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EFFECTS OF DELIVERY RISK ON FUTURES MARKETS: THEORY AND EMPIRICAL EVIDENCE

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Effects of Delivery Risk on Futures Markets: Theory and Empirical Evidence

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DEDICATION

I would like to dedicate this dissertation to my parents, Lawun and Kiat Roongsangmanoon, whose achievements and struggles in lives were second to none and who taught me to believe in the power of knowledge and education as well as to appreciate the accomplishments by hard work and self-discipline.

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ABSTRACT

This dissertation consists of four essays on the effects of delivery risk in futures markets. In this dissertation, the delivery risk is defined as the uncertainty arising from the multiple delivery specifications that causes futures price to diverge from the spot price of the par-delivery asset underlying the futures contract.

The first essay (*Models of Futures Risk Premium: A Critical Review*) provides a critical review of the extant literature on futures risk premiums that are defined as the difference between the current futures price and the expected spot price of the asset underlying the futures contract.

The second essay (*Delivery Risk and Hedging Role of Futures Options in the Futures Market Equilibrium*) contains a mathematical analysis that demonstrates that the delivery risk gives rise to a complimentary hedging role of futures options in the context of futures market equilibrium. The analysis also provides a mathematical relationship between the futures risk premium and the futures option's payoff in the presence of delivery risk, which indicates a role of futures option in explaining the futures risk premiums.

The third essay (*Futures Risk Premium in the Presence of Delivery Risk: Theory*) extends Hirshleifer's (1988) two-factor model of futures risk premiums to a three-factor model by incorporating the expected returns on futures option as an additional determinant of the futures risk premium.

The fourth essay (*Futures Risk Premium in the Presence of Delivery Risk: Evidence*) contains an empirical analysis of the hypotheses generated by the three-

factor model of futures risk premiums. Empirical findings based on the two-step regression analysis employing weekly settlement price and hedging pressure data on 13 highly liquid futures and their corresponding futures options indicate that futures option returns significantly affect the futures risk premiums.

Collectively, this dissertation provides original explanations on the hedging role of futures options in the context of futures market equilibrium and proposes a three-factor model of futures risk premiums and documents new evidence on the importance of futures options as a determinant of the futures risk premiums.

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CHAPTHER 1

INTRODUCTION

The dissertation contains theoretical and empirical analyses of the effects of delivery risk on futures demand, futures price, and futures risk premium. Chapter 2 reviews the models of futures risk premium. The review emphasizes on the interplay between theory and empirical work to understanding the behavior of risk factors that are priced in futures risk premiums. Current theoretical analyses of futures risk premiums identify two main types of risk factors: systematic risk and hedging pressure. The models integrating systematic risk and hedging pressure, however, do not completely explain variation in futures risk premiums and abstract from multiple delivery specifications in the market practice.

Futures contracts with multiple delivery specifications allow the sellers to deliver any of the deliverable grades of the underlying asset at any designated locations.[1] Although the grades are accompanied by a schedule of discounts and premiums allowable for the delivery of lesser or greater quality than the par-delivery grade, the realized prices of these grades can be significantly different following adjustment for these discounts or premiums. Given these different prices, sellers will choose the grade with the cheapest delivery cost. Consequently, futures price on the delivery day converges to the spot price of the cheapest-to-deliver grade, which is currently unknown. This causes futures traders to face delivery risk in addition to the price risk of the underlying asset.

[1] The purpose of multiple delivery specifications is to reduce the problem that arises from short squeeze, i.e., the contract sellers competing for inadequate deliverable grades of the asset underlying the futures contract. Other types of delivery specifications, e.g., timing options, are not so important, as empirically documented by Gay and Manaster (1984; 1986).

In the absence of delivery risk (i.e., for futures contracts that are cash-settled), an assumption employed in the literature on equilibrium futures pricing, futures price converges to the spot price of the certain asset underlying the contract (i.e., the par-delivery grade) on the delivery day.[2] Hence, the hedgers' revenues, which are risky but linear in the spot price, can be optimally hedged by taking positions in futures contracts, of which the gains or losses are also linear in the spot price.

The presence of delivery risk, however, causes futures price to converge to the cheapest-to-deliver price on the delivery day. Thus, hedgers, whose revenues are linearly related to the spot price of par-delivery grade, cannot precisely hedge the revenue risk using merely futures contracts, of which the payoffs are not linear in the spot price. The nonlinearity of futures payoff gives rise to some residual revenue risk of hedgers to be hedged with futures options.[3]

Chapter 3 addresses the effect of delivery risk on the futures market equilibrium when futures options are present. A comparative-static analysis shows that the presence of delivery risk affects the net hedging demand for futures and generates the net hedging demand for futures options. These results indicate that the delivery risk causes futures options to become a nonredundant

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[2] This literature includes the well-known models of Dusak (1973), Stoll (1979), Hirshleifer (1988) and DeRoon Nijman and Veld (2000). These models are based on an implicit assumption of no delivery risk, so futures price converges to the spot price of the underlying asset at the end of the period. Thus, futures payoff in these models is defined as the end-of-period spot price minus current futures price.

[3] The analysis of the hedging role of futures options is not original to this research. Lien and Wong (2002) first derived an individual agent's optimal hedging strategy involving futures and futures options in the presence of delivery risk. Our analysis of the effect of delivery risk is implemented in the context of futures market equilibrium, which is the main contribution of this dissertation.

hedging instrument and hence relates the futures options to the futures market equilibrium. A further analysis shows that the equilibrium of futures market is also explained by the payoff of futures options. This implies a role of futures options in explaining futures risk premiums.

Chapter 4 formulates the implication of the nonredundant futures options for pricing futures contracts.[4] Specifically, we extend the two-factor (systematic risk and residual risk conditional on hedging pressure) model of Hirshleifer (1988) to a three-factor model by incorporating the expected returns on futures option as an additional determinant of the futures risk premiums.

Chapter 5 examines the hypotheses generated by the three-factor model of futures risk premiums developed in Chapter 4. Empirical evidence based on two-step regressions employing weekly data of 13 highly liquid U.S.A. futures contracts and their corresponding futures options indicates that the expected returns of futures option significantly affect the average futures returns and futures risk premiums. This result is obtained after controlling for systematic risk and residual risk conditional on hedging pressure in the futures market and remains significant even after purging the component of futures option returns that may be explained by futures returns and stock market returns. These results

[4] This pricing implication seemingly counters the belief that the option price is determined by the behavior of the underlying price. However, this unidirectional relationship is only true in complete markets. When the markets are incomplete, as shown by DeTemple and Selden (1991), derivatives may change the hedging opportunities available to traders, thereby introducing price effects on the security underlying the derivatives. Back (1993) analyzed the effect of asymmetric and non-unidirectional on the option pricing framework. Vanden (2004) showed that, when options are nonredundant in the economy, the option returns significantly explain the cross-section of the underlying security returns. Based on this premise, returns on futures options would thus influence futures returns and risk premiums in the presence of delivery risk.

suggest the importance of futures option returns in explaining futures risk premiums.

Chapter 6 summarizes the theoretical and empirical results and presents some conclusions. Collectively, this dissertation provides a rationale for the hedging role of futures options in the futures market equilibrium and documents the importance of futures options in pricing futures contracts. By providing analyses on the effects of delivery risk in the context of futures market equilibrium, this dissertation contributes to both the literature on the hedging benefit in futures options (e.g., Lien and Wong (2002)) and the literature on the informational role of options in pricing the underlying security (e.g., DeTemple and Selden (1991), Vanden (2004)).

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CHAPTHER 2

MODELS OF FUTURES RISK PREMIUM: A CRITICAL REVIEW

This chapter reviews the literature on modeling futures risk premium. While emphasizing on the interplay between theory and empirical work, the review provides background material, summarizes findings, appraises the extant theories on futures risk premiums. Detailed discussions are provided for several underexplored issues that arise in the presence of delivery risk. The discussions also confine the scope for research implemented in this dissertation.

1. Introduction

Risk premium in futures is traditionally known as the difference between the current futures price and the expected spot price of the asset underlying the futures contract. The risk premium roughly reflects an equilibrium price for transferring the underlying asset's price risk with the use of a futures contract. Measuring futures risk premium is important in estimating hedging costs and benefits, making arbitrage and speculative investment decisions, and understanding the dynamics of both spot and futures prices.

Futures risk premium is related to the underlying asset's risk premium, but the nature of these premiums can differ substantially, as shown by Fama and French (1987). The basis (i.e., the difference between futures price and the current spot price of the underlying asset) can be viewed as the sum of futures risk premium and expected change in the underlying asset prices. This view indicates a relationship between futures risk premium and the underlying asset's risk premium, which is a part of the asset's return. However, futures risk premium cannot be inferred directly from the underlying asset's return because the relationship is smeared by the uncertainty of basis. Due to the aforementioned importance of futures risk premium, coupled with its obscure relation with the risk premium of underlying asset, futures risk premium is one of the central research issues in the study of futures markets.

This chapter provides a critical review of both theoretical and empirical models of the futures risk premiums. The review focuses on the interplay between theory and empirical work for the purpose of understanding the behavior of priced risk factors that constitute the futures risk premiums. The review is carried out through a general mathematical framework for comparability. Extant models identified two types of priced risk factors: namely, the systematic risk and the hedging pressure (as a non-systematic risk).

Recent empirical evidence[5] generally supports the models integrating both systematic risk and hedging pressure and hence refines the determination of futures risk premiums. Among several competing models, Hirshleifer (1988) model is the most appropriate model for theoretical extension and empirical testing. This is because Hirshleifer model is developed with solid theoretical ground, relatively parsimonious (two-factor model) and empirically supported.

Extant literature on modeling futures risk premium does not address the futures-pricing effect of the delivery risk[6], although the importance of delivery specifications in the pricing futures contracts has been examined extensively[7]. For example, Lien and Wong (2002) demonstrate that a hedging role of futures options arises from the presence of delivery risk. As implied by such studies, the consideration of delivery risk leads to several underexplored interesting

[5] See for example, Carter Rausser and Schmitz (1983), Bessembinder (1992), DeRoon, Nijman and Veld (2000). Gorton and Rouwenhorst (2004) and Erb and Harvey (2005) provided updated empirical studies of returns and risk premiums on commodity futures contracts.
[6] The delivery risk arises from the multiple delivery specifications that are often provided in futures contracts. In essence, the specifications allow the sellers to choose any of listed deliverable goods to satisfy the delivery requirement. The uncertainty in identifying the cheapest-to-deliver goods is referred to as delivery risk.
[7] The literature includes Garbade and Silber (1983), Gay and Manaster (1984, 1986), Kamara (1990), Kane and Marcus (1986), and Kilcollin (1982), Adam-Mueller and Wong (2003). Chance and Hemler (1993) provided a rather complete survey on the impact of delivery risk on the pricing futures contracts.

implications on futures risk premiums and other characteristics of the futures market equilibrium.

Specifically, this dissertation examines the following issues:

1. To address the hedging role of futures option in the context of futures market equilibrium.

2. To re-examine the effect of futures options on the equilibrium futures price, demand for futures, and futures risk premium.

3. To examine whether the expected returns on futures option could also explain the determination of the futures risk premiums.

The remaining sections are organized as follows. Section 2 discusses the unique concept of futures return and risk premium and a general framework to review the major theoretical and empirical models of futures risk premium. Section 3 surveys one-factor models. Two-factor models are reviewed in section 4 and multifactor models are in section 5. Section 6 discusses and assesses the models. Section 7 confine the scope of this research in the presence of delivery risk. Section 8 provides a summary and conclusions.

2. Futures Risk Premium: Integration

2.1 Futures risk premium and its relation to futures return

Due to the absence of initial investment, return on futures position held by an investor cannot be defined as a proportional change in asset value to the investment cost. Therefore, futures return and risk premium must be defined in dollar term (Black, 1976).

Futures risk premium, π_t, is traditionally defined as the expected spot price at the contract maturity T, $E_t[S_T]$, minus current futures price, F_t. This definition is based on an implicit assumption of no delivery risk; thus, futures price converges to the spot price of the underlying asset at maturity.

$$\pi_t \equiv E_t[S_T] - F_t \tag{1}$$

where $E_t[.]$ denotes an expectation operator conditional on the information available at time t. Equation (1) states that futures risk premium is the deviation of futures price from the expected spot price and that the premium is conditional on the common information of investors at time t.

Positive futures risk premium can be interpreted as a compensation that a futures investor who holds a short position expects to pay to his counter party for taking a long futures position. Conversely, when futures risk premium is negative, a long-position investor expects to compensate his counter party for holding a short position in futures.

Another view of the relation between futures return and risk premium is that, given the definition in Equation (1), dollar futures return of a long-position investor for n holding-periods can be expressed as a change in futures risk premiums.

$$R_{t+n} = F_{t+n} - F_t = \pi_t - \pi_{t+n} \tag{2}$$

Note that expected futures return can be positive or negative. The expected return reflects that an investor with a futures position expects to compensate his counter party for holding the opposite futures position. For example, the positive expected return means that a short-position investor expected to pay his counter long-position party.

Futures risk premium can be expressed as a change in spot prices minus the contemporaneous spot-futures basis: $\pi_t = E_t[S_T - S_t] - (F_t - S_t)$. This expression together with Equation (2) implies that a change in futures risk premiums reflects spot price risk (the uncertainty of spot price change) as well as basis risk (the uncertainty of basis change). Thus, futures investors are exposed to spot price risk and basis risk.

2.2 A unified framework

A general framework for theoretical models is that the expected change in futures prices in percentage, $E_t[r_{t+n}]=[F_{t+n}-F_t]/F_t$, equals to the sum of the multiplication of betas and expected determinants.

$$E_t[r_{t+n}] = \sum_{j=1}^{k} \beta_{j,t} E_t[\gamma_{j,t+n}] \quad (3)$$

where $E_t[.]$ is an expectation operator conditional on the information available at time t; $j \in [1, 2, \ldots, k]$; $\beta_{j,t}$ is the jth beta or the conditional sensitivity of the futures price change to the jth determinant in effect at time $t+n$ ($\gamma_{j,t+n}$). Thus, $\beta_{j,t}$ is defined as

$$\beta_{j,t} = \text{cov}_t[r_{t+n}, \gamma_{j,t+n}] / \text{var}_t[\gamma_{j,t+n}] \quad (4)$$

where $\text{cov}_t[.]$ and $\text{var}_t[.]$ are operators of covariance and variance conditional on the information available at time t respectively.

The econometric specification of (3) is that the realized change in futures prices at time $t+n$, r_{t+n}, equates the sum of the multiplication of $\hat{\beta}_{j,t}$ and $\gamma_{j,t+n}$, plus an intercept[8], α_{t+n}, and an error term, ε_{t+n}.

$$r_{t+n} = \alpha_{t+n} + \sum_{j=1}^{k} \hat{\beta}_{j,t} \gamma_{j,t+n} + \varepsilon_{t+n} \quad (5)$$

[8] In a general asset-pricing model, the intercept term reflects risk-free return on an asset: i.e., the total expected return net of the return required to compensate the investor for taking risky investment position in the asset. In futures pricing model, the intercept is expected to be zero because taking a position in futures contract essentially requires zero initial investment. Hence, for futures traders, the total expected return reflects only the risk premium.

where $\hat{\beta}_{j,t}$ is the conditionally estimated jth beta; $\gamma_{j,t+n}$ is the realized jth determinant at time $t+n$.

The following sections review the theory and empirical work of futures risk premium within the unified framework (3) and (5). The models of futures risk premiums are categorized into three classes: one-factor, two-factor and multifactor models respectively.

3. One-factor Models

The literature on futures risk premium illustrates that the premium attributes to the risk aversion of futures traders and the assumptions of market imperfection. The different assumptions cause various one-factor models as follows.

3.1 Theories of hedging and returns to speculators

Theories of hedging are established in an economy with risk-averse investors and market imperfection (i.e., the existence of the non-marketable assets). Hedgers possess businesses generating uncertain revenue that relates to the asset underlying the futures contract. In the imperfect market, hedgers cannot trade ownership claims on certain assets or production techniques. Thus, hedgers need to transfer the risk on their revenue through futures market.

Keynes (1930) originally related futures risk premium to the net futures positions of hedgers, called hedging pressure. Regarding a futures market as an insurance scheme, Keynes viewed futures risk premium as an insurance premium that the hedger compensates his insurer, called speculator, for assuming the transferred price risks of the underlying asset. Thus, consistent with Keynes' theory, average futures returns are conditioned on the sign of net futures positions of hedgers and average futures returns are significantly larger when hedgers are net short than when they are net long. This can be formalized as

$$E_t[r_{t+1}] = \beta\gamma_t \quad (6)$$

where $E_t[r_{t+1}]$ denotes expected percentage change in futures prices at time $t+1$; β is constant and the determinant (γ_t) is hedging pressure[9]. The model in (6) is analogous to (3) given n=1, k=1.

In contrast to Keynes' theory, Hardy (1940) attested no futures risk premium. According to Hardy, speculators, as a class, do not make money on average at expense of the hedgers. The profits of the successful speculators come from the other less skillful speculators, who stay in the game presumably for its entertainment value.

[9] Hedging pressure is measured by the net hedging positions on a futures contract that expires on a later date. The net hedging positions (i.e., the total positions held by hedgers minus the total positions held by speculators) are deemed to be currently known to all traders as this (and other) weekly information is published in the COT reports [Commitments of Traders in Commodity Futures reports: www.cftc.gov/cftccotreports.htm] and assessable by general public.

Although a lot of researches devoted to verify the Keynes-Hardy controversy, the conflict remains unsolved. The most influential support of Keynes' view has been Cootner (1960a; 1960b) whereas that of Hardy's view has been Telser (1958;1960;1967). Other empirical findings consistent with the Keynes' view include Houthakker (1957; 1968), Stevenson and Bear (1970), Chang (1985), Bessembinder (1992), DeRoon, Nijman, and Veld (2000) while those with Hardy's view include Rockwell (1967), Hartzmark (1987) and Kolb (1992).

An alternative theory (Gray (1960)) argued that futures risk premium tend to be small in the mature market while the risk premium would be large in the thin market, in which speculators are less interested. Results from the study suggest additional insights of the behavior of futures risk premiums and provide some basics for generalizing the hedging role in the equilibrium pricing of futures contract.

In summary, along the theories of hedging, futures risk premium has been debated between two contending theories of Keynes and Hardy. Despite the considerable research attempts to verify the theories, the conflict between the two theories remains unresolved. Each theory appears to be valid only for some markets under certain conditions or time intervals.

3.2 Theories of asset pricing models in futures markets

Theories of asset pricing models are also established in an economy with risk-averse investors. Unlike the theories of hedging, the theories of asset pricing models conventionally assume all claims are freely marketable (market perfection). Thus the futures risk premium required on a futures contract should depend on only the non-diversifiable risk, which is systematically related to the degree of covariation between futures prices and economic state variables.

In applying the asset pricing models to futures contracts, a crucial assumption of the market integration is that all markets are subject to the same economic variables and have a uniform risk-return trade-off. If both equity and futures markets are integrated, the asset pricing models for equity markets are applicable to futures markets also.

3.2.1 Futures Risk Premium Model based on Capital Asset Pricing Model (CAPM)

Dusak (1973) first analyzed futures risk premium in context of the CAPM of Sharpe (1964). The model assumes no taxes, no limitation of borrowing, market frictionless, non-stochastic interest rate, no basis risk, and no business risk of the storage and processing industries. Futures risk premium depends only on the systematic risk, the covariance of change in futures prices with return on total wealth. The model can be written as

$$E_t[r_{t+1}] = \beta E_t[\gamma_{t+1}] \qquad (7)$$

where $E_t[.]$ denotes an expectation operator ; r_{t+1} is the percentage change in futures prices at time t+1; γ_{t+1} is excess stock index return over risk-free return at time t+1; β is the sensitivity of the futures return to γ_{t+1}. The model in (7) is analogous to (3) given n=1, k=1.

The application of CAPM to futures markets is theoretically intuitive; however, the model is partly supported by empirical tests (e.g., Dusak (1973), Bodie and Rosansky (1980), Carter Rausser and Schmidt (1983), Marcus (1984), and Kolb (1996)). Junkus (1991) documented that the testing results of CAPM were sensitive to the proxies of market return and testing periods. The evidence indicates that the asset and the futures markets are not fully integrated and suggests that the futures risk premium should account for non-systematic risk(s).

Apart from the mixed and sensitive results, perhaps the assumptions of a stationary investment opportunity and consumption set are the main causes to the unsuccessful application of CAPM to futures markets. These assumptions imply constant relative prices and interest rates conditions that would make any futures contract unrealistically risk-free. Such problems cause researchers to consider more general asset pricing models, which are discussed in the following subsection.

3.2.2 Futures Risk Premium Model based on the Consumption-based CAPM framework

Many studies consider the risk of futures contract in context of intertemporal asset pricing models with stochastic investment opportunities and real consumption. Grauer and Litzenberger (1979) developed a single-period model that the risk premium is proportional to the covariance between the commodity price and the social marginal utility of real wealth.

Breeden (1980) analyzed futures risk premiums within the framework of the multi-period consumption based CAPM of Breeden (1979). The model states that futures risk premium is proportional to the covariance of futures returns with changes in aggregate consumption, i.e., the consumption beta. The consumption beta depends on the commodity's price elasticity of supply and demand. The model is written as

$$E_t[r_{t+1}] = \beta E_t[\gamma_{t+1}] \qquad (8)$$

where $E_t[.]$ denotes an expectation operator ; r_{t+1} is the percentage change in futures prices at time $t+1$; γ_{t+1} is excess real returns on the aggregate consumption portfolio over real risk-free return; β is the sensitivity of the futures return to γ_{t+1}. The model in (8) is comparable to (3) given $n=1$, $k=1$.

The Breeden (1980) model was examined in several studies. Breeden (1980) showed that estimated betas were consistent with those theoretically predicted by the model. Hazuka (1984) found significant risk premium in the predicted direction but the coefficient estimates were different from their theoretical value.

Jagannathan (1985) extended Grauer and Litzenberger (1979) model to a multi-period model with time-varying consumption beta. The evidence on corn, wheat, and soybean futures rejected the model. Nevertheless, the justification of these results is subject to two critical reasons: the small size and the error of the assumptions underlying the model (e.g., agents do not possess the same information, utility function is not time separable, some agents face liquidity constraints).

Using a time-varying consumption-based CAPM model, Kaminsky and Kumar (1990) documented the significant futures risk premiums for corn, soybeans, wheat, cocoa, coffee, and copper markets. Similarly, McCurdy and Morgan (1992) found strong evidence of time-varying risk premiums for five currency futures. The risk premiums were estimated with respect to the covariances of futures returns with the portfolio returns representing both consumption and wealth.

To summarize, the theories of asset pricing models (i.e., CAPM and consumption based CAPM) have mixed empirical results. The conflicting results in the aforementioned studies may arise from the dissimilar proxies of stock market return, different testing periods and various futures markets. However, the more critical justification of these models is whether their implicit assumptions are valid. Among other assumptions, the integration of futures and asset markets and the time variation of the asset betas should be carefully examined. Table 1 summarizes the theoretical and empirical one-factor models of futures risk premium.

4. Two-factor Models

The unsatisfied performance of the one-factor models motivated researchers to seek more general models, i.e., two-factor models[10]. The well-known models of integrating hedging pressure and systematic risk are established in an economy with risk-averse investors and market imperfection. The models generalize the role of hedging into the equilibrium-pricing framework.

Stoll (1979) initially presented a model that relates both systematic risk and nonmarketable risks born by hedgers (e.g. storage risk or processing risk) to futures risk premiums. According to Stoll, futures risk premium exists because hedgers are unable to trade ownership claims on certain assets or production techniques. Stoll (1979) model suggests that futures price is naturally downward biased (normal backwardation), given positive storage costs and a positive covariance between commodity and stock prices[11]. However, the upward price bias (contango) is possible, even with positive storage costs if the covariance is sufficiently negative.

[10] Another sub-class of two-factor models characterizes futures risk premium under the rational expectations equilibrium models. The model of Richard and Sundaresan (1981) identifies two priced factors of futures risk premium, namely the usefulness of hedging and interest rate risk. Anderson and Danthine (1983) model and Britto (1984) model indicate that quantity risk (uncertainty output) in additional to price risk affects futures risk premium; thus futures price bias also depends on the elasticity of demand. Kawai (1983) model focuses on futures risk premium for non-storable commodities with no price uncertainty. Turnovsky (1983) model indicates that futures risk premium depends on both the trading-inventory and producing cost functions. In short, the theoretical works, which characterize the spot-futures price relation under rational expectations, have shed the light to additional factors influencing on futures risk premium. However, these models have not been verified empirically and focus on the individuals' wealths concerning only spot and futures.

[11] As Stoll stated in his article (Journal of Financial Quantitative and Analysis, 1979, page 892.), the covariance between commodity and stock prices are the relevant covariance risk facing futures traders who are assumed to hold positions in the general assets (e.g., stocks and their portfolios) as well. The positive covariance increases with futures risk premium.

Carter, Rausser and Schmitz (1983) empirically examined a two-factor (systematic risk and hedging pressure) model of futures risk premiums. The model is described as

$$E_t[r_{t+1}] = \beta_1 E_t[\gamma_{1,t+1}] + \beta_2 E_t[\gamma_{2,t+1}] \qquad (9)$$

where $E_t[.]$ denotes an expectation operator ; r_{t+1} is the percentage change in futures prices at time $t+1$; β_1 and β_2 are the sensitivities of futures return to efficient portfolio return ($\gamma_{1,t+1}$) and hedging pressure ($\gamma_{2,t+1}$) respectively. The model in (9) is comparable to (3) given n=1, k=2.

Using return on the equal-weighted index of the S&P 500 and the Dow Jones Commodity Futures as a proxy for efficient portfolio return, Carter et al. (1983) found the evidence on five commodity futures supported the model in equation (9). However, this study was criticized by Marcus (1984) for over-weighting commodities. Baxter, Conine, and Tamarkin (1985) used a smaller weight of commodity index in the efficient portfolio proxy and found no risk premium for commodity futures.

Hirshleifer (1988) proposed a model that relates futures risk premium to systematic risk and residual risk conditional on the net futures positions of hedgers. The rationale of the model is that, in equilibrium, futures risk premium is set to compensate the marginal speculator for his setup costs and the incremental risk associated with his futures positions. The assumptions of the model include no taxes, no market friction for stocks, the hedgers holding assets associated with nonmarketable risks, and fixed transaction costs deterring some speculators from participating in the futures market. The model can be written as

$$E_t[r_{t+1}] = \beta_{1,t} E_t[\gamma_{1,t+1}] + \beta_{2,t} h_t \gamma_{2,t} \tag{10}$$

where $E_t[.]$ denotes an expectation operator ; r_{t+1} is the percentage change in futures prices at time $t+1$; $\beta_{1,t}$ and $\beta_{2,t}$ are the time-dependent sensitivities of the futures return to return on a well-diversified portfolio of assets ($\gamma_{1,t+1}$) and residual risk ($\gamma_{2,t}$) conditional on hedging indicator variable (h_t). h_t is equal to 1 for the net long futures positions of hedgers and to –1 for the net short positions. The model in (10) is comparable to (3) given n=1, k=2.

The evidence, found by Bessembinder (1992), supports Hirshleifer (1988) model. After controlling for the systematic risk, hedging pressure additional explained variation in risk premiums for currency and agricultural futures markets.

Hirshleifer (1989, 1990) showed that, in general equilibrium setting, given no transactions costs, there is no futures risk premium regardless of the demand elasticity. On the other hand, given fixed transaction costs, small traders (consumers) are deterred from the futures market, leaving only large traders (producers), and therefore, leading futures price to be upward biased for elastic demand and downward biased for inelastic demand.

In summary, the two-factor models integrating systematic risk and hedging pressure provide a useful intuitive perspective and are empirically supported in some futures markets. However, the models do not completely explain the behavior of futures risk premium. Table 2 presents the theoretical and empirical two-factor models of futures risk premium.

5. Multifactor Models

5.1 Arbitrage Pricing Theory (APT) Model

The Arbitrage Pricing Theory (APT) of Ross (1976), which offers a multifactor version of pricing models, has used to determine futures risk premium. APT predicts the linear relation between futures return and economic state variables. Thus, futures risk premium is proportional to the covariance of futures price with economic state variables. The assumptions of the model include no taxes and market frictionless (e.g., no transaction cost and no limitation on borrowing). The model is described as

$$E_t[r_{t+1}] = \sum_{j=1}^{k} \beta_{j,t} E_t[\gamma_{j,t+1}] \tag{11}$$

where $E_t[.]$ denotes an expectation operator ; r_{t+1} is the percentage change in futures prices at time $t+1$; $\beta_{j,t}$ is conditional sensitivities to the jth determinant; $E_t[\gamma_{j,t+1}]$ is the expected jth determinant in effect at time $t+1$.

However, the empirical results have been mixed. Bessembinder (1993) documented evidence supporting a multifactor model like APT in financial and currency futures markets but objecting the model in agricultural and metal futures markets. Young (1991) provided the oppose evidence in soybean and wheat futures while Chen, Cornett, and Nabar (1993) reported the support evidence in Treasury futures. Meanwhile, Bessembinder and Chan (1992) and Miffre (2000) tested APT models and documented some strong evidence of time-varying futures risk premium for commodity futures.

5.2 DeRoon, Nijman and Veld (2000) model

DeRoon Nijman and Veld (2000) presented a model allowing futures return and risk premium to vary with both systematic risk and hedging pressures. The distinguishing feature of the model is that futures risk premium depends on additional hedging pressures from other futures, termed cross-hedging pressures. The assumptions of the model include no taxes, no market friction for marketable assets, only nonmarketable risk associated with hedgers' endowment. The model is

$$E_t[r_{t+1}] = \beta_1 E_t[\gamma_{1,t+1}] + \sum_{j=2}^{k} \beta_j \gamma_{j,t} \qquad (12)$$

where $E_t[.]$ denotes an expectation operator ; r_{t+1} is the percentage change in futures prices at time $t+1$; $\beta_{1,t}$ is the sensitivity of futures return to the expected systematic risk $E_t[\gamma_{1,t+1}]$; β_j is the sensitivity of futures return to the net hedging positions at time *t*; $\gamma_{j,t}$ is hedging pressure for futures market j; $j = 2,3,\ldots,k$.

Although the cross-hedging pressures are statistically significant, at least one, in each group of futures contracts, the incremental explanatory power of these cross-hedging pressures should be carefully interpreted. The model may be less parsimonious since the number of cross-hedging pressure variables added to the model is arbitrarily determined.

In short, multifactor models include additional factors to explain futures risk premiums. While the results of testing APT models show evidence of systematic risk for some financial and currency futures and evidence of time varying risk premium for commodity futures, the evidence on DeRoon et al. (2000) model indicates cross-hedging pressures in futures markets as additional priced factors of futures risk premium. However, the multifactor models may be less preferable due to less parsimony and insignificant contribution to the power in explaining futures risk premiums. Table 3 summarizes the theoretical and empirical multifactor models of futures risk premium.

6. Discussion

This section assesses the futures risk premium models and identifies the most appropriate model that coincides with the recent evidence on futures risk premium. The appropriate model can be used as a benchmark model for theoretical extension and empirical testing. Recent empirical studies generally supported the models integrating two priced factors: systematic risk and hedging pressure (Carter Rausser and Schmitz (1983), Bessembinder (1992), DeRoon, Nijman and Veld (2000)). The three existing models in this category are accessed based on strong theoretical ground, parsimony, testability, empirical support, and applicability to general futures contracts. The results are as follows.

1) Stoll (1979) model: a Keynes insurance justification of hedging was initially developed in the framework of capital market equilibrium. The model is parsimonious because it relates futures risk premium to only two risk factors, namely systematic risk and storage (or processing) risk. Stoll model, however, has no empirical support due partly to the difficulty to observe storage risk. Moreover, Stoll model is applicable to certain commodity futures markets, in which storage risk is prevail.

