Quantitative imaging using ultrasonic tomography based on full waveform inversion

Rao, Jing

2018

Rao, J. (2018). Quantitative imaging using ultrasonic tomography based on full waveform inversion. Doctoral thesis, Nanyang Technological University, Singapore.

<http://hdl.handle.net/10356/75928>

<https://doi.org/10.32657/10356/75928>

Quantitative Imaging Using Ultrasonic Tomography Based on Full Waveform Inversion

Jing Rao

School of Mechanical & Aerospace Engineering

A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirements for the degree of
Doctor of Philosophy

2018

Acknowledgements

I would like to express my gratitude to my supervisor, Assistant Professor Zheng Fan, for his professional guidance and for creating a collaborative environment. I also wish to give my gratitude to Dr. Madis Ratassepp, for his unconditional help in my PhD study. Their attitudes towards research set a good example of what a true researcher should be.

My special thanks go to my colleagues and friends. In particular, I would like to thank Mr Danylo Lisevych, Mr Ngee Kwan Tan and Mr Hang Kit Wong for supporting my experimental work, and Dr. Jizhong Yang for collaborating the multi-parameter full waveform inversion in Chapter 8.

I would also like to thank Dr. Peter Huthwaite at Imperial College London for many critical discussions and sharing the code used in the helical path separation in Chapter 7, Associate Professor Romain Brossier at University Grenoble Alpes for helpful suggestions, Dr. Christian Bohm at ETH Zurich for supporting new computing methods and Professor Ernst Rank at Technical University of Munich for his generous help.

Finally, and most importantly, I thank my family for understanding my work, especially for Dr. Pengxiao Li. They never put pressure on me and support me unconditionally. I love you all!

Jing Rao, March 2018

Abstract

Quantitative imaging can provide quantitative estimation of physical properties such as velocity, density and attenuation. The ability to reconstruct physical properties from wave measurements is very valuable in many applications as it could allow diagnostic methods with high sensitivity. For example, numerous defects are reflected in acoustic velocity variations, and thus velocity reconstruction from wave measurements can be used for defect detection. Based on this concept, ultrasonic tomography provides a promising alternative for defects/inclusions diagnosis. The key thing in ultrasonic tomography is the applicability and usefulness of accurate and high resolution algorithms for reconstructions of physical properties. In this thesis, a quantitative ultrasonic imaging approach based on full waveform inversion (FWI) is developed for accurate reconstructions of physical properties in isotropic structures. The forward model is computed in the frequency domain by solving a full-wave equation in a two dimensional (2D) acoustic model, accounting for higher order effects such as diffractions and multiple scattering. The inversion is based on local optimization of a waveform misfit function between modeled and measured data, and is applied iteratively to discrete frequency components from low to high frequencies.

The mono-parameter FWI combined with ultrasonic guided waves is used for the application of corrosion mapping. In guided wave tomography (GWT), the resulting wave velocity maps can be converted into thickness maps by the dispersion characteristics of selected guided modes. The results suggest that the FWI algorithm is capable of reconstructing the thickness map of an irregularly shaped defect accurately on a 10 mm thick plate with the thickness error within 1 mm. This thesis discusses the reconstruction accuracy of FWI on plate-like structures by using simulations as well as experiments. FWI can obtain a resolution around 1.5-2 wavelengths from the realistic data compared to 0.7 wavelengths from the idealized case. The resolution loss with the realistic test data indicates the acoustic model cannot accurately capture the scattering of guided waves and the elastic inversion approach can achieve resolution improvements. The thesis also investigates

the performance of FWI with limited view configurations, and applies the regularization technique performed by an adaptive threshold approach to synthesize the missing components, improving the image quality. Then, this thesis extends the application of GWT to liquid-loading plates. A significant challenge is the energy of guided waves leaking into the liquid. By considering attenuation effects through complex velocity in frequency-domain FWI, the validation experiment performed on a water-loaded plate with an irregularly shaped defect shows excellent performance of the reconstruction algorithm. A new GWT system based on customized piezoelectric transducers is also presented for corrosion monitoring of plate-like and pipe structures. The efficiency and the accuracy of this system have been demonstrated through continuous forced corrosion experiments.

Finally, the multi-parameter FWI is applied to ultrasonic tomography for the application of density mapping, which aims to simultaneously reconstruct density and velocity by using ultrasonic bulk waves. Density is very difficult to reconstruct in multi-parameter FWI due to the trade-off effects between velocity and density, and the difference in amplitude of different parameters in the wave-field. The inverse Hessian has been shown to be able to mitigate these effects and rescale the amplitudes. The results show that good velocity reconstruction is achieved and well-resolved density reconstruction is also obtained.

Contents

Acknowledgements	iii
Abstract	v
List of Figures	xi
List of Tables	xix
1 Introduction	1
1.1 Motivation	1
1.1.1 Thickness mapping	1
1.1.2 Density mapping	4
1.2 Outline	6
2 Literature Review on Ultrasonic Tomographic Methods for NDT	9
2.1 Introduction	9
2.2 Ultrasonic bulk waves and tomographic methods	10
2.2.1 Ultrasonic bulk waves	10
2.2.2 Tomographic methods	11
2.2.2.1 Traveltime tomography	12
2.2.2.2 Diffraction tomography	12
2.3 Ultrasonic guided wave tomography	14
2.3.1 Guided waves	14
2.3.2 Guided wave tomography	15
2.3.3 Mode selection for GWT	17
2.4 Potential approaches for higher resolution	19
2.4.1 Inverse scattering approach under the Born approximation	19
2.4.2 Non-linear inverse methods	21
2.4.2.1 Linear sampling method	21
2.4.2.2 Iterative methods	23
2.5 Summary	27
3 Ultrasonic Tomography Based on Full Waveform Inversion	29
3.1 Introduction	29

3.2	Theory	30
3.2.1	Forward modeling in the space-frequency domain	30
3.2.2	Inverse problem in the space-frequency domain	31
3.3	Numerical modeling	36
3.3.1	Elastic model based on FE method	36
3.3.2	Acoustic model based on FD method	36
3.3.3	Data processing	37
3.3.4	Frequency selection and convergence	38
3.4	Experiments	39
3.5	Reconstruction results	40
3.5.1	Single regular defect	40
3.5.2	Irregularly shaped defect	44
3.6	Discussion	46
3.7	Summary	47
4	Investigation of Reconstruction Accuracy of FWI	49
4.1	Introduction	49
4.2	Validation methods	50
4.2.1	Defects	50
4.2.2	Numerical modeling	51
4.2.3	Experimental procedures	52
4.3	Results	53
4.3.1	Depth estimation of the Hann-shaped defect	53
4.3.2	Imaging of multiple defects	56
4.4	Analysis and discussion	57
4.4.1	Resolution analysis of the Hann-shaped defect	57
4.4.2	Effect of the bottom width on the reconstruction accuracy	60
4.4.3	Comparison with DT on the imaging of multiple defects	62
4.5	Summary	64
5	FWI on Limited View Problem	65
5.1	Introduction	65
5.2	Theory	67
5.2.1	Analysis of the limited view problem	67
5.2.2	Regularization to synthesize unknown components	68
5.2.3	Adaptive thresholding regularization method	69
5.2.4	Implementation of the regularization method in guided wave tomography	72
5.3	Validation methods	74
5.3.1	Problems studied	74
5.3.2	Numerical modeling	74
5.3.3	Experimental procedures	75
5.4	Results and analysis	76
5.4.1	Irregularly shaped defect	76

5.4.2	Two defects	79
5.5	Discussions	81
5.6	Summary	84
6	GWT on Liquid-Loaded Plates	85
6.1	Introduction	85
6.2	Properties of low frequency Lamb modes of liquid-loaded plates . .	86
6.3	FWI with attenuation	87
6.4	Test approach	87
6.4.1	Configuration	87
6.4.2	FE simulations	88
6.4.3	Experimental setup and procedures	89
6.5	Effect of the attenuation on the reconstruction accuracy	91
6.6	Experimental validation	96
6.7	Summary	98
7	Corrosion Mapping Using GWT	101
7.1	Introduction	101
7.2	Monitoring system	103
7.2.1	Array system and experimental setup	103
7.2.2	Data Processing	105
7.2.3	Scattering from transducers	109
7.2.4	Scattering effect on the thickness reconstruction	111
7.3	Experimental simulation of corrosion damage	112
7.3.1	Forced electrochemical corrosion	112
7.3.2	Thickness measurement using laser profilometry	114
7.4	Results and discussions	114
7.4.1	Corrosion monitoring of plate	115
7.4.1.1	Wall loss monitoring	115
7.4.1.2	Thickness reconstruction	115
7.4.2	Corrosion mapping of pipe	118
7.4.2.1	Empty pipes	118
7.4.2.2	Water-filled pipes	121
7.5	Summary	122
8	Density reconstruction using multi-parameter FWI	123
8.1	Introduction	123
8.2	Theory	125
8.3	Numerical results	131
8.3.1	Numerical model	131
8.3.2	Data processing	132
8.3.3	Reconstructions	132
8.4	Experiments	135
8.4.1	Experimental setup	135

8.4.2	Data processing	136
8.4.3	Experimental results	137
8.5	Summary	139
9	Conclusions	141
9.1	Thesis review	141
9.2	Main findings	143
9.3	Future Work	145
A	Derivation of sensitivity kernels	147
	Author's Publications	151
	Bibliography	153

List of Figures

1.1	(a) Corrosion on the bottom of a crude oil storage tank and (b) corrosion on the gas oil aboveground storage tank [1].	2
1.2	The configuration of a circular transducers array for guided wave tomography on a plate.	3
2.1	Illumination of the object and the resulting scattered field. (a) The object is illuminated from the $\bar{r}_0$ direction and the scattered field is measured from the $\bar{r}$ direction; (b) Under Born approximation, the measured scattered field in $\bar{r}$ direction from the illumination in $\bar{r}_0$ direction is mapped to the point $\Omega = k_0(\bar{r}_0 - \bar{r})$ in the spatial frequency domain.	13
2.2	Dispersion curves of Lamb modes in an aluminum plate. (a) Phase velocity and (b) group velocity.	15
2.3	The effect of one-sided water-loading on a steel plate. (a) Phase velocity, (b) group velocity and (c) attenuation dispersion curves for a steel plate without and with liquid loading presenting on one side of the steel plate, represented by solid and dash lines, respectively.	18
2.4	Super resolved images at 42 kHz around two scatterers. (a) FM and (b) cross section of FM pseudo spectrum along the horizontal centre line [78].	23
2.5	(a) Permittivity reconstruction of a plastic pipe with the outer diameter of 4.8 cm and the inner diameter of 3.7 cm by using distorted Born iterative method (DBIM). (b) Reconstruction of this plastic pipe by using Born approximation [89].	25
2.6	(a) Photography of a real defect, (b) estimated defect profile from the finite element model and (c) cross section of (b) along X=0 mm (solid line) and the standard deviation (dotted lines) [93].	26
3.1	A schematic discrete representation of the $l = n_x \times n_y$ finite difference grid points for forward modeling.	31
3.2	Structure of the mono-parameter FWI algorithm.	32
3.3	(a) Experimental setup with a complex shaped defect, (b) a zoomed picture of the defect, and (c) a typical time trace from the experiment with chosen time window.	40

3.4	FWI reconstructions with a single central defect. (a) Original model; (b) monochromatic 35 kHz in the acoustic model, (c) the elastic model, (d) the experimental data; (e) polychromatic 60 kHz in the acoustic model, (f) the elastic model, and (g) the experimental data.	41
3.5	Cross sections of thickness reconstructions of a single central defect along the central line in (a) the acoustic model, (b) the elastic model, and (c) the experimental data.	43
3.6	Polychromatic FWI reconstructions applied to irregular defects at 60 kHz. (a) Original model, (b) the acoustic model, (c) the elastic model, and (d) the experimental data.	45
3.7	Cross sections of the thickness reconstruction of a complex defect along the lines shown in Figure 3.6 along (a) the vertical line and (b) the horizontal line.	45
4.1	(a) Cross section of a Hann-shaped defect and (b) cross section of a flat-bottom defect along the central line.	51
4.2	Experimental setup with three flat-bottom defects with different sizes and depths. The transducer was used to generate the guided wave and measurements were taken by the laser vibrometer.	53
4.3	Cross sections of the thickness reconstructions of Hann-shaped defects along the central line from the acoustic and elastic modeling data at 60 kHz. The first row (a)-(c) has surface width of 120 mm, the middle (d)-(f) has width of 60 mm, and the final (g)-(i) has width of 30 mm; The left column (a), (d) and (g) is 60% deep, the middle (b), (e) and (h) is 30% deep, and the right (c), (f) and (i) is 10% deep.	54
4.4	(a) The maximum depth error of reconstruction from the elastic model as a percentage of the nominal thickness. Errors are given for Hann-shaped defects with surface diameter of 60 mm. (b) Cross sections along the central line of the reconstructed defects for varying depths at 60 kHz from (a).	55
4.5	FWI reconstructions with three flat-bottom defects at 60 kHz. (a) Original model, (b) the acoustic model, (c) the elastic model, and (d) the experiment.	56
4.6	(a) Cross sections of thickness reconstructions of three flat-bottom defects at different frequencies based on FWI. Acoustic model along centre line of A and B, and (b) along centre line of C and A; (c) Elastic model along centre line of A and B, and (d) along centre line of C and A; (e) Experiment along centre line of A and B, and (f) along centre line of C and A.	58
4.7	The acceptable resolution limit for different defect depths and sizes from the acoustic model and the elastic model at 60 kHz.	59
4.8	Resolution limit for the A_0 mode as a function of frequency obtained from the elastic model. (a) An absolute value, mm, and (b) relative to wavelengths λ for the 30% deep defect.	60

4.9	The maximum depth errors of flat-bottom defects with different ratio of bottom diameter W_2 to surface diameter W_1 using data from the elastic model for depths of (a) 6 mm, and (b) 3 mm at 60 kHz. . . .	62
4.10	(a) DT reconstruction with three flat-bottom defects from the elastic model at 60 kHz, (b) cross sections of thickness reconstruction along centre line of A and B, and (c) along centre line of C and A.	63
5.1	2D Fourier transform based on Born approximation. (a) The full view array and (b) the corresponding Fourier components obtained from each layout; (c) a pair of parallel linear arrays with a source array above the defect and a receiver array below it and (d) the corresponding Fourier components.	68
5.2	Applying the approach used by Huthwaite [145] and Sung <i>et al.</i> [146] to synthesis unknown Fourier components by regularization. (a) Fourier components from a limited view, (b) the reconstruction obtained by using Fourier transforming, then the regularization is applied in (c), and (d) the additional information can be obtained by regularization. The estimations of the unknown components in (e) are combined with the known components.	70
5.3	1D example illustrating the schematic outline of the method of the thresholding regularization. (a) The standard limited view reconstruction, (b) a threshold used in the image and (c) the next regularized reconstruction.	71
5.4	The flowchart of regularization with the FWI algorithm for a limited view configuration.	73
5.5	The configuration of a pair of parallel linear transducer arrays for GWT on a plate.	75
5.6	The experimental setup on an irregularly shaped defect.	76
5.7	Reconstructions with an irregularly shaped defect by a pair of parallel linear transducer arrays. Original model (a), the standard FWI limited view reconstruction (b), and the regularized reconstruction (c) based on the FE model; the standard limited view reconstruction (d), and the regularized reconstruction (e) in the experiment. . . .	77
5.8	Cross-sections of standard limited view FWI and regularized reconstructions of an irregularly shaped defect along the horizontal direction in (a) the FE model and (b) the experiment.	79
5.9	Reconstructions with two flat-bottom defects by a pair of parallel linear transducer arrays. (a) Original model, (b) the standard limited view reconstruction, and (c) the regularized reconstruction based on the FE model; (d) the standard limited view reconstruction, and (e) the regularized reconstruction based on the experiment.	80
5.10	Cross-sections of standard limited view FWI and regularized reconstructions of two flat-bottom defects along the horizontal direction in (a) the FE model and (b) the experiment.	81

5.11	Limited view reconstructions of a flat-bottom defect based on the FE model at 60 kHz. The first column shows the original model with defect depth of 5 mm and the surface diameter of 60 mm (a), the second column, i.e. (b),(d),(f),(h) and (j) present the standard limited view reconstructions with FWI and the third column, i.e. (c),(e),(g),(i) and (k) show FWI with regularization. In (b) and (c), the ratio L/D is 4; (d) and (e) reduce the ratio to 3.2 and (f) and (g) show ratio of 2. In (h) and (i), ratio is equal to 0.8 and (j) and (k) reduce that to 0.4.	83
5.12	The relationship between the ratio L/D and the maximum depth error e_{depth} with a single flat-bottom defect	84
6.1	The attenuation dispersion curves for an aluminum plate with fluid loading. The dash dot line presents the attenuation for a synthetic liquid with the density of 1500 kg/m ³	86
6.2	Configuration of the fluid-tank model with a defect located at the center of the plate and a transducer array.	88
6.3	The experimental setup. An irregularly shaped defect was machined in the aluminum plate.	90
6.4	(a) Contour plot of measured out-of-plane displacement excited in the transducer shown in Figure 6.3. The solid curves present the adaptive time gating according to different propagation distances. (b) Time trace from one of receivers and the signal after time gating.	90
6.5	Thickness reconstructions of a Hann-shaped defect with a surface width of 100 mm and a depth of 3 mm in a 10 mm thick aluminum plate across different frequencies from the FE model. The images along different rows represent reconstruction results at 130 kHz, 150 kHz and 180 kHz, respectively. The images along different columns represent reconstruction results in free plate, water loaded plate with and without considering the attenuation effect. The dashed curves indicate the position of the tank wall.	92
6.6	Cross sections of the thickness reconstructions from Figure 6.5 along the center line at different frequencies of (a) 130 kHz, (b) 150 kHz and (c) 180 kHz.	93
6.7	The maximum depth error of a defect with a width of 100 mm and a depth of 3 mm at varying frequencies. The reconstructions of a free aluminum plate, water-loaded plate and higher attenuation liquid-loaded plate with and without the consideration of the attenuation in the FWI algorithm.	94
6.8	The maximum depth error of a Hann-shaped defect with a fixed depth of 3 mm and varying surface widths across different frequencies in water-loaded plates with the consideration of the attenuation with the S_0 mode.	95

6.9	The maximum depth error of a Hann-shaped defect with a fixed width of 100 mm and varying depths across different frequencies in the water-loaded plates with the consideration of the attenuation with the S_0 mode.	96
6.10	FWI thickness reconstructions of an irregularly shaped defect at 150 kHz. (a) Original model, and (b) the experiment; (c) and (d) zoomed pictures of the defect from (a) and (b). The dotted curve and dashed curve represent the positions of a transducer array and the tank wall, respectively.	97
6.11	Cross sections of the thickness reconstruction of the irregularly shaped defect along the horizontal and vertical line in the experiment from Figure 6.10(c) and (d) at 150 kHz.	98
7.1	(a) Setup of the guided wave tomography measurements of the steel plate. All experiments were performed at room temperature. (b) Time trace and (c) frequency spectrum of a received signal from the experiment without defects in the steel plate. The source was at point 1 and the measurement was taken at point 17.	104
7.2	Experimental setup for measurements, including the mild steel pipe (a diameter of 12 inch and the thickness of 10.3 mm) containing a corrosion site, two ring arrays of transducers and schematic instrumentation.	105
7.3	(a) 3D physical model showing transmitter and receiver arrays and higher order helical paths, and (b) order 0 is the unwrapped section of pipe and other orders are added to both directions along x axis to replicate the cyclic characteristic of the pipe. In the pipe, the measured signals at R_0 correspond to a superposition of signals from each path in the flatten plate. The arrows present various propagation path between a transmitter and a receiver.	106
7.4	Separation of the experimental time traces for one iteration. The measured time traces are replicated two times (from $-1th$ to $+1th$ order) in (a) the empty pipe and (b) the water-filled pipe; (c) and (d) the separated signals extracted by the separation algorithm from (a) the empty pipe and (b) the water-filled pipe, respectively. . . .	108
7.5	(a) Schematic of measurement for the scattering transducer in the center of the array by using the symmetry in this case with one source and 13 monitor points, (b) cross section of modeled transducer, the scattering transducer, in FE model, (c) scattered wavefield obtained by subtracting the incident wavefield of the FE simulation with and without the scatterer in measured point 1, (d) scattered wavefield obtained from the experiment in measured point 1 and (e) the normalized amplitude varying with the measured points in the simulation and the experiment.	110

7.6	Thickness profiles across the reconstructions of a Hann-shaped defect with the surface diameter of 120 mm and the depth of 3 mm from the FE model in the plate without and with an array of modeled transducers. h is the thickness of the coupling layer.	112
7.7	(a) The experimental setup of the forced corrosion and (b) a laser profile scan of the corrosion defect in a 10 mm steel plate.	113
7.8	The comparison of the average reconstructed thickness of the defect obtained by FWI at 60 kHz and predictions by Faraday's law in different corrosion periods of a mild steel plate. The upper and lower limits of measured thickness of defect A and B at corrosion time of 74 and 110 hours obtained by FWI and laser profilometry are shown in the rectangular boxes, respectively. The results of laser profilometry will be explained in Section 7.4.1.2.	116
7.9	Thickness mapping of the corrosion defect A and B in the steel plate. (a) Laser measured profile of defect A after post-processing, (b) thickness reconstruction from FWI at 60kHz and (c) a zoomed-in view of defect A from (b). The dashed curve presents the position of the array. (d) Laser measured profile of defect B after post-processing, (e) thickness reconstruction from FWI at 60 kHz and (f) a zoomed picture of defect B from (e).	117
7.10	Cross sections of FWI reconstruction and laser measurement of (a) defect A and (b) defect B along the vertical line in the plate. . . .	118
7.11	Thickness mapping of an irregularly shaped defect in the empty pipe. (a) The profile of the defect measured by a laser scanner, (b) the FWI reconstruction obtained considering four replicas and (c) the FWI reconstruction obtained considering two replicas at 60 kHz. (unit=mm)	119
7.12	Cross sections of FWI reconstructions of an irregular defect and laser measurement along the line marked in Figure 7.11 along (a) the horizontal line and (b) the vertical line in the empty pipe. (unit=mm)	120
7.13	Thickness mapping of an irregularly shaped defect in the water-filled pipe. (a) The profile of the defect measured by a laser scanner (the same as in Figures 7.11(a)), and (b) the FWI reconstruction obtained considering two replicas with the attenuation in the inversion model at 60 kHz. (unit=mm)	121
7.14	Cross sections of the thickness reconstruction from Figure 7.13 along (a) the horizontal line and (b) the vertical line in the water-filled pipe. (unit=mm)	122
8.1	The work-flow of the truncated Gauss-Newton FWI.	130
8.2	One central inclusion model with a 64 element circular array. . . .	132

8.3	Velocity and density reconstruction of a single central inclusion with the diameter of 100 mm in an aluminum block from the elastic model. The images along different rows represent the velocity reconstruction and the density reconstruction. (a) The true model of velocity and (b) the velocity reconstruction; (c) the true model of density and (d) the density reconstruction.	133
8.4	Cross sections of velocity and density reconstruction of a single central inclusion along $y=500$ mm from (a)-(b) the elastic model and (c)-(d) the acoustic model.	134
8.5	(a) The experimental setup, including the PZT source, laser vibrometer, the sample and rotational stage, and (b) the geometry of the experimental setup from the top view.	136
8.6	(a) Displacement field for the brass/steel cylinder and (b) a typical time trace from one receiver.	137
8.7	The velocity and density mapping of the brass/steel cylinder at 1 MHz in the experiment. Two rows represent velocity and density reconstructions from the multi-parameter FWI.	138
8.8	Cross sections of velocity and density reconstruction along $y= 50$ mm from Figure 8.7.	139

List of Tables

3.1	Material properties	36
5.1	Comparison of the maximum depth errors (e_{depth}) between the re- construction with and without regularization.	84
7.1	Material properties of Alumina and PZT	110
8.1	Conjugate gradient algorithm for solving linear system $Ax = b$	129
8.2	Material properties	136

Chapter 1

Introduction

1.1 Motivation

The use of thickness mapping and density mapping is ever increasing in the fields of petrochemical and nuclear equipments, and forged and powder metallurgy components.

1.1.1 Thickness mapping

Pressure vessels, storage tanks and pipelines have been widely used in petrochemical and nuclear industry. Due to humidity or chemically aggressive environment, these structures are prone to corrosion. Corrosion gives rise to the reduction of the wall thickness and may lead to leakage. Two examples of corrosion on storage tanks are presented in Figure 1.1 [1]. In 2006, a leak from corroded pipe spilled more than 200,000 gallons of oil in Alaska [2], causing serious contamination to the environment and leading to a \$20 million fine to the company involved.

Accurate thickness mapping of large engineering structures containing corrosion damages is critical to assess the integrity and residual life of mechanical components. There are several non-destructive testing (NDT) techniques for the inspection of corrosion in these structures, such as eddy current testing [3], magnetic flux leakage techniques [4, 5], and ultrasonic thickness-gauging methods [6]. All of these

FIGURE 1.1: (a) Corrosion on the bottom of a crude oil storage tank and (b) corrosion on the gas oil aboveground storage tank [1].

approaches are based on local thickness measurements, and thus require direct access to surfaces. However, for the inspection areas with restricted access, such as supports, under coatings or clamps, these methods are difficult to implement without additional effort. An alternative method for quick inspection of large areas is to use ultrasonic guided waves [7, 8], which can propagate along structures from remote locations and offer the potential to inspect structures. While this method can be used to estimate locations of defects, quantitative information about defects is difficult to obtain. Also, the constant group velocity approach is proposed to estimate the average wall thickness loss between two guided wave transducers arranged in the pitch-catch configuration, typically for the distance of a few diameters of the pipe [9]. Although the average thickness may be sufficient to estimate relatively uniform damages, more quantitative information is required to assess the maximum depth of irregularly shaped defects.

More recently, guided wave tomography (GWT), which is an important NDT application for quantitative imaging, has been developed for more accurate sizing of defects in plate-like structures and pipelines [10–14]. The principle of GWT is to image spatially localized wave propagation data, such as the thickness of the waveguide, by converting the measured signals captured by a transducer array around the inspection area. An example of the application of GWT to the plate is shown in Figure 1.2. A widely used approach for the inversion has been the probability-based imaging [15, 16]. It is based on the probability of defect occurrence at a certain point, which is directly related to the severity of the signal change from the

reference. Although such techniques can successfully locate thickness loss, they are unable to accurately size damages. More quantitative tomographic reconstructions can be achieved by using velocity-to-thickness mapping algorithms [11, 12, 17]. These are based on the dispersive nature of Lamb waves, which allows for the mapping of the thicknesses of structures through measuring velocity variations. These quantitative approaches therefore aim to use tomographic methods to quantify the remaining thickness, which is used to evaluate the remnant life of structures.

FIGURE 1.2: The configuration of a circular transducers array for guided wave tomography on a plate.

The main purpose of this thesis is to develop a new quantitative imaging method for GWT. Although quantitative imaging approaches for GWT show the capability to reconstruct defect depths, it is relatively underdeveloped as the majority of quantitative imaging applications are based on ray assumptions. These assumptions ignore diffraction and thus severely reduce the resolution of the reconstruction [18]. Only large and smoothly varying defects can be successfully reconstructed. Better resolution can be achieved by diffraction tomography (DT) because effects of diffraction are taken into account. However, DT is based on the linearized scattering model such as the Born approximation or the Rytov approximation. Under the Born approximation, DT is suitable for small and low contrast defects. The Rytov approximation is limited to simple and low contrast defects with little noise due to the limitation of phase unwrapping [12]. There are also methods such as the combination of bent-ray tomography (BRT) and DT [17], and the Mindlin plate theory for GWT [19, 20] to improve the quality of the reconstruction.

One aim of this thesis is to develop a new quantitative ultrasonic imaging technique in GWT for the thickness mapping of corrosion damages, which addresses the requirement for an accurate and high resolution approach. Accurate thickness reconstruction of corrosion damages allows to evaluate the remnant life of mechanical components. Ideally, the reconstruction can be obtained when the measurements are conducted from all possible angles around the inspection area - a full view configuration. However, in many applications, only the limited range of viewing angles is available. The missing angles lead to artifacts which degrade the image quality. Many solutions have been proposed to solve the limited view problem [21–23], however, the challenge of this new quantitative ultrasonic imaging technique for dealing with the limited view problem needs to be resolved.

1.1.2 Density mapping

The quantitative ultrasonic imaging technique can also be used for quantitative material characterization. Forged components and powder metallurgy components are commonly used in aviation and automotive industry, such as forged engine and powertrain components used in cars, forged bulkheads and shafts for many aircraft, and powder metallurgy parts of gear shift components and pulleys for cars. In the quality control of these components, density is a major parameter that needs to be monitored, as it correlates with their mechanical properties (e.g. hardness and strength) [24]. For example, the fatigue strength of powder metal components is improved by increasing the density. During the stage of compaction process, powders redistribute themselves by flowing into a die cavity. When the pressure increases, the movement of powders is restricted and then the density gradient may occur. If a low density region is around the internal corner, it could be a fatal flaw. Since the corner is typically a stress concentration point when this component is loaded in service [25]. Therefore, the characterization of density distribution is important in quality control.

There are many NDT techniques to characterize density variations. Eddy current (EC) is a NDT technique that can be used to evaluate the near surface density distribution in conductive metal parts. It has been shown that the density can affect the effective electrical conductivity of metal materials, which causes the change in

the EC response [26]. However, only near surface density can be quantified due to the skin depth of the current.

X-ray methods can also be used to characterize the density variation of a component by measuring the transmitted intensity of the X-ray. The image of attenuation can be reconstructed, which depends on the mass density, mass attenuation coefficient and object thickness [27]. Computed tomography (CT) can be combined with X-ray to enable the 3D reconstruction of the object. Similar to X-ray, gamma-ray can also be used for the characterization of the density distribution with even better penetration depth due to its higher energy. It should be noted that both X-ray and gamma-ray methods suffer from expensive systems and radioactive hazards.

A non-hazardous and low-cost alternative for the density characterization is to use ultrasound, as its velocity is dependent on the density and the elastic modulus of the material [28, 29]. The relationship between density ρ and other parameters for a longitudinal wave is expressed as [30]:

$$\rho = \frac{E_l(1 - \nu)}{v_l^2(1 + \nu)(1 - 2\nu)}, \quad (1.1)$$

where v_l is the ultrasonic velocity, E_l is the Young's modulus and ν is the Poisson's ratio. Although the Poisson's ratio depends weakly on the density, the Young's modulus is strong density dependent. Therefore, the coupled effect largely affects the accuracy of density. Also, this method is usually laborious, especially for inaccessible regions.

CT can also be combined with ultrasound [31], which offers a good potential to characterize the density variation without accessing all points on the surface. An ultrasonic tomography method was used to predict density variation of powder metal components by using a filtered back-projection algorithm based on the traveltime of ultrasonic bulk longitudinal waves [32]. Only velocity reconstruction, which serves as an indicator of the density distribution, is directly obtained. The physical properties such as velocity, density and attenuation affect the wave propagation, however, these coupled effects between parameters are not considered in this work. Also, this algorithm is only valid when inhomogeneities are much larger than the wavelength by ignoring diffraction. An ultrasonic DT approach was generalized to reconstruct the density [33]. However, the coupled effects are also not

taken into account in this work. The DT algorithm is limited by linear scattering, and thus it is suitable for small or low contrast inhomogeneities.

The ultrasonic imaging algorithm mentioned in Section 1.1.1, which is based on the general principle, has wide applications. Therefore, the modification to the algorithm is then introduced to simultaneously reconstruct density and velocity. Accurate and high resolution density reconstruction can provide useful information for quality control of engineering structures.

1.2 Outline

A new quantitative ultrasonic imaging approach based on full waveform inversion (FWI) is developed in this thesis. The theory of FWI is outlined and the performance of this quantitative technique for two different applications of thickness mapping and density mapping is investigated. In the application of corrosion mapping, the mono-parameter FWI in guided wave tomography (GWT) has been introduced to achieve accurate and high resolution reconstructions of thicknesses of corrosion damages in plate-like and pipe structures. This algorithm is further demonstrated on liquid-loaded plates and corrosion monitoring of plate-like and pipe structures. In the application of density mapping, simultaneous reconstructions of physical properties of velocity and density in metal parts are carried out by multi-parameter FWI in ultrasonic tomography.

In Chapter 2, a review of ultrasonic bulk waves and tomographic methods used in geophysical and medical imaging is undertaken. After a short introduction to ultrasonic guided waves, tomographic methods combined with guided waves are presented, followed by model selection for GWT. Finally, potential approaches for achieving higher resolution are outlined.

Chapter 3 presents the theory of the mono-parameter FWI using the finite-difference frequency-domain method, including forward modeling as well as inversion method. This method is applied to GWT in this chapter. The performance of the mono-parameter FWI in GWT is investigated by a simple circular hole and an irregularly shaped defect based on both numerical simulations and experiments. The FWI method allows higher order scattering effects to be considered in the model, and thus it could lead to improved resolution.

Chapter 4 shows the resolution analysis of the mono-parameter FWI algorithm in GWT. Simulation studies of smoothly varying defects with different widths and depths across a range of frequencies demonstrate the achievable resolution based on the acoustic model and the more realistic elastic model. The reconstruction of multiple defects is validated by the experiment and compared with the results obtained from diffraction tomography.

In many practical situations, limited access to corrosion structures restricts the number and range of viewing angles. In Chapter 5, an adaptive thresholding regularization approach, which predicts missing components, combined with the FWI algorithm with limited view configurations is used to improve the image quality. This algorithm is validated by both numerical and experimental data obtained from an irregularly shaped defect and two flat-bottom defects. The effect of the array length on the image quality is also discussed in this chapter.

