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Improved Lithium-Ion Battery Model with Hysteresis Effect

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Abstract—This paper presents an improved method of modelling the hysteresis effect in batteries using the discrete preisach model. The hysteresis effect can be seen as a path-dependent effect or history-dependent effect on the lithium ion battery. This effect complicates the relationship between State of Charge (SOC) and Open Circuit Voltage (OCV) as it is no longer simply a one to one relationship, but rather it is depending on history of the battery. To solve the aforementioned effect, a discrete preisach model is implemented in an improved battery model to encapsulate the SOC-OCV relationship.

I. INTRODUCTION

The hysteresis effect is one of the complex dynamic behavioural characteristics of batteries especially for those which are based on intercalation materials. The hysteresis effect shows different equilibrium potentials at the same State of Charge (SOC) depending on whether the battery was previously charged (intercalated) or discharged (deintercalated). The Open Circuit Voltage (OCV) after a previous charge is higher than the OCV after discharge at the same SOC value [1]. The extent of the hysteresis effect is different in different types of batteries. The hysteresis effect is extremely significant in Nickel-Metal Hydride (NiMH) batteries [2]. Lithium-ion (Li-ion) cells based on Lithium Iron Phosphate (LiFePO₄) [2] and graphite [3] have a very flat OCV charge discharge profile with a pronounced OCV hysteresis phenomena. The voltage plateau exists in the SOC range from 20% to 80%. Very accurate measurements of the terminal voltage are required in this range as any errors in the measurement would be magnified many times as compared to cells from other chemistries. This aspect makes it more difficult to design a Battery Management System (BMS) for cells based on the LiFePO₄ chemistry as accurate estimations of the SOC are needed in the mid-range.

A. Electrochemistry explanation of hysteresis

The origin of hysteresis is not really fully understood. However, there exist several possible theories to address hysteresis. The detailed electrochemical explanation given by researchers for the hysteresis observation in Li-ion batteries can be grouped into three main groups.

The first group focuses their explanations based on the kinetics of insertion batteries. Yamada, et al [4] have stated that lithium has two coexisting states/phases at any time when it is charged or discharged except at the initial start and at the end of the charge/discharge reaction when it exists in the monophasic region. The two phases gives the plateau to a lithium profile that can be attributed to the graphite material in the anode of the lithium ion battery.

Srinivasan and Newman [5] have noted that the existence of an asymmetric behavior between charge and discharge in the LiFePO₄ cathode, whereby the utilization on charge is considerably larger than that on discharge under current densities where transport limitations are important. They have described how the existence of a path-dependence in the system whereby the high-rate electrochemical behavior of the electrode at a particular SOC depends on the path by which the electrode was brought to that SOC. The “shrinking core model” proposed by them [6] is used to describe the voltage plateau in the lithium profile. The juxtaposition of the two phases is assumed to be in the form of a shrinking core, where a shell of one phase covers a core of the second phase.

In a work by Morgan [7], the activation barriers to Li ion motion are calculated in the absence of electrical conductivity constraints. The authors conclude that the activation barrier for Li hopping in the lithiated material is larger than that in the delithiated material suggesting that the transport properties depend on the material through which conduction occurs.

The second group attribute the hysteresis to a thermodynamic origin. Dreyer et al [8] have described the LiFePO₄ as a many-particle system with interconnected storage particles where the individual particles show a non-monotone potential. They propose that at least part of the hysteresis is not of kinetic origin—contrary to the general belief in the battery community. They have presented experimental evidence that proposes a new model that relies on the intrinsic thermodynamic properties rather than the kinetics of the insertion battery systems [9].

The last group attribute an electrochemistry explanation for the hysteresis phenomenon. Zheng [3, 10] has observed hysteresis in quasi open-circuit voltage measurements of lithium insertion in hydrogen-containing carbons in graphite. He has given an explanation that the lithium atoms bind near the aromatic hydrogen at the edge of each graphene layer in the

materials. This activated process involves a carbon-carbon bonding change from $(sp)^2$ to $(sp)^3$ leading to large hysteresis during lithium insertion.

B. How Hysteresis is modelled by others

While the explanations for the occurrence of hysteresis are varied, there have also been various attempts by reseachers in recent years to model the hysteresis phoenomenon in literature.

One direct approach in battery hysteresis modeling is to average the major loops and to relate the battery's OCV with its SOC through the average. Though simple, this method induces a bias and suffers from a large hysteresis error introduced into the SOC estimation.

Windarko and Choi employ the Takacs model in battery hysteresis modeling. This approach is basically an Input-Output (I/O) mapping of the major loops and minor loops of the hysteresis [11]. Windarko improved the original Takacs model by introducing a correcting function into the model which takes into account the temperature effect. However, this model only works on major hysteresis OCV-SOC loops, and is not able to describe minor hysteresis loop.