2) Hirshleifer (1988) model was developed in the capital market equilibrium, where setup costs or informational barriers that reduce the speculators' participation on the futures market increase the impact of hedging on the risk premium. In the model, futures risk premium is explained by two risk factors: systematic risk and residual risk conditional on the net hedging pressure. The two factors are considered parsimonious and general so that they are applicable to explaining risk premiums of various futures contracts, e.g., commodity, index, and currency futures. Moreover, the risk factors are testable because they can be measured from returns on traded securities, not the returns on the market portfolio of all invested wealth. Moreover, Bessembinder (1992) found evidence support to Hirshleifer model in currency and agricultural futures markets.

3) DeRoon Nijman and Veld (2000) model proposes a number of cross-hedging pressures as additional price factors after controlling for the two well-known factors: systematic risk and own hedging pressure. Despite evidence support, DeRoon et al. (2000) model may be less preferable and parsimonious since the number of cross-hedging pressure variables is arbitrarily determined. Consequently, the incremental power of these variables to explain the variation in futures risk premium should be carefully considered.

These alternative theories and mixed evidence suggest that (1) the asset and futures markets are not fully integrated especially in the case of futures contracts that are only physically settled; (2) hence, the systematic risk premium tends to be insignificant; and (3) hedging pressure appears to be the non-systematic risk that mainly determines the futures risk premium. Given these suggestions, Hirshleifer (1988) model becomes the most appropriate model for theoretical extension and empirical testing for the study of futures risk premiums in the presence of delivery risk.

7. Does Delivery Risk Explain Futures Risk Premium?

This section discusses an ignorance of the presence of delivery risk from the literature of modeling futures risk premiums. Although the importance of delivery specifications in the pricing futures contracts has been examined extensively[12], the literature on modeling futures risk premium abstracts from delivery risk. This abstract reflects the wide beliefs that deliveries on futures contracts are rarely made and taken and that delivery risk is relevant to only participants who make or take delivery. However, several studies documented significant deliveries on futures contracts. For example, Peck and Williams (1991; 1992) showed that, for agricultural and mineral futures, about 15 percent of the peak open interest is delivered in each contract month. To the relevancy of all futures market participants, the futures-spot price relation prior to delivery largely depends on the grade that is likely to be delivered. Therefore, the effects of delivery risk are important to any participant in futures markets even though he or she never makes or takes delivery.

A striking study by Lien and Wong (2002) shows that the presence of delivery risk gives rise to the hedging role of futures options and thereby causing the options to be non-redundant in the economy. This finding induces several unexplored implications of delivery risk for futures risk premium and the equilibrium of futures market, in which futures options are available. The implications, which are examined in this thesis, include.

1. Although the effect of delivery risk on futures contract and the hedging role of futures options have been investigated at an individual level, the effect at the futures market equilibrium level remain unexplored. Examining the

[12] The literature on the importance of delivery specifications in the pricing of futures contracts includes Garbade and Silber (1983), Gay and Manaster (1984, 1986), Kamara (1990), among others.

delivery risk effect at equilibrium is necessary to evaluate whether the proper analyses of the futures market equilibrium need to account for the availability of futures options.

2. If the presence of delivery risk gives rise to a hedging role of the futures option in the equilibrium of futures market, there is a need to re-examine how this phenomenon changes the equilibrium of futures price, demand, and risk premium.

3. A general conclusion drawing from research on delivery risk indicates that failure to incorporate delivery risk in futures pricing models lead to incorrect inferences about equilibrium futures pricing[13]. The existing models of futures risk premiums abstract from the presence of delivery risk and, therefore, regard futures options as redundant derivatives. If delivery risk induces a non-redundant role of futures option in the futures market equilibrium, at least two analyses are required. The first analysis is to identify the exact nature of futures pricing relation when the non-redundant futures options are present in the futures market equilibrium. The other is to examine whether or not the variation in futures options helps to explain the variation in futures risk premium.

[13] This conclusion was addressed in Chance and Hemler (1993). Kamara (1990) found that the market inefficiency disappeared when the equilibrium pricing model allows for delivery options.

8. Summary and Conclusions

Futures risk premium, or market price for the risk transfer, is known as the deviation of futures price from the expected spot price. The price deviation attributes to the risk aversion of futures traders and the assumption of market perfection.

This study reviews three classes of models of futures risk premium. A class of the models is explained by one priced factor, which is either hedging pressure or systematic risk. Keynes (1930) initially viewed that hedging pressure led to futures risk premium because hedgers would need to pay futures risk premium to speculators for bearing the transferred price risks. On the other hand, Hardy (1940) attested no risk premium, on average, for futures trading because the large number of outside speculators is available to bear risk. Although these models have been examined the issues, the Keynes-Hardy controversy remains unresolved.

Other one-factor models assume the claims on hedgers' assets are freely marketable (market perfection) so that futures risk premium depends only on systematic risk related to the aggregate economic variables. Empirical evidence on these models is mixed, however. The mixed evidence is due partly to the dissimilar proxies of systematic risk, different testing periods and various futures markets. The more critical justification of these models is based on the underlying assumptions, e.g., the integration of futures and asset markets and the time variation of the asset betas. This incongruous evidence causes the validation of the equilibrium pricing models to be an unsettled and interesting issue.

The second class is the two-factor models generalize the role of hedging into the equilibrium-pricing framework. This yields the two-factor models integrating systematic risk and hedging pressure.

The last class is the multifactor models, which are APT models and the model of DeRoon, Nijman and Veld (2000). While the evidence of APT model is inconclusive, DeRoon et al. model is empirically confirmed. Nevertheless the multifactor models are less preferable unless the additional factors offer a significant improvement in explaining futures risk premiums.

Collectively, the literature on modeling futures risk premiums is usually based on two types of risk factors: systematic risk and hedging pressure (nonmarketable risk). Recent empirical evidence generally supports the models integrating both systematic risk and hedging pressure (e.g. Hirshleifer (1988)). However, the models are still far from perfectly explaining futures risk premiums and abstract from the presence of multiple delivery specification in the market practice.

Apart from extensive evidence on the importance of delivery specifications in the pricing of futures contracts, the hedging role of futures options arising from the presence of delivery risk, as shown by Lien and Wong

(2002), should affect the analysis of futures risk premium. This creates the following unexplored issues.

Does the role of futures options in the presence of delivery risk still remain in the equilibrium of futures market? Do the non-redundant futures options (if it exists) alter the equilibrium price, demand and risk premium? What is the exact nature of futures pricing relation when the non-redundant futures options are present in the futures market equilibrium? Does the variation in futures options explain the variation in futures risk premium? All mentioned issues are addressed in the following chapters of this dissertation.

Table 1 One-factor Models of Futures Risk Premium

General Theoretical models	Empirical models
Unified model (3) $$E_t[r_{t+n}] = \sum_{j=1}^{k} \beta_{j,t} E_t[\gamma_{j,t+n}]$$ $\beta_{j,t}$ is the conditional sensitivity to the jth determinant. $E_t[\gamma_{j,t+n}]$ is the expected jth determinant in effect at time $t+n$.	Unified model (5) $$r_{t+n} = \alpha_{t+n} + \sum_{j=1}^{k} \hat{\beta}_{j,t} \gamma_{j,t+n} + \varepsilon_{t+n}$$ $\hat{\beta}_{j,t}$ is the estimated conditional slope coefficient to the jth determinant. $\gamma_{j,t+n}$ is the realized jth determinant at time $t+n$.
Model of Keynes (1930), Hicks (1939), Cootner(1960) $$E_t[r_{t+1}] = \beta\gamma_t$$ The model is analogous to (3) given n=1, k=1 and constant β. Factor (γ_t) is hedging pressure.	Empirical model: $$r_{t+1} = \alpha + \hat{\beta}\gamma_t + \varepsilon_{t+1}$$ Empirical studies include DeRoon Nijman and Veld (2000) using net hedging positions in futures as the proxy of the factor.
CAPM: Dusak (1973) $$E_t[r_{t+1}] = \beta E_t[\gamma_{t+1}]$$ The model is parallel to (3) given n =1, k =1 and constant β. The factor, γ_{t+1}, is excess market return over risk-free return.	Empirical model: $$r_{t+1} = \alpha + \hat{\beta}\gamma_{t+1} + \varepsilon_{t+1}$$ Empirical studies include Dusak (1973), Bodie and Rosansky (1980) using S&P 500 stock index as the market proxy.
Consumption CAPM: Breeden (1980) $$E_t[r_{t+1}] = \beta E_t[\gamma_{t+1}]$$ The model is parallel to (3) given n =1, k =1 and constant β. γ_{t+1} is an aggregate consumption risk premium .	Empirical model: $$r_{t+1} = \alpha + \hat{\beta}\gamma_{t+1} + \varepsilon_{t+1}$$ Empirical studies include Breeden (1980) using forecasts for the growth rate of aggregate real consumption from U.S.Conference Board's Economic Forum.

Table 2 Two-factor Models of Futures Risk Premium

Theoretical model	Empirical model
Unified model (3) $$E_t[r_{t+n}] = \sum_{j=1}^{k} \beta_{j,t} E_t[\gamma_{j,t+n}]$$ $\beta_{j,t}$ is the conditional sensitivity to the *j*th determinant. $E_t[\gamma_{j,t+n}]$ is the expected *j*th determinant in effect at time $t+n$.	Unified model (5) $$r_{t+n} = \alpha_{t+n} + \sum_{j=1}^{k} \hat{\beta}_{j,t} \gamma_{j,t+n} + \varepsilon_{t+n}$$ $\hat{\beta}_{j,t}$ is the estimated conditional slope coefficient to the *j*th determinant. $\gamma_{j,t+n}$ is the realized *j*th determinant at time $t+n$.
A combined-role model of systematic risk and hedging pressure: $$E_t[r_{t+1}] = \beta_1 E_t[\gamma_{1,t+1}] + \beta_2 E_t[\gamma_{2,t+1}]$$ The model is corresponding to (3) given n=1, k =2, constant betas. Factors: efficient portfolio return and hedging pressure.	Empirical model: $$r_{t+1} = \alpha + \hat{\beta}_1 \gamma_{1,t+1} + \hat{\beta}_2 \gamma_{2,t+1} + \varepsilon_{t+1}$$ Empirical studies include Carter Rausser and Schmidt (1983) using following proxies 1. return on the equal-weighted index of the S&P 500 and the Dow Jones Commodity Futures. 2. net positions of speculators reported by Commodity Futures Trading Commission (CFTC).
Hirshleifer (1988) model $$E_t[r_{t+1}] = \beta_{1,t} E_t[\gamma_{1,t+1}] + \beta_{2,t} h_t \gamma_{2,t}$$ The model corresponds to (3), n=1, k =2. Factors: stock market return and standard deviation of residuals conditional on hedging positions in futures contracts.	Empirical model: $$r_{t+1} = \alpha_t + \hat{\beta}_{1,t} \gamma_{1,t+1} + \hat{\beta}_{2,t} h_t \gamma_{2,t} + \varepsilon_{t+1}$$ Bessembinder (1992) uses two proxies: return on CRSP value-weighted equity index, standard deviation of residuals from the regression of futures returns on the index returns. The sign of this standard deviation is conditional on net hedging positions in futures.

Table 3 Multifactor Models of Futures Risk Premium

Theoretical model	Empirical model
Unified model (3) $$E_t[r_{t+n}] = \sum_{j=1}^{k} \beta_{j,t} E_t[\gamma_{j,t+n}]$$ $\beta_{j,t}$ is the conditional sensitivity to the jth determinant. $E_t[\gamma_{j,t+n}]$ is the expected jth determinant in effect at time $t+n$.	Unified model (5) $$r_{t+n} = \alpha_{t+n} + \sum_{j=1}^{k} \hat{\beta}_{j,t} \gamma_{j,t+n} + \varepsilon_{t+n}$$ $\hat{\beta}_{j,t}$ is the estimated conditional slope coefficient to the jth determinant. $\gamma_{j,t+n}$ is the realized jth determinant at time $t+n$.
APT model by Ross (1976) $$E_t[r_{t+1}] = \sum_{j=1}^{k} \beta_{j,t} E_t[\gamma_{j,t+1}]$$ The model is corresponding to (3) given n=1, the number of factors γ_t is k.	Empirical model: $$r_{t+1} = \alpha_{t+1} + \sum_{j=1}^{k} \hat{\beta}_{j,t} \gamma_{j,t+1} + \varepsilon_{t+1}$$ Empirical studies include Bessembinder (1992, 1993) using seven factors: return on CRSP value-weighted index; change in expected inflation; unexpected inflation; change in 3-month Treasury yield; change in spreads of 30-year and 3-month Treasury yields; change in the default risk premiums (spreads of yield on U.S. corporate BAA bond and 30-year U.S. Treasury yield); unexpected change in industrial production.
DeRoon, Nijman and Veld (2000) model $$E_t[r_{t+1}] = \beta_1 E_t[\gamma_{1,t+1}] + \sum_{j=2}^{k} \beta_j \gamma_{j,t}$$ The model is corresponding to (3) given n=1, the k factors are stock market return and $k-1$ hedging pressure variables in the related futures markets.	Empirical model: $$r_{t+1} = \alpha + \hat{\beta}_1 \gamma_{1,t+1} + \sum_{j=2}^{k} \hat{\beta}_j \gamma_{j,t} + \varepsilon_{t+1}$$ DeRoon, Nijman and Veld (2000) use following proxies of factors: 1. return on S&P500 index 2. hedging pressure variables in the related futures markets.

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CHAPTHER 3

DELIVERY RISK AND HEDGING ROLE OF FUTURES OPTIONS IN THE FUTURES MARKET EQUILIBRIUM

This study analyzes the equilibrium demand for futures contracts in the presence of delivery risk and in the availability of futures options. We show that the delivery risk changes the net hedging demand for futures and induces the net hedging demand for futures options. The analysis of the futures equilibrium, in which futures options are present, shows that the equilibrium futures price is partly explained by the payoff of futures options. Collectively, these results indicate that a role of futures options in explaining futures risk premiums.

1. Introduction

Most futures contracts allow the sellers to deliver any of the deliverable grades of the underlying asset at any designated locations. Although the grades are accompanied by a schedule of discounts and premiums allowable for the delivery of lesser or greater quality than the par-delivery grade, the realized prices of these grades can be significantly different following adjustment for these discounts or premiums. Given these different prices, sellers will choose the grade with the cheapest delivery cost. Consequently, the futures price on the delivery day converges to the spot price of the cheapest-to-deliver grade, rather than the spot price of the par-delivery grade. The fact that the cheapest-to-deliver grade is uncertain until it is realized on the delivery day causes futures traders to face delivery risk in addition to the underlying price risk.

Despite the importance of delivery risk in the hedging with futures contracts as extensively documented in the literature[14], a question concerning the effect of delivery risk on the futures market equilibrium remains largely unexplored. If the role of futures options is present in the equilibrium, it would be useful to examine how the non-redundant futures options alter the futures price and demand. These unexplored issues have important implications for the futures pricing relation and the hedging demands for futures contracts. The analysis of the implications would benefit not only futures traders but also regulators and policy makers.

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[14] This literature includes Kamara and Siegel (1987), Lien (1988; 1991), Viswanathan and Chatterjee (1992), Lien and Wong (2002), among others.

In an attempt to answer this question, this study analyzes the equilibrium demand for futures contracts in the presence of delivery risk and the availability of futures options. The main result indicates that the delivery risk affects the net hedging demand for futures and induces the net hedging demand for futures options. The result implies that the futures option, as a non-redundant hedging instrument, affects the equilibrium of its underlying market.

We further analyze the futures market equilibrium in which futures options are present. The results show that the equilibrium futures price and demand are partly explained by the premium and the net hedging demand of futures options. The relation between futures option payoffs and futures payoffs indicates a role of futures options in explaining futures risk premiums.

The effect of the futures option on the futures contract is consistent with Black's argument (1975) that transactions in derivative markets may contain information of impending changes in the markets of their underling securities. This argument has been investigated extensively in equity and equity option markets (see, for example, Manaster and Rendleman (1982), Stephan and Whaley (1990), Easley, O'Hara and Srinivas (1998), Vanden (2004)).

This study is contrasted with the existing literature on the importance of delivery specifications in futures markets. The literature focuses on the analysis of hedging strategies with only futures contract in the presence of delivery risk (Kamara and Siegel (1987), Lien (1988; 1991), and Viswanathan and Chatterjee (1992)). The analyses in this study, however, suggest that the hedgers optimally hold both futures and futures options. This result is similar to that of Lien and

Wong (2002), which highlights the hedging role for options on futures in the presence of delivery risk. Nevertheless, the emphases of the two studies are different. Lien and Wong focus on the optimal hedging strategy for an individual whereas this study concentrates on the effect of delivery risk on the futures market equilibrium.

The remainder of this study is organized as follows. Section 2 describes the economic setting and a succinct derivation of the futures market equilibrium while the full derivation is provided in Appendix. Section 3 analyzes the effects of delivery risk on the net hedging demands for futures and futures options. Section 4 provides a graphical illustration of the hedging role of futures options in the presence of delivery risk. Section 5 analyzes the equilibrium futures price and risk premium in the presence of delivery risk and the availability of futures options. Section 6 provides concluding and remarks.

2. Futures Market Equilibrium in the Presence of Delivery Risk

2.1. Economic Setting

Consider a one-period (two dates, 0 and 1) mean-variance model with two groups of risk-averse investors: hedgers and outside investors, in the competitive markets of futures and options on futures. [15] All investors have homogenous

[15] The two-date mean variance model is widely used in futures pricing literature (e.g., Stoll (1979), Hirshleifer (1988)). The two-date model abstracts from the daily settlement characteristic of futures contracts, which can induce the difference between futures and forward prices given stochastic interest rates. Nevertheless, a number of studies have concluded that the forward and futures prices are not significantly different, e.g., Cornell

beliefs about the distributions of all variables. There are fixed setup costs for trading derivatives. Trading derivatives is assumed to be costly in that the fixed setup cost of the trading deters some outside investors from participation in the derivative markets. This assumption is sensible because the number of futures traders is small relative to those in other markets, e.g., stock investors. The setup cost may be understood as the time investment required for learning about derivative markets, i.e., the trading mechanism, principle of futures pricing, and the factors influencing supply-and-demand conditions.

The investors decide now to take positions, in these markets, that affect their wealths at date 1. Each hedger is defined to possess a business generating uncertain revenue ($\tilde{g}_h$) that is linearly related to the date 1 price of the par-delivery grade underlying the futures contract ($\tilde{p}_A$). Market imperfections (that lead to, e.g., adverse selection, moral hazard) cause hedgers to feel reluctant or to be unable to trade the equity shares in their businesses through the equity market. Thus, hedgers are motivated to hedge their risky revenues with derivatives, i.e., trading infinitely divisible futures contracts and call options on the futures.[16] On the other hand, other investors, whose wealths are independent of the price risk underlying the futures contract, will trade derivatives only if the trading does not make their expected

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and Reinganum (1981) on foreign exchange, MacKinlay and Ramaswamy (1988) on stock indices and Grinblatt and Jegadeesh (1996) on Eurodollar contracts.

The analysis in this study is conducted in the context of futures and futures option markets. A more general analysis that includes the stock market is available upon request. Although the results of the analysis in context of the three markets are more complex than the results in this study, the conclusions inferred from both analyses are same.

[16] Many options on futures contracts are available and actively traded in U.S. derivative exchanges. Without loss of generality, the analysis focuses on the hedging role of call option on futures. The analysis on the role of put futures options can be conducted in the similar way.

utilities lower than not trading at all. Hereafter, investors who trade futures and futures options are called speculators and those who trade futures options only are called outside investors. [17] Each investor maximizes the expected utility of the wealth at date 1.[18]

$$E(\tilde{U}_i) = E(\tilde{W}_i) - \frac{\lambda_i}{2}\,\mathrm{var}(\tilde{W}_i) \qquad (1)$$

where subscript i indicates investor type, i.e., h, s, r: h for a hedger, s for a speculator, r for an outside investor who trades only futures options. $E(.)$ and var(.) denote expectation and variance operators respectively; A tilde (~) indicates a random variable; $\tilde{U}_i$ denotes utility at date 1; $\tilde{W}_i$ denotes wealth at date 1; λ_i denotes coefficient of absolute risk aversion. The wealth functions of each investor type at date 1 are described as follows.

$$\tilde{W}_h = a_h - t + \tilde{g}_h + [\tilde{f} - f]y_h + \tilde{R}_o z_h \qquad (2h)$$

$$\tilde{W}_s = a_s - t + [\tilde{f} - f]y_s + \tilde{R}_o z_s \qquad (2s)$$

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[17] The setting that speculators trade both futures and futures options while some investors trade only futures options may not be unrealistic. As documented by Daigler and Wiley (1998; 1999) and Kodres and Pritsker (1997), the clearing members, who are the major speculators, trade futures and other related derivatives. Meanwhile, a number of investors may trade only futures options because the options tend to entail lower transaction costs than futures due partly to the leverage effect. Moreover, futures options are settled in cash and the exercise of the options does not usually lead to the delivery of the commodity underlying the derivatives.

[18] This expected utility function is applied under the assumptions of the exponential preference with the constant absolute risk aversion and the multivariate normality of the investor's terminal wealth. See Appendix for details.

$$\widetilde{W}_r = a_r - t` + \widetilde{R}_o z_r \tag{2r}$$

The notations in Equations (2h), (2s), and (2r) are described as follows. $\widetilde{g}_h$ denotes hedger's revenue, which is linear in the date 1 price of the par-delivery grade underlying the futures contract. a_i is an initial wealth of investor i (units of consumption good at date 1). y_i, and z_i are the positions of futures contracts, and call options on futures respectively. A positive (negative) position indicates a long (short) position. t is a fixed setup cost for trading derivatives: both futures and futures options. $t`$ is a fixed setup cost for trading futures options, which is significantly lower than that for futures. f is current futures price. $\widetilde{f}$ is the date 1 settlement price of futures contract. $\widetilde{R}_o (= \widetilde{o} - o)$ is payoff of a call option on futures contract buying at date 0 and receiving the payoff (per dollar) at date 1, where o is unit price of the futures option at date 0; $\widetilde{o}\,(= \max(\widetilde{f} - k, 0))$ is unit price of the option at date 1; k = the option exercise price.

Some further definitions follow: Futures risk premium (Π) defined as $\Pi \equiv E[\widetilde{f} - f]$ and futures payoff $\widetilde{\Pi} = \widetilde{f} - f$. Due to delivery risk, futures risk premium defined here is slightly different from the traditional one, which is the difference between current futures price and expected spot price of asset underlying futures contract at maturity. In the absence of delivery risk, the expected futures price converges to the expected spot price on the delivery date. Thus the proposed definition of futures risk premium coincides with the traditional risk premium.

For the analysis of delivery risk in this setting, the sellers of futures contract can deliver any of two deliverable grades, named grade A (the par-delivery grade) and grade B. Logically, the sellers will deliver the cheaper grade; thus the date 1 futures price equals the minimum of the delivery-adjusted prices of grade A and grade B ($\tilde{f} = \min(\tilde{P}_A, \tilde{P}_B)$). The uncertainty of which grade will be cheaper to deliver is referred to as delivery risk.

2.2. Equilibrium demand for futures contracts

The concavity of utility function with respect to futures position (y) and futures option position (z) ensures that the optimal positions for an investor satisfy the following first order conditions.[19]

$$\Pi = \lambda \operatorname{cov}(\tilde{\Pi}, \tilde{g} + y\tilde{\Pi} + z\tilde{R}_o) \qquad (3)$$

$$E(\tilde{R}_o) = \lambda \operatorname{cov}(\tilde{R}_o, \tilde{g} + y\tilde{\Pi} + z\tilde{R}_o) \qquad (4)$$

Since covariance (cov) is a linear operator, the first order conditions still hold when the investor's wealth is understood to be the average wealth of all futures traders[20]. Equation (3) states that futures risk premium is proportional to the covariance of futures payoff with the average wealth of futures traders.

[19] The detailed derivation is in Appendix. In Equations (3) and (4), the subscript i is dropped because the first order conditions are general for all investor types. The first order conditions (3)-(4) apply directly to a hedger. The conditions apply to a speculator with a restriction of $\tilde{g} \equiv 0$, to an outside investor, who trades only futures options with restrictions of $\tilde{g} \equiv 0$ and $y = 0$.

[20] The covariance of future payoff with change in wealth and the covariance with the level of wealth are identical since the initial wealth is known.

Equation (4) shows that expected return on futures option depends on the covariance of option return with the average wealth.

The optimal futures position for a hedger results from the first order condition in Equation (3)

$$y_h = \frac{\Pi / \lambda_h}{\mathrm{var}(\widetilde{\Pi})} - \frac{\mathrm{cov}(\widetilde{\Pi}, \widetilde{g}_h + z_h \widetilde{R}_o)}{\mathrm{var}(\widetilde{\Pi})} \tag{5}$$

For a speculator, who has no revenue related to the price risk of the asset underlying futures contract, the optimal futures position is

$$y_s = \frac{\Pi / \lambda_s}{\mathrm{var}(\widetilde{\Pi})} - \frac{\mathrm{cov}(\widetilde{\Pi}, z_s \widetilde{R}_o)}{\mathrm{var}(\widetilde{\Pi})} \tag{6}$$

Both optimal futures positions in Equations (5)-(6) can be divided into the speculative component, which is the first term on the right-hand side of each equation and the hedging component, which is the remaining term of each equation respectively. The speculative component decreases with risk aversion, λ_i , and with the variance of futures and increases with the net effect of expected change in the futures prices. The hedging component is negatively proportional to the covariance of the hedger's revenue with futures payoff and also decreases with the variance of futures payoff.

Aggregating the optimal futures positions for all hedgers according to Equation (5) and those for all speculators according to Equation (6) respectively yields

$$y_H = \frac{\Pi / \lambda_H}{\mathrm{var}(\widetilde{\Pi})} - \frac{\mathrm{cov}(\widetilde{\Pi}, \widetilde{G})}{\mathrm{var}(\widetilde{\Pi})} - \frac{H}{H+S} Z \frac{\mathrm{cov}(\widetilde{\Pi}, \widetilde{R}_o)}{\mathrm{var}(\widetilde{\Pi})} \tag{7}$$

$$y_S = \frac{\Pi / \lambda_S}{\mathrm{var}(\widetilde{\Pi})} - \frac{S}{H+S} Z \frac{\mathrm{cov}(\widetilde{\Pi}, \widetilde{R}_o)}{\mathrm{var}(\widetilde{\Pi})} \tag{8}$$

where $\frac{1}{\lambda_H} = \sum_{h=1}^{H} \frac{1}{\lambda_h}$; $\frac{1}{\lambda_S} = \sum_{s=1}^{S} \frac{1}{\lambda_s}$; $\widetilde{G} = \sum_{h=1}^{H} \widetilde{g}_h$; $Z = \sum_{h=1}^{H} z_h + \sum_{s=1}^{S} z_s$; $\frac{S}{H+S} Z = \sum_{s=1}^{S} z_s$; $\frac{H}{H+S} Z = \sum_{h=1}^{H} z_h$; H and S denote the total numbers of hedgers and speculators in the futures market, respectively.

The equilibrium, where the net aggregate demand of futures contracts is equal to zero, is established as follows. Summing the aggregate positions in Equations (7) and (8), equating the result to zero.

3. Effects of Delivery Risk: Comparative Statics

This section describes how futures options become a nonredundant hedging instrument in the presence of delivery risk. The section first presents the economy with the absence of delivery risk. Then, the effects of delivery risk on the hedging demands for futures and futures options are analyzed using comparative statics.

3.1. The absence of delivery risk

In the absence of delivery risk, futures price converges to the spot price of the par-delivery grade $(\tilde{p}_A)$ on the delivery date. The hedger's revenue $(\tilde{g}_h)$ can be optimally hedged with a futures position (y) by an amount negatively proportional to $\text{cov}(\tilde{\Pi}, \tilde{g}_h)/\text{var}(\tilde{\Pi})$, which is the covariance of futures payoff with the hedger's revenue divided by the variance of futures payoff. The basic concept is straightforward. Given no delivery risk, the hedger's revenue is a linear function of $\tilde{p}_A$, so hedging this risky revenue with a futures contract, whose payoff is also linear in $\tilde{p}_A$, strictly dominates hedging with a futures option, whose payoff is piecewise linear in $\tilde{p}_A$. Thus, there is no hedging demand of futures options in the absence of delivery risk.

3.2. The presence of delivery risk

To trace how the hedging demand for the futures options arises in the presence of delivery risk, we adopt a methodology of Lien and Wong (2002). A joint-probability distribution of $\tilde{p}_A$ and $\tilde{p}_B$ is assumed. Let ϕ be a probability that the price of the par-delivery grade A will be higher than the delivery-adjusted price of the deliverable grade B on the delivery day. This can be written in symbolic forms as $\tilde{p}_B = \tilde{p}_A - \delta$ with probability ϕ and $\tilde{p}_B > \tilde{p}_A$ with probability 1- ϕ, where δ is a positive constant.

Let Y denote y_H or $-y_S$. Let $Z\left(=\sum_{h=1}^{H} z_h + \sum_{s=1}^{S} z_s\right)$ denote the net positions of futures options held by all futures traders. The demand Equations (7) and (8) can be written in the futures market equilibrium as

$$L_H = \frac{\Pi/\lambda_H}{\mathrm{var}(\tilde{\Pi})} - \frac{\mathrm{cov}(\tilde{\Pi},\tilde{G})}{\mathrm{var}(\tilde{\Pi})} - \frac{H}{H+S} Z \frac{\mathrm{cov}(\tilde{\Pi},\tilde{R}_o)}{\mathrm{var}(\tilde{\Pi})} - Y = 0 \tag{9}$$

$$L_S = \frac{\Pi/\lambda_S}{\mathrm{var}(\tilde{\Pi})} - \frac{S}{H+S} Z \frac{\mathrm{cov}(\tilde{\Pi},\tilde{R}_o)}{\mathrm{var}(\tilde{\Pi})} + Y = 0 \tag{10}$$

The condition of the futures market equilibrium is imposed by combining Equations (9) and (10). In the futures equilibrium, the net positions of futures options held by all futures traders, Z, is

$$Z = \frac{(H+S)(\Pi - \theta\,\mathrm{cov}(\tilde{\Pi},\tilde{G}))}{\theta\,\mathrm{cov}(\tilde{\Pi},\tilde{R}_o)} \tag{11}$$

where $\theta\left(=\frac{(\lambda_H\lambda_S)/(\lambda_H+\lambda_S)}{H+S}\right)$ denotes the average coefficient of absolute risk aversion.