The application of the mono-parameter FWI in GWT on a liquid-loaded plate is presented in Chapter 6. The improved FWI approach with the consideration of attenuation for quantifying the thickness loss is investigated. This study discusses the influence of attenuation on the reconstruction accuracy by using different frequencies and defect dimensions. The validation experiment is performed on a water-loaded plate with an irregularly shaped defect, showing good performance.

Chapter 7 presents a new GWT system based on customized piezoelectric transducers for corrosion monitoring of plate-like and pipe structures. The effect of the scattering from the transducer array on thickness reconstruction accuracy is discussed. The efficiency and the accuracy of this new system have been demonstrated through continuous forced corrosion experiments in plates. The suitability of this system as a practical solution to reconstruct the thickness of the defect in liquid-loaded pipes with two ring transducer arrays, which is a typical limited view problem, is also studied in this chapter.

Chapter 3 to Chapter 7 are within the framework of the mono-parameter FWI in GWT, which provides an attractive solution to map thickness variation of corrosion damages. In Chapter 8, the multi-parameter FWI in ultrasonic tomography is introduced to obtain high resolution reconstruction of velocity and density in metal structures. Compared with the mono-parameter inversion, the multi-parameter

inversion increases the indetermination. In this chapter, the truncated Gauss-Newton method, which is used to estimate the inverse Hessian, can help to mitigate the effects of cross-talk between velocity and density. This algorithm is validated by both simulation and experiment.

Chapter 9 summarizes the contributions, followed by future research directions.

Chapter 2

Literature Review on Ultrasonic Tomographic Methods for NDT

2.1 Introduction

Ultrasonic inspection of engineering structures has been widely used in aviation, automotive and petrochemical industry. Conventional inspection techniques are largely used to determine locations and sizes of defects/inclusions without providing material properties. As the use of high-performance engineering components increases substantially, more detailed information of material properties is needed to assess the failure probability of structures used in these applications. Ultrasonic tomography can be a promising technique to obtain quantitative information of material properties.

In ultrasonic tomography, there are two major steps to be considered: first, quantitative information is extracted from ultrasonic waveforms by different techniques; second, this information must be appropriately processed to produce the mapping of the object parameter by using tomographic methods [34]. The focus of this chapter is to provide a literature review of ultrasonic tomographic methods for non-destructive testing (NDT).

Section 2.2 reviewed ultrasonic bulk waves and tomographic methods. In Section 2.3, after a short introduction to ultrasonic guided waves, tomographic methods based on guided waves were presented, including mode selection for guided

wave tomography. Section 2.4 then outlined potential approaches for improving the resolution, including inverse scattering approaches under the Born approximation and nonlinear inverse methods. Conclusions were summarized in Section 2.5.

2.2 Ultrasonic bulk waves and tomographic methods

2.2.1 Ultrasonic bulk waves

The elastic waves propagating in an infinite isotropic medium have been presented in details in [35–37]. Therefore, only principal equations are summarized here. The equation of motion for isotropic elastic media in the absence of body forces is given by (Navier’s equations)

$$\mu \nabla^2 \mathbf{U} + (\lambda + \mu) \nabla \nabla \cdot \mathbf{U} = \rho \frac{\partial^2 \mathbf{U}}{\partial t^2}, \quad (2.1)$$

where ∇ and ∇^2 are the gradient operator and the Laplace operator, respectively. λ and μ are Lamé constants, $\mathbf{U}$ being 3D displacement vector, and ρ is the density.

The displacement vector $\mathbf{U}$ can be expressed as a superposition of gradient of a scalar potential (longitudinal wave components) ϕ and curl of a vector potential (shear wave components) Ψ by using the Helmholtz decomposition,

$$\mathbf{U} = \nabla \phi + \nabla \times \Psi, \quad \text{where} \quad \nabla \cdot \Psi = 0. \quad (2.2)$$

By substituting equation (2.2) into (2.1) and ignoring body forces, the Navier’s equation can be rewritten as two separate equations: one governs longitudinal waves

$$\frac{\partial^2 \phi}{\partial t^2} = c_l^2 \nabla^2 \phi, \quad (2.3)$$

and the other governs shear waves

$$\frac{\partial^2 \Psi}{\partial t^2} = c_s^2 \nabla^2 \Psi, \quad (2.4)$$

where c_l is the velocity of longitudinal waves and c_s is the velocity of shear waves in the infinite isotropic medium. The velocities are linked to Lamé constants:

$$c_l^2 = \frac{\lambda + 2\mu}{\rho}, \quad (2.5)$$

$$c_s^2 = \frac{\mu}{\rho}. \quad (2.6)$$

Two independent equations of (2.3) and (2.4) mean that longitudinal waves and shear waves travel without interaction in unbounded isotropic media. The general solutions to these two equations are given by

$$\phi = \phi_0 e^{i(k_l \mathbf{r} - \omega t)}, \quad (2.7)$$

$$\Psi = \Psi_0 e^{i(k_s \mathbf{r} - \omega t)}, \quad (2.8)$$

where ϕ_0 and Ψ_0 are initial wave amplitude constants. ω is the angular frequency, t being time and $\mathbf{r}$ is the spatial coordinate of the wave. k_l and k_s represent the wavenumbers of longitudinal waves and shear waves, which can be expressed as

$$k_{l,s}^2 = \frac{\omega^2}{c_{l,s}^2}. \quad (2.9)$$

2.2.2 Tomographic methods

Tomography aims to recover the cross-sectional images of an object from projected data. In ideal cases, the projection corresponds to a set of line integrals of some parameter of the object at a given angle [38]. Tomographic imaging was first introduced by Radon [39] and then successfully performed through the first X-ray computed tomography (CT) scanner by Hounsfield in 1972 [40]. Since then, CT has been widely used in medical imaging [41], and then was applied to geophysical imaging [42] and NDT [43]. This section focused on the review of NDT related applications.

2.2.2.1 Traveltime tomography

Traveltime tomography (TT) method is based on ray model, in which diffraction is ignored when the size of the defect is larger than the wavelength. TT uses arrival times of wave-packets to reconstruct slowness (reciprocal of velocity) distributions. Therefore, accurately extracting the arrival times from the time traces is an important step in TT reconstruction, and various approaches have been developed, including cross correlation, thresholding and wavelet transforms [44–46]. Tomographic methods are then used to generate the velocity map based on direct methods (e.g. filtered back-projection method [38]) and iterative gradient-based methods [47]. Besides straight ray methods, bent ray tomography (BRT) has also been developed by taking refraction into account [48]. As the tomographic problem becomes nonlinear, conjugate gradient methods are usually implemented [49, 50].

The resolution limit of TT depends on the accuracy of the arrival time extraction. When the cross correlation method is used to pick arrival times, the resolution is of the size of the first Fresnel zone $\sqrt{L\lambda}$, where λ is the wavelength of the illuminating wave-field and L is the distance between the source and the receiver [18]. The resolution can be improved when arrival times can be estimated from the signal first break [51], however in practice the accuracy of first break arrival measurements is affected by noise. In general, TT reconstruction works best with objects which are low contrast and smoothly varying, with the size much larger than the wavelength.

2.2.2.2 Diffraction tomography

Diffraction tomography (DT) is based on a linearized scattering model such as the Born approximation or the Rytov approximation [52]. For the discussion here, only the Born approximation is considered. By taking diffraction into account, the resolution limit of DT is significantly improved compared with TT.

However, the Born approximation assumes that the scatterer is small and low contrast, i.e. weakly scattering object, meaning that the phase change between the incident field and the field inside the scatterer is small. Under this approximation, the field scattered by the scatterer can be mapped to the spatial frequency domain Ω , as shown in Figure 2.1. An array is illuminating a scatterer from $\bar{r}_0$ direction and the scattered field is measured in $\bar{r}$ direction (Figure 2.1(a)), which is mapped

to the point $k_0(\bar{r}_0 - \bar{r})$ in the Ω domain (k_0 is the wavenumber), as shown in Figure 2.1(b). As the transmitter location rotates around this circular array, a circle in the Ω domain with the radius $2k_0$ is obtained, namely the Ewald limiting circle [53]. It means that the highest spatial frequency component is $2k_0$, i.e. the resolution of $\lambda/2$. However, when limited access to the scatterer restricts the range of viewing angles, the coverage region is no longer entire circle, leading to the reduction in resolution. In the transmission mode DT, the scattered field contains information only from the transmission zone, which maps in the Ω domain on the transmission arc, and thus the maximum reconstruction resolution reduces to $\sqrt{2}k_0$ [54]. In the reflection mode DT, the reflection zone maps on the reflection arc in Ω domain and the resolution is from $\sqrt{2}k_0$ to $2k_0$ where only high frequencies are available in this coverage region [55]. DT only works with small or low contrast defects where the phase shift traveling through the defect is small.

FIGURE 2.1: Illumination of the object and the resulting scattered field. (a) The object is illuminated from the $\bar{r}_0$ direction and the scattered field is measured from the $\bar{r}$ direction; (b) Under Born approximation, the measured scattered field in $\bar{r}$ direction from the illumination in $\bar{r}_0$ direction is mapped to the point $\Omega = k_0(\bar{r}_0 - \bar{r})$ in the spatial frequency domain.

More recently, Huthwaite *et al.* [56] combined the BRT and DT algorithm and proposed a HARBUT algorithm. This was initially developed for medical applications and then extended for NDT applications [17]. HARBUT algorithm uses the BRT as the background estimation to correct the Born approximation used in DT algorithm. The algorithm can enlarge the range of applicability of DT reconstruction, since the phase distortion through defects can be estimated and taken into account. However, the reconstruction of small and high contrast defects still

suffers degradation. For such defects, iterative HARBUT is used to improve the quality of reconstructions [17], which uses the result of the last HARBUT as the starting model for the next HARBUT stage and repeats this process until the convergence criterion is reached. This hybrid algorithm provides a fast and high resolution reconstruction of ultrasonic velocity, with the same resolution limit as the DT algorithm of $\lambda/2$.

Based on tomographic methods, an ideal reconstruction of ultrasound velocity using full view measurements collected by ultrasonic transducers deployed around the defect can be achieved. However, there are many ultrasonic applications in which such access is not practical. The problem of the reconstruction from incomplete data is ill-posed. The missing components cause noticeable artifacts, which degrade the imaging by varying values of velocity and generating ghost features. Many solutions have been proposed to solve this problem and more details can be found in Section 5.1.

2.3 Ultrasonic guided wave tomography

This section starts with a brief review of ultrasonic guided waves, followed by reviewing the development of guided wave tomography (GWT) methods. As an important part of GWT, model selection criteria are also discussed in this section.

2.3.1 Guided waves

A guided wave is a wave whose energy is concentrated between boundaries and is guided along the waveguide. It may be considered as the superposition of bulk waves reflected back and forth between boundaries of the structure. In plates, there are two possible guided wave types, Lamb waves and shear horizontal waves. The plate represents the waveguide and the elastic wave travels parallel to bounding surfaces. In the direction of thickness (perpendicular to the bounding surfaces), guided waves behave as standing waves, which determine antisymmetric, symmetric and shear horizontal properties. Lamb waves are briefly reviewed in the following because only these are considered in this thesis. More details of Lamb waves can be found in [35, 36, 57].

In an infinite plate, the Rayleigh-Lamb frequency equations can be obtained by solving Navier's equations (2.1) for traction-free boundary conditions on both surfaces of the plate. These equations describe the dependence of the speed of wave on the frequency, which is referred as the dispersion curve. The solutions of these equations are obtained numerically, using transfer matrix or global matrix methods [58, 59]. Figure 2.2 shows dispersion curves of anti-symmetric (A modes) and symmetric Lamb modes (S modes) for an aluminum plate, using a commercially available software package [60]. It can be seen that only fundamental A_0 and S_0 modes exist through all frequencies. The wave modes become complex with increasing frequency-thickness products. When the frequency is fixed, the phase velocity is the function of the thickness, as shown in Figure 2.2(a). These properties of guided waves can be used for the thickness estimation. Also, the phase velocity is associated to elastic constants (e.g. Young's modulus) via the dispersion characteristics of the selected guided mode [61], which can be applied to obtain the distribution of elastic constants.

FIGURE 2.2: Dispersion curves of Lamb modes in an aluminum plate. (a) Phase velocity and (b) group velocity.

2.3.2 Guided wave tomography

The principle of guided wave tomography (GWT) is to image spatially localized wave propagation data, such as thickness and elastic constants of the waveguide, by converting the measured signals captured by a transducer array around the inspected region. Compared with ultrasonic tomography, GWT can exploit the

dispersion characteristics of guided waves, which provide the opportunity to correlate the properties of the waveguide to the velocity map of the selected guided mode. GWT is mainly used to estimate the remnant thickness of the waveguide. The velocity map is achieved by using inversion algorithms, which is converted into the thickness map based on dispersion curves. GWT is also extended to image nonuniformly distributed elastic constants in plate-like structures [61]. The mapping is based on the scattering field caused by the velocity variation of the guided mode when traveling through the area with the change in elastic constants. The velocity reconstruction carried out by inversion approaches is also linked to elastic constants via dispersion relations of guided modes.

This section will focus on the application of GWT for thickness mapping. The majority of quantitative tomographic reconstructions use velocity-to-thickness mapping algorithms. It has been shown that the elastic scattering of guided waves from wall thinning can be described by the acoustic model based on dispersion relations of Lamb modes [11, 12, 17]. When the mode and the frequency are selected, it can be assumed that the interaction of guided waves with the inspected area of wall thinning is modeled as the perturbation obtained by acoustic pressure waves traveling in the medium with varying velocity. The velocity is derived from the defect depth by determining the corresponding phase velocity at that product of frequency and thickness at each point of the defect.

Based on this assumption, acoustic velocity reconstruction algorithms (Section 2.2.2) combined with guided waves have been used to produce thickness maps. McKeon *et al.* [62] and Malyarenko *et al.* [63] used arrival times of low frequency Lamb modes to reconstruct thickness with different acquisitions, such as cross-hole and fan beam. Belanger *et al.* [64] also investigated the feasibility of TT reconstruction using low frequency Lamb modes to estimate the maximum depth of the defect. When the ray assumption is violated, the incident and the scattered signals interfere with each other, affecting the measured guided wave-packets, and therefore the obtained traveltime is inaccurate. In order to improve the resolution of straight ray algorithm, Malyarenko *et al.* [65] carried out a curved ray tracing technique to take refraction into account. In addition, the applicable range of BRT can be enlarged by iterative BRT [51, 66, 67]. However, in order to increase resolution further, it is necessary to take into account diffraction.

Better resolution can be achieved by using guided wave DT for thickness mapping. Belanger *et al.* [11] investigated simple defects under the Born approximation, which relied on weakly scattering objects. Then the HARBUT algorithm was applied to GWT in [17], enabling higher resolution to be achieved for a wider range of defects. This achieves a resolution of around $\lambda/2$ from idealized cases, and the achievable resolution drops to $1.5-2\lambda$ from test data produced by the elastic model [50]. The loss in resolution is caused by the inversion which is based on 2D acoustic model and therefore it cannot fully capture the behavior of the scattering of guided wave in 3D elastic model. In order to further improve accuracy, methods that more fully capture the guided wave scattering are required. The resolution accuracy can be improved by considering density variation due to thickness loss in the acoustic model, but it cannot fully capture the scattering of guided waves [68]. In a more sophisticated method, Mindlin plate theory was applied to GWT [54, 69]. This model can capture the scattering of guided waves and then a high resolution map of thickness is achieved. However, this algorithm is still based on the Born approximation, and assumes a small perturbation in the local property. GWT with an improved scattering model can more accurately capture the scattering behavior and then lead to significant improvement in reconstruction accuracy [70]. It can achieve a resolution better than λ .

2.3.3 Mode selection for GWT

To obtain accurate reconstruction of remnant wall thickness, selection of suitable Lamb modes for GWT is very important. In guided waves, many modes can be used to identify damages in plate-like structures. In some frequency-thickness products, the propagation of multiple modes has almost the same velocity. Also, in high frequency-thickness products, many modes exist simultaneously, which result in complex waveforms. In order to avoid multiple modes overlapping, only fundamental modes of A_0 and S_0 below the cut-off frequency-thickness product are considered in this thesis.

In order to choose an appropriate fundamental Lamb modes A_0 and S_0 for tomography, it is important to understand its characteristics which affect the performance of GWT. More details of A_0 and S_0 used in different GWT applications can be found in [71]. Figure 2.3(a) shows the relationship between the phase velocity and

frequency-thickness product for A_0 and S_0 for a free and one-sided liquid-loaded (in this discussion we use water for simplicity) steel plate. It is clear that the fluid-loading does not affect the velocities much since the curves overlap, suggesting the similar capability of thickness estimation as in a free plate. Most suitable regions for the thickness estimation are the points on dispersion curves where the velocity changes rapidly with the thickness. Such sensitive region for the A_0 mode exists below around 1.5 MHz-mm (between 0.2-1.5 MHz-mm if under water loading), while at higher frequencies the curve becomes more flat and thus losing its sensitivity. In contrast, the S_0 mode has practically no dispersion at low frequencies and the sensitive region is around 1.3 to 2.1 MHz-mm. The group velocities are shown in Figure 2.3(b). There exists a range in the curves, where S_0 is much faster than A_0 , and thus can be separated easily.

FIGURE 2.3: The effect of one-sided water-loading on a steel plate. (a) Phase velocity, (b) group velocity and (c) attenuation dispersion curves for a steel plate without and with liquid loading presenting on one side of the steel plate, represented by solid and dash lines, respectively.

The attenuation of fundamental modes is presented in Figure 2.3(c). The vertical

axis is given as the product of thickness and attenuation to present the results for the plate of arbitrary thickness. Large attenuation of the A_0 mode prevents its use for long range GWT in a liquid-loaded case. The preferred mode is the S_0 mode which has very small attenuation at low frequencies. However, as the frequency increases, the attenuation of S_0 also gradually increases and becomes larger than that of A_0 above 2.2 MHz-mm.

In summary, the advantage of A_0 is the steep gradient of phase velocity in the low frequency-thickness product region, which results in a greater velocity variation with the thickness, thus a better sensitivity. Compared with the A_0 mode, the S_0 mode can propagate faster below 2.2 MHz-mm, making it easier to separate measured signals and avoid reflections from boundaries in some cases. When liquid loading is present, the S_0 mode performs better than the A_0 mode due to the significantly lower attenuation below 1.5 MHz-mm. It needs to be mentioned that these results are also valid for pipes with large diameters for which the curvature of the wall can be neglected [72–74].

2.4 Potential approaches for higher resolution

The resolution of most imaging techniques is imposed by the wavelength which could be propagated based on diffraction limit [75, 76]. This limits the possibility of obtaining subwavelength resolution. Therefore, to improve the resolution, the wavelength has to be reduced. However, as the wavelength decreases, the attenuation of waves increases, leading to the reduction in the maximum imaging depth [76]. In this section, some potential imaging approaches for achieving higher resolution were presented, including the inverse scattering approach under the Born approximation and non-linear inverse methods.

2.4.1 Inverse scattering approach under the Born approximation

In inverse scattering problems, the incident field is known and the measured scattered field is obtained, however the object function is unknown. Therefore, this problem is to determine the object function based on the measured scattered field.

Reconstruction of the object function can generate the image of object properties, thus this problem can be considered to produce the image of the object from the scattered field [77].

The inverse scattering problem is ill-posed. Under the Born approximation, the forward scattering problem ignores the effects of multiple scattering, and thus this problem becomes linear and the inverse problem is also a linear problem about the object function. This approximation leads to one-to-one relationship between the scattering amplitude and spatial Fourier transform of the object function, which is limited to Ewald limiting circle of radius $2k_0$ in the Ω domain, as presented in Figure 2.1. Therefore, the determination of the object function is only based on the information within this circle. The information outside this circle contained in the object function is not taken into account under the Born approximation. A potential method for dealing with this is to set the information outside this circle to zero, as follows:

$$\tilde{O}_{ba}(\Omega) = H(\Omega)\tilde{O}(\Omega), \quad (2.10)$$

where $\tilde{O}_{ba}(\Omega)$ is the Born approximation of the object function $\tilde{O}(\Omega)$ in Ω domain. $H(\Omega)$ presents a low-pass filter, $H(\Omega) = 1$ for $|\Omega| \leq 2k_0$ and $H(\Omega) = 0$ for $|\Omega| > 2k_0$. This means that spatial frequencies larger than $2k_0$ are not recovered, thus the classical resolution limit is $\lambda/2$.

Equation (2.10) implies that $\tilde{O}_{ba}(\Omega)$ is defined over the entire spatial frequency domain Ω . Then the inverse Fourier transform of $\tilde{O}_{ba}(\Omega)$ is taken to achieve the reconstructed image of $O_{ba}(r)$ in the geometric space, which is given by the convolution of the point spread function $h(r)$ and the real object function $O(r)$ [78],

$$O_{ba}(r) = \int_D d^2r' h(r - r') O(r'), \quad (2.11)$$

where

$$h(r) = k_0 \frac{J_1(2k_0|r|)}{\pi|r|}. \quad (2.12)$$

$h(r)$ is the inverse Fourier transform of low-pass filter $H(\Omega)$ and J_1 is the Bessel function of the first order. $O_{ba}(r)$ is a low-pass filter version of $O(r)$ without spatial frequencies larger than $2k_0$.

Under the Born approximation, one potential solution for obtaining the super resolution is based on the analytic function $\tilde{O}(\Omega)$ (see Equation (2.10)). This function $\tilde{O}(\Omega)$ can be extrapolated to points outside Ewald limiting disc by using analytic continuation [79], and the inverse Fourier Transform of the data can be taken to obtain the exact object function instead of the low-pass filtered version [77]. It is equivalent to deconvolution of $O(r)$ in Equation (2.11) based on the information of the point spread function. In this way, it can achieve unlimited resolution. However, several authors [80–82] have demonstrated that analytic continuation is severely unstable and sensitive to the noise, and thus stable solutions to inverse scattering problems cannot be provided.

The other option for achieving super resolution under the Born approximation is to include information of object properties (such as the structure or material properties) combined with regularized data. However, this information is not usually known in advance.

2.4.2 Non-linear inverse methods

Reconstructing the object function by considering the effects of multiple scattering is an ill-posed non-linear problem. Two main approaches for dealing with the non-linear inverse problem, including linear sampling methods (LSM) and iterative methods, were presented [83, 84].

2.4.2.1 Linear sampling method

Colton *et al.* [85] have proposed the linear sampling method (LSM), which replaces a non-linear inverse problem with a linear integral problem. LSM reconstructs the object shape from the far-field T -matrix operator $T_\infty: L^2(s) \rightarrow L^2(s)$,

$$T_\infty|y\rangle = \int_s ds(\bar{r}_0) f(k_0\bar{r}, k_0\bar{r}_0) y(k_0\bar{r}_0), \quad (2.13)$$

where L^2 is the Hilbert space; s represents the unit shell and the ket symbol is used based on Dirac bra-ket notation [79]. $f(k_0\bar{r}, k_0\bar{r}_0)$ is the scattering amplitude, which is obtained by illuminating the object from the incident directions r_0 and detecting scattered field in the r directions (see Figure 2.1). In LSM, the scattering

amplitude includes the multiple scattering effects. $y(k_0\bar{r}_0)$ represents the incident wave amplitude.

LSM considers the solution to this equation

$$T_\infty|y\rangle = |g_\infty(\bar{r}, z)\rangle, \quad (2.14)$$

where $g_\infty(\bar{r}, z) = e^{-ik_0\bar{r}\cdot z}$ is the steering function and proportional to the far field pattern of the source at a point z in $\bar{r}$ direction. The $L^2(s)$ norm of the solution $\sqrt{\langle y_z|y_z\rangle}$ becomes unbounded as the point z goes towards the boundary ∂D of the support D (i.e. volume occupied by the object) of the object function [78]. Therefore, ∂D is obtained by estimating $\sqrt{\langle y_z|y_z\rangle}$ and expressed as the locus of points z where $\sqrt{\langle y_z|y_z\rangle}$ increases significantly.

Later, Factorization Method (FM) [86] is a modified version of LSM, which considers the following equation:

$$(T_\infty^\dagger T_\infty)^{1/4}|y\rangle = |g_\infty(\bar{r}, z)\rangle, \quad (2.15)$$

where $T_\infty^\dagger$ is the adjoint of T_∞ and $T_\infty^\dagger T_\infty$ represents the time reversal operator [87]. There exists z belonging to the support D if and only if Equation (2.15) is solvable. Based on Picard's theorem [88], Equation (2.15) is solvable if and only if

$$\sum_{j=1}^{\infty} \frac{1}{|\mu_j|} |\langle g_\infty(\bar{r}, z)|v_j\rangle|^2 < \infty, \quad (2.16)$$

where μ_j and v_j are eigenvalues and eigenfunctions of T_∞ . Similarly as in LSM, the support D is the locus of points z for which Equation (2.16) is finite,

$$FM(z) = \left(\sum_{j=1}^{\infty} \frac{1}{|\mu_j|} |\langle g_\infty(\bar{r}, z)|v_j\rangle|^2 \right)^{-1} > 0. \quad (2.17)$$

FM applies Equation (2.17) to each coordinate of the area of interest. If a coordinate is the part of the support of the object D , this equation will be non-zero; otherwise it will be zero. Therefore, D will be presented by points with non-zero values. According to Equation (2.17), it can theoretically achieve the reconstruction of the object shape in unlimited resolution by FM. In the presence of noise,

FM has been shown to obtain super resolution of $\lambda/3$ by the real experiment [78], as shown in Figure 2.4.

In LSM and FM, no assumption is made about the Born approximation and thus multiple scattering within the object is taken into account, which leads to super resolution [78]. However, LSM and FM cannot be used to reconstruct the material properties of the object. Also, they are sensitive to noise and experimental uncertainties.

FIGURE 2.4: Super resolved images at 42 kHz around two scatterers. (a) FM and (b) cross section of FM pseudo spectrum along the horizontal centre line [78].

2.4.2.2 Iterative methods

The iterative algorithms are optional approaches to solve the non-linear inverse scattering problems. The measured scattered data are used to construct approximation solutions to inverse scattering problems. Then the calculated scattered field from the approximation solution in the forward problem is obtained and compared with the measured scattered field. The comparative results are used to update the approximation. The process is repeated until calculated data converges to measured data.

Distorted Born iterative method

The distorted Born iterative method (DBIM) calculates iterative solutions to the non-linear inverse scattering problem by using successive linear approximations [89,

90], which allows multiple scattering effects to be considered. In order to better understand DBIM, the main principle of Born iterative method (BIM) is also presented here. More details of DBIM and BIM can be found in [89, 91].

When the incident field (pressure) is $U_{inc}(r)$, the scattered field $U_{sca}(r)$ can be written as the integral equation [92]

$$U_{sca}(r) = \int_S dr' G_{reen0}(k_{bac}, r - r') O(r') U(r'), \quad (2.18)$$

where the total field $U(r)$ is equal to $U_{inc}(r) + U_{sca}(r)$ in the inhomogeneous area S and $G_{reen0}(k_{bac}, r - r')$ represents Green's function for the background. k_{bac} is the wave number and $O(r)$ represents the object function. Equation (2.18) can also be given by

$$U(r) = U_{inc}(r) + \int_S dr' G_{reen0}(k_{bac}, r - r') O(r') U(r'). \quad (2.19)$$

Equation (2.19) is a non-linear integral expression of the object function $O(r)$. Solving $O(r)$ based on the known scattered field is the inverse problem.

The basic principle of BIM is presented first. The unknown $O(r)$ is assumed to be zero, corresponding to $U(r) = U_{inc}(r)$. Then, the total field $U(r)$ in the integrand in Equation (2.19) is replaced by $U_{inc}(r)$, and $O(r) \simeq O_0(r)$ is solved. Next, $O(r)$ is substituted by $O_0(r)$ in the integrand and $U(r)$ is obtained. $O(r) \simeq O_1(r)$ is solved based on the obtained total field $U(r)$. By repeating this process until the convergence criterion is achieved. In all iterations, Green's function G_{reen0} is unchanged in BIM. In DBIM, the difference is $G_{reen0}(k_{bac}, r - r')$, which is renewed to $G_{reen}(k_1, r - r')$ in the first iteration and then to $G_{reen}(k_2, r - r')$ in the second iteration, and so on [92]. The k_1 and k_2 correspond to the object function O_1 and O_2 . It needs to be noted that the total field is updated at each run based on the last obtained object function. In summary, in BIM, the background is fixed and only the total field is renewed; in DBIM, the background is not limited to be homogeneous and is updated at each run, thus both the total field and Green's function are updated.

DBIM can achieve a super resolution, which is attributed to effects of multiple scattering within an inhomogeneous object [89]. The information of the high resolution of the object is carried by evanescent waves when single scattering physics is taken

into account. The conversion occurs between evanescent waves and propagating waves in the multiply scattered field. Therefore, the measured scattered waves have the information of the high resolution of the object due to this conversion. With the consideration of multiple scattering, DBIM can obtain these information of the object. The super resolution in DBIM has been validated by the experiment [89], as shown in Figure 2.5. However, this method is computationally expensive because it needs to repeatedly compute solutions to each of required forward scattering problems.

FIGURE 2.5: (a) Permittivity reconstruction of a plastic pipe with the outer diameter of 4.8 cm and the inner diameter of 3.7 cm by using distorted Born iterative method (DBIM). (b) Reconstruction of this plastic pipe by using Born approximation [89].

Markov-Chain Monte Carlo algorithm

A Bayesian method was used to reconstruct subwavelength defects from their scattered fields in elastic waveguides [93]. The main idea of this method is to match the simulated scattered field to the measured scattered field by using a Markov-Chain Monte Carlo (MCMC) algorithm [94] combined with efficient models. The output data of the model is obtained by using an analytical model [95] and/or an efficient finite element model [96], which are used to estimate the full 3D geometry of subwavelength defects in elastic waveguides. Then statistical information of defect properties (e.g. shape, depth and size) can be obtained. Compared with velocity inversion approaches, this algorithm can capture guided wave scattering, rather than acoustic scattering, and thus it can achieve higher resolution accuracy.

This MCMC algorithm has been shown to accurately reconstruct different sub-wavelength defects via both simulations and experiments. In addition, accurate reconstruction can also be obtained when partial data on the scattered field are used, as shown in Figure 2.6 [93]. However, this MCMC algorithm is low efficient because usually thousands of iterations are required, and also dependent on the efficiency of the forward model.

FIGURE 2.6: (a) Photography of a real defect, (b) estimated defect profile from the finite element model and (c) cross section of (b) along $X=0$ mm (solid line) and the standard deviation (dotted lines) [93].

Full waveform inversion

Full waveform inversion (FWI) is an iterative optimization approach which yields high resolution models [97–99]. FWI is to estimate model parameters by fitting the modeled data to the observed data. The modeled data are computed by solving a full-wave equation. Ray assumptions used in TT are avoided, and higher order diffraction and scattering are automatically taken into account in FWI to improve the resolution when compared with DT limited by linear scattering.

In FWI method, the forward model can be performed in time or frequency domain. The time domain FWI, covering in principle the whole frequency spectrum, provides a precise solution and can be easily run in parallel [97, 100, 101]. Temporal windowing in the time domain FWI can be used to reduce the computation. By contrast, multiple sources are easier to be solved in frequency domain FWI because the forward modeling operator can be reused at each iteration [98, 102, 103]. Also, the multi-scale strategy can mitigate the nonlinearity of the inverse problem through moving from low to high frequencies, and therefore, more likely to obtain the global optimization [104]. Finally, dispersion and attenuation can be easily

incorporated through complex velocity without extra cost in the formulation [45]. For these reasons the frequency domain FWI will be considered in this thesis.

2.5 Summary

A brief overview of bulk waves, which can propagate in two types of longitudinal waves and shear waves that are uncoupled in an infinite medium, is presented. Then the literature review describes two main tomographic methods used in geophysical and medical imaging. Travetime tomography (TT) based on the ray assumption is suitable for objects with size larger than the first Fresnel zone. Bent-ray tomography (BRT) can achieve a slight resolution improvement by accounting for refraction. Diffraction tomography (DT) takes into account diffraction and thus improves the resolution limit over TT. However, DT is limited to the Born approximation, which is only valid for small or low contrast defects.

Ultrasonic guided wave tomography (GWT) uses guided waves, which can be considered as a superposition of bulk waves reflected back and forth between boundaries of the structure, along with tomographic methods. GWT provides a solution to quantify thickness losses by corrosion. The dispersion characteristics of selected guided modes can be exploited to achieve thickness reconstructions to be generated from velocity reconstructions based on acoustic velocity inversion algorithms. Due to the presence of multiple modes, it is necessary to select suitable modes for GWT.

To obtain higher resolution, a review of potential approaches is presented, including the inverse scattering approaches under the Born approximation and non-linear inverse methods. Under the Born approximation, the inverse scattering problems become linearization by ignoring multiple scattering effects. Analytic continuation can be used to extrapolate the object function to the exterior of the Ewald limiting circle, thus achieving unlimited resolution. However, analytic continuation is not stable and is very sensitive to noise. To solving the non-linear inverse problem, linear sampling methods (LSM) and iterative methods are discussed. A modified version LSM known as Factorization Method (FM) is described. FM, which considers multiple scattering effects, can achieve a super resolution image of the object shape. However, FM cannot be able to provide the material properties of

the object and is sensitive to noise. Then, three iterative approaches are discussed, including distorted Born iterative method (DBIM), Markov-Chain Monte Carlo algorithm (MCMC) and full waveform inversion (FWI). Although DBIM can achieve super resolution in the experiment, it is very computationally expensive. MCMC in GWT can capture guided wave scattering behavior to improve the resolution, but it usually needs thousands of iterations for the convergence. FWI overcomes the limitation imposed by ignoring crucial low frequency effects in TT. Compared with other tomography methods which are limited by linear scattering, FWI allows higher order diffraction and scattering to be considered in its numerical solver, thus it is possible to capture more of the wave scattering physics which could lead to more accurate inversion results. The FWI algorithm will be investigated in this thesis.

