The proposed scheme combines the RWAPPM and the noise-shaper to generate a low-harmonics low-noise input current. The proposed pulse generator structure that embodies the noise-shaped randomized modulation scheme has been presented. As an integrator and a feedback noise-shaper is added, the computational complexity of the hybrid scheme is slightly increased as compared to the RWAPPM. The analytical expression for the input current spectrum of the proposed scheme has been derived. The proposed scheme has been benchmarked against other modulation schemes. The proposed scheme features lower peak spectral power against the PWM, a comparable input noise current against the PWM, and a lower input noise current against the RWAPPM without the noise-shaper. A dc-dc converter employing the pulse generator has been realized and measured. The derived expression of the proposed scheme has been verified by measuring the converter in the steady-state. The output voltage spectrum, the transient-response, and the operating range of the converter have been measured.

CHAPTER 4

RWAPPM WITH ADJUSTABLE LIMITS

4.1 Introduction

It has been discussed in Chapter 1 and Chapter 3 that the randomized modulation schemes are a great replacement of the conventional PWM in order to mitigate the switching-frequency harmonics, thereby reducing the conducted and radiated EMI. As discussed in Chapter 2 and Chapter 3, the RWAPPM is one of the randomized modulation schemes that features low switching-harmonics in both its input current and output voltage power spectra, low hardware complexity, and simple algorithm. Nonetheless, the RWAPPM was proposed, at the outset, for mitigating the output voltage harmonics without optimizing the output ripple noise. Due to the randomization process, the RWAPPM has the drawback of higher output ripple noise voltage compared to that of the PWM as well. Since the output ripple noise is also an important parameter/specification in switched-mode dc-dc converter design, it would be desirable if there is a method to improve/reduce the output ripple noise of the RWAPPM.

In this chapter, it is proposed to improve the output ripple noise of the RWAPPM by adjusting the limits, or the range, of the possible values of the random parameter (pulse position) of the RWAPPM. An expression for the output

voltage spectrum of the modified RWAPPM is derived, and the effects of varying the limit values onto the output voltage spectrum of the RWAPPM are analyzed. It is found that the adjustable limits of the RWAPPM provides a means to trade-off the ripple noise with the peak spectral power of the harmonics. When the minimum (or maximum) limit is increased (or decreased) by 1% of the switching-period, the peak spectral power increases by 1 dB, whereas the ripple noise decreases by 0.6 mV. This trade-off can be useful to optimize the output ripple noise, at the cost of higher peak spectral power, when designing a switched-mode dc-dc converter. The calculated trade-off and the analytical output voltage spectra are verified by means of SPICE simulations and experimental measurements at 3.3 V input voltage, 0.75 duty cycle, and 100 kHz switching-frequency. The results show that when the extreme limit values are used for the RWAPPM, the ripple noise is the highest at 56.8 mV. The ripple noise is improved by 9.1 mV and 10.7 mV respectively for the minimum limit value at 15% of the switching-period and the maximum limit value at 85% of the switching-period, as compared to that for the RWAPPM without adjustable limits.

This chapter is organized in the following manner. Section 4.2 presents the derivation for the output voltage spectrum with the adjustable limits. Section 4.3 shows the analytical output voltage spectra and ripple noise with various range of the random parameter. Section 4.4 presents the verification of the derived expression by computer simulations and on the basis of experimental measurements. The conclusions of this chapter are drawn in Section 4.5.

4.2 Output Voltage Spectrum of the RWAPPM with the Adjustable Limits

In Chapter 1, it has been discussed that the randomized modulation schemes have one or more of their modulation parameters randomized. The values of the randomized modulation parameters are bounded by minimum and maximum limits. The difference between these limits is also known as the randomness level [53]. As reviewed earlier in Chapter 2, the RWAPPM utilizes ε_k to randomize the pulse position in each switching period. In [70], ε_k is limited by its extreme values as follows

$$0 \leq \varepsilon_k \leq \tau. \quad (4.1)$$

Using both extreme values, the RWAPPM scheme achieves the lowest peak spectral harmonic power in the output voltage when compared to that of the PWM and of the reported randomized modulation schemes (-26.6 dBFS peak at $V_{in} = 3.6$ V, $V_{out} = 2.5$ V and average $f_{sw} = 100$ kHz) [70]. Nonetheless, the ripple noise of the RWAPPM is higher than that of the PWM (higher by 1.1 mV at $V_{in} = 3.6$ V, $V_{out} = 2.5$ V and $f_{sw} = 100$ kHz). In this section, it is proposed to vary the ε_k limits, or the randomness level, of the RWAPPM so that the high harmonic attenuation can be traded-off to improve the output ripple noise.

In order to derive an analytical expression for the output voltage spectrum with adjustable limits, the adjustable minimum and maximum limits of ε_k can be defined as ε_{min} , and ε_{max} , respectively as depicted in Figure 4.1. The ε_{min} and

$\varepsilon_{\max}$ that ensure Case 1 and Case 2 of RWAPPM can occur are given by

$$\begin{aligned} \text{Case 1: } & 0 \leq \varepsilon_{\min} \leq \varepsilon_k \leq (\tau - \alpha), \text{ and} \\ \text{Case 2: } & (\tau - \alpha) < \varepsilon_k \leq \varepsilon_{\max} \leq \tau. \end{aligned} \quad (4.2)$$

The output voltage waveforms of the two cases are also depicted in Figure 4.1.

Figure 4.1: Output voltage waveforms with the adjustable limits in

(a) Case 1, and (b) Case 2

Assuming ε_k is stationary and uniformly distributed in $[\varepsilon_{\min}, \varepsilon_{\max}]$ [109], the probability for Case 2 to occur can be derived as

$$p = \frac{\varepsilon_{\max} - (\tau - \alpha)}{\varepsilon_{\max} - \varepsilon_{\min}}. \quad (4.3)$$

With the new p , the Probability Mass Function (PMF) [110] of χ_k is consequently different than that in (A.8) shown in Appendix A for the RWAPPM without the adjustable limits. Similarly, the Probability Distribution Functions (PDFs) [110] of ψ_k and Δ_k are different as well. The PDF of ψ_k can be determined from (5.2) as follows

$$w(\psi) = \begin{cases} \frac{1}{((\tau - \alpha) - \varepsilon_{\min})} & \varepsilon_{\min} \leq \psi_k \leq (\tau - \alpha) \\ 0 & \text{otherwise.} \end{cases} \quad (4.4)$$

Similarly, the PDF of Δ_k can be determined as

$$w(\Delta) = \begin{cases} \frac{1}{\varepsilon_{\max} - (\tau - \alpha)} & (\tau - \alpha) < \Delta_k \leq \varepsilon_{\max} \\ 0 & \text{otherwise.} \end{cases} \quad (4.5)$$

Following similar steps in [70], the time-domain output voltage pulse can be obtained as follows

$$v(t - k\tau; \chi_k, \psi_k, \Delta_k) = H \cdot \left[(1 - \chi_k) \Pi_A + \chi_k (\Pi_V - \Pi_W) \right], \quad (4.6)$$

where H is the amplitude of the voltage pulse, Π_V and Π_W are the rectangular functions depicted in Figure 4.1. The expressions for Π_V and Π_W are given as follows

$$\Pi_V = \Pi\left(\frac{t - k\tau}{\tau}\right) = \int_{-\infty}^{\infty} V_V(f) e^{-j\omega k\tau} e^{j\omega t} df, \text{ and} \quad (4.7)$$

$$\Pi_W = \Pi\left(\frac{t - k\tau - \Delta_k}{\alpha - \tau}\right) = \int_{-\infty}^{\infty} V_W(f) e^{-j\omega(k\tau + \Delta_k)} e^{j\omega t} df, \quad (4.8)$$

where $V_V(f)$ and $V_W(f)$ are the Fourier Transforms of the rectangular functions Π_V

and Π_W respectively, and are given by

$$V_V(f) = F \{u(t) - u(t - \tau)\} = \frac{1 - e^{-j\omega\tau}}{j\omega}, \text{ and} \quad (4.9)$$

$$V_W(f) = F \{u(t) - u(t - (\alpha - \tau))\} = \frac{1 - e^{-j\omega(\alpha - \tau)}}{j\omega}. \quad (4.10)$$