Given the joint-probability distribution function of $\tilde{p}_A$ and $\tilde{p}_B$, the system equations (9) and (10), futures risk premium, Π, expected option payoff, $E(\tilde{R}_o)$, their covariance, and variance terms can be expressed as follows.

$$L_H = 0 = \phi\left\{\frac{E(\tilde{p}_A) - f - \delta}{\lambda_H \operatorname{var}(\tilde{p}_A - f)} - \frac{\operatorname{cov}(\tilde{p}_A - f, \tilde{G})}{\operatorname{var}(\tilde{p}_A - f)} - \frac{H}{H+S} Z \frac{\operatorname{cov}(\tilde{p}_A - f, \max(\tilde{p}_A - k - \delta, 0))}{\operatorname{var}(\tilde{p}_A - f)} - Y\right\}$$
$$+ (1-\phi)\left\{\frac{E(\tilde{p}_A) - f}{\lambda_H \operatorname{var}(\tilde{p}_A - f)} - \frac{\operatorname{cov}(\tilde{p}_A - f, \tilde{G})}{\operatorname{var}(\tilde{p}_A - f)} - \frac{H}{H+S} Z \frac{\operatorname{cov}(\tilde{p}_A - f, \max(\tilde{p}_A - k, 0))}{\operatorname{var}(\tilde{p}_A - f)} - Y\right\}$$

(12)

$$L_S = 0 = \phi\left\{\frac{E(\tilde{p}_A) - f - \delta}{\lambda_S \operatorname{var}(\tilde{p}_A - f)} - \frac{\operatorname{cov}(\tilde{p}_A - f, \tilde{G})}{\operatorname{var}(\tilde{p}_A - f)} - \frac{S}{H+S} Z \frac{\operatorname{cov}(\tilde{p}_A - f, \max(\tilde{p}_A - k - \delta, 0))}{\operatorname{var}(\tilde{p}_A - f)} - Y\right\}$$
$$+ (1-\phi)\left\{\frac{E(\tilde{p}_A) - f}{\lambda_S \operatorname{var}(\tilde{p}_A - f)} - \frac{\operatorname{cov}(\tilde{p}_A - f, \tilde{G})}{\operatorname{var}(\tilde{p}_A - f)} - \frac{S}{H+S} Z \frac{\operatorname{cov}(\tilde{p}_A - f, \max(\tilde{p}_A - k, 0))}{\operatorname{var}(\tilde{p}_A - f)} - Y\right\}$$

(13)

The presence of delivery risk implies positive probability ϕ; thus the following comparative statics with respect to ϕ ($dY^*/d\phi$ and $dZ^*/d\phi$) indicate how delivery risk affects the net hedging demand for futures contracts (Y) and the net hedging demand for futures options (Z). Totally differentiating the demand equations with respect to ϕ, and evaluating at $\phi = 0$ give[21]

$$\frac{dY^*}{d\phi} = \frac{\delta}{\operatorname{var}(\tilde{p}_A - f)}\left(\frac{\lambda_H - \lambda_S}{(H+S)\lambda_H \lambda_S}\right) \quad (14)$$

$$\frac{dZ^*}{d\phi} = \frac{\delta(\lambda_H + \lambda_S)[(H+S) - 1]}{\lambda_H \lambda_S \operatorname{cov}(\tilde{p}_A - f, \max(\tilde{p}_A - k, 0))} \quad (15)$$

[21] The derivation is provided in Appendix.

Note: $\frac{dZ^*}{d\phi} > 0$ because all of δ, λ_H, λ_S, H, S and $\mathrm{cov}(\tilde{p}_A - f, \max(\tilde{p}_A - k, 0))$ are positive and $(H + S) > 1$. Meanwhile, the sign of $\frac{dY^*}{d\phi}$ depends on the amounts of λ_H and λ_S as follows.

1) If the aggregate risk aversion of hedgers is greater than that of speculators ($\lambda_H > \lambda_S$), then $\frac{dY^*}{d\phi} > 0$.
2) If the aggregate risk aversion of hedgers is equal to that of speculators ($\lambda_H = \lambda_S$), then $\frac{dY^*}{d\phi} = 0$.
3) If the aggregate risk aversion of hedgers is less than that of speculators ($\lambda_H < \lambda_S$), then $\frac{dY^*}{d\phi} < 0$.

As mentioned above, H and S are the total numbers of hedgers and speculators in the futures market, respectively. λ_H and λ_S are the aggregate risk aversions of hedgers and speculators, respectively.

In a further investigation, the net hedging demand for futures contracts ($dY^*/d\phi$) in Equation (14) can be written in terms of $dZ^*/d\phi$ as

$$\frac{dY^*}{d\phi} = \frac{dZ^*}{d\phi} + \alpha \qquad (16)$$

where

$$\alpha = \frac{\delta\,\mathrm{cov}(\tilde{p}_A - f, \max(\tilde{p}_A - k, 0))(\lambda_H - \lambda_S) - \delta\,\mathrm{var}(\tilde{p}_A - f)(\lambda_H + \lambda_S)[(H+S)^2 - (H+S)]}{(H+S)\lambda_H \lambda_S\,\mathrm{var}(\tilde{p}_A - f)\,\mathrm{cov}(\tilde{p}_A - f, \max(\tilde{p}_A - k, 0))}$$

Equation (16) indicates two components of the delivery risk effect on the net hedging demand for futures ($dY/d\phi$). The first component is the net hedging demand of futures options ($dZ/d\phi$), which is positive. This component implies that the availability of the futures options mitigates negative effect of delivery risk on the futures demand. The sign of the second component (α) depend on whether the amount of (λ_H - λ_S) is greater than (equal to, or less than) zero. α reflects a response of the futures demand to the presence of delivery risk when futures options are not available. Collectively, Equation (16) shows that the presence of delivery risk affects the net hedging demand for futures options and induces the net demand of all futures traders for futures options. In an aspect of hedging role in the presence of delivery risk, futures option should be viewed as a complement to the futures contract, rather than a substitute for the contract. The following proposition is established from the aforementioned results.

Proposition 1. Suppose that the prices of deliverable grades underlying futures contracts are jointly distributed. Delivery risk causes the net hedging demand for futures contracts to increase (unchanged, decrease) depending on whether the aggregate risk aversion of hedgers (λ_H) is greater than (equal to, less than) the aggregate risk aversion of speculators (λ_S), respectively.

As indicated by the comparative statics, the hedging demand for futures options arises in the presence of delivery risk. The basic rationale is that delivery risk causes futures payoff to be nonlinear in its underlying value. The hedgers cannot exactly hedge their uncertain revenues with futures contracts. The remaining exposure, however, can be effectively hedged using futures options because of two striking option characteristics: the curvature of option payoff and the leverage. Since the option value is piecewise-linear in the underlying value, the curvature of option payoff can be used to tailor the exposure of the hedgers' payoffs, thereby suiting their needs. Moreover, the degree of leverage causes the additional hedging cost due to futures options to be relatively marginal. The leverage arises from the fact that an option allows investors to assume much of the risk of the option's underlying asset with a relatively small investment.

4. Hedging Role of Futures Options: Graphical Illustration

This section illustrates the effects of delivery risk on the hedging demand for futures and futures options through a graphical analysis. The following analysis accounts for the payoffs of derivatives and the underlying assets to an investor at maturity. For simplicity, the cost of futures option is not included in the calculation. Each of the following figures graphs the derivative and underlying payoffs on the vertical axis against the terminal price of the par-delivery grade ($\widetilde{p}_A$) on the horizontal axis.

Figure 1 represents futures payoff ($\widetilde{f} - f$) in the absence of delivery risk. The futures payoff is linear in $\widetilde{p}_A$ on the delivery day; thus the hedger's revenue, which also is linear in $\widetilde{p}_A$, can be optimally hedged by using only futures contracts.

For the analysis of delivery risk, the sellers of futures contract are allowed to deliver either the par-delivery grade, named grade A, or the deliverable grade B. Figure 2 describes the geometry of the deliverable grade payoffs. The payoff of deliverable grade A is obviously linear in $\tilde{p}_A$ with an angle of 45^{o} to the horizontal axis. The delivery-date prices of grade A and grade B are positively correlated since they are underlying the same futures contract. For analytic simplicity, the payoff of deliverable grade B is assumed to be linear in $\tilde{p}_A$ with a specific angle, e.g., 22.5^{o}.[22]

Figure 3 portrays the futures payoff ($\tilde{f} - f$) in the presence of delivery risk. Futures payoff (the solid line) tracks the payoff of the cheaper-to-deliver grade. The different sensitivities of the prices of deliverable grades to a change in $\tilde{p}_A$ cause the deliverable grade A to be cheaper to deliver when $\tilde{p}_A$ is lower than k and the deliverable grade B to be cheaper when $\tilde{p}_A$ is higher than k. k indicates $\tilde{p}_A$ at the crossover point of the deliverable-grade payoffs. This figure demonstrates that, in the presence of delivery risk, the effectiveness of hedging with futures contract decreases because the futures payoff is not linear in $\tilde{p}_A$.

Figure 4 shows the payoff of a long position in call futures option with the exercise price k. The call option payoff ($\tilde{c}$) is equal to max($\tilde{f} - k, 0$) because the call futures option is exercised if futures price is greater than k. Similarly, Figure 5 depicts the payoff of a long position in put futures option with exercise price k.

[22] The conclusion of the analysis is unaffected by the degree of the angle. Changing the degree of the angle affects the ratio of futures to futures options and the option types (e.g., call, put) which are used to hedge against the hedger's revenue.

Since a holder of the put option will exercise only if futures price is lower than k, the put option payoff ($\tilde{p}'$) is equal to max($k - \tilde{f}$,0).

Figure 6 represents the payoff of long positions in futures and futures options, with a ratio of one to one, in the presence of delivery risk. The payoff ($\tilde{f} - f + \tilde{c}$) is linear in $\tilde{p}_A$ with an angle of 45° to the horizontal axis. The hedger's revenue, which is linear in $\tilde{p}_A$, can be optimally hedged by holding positions in futures and futures options. Thus, in the presence of delivery risk, using call futures option and futures contract can improve the effectiveness of hedging against the risky revenue of hedgers.

In another example, Figure 7 depicts the futures payoff ($\tilde{f}' - f'$) in the presence of delivery risk. Futures payoff (the solid line) tracks the payoff of the cheaper-to-deliver grade. The payoff of deliverable grade A is linear in $\tilde{p}_A$ with an angle of 45° to the horizontal axis. The payoff of deliverable grade B′ is assumed to be linear in $\tilde{p}_A$ with an angle of 67.5°. k' indicates $\tilde{p}_A$ at the crossover point of the deliverable-grade payoffs.

Figure 8 represents the payoff of long positions in a futures contract and a half of put futures, in the presence of delivery risk. The payoff ($\tilde{f}' - f' + 0.5\tilde{p}'$) is linear in $\tilde{p}_A$ with an angle of 45° to the horizontal axis. The hedger's revenue, which is linear in $\tilde{p}_A$, can be optimally hedged by holding positions in futures and put futures (with the exercise price of k'). Thus, in this case, using put futures option and futures contract can improve the effectiveness of hedging against the risky revenue of hedgers.

5. Futures Market Equilibrium with Futures Options

The section analyzes the futures equilibrium, in which futures options are present. The analysis is based on an assumption of at-the-money futures options. The covariance of futures payoff with futures option payoff is positive $\left(\text{cov}(\tilde{f}-f,\tilde{o}-o)>0\right)$ because of the positive correlation between option and underlying payoffs when the call option is in the money; and no correlation between option and underlying payoffs when the call option is out of the money.

The equilibrium, where the net aggregate demand of futures contracts is equal to zero, is established as follows. Summing the aggregate positions in Equations (7) and (8), equating the result to zero and rearranging the equality give the equilibrium futures price, and risk premium as follows.

Futures price:

$$f^{*}=E(\tilde{f})-\theta\,\text{cov}(\tilde{f}-f,\tilde{G})-\text{cov}\left(\tilde{f}-f,\frac{(E(\tilde{f})-f-\theta\,\text{cov}(\tilde{f}-f,\tilde{G}))(\tilde{o}-o)}{\text{cov}(\tilde{f}-f,\tilde{o}-o)}\right) \quad (17)$$

Futures risk premium:

$$\Pi^{*}=\theta\,\text{cov}(\tilde{f}-f,\tilde{G})+\text{cov}\left(\tilde{f}-f,\frac{(E(\tilde{f})-f-\theta\,\text{cov}(\tilde{f}-f,\tilde{G}))(\tilde{o}-o)}{\text{cov}(\tilde{f}-f,\tilde{o}-o)}\right) \quad (18)$$

where $\theta \left(= \frac{(\lambda_H \lambda_S)/(\lambda_H + \lambda_S)}{H+S} \right)$ denotes the average coefficient of absolute risk aversion of futures traders.

Equations (17) and (18) state that the equilibrium futures price deviates from the expected futures price. The deviation, defined as futures risk premium, has two components. The first is a hedging component, which reflects hedgers' incentive to hedge against underlying assets with futures contract. The hedging component is proportional to the covariance of futures payoff and the non-marketable revenues of hedgers. The component also increases with the average risk aversion of futures traders, θ. The second term reflects the relation of futures option payoff and futures risk premium. Since futures payoff is partially and positively correlated with futures option payoff, By Equation (18) participating on futures options of futures traders can substantially affect futures risk premium. The following proposition summarizes the aforementioned results.

Proposition 2. Consider the futures market equilibrium in which delivery risk and futures options are present. Then the equilibrium futures price and risk premium are partly explained by the futures option payoffs.

6. Concluding Remarks

For most futures contracts, traders face delivery risk in addition to underlying price risk. This study analyzes the equilibrium demand for futures contracts in the presence of delivery risk and the availability of futures options. It is shown that the delivery risk changes the net hedging demand for futures and generates the net hedging demand for futures options. These results indicate that the presence of delivery risk causes options on futures to become a nonredundant hedging instrument in the futures market equilibrium.

The rationale is that in the absence of delivery risk, futures price converges to the spot price of the certain asset underlying the contract (i.e., the par-delivery grade) on the delivery day. Hence, the hedgers' revenues, which are risky but linear in the spot price, can be optimally hedged by using only futures contracts, of which the gains or losses are also linear in the spot price.

The presence of delivery risk, however, causes futures price to converge to the cheapest-to-deliver price on the delivery day. Thus, hedgers, whose revenues are linearly related to the spot price of par-delivery grade, cannot precisely hedge the revenue risk using merely futures contracts, of which the payoffs are not linear in the spot price. The nonlinearity of futures payoff gives rise to some residual revenue risk of hedgers to be hedged with futures options.

This study further analyzes the relation between the nonredundant futures options and the equilibrium futures price and risk premium. The result shows that the equilibrium futures price and risk premium are also explained by the futures option payoffs. Collectively, the relation between futures option payoffs and futures risk premiums indicates a role of futures options in explaining futures risk premiums.

Nevertheless the result of relation between futures price and futures option payoff should be interpreted carefully since futures price and futures option payoff are endogenously determined. While the derivations and their discussions of section 5 appear to follow the causal chains among these variables, we should remind that any change in these endogenous variables is due to some exogenous variables, e.g., delivery risk.

Appendix

1. The Mean-Variance Model

Assume that an investor *i* has exponential utility function, $U(\widetilde{W}_i) = -e^{-\lambda_i \widetilde{W}_i}$, with a constant absolute risk aversion (λ_i) and his terminal wealth $(\widetilde{W}_i)$ is normally distributed.

Then $$EU(\widetilde{W}_i) = -E[e^{-\lambda_i \widetilde{W}_i}] = -e^{-\lambda_i [E(\widetilde{W}_i) - \frac{\lambda_i}{2}\mathrm{var}(\widetilde{W}_i)]}$$

where $U'(\widetilde{W}_i) = \lambda_i e^{-\lambda_i \widetilde{W}_i}$ and $U''(\widetilde{W}_i) = -\lambda_i^2 e^{-\lambda_i \widetilde{W}_i}$.

If the investor's wealth $(\widetilde{W}_i)$ is a function of decision variables, *y* and *z*, then

$$\arg\max_{y,z} EU(\widetilde{W}_i) = \arg\max_{y,z} E(\widetilde{W}_i) - \frac{\lambda_i}{2}\mathrm{var}(\widetilde{W}_i)$$

Thus, each investor maximizes the expected utility of his terminal wealth.

$$\max_{y,z} EU(\widetilde{W}_i) = \max_{y,z} E(\widetilde{W}_i) - \frac{\lambda_i}{2}\mathrm{var}(\widetilde{W}_i) \quad (1)$$

where the following equations describe the wealth of investor *i* = *h* (hedger), *s*(speculator) and *r* (an outside investor who trades the futures option), respectively.

$$\widetilde{W}_h = a_h - t + \widetilde{g}_h + [\widetilde{f} - f]y_h + \widetilde{R}_o z_h \quad (2h)$$

$$\widetilde{W}_s = a_s - t + [\widetilde{f} - f]y_s + \widetilde{R}_o z_s \quad (2s)$$

$$\widetilde{W}_r = a_r - t` + \widetilde{R}_o z_r \quad (2r)$$

Notations

$\tilde{g}_h$ denotes hedger's revenue which is linear in the date 1 price of the par-delivery grade (named grade A) underlying the futures contract.

a_i is an initial wealth of investor i (units of consumption good at date 1).

y_i, and z_i are the positions of futures contracts, and call options on futures respectively. A positive (negative) position indicates a long (short) position.

t is a fixed setup cost for trading derivatives: both futures and futures options.

$t`$ is a fixed setup cost for trading futures options, which is significantly lower than that for futures due partly to the leverage effect.

f is current futures price.

$\tilde{f}$ is the date 1 settlement price of futures contract, which is equal to the minimum of the delivery-adjusted prices of grade A and grade B $(\tilde{f} = \min(\tilde{P}_A, \tilde{P}_B))$.

$\tilde{R}_o (= \tilde{o} - o)$ is payoff of a call option on futures contract buying at date 0 and receiving the payoff (per dollar) at date 1, where o is unit price of the futures option at date 0; $\tilde{o} (= \max(\tilde{f} - k, 0))$ is unit price of the option at date 1; k = the option's exercise price.

Futures risk premium (Π) is defined as $\Pi \equiv E[\tilde{f} - f]$ and the percentage of risk premium as $\pi \equiv E[\tilde{f} - f] / f$; futures payoff $\tilde{\Pi} = \tilde{f} - f$; the percentage change in futures prices $\tilde{\pi} = (\tilde{f} - f) / f$.

The expected utility of each investor can be expressed as

$$E(\tilde{U}_h) = E[a_h - t + \tilde{g}_h + y_h \tilde{\Pi} + z_h \tilde{R}_o] - \frac{\lambda_h}{2} \operatorname{var}\left[a_h - t + \tilde{g}_h + y_h \tilde{\Pi} + z_h \tilde{R}_o\right] \quad \text{(A1h)}$$

$$E(\tilde{U}_s) = E[a_s - t + y_s \tilde{\Pi} + z_s \tilde{R}_o] - \frac{\lambda_s}{2} \operatorname{var}[a_s - t + y_s \tilde{\Pi} + z_s \tilde{R}_o] \quad \text{(A1s)}$$

$$E(\tilde{U}_r) = E[a_r - t` + z_r \tilde{R}_o] - \frac{\lambda_r}{2} \operatorname{var}[a_r - t` + z_r \tilde{R}_o] \quad \text{(A1r)}$$

2. Equilibrium demand for futures contracts

Derive the first order conditions (FOCs) by differentiating the expect utility in Equations (A1) with respect to futures position (*y*) and futures option position (*z*) respectively.

$$\frac{dE(\tilde{U})}{dy} = 0 = \Pi - \lambda \operatorname{cov}(\tilde{\Pi}, \tilde{g} + y\tilde{\Pi} + z\tilde{R}_o)$$

$$\frac{dE(\tilde{U})}{dz} = 0 = E(\tilde{R}_o) - \lambda \operatorname{cov}(\tilde{R}_o, \tilde{g} + y\tilde{\Pi} + z\tilde{R}_o)$$

Rearranging the derivatives gives

$$\Pi = \lambda \operatorname{cov}(\tilde{\Pi}, \tilde{g} + y\tilde{\Pi} + z\tilde{R}_o) \qquad (3)$$

$$E(\tilde{R}_o) = \lambda \operatorname{cov}(\tilde{R}_o, \tilde{g} + y\tilde{\Pi} + z\tilde{R}_o) \qquad (4)$$

Note: the subscripts *h*, *s*, and *r* in Equations (A1) are dropped because the first order conditions are general for all investor types. Thus, the FOCs in Equations (3)-(4) can apply directly to a hedger while the FOCs for a speculator with a restriction of $\tilde{g} \equiv 0$ and the FOCs for an outside investor, who trades the futures option with restrictions of $\tilde{g} \equiv 0$ and $y = 0$.

The optimal futures position in Equation (3) can be specified for a hedger and a speculator respectively.

$$y_h = \frac{\Pi / \lambda}{\operatorname{var}(\tilde{\Pi})} - \frac{\operatorname{cov}(\tilde{\Pi}, \tilde{g}_h + z_h \tilde{R}_o)}{\operatorname{var}(\tilde{\Pi})} \qquad (5)$$

$$y_s = \frac{\Pi / \lambda}{\operatorname{var}(\tilde{\Pi})} - \frac{\operatorname{cov}(\tilde{\Pi}, z_s \tilde{R}_o)}{\operatorname{var}(\tilde{\Pi})} \qquad (6)$$

Sum the positions for all hedgers and all speculators, respectively. Then divide the equations by total number of hedgers and speculators.

$$y_H = \frac{\Pi / \lambda_H}{\mathrm{var}(\tilde{\Pi})} - \frac{\mathrm{cov}(\tilde{\Pi}, \tilde{G})}{\mathrm{var}(\tilde{\Pi})} - \frac{H}{H+S} Z \frac{\mathrm{cov}(\tilde{\Pi}, \tilde{R}_o)}{\mathrm{var}(\tilde{\Pi})} \tag{7}$$

$$y_S = \frac{\Pi / \lambda_S}{\mathrm{var}(\tilde{\Pi})} - \frac{S}{H+S} Z \frac{\mathrm{cov}(\tilde{\Pi}, \tilde{R}_o)}{\mathrm{var}(\tilde{\Pi})} \tag{8}$$

where $\frac{1}{\lambda_H} = \sum_{h=1}^{H} \frac{1}{\lambda_h}$; $\frac{1}{\lambda_S} = \sum_{s=1}^{S} \frac{1}{\lambda_s}$; $\tilde{G} = \sum_{h=1}^{H} \tilde{g}_h$; $Z = \sum_{h=1}^{H} z_h + \sum_{s=1}^{S} z_s$; $\frac{S}{H+S} Z = \sum_{s=1}^{S} z_s$; $\frac{H}{H+S} Z = \sum_{h=1}^{H} z_h$; H and S denote the total numbers of hedgers and speculators in the futures market, respectively.

3. Proof of the comparative statics associated with Proposition 1

In the futures market equilibrium, $y_H + y_S = 0$ and let Y denote y_H and Z denote z_H. Equations (7) and (8) can be rewritten as the following system of the demand equations.

$$L_H = \frac{\Pi / \lambda_H}{\mathrm{var}(\tilde{\Pi})} - \frac{\mathrm{cov}(\tilde{\Pi}, \tilde{G})}{\mathrm{var}(\tilde{\Pi})} - \frac{H}{H+S} Z \frac{\mathrm{cov}(\tilde{\Pi}, \tilde{R}_o)}{\mathrm{var}(\tilde{\Pi})} - Y = 0 \tag{9}$$

$$L_S = \frac{\Pi / \lambda_S}{\mathrm{var}(\tilde{\Pi})} - \frac{S}{H+S} Z \frac{\mathrm{cov}(\tilde{\Pi}, \tilde{R}_o)}{\mathrm{var}(\tilde{\Pi})} + Y = 0 \tag{10}$$

The condition of the futures market equilibrium is imposed by combining Equations (9) and (10). In the futures equilibrium, the net positions of futures options held by all futures traders, $Z \left(= \sum_{h=1}^{H} z_h + \sum_{s=1}^{S} z_s \right)$, is

$$Z = \frac{(H+S)(\Pi - \theta \operatorname{cov}(\tilde{\Pi}, \tilde{G}))}{\theta \operatorname{cov}(\tilde{\Pi}, \tilde{R}_o)} \tag{11}$$

where $\theta \left(= \dfrac{(\lambda_H \lambda_S)/(\lambda_H + \lambda_S)}{H+S} \right)$ denotes the average coefficient of absolute risk aversion.

Substituting Equation (11) into Z in Equations (9) and (10) yields

$$L_H = \frac{\Pi / \lambda_H}{\operatorname{var}(\tilde{\Pi})} - \frac{\operatorname{cov}(\tilde{\Pi}, \tilde{G})}{\operatorname{var}(\tilde{\Pi})} - \frac{H(\Pi - \theta \operatorname{cov}(\tilde{\Pi}, \tilde{G}))}{(H+S)\theta \operatorname{var}(\tilde{\Pi})} - Y = 0 \tag{A2}$$

$$L_S = \frac{\Pi / \lambda_S}{\operatorname{var}(\tilde{\Pi})} - \frac{S(\Pi - \theta \operatorname{cov}(\tilde{\Pi}, \tilde{G}))}{(H+S)\theta \operatorname{var}(\tilde{\Pi})} + Y = 0 \tag{A3}$$

Given the joint-probability distribution function of $\tilde{p}_A$ and $\tilde{p}_B$, we can rewrite Equations (9) and (10) futures risk premium (Π), expected option payoff $E(\tilde{R}_o)$, their covariance and variance terms as

$$L_H = 0 = \phi \left\{ \frac{E(\tilde{p}_A - f - \delta)}{\lambda_H \operatorname{var}(\tilde{p}_A - f)} - \frac{\operatorname{cov}(\tilde{p}_A - f, \tilde{G})}{\operatorname{var}(\tilde{p}_A - f)} - \frac{H}{H+S} Z \frac{\operatorname{cov}(\tilde{p}_A - f, \max(\tilde{p}_A - k - \delta, 0))}{\operatorname{var}(\tilde{p}_A - f)} - Y \right\}$$

$$+ (1-\phi) \left\{ \frac{E(\tilde{p}_A - f)}{\lambda_H \operatorname{var}(\tilde{p}_A - f)} - \frac{\operatorname{cov}(\tilde{p}_A - f, \tilde{G})}{\operatorname{var}(\tilde{p}_A - f)} - \frac{H}{H+S} Z \frac{\operatorname{cov}(\tilde{p}_A - f, \max(\tilde{p}_A - k, 0))}{\operatorname{var}(\tilde{p}_A - f)} - Y \right\} \tag{12}$$

$$L_S = 0 = \phi \left\{ \frac{E(\tilde{p}_A - f - \delta)}{\lambda_S \operatorname{var}(\tilde{p}_A - f)} - \frac{S}{H+S} Z \frac{\operatorname{cov}(\tilde{p}_A - f, \max(\tilde{p}_A - k - \delta, 0))}{\operatorname{var}(\tilde{p}_A - f)} + Y \right\}$$

$$+ (1-\phi) \left\{ \frac{E(\tilde{p}_A - f)}{\lambda_S \operatorname{var}(\tilde{p}_A - f)} - \frac{S}{H+S} Z \frac{\operatorname{cov}(\tilde{p}_A - f, \max(\tilde{p}_A - k, 0))}{\operatorname{var}(\tilde{p}_A - f)} + Y \right\} \tag{13}$$

Note: Since δ is constant, $\mathrm{var}(\tilde{p}_A - f - \delta) = \mathrm{var}(\tilde{p}_A - f)$ and

$$\mathrm{cov}(\tilde{p}_A - f - \delta, \max(\tilde{p}_A - k - \delta, 0)) = \mathrm{cov}(\tilde{p}_A - f, \max(\tilde{p}_A - k - \delta, 0))$$

Similarly we can rewrite Equations (A2) and (A3) as

$$L_H = 0 = \phi\left\{\frac{E(\tilde{p}_A - f - \delta)}{\lambda_H \mathrm{var}(\tilde{p}_A - f)} - \frac{\mathrm{cov}(\tilde{p}_A - f, \tilde{G})}{\mathrm{var}(\tilde{p}_A - f)} - \frac{H(E(\tilde{p}_A - f - \delta) - \theta \mathrm{cov}(\tilde{p}_A - f, \tilde{G}))}{(H+S)\theta \mathrm{var}(\tilde{p}_A - f)} - Y\right\}$$
$$+ (1-\phi)\left\{\frac{E(\tilde{p}_A - f)}{\lambda_H \mathrm{var}(\tilde{p}_A - f)} - \frac{\mathrm{cov}(\tilde{p}_A - f, \tilde{G})}{\mathrm{var}(\tilde{p}_A - f)} - \frac{H(E(\tilde{p}_A - f) - \theta \mathrm{cov}(\tilde{p}_A - f, \tilde{G}))}{(H+S)\theta \mathrm{var}(\tilde{p}_A - f)} - Y\right\} \quad \text{(A4)}$$

$$L_S = 0 = \phi\left\{\frac{E(\tilde{p}_A - f - \delta)}{\lambda_S \mathrm{var}(\tilde{p}_A - f)} - \frac{S(E(\tilde{p}_A - f - \delta) - \theta \mathrm{cov}(\tilde{p}_A - f, \tilde{G}))}{(H+S)\theta \mathrm{var}(\tilde{p}_A - f)} + Y\right\}$$
$$+ (1-\phi)\left\{\frac{E(\tilde{p}_A - f)}{\lambda_S \mathrm{var}(\tilde{p}_A - f)} - \frac{S(E(\tilde{p}_A - f - \delta) - \theta \mathrm{cov}(\tilde{p}_A - f, \tilde{G}))}{(H+S)\theta \mathrm{var}(\tilde{p}_A - f)} + Y\right\} \quad \text{(A5)}$$

Totally differentiating the demand equations (A4) and (A5) with respect to ϕ and evaluating at $\phi = 0$ yield

$$\begin{bmatrix} \frac{\partial L_H}{\partial Y} & \frac{\partial L_H}{\partial Z} \\ \frac{\partial L_S}{\partial Y} & \frac{\partial L_S}{\partial Z} \end{bmatrix} \begin{bmatrix} \frac{dY^*}{d\phi} \\ \frac{dZ^*}{d\phi} \end{bmatrix} = \begin{bmatrix} -\frac{\partial L_H}{\partial \phi} \\ -\frac{\partial L_S}{\partial \phi} \end{bmatrix} \quad \text{(A6)}$$

$$\frac{dY^*}{d\phi} = \frac{1}{J}\left[-\frac{\partial L_H}{\partial \phi}\frac{\partial L_S}{\partial Z} + \frac{\partial L_S}{\partial \phi}\frac{\partial L_H}{\partial Z}\right] \quad \text{(A7)}$$

$$\frac{dZ^*}{d\phi} = \frac{1}{J}\left[-\frac{\partial L_S}{\partial \phi}\frac{\partial L_H}{\partial Y} + \frac{\partial L_H}{\partial \phi}\frac{\partial L_S}{\partial Y}\right] \quad \text{(A8)}$$

$$J = \left[\frac{\partial L_H}{\partial Y}\frac{\partial L_S}{\partial Z} - \frac{\partial L_H}{\partial Z}\frac{\partial L_S}{\partial Y}\right] \quad \text{(A9)}$$

The relevant derivatives evaluating at $\phi = 0$ are as follows.

$$\left.\frac{\partial L_H}{\partial Y}\right|_{\phi=0} = -1 \tag{A10}$$

$$\left.\frac{\partial L_S}{\partial Y}\right|_{\phi=0} = 1 \tag{A11}$$

$$\left.\frac{\partial L_H}{\partial Z}\right|_{\phi=0} = -\frac{\bar{h}\,\mathrm{cov}(\tilde{p}_A - f, \max(\tilde{p}_A - f - k, 0))}{\mathrm{var}(\tilde{p}_A - f)} \tag{A12}$$

$$\left.\frac{\partial L_S}{\partial Z}\right|_{\phi=0} = -\frac{(1-\bar{h})\,\mathrm{cov}(\tilde{p}_A - f, \max(\tilde{p}_A - f - k, 0))}{\mathrm{var}(\tilde{p}_A - f)} \tag{A13}$$

$$\left.\frac{\partial L_H}{\partial \phi}\right|_{\phi=0} = \frac{\delta}{\mathrm{var}(\tilde{p}_A - f)}\left(\frac{H(\lambda_H + \lambda_S) - \lambda_S}{\lambda_H \lambda_S}\right) \tag{A14}$$

$$\left.\frac{\partial L_S}{\partial \phi}\right|_{\phi=0} = \frac{\delta}{\mathrm{var}(\tilde{p}_A - f)}\left(\frac{S(\lambda_H + \lambda_S) - \lambda_H}{\lambda_H \lambda_S}\right) \tag{A15}$$

Substituting the derivatives in Equations (A10) - (A15) into Equations (A7) - (A9) gives

$$J = \frac{\mathrm{cov}(\tilde{p}_A - f, \max(\tilde{p}_A - f - k, 0))}{\mathrm{var}(\tilde{p}_A - f)} \tag{A16}$$

$$\frac{dY^*}{d\phi} = \frac{\delta}{\mathrm{var}(\tilde{p}_A - f)}\left(\frac{\lambda_H - \lambda_S}{(H+S)\lambda_H \lambda_S}\right) \tag{14}$$

$$\frac{dZ^*}{d\phi} = \frac{\delta(\lambda_H + \lambda_S)[(H+S) - 1]}{\lambda_H \lambda_S\,\mathrm{cov}(\tilde{p}_A - f, \max(\tilde{p}_A - k, 0))} \tag{15}$$

Note: $\frac{dZ^*}{d\phi} > 0$ because all of δ, λ_H, λ_S, H, S and $\mathrm{cov}(\tilde{p}_A - f, \max(\tilde{p}_A - k, 0))$ are positive and $(H + S) > 1$. Meanwhile, the sign of $\frac{dY^*}{d\phi}$ depends on the amounts of λ_H and λ_S as follows

If $\lambda_H > \lambda_S$, then $\frac{dY^*}{d\phi} > 0$.

