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2015

Lin, C. S., Chua, C. K., & Yeo, J. H. (2015). Badminton shuttlecock stability: Modelling and simulating the angular response of the turnover, Proceedings of the Institution of Mechanical Engineers, Part P: Journal of Sports Engineering and Technology.

<https://hdl.handle.net/10356/79332>

<https://doi.org/10.1177/1754337115596481>

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Badminton shuttlecock stability: Modelling and simulating the angular response of the turnover

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ABSTRACT

Turnover of a badminton shuttlecock is the flipping motion of the shuttlecock after its initial contact with the racket. During the process, the shuttlecock experiences a large change in heading. In this paper, the turnover stability of the shuttlecock is investigated through experiment and simulation. Three types of badminton shuttlecocks are experimentally evaluated: one feather shuttlecock (Li-Ning A+600) and two synthetic ones (Yonex Mavis 350 and Mizuno NS-5). The experimental results are applied to a response model that takes the form of an under-damped second order transfer function. This angular response model is then used for the identification of the turnover parameters: the damping ratio and the time constant. The identified parameters are subsequently used as input for building a response function to predict the turnover angular behaviour of the shuttlecock. The feather shuttlecock, which has the highest damping ratio and the lowest time constant, is the shuttlecock with the best turnover stability. Finally, the simulated pitching moment components of the feather shuttlecock are evaluated.

Keywords: badminton, shuttlecock, turnover, simulation, modelling

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1. Introduction

Badminton is a sport which is played with rackets and shuttlecocks. The shuttlecock is comprised of a cork and a skirt that is formed by 16 feathers. For a synthetic shuttlecock, the synthetic materials are substituted for the feather skirt. Regardless of the construction or material, the standard shuttlecock weighs approximately 5g. The mass is concentrated around the cork, such that the centre of gravity (C.G.) is just behind the base of the cork. The thin-walled conical skirt of the badminton shuttlecock generates large aerodynamic forces that act through the centre of pressure (C.P.). The combination of a forward C.G. and a rearward C.P. gives the badminton shuttlecock a natural tendency to fly nose first and in a stable manner.¹

Fig.1 shows the shuttlecock flying at an angle to the airflow. The resultant aerodynamic forces (lift and drag) acting through the C.P. produce a resultant clockwise torque. This torque, which acts through the C.G., reduces the flight angle with respect to the airflow direction and aligns the shuttlecock to the flight path. Consequently, when it is perturbed, the shuttlecock always tries to self-correct and align back to the flight path. Thus, it has a stable heading.

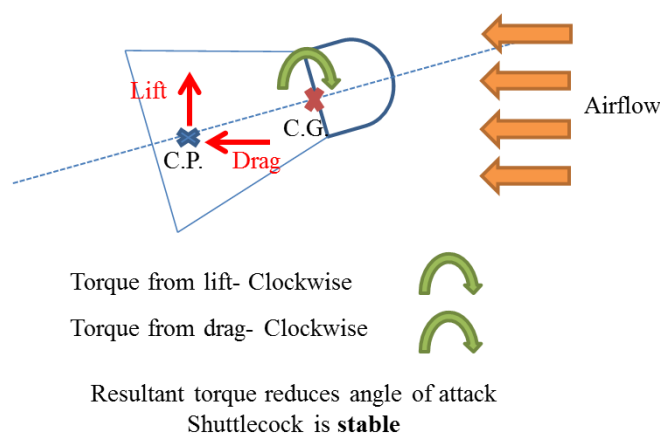


Fig.1 The combination of a forward C.G. and a rearward C.P. gives the shuttlecock a natural tendency to fly nose first in a stable manner.

During play, the badminton shuttlecock flies nose first towards a player. When the player returns the shot to his opponent, the motion of the stroke imparts to the shuttlecock a

translational velocity opposite to the shuttlecock initial velocity. Due to a tendency to fly nose first, the shuttlecock flips over to reduce the angle between its heading and the flight path. This behaviour of flipping and change in heading is commonly referred to as the turnover of a shuttlecock.^{2, 3} The turnover of the shuttlecock occurs in the unsteady-flight state, which refers to the flight segment from the moment the racket hits the shuttlecock until the time at which the shuttlecock has achieved a stable flight heading.

Upon completion of the turnover, the flight transitions into the steady-flight state. In the steady-flight state, the shuttlecock experiences a large change in heading. However, the angle of attack remains small because the shuttlecock heading follows closely to the flight path heading. While the steady-flight state is commonly studied through work on flight trajectory⁴⁻⁸, wind tunnel evaluation⁹⁻¹¹ and flow simulation¹²⁻¹⁴, the unsteady-flight state and the turnover are much less studied.

The turnover process is only relevant for a projectile which is non-spherical and has a flight heading of significance. Characterised by the heading, the turnover phenomenon is very different from the angular motion (spin) of sport balls.¹⁵⁻¹⁷ However, the lateral stability, which is the equivalent of the turnover stability, is a much studied topic in other flight applications such as bombs^{18, 19}, arrows²⁰, aircraft²¹ and wingsuits.²² In the open literature, examples of work on shuttlecock turnover are limited¹⁻³.

The turnover behaviour of a badminton shuttlecock was first discussed by Cooke³ who determined that the duration of the turnover was 0.03s. Although both the feather and synthetic shuttlecocks were tested, there was no difference recorded. In another study, the turnover duration observed by Texier et al.¹ was 0.2s and very much longer than what Cooke³ had observed. This difference may have been due to changes in the launch conditions and the definition of the turnover duration. The chrono-photograph presented by Texier et al.¹

showed that the turnover process was an under-damped angular motion. The turnover performance of the feather and synthetic shuttlecocks was further evaluated by Lin et al.² using a mechanical racket-based launcher. The launch condition of the shuttlecocks was controlled to give comparable results between the shuttlecocks. Similar to the observation by Cooke³, it was determined that the synthetic shuttlecocks had similar angular response time as the tested feather shuttlecocks. However, the tested synthetic samples had much lower damping in the angular response. This lower damping led to a larger amplitude of oscillation in the turnover. The observed shuttlecock turnover behaviour had an angular response that was similar to the harmonic motion of an under-damped mass-spring-damper setup.