Chapter 3

Ultrasonic Tomography Based on Full Waveform Inversion

3.1 Introduction

A quantitative ultrasonic imaging technique based on full waveform inversion (FWI) is developed in this thesis. Corrosion mapping and quality control are two important applications which benefit from this quantitative imaging method based on FWI. In the application of corrosion mapping, the mono-parameter FWI in guided wave tomography (GWT) is proposed to reconstruct the remaining thickness of corrosion damages in plate-like and pipe structures based on velocity-to-thickness mapping employing the dispersive curves. Chapter 3 to Chapter 7 are within the framework of the mono-parameter FWI in GWT. In the quality control application, the multi-parameter FWI in ultrasonic tomography is developed for simultaneous reconstructions of velocity and density of metal parts. More details about the multi-parameter FWI in ultrasonic tomography will be presented in Chapter 8.

In this chapter, the mono-parameter FWI in GWT is introduced to reconstruct the velocity map from full view configurations and then convert this to thickness based on dispersion curves. The theory of the mono-parameter FWI based on the finite-difference frequency-domain method is introduced in Section 3.2, including the forward modeling as well as the inversion method. Section 3.3 presents the numerical simulations based on a simplified acoustic model using finite difference (FD) method and on a more realistic elastic model using finite element

(FE) method. The choice of frequencies and the calibration method are also introduced in this section. Experimental setup and data processing are demonstrated in Section 3.4. The performance of the mono-parameter FWI including its sensitivity is discussed in Section 3.5 by two representative examples. The discussion on multi-scale inversion and computational aspects is followed in Section 3.6.

3.2 Theory

3.2.1 Forward modeling in the space-frequency domain

The frequency domain 2D acoustic wave equation in the constant density media can be given by

$$(\nabla^2 + K^2)p(x, y, \omega) = -s(x, y, \omega), \quad (3.1)$$

where $p(x, y, \omega)$ is the pressure (or displacement) wave-field, $s(x, y, \omega)$ being the source, ω being the angular frequency and $K(x, y, \omega) = \omega/v(\omega)$ is the wavenumber linked with the phase velocity v .

Equation (3.1) is discretized with the FD method using the mixed-grid approach [105–107]. Repeating this method at all grid points leads to a large system of linear equations. In order to solve these equations, they are rewritten in the matrix form

$$AP = S \quad \text{or} \quad P = A^{-1}S, \quad (3.2)$$

where the complex-valued impedance matrix, also the forward modeling operator, is given by $A = \nabla^2 + K^2$, which depends on frequency and physical properties of the media. We now introduce 2D discretization, as presented in Figure 3.1. The pressure wave-field is calculated at $l = n_x \times n_y$ grid points on the 2D regular grid, where n_x and n_y represent the number of grids in the horizontal and the vertical direction, respectively. The pressure wave-field P and the source term S at one frequency are mapped into $l \times 1$ column vectors, and the complex-valued impedance matrix A is a $l \times l$ matrix.

Equation (3.2) is often solved by the direct method of LU factorization. After LU decomposition of the A matrix, the factored A matrix can be reused to solve the

FIGURE 3.1: A schematic discrete representation of the $l = n_x \times n_y$ finite difference grid points for forward modeling.

forward problem for other source vectors, so that the multiple sources problem can be efficiently computed. This plays a key role in iterative solutions of inverse problems, because the forward solutions for real and “virtual” sources (explained later) are needed at each iteration. Equation (3.2) can be given as

$$\text{LU}[P_1 P_2 \cdots P_n] = [S_1 S_2 \cdots S_n], \quad (3.3)$$

where n is the number of sources.

The absorbing boundaries along the model edges can be utilized to avoid the reflections from edges [107–109], which allows the reduction of the model size.

3.2.2 Inverse problem in the space-frequency domain

Figure 3.2 outlines the structure of the FWI algorithm. The multi-resolution nature of the reconstruction is controlled by loop 1 over several discrete frequencies starting from low and moving towards higher frequencies. At each frequency several iterations are performed, corresponding to loop 2. At the end of each iteration at a given frequency, a new model parameter is calculated and is reset as the initial model for the next iteration until a predefined maximum iteration number is reached. This number is chosen between 20 to 40 in this study. At each iteration, the residual data (the difference between computed results from the current

FIGURE 3.2: Structure of the mono-parameter FWI algorithm.

model and the observed data from experiments) is minimized in the sense of least squares. The velocity map obtained from the last iteration of the current frequency becomes the initial model for the next frequency. This process is repeated until the convergence criterion is reached at the highest frequency. The convergence criterion will be discussed in details in Section 3.3.4. The aim of the inversion is to create a set of model parameters $\mathbf{m}$ which can reproduce the observations by using the forward modeling. Such model parameters consist of the values of the squared slowness [104]. The theory of the FWI has been introduced in details in seismology [102, 110, 111], thus only principle equations are given here.

The weighted least squares norm of the objective function, i.e. the L_2 norm of the data residuals, is defined as

$$C^{(k)}(\mathbf{m}) = \frac{1}{2} \Delta \mathbf{d}^\dagger \mathbf{W}_d \Delta \mathbf{d}, \quad (3.4)$$

where $\Delta \mathbf{d} = d_{calc}^{(k)} - d_{obs}$ is the data residual (difference between data $d_{calc}^{(k)}$ calculated in the current model $\mathbf{m}^{(k)}$ and the observed data d_{obs}). The superscript $\dagger$ represents the transposed conjugate. $\mathbf{W}_d$ is a weighting vector that is used in the data residual to scale the relative contribution of each component in the inversion [112], and k is the iteration number.

The inverse problem is to minimize the objective function, and it can be computed by using the gradient

$$G^{(k)} = Re\{\mathbf{J}^t \mathbf{W}_d \Delta \mathbf{d}^*\}, \quad (3.5)$$

where $\mathbf{J}^t$ is the transpose of the Fréchet derivative matrix (i.e. Jacobian matrix, $\mathbf{J} = \frac{\partial \mathbf{P}}{\partial \mathbf{m}}$). $\Delta \mathbf{d}^*$ is the conjugate of the data residual and Re is the real part of a complex number [111].

Furthermore, in order to derive the expression for any of partial derivatives $\frac{\partial \mathbf{P}}{\partial \mathbf{m}}$ in equation (3.5), I take the partial derivatives of both sides of the forward equation (3.2) with respect to the i th model parameter $\mathbf{m}_i$

$$\mathbf{A} \frac{\partial \mathbf{P}}{\partial \mathbf{m}_i} = -\frac{\partial \mathbf{A}}{\partial \mathbf{m}_i} \mathbf{P}. \quad (3.6)$$

A “virtual” source term $f^{(i)} = -\frac{\partial A}{\partial m_i}P$ can be introduced as

$$\frac{\partial P}{\partial m_i} = A^{-1}f^{(i)}. \quad (3.7)$$

An analogy between equation (3.2) and equation (3.7) indicates that partial derivatives $\frac{\partial P}{\partial m}$ are the solutions of a new forward model of the “virtual” source $f^{(i)}$. Thus, the link between the gradient vector (including the partial derivatives, $\frac{\partial P}{\partial m}$) and a new forward modeling can be established.

Since the equation similar to equation (3.7) for any value of i can be generated, all partial derivatives by the matrix equation can be given by

$$J = \left[\frac{\partial P}{\partial m_1} \frac{\partial P}{\partial m_2} \cdots \frac{\partial P}{\partial m_q} \right] = A^{-1}[f^{(1)}f^{(2)} \cdots f^{(q)}], \quad (3.8)$$

where q is the number of model parameters, $q \leq l$. The computation of elements of J requires to solve q forward-propagation problems using virtual sources.

Thus, in order to obtain the gradient using equation (3.5), it is unnecessary to calculate elements of partial derivatives J directly. Substituting equation (3.8) and $f^{(i)}$ into equation (3.5), it is obtained

$$G^{(k)} = Re \left\{ -P^t \left[\frac{\partial A^t}{\partial m_i} \right] A^{-1t} W_d \Delta d^* \right\}. \quad (3.9)$$

Since A^{-1} is symmetric in the acoustic problem, i.e. $[A^{-1}]^t = A^{-1}$, equation (3.9) can be given by

$$G^{(k)} = Re \left\{ -P^t \left[\frac{\partial A^t}{\partial m_i} \right] A^{-1} W_d \Delta d^* \right\}, \quad (3.10)$$

where $A^{-1}W_d\Delta d^*$ is defined as backward propagated wave-field. The matrix $\frac{\partial A^t}{\partial m_i}$ can be easily computed by using the coefficients of the matrix A . Based on the reciprocity principle, the gradient is computed by zero-lag convolution of the forward-modeled wave-field P with the backward-propagated residual wave-field $A^{-1}W_d\Delta d^*$ (i.e. the adjoint-state method). It means that only two forward models per source are required. The first forward problem is to obtain the wave-field P for a source location. The second forward problem calculates the backward-propagated residual wave-field using a “composite” source formed by assembling data residuals.

To obtain reliable and stable results, the regularization and scaling are applied to the gradient equation (3.10). It can be modified to

$$G^{(k)} = (\text{diag}H_a + \epsilon I)^{-1} \mathcal{G}_{2D} \text{Re} \left\{ -P^t \left[\frac{\partial A^t}{\partial \mathbf{m}_i} \right] A^{-1} W_d \Delta \mathbf{d}^* \right\}, \quad (3.11)$$

where $\text{diag}H_a = \text{diagRe}\{J^t W_d \Delta J^*\}$ indicates the diagonal elements of weighted approximate Hessian H_a ; ϵ is the damping factor and $\mathcal{G}_{2D}$ is the spatial smoothing operator.

The diagonal of H_a provides a proper preconditioner for the gradient which scales the tomographic model. The diagonal element of H_a is the scalar product of two partial derivative wave-fields. Furthermore, as the scatterers are removed from the source position, the amplitudes of the partial derivative wave-fields and the corresponding elements of H_a will decrease. Dividing data residuals by these squared amplitude terms is equivalent to remove effects of geometrical amplitudes decreasing of partial derivative wave-fields from data residuals [112]. The damping factor ϵ is added to the diagonal elements of H_a , which can stabilize the inversion process because the H_a may be ill conditioned or singular. The smoothing operator $\mathcal{G}_{2D}$ is in the form of a 2D Gaussian spatial filter in this thesis and its correlation length is adapted to inverted frequency components.

Finally, the model parameter vector is updated iteratively according to

$$\mathbf{m}^{(k+1)} = \mathbf{m}^{(k)} - \alpha^{(k)} g^{(k)} \quad (3.12)$$

where α is a scalar step length and g indicates the search direction of the objective function. In this thesis, the inverse problem is carried out by the iterative linearized approach using steepest-descent algorithm, which means that the direction of the gradient is the search direction $g^{(k)} = -G^{(k)}$. Moreover, the step length can be obtained to minimize the objective function along the search direction [113].

TABLE 3.1: Material properties

Material	Young's modulus (GPa)	Poisson's ratio	Density (kg/m ³)
Aluminum	70	0.33	2700
Steel	210	0.287	7850

3.3 Numerical modeling

3.3.1 Elastic model based on FE method

FE simulations were carried out on a 10 mm thick aluminum plate with the size of 1100×1100 mm² and the material property is given in Table 3.1. Cubic shape eight node elements with the size of 1 mm were used in the mesh which ensured that more than 30 elements per wavelength at the highest frequency was exploited for accurate modeling [114]. The corrosion patches with desired shapes were modeled by removing the elements from the mesh. The plate was surrounded by absorbing region to avoid reflections coming from the edges [115]. The waves were monitored by a circular array with a diameter of 700 mm consisting of 64 nodes acting as transducers as shown in Figure 1.2. Out-of-plane displacement was applied in one of the nodes and the wave-fields were measured by the other 63 nodes. This was repeated for all the source-receive combinations resulting in a 64×63 signal matrix. The source excitation used in simulations was a 5 cycle Hann-windowed tone-burst signal at a central frequency of 50 kHz, which had around 15 dB bandwidth from 35 to 65 kHz. By using the normal force at this frequency it ensures that nearly pure A_0 mode is generated [116]. Also, around this frequency the A_0 mode is highly dispersive and thus sensitive to thickness change, as shown in Figure 2.3(a). More details about the mode selection for guided wave tomography have been discussed by Huthwaite [71].

3.3.2 Acoustic model based on FD method

FD simulations were performed in a 2D space domain without considering the thickness of the plate. Under this approximation, the 2D acoustic model approximates the behavior of dispersive guided waves by using the equivalent velocities in the model. The thickness variations in the plate are described by the respective velocity variations calculated from the dispersion relations characterizing the

guided wave in the model. In this model, mode conversions do not occur. This is consistent with the 3D FE model, where the thickness variations of the defect are smooth, thus causing less mode conversions [117]. However, in the case of sharp discontinuities in the wave-guide, the mismatch between the two models increases which could result in larger thickness reconstruction errors. Plate with the same dimension as in FE models was meshed by grid points with 2 mm spacing. This guarantees the calculation accuracy which requires at least 4 grid points per shortest wavelength [107]. The plate was surrounded by absorbing areas with the same width as in FE models. Omni-directional pressure waves were excited and monitored with the similar array configuration as described in the FE setup.

3.3.3 Data processing

As the inversion algorithm is based on the FD acoustic modeling, the input scattered signals from the FE elastic modeling should be calibrated to account for possible deviations between the two models. One way to reduce the amount of phase and amplitude errors of the input is to match wave propagation results of two homogeneous models (without scatterers) by introducing the calibration factor defined as [17]

$$Q = \frac{fft^*(B_{FD})}{fft^*(B_{FE})}, \quad (3.13)$$

where fft represents the fast Fourier transform, $*$ being the complex conjugate, and B_{FD} and B_{FE} are the data from receivers in homogeneous models by FD and FE methods, respectively.

Additionally, the Gaussian filter is used to smooth the background before subsequent iterations. The aim of this filter is to minimize the effects of artifacts from each iteration. The selection for correlation length of the Gaussian filter is around $\lambda/2$.

After the inversion the reconstructed velocity map from FD and FE models are converted to thickness maps by the known dispersion relationship between phase velocity, thickness and frequency.

3.3.4 Frequency selection and convergence

The iteration over sequential frequencies enables the inversion to start from a homogeneous velocity model and moving towards the desired high-resolution true velocity model. At each frequency the iterated model should provide the updated model with a close neighborhood to the global minimum for the next frequency. Global minima can be found more easily at lower frequencies where the velocity model resembles to the homogeneous background model [118]. However, the well-defined strategy for selecting appropriate frequencies in waveform tomography has not yet been established [119]. Here we propose the largest frequency step between the frequency iterations according to the wavenumber k distribution of the scattered wave field from diffraction tomography [120]. The scattered field contains the transmission part which is limited within the circle of radius $\sqrt{2}k_1$ in the K space and the reflection subset which is contained between $\sqrt{2}k_1$ and $2k_1$ [11], as shown in Figure 2.1. Transmission wavenumbers at the lower frequency f_1 are always included in the larger wavenumber subset at the frequency f_2 within the circle of radius $\sqrt{2}k_2$. In order to account for all the reflection wavenumbers in the inversion, the relationship between two subsequent modelings at wavenumbers k_1 and k_2 should satisfy the relation $k_2 \leq \sqrt{2}k_1$. Thus for the inversion based on non-dispersive waves the relationship between the frequencies is directly $f_2 \leq \sqrt{2}f_1$. This is slightly a stricter criterion for the current study as the wave propagation is dispersive and the growing rate of wavenumber in frequency is slower than non-dispersive waves.

The convergence of the results over iterations is examined by comparing the average relative thickness change E around the thinnest area with respect to the nominal thickness, similarly as in [17],

$$E_j^{(i)} = \frac{\overline{|T_j^{(i)}(x, y) - T_j^{(i-1)}(x, y)|}}{T_0}, \quad (3.14)$$

where $T_j^{(i)}(x, y)$ is the thickness at location (x, y) for the current iteration i at j th frequency component, and T_0 is the nominal thickness. The value of $E \leq 10^{-2}$ is accurate for the thickness estimation in engineering applications. However, the threshold for stopping the inversion is set to be $E \leq 0.5 \times 10^{-3}$ which is a stricter limit due to the slow conversion rate of the FWI algorithm. It should be noted that at lower frequencies a fixed number of iterations are carried out and the value

E is approaching the convergence, while the above convergence criterion is only checked at the highest frequency. There are possibilities to optimize the choice of the frequencies and the number of iterations, but this is beyond the scope of the current study.

3.4 Experiments

The experimental setup is presented in Figure 3.3(a). Experimental measurements were performed on two 1100 mm×1100 mm×10 mm aluminum plates. A flat-bottomed circular hole with a wall boundary having an angle of 30° to the plate surface was machined in the first plate. Its diameter on the plate surface is 60 mm and its depth is 50% of the thickness. The second plate contains an irregularly shaped defect with the maximum depth of 5 mm, which was produced by a computer numerical control (CNC) milling machine, and defect was constructed by removing the material layer by layer. A zoomed picture of the defect is shown in Figure 3.3(b).

The measurement was performed on a circle with a diameter of 700 mm around the defect, with 64 generator/monitor positions equally spaced along the circle. The A_0 guided mode was generated by a PZT transducer (Panametrics V1011) at one position, and measured at other positions by a Polytec OFV-505 laser vibrometer. In each measurement, a 5 cycle Hann-windowed tone-burst signal at 50 kHz was generated by a Tiepie Handyscope HS3. Figure 3.3(c) shows a typical signal measured by the vibrometer. A gating function similar to [17] was applied to remove unwanted components and obtain the first arrival wavepacket. To avoid the reflection from the edges, only transmitted signals were used and thus measurements were taken on half of the circle with 33 signals measured, as shown in Figure 3.3(a). This process was repeated 64 times to build up a matrix of 64×33 signals.

Similarly as in the simulations, the measured signals need to be calibrated using equation (3.13) before they can be used for inversion. However in the experiment, due to the presence of the defect, signals in a homogeneous medium can only be obtained from limited positions. Therefore, we only measured the waveform in one of the receiver position (shown in Figure 3.3(a)), which was not affected by the scatterer, and mapped it to all other receivers according to their positions. This

FIGURE 3.3: (a) Experimental setup with a complex shaped defect, (b) a zoomed picture of the defect, and (c) a typical time trace from the experiment with chosen time window.

was a reasonable approach, given that the anisotropy in sound speed is limited in this plate.

3.5 Reconstruction results

3.5.1 Single regular defect

The first modeling was performed with a circular defect situated at the center of a 64 element circular array, as shown in Figure 3.4(a). The defect is a flat-bottomed circular hole with a stepped wall boundary. It has the surface diameter of 60 mm ($\simeq 1.3\lambda$ at 35 kHz, 1.8λ at 60 kHz) and its thickness reduction is 50%.

Figures 3.4(b), (c) and (d) show the monochromatic reconstruction of the thickness at 35 kHz obtained after 40 iterations for the FD model, the FE model and the experiment, respectively. Figures 3.4(e), (f) and (g) show the polychromatic reconstruction of the plate thickness at 60 kHz. Sequential frequency group with

FIGURE 3.4: FWI reconstructions with a single central defect. (a) Original model; (b) monochromatic 35 kHz in the acoustic model, (c) the elastic model, (d) the experimental data; (e) polychromatic 60 kHz in the acoustic model, (f) the elastic model, and (g) the experimental data.

frequencies 35, 46 and 60 kHz was used according to the frequency selection criterion discussed in Section 3.3.4. Homogeneous background was used as the starting model at the lowest frequency and 20 iterations at each frequency. The velocity map obtained in the final iteration at each frequency was used as a starting model for the next frequency. This work was carried out on a HP Z820 work station with 1-core and 128G memory. Each forward model in the FD simulation took about 25 seconds to solve, while the inversion took around 1.5 hours to reach the convergence.

It can be seen from the figures that the defect was reconstructed in all cases, with sharper images at higher frequencies. Very clear images were obtained from the acoustic modeling. Some artifacts at the location of an array can be seen in the images from elastic models but they do not degrade the quality of the interior of the resolvable area. This is caused by the under-sampling of the wave-field in this circular array. According to Simonetti *et al.* [121], the minimum number of transducers in a circular array to correctly sample the wave-field can be expressed as

$$N > \frac{4\pi r}{\lambda}, \quad (3.15)$$

where r is the radius of the image to be free from grating lobes. At 60 kHz ($\lambda = 33.4$ mm), around 130 transducers would be needed to correctly reconstruct the image for an area of 700 mm in diameter. However, it is not practical to have too many transducers in the experiment and therefore the number of transducers is limited to 64 in this study with expected reduction in the imaging area that is free from grating lobes. It needs to be mentioned that only limited range of angles can be used to place transducers in many field applications, which further limits the information that can be retrieved. Another possible reason for the artifact at the position of the array could be the inability to cancel entirely the source contribution in the models during the inversion which arise from the singularities in Green's function and its near-field effects [45]. Notable artifacts are observed in the images reconstructed from the experimental data. Despite the reason of the under-sampling as discussed above, the excited wave-fields may not be ideally omni-directional which means that the amplitude and phase of the signal may vary with respect to the excitation angle. As in the calibration, the signals without the scatterer are generated synthetically, and therefore this mismatch between

the true and synthesized signal leads to the phase and amplitude errors in the inversion. In addition, the results can be affected by the noise, the measurement positioning error, the length of the gating function, and the slight anisotropy of the material [122].

FIGURE 3.5: Cross sections of thickness reconstructions of a single central defect along the central line in (a) the acoustic model, (b) the elastic model, and (c) the experimental data.

Comparisons between the reconstructions and the original thickness profile across the defect extracted from Figures 3.4(b)-(g) are shown in Figure 3.5. It can be seen that in both FD and FE cases the hole's depth is reconstructed already at 35 kHz. The reconstruction from experimental data slightly underestimates the depth by 0.4 mm. The reconstruction of all cases at 60 kHz slightly overestimates the depth but the largest error obtained by experimental data is 0.3 mm from the true value. The thickness reconstruction from the experiment by using only the transmission part (a matrix of 64×33 signals) is similar to the simulation results based on

both reflection and transmission parts (a matrix of 64×63 signals). These results clearly demonstrate that the resolution compared to the resolution of traveltime tomography, i.e. $\sqrt{L\lambda} = 153$ mm (at 60 kHz) is improved significantly.

It is worth mentioning that an attempt was made to carry out the inversion directly at 60 kHz from a homogeneous velocity model but the convergence was not achieved indicating that the starting model was not accurate enough for the inversion. The reason is that at the lower frequency, the initial estimate is obtained at the longer scale component, which has slow varying features and fewer minima [118]. Therefore, the neighborhood of the global minimum is more likely to be captured at such scale.

3.5.2 Irregularly shaped defect

The reconstruction of the irregularly shaped defect as in the experiment was carried out in Figure 3.6(a). The depth of the defect varies randomly up to 5 mm and its largest extent is around 220 mm. The defect is characterized by complicated shape and smooth variations in thickness. Three sequential frequencies 35, 46 and 60 kHz were used for the inversion, and at each frequency 30 iterations were applied. The forward model in this case took 50 seconds to solve, slightly longer than the previous case due to the complication in the velocity map. The inversion process took around 4.5 hours to get the results presented in Figures 3.6(b), (c) and (d).

It can be seen that the original shape with fine details of the defect is reconstructed very well from the acoustic model. The reconstruction based on the elastic model can also accurately capture almost all details of the defect. The image from the experimental data is more noisy due to the reasons explained previously. However, the overall shape of the defect and thinnest parts match the original image very well.

The cross-sections of Figure 3.6 with the largest corrosion depths are shown in Figures 3.7(a) and (b). It can be seen that the profiles of the defect are very accurately reconstructed by using the data from the acoustic model. The reconstructions using the data from the elastic model and the experiment are also very good although the deepest points in the defect are slightly underestimated, with an error of about 0.5 mm. It is worth noting that the reconstruction results between

FIGURE 3.6: Polychromatic FWI reconstructions applied to irregular defects at 60 kHz. (a) Original model, (b) the acoustic model, (c) the elastic model, and (d) the experimental data.

FIGURE 3.7: Cross sections of the thickness reconstruction of a complex defect along the lines shown in Figure 3.6 along (a) the vertical line and (b) the horizontal line.

the experimental data and the elastic model using FE simulations are very close to each other, suggesting that the inversion can be reliably performed with only the transmitted signals. This is also observed by other researchers in the previous work [11].

3.6 Discussion

The results presented in this chapter for two different defects have demonstrated that the FWI is a useful tool to obtain high resolution thickness mapping for plate-like structures by using only transmission measurement of the total field. The results of the irregular defect also indicate that the resolution of FWI is slightly compromised when defects were reconstructed from the elastic model and experimental data compared with the equivalent reconstruction by using the simplified acoustic model. The reason comes from the fact that 2D FD model is an approximation for the realistic 3D wave propagation model and can yield some restrictions in the inversion if the modeling data severely deviate from each other. This has been shown by Huthwaite [12] where the scattering from small scatterers of the two models within the Born approximation was compared. It was found that the scattered waves from 2D acoustic and 3D elastic models behave similarly only in a limited scattering region in the transmission zone.

One of the benefits of FWI lies in its multiple frequency strategy that helps to process data subsets of improving the resolution to incorporate smaller wavenumbers in tomography models [99]. The inversion can be started from the homogeneous background model at low frequency where the global minimum can be more easily found as the velocity errors in the waveforms remain below a half-cycle [45]. As mentioned before, traveltimes tomography provides accurate reconstructions when the object size to be imaged is much larger than λ and the size of the first Fresnel zone $\sqrt{L\lambda}$ [11]. For the diffraction tomography to be valid the phase shift travelling through the defect must be small. The FWI is an alternative approach to fill the gap between traveltimes tomography and diffraction tomography. The results of the irregular defect demonstrated that the inversion is successful even when the defect is larger than the first Fresnel zone and its smaller details around the size of the wavelength can be determined.

Compared with existing methods, the major limitation of the FWI approach is its computational complexity and cost. The relationship between the model and the measurement data is nonlinear and the inversion requires to be iterated many times before it converges [99]. The calculation results showed that the convergence for a single frequency can be obtained in 20 to 40 iterations. Depending on the total number of frequencies needed for the inversion, much more iterations would be required. Although the current computational cost is acceptable as an off-line imaging method, it is possible to improve the time efficiency of the FWI. In this work the conventional steepest descent algorithm was used for numerical optimization. The convergence rate could be improved significantly if more advanced optimization methods, such as quasi-Newton algorithm [123], are used. Another solution for reducing iterations is to use a low resolution tomography algorithm to build a more accurate starting model that will subsequently be refined by FWI. Finally, the sequential execution used to solve equations of the forward problem in this work can be replaced by the massively parallel solver MUMPS algorithm [124], which distributes the solving processes over the processors and improves the calculation speed by more than one order.

3.7 Summary

In this chapter, the mono-parameter FWI method is developed for guided wave tomography on plate-like structures. It includes a forward solver to predict the scattering measurements in a 2D acoustic model and an inverse model to update the velocity map iteratively, which is then linked to the thickness map via the dispersion relations of selected guided wave modes. The algorithm uses multiple frequency strategy, which applies the image generated at the lower frequency as the starting model for the higher frequency. Consequently, the reconstruction of the thickness map is less dependent on the initial model, while keeps high resolution to small features.

In this work, a simple circular hole and a complex shaped defect were used for the demonstration of the algorithm via both simulations and experiments. The results showed that the minimum thickness can be reconstructed to be within 1 mm for a 10 mm thick plate. The FWI method allows higher order scattering effects to be

considered in the model, which could lead to improved resolution compared with other GWT methods, and the details will be investigated in the next chapter.

Chapter 4

Investigation of Reconstruction Accuracy of FWI

4.1 Introduction

A number of studies have been carried out to investigate GWT algorithms, such as traveltime tomography (TT) [63], diffraction tomography (DT) [11], HARBUT algorithm [56] and the FWI approach introduced in Chapter 3. One of the major concerns for GWT is its reconstruction resolution and accuracy, and research work has been carried out to evaluate the performance among various algorithms.

TT based on ray assumptions is only able to successfully reconstruct the large and smooth variation defects, and the minimum resolvable size is often described by the first Fresnel zone. Better resolution can be obtained by DT, which is based on a linearized scattering model. Although the theoretical resolution of DT is $\lambda/2$, by ignoring multiple scattering effects, DT can only reconstruct small or low contrast defects. HARBUT has been developed to reconstruct complex shaped defects. The theoretical resolution limit of HARBUT is the same as that of DT. However, the achievable resolution based on the elastic model drops to around $1.5-2\lambda$ due to the simplification in the description of guided waves propagation using the acoustic inversion model. To improve the accuracy beyond this, Mindlin plate theory-based approach is developed to take into account the scattering from guided waves in the inversion [19]. However, this model still relies on the Born approximation and assumes small perturbations in local properties (i.e. small thickness loss), which

limits the range of defects. GWT with an improved scattering model can more accurately capture the scattering behavior and then leads to significant improvement in reconstruction accuracy [70].

This chapter provides a further discussion on the reconstruction accuracy of the mono-parameter FWI. Compared with DT, FWI allows higher order diffraction and scattering to be considered in its numerical solver, thus it has the potential to achieve more accurate inversion results, especially when multiple defects exist. The accuracy of reconstruction is also affected by defect shapes, and therefore defects with various shapes have been studied. Section 4.2 introduces modeling and experimental procedures. The performance of FWI is discussed in Section 4.3 based on single and multiple defects. The results of multiple defects are validated by experimental measurements, and they are compared with the results obtained from DT in Section 4.4. Conclusions are summarized in Section 4.5.

4.2 Validation methods

4.2.1 Defects

The configuration is presented in Figure 1.2. A circular array of 64 transducers surrounds the defect. Two shapes of artificial defects were considered in this chapter. One has an axisymmetric Hann-shaped profile, as shown in Figure 4.1(a). It can be described mathematically by

$$T(r) = \begin{cases} T_0 - \frac{D}{2} \left[1 + \cos \left(\frac{2\pi r}{W} \right) \right] & r < \frac{W}{2} \\ T_0 & r \geq \frac{W}{2}, \end{cases} \quad (4.1)$$

where T_0 denotes the nominal thickness, and r is the radius of the defect at certain depth. This profile has a smoothly varying shape which is characterized by the deepest depth D and the surface diameter W . The smooth change of the thickness allows for a better understanding of the performance and the limitations of tomographic algorithm itself, because the complexity of actual corrosion patches can be largely suppressed. Also, no mode conversions are expected from smooth thickness changes which cannot be modeled in acoustic model. Although such defects cannot represent the real defects in practice, the behavior of complex defects is likely to

be similar to the superposition of simpler defects [12, 125]. It would be expected that identified trends for simpler defects could be present for complex defects.

The other defect shape is the flat-bottom defect with a varying wall angle. It is characterized by the surface diameter W_1 , the bottom diameter W_2 and the defect depth D , as shown in Figure 4.1(b). This model is useful for describing the abrupt thickness changes. In this case mode conversions could take place and affect the inversion performance [116, 126, 127]. Additionally, three flat-bottom defects with different sizes and depths were modeled in the same plate, which simulated more complicated case of closely positioned defect cluster. Here multiple scattering effects may occur which might affect the FWI reconstructions. Defect A was modeled at the center of the plate, with $W_1=60$ mm, $W_2=44$ mm and $D=5$ mm. Two smaller flat-bottom circular defects B and C were located at 100 mm from defect A in the vertical direction and the horizontal direction, respectively. Defect B had $W_1=42$ mm, $W_2=30$ mm and $D=4$ mm. The smallest defect C had $W_1=24$ mm, $W_2=16$ mm and $D=3$ mm.

FIGURE 4.1: (a) Cross section of a Hann-shaped defect and (b) cross section of a flat-bottom defect along the central line.

4.2.2 Numerical modeling

FE simulations were carried out on two different cases to analyze the resolution of FWI. The first case is a series of single Hann-shaped defects with different sizes and depths located at the center of an aluminum plate (see Table 3.1) with the dimension of $400 \text{ mm} \times 400 \text{ mm} \times 10 \text{ mm}$. We chose the cubic-shaped brick elements with the edge length of 0.5 mm, which was fine enough to accurately model this case [114]. Absorbing region was introduced to all sides of the plate to remove the reflections from the edges [128]. An array of 64 transducers with the diameter of 360 mm was modeled, as shown in Figure 1.2. For a given excitation point, the A_0

mode was generated by using an out-of-plane force and out-of-plane displacements were measured at the other 63 receivers. By repeating this process, a signal matrix of 64×63 can be recorded by combining all source-receive pairs. The input signal was a 2 cycle Hanning-windowed tone-burst signal centred at 70 kHz, with around 23 dB bandwidth from 10 to 130 kHz. In this frequency range the phase velocity of the A_0 mode is frequency dependent and thus could be sensitive to the thickness variations in the plate.

The second case considered three flat-bottom defects located at the center of an aluminum plate with the size of 1100 mm $\times$ 1100 mm $\times$ 10 mm. A circular array of 64 transducers with a diameter of 700 mm was used in this simulation. The edge length of the cubic shape eight node element is 1 mm. A 5 cycle Hanning-windowed tone-burst signal centred at 50 kHz was used in this case, with around 15 dB bandwidth between 35 and 65 kHz. The signals from the simulations were collected similarly as in the model with a single Hann-shaped defect.

The FD model was meshed by no less than 4 grid points per shortest wavelength [107]. The absorbing regions and the array configurations were modeled as described in the corresponding FE models. More details can be found in Section 3.3.

4.2.3 Experimental procedures

The experimental setup comprised an aluminum plate of 1100 mm $\times$ 1100 mm $\times$ 10 mm with the same set of three defects as was described in Section 4.2.1, with a 64 element array with a diameter of 700 mm, as shown in Figure 4.2. The PZT transducer was used to excite A_0 at one position and a laser vibrometer was used to measure the displacement normal to the surface. In each measurement, a 5 cycle Hann-windowed tone-burst signal centred at 50 kHz was generated by a Tiepie Handyscope HS3.