If $\lambda_H = \lambda_S$, then $\frac{dY^*}{d\phi} = 0$.

If $\lambda_H < \lambda_S$, then $\frac{dY^*}{d\phi} < 0$.

Equation (14) can be rewritten as

$$\frac{dY^*}{d\phi} = \frac{dZ^*}{d\phi} + \alpha \qquad (16)$$

where

$$\alpha = \frac{\delta \,\mathrm{cov}(\tilde{p}_A - f, \max(\tilde{p}_A - k, 0))(\lambda_H - \lambda_S) - \delta \,\mathrm{var}(\tilde{p}_A - f)(\lambda_H + \lambda_S)[(H+S)^2 - (H+S)]}{(H+S)\lambda_H \lambda_S \,\mathrm{var}(\tilde{p}_A - f)\,\mathrm{cov}(\tilde{p}_A - f, \max(\tilde{p}_A - k, 0))}$$

The proof is completed.

4. Derivation of futures price and risk premium for Proposition 2

Combining Equations (A2) and (A3) yield

$$\frac{\Pi}{\theta \,\mathrm{var}(\tilde{\Pi})} - \frac{\mathrm{cov}(\tilde{\Pi}, \tilde{G})}{\mathrm{var}(\tilde{\Pi})} - \frac{(\Pi - \theta \,\mathrm{cov}(\tilde{\Pi}, \tilde{G}))}{\theta \,\mathrm{var}(\tilde{\Pi})} = 0 \qquad (A16)$$

where $\theta \left(= \frac{(\lambda_H \lambda_S)/(\lambda_H + \lambda_S)}{H + S} \right)$ denotes the average coefficient of absolute risk aversion.

Multiplying the equality by $\theta\,\mathrm{var}(\tilde{\Pi})$ yields

$$\Pi - \theta\,\mathrm{cov}(\tilde{\Pi}, \tilde{G}) - (\Pi - \theta\,\mathrm{cov}(\tilde{\Pi}, \tilde{G})) = 0 \qquad \text{(A17)}$$

Using Equation (A17) and $\Pi \equiv \tilde{f} - f$, we can derive futures price and risk premium in Equations (15) and (16), respectively as

$$f^* = E(\tilde{f}) - \theta\,\mathrm{cov}(\tilde{f} - f, \tilde{G}) - \mathrm{cov}\left(\tilde{f} - f, \frac{(E(\tilde{f}) - f - \theta\,\mathrm{cov}(\tilde{f} - f, \tilde{G}))(\tilde{o} - o)}{\mathrm{cov}(\tilde{f} - f, \tilde{o} - o)}\right) \qquad (15)$$

$$\Pi^* = \theta\,\mathrm{cov}(\tilde{f} - f, \tilde{G}) + \mathrm{cov}\left(\tilde{f} - f, \frac{(E(\tilde{f}) - f - \theta\,\mathrm{cov}(\tilde{f} - f, \tilde{G}))(\tilde{o} - o)}{\mathrm{cov}(\tilde{f} - f, \tilde{o} - o)}\right) \qquad (16)$$

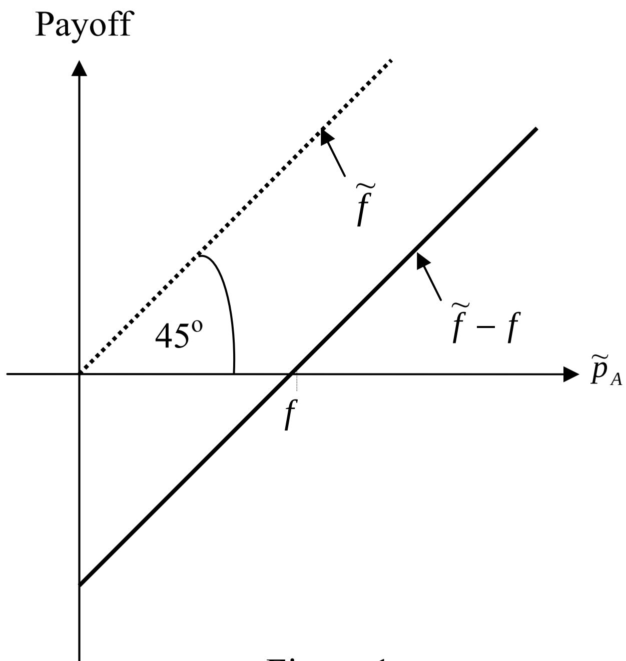

Figure 1
Futures Payoff in the Absence of Delivery Risk

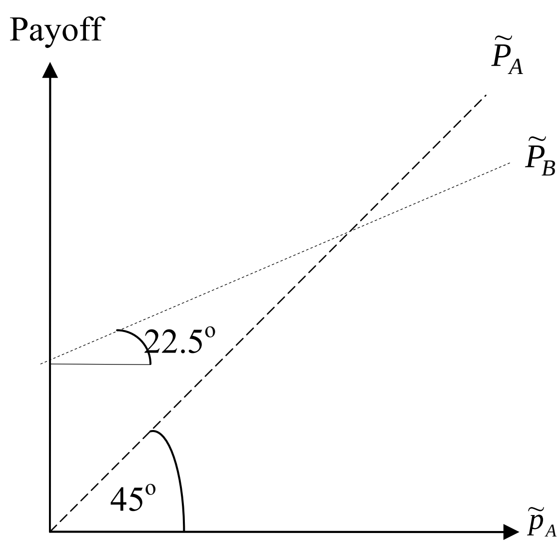

Figure 2
Payoffs of Deliverable Grades

Remark: $\tilde{p}_A$ is the terminal price of the par-delivery grade.
f is current futures price.

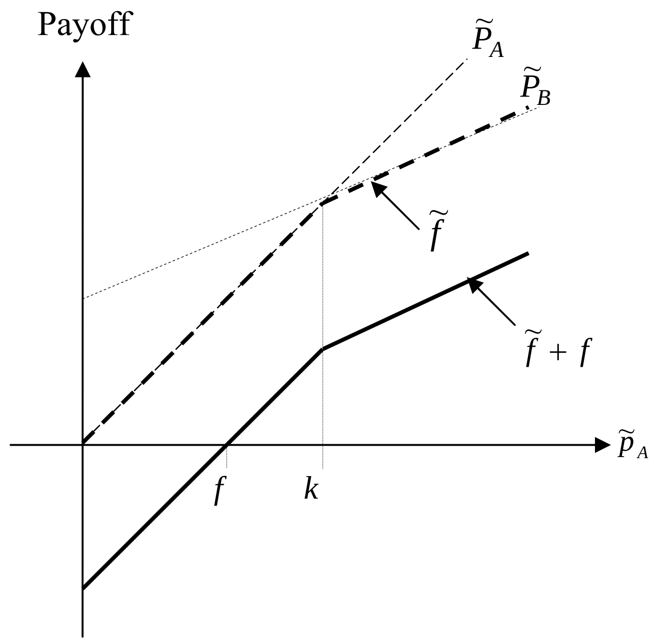

Figure 3
Futures Payoff in the Presence of Delivery Risk

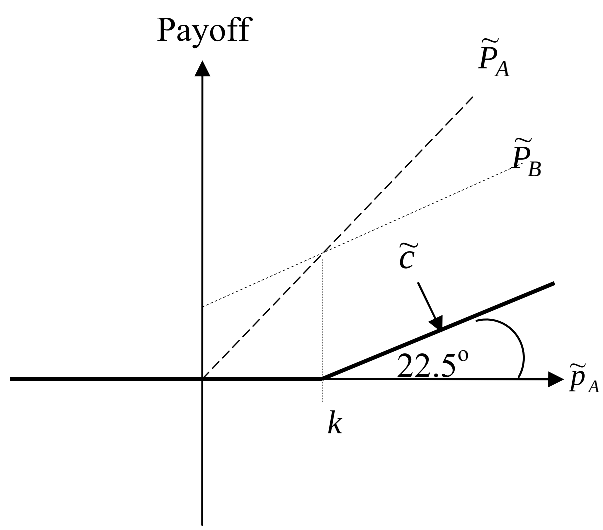

Figure 4
Payoff of Call Futures Option

Remark: $\tilde{p}_A$ is the terminal price of the par-delivery grade. k is the exercise price of the call futures option. f is current futures price.

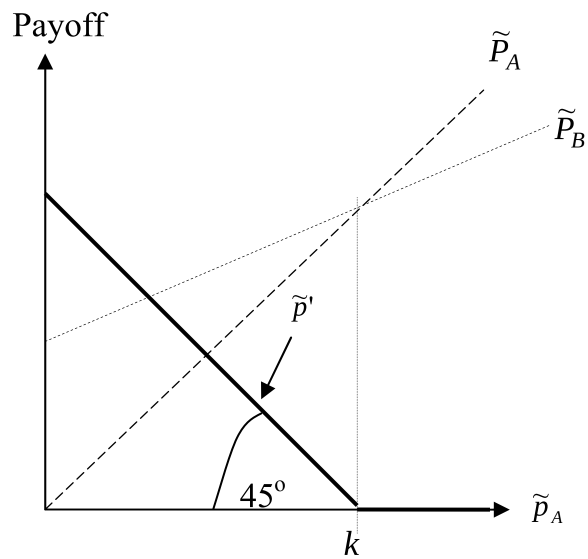

Figure 5
Payoff of Put Futures Option

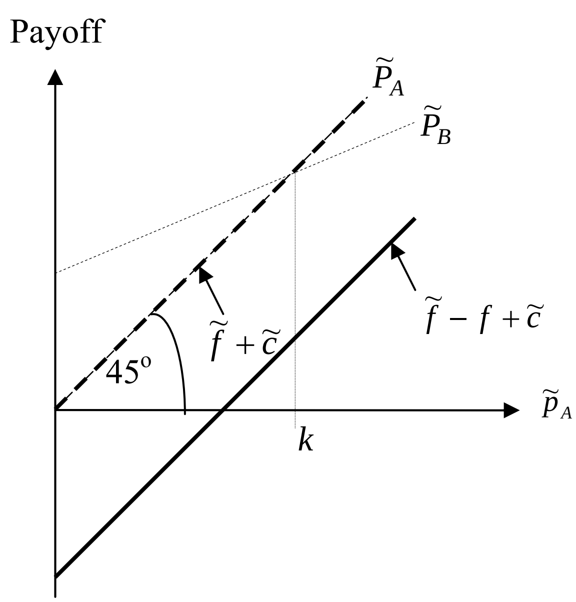

Figure 6
Payoff of Long Positions in Futures and Call Futures

Remark: $\tilde{p}_A$ is the terminal price of the par-delivery grade. k is the exercise price of the call and put futures options. f is current futures price.

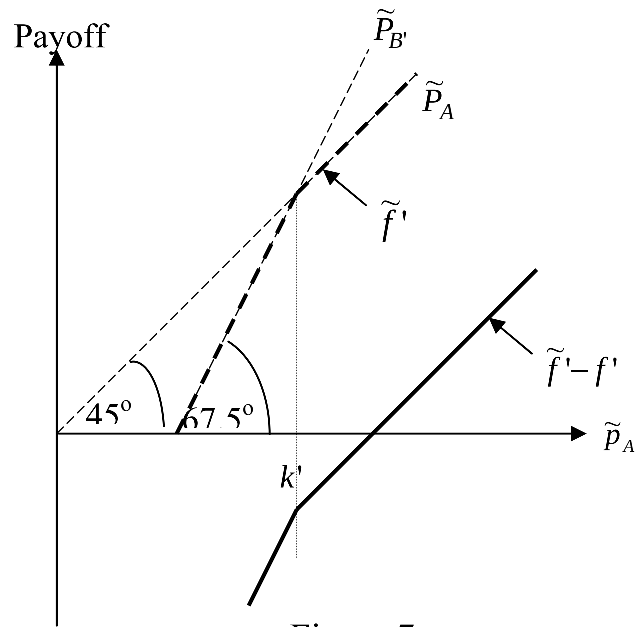

Figure 7
Futures Payoff in the Presence of Delivery Risk (2)

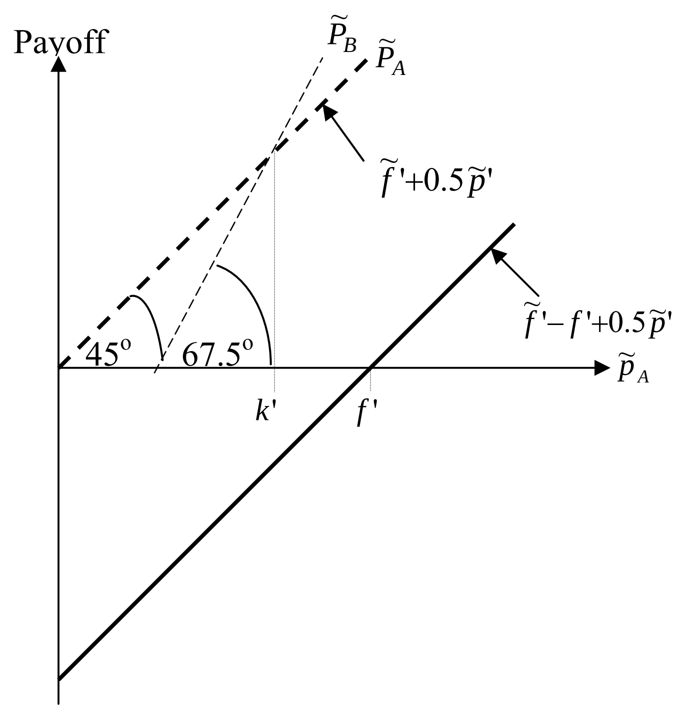

Figure 8
Payoff of Long Positions in Futures and Put Futures

Remark: $\widetilde{p}_A$ is the terminal price of the par-delivery grade. k' is the exercise price of the put futures options. f' is current futures price.

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CHAPTER 4

FUTURES RISK PREMIUM

IN THE PRESENCE OF DELIVERY RISK: THEORY

This chapter develops an equilibrium futures pricing model in spirit of Hirshleifer (1988) in which futures traders face delivery risk and futures options are available. The two-factor model of Hirshleifer is extended to a three-factor model by incorporating the expected return on futures option as an additional determinant of futures risk premium.

1. Introduction

Futures risk premium is traditionally known as the difference between the current price of futures and expected spot price of the asset underlying the futures contract.[23] The premium roughly reflects an equilibrium price for transferring the underlying asset's price risk with the use of futures contracts. Measuring futures risk premium is important in estimating hedging costs and benefits, making arbitrage and speculative investment decisions, and understanding the dynamics of both spot and futures prices. Due to its importance, the futures risk premium has attracted much attention from financial economists, who attempt to construct futures pricing models.

The analysis of futures risk premium identifies two determinants: hedging pressure and systematic risk. Early studies focused on the effect of each determinant separately and provided mixed evidence.[24] Recent models (e.g., Stoll (1979), Hirshleifer (1988)) combined the roles of hedging pressure and systematic risk in determining futures risk premiums and were empirically supported (Cater, Rausser, and Schmitz (1983), Bessembinder (1992), DeRoon, Nijman, and Veld (2000)). These combined-role models, however, do not completely explain the cross-sectional and time-series variations in futures risk premiums. Furthermore,

[23] This traditional definition is based on an implicit assumption of no delivery risk; thus, futures price certainly converges to the spot price of the underlying asset at maturity. However, the definition of futures risk premium proposed here ($\Pi \equiv E[\tilde{f} - f]$) not only coincides with the traditional one but also is general in that it accounts for delivery risk that can make the futures price at maturity diverged from the underlying spot price.

[24] For evidence on hedging pressure see, for example, Chang (1985), Kolb (1992), Bessembinder (1992), and DeRoon, Nijman, and Veld (2000). For evidence on systematic risk see, e.g., Dusak (1973), Bodie and Rosansky (1980), and Kolb (1996).

these models abstract from multiple delivery specifications that are provided in most futures contracts.

This study examines the theoretical implication of the delivery specifications for determining futures risk premiums. Futures contracts with multiple delivery specifications allow the sellers to deliver any of the deliverable grades of the underlying asset at any designated locations. Since the realized prices of these grades can be significantly different, the futures price on the delivery day converges to the spot price of the cheapest-to-deliver grade, which is currently unknown. This causes futures traders to face delivery risk in addition to the price risk in the underlying asset.

In the absence of delivery risk, an assumption employed in the literature on equilibrium futures pricing, the futures price converges to the spot price of the certain asset underlying the contract (i.e., the par-delivery grade) on the delivery date.[25] Hence, the hedgers' revenues, which are risky but linear in the spot price, can be optimally hedged by taking positions in futures contracts, of which the gains or losses are also linear in the spot price.

The presence of delivery risk, however, causes the futures price to converge to the cheapest-to-deliver price rather than the spot price of par-delivery grade on the delivery day. Thus, hedgers, whose revenues are linearly related to the spot price, cannot precisely hedge the revenue risk using merely futures contracts, of which the payoffs are not linear in the spot price of the par-delivery

[25] This literature includes the well-known models of Dusak (1973), Stoll (1979), Hirshleifer (1988) and DeRoon Nijman and Veld (2000), among others. These models are based on an implicit assumption of no delivery risk, so futures payoff in the models is defined as expected spot price minus current futures price. This definition implies that futures price converges to the spot price of the underlying asset at the end of the period.

grade. The nonlinearity of futures payoff gives rise to some residual revenue risk to be hedged by futures options.

It is shown that, in the futures market equilibrium, the delivery risk affects the net hedging demand for futures, and induces the hedging demand for futures options. Hence, the presence of delivery risk causes futures options to become a nonredundant hedging instrument and affect the equilibrium futures price. This implies the futures options should be significant in pricing futures contracts.

The implication of futures options for pricing futures contracts seemingly counters the belief that the option price is determined by the behavior of the underlying price. However, this unidirectional relationship is only true in complete markets. When the markets are incomplete, as shown by DeTemple and Selden (1991), derivatives may change the hedging opportunities available to traders, thereby introducing price effects on the security underlying the derivatives. Vanden (2004) shows that, when options are nonredundant in the economy, the option returns should explain the cross-section of the underlying security returns. Based on this premise, returns on futures options would thus influence futures returns and risk premiums in the presence of delivery risk.

To formulate the futures-pricing implication of futures options, this study develops an equilibrium futures-pricing model by extending Hirshleifer (1988) model to account for the delivery risk and availability of options on futures. The main thrust of this extension is that when only some of potential speculators trade derivatives, the futures risk premium is set to compensate the marginal speculator for trading costs and incremental risks, including delivery risk, that are associated with his futures position.

The extended model shows that the additional risks cause futures risk premiums to deviate from proportionality to the two risk components characterized in Hirshleifer model (that is, systematic risk related to the stock market return and residual risk conditional upon hedging pressure in the futures market). The deviation increases with expected return on the futures option. This leads to a futures pricing model, which relates futures risk premium to expected return on stock market portfolio, residual risk premium, and expected return on futures option.

The pricing model presented here is distinctive from the existing literature on valuing the impact of delivery specifications on futures price[26]. The previous studies employ several approaches to estimate the value of delivery options. These include option pricing model (Gay and Manaster (1984), Hemler (1990)), Monte-Carlo simulations (Kane and Marcus (1986)), central moment approximation (Boyle 1989), among others. In contrast to these approaches, the model in this study is developed in context of the mainstream asset pricing theory. Particularly, the model, in spirit of Hirshleifer (1988), predicts that expected return on futures option captures the delivery risk and other incremental risks associated with the futures contract, so the variation in the futures option returns has power to explain average futures risk premiums.

[26] See Chance and Hemler (1993) for a survey on the impact of delivery specifications on futures prices.

The remaining sections are organized as follows. Section 2 describes the economic setting and derives the futures market equilibrium. Given the number of futures traders, Section 3 relates futures risk premium to the futures covariation with systematic risk, nonmarketable risk and the incremental risks captured by expected return on futures option. To yield a testable futures pricing model, Section 4 further models futures risk premium with an endogenous number of traders. Section 5 concludes the study. The detailed derivations are provided in Appendix.

2. Futures Market Equilibrium

2.1 Economic Setting

Consider a two-date mean variance model. There are two groups of risk-averse investors, hedgers and outside investors, in three competitive markets (equities, futures and options on futures). All investors have homogenous beliefs about the distributions of all variables. Equities are traded freely but there are fixed setup costs for trading derivatives.[27] The investors decide now to take positions, in these markets, that affect their wealths at date 1. Each hedger is defined to possess a business generating uncertain revenue ($\tilde{g}_h$) that is linearly related to the date 1 price of the par-delivery grade underlying the futures contract

[27] The cost of trading derivatives is assumed in order to deter some outside investors from participation in the derivative markets. This assumption reflects the fact that the number of futures traders is small relative to that of investors in stock market. For stock, these setup costs can be avoided by investing in a passively managed mutual fund. In contrast, investing in futures mutual funds still incurs the cost of monitoring and assessing because the funds are actively managed due partly to the short expiration of futures.

($\widetilde{P}_A$). Market imperfections (that lead to, e.g., adverse selection, moral hazard) cause hedgers to be unable to trade the equity shares in their businesses through the equity market. Thus, they need to hedge their risky revenues with derivatives, i.e., trading infinitely divisible futures and call options on futures.[28] On the other hand, outside investors, whose wealths are independent of the price risk underlying futures contract, will trade derivatives only if the trading does not make their expected utilities lower than not trading at all. Hereafter, outside investors who trade futures are called speculators. Each investor maximizes the expected utility of the wealth at date 1.[29]

$$E(\widetilde{U}_i) = E(\widetilde{W}_i) - \frac{\lambda_i}{2}\mathrm{var}(\widetilde{W}_i) \tag{1}$$

where subscript i indicates investor type: h for a hedger, s for a speculator, $r1$ for an outside investor who trades only futures options, or $r2$ for an outside investor who refrains from trading any derivatives;[30] $E(.)$ and var(.) denote expectation and variance operators respectively; A tilde (~) indicates a random

[28] Many options on futures contracts are available and actively traded in U.S. derivative exchanges. Without loss of generality, the analysis focuses on the hedging role of call option on futures. The analysis on the role of put futures options can be conducted in the similar way.

[29] This expected utility function is applied under the assumption of the exponential preference with constant absolute risk aversion and the multivariate normality of the wealth at date 1.

[30] The setting that speculators trade both futures and futures options while some investors trade only futures options may not be unrealistic. As documented by Daigler and Wiley (1999) and Kodres and Pritsker (1997), the clearing members, who are the major speculators, trade futures and other related derivatives. Meanwhile, a number of investors may trade only futures options because the options tend to entail lower transaction costs than futures. Moreover, futures options are settled in cash and do not usually lead to delivery of the commodity underlying the derivatives.

variable; $\widetilde{U}_i$ denotes utility at date 1; $\widetilde{W}_i$ denotes wealth at date 1; λ_i denotes coefficient of absolute risk aversion. The wealth functions of hedger ($\widetilde{W}_h$) and speculator ($\widetilde{W}_s$) at date 1 are described as follows.[31]

$$\widetilde{W}_h = a_h - t + \widetilde{g}_h + \widetilde{R}_m x_h + [\widetilde{f} - f] y_h + \widetilde{R}_o z_h \quad \text{(2h)}$$

$$\widetilde{W}_s = a_s - t + \widetilde{R}_m x_s + [\widetilde{f} - f] y_s + \widetilde{R}_o z_s \quad \text{(2s)}$$

The notations in Equations (2h) and (2s) are described as follows. $\widetilde{g}_h$ denotes hedger's revenue which is linear in the date 1 price of the par-delivery grade underlying the futures contract. a_i is an initial wealth of investor i (units of consumption good at date 1). x_i, y_i, and z_i are the positions of stocks, futures contracts, and call options on futures respectively. A positive (negative) position indicates a long (short) position. t denotes fixed setup costs for trading derivatives: both futures and futures options. $t`$ denotes fixed setup costs for trading futures options, which are significantly lower than those for futures due partly to the leverage effect. f is current futures price. $\widetilde{f}$ is the date 1 settlement price of futures contract. $\widetilde{R}_m$ is net return on the stock market portfolio by buying one dollar of option at date 0 and receiving the random payoff of $\widetilde{R}_m$ at date 1. $\widetilde{R}_o$ is

[31] The wealth of an outside investor, who trades only the futures option ($r1$), can be described in equation (2s) with a restriction of zero futures position ($y_s = 0$) and by replacing trading cost of derivatives (t) by trading cost of futures options ($t`$). Similarly, the wealth of an outside investor, who refrains from trading derivatives, can be described in equation (2s) with restrictions of $y_s = z_s = 0$ and dropping trading cost of derivatives (t).

net return on a call option on futures contract by buying one dollar of option at date 0 and receiving the random payoff of $\widetilde{R}_o$ at date 1.

Some further definitions follow: Futures risk premium (Π)[32] defined as $\Pi \equiv E[\widetilde{f} - f]$ and the percentage of risk premium as $\pi \equiv E[\widetilde{f} - f]/f$; futures payoff $\widetilde{\Pi} = \widetilde{f} - f$; the percentage change in futures prices $\widetilde{\pi} = (\widetilde{f} - f)/f$. To reflect the presence of delivery risk in this setting, the sellers of futures contract can deliver any of two deliverable grades, named grade A (the par-delivery grade) and grade B. Logically, the sellers will deliver the cheaper grade; thus the date 1 futures price must equal the minimum of the delivery-adjusted prices of grade A and grade B ($\widetilde{f} = \min(\widetilde{P}_A, \widetilde{P}_B)$). The uncertainty of which grade will be cheaper to deliver is referred to as delivery risk.

2.2 Equilibrium futures price and risk premium

The concavity of utility function with respect to with respect to stock position (*x*), futures position (*y*) and futures option position (*z*) respectively ensures that the optimal positions for an investor satisfy the following first order conditions.[33]

[32] Due to delivery risk, futures risk premium defined here is slightly different from the traditional one, which is the difference between current futures price and expected spot price of asset underlying futures contract at contract maturity. In the absence of delivery risk, the expected futures price converges to the expected spot price on the delivery date. Thus the proposed definition of futures risk premium coincides with the traditional risk premium.

[33] The subscript *i* is dropped because the first order conditions are general for all investor types. The first order conditions (3)-(5) apply directly to a hedger while they apply to a speculator with a restriction of $\widetilde{g} = 0$, to an outside investor, who trades the futures

$$E(\widetilde{R}_m) = \lambda \operatorname{cov}(\widetilde{R}_m, \widetilde{g} + x\widetilde{R}_m + y\widetilde{\Pi} + z\widetilde{R}_o) \tag{3}$$

$$\Pi = \lambda \operatorname{cov}(\widetilde{\Pi}, \widetilde{g} + x\widetilde{R}_m + y\widetilde{\Pi} + z\widetilde{R}_o) \tag{4}$$

$$E(\widetilde{R}_o) = \lambda \operatorname{cov}(\widetilde{R}_o, \widetilde{g} + x\widetilde{R}_m + y\widetilde{\Pi} + z\widetilde{R}_o) \tag{5}$$

Since covariance (cov) is a linear operator, the conditions still hold when the investor's wealth is understood to be the average wealth of all futures traders. The conditions (3) and (5) state that expected stock return and expected return on futures option depend on the covariance of their returns with the average wealth. As in the consumption-based CAPM (Breeden 1980), the condition (4) shows that futures risk premium is proportional to the covariance of futures payoff with the average wealth of futures traders. However, that average wealth does not include outside investors, who refrains from trading futures, causing the following predictions to deviate from those of the CAPM.

The optimal futures position for a hedger (y_h) results from Equation (4). The optimal futures position for a speculator (y_s), who has no revenue related to the price risk of the asset underlying the futures contract, is obtained by setting $\widetilde{g} = 0$ in Equation (4).

$$y_h = \frac{\Pi / \lambda_h}{\operatorname{var}(\widetilde{\Pi})} - \frac{\operatorname{cov}(\widetilde{\Pi}, \widetilde{g}_h + x_h\widetilde{R}_m + z_h\widetilde{R}_o)}{\operatorname{var}(\widetilde{\Pi})} \tag{6}$$

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option with restrictions of $\widetilde{g} = 0$ and $y = 0$, and to an outside investor, who refrains from trading derivatives with restrictions of $\widetilde{g} = 0$ and $y = z = 0$.

$$y_s = \frac{\Pi / \lambda_s}{\mathrm{var}(\tilde{\Pi})} - \frac{\mathrm{cov}(\tilde{\Pi}, x_s \tilde{R}_m + z_s \tilde{R}_o)}{\mathrm{var}(\tilde{\Pi})} \tag{7}$$

The first term on the right-hand side of both Equations (6) and (7) increases with risk premium (Π) but decreases with risk aversion λ_i while the remaining term is proportional to the covariance of futures payoff with the investor's wealth. Equation (6) suggests that, when payoffs of futures, futures options, and stocks are correlated, the hedger's positions in futures option and stock significantly affect optimal futures position.

Equation (7) states that given no covariance of futures payoff ($\tilde{\Pi}$) with stock market return ($\tilde{R}_m$) and option return ($\tilde{R}_o$), a speculator will trade futures only if he earns sufficient risk premium (Π) for the incremental risks imposed by holding a futures position. Conversely, given nontrivial covariance of $\tilde{\Pi}$ with $\tilde{R}_m$ and $\tilde{R}_o$ but no risk premium ($\Pi = 0$), a speculator may take a futures position to hedge positions in stocks and/or futures options.

It is noteworthy that trading cost, *t*, does not explicitly affect the size of an optimal futures position. However, *t* determines the number of speculators in equilibrium, thereby influencing the position size through futures risk premium, Π.

Equilibrium futures price (f^*) and risk premium (Π^*) can be derived as follows. First, the optimal futures positions for all hedgers are aggregated according to Equation (6) and those for all speculators according to Equation (7). Then, the futures market equilibrium is established by equating the sum of the aggregate positions to zero. Rearranging the equality gives

$$f^* = E(\tilde{f}) - \theta \operatorname{cov}(\tilde{\Pi}, b\bar{g} + \bar{x}\tilde{R}_m + \bar{z}\tilde{R}_o) \tag{8}$$

$$\Pi^* = E(\tilde{f}) - f = \theta \operatorname{cov}(\tilde{\Pi}, b\bar{g} + \bar{x}\tilde{R}_m + \bar{z}\tilde{R}_o) \tag{9}$$

where $\theta = \frac{(\lambda_H \lambda_S)/(\lambda_H + \lambda_S)}{H+S}$; $\frac{1}{\lambda_H} = \sum_{h=1}^{H} \frac{1}{\lambda_h}$; $\frac{1}{\lambda_S} = \sum_{s=1}^{S} \frac{1}{\lambda_s}$; $b = \frac{H}{H+S}$; $\bar{g} = \frac{1}{H}\sum_{h=1}^{H} \tilde{g}_h$; $\bar{x} = \frac{x_H + x_S}{H+S}$; $x_H = \sum_{h=1}^{H} x_h$; $x_S = \sum_{s=1}^{S} x_s$; $\bar{z} = \frac{z_H + z_S}{H+S}$; $z_H = \sum_{h=1}^{H} z_h$; $z_S = \sum_{s=1}^{S} z_s$; θ is the average coefficient of absolute risk aversion; $\bar{g}$ is the average of the hedgers' nonmarketable revenues; b is the proportion of hedgers to total futures traders; $\bar{x}$ and $\bar{z}$ denote the average positions of stock and futures option held by futures traders respectively; H and S denote the total numbers of hedgers and speculators in the futures market respectively.