The Badminton World Federation (BWF) has termed the turnover stability as tumbling stability.²³ Its testing to define shuttlecock stability took a different approach and was based on the flight distance travelled before the turnover was completed. Despite the approach difference, its definition of good turnover stability is the same as those from¹⁻³. The desired turnover response is one which is completed in the shortest time and with the smallest oscillation amplitude. However, this evaluation criterion was insufficient to provide a thorough comparison between the synthetic and the feather shuttlecocks. To provide such a comparison, the entire angular response of each shuttlecock, which is in addition to the flight parameters such as the duration of turnover and the amplitude of oscillation, should be analysed. Moreover, the pitching moments involved in the turnover process should be discussed.

It was observed that the existing work on turnover used measurement indicators that did not thoroughly account for the quality of the turnover behaviour. The purpose of this research is to further enhance the understanding of the turnover behaviour by extending the work that was presented in². This paper aims to use the experimental data and a flight model to simulate the turnover response of feather and synthetic shuttlecocks. The simulation of the

angular response will provide information on the significance of the damping and oscillation period for the turnover process. The pitching moments will also be studied.

2. Method

2.1 Flight model

The change in the flight angle of a shuttlecock during the turnover process is very similar to the harmonic oscillation of a mass-spring-damper setup.² Therefore, the turnover process can be modelled by a second-order under-damped transfer function which is similar to that for the classical mass-spring-damper. The validity of this approach can be shown through the further derivation of the equation of angular motion that was presented by Cooke.³ By taking the summation of the aerodynamic moments in the pitch axis, the moments acting on the shuttlecock are given as:

$$I_{yy}\dot{q} + cq + \left| \frac{dM}{d\alpha} \right| \alpha = 0 \quad (1)$$

The symbol I_{yy} is the moment of inertia; q is the angular rate of heading change; \dot{q} is the first time derivative of q ; c is the damping constant of the rotational motion in pitch; $\frac{dM}{d\alpha}$ is the rate of change in the pitching moment with respect to the angle of attack; and α is the angle of attack with respect to the flight path. In equation (1), there are two moments acting on the shuttlecock to correct the heading: the moment from translation and the moment from rotation. The moment from translation is the pitching moment induced on the shuttlecock when it is flying at an angle to the free-stream. This moment is represented by the term $\left| \frac{dM}{d\alpha} \right| \alpha$. The moment from rotation is represented by the term cq , which is the moment due to the flipping motion (angular velocity) that occurs during the turnover.

Although the pitch angle is different from the angle of attack, the change rates of these two variables are always the same. This equality is because a change in the pitch angle results in an equal change of the angle of attack in the same direction. Therefore, the angular rates of both the pitch angle and the angle of attack can be related by:

$$\dot{\alpha} = \dot{q} \quad (2)$$

$$\ddot{\alpha} = \ddot{q} \quad (3)$$

The turnover process is initiated by an external moment from the racket, $M_{racket}(t)$, which is a short impulse-like input to the shuttlecock. By substituting (2) and (3) into (1) and then taking $M_{racket}(t)$ into consideration, the turnover process is modelled as:

$$I_{yy}\ddot{\alpha} + c\dot{\alpha} + \left|\frac{dM}{d\alpha}\right|\alpha = M_{racket}(t) \quad (4)$$

Applying a Laplace transformation to (4), the turnover dynamics are then given as:

$$I_{yy}S^2\alpha + cS\alpha + \left|\frac{dM}{d\alpha}\right|\alpha = M_{racket}(S) \quad (5)$$

$$\frac{\alpha}{M_{racket}(S)} = \frac{\left|\frac{d\alpha}{dM}\right|}{\frac{I_{yy}}{\left|\frac{dM}{d\alpha}\right|}S^2 + \frac{c}{\left|\frac{dM}{d\alpha}\right|}S + 1} \quad (6)$$

Equation (6) is the second-order transfer function that describes the oscillatory behaviour of the turnover process. It is also in the same form as the second-order system that is used for describing the linear oscillation of a mass-spring-damper setup. This system is represented by equation (7).

$$\frac{X}{F(S)} = \frac{\frac{1}{k}}{\frac{m}{k}S^2 + 2\frac{c}{k}S + 1} = \frac{\frac{1}{k}}{(T_w S)^2 + 2\zeta T_w S + 1} \quad (7)$$

The symbol X refers to the displacement in the Laplace domain, k is the spring stiffness and ζ is the damping ratio. The time constant, T_w , is the inverse of the natural frequency.

Therefore, the P2U (two poles, under-damped) structure that is used for describing a mass-spring-damper setup can also be applied to describe the turnover performance. By comparing (6) to (7), it can be shown that c and $\left|\frac{dM}{d\alpha}\right|$ of a shuttlecock can be represented by I_{yy} , ζ and T_w .

$$\left|\frac{dM}{d\alpha}\right| = \frac{I_{yy}}{(T_w)^2} \quad (8)$$

$$c = 2 \left|\frac{dM}{d\alpha}\right| \zeta T_w = 2 \frac{I_{yy}}{T_w} \zeta \quad (9)$$

The turnover parameters, ζ and T_w , are variables that can be obtained through analysis of the shuttlecock heading. From the perspective of the actual response, ζ symbolises the amount of overshoot where a larger value of ζ means more damping and smaller amplitude of oscillation. The time constant, T_w , refers to the period of the oscillation where a smaller value of T_w means that the oscillation period is shorter.