Gregory Plett has proposed an Enhanced Self-Correcting (ESC) model [12] with a new state called hysteresis state in the battery model to take into account the hysteresis phenomenon. The state is corresponding to voltage hysteresis and it is as function of SOC and time. The voltage hysteresis rate of change is proportional to the difference between half of major hysteresis loop and current voltage hysteresis. He has mentioned that battery models with a single-state are very simple, but perform the poorest. Adding hysteresis and filter states to the model aids performance but at the cost of greater complexity.

Shiqi [13] has used an extended kalman filter to model battery hysteresis effects and validated the proposed method using data acquired from two different batteries. Gagneur [14] has modelled the OCV hysteresis with a SOC observer.

The mathematical technique of Preisach modelling has been adopted in this paper to represent the nonlinearities of OCV hysteresis phenomena. This method is widely used in magnetic applications and shape memory alloy actuators [15-17].

II. PREISACH MODELLING

A. Classical Preisach Model

The basic idea of classical preisach model is to construct the hysteresis curve from its basic elementary units, called hysteron. A hysteron can be represented as a loop on input-output diagram with x-axis corresponding to input data value and y-axis corresponding to the hysteron state as shown in Fig. 1(a). The value of α and β in the x-axis correspond to switch on and switch off value of the hysteron respectively. The hysteron state is depending on input data value. If the input data value is less than β , the output of the switch is equal to -1 and if the input data value is more than α , the output of the switch is equal to 1. For the input data value between α and β , the output remains the same as the previous value. In the mathematical form, it can be written as

$$\hat{\gamma}_{\alpha\beta}[u(t)] = \begin{cases} 1, & \text{if } u(t) > \alpha \\ -1, & \text{if } u(t) < \beta \\ \text{remain unchanged,} & \text{if } \beta \leq u(t) \leq \alpha \end{cases} \quad (1)$$

where $u(t)$ is input, $\hat{\gamma}_{\alpha\beta}$ is hysteron state, also known as the hysteresis operator.

Fig. 1(b) shows one hysteron which has α_l and β_k in the α - β plane. This plane is also known as the preisach plane, which is used to store all the hysterons' switch on and off values. The plane is bounded by line of $\alpha=\beta$, maximum input value ($u_{\max}$), and minimum input value ($u_{\min}$). The horizontal axis is corresponding to β (switch off) value. The vertical axis is corresponding to α (switch on) value.

Fig. 1: (a) Representation of a hysteron with parameters of α_l and β_k in input-output diagram and (b) Representation of a hysteron in preisach plane.

Classical preisach model consists of set of hysteron state $\hat{\gamma}_{\alpha\beta}$ and weight function $\mu(\alpha, \beta)$, in mathematical form, it can be shown as

$$f(t) = \iint_P \mu(\alpha, \beta) \cdot \hat{\gamma}_{\alpha\beta} [u(t)] d\alpha d\beta \quad (2)$$

P represents the preisach plane. It is defined mathematically as

$$P = \{(\alpha, \beta) | \alpha \geq \beta, \alpha \leq u_{max}, \beta \geq u_{min}\} \quad (3)$$

In classical preisach model, weight function is determined by twice differentiation of First Order Reversal Curve (FORC) [18, 19]. FORC is curve formed after the first reversal/transition of input as shown in Fig. 2. The formulation of weight function is given as the following

$$u(\alpha', \beta') = \frac{1}{2} \frac{\partial^2 f_{\alpha'\beta'}}{\partial \alpha' \partial \beta'} \quad (4)$$

B. Geometric Interpretation of Preisach Model

In this section, geometric interpretation of the model is subjected to input variation as discussed. The formation of memory in preisach plane which relates to the arrangement of all hysteron states in the plane, is discussed as well. The vertical axis in preisach plane corresponds to increasing input is denoted by α , whereas the horizontal axis in the plane corresponds to decreasing input denoted by β . When the input increases, its corresponding value in the plane is represented as a moving horizontal line along α axis in upward direction. Whereas, when the input decreases, its corresponding value in the plane is represented to the moving vertical line along β axis in the leftward direction.

Fig. 2: First order reversal/transition curve in input (u) - output (f) diagram [19]. f_α refers to a point where the input reversal occurred. $f_{\alpha\beta}$ refers to curve after input reversal.