Equations (8) and (9) show that the equilibrium futures price deviates from the expected settlement price of a futures contract due to the futures risk premium. The premium is linearly related to the covariance of futures payoff with average wealth.

3. Futures Risk Premium Given the Number of Traders

This section derives futures risk premium by taking as given the number of traders. Dividing Equation (9) with the futures price and decomposing the covariance term give the percentage risk premium as

$$\pi^* = \theta\, b \operatorname{cov}(\tilde{\pi}, \bar{g}) + \theta\, \bar{x} \operatorname{cov}(\tilde{\pi}, \tilde{R}_m) + \theta\, \bar{z} \operatorname{cov}(\tilde{\pi}, \tilde{R}_o) \qquad (10)$$

Equation (10) characterizes the futures risk premium, given the number of futures traders (as reflected in *b*, the proportion of hedgers to total futures traders). The risk premium has three components.[34] The first, nonmarketable risk component, reflects the hedger's motive to trade futures contracts. This component is proportional to the covariance of futures payoff with nonmarketable revenue. The second, marketable risk component, is related systematically to the covariance between futures payoff and stock market payoff. The third component compensates for the other incremental risks including delivery risk, which are captured by the covariance of futures risk premium and futures option return.

To express futures risk premium in terms of the contract's beta and its residual risk, the study assumes the following models, which systematically relate $\tilde{\pi}$, $\bar{g}$ and $\tilde{R}_o$ to stock market return:

$$\begin{aligned} \tilde{\pi} &= \alpha_{\pi m} + \beta_{\pi m}\tilde{R}_m + \varepsilon_{\pi m} \\ \bar{g} &= \alpha_{gm} + \beta_{gm}\tilde{R}_m + \varepsilon_{gm} \\ \tilde{R}_o &= \alpha_{om} + \beta_{om}\tilde{R}_m + \varepsilon_{om} \end{aligned} \qquad (11)$$

[34] Stoll (1979) first decomposed risk premium into a marketable risk component and a nonmarketable risk component in the context of equilibrium futures pricing. The decomposition is also described in the model of Hirshleifer (1988).

Replacing $\tilde{\pi}$, $\bar{g}$ and $\tilde{R}_o$ in Equation (10) by the models in Equation (11) and rearranging the equation give proposition 1:

Proposition 1 With a given number of futures traders, futures risk premium is

$$\pi = \theta \operatorname{var}(\tilde{R}_m)(\bar{x} + \bar{z}\beta_{om} + b\beta_{gm})\beta_{\pi m} \tag{12}$$

$$+ [\theta\,\bar{z}\sigma(\varepsilon_{om})\operatorname{corr}(\varepsilon_{\pi m}, \varepsilon_{om}) + \theta\, b\sigma(\varepsilon_{gm})\operatorname{corr}(\varepsilon_{\pi m}, \varepsilon_{gm})]\sigma(\varepsilon_{\pi m})$$

where corr(.) denotes a correlation coefficient, and σ(.) denotes a standard deviation.

Proposition 1 shows that futures risk premium has two additive components. First, the systematic risk component (consisting of three beta terms) depends on $\beta_{\pi m}$. Second, the residual risk component is proportional to the residual standard deviation $\sigma(\varepsilon_{\pi m})$.

However, the model is difficult to test for three reasons. First, the residual risk, $\sigma(\varepsilon_{\pi m})$, is multiplicative with other security-specific variables: corr($\varepsilon_{\pi m}$,ε_{gm}) and corr($\varepsilon_{\pi m}$,ε_{om}). Second, some security-specific variables, e.g., nonmarketable revenue and its correlation, are unobservable thus the proxies are needed for these variables. Third, the cross-section regression of average futures risk premiums in Proposition 1 will be misspecified, if the proportion of hedgers to total futures traders (*b*), the average positions of futures traders in stock market and futures option market (i.e., $\bar{x}$ and $\bar{z}$, respectively) are endogenously determined and systematically related to the other right-hand side variables.

To yield a testable pricing model, the next section further derives the number of speculators endogenously and eliminates both *b* and the unobserved correlation with nonmarketable revenue.

4. Futures Risk Premium with an Endogenous Number of Traders

This section derives futures risk premium and the number of traders simultaneously. Speculators will trade derivatives only if trading derivatives with additional cost *t* today generates expected utilities at least equal to refraining from trade (y_{r2} =0 and z_{r2}=0). Given this condition, the equilibrium of futures market is determined by the marginal speculator, who has indifferent satisfaction either trading derivatives or refraining.

$$E(\widetilde{U}_s; trade) = E(\widetilde{U}_{r2}; refrain) \tag{13}$$

where $E(\widetilde{U}_s)$ and $E(\widetilde{U}_{r2})$ are referred from Equations (1) and (2s):

$$E(\widetilde{U}_s) = E[a_s - t + x_s \widetilde{R}_m + y_s \widetilde{\Pi} + z_s \widetilde{R}_o] - \frac{\lambda_s}{2} \mathrm{var}[a_s - t + x_s \widetilde{R}_m + y_s \widetilde{\Pi} + z_s \widetilde{R}_o]$$
$$E(\widetilde{U}_{r2}) = E[a_{r2} + x_{r2} \widetilde{R}_m] - \frac{\lambda_{r2}}{2} \mathrm{var}[a_{r2} + x_{r2} \widetilde{R}_m]$$

The equilibrium condition implies that the setup cost for trading derivatives (*t*) determines the number of futures traders. An additional assumption required is that cov(g,π) ≠ 0 so that hedgers have an incentive to reduce their nonmarketable risk by trading derivatives and pay the risk premium to speculators. Thus, the following proposition for futures risk premium is derived:

Proposition 2 With nonzero covariance of futures payoff with nonmarketable revenues and a fixed setup cost that deters some potential speculators from trading derivatives, futures risk premiums (π) satisfy

$$\pi = \beta' E(\tilde{R}_m) + d\,\sigma'\sqrt{2\lambda' t} + \Gamma E(\tilde{R}_o) \tag{14}$$

$$\beta' = \frac{(\beta_{\pi m} - \beta_{\pi o}\beta_{om})}{1-\rho_{om}^2}\ ;\ \sigma' = \sigma_\pi\sqrt{\frac{\gamma}{1-\rho_{om}^2}}\ ;\ \Gamma = \frac{(\beta_{\pi o} - \beta_{\pi m}\beta_{mo})}{1-\rho_{om}^2}$$

where π is the percentage of futures risk premium; $E(\tilde{R}_m)$ is expected return on stock market portfolio; $E(\tilde{R}_o)$ is expected return on futures options; $\beta_{\pi m}$, $\beta_{\pi o}$, β_{om} and β_{mo} denote appropriately defined betas; σ_π is the standard deviation of futures risk premiums; *d* denotes an indicator variable equal to 1 for the net long futures positions of hedgers and -1 for the net short positions; λ' denotes the risk aversion of the marginal speculator; *t* denotes costs of trading derivatives; $\rho_{ab}^2 = \beta_{ab}\beta_{ba}$ and ρ_{ab} denotes the correlation coefficient of *a* and *b* ; $\gamma = 1 - \rho_{om}^2 - \rho_{o\pi}^2 - \rho_{\pi m}^2 + 2\beta_{mo}\beta_{o\pi}\beta_{\pi m}$.

The model in Proposition 2 is a generalization of Hirshleifer (1988) model to account for delivery risk and the availability of options on futures. In the absence of delivery risk or when futures options are not available, futures risk premium will reduce to be the risk premium exactly described in Hirshleifer model. The proposition thus shows precisely how futures risk premium is altered when the futures option is nonredundant in the economy.

Hirshleifer (1988) model: $$\pi = \beta_{\pi m} E(\tilde{R}_m) + d\sigma_\pi \sqrt{2\lambda' t(1-\rho_{\pi m}^2)} \qquad (15)$$

As shown in the proposition, futures risk premium contains three components: the first is systematic risk premium, which is proportional to the net sensitivity of futures risk premium to stock market return ($\beta_{\pi m}$ - $\beta_{\pi o}\beta_{om}$). The second is residual risk premium, which depends on the investor's risk aversion and a setup cost for trading derivatives. The third component depends on the net sensitivity of futures risk premium to the futures option return ($\beta_{\pi o}$ - $\beta_{\pi m}\beta_{mo}$).

1) Systematic risk premium: The basic rationale is straightforward. If the asset underlying the futures contract is correlated with the stock market, a speculator can remove the risk associated with a futures position by shifting the stock market position by an amount negatively proportional to β', thereby earning the premium of $\beta' E(R_m)$ for this shift.

2) Residual risk premium: The premium for residual risk rises in absolute value with the standard deviation of residual risk, $\sigma_\pi \sqrt{\gamma/(1-\rho_{om}^2)}$. This result, first proposed by Hirshleifer (1988), contrasts with the general presumption (refer Arrow and Pratt) and empirical findings (e.g., Grinblatt and Titman (1983)) that premium for bearing idiosyncratic risk is linear in residual variance. However, Bessembinder (1992) documents that the standard deviation of residual risk has incremental power, beyond the stock market beta, to explain the risk premiums for currency and agricultural futures contracts.

Residual risk premium is proportional to the square root of the risk-aversion coefficient λ' and trading cost t. The sign of this premium is conditional on net hedging demand in the futures market. The marginal speculator is concerned whether the residual risk premium is sufficient to compensate for the dispersion of the residual futures return. However, the residual risk premium reduces proportionally to a decrease in either trading cost or investors' risk aversion because this decrease motivates more outside investors to trade futures for a smaller premium.

3) Risk premium captured by the futures option return: Unlike underlying price risk, delivery risk arises from uncertainty imposed by the contract specification. It is shown that futures options are used to hedge against delivery risk associated with holding a futures position. This fact implies that delivery risk is systematically captured by Γ and thus delivery risk premium should be proportional to Γ. However, the option returns may also capture other risks associated with futures contracts in addition to those captured by the two pricing factors, namely systematic risk and residual risk. This is because futures options are also exposed to the common risks associated with the underlying futures contract. Consequently, the premium of $\Gamma E(R_o)$ compensates the marginal speculator for bearing the incremental risks that are captured by the variation in futures option returns.

The model in Proposition 2 is empirically testable since all risk variables are observable. The stock market portfolio here refers only to tradable endowments. In equilibrium, futures risk premium is independent of the contract's covariance with the unobserved nonmarketable risk.[35] This seemingly counters the intuition that the risk premium is driven by the incentive to hedge the nonmarketable revenue. It is tempting to assume that the higher the covariance of futures payoff with risky revenue, the larger the premiums that hedgers would be willing to pay. However, the rationale for eliminating the nonmarketable risk from futures risk premiums is clear; the marginal speculators, who determine the equilibrium risk premium, are only interested in their own payoffs, not those of hedgers. Since speculators do not face the nonmarketable risk, any covariance with that risk is irrelevant to their decisions and thus to futures risk premium. Although the nonmarketable revenue of hedgers does not influence the risk premium, hedgers still play an important role in determining futures risk premium. Specifically, hedging pressure in the futures market determines the sign of residual risk premium.

Proposition 2 would be valid under more general assumptions that speculators have different trading costs and beliefs. The higher-cost speculators require larger risk premiums to trade derivatives. However, the sensitivity of futures risk premium to the trading costs can be considerably reduced if a large number of potential speculators are available to enter the derivative markets. Under rational expectations with a limited range of different beliefs of the speculators, the

[35] This might seem inconsistent with equation (10). However, Proposition 2 implies that the covariance of nonmarketable risk offsets the variation in the endogenously determined proportion of hedgers (b).

proposition should be valid to predict the behavior of futures returns and risk premiums.

5. Conclusions

For most futures contracts, traders face delivery risk in addition to underlying price risk. It is shown that the presence of delivery risk causes options on futures to become a nonredundant hedging instrument and to influence the equilibrium futures prices.

This study formulates the implication of delivery risk for pricing futures contracts by extending the Hirshleifer (1988) model to account for the delivery risk and the availability of futures options. The rationale of this extension is that, when only some of the potential speculators trade derivatives, futures risk premium is set to compensate the marginal speculator for the additional trading cost and the incremental risks, including delivery risk, that are associated with a futures position.

The extended model predicts that the incremental risks cause futures risk premium to deviate from the proportionality to the two risk components characterized in the Hirshleifer model (systematic risk related to the stock market return and residual risk conditional on hedging pressure in the futures market). The deviation increases with expected return on futures option. The model suggests that the additional risks captured by futures option returns should be important in explaining futures risk premiums.

Appendix

The appendix contains a theoretical extension of Hirshleifer (1988) model, in which futures traders face delivery risk and futures options are available. The two-factor model of Hirshleifer is extended to a three-factor model, in which incorporates the expected returns on futures option as an additional determinant of futures risk premium.

The theoretical extension is organized as follows. First, futures risk premium is conventionally derived by assuming a constant number of futures traders and imposing an equilibrium condition of the aggregate futures positions held by the traders equal to zero. This yields a model of futures risk premium, stated in Proposition 1. The model shows that the effects on futures risk premium of the systematic risk and residual risk components of futures returns separate additively; however, the model is difficult to test empirically.[36]

To yield a testable model, we further derives futures risk premium and the number of futures traders endogenously and eliminates the unobserved variables, characterized in Proposition 1. To do so, futures risk premium is derived from an alternative equilibrium that is determined by the marginal speculator, who has indifferent satisfaction either trading derivatives or refraining.[37] This yields a three-factor model of futures risk premium, stated in Proposition 2. The three-factor model consists of three risk components: systematic risk, residual risk conditional on hedging pressure, and risk captured by futures option returns.

[36] The model is difficult to test because the residual risk, $\sigma(\varepsilon_{\pi m})$, is multiplicative with other security-specific variables: corr($\varepsilon_{\pi m}$,ε_{gm}) and corr($\varepsilon_{\pi m}$,ε_{om}). Some security-specific variables, e.g., nonmarketable revenue and its correlation, are also unobservable thus the proxies are needed for these variables. Moreover, the cross-section regression of average futures risk premiums in Proposition 1 will be misspecified, if the proportion of hedgers to total futures traders (b), the average positions of futures traders in stock market and futures option market (i.e., $\bar{x}$ and $\bar{z}$, respectively) are endogenously determined and systematically related to the other right-hand side variables.

[37]Speculators will trade derivatives only if trading derivatives with additional cost t today generates expected utilities at least equal to refraining from trade. Thus the equilibrium futures risk premium is just sufficient to compensate the marginal speculator for the costs and the incremental risk imposed by holding positions of futures and futures option.

The additional assumptions of the proposed three-factor model are parsimonious since in the absence of delivery risk, the futures options are redundant (i.e., they have no role in the futures market equilibrium). Thus the betas of futures options vanish and the proposed model reduces to the two-factor model of Hirshleifer (1988).

1. The First Order Conditions of a Mean-Variance Model

Assume that an investor *i* has exponential utility function, $U(\widetilde{W}_i) = -e^{-\lambda_i \widetilde{W}_i}$, with a constant absolute risk aversion (λ_i) and his terminal wealth ($\widetilde{W}_i$) is normally distributed. Then each investor maximizes the expected utility of his terminal wealth.

$$\max_{y,z} EU(\widetilde{W}_i) = \max_{y,z} E(\widetilde{W}_i) - \frac{\lambda_i}{2} \operatorname{var}(\widetilde{W}_i) \tag{1}$$

where subscript *i* indicates investor type: *h* for a hedger, *s* for a speculator, *r*1 for an outside investor who trades only futures options, or *r*2 for an outside investor who refrains from trading any derivatives;[38] *E*(.) and var(.) denote expectation and variance operators respectively; A tilde (~) indicates a random

[38] Each investor type plays a different role in the economy. Hedgers have an incentive to hedging against their risky revenue with futures and futures options. Speculators enter the derivative markets only if trading the derivatives generates their satisfaction at least equal to refraining from doing so. The presence of outsiders who trade only futures options causes participants in the futures market to be different from those in the futures option market, and thereby, the equilibrium of the two markets are not jointly determined. Although outsiders who refrain from trading any derivatives seem irrelevant in the future market equilibrium, one of these outsiders potentially enters the derivative markets as the marginal speculator, who determines the equilibrium futures risk premium.

The setting that speculators trade both futures and futures options while some investors trade only futures options may not be unrealistic. As documented by Daigler and Wiley (1999) and Kodres and Pritsker (1997), the clearing members, who are the major speculators, trade futures and other related derivatives. Meanwhile, a number of investors may trade only futures options because the options tend to entail lower transaction costs than futures and futures options are cash settlement.

variable; $\widetilde{U}_i$ denotes utility at time 1; $\widetilde{W}_i$ denotes wealth at time 1; λ_i denotes coefficient of absolute risk aversion.

$$\widetilde{W}_h = a_h - t + \widetilde{g}_h + \widetilde{R}_m x_h + [\widetilde{f} - f] y_h + \widetilde{R}_o z_h \quad (2h)$$

$$\widetilde{W}_s = a_s - t + \widetilde{R}_m x_s + [\widetilde{f} - f] y_s + \widetilde{R}_o z_s \quad (2s)$$

$$\widetilde{W}_{r1} = a_{r1} - t` + \widetilde{R}_m x_{r1} + \widetilde{R}_o z_{r1} \quad (2r)$$

$$\widetilde{W}_{r2} = a_{r2} + \widetilde{R}_m x_{r2} \quad (2q)$$

The notations in the above equations are described as follows.

- $\widetilde{g}_h$ denotes hedger's revenue which is linear in the price of the par-delivery grade underlying the futures contract at time 1.
- a_i is an initial wealth of investor i (units of consumption good at time 1).
- x_i, y_i, and z_i are the positions of stocks, futures contracts, and their corresponding call futures options respectively. A positive (negative) position indicates a long (short) position.
- t denotes fixed setup costs for trading derivatives: both futures and futures options. Meanwhile, $t`$ denotes fixed setup costs for trading futures options, which are significantly lower than those for futures due partly to the leverage effect.
- f is current futures price and $\widetilde{f}$ is the contract settlement price at time 1.
- $\widetilde{R}_m$ is net return on the stock market portfolio by buying one dollar of option at time 0 and receiving the random payoff of $\widetilde{R}_m$ at time 1.

- $\tilde{R}_o$ is net return on a call option on futures contract by buying one dollar of option at time 0 and receiving the random payoff of $\tilde{R}_o$ at time 1.
- Futures risk premium (Π)defined as the difference between expected futures price and current futures price ($\Pi \equiv E[\tilde{f} - f]$). Due to delivery risk, futures risk premium defined here is slightly different from the traditional one, which is the difference between current futures price and expected spot price of asset underlying futures contract at maturity. In the absence of delivery risk, the expected futures price converges to the expected spot price on the delivery date. Thus the proposed definition of futures risk premium coincides with the traditional risk premium.
- The percentage of risk premium is $\pi \equiv E[\tilde{f} - f]/f$; futures payoff is $\tilde{\Pi} = \tilde{f} - f$; the percentage change in futures prices is $\tilde{\pi} = (\tilde{f} - f)/f$.

The expected utility of each investor can be expressed as

$$E(\tilde{U}_h) = E[a_h - t + \tilde{g}_h + x_h\tilde{R}_m + y_h\tilde{\Pi} + z_h\tilde{R}_o] - \frac{\lambda_h}{2}\mathrm{var}\left[a_h - t + \tilde{g}_h + x_h\tilde{R}_m + y_h\tilde{\Pi} + z_h\tilde{R}_o\right] \quad \text{(A1h)}$$

$$E(\tilde{U}_s) = E[a_s - t + x_s\tilde{R}_m + y_s\tilde{\Pi} + z_s\tilde{R}_o] - \frac{\lambda_s}{2}\mathrm{var}[a_s - t + x_s\tilde{R}_m + y_s\tilde{\Pi} + z_s\tilde{R}_o] \quad \text{(A1s)}$$

$$E(\tilde{U}_{r1}) = E[a_{r1} - t` + x_{r1}\tilde{R}_m + z_{r1}\tilde{R}_o] - \frac{\lambda_{r1}}{2}\mathrm{var}[a_{r1} - t` + x_{r1}\tilde{R}_m + z_{r1}\tilde{R}_o] \quad \text{(A1r)}$$

$$E(\tilde{U}_{r2}) = E[a_{r2} + x_{r2}\tilde{R}_m] - \frac{\lambda_{r2}}{2}\mathrm{var}[a_{r2} + x_{r2}\tilde{R}_m] \quad \text{(A1q)}$$

Derive the first order conditions (FOCs) by differentiating the expect utility in Equations (A1) with respect to stock position (*x*), futures position (*y*) and futures option position (*z*) respectively.

$$\frac{dE(\widetilde{U})}{dx} = 0 = E(\widetilde{R}_m) - \lambda \operatorname{cov}(\widetilde{R}_m, \widetilde{g} + x\widetilde{R}_m + y\widetilde{\Pi} + z\widetilde{R}_o)$$

$$\frac{dE(\widetilde{U})}{dy} = 0 = \Pi - \lambda \operatorname{cov}(\widetilde{\Pi}, \widetilde{g} + x\widetilde{R}_m + y\widetilde{\Pi} + z\widetilde{R}_o)$$

$$\frac{dE(\widetilde{U})}{dz} = 0 = E(\widetilde{R}_o) - \lambda \operatorname{cov}(\widetilde{R}_o, \widetilde{g} + x\widetilde{R}_m + y\widetilde{\Pi} + z\widetilde{R}_o)$$

Rearranging the derivatives gives

$$E(\widetilde{R}_m) = \lambda \operatorname{cov}(\widetilde{R}_m, \widetilde{g} + x\widetilde{R}_m + y\widetilde{\Pi} + z\widetilde{R}_o) \quad (3)$$

$$\Pi = \lambda \operatorname{cov}(\widetilde{\Pi}, \widetilde{g} + x\widetilde{R}_m + y\widetilde{\Pi} + z\widetilde{R}_o) \quad (4)$$

$$E(\widetilde{R}_o) = \lambda \operatorname{cov}(\widetilde{R}_o, \widetilde{g} + x\widetilde{R}_m + y\widetilde{\Pi} + z\widetilde{R}_o) \quad (5)$$

Note: the subscripts *h*, *s*, *r*1 and *r*2 in Equations (A1) are dropped because the first order conditions are general for all investor types. Thus, the FOCs in Equations (3)-(5) can apply directly to a hedger while the FOCs for a speculator with a restriction of $\widetilde{g} = 0$ and the FOCs for an outside investor, who trades the futures option with restrictions of $\widetilde{g} = 0$ and $y = 0$, and for an outside investor, who refrains from trading derivatives with restrictions of $\widetilde{g} = 0$ and $y = z = 0$.

2. The derivation of the futures market equilibrium

The optimal futures position in Equation (4) can be specified for a hedger and a speculator, respectively.

$$y_h = \frac{\Pi / \lambda_h}{\operatorname{var}(\widetilde{\Pi})} - \frac{\operatorname{cov}(\widetilde{\Pi}, \widetilde{g}_h + x_h\widetilde{R}_m + z_h\widetilde{R}_o)}{\operatorname{var}(\widetilde{\Pi})} \quad (6)$$

$$y_s = \frac{\Pi / \lambda_s}{\operatorname{var}(\widetilde{\Pi})} - \frac{\operatorname{cov}(\widetilde{\Pi}, x_s\widetilde{R}_m + z_s\widetilde{R}_o)}{\operatorname{var}(\widetilde{\Pi})} \quad (7)$$

Sum the positions for all hedgers and all speculators respectively.

$$y_H = \frac{\Pi / \lambda_H}{\operatorname{var}(\tilde{\Pi})} - \frac{\operatorname{cov}(\tilde{\Pi}, \tilde{G} + x_H \tilde{R}_m + z_H \tilde{R}_o)}{\operatorname{var}(\tilde{\Pi})} \qquad \text{(A2)}$$

$$y_S = \frac{\Pi / \lambda_S}{\operatorname{var}(\tilde{\Pi})} - \frac{\operatorname{cov}(\tilde{\Pi}, x_S \tilde{R}_m + z_S \tilde{R}_o)}{\operatorname{var}(\tilde{\Pi})} \qquad \text{(A3)}$$

where $\frac{1}{\lambda_H} = \sum_{h=1}^{H} \frac{1}{\lambda_h}$; $\frac{1}{\lambda_S} = \sum_{s=1}^{S} \frac{1}{\lambda_s}$; $\tilde{G} = \sum_{h=1}^{H} \tilde{g}_h$; $x_H = \sum_{h=1}^{H} x_h$; $x_S = \sum_{s=1}^{S} x_s$; $z_H = \sum_{h=1}^{H} z_h$; $z_S = \sum_{s=1}^{S} z_s$; H and S denote the total numbers of hedgers and speculators in the futures market respectively. Combining Equations (A2) and (A3) and using equilibrium condition, which the combination equates zero ($y_H + y_S = 0$), give us

$$0 = \frac{(\lambda_H + \lambda_S)\Pi}{\lambda_H \lambda_S \operatorname{var}(\tilde{\Pi})} - \frac{\operatorname{cov}(\tilde{\Pi}, \tilde{G} + (x_H + x_S)\tilde{R}_m + (z_H + z_S)\tilde{R}_o)}{\operatorname{var}(\tilde{\Pi})}$$

Multiplying the equality by $\frac{\lambda_H \lambda_S}{\lambda_H + \lambda_S} \operatorname{var}(\tilde{\Pi})$ yields

$$\Pi^* = E(\tilde{f}) - f = \theta \operatorname{cov}(\tilde{\Pi}, b\bar{g} + \bar{x}\tilde{R}_m + \bar{z}\tilde{R}_o) \qquad (9)$$

where $\theta = \frac{(\lambda_H \lambda_S)/(\lambda_H + \lambda_S)}{H + S}$; $b = \frac{H}{H + S}$; $\bar{g} = \frac{1}{H} \sum_{h=1}^{H} \tilde{g}_h$; $\bar{x} = \frac{x_H + x_S}{H + S}$; $\bar{z} = \frac{z_H + z_S}{H + S}$.

Rearranging (9) yields

$$f^* = E(\tilde{f}) - \theta \operatorname{cov}(\tilde{\Pi}, b\bar{g} + \bar{x}\tilde{R}_m + \bar{z}\tilde{R}_o) \qquad (8)$$

3. Proof of Proposition 1

From Equation (9) dividing both side with futures price *f* give us,

$$\pi^{*} = \theta \operatorname{cov}(\tilde{\pi}, b\bar{g} + \bar{x}\tilde{R}_m + \bar{z}\tilde{R}_o) \tag{A4}$$

$$\pi^* = \theta\, b \operatorname{cov}(\tilde{\pi}, \bar{g}) + \theta\, \bar{x} \operatorname{cov}(\tilde{\pi}, \tilde{R}_m) + \theta\, \bar{z} \operatorname{cov}(\tilde{\pi}, \tilde{R}_o) \tag{10}$$

Assume the following models, which systematically relate the asset returns to the stock market return:

$$\begin{aligned} \tilde{\pi} &= \alpha_{\pi m} + \beta_{\pi m}\tilde{R}_m + \varepsilon_{\pi m} \\ \bar{g} &= \alpha_{gm} + \beta_{gm}\tilde{R}_m + \varepsilon_{gm} \\ \tilde{R}_o &= \alpha_{om} + \beta_{om}\tilde{R}_m + \varepsilon_{om} \end{aligned} \tag{11}$$

Substitute the relations in Equation (11) into (10) and assume that $\operatorname{cov}(\tilde{R}_m, \varepsilon_{om}) = 0$; $\operatorname{cov}(\tilde{R}_m, \varepsilon_{rm}) = 0$; $\operatorname{cov}(\tilde{R}_m, \varepsilon_{\pi m}) = 0$.

$$\pi = \theta\, b \operatorname{cov}(\alpha_{\pi m} + \beta_{\pi m}\tilde{R}_m + \varepsilon_{\pi m}, \alpha_{gm} + \beta_{gm}\tilde{R}_m + \varepsilon_{gm})$$

$$+\theta \bar{x} \operatorname{cov}(\alpha_{\pi m} + \beta_{\pi m}\tilde{R}_m + \varepsilon_{\pi m}, \tilde{R}_m) + \theta \bar{z} \operatorname{cov}(\alpha_{\pi m} + \beta_{\pi m}\tilde{R}_m + \varepsilon_{\pi m}, \alpha_{om} + \beta_{om}\tilde{R}_m + \varepsilon_{om})$$

$$\pi = \theta b[\beta_{\pi m}\beta_{gm} \operatorname{var}(\tilde{R}_m) + \sigma_{\pi m}\sigma_{gm}\operatorname{corr}(\varepsilon_{\pi m}, \varepsilon_{gm})] \tag{A5}$$

$$+\theta \bar{x}\beta_{\pi m} \operatorname{var}(\tilde{R}_m) + \theta \bar{z}[\beta_{om}\beta_{\pi m} \operatorname{var}(\tilde{R}_m) + \sigma_{om}\sigma_{\pi m}\operatorname{corr}(\varepsilon_{\pi m}\varepsilon_{om})$$

Rearranging (A5) gives

$$\pi = \theta \operatorname{var}(\tilde{R}_m)(\bar{x} + \bar{z}\beta_{om} + b\beta_{gm})\beta_{\pi m} \tag{12}$$

$$+[\theta\, \bar{z}\sigma(\varepsilon_{om})\operatorname{corr}(\varepsilon_{\pi m}, \varepsilon_{om}) + \theta\, b\sigma(\varepsilon_{gm})\operatorname{corr}(\varepsilon_{\pi m}, \varepsilon_{gm})]\sigma(\varepsilon_{\pi m})$$

This completes the proof of Proposition 1.

4. Proof of Proposition 2

Assume cov(g,π) ≠ 0 and refer to the FOCs in Equations (3)-(5). Since speculators have no revenue related to the variation in futures prices, g = 0, the optimal positions of a speculator can be written as

$$x_s = \frac{E(\tilde{R}_m)}{\lambda_s \sigma_m^2} - y_s \beta_{\pi m} - z_s \beta_{om} \tag{A6}$$

$$y_s = \frac{\pi}{\lambda_s \sigma_\pi^2} - x_s \beta_{m\pi} - z_s \beta_{o\pi} \tag{A7}$$

$$z_s = \frac{E(\tilde{R}_o)}{\lambda_s \sigma_o^2} - x_s \beta_{mo} - y_s \beta_{\pi o} \tag{A8}$$

where $\beta_{\pi m}$, $\beta_{\pi o}$, $\beta_{o\pi}$ and β_{mo} denote appropriately defined betas: β_{ab} = cov(a,b)/var(b) for random variables a and b and $\sigma_m^2 = \text{var}(\tilde{R}_m); \sigma_\pi^2 = \text{var}(\pi); \sigma_o^2 = \text{var}(\tilde{R}_o)$ It is easy to show that the optimal positions of outsiders who do not trade derivatives are $x_{r2} = E(\tilde{R}_m)/\lambda\sigma_m^2$; y_{r2} = 0 ; z_{r2} = 0. ρ_{ab} denotes the correlation coefficient of a and b: $\rho_{ab}^2 = \beta_{ab}\beta_{ba}$.