As the plate size was limited, to avoid the reflections from the edges, only transmitted signals were used and therefore measurements were taken around half of the array, as shown in Figure 4.2. This process was repeated 64 times, generating a 64×33 matrix of all source-receiver combinations. To remove unwanted components from the measured signals, a gating function was applied to obtain the first arrival

wavepacket. The signals were calibrated before they were used for the inversion by using the same method as in Section 3.3.3.

FIGURE 4.2: Experimental setup with three flat-bottom defects with different sizes and depths. The transducer was used to generate the guided wave and measurements were taken by the laser vibrometer.

4.3 Results

4.3.1 Depth estimation of the Hann-shaped defect

The first case considers the reconstructions of the axisymmetric Hann-shaped defects at the center of the circular array, in a similar approach as demonstrated in [12]. Various defects were simulated, with depths of 1, 3 and 6 mm and surface diameters of 30, 60 and 120 mm, corresponding to 0.9λ , 1.8λ and 3.6λ at 60 kHz. Figure 4.3 presents the cross-sectional reconstructions from the acoustic model and the elastic model at 60 kHz. Homogeneous background at the lowest frequency of 10 kHz is set as the initial model, with 40 iterations per frequency.

It can be seen that the reconstructions of the defects based on the acoustic model are very accurate. The largest error is obtained for the narrowest and deepest

FIGURE 4.3: Cross sections of the thickness reconstructions of Hann-shaped defects along the central line from the acoustic and elastic modeling data at 60 kHz. The first row (a)-(c) has surface width of 120 mm, the middle (d)-(f) has width of 60 mm, and the final (g)-(i) has width of 30 mm; The left column (a), (d) and (g) is 60% deep, the middle (b), (e) and (h) is 30% deep, and the right (c), (f) and (i) is 10% deep.

defect (Figure 4.3(g)) being 0.4 mm from the true thickness. On the other hand, larger reconstruction errors can be observed for defects simulated in the elastic model. The smallest error is around 0.2 mm in the thickness for the large and shallow defect seen in Figure 4.3(c) and the error increases gradually when the defect becomes smaller and deeper, and in Figure 4.3(g) the reconstruction is not successful. The lack of the resolution can be caused by the different scattering behavior between the guided waves with varying thickness and the acoustic waves with varying velocity. The 2D approximation used in the inversion leads to the inconsistency with the signals measured from the 3D elastic model [12].

One of the major interest for industries to evaluate the remnant life of the structure is to find out the minimum thickness in the corrosion profile. Therefore in

FIGURE 4.4: (a) The maximum depth error of reconstruction from the elastic model as a percentage of the nominal thickness. Errors are given for Hann-shaped defects with surface diameter of 60 mm. (b) Cross sections along the central line of the reconstructed defects for varying depths at 60 kHz from (a).

the guided wave tomography, a quality parameter e_{depth} is used to describe the maximum depth error, which can be defined as [12]

$$e_{depth} = \frac{T_{min} - T_{min}^{res}}{T_0}, \quad (4.2)$$

where T_{min} and T_{min}^{res} represent the minimum thickness of the original and reconstructed thickness maps, respectively. T_0 is the nominal thickness. It needs to be noted that the maximum depth error is calculated by using the nominal thickness of the plate, rather than the defect depth, which results in smaller errors for lower contrast defects. Figure 4.4(a) presents the maximum depth error of reconstruction from the elastic model with respect to different frequencies and depths with the surface diameter of 60 mm. It can be seen that the maximum depth error reduces with the increase of the frequency, and increases with the increase of the defect depth. When the depth is fixed, the accuracy of the reconstruction improves as frequency increases. This is primarily due to the improvement in the resolution at higher frequencies. The cross-sectional reconstructions for the defect with the surface diameter of 60 mm at a single frequency of 60 kHz are presented in Figure 4.4(b). This plot shows that the reconstructed depth increases as the true depth of the defect increases. However, it should be noted that when the thickness loss is used to represent the reconstruction and divided by the true defect depth, the curves almost overlap, as it has already been demonstrated in [125]. From this, it

can be seen that the maximum depth error is proportional to the defect depth.

4.3.2 Imaging of multiple defects

The second case investigates multiple defects, in which three flat-bottom holes with different sizes and depths are placed close to each other on a plate, as shown in Figure 4.5(a). The surface diameter of the defect A is 60 mm ($\simeq 1.3\lambda$ at 35 kHz, $\simeq 1.5\lambda$ at 46 kHz, and $\simeq 1.8\lambda$ at 60 kHz) and the depth is 50% of the thickness. Defect B has the surface diameter of 42 mm ($\simeq 0.89\lambda$ at 35 kHz, $\simeq 1.1\lambda$ at 46 kHz, and $\simeq 1.3\lambda$ at 60 kHz) and its depth is 40% of the thickness. Defect C has the surface diameter of 24 mm ($\simeq 0.51\lambda$ at 35 kHz, $\simeq 0.6\lambda$ at 45 kHz, and $\simeq 0.72\lambda$ at 60 kHz) and its thickness reduction is 30%.

FIGURE 4.5: FWI reconstructions with three flat-bottom defects at 60 kHz. (a) Original model, (b) the acoustic model, (c) the elastic model, and (d) the experiment.

Figures 4.5(b), (c) and (d) show the polychromatic reconstructed thickness mapping from the acoustic model, elastic model and experiment at 60 kHz, respectively.

It can be clearly seen that the images with very good quality can be obtained for all three defects in the acoustic model. The larger defects A and B are also well reconstructed from the elastic modeling but the reconstructed image of the smallest defect C is much fainter and its diameter seems much larger than the original defect size. The images of defects A and B based on the experiment also match the true thickness mapping well, however the smallest defect C is faintly seen. This indicates compromised resolution when performing the inversion with elastic data and experimental data compared to acoustic data. In the elastic model, some artifacts can be observed near the position of the array. Also, there are notable artifacts in the reconstructed image from the experimental data. The reasons for emergence of artifacts have been discussed in Section 3.5.1. Figures 4.6(a)-(f) show the comparison between the reconstructions of the thickness along two cross-sections across defects A, B and C, A for separate frequencies at 35, 46 and 60 kHz, respectively. It can be seen that all defects are very accurately reconstructed in the acoustic model: defect A at 35 kHz ($\simeq 1.3\lambda$), defect B at 46 kHz ($\simeq 1.1\lambda$) and the smallest defect C at 60 kHz ($\simeq 0.72\lambda$). The reconstruction error increases when the realistic data from the elastic model and the experiment are used, compared with the acoustic modeling data. The depth of the defect A is underestimated at 35 kHz in the experiment and slightly overestimated at 60 kHz in the elastic model and the experiment. Defect B is reconstructed at 46 kHz ($\simeq 1.1\lambda$) in the elastic model and in the experiment at 60 kHz ($\simeq 1.3\lambda$), with the depth error less than 1 mm. Finally, it can be seen that both the elastic model and the experiment are not able to reconstruct the defect C, with the depth error of around 2 mm. This indicates again that the resolution achieved in the elastic model and experiment is compromised compared to the acoustic model, and the possible reason will be discussed in the next section.

4.4 Analysis and discussion

4.4.1 Resolution analysis of the Hann-shaped defect

The main interest in GWT is to determine the maximum depth of the defect in a corrosion profile. In this chapter, the reconstruction of the defect is considered to be accurate when the error in the maximum depth is less than 10%, i.e. $|e_{depth}| <$

FIGURE 4.6: (a) Cross sections of thickness reconstructions of three flat-bottom defects at different frequencies based on FWI. Acoustic model along centre line of A and B, and (b) along centre line of C and A; (c) Elastic model along centre line of A and B, and (d) along centre line of C and A; (e) Experiment along centre line of A and B, and (f) along centre line of C and A.

10%. Hence, the resolution limit is defined here as the wavelength corresponding to the maximum wavenumber which is accurately reconstructed in the image [12].

The reconstructions of defects with different depths (0.5, 1, 2, 3, 4, 5, 6 mm) and surface widths (5, 10, 15, 20, 30, 40, 60, 80, 120, 160, 240 mm) were analyzed using both the acoustic model and the elastic model at 60 kHz. The resolvable defect sizes of two models are shown in Figure 4.7. It can be clearly seen that defects can be accurately reconstructed when their sizes are around 0.7λ and their depths are around 3 to 6 mm. Smaller defects with shallower depths can also be reconstructed from the acoustic model. Compared to the resolution achieved from the acoustic modeling, the resolution obtained from the more realistic FE modeling data drops to around $1.5-2\lambda$ for the majority of defect depths. This result is similar to previous studies using other tomography algorithm [12]. This suggests that, regardless of inversion approach, the resolution limit is mainly constrained by the mapping from the simplified 2D acoustic model to the realistic 3D elastic model.

FIGURE 4.7: The acceptable resolution limit for different defect depths and sizes from the acoustic model and the elastic model at 60 kHz.

For further analysis of the resolution from the realistic test data, the relationship between the resolution limit and frequency for different Hann-shaped defects with

the depth of 3 mm is presented in Figure 4.8(a). It can be seen from the figure that the resolution improves with the increasing of the frequency. This could be expected because the wavelength decreases as the frequency increases, and the resolution limit is usually proportional to the wavelength [38]. In Figure 4.8(b), the achievable resolution limit normalized by the wavelength shows slight upward trend at higher frequencies in general, indicating decreasing accuracy of the algorithm at higher frequencies. The reason could be that the phase velocity of A_0 is approaching to plateau at higher frequencies thus losing its sensitivity to the thickness. This suggests that a small error in the velocity will lead to a large error in the reconstructed thickness. Similar trends shown in Figure 4.8 were observed in other defect depths.

FIGURE 4.8: Resolution limit for the A_0 mode as a function of frequency obtained from the elastic model. (a) An absolute value, mm, and (b) relative to wavelengths λ for the 30% deep defect.

4.4.2 Effect of the bottom width on the reconstruction accuracy

In the previous section the resolution limit was predicted for the defect with smooth thickness variation and single deepest point. Results from Section 4.3.2 indicate that different accuracy can be achieved for flat-bottom defects. It was found that the defects A and B were resolved at 1.3λ and 1.1λ which are much better results

compared with $1.5-2\lambda$ for the Hann-shaped defect from the realistic elastic data. It is clear that the shape of the defect profile affects the resolvability, and therefore in this section investigations are carried out to understand the influence to the reconstruction accuracy from the surface diameter W_1 and the bottom diameter W_2 of the flat-bottom defect. Figures 4.9(a) and (b) show the maximum depth errors as a function of the ratio of W_2/W_1 for depths of $D=6$ mm and $D=3$ mm, respectively. Three surface diameters of $W_1=42, 60$ and 75 mm have been selected and the reconstructions have been performed at 60 kHz to obtain the defect sizes of around 1.25λ , 1.8λ and 2.25λ . In general, trends of e_{depth} are similar for different surface diameters at the corresponding depth, while smaller errors were observed in the shallower defects, as shown in Figure 4.9(b). When the ratio of W_2/W_1 is small, the defect depth is underestimated, similarly as it was shown in the reconstructions of Hann-shaped defects. For example, in Figure 4.3(d), the reconstruction of the defect with $W=60$ mm and $D=6$ mm at 60 kHz gives the depth error of 15% which is the same as in Figure 4.9(a), when the ratio is 0.12. The error is caused by neglecting the presence of additional scattering term related to density variation in the 2D inversion model [68, 129]. For a fixed surface diameter, it can be seen that the depth error is decreasing with the bottom diameter increasing. After the ratio in a range of $W_2/W_1=0.4-0.8$, the error becomes negative which means that the reconstruction overestimates the depth. This can be identified from the defect A (ratio $\simeq 0.7$) shown in Figure 4.6(c). Finally, a considerable overestimation of the thickness at $W_2/W_1=1$ can be seen. A low pass filtering is a feature of the image algorithm, which leads to a diffraction-limited resolution limit proportional to the wavelength in the reconstruction. All spatial frequency components above a certain value are removed, which results in the overestimation of defect depth, as similar results have also been discussed in ref [125].

In summary, when the surface diameter is close to the resolution limit from the elastic data ($\simeq 1.8\lambda$), the reconstruction error of the defect with sharp thickness variation reduces compared to the smoothly varying defect. The larger defects with sharper change in thickness are more likely to cause overestimation in depth compared to smaller defects with the same large ratio.

FIGURE 4.9: The maximum depth errors of flat-bottom defects with different ratio of bottom diameter W_2 to surface diameter W_1 using data from the elastic model for depths of (a) 6 mm, and (b) 3 mm at 60 kHz.

4.4.3 Comparison with DT on the imaging of multiple defects

One of the important cases encountered in practice is the imaging of multiple defects which was investigated in Section 4.3.2. In this section, the same defects are reconstructed using DT which is based on the Born approximation, compared with the results from FWI. Figure 4.10 presents the reconstruction of three flat-bottom defects by using DT from the elastic data at 60 kHz. The cross-sectional plots along the vertical and horizontal central lines are shown in Figures 4.10(b) and (c). Compared with the results using FWI in Figure 4.5(c) and Figures 4.6(c)-(d), although defect A and B can still be identified, very strong artifacts can be observed which completely overshadow defect C. One of the reasons is the violation of the Born approximation which needs the phase shift between the incident field and the wave traveling through the defect be less than π [130]. As the wave travels through several relatively large defects comparable in size to the wavelength the accumulated phase delay can be much larger than this. The advantage of FWI is the possibility to use the low frequency homogeneous background model in the inverse procedure which ensures the phase difference less than a half cycle [131]. The other reason is that the higher order scattering information in the inversion in DT is ignored, which can result in the form of coherent noise [132, 133]. The strength of the higher order scattering depends on the size, contrast and position of defects, as well as the distance between them. Specifically, it has been shown that even when

FIGURE 4.10: (a) DT reconstruction with three flat-bottom defects from the elastic model at 60 kHz, (b) cross sections of thickness reconstruction along centre line of A and B, and (c) along centre line of C and A.

each component of a multi-component object is weakly scattering, the higher order scattering effects become important if the components are in line with the direction of illumination [133, 134]. In the current case, the second order scattering fields are mainly generated from the blocking component, which illuminate the component in the shadow area of the forward scattering wavefield. Thus, additional scatterer information encoded by higher order scattering which is taken into account in FWI can be exploited to obtain a more accurate reconstruction than DT based on Born approximation.

4.5 Summary

In this chapter, the reconstruction accuracy analysis of GWT based on mono-parameter FWI has been carried out to map the thickness of plate-like structures containing corrosion defects. Simulation studies of smoothly varying defects with various widths and depths across a range of frequencies showed that the achievable resolution is 0.7 wavelengths from the acoustic modeling data but it drops to around 1.5-2 wavelengths from the real elastic modeling data. The resolution limit matches with previous studies using other guided wave tomography methods based on the velocity-to-thickness mapping approaches. The loss in accuracy is caused by implementing the approximate 2D inversion algorithm which limits the complete exploitation of data from 3D elastic model. Analysis has also shown that the reconstruction accuracy of FWI is affected by the shape of the defect. The abrupt change in the wall thickness has been shown to decrease the reconstruction error of small defects compared to the smooth variation of thickness. For larger defects with sharper change in thickness, they are more likely to lead to overestimation in depth. The imaging results of multiple defects validated by experiments demonstrated the good performance of FWI, because the reconstruction starting from the low frequency allows phase distortion to be estimated and taken into account.

Chapter 5

FWI on Limited View Problem

5.1 Introduction

Ideal quantitative reconstruction using GWT can be obtained when the measured data is collected from as many locations around the object as possible - a full view configuration. So far this thesis has focused on full view problems. However, in many cases, only small view angles are available. For example, the assessment of pipelines using GWT is usually carried out by two transducer rings on each side of the inspection area, thus limiting the viewing angles. Reconstruction from incomplete data is ill-posed, which suggests that more than one solution can fit the measured data [135]. The missing angles will typically result in noticeable artifacts, which degrade the quality of reconstruction by varying reconstructed velocities and generating ghost features [136]. Such problems also widely exist in other related areas, such as X-ray tomography [137], seismology [138] and medical ultrasound applications [139]. Efforts have been made to improve the reconstruction from the limited view, which can be broadly divided into three groups, including statistical, probabilistic algorithms [21], minimization [22, 23] and projection regularization methods [136, 140].

Two typical approaches of statistical, probabilistic algorithms are maximum entropy techniques [141] and Bayesian methods [142]. These two approaches allow the incorporation of the prior information about the maximum entropy and the general shape or structure of the object, respectively. When the prior information is restrictive enough, good estimations of the unknown components can be

obtained, thereby reducing the image artifacts. The main challenge of these algorithms is the high computational complexity and the sensitivity to errors in the data [143].

The minimization method obtains a solution that can best fit to the measured data and also match a particular constraint. Weighting variables are used to achieve the balance between the measured data and the constraint. At each iteration in the inverse model, the residual data (difference between computed results and the measured data) is minimized. Ultimately, they will converge to a single fitting minimum. This method is widely used for the total variation minimization and also combined with regularization [22, 23]. However, the performance of minimization algorithms is largely dependent on the choice of appropriate constraints, which are complex to choose. Moreover, these methods are also computational expensive [144].

The projection regularization method can be used in combination with direct imaging approaches in the reconstructions. This projection process indicates that in each step a regularization restriction can be carried out by directly applying particular constraints to the image. Positivity is a common example of this method [140]. It enforces the constraint of positivity on each pixel of the estimated image and penalizes negative values to be zero. Huthwaite [136] introduced a projection-based regularization approach of Virtual Image Space Component Iterative Technique, which combined the image regularization approach with a linearized forward solver to determine the unknown components. Compared with a static regularization like positivity, better values of the unknown components can be obtained by iteratively using the thresholding to the reconstruction. This approach overcomes the limitation of complex implementation and intensive computation in the statistical, probabilistic algorithms. Compared with the minimization algorithms which work on the continuous functions to match the measured data with the restriction, the restriction of the projection regularization method does not rely on continuous defining of matching degree with the image.

A nonlinear inverse scattering model has been developed for GWT in Chapter 3. This chapter combines the FWI method with the adjustable thresholding regularization method to address the challenges in the limited view GWT. In this chapter, the A_0 mode was employed for defect identification [71].

This chapter is organized as follows. In Section 5.2, after a brief overview of the challenges in the limited view problem, the theory of the FWI with adaptive thresholding regularization is introduced. This method is validated by both FE simulations and experiments. The setup is introduced in Section 5.3 and the reconstructions of an irregularly shaped defect as well as two defects are shown in Section 5.4. In Section 5.5, discussions are carried out to explore the effect of the array length to the reconstructed image. Conclusions are summarized in Section 5.6.

5.2 Theory

5.2.1 Analysis of the limited view problem

For simplicity, the limited view problem is analyzed in the framework of a Born model with the far field assumption. Under these assumptions, the scattered field U_s can be described by the following equation [52, 145]

$$U_s \approx - \int_{\Omega} e^{ik(\hat{r}_0 - \hat{r})x} O(x) dx, \quad (5.1)$$

where Ω represents the domain of the scatterer and k is the wavenumber. $\hat{r}_0$ and $\hat{r}$ are the directions of the incident illumination and the reception, respectively. $O(x)$ is the object function, which is the mathematical representation of the scatterer. The scattered field can be effectively given by a Fourier transform of the object function $\tilde{O}$ in the spatial frequency domain:

$$U_s(\hat{r}_0, \hat{r}) \approx -\tilde{O}[k(\hat{r}_0 - \hat{r})]. \quad (5.2)$$

It means that the signal obtained from each source-receiver corresponds to the Fourier component of the object function at $k(\hat{r}_0 - \hat{r})$. As shown in Figures 5.1(a) and (b), the ideal reconstruction of the object can be obtained by using the information from all possible illumination and reception angles for all frequencies within a disk [52]. The effect of the limited view is to leave sections where the estimation of the Fourier transform of the object cannot be obtained. This introduces a low pass filter for the areas where there is no data. Typical limited view problem with two parallel linear arrays measuring the transmission data and its

filtered wavenumber spectrum are shown in Figures 5.1(c) and (d). More detailed discussions can be found in [138, 145].

FIGURE 5.1: 2D Fourier transform based on Born approximation. (a) The full view array and (b) the corresponding Fourier components obtained from each layout; (c) a pair of parallel linear arrays with a source array above the defect and a receiver array below it and (d) the corresponding Fourier components.

Therefore, the key issue in the limited view reconstruction is to have a good estimation of the missing components, and this could be obtained from the prior information. For example, the reconstructed values of the scatterer should be equal or larger than the background value [146].

5.2.2 Regularization to synthesize unknown components

In order to reduce artifacts caused by the limited view, the regularization approach based on a prior information via the iterative algorithm is applied to obtain more

suitable data for unknown components. An example of this process used in references [145, 146] is illustrated in Figure 5.2. The regularization assumption is made that the values of the scatterer are the same or larger than the background. Based on the far field assumption, the known Fourier components can be directly obtained from the data, as shown in Figure 5.2(a), which is similar as the Figure 5.1(d). The values outside the known components are zero. The image shown in Figure 5.2(b) can be obtained directly from the Fourier components shown in Figure 5.2(a) by taking inverse Fourier transform. Since unknown components are set to zero, the reconstruction will become a low-pass filtered version of the true image. This will result in “x” shaped artifacts [138]. Then, the reconstruction is regularized by setting the negative values to the background value of zero and the forward Fourier transform of this image is taken, as illustrated in Figures 5.2(c) and (d), respectively. It can be seen that the unknown components that are zero in the wavenumber space image become now non-zero values, which suggest that additional information from the missing components is obtained. Then, the estimations of the unknown components (see Figure 5.2(e)) are combined with the known components. The process is iterated until the convergence is achieved.

5.2.3 Adaptive thresholding regularization method

In many cases, although the artifacts linked with the limited-view problem mask the lower amplitude features, the peak features are relatively well reconstructed [138], as demonstrated in Figure 5.3(a). This characteristic can be exploited by an adaptive thresholding approach [136]. In this method, a threshold is applied to the image, which is below the artifacts level but above the level of peak feature, as shown in Figure 5.3(b). Below this threshold level, all values are maintained and values above this threshold are set to the background. Such process can be used to populate the unmeasurable components due to the limited view. The extra information from the missing angles obtained by using this regularization is combined with the measured data, and thus the artifacts above this threshold will be reduced, as shown in Figure 5.3(c). As the artifacts reduce at later iterations, the threshold level can be gradually increased towards the background level until most of the artifacts are removed.

FIGURE 5.2: Applying the approach used by Huthwaite [145] and Sung *et al.* [146] to synthesis unknown Fourier components by regularization. (a) Fourier components from a limited view, (b) the reconstruction obtained by using Fourier transforming, then the regularization is applied in (c), and (d) the additional information can be obtained by regularization. The estimations of the unknown components in (e) are combined with the known components.

It should be noted that this threshold method can be used in either case of the reconstructed value above or below the background. The threshold can be defined as [136]

$$m_{reg}(x) = \begin{cases} m(x) & \text{if } \frac{|m(x) - m_0|}{\max|m(x) - m_0|} < l_e \\ m_0 & \text{otherwise,} \end{cases} \quad (5.3)$$

FIGURE 5.3: 1D example illustrating the schematic outline of the method of the thresholding regularization. (a) The standard limited view reconstruction, (b) a threshold used in the image and (c) the next regularized reconstruction.

where $m(x)$ is the model parameter at x , m_0 being the level of the background, and l_e is the threshold fraction.

5.2.4 Implementation of the regularization method in guided wave tomography

Figure 5.4 outlines the flowchart of the regularization method in combination with the FWI algorithm. Initially, the unmeasurable components are set to zero. The image is obtained by using the measured, limited view data based on the FWI algorithm. Then the regularization based on the adaptive thresholding method is applied. At each threshold level, a non-linear forward solver of the current regularized image is used to generate unknown data in the FWI algorithm, then the inversion is followed to perform a reconstruction from that. The information included in the unknown components contributes to the improvement in the image caused by the regularization. Thus, the solution incorporates this additional information into the measured, limited-view data, which can achieve a better estimation of the image for the next iteration. This process is repeated until the convergence is achieved. The mean square error (MSE) can be used as a criterion of convergence, similarly as in references [136, 147]

$$MSE_{(FWI)} = \sqrt{\frac{\sum_{pi=1}^{N_{pi}} (T_{pi}^{(i)} - T_{pi})^2}{\sum_{pi=1}^{N_{pi}} T_{pi}^2}} \quad (5.4)$$

where T_{pi} is the original thickness at pixel pi , $T_{pi}^{(i)}$ is the reconstruction of the thickness at pixel pi for the i th iteration and N_{pi} represents all pixel number in the reconstructed image. The value of MSE is decreasing with the increasing of the number of the iteration, and then it can converge to a single value. Experience showed that the convergence for a single frequency can be achieved in 10 to 20 iterations.

It is important to set an appropriate threshold fraction in the regularization, and in this chapter, it can be expressed as $f^{(n+1)} = p(f^{(n)} + q) - q$, similar to the literature [136]. The parameter p is used to define the degree of threshold increase at each iteration and q is the asymptotic limit. In this chapter, $p=0.8$, $q=0.2$ and $f^{(0)}=-0.6$ were used. The background level is set to zero and so the threshold

FIGURE 5.4: The flowchart of regularization with the FWI algorithm for a limited view configuration.

fraction below this level is negative. The initial value $f^{(0)}$ can be adjusted according to the particular case. The standard limited view FWI algorithm was firstly carried out by using multiple frequencies. Based on the obtained results from the final frequency, we chose to perform each regularization iteration for several FWI iterations. In this chapter, after one regularization iteration, around 10 FWI iterations are performed to update the background. This process is iterated through repeating application of the regularization until convergence.

5.3 Validation methods

5.3.1 Problems studied

In this chapter, two types of defects: irregularly shaped defect and axisymmetric flat-bottom circular ones, are considered on an aluminum plate (see Table 3.1) with the dimension of 1100 mm×1100 mm×10 mm. The irregularly shaped defect has the largest size of around 100 mm, with depth of 3 mm. The flat-bottom defect can be described by the surface diameter, the bottom diameter and the defect depth [11]. In the case of two flat-bottom defects, each defect is offset by 40 mm from the center of the plate in the horizontal direction. The larger defect has the surface diameter of 60 mm, the bottom diameter of 44 mm and the depth of 50% of the nominal plate thickness. The smaller defect has the surface and bottom diameters of 42 mm and 34 mm and its thickness reduction is 20%.

5.3.2 Numerical modeling

To validate the proposed algorithm, FE simulations were performed for the above two cases. The cubic-shaped brick elements were used to model the plate, with the element size of 1 mm. Absorbing layers were applied to minimize reflections from the edges [128]. A pair of parallel linear transducer arrays were modeled, as shown in Figure 5.5. A 5 cycle Hanning-windowed tone-burst signal at 50 kHz was simulated as the input signal. For a given excitation point in one array, the A_0 mode was generated by applying an out-of-plane force and the wave-fields were measured by all receivers in the other array. The setup of the arrays is slightly different in the two cases. For the irregularly shaped defects, each array consists of

35 different generator/monitor positions equally separated by 20 mm ($\simeq 0.6\lambda$ at 60 kHz), and the space between two parallel linear transducer arrays is 150 mm. For the cases of two defects, each array has 17 generator/monitor positions separated by 40 mm ($\simeq 1.2\lambda$ at 60 kHz) and the space between the two arrays is 200 mm.

FIGURE 5.5: The configuration of a pair of parallel linear transducer arrays for GWT on a plate.

5.3.3 Experimental procedures

The experimental setup is presented in Figure 5.6, with the defects introduced by a CNC milling machine in the same dimension as in FE simulations. The A_0 mode was excited by the PZT transducer (Panametrics V1011) at one position, which was coupled to the plate through a small area by an aluminum disk with the diameter of 10 mm and the thickness of 0.5 mm [148]. The displacement normal to the surface was measured by a Polytec OFV-505 laser vibrometer. In the measurement, a 5 cycle Hanning-windowed tone-burst signal centred at 50 kHz was generated by a Tiepie Handyscope HS3.

To avoid reflections from plate edges, only transmitted signals were measured and thus the measurements were performed along one linear transducer array, as shown in Figure 5.6. By repeating this process, a matrix of 70×35 signals obtained from the irregularly shaped defect and a 34×17 matrix of all source-receiver combinations based on two flat-bottom defects can be generated from two measurements.

The signal-to-noise ratio (SNR) greater than 35 dB can be attained after 100 averages. The unwanted parts from the measured signals can be removed by using a gating function [17]. Before using these signals in the inversion, the calibration of obtained signals was conducted in the similar way as in Section 3.3.3.

FIGURE 5.6: The experimental setup on an irregularly shaped defect.

5.4 Results and analysis

5.4.1 Irregularly shaped defect

Figure 5.7(a) presents the original model of an irregularly shaped defect as in the experiment by using a pair of parallel linear transducer arrays. Each array has 35 transducers separated by 20 mm. The defect has a complex shape and smooth variations in the thickness. The largest size of this defect is about 100 mm and its depth is 3 mm. Three sequential frequencies of 35, 46 and 60 kHz were used in the standard limited view FWI algorithm, with 40 iterations per frequency. Then, based on the results obtained from 60 kHz, the regularization is applied and 10 FWI iterations are used to update the background. As the artifacts decrease with increasing number of iterations, the threshold level can be slightly increased. This

process is repeated until it converges. The work was run on a HP Z820 work station with 1-core and 128G memory. The total process took around 2 hours to generate regularized results shown in Figure 5.7(c).

FIGURE 5.7: Reconstructions with an irregularly shaped defect by a pair of parallel linear transducer arrays. Original model (a), the standard FWI limited view reconstruction (b), and the regularized reconstruction (c) based on the FE model; the standard limited view reconstruction (d), and the regularized reconstruction (e) in the experiment.

Figure 5.7(b)-(e) are on the same color bar and scales as in Figure 5.7(a). It can be seen from Figure 5.7(b) that the standard FWI limited view reconstruction based

on the FE simulation has slight elongation in the shape of the defect and clear “x” shaped artifacts. Compared to the full view configuration, the relatively small range of viewing angles can slightly distort the defect. However, the quality of the resolvable area is not degraded in general. These diagonal artifacts are common results in a limited view configuration [149]. The reason is that the unknown components are set to zero in the standard limited view reconstruction, thus the limited view reconstruction will lead to a filtered version of the true image and this filtering will cause artifacts in the reconstruction. Some artifacts observed at locations of two arrays are mainly caused by the undersampling of the wavefield as the separation of the transducer locations is below the Nyquist requirement of half wavelength [131]. Figure 5.7(c) shows the image after regularization which slightly improves the defect representation, and more importantly, artifacts can hardly be observed. In the experimental results, significant artifacts are observed in the standard limited view reconstruction, as shown in Figure 5.7(d). Besides the undersampling of the wave-field, the noise as well as a wide range of the uncertainties in the experiment can also corrupt the measured data [131]. Reasonable estimation of the overall shape of this defect can still be achieved. The artifacts are significantly reduced by the regularization method, as shown in Figure 5.7(e).

Figures 5.8(a) and (b) show the cross-sections of the defect at the deepest depths along the horizontal direction in Figure 5.7. The reconstructions using the data from the FE model and experiment slightly underestimate the depths of the defect by around 0.1 mm and 0.3 mm, respectively, as shown in Figures 5.8(a) and (b). It is clear that by using regularization method a slightly better estimation of the defect profile and a reduction in artifacts can be achieved. In Figure 5.8(b), the profiles of the defect by using the experimental data show slightly narrower than the true value, which are due to the limited accuracy of the standard limited view reconstruction and the choice of the thresholding level. It is worth mentioning that the overall level of artifacts shown in reconstructions of the irregularly shaped defect is relatively low, since relatively large range of viewing angles and small sampling spacing used in the measurement [38].

FIGURE 5.8: Cross-sections of standard limited view FWI and regularized reconstructions of an irregularly shaped defect along the horizontal direction in (a) the FE model and (b) the experiment.

5.4.2 Two defects

To demonstrate the suitability of this algorithm for multiple defects, two flat-bottom defects with relatively large difference in the depth are considered in the second model. This modeling is also carried out with a pair of parallel linear transducer arrays. The array consists 17 transducers with the separation distance of 40 mm, as shown in Figure 5.9(a). The larger defect has the surface diameter of 60 mm ($\simeq 1.8\lambda$ at 60 kHz) and the bottom diameter of 44 mm ($\simeq 1.32\lambda$ at 60 kHz), with the deepest depth of 5 mm. The shallower one has the surface diameter and the bottom diameter of 42 mm ($\simeq 1.3\lambda$ at 60 kHz) and 34 mm ($\simeq 1\lambda$ at 60 kHz) respectively, with 20% deep of the thickness of the plate. In this model, the choice of frequencies and iterations are the same as in the previous example. In the regularization, the starting level of threshold slightly below the peak value of the shallower defect is used to verify the capability of the regularization to distinguish shallower defect and the artifacts. It means that the starting model after the first regularization only contains the information of the larger defect without any information of the shallow defect.

The same color bar and scales are used in Figures 5.9(b)-(e). Figures 5.9(b) and (d) show the standard limited view reconstruction based on the FE simulation and the experiment. It can be seen that although the larger defects can be reasonably reconstructed, the smaller defects are almost shadowed by the artifacts. Significant improvement can be achieved by several iterations of regularization, as presented in Figures 5.9(c) and (e). Both defects are clearly reconstructed, with the peak values

FIGURE 5.9: Reconstructions with two flat-bottom defects by a pair of parallel linear transducer arrays. (a) Original model, (b) the standard limited view reconstruction, and (c) the regularized reconstruction based on the FE model; (d) the standard limited view reconstruction, and (e) the regularized reconstruction based on the experiment.

captured initially and the low contrast features obtained by gradually increasing the threshold level. However, the diameter of the shallower defects seem to be smaller than the original defect size due to the reasons explained previously. There are obvious artifacts in the reconstruction obtained from the experiment and the main reasons were the under-sampling of the wave-field and the noise.