2.2 Flight experiment

The turnover behaviours of three different types of shuttlecocks were observed experimentally. The shuttlecocks used are: a Li-Ning A+600, which is a tournament-grade feather shuttlecock; a Yonex Mavis 350, which is a nylon synthetic shuttlecock; and a Mizuno NS-5, which is constructed from artificial feathers. The shuttlecocks were launched using a shuttlecock launcher which is based on a rotating racket. Because the racket is only rotated about a single axis, the resultant moment, $M_{racket}(t)$, can be assumed to be acting only in the pitch direction of the shuttlecock. With a consistent launcher setting, the resultant launches had an initial speed of 22-24 m/s at an angle of approximately 31 degrees to the

horizon. This initial flight condition produced a flight path that is similar to a clear shot. A high-speed camera (Photron Fastcam 1024PCI) was used to record the initial turnover motion of the shuttlecocks at 1000 frames per second (fps). The camera was positioned at a distance away from the launcher such that the resultant image contained more than 1.5m of the initial flight path. This 1.5m flight distance ensured that the first oscillation of each launch was fully captured. The camera was aligned such that the image plane was parallel to the flight path. A high-speed chrono-photograph of one of the launches is shown in Fig.2 and demonstrates the resultant field of view from the camera.

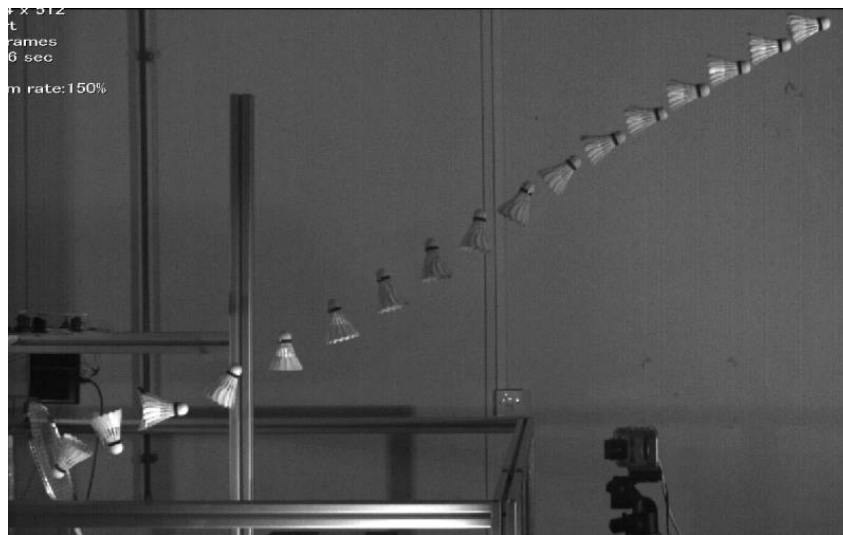


Fig.2 A high-speed chrono-photograph of the turnover process in one of the launches

The experiment was repeated eight times for each shuttlecock. The captured high-speed videos were then digitised at a sampling interval of 0.005s to obtain the shuttlecock heading angles with respect to time. The flight path angle of each launch was also measured and subtracted from the shuttlecock heading angles to obtain the angles of attack. This approach of analysing the angle of attack with respect to the flight path eliminated the bias that might arise from variations in flight path angles. The flight path angle was assumed to be constant for each launch. This assumption is probably valid because the time span of analysis was too short for gravitational acceleration to produce a curved flight path. Each shuttlecock took

between 0.05-0.06s to complete the first oscillation in turnover. Based on the gravitational acceleration (g) of 9.81 m/s and the time duration (t) of 0.06s, the resultant flight path displacement due to gravity was calculated to be 0.0177m (10). This 0.0177m was insignificant compared to the distance travelled in the same 0.06s. Therefore, the effect of gravity on the initial flight path was neglected and a straight line flight path was assumed.

$$Deviation = 0.5gt^2 = 0.5(9.81)(0.06^2) = 0.0177m \quad (10)$$

The time period of stabilisation was measured for each run. This period was taken as the time from which the turnover was initiated to the time when the shuttlecock attained a zero-degree angle of attack after the first oscillation. The overshoot angles were also measured. These provided physical explanations of the ζ and ω_n that were to be identified in the next step.

2.3 Parameter identification

Using the model identification function on MatlabTM, ζ and T_w of each of the experimental runs were estimated from the data of the angle of attack. The P2U structure, which is appropriate for an under-damped second-order response, was applied as the identification model. The identification tool on MatlabTM uses the model response data (the angles of attack) and the predetermined identification model (P2U) to estimate the parameters (ζ and T_w). Therefore, the input is an iddata object consisting of the angles of attack and the time, while the outputs are the estimated ζ and T_w . The estimated ζ and T_w form a second-order transfer function that has the same output response as the shuttlecock heading during turnover.

Because only one pair of ζ and T_w was estimated for the whole process of each turnover response, the obtained values were the estimated means. The large change in heading during the turnover meant that ζ and T_w were unlikely to remain as constants for the entire process. Therefore, their values should ideally be identified at every time step for improved accuracy.

However, such an approach is complicated and tedious. Although the applied P2U fitting method has a disadvantage in accuracy, it is fast and feasible. Moreover, comparisons of the simulated response and experimental data showed reasonable agreement. This reasonable agreement is demonstrated in section 3.2 of this paper.

From the eight sets of data for each of the shuttlecocks, eight independent pairs of ζ and T_w were identified. From those, the four median experimental runs were used to compute the mean ζ and T_w of each of the shuttlecocks. The four median runs were decided by first selecting the median six runs based on the values of ζ and then the median four runs from the remaining six by eliminating the runs with the smallest and the largest T_w . This process was done for all three tested shuttlecocks to eliminate any bias due to uncontrolled launch variations.