Step1) The speculator's positions in terms of expectations and covariances

1.1) Solve for y_s . First, rewrite x_s in term of y_s by substituting (A8) into (A6)

$$x_s = \frac{E(\tilde{R}_m)}{\lambda_s \sigma_m^2} - y_s \beta_{\pi m} - \left(\frac{E(\tilde{R}_o)}{\lambda_s \sigma_o^2} - x_s \beta_{mo} - y_s \beta_{\pi o} \right) \beta_{om}$$

$$x_s = \frac{E(\tilde{R}_m)}{\lambda_s \sigma_m^2} - y_s \beta_{\pi m} - \beta_{mo} \frac{E(\tilde{R}_o)}{\lambda \sigma_m^2} + x_s \beta_{mo}\beta_{om} + y_s \beta_{\pi o}\beta_{om}$$

$$(1-\rho_{mo}^2)x_s = \frac{E(\tilde{R}_m) - \beta_{mo}E(\tilde{R}_o)}{\lambda_s \sigma_m^2} - y_s(\beta_{\pi m} - \beta_{\pi o}\beta_{om})$$

$$x_s = \frac{E(\tilde{R}_m) - \beta_{mo}E(\tilde{R}_o)}{\lambda_s \sigma_m^2 (1-\rho_{mo}^2)} - \frac{y_s(\beta_{\pi m} - \beta_{\pi o}\beta_{om})}{(1-\rho_{mo}^2)} \tag{A9}$$

Rewrite z_s in term of y_s by substituting (A9) into (A8).

$$z_s = \frac{E(\tilde{R}_o)}{\lambda_s \sigma_o^2} - y_s \beta_{\pi o} - \left(\frac{E(\tilde{R}_m) - \beta_{mo} E(\tilde{R}_o)}{\lambda_s \sigma_m^2 (1-\rho_{mo}^2)} - \frac{y_s (\beta_{\pi m} - \beta_{\pi o} \beta_{om})}{(1-\rho_{mo}^2)} \right) \beta_{mo}$$

$$z_s = \frac{E(\tilde{R}_o)}{\lambda_s \sigma_o^2} - y_s \beta_{\pi o} - \frac{\beta_{mo} E(\tilde{R}_m)}{\lambda_s \sigma_m^2 (1-\rho_{mo}^2)} + \frac{\beta_{om} \beta_{mo} E(\tilde{R}_o)}{\lambda_s \sigma_o^2 (1-\rho_{mo}^2)} + \frac{y_s (\beta_{mo} \beta_{\pi m} - \beta_{\pi o} \beta_{om} \beta_{mo})}{(1-\rho_{mo}^2)}$$

$$z_s = \frac{E(\tilde{R}_o)}{\lambda_s \sigma_o^2} \left(1 + \frac{\rho_{mo}^2}{(1-\rho_{mo}^2)} \right) - \frac{\beta_{mo} E(\tilde{R}_m)}{\lambda_s \sigma_m^2 (1-\rho_{mo}^2)} + \frac{y_s}{(1-\rho_{mo}^2)} (-\beta_{\pi o}(1-\rho_{mo}^2) + \beta_{mo} \beta_{\pi m} - \beta_{\pi o} \rho_{mo}^2)$$

$$z_s = \frac{E(\tilde{R}_o)}{\lambda_s \sigma_o^2 (1-\rho_{mo}^2)} - \frac{\beta_{mo} E(\tilde{R}_m)}{\lambda_s \sigma_m^2 (1-\rho_{mo}^2)} + \frac{y_s}{(1-\rho_{mo}^2)} (\beta_{mo} \beta_{\pi m} - \beta_{\pi o}) \tag{A10}$$

Replace x_s and z_s in Equation (A7) by the right-hand side of Equations (A9) and (A10) and solve for y_s.

$$y_s = \frac{\pi}{\lambda_s \sigma_\pi^2} - \left(\frac{E(\tilde{R}_m) - \beta_{mo} E(\tilde{R}_o)}{\lambda_s \sigma_m^2 (1-\rho_{mo}^2)} - \frac{y_s (\beta_{\pi m} - \beta_{\pi o} \beta_{om})}{(1-\rho_{mo}^2)} \right) \beta_{m\pi}$$
$$- \left(\frac{E(\tilde{R}_o)}{\lambda_s \sigma_o^2 (1-\rho_{mo}^2)} - \frac{\beta_{mo} E(\tilde{R}_m)}{\lambda_s \sigma_m^2 (1-\rho_{mo}^2)} + \frac{y_s (\beta_{mo} \beta_{\pi m} - \beta_{\pi o})}{(1-\rho_{mo}^2)} \right) \beta_{o\pi}$$

$$y_s = \frac{\pi}{\lambda_s \sigma_\pi^2} - \frac{\beta_{m\pi} E(\tilde{R}_m)}{\lambda_s \sigma_m^2 (1-\rho_{mo}^2)} + \frac{\beta_{m\pi} \beta_{mo} E(\tilde{R}_o)}{\lambda_s \sigma_m^2 (1-\rho_{mo}^2)} + \frac{y_s (\rho_{m\pi}^2 - \beta_{\pi o} \beta_{om} \beta_{m\pi})}{(1-\rho_{mo}^2)}$$
$$- \frac{\beta_{o\pi} E(\tilde{R}_o)}{\lambda_s \sigma_o^2 (1-\rho_{mo}^2)} + \frac{\beta_{o\pi} \beta_{mo} E(\tilde{R}_m)}{\lambda_s \sigma_m^2 (1-\rho_{mo}^2)} - \frac{y_s (\beta_{o\pi} \beta_{mo} \beta_{\pi m} - \rho_{\pi o}^2)}{(1-\rho_{mo}^2)}$$

$$y_s \left(1 - \frac{(\rho_{m\pi}^2 + \rho_{\pi o}^2 - 2\beta_{\pi o} \beta_{om} \beta_{m\pi})}{(1-\rho_{mo}^2)} \right) = \frac{\pi}{\lambda_s \sigma_\pi^2} - \frac{\beta_{m\pi} E(\tilde{R}_m)}{\lambda_s \sigma_m^2 (1-\rho_{mo}^2)} + \frac{\beta_{m\pi} \beta_{mo} E(\tilde{R}_o)}{\lambda_s \sigma_m^2 (1-\rho_{mo}^2)}$$
$$- \frac{\beta_{o\pi} E(\tilde{R}_o)}{\lambda_s \sigma_o^2 (1-\rho_{mo}^2)} + \frac{\beta_{o\pi} \beta_{mo} E(\tilde{R}_m)}{\lambda_s \sigma_m^2 (1-\rho_{mo}^2)}$$

$$y_s \left(1 - \rho_{\pi o}^2 - \rho_{\pi m}^2 - \rho_{mo}^2 + 2\beta_{mo} \beta_{\pi m} \beta_{o\pi} \right) = \frac{\pi(1-\rho_{mo}^2)}{\lambda_s \sigma_\pi^2} - \frac{\beta_{\pi m} E(\tilde{R}_m)}{\lambda_s \sigma_\pi^2} + \frac{\beta_{mo} \beta_{\pi m} E(\tilde{R}_o)}{\lambda_s \sigma_\pi^2}$$
$$- \frac{\beta_{\pi o} E(\tilde{R}_o)}{\lambda_s \sigma_\pi^2} + \frac{\beta_{om} \beta_{\pi o} E(\tilde{R}_m)}{\lambda_s \sigma_\pi^2}$$

$$y_s = \frac{\pi(1-\rho_{mo}^2) - (\beta_{\pi m} - \beta_{\pi o} \beta_{om}) E(\tilde{R}_m) - (\beta_{\pi o} - \beta_{\pi m} \beta_{mo}) E(\tilde{R}_o)}{\lambda_s \sigma_\pi^2 \left(1 - \rho_{\pi o}^2 - \rho_{\pi m}^2 - \rho_{mo}^2 + 2\beta_{mo} \beta_{\pi m} \beta_{o\pi} \right)} \tag{A11}$$

1.2) Solve for z_s , similar to step (1.1).

$$z_s = \frac{E(\tilde{R}_o)(1-\rho_{m\pi}^2) - (\beta_{om} - \beta_{o\pi} \beta_{\pi m}) E(\tilde{R}_m) - (\beta_{o\pi} - \beta_{om} \beta_{m\pi}) \pi}{\lambda_s \sigma_o^2 \left(1 - \rho_{\pi o}^2 - \rho_{\pi m}^2 - \rho_{mo}^2 + 2\beta_{mo} \beta_{\pi m} \beta_{o\pi} \right)} \tag{A12}$$

1.3) Solve for x_s, similar to step (1.1).

$$x_s = \frac{E(\tilde{R}_m)(1-\rho_{\pi o}^2) - (\beta_{mo} - \beta_{m\pi}\beta_{\pi o})E(\tilde{R}_o) - (\beta_{m\pi} - \beta_{mo}\beta_{o\pi})\pi}{\lambda_s \sigma_m^2 \left(1 - \rho_{\pi o}^2 - \rho_{\pi m}^2 - \rho_{mo}^2 + 2\beta_{mo}\beta_{\pi m}\beta_{o\pi}\right)} \qquad \text{(A13)}$$

Step 2) The futures market equilibrium condition

The following notations are used to simplify the terms in Equations (A11)-(A13).

$$\gamma = 1 - \rho_{om}^2 - \rho_{o\pi}^2 - \rho_{\pi m}^2 + 2\beta_{mo}\beta_{o\pi}\beta_{\pi m} \; ;$$

$$c_1 = (1 - \rho_{mo}^2);$$

$$d_1 = -(\beta_{\pi m} - \beta_{\pi o}\beta_{om})E(\tilde{R}_m) - (\beta_{\pi o} - \beta_{\pi m}\beta_{mo})E(\tilde{R}_o);$$

$$c_2 = -(\beta_{o\pi} - \beta_{om}\beta_{m\pi});$$

$$d_2 = E(\tilde{R}_o)(1 - \rho_{m\pi}^2) - (\beta_{om} - \beta_{o\pi}\beta_{\pi m})E(\tilde{R}_m);$$

$$c_3 = -(\beta_{m\pi} - \beta_{mo}\beta_{o\pi});$$

$$d_3 = E(\tilde{R}_m)(1 - \rho_{\pi o}^2) - (\beta_{mo} - \beta_{m\pi}\beta_{\pi o})E(\tilde{R}_o).$$

Thus Equations (A11), (A12), and (A13) can be rewritten as

$$y_s = \frac{c_1\pi + d_1}{\gamma\lambda_s\sigma_\pi^2} \qquad \text{(A14)}$$

$$z_s = \frac{c_2\pi + d_2}{\gamma\lambda_s\sigma_o^2} \qquad \text{(A15)}$$

$$x_s = \frac{c_3\pi + d_3}{\gamma\lambda_s\sigma_m^2} \qquad \text{(A16)}$$

where $\gamma = 1 - \rho_{om}^2 - \rho_{o\pi}^2 - \rho_{\pi m}^2 + 2\beta_{mo}\beta_{o\pi}\beta_{\pi m}$.

In the futures market equilibrium, the marginal speculator has indifferent satisfaction either trading derivatives or refraining from trade. This condition is formalized as

$$E(\widetilde{U}_s;trade) = E(\widetilde{U}_{r2};refrain) \tag{13}$$

Recall Equations (1), (2s) and (2q):

$$E(\widetilde{U}_s) = E[a_s - t + x_s\widetilde{R}_m + y_s\widetilde{\Pi} + z_s\widetilde{R}_o] - \frac{\lambda_s}{2}\mathrm{var}[a_s - t + x_s\widetilde{R}_m + y_s\widetilde{\Pi} + z_s\widetilde{R}_o]$$

$$E(\widetilde{U}_{r2}) = E[a_{r2} + x_{r2}\widetilde{R}_m] - \frac{\lambda_{r2}}{2}\mathrm{var}[a_{r2} + x_{r2}\widetilde{R}_m]$$

Equating the right-hand sides of these two equalities, according to (13), gives us

$$\begin{aligned} 0 = &-t + y_s\pi + (x_s - x_{r2})E(\widetilde{R}_m) + z_sE(\widetilde{R}_o) \\ &- \frac{\lambda_s}{2}\left[y_s^2\sigma_\pi^2 + z_s^2\sigma_o^2 + (x_s^2 - x_r^2)\sigma_m^2 + 2y_sz_s\sigma_{\pi o} + 2x_sy_s\sigma_{m\pi} + 2x_sz_s\sigma_{mo}\right] \end{aligned} \tag{A17}$$

Note: a_s is equal to a_{r2} since it belongs to the marginal speculator. Let λ' denote the risk-aversion coefficient of the marginal speculator, (i.e., $\lambda' = \lambda_s = \lambda_{r2}$). $\sigma_m^2 = \mathrm{var}(\widetilde{R}_m); \sigma_\pi^2 = \mathrm{var}(\pi); \sigma_o^2 = \mathrm{var}(\widetilde{R}_o); \sigma_{m\pi} = \mathrm{cov}(\widetilde{R}_m, \pi)$ $\sigma_{mo} = \mathrm{cov}(\widetilde{R}_m, \widetilde{R}_o)$

Substituting the optimal positions of speculators (i.e., x_s, y_s, z_s) and those of outside investors (i.e., x_{r2}, y_{r2}, z_{r2}) into (A17) gives

$$\begin{aligned} 0 = &-t + \left(\frac{c_1\pi + d_1}{\gamma\lambda'\sigma_\pi^2}\right)\pi + \left(\frac{c_3\pi + d_3}{\gamma\lambda'\sigma_m^2} - \frac{E(\widetilde{R}_m)}{\lambda'\sigma_m^2}\right)E(\widetilde{R}_m) + \left(\frac{c_2\pi + d_2}{\gamma\lambda'\sigma_o^2}\right)E(\widetilde{R}_o) \\ &- \frac{\lambda'}{2}\left(\frac{(c_1\pi + d_1)^2}{\gamma^2\lambda'^2\sigma_\pi^4}\right)\sigma_\pi^2 - \frac{\lambda'}{2}\left(\frac{(c_2\pi + d_2)^2}{\gamma^2\lambda'^2\sigma_o^4}\right)\sigma_o^2 - \frac{\lambda'}{2}\left(\frac{(c_3\pi + d_3)^2}{\gamma^2\lambda'^2\sigma_m^4}\right)\sigma_m^2 + \frac{\lambda'}{2}\frac{E(\widetilde{R}_m)}{\lambda'^2\sigma_m^4}\sigma_m^2 \end{aligned}$$

$$-\lambda'\left(\frac{c_1\pi+d_1}{\gamma\lambda'\sigma_\pi^2}\right)\left(\frac{c_2\pi+d_2}{\gamma\lambda'\sigma_o^2}\right)\sigma_{\pi o}-\lambda'\left(\frac{c_1\pi+d_1}{\gamma\lambda'\sigma_\pi^2}\right)\left(\frac{c_3\pi+d_3}{\gamma\lambda'\sigma_m^2}\right)\sigma_{\pi m}-\lambda'\left(\frac{c_2\pi+d_2}{\gamma\lambda'\sigma_o^2}\right)\left(\frac{c_3\pi+d_3}{\gamma\lambda'\sigma_m^2}\right)\sigma_{om}$$

Multiplying with λ' both sides of this equation gives

$$0=-\lambda' t-\frac{1}{2}\frac{E(\widetilde{R}_m)}{\sigma_m^2}+\frac{c_1\pi^2}{\gamma\sigma_\pi^2}+\frac{d_1\pi}{\gamma\sigma_\pi^2}+\frac{c_3E(\widetilde{R}_m)\pi}{\gamma\sigma_m^2}+\frac{d_3E(\widetilde{R}_m)}{\gamma\sigma_m^2}+\frac{c_2E(\widetilde{R}_o)\pi}{\gamma\sigma_o^2} \tag{A18}$$

$$+\frac{d_2E(\widetilde{R}_o)}{\gamma\sigma_o^2}-\frac{c_1^2\pi^2+2c_1d_1\pi+d_1^2}{2\gamma^2\sigma_\pi^2}-\frac{c_2^2\pi^2+2c_2d_2\pi+d_2^2}{2\gamma^2\sigma_o^2}-\frac{c_3^2\pi^2+2c_3d_3\pi+d_3^2}{2\gamma^2\sigma_m^2}$$

$$-\frac{\left(c_1c_2\pi^2+(c_1d_2+c_2d_1)\pi+d_1d_2\right)\sigma_{\pi o}}{\gamma^2\sigma_\pi^2\sigma_o^2}-\frac{\left(c_1c_3\pi^2+(c_1d_3+c_3d_1)\pi+d_1d_3\right)\sigma_{\pi m}}{\gamma^2\sigma_\pi^2\sigma_m^2}$$

$$-\frac{\left(c_2c_3\pi^2+(c_2d_3+c_3d_2)\pi+d_2d_3\right)\sigma_{om}}{\gamma^2\sigma_o^2\sigma_m^2}$$

Step 3) Solve for futures risk premium (in percentage)

Arrange Equation (A18) in a quadratic form, i.e.,

$$0=\mathrm{A}\pi^2+\mathrm{B}\pi+\mathrm{C} \tag{A19}$$

3.1) Solve for A

$$\mathrm{A}=\frac{c_1}{\gamma\sigma_\pi^2}-\frac{c_1^2}{2\gamma^2\sigma_\pi^2}-\frac{c_2^2}{2\gamma^2\sigma_o^2}-\frac{c_3^2}{2\gamma^2\sigma_m^2}-\frac{c_1c_2\sigma_{\pi o}}{\gamma^2\sigma_\pi^2\sigma_o^2}-\frac{c_1c_3\sigma_{\pi m}}{\gamma^2\sigma_\pi^2\sigma_m^2}-\frac{c_2c_3\sigma_{om}}{\gamma^2\sigma_o^2\sigma_m^2}$$

$$\mathrm{A}=\frac{(1-\rho_{mo}^2)}{\gamma\sigma_\pi^2}-\frac{(1-\rho_{mo}^2)^2}{2\gamma^2\sigma_\pi^2}-\frac{(\beta_{o\pi}-\beta_{om}\beta_{m\pi})^2}{2\gamma^2\sigma_o^2}-\frac{(\beta_{m\pi}-\beta_{mo}\beta_{o\pi})^2}{2\gamma^2\sigma_m^2}+\frac{(1-\rho_{mo}^2)(\beta_{o\pi}-\beta_{om}\beta_{m\pi})\sigma_{\pi o}}{\gamma^2\sigma_\pi^2\sigma_o^2}$$

$$+\frac{(1-\rho_{mo}^2)(\beta_{m\pi}-\beta_{mo}\beta_{o\pi})\sigma_{\pi m}}{\gamma^2\sigma_\pi^2\sigma_m^2}-\frac{(\beta_{o\pi}-\beta_{om}\beta_{m\pi})(\beta_{m\pi}-\beta_{mo}\beta_{o\pi})\sigma_{om}}{\gamma^2\sigma_o^2\sigma_m^2}$$

$$\mathrm{A}=\frac{(1-\rho_{mo}^2)}{\gamma\sigma_\pi^2}-\frac{(1-\rho_{mo}^2)^2}{2\gamma^2\sigma_\pi^2}-\frac{(\beta_{o\pi}-\beta_{om}\beta_{m\pi})(\beta_{\pi o}-\beta_{mo}\beta_{\pi m})}{2\gamma^2\sigma_\pi^2}-\frac{(\beta_{m\pi}-\beta_{mo}\beta_{o\pi})(\beta_{\pi m}-\beta_{om}\beta_{\pi o})}{2\gamma^2\sigma_\pi^2}$$

$$+\frac{(1-\rho_{mo}^2)(\beta_{o\pi}-\beta_{om}\beta_{m\pi})\beta_{\pi o}}{\gamma^2\sigma_\pi^2}+\frac{(1-\rho_{mo}^2)(\beta_{m\pi}-\beta_{mo}\beta_{o\pi})\beta_{\pi m}}{\gamma^2\sigma_\pi^2}$$

$$-\frac{(\beta_{o\pi}-\beta_{om}\beta_{m\pi})(\beta_{\pi m}-\beta_{om}\beta_{\pi o})\beta_{mo}}{\gamma^2\sigma_\pi^2}$$

Multiplying with $2\gamma^2\sigma_\pi^2$ both sides of this equation gives

$$2\gamma^2\sigma_\pi^2 \mathrm{A} = 2\gamma(1-\rho_{mo}^2) - (1-\rho_{mo}^2)^2 - (\beta_{o\pi}-\beta_{om}\beta_{m\pi})(\beta_{\pi o}-\beta_{mo}\beta_{\pi m}) - (\beta_{m\pi}-\beta_{mo}\beta_{o\pi})(\beta_{\pi m}-\beta_{om}\beta_{\pi o})$$
$$+2(1-\rho_{mo}^2)(\beta_{o\pi}-\beta_{om}\beta_{m\pi})\beta_{\pi o} + 2(1-\rho_{mo}^2)(\beta_{m\pi}-\beta_{mo}\beta_{o\pi})\beta_{\pi m}$$
$$-2(\beta_{o\pi}-\beta_{om}\beta_{m\pi})(\beta_{\pi m}-\beta_{om}\beta_{\pi o})\beta_{mo}$$
$$2\gamma^2\sigma_\pi^2 \mathrm{A} = (1-\rho_{mo}^2)\left(1-\rho_{mo}^2-\rho_{\pi o}^2-\rho_{m\pi}^2+2\beta_{m\pi}\beta_{om}\beta_{\pi o}\right)$$

$$\mathrm{A} = \frac{(1-\rho_{mo}^2)}{2\gamma\sigma_\pi^2} \qquad \text{(A20)}$$

3.2) Solve for B

$$\mathrm{B} = \frac{d_1}{\gamma\sigma_\pi^2} + \frac{c_3 E(\tilde{R}_m)}{\gamma\sigma_m^2} + \frac{c_2 E(\tilde{R}_o)}{\gamma\sigma_o^2} - \frac{c_1 d_1}{\gamma^2\sigma_\pi^2} - \frac{c_2 d_2}{\gamma^2\sigma_o^2} - \frac{c_3 d_3}{\gamma^2\sigma_m^2} - \frac{(c_1 d_2 + c_2 d_1)\sigma_{\pi o}}{\gamma^2\sigma_\pi^2\sigma_o^2} - \frac{(c_1 d_3 + c_3 d_1)\sigma_{\pi m}}{\gamma^2\sigma_\pi^2\sigma_m^2} - \frac{(c_2 d_3 + c_3 d_2)\sigma_{om}}{\gamma^2\sigma_o^2\sigma_m^2}$$

$$\mathrm{B} = \frac{d_1}{\gamma^2\sigma_\pi^2}(\gamma - c_1 - c_2\beta_{\pi o} - c_3\beta_{\pi m})$$
$$+\frac{c_2 E(\tilde{R}_o)}{\gamma\sigma_o^2} - \frac{d_2}{\gamma^2\sigma_o^2}(c_2 + c_1\beta_{o\pi} + c_3\beta_{om}) + \frac{c_3 E(\tilde{R}_m)}{\gamma\sigma_m^2} - \frac{d_3}{\gamma^2\sigma_m^2}(c_3 + c_1\beta_{m\pi} + c_2\beta_{mo})$$

Substituting c_1, c_2, c_3 into this equation gives

$$\mathrm{B} = \frac{d_1}{\gamma^2\sigma_\pi^2}(\gamma - 1 + \rho_{mo}^2 + (\beta_{o\pi}-\beta_{om}\beta_{m\pi})\beta_{\pi o} + (\beta_{m\pi}-\beta_{mo}\beta_{o\pi})\beta_{\pi m})$$
$$+\frac{-(\beta_{o\pi}-\beta_{om}\beta_{m\pi})E(\tilde{R}_o)}{\gamma\sigma_o^2} - \frac{d_2}{\gamma^2\sigma_o^2}\left(-(\beta_{o\pi}-\beta_{om}\beta_{m\pi}) + (1-\rho_{mo}^2)\beta_{o\pi} - (\beta_{m\pi}-\beta_{mo}\beta_{o\pi})\beta_{om}\right)$$
$$+\frac{-(\beta_{m\pi}-\beta_{mo}\beta_{o\pi})E(\tilde{R}_m)}{\gamma\sigma_m^2} - \frac{d_3}{\gamma^2\sigma_m^2}\left(-(\beta_{m\pi}-\beta_{mo}\beta_{o\pi}) + (1-\rho_{mo}^2)\beta_{m\pi} - (\beta_{o\pi}-\beta_{om}\beta_{m\pi})\beta_{mo}\right)$$
$$\mathrm{B} = -\frac{(\beta_{o\pi}-\beta_{om}\beta_{m\pi})E(\tilde{R}_o)}{\gamma\sigma_o^2} - \frac{(\beta_{m\pi}-\beta_{mo}\beta_{o\pi})E(\tilde{R}_m)}{\gamma\sigma_m^2}$$

$$\mathrm{B} = -\frac{(\beta_{\pi o}-\beta_{mo}\beta_{\pi m})E(\tilde{R}_o) + (\beta_{\pi m}-\beta_{om}\beta_{\pi o})E(\tilde{R}_m)}{\gamma\sigma_\pi^2} \qquad \text{(A21)}$$

3.3) Solve for C

$$\mathrm{C} = -\lambda' t - \frac{1}{2}\frac{[E(\tilde{R}_m)]^2}{\sigma_m^2} + \frac{d_3[E(\tilde{R}_m)]}{\gamma\sigma_m^2} + \frac{d_2[E(\tilde{R}_o)]}{\gamma\sigma_o^2} - \frac{d_1^2}{2\gamma^2\sigma_\pi^2} - \frac{d_2^2}{2\gamma^2\sigma_o^2} - \frac{d_3^2}{2\gamma^2\sigma_m^2}$$
$$-\frac{d_1 d_2\sigma_{\pi o}}{\gamma^2\sigma_\pi^2\sigma_o^2} - \frac{d_1 d_3\sigma_{\pi m}}{\gamma^2\sigma_\pi^2\sigma_m^2} - \frac{d_2 d_3\sigma_{om}}{\gamma^2\sigma_o^2\sigma_m^2}$$
$$\mathrm{C} = -\lambda' t - \frac{1}{2}\frac{[E(\tilde{R}_m)]^2}{\sigma_m^2} + \frac{\left(E(\tilde{R}_m)(1-\rho_{\pi o}^2) - (\beta_{mo}-\beta_{m\pi}\beta_{\pi o})E(\tilde{R}_o)\right)[E(\tilde{R}_m)]}{\gamma\sigma_m^2}$$

$$+\frac{\left(E(\tilde{R}_o)(1-\rho_{m\pi}^2)-(\beta_{om}-\beta_{o\pi}\beta_{\pi m})E(\tilde{R}_m)\right)[E(\tilde{R}_o)]}{\gamma\sigma_o^2}-\frac{\left(-(\beta_{\pi m}-\beta_{\pi o}\beta_{om})E(\tilde{R}_m)-(\beta_{\pi o}-\beta_{\pi m}\beta_{mo})E(\tilde{R}_o)\right)^2}{2\gamma^2\sigma_\pi^2}$$

$$-\frac{\left(E(\tilde{R}_o)(1-\rho_{m\pi}^2)-(\beta_{om}-\beta_{o\pi}\beta_{\pi m})E(\tilde{R}_m)\right)^2}{2\gamma^2\sigma_o^2}-\frac{\left(E(\tilde{R}_m)(1-\rho_{\pi o}^2)-(\beta_{mo}-\beta_{m\pi}\beta_{\pi o})E(\tilde{R}_o)\right)^2}{2\gamma^2\sigma_m^2}$$

$$-\frac{\left(-(\beta_{\pi m}-\beta_{\pi o}\beta_{om})E(\tilde{R}_m)-(\beta_{\pi o}-\beta_{\pi m}\beta_{mo})E(\tilde{R}_o)\right)\left(E(\tilde{R}_o)(1-\rho_{m\pi}^2)-(\beta_{om}-\beta_{o\pi}\beta_{\pi m})E(\tilde{R}_m)\right)\sigma_{\pi o}}{\gamma^2\sigma_\pi^2\sigma_o^2}$$

$$-\frac{\left(-(\beta_{\pi m}-\beta_{\pi o}\beta_{om})E(\tilde{R}_m)-(\beta_{\pi o}-\beta_{\pi m}\beta_{mo})E(\tilde{R}_o)\right)\left(E(\tilde{R}_m)(1-\rho_{\pi o}^2)-(\beta_{mo}-\beta_{m\pi}\beta_{\pi o})E(\tilde{R}_o)\right).\sigma_{\pi m}}{\gamma^2\sigma_\pi^2\sigma_m^2}$$

$$-\frac{\left(E(\tilde{R}_o)(1-\rho_{m\pi}^2)-(\beta_{om}-\beta_{o\pi}\beta_{\pi m})E(\tilde{R}_m)\right)\left(E(\tilde{R}_m)(1-\rho_{\pi o}^2)-(\beta_{mo}-\beta_{m\pi}\beta_{\pi o})E(\tilde{R}_o)\right)\sigma_{om}}{\gamma^2\sigma_o^2\sigma_m^2}$$

Arrange C in a form of quadratic equation as

$$C=-\lambda' t+D[E(\tilde{R}_m)]^2+F[E(\tilde{R}_m)E(\tilde{R}_o)]+G[E(\tilde{R}_o)]^2$$

Then D, F, and G can be solved as follows.

$$D=\frac{(\beta_{\pi m}-\beta_{\pi o}\beta_{om})^2}{2\gamma\sigma_\pi^2(1-\rho_{mo}^2)}$$

$$F=\frac{(\beta_{\pi m}-\beta_{\pi o}\beta_{om})(\beta_{\pi o}-\beta_{\pi m}\beta_{mo})}{\gamma\sigma_\pi^2(1-\rho_{mo}^2)}$$

$$G=\frac{(\beta_{\pi o}-\beta_{\pi m}\beta_{mo})^2}{2\gamma\sigma_\pi^2(1-\rho_{mo}^2)}$$

$$C=-\lambda' t+\frac{(\beta_{\pi m}-\beta_{\pi o}\beta_{om})^2}{2\gamma\sigma_\pi^2(1-\rho_{mo}^2)}[E(\tilde{R}_m)]^2+\frac{(\beta_{\pi m}-\beta_{\pi o}\beta_{om})(\beta_{\pi o}-\beta_{\pi m}\beta_{mo})}{\gamma\sigma_\pi^2(1-\rho_{mo}^2)}E(\tilde{R}_m)E(\tilde{R}_o)+\frac{(\beta_{\pi o}-\beta_{\pi m}\beta_{mo})^2}{2\gamma\sigma_\pi^2(1-\rho_{mo}^2)}[E(\tilde{R}_o)]^2$$

$$C=-\lambda' t+\frac{[(\beta_{\pi m}-\beta_{\pi o}\beta_{om})E(\tilde{R}_m)+(\beta_{\pi o}-\beta_{\pi m}\beta_{mo})E(\tilde{R}_o)]^2}{2\gamma\sigma_\pi^2(1-\rho_{mo}^2)} \tag{A22}$$

To summarize the futures risk premium in the quadratic equation (A19):

$$0=A\pi^2+B\pi+C \tag{A19}$$

where

$$A=\frac{(1-\rho_{mo}^2)}{2\gamma\sigma_\pi^2} \tag{A20}$$

$$B=-\frac{(\beta_{\pi o}-\beta_{mo}\beta_{\pi m})E(\tilde{R}_o)+(\beta_{\pi m}-\beta_{om}\beta_{\pi o})E(\tilde{R}_m)}{\gamma\sigma_\pi^2} \tag{A21}$$

$$C = -\lambda' t + \frac{[(\beta_{\pi m} - \beta_{\pi o}\beta_{om})E(\tilde{R}_m) + (\beta_{\pi o} - \beta_{\pi m}\beta_{mo})E(\tilde{R}_o)]^2}{2\gamma\sigma_\pi^2(1-\rho_{mo}^2)} \tag{A22}$$

Substituting A, B and C into Equation (A19) and solving the equation yields

$$\pi = \frac{(\beta_{\pi m} - \beta_{\pi o}\beta_{om})}{1-\rho_{om}^2} E(\tilde{R}_m) + \frac{(\beta_{\pi o} - \beta_{\pi m}\beta_{mo})}{1-\rho_{om}^2} E(\tilde{R}_o) \pm \sigma_\pi \sqrt{\frac{2\lambda t\gamma}{1-\rho_{om}^2}} \tag{A23}$$

Equation (A23) can be rewritten as

$$\pi = \beta' E(\tilde{R}_m) + d\,\sigma' \sqrt{2\lambda' t} + \Gamma E(\tilde{R}_o) \tag{14}$$

where $\beta' = \frac{(\beta_{\pi m} - \beta_{\pi o}\beta_{om})}{1-\rho_{om}^2}$; $\sigma' = \sigma_\pi \sqrt{\frac{\gamma}{1-\rho_{om}^2}}$; $\Gamma = \frac{(\beta_{\pi o} - \beta_{\pi m}\beta_{mo})}{1-\rho_{om}^2}$; d denotes an indicator of net futures positions of hedgers, which equals to 1 for net long positions and -1 for net short positions.