After applying the Fourier Transform to (4.6), the frequency-domain expression $V(f)$ can be obtained. Subsequently, the output voltage spectrum can be obtained by substituting the $V(f)$ into the following autocorrelation of the output voltage equation [91],

$$S(f) = \frac{1}{\tau} [E \{|V(f)|^2\} - |E \{V(f)\}|^2] + \frac{1}{\tau} |E \{V(f)\}|^2 \cdot \sum_{q=-\infty}^{\infty} \delta(f - \frac{q}{\tau}). \quad (4.11)$$

From (4.11), the output voltage spectrum, $S_V(f)$, is obtained as follows

$$S_V(f) = W_V(f) + P_V(f) + P_{VDC}, \quad (4.12)$$

where $W_V(f)$ is a continuous spectrum and is given by

$$W_V(f) = \frac{H^2}{\tau} [N_{1A}(N_{1B} - N_{1C}) + N_{2A}(N_{2B} - N_{2C}) + N_{4A}(N_{4B} - N_{4C})]_{f \neq 0}, \quad (4.12a)$$

$P_V(f)$ is the power of the discrete harmonics and is given by

$$P_V(f) = \frac{H^2}{\tau^2} (N_{1A}N_{1C} + N_{2A}N_{2C} + N_{3A}N_{3C} + N_{4A}N_{4C}) \sum_{\substack{q=-\infty, \\ q \neq 0}}^{q=\infty} \delta\left(f - \frac{q}{\tau}\right), \quad (4.12b)$$

P_{VDC} is the dc power and is given by

$$P_{VDC} = \left(\frac{\alpha}{\tau}\right)^2 H^2. \quad (4.12c)$$

The expressions defining the symbols in the above equations are given as follows

$$N_{1A} = |V_A(f)|^2, \quad (4.13)$$

$$N_{1B} = E\{\chi^2\} - 2E\{\chi\} + 1, \quad (4.14)$$

$$N_{1C} = (E\{\chi\})^2 \left| E\{e^{-j\omega\psi}\} \right|^2 - 2 \left| E\{e^{-j\omega\psi}\} \right|^2 E\{\chi\} + \left| E\{e^{-j\omega\psi}\} \right|^2, \quad (4.15)$$

$$N_{2A} = |V_W(f)|^2, \quad (4.16)$$

$$N_{2B} = N_{4B} = E\{\chi^2\}, \quad (4.17)$$

$$N_{2C} = (E\{\chi\})^2 \left| E\{e^{-j\omega\Delta}\} \right|^2, \quad (4.18)$$

$$N_{3A} = 2 \operatorname{Re} \left[V_V(f) V_A(-f) E\{e^{j\omega\psi}\} \right] + 2 \operatorname{Re} \left[V_W(f) V_A(-f) E\{e^{j\omega\psi}\} E\{e^{-j\omega\Delta}\} \right], \quad (4.19)$$

$$N_{3C} = E\{\chi\}, \quad (4.20)$$

$$N_{4A} = |V_V(f)|^2 + 2 \operatorname{Re} \left[V_V(f) V_W(-f) E\{e^{j\omega\Delta}\} \right] - 2 \operatorname{Re} \left[V_V(f) V_A(-f) E\{e^{j\omega\psi}\} \right] - 2 \operatorname{Re} \left[V_W(f) V_A(-f) E\{e^{j\omega\psi}\} E\{e^{-j\omega\Delta}\} \right], \quad (4.21)$$

$$N_{4C} = (E\{\chi\})^2. \quad (4.22)$$

where $\operatorname{Re}[\cdot]$ is the real part of a complex number.

By means of Equation (4.12), the effects of varying $\varepsilon_{\min}$ and $\varepsilon_{\max}$, and also the parameters H , τ (or f_{sw}), α on the output spectrum and on the output ripple noise now can be analyzed.

4.3 Output Voltage Spectra and Ripple Noise with Various $\varepsilon_{\min}$ and $\varepsilon_{\max}$

Using the expression derived in (4.12), the output voltage power spectra before the low-pass filter are plotted by varying $\varepsilon_{\min}$ and $\varepsilon_{\max}$. The following test conditions are used: H (or V_{in}) = 3.3 V, D (or V_{out}/V_{in}) = 0.75, and average f_{sw} = 100 kHz. Note that both $\varepsilon_{\min}$ and $\varepsilon_{\max}$ can be varied simultaneously; however, for simplicity, only either one of them will be varied here while the other is set at the minimum/maximum value. Figure 4.2(a) depicts a 3-D plot for various output voltage spectra when $\varepsilon_{\min}$ is swept from 0 to 15% τ while $\varepsilon_{\max}$ is kept constant at τ . Similarly, Figure 4.2(b) is the 3-D plot when $\varepsilon_{\max}$ is swept from 85% τ to τ while $\varepsilon_{\min}$ is kept at 0. Figures 4.3(a) and (b) show the plots of the output ripple noise when $\varepsilon_{\min}$ is swept from 0 to 15% τ and when $\varepsilon_{\max}$ is swept from 85% τ to τ respectively. The peak spectral power and the ripple noise for different $\varepsilon_{\min}$ and $\varepsilon_{\max}$ values are tabulated in Table 4.1. For completeness, the resulting effects on the peak spectral power and the ripple noise of the input current are tabulated in Table 4.1 as well. The peak spectral power and the ripple noise obtained under the same test conditions for the PWM and the RPPM are also tabulated in Table 4.1 for benchmarking purpose.

From Figure 4.2, Figure 4.3, and Table 4.1, it can be concluded that whenever $\varepsilon_{\min}$ is increased from 0, or when $\varepsilon_{\max}$ is decreased from τ , the discrete harmonics at multiples of the switching frequency gradually become more significant in the output voltage spectra. In other words, as the randomness level decreases, the

power of the discrete harmonics increases. It is only at the extreme limit values, i.e. $\varepsilon_{\min}$ is 0 and $\varepsilon_{\max}$ is τ , the peak spectral power of the RWAPPM is the lowest and buried under the continuous spectrum. On the other hand, it can be observed from Table 4.1 that the output ripple noise decreases along with the decrease in the limit values. Compared to the output voltage, similar results can be observed in the input current; however, the peak spectral power increases less when the limit values are reduced (in order to lower the ripple noise), and this is a desirable effect/trade-off. This fact, therefore, can be exploited by adjusting the limit values to trade-off the peak spectral power of the RWAPPM with the output ripple noise to meet the converter specifications during the design process. From Table 4.1, it can be calculated that, at the given test conditions, for every 1% decrease in the limit values, the peak spectral power of the output voltage decreases by 1 dB and the ripple noise voltage improves by 0.6 mV, whereas the peak spectral power of the input current decreases by 0.15 dB and the ripple noise current improves by 0.8 mA.

Furthermore, it can be observed from Table 4.1 that by varying the limit values of the proposed scheme, the ripple noise voltage can be reduced to 45.3 mV rms and is comparable to those of the PWM (35.5 mV rms). The ripple noise voltage of the proposed scheme is much lower as compared to that of the RPPM. Although the voltage peak spectral power of the proposed scheme increases as the limit values varies, it is still lower as compared to that of the PWM and the RPPM. In the worst case that $\varepsilon_k \in [15\% \tau, \tau]$, the voltage peaks spectral power of the proposed scheme is still lower by 8.6 dB and 7.7 dB respectively as compared to that of the PWM and the RPPM.