2.4 Response simulation

The turnover angular response was simulated by using the MatlabTM impulse response function. The impulse response simulation was applied to transfer functions which were based on the structure of equation (7) and the obtained values of ζ and T_w . The output of the simulation was the simulated angles of attack in the time-domain.

The simulation approach was first validated by comparing the experimental result and the simulated turnover response of one of the experimental runs. Upon proving the accuracy of the approach, the same simulation methodology was used to obtain the mean turnover response of each of the shuttlecocks. The simulated mean responses provided a comparison between the shuttlecocks. Moreover, the simulated responses also gave insights on the subsequent oscillations of the turnover which were not captured with the high-speed camera. Using the simulated angular response of the A+600, the pitching moments acting on the

feather shuttlecock were also calculated. They were used to explain the contribution of each component of the pitching moment.

3. Results

3.1 Flight experiment

The mean identified values of T_w and ζ and their standard deviations (std. dev.) are shown in Table 1. These mean values are the average of the four median runs of each shuttlecock. The result shows the A+600 feather shuttlecock had the largest damping ratio and the smallest time constant. In practice, the identified values mean that the tested feather shuttlecock had less overshoot (smaller oscillation amplitude) than the tested Mavis 350 and NS-5. The A+600 also completed each oscillation within a shorter duration. The mean identified values of T_w and ζ agreed with the mean measured overshoot and the period, which are also shown in Table 1. The amplitude of the mean overshoot on the A+600 was only 67° as compared to 114° observed for the tested Mavis 350 synthetic. The large amount of overshoot observed for the Mavis 350 is presented in Fig.3.

Table 1 The mean values of the identified natural frequencies and damping ratios, and also the measured mean overshoot and time to complete turnover.

	Mean time constant, T_w ($s\ rad^{-1}$)		Mean damping Ratio, ζ		Mean overshoot (degree)		Approximate period (s)	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
Li-Ning A+600	0.0102	0.00109	0.390	0.0111	67	7.2	0.050	0.0031
YonexMavis350	0.0114	0.00033	0.282	0.0119	114	14.2	0.050	0.0028
Mizuno NS-5	0.0129	0.00156	0.368	0.0649	95	8.9	0.055	0.0011

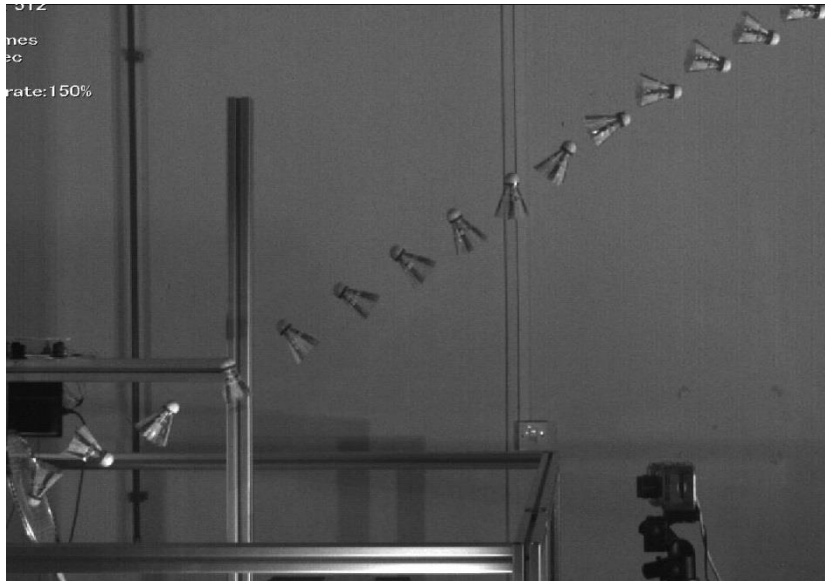


Fig.3 Chrono-photograph of the turnover process of a Yonex Mavis 350 synthetic shuttlecock.

The standard deviation of the mean overshoot angle shows the variation in overshoot. This variation in overshoot may have been caused by the difference in the initial racket-shuttlecock contact angle among the experimental runs, which are presented in Table 2. The different initial contact angles could have resulted in variations between the initial angular velocities, which have led to different overshoot angles. However, this difference in the initial contact angles was unlikely to have affected the response analysis because the analysis took into account the angular velocity.

Table 2 Initial angle at launch.

	Initial angle (deg)	
	Mean	Std Dev.
A600	147.7	3.015792
Mavis 350	145.55	4.025233
NS-5	148.575	3.757908

3.2 Response simulation

The simulated responses were obtained by using results from one of the runs of each shuttlecock. The results from an individual experimental run were chosen over the means because the individual experimental run provided experimental flight angle data for

validating the simulated response. The parameters that were used for simulation are listed in Table 3. The simulated angular response of each shuttlecock was plotted against the experimental data, and the data are presented in Fig.4 to Fig.6. The simulated angular responses showed good agreement with the experimental data.

Table 3 The time constants and the damping ratios that were identified from the experimental runs and used for simulation.