This completes the proof of Proposition 2.

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CHAPTER 5

FUTURES RISK PREMIUM IN THE PRESENCE OF DELIVERY RISK: EVIDENCE

This chapter contains an empirical analysis of the hypotheses generated by the three-factor model of futures risk premiums. Empirical findings based on the two-step regression analysis employing weekly settlement price and hedging pressure data on 13 highly liquid futures and their corresponding futures options indicate that futures option returns significantly affect the futures risk premiums. This result is obtained after controlling for systematic risk and residual risk conditional on hedging pressure in the futures market and remains significant even after purging the component of futures option returns that may be explained by futures returns and stock market returns. These results suggest the importance of futures option returns in explaining futures risk premiums.

1. Introduction

This chapter investigates equilibrium futures risk premium in the presence of delivery risk. The delivery risk, which arises from multiple delivery specifications associated with futures contracts, causes options on futures to become a nonredundant hedging instrument and to affect the equilibrium futures prices. This implies that futures options should play a role in pricing futures contract. The pricing implication is examined using an extension of Hirshleifer (1988) model that accounts for the delivery risk and the availability of options on futures. The extended model predicts that futures risk premium is related to expected return on stock market portfolio, residual risk premium and expected return on futures option.

Empirical evidence on the weekly data of 13 U.S. futures contracts indicates that the futures option returns significantly affect average futures returns and risk premiums, after controlling for systematic risk and residual risk conditional on hedging pressure in the futures market. These effects remain significant even after purging the component of futures option returns that may be explained by futures returns and stock market returns. These results suggest the importance of futures option returns in explaining futures risk premiums.

The futures pricing literature (e.g., Stoll (1979), Hirshleifer (1988), Cater, Rausser, and Schmitz (1983), Bessembinder (1992), DeRoon Nijman and Veld (2000)) documents that futures risk premiums are usually related to systematic risk and to hedging pressure in the market. However, this study shows that returns on futures options, arising from the presence of delivery risk, additionally explain average futures risk premiums.

This study is also contrasted with the empirical literature on the impact of delivery specifications on futures price.[39] For example, Gay and Manaster (1984) values delivery options using the option pricing model of Margrabe (1978). Kane and Marcus (1986) employ Monte-Carlo simulations to estimate the value of delivery specification. Barnhill (1990) examines the delivery option based on trading strategies involving short futures and successive rolling over of the current cheapest-to-deliver. In contrast, this study examines the effect of delivery risk from an equilibrium futures pricing model in spirit of Hirshleifer (1988).

The empirical results in this study also add to the growing literature on the linkage between futures option and futures markets (e.g., Hilliard and Reis (1998), Bellalah (1999)). The related evidence includes Hwang and Satchell (2000) who find that the price volatility of FTSE 100 index futures decreases after the introduction of the futures options. Chan and Lien (2002) show that the introduction of the currency futures options significantly improve the price formation process of the futures contracts. Tompkins (2003) document that risk premiums (defined as the difference between estimated and observed option prices) on Treasury futures options are similar to those on Treasury futures.

The remaining sections are as follows. Section 2 illustrates the model and methodology. The data are described in Section 3. Section 4 and Section 5

[39] See Chance and Hemler (1993) for a survey on the impact of delivery specifications on futures prices.

discuss the results and the robustness of the results respectively. Section 6 concludes the study.

2. Model and Methodology

The existing literature identifies two determinants of futures risk premium: systematic risk and hedging pressure. Among the well-known models combining the roles of the two determinants, Hirshleifer (1988) model is developed with a solid theoretical ground and supported by the empirical evidence (e.g., Bessembinder (1992)). However, Hirshleifer model abstracts from the delivery risk in the real futures market. The presence of delivery risk causes options on futures to be nonredundant in the economy, thereby implying a role of the futures option in pricing futures contract.

To test the pricing implication, this study extends the Hirshleifer model in the case that futures traders face delivery risk in addition to underlying price risk and futures options are available. The extended model[40] is

$$\pi = \beta' E(\tilde{R}_m) + d\,\sigma' \sqrt{2\lambda' t} + \Gamma E(\tilde{R}_o) \quad (1)$$

[40] The derivation of the model is shown in Appendix 4, Chapter 4. The model is developed in the context of the two-date model, which abstracts from the daily settlement characteristic of futures contracts, and thus the difference between futures and forward prices given stochastic interest rates. Nevertheless, a number of researches have concluded that the forward and futures prices are not significantly different: Cornell and Reinganum (1981) on foreign exchange, French (1983) on copper and silver, MacKinlay and Ramaswamy (1988) on stock indices and Grinblatt and Jegadeesh (1996) on Eurodollar contracts.

$$\beta' = \frac{(\beta_{\pi m} - \beta_{\pi o}\beta_{om})}{1-\rho_{om}^2} \; ; \; \sigma' = \sigma_{\pi}\sqrt{\frac{\gamma}{1-\rho_{om}^2}} \; ; \; \Gamma = \frac{(\beta_{\pi o} - \beta_{\pi m}\beta_{mo})}{1-\rho_{om}^2}$$

where π denotes the percentage of futures risk premium; $E(\widetilde{R}_m)$ denotes the expected return on stock market portfolio; $E(\widetilde{R}_o)$ denotes the expected return on futures options; $\beta_{\pi m}$, $\beta_{\pi o}$, β_{om} and β_{mo} denote appropriately defined betas; σ_{π} denotes the standard deviation of futures risk premiums; *d* denotes an indicator variable equal to 1 for the net long futures positions of hedgers and -1 for the net short positions; λ' denotes the risk-aversion coefficient of the marginal speculator, *t* denotes a fixed cost for trading derivatives; $\gamma = 1 - \rho_{om}^2 - \rho_{o\pi}^2 - \rho_{\pi m}^2 + 2\beta_{mo}\beta_{o\pi}\beta_{\pi m}$, $\rho_{ab}^2 = \beta_{ab}\beta_{ba}$ and ρ_{ab} denotes the correlation coefficient of *a* and *b*.

Futures risk premium in Equation (1) is set to compensate the marginal speculator, who determines the futures market equilibrium, for the additional trading costs and for the incremental risks associated with a futures position. Thus, futures risk premium can be decomposed into three components. The first is systematic risk premium, which is related to expected return on stock market portfolio. The second is residual risk premium that depends on the investor's risk aversion and on the magnitude of barriers to derivatives trading, i.e., trading costs. The third premium is captured by expected return on the futures option.

The first two risk components in Equation (1) are in the same fashion as the Hirshleifer (1988) model, while the third component arises from the presence of delivery risk and the availability of options on futures. The absence of delivery risk implies no hedging role for the options; therefore, the betas related to the options vanish and the extended model reduces to the Hirshleifer model. The above pricing relation shows precisely how futures risk premium is altered when options on the futures contract are nonredundant in the economy.

Hirshleifer (1988) model: $\pi = \beta_{\pi m} E(\tilde{R}_m) + d\sigma_\pi \sqrt{2\lambda' t(1-\rho_{\pi m}^2)}$ (2)

The econometric specification of the pricing model in Equation (1) is described as follows. Without loss of generality, replacing futures risk premium (π) in the pricing model by realized futures return (R_F)[41] and expectations by realizations yields

$$R_F = \delta_0 + \beta' R_m + d\,\sigma' \sqrt{2\lambda' t} + \Gamma R_o + \varepsilon$$

Then substituting R_m, $\sqrt{2\lambda' t}$, R_o with δ_{1t}, δ_{2t}, δ_{3t} respectively and indicating the variables in effect at time t with subscript t give

$$R_{Ft} = \delta_{0t} + \beta'_t\, \delta_{1t} + \sigma'_t\, d_t \delta_{2t} + \Gamma_t \delta_{3t} + \varepsilon_t \quad (3)$$

[41] It is well known that empirical studies measure the percentage change in futures risk premiums as the realized return (e.g., Dusak (1973), Bessembinder (1992), and DeRoon Nijman and Veld (2000)). The rationale is as follows. Generally, expected return on an asset can be empirically replaced by realized asset return. The expected return can be decomposed into risk free return and risk premium. This is because the asset return compensates the investor for financing the initial payment to acquire the asset via risk free return and the risk premium compensates the investor for bearing the risk associated with holding (or bearing) the asset. In a case of futures contracts, the zero initial investment in the contract implies no risk free return. Expected return on futures contract, thus, is equal to futures risk premium and replaced by realized futures return.

$$\beta'_t = \frac{(\beta_{Fm,t} - \beta_{Fo,t}\beta_{om,t})}{1-\rho_{om,t}^2}\ ;\ \sigma'_t = \frac{\sigma_{Fmo,t}}{\sqrt{1-\rho_{om,t}^2}};\ \Gamma_t = \frac{(\beta_{Fo,t} - \beta_{Fm,t}\beta_{mo,t})}{1-\rho_{om,t}^2} \quad (4)$$

β'_t, σ'_t, Γ_t denote the coefficients as indicated in (4) constructed from the estimated betas and/or the standard deviation of residuals from the regression of futures returns on stock market returns and futures option returns ($\sigma_{Fmo,t}$). δ_{1t}, δ_{2t}, δ_{3t} denote factor premiums for systematic risk, residual risk, and the risks captured by the futures option return respectively. An indicator is denoted as d_t, which equals to 1 for the net long futures positions of hedgers and -1 for the net short positions. δ_{0t} denotes an intercept and ε_t denotes a random error.

To evaluate the marginal contribution of futures option returns to explain average futures returns and risk premiums, the performance of Hirshleifer (1988) model is used as a benchmark. The specification of Hirshleifer model is

$$R_{Ft} = \delta_{0t} + \beta_{Fm,t}\delta_{1t} + \sigma_{Fm,t}d_t\delta_{2t} + \varepsilon_t \quad (5)$$

R_{Ft} denotes futures return, with percentage change in futures prices at time t. $\beta_{Fm,t}$ is the coefficient obtained from a regression of futures returns on stock market returns. $\sigma_{Fm,t}$ is the standard error of residuals from the regression. Premiums for systematic risk and residual risk factors at time t are denoted by δ_{1t} and δ_{2t} respectively. d_t denotes an indicator variable equal to 1 for the net long futures positions of hedgers and -1 for the net short positions. An intercept and a random error at time t are denoted by δ_{0t} and ε_t respectively.

As mentioned above, the premiums for systematic risk and residual risk in both futures-pricing specifications in Equations (3) and (5) are comparable. Thus β'_t is analogous to $\beta_{Fm,t}$ and σ'_t is parallel to $\sigma_{Fm,t}$. In addition, both models specify a zero intercept ($\delta_{0t} = 0$), which reflects zero initial investment in futures contracts. This specification implies a simple test of whether the premiums associated with any set of explanatory returns suffice to describe the cross-section of average futures returns: the intercepts in the time-series regressions of the excess returns should be insignificantly different from zero.

The two-step regressions of Fama and MacBeth (1973) were adapted to estimate factor risk premiums for each period. In the first step, conditional slope coefficients (hereafter betas β_{Fm}, β_{mF}, β_{Fo}, β_{oF}, β_{om} and β_{mo}) were estimated from time-series regressions which the variables are weekly futures returns[42], stock market returns (returns on value-weighted CRSP index) and futures option returns, using the data for the prior 50 weeks. The betas were used to compile the estimated factors (β'_t, σ'_t, Γ_t) according to Equations (4) and (5). In the second step, for each day, the cross-section of futures returns is regressed on the estimated factors, thereby yielding time-series estimates of factor risk premiums (i.e., δ_{1t}, δ_{2t}, and δ_{3t}). The final estimated premium for each factor is the average of the daily premiums. Each of the final estimated premiums was used to test whether the factor on average has a non-zero expected premium. As shown by Shanken (1992), the calculated standard errors of the final estimate from simple Ordinary Least

[42] Weekly returns were calculated as percentage changes in futures prices settled on every Tuesday. Thus, the returns were matched with the weekly data (on Tuesday) of futures positions held by futures traders, reported in Commitments of Traders report. (The data of futures positions is available via URL: www.cftc.gov/cftccotreports.htm.)

Squares (OLS) understate the true standard error due to estimation error in the estimated betas. Thus, to correct for the estimation error, each regression was estimated with Newey-West heteroskedasticity and autocorrelation consistent estimates of covariance matrices.

3. Data Description

The data set for the analysis, covering the period January 2000 - February 2004, consists of settlement prices of 13 U.S. futures and options on futures contracts obtained from the Datastream International Database and New York Board of Trade (NYBOT), total return on CRSP value-weighted equity index obtained from the Center for Research in Security Prices (CRSP), net futures positions of large traders in the Commitments of Traders reports provided by Commodity Futures Trading Commission (CFTC). The selected contracts were classified into two groups: contracts with delivery risk (Treasury bonds, 10-year Treasury notes, cocoa, coffee, cotton, and world sugar) and contracts without delivery risk (three-month Eurodollars, S&P500 index, Japanese yen, Swiss franc, Canadian dollar, Australian dollar, British pound). [43]

The time-series returns on futures and futures options were computed as percentage changes in the contracts' settlement prices. Although computing

[43] Both three-month Eurodollars and S&P500 index contracts are cash settled based on the Final Settlement Rule of the Chicago Mercantile Exchange (CME), while each of the five currency contracts is physically delivered at a specific final settlement price determined by the Trading Floor Pit Committee, CME. The futures and futures option contracts were selected on various assets underlying the contracts and their economic importance, as evidenced by the relatively large trading volumes and open interests.

futures return as percentage change in futures prices is consistent with the empirical literature on pricing, it is noteworthy that this futures return is a misnomer. Due to zero initial investment cost, futures return cannot be defined as a percentage change in futures value with respect to its investment cost but should be defined as dollar change in futures prices (Black (1976)). Nevertheless, measuring futures return as a percentage is widely used because it makes return analysis comparable across futures contracts as well as other assets.

To prevent the effects of stale prices and microstructure, the following procedure was applied to generate the returns on futures options (call, put, and straddle).[44] First, the price data of near-expiration options were collected from the option prices with the nearest expiry month, except within the expiry month when the prices of the second-nearest contract were used. Then, the moneyness of each option was determined by using its underlying price (the futures price) from the beginning of the *t*th week and the nearest in-the-money and the nearest out-of-the-money options were selected from the data collection. Finally, the weekly option returns from the equal-weighted prices of two nearest-the-money options were calculated. Meanwhile the time-series of futures returns was calculated from the corresponding futures prices, which were underlying the futures options.

Additional details should be mentioned. For both futures and futures options, each return was computed using successive prices on a contract for a specific expiry date and never across contracts with different dates of expiry. For Eurodollar futures, the quoted settlement price does not reflect the delivery price.

[44] Straddle is a long position in one call and one put at the same exercise price.

The index price basis in which the contract is quoted is equal to 100 minus the annualized futures LIBOR (London Interbank Offered Rate). These quoted prices were converted to implied delivery prices; thus the return on Eurodollar futures was computed as the percentage change in these implied delivery prices.

The summary statistics of weekly returns on futures and futures options in table 1, by and large, confirm some stylized facts about the returns reported in previous studies. Average futures returns in panel A are relatively small, ranging from -0.50% to 0.22% per week, and are significant at the 5% level for two contracts. These results are largely consistent with those in Bessembinder (1992) and DeRoon, Nijman and Veld (2000).

The average returns on near-the-money futures options are substantially large in absolute value than futures returns. Most call futures returns are positive while most put futures returns are negative. Average returns on call futures options, excluding the call returns on S&P500, Coffee, and Cotton, considerably exceed the average returns on their underlying securities (futures returns) of 3% - 10.5% per week. Conversely, all average put option returns, except for those returns on S&P500, Coffee, and Cotton, are considerably lower than average risk-free returns (i.e., returns on one-month U.S. Treasury bill), the difference ranging from 0.8% to 6.4% per week. The futures option returns are, by and large, congruous with the theory of expected option returns proposed by Coval and Shumway (2001) and the magnitudes of the futures option returns are comparable to those reported by Coval and Shumway.[45]

[45] Coval and Shumway (2001) show that, if call options are written on securities with expected returns above the risk free rate, the expected call returns should exceed expected

Nevertheless, the discrepancy of the average option returns on S&P500, Coffee, and Cotton probably arises from the negative mean returns on futures underlying the options. Then the positive correlation between call futures options and futures contracts leads to the negative call returns on these contracts. Similarly, the negative correlation between put futures options and their underlying security causes the average put returns on these contracts to be positive.

Overall, the return behavior of futures options reflects an option's characteristic that can be used to lever positions in the underlying asset. The leverage effect arises from the fact that, with a relatively small investment, an option allows investors to assume much of the risk associated with the option's underlying asset. As implied by the pricing model of Black and Scholes (1973), this implicit leverage should be priced in option returns.

Average returns on straddle futures options are also reported in table 1. Each straddle is an equal-weighted combination of at-the-money put futures and at-the-money call futures the same maturity. Average returns on straddle options, excluding straddle returns on coffee and Swiss franc contracts, are positive, ranging from 0.02% to 2.98% per week and mostly insignificant.

Panel B in table 1 shows the return correlation matrices for futures, calls, puts, straddle options, and the value-weighted CRSP stock index. As expected, the futures returns are positively correlated with call returns and negatively correlated

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returns on the underlying securities. Conversely, put options should earn expected returns below that of the underlying security. Coval and Shumway also report average daily call and put returns on near-the-money options on futures contracts for the period of October 1988 to August 1999. Call and put returns on Treasury futures are 0.53 % and -1.51 % while the Eurodollar call and put returns are 1.20% and -1.51% respectively.

with put returns. These correlations confirm the characteristics of the call options and the usefulness of the put options as instruments for portfolio insurance.

4. Risk Effect Captured by Futures Option Returns

The following analyses focus on testing the hypothesis of the model in Equation (3) that futures option returns can explain average futures returns and risk premiums. This hypothesis should be valid regardless of whether call futures, put futures, or both futures options are available in the economy. Therefore, the following tables present the regression results of the model in (3) using different types of futures option returns: call, put, straddle and call-put spread respectively. The straddle is created by buying an at-the-money call and an at-the-money put. The call-put spread is created by buying an at-the-money call and selling an at-the-money put.

The regression results of the Hirshleifer model in Equation (5) are provided in columns 3 to 6 in each of tables 2 -5. These results are used as a benchmark against which the other regressions can be evaluated in order to illustrate the marginal contribution of the futures option returns. Panel A presents averages of the two estimated risk factors, which are the slope coefficients, representing systematic risk, and standard error of residuals, representing residual risk. These two risk factors were estimated from a time-series regression of futures returns on stock market returns (value-weighted CRSP index) using data for the prior 50 weeks. Panel B shows the average factor risk premiums estimated from the day-by-day cross-sectional regressions of futures returns on the two estimated factors.

The regression results indicate a significant relation (*t*-statistic of 2.44) between average futures returns and residual risk but no reliable relation exists between futures returns and systematic risk. Average premiums for systematic risk are relatively small and insignificant, ranging from -0.04% to 0.00% per week. In contrast, average premiums for hedging-conditioned residual risk are economically and statistically significant, ranging from -0.13% to 0.14% per week. Residual risk premiums for all agricultural futures, except Cocoa, are relatively higher than those for financial and currency futures, except for Eurodollars. These results are largely consistent with the findings of Bessembinder (1992).

Despite the evidence supporting the residual risk, the significant (at the 1% level) average intercept in the regressions suggests that the set of explanatory variables is not sufficient to describe the cross-section of average futures returns. The Hotelling T^2 statistic[46] fails to reject the hypothesis that both estimated premiums are jointly zero at any conventional significance level. Collectively, the statistic tests indicate that the risk factors of Hirshleifer model do not suffice to explain the cross-section of futures returns, at least over the period of 2000 to 2004.[47]

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[46] The joint hypothesis was tested using Hotelling's T^2 test, with Newey-West heteroskedasticity and autocorrelation consistent estimates of covariance matrices to correct for the estimation error in the conditional parameters. For details of the Hotelling's T^2 test, see Campbell, Lo, and MacKinlay (1997, pp.231-233).

[47] This conclusion is not inconsistent with the evidence provided by Bessembinder (1992) Bessembinder found a significant effect on average futures returns of hedging-conditioned residual risk in the Hirshleifer's (1988) model for agricultural and foreign currency futures. Like Bessembinder' s finding, this study also shows a significant effect from the residual

Table 2 reports the results of the model in Equation (3) using call futures returns. Panel A contains averages of the three estimated factors compiled from the time-series regressions, which the variables are returns on futures, call futures, and stock market, over the prior 50 weeks and standard deviations of residuals from the 50-week rolling regressions of futures returns on call futures returns and stock market returns (the first step of Fama-MacBeth regressions). Panel B presents the average factor risk premiums estimated from the cross-sectional regressions of futures returns on the three estimated factors (the second step of Fama-MacBeth regressions). The average factor risk premiums captured by the futures option returns are reported separately for the futures contracts with delivery risk and those without delivery risk.

The regression results indicate the importance of call returns in explaining the cross-section of average futures returns. A significant effect of call futures returns on futures returns is found for the contracts with delivery risk. Average of factor risk premiums captured by call futures returns is 4.3% per week, with *t*-statistics of 2.20, for the contracts with delivery risk while the average factor premiums for the contracts without delivery risk are relatively small and insignificant. The average risk premiums captured by call returns are substantial for the contracts with delivery risk, ranging from 0.10% to 0.32% per week. Among these contracts, the risk premiums for agricultural futures are larger than those for Treasury futures.

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risk, however, Hirshleifer's model as a whole does not have the significant power to explain average futures returns. Moreover, unlike Bessembinder's results, results from this study are based on the sample set that includes financial futures contracts in addition to agricultural and currency futures.

A considerable increase in average adjusted *R-square* of the regressions indicates some improvement from Hirshleifer (1988) model. However, the average intercept is still significant at the 1% level and the test for the hypothesis that all factor premiums are jointly zero, indicated by Hotelling T^2 statistic, suggests that the model in Equation (3) using call option returns offers a statistically insignificant improvement.

Table 3 shows the results of the model in Equation (3) using put futures returns. The results indicate the significant effect of put returns on the cross-section of the average futures returns. For all contracts with delivery risk, except Treasury bond futures, the average risk premiums captured by put returns are large in both practical and statistical terms, ranging from 0.11% to 0.41% per week. Again, the premiums for agricultural futures are substantially larger than those for interest rate futures. For the futures contracts without delivery risk, the average premiums captured by put returns are relatively small and statistically insignificant, ranging from -0.07% to -0.02% per week. Residual risk premiums, on average, are also economically and statistically significant. The average adjusted *R-square* from the regressions in Equation (3) using put futures returns increases considerably relative to the *R-square* the regressions of Hirshleifer (1988). However, the average intercept is still significant and Hotelling T^2 statistic indicates all factor premiums are insignificantly different from zero. This suggests a similar conclusion to the regression results using call futures returns in table 2.

Table 4 provides the results of the model in Equation (3) using returns on straddle futures options. Using the straddle returns provides a test of whether volatility risk is priced in futures returns.[48] The results show that the straddle returns have significant power to explain average futures returns. The average risk premiums captured by the straddle returns are statistically significant, at the 1% level, and large in absolute value, ranging from -0.35% to 0.14% per week, for the contracts with delivery risk. For the futures contracts without delivery risk, the average risk premiums captured by straddle returns are also significant but relatively smaller, ranging from -0.08% to 0.09% per week.

Average adjusted *R-square* of the regressions suggests an improvement over Hirshleifer (1988) model. The average premiums for the systematic risk and the hedging-conditioned residual risk are also statistically significant. The Hotelling T^2 statistic confirms the test results by rejecting the hypothesis that all factor premiums are jointly zero at the 1% significance level. Collectively, the statistical tests indicate the model in Equation (3), using straddle returns, has significant power to explain the cross-section of average futures returns. In addition, these results would suggest that volatility risk is priced in average futures returns.

[48] As addressed by Coval and Shumway (2001), straddle returns are useful for capturing the volatility of underlying returns. A straddle's characteristic allows us to detach the leverage effect and focus on the pricing of volatility of the security return. This is because the straddle positions are formed by buying an at-the-money call and an at-the-money put with the same maturity. Due to the offset between the payoffs of near-the-money call and put options caused by their underlying price change, straddle return is not sensitive to futures return per se but sensitive to the volatility of futures return. Specifically, when volatility is higher than expected, the straddles have positive returns. When volatility is lower than expected, the straddles have negative returns. Since straddle has a large, positive volatility beta, it allows investors to hedge the volatility of the underlying returns. This would make the straddle return capture the volatility risk associated with a futures position.

Table 5 reports the results of the model in Equation (3) using returns on the call-put spreads. The results show that the spread returns have significant power to explain average futures returns. The average risk premiums captured by the spread returns are statistically significant, at the 1% level, and large in absolute value, ranging from -0.16% to -0.47% per week, for the contracts with delivery risk. For the futures contracts with no delivery risk, the average risk premiums captured by the spread returns are also insignificant and relatively smaller in absolute value, ranging from -0.04% to -0.14% per week.

Average adjusted *R-square* of the regressions indicates an improvement over Hirshleifer (1988) model. The average premiums for the other two risk factors as well as the average intercept are also statistically significant. The Hotelling T^2 statistic confirms the test results by rejecting the hypothesis that all premiums are jointly zero at the 5% significance level. This suggests that the model in Equation (3) significantly explains the cross-section of average futures returns.

It is noteworthy that the estimated premiums of delivery risk for interest rate futures are consistently less significant than those for agricultural futures, as shown in Tables 2-5. The discrepancy in the estimated premiums can be explained by the following proposed conjecture. In interest rate futures, e.g., T-Bond, the cheapest-to-deliver (and next-to cheapest-to-deliver) can be identified with ease. A standard database used by a bond dealer would be sufficient to identify the cheapest-to-deliver T-bond in the bond market. Consequently, the delivery risk in T-bond futures market imposes less impact on futures risk premiums. By contrast, the identification of the cheapest-to-deliver agricultural commodity goods is not straightforward. Due to various quality grades, locations, and convenience yields associated with deliverable commodity goods. Thus, the delivery risk in the commodity goods is more significant than it is in the interest rate futures contract.

5. Effect of Residual Futures Option Returns on Futures Risk Premium

The empirical evidence presented so far indicates the importance of futures option returns in explaining average futures returns and risk premiums. However, an alternative explanation of these results in section 4.1 might be given by the high correlation between derivatives and their underlying assets. In other words, the significant effect of futures options on futures risk premiums may arise because of the interaction between futures returns and futures option returns.

To see whether futures option returns indeed contains an additional power to explain futures risk premiums, we test the hypothesis of the model in Equation (3) using residual returns on futures options, which are orthogonal to futures returns and stock market returns. By doing so, we abstract from the correlation between futures option and futures contract and focus on the marginal contribution of residual returns on futures options to explain average futures risk premiums.

The residual returns on futures options ($\varepsilon_{o,t}$) are obtained from the equation

$$\varepsilon_{o,t} = R_{o,t} - \alpha - \beta_{om} R_{m,t} - \beta_{oF} R_{F,t} \tag{6}$$

where $R_{o,t}$, $R_{m,t}$ and $R_{F,t}$ are futures option return, stock market return and futures return at time t respectively; β_{om} and β_{oF} denote unconditional slope coefficients estimated from a time-series regression of futures option returns on futures returns and stock market returns; α is an intercept of the regression.

Table 6 reports average factor risk premiums estimated from the Fama-MacBeth regressions in Equation (3) using different types of residual returns on futures options: call, put, straddle and call-put spread respectively. The results show that, the significant effects of futures option returns exist, even after purging the component of futures option returns that may be explained by futures returns and stock market returns. Although the pricing relations between futures returns and residual returns on futures options are somewhat weaker than the relations between futures returns and futures option returns reported in tables 2 – 5, these results still provide convincing evidence for the role of futures options in determining futures returns and risk premiums.

6. Conclusions

This study analyzes equilibrium futures pricing in the presence of delivery risk in addition to underlying price risk. The underlying premise is that the presence of delivery risk causes options on futures to be nonredundant hedging instruments and to influence the equilibrium futures prices, thereby implying a role of the futures options in pricing futures contracts.

To test this implication, the futures pricing model of Hirshleifer (1988) is extended to account for delivery risk and the availability of options on futures. The extended model relates equilibrium futures risk premium to expected return on stock market portfolio, expected return on futures option, and residual risk premium conditional on the net futures positions of hedgers.

Empirical tests of the extended model with weekly data for 13 U.S. futures contracts offer limited support for the role of futures option returns in explaining average futures returns and risk premiums. The significance of the estimated risk premiums captured by the option returns indicates the importance of futures options in explaining the cross-section of average futures returns. The results from the model’s performance, however, are mixed. The extended model, which incorporates either put returns or call returns, offers only a slight improvement over the benchmark model of Hirshleifer (1988). In contrast, the extended model that includes either straddle returns or returns on the call-put spreads has significant power to explain average futures risk premiums. The effects of futures option returns on futures risk premiums are robust, even after purging the component of futures option returns that may be explained by futures returns and stock market returns. Overall, the results suggest the importance of futures option returns in explaining futures risk premiums.

Table 1 Summary Statistics

Panel A reports sample mean returns (% per week) and standard deviation for futures and three types of options on futures: call , put, and straddle. Straddle is created by taking long positions in an at-the-money call and an at-the-money put. Panel B shows the return correlation matrix for all four types of contracts and value-weighted CRSP stock index (VW-CRSP). *n* denotes the sample size in weeks.The sample period is from January 2000 (May 2000 for Cocoa, Coffee, Cotton, and Sugar) to February 2004. * denotes statistical significance at the 5% level.