Table 4.1: Analytical peak spectral power and ripple noise in the output voltage and the input current of the RWAPPM at different limit values

for $V_{in} = 3.3$ V, $V_{out} = 2.5$ V, and average $f_{sw} = 100$ kHz

Modulation Schemes		Output Voltage		Input Current	
RWAPPM with Adjustable Limits	Limit Values	Peak Spectral Power (dBFS)	rms Ripple Noise (mV)	Peak Spectral Power (dBFS)	rms Ripple Noise (mA)
	$\varepsilon_k \in [0, \tau]$	-28.1	56.6	-21.1	57.5
	$\varepsilon_k \in [5\% \tau, \tau]$	-19.7	51.6	-20.8	54.2
	$\varepsilon_k \in [10\% \tau, \tau]$	-13.6	47.9	-20.6	51.3
	$\varepsilon_k \in [15\% \tau, \tau]$	-9.8	45.3	-18.8	45.3
	$\varepsilon_k \in [0, 85\% \tau]$	-16.3	49.7	-18.9	46.8
	$\varepsilon_k \in [0, 90\% \tau]$	-19.7	51.6	-20.7	54.1
	$\varepsilon_k \in [0, 95\% \tau]$	-24.4	53.8	-21.0	54.7
PWM		-1.2	35.5	-2.2	31.4
RPPM		-2.1	83.3	-3.1	75.1

Figure 4.2: Analytical output voltage spectrum of the RWAPPM at

(a) $\varepsilon_k \in [\varepsilon_{\min}, \tau]$, where $\varepsilon_{\min}$ is swept from 0 to 15% τ , and

(b) $\varepsilon_k \in [0, \varepsilon_{\max}]$, where $\varepsilon_{\max}$ is swept from 85% τ to τ

(a)

(b)

Figure 4.3: Analytical output ripple noise when
 (a) $\varepsilon_{\min}$ is swept from 0 to 15% τ and (b) $\varepsilon_{\max}$ is swept from 85% τ to τ

4.4 Simulation and Experimental Results

In this section, the simulation and experimental results are presented to verify the analytical expression derived in Section 4.2. The simulations and measurements are performed using the dc-dc buck converter depicted in Figure 4.4 under the same test conditions used in Section 4.2. Figures 4.5(a), (b) and (c) depict the plots of the simulated and measured spectra at the node V_x in Figure 4.4 for $\varepsilon_k \in [0, \tau]$, $[15\% \tau, \tau]$, and $[0, 85\% \tau]$ respectively. All spectra are obtained using an RBW of 200 Hz. Each plot is superimposed with the corresponding analytical spectrum obtained by means of (4.12). The output ripple noise voltages are calculated from the sampled voltage waveforms and tabulated in Table 4.2.

Figure 4.4: The schematic of the dc-dc buck converter used in simulations and experimental measurements

It can be observed from each figure that all of the three spectra are in close agreement with each other, thereby verifying the derived expression. The measured output ripple noise voltages in Table 4.2 show that when the extreme

limit values are used for the RWAPPM, the ripple noise is the highest at 56.8 mV. When either $\varepsilon_{\min}$ increases or $\varepsilon_{\max}$ decreases, the ripple noise decreases. The ripple noise for $\varepsilon_k \in [15\% \tau, \tau]$, and $[0, 85\% \tau]$ are improve by 16% (9.1 mV) and ~19% (10.7 mV), respectively, compared to those obtained for $\varepsilon_k \in [0, \tau]$. The measured ripple noise values in Table 4.2 are also in close agreement with the ripple noise values obtained in Table 4.1, thereby verifying further the derived expression.

Table 4.2: Measured ripple noise in the output voltage of the RWAPPM
at different limit values

Limit Values	Measured rms Ripple Noise (mV)
$\varepsilon_k \in [0, \tau]$	56.8
$\varepsilon_k \in [5\% \tau, \tau]$	52.2
$\varepsilon_k \in [10\% \tau, \tau]$	48.1
$\varepsilon_k \in [15\% \tau, \tau]$	47.7
$\varepsilon_k \in [0, 85\% \tau]$	46.1
$\varepsilon_k \in [0, 90\% \tau]$	48.5
$\varepsilon_k \in [0, 95\% \tau]$	51.4

Figure 4.5: Simulated and measured output voltage spectra of the RWAPPM at

(a) $\varepsilon_k \in [0, \tau]$, (b) $\varepsilon_k \in [15\% \tau, \tau]$ and (c) $\varepsilon_k \in [0, 85\% \tau]$

4.5 Conclusions

In this chapter, it has been proposed to adjust the limits of the randomized pulse position of the RWAPPM in order to optimize the output ripple noise. The increased computational complexity for the proposed scheme is negligible as

only two extra adders and one multiplier are needed. An expression for the output voltage spectrum of the modified RWAPPM has been derived. By means of the derived expression, it has been shown that the adjustable limits provides a means to improve the ripple noise at the cost of higher peak spectral power of the harmonics. All of the derived expressions have been verified by means of computer simulations and experimental measurements.

CHAPTER 5

A LOW-HARMONIC LOW-NOISE RANDOMIZED MODULATION SCHEME FOR MULTI-PHASE DC-DC CONVERTERS

5.1 Introduction

A large portion of this chapter is extracted from a paper published in the IEEE International New Circuits and Systems Conference [84] that is co-authored by the author of this thesis.

As discussed in Chapters 1 and 2, the PWM is routinely employed in conventional single-phase switched-mode dc-dc buck converters to modulate the gate voltages for the power switches. The constant switching-frequency of the PWM output pulses from the power stage generates undesirable high harmonics at multiples of the switching-frequency in the input current and in the output voltage. When the input of the converter that supplies the power transistors is connected to a common power line, the input current harmonics can cause conducted EMI emissions that interfere with the operation of other electronic circuitries [9]. On the other hand, the harmonics at the output voltage translate to the ripple noise at the output voltage signal.

As introduced in Chapter 2, to better reduce the ripple noise at the output voltage, the multi-phase configuration can be employed, especially for the applications where load current demand is relatively high, such as microprocessors. The interleaving of the multi-phase power stages achieves a cancellation effect that results in a smaller combined ripple current at the output capacitor. Thereby improving the transient response at the output voltage as compared to that in single-phase converters. However, the input current that is degraded by the switching-frequency harmonics remains to be an unresolved the problem in multi-phase dc-dc converters, and it is desirable to mitigate the harmonics.

Randomized modulation schemes are an effective method to mitigate the switching-frequency harmonics, and is often used in single-phase converters [91], as discussed in Chapter 1. The randomized modulation schemes, nonetheless, generally have a drawback of a higher output ripple noise voltage compared to that of the PWM. This is due to the additional noise introduced by the randomization (and due to the residual output harmonics). These schemes are yet to find applications in multi-phase converters. It would be desirable if there is a randomized modulation scheme that features low harmonics at the input current, and simultaneously, also has a low output ripple noise voltage. Such a scheme is desirable for low-harmonics low-noise multi-phase converters.

In this chapter, it is proposed to employ the RWAPPM [70], to mitigate the PWM harmonics in the input current and in the output voltage of multi-phase converters. The RWAPPM was proposed to delay the output pulses from the power transistors by a random delay value in its original single-phase converter

applications. In this chapter, the RWAPPM is adopted for multi-phase converters by means of generating multi-phase RWAPPM pulses using shift-registers. The multi-phase RWAPPM results in low harmonics at the input current and at the output voltage when employed in multi-phase converters; nonetheless, it negates the ripple current cancellation effect. To improve the ripple cancellation, a Wrapped-Around Phase-Shift (WAPS) method is proposed for the multi-phase RWAPPM. This new scheme is denoted as the RWAPPM+WAPS. A complete analysis of the input current spectrum of the RWAPPM+WAPS is presented in this chapter. The derived analytical expression for the input current spectrum is useful to elucidate harmonics and other non-linearities that are present in the input current, and to identify the parameters that affect the said non-linearities. Subsequently, simulations are done for single-, two-, and three-phase dc-dc converters operating based on the PWM, the RWAPPM, and the proposed RWAPPM+WAPS, and thereafter, the derived analytical input current spectrum is verified and their input current spectra and output ripple noise voltages are compared.