Figures 5.10(a) and (b) present cross-sectional reconstructions across the center of the defects along the horizontal direction. In Figures 5.10(a) and (b), the larger defects can be accurately reconstructed in both standard limited view and regularized reconstructions from the FE simulation and the experiment, respectively. The regularized reconstructions of the shallower ones significantly improve the depth estimations compared to the standard limited view reconstructions, despite the sizes are smaller than the original one. Very few artifacts are visible in regularized reconstructions, which are closer to true values.

FIGURE 5.10: Cross-sections of standard limited view FWI and regularized reconstructions of two flat-bottom defects along the horizontal direction in (a) the FE model and (b) the experiment.

5.5 Discussions

In Section 5.4, the work has focused on different types of defects with a relatively large range of angles. It enables a general analysis of the performance of the regularization to be identified. However, in reality, only a limited area is often allowed to place transducers, which significantly limits the range of viewing angles. In this section, a single flat-bottom defect was used to evaluate the influence of different

array lengths with the data obtained from FE simulations and demonstrate the effectiveness of regularization. The surface and bottom diameters of the defect is 60 mm and 44 mm and its thickness reduction is 50%, as shown in Figure 5.11(a). The varying ratio of the array length L to the fixed distance $D = 200$ mm between two arrays and the fixed sampling interval of 40 mm are considered. Figures 5.11(b)-(k) are on the same color bar and scales as in Figure 5.11(a). Figures 5.11(b),(d),(f),(h) and (j) show the standard limited view reconstructions using a pair of linear arrays on each side of the defect, and Figures 5.11(c),(e),(g),(i) and (k) show FWI with the adaptive thresholding regularization method.

It can be observed from the Figure 5.11(b) that when the ratio L/D is 4, good reconstruction of the defect can be obtained by using the standard limit view method with noticeable artifacts. Similar result can be achieved when the ratio L/D is equal to 3.2, as shown in Figure 5.11(d). As the array reduces in length, the artifacts around the defect gradually expand. More clear distortions of the shape of the defect and underestimations in the peak contrasts can be observed in Figures 5.11(f), (h) and (j). For this defect, the ratio L/D needs to be larger than 3.2 in order to obtain satisfactory results. Compared with the standard limited view results, the reconstructions after regularization can achieve improved images, with much fewer artifacts and better estimations of the contrast of defects. The improvement is still visible in the regularized image even the ratio is very small, as shown in Figure 5.11(k).

The critical information in the reconstruction of thickness maps is the maximum depth of the defect. Therefore, it is necessary to calculate the maximum depth error, which is defined as the difference between reconstruction depth and true depth divided by the nominal plate thickness [131]. Figure 5.12 shows the maximum depth errors of FWI with and without regularization with respect to the ratio L/D . It is clear that the maximum depth errors are slightly reduced due to the additional information obtained from the regularization method. As the length of array increases, the viewing angles become larger, thus reducing the reconstruction errors. The difference in the maximum depth errors of FWI with and without regularization extracted from Figure 5.12 is listed in Table 5.1. It is worth mentioning that the similar improvement trend can also be observed in the cases of irregularly shaped defect and two flat-bottom defects when applying the regularization approach.

FIGURE 5.11: Limited view reconstructions of a flat-bottom defect based on the FE model at 60 kHz. The first column shows the original model with defect depth of 5 mm and the surface diameter of 60 mm (a), the second column, i.e. (b),(d),(f),(h) and (j) present the standard limited view reconstructions with FWI and the third column, i.e. (c),(e),(g),(i) and (k) show FWI with regularization. In (b) and (c), the ratio L/D is 4; (d) and (e) reduce the ratio to 3.2 and (f) and (g) show ratio of 2. In (h) and (i), ratio is equal to 0.8 and (j) and (k) reduce that to 0.4.

FIGURE 5.12: The relationship between the ratio L/D and the maximum depth error e_{depth} with a single flat-bottom defect

TABLE 5.1: Comparison of the maximum depth errors (e_{depth}) between the reconstruction with and without regularization.

ratio L/D	0.4	0.8	2	3.2	4
standard limited view	26%	17.5%	2.9%	0%	-2%
limited view with regularization	21.8%	15.1%	1.3%	-0.3%	-3%

5.6 Summary

This chapter has introduced the regularization technique to predict the missing components in the FWI algorithm with limited view configuration. The regularization is carried out by using an adaptive thresholding approach to the limited view reconstruction. The performance of FWI algorithm with regularization to solve an irregular defect and two defects was demonstrated by using simulations as well as experiments. The results showed that the image quality is improved by combining the extra information through the regularization with measured data from the limit view configuration. When testing the regularization approach with limited view elastic data via the variation in the array length, similar improvement can be achieved. This limited view configuration is very useful for pipes which can be assessed by two arrays on either side of the defect region. This will be investigated in Chapter 7.

Chapter 6

GWT on Liquid-Loaded Plates

6.1 Introduction

Large-scale storage tanks have been widely used in the petrochemical industry. Liquid-loaded tank floors can corrode under humidity or chemically aggressive environment, resulting in wall thickness losses. In this chapter, this study extends the application of GWT on a liquid-loaded plate. Compared to the application on free plates, the thickness reconstruction of liquid-loaded plates becomes more challenging due to the leakage of guided wave energy into liquid. However, most of the existing work on GWT was dealing with waveguides with free boundaries, and it remains challenging for waveguides with liquid loading due to the strong attenuation of guided waves leading to low signal-to-noise ratio. An ultrasonic guided wave attenuation tomography, which considered signal losses due to liquid loading, was used for the localization of defects in a storage tank floor [150]. However, quantitative information of thickness losses has not been successfully obtained.

This chapter presents an improved GWT method with the consideration of attenuation for quantifying the remaining thicknesses of liquid-loaded plates. The thickness mapping is performed using full waveform inversion (FWI) which accounts for higher order scattering. The attenuation of waves is implemented via complex velocity in the forward modeling, which is approximated by a 2D acoustic model. Section 6.2 discusses the suitability of low frequency Lamb modes for GWT in the presence of liquid loading, and the imaging algorithm with the consideration of the attenuation is followed in Section 6.3. FE modeling and experimental

procedures are presented in Section 6.4. Section 6.5 studies the influence of the attenuation on the reconstructions accuracy over various defect widths and depths at different frequencies. The reconstruction of an irregularly shaped defect in a water tank floor is validated by an experiment in Section 6.6. Conclusions are drawn in Section 6.7.

6.2 Properties of low frequency Lamb modes of liquid-loaded plates

The selection of suitable mode for tomography of fluid-loaded structures mainly depends on the sensitivity of the mode to thickness variations and attenuation due to fluid loading. The behavior of the fundamental Lamb modes A_0 and S_0 has been discussed in Section 2.3.3. It has been shown that the sensitivity of the A_0 mode is better than that of the S_0 mode, while much higher attenuation of the A_0 mode limits its propagation range compared to the S_0 mode.

FIGURE 6.1: The attenuation dispersion curves for an aluminum plate with fluid loading. The dash dot line presents the attenuation for a synthetic liquid with the density of 1500 kg/m^3 .

Figure 6.1 shows the attenuation of A_0 and S_0 when fluid is present on one side of an aluminum plate. It can be clearly seen that the S_0 mode has much smaller attenuation in the liquid-loaded plate than the A_0 mode below 1.5 MHz-mm , and thus the S_0 mode is chosen to be used in this chapter. A synthetic liquid with

density of 1500 kg/m³, which will result in larger attenuation, is also calculated for the investigation of the attenuation effect on the reconstruction accuracy. The details will be discussed in Section 6.5.

6.3 FWI with attenuation

The velocity mapping is produced prior to being converted to the thickness mapping. This is achieved by the FWI method, which has been discussed in details in Chapter 3. In the presence of liquid loading, the attenuation of the waves can be presented by complex velocity $\tilde{v}(\omega)$ [151] in the frequency domain FWI, as follows

$$\tilde{v}(\omega) = v(\omega)\left(1 - \frac{i}{2Q_{atten}}\right), \quad (6.1)$$

where v is the real-valued phase velocity, as shown in Figure 2.3(a). Q_{atten} is the attenuation factor and is expressed as

$$Q_{atten}(\omega) = \frac{\omega}{2\alpha_{atten}v(\omega)}, \quad (6.2)$$

where α_{atten} is the attenuation given in Np/m.

6.4 Test approach

6.4.1 Configuration

General configuration of the model as well as the experiment carried out in this work is presented in Figure 6.2. It consists of a circular water layer with a diameter of 600 mm on the top of a plate. A surface defect is located at the center of the plate. A 700 mm diameter circular array consisting of 64 equally spaced transducers is placed on the plate outside the area with water loading. The S_0 mode is excited on each of the transducers and received by the rest of the transducers, although only the transmitted signals are used for the reconstruction.

FIGURE 6.2: Configuration of the fluid-tank model with a defect located at the center of the plate and a transducer array.

6.4.2 FE simulations

Hann-shaped defects with different sizes and depths located in the center of a 1100 mm $\times$ 1100 mm $\times$ 10 mm aluminum plate (see Section 3.1) coupled with a liquid layer (Bulk modulus=2.2 GPa, water density=1000 kg/m³ and a synthetic liquid density=1500 kg/m³) have been modeled by using Abaqus, as shown in Figure 6.2. The full-scale modeling of the problem is computationally very demanding, thus the parts of the model should be optimally designed with coarser meshes. The whole model was meshed with 8-noded linear cubic-shaped elements with the size of 1 mm, resulting in around 30 million elements. The selected element size of 1 mm ensures that more than 30 elements are used per wavelength at 150 kHz ($\lambda=34.4$ mm), which is enough for the accurate simulation of elastic waves in the plate [114]. Although much finer mesh is needed for the acoustic waves propagating in the liquid layer due to the smaller wavelength, the reconstruction procedure is only based on the waveforms measured in the plate, and thus only the attenuation of the waves propagating in the plate due to energy leakage should be accurately modelled. Additional studies on 2D models of plates with water loading and varying mesh densities showed that eight elements per wavelength in the water layer is sufficient to accurately model the attenuation of plate waves. To reduce the height of the water layer, an absorbing region with increasing damping [152] was used. This also helps to avoid the reflection from the upper surface of the water, which can interfere with the plate waves and complicate the measurements.

The plate-water interface was simulated by a surface-based coupling constraint in the out-of-plane direction. Thickness reductions were modelled by reducing the out-of-plane dimension of the elements [12]. To reduce the model size in the plate, the absorbing boundary around the plate was used in a similar way as that of the water layer to avoid boundary reflections.

The signals were excited and measured with a 64 transducer array. The input signal was a 5 cycle Hanning-windowed tone-burst centered at 150 kHz. Around this frequency, the S_0 mode is characterized by both considerable in-plane and out-of-plane displacement components. To excite the pure S_0 mode, symmetric out-of-plane forces with respect to the mid-plane of the plate were applied on the top and bottom surface of the plate in the out-of-plane direction [68]. In the reception, the S_0 waves were obtained by the difference of the out-of-plane components at both surfaces of the plate.

6.4.3 Experimental setup and procedures

The experimental setup is presented in Figure 6.3. The experiments were carried out on an aluminum plate with sizes of 1100 mm $\times$ 1100 mm $\times$ 10 mm. An irregularly shaped defect was machined in the center of the plate. The tank wall on the plate is made of a 2 mm thick aluminum sheet, which is rolled into a cylinder with an inner diameter of 600 mm. It is attached to the plate by silicon and the tank is filled with water to a level of 200 mm. A PZT transducer (Panametrics V1011) was used to excite the plate in the out-of-plane direction. A 5 cycle Hanning-windowed tone-burst signal centered at 150 kHz was generated by RITEC RAM-5000 high power ultrasonic pulser (Ritec Inc., USA). The out-of-plane components of the waves at the plate surface were collected by a laser vibrometer. A reflective strip was applied to the surface of the plate to reflect signals back to the laser receiver.

The measurements were repeated for 64 excitation locations on an array and the signals were post-processed before using them in the imaging algorithm. In the experiment, both S_0 and A_0 mode are generated under out-of-plane excitation. However, they can be separated by applying appropriate time gating. Figure 6.4(a) shows a contour plot of the typical out-of-plane displacement signal measured over receivers. The first arrival of the S_0 mode was separated from the signals by

FIGURE 6.3: The experimental setup. An irregularly shaped defect was machined in the aluminum plate.

removing unwanted components with a gating function, as shown in Figure 6.4(b). Gating function is a rectangular window smoothed with rising and falling sinusoid at the edges on both sides. The length of the gate was varied with the propagation distance due to the dispersion of S_0 . The starting point of the gate was set by the arrival of S_0 and the gate end by the arrival of the slower A_0 mode. All signals after time gating were calibrated before using them in the inversion by applying the similar method introduced in Section 3.3.3.

FIGURE 6.4: (a) Contour plot of measured out-of-plane displacement excited in the transducer shown in Figure 6.3. The solid curves present the adaptive time gating according to different propagation distances. (b) Time trace from one of receivers and the signal after time gating.

6.5 Effect of the attenuation on the reconstruction accuracy

As explained in Section 6.2, the S_0 mode propagating in a liquid-loaded plate has considerable attenuation at frequencies where its dispersive nature can be used for the thickness reconstruction. The impact of this is investigated by a range of defect widths and depths across different frequencies. These results are compared with the reconstructions obtained from a free plate and from a liquid-loaded plate neglecting the consideration of attenuation in the FWI algorithm. The aim is to understand the improvement in the reconstruction accuracy when the attenuation has been considered in the inversion.

Figure 6.5 shows the plate thickness reconstructions of a Hann-shaped defect at the center of a circular array from the FE modelling data. The surface width of the defect is 100 mm and its depth is 3 mm in a 10 mm thick aluminum plate. Sequential frequencies of 130, 150, and 180 kHz were used for the inversion. Homogeneous background was used as the starting model at the lowest frequency and 20 iterations at each frequency were implemented. It can be seen from the figures that the defect is reconstructed in all cases in the center of the plate and the contrast is slightly improved at higher frequencies. No artifacts are seen in the cases of free plates. In the thickness reconstructions of the defects on water-loaded plates, the water-loaded areas remain free of artifacts in almost all images. There are artifacts at the location of the array and they become more apparent with the increase of the frequency. This can be attributed to the combination of under-sampling of the measured wavefield in the circular array and the larger mismatch between the amplitudes of the modelled and observed wave data [121, 131]. More notable artifacts can be observed when the inversion is performed without consideration of the attenuation. It should be noted that artifacts do not disappear completely when the attenuation has been considered in the inversion. This suggests that attenuation used in the inversion algorithm might differ from the one in the FE data, which leads to some unwanted data residuals in the inversion.

Figure 6.6 presents the cross-sections of the reconstructions from Figure 6.5 along the central line of the plate at different frequencies of 130, 150 and 180 kHz. At 130 and 150 kHz, all reconstructions slightly underestimate the true defect depth, while at 180 kHz a slight overestimation can be observed. The major reason is that the

FIGURE 6.5: Thickness reconstructions of a Hann-shaped defect with a surface width of 100 mm and a depth of 3 mm in a 10 mm thick aluminum plate across different frequencies from the FE model. The images along different rows represent reconstruction results at 130 kHz, 150 kHz and 180 kHz, respectively. The images along different columns represent reconstruction results in free plate, water loaded plate with and without considering the attenuation effect. The dashed curves indicate the position of the tank wall.

FIGURE 6.6: Cross sections of the thickness reconstructions from Figure 6.5 along the center line at different frequencies of (a) 130 kHz, (b) 150 kHz and (c) 180 kHz.

2D acoustic model is an approximation for the 3D wave propagation model, causing some accuracy loss in the inversion [12]. Also, it can be seen that when attenuation is small at low frequency, the difference between the reconstructions obtained from different approaches is very small. In contrast, when the attenuation increases with the frequency, the results obtained from the inversion without consideration of the attenuation deviate more from the true value of the depth. The reconstructions of the defect on a free plate and on a water-loaded plate with the consideration of attenuation in the inversion match very well.

Figure 6.7 shows the maximum depth error of a Hann-shaped defect with a surface width of 100 mm and a depth of 3 mm as the frequency varies. The results for liquid-loaded plates with larger attenuation are also shown to demonstrate the influence of different levels of attenuation on the reconstruction accuracy. Similar trends can be seen for all of the results: the maximum defect depths are slightly underestimated (negative value) at lower frequencies. As the frequency increases, the

FIGURE 6.7: The maximum depth error of a defect with a width of 100 mm and a depth of 3 mm at varying frequencies. The reconstructions of a free aluminum plate, water-loaded plate and higher attenuation liquid-loaded plate with and without the consideration of the attenuation in the FWI algorithm.

reconstructed depths become overestimated (positive value). At lower frequencies, when the attenuation is small, the reconstructions with and without the consideration of attenuation in the inversion are very close to each other. However, as the attenuation increases with the frequency, more divergence in the results can be observed. For example, it can be seen that at 150 kHz, by considering the attenuation, the reconstruction results are closer to the true depths. At higher frequencies when the reconstructed results show overestimation of the wall loss, the errors are larger for the cases in which the attenuation is not considered. Importantly, it can be clearly seen that the errors increase when the liquid results in larger attenuation. This error is caused by the increase of the attenuation which increases the differences between the amplitudes of the waves that 1) propagate directly from the source to the receiver and 2) are scattered from the defect, compared to the plate without liquid loading. Also, this error increases with propagation distances. In general, when attenuation has been considered in the reconstructions, much closer results to those of the free plates can be obtained. Only at 160 kHz the results show some deviations. This may be due to the combined effect of attenuation and the density correction for the S_0 mode. Since the S_0 mode tends to be more sensitive to errors than the A_0 mode given the nature of the dispersion curves [125], applying density correction to the FWI reconstruction with the consideration of

attenuation at some frequencies affects the reconstruction accuracy.

The accuracy of the thickness reconstructions of water-loaded plates with the consideration of attenuation in the inversion is investigated over different defect widths and frequencies for a fixed depth of 3 mm, as shown in Figure 6.8. In this case, the inversion was performed over sequential frequencies from 130 to 180 kHz with a step of 10 kHz. The inversion was started from the homogeneous background model at 130 kHz, with 20 iterations at each frequency. The surface width of the defect was varied from 40 to 120 mm with a step of 20 mm. Some trends can be identified. At a fixed frequency, the depth of defect is underestimated for smaller widths and as the width increases, the reconstruction accuracy is improved. At a fixed width, the accuracy of the reconstruction improves when the frequency increases. The maximum depth error in the reconstruction of water-loaded plates with the consideration of attenuation agrees well with the previous findings of free plates [68, 153], which demonstrated that the resolution improves with the increase of the frequency due to smaller wavelengths at higher frequencies, leading to better accuracy of the reconstruction.

FIGURE 6.8: The maximum depth error of a Hann-shaped defect with a fixed depth of 3 mm and varying surface widths across different frequencies in water-loaded plates with the consideration of the attenuation with the S_0 mode.

Figure 6.9 shows the error across a range of frequencies and different depths of the defect in the water-loaded plate with the consideration of attenuation, when the surface width is fixed at 100 mm. At lower frequencies, the depth of defect is underestimated and the error has an increasing trend with the increase of the

depth. As the frequency increases, the depth error becomes smaller due to the resolution improvement.

FIGURE 6.9: The maximum depth error of a Hann-shaped defect with a fixed width of 100 mm and varying depths across different frequencies in the water-loaded plates with the consideration of the attenuation with the S_0 mode.

In summary, the results have shown that the attenuation affects the reconstruction accuracy. More accurate thickness reconstruction can be achieved with the S_0 mode at higher frequencies as higher attenuation has been considered in the inversion.

6.6 Experimental validation

Figure 6.10(a) shows the original model of an irregularly shaped defect located at the center of a 64-transducer circular array. This complex shaped defect has the maximum surface width of around 100 mm ($\simeq 2.9\lambda$ at 150 kHz), a flat bottom width of around 25 mm ($\simeq 0.73\lambda$ at 150 kHz) and a defect depth of 3 mm. The reconstruction algorithm with the consideration of the attenuation of the waves due to liquid loading was used. Homogeneous background was used as the starting model and the inversion was performed at a single frequency of 150 kHz, with 20 iterations. It should be noted from previous discussions that more accurate results can be obtained at higher frequency. However, due to the constraint of propagation distance, it is difficult to separate the S_0 and A_0 mode at the higher frequencies, and therefore the reconstruction was carried out at 150 kHz. Figure 6.10(b) presents

the reconstruction of the complex defect from the experiment. It can be seen from the image that the defect is very well reconstructed. The zoomed defects from Figures 6.10(a) and (b) presented in Figures 6.10(c) and (d) agree well with each other. Notable artifacts are observed in the image obtained from the experiment. Factors which can affect the image quality include under-sampling of the wavefield, noise in the measurements, unwanted directionality of the transducer due to uneven coupling, and the additional scattering from the attached tank wall.

FIGURE 6.10: FWI thickness reconstructions of an irregularly shaped defect at 150 kHz. (a) Original model, and (b) the experiment; (c) and (d) zoomed pictures of the defect from (a) and (b). The dotted curve and dashed curve represent the positions of a transducer array and the tank wall, respectively.

Figures 6.11(a) and (b) show the cross-sections through the horizontal and the vertical line of the irregularly shaped defect from Figures 6.10(c) and (d), respectively. It can be seen that the reconstructed profile matches very well with the true one and the deepest depth of the defect is accurately reconstructed. It should be noted from the figure that a slight overestimation of the maximum depth of

the defect can be observed, which is different from the prediction shown in Figure 6.7 for a Hann-shaped defect with similar size. This is mainly due to the effect of the bottom profile on the reconstruction accuracy. As discussed in [153], when the bottom width of a flat-bottom defect is small, the depth estimation resembles that of a Hann-shaped defect and the defect depth is underestimated. When the bottom width increases, the reconstruction overestimates the defect depth. The reason has been explained in [125, 153].

FIGURE 6.11: Cross sections of the thickness reconstruction of the irregularly shaped defect along the horizontal and vertical line in the experiment from Figure 6.10(c) and (d) at 150 kHz.

6.7 Summary

In this chapter, GWT is applied on liquid-loaded plate-like structures exploiting the FWI method. Using the S_0 mode, the effect of attenuation arising from the presence of liquid loading on the thickness reconstruction is investigated. The simulations of a range of defect widths and depths in the water tank floor with the consideration of attenuation across different frequencies have demonstrated the suitability of the S_0 mode for accurate reconstruction of the defect with improving sensitivity at higher frequencies. It is found that when the attenuation is small, the influence of the consideration of the attenuation on the thickness reconstructions in the liquid-loaded tank floor for S_0 is generally small. As the attenuation increases with the increase of the frequency, especially in the presence of a liquid resulting in high attenuation, the attenuation effect cannot be neglected for more

accurate reconstruction. The experimental results of an irregularly shaped defect on a water tank floor demonstrate that the reconstructed thickness of this defect at the frequency of 150 kHz with the attenuation considered in the inversion model agree well with its actual depth.

Chapter 7

Corrosion Mapping Using GWT

7.1 Introduction

Corrosion is a dynamic process which can be influenced by many factors including time and process variables. Therefore, the monitoring of corrosion processes has become highly sought-after to enhance the safety and the profitability of petrochemical plants. The relatively long time intervals associated with off-line inspections [154] render these techniques incapable of revealing unforeseeable changes which could have developed during corrosion progression. Ultrasonic techniques provide a non-intrusive means for the on-line monitoring of corrosion [155, 156].

A small number of permanently installed point thickness gauges has been used for monitoring the average thickness loss over a moderate area [157, 158], which provides high probability of detection. However, in order to detect non-uniform and highly localized defects, the number of transducers required substantially increases. In these cases, the area monitoring systems (e.g. the field signature approach [159] or dispersive guided wave velocity measurements [160, 161]) appear to be more attractive. Nonetheless, the area monitoring systems are only sensitive to average wall thickness loss over a monitored area. If a wall loss is not uniform, these systems would either not be able to detect it or would underestimate the severity of non-uniform corrosion since the average thickness loss is taken as representative. In cases of severe wall loss over a large area coverage, the GWT system has demonstrated the capability and sensitivity of mapping non-uniform, localized corrosion damages [17, 162–164].

Generally speaking, a GWT system consists of an array of transducers, a data acquisition system, data processing techniques and a reconstruction algorithm. A successful GWT system ought to have good sensitivity to corrosion damages, high localization accuracy and imaging quality, and a low transducer density [165]. Ultrasonic NDT systems based on the guided waves generated by arrays of electromagnetic acoustic transducers (EMATs) have been used to the corrosion monitoring of pipes and plate-like structures [166]. For example, the received signals from arrays of EMATs encircled the pipe were processed by using ultrasonic computerized tomography [167, 168] to generate the maps of thickness loss. However, most EMATs have a relatively low SNR and their performance relies on the electromagnetic properties of the tested objects such as the presence of a highly magnetostrictive oxide layer [169]. By contrast, piezoelectric transducers have high SNR, relatively low cost and small energy consumption which are more appropriate for long-term monitoring [170–173]. However, it has previously been shown that the scattering from transducers can affect the performance of the thickness reconstruction and it needs to be minimized for more accurate reconstruction [162]. In this chapter, an optimized low frequency A_0 mode transducer is designed for the GWT system and the effect of transducer array scattering on the accuracy of the thickness reconstruction is also investigated.

The GWT system presented in this chapter consists of customized piezoelectric transducer arrays, automated signal generation, acquisition and processing, and the FWI algorithm. As such, the automated on-line monitoring of corrosion progression can be done efficiently and with little manual labor. The performances of the system are examined in the lab-based forced corrosion experiments. In Section 7.2, a GWT system is described, including the array system, data processing method and the scattering from transducers. In Section 7.3, forced corrosion and thickness measurements using laser profilometry are presented. The results from experiments and discussions are followed in Section 7.4. Conclusions are summarized in Section 7.5.

7.2 Monitoring system

7.2.1 Array system and experimental setup

Two experimental setups for the GWT system for corrosion monitoring of the steel plate and pipe are carried out in this chapter. The first case considers an array consisting of 64 piezoelectric transducers used in the steel plate, which were uniformly distributed on a circular array of diameter of 700 mm, as presented in Figure 7.1(a). The transducer was designed to generate the A_0 guided wave between 30 kHz to 90 kHz. The design was based on the stacking of piezoelectric elements to maximize the generation and receiving efficiency of the selected guided mode and meanwhile minimize the dimensions of the transducer [174]. Piezoelectric ceramic PZT-5A element with 2 mm in thickness and 10 mm in diameter was used. The waveform generator was controlled by an ultrasonic phased array controller (Lecoeur électronique, Chuelles, France). A 5-cycle Hanning windowed tone-burst signal with the central frequency of 60 kHz was selected for the excitation. Figures 7.1(b) and (c) show a typical time trace of a received A_0 mode, excited at point 1 and measured at point 17 in a 10 mm thick steel plate and its frequency spectrum. The 64 transducers were held in place by a support ring to ensure the accuracy in their positioning in Figure 7.1(a). In order to ensure good contact between the plate and each of the transducers, couplant was used and then weights were evenly distributed on the mounting bracket of the support ring, with a spring fitted between the support ring and each individual transducer. In this experiment, only transmitted signals from the transmission subset were used here to avoid reflections from edges of the plate [175]. Using the current system, SNR greater than 30 dB can be attained after five averages.

In the first case, a 1500 mm×1500 mm×10 mm mild steel plate (see Table 3.1) was selected for the experiment, and a corrosion site was designed at the center of the plate, as shown in Figure 7.1(a). It is worth mentioning that prior to exposing the steel plate to corrosion, an artificial defect with the depth of 1 mm and the diameter of 80 mm was milled at the center of the plate. Such approach will lead the subsequent corrosion to a smooth transition rather than a step change in depth from the top of the plate to the bottom of the defect. Less mode conversions (the A_0 to S_0 mode) are expected from relatively smooth thickness changes, favoring the acoustic assumption of the thickness reconstruction algorithm [50].

FIGURE 7.1: (a) Setup of the guided wave tomography measurements of the steel plate. All experiments were performed at room temperature. (b) Time trace and (c) frequency spectrum of a received signal from the experiment without defects in the steel plate. The source was at point 1 and the measurement was taken at point 17.

In the second case, the experimental setup for corrosion monitoring of the quiescent water-filled steel pipe is shown in Figure 7.2. This experiment was carried out on a 12 inch pipe (outer diameter is 323.85 mm) with a wall thickness of 10.3 mm and a length of 2 m. An irregularly shaped defect was induced by forced electrochemical corrosion at the middle of the pipe. Two ring arrays of transducers were attached on either side of the defect with the separation distance of 600 mm. Each array has 24 piezoelectric transducers, which are also used to excite A_0 at the center frequency of 60 kHz by applying an out-of-plane force. Two adjacent transducers are spaced around 42.4 mm in the circumferential direction, which is equal to 1.25λ ($\lambda=34$ mm at 60 kHz). This separation distance is above the Nyquist requirement of half a wavelength, thus making this system under-sampling [121].

In each dataset for the thickness mapping of the pipe, a matrix of 48×24 source-receiver combinations is recorded. It is worth noting that the measurements were performed first on a defect free pipe with the same setting before testing on the pipe with a defect. A total of four tests were carried out at room temperature,

FIGURE 7.2: Experimental setup for measurements, including the mild steel pipe (a diameter of 12 inch and the thickness of 10.3 mm) containing a corrosion site, two ring arrays of transducers and schematic instrumentation.

including a free and water-filled pipe without a defect, and a free and water-filled pipe with a defect.

7.2.2 Data Processing

In the first case of plates, data processing is in the same way as in Section 3.3.3. In the second case of pipes, the measured signals need to be separated in the different way based on the helical path separation algorithm proposed by Huthwaite [176]. The background and this algorithm are only briefly presented here.

The cyclic nature of pipe geometry allows infinite helical paths to travel from a transmitter to a receiver because the waves can propagate many times around the circumferential direction of a pipe. Figure 7.3(a) shows the configuration of the pipe, with N number of transducers in both the transmitter array and the receiver array. When the curvature of a pipe can be neglected [73, 74, 177], the pipe can be presented as a 2D domain plate with x -axis denoting the circumferential direction and z -axis representing the axial direction, as shown in Figure 7.3(b). The effect of the circular-symmetric nature of the pipe can be described by replicating the unwrapped pipe (order 0) multiple times and extending the replications (higher orders) in both sides to infinity. The signal transmitted from T_0 and measured

at R_0 in the pipe corresponds to a superposition of the signals measured at all receivers R_{-2} , R_{-1} , R_0 , R_1 , and R_2 on the flattened plate, as illustrated in Figure 7.3(b). It needs to be noted that the $\pm n$ th replica is referred to the $\pm n$ th order of the helical mode. It can be seen from Figure 7.3(b) that the separation of various wave-packets, which corresponds to the respective helical order, increases the length of two parallel linear arrays and the missing viewing angles decrease, resulting in the less ill-posed problem.

FIGURE 7.3: (a) 3D physical model showing transmitter and receiver arrays and higher order helical paths, and (b) order 0 is the unwrapped section of pipe and other orders are added to both directions along x axis to replicate the cyclic characteristic of the pipe. In the pipe, the measured signals at R_0 correspond to a superposition of signals from each path in the flatten plate. The arrows present various propagation path between a transmitter and a receiver.

These multiple helical signals need to be separated. This can be achieved by using the helical path separation algorithm [176], which allows entire wave-packets, not

only the first arrival times [66], to be separated even when waves overlap. Firstly, the measured time traces for one source and all receivers are replicated $2n$ times (from $-nth$ to $+nth$ order) in order to expand the aperture of the two parallel arrays and extract the information contained in higher order modes. Secondly, the replicated signals which contain the wavefront of interest are back-propagated to the transmitter location by making use of the known receiver locations, the associated propagation distances and the dispersion curves of the steel plate. After the back-propagation step, the wavefront of interest is almost uniform for all receivers in time domain and then is windowed to remove the unwanted signals. A low pass filter is used over all receivers in Fourier domain and the filtered wavefront is forward propagated back to the original locations of receivers. Finally, the wavefront is shifted to all the non-zeroth orders and then subtracted from all the replicated signals, removing the unwanted signals. This process is iterated using the obtained signals as the new original signals and adjusting the low pass filter in each iteration. This approach has been validated by empty pipes with defects via simulations as well as experiments in [168, 176].

The separation of the experimental time traces of the water-filled pipe and the empty pipe is presented in Figure 7.4. Firstly, the measured time traces from one source and all receivers are replicated several times to consider different orders. Figures 7.4(a) and (b) show the replicated signals for two times (from $-1th$ to $+1th$ order) obtained from a free pipe and a water-filled pipe without the defect. The wavefronts of the A_0 mode obtained from the free pipe are clearly observed. In contrast, there is less energy in the signals obtained from the water-filled pipe, especially in higher orders, which is due to higher attenuation with increasing propagation path. Also, it can be seen from Figure 7.4(b) that very strong signals appear after 0.4 ms, which are caused by the direct water waves due to the A_0 mode leakage.

Secondly, the replicated signals are separated by the helical separation algorithm [176]. The separated time traces are presented in Figures 7.4 (c) and (d). It further confirms that the water loading exerts obvious influence on the A_0 mode, especially in the amplitude. The higher order signals have more energy leakage to the water medium, resulting in lower SNR because the noise level remains constant. In Figure 7.4(d), the water waves are removed after the separation algorithm and basically do not interfere with A_0 . It needs to be mentioned that the separated signals show

FIGURE 7.4: Separation of the experimental time traces for one iteration. The measured time traces are replicated two times (from -1^{th} to $+1^{th}$ order) in (a) the empty pipe and (b) the water-filled pipe; (c) and (d) the separated signals extracted by the separation algorithm from (a) the empty pipe and (b) the water-filled pipe, respectively.

less information (Figures 7.4(c) and (d)) compared with corresponding parts in the replicated signals (Figures 7.4(a) and (b)). This is caused by the window applied in the receiver direction. This smoothing window is used to minimize the spectral leakage, which could occur with the discrete receiver Fourier transform [176].