	Time constant, T_w ($s \text{ rad}^{-1}$)	Damping ratio, ζ
A+600	0.0107	0.381
Mavis 350	0.0118	0.268
NS-5	0.0137	0.335

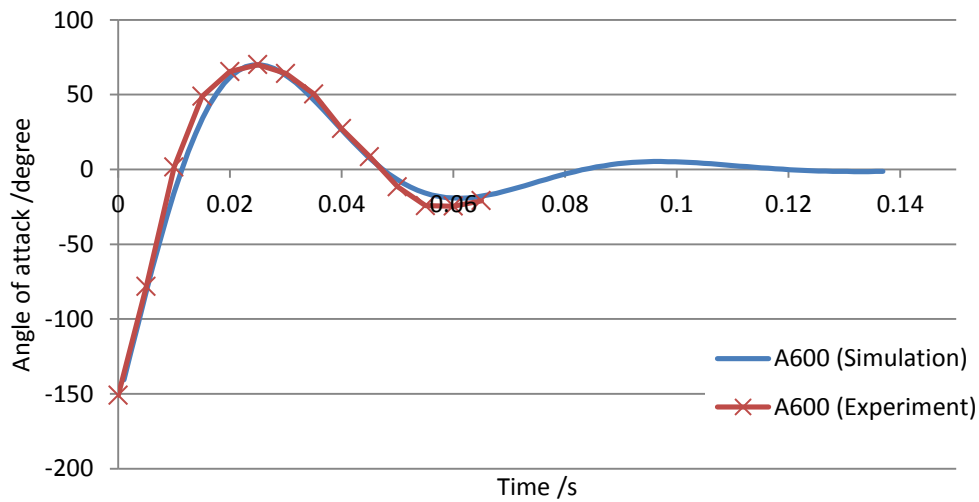


Fig.4 Comparison between the experimental and the simulated turnover response of the A+600 feather shuttlecock.

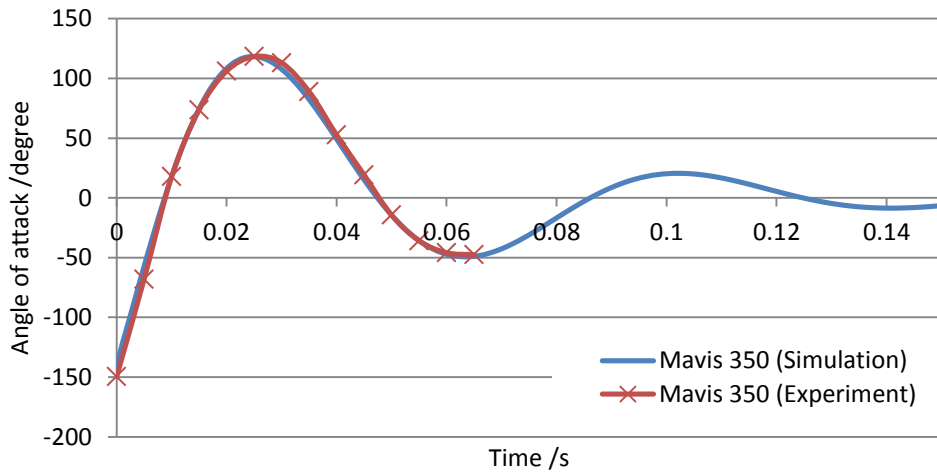


Fig.5 Comparison between the experimental and the simulated turnover response of the Mavis 350 synthetic shuttlecock.

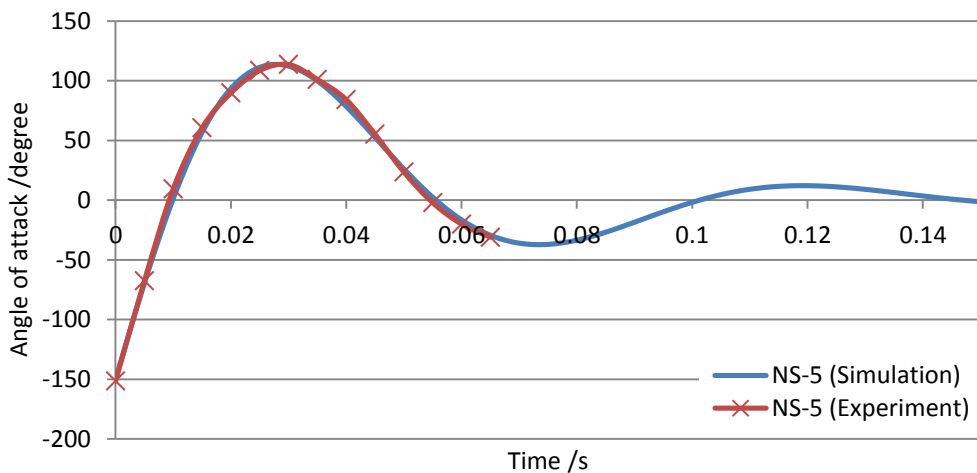


Fig.6 Comparison between the experimental and the simulated turnover response of the NS-5 artificial feather shuttlecock.

Using the mean values of T_w and ζ that were presented in Table 1, the mean angular response of each shuttlecock was simulated. The decay responses of the shuttlecocks were plotted and they are presented in Fig.7. The angular response of the A+600 corresponds to the response of a good turnover which has a large ζ and a small T_w . This response can be viewed as the reference response. The angular behaviour of the Mavis 350 corresponds to the response obtained when damping is insufficient. The response of the NS-5 is that of a shuttlecock which has a larger response period T_w .