This chapter is organized as follows. Section 5.2 describes the proposed WAPS for multi-phase converters and RWAPPM+WAPS. Section 5.3 presents the derivation for the input current spectrum of a general N -phase dc-dc converter. Section 5.4 presents the simulation results to verify the analytical input current spectrum and compare the performance of the RWAPPM+WAPS against the multi-phase PWM and the RWAPPM. Finally, Section 5.5 draws the conclusions.

5.2 The RWAPPM with Wrapped-Around Phase-Shift (RWAPPM+WAPS)

The RWAPPM was originally proposed for single-phase converters. It is now adopted for multi-phase converters. This is done by generating first the RWAPPM output pulses for Phase 1, and thereafter, creating phase-shifted copies of these pulses for the other phases. The copies are generated by means of a simple phase-shift method using shift-registers. The multi-phase RWAPPM generated using this simple phase-shift method, nonetheless, has a drawback compared to the multi-phase PWM that will be discussed below. For ease of illustration in this chapter, Figure 2.3 in Chapter 2 is reproduced below as Figure 5.1.

Figure 5.1: Switched-mode dc-dc buck converter with an N -phase output stage

In the multi-phase PWM, as discussed earlier in Chapter 2, the rising part of the ripple current in a particular phase overlaps with the falling part of the ripple currents in the other phases such that there is a ripple cancellation (subtractive) effect at i_C in Figure 5.1. The starting position of each PWM pulse is always at the beginning of its switching cycle—no random delay, and this ensures that the multi-phase PWM always achieves the ripple cancellation (ideally). However, this is not the case in the multi-phase RWAPPM, where the starting position of each RWAPPM pulse is randomized. Figure 5.2 depicts an example using a two-phase RWAPPM to illustrate how the randomization of the RWAPPM negates/degrades the ripple current cancellation at i_C . The rising part of the ripple current of a phase overlaps with the rising part of other ripple currents in the other phases, and vice-versa with the falling parts. Therefore, the net effect is additive, instead of subtractive, and the ripple current is larger compared to that of the two-phase PWM. In short, the simple phase-shift method is inadequate for the multi-phase RWAPPM, and a better method is desirable.

In order to address the ripple cancellation problem in the multi-phase RWAPPM, the WAPS method is proposed. Figure 5.3 illustrates the multi-phase RWAPPM+WAPS. In this method, V_{GP1} for Phase-1 is the same as that of the multi-phase RWAPPM described above, where V_{GP1} is only delayed by ε . The gate voltage signals for the other phases are generated in two steps as follows:

- 1) The phase-shifted versions of V_{GP1} are generated for the other gate voltage signals. In Figure 5.3, this is illustrated using Phase-2, where the said phase-shifted gate voltage is denoted by V_{GP2}^* .

2) If any one of the phase-shifted gate voltage signals (V_{GP}^*) exceeds the switching period, the overrun portion is moved to the beginning of the original switching period. Figure 5.3 depicts this for V_{GP2} of Phase-2.

Figure 5.3 shows that the ripple cancellation at i_C of the multi-phase RWAPPM+WAPS is achieved within each switching cycle. The resultant i_C of the multi-phase RWAPPM+WAPS in Figure 5.3 is smaller as compared to that shown in Figure 5.2 of the multi-phase RWAPPM, where both of them are given the same input V_{GP1} pulses. This concludes that the multi-phase RWAPPM+WAPS improves the i_C ripple cancellation effect of the multi-phase RWAPPM.

Figure 5.2: Negation of the ripple current cancellation for a two-phase RWAPPM

Figure 5.3: Cancellation of the ripple currents at i_C for a two-phase

RWAPPM+WAPS

5.3 Input Current Spectrum of the RWAPPM+WAPS

5.3.1 Derivation of an N -phase Converter Input Current

Spectrum

This subsection presents the derivation of the analytical spectrum of the input current (i_{in} in Figure 5.1) for an N -phase converter employing RWAPPM+WAPS. Following the same method described in [70], the time-

domain expression of i_{in} is derived before being transformed to the frequency-domain. The time-domain expression of i_{in} can be obtained by summing up the input phase-currents i_{phn} flowing in each phase. As there are two possible cases for each of the phase-current waveform, as depicted in Figure 5.4, the symbol χ_{nk} can be used to denote the random variable for the occurrence of Case 2 in Phase- n , k -th switching-period, and ϕ_{nk} is used to denote the delay from the beginning of the k -th switching-period to the starting position of either pulse R_A in Case 1 or pulse R_C in Case 2, as depicted in Figure 5.4. The expressions of ϕ_{nk} and χ_{nk} are given by

$$\phi_{nk} = \begin{cases} \varepsilon_{1k} + \frac{n-1}{N} \cdot T, & \varepsilon_{1k} + \frac{n-1}{N} \cdot T < T \\ \varepsilon_{1k} + \frac{n-1}{N} \cdot T - T, & \varepsilon_{1k} + \frac{n-1}{N} \cdot T \geq T \end{cases} \quad (5.1)$$

$$\chi_{nk} = \begin{cases} 1, & \phi_{nk} \in [0, T - \alpha) \\ 0, & \phi_{nk} \in [T - \alpha, T] \end{cases} \quad (5.2)$$

where α is the pulse width in each switching-period, T is the switching-period.

Figure 5.4: The possible two cases in each switching-period

Following the single-phase RWAPPM input current spectrum derivations in [102], the currents, i_{An} in Case 1, i_{Bn} and i_{Cn} in Case 2, can be obtained as

$$i_{An}(t) = \left(I_{pho} - \frac{V_{out}}{L} \phi_{nk} \right) \cdot R_A + \frac{V_{in} - V_{out}}{L} \left[\int_{-\infty}^t R_A dt - \alpha \cdot u(t - \alpha - \phi_{nk}) \right], \quad (5.3)$$

$$i_{Bn}(t) = I_{pho} \cdot R_B + \frac{V_{in} - V_{out}}{L} \cdot \left\{ \int_{-\infty}^t R_B dt - (\phi_{nk} + \alpha - T) \cdot u[t - (\phi_{nk} + \alpha - T)] \right\}, \text{ and} \quad (5.4)$$

$$i_{Cn}(t) = \left(I_{pho} + \frac{V_{in}}{L} \cdot (\phi_{nk} + \alpha - T) - \frac{V_{out}}{L} \cdot \phi_{nk} \right) \cdot R_C + \frac{V_{in} - V_{out}}{L} \cdot \left[\int_{-\infty}^t R_C dt - (T - \phi_{nk}) \cdot u(t - T) \right], \quad (5.5)$$

where V_{in} , V_{out} , and L are the input voltage, the output voltage and the phase inductance respectively. The current I_{pho} is the nominal output current in each phase and it is given by I_o/N , where I_o is the total nominal output current. The function $u(t)$ is the Heaviside step function. The functions R_A , R_B , and R_C are rectangular functions depicted in Figure 5.4 and they are given by [70]

$$R_A = \int_{-\infty}^{\infty} V_A(f) e^{-j\omega(kT + \phi_{nk})} e^{j\omega t} df, \quad (5.6)$$

$$R_B = \int_{-\infty}^{\infty} V_B(f) e^{-j\omega kT} e^{j\omega t} df, \quad (5.7)$$

$$R_C = \int_{-\infty}^{\infty} V_C(f) e^{-j\omega(kT + \phi_{nk})} e^{j\omega t} df, \quad (5.8)$$

$$V_A(f) = F \{u(t) - u(t - \alpha)\} = \frac{1 - e^{-j\omega\alpha}}{j\omega}, \quad (5.9)$$

$$V_B(f) = F \{u(t) - u(t - (\phi_{nk} + \alpha - T))\} = \frac{1 - e^{-j\omega(\phi_{nk} + \alpha - T)}}{j\omega}, \text{ and} \quad (5.10)$$

$$\begin{aligned}
V_C(f) &= F \{u(t) - u(t - (T - \phi_{nk}))\} \\
&= \frac{1 - e^{-j\omega(T - \phi_{nk})}}{j\omega}.
\end{aligned} \tag{5.11}$$

The function $\Phi\{.\}$ denotes the Fourier transform operation.