The separated signals are then calibrated before using them in the inversion by the similar method described in Section 3.3.3. Based on the results obtained from the standard limited view FWI algorithm, the adaptive thresholding regularization method is applied. The regularized velocity map is converted into the thickness map via the dispersion curves for a steel plate with/without water loading, as shown in Figure 2.3. The pipe curvature is ignored here according to [72–74].

7.2.3 Scattering from transducers

In the imaging algorithm, the forward modeling is based on the assumption that the incident wavefield is generated by a point excitation transducer and the scattering from the defect is produced by this incident wavefield. However, in the case of contact transducers array, the incident wave can interact with other transducers, causing secondary incident wavefields that can produce unwanted scattering components from the defect. This can affect the reconstruction of the defect [162]. Therefore, it is important to evaluate the scattering from transducers and its impact on the thickness reconstruction.

FE simulations and experiments were used to investigate the scattering from a single transducer attached to the steel plate. The wave generated from a transducer interacting with its neighbour in the designed array was modeled. The setup is shown in Figure 7.5(a). The scattering transducer is located at the center of a 300 mm diameter array and the source transducer is located 35 mm away from the center.

The cross section of the scattering transducer attached to the steel plate used in the FE model is shown in Figure 7.5(b) and the properties of materials are given in Table 3.1 and Table 7.1. The transducer is composed of an aluminum cylinder, alumina front layer and a stack of PZT discs. The contact between the transducer and the plate was modeled by a thin coupling layer (density=1100 kg/m³ and acoustic impedance=1.84 × 10⁶ kg/m² s). The model was meshed with cubic eight node elements with the size of 1 mm, and smaller elements for the coupling and the front layer. The A_0 mode was excited by an out-of-plane force at the plate surface and out-of-plane displacements were measured at 13 monitor points equally spaced along a half circular array, as shown in Figure 7.5(a). The excitation signal was a 5-cycle Hanning windowed tone-burst at the central frequency of 60 kHz, the same as the one used in the thickness reconstruction. The scattered wavefield was obtained by subtracting the incident wavefield of the model with and without the scatterer, as shown in Figure 7.5(c). The experiments were performed on the steel plate described in Section 7.2.1 and two transducers from the array were used for the measurements. The scattering transducer was attached to the plate by adding weight on it. The A_0 mode was excited by using Tiepie Handyscope HS3 and the out-of-plane displacements were measured by a Polytec OFV-505 laser vibrometer.

FIGURE 7.5: (a) Schematic of measurement for the scattering transducer in the center of the array by using the symmetry in this case with one source and 13 monitor points, (b) cross section of modeled transducer, the scattering transducer, in FE model, (c) scattered wavefield obtained by subtracting the incident wavefield of the FE simulation with and without the scatterer in measured point 1, (d) scattered wavefield obtained from the experiment in measured point 1 and (e) the normalized amplitude varying with the measured points in the simulation and the experiment.

TABLE 7.1: Material properties of Alumina and PZT

Material	Young's modulus (GPa)	Poisson's ratio	Density (kg/m^3)
Alumina	300	0.21	3860
PZT	50	0.35	7750

The scattered wavefield was extracted in the same manner as in FE modeling, as shown in Figure 7.5(d).

Figure 7.5(c) shows the amplitude of the scattered wavefield from the transducer at 60 kHz with respect to different monitoring positions. It is normalized by the incident wave amplitude with the same propagation distance. Since the exact thickness

of the coupling layer in the experiment is unknown, two reasonable assumptions of 0.2 mm and 0.4 mm were used in the FE model for comparison. It can be seen that overall the scattered wavefield is weak, with the amplitude decreasing from the transmission part (closer to monitor point 1) to the reflection part (closer to monitor point 13). It can also be noted from the figure that thinner coupling layer results in stronger scattering. The reason is that in the case of thinner layer, more energy from the wave propagating in the plate is transmitted to the couplant [178] and thus more energy is converted into the scattering. It can be seen that the trend shown in the experimental results agrees well with the predictions from the FE simulation, despite some variations due to the noise.

7.2.4 Scattering effect on the thickness reconstruction

The transducer scattering effect on the thickness reconstruction of plates was investigated by FE modeling. A Hann-shaped defect with the surface diameter of 120 mm and the depth of 3 mm was modeled at the center of a steel plate with the dimension of 1100 mm×1100 mm×10 mm, the same as the one shown in Section 4.3.1. A 700 mm diameter array with 64 transducers was simulated. Two cases were considered: the array with artificial transducers (waves generated on the nodal position) and the one with real transducers introduced in Section 7.2.3. The same input signal and reconstruction method described were applied in the simulation.

The cross sections of the reconstructed thickness map are presented in Figure 7.6. Two coupling layers of 0.2 mm and 0.4 mm thick were also modeled. It can be seen that the reconstructed thickness profiles obtained from artificial and real transducers are almost superposed, indicating that the scattering from the transducers has minimal effect on the thickness reconstruction.

FIGURE 7.6: Thickness profiles across the reconstructions of a Hann-shaped defect with the surface diameter of 120 mm and the depth of 3 mm from the FE model in the plate without and with an array of modeled transducers. h is the thickness of the coupling layer.

7.3 Experimental simulation of corrosion damage

7.3.1 Forced electrochemical corrosion

While the unforced corrosion is more representative of real-world cases, the forced corrosion, which is driven by an applied electric current, is used for accelerating corrosion damage for the purposes of obtaining results in a relatively short time [179]. For the corrosion of steel, this reaction is dominated by the two-electron oxidation of elemental iron, $Fe \rightarrow Fe^{2+} + 2e^-$. In the forced corrosion experiments carried out in this chapter, the mild steel plate/pipe was made the anode on which the oxidation of element iron occurs, and a steel thin disk inserted into the chamber (an example shown in Figure 7.7(a)) acts as the cathode.

For example, Figure 7.7(a) shows the experimental setup of forced electrochemical corrosion on the mild steel plate. The corrosion chamber with a diameter of 130 mm is attached to the surface of the steel plate, which is the concentric location

FIGURE 7.7: (a) The experimental setup of the forced corrosion and (b) a laser profile scan of the corrosion defect in a 10 mm steel plate.

of the artificial defect. In the corrosion process, a brine electrolyte is circulated through the chamber by a pump (Verderflex M025, Verder Ltd., Castleford, UK) in order to prevent the precipitation of corrosion product iron oxide. This will help to maintain the rate of corrosion and also to prevent the further oxidation of iron oxide into iron hydroxide which is insoluble and rather protective against further corrosion. Also, it can minimize the temperature change during the electrochemical corrosion process because the obvious increase in temperature causes the reductions in ultrasonic velocities[28]. In the forced corrosion experiment, the electric current is supplied by a constant current source (IPS 303DD, ISO TECH, UK).

Faraday's law of electrolysis is used to predict the average wall loss due to an applied current in the forced corrosion experiment. According to this law, the average wall loss ΔT_h of the steel plate can be calculated by

$$\Delta T_h = -\frac{M_{Fe}It}{2FA_c\rho_{Fe}}, \quad (7.1)$$

where M_{Fe} is molar mass of steel (0.055845 mol/kg), I being applied current, and t is the elapsed time. F is the Faraday's constant (96485.3329), A_c being the area of the corrosion surface (diameter of 0.13 m) and ρ_{Fe} is the density of steel (7850 kg/m³).

In this chapter, the constant current of 3 A was used and thus the relationship between the wall loss of steel plate and time is given by

$$\Delta T_h = -8.3324 \times 10^{-9}t. \quad (7.2)$$

7.3.2 Thickness measurement using laser profilometry

In this chapter, a blue laser scanner (LJ-V7080, KEYENCE, Japan) was employed to measure the wall losses and profiles of the controlled corrosion damage. An example is shown in Figure 7.7(b). It has the repeatability precision of 10 μm along the width of the laser beam, and 0.5 μm along the thickness of the plate. The results are post-processed by using the software supplied by the manufacturer. It needs to be noted that the largest measuring range of laser in width is around 40 mm and thus multiple measurements should be taken and merged together for a larger size of the corrosion damage.

Before measuring the corrosion profile of the plate/pipe with the laser scanner, the forced corrosion setup is removed and the residual corrosion products on the corrosion surface are carefully cleaned away.

7.4 Results and discussions

The focus of this chapter lies on the performance of the GWT system for corrosion monitoring of the plate and the pipe. In the case of the plate, firstly, the reconstructed depths of the corrosion defect at different stages using the FWI algorithm are compared with the values predicted by Faraday's law. This is followed by a detailed comparison of thickness maps of the plate, obtained by the FWI algorithm and laser profilometry, at two different corrosion periods. In corrosion monitoring of the pipe, the thickness reconstruction of an irregularly shaped defect in the empty pipe and the water-filled pipe is investigated.

7.4.1 Corrosion monitoring of plate

7.4.1.1 Wall loss monitoring

In this work, measurements of the corrosion damage induced by forced corrosion were carried out over a period of approximately 110 hours. According to equation (7.2), the average thickness loss of 1 mm can be achieved around 36 hours.

To compare the average reconstruction by FWI with the average wall loss calculated by Faraday's law, an alternative metric for quantifying the average reconstructed thickness from FWI is to take the mean value of all reconstructed thickness within the upper limit of the minimum reconstructed thickness $(1 + 5\%)T_{min}$ [50]. The average reconstructed thickness $\bar{T}$ can be given by

$$\bar{T} = \sum_{n=1}^N \frac{T(x_n, y_n)}{N} \quad \text{when} \quad T(x_n, y_n) \leq (1 + 5\%) \times T_{min}, \quad (7.3)$$

where $T(x_n, y_n)$ is the reconstructed thickness at coordinate x_n and y_n ; T_{min} is the minimum reconstructed thickness and N is the reconstructed number according to the range within $(1 + 5\%)T_{min}$.

Figure 7.8 shows the average reconstructed thicknesses of defects at different stages during the corrosion process by the FWI algorithm, together with analytical predictions by Faraday's law. It can be observed that all average reconstructed thicknesses are in good agreement with analytical predictions in the continuous corrosion.

7.4.1.2 Thickness reconstruction

In Chapter 4, it has been shown that the FWI algorithm achieves a resolution around $1.5-2\lambda$ for the majority of defect depths from the experiment. Also, the shape of the defect profile has the influence on the resolvability. Here, the verification of the effect on reconstruction accuracy is carried out by different surface and bottom diameters, and the depths of flat-bottom defects in different corrosion progression. The results of corrosion defect A and B at the corrosion time of 74 and 110 hours based on the GWT and a laser profile scan in two dotted boxes are

FIGURE 7.8: The comparison of the average reconstructed thickness of the defect obtained by FWI at 60 kHz and predictions by Faraday's law in different corrosion periods of a mild steel plate. The upper and lower limits of measured thickness of defect A and B at corrosion time of 74 and 110 hours obtained by FWI and laser profilometry are shown in the rectangular boxes, respectively.

The results of laser profilometry will be explained in Section 7.4.1.2.

shown in Figure 7.8. The wall losses are presented as error bars, showing the upper and lower limit of the measured thickness. Excellent agreements between the reconstructions from GWT and the laser profile measurements, even closer than the predictions from the analytical model.

Figure 7.9(a) shows the profile measured by laser scans after 74 hours of corrosion. As mentioned earlier, due to the limitation of the laser scanner, the profile of the defect needs to be scanned by separate strips, and therefore the profile spliced from the scans is not perfectly smooth. However, it is clear from the figure that the quality of the interior of the resolvable area has not been degraded. The defect has a regular shape and relatively smooth variations in the thickness. The largest extent is around 130 mm ($\simeq 3.9\lambda$ at 60 kHz) and the deepest depth is about 3 mm. In this study, sequential frequencies of 35, 46 and 60 kHz are used in the inversion. Figures 7.9(b) and (c) present the thickness reconstruction results and the zoomed image of this defect at 60 kHz. It can be seen that the overall shape of defect A reconstruction is well approximated, when compared with the measured corrosion profile. However, the experimental reconstruction has slight

deformation in the shape of the defect and the contrast of the defect is also slightly underestimated. The reason is that the reconstructed thickness is affected by the effect of different scattering behavior between guided waves with thickness variation and acoustic waves with velocity variation [70]. Some artifacts can be observed in the reconstructed image, especially at the positions of the transducers array and around the defect. There could be a few sources for the artifacts. Besides the environmental noise in the experiment, artifacts at the array position can be affected by the undersampling of the wavefield [180] and the scattering from the array of transducers. The artifacts around the defects could mainly be produced by the attached chamber wall and silicon used in the bond of the chamber wall and the plate, which have not been taken into account in the reconstruction algorithm.

FIGURE 7.9: Thickness mapping of the corrosion defect A and B in the steel plate. (a) Laser measured profile of defect A after post-processing, (b) thickness reconstruction from FWI at 60kHz and (c) a zoomed-in view of defect A from (b). The dashed curve presents the position of the array. (d) Laser measured profile of defect B after post-processing, (e) thickness reconstruction from FWI at 60 kHz and (f) a zoomed picture of defect B from (e).

The thickness profiles of a deeper defect B after 110 hours of corrosion are shown in Figures 7.9(d) and (f). It is clear that the reconstruction can still capture

most features of the defect, although the discrepancy compared with the measured thickness profile is larger than the previous case, as the error of the reconstruction increases with the depth of the defect [168].

The cross sections of the thickness profile along the vertical direction (dashed lines in Figure 7.9) for both defects are plotted in Figure 7.10. It can be seen that the location of the defect and the minimum thickness have been successfully identified for both cases. It can also be noted that the reconstructed defects are slightly narrower than the measurement from the laser scanner, which is believed to be caused by the difference in the scattering by the acoustic and the elastic model [70].

FIGURE 7.10: Cross sections of FWI reconstruction and laser measurement of (a) defect A and (b) defect B along the vertical line in the plate.

7.4.2 Corrosion mapping of pipe

The thickness reconstruction of an irregularly shaped defect in the empty pipe provides the ability to identify the achievable reconstruction accuracy. In order to analyze the effect of attenuation due to the presence of water loading on the reconstruction accuracy, the water-filled pipe is investigated.

7.4.2.1 Empty pipes

The profile of the irregularly shaped defect induced by forced corrosion is measured by the laser scanner, and the result is shown in Figure 7.11(a). This defect has

FIGURE 7.11: Thickness mapping of an irregularly shaped defect in the empty pipe. (a) The profile of the defect measured by a laser scanner, (b) the FWI reconstruction obtained considering four replicas and (c) the FWI reconstruction obtained considering two replicas at 60 kHz. (unit=mm)

the largest width of around 150 mm ($\simeq 4.4\lambda$ at 60 kHz) and the depth of around 4 mm. To examine the benefits of the replications (higher order modes), Figure 7.11 compares the thickness reconstructions of the empty pipe obtained by considering four replicas (Figure 7.11(b)) and two replicas (Figure 7.11(c)). As expected, the larger aperture leads to the reconstruction with better quality in terms of the defect shape and the depth estimation. However, both reconstructions slightly underestimate the size of the defect. The discrepancy is mainly attributed to the limited view configuration of the GWT system. Since these data are measured from the pipe with two ring arrays, some information is missed out on certain viewing angles even with the arrays over four replicas. The remaining discrepancy is due to the difference between the acoustic and the elastic scattering, since the acoustic reconstruction algorithm is used to reconstruct the elastic scattering data from the experiment [68, 168].

The cross sections of Figure 7.11 along the horizontal and vertical lines are presented in Figures 7.12(a) and (b). The whole profiles of the defect with four and two replicas are narrower than that measured by the laser scanner due to the reasons explained previously. However, it is again shown that the reconstruction with four replicas can achieve slightly better estimations of the defect profile and depth than that with two replicas due to a larger range of angles available for the inversion. The maximum depths of the defect with four and two replicas are underestimated by around 0.2 mm and 0.7 mm, respectively.

FIGURE 7.12: Cross sections of FWI reconstructions of an irregular defect and laser measurement along the line marked in Figure 7.11 along (a) the horizontal line and (b) the vertical line in the empty pipe. (unit=mm)

7.4.2.2 Water-filled pipes

In Figure 7.13, the defect profile measured by the laser scan is compared with the reconstruction thickness of the water-filled pipe. Only two replicas are used in the inversion, because the attenuated signals which gradually leak energy into water with increasing helical path cannot be identified in higher order paths (for example, ± 2 th order). Compared with the empty pipe results shown in Figure 7.11(c), the defect size is slightly smaller and its shape is distorted. Also, more noticeable artifacts can be observed here even when the attenuation has been taken into account in the regularized image. The main reason is that the effective array aperture in the water-filled pipe is smaller than that in the empty pipe due to the energy leakage to the water medium, as shown in Figures 7.4(c) and (d), which affects the reconstruction accuracy.

FIGURE 7.13: Thickness mapping of an irregularly shaped defect in the water-filled pipe. (a) The profile of the defect measured by a laser scanner (the same as in Figures 7.11(a)), and (b) the FWI reconstruction obtained considering two replicas with the attenuation in the inversion model at 60 kHz. (unit=mm)

Figures 7.14(a) and (b) show the cross sections of Figure 7.13 along the horizontal and vertical lines. The thickness reconstructions of the deepest points with the consideration of attenuation are slightly underestimated, with the thickness error around 0.7 mm. In general, the estimations of the maximum defect depth in the empty pipe and the water-filled pipe with the consideration of attenuation in the inversion model can match well.

FIGURE 7.14: Cross sections of the thickness reconstruction from Figure 7.13 along (a) the horizontal line and (b) the vertical line in the water-filled pipe. (unit=mm)

7.5 Summary

In this chapter, a GWT system for the efficient and accurate corrosion monitoring of plate-like and pipe structures has been presented. The array of piezoelectric transducers has been designed to generate and receive the A_0 Lamb wave at low frequencies. It was found that the influence of the scattering from the selected transducers on the accuracy of the thickness reconstruction is minimum.

In the forced corrosion experiment of the plate, the average wall losses that were reconstructed by FWI agreed well with the analytical predictions obtained by Faraday's law. The reconstructed thickness maps of the plate at two different stages matched well with the measurements from a laser scanner. Due to the pipe with the limited view configuration, an adaptive regularization method and the information carried by higher order modes obtained by the helical path separation method are used to improve the reconstruction accuracy. In general, the results show that the maximum depth estimation of the defect in a water-filled pipe with the consideration of the attenuation agrees well with that of an empty pipe.

Chapter 8

Density reconstruction using multi-parameter FWI

8.1 Introduction

The quality control of large forging components and powder metal compacts is very important in the aviation and automotive industry. Quality assurance of these structures requires the control of various parameters, e.g. density, elastic moduli and tensile strength, all of which can provide complementary information on mechanical performance. Density is an important quality control parameter, because it correlates to elastic moduli and strength. For example, the fatigue strength of powder metal components is usually improved by increasing the density [32]. Significant density fluctuations affect mechanical performance. For example, in powder metallurgy, the powder can flow into the die cavity at the early stage of the compaction. However, as the pressure increases and the movement of powder is restricted, uneven distribution of density may occur. If a low density region exists the internal corner of the part, it can be a fatal flaw. Because the corner is always the stress concentration point when the part is loaded in service [25]. Currently, the standardized density tests are gravimetric measurements in specimens that are cut out from components [181], which are intrusive. Therefore, it is necessary to develop NDT techniques for determining the density distribution.

Computed tomography (CT) is an NDT technique which can be used to characterize the density distribution. CT can be carried out by X-rays or gamma-rays. The

attenuation of X-rays/gamma-rays passing through the sample depends on mass density, mass attenuation coefficient and sample thickness [27]. Compared with X-ray CT, gamma-ray CT can characterize thicker samples because the energy of gamma-rays are higher than X-rays. However, the cost of a CT unit is high and it requires a radioactive source.

Ultrasonic testing is a low-cost alternative for NDT of engineering components. When ultrasonic waves propagate through metal parts, the physical properties of objects such as velocity, density and attenuation can be determined by using the information carried by the waves [29]. Most work of characterizing density distribution is based on ultrasonic velocity measurements [182, 183]. The density of the sample is indirectly determined from the velocity of ultrasonic waves propagating through the sample, assuming Young's modulus and Poisson's ratio are known. However, Young's modulus and density are always coupled together and therefore it is difficult to ensure the accuracy in the density reconstruction. Also, these methods are usually tedious, especially for inaccessible regions.

Ultrasonic tomography offers a good potential to estimate the density distribution of metal parts without accessing all points on the surface. An ultrasonic tomography method was used to determine density variation of powder metal components by using a filtered back-projection algorithm based on the traveltime of ultrasonic bulk longitudinal waves [32]. The obtained velocity reconstruction lays the foundation for predicting the density distribution. However, only velocity can be reconstructed in this work, which ignores coupled effects between different parameters. Also, this method is only valid when inhomogeneities are much larger than the wavelength because diffraction effects are ignored [32].

In this work, a new ultrasonic tomography method for simultaneously quantifying density variation and velocity distribution of metal parts is developed. The reconstruction of velocity and density is performed by using multi-parameter FWI, which can take into account higher order diffraction and scattering. Comparing to the mono-parameter FWI, the multi-parameter FWI considers the density term in 2D acoustic model in this work. However, moving from the mono-parameter inversion to the multi-parameter inversion increases the indetermination. Different parameters with different natures can have coupled effects on the response for the propagation region from transmission to reflection (referred as cross-talk between

different parameters). Moreover, the difference in amplitude of different parameters in the wave-field is also a difficulty, which should be properly scaled otherwise it can make the inversion ill conditioned [29].

To mitigate the effects of cross-talk and rescale the amplitudes of different parameters, a robust estimation of the inverse Hessian operator plays a very important role in the multi-parameter inversion [29, 184]. In the multi-parameter inversion, the off-diagonal blocks of the Hessian operator provide the coupling information between different parameters. Therefore, accurate estimation of the inverse Hessian in the inversion can mitigate the trade-off effect in the reconstruction. However, in practical applications, the inverse Hessian is too large to be calculated or stored. The approximation of inverse Hessian can be obtained by using the limited memory Broyden-Fletcher-Goldfarb-Shanno method [185], but this method accurately relied on the initial model and iterative scheme [123]. Sheen [186] used low-rank approximation to decompose the Hessian in the time domain FWI. However, it needs large memory to store this Hessian. In this work, the truncated Gauss-Newton method is introduced to better estimate the inverse Hessian in multi-parameter FWI for simultaneous reconstruction of velocity and density. The approximate Hessian does not need to be stored in memory in advance [187].

In this chapter, the theory of multi-parameter FWI based on the truncated Gauss-Newton method is described in Section 8.2. In Section 8.3, simultaneous reconstruction of velocity and density from the simulated data is presented. Section 8.4 then applies the multi-parameter FWI to the real experimental data from a diffusion bonded cylinder of brass/steel. The conclusions are summarized in Section 8.5.

8.2 Theory

The frequency-domain 2D acoustic wave equation in the isotropic heterogeneous media can be expressed as

$$\frac{\omega^2}{\rho(x, y)v^2(x, y)}p(x, y, \omega) + \nabla \cdot \left(\frac{1}{\rho(x, y)} \nabla p(x, y, \omega) \right) = -s(x, y, \omega), \quad (8.1)$$

where $p(x, y, \omega)$ is the frequency-domain pressure (or displacement) wave-field and ω is the angular frequency. $\rho(x, y)$, $v(x, y)$ and $s(x, y, \omega)$ are the density, the velocity and the source, respectively.

The aim of multi-parameter FWI is to find the model $\mathbf{m}$ by minimizing the data residual $\Delta \mathbf{d}$ between the data $d_{calc}^{(k)}$ computed in the current model $\mathbf{m}^{(k)}$ and the observed data d_{obs} . A least-squares norm of the objective function is given by

$$C^{(k)}(\mathbf{m}) = \frac{1}{2} \Delta \mathbf{d}^\dagger \Delta \mathbf{d}, \quad (8.2)$$

where $\Delta \mathbf{d} = d_{calc}^{(k)} - d_{obs}$ is the data residual and k is the iteration number. The superscript $\dagger$ represents the transposed conjugate. In this chapter, the model $\mathbf{m}$ is defined as

$$\mathbf{m}(x, y) = [v(x, y) \quad \rho(x, y)]^T. \quad (8.3)$$

Local minimization of the misfit function is given by

$$G^{(k)}(\mathbf{m}) = \nabla C^{(k)}(\mathbf{m}) = -H^{(k)}(\mathbf{m}) \Delta \mathbf{m}^{(k)}, \quad (8.4)$$

$$i.e. \quad \Delta \mathbf{m}^{(k)} = -(H^{(k)}(\mathbf{m}))^{-1} G^{(k)}(\mathbf{m}), \quad (8.5)$$

where $\Delta \mathbf{m}$ represents the descent direction. $G(\mathbf{m})$ and $H(\mathbf{m})$ are the first and second derivatives of the misfit function with respect to the model $\mathbf{m}$, i.e. the gradient and the Hessian

$$G(\mathbf{m}) = \frac{\partial C}{\partial \mathbf{m}} = \left(\frac{\partial d_{calc}}{\partial \mathbf{m}} \right)^\dagger \Delta \mathbf{d} = J^\dagger \Delta \mathbf{d}, \quad (8.6)$$

$$H(\mathbf{m}) = \frac{\partial^2 C}{\partial \mathbf{m}^2} = J^\dagger J + \left(\frac{\partial J}{\partial \mathbf{m}} \right)^\dagger \Delta \mathbf{d}, \quad (8.7)$$

where J is the Fréchet derivative matrix (i.e. the sensitivity kernel). It needs to be mentioned that the second term in equation (8.7) is usually difficult to calculate and it has relatively small values [188]. Thus, this is commonly dropped off and then the approximate Hessian H_a can be written as

$$H_a(\mathbf{m}) = J^\dagger J. \quad (8.8)$$

The model $\mathbf{m}^{(k+1)}$ is updated iteratively along the descent direction $\Delta\mathbf{m}$ according to

$$\mathbf{m}^{(k+1)} = \mathbf{m}^{(k)} + \alpha^{(k)} \Delta\mathbf{m}_k, \quad (8.9)$$

where α is a step length using the linear-search method and $\alpha\Delta\mathbf{m}$ is the perturbation model.

In this chapter, the descent direction $\Delta\mathbf{m}$ and kernel matrix J are given by

$$\Delta\mathbf{m} = \begin{pmatrix} \Delta v \\ \Delta\rho \end{pmatrix}, \quad (8.10)$$

$$J = \begin{pmatrix} J_v & J_\rho \end{pmatrix}, \quad (8.11)$$

where J_v and J_ρ are the sensitivity kernels of velocity and density. Therefore, the Gauss-Newton normal equation can be written as [189]

$$\begin{pmatrix} J_v^\dagger \\ J_\rho^\dagger \end{pmatrix} \begin{pmatrix} J_v & J_\rho \end{pmatrix} \begin{pmatrix} \Delta v \\ \Delta\rho \end{pmatrix} = - \begin{pmatrix} J_v^\dagger \\ J_\rho^\dagger \end{pmatrix} \Delta\mathbf{d}, \quad (8.12)$$

where

$$\begin{pmatrix} J_v^\dagger \\ J_\rho^\dagger \end{pmatrix} \begin{pmatrix} J_v & J_\rho \end{pmatrix} = \begin{pmatrix} J_v^\dagger J_v & J_v^\dagger J_\rho \\ J_\rho^\dagger J_v & J_\rho^\dagger J_\rho \end{pmatrix}. \quad (8.13)$$

This Gauss-Newton approximate Hessian includes a 2×2 block matrix. The diagonal blocks represent the correlations of the sensitivity kernel J with parameters of the same nature. In each sub-matrix, the diagonal elements are calculated by the zero-lag auto-correlation of J at the same spatial position, which provide information of coincident-in-space model parameters of different natures. The off-diagonal blocks are the correlations of J with the parameters of different natures and thus reflect the effects of cross-talk between velocity and density. These off-diagonal elements in each sub-matrix are obtained from the zero-lag cross-correlation of J at different spatial positions, which contain the information of the interaction between parameters of different natures [189]. The inverse of Gauss-Newton approximate Hessian can be used to mitigate the trade-off effects and rescale the magnitudes of velocity and density.

The calculation of the misfit function gradient in equation (8.6) is a key step in

FWI, which can be carried out by the adjoint-state (AS) method [190, 191] or the scattering-internal (SI) method [187, 192, 193]. The AS approach used in Chapter 3 is based on cross-correlating the forward wave-field with the back propagated residual wave-field to obtain the gradient. In this chapter, the SI method proposed by Liu et al. [187] evaluates kernel-vector products in equation (8.6) by accumulating decomposed vector-scalar product without explicitly calculating the matrix (see equation (8.14)). It needs to be noted that the SI method is a useful alternative, which is not a replacement of the AS method [189].

Assume that the data residual Δd has the size of $l \times 1$ and the model parameters of velocity and density have the dimension of $n \times 1$, then kernel matrix and its conjugate transpose have the sizes of $l \times n \times 2$ and $2 \times n \times l$, respectively. The gradient of the misfit function in equation (8.6) can be given by:

$$G = J_m^\dagger \Delta d = \quad (8.14)$$

$$\begin{pmatrix} \overline{J_{v11}} \cdots \overline{J_{vi1}} \cdots \overline{J_{vl1}} \\ \vdots \quad \ddots \quad \vdots \quad \ddots \quad \vdots \\ \overline{J_{v1j}} \cdots \overline{J_{vij}} \cdots \overline{J_{vlj}} \\ \vdots \quad \ddots \quad \vdots \quad \ddots \quad \vdots \\ \overline{J_{v1n}} \cdots \overline{J_{vin}} \cdots \overline{J_{vln}} \\ \overline{J_{\rho11}} \cdots \overline{J_{\rho i1}} \cdots \overline{J_{\rho l1}} \\ \vdots \quad \ddots \quad \vdots \quad \ddots \quad \vdots \\ \overline{J_{\rho1j}} \cdots \overline{J_{\rho ij}} \cdots \overline{J_{\rho lj}} \\ \vdots \quad \ddots \quad \vdots \quad \ddots \quad \vdots \\ \overline{J_{\rho1n}} \cdots \overline{J_{\rho in}} \cdots \overline{J_{\rho ln}} \end{pmatrix} \begin{pmatrix} \Delta d_1 \\ \vdots \\ \Delta d_i \\ \vdots \\ \Delta d_l \end{pmatrix} = \sum_{i=1}^l \begin{pmatrix} \overline{J_{vi1}} \\ \vdots \\ \overline{J_{vij}} \\ \vdots \\ \overline{J_{vin}} \\ \overline{J_{\rho i1}} \\ \vdots \\ \overline{J_{\rho ij}} \\ \vdots \\ \overline{J_{\rho in}} \end{pmatrix} \Delta d_i,$$

where the over-line represents the complex conjugate. J_{vij} ($J_{\rho ij}$) is the sensitivity of the i th component of data d_i with respect to the j th model parameters m_j of velocity (or density).

In equation (8.4), the Hessian is associated with the descent direction Δm . The explicit calculation of the Hessian or the inverse Hessian is very difficult for a large scale inverse problem. Due to the linearity in this equation, the descent direction can be solved by a matrix-free conjugate gradient method [185], as shown in Table 8.1. In this algorithm, instead of direct calculation of the inverse Hessian, the Hessian-vector products can be used to obtain the descent direction Δm . The

TABLE 8.1: Conjugate gradient algorithm for solving linear system $Ax = b$.

-
-
1. Starting from initial vector v_0 and setting the residual vector $r_0 = b - Ax_0$, initial search direction q_0 is set as r_0 , i.e. $q_0 = -r_0$.
 2. The scalar α_k is computed using the relationship $\alpha_k = \frac{r_k^T r_k}{q_k^T A q_k} = \frac{r_k^T r_k}{q_k^T [J^T (J q_k)]}$ where k is the number of iterations.
 3. The calculation of x_{k+1} is expressed as $x_{k+1} = x_k + \alpha_k q_k$.
 4. Then, the next residual vector r_{k+1} is given by $r_{k+1} = r_k - \alpha_k A q_k = r_k - \alpha_k [J^T (J q_k)]$.
 5. The next step is to compute β_k , which is used to find the next search direction $\beta_k = \frac{r_{k+1}^T r_{k+1}}{r_k^T r_k}$.
 6. The next search direction q_{k+1} is obtained by using this scalar β_k , $q_{k+1} = -r_{k+1} + \beta_k q_k$.
 7. Then, the next scale α_{k+1} is computed by using the obtained search direction q_{k+1} and this process is repeated.
-
-

Hessian-vector products can be calculated by using two kernel-vector products. First, the kernel-vector products Jq are computed by using the row decomposition

$$Jq = \tag{8.15}$$

$$\begin{pmatrix} J_{v11} & \cdots & J_{v1j} & \cdots & J_{v1n} & \cdots & J_{\rho11} & \cdots & J_{\rho1j} & \cdots & J_{\rho1n} \\ \vdots & & \ddots & & \vdots & & \vdots & & \ddots & & \vdots \\ J_{vi1} & \cdots & J_{vij} & \cdots & J_{vin} & \cdots & J_{\rho i1} & \cdots & J_{\rho ij} & \cdots & J_{\rho in} \\ \vdots & & \ddots & & \vdots & & \vdots & & \ddots & & \vdots \\ J_{vl1} & \cdots & J_{vlj} & \cdots & J_{vln} & \cdots & J_{\rho l1} & \cdots & J_{\rho lj} & \cdots & J_{\rho ln} \end{pmatrix} \begin{pmatrix} q_{v1} \\ \vdots \\ q_{vj} \\ \vdots \\ q_{vn} \\ q_{\rho 1} \\ \vdots \\ q_{\rho j} \\ \vdots \\ q_{\rho n} \end{pmatrix} = \begin{pmatrix} J_{v1}q_v + J_{\rho 1}q_\rho \\ \vdots \\ J_{vi}q_v + J_{\rho i}q_\rho \\ \vdots \\ J_{vl}q_v + J_{\rho l}q_\rho \end{pmatrix},$$

where q is an arbitrary vector. Second, the kernel-vector products $J^\dagger J q_k$ can be obtained based on equation (8.14).