Panel A:	*n*	Futures (%)		Call (%)		Put (%)		Straddle (%)	
		Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.
Contract with Delivery Risk									
30-Year T-Bond	215	0.16	1.43	6.81	50.84	-5.30	42.31	1.17	9.73
10-Year T-Note	215	0.13	0.99	6.91*	43.90	-6.29*	40.34	0.38	10.39
Cocoa	196	0.04	0.50	3.61	49.70	-0.71	51.22	0.02	15.54
Coffee	196	-0.50	4.94	-2.25	82.46	1.79	36.34	-1.59	22.24
Cotton	196	-0.20	3.72	-1.22	51.84	3.23	44.84	0.76	11.36
Sugar	196	0.30	4.43	3.77	47.31	-0.41	40.75	0.13	11.82
Contract without Delivery Risk									
Eurodollar	215	0.01*	0.03	5.96*	39.09	-3.79	40.34	0.14	9.76
S&P 500	215	-0.04	2.86	-0.33	32.00	0.48	48.83	0.21	9.20
Japanese Yen	215	0.01	1.34	2.23	42.25	-1.14	21.18	0.54	11.44
Swiss Franc	215	0.18	1.48	3.88*	24.98	-5.74	45.97	-1.02	10.43
Canadian Dollar	215	0.05	0.94	3.17	40.97	-1.61	31.26	0.31	6.07
Australian Dollar	215	0.22*	1.54	10.73*	50.79	-5.47*	33.33	2.98*	14.01
British Pound	215	0.07	1.14	3.60	39.16	-1.83	20.28	0.74	9.64

Panel B: Correlation matrix		Futures	Call	Put	Straddle	VW CRSP
30-Year T-Bond	Futures	1	0.859	-0.895	0.284	-0.230
	Call		1	-0.875	0.523	-0.198
	Put			1	-0.208	0.203
	Straddle				1	-0.071
	VW CRSP					1
10-Year T-Note	Futures	1	0.808	-0.826	0.100	-0.362
	Call		1	-0.806	0.315	-0.319
	Put			1	0.069	0.315
	Straddle				1	-0.076
	VW CRSP					1
Cocoa	Futures	1	0.763	-0.798	-0.015	-0.028
	Call		1	-0.588	0.371	-0.043
	Put			1	0.210	0.023
	Straddle				1	0.003
	VW CRSP					1
Coffee	Futures	1	0.844	-0.787	0.761	0.128
	Call		1	-0.649	0.828	0.096
	Put			1	-0.530	-0.100
	Straddle				1	0.105
	VW CRSP					1

Table 1 Summary Statistics

Panel B: (continued)		Futures	Call	Put	Straddle	VW CRSP
Cotton	Futures	1	0.907	-0.892	0.161	0.051
	Call		1	-0.836	0.274	0.043
	Put			1	0.083	-0.066
	Straddle				1	0.025
	VW CRSP					1
Sugar	Futures	1	0.840	-0.839	0.063	-0.006
	Call		1	-0.741	0.306	-0.001
	Put			1	0.100	0.031
	Straddle				1	0.043
	VW CRSP					1
Eurodollar	Futures	1	0.791	-0.767	0.127	-0.282
	Call		1	-0.739	0.269	-0.194
	Put			1	0.123	0.233
	Straddle				1	0.027
	VW CRSP					1
S&P500	Futures	1	0.929	-0.932	-0.857	0.976
	Call		1	-0.981	-0.875	0.910
	Put			1	0.948	-0.915
	Straddle				1	-0.845
	VW CRSP					1
Japanese Yen	Futures	1	0.920	-0.921	0.860	-0.024
	Call		1	-0.972	0.967	-0.008
	Put			1	-0.907	0.007
	Straddle				1	-0.007
	VW CRSP					1
Swiss Franc	Futures	1	0.912	-0.908	-0.863	-0.252
	Call		1	-0.923	-0.805	-0.220
	Put			1	0.904	0.240
	Straddle				1	0.218
	VW CRSP					1
Canadian Dollar	Futures	1	0.896	-0.872	0.837	0.246
	Call		1	-0.921	0.924	0.237
	Put			1	-0.884	-0.240
	Straddle				1	0.240
	VW CRSP					1
Australian Dollar	Futures	1	0.876	-0.800	0.876	0.237
	Call		1	-0.842	-0.800	0.188
	Put			1	0.825	-0.211
	Straddle				1	0.195
	VW CRSP					1
British Pound	Futures	1	-0.930	0.963	0.903	-0.090
	Call		1	-0.907	-0.890	-0.117
	Put			1	0.898	0.114
	Straddle				1	-0.101
	VW CRSP					1

Table 2 The Risk Effect Captured by Returns on Call Futures Options

The table presents (in Panel B) average factor risk premiums estimated from the day-by-day Fama-MacBeth (1973) regressions of the cross-section of futures returns (R_{Ft}) on the estimated factors of two following models.

Hirshleifer's (1988) model : $$R_{Ft} = \delta_{0t} + \beta_{Fm,t}\delta_{1t} + \sigma_{Fm,t} d_t \delta_{2t} + \varepsilon_t$$

The extended model with futures options: $$R_{Ft} = \delta_{0t} + \beta'_t \delta_{1t} + \sigma'_t d_t \delta_{2t} + \Gamma_t \delta_{3t} + \varepsilon_t$$

$$\beta'_t = \frac{(\beta_{Fm,t} - \beta_{Fo,t}\beta_{om,t})}{1-\rho^2_{om,t}}\ ;\ \sigma'_t = \frac{\sigma_{Fmo,t}}{\sqrt{1-\rho^2_{om,t}}}\ ;\ \Gamma_t = \frac{(\beta_{Fo,t} - \beta_{Fm,t}\beta_{mo,t})}{1-\rho^2_{om,t}}$$

δ_{1t}, δ_{2t}, δ_{3t} are factor premiums for systematic risk, residual risk and the risks captured by call futures returns respectively. The estimated factors are slope coefficient ($\beta_{Fm,t}$) and standard error of residuals ($\sigma_{Fm,t}$) from time-series regressions of futures returns on stock market returns over the prior 50 weeks; and the factors of β'_t, σ'_t and Γ_t compiled from the parameters estimated from regressions, which the variables are returns on futures, call futures, stock market, using data for the prior 50 weeks. d_t denotes an indicator equal to 1 for net long futures positions of hedgers and -1 for net short positions. Panel A reports average estimated factors and average estimated premiums for each futures contract. t-statistics in parentheses are for the hypothesis that average factor premium is zero. Hotelling's T^2 statistics in square brackets are for the hypothesis that all factor premiums are jointly zero. **, * denote significance at the 1 % and 10% levels. All test statistics are based on Newey-West heteroskedasticity and autocorrelation estimates of covaraince matrices. R^2 denotes the average adjusted *R-square* of the regressions. n denotes weekly observations over the period of January 2000 (May 2000 for Cocoa, Coffee, Cotton, Sugar) to February 2004.

Panel A	n	Hirshleifer's Model				The Extended Model with Call Futures Options					
		Esimated Factor		Estimated Premium (%)		Estimated Factor			Estimated Premium (%)		
		$\beta_{Fm,t}$	$d_t\sigma_{Fm,t}$	Sys. risk	Res. risk	β'_t	$d_t\sigma'_t$	Γ_t	Sys. Risk	Res. Risk	Options
Contract with Delivery Risk											
30-Year T-Bond	215	-0.02	0.00	0.00	-0.01	-0.02	0.00	0.03	0.00	-0.01	0.11
10-Year T-Note	215	-0.03	0.00	0.00	0.01	-0.03	0.00	0.02	0.00	0.01	0.10
Cocoa	196	0.04	0.01	0.00	0.02	0.13	0.01	0.04	0.01	0.04	0.19
Coffee	196	0.03	-0.01	0.00	-0.03	0.01	-0.01	0.07	0.00	-0.04	0.30
Cotton	196	0.07	0.04	-0.01	0.14	0.12	0.02	0.05	0.00	0.13	0.23
Sugar	196	0.01	0.03	0.00	0.09	-0.03	0.01	0.08	0.00	0.07	0.32
Contract without Delivery Risk											
Eurodollar	215	0.10	-0.04	-0.01	-0.13	0.11	-0.02	0.07	0.00	-0.15	0.11
S&P 500	215	0.28	-0.01	-0.02	-0.03	0.35	-0.01	0.03	0.01	-0.05	0.04
Japanese Yen	215	0.45	0.00	-0.04	0.01	0.54	0.00	0.06	0.02	0.01	0.10
Swiss Franc	215	-0.02	-0.01	0.00	-0.02	-0.04	0.00	0.03	0.00	-0.02	0.06
Canadian Dollar	215	-0.04	0.00	0.00	-0.02	-0.09	0.00	0.04	0.00	-0.02	0.07
Australian Dollar	215	0.02	-0.01	0.00	-0.02	0.01	0.00	0.02	0.00	-0.02	0.04
British Pound	215	0.03	0.00	0.00	-0.02	0.03	0.00	0.03	0.00	-0.02	0.05
Panel B		δ_{0t}	δ_{1t}	δ_{2t}	R^2	δ_{0t}	δ_{1t}	δ_{2t}	$\delta_{3t\ w\ del.risk}$	$\delta_{3t\ wo\ del.risk}$	R^2
Factor risk premium		0.001	-0.001	0.035	0.01	0.002	0.000	0.064	0.043	0.017	0.12
		(2.87)**	(-0.64)	(2.44)*	[1.48]	(4.16)**	(0.27)	(1.97)*	(2.20)*	(1.04)	[0.69]

Table 3 The Risk Effect Captured by Returns on Put Futures Options

The table presents (in Panel B) average factor risk premiums estimated from the day-by-day Fama-MacBeth (1973) regressions of the cross-section of futures returns (R_{Ft}) on the estimated factors of two following models.

Hirshleifer's (1988) model : $$R_{Ft} = \delta_{0t} + \beta_{Fm,t}\delta_{1t} + \sigma_{Fm,t} d_t \delta_{2t} + \varepsilon_t$$

The extended model with futures options: $$R_{Ft} = \delta_{0t} + \beta'_t\, \delta_{1t} + \sigma'_t\, d_t \delta_{2t} + \Gamma_t \delta_{3t} + \varepsilon_t$$

$$\beta'_t = \frac{(\beta_{Fm,t} - \beta_{Fo,t}\beta_{om,t})}{1-\rho^2_{om,t}}\ ;\ \sigma'_t = \frac{\sigma_{Fmo,t}}{\sqrt{1-\rho^2_{om,t}}}\ ;\ \Gamma_t = \frac{(\beta_{Fo,t} - \beta_{Fm,t}\beta_{mo,t})}{1-\rho^2_{om,t}}$$

δ_{1t}, δ_{2t} , δ_{3t} are factor premiums for systematic risk, residual risk and the risks captured by put futures returns respectively. The estimated factors are slope coefficient ($\beta_{Fm,t}$) and standard error of residuals ($\sigma_{Fm,t}$) from time-series regressions of futures returns on stock market returns over the prior 50 weeks; and the factors of β'_t, σ'_t and Γ_t compiled from the parameters estimated from regressions, which the variables are returns on futures, put futures, stock market, using data for the prior 50 weeks. d_t denotes an indicator equal to 1 for net long futures positions of hedgers and -1 for net short positions. Panel A reports average estimated factors and average estimated premiums for each futures contract. t-statistics in parentheses are for the hypothesis that average factor premium is zero. Hotelling's T^2 statistics in square brackets are for the hypothesis that all factor premiums are jointly zero. **, * denote significance at the 1 % and 10% levels. All test statistics are based on Newey-West heteroskedasticity and autocorrelation estimates of covaraince matrices. R^2 denotes the average adjusted *R-square* of the regressions. n denotes weekly observations over the period of January 2000 (May 2000 for Cocoa, Coffee, Cotton, Sugar) to February 2004.

Panel A	n	Hirshleifer's Model				The Extended Model with Put Futures Options					
		Esimated Factor		Estimated Premium (%)		Estimated Factor			Estimated Premium (%)		
		$\beta_{Fm,t}$	$d_t \sigma_{Fm,t}$	Sys. risk	Res. risk	β'_t	$d_t \sigma'_t$	Γ_t	Sys. Risk	Res. Risk	Options
Contract with Delivery Risk											
30-Year T-Bond	215	-0.02	0.00	0.00	-0.01	-0.08	0.00	0.01	0.02	-0.02	-0.03
10-Year T-Note	215	-0.03	0.00	0.00	0.01	-0.03	0.00	-0.03	0.01	0.01	0.11
Cocoa	196	0.04	0.01	0.00	0.02	0.10	0.00	-0.04	-0.03	0.04	0.17
Coffee	196	0.03	-0.01	0.00	-0.03	-0.01	-0.01	-0.08	0.00	-0.05	0.33
Cotton	196	0.07	0.04	-0.01	0.14	0.05	0.02	-0.11	-0.02	0.21	0.41
Sugar	196	0.01	0.03	0.00	0.09	-0.05	0.01	-0.09	0.02	0.10	0.35
Contract without Delivery Risk											
Eurodollar	215	0.10	-0.04	-0.01	-0.13	0.18	-0.03	-0.07	-0.06	-0.22	-0.07
S&P 500	215	0.28	-0.01	-0.02	-0.03	0.32	-0.01	-0.02	-0.10	-0.07	-0.02
Japanese Yen	215	0.45	0.00	-0.04	0.01	0.18	0.00	-0.05	-0.06	0.01	-0.04
Swiss Franc	215	-0.02	-0.01	0.00	-0.02	0.01	0.00	-0.04	0.00	-0.03	-0.04
Canadian Dollar	215	-0.04	0.00	0.00	-0.02	-0.02	0.00	-0.03	0.01	-0.02	-0.03
Australian Dollar	215	0.02	-0.01	0.00	-0.02	0.00	0.00	-0.03	0.00	-0.03	-0.03
British Pound	215	0.03	0.00	0.00	-0.02	0.02	0.00	-0.04	-0.01	-0.03	-0.04
Panel B		δ_{0t}	δ_{1t}	δ_{2t}	R^2	δ_{0t}	δ_{1t}	δ_{2t}	$\delta_{3t\ w\ del.risk}$	$\delta_{3t\ wo\ del.risk}$	R^2
Factor risk premium		0.001	-0.001	0.035	0.01	0.002	-0.003	0.089	-0.039	0.010	0.15
		(2.87)**	(-0.64)	(2.44)*	[1.48]	(4.35)**	(-1.10)	(2.63)*	(-3.10)**	(0.59)	[1.64]

Table 4 The Risk Effect Captured by Returns on Straddle Futures Options

The table presents (in Panel B) average factor risk premiums estimated from the day-by-day Fama-MacBeth (1973) regressions of the cross-section of futures returns (R_{Ft}) on the estimated factors of two following models.

Hirshleifer's (1988) model : $$R_{Ft} = \delta_{0t} + \beta_{Fm,t}\delta_{1t} + \sigma_{Fm,t} d_t \delta_{2t} + \varepsilon_t$$

The extended model with futures options: $$R_{Ft} = \delta_{0t} + \beta'_t\,\delta_{1t} + \sigma'_t\, d_t \delta_{2t} + \Gamma_t \delta_{3t} + \varepsilon_t$$

$$\beta'_t = \frac{(\beta_{Fm,t} - \beta_{Fo,t}\beta_{om,t})}{1-\rho^2_{om,t}}\ ;\ \ \sigma'_t = \frac{\sigma_{Fmo,t}}{\sqrt{1-\rho^2_{om,t}}};\ \ \Gamma_t = \frac{(\beta_{Fo,t} - \beta_{Fm,t}\beta_{mo,t})}{1-\rho^2_{om,t}}$$

Straddle is created by buying an at-the-money call and an at-the-money put. δ_{1t}, δ_{2t} , δ_{3t} are factor premiums for systematic risk, residual risk and the risks captured by straddle futures option returns respectively. The estimated factors are slope coefficient ($\beta_{Fm,t}$) and standard error of residuals ($\sigma_{Fm,t}$) from time-series regressions of futures returns on stock market returns over the prior 50 weeks; and the factors of β'_t, σ'_t and Γ_t compiled from the parameters estimated from regressions, which the variables are returns on futures, the straddle, stock market, using data for the prior 50 weeks. d_t denotes an indicator equal to 1 for net long futures positions of hedgers and -1 for net short positions. Panel A reports average estimated factors and average estimated premiums for each futures contract. t-statistics in parentheses are for the hypothesis that average factor premium is zero. Hotelling's T^2 statistics in square brackets are for the hypothesis that all factor premiums are jointly zero. **, * denote significance at the 1 % and 10% levels. All test statistics are based on Newey-West heteroskedasticity and autocorrelation estimates of covaraince matrices. R^2 denotes the average adjusted *R-square* of the regressions. n denotes weekly observations from January 2000 (May 2000 for Cocoa, Coffee, Cotton, Sugar) to February 2004.

		Hirshleifer's Model				**The Extended Model with Straddle Futures Options**					
Panel A	***n***	Esimated Factor		Estimated Premium (%)		Estimated Factor			Estimated Premium (%)		
		$\beta_{Fm,t}$	$d_t\sigma_{Fm,t}$	Sys. risk	Res. risk	β'_t	$d_t\sigma'_t$	Γ_t	Sys. Risk	Res. Risk	Options
Contract with Delivery Risk											
30-Year T-Bond	215	-0.02	0.00	0.00	-0.01	-0.06	0.00	-0.04	0.02	-0.02	0.08
10-Year T-Note	215	-0.03	0.00	0.00	0.01	-0.13	0.00	0.07	0.04	0.02	-0.15
Cocoa	196	0.04	0.01	0.00	0.02	-0.12	0.01	-0.07	0.04	0.04	0.14
Coffee	196	0.03	-0.01	0.00	-0.03	0.02	-0.01	0.06	-0.01	-0.06	-0.13
Cotton	196	0.07	0.04	-0.01	0.14	0.09	0.03	0.17	-0.03	0.20	-0.35
Sugar	196	0.01	0.03	0.00	0.09	0.06	0.03	0.07	-0.02	0.17	-0.14
Contract without Delivery Risk											
Eurodollar	215	0.10	-0.04	-0.01	-0.13	0.06	-0.04	0.01	-0.02	-0.26	0.01
S&P 500	215	0.28	-0.01	-0.02	-0.03	0.21	-0.01	-0.08	-0.06	-0.06	-0.07
Japanese Yen	215	0.45	0.00	-0.04	0.01	0.68	0.00	0.01	-0.21	0.01	0.01
Swiss Franc	215	-0.02	-0.01	0.00	-0.02	-0.08	0.00	0.06	0.02	-0.03	0.05
Canadian Dollar	215	-0.04	0.00	0.00	-0.02	0.01	0.00	-0.09	0.00	-0.03	-0.08
Australian Dollar	215	0.02	-0.01	0.00	-0.02	0.02	0.00	0.10	-0.01	-0.03	0.09
British Pound	215	0.03	0.00	0.00	-0.02	0.02	0.00	0.10	-0.01	-0.02	0.09
Panel B		δ_{0t}	δ_{1t}	δ_{2t}	R^2	δ_{0t}	δ_{1t}	δ_{2t}	$\delta_{3t\,w\,del.risk}$	$\delta_{3t\,wo\,del.risk}$	R^2
Factor risk premium		0.001	-0.001	0.035	0.01	0.002	-0.003	0.069	-0.020	0.009	0.03
		(2.87)**	(-0.64)	(2.44)*	[1.48]	(4.77)**	(-2.71)**	(3.81)**	(-3.00)**	(2.51)*	[4.42]**

Table 5 The Risk Effect Captured by Returns on Call-Put Spreads

The table presents (in Panel B) average factor risk premiums estimated from the day-by-day Fama-MacBeth (1973) regressions of the cross-section of futures returns (R_{Ft}) on the estimated factors of two following models.

Hirshleifer's (1988) model :

The extended model with futures options:

Call-put spread is created by buying an at-the-money call and selling an at-the-money put. δ_{1t}, δ_{2t}, δ_{3t} are factor premiums for systematic risk, residual risk and the risks captured by returns on call-put spreads respectively. The estimated factors are slope coefficient ($\beta_{Fm,t}$) and standard error of residuals ($\sigma_{Fm,t}$) from time-series regressions of futures returns on stock market returns over the prior 50 weeks; and the factors of β'_t, σ'_t and Γ_t compiled from the parameters estimated from regressions, which the variables are returns on futures, the spread, stock market, using data for the prior 50 weeks. d_t denotes an indicator equal to 1 for net long futures positions of hedgers and -1 for net short positions. Panel A reports average estimated factors and average estimated premiums for each futures contract. t-statistics in parentheses are for the hypothesis that average factor premium is zero. Hotelling's T^2 statistics in square brackets are

Panel A	n	Hirshleifer's Model				The Extended Model with Call-Put Spread Futures Options					
		Esimated Factor		Estimated Premium (%)		Estimated Factor			Estimated Premium (%)		
		$\beta_{Fm,t}$	$d_t \sigma_{Fm,t}$	Sys. risk	Res. risk	β'_t	$d_t \sigma'_t$	Γ_t	Sys. Risk	Res. Risk	Options
Contract with Delivery Risk											
30-Year T-Bond	215	-0.02	0.00	0.00	-0.01	-0.03	0.00	0.04	-0.01	-0.03	-0.16
10-Year T-Note	215	-0.03	0.00	0.00	0.01	-0.02	0.00	0.03	-0.01	0.02	-0.12
Cocoa	196	0.04	0.01	0.00	0.02	0.17	0.00	0.06	0.04	0.06	-0.23
Coffee	196	0.03	-0.01	0.00	-0.03	-0.03	0.00	0.11	-0.01	-0.05	-0.47
Cotton	196	0.07	0.04	-0.01	0.14	0.01	0.01	0.10	0.00	0.19	-0.41
Sugar	196	0.01	0.03	0.00	0.09	-0.05	0.01	0.09	-0.01	0.12	-0.39
Contract without Delivery Risk											
Eurodollar	215	0.10	-0.04	-0.01	-0.13	0.16	-0.02	0.09	0.04	-0.29	-0.14
S&P 500	215	0.28	-0.01	-0.02	-0.03	0.33	-0.01	0.03	0.09	-0.11	-0.04
Japanese Yen	215	0.45	0.00	-0.04	0.01	0.32	0.00	0.05	0.08	0.01	-0.09
Swiss Franc	215	-0.02	-0.01	0.00	-0.02	-0.02	0.00	0.04	0.00	-0.03	-0.07
Canadian Dollar	215	-0.04	0.00	0.00	-0.02	-0.05	0.00	0.04	-0.01	-0.03	-0.06
Australian Dollar	215	0.02	-0.01	0.00	-0.02	0.00	0.00	0.03	0.00	-0.04	-0.04
British Pound	215	0.03	0.00	0.00	-0.02	0.02	0.00	0.04	0.00	-0.03	-0.06
Panel B		δ_{0t}	δ_{1t}	δ_{2t}	R^2	δ_{0t}	δ_{1t}	δ_{2t}	$\delta_{3t\,w\,del.risk}$	$\delta_{3t\,wo\,del.risk}$	R^2
Factor risk premium		0.001	-0.001	0.035	0.01	0.003	0.003	0.131	-0.042	-0.016	0.12
		(2.87)**	(-0.64)	(2.44)*	[1.48]	(5.06)**	(1.11)	(2.92)**	(-3.44)**	(-0.95)	[1.36]

Table 6 Effect of Residual Futures Option Returns on Futures Returns

The table reports average factor risk premiums estimated from the day-by-day Fama-MacBeth (1973) regressions of the cross-section of futures returns (R_{Ft}) on the estimated factors of stock market returns (β'_t), standard deviation of residual risk for futures market (σ'_t) and residual returns on futures options (Γ_t).

$$R_{Ft} = \delta_{0t} + \beta'_t \delta_{1t} + \sigma'_t d_t \delta_{2t} + \Gamma_t \delta_{3t} + \varepsilon_t$$

$$\beta'_t = \frac{(\beta_{Fm,t} - \beta_{Fo,t}\beta_{om,t})}{1-\rho^2_{om,t}} \; ; \; \sigma'_t = \frac{\sigma_{Fmo,t}}{\sqrt{1-\rho^2_{om,t}}} \; ; \; \Gamma_t = \frac{(\beta_{Fo,t} - \beta_{Fm,t}\beta_{mo,t})}{1-\rho^2_{om,t}}$$

δ_{1t}, δ_{2t} and δ_{3t} denote the factor premiums for systematic risk, residual risk and the risks captured by futures option returns respectively. The estimated factors are compiled from the time-series regressions which the variables are futures returns, stock market returns (value-weighted CRSP index) and residual returns on futures options, using data for the prior 50 weeks. d_t denotes an indicator equal to 1 for net long futures positions of hedgers and equal to -1 for net short positions. t-statistics in parentheses are for the hypothesis that average of factor premiums is zero. Hotelling's T^2 statistics in square brackets are for the hypothesis that all factor premiums are jointly zero. **, * denote significance at the 1 % and 10% levels respectively. All test statistics are based on Newey-West heteroskedasticity and autocorrelation estimates of covaraince matrices. The testing period is from January 2000 to February 2004.

Type of Futures Options	δ_{0t}	δ_{1t}	δ_{2t}	$\delta_{3t\ w\ del.risk}$	$\delta_{3t\ wo\ del.risk}$
Call	0.002	-0.003	0.056	0.082	-0.012
	(2.82)**	(-1.16)	(1.88)*	(2.13)*	(-0.67)
Put	0.002	-0.001	0.090	-0.023	0.018
	(2.75)**	(-1.22)	(1.93)*	(2.04)*	(1.02)
Straddle	0.001	-0.004	0.036	-0.018	-0.004
	(1.86)*	(-2.88)**	(1.89)*	(-1.65)*	(-0.22)
Call-Put Spreads	0.002	-0.002	0.006	-0.053	-0.021
	(2.85)**	(-1.50)	(2.25)*	(-2.17)*	(-1.07)

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CHAPTER 6

CONCLUDING REMARKS

This dissertation contains the theoretical and empirical analyses of the effects of delivery risk on futures demand, futures price, and futures risk premium. Most economists have ignored the deliver risk from the analyses of the futures market equilibrium. However, the presence of delivery risk causes options on futures to become a nonredundant hedging instrument and hence influences the equilibrium futures demand, price and risk premium.

Chapter 1 introduces the research issues and synthesizes the theoretical and empirical results obtained in various chapters. Chapter 2 reviews models of futures risk premium within a mathematical framework.

Chapter 3 provides a theoretical framework for justifying and analyzing the hedging role of futures options. The results show that the delivery risk decreases the net hedging demand for futures and induces the net hedging demand for futures options. Our further analysis shows that, in the presence of delivery risk, the equilibrium futures price and demand are partly explained by payoff of futures options. This implies that the futures options should play a role in pricing futures contracts.

Chapter 4 formulates the implication of futures options for pricing futures contracts by extending the pricing model of Hirshleifer (1988) to account for the delivery risk and the availability of futures options. The extended model predicts that the incremental risks captured by expected returns on futures options should be priced in futures risk premiums.

Chapter 5 tests the extended model's prediction with weekly data for 13 U.S. futures contracts. The results show that futures option returns significantly explain average futures returns and risk premiums. The explanatory power of the option return remains even after purging the component of futures option returns that may be explained by futures returns and stock market returns. The results suggest that futures option returns are important in explaining futures risk premiums.

Although the empirical results verify the general prediction of the extended model, the results should be carefully interpreted. The model focuses only on the role of the futures option in pricing futures contracts, which arises from the presence of delivery risk. As shown in this study, the model is indeed rational and meaningful, but it abstracts from other potential factors that may also influence the real futures market. It is possible that the pricing relationship documented here is due to factors other than those addressed in the model.

Moreover, the risks associated with futures contracts captured by the option return are not limited to only delivery risk. The futures option returns, which are also exposed to the common risks associated with their underlying contracts, possibly capture additional risks that are priced in futures risk premiums but not captured by the two pricing factors of Hirshleifer (1988) model. This conjecture could be addressed in further research and testing of the real market. Regardless of the causes of the pricing role of the futures options, this study shows that the futures option returns are relevant for explaining futures risk premiums.

APPENDICES

Appendix A. References Classified into Subheadings

I. One-factor models of futures risk premium

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Appendix B. Futures Data Sources

	Futures Industry Institute	Datastream	Bloomberg
Website	www.fiafii.org	www.datastream.com	www.bloomberg.com
e-mail address	data@fiafii.org	tf_helpdesk@tfn.com.ph	Help Desk Bloomberg
Average Data Range	20 Years	15 Years	15 Years
Frequency of Data	Daily	Daily	Daily
No. of Futures Contracts	121	125	183
U.S.A.	78	58	79
Non U.S.A.	43	67	104
Cost	1,770 US$	N.A.	N.A.
Advantages	Reliable data	Reliable data	Extensive data
Disadvantages	Expensive	Download time	Download time

Remark

1) This information is as of 30 December 2003

2) All vendors provide at least 20 years data for any contract that has been trading more than 20 years. and the data are available for the contracts have been trading less than 20 years.

3) The following major U.S. futures contracts are available for all data vendors.

Financial Futures: S&P500, Eurodollar, T-Bill, T-Note, and T-Bond

Currency Futures: Canadian dollar, British pound, Euro, and Japanese Yen

Energy Futures: Crude oil, Heating oil, Natural gas, Unlead gas

Agricultural Futures: Corn, Soybeans, Wheat, Soy meal, Soybean oil, World sugar, Coffee, Cocoa, Cotton, Live cattle, and Lean Hog

Metal Futures: Gold, Silver, Aluminium, Copper, Nickel, Zinc

Appendix C. Futures Option Data Sources

	Chicago Board of Trade	New York Mercantile Exchange	New York Board of Trade
Website	www.cbot.com	www.nymex.com	www.nybot.com
e-mail address	cbot_historical_data@cbot.com	exchangeinfo@nymex.com	webmaster@nybot.com
Average Data Range	10 Years	10 Years	4 Years
Frequency of Data	Daily	Daily	Daily
No. of Futures Options	39	10	38
U.S.A.	39	10	38
Non U.S.A.	0	0	0
Cost	40 US$ per year per commodity	125 US$ per year per commodity	Free
Advantages	Reliable data	Reliable data	Reliable data
Disadvantages	Expensive	Expensive	Download time

	Datastream	Bloomberg
Website	www.datastream.com	www.bloomberg.com
e-mail address	tf_helpdesk@tfn.com.ph	Help Desk Bloomberg
Average Data Range	2 Years	1 Years
Frequency of Data	Daily	Daily
No. of Futures Options	33	113
U.S.A.	23	91
Non U.S.A.	10	22
Cost	N.A.	N.A.
Advantages	Reliable data	Extensive data
Disadvantages	Short data range Download time	Short data range Download time

Remark: This information is as of 30 December 2003

Appendix D. Sources of Information on Derivatives

Organizations

- Commodity Futures Trading Commission : www.cftc.gov
- Federal Reserve Board : www.bog.frb.fed.us
- Futures Industry Institute : www.fiafii.org
- Global Association of Risk Professionals : www.garp.com
- International Association of Financial Engineers : www.iafe.org/home.php
- International Organization of Securities Commissions : www.iosco.org
- International Swaps and Derivatives Association : www.isda.org
- National Futures Association: www.nfa.futures.org
- Professional Risk Managers' International Association : www.prmia.org

Specialized Trade Publications

- Derivatives Strategy : www.derivatives.com
- Futures and Options World : www.fow.com
- Futures Industry : www.futuresindustry.org/fimagazi-1929.asp
- Futures Magazine : www.futuresmag.com
- Risk : www.riskpublications.com

Specialized Derivatives Journals

- Risk magazine and sister publications : www.riskwaters.com
- Review of Derivative Research : www.kluweronline.com/issn/1380-6645
- Journal of Derivatives : www.iijournals.com/JOD/default.asp
- Journal of Futures Markets : www3.interscience.wiley.com/cgi-bin/jhome/34434
- Journal of Risk Finance : www.iijournals.com/JRF/default.asp
- Mathematical Finance : www.blackwellpublishing.com/journal.asp?ref=0960-1627

- Finance and Stochastics : springerlink.metapress.com/app/home/journal.asp?wasp=p621a3wqym3txmaq9evl&referrer=parent&backto=linkingpublicationresults,1:101164,1
- Journal of Computational Finance : www.thejournalofcomputationalfinance.com
- Journal of Risk : www.thejournalofrisk.com

Miscellaneous Sites

- Contingency Analysis (includes discussion group) : www.contingencyanalysis.com
- FinanceWise : www.financewise.com
- Finance Wat.ch : www.finance.wat.ch
- INO : www.ino.com
- Bill Margrabe's Derivatives' Zine Site : www.margrabe.com
- Numa Web Internet Resource Center for Derivatives : www.numa.com
- Risk Management Digest : www.riskmanagementdigest.com
- Mark Rubinstein's In-the-Money : www.in-the-money.com

Derivative Collections of Sites

- Google's Listing for Commodities and Futures : directory.google.com/Top/Business/Investing/Commodities_and_Futures
- Yahoo's Listing of Futures and Options Sites : dir.yahoo.com/Business_and_Economy/Finance_and_Investment/Futures_and_Options