For example, the preisach model is subjected to varying input values as illustrated sequentially in Figs. 3(a)-(j). Fig. 3 (a) illustrates a monotonically increasing input from value of 0 at t_0 to value of U_1 at time of t_1 . At the beginning (at time t_0), all hysterons are initially in a switched-off condition (hysteron state is -1) as the input value is 0. As the input increases, all hysterons with α value less than the input value will switch on (hysteron state is +1) according to (1). The rest of hysterons with α value more than or equal to input value will remain unchanged according to (1). Fig. 3(b) illustrates preisach plane at time t_1 . The plane is divided into two regions, S+ region (switched-on hysteron region) and S- region (switched-off hysteron region) by line $\alpha = U_1$. The line acts as boundary between these two regions and only goes upward when input increases.

Subsequently, input is decreasing monotonically from value of U_1 at time t_1 to value of U_2 at time t_2 as illustrated in Fig. 3(c). As the input decreases, all hysterons with β value more than the input value will switch off (hysteron state is -1) according to (1). Whereas, the rest of hysterons with β value which are less than or equal to the input value will remain unchanged according to (1). Therefore, in preisach plane, the negative region S- is growing, whereas the positive region S+ is shrinking as illustrated in movement of vertical line toward left hand side in Fig. 3(d). Boundary line between S+ and S- region moves from $\beta = U_1$ to $\beta = U_2$. The region or hysteron on the right hand side of the line will switch off. The boundary line between positive and negative region is no longer only line $\alpha = U_1$, but also line $\beta = U_2$.

Subsequently, from time t_2 to t_3 , the input is increasing monotonically from value of U_2 to value of U_3 and value of U_3 is less than value of U_1 as shown in Fig. 3(e). The same principle is implemented during t_0 - t_1 to determine the value of hysteron state. In preisach plane, the boundary line will move upward from $\alpha = U_2$ to $\alpha = U_3$ as the input increases. Any negative

region, S- which has been passed by the boundary line, will turn into positive area S+. Fig. 3(f) illustrates S+ and S- in preisach plane at time t3 and the corresponding region boundary lines namely line $\alpha = U1$, $\beta = U2$ and $\alpha = U3$.

Subsequently, from time t3 to t4, the input is decreasing monotonically from value of U3 to value of U4 and value of U4 is more than value of U2 as shown in Fig. 3(g). The same principle in time t1-t2 can be implemented to determine the value of hysteron state. In preisach plane, a boundary line will move toward left hand side from $\beta = U3$ to $\beta = U4$ as the input decreases. Any positive region, S+ which has been passed by the boundary line, will turn into negative region S-. Fig. 3(h) illustrates S+ and S- in preisach plane at time t4 and the corresponding region boundary lines namely line $\alpha = U1$, $\beta = U2$, $\alpha = U3$ and $\beta = U4$. Intersection of boundary line α and β make a vertex, $V(\alpha, \beta)$. The vertices are related to memory formation in the plane and formed by a pair of local maximum (increasing input)-local minimum (decreasing input). In Fig. 3(h), two vertices can be identified, namely $V1(U1, U2)$ and $V2(U3, U4)$.

The subsequent input from time t4 to t5 is increasing monotonically to value of U5 which is larger than value of U3 as illustrated in Fig. 3(i). The boundary line will move upward starting from value of U4 to value of U5. As value of U5 is larger than value of U3, there is an instance when the increasing input has the same value with value of U3, at this critical instance, the vertex $V2(U3, U4)$ corresponding to $\alpha = U3$ and $\beta = U4$ in the plane is wiped out by the boundary line and only vertex $V1$ still remains. Fig. 3(j) illustrates S+ and S- in preisach plane at time t5 and the corresponding region boundary lines in the plane namely line $\alpha = U1$, $\beta = U2$, and $\alpha = U5$. Similarly, this mechanism is occurred in decreasing input as well. This mechanism in preisach model is known as Wiping-out property. The properties is formulated as any vertices whose α value less than maximum local input value or vertices whose β value more than minimum local input value will be wiped out from the plane. As the result, the associated histories stored in vertices are being wiped out as well.

C. Discrete Preisach Model

In practice, the implementation of classical preisach model is not really desirable due to the required computing resources to perform double integration calculation and also the error introduced by the second derivative [18, 20]. To solve the issue, discrete preisach model is implemented for hysteresis model. Unlike classical preisach model which has an infinite number of hysterons, the discrete preisach model has finite (discrete) number of hysterons which are represented as points in the plane as shown in Fig. 4. Each point has its local weight value which is represented by a weight function in discrete form.

The output of the model is the summation of all hysteron states, multiplied with local weight. The output can be calculated as following,

$$y(t) = \sum_{k=1}^n w_k(t) \cdot \mu_k + c \quad (5)$$

where n, w_k , μ_k , and c are number of hysteron, hysteron state in n-dimensional vector form, local weight in n-dimensional vector form, and constant term, respectively.