Therefore, the truncated Gauss-Newton method used in multi-parameter FWI consists of two nested loops. First, the external loop is the model iterative update based on equation (8.9). Second, in order to obtain the descent direct, the internal loop

is iteration of the linear system following equation (8.4) by using the conjugate gradient algorithm shown in Table 8.1. The work-flow of the truncated Gauss-Newton FWI is shown in Figure 8.1. For simplicity, the frequency loop and the iteration loop for searching the descent direction are omitted in the work-flow. In general, there are three steps in this work-flow: solving the forward modeling, calculating sensitivity kernels and updating the model. The overall process is similar to Figure 3.2 shown in Chapter 3. The differences are the methods for obtaining the gradient and the descent direction: the mono-parameter FWI uses the AS method to calculate the gradient and the steepest descent algorithm to search the descent direction; and the multi-parameter FWI uses the SI approach to obtain the gradient and the matrix-free conjugate gradient algorithm to find the descent direction.

FIGURE 8.1: The work-flow of the truncated Gauss-Newton FWI.

In Figure 8.1, the computation of sensitivity kernels is an important step in multi-parameter FWI. The sensitivity kernel represents the sensitivity of the data with respect to the model parameters. Based on the same model perturbation, a larger sensitivity kernel for a parameter denotes a larger sensitivity of the data to this parameter. The frequency-domain sensitivity kernels of velocity and density in the homogeneous background can be given by [189]

$$\begin{cases} J_v(x) = \frac{-2\omega^2}{\rho_0(x)v_0^2(x)}G_{reen0}(r, \omega; x)p_0(x, \omega) \\ J_\rho(x) = -\frac{\omega^2}{\rho_0(x)v_0^2(x)}G_{reen0}(r, \omega; x)p_0(x, \omega) + \frac{1}{\rho_0(x)}\nabla G_{reen0}(r, \omega; x) \cdot \nabla p_0(x, \omega) \end{cases} \quad (8.16)$$

where $p_0(x, \omega)$ is the pressure wave-field from a transmitter position in the background model and $G_{reen0}(r, \omega; x)$ is the Green's function at the receiver position in the background medium. More details about the derivation of sensitivity kernels can be found in Appendix A.

8.3 Numerical results

8.3.1 Numerical model

2D FE simulations carried out by Abaqus are performed in aluminum blocks (density=2700 kg/m³, longitudinal wave velocity=6198 m/s and Poisson's ratio=0.33) with the dimension of 1000 mm × 1000 mm. Four-node plane strain elements with the size of 1 mm are used in the mesh, which can ensure more than 30 elements per wavelength [114]. A 100 mm diameter inclusion of simultaneously varying velocity of 5843 m/s and density of 2430 kg/m³ (bulk modulus=54.9 GPa) located in the center of the plane is modeled, as shown in Figure 8.2. The absorbing regions around the plane are used to avoid the boundary reflections. The signals are monitored by a 700 mm circular array with 64 nodes as transducers, as shown in Figure 8.2. The input signal is a 2 cycle Hanning-windowed tone-burst centered at the frequency of 200 kHz.

FD simulations are performed in a 2D space domain, with the same size as in FE models. The plane is meshed by grid points with 2 mm spacing, which satisfies the calculation accuracy requirement of at least 4 grid points per shortest wavelength [107]. The absorbing areas with the same width as in FE models are

FIGURE 8.2: One central inclusion model with a 64 element circular array.

used around the plane edges. Omni-directional pressure waves are generated and monitored with the same array configuration as presented in the FE setup.

8.3.2 Data processing

Since the multi-parameter FWI is based on the FD acoustic model, the scattered signals from the FE elastic model need to be calibrated in order to consider deviations between the two models. The calibration factor is defined as the ratio of the required frequency domain data of the homogeneous background model (without defects) by the FD method and the FE method on the homogeneous structure, as discussed in Section 3.3.3. A contour plot of the typical displacement signal measured over receivers will be shown in Section 8.4.2.

8.3.3 Reconstructions

The model is performed with a circular inclusion with different velocity and density situated in the center of a 64 node circular array, as shown in Figures 8.3(a) and (c). The diameter of the inclusion is 100 mm (around 3.4λ at 200 kHz). Eight consecutive frequencies between 40 kHz and 180 kHz with the same interval are used. A maximum of 40 external iterations for the frequency loop and 20 internal iterations for searching the descent direction are performed for each frequency. The homogeneous backgrounds of velocity and density are used as the starting models

at the lowest frequency. The obtained results in the final iterations at previous frequency are chosen as the starting models for the next higher frequency.

FIGURE 8.3: Velocity and density reconstruction of a single central inclusion with the diameter of 100 mm in an aluminum block from the elastic model. The images along different rows represent the velocity reconstruction and the density reconstruction. (a) The true model of velocity and (b) the velocity reconstruction; (c) the true model of density and (d) the density reconstruction.

Figures 8.3(b) and (d) outline the velocity and density reconstruction of a single central inclusion based on the elastic model at 180 kHz. It can be clearly seen from the figure that the velocity reconstruction can capture almost details of the inclusion, however, the reconstructed density image has smaller size than the original model. The reason is that the velocity can govern both traveltimes and amplitudes of the waves at all angles, while the density has a dominant effect on the reflection amplitudes at smaller angles. This is consistent with the fact that the wave-field scattered by the velocity perturbation has an isotropic diffraction pattern, which corresponds to the virtual source behaving as a pressure source; amplitudes of

the wave-field scattered by density decrease progressively from transmission (low frequencies in the spatial frequency domain) to reflection (high frequencies in the spatial frequency domain)(see Figure 2.1), which corresponds to the virtual source behaving as a vertical force pointing upward in the excitation direction (reflection part) [29, 194]. The velocity reconstruction can be obtained from all acquisition due to no filtering applied on the velocity gradient by the virtual pressure source; however, the short-to-intermediate wavelengths can be achieved in the density reconstruction because the vertical force virtual source applies the high-pass filtering on the density gradient [29]. Also, the parameter of velocity has a dominant effect on the data residual, thus this parameter is reconstructed before the parameter of density. The accuracy of velocity reconstruction can also affect the density reconstruction. The remaining discrepancy is attributed to the difference between acoustic scattering and elastic scattering, since the acoustic inversion approach is used to reconstruct the elastic data.

FIGURE 8.4: Cross sections of velocity and density reconstruction of a single central inclusion along $y=500$ mm from (a)-(b) the elastic model and (c)-(d) the acoustic model.

The cross sections of reconstructions from Figure 8.3 along the horizontal line in the elastic model are shown in Figures 8.4(a) and (b). It can be seen that the velocity reconstruction is slightly overestimated and the density reconstruction is underestimated. When the 2D acoustic inversion is applied to the data from the 2D acoustic forward model, the cross sections of reconstructions of velocity and density are presented in Figures 8.4(c) and (d). It can be seen that the accuracy of the reconstructions still suffers, mainly reflecting the trade-off effects. Compared with reconstructions using the data from the elastic model shown in Figures 8.4(a) and (b), reconstructions from the acoustic model shown in Figures 8.4(c) and (d) are slight better.

8.4 Experiments

8.4.1 Experimental setup

The experimental setup is shown in Figure 8.5(a). The experiment is performed on a diffusion bonded cylinder of brass (outer)/stainless steel (inner), with an outer diameter of 100 mm, an inner diameter of 10 mm and a height of 100 mm. Compared with sizes used in numerical models, the dimensions are scaled down in the ratio of around one to ten in the experiment due to easier manufacturing and lower cost. The material properties are shown in Table 8.2. A disk-shaped piezoelectric transducer (PZT5, Sinoceramics Inc., China) with a diameter of 5 mm and a thickness of 1 mm is attached to the surface of the cylinder using the conductive epoxy. A 5 cycle Hanning-windowed tone-burst signal centered at 1 MHz was generated by RITEC RAM-5000 high power ultrasonic pulser (Ritec Inc., USA). It needs to be noted that the size of the numerical model is scaled from 1000 mm to 100 mm (i.e. a scaling factor of 10), and the frequency changes from 200 kHz (used in Section 8.3.1) to 2 MHz. The lower central frequency of 1 MHz is used in the experiment compared with the corresponding frequency of 2 MHz used in the scaling sizes of numerical models. The reasons are: (1) at the lower frequency, the minimum number of transducers in the circular array is reduced, because it is not practical to have too many measured points in this experiment; (2) the attenuation of the ultrasound at lower frequency is less in the brass/steel cylinder. The sample is mounted on a computer-controlled rotational

TABLE 8.2: Material properties

Material	Young's modulus (GPa)	Poisson's ratio	Density (kg/m ³)
Aluminum	70	0.33	2700
Stainless steel	210	0.287	7850
Brass	110	0.34	8700

stage, whereas the positions of source and receiver are fixed. The displacements are measured by a Polytec OFV-505 laser vibrometer normal to the cylinder axis. The source and receiver are located in the same plane. By rotating the computer-controlled stage, measurements are taken along three quarters of the circle with 123 receiver points, excluding the small range of angles close to the PZT source, as shown in Figure 8.5(b). Since the sample is axisymmetric, the measurements are only taken from a single source and then copied to generate a signal matrix of 164×123 .

FIGURE 8.5: (a) The experimental setup, including the PZT source, laser vibrometer, the sample and rotational stage, and (b) the geometry of the experimental setup from the top view.

8.4.2 Data processing

The experimental signals are post-processed before using them in the image algorithm. Figures 8.6(a) and (b) show a contour plot of the normal displacement field for all receivers and a typical signal measured by the laser vibrometer in the

experiment. It can be seen from Figure 8.6(a) that the direct longitudinal wave is with a curved moveout and the Rayleigh wave propagating around the cylinder with a linear moveout. The direct longitudinal wave is separated from signals by removing the slower Rayleigh wave and the unwanted components with the gating function (see Section 3.4). All signals after time gating are calibrated before using them in the inversion based on similar method introduced in Section 8.3.2.

FIGURE 8.6: (a) Displacement field for the brass/steel cylinder and (b) a typical time trace from one receiver.

8.4.3 Experimental results

Figures 8.7(a) and (c) show the true velocity and density models of a single circular inclusion located at the centre of a 164-transducer circular array with the diameter of 100 mm. The steel inclusion has the diameter of 10 mm (around 1.7λ at 1 MHz). The inversion is carried out by eight consecutive frequencies from 650 kHz to 1 MHz with the same interval. The homogeneous background models at the lowest frequency are used as the starting models for velocity and density, with 40 external iterations for the frequency loop and 20 internal iterations for searching the descent direction at each frequency. Figures 8.7(b) and (d) present the reconstructed velocity and density of the inclusion from the experiment by the multi-parameter FWI. It can be seen from Figure 8.7(b) that the diameter of velocity reconstruction of the inclusion is smaller than the true velocity size. The overall shape of density reconstruction of the inclusion is also narrow and the

FIGURE 8.7: The velocity and density mapping of the brass/steel cylinder at 1 MHz in the experiment. Two rows represent velocity and density reconstructions from the multi-parameter FWI.

contrast of the inclusion is underestimated. This is mainly due to the limited view configuration of the ultrasonic tomography system, constraint from the 2D acoustic model applied to the realistic elastic data and trade-off effects between velocity and density. It needs to be mentioned that the regularization has not been used in this study. When the noise is in the measured data, the appropriate regularization introduced in the misfit function deals with the ill-posedness of the FWI algorithm. The regularization, which helps to alleviate the noise effect, can affect the behavior of the truncated Gauss-Newton method by damping of the Hessian. More details of the Hessian used to improve the descent direction and the smoothing regularization constraint can be found in [189, 195, 196]. Noticeable artifacts are observed from both imaging in the experiment. Besides the reason discussed above, the excitation wave-fields cannot be ideally omni-directional, which cause the amplitude and phase of signals vary with the excited angle. In the calibration, the signals without the defect are obtained synthetically, which mean that

the mismatch between the synthesized and the true signals results in the errors of the amplitude and phase in the inversion. In addition, measuring position error, the slight anisotropy of the sample material and the length of the gate function can affect the reconstruction quality. In this experiment, the axisymmetric ring artifacts are expected due to the 164×123 matrix of source-receiver combinations by repeating only 123 measurements from one source [17].

The cross sections of Figure 8.7 along $y=50$ mm are shown in Figures 8.8(a) and (b). It confirms that the size of the reconstructed velocity is smaller than the original one and the FWI tomography overestimates the velocity reconstruction. The size of the obtained density of the inclusion is narrower than the true one and the density reconstruction of the lowest value is underestimated.

FIGURE 8.8: Cross sections of velocity and density reconstruction along $y=50$ mm from Figure 8.7.

8.5 Summary

In this chapter, an ultrasonic tomography approach based on the acoustic multi-parameter FWI using the truncated Gauss-Newton method is developed for velocity and density reconstructions of metal parts. Accurate estimation of the inverse Hessian is very important in the multi-parameter FWI. Instead of direct calculation of the inverse Hessian, it can be taken into account through a matrix-free conjugate gradient solution of the Gauss-Newton normal equation. After considering this, trade-off effects between velocity and density are effectively alleviated and

the reconstructions of velocity and density are expected. The centred inclusions are used to demonstrate the multi-parameter FWI algorithm via simulation and experiment. It is shown that the velocity reconstruction agrees well with the true model and the estimated density is well achieved. The method, which can also be extended to guided wave tomography, would be useful for quality control in the aviation and automotive industry.

Chapter 9

Conclusions

9.1 Thesis review

A new algorithm for quantitative ultrasonic imaging has been investigated in this thesis. This technique can be used to achieve accurate and high resolution mapping of material properties of objects. Chapter 3 to Chapter 7 are within the framework of the mono-parameter FWI in guided wave tomography (GWT) for corrosion mapping. Chapter 8 focuses on the multi-parameter FWI in ultrasonic tomography for density mapping.

Chapter 2 reviewed the ultrasonic bulk waves and the majority of tomographic methods used in geophysical and medical imaging. Then, these tomographic methods applied to guided waves were presented, including model selection for GWT. Finally, potential approaches for achieving higher resolutions were introduced to solve the inverse scattering problems.

Chapter 3 outlined the theory of the mono-parameter FWI. It uses a 2D acoustic forward model to predict the scattering measurements, and the inverse model to obtain velocity maps. In GWT, the obtained velocity reconstruction can be converted into thickness maps based on the dispersion curves of selected guided wave modes. This algorithm can be iterated through finite discrete frequencies from low to high frequencies and can easily incorporate the dispersion laws and attenuation. A simple circular defect and an irregularly shaped defect were used to validate

this algorithm by simulations as well as experiments. It was shown that minimum thickness can be estimated to less than 1 mm for a 10 mm thick plate.

The reconstruction accuracy analysis of GWT based on the mono-parameter FWI in mapping the thickness of plate-like structures was investigated in Chapter 4. When the simulated data were obtained from the acoustic model, this chapter has shown that the achievable resolution is about 0.7λ but it drops to around $1.5-2\lambda$ from the realistic elastic modeling data. The accuracy loss is caused by implementing the approximate 2D inversion algorithm which limits the complete exploitation of data from 3D elastic model. Analysis results have also shown that the defect shape can affect the reconstruction accuracy. The reconstruction of multiple defects by using FWI validated by experiments is more accurate than the results obtained from diffraction tomography.

Ultrasonic guided wave imaging from a limited view configuration was considered in Chapter 5. Quantitative imaging is very sensitive to the limited view data. The adaptive threshold regularization method was introduced to predict the missing components in the FWI algorithm with the limited view configuration. The results showed that the image quality is improved by combining the extra information obtained through the regularization with the measured data from the limited view configuration.

The mono-parameter FWI was applied to the application of GWT on liquid-loaded plates in Chapter 6. The effect of attenuation due to energy leakage to liquid on thickness reconstruction was studied. Simulations of various defects in the water tank floor with the consideration of attenuation across different frequencies have demonstrated the suitability of the S_0 mode for accurate mapping of the defect with improving sensitivity at higher frequencies. It was shown that as the attenuation increases with the increase of the frequency, more accurate reconstruction can be achieved by considering the attenuation effect. This was demonstrated by simulated and experimental data.

In Chapter 7, a new GWT system based on customized piezoelectric transducers for corrosion monitoring of plates and pipes was presented. It has been shown that the effect of scattering from the selected transducers on the accuracy of the thickness reconstruction was negligible. In the forced corrosion experiments conducted in plates, the reconstructed average thickness losses matched well with the

analytical predictions by Faraday's law. The reconstructed thickness maps at two different stages showed good agreement with the measurements from a laser scanner. Due to the limited view configuration of two ring transducers used in pipes, the regularization method combined with the helical paths separation approach was used to improve the reconstruction accuracy.

In Chapter 8, the multi-parameter FWI was introduced to ultrasonic tomography for density mapping. The method was shown to simultaneously reconstruct density and velocity of metal parts. In multi-parameter FWI, the accurate consideration of the inverse Hessian is very important. In this chapter, the inverse Hessian was indirectly accounted for through a matrix-free conjugate gradient solution of the Gauss-Newton normal equation. After considering this, the trade-off effects between velocity and density were mitigated and thus reconstructions of velocity and density were expected. This algorithm was demonstrated by simulation and experiment.

9.2 Main findings

Guided wave tomography is used to obtain the thickness map of defects in plate-like and pipe structures, which is commonly achieved by exploiting the dispersion characteristics of selected guided modes to convert velocity into thickness. Traveltime tomography is usually limited to large and smooth defects. Other tomography methods based on the Born approximation are usually suitable for small or low contrast defects. This thesis introduces a new quantitative imaging technique for thickness mapping of corrosion damages in plate-like and pipe structures by combining guided waves with the mono-parameter FWI to improve the resolution. By using multiple frequencies from low to high in FWI, it has been shown to accurately reconstruct the minimum thickness of corrosion damage within 1 mm for a 10-mm-thick plate. This performance was demonstrated by both numerical simulations and experiments (as published in [A1]). Compared with diffraction tomography, FWI allows higher order diffraction and scattering to be considered in its numerical solver, thus it has the potential to achieve more accurate inversion results, especially when multiple defects exist. This was demonstrated by both simulated and experimental data obtained from multiple defects. The reconstruction accuracy of this algorithm is mainly constrained by the mapping from 2D acoustic model to

3D elastic model. Therefore, although in theory FWI can lead to more accurate reconstruction, the current resolution is the same as other guided wave tomography based on velocity-to-thickness mapping methods. This was validated by using various shapes of defects across a range of frequencies (as published in [A3]).

In many applications only limited locations are available to place transducers. Reconstruction from the limited view is ill-posed. Efforts have been made to solve this problem, however little work has been done on a simple solution for solving nonlinear inverse scattering problems. This thesis has combined the FWI method with the adjustable thresholding regularization method to address the challenges in the limited view tomography. This algorithm for solving an irregular defect and two defects was demonstrated by simulations and experiments. The improvement of the image quality achieved by this algorithm was also confirmed when using it to reconstruct limited view elastic data via the variation in the array length (as published in [A2]).

A significant challenge when the application of guided wave tomography on fluid-loaded plates is the energy of the guided waves leaking into the liquid. Most of the existing work has been proposed to deal with waveguides with free boundaries, little work has been shown to solve waveguides with liquid loading due to the strong attenuation leading to low signal-to-noise ratio. The advanced forward model was developed in this thesis by considering attenuation effects using complex velocities. This was demonstrated to be efficient for a range of defect widths and depths in the water tank floor, providing more accurate reconstruction compared with the case without the consideration of the attenuation, especially in the presence of liquid resulting high attenuation. This algorithm was confirmed by experimental data obtained from an irregularly shaped defect on a water tank floor (as published in [A4]).

The practical implementation of the guided wave tomography system for corrosion monitoring poses many challenges. In contact transduction, the scattering from transducers can affect the performance of the thickness reconstruction. In this thesis, an optimized low frequency A_0 mode piezoelectric transducer has been designed for the guided wave tomography system and the scattering from transducer array has been demonstrated to have a small effect on the thickness reconstruction. The efficiency and the accuracy of this guided wave system have been demonstrated

through continuous forced corrosion experiments on plates and pipes (as published in [A5]).

In the quality control application, the density distribution is usually indirectly determined by the wave velocity. Only velocity is obtained which ignores the coupled effects between different parameters, but the propagation of waves is affected by different parameters such as velocity, density and attenuation. The multi-parameter FWI in ultrasonic tomography developed in this thesis has been shown to simultaneously reconstruct velocity and density. The algorithm was validated by the simulated and experimental data (to be submitted in [A8]).

9.3 Future Work

In this thesis, GWT based on acoustic mono-parameter FWI has been demonstrated to be efficient for the reconstruction of velocity with high resolution using the data from the acoustic model. However, there is a loss of reconstruction resolution when realistic test data are used. An alternative approach is to develop an elastic inversion technique, which can consider guided wave scattering more accurately. Also, the mode conversion can be considered in this inversion to improve the reconstruction accuracy.

In acoustic multi-parameter inversion, only the reconstruction of velocity and density under the acoustic approximation has been achieved. Therefore, the inversion that involves more parameters (e.g. velocity, density and attenuation) will be investigated. Also, the elastic inversion will be applied in multi-parameter FWI.

The methods presented in this thesis can be extended to the medical ultrasound imaging, such as the breast tomography, bone imaging and brain imaging. For example, in bone imaging, besides the quantitative mapping of the velocity of the bone, multi-parameter FWI allows the assessment of the density of the bone.

Appendix A

Derivation of sensitivity kernels

For simplicity, the derivation of monochromatic sensitivity kernels is presented here. Under the Born approximation, the model $m(x)$ can be expressed as a background model $m_0(x)$ and a model perturbation $\delta m(x)$

$$m(x) = m_0(x) + \delta m(x). \quad (\text{A.1})$$

Also, the pressure wave-field $p(x, \omega)$ can be given by

$$p(x, \omega) = p_0(x, \omega) + \delta p(x, \omega), \quad (\text{A.2})$$

where $p_0(x, \omega)$ and $\delta p(x, \omega)$ are the reference pressure wave-field and the wave-field perturbation. The reference pressure wave-field satisfies equation (8.1), as follows

$$\frac{\omega^2}{\rho_0 v_0^2(x)} p_0(x, \omega) + \nabla \cdot \left(\frac{1}{\rho_0(x)} \nabla p_0(x, \omega) \right) = -\delta(x - x_s), \quad (\text{A.3})$$

After introducing the Green's function

$$\frac{\omega^2}{\rho_0 v_0^2(x)} G_{reen0}(x, \omega) + \nabla \cdot \left(\frac{1}{\rho_0(x)} \nabla G_{reen0}(x, \omega) \right) = -\delta(x - x_s), \quad (\text{A.4})$$

the solution of equation (A.3) can be given by

$$p_0(r, \omega) = \int_V G_{reen0}(r, \omega; x) s(\omega) d^3x. \quad (\text{A.5})$$

Substituting equation (A.1) and (A.2) into equation (8.1), it can be written as

$$\frac{\omega^2}{\rho_0 v_0^2(x)} \delta p(x, \omega) + \nabla \cdot \left(\frac{1}{\rho_0(x)} \nabla \delta p(x, \omega) \right) = -\Delta s(x, \omega), \quad (\text{A.6})$$

where Δs is the secondary source. This source can be given by [189]

$$\Delta s(x, \omega) = \frac{-2\omega^2 \delta v(x)}{\rho_0(x) v_0^3(x)} p_0(x, \omega) - \frac{\omega^2 \delta \rho(x)}{\rho_0^2(x) v_0^2(x)} p_0(x, \omega) - \nabla \cdot \left(\frac{\delta \rho(x)}{\rho_0^2(x)} \nabla p_0(x, \omega) \right) \quad (\text{A.7})$$

Equation (A.5) is the solution of equation (A.3). Similarly, the solution of equation (A.6) can be written as

$$\delta p_0(r, \omega) = \int_V G_{reen0}(r, \omega; x) \Delta s(x, \omega) d^3 x. \quad (\text{A.8})$$

According to the relationship

$$\alpha \operatorname{div}(v) = -(\operatorname{grad} \alpha) v + \operatorname{div}(\alpha v) \quad (\text{A.9})$$

and

$$\int_V dr \operatorname{div}(v) = \int_S v \cdot ds, \quad (\text{A.10})$$

we can deduce that

$$\begin{aligned} \delta p(r, \omega) = & \int_V \left[\frac{-2\omega^2 \delta v(x)}{\rho_0(x) v_0^3(x)} G_{reen0}(r, \omega; x) p_0(x, \omega) \right. \\ & - \frac{\omega^2 \delta \rho(x)}{\rho_0^2(x) v_0^2(x)} G_{reen0}(r, \omega; x) p_0(x, \omega) \\ & \left. + \frac{\delta \rho(x)}{\rho_0^2(x)} \nabla G_{reen0}(r, \omega; x) \cdot \nabla p_0(x, \omega) \right] d^3 x. \end{aligned} \quad (\text{A.11})$$

According to [197, 198], the relationship between the wave-field residual and model perturbation can be given by

$$\delta p(x) = \int_V J(x) \delta \ln m(x) d^3 x, \quad (\text{A.12})$$

where $\delta \ln m = \delta m / m_0$ is the relative model perturbation. Thus, equation (A.11) can be rewritten as

$$\delta p(r, \omega) = \int_V [J_v(x) \delta \ln v(x) + J_\rho(x) \delta \ln \rho(x)] d^3 x. \quad (\text{A.13})$$

where

$$\begin{cases} J_v(x) = \frac{-2\omega^2}{\rho_0(x)v_0^2(x)} G_{\text{reen}0}(r, \omega; x) p_0(x, \omega) \\ J_\rho(x) = -\frac{\omega^2}{\rho_0(x)v_0^2(x)} G_{\text{reen}0}(r, \omega; x) p_0(x, \omega) + \frac{1}{\rho_0(x)} \nabla G_{\text{reen}0}(r, \omega; x) \cdot \nabla p_0(x, \omega) \end{cases}$$

Author's Publications

Journal Articles

- [A1] **J. Rao**, M. Ratsassepp and Z. Fan. “Guided Wave Tomography Based on Full-Waveform Inversion.” IEEE transactions on Ultrasonics, Ferroelectrics, and Frequency Control, 63(5):737-745, 2016.
- [A2] **J. Rao**, M. Ratsassepp and Z. Fan. “Limited-View Ultrasonic Guided Wave Tomography Using An Adaptive Regularization Method.” Journal of Applied Physics, 120(19):194902, 2016. (**Featured cover paper**)
- [A3] **J. Rao**, M. Ratsassepp and Z. Fan. “Investigation of The Reconstruction Accuracy of Guided Wave Tomography Using Full Waveform Inversion.” Journal of Sound and Vibration, 400:317-328, 2017.
- [A4] **J. Rao**, M. Ratsassepp and Z. Fan. “Quantification of Thickness Loss in A Liquid-Loaded Plate Using Ultrasonic Guided Wave Tomography.” Smart Materials and Structures, 26:1-10, 2017.
- [A5] **J. Rao**, M. Ratsassepp, D. Lisevych, M.H. Caffoor and Z. Fan. “On-Line Corrosion Monitoring of Plate Structures Based on Guided Wave Tomography Using Piezoelectric Sensors.” Sensors, 17, 2882, 2017.
- [A6] F. Zou, **J. Rao***, and M.H. Aliabadi. “Highly Accurate Online Characterisation of Cracks in Plate-Like Structures.” NDT & E International, 94:1-12, 2017.
- [A7] M. Ratsassepp, **J. Rao** and Z. Fan. “Quantitative Imaging of Young’s Modulus in Plates Using Guided Wave Tomography.” NDT & E International, 94:22-30, 2017.

- [A8] **J. Rao**, J.Z. Yang, M. Ratassepp and Z. Fan. “Multi-Parameter Reconstruction of Velocity And Density Using Ultrasonic Tomography Based on Full Waveform Inversion.” *Journal of Sound and Vibration*, submitted, 2018.
- [A9] J.Y. Zheng, J. Shi, J.F. Shi, S.J. Zhong and **J. Rao**. “Short-Term Burst Pressure of Polyethylene Pipe Reinforced by Winding Steel Wires Under Various Temperatures.” *Composite Structures*, 121:163-171, 2015.

Conference Proceedings

- [A10] **J. Rao**, M. Ratassepp and Z. Fan. “Guided Wave Tomography for Quantification of Corrosion Damage in Fluid-Filled Pipes.” *awarded **Best Paper** in IEEE Far East NDT New Technology & Application Forum, 2018.*
- [A11] **J. Rao**, M. Ratassepp and Z. Fan. “An Adaptive Regularization Method for Limited View Thickness Reconstruction.” *awarded **Second-prize** in Student Best Paper Competition of IEEE Far East NDT New Technology & Application Forum, 2017.*
- [A12] **J. Rao**, M. Ratassepp and Z. Fan. “Quantitative Corrosion Mapping in Ship Structures Using Ultrasonic Guided Wave Tomography.” *Top research project from Singapore Maritime Institution, 2017, Singapore.*
- [A13] **J. Rao**, M. Ratassepp and Z. Fan. “Online Monitoring of Corrosion Damage Using Ultrasonic Guided Wave Tomography.” in *Structural Health Monitoring, 2017, San Francisco.*
- [A14] **J. Rao**, M. Ratassepp and Z. Fan. “Quantitative Imaging of Thickness Maps Using Ultrasonic Guided Wave Tomography Based on Full-Waveform Inversion.” in *6th Asia Pacific Workshop on Structural Health Monitoring, 2016, Hobart.*
- [A15] **J. Rao**, M. Ratassepp and Z. Fan. “Performance Analysis of Guided Wave Tomography Based on Full Waveform Inversion.” in *Review of Progress in Quantitative Non-Destructive Evaluation, 2016, Atlanta.*

Bibliography

- [1] A. Groysman. *Corrosion for Everybody*. Springer, New York, 2010.
- [2] L. Demer. Appeals court gives go-ahead to BP investor suit over 2006 alaska oil spills. <http://www.adn.com/article/20140213/appeals-court-gives-go-ahead-bp-investor-suit-over-2006-alaska-oil-spills>, 2014.
- [3] N. Kasai, Y. Fujiwara, K. Sekine, and T. Sakamoto. Evaluation of back-side flaws of the bottom plates of an oil-storage tank by the RFECT. *NDT & E. Int.*, 41(7):525–529, 2008.
- [4] D. M. Amos. Magnetic flux leakage as applied to aboveground storage tank flat bottom tank floor inspections. *Mater. Eval.*, 54(1):26–28, 1996.
- [5] N. Kasai, K. Sekine, and H. Maruyama. Non-destructive evaluation method for far-side corrosion type flaws in oil storage tank bottom floors using the magnetic flux leakage technique. *J. Japan Petroleum Inst.*, 46(2):126–132, 2003.
- [6] J. Krautkramer and H. Krautkramer. *Ultrasonic Testing of Materials*. Springer, Berlin, 1969.
- [7] D. N. Alleyne, B. Pavlakovic, M. J. S. Lowe, and P. Cawley. Rapid long-range inspection of chemical plant pipework using guided waves. *Insight*, 43:93–96, 2001.
- [8] P. Cawley and D. Alleyne. The use of lamb waves for the long range inspection of large structures. *Ultrasonics*, 34(2-5):287–290, 1996.
- [9] P. B. Nagy, F. Simonetti, and G. Instanes. Corrosion and erosion monitoring in plates and pipes using constant group velocity lamb wave inspection. *Ultrasonics*, 54(7):1832–1841, 2014.

- [10] X. Zhao, R. L. Royer, S. E. Owens, and J. L. Rose. Ultrasonic lamb wave tomography in structural health monitoring. *Smart Mater. Struct.*, 20(10):105002, 2011.
- [11] P. Belanger, P. Cawley, and F. Simonetti. Guided wave diffraction tomography within the Born approximation. *IEEE Trans. Ultrason. Ferroelectr. Freq. Control*, 57(6):1405–1418, 2010.
- [12] P. Huthwaite. Evaluation of inversion approaches for guided wave thickness mapping. *Proc. R. Soc. A.*, 470:20140063, 2014.
- [13] J. Z. He and F. G. Yuan. Lamb wave-based subwavelength damage imaging using the DORT-MUSIC technique in metallic plates. *Structural Health Monitoring*, 15(1):65–80, 2016.
- [14] J. Z. He and F. G. Yuan. A quantitative damage imaging technique based on enhanced CCRTM for composite plates using 2D scan. *Smart Mater. Struct.*, 25(10):105022, 2016.
- [15] J. Chen, Z. Q. Su, and L. Cheng. Identification of corrosion damage in submerged structures using fundamental anti-symmetric lamb waves. *Smart Mater. Struct.*, 19(1):015004, 2009.
- [16] J. P. Koduru and Rose J. L. Mode controlled guided wave tomography using annular array transducers for shm of water loaded plate like structures. *Smart Mater. Struct.*, 22(12):125021, 2013.
- [17] P. Huthwaite and F. Simonetti. High-resolution guided wave tomography. *Wave Motion*, 50(5):979–993, 2013.
- [18] P. R. Williamson. A guide to the limits of resolution imposed by scattering in ray tomography. *Geophysics*, 56(2):202–207, 1991.
- [19] L. R. F. Rose and C. H. Wang. Mindlin plate theory for damage detection: Imaging of flexural inhomogeneities. *J. Acoust. Soc. Am.*, 127(2):754–763, 2010.
- [20] C. H. Wang and L. R. F. Rose. Wave reflection and transmission in beams containing delamination and inhomogeneity. *J. Sound Vib.*, 264:851–872, 2003.