With the above expressions of currents in each case, the phase-current can be expressed as

$$i_{phn}(t; \chi_n, \phi_n) = \begin{cases} i_{An}, & \text{if } C_n = 1 - \chi_n \\ i_{Bn} + i_{Cn}, & \text{if } C_n = \chi_n \end{cases}, \tag{5.12}$$

where

$$C_n = \begin{cases} 1 - \chi_n, & \text{if Case 1 occurs} \\ \chi_n, & \text{if Case 2 occurs} \end{cases}. \tag{5.13}$$

The total input current can be expressed by a combination of the input phase-currents in each phase. Since two cases may appear in each phase, there will be totally 2^N possible combinations (terms) in the total input current expression.

The expression is derived as follows

$$i(t - kT; \chi_{nk}, \phi_{nk}) = \sum \left(\left(\prod_{q=1}^N C_q \right) \cdot \left(\sum_{q=1}^N i_{phq}(t; \chi_{qk}, \phi_{qk}) \right) \right). \tag{5.14}$$

The exact time-domain input current expression can be obtained by substituting (5.1) – (5.13) into (5.14). The time-domain expression of $i(t - kT; \chi_{nk}, \phi_{nk})$ can be transformed to its frequency-domain, $I(f)$, using the Fourier Transform. Subsequently, the input current spectrum can be obtained by substituting $I(f)$ to the following equation [91]:

$$S(f) = \frac{1}{T} \cdot \left[E \left\{ |I(f)|^2 \right\} - |E \{ I(f) \}|^2 + \frac{1}{T} |E \{ I(f) \}|^2 \cdot \sum_{q=-\infty}^{\infty} \delta(f - \frac{q}{\tau}) \right], \quad (5.15)$$

where $E\{.\}$ is the expectation (statistical average) operation.

The expression of $S(f)$ provides a means to analyze the input current spectrum of the RWAPPM+WAPS in an N -phase converter. It identifies the non-linearities in the input current as well as the parameters that affect them. With this expression, the peak power in the input current spectrum that is important in conducted EMI tests for compliance to regulations [103] can be easily estimated.

5.3.2 Analytical Input Current Spectra

As the single-phase input current spectrum has been shown in Chapter 3, Figure 5.5 only depicts the plot of the analytical input current spectra of the RWAPPM+WAPS in two- and three-phase converters. The plots are obtained using the expression in (5.15) at $V_{in} = 3.3$ V, $V_{out} = 2.5$ V (duty cycle = 0.75), load $R_{load} = 4 \Omega$, and average switching-frequency $f_{sw} = 100$ kHz. The L and C values for each phase are calculated according to [108] at a ripple current of 20% $\times$ each phase current, and a ripple noise voltage of 10% $\times$ the output voltage. The RBW is 200 Hz. The peak spectral power is -27.09 dBFS and -29.57 dBFS at 100 kHz for two- and three-phase spectra respectively. It can be observed that when the number of phase increases, not only the peak spectral power decreases slightly, but the noise floor is also lowered.

(a)

(b)

Figure 5.5: Analytical input current spectra of the (a) two-phase and (b) three-phase RWAPPM+WAPS at $V_{in} = 3.3$ V, $V_{out} = 2.5$ V, $R_{load} = 4$ Ω , and average

$$f_{sw} = 100 \text{ kHz}$$

5.4 Simulation Results

5.4.1 Verification of the Analytical Input Current Spectra

In this subsection, the analytical input current spectra are verified by means of SPICE simulations of multi-phase dc-dc buck converters operating based on the RWAPPM+WAPS. The power transistors used are the Si3585 N-channel and P-channel MOSFET [107]. A pseudorandom number generator based on the LFSR method is employed to generate $N = 64$ pseudorandomized ε_{lk} values. Figure 5.6 depicts the plots of the simulated input current spectra of the RWAPPM+WAPS in two- and three-phase converters. They are obtained under the same conditions as stated in Section 5.3.2, i.e. $V_{in} = 3.3$ V, $V_{out} = 2.5$ V (duty cycle = 0.75), load $R_{load} = 4 \Omega$, average switching-frequency $f_{sw} = 100$ kHz and RBW of 200 Hz. It can be observed that both the simulated two- and three-phase spectra match well with the analytical spectra, thereby verifying the derived expression.

Figure 5.6: Analytical and simulated input current spectra of the (a) two-phase and (b) three-phase RWAPPM+WAPS at $V_{in} = 3.3$ V, $V_{out} = 2.5$ V, $R_{load} = 4 \Omega$, and average $f_{sw} = 100$ kHz

5.4.2 Performance of the RWAPPM+WAPS

This subsection presents simulated performance of the dc-dc converter based on the RWAPPM+WAPS, the multi-phase RWAPPM, and the multi-phase PWM. The simulations are performed using SPICE. The power transistor Si3585DV [107] is used in the simulations. Single-, two-, and three-phase switched-mode dc-dc buck converters are simulated using the multi-phase PWM, the multi-phase RWAPPM, and the proposed multi-phase RWAPPM+WAPS. The single-phase simulation results are used for reference in the tables and some of the figures presented later. The configurations for the converters are: 3.3 V input, 100 kHz switching-frequency, and 0.75 duty cycle. The L and C values for each phase are calculated in the same way as discussed in Section 5.3.2 at a load current of 600 mA, a ripple current of $20\% \times$ each phase current, and a ripple noise voltage of $10\% \times$ the output voltage.

Figure 5.7 depicts the input current spectra for three-phase converters operating based on the multi-phase PWM and the proposed multi-phase RWAPPM+WAPS. It can be observed that there are strong undesirable switching-frequency harmonics in the spectrum of the PWM. On the other hand, the harmonics in the spectrum of the RWAPPM+WAPS are attenuated effectively.

(a)

(b)

Figure 5.7: Input current spectra of (a) multi-phase PWM, and (b) multi-phase

RWAPPM+WAPS in three-phase converters

Figure 5.8 depicts the i_C waveform plots for single-, two-, and three-phase converters using the different modulation schemes. The i_C of the single- and multi-phase RWAPPM has an undesirably large ripple. On the other hand, the i_C of the RWAPPM+WAPS has a smaller ripple, and is comparable to that of the PWM.

Figure 5.8: i_C waveforms of single-, two-, and three-phase converters using PWM, multi-phase RWAPPM, and multi-phase RWAPPM+WAPS

Table 5.1 tabulates the peak spectral power at the input current, and Table 5.2 tabulates the output ripple noise voltages and i_C currents obtained from the simulation results of the different converters using the different modulation schemes; the results in Table 5.2 are in rms values. On the basis of Table 5.1 and Table 5.2, the following observations can be made:

- i. All modulation schemes have smaller peak spectral power when the number of phases is increased as expected. The three-phase RWAPPM+WAPS achieves the lowest peak spectral power at -29.9 dBFS, and this is lower by ~ 19 dB compared to that of the PWM.
- ii. All modulation schemes have smaller output ripple noise voltages and smaller i_C as the number of phases is increased. When the number of phase is increased to 3, the ripple noise of the RWAPPM+WAPS is comparable to that of the multi-phase PWM. The output ripple noise voltage and i_C of the three-phase RWAPPM+WAPS outperforms those of the three-phase RWAPPM by ~ 30 mV and ~ 31 mA respectively.