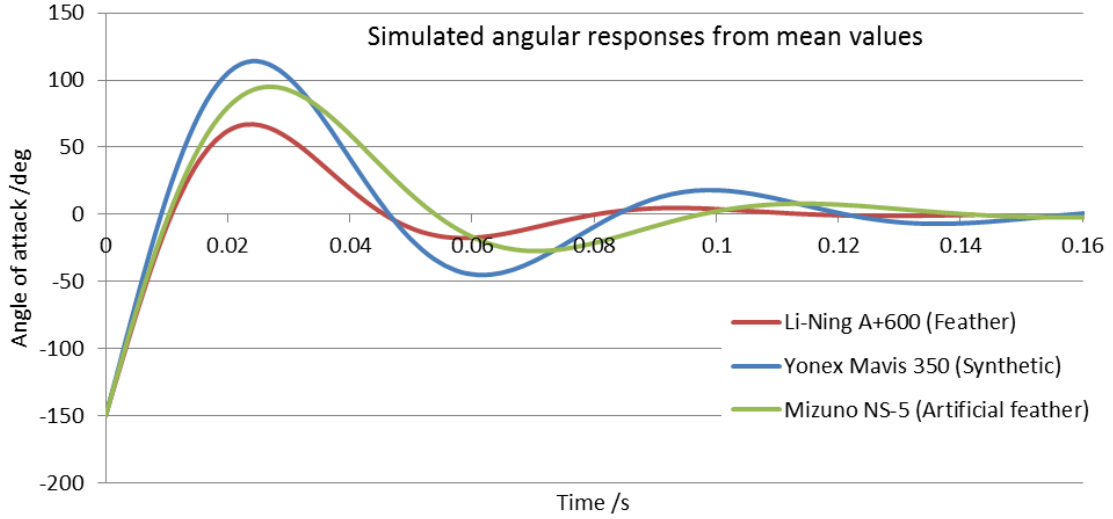


Fig.7 Simulated turnover response based on the mean identified values of the shuttlecock parameters.

3.3 Feather shuttlecock- pitching moments

The pitching moments acting on the A+600 were calculated from the simulated mean response. They are presented in Fig.8 with the simulated mean angular response. The moment from translation is referred to as the “moment (translation)”, while the “moment (rotation)” refers to the moment that is produced from the rotational motion (pitching) of the shuttlecock.

Using the bifilar swing method that was described by Cooke³, five measurements of the moment of the inertia were taken for the A+600. The mean value is $2.92 \times 10^{-6} \text{ kg}\cdot\text{m}^2$ with a standard deviation of $5.02 \times 10^{-8} \text{ kg}\cdot\text{m}^2$. Applying the moment of inertia and the mean T_w and ζ to equation (9) and (10), $\left| \frac{dM}{d\alpha} \right|$ and c were calculated as $2.82 \times 10^{-2} \text{ Nm}\cdot\text{rad}^{-1}$ and $2.24 \times 10^{-4} \text{ Nm}\cdot\text{rad}^{-1}\text{s}$, respectively. The calculated $\left| \frac{dM}{d\alpha} \right|$ is equivalent to a $\left| \frac{dCm}{d\alpha} \right|$ value of 0.28 rad^{-1} . This value agrees with the value obtained by Cooke³ (0.25 rad^{-1}) but is much lower than the result of Hasegawa et al.¹⁰ (0.72 rad^{-1} for no axial spin). Multiplying $\left| \frac{dM}{d\alpha} \right|$ with the angle of attack gave the moment from translation, while the product of c and the angular velocity gave the moment from rotation.

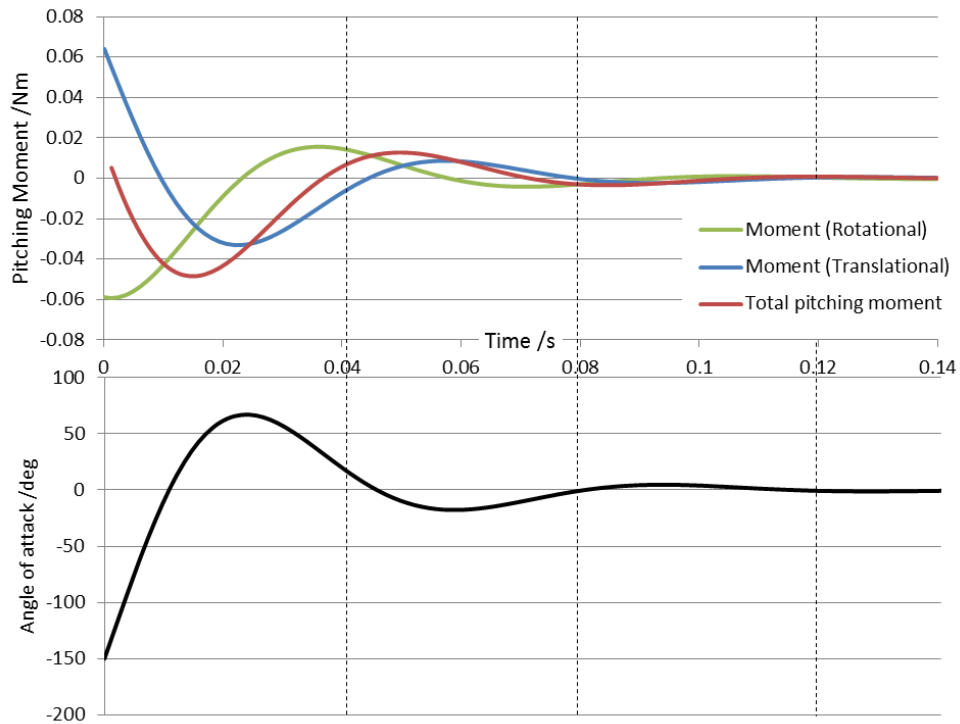


Fig.8 The simulated pitching moments of the feather shuttlecock plotted against the angular behaviour.

4. Discussion

4.1 Damping ratio and time constant

The Mavis 350 and the NS-5 had inferior stability at turnover when compared against the tested feather shuttlecock. Although the Mavis 350 and the NS-5 are both synthetic shuttlecocks, their turnover performance deviated from the tested A+600 in different forms. It was observed that the Mavis 350 had a much smaller ζ than the NS-5 and the A+600, which can be correlated with its significant larger overshoot compared with that of NS-5 and A+600. Comparing the chrono-photographs for A+600 (Fig.2) and the Mavis 350 (Fig.3), it was observed that the Mavis 350 had increased oscillation amplitude.