- [21] M. C. Lawrence, M. A. Jaffer, and B. T Sewell. The application of the maximum entropy method to electron microscopic tomography. *Ultramicroscopy*, 31:285–301, 1989.
- [22] M. Persson, D. Bone, and H. Elmqvist. Total variation norm for three-dimensional iterative reconstruction in limited view angle tomography. *Phys. Med. Biol.*, 46(3):853–866, 2001.
- [23] S. LaRoque, E. Sidky, and X. Pan. Accurate image reconstruction from few-view and limited-angle data in diffraction tomography. *J. Opt. Soc. Am.*, 25(7):1772–1782, 2008.
- [24] H. Danninger, Z-O. Zengin, and M. Drozda. High pressure compaction of ferrous PM parts. *Metal Powder Report*, 41:833–838, 1986.
- [25] R. M. German. *Powder Metallurgy Science*. NJ: Metal Powder Industries Federation, Princeton, 1994.
- [26] P. K. Liaw, R. E. Shannon, and Clark Jr. W. G. et al. Determining material properties of metal-matrix composites by NDE. *JOM*, 44:36–40, 1992.
- [27] D. H. Phillips and J. J. Lannutti. Measuring physical density with X-ray computed tomography. *NDT & E. Int.*, 30(6):339–350, 1997.
- [28] C. B. Scruby and B. C. Moss. Non-contact ultrasonic measurements on steel at elevated temperatures. *NDT & E. Int.*, 26:177–188, 1993.
- [29] S. Operto, Y. Gholami, V. Prieux, A. Ribodetti, R. Brossier, L. Metivier, and J. Virieux. A guided tour of multiparameter full waveform inversion for multicomponent data: from theory to practice. *The Leading Edge, September, Special section Full Waveform Inversion*, pages 1040–1054, 2013.
- [30] T. Garino, M. Mahoney, and M. Readey. Characterization techniques to validate models of density variations in pressed powder compacts. *In Proceedings of the 27th International SAMPE Technical Conference*, 1995.
- [31] J. F. Greenleaf, S. A. Johnson, and S. L. et al. Lee. Algebraic reconstruction of spatial distributions of acoustic absorption with tissues from their two-dimensional acoustic projection. *Acoustical Holograp*, 5:591–603, 1974.

- [32] X. Zhao, J. L. Rose, and H. D. Gao. Determination of density distribution in ferrous powder compacts using ultrasonic tomography. *IEEE Trans. Ultrason. Ferroelectr. Freq. Control*, 53(2):360–369, 2006.
- [33] A. J. Devaney. Variable density acoustic tomography. *J. Acoust. Soc. Am.*, 78(120):120–130, 1985.
- [34] J. W. Eberhard. Ultrasonic tomography for nondestructive evaluation. *Ann. Rev. Mater. Sci.*, 12:1–21, 1982.
- [35] J. L. Rose. *Ultrasonic waves in solid media*. Cambridge University Press, Cambridge, 1999.
- [36] B. A. Auld. *Acoustic Fields and Waves in Solids*. Krieger, Malabar, FL, 1990.
- [37] L. M. Brekhovskikh and V. Goncharov. *Mechanics of Continua and Wave Dynamics*. Springer-Verlag, 1985.
- [38] A. C. Kak and M. Slaney. *Principles of Computerized Tomographic Reconstruction*. IEEE Press, New York, 1998.
- [39] J. Radon. On the determination of functions from their integral values along certain manifolds. *IEEE Trans. Med. Imag.*, 5:170–176, 1986.
- [40] G. N. Hounsfield. Computerized transverse axial scanning (tomography): Part 1. description of system. *Br. J. Radiol.*, 46(552):1016–1022, 1973.
- [41] W. A. Kalender. *Computed Tomography: Fundamentals, System Technology, Image Quality, Applications*. Publicis Publishing, Erlangen, 2005.
- [42] K. A. Dines and R. J. Lytle. Computerized geophysical tomography. *Proceedings of the IEEE*, 67(7):1065–1073, 1979.
- [43] Y. Nagata, J. Huang, J. Achenbach, and S. Krishnaswamy. Lamb wave tomography using laser-based ultrasonics. *Rev. Prog. Quant. Nondestruct. Eval.*, 14:561–568, 1995.
- [44] P. S. Earle and P. M. Shearer. Characterization of global seismograms using an automatic-picking algorithm. *Bull. Seismol. Soc. Am.*, 84:366–376, 1994.

- [45] R. G. Pratt. Seismic waveform inversion in the frequency domain, Part 1: Theory and verification in a physical scale model. *Geophysics*, 64(3):888–901, 1999.
- [46] H. Zhang, C. Thurber, and C. Rowe. Automatic P-wave arrival detection and picking with multiscale wavelet analysis for single-component recordings. *Bull. Seismol. Soc. Am.*, 84:1904–1912, 2003.
- [47] A. H. Hielscher, A. D. Klose, and K. M. Hanson. Gradient-based iterative image reconstruction scheme for time-resolved optical tomography. *IEEE Trans. Med. Imag.*, 18(3):262–271, 1999.
- [48] S. Li, M. Jackowski, and D. et al. Dione. Refraction corrected transmission ultrasound computed tomography for application in breast imaging. *Med. Phys.*, 37:2233–2246, 2010.
- [49] S. Li, K. Mueller, and M. et al. Jackowski. Fast marching method to correct for refraction in ultrasound computed tomography. In *3rd IEEE Int. Symp. on Biomedical Imaging: Nano to Macro*, pages 896–899, 2006.
- [50] P. Huthwaite. Evaluation of inversion approaches for guided wave thickness mapping. *Proc. R. Soc. A.*, 470:20140063, 2014.
- [51] C. L. Willey and F. Simonetti. A two-dimensional analysis of the sensitivity of a pulse first break to wave speed contrast on a scale below the resolution length of ray tomography. *J. Acoust. Soc. Am.*, 139(6):3145–3158, 2016.
- [52] M. Born and E. Wolf. *Principles of Optics*. Cambridge University Press, Cambridge, UK, 1999.
- [53] P. P. Ewald. Introduction to the dynamical theory of x-ray diffraction. *Acta Crystallogr. Sect. A*, 25(1):103–108, 1969.
- [54] A. H. Rohde, M. Veidt, L. R. F. Rose, and J. Homer. A computer simulation study of imaging flexural inhomogeneities using plate-wave diffraction tomography. *Ultrasonics*, 48(1):6–15, 2008.
- [55] M. A. Anastasio and X. Pan. A new reconstruction approach for reflection mode diffraction tomography. *IEEE Trans. Image Process.*, 9(7):1262–1271, 2000.

- [56] P. Huthwaite and F. Simonetti. High-resolution imaging without iteration: a fast and robust method for breast ultrasound tomography. *J. Acoust. Soc. Am.*, 130(3):1721–1734, 2011.
- [57] H. Lamb. On waves in an elastic plate. *Proc. R. Soc. London, Ser. A*, 93 (648):114–128, 1917.
- [58] W. T. Thomson. Transmission of elastic waves through a stratified solid medium. *J. Appl. Phys.*, 21:89–93, 1950.
- [59] M. J. S. Lowe. Matrix techniques for modelling ultrasonic waves in multilayered media. *IEEE Trans. Ultrason. Ferroelectr. Freq. Control*, 42:525–542, 1995.
- [60] B. Pavlakovic, , M. J. S. Lowe, D. Alleyne, and P. Cawley. Disperse: A general purpose program for creating dispersion curves. *Rev. Prog. Quant. Nondestruct Eval.*, 16:185–192, 1997.
- [61] M. Ratsassepp, J. Rao, and Z. Fan. Quantitative imaging of Young’s modulus in plates using guided wave tomography. *NDT & E. Int.*, 94:22–30, 2018.
- [62] J. C. P. McKeon and M. K. Hinders. Parallel projection and crosshole lamb wave contact scanning tomography. *J. Acoust. Soc. Am.*, 106(5):2568–2577, 1999.
- [63] M. Malyarenko and M. Hinders. Fan beam and double crosshole lamb wave tomography for mapping flaws in aging aircraft structures. *J. Acoust. Soc. Am.*, 108(4):1631–1639, 2000.
- [64] P. Belanger and P. Cawley. Feasibility of low frequency straight-ray guided wave tomography. *NDT & E Int.*, 42(2):113–119, 2009.
- [65] E. V. Malyarenko and M. K. Hinders. Ultrasonic lamb wave diffraction tomography. *Ultrasonics*, 39(4):269–281, 2001.
- [66] A. Volker and J. Bloom. Experimental results of guided wave travel time tomography. *Rev. Prog. Quant. Nondestruct. Eval.*, 30:215–222, 2011.
- [67] C. L. Willey, F. Simonetti, P. B. Nagy, and G. Instanes. Guided wave tomography of pipes with high-order helical modes. *NDT & E. Int.*, 65:8–21, 2014.

- [68] P. Huthwaite. Improving accuracy through density correction in guided wave tomography. *Proc. R. Soc. A.*, 472:20150832, 2016.
- [69] A. H. Rohde, L. R. F. Rose, M. Veidt, and C Wang. Two inversion strategies for plate wave diffraction tomography. *Mater. Forum.*, 33:489–495, 2009.
- [70] P. Huthwaite. Guided wave tomography with an improved scattering model. *Proc. R. Soc. A: Mathematical, Physical and Engineering sciences*, 472:20160643, 2016.
- [71] P. Huthwaite, R. Ribichini, P. Cawley, and M. J. S. Lowe. Mode selection for corrosion detection in pipes and vessels via guided wave tomography. *IEEE Trans. Ultrason. Ferroelectr. Freq. Control*, 60(6):1165–1177, 2013.
- [72] J. Li and J. L. Rose. Natural beam focusing of non-axisymmetric guided waves in large-diameter pipes. *Ultrasonics*, 44:35–45, 2006.
- [73] G. Liu and J. Qu. Guided circumferential waves in a circular annulus. *J. Appl. Mech.*, 65(2):424–430, 1998.
- [74] A. J. Brath, F. Simonetti, and P. B. Nagy. Acoustic formulation of elastic guided wave propagation and scattering in curved tubular structures. *IEEE Trans. Ultrason. Ferroelectr. Freq. Control*, 61(5):815–829, 2014.
- [75] J. W. Goodman. *Fourier Optics*. McGraw-Hill, London, 1996.
- [76] F. Simonetti. Breaking the resolution limit: A new perspective for imaging in NDT. *Rev. Prog. Quant. Nondestruct Eval.*, 25:700–707, 2006.
- [77] M. J. Fleming. *Far-field super resolution imaging*. PhD thesis, Univ. of Imperial College, 2008.
- [78] F. Simonetti. Multiple scattering: The key to unravel the subwavelength world from the far-field pattern of a scattered wave. *Phys. Rev. E*, 73:036619, 2006.
- [79] G. B. Aarfken and H. J. Weber. *Mathematical Methods for Physicists*. Academic Press, London, 2001.
- [80] C. K. Rushforth and R. W. Harris. Restoration, resolution and noise. *J. Opt. Soc. Am.*, 58(4):539–545, 1968.

- [81] D. Slepian and H. O. Pollak. Prolate spheroidal wave functions, Fourier analysis and uncertainty-I. *Bell Syst. Tech. J.*, 40:43–63, 1961.
- [82] G. Toraldo di Francia. Degrees of freedom of an image. *J. Opt. Soc. Am.*, 59(7):799–804, 1969.
- [83] R. Potthast. *Point Sources and Multipoles in Inverse Scattering Theory*. Chemical Rubber Company Press, Boca Raton, FL, 2001.
- [84] R. D. Luke and R. Potthast. The no response test - a sampling method for inverse scattering problems. *SIAM J. Appl. Math.*, 63(4):1292–1312, 2003.
- [85] D. Colton and A. Kirsch. A simple method for solving inverse scattering problems in the resonance region. *Inverse Probl.*, 12(4):383–393, 1996.
- [86] A. Kirsch. Characterization of the shape of a scattering obstacle using the spectral data of the far field operator. *Inverse Probl.*, 14:1489–1512, 1998.
- [87] C. Prada, J. L. Thomas, and M. Fink. Decomposition of the time reversal operator: detection and selective focusing on two scatterers. *J. Acoust. Soc. Am.*, 99(4):2067–2076, 1996.
- [88] D. Colton and R. Kress. *Inverse Acoustic and Electromagnetic Scattering Theory*. Springer, Heidelberg, 1998.
- [89] F. C. Chen and W. C. Chew. Experimental verification of super resolution in nonlinear inverse scattering. *Appl. Phys. Lett.*, 72(23):3080–3082, 1998.
- [90] A. J. Hesford and W. C. Chew. Fast inverse scattering solutions using the distorted Born iterative method and the multilevel fast multipole algorithm. *J. Acoust. Soc. Am.*, 128(2):679–690, 2010.
- [91] Y. M. Wang and W. C. Chew. An iterative solution of the two-dimensional electromagnetic inverse scattering problem. *Int. J. Imaging Syst. Technol.*, 1:100–108, 1989.
- [92] C. Lu, J. Lin, W. Chew, and G. Otto. Image reconstruction with acoustic measurement using distorted Born iteration method. *Ultrason. Imaging*, 18:140–156, 1996.

- [93] L. Moreau, A. J. Hunter, A. Velichko, and P. D. Wilcox. 3-D reconstruction of sub-wavelength scatterers from the measurement of scattered fields in elastic waveguides. *IEEE Trans. Ultrason. Ferroelectr. Freq. Control*, 61(11):1864–1879, 2014.
- [94] N. Metropolis, A. W. Rosenbluth, and M. N. et al. Rosenbluth. Equation of state calculations by fast computing machines. *J. Chem. Phys.*, 21(6):1087–1091, 1953.
- [95] L. Moreau, M. Caleap, A. Velichko, and P. D. Wilcox. Scattering of guided waves by through-thickness cavities with irregular shapes. *Wave Motion*, 48(7):586–602, 2011.
- [96] A. Velichko and P. D. Wilcox. A generalized approach for efficient finite element modeling of elastodynamic scattering in two and three dimensions. *J. Acoust. Soc. Am.*, 128(3):1004–1014, 2010.
- [97] A. Tarantola. A strategy for nonlinear elastic inversion of seismic reflection data. *Geophysics*, 51(10):1893–1903, 1986.
- [98] R. G. Pratt and M. H. Worthington. Inverse theory applied to multisource cross-hole tomography. Part 1: acoustic wave-equation method. *Geophys. Prosp.*, 38:287–310, 1990.
- [99] J. Virieux and S. Opertor. An overview of full-waveform inversion in exploration geophysics. *Geophysics*, 74(6):WCC1–WCC26, 2009.
- [100] P. Mora. Nonlinear two-dimensional elastic inversion of multioffset seismic data. *Geophysics*, 52(9):1211–1228, 1987.
- [101] R. Seidl and E. Rank. Iterative time reversal based flaw identification. *Comput. Math. Appl*, 72:879–892, 2016.
- [102] R. G. Pratt, C. Shin, and G. J. Hicks. Gauss-newton and full newton methods in frequency-space seismic waveform inversion. *Geophys. J. Int.*, 133:341–362, 1998.
- [103] K. J. Marfurt. Accuracy of finite-difference and finite-element modeling of the scalar and elastic wave-equations. *Geophysics*, 49(5):533–549, 1984.

- [104] L. Sirgue and R. G. Pratt. Efficient waveform inversion and imaging: A strategy for selecting temporal frequencies. *Geophysics*, 69(1):231–248, 2004.
- [105] C. H. Jo, C. Shin, and J. H. Suh. An optimal 9-point, finite-difference, frequency-space 2-D scalar extrapolator. *Geophysics*, 61(2):529–537, 1996.
- [106] I. Stekl and R. G. Pratt. Accurate viscoelastic modeling by frequency-domain finite differences using rotated operators. *Geophysics*, 63(5):1779–1794, 1998.
- [107] B. Hustedt, S. Operto, and J. Virieux. Mixed-grid and staggered-grid finite-difference methods for frequency-domain acoustic wave modelling. *Geophys. J. Int.*, 157:1269–1296, 2004.
- [108] J. P. Berenger. A perfectly matched layer for absorption of electromagnetic waves. *J. Comput. Phys.*, 114:185–200, 1994.
- [109] R. Clayton and B. Engquist. Absorbing boundary conditions for acoustic and elastic wave equations. *Bull. Seism. Soc. Am.*, 67(6):1529–1540, 1977.
- [110] C. Shin, K. Yoon, K. J. Marfurt, and K. Park. Efficient calculation of a partial derivative wavefield using reciprocity for seismic imaging and inversion. *Geophysics*, 66(6):1856–1863, 2001.
- [111] C. Ravaut, S. Operto, L. Improta, and J. Virieux. Multiscale imaging of complex structures from multifold wide-aperture seismic data by frequency-domain full-waveform tomography: Application to a thrust belt. *Geophys. J. Int.*, 159:1032–1056, 2004.
- [112] S. Operto, J. Virieux, J. X. Dessa, and G. Pascal. Crustal seismic imaging from multifold ocean bottom seismometer data by frequency domain full waveform tomography: Application to the eastern Nankai trough. *J. Geophys. Res.*, 111:2005JB003835, 2006.
- [113] D. Vigh, E. W. Starr, and J. Kapoor. Developing earth model with full waveform inversion. *The Leading Edge*, 28:432–435, 2009.
- [114] P. Huthwaite, F. Simonetti, and M. J. S. Lowe. On the convergence of finite element scattering simulations. *Rev. Prog. Quant. Nondestruct Eval.*, 29: 65–72, 2010.

- [115] M. Castaings and C. Bacon. Finite element modeling of torsional wave modes along pipes with absorbing materials. *J. Acoust. Soc. Am.*, 119(6):3741–3751, 2006.
- [116] P. Belanger and P. Cawley. Lamb wave tomography to evaluate the maximum depth of corrosion patches. *Rev. Prog. Quant. Nondestruct Eval.*, 27:1290–1297, 2008.
- [117] Y. Cho. Estimation of ultrasonic guided wave mode conversion in a plate with thickness variation. *IEEE Trans. Ultrason. Ferroelectr. Freq. Control*, 47(3):591–603, 2000.
- [118] C. Bunks, F. M. Saleck, S. Zaleski, and G. Chavent. Multiscale seismic waveform inversion. *Geophysics*, 60(5):1457–1473, 1995.
- [119] E. M. T. Takougang and A. J. Calvert. Application of waveform tomography to marine seismic reflection data from the Queen Charlotte Basin of western Canada. *Geophysics*, 76(2):B55–B70, 2011.
- [120] F. Simonetti, L. Huang, N. Duric, and P. Littrup. Diffraction and coherence in breast ultrasound tomography: A study with toroidal array. *Med. Phys.*, 36(7):2955–2965, 2009.
- [121] F. Simonetti, L. Huang, and N. Duric. On the spatial sampling of wave fields with circular ring apertures. *J. Appl. Phys.*, 101(8):083103, 2007.
- [122] P. Belanger. *Feasibility of thickness mapping using ultrasonic guided waves*. PhD thesis, Univ. of Imperial College, London, 2009.
- [123] R. Brossier, S. Operto, and J. Virieux. Seismic imaging of the complex on shore structures by 2D elastic frequency-domain full-waveform inversion. *Geophysics*, 74(6):WCC105–WCC118, 2009.
- [124] P. R. Amestoy, A. Guermouche, J. Y. L’Excellent, and S. Pralet. Hybrid-scheduling for the parallel solution of linear systems. *Parallel Comput.*, 32:136–156, 2006.
- [125] P. Huthwaite, M. J. S. Lowe, and P. Cawley. Guided wave tomography performance analysis. *AIP Conference Proceedings*, 1706:020021, 2016.

- [126] M. Lemistre and D. Balageas. Structural health monitoring system based on diffracted lamb wave analysis by multiresolution processing. *Smart Mater. Struct.*, 10(3):504–511, 2001.
- [127] F. B. Cegla, A. Rohde, and M. Veidt. Analytical prediction and experimental measurement for mode conversion and scattering of plate waves at non-symmetric circular blind holes in isotropic plates. *Wave Motion*, 45(3):162–177, 2008.
- [128] P. Rajagopal, M. Drozd, E. Skelton, M. J. S. Lowe, and R. Craster. On the use of absorbing layers to simulate the propagation of elastic waves in unbounded isotropic media using commercially available finite element packages. *NDT & E. Int.*, 51:30–40, 2012.
- [129] P. Huthwaite. Guided wave tomography with an improved scattering model. *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, 472:20160643, 2016.
- [130] M. Slaney, A. C. Kak, and L. E. Larsen. Limitations of imaging with first-order diffraction tomography. *IEEE Trans. Microwave Theory Tech.*, 32(8):860–874, 1984.
- [131] J. Rao, M. Ratassepp, and Z. Fan. Guided wave tomography based on full-waveform inversion. *IEEE Trans. Ultrason. Ferroelectr. Freq. Control*, 63:737–745, 2016.
- [132] F. Simonetti, M. Fleming, and E A. Marengo. Illustration of the role of multiple scattering in subwavelength imaging from far-field measurements. *J. Opt. Soc. Am. A*, 25(2):292–303, 2008.
- [133] J. W. Young and J. C. Bertrand. Multiple scattering by two cylinders. *J. Acoust. Soc. Am.*, 58(6):1190–1195, 1975.
- [134] M. Azimi and A. C. Kak. Distortion in diffraction tomography caused by multiple scattering. *IEEE Trans. Med. Imaging*, MI-2(4):176–195, 1983.
- [135] M. Davison. The ill-conditioned nature of the limited angle tomography problem. *SIAM J. Appl. Math.*, 43(2):428–448, 1983.

- [136] P. Huthwaite, A. A. Zwiebel, and F. Simonetti. A new regularization technique for limited-view sound-speed imaging. *IEEE Trans. Ultrason. Ferroelectr. Freq. Control*, 60(3):603–613, 2013.
- [137] A. Andersen. Algebraic reconstruction in CT from limited views. *IEEE Trans. Med. Imag.*, 8(1):50–55, 1989.
- [138] A. Devaney. Geophysical diffraction tomography. *IEEE Trans. Geosci. Remote Sens.*, 22(1):3–13, 1984.
- [139] N. Duric, P. Littrup, L. Poulo, and A. Babkin. Detection of breast cancer with ultrasound tomography: First results with the computed ultrasound risk evaluation (CURE) prototype. *Med. Phys.*, 34:773, 2007.
- [140] E. Sidky and X. Pan. Image reconstruction in circular cone-beam computed tomography by constrained, total-variation minimization. *Phys. Med. Biol.*, 53(17):4777–4807, 2008.
- [141] M. L. Reis and N. C. Roberty. Maximum entropy algorithms for image reconstruction from projections. *Inverse Probl.*, 8:623–644, 1992.
- [142] K. M. Hanson. *Image Recovery: Theory and Application*. Academic, New York, 1987.
- [143] A. P. Dhawan, R. M. Rangayyan, and R. Gordon. Image restoration by Wiener deconvolution in limited-view computed tomography. *Appl. Opt.*, 24:4013–4020, 1985.
- [144] A. Delaney and Y. Bresler. Globally convergent edge-preserving regularized reconstruction: An application to limited-angle tomography. *IEEE Trans. Image Process.*, 7(2):204–221, 1998.
- [145] P. Huthwaite. *Quantitative imaging with mechanical waves*. PhD thesis, Univ. of Imperial College, 2009.
- [146] Y. Sung, W. Choi, C. Fang-Yen, and K. Badizadegan. Optical diffraction tomography for high resolution live cell imaging. *Optics express*, 17(1):266–277, 2009.
- [147] W. C. Chew and Y. M. Wang. Reconstruction of two-dimensional permittivity distribution using the distorted Born iterative method. *IEEE Trans. Med. Imag.*, 9(2):218–225, 1990.

- [148] M. Ratssepp, M. J. S. Lowe, P. Cawley, and A. Klauson. Scattering of the fundamental shear horizontal mode in a plate when incident at a through-crack aligned in the propagation direction of the mode. *J. Acoust. Soc. Am.*, 124(5):2873–2882, 2008.
- [149] G. McMechan, J. Harris, and L. Anderson. Cross-hole tomography for strongly variable media with applications to scale model data. *Bull. Seismol. Soc. Am.*, 77(6):1945–1960, 1987.
- [150] L. Mazeika, R. Kazys, R. Raisutis, and R. Sliteris. Ultrasonic guided wave tomography for the inspection of the fuel tanks floor. *International Journal of Materials and Product Technology*, 41(1-4):128–139, 2011.
- [151] M. N. Toksoz and D. H. Johnston. *Seismic Wave Attenuation*. Society of Exploration Geophysicists, Tulsa, Okla, 1981.
- [152] M. B. Drozd. *Efficient finite element modelling of ultrasound waves in elastic media*. Imperial College, London, 2008.
- [153] J. Rao, M. Ratssepp, and Z. Fan. Investigation of the reconstruction accuracy of guided wave tomography using full waveform inversion. *J. Sound Vib.*, 400:317–328, 2017.
- [154] P. R. Roberge. *Corrosion Inspection and Monitoring*. New Jersey, John Wiley, 2007.
- [155] F. Yan, R. L. Royer, and J. L. Rose. Ultrasonic guided wave imaging techniques in structural health monitoring. *J. Intell. Mater. Syst. Struct.*, 21:377–384, 2010.
- [156] D. J. Wang, W. F. Zhang, X. Y. Wang, and B. Sun. Lamb-wave-based tomographic imaging techniques for hole-edge corrosion monitoring in plate structures. *Materials*, 9:916, 2016.
- [157] F. B. Cegla, P. Cawley, J. Allin, and J. Davies. High-temperature ($>500^{\circ}\text{C}$) wall thickness monitoring using dry-coupled ultrasonic waveguide transducers. *IEEE Trans. Ultrason. Ferroelectr. Freq. Control.*, 58:158–167, 2010.
- [158] F. B. Cegla and J. Allin. Wireless ultrasonic thickness monitoring at elevated temperatures. *Mater. Eval.*, 69:26–31, 2011.

- [159] G. Sposito, P. Cawley, and P. B. Nagy. Potential drop mapping for the monitoring of corrosion or erosion. *NDT & E Int.*, 43:394–402, 2010.
- [160] I. Geir, M. A. Toppe, and P. B. Nagy. Method and system of measuring material loss from a solid structure. *U.S. Patent No. 8,225,665*, 2012.
- [161] G. Instanes, M. Toppe, B. Lakshminarayan, and P. B. Nagy. Corrosion and erosion monitoring of pipes by an ultrasonic guided wave method. *Advanced Ultrasonic Methods for Material and Structure Inspection*, pages 115–157, 2007.
- [162] P. Belanger, P. Cawley, and F. Simonetti. Guided wave diffraction tomography within the Born approximation. *IEEE Trans. Ultrason. Ferroelectr. Freq. Control*, 57:1405–1418, 2010.
- [163] X. Zhao, R. L. Royer, S. E. Owens, and J. L. Rose. Ultrasonic lamb wave tomography in structural health monitoring. *Smart Mater. Struct.*, 20:105002, 2011.
- [164] A. C. Cobb and J. L. Fisher. Flaw depth sizing using guided waves. *AIP Conf. Proc.*, 1706:030013, 2016.
- [165] T. Clarke, P. Cawley, P. D. Wilcox, and A. J. Croxford. Evaluation of the damage detection capability of a sparse-array guided-wave SHM system applied to a complex structure under varying thermal conditions. *IEEE Trans. Ultrason. Ferroelectr. Freq. Control*, 56:2666–2678, 2009.
- [166] S. Vinogradov, C. Duffer, and G. M. Light. Magnetostrictive sensing probes for guided wave testing of high temperature pipes. *Mater. Eval.*, 72:803–811, 2014.
- [167] G. Instanes, A. O. Pedersen, and A. Brath. Ultrasonic computerized tomography for continuous monitoring of corrosion and erosion damage in pipelines. *CORROSION 2015 NACE International*, 2015.
- [168] M. Seher, P. Huthwaite, and M. J. S. Lowe. Experimental studies of the inspection of areas with restricted access using A0 lamb wave tomography. *IEEE Trans. Ultrason. Ferroelectr. Freq. Control*, 63:1455–1467, 2016.

- [169] R. Ribichini, F. Cegla, P. B. Nagy, and P. Cawley. Experimental and numerical evaluation of electromagnetic acoustic transducer performance on steel materials. *NDT & E Int.*, 45:32–38, 2012.
- [170] H. Gao, Y. Shi, and J. L. Rose. Guided wave tomography on an aircraft wing with leave in place sensors. *AIP Conf. Proc.*, 760:1788–1794, 2004.
- [171] P. Fromme, P. D. Wilcox, M. J. S. Lowe, and P. Cawley. On the development and testing of a guided ultrasonic wave array for structural integrity monitoring. *IEEE Trans. Ultrason. Ferroelectr. Freq. Control*, 53:777–785, 2006.
- [172] T. Wandowski, P. Malinowski, and W. M. Ostachowicz. Damage detection with concentrated configurations of piezoelectric transducers. *Smart Mater. Struct.*, 20:025002, 2011.
- [173] X. Wang, X. Lu, and J. Tang. Damage detection using piezoelectric transducers and the lamb wave approach: I. System analysis. *Smart Mater. Struct.*, 17:025033, 2008.
- [174] <http://www.piezomechanik.com> PiezoMechanik GmbH, "Piezo-Mechanics: An Introduction".
- [175] J. Rao, M. Ratassepp, and Z Fan. Guided wave tomography based on full-waveform inversion. *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency*, 63(5):737–745, 2016.
- [176] P. Huthwaite and M. Seher. Robust helical path separation for thickness mapping of pipes by guided wave tomography. *IEEE Trans. Ultrason. Ferroelectr. Freq. Control*, 62(5):927–938, 2015.
- [177] A. Velichko and P. D. Wilcox. Excitation and scattering of guided waves: relationships between solutions for plates and pipes. *J. Acoust. Soc. Am.*, 125(6):3623–3631, 2009.
- [178] L. E. Kinsler and A. R. Frey. *Fundamentals of Acoustics*. John Wiley and Sons, New York, 1962.
- [179] F. Zou and F. B. Cegla. High accuracy ultrasonic monitoring of electrochemical processes. *Electrochemistry Communications*, page accepted, 2017.

- [180] F. Simonetti, L. Huang, and N. Duric. On the spatial sampling of wave fields with circular ring apertures. *J. Appl. Phys.*, 101:083103, 2007.
- [181] ASTM B703-17. Standard test method for apparent density of metal powders and related compounds using the Arnold meter.
- [182] J. P. Panakkal, J. K. Ghosh, and P. R. Roy. Use of ultrasonic velocity for measurement of density of sintered fuel pellets. *J. Phys. D: Appl. Phys.*, 17: 1791–1795, 1984.
- [183] S. Howard, J. Tani, H. Arnold, and W. Sachse. Ultrasonic characterization of porosity in powder metals. *NDE of Microstructure for Process Control ed H.N.G. Wadley, ASM, PA*, pages 81–89, 1985.
- [184] L. Metivier, R. Brossier, S. Operto, and J. Virieux. Acoustic multiparameter FWI for the reconstruction of P-wave velocity, density and attenuation: Pre-conditioned truncated Newton approach. *85th Annual International Meeting, SEG, Expanded Abstracts*, pages 1198–1203, 2015.
- [185] J. Nocedal and S. J. Wright. *Numerical optimization (2nd ed.): Springer Science + Business Media*. Springer Series in Operations Research and Financial Engineering, LLC, 2006.
- [186] D.H. Sheen, K. Tuncay, C.E. Baag, and P.J. Ortoleva. Time domain Gauss-Newton seismic waveform inversion in elastic media. *Geophys. J. Int.*, 167: 1373–1384, 2006.
- [187] Y. Liu, J. Yang, B. Chi, and L. Dong. An improved scattering-integral approach for frequency-domain full waveform inversion. *Geophys. J. Int.*, 202:1827–1842, 2015.
- [188] A. Tarantola. *Inverse Problem Theory: Methods for Data Fitting and Parameter Estimation*. Elsevier, 1987.
- [189] J. Yang, Y. Liu, and L. Dong. Simultaneous estimation of velocity and density in acoustic multiparameter full-waveform inversion using an improved scattering-integral approach. *Geophysics*, 81(6):R399–R415, 2016.
- [190] A. Tarantola. Inversion of seismic-reflection data in the acoustic approximation. *Geophysics*, 49(8):1259–1266, 1984.

-
- [191] R. E. Plessix. A review of the adjoint-state method for computing the gradient of a functional with geophysical applications. *Geophys. J. Int.*, 167:495–503, 2006.
 - [192] P. Chen, T. H. Jordan, and L. Zhao. Full three-dimensional tomography: A comparison between the scattering-integral and adjoint-wavefield methods. *Geophys. J. Int.*, 170:175–181, 2007.
 - [193] F. Schumacher, W. Friederich, and S. Lamara. A flexible, extendable, modular and computationally efficient approach to scattering-integral based seismic full waveform inversion. *Geophys. J. Int.*, 204:1100–1119, 2016.
 - [194] E. Forgues and G. Lambare. Parameterization study for acoustic and elastic ray plus Born inversion. *J. Seism. Explor.*, 6:253–278, 1997.
 - [195] A. Y. Anagaw and M. D. Sacchi. Comparison of multifrequency selection strategies for simultaneous-source full-waveform inversion. *Geophysics*, 79(5):R165–R181, 2014.
 - [196] L. Metivier, F. Breteau, R. Brossier, S. Operto, and J. Virieux. Full waveform inversion and the truncated newton method: Quantitative imaging of complex subsurface structures. *Geophysics*, 62:1353–1375, 2014.
 - [197] M. J. Woodward. Wave-equation tomography. *Geophysics*, 57:15–26, 1992.
 - [198] J. Tromp, C. Tape, and Q. Liu. Seismic tomography, adjoint methods, time reversal and banana-doughnut kernels. *Geophys. J. Int.*, 160:195–216, 2005.