Table 5.1: Peak spectral power at the input current for single-, two-, and three-phase converters using different modulation schemes

Modulation Scheme	Peak spectral power (dBFS)		
	single-phase	two-phase	three-phase
Multi-phase PWM	-2.1	-4.9	-11.0
Multi-phase RWAPPM	-20.0	-24.1	-26.8
Multi-phase RWAPPM + WAPS	N.A.	-24.8	-29.9

Table 5.2: Output ripple noise voltages and i_C for single-, two-, and three-phase converters using different modulation schemes

Modulation Scheme	Output ripple noise voltage (mV rms)			i_C (mA rms)		
No. of Phases	single-phase	two-phase	three-phase	single-phase	two-phase	three-phase
Multi-Phase PWM	33.9	7.8	4.4	34.3	11.7	4.0
Multi-Phase RWAPPM	38.0	34.8	35.3	51.2	39.0	36.9
Multi-Phase RWAPPM+WAPS	N.A.	12.9	4.9	N.A.	17.5	6.0

5.5 Conclusions

In this chapter, the multi-phase RWAPPM+WAPS has been proposed for low-harmonics low-noise multi-phase switched-mode dc-dc converters. To perform the WAPS operation in each phase, two adders and one multiplier are needed in each phase, the computational complexity is increased linearly with the increase in the number of phases. An analytical expression for the input current spectrum has been derived to elucidate the harmonics and other non-linearities in the input current. By means of the derived expression, it has been shown that the input current spectrum of the RWAPPM+WAPS features very low peak spectral power and low discrete harmonics. The derived analytical expression has been verified by means of computer simulations. The multi-phase RWAPPM+WAPS has been simulated and compared against the multi-phase PWM and the multi-phase RWAPPM without the WAPS. Simulation results show that the multi-phase

RWAPPM+WAPS features the lowest peak spectral power in its input current spectrum, a low ripple current at the output capacitor, and a low output ripple noise voltage.

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

6.1 Conclusions

This thesis has described the proposal, analysis and verification of randomized modulation schemes for switched-mode dc-dc converters. The intended applications of the proposed randomized modulation schemes are portable devices that the circuit area is scarce, low level of EMI is mandatory, and power source is limited.

The randomized modulation schemes usually feature low switching-frequency harmonics in their input current and output voltage spectra. This is due to the dithering/randomization process that spreads the power of the switching-frequency harmonics over the frequency spectrum; nonetheless, the dithering/randomization also has a drawback of generating extra ripple noise at the input current and the output voltage. In view of the feature and the drawback of the randomized modulation schemes and the intended applications, the objectives of the proposed randomized modulations schemes are low computational complexity (simple algorithms), low harmonics in the input

current and the output voltage spectra, and low ripple noise in the input current and in the output voltage.

The specific modulation schemes are the proposed noise-shaped randomized modulation scheme for the applications in low harmonics, low input noise current single-phase switched-mode dc-dc converters; the RWAPPM with adjustable limits for the applications in single-phase switched-mode dc-dc converters that require relatively low load current, low harmonics, and low output ripple noise voltage; and the RWAPPM+WAPS for the applications in multi-phase dc-dc converters that require relatively high load current, low harmonics, and low output ripple noise voltage.

The analysis of the randomized modulation schemes pertains to the derivation of analytical expressions for the input current spectra and output voltage spectra, so as to identify the components that present in the input current and output voltage, and the parameters affecting them, to determine the peak power that is important in EMI test, and to calculate the ripple noise from the low-frequency components.

The randomized modulation schemes implemented either in FPGA, discrete electronic components, or in SPICE simulations provide a means to verify the features of the proposed randomized modulation schemes, and the corresponding derived analytical expressions of the power spectra.

In Chapter 2, different configurations of switched-mode dc-dc converters have been reviewed. The buck, boost, and buck-boost configuration are the most basic

ones of the switched-mode dc-dc converter. Amongst the three configurations, the buck converter is the most commonly used in verifying the performance of various modulation schemes. This is due to the simple relation between its input and output voltages. The multi-phase dc-dc converter is a configuration that is often used in applications that have high current demands. The current flowing in multi-phase converters are shared among different phases, thereby reducing the current stress in each phase, and increases its current carrying capability. Further, the output ripple noise can be cancelled among different phases. This attractive attribute inspired the proposal of RWAPPM+WAPS in Chapter 5.

In Chapter 2, a literature review on various modulation schemes have been provided. All the reviewed randomized modulation schemes can spread the power of the switching-frequency harmonics over the power spectrum to reduce the peak spectral power, whereas the RWAPPM features the lowest peak spectral power, simple algorithm, and low hardware complexity. This is because the constant switching frequency of the RWAPPM renders easier realization. Therefore, the RWAPPM is also adopted as the basis of the randomized modulation schemes proposed in other chapters of this thesis.

In Chapter 3, a novel noise-shaped randomized modulation scheme for low-harmonics low-noise single-phase switched-mode dc-dc converters has been proposed. The proposed scheme is a hybrid of the RWAPPM and a noise-shaper. The RWAPPM mitigates the switching-frequency harmonics in the input current, whereas the noise-shaper mitigates the low-frequency noise therein. An analytical expression for the input current spectrum of the proposed scheme has

been derived to analyze the harmonics and low-frequency noise that are present in the input current. A novel pulse generator structure embodying the noise-shaped randomized modulation scheme has also been proposed in Chapter 3. A dc-dc converter employing the proposed scheme has been realized using the proposed pulse generator structure. The derived analytical expression has been verified by means of experimental measurements on the input current spectrum, the output voltage spectrum, the transient-response and the operating range of the realized converter. The results have shown that the proposed scheme features very low peak spectral power in the input current spectrum (18.1 dB lower than the PWM at 3.3 V input voltage, 0.5 duty cycle, and 100 kHz average switching-frequency). The input noise current of the proposed scheme, have been obtained at $\sim 73\text{mA}$ rms (integrated over a 200 kHz bandwidth without an input filter), is comparable to that of the PWM, and is lower by $\sim 16\text{ mA}$ rms compared to that of the RWAPPM without the noise-shaper. The measurement results have also shown that the conventional PWM can be readily replaced with the proposed noise-shaped randomized modulation scheme.

The noise-shaped randomized modulation scheme proposed in Chapter 3 is able to suppress the low-frequency input noise current, whereas the output ripple noise voltage remains an issue. This inspired the proposal of adjustable limit RWAPPM in Chapter 4. The proposed RWAPPM with adjustable limits can trade-off the output ripple noise voltage and the peak spectral power by varying/adjusting the limits on the pulse position. An analytical expression of the output voltage spectrum of the RWAPPM with adjustable limits has been derived. This expression provides an analytical means of calculating the trade-

offs between the peak spectral power and the output ripple noise voltage. The derived expression has been verified by means of simulations and experimental measurements.

The simulations and experimental measurements has been conducted at 3.3 V input voltage, 0.75 duty cycle, and 100 kHz switching-frequency. The minimum limit value of the pulse position has been varied from 0 to 15% of the switching-period, and the maximum limit value of the pulse position has been varied from 85% to 100% of the switching-period. It has been shown that when the extreme limit values are used for the RWAPPM, the ripple noise is the highest at 56.8 mV. While when the minimum (or maximum) limit is increased (or decreased) by 1% of the switching-period, the peak spectral power increases by 1 dB, whereas the ripple noise decreases by 0.6 mV. The ripple noise is improved by 9.1 mV and 10.7 mV respectively for the minimum limit value at 15% of the switching-period and the maximum limit value at 85% of the switching-period, as compared to that for the RWAPPM without adjustable limits.

To further improve the output ripple noise voltage in applications where the load current is relatively high, the RWAPPM has been proposed to be applied in multi-phase switched-mode dc-dc converters in Chapter 5 to benefit from its output ripple cancellation effect. None of the randomized modulation schemes has been employed in multi-phase converters before, because the dithering/randomization in theses modulation schemes negates the ripple cancellation effect in the multi-phase dc-dc converters. In order to resolve this issue that also happened on the RWAPPM when being employed in multi-phase converters, the WAPS method

has been proposed in Chapter 5. A general N -phase analytical expression of the input current spectrum of the multi-phase converters have been derived to show that the RWAPPM+WAPS reserves the attribute of low peak spectral power in the input current spectra. The performance of the proposed RWAPPM+WAPS has been verified and benchmarked against the multi-phase PWM and the multi-phase RWAPPM by means of SPICE simulations.

It has been shown in Chapter 5 that the proposed RWAPPM+WAPS features a low output ripple noise voltage at 4.9 mV, and is comparable to that of the PWM (4.4 mV) under the condition of 3.3 V input, 100 kHz switching-frequency, 0.75 duty cycle, and with a three-phase converter. The RWAPPM+WAPS has also been shown to feature very low peak spectral power of the input current harmonics at -29.9 dBFS, and this is lower by ~ 19 dB compared to that of the PWM at the same conditions.