In contrast to the Mavis 350, the mean ζ of the NS-5 was 0.368, and it was only 5.5% lower than that of the tested feather shuttlecock (0.390). However, the NS-5 had 41.8% more

overshoot than the A+600. This overshoot was because the T_w of the NS-5 was significantly larger than the other two shuttlecocks. The equations of motion show that the moment due to translation is a function of the T_w , while the moment due to rotation is a function of both T_w and ζ . Therefore, an increase in T_w would reduce both the $\left|\frac{dM}{d\alpha}\right|$ and c , leading to larger overshoot observed for the NS-5. In practice, the increase in T_w meant that the oscillation cycles in the turnover process took longer to dissipate. Because the turnover stability can influence the perception of shuttlecock quality, it is likely that an observer will find the A+600 to be the better shuttlecock.

The difference between the Mavis 350 and the NS-5 was likely due to the difference in construction between the two shuttles. The Mavis 350 is a synthetic nylon skirt shuttlecock with a relatively porous skirt. The increase in porosity would have increased the air bleeding through the skirt. Consequently, the resultant correcting moment acting on Mavis 350 when the shuttlecock was flying at an angle would have been reduced. Moreover, the increase in skirt porosity may have reduced the flipping resistance and that would have resulted in a smaller c . Compared to the Mavis 350, the foam vanes on the artificial feathers of the NS-5 formed a relatively non-porous skirt, much like the skirt on the feather shuttlecock. This difference between the skirts may also account for the higher damping ratio that was observed on the NS-5.

4.2 Simulation

Despite the simplified method of P2U fitting, good agreement was obtained between the experimental data and the simulated results as shown in Fig.4 to Fig.6. This good agreement shows that the assumption of constant ζ and T_w did not result in large deviations between the simulated response and the experimental result. The flight modelling and simulation approach is thus likely to be sufficient for describing the shuttlecock turnover process. This sufficiency

was possible because the angular response of the turnover process was fundamentally similar to that of a P2U structure.

The trend observed for the plots was the same as that predicted through the discussed mean values. A similar trend was also observed for the simulated responses that were obtained with the mean values. The trend was described using the plot presented in Fig.7. The feather A+600 had the least amount of overshoot in the turnover response and was able to complete turnover in the shortest time. The Mavis 350 and the NS-5 had poorer angular stability and significantly larger overshoot than the feather shuttlecock.

The NS-5 demonstrated smaller amplitude in the residual oscillations due to better damped angular response. The observer who tests the shuttlecock in actual use is likely to get a visual effect that the oscillations on the NS-5 are smaller and thus grade the NS-5 as having better turnover stability than the Mavis 350. This observation suggests that the damping ratio is an important parameter.

4.3 Pitching moments

In addition to providing angular response, the simulation also provided information on the pitching moments. Due to the small magnitude of torque involved and the complexity of the dynamic flight condition required, it is difficult to obtain these pitching moments experimentally. From the simulated pitching moments presented in Fig.8, it was observed that the moment due to translation and rotation had similar orders of magnitude. This similarity of order suggests that both components of the pitching moment equation in (1) are of equal significance. This equal significance implies that the traditional wind tunnel analysis of the pitching moment coefficient is insufficient for understanding the turnover of the shuttlecock. This equal significance is because the pitching moment coefficient from the static wind tunnel analysis is only comprised of the moment from translation.

In the initial phase of the turnover, the change in heading was large. The combination of the moment components corrected the heading and the angular velocity: the moment due to translation reduced the magnitude of the angle of attack and the moment due to the angular motion reduced the magnitude of the angular velocity. Due to the interaction of these two moment components, there were instances where one counteracted the other. This counteraction can be seen in Fig.8 for the time before 0.01s. During this initial phase of the turnover, the shuttlecock had a negative angle of attack that resulted in a correcting moment, $\left|\frac{dM}{d\alpha}\right|\alpha$. This moment pushed the nose of the shuttlecock up to reduce the angle of attack. However, the angular velocity of the shuttlecock at the mean time was in the direction of an increasingly positive angle of attack, and the moment, $c\dot{\alpha}$, induced by the angular velocity would be in the opposite direction of $\left|\frac{dM}{d\alpha}\right|\alpha$. Therefore, it can be said that these moment components worked against each other in a portion of the turnover process.

As the shuttlecock heading stabilised, the angular velocity was greatly reduced. This reduction corresponded to the time after 0.05s in Fig.8. The decrease in angular velocity also reduced the contribution of $c\dot{\alpha}$ to the total moment. This result suggests that the elimination of small amplitude oscillation was largely a consequence of the moment from translational motion. It was likely that the small amplitude oscillations – observed in the later segments of the turnover for the synthetic shuttlecocks – could be mitigated with an increase in the $\left|\frac{dM}{d\alpha}\right|$.

4.4 Implications

The experimental angular response of the feather shuttlecock was compared to the response of three mass-spring-damper systems with differing damping factors. The damping ratios of these systems with respect to the critical damping ratio, c_{critical} , were:

- ($c = 0$) for the case of zero damping

- ($c < c_{critical}$) for the case of under-damped
- ($c > c_{critical}$) for the case of over-damped

The responses are presented in Fig.9. The symbol A_i / A_o refers to the ratio of the instantaneous amplitude to the original amplitude. It is clearly shown that the shuttlecock angular response resembled that of the under-damped system. The decay in amplitude signifies the importance of damping for obtaining good turnover stability. Because the moment from damping was usually not considered in wind tunnel studies, it is likely that experimental data from the wind tunnel are not representative of the shuttlecock turnover stability.

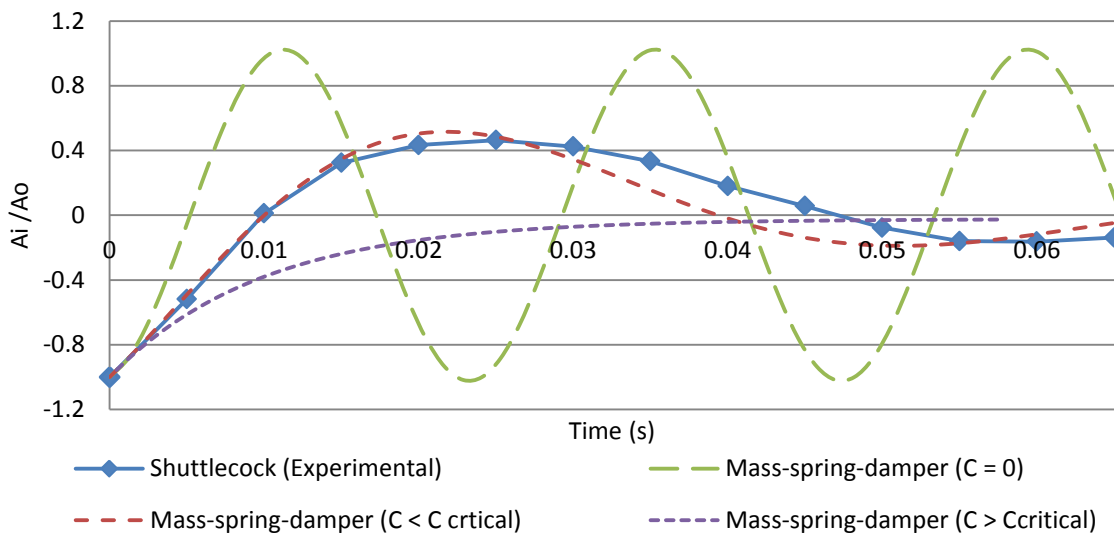


Fig.9 Comparison of shuttlecock angular response with mass-spring-damper system responses.

The response for the system with zero damping can be seen as the expected response from a shuttlecock when the angular motion is not damped. This will also be the response obtained if the wind-tunnel-measured pitching moment data is used for simulating the turnover response. Since the output amplitude does not increase with time, this is a stable response in the context of system modeling. Such behavior is classified as an unstable turnover and is highly undesired. Thus, the pitch damping is a critical component in angular stability. From the free-

body diagram in Fig.1, it can be deduced that such behavior may occur when the C.G. is too rearward and the C.P. is too forward, or when there is an insufficient restoring moment.

In practice, the aforementioned points suggest that the turnover of a shuttlecock can be enhanced through the following changes:

- A heavier head (cock) with a lighter skirt to bring the C.G. forward.
- A skirt which has the aerodynamic surfaces concentrated at the trailing edge of the skirt. This skirt configuration is the case of the current feather shuttlecocks which have the feather vanes on the trailing edge. The result is a rearward C.P. that increases the moment arm of the restoring moments.
- A larger cross-sectional profile of the shuttlecock such that the rotational resistance (damping ratio, c) is increased. This increase in the rotational resistance corresponds to an increase in the pitching moment from damping.

4.5 Limitations and future work

After the first overshoot, the amplitudes of the second and subsequent oscillations are much smaller than the initial oscillation. These small angles of attack are difficult to be measured accurately. Further simulation study is suggested to monitor the longer segment of flight and obtain more information on this segment.

While the experimental results were only obtained for the initial 0.065s of flight, they were able to characterize the turnover performance of the three types of shuttlecocks tested and determine the deficiency in the turnover performance of the synthetics. Nonetheless, it is proposed that experiments in future work should attempt to monitor the full process of the turnover.

In the simulation conducted in this study, the damping ratio and the time constants were assumed to be independent of the air speed. This independence was because the controlled launch condition meant that the initial speed of the shuttlecocks were similar. Moreover, the flight segment of measurement was assumed to be too short for large changes in flight velocity. Future work to address these assumptions will further enhance the understanding of the turnover oscillations of a badminton shuttlecock.

In the future, the launching velocities of the shuttlecock can be varied in a controlled manner and the identification process for T_w and ζ can be experimented using a piece-wise approach. The piece-wise approach will also be beneficial in understanding the change in the constants with respect to the heading change.

5. Conclusion

The turnover of the shuttlecock has traditionally been quantified through the time duration or the flight distance to achieve a stable heading. This approach has given a limited understanding of the turnover stability. In this study, the angular response of the shuttlecock in turnover was modelled and studied using a second-order approach that is derived from first principles. This approach has the advantage of being able to compare the turnover quality of the shuttlecocks. It also provided an understanding of the turnover phenomenon through analysis of the fundamental parameters.

The angular response model was similar to that of the classical mass-spring-damper example and was validated with experimental data from the testing of three different types of badminton shuttlecocks. Mean values of the turnover parameters were also identified from the experimental data and used to simulate the angular responses. The differences in angular response among the three shuttlecocks were explained through the various parameters. Although the shuttlecocks took similar time to attain a stable heading, the feather shuttlecock

had a smaller oscillation amplitude. Therefore, an observer may identify the feather shuttlecock to be superior. The pitching moment components of the feather shuttlecock suggested that the moment from rotational motion (damping) is as important as that from translational motion (pitching). Thus, the common wind tunnel measurement of the translational (pitching) moment is insufficient to predict turnover performance. The turnover performance of the shuttlecock can be enhanced by shifting the C.G. forward to the tip of the cock, shifting the C.P. rearward closer to the trailing edge of the skirt or increasing the effective cross-sectional profile.

Acknowledgement: The authors thank Dr. Liu Dan for her assistance in language editing and proofreading.

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