As an overall conclusion, the work in this thesis pertains to the design, analyse, and verify of randomized modulation schemes for switched-mode dc-dc converters. The intended applications of the switched-mode dc-dc converters are the portable devices that the circuit area is scarce, low level EMI is mandatory, and power source is limited. The proposed randomized modulation schemes in this thesis have been analyzed by deriving analytical expressions for the input current and output voltage spectra. The derived analytical expressions have been verified by computer simulations and on the basis of experimental measurements.

6.2 Recommendations for Future Work

The following directions are recommended for future work pertaining to randomized modulation schemes for switched-mode dc-dc converters.

- (a) Closed-loop multi-phase dc-dc converter with a randomized modulation scheme

Chapter 5 presented the steady-state analyses of multi-phase dc-dc converter employing the RWAPPM+WAPS. It was assumed that there is no perturbation or interference under the steady-state analyses. This is equivalent to an open-loop analysis. Nonetheless, switched-mode dc-dc converters are usually implemented in a closed-loop configuration to regulate the output voltage and reduce the influence caused by perturbations and noise. To apply the RWAPPM+WAPS in practical applications, it would be worthwhile to design a closed-loop multi-phase dc-dc converter IC that embodies the proposed randomized modulation scheme, and analyze the effects of the perturbations and noise on the response and performance of the randomized modulation scheme in the closed-loop multi-phase dc-dc converter. Further, when designing the custom IC, the jittering of the clock may cause severe mismatch issues among different phases. The area and cost of the output stage will also increase due to the multi-phase configuration. It would be challenging to resolve the mismatch issue and devise a method to keep the area and cost as low as possible.

- (b) A low harmonic, low ripple noise, high power-efficiency, and wide load current range multi-mode randomized modulation scheme

In view of the intended applications in portable devices, another important requirement of switched-mode dc-dc converters is a high power-efficiency. Portable devices generally spend most of the time in the standby mode instead of the usual operating mode. Hence, the power-efficiency at the standby mode is critical. The Pulse Frequency Modulation (PFM) is another randomized modulation scheme that is often employed to maintain high power-efficiency when the device is in the standby mode. The PFM varies its switching-frequency according to the load current thereby reducing the switching loss in the switched-mode converter, and can result in a high efficiency when the load current is low. For high load currents, however, the PFM is not optimum. It would be interesting to devise a multi-mode modulation scheme that combines an optimized PFM and other randomized modulation schemes for switched-mode dc-dc converters. It would be desirable that this randomized modulation scheme can switch between the different randomized modulation schemes according to the load current and switching-frequency requirements, and at the same time, maintain low harmonic, low ripple noise, high power-efficiency through a wide range of load current.

- (c) Fully-integrated switched-mode dc-dc converter with a randomized modulation scheme for high switching-frequency applications

By switching at very high frequencies, the size of the inductor and output capacitor can be reduced tremendously and it will be feasible to realize a

fully-integrated dc-dc converter. The proposed randomized modulation schemes described herein have been shown to outperform the PWM. However, state-of-the-art fully-integrated dc-dc converters still operate based on the PWM. It would be worthwhile to realize the first fully-integrated converter with a randomized modulation scheme.

APPENDIX A

DERIVATION OF THE INPUT CURRENT SPECTRUM OF THE RWAPPM

The detailed derivation of the input current spectrum (spectrum of i_{in} in Figure 3.1) of the RWAPPM without the noise-shaper is presented in this appendix. This spectrum is required to obtain the spectrum of the noise-shaped randomized modulation scheme proposed in Chapter 3. The rectangular functions Π_A , Π_B , and Π_C in Figure A.1 will be used frequently in later derivations. The following expressions for Π_A , Π_B , and Π_C can be obtained [70]:

$$\Pi_A = \int_{-\infty}^{\infty} V_A(f) e^{-j\omega(k\tau + \psi_k)} e^{j\omega t} df, \quad (\text{A.1})$$

$$\Pi_B = \int_{-\infty}^{\infty} V_B(f) e^{-j\omega k\tau} e^{j\omega t} df, \text{ and} \quad (\text{A.2})$$

$$\Pi_C = \int_{-\infty}^{\infty} V_C(f) e^{-j\omega(k\tau + \Delta_k)} e^{j\omega t} df, \quad (\text{A.3})$$

where

$$\begin{aligned} V_A(f) &= \mathcal{F} \{u(t) - u(t - \alpha)\} \\ &= \frac{1 - e^{-j\omega\alpha}}{j\omega}, \end{aligned} \quad (\text{A.4})$$

$$\begin{aligned} V_B(f) &= \mathcal{F} \{u(t) - u(t - (\Delta_k + \alpha - \tau))\} \\ &= \frac{1 - e^{-j\omega(\Delta_k + \alpha - \tau)}}{j\omega}, \text{ and} \end{aligned} \quad (\text{A.5})$$

$$\begin{aligned} V_C(f) &= \mathcal{F} \{u(t) - u(t - (\tau - \Delta_k))\} \\ &= \frac{1 - e^{-j\omega(\tau - \Delta_k)}}{j\omega}. \end{aligned} \quad (\text{A.6})$$

The ψ_k and Δ_k are the random delays in Case 1 and Case 2 of the RWAPPM respectively, $\Phi\{.\}$ denotes the Fourier transform operation and $u(t)$ is the Heaviside step function.

Figure A.1: Input current (i_{in}) waveforms with the corresponding V_x waveforms for (a) Case 1, and (b) Case 2 of the RWAPPM

The time-domain expressions of i_{in} is derived before transforming them to the frequency-domain. Figure A.1 depicts the time-domain waveforms of V_x and the corresponding i_{in} drawn by the converter. Since i_{in} can be either i_A , or combined i_B and i_C in Figure A.1 at any given time, i_{in} can be expressed by the products of the currents and their corresponding probabilities of occurrence as

follows

$$i(t - k\tau; \chi_k, \psi_k, \Delta_k) = (1 - \chi_k) \cdot i_A(t) + \chi_k \cdot [i_B(t) + i_C(t)], \quad (\text{A.7})$$

where χ_k is the random variable for the occurrence of Case 2, and is assumed to be stationary and statistically independent from one output pulse to another. The PMF of χ_k is

$$P(\chi) = \begin{cases} 1 - p & \chi_k = 0 \quad (\text{Case 1 occurs}) \\ p & \chi_k = 1 \quad (\text{Case 2 occurs}) \\ 0 & \text{otherwise} \end{cases} \quad (\text{A.8})$$

where $p = \alpha/\tau$ is the probability of the occurrence of Case 2.

In Case 1, when V_x is high, L in Figure 3.1 is charged, and consequently, i_{in} drawn by the converter ramps up. This i_{in} is depicted in Figure A.1(a) as i_A . Note that the inductor current at the beginning of each randomized switching period is at its nominal value I_o . The waveform of i_A can be resolved into a rectangular waveform and a ramp-up triangle waveform, and both waveforms can be expressed in terms of Π_A introduced earlier. Using the slope indicated in Figure A.1(a), the expression of i_A can be obtained as follows

$$i_A(t) = i_1 \cdot \Pi_A + \frac{V_{in} - V_{out}}{L} \left[\int_{-\infty}^t \Pi_A dt - \alpha \cdot u(t - \alpha - \psi_k) \right], \quad (\text{A.9})$$

where V_{in} , V_{out} and L are the input voltage, the (desired) output voltage and the inductor value respectively, i_1 is the initial value of i_A . The current i_1 can be derived by knowing I_o and the slope indicated in Figure A.1(a), and is given by

$$i_1 = I_o - \frac{V_{out}}{L} \psi_k. \quad (\text{A.10})$$

In Case 2, V_x is high at the beginning of the switching period, L is charged and

draws i_B as depicted in Figure A.1(b). When V_x toggles low, L is discharged and the inductor current ramps down to i_3 . At $t = \Delta_k$, V_x toggles high again, L is charged and draws the input current i_C . The current i_B and i_C can be derived as

$$i_B(t) = I_o \cdot \Pi_B + \frac{V_{in} - V_{out}}{L} \cdot \left\{ \int_{-\infty}^t \Pi_B dt - (\Delta_k + \alpha - \tau) \cdot u[t - (\Delta_k + \alpha - \tau)] \right\}, \text{ and} \quad (\text{A.11})$$

$$i_C(t) = i_3 \cdot \Pi_C + \frac{V_{in} - V_{out}}{L} \cdot \left[\int_{-\infty}^t \Pi_C dt - (\tau - \Delta_k) \cdot u(t - \tau) \right], \quad (\text{A.12})$$

where i_3 is the initial value of i_C , and can be derived from I_o and the slope indicated in Figure A.1(b) as follows

$$\begin{aligned} i_3 &= i_2 - \frac{V_{out}}{L} \cdot (\tau - \alpha) \\ &= I_o + \frac{V_{in}}{L} \cdot (\Delta_k + \alpha - \tau) - \frac{V_{out}}{L} \cdot \Delta_k. \end{aligned} \quad (\text{A.13})$$

The overall time-domain input current expression can be obtained by substituting (A.9) – (A.13) into (A.7). The time-domain expression of $i(t - k\tau, \chi_k, \psi_k, \Delta_k)$ can be transformed to its frequency-domain, $I(f)$, using the Fourier Transform. Subsequently, the input current spectrum can be obtained by substituting $I(f)$ to the following autocorrelation of the input current equation [91],

$$S(f) = \frac{1}{\tau} [E\{|I(f)|^2\} - |E\{I(f)\}|^2 + \frac{1}{\tau} |E\{I(f)\}|^2 \cdot \sum_{q=-\infty}^{\infty} \delta(f - \frac{q}{\tau})], \quad (\text{A.14})$$

where $E\{.\}$ is the expectation (statistical average) operation.

From (A.14), the final expression for the input current spectrum of the RWAPPM is obtained as Equation (3.3) in Chapter 3.

APPENDIX B

DERIVATION OF THE INPUT CURRENT SPECTRA OF THE PWM, THE RCFMFD, AND THE PCFM

This appendix presents the derivation of the input current spectra of the modulation schemes that are used for benchmarking in Chapter 3.

B.1 Derivation of the Input Current Spectra of the PWM

The waveform of i_{in} drawn by a switched-mode dc-dc buck converter operating based on the PWM is similar to the waveform shown in Figure A.1(a), except that ψ_k is 0. Therefore, the input current expression in the time domain can be obtained by substituting $\psi_k = 0$ into (A.9):

$$i_{PWM}(t) = I_{\min} \cdot \Pi_A + \frac{V_{in} - V_{out}}{L} \left[\int_{-\infty}^t \Pi_A dt - \alpha \cdot u(t - \alpha) \right], \quad (B.1)$$

where $I_{\min}$ is the initial value of $i_{PWM}(t)$, and is given by

$$I_{\min} = I_o - \frac{V_{in} - V_{out}}{L} \cdot \frac{\alpha}{2}. \quad (B.2)$$

After applying the Fourier Transform to (B.1) and substituting the result $I_{PWM}(f)$ into (A.14), the input current spectrum of the PWM is obtained as follows

$$PWM(f) = P_{PWM}(f) + P_{PWM_DC}, \quad (B3)$$

where $P_{PWM}(f)$ is the power of the discrete harmonics and is given by

$$P_{PWM}(f) = \left(\frac{1}{\tau}\right)^2 \cdot \left|E\{I_{PWM}(f)\}\right|^2 \cdot \sum_{\substack{q=-\infty \\ q \neq 0}}^{q=\infty} \delta(f - \frac{q}{\tau}), \quad (\text{B.4})$$

P_{PWM_DC} is the dc power and is given by

$$P_{PWM_DC} = \left(\frac{\alpha}{\tau}\right)^2 \cdot \frac{V_{in}}{R_{load}}. \quad (\text{B.5})$$

B.2 Derivation of the Input Current Spectra of the RCFMFD

The waveform of i_{in} drawn by a switched-mode dc-dc buck converter operating based on the RCFMFD is similar to the waveform of the PWM, except that the pulse width changes for each switching-period. Therefore, the input current expression in the time domain can be obtained by substituting $\psi_k = 0$ into (A.9):

$$i_{RCFMFD}(t) = I_{\min} \cdot \Pi_A + \frac{V_{in} - V_{out}}{L} \left[\int_{-\infty}^t \Pi_A dt - \alpha \cdot u(t - \alpha) \right], \quad (\text{B.6})$$

where $I_{\min}$ is the initial value of $i_{RCFMFD}(t)$, and is given by

$$I_{\min} = I_o - \frac{V_{in} - V_{out}}{L} \cdot \frac{\alpha}{2}. \quad (\text{B.7})$$

The derivation of the expression for the RCFMFD voltage spectrum can be found in [46]. By substituting the Fourier Transform of the input current expression in (B.6) to the general expression for RCFMFD spectrum, the input current spectrum of the RCFMFD, $RCFMFD(f)$, can be obtained by means of the following expression

$$RCFMFD(f) = \frac{1}{E[T_k]} \cdot E[|I_{RCFMFD}(f)|]^2 + \frac{1}{E[T_k]} \cdot 2 \operatorname{Re} \left\{ \frac{E[I_{RCFMFD}(f) e^{j2\pi f T_k}] E[I_{RCFMFD}^*(f)]}{1 - E[e^{j2\pi f T_k}]} \right\}, \quad (\text{B.8})$$

where T_k is the randomized switching period.

B.3 Derivation of the Input Current Spectra of the PCFM

Following the method described in [58], the input current of the PCFM can be represented using the Fourier series and expressed as follows,

$$i_{PFM} = \sum_{n=-\infty}^{+\infty} a_n \cdot e^{j2\pi f_m t}, \quad (\text{B.9})$$

where f_m is the modulation frequency,

$$a_n = \frac{1}{j2\pi n} \left(I_o - \frac{V_{in} - V_{out}}{j2\pi n L} \right) \cdot \sum_{k=1}^N \left(\exp \left(-j \left(\frac{2\pi n \xi_k}{N \cdot T_s} \right) \right) - \exp \left(-j \left(\frac{2\pi n (\xi_k + d_k \cdot T_k)}{N \cdot T_s} \right) \right) \right) - \frac{V_{in} - V_{out}}{L} d_k T_k \sum_{k=1}^N \left(\exp \left(-j \left(\frac{2\pi n (\xi_k + d_k \cdot T_k)}{N \cdot T_s} \right) \right) \right), \quad (\text{B.10})$$

d_k is the duty cycle of k -th switching cycle,

N is the number of switching-cycles in a modulation period,

T_s is the non-modulated switching period,

ξ_k is the time at which the k -th switching-cycle starts, and

T_k is the k -th switching-period and is generated by adding a period jitter ΔT_k to

T_S as follows

$$\begin{aligned} T_k &= T_S + \Delta T_k, \\ \sum_{k=1}^N \Delta T_k &= 0. \end{aligned} \tag{B.11}$$

The discrete harmonic power of the input current at multiples of f_m can be obtained by means of (B.9).

AUTHOR'S PUBLICATIONS

Journal Papers

1. K. Cui, V. Adrian, B. H. Gwee, J. S. Chang, "A noise-shaped randomized modulation for switched-mode DC-DC converters," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 65, no. 1, pp. 394–405, Jan. 2018.
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1. V. Adrian, K. Cui, B. H. Gwee, J. S. Chang, "A randomized modulation scheme for filterless digital class D audio amplifiers," *IEEE International Symposium on Circuits and Systems*, pp.774–777, Jun. 2014.
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3. K. Cui, V. Adrian, B. H. Gwee, J. S. Chang, "A low-harmonics low-noise randomized modulation scheme for multi-phase DC-DC converters," 2017 *15th IEEE International New Circuits and Systems Conference*, Strasbourg, France, pp. 165–168, Jun. 2017.

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