Fig. 3: Discrete preisach model.

Fig. 4: Time varying input (in (a), (c), (e), (g) and (i)) and its representation in preisach plane (in (b), (d), (f), (h) and (j)).

D. Implementation of Preisach Model

As discussed in Section II B, the history of input variation is memorized in preisach plane in the form of vertices and not all input history are taken into account. The model only takes into account local maximum and minimum of the input. In the implementation, stack data structure is used in storing the local maximum and minimum. In each data in stack, it contains a

pair of local maximum and local minimum as illustrated in Fig. 5(a). The pair (X_k, Y_k) means the input is increased to state X_k and followed by a decrease to state Y_k . Supposedly, pairs of $(X_1, Y_1), \dots, (X_k, Y_k)$ exist in the stack and new input is fed into the stack, there are four possible cases

1. The input is in increasing trend and its value is less than X_k value. The input is stacked on the top of pair (X_k, Y_k) as shown in Fig. 5(b).
2. Supposedly after case 1, the subsequent input is in decreasing trend and its value is more than Y_k value. The input is stacked on the top of pair (X_k, Y_k) as shown in Fig. 5(c).
3. The input is in increasing trend and its value is more than or equal to X_{k-i} value. All Pairs between (X_k, Y_k) and (X_{k-i}, Y_{k-i}) are wiped out from the stack. The input is stacked on the top of pair (X_{k-i-1}, Y_{k-i-1}) as shown in Fig. 5(d).
4. The input is in decreasing trend and its value is less than or equal to Y_{k-i} value. All Pairs between (X_k, Y_k) and (X_{k-i-1}, Y_{k-i-1}) are wiped out from the stack. The input replaces Y_{k-i} as local minimum as shown in Fig. 5(e).

E. Incorporation of Preisach Model into Second Order Equivalent Circuit (SOEC) model

In our study, we considered a second order equivalent circuit model as shown in Fig. 7. The model consists of a voltage controlled source, series resistance (R_i) and two parallel Resistor-Capacitor (RC) branches. The voltage controlled source represents OCV of the battery. R_i represents the ohmic resistance of the battery while the RC networks represent transient dynamic. One RC represents the fast dynamics and another RC represents the slow dynamics of the battery. For parameterization of R_i , R_1 , R_2 , C_1 , and C_2 , it is implemented based on the work presented in [21]. All components in the equivalent circuit model are SOC and temperature dependent, however, currently, we only considered the components to be SOC dependent as temperature is taken to be constant at room temperature. For the controlled-voltage source, it is normally implemented by using a look-up table [21] or described by a polynomial-exponential function [22] fitted with experimental data. This only maps the current SOC into OCV; however there is no associated history of the battery. Hence, in [21], only the major hysteresis OCV curve is modelled, but not the minor hysteresis OCV curve. In the improved battery model presented in this work, the controlled voltage source is implemented with discrete preisach model while the series resistor and the two RC networks remain unchanged.

Fig. 5: Second Order Equivalent Circuit.

Fig. 6: Implementation of memory formation in stack data structure

III. EXPERIMENTAL

A. Experimental Setup

The tests were carried out by using Basytec CTS (cell test system). Basytec CTS has 4 independent channels with maximum rating of current $\pm 5A$ (maximum amount of charge/discharge current) and voltage 0-5V. For voltage measurement, it has resolution of 0.25mV and accuracy of 0.05%. For current measurement and control, it has resolution of 0.005% of controlled current magnitude (e.g. for 5A charge/discharge current, it has resolution of 0.25mA, whereas for 15mA charge/discharge, it has resolution of 0.75mA) and accuracy of 0.05%.

The cell under test is a 18650 cylindrical battery cell with LiFePO4 chemistry. It has nominal voltage of 3.3V and capacity of 1.1Ah.

The connection from test cell to the battery tester is established by using kelvin sensing technique (four wire measurement method). During testing, the test cell is maintained constant temperature of 25 degree celsius inside a Memmert thermal chamber.

B. Parameterization Test Procedure

The test procedure is based on OCV test. OCV test consists of major OCV test and minor OCV test. During major OCV test, the cell is gradually discharged with 5% step of SOC step from fully-charged (SOC 100%) to fully-discharged (SOC 0%). Then, the cell is charged gradually with 5% change of SOC step from fully discharged (SOC 0%) to fully-charged (SOC 100%). One hour rest is inserted in each SOC step to obtain OCV.

During minor OCV test, the cell is partially discharged/charged followed by reversal of current direction. The test is done to obtain intermediate OCV values in between major OCV charge and discharge. The table below shows the test procedure of major and minor OCV test.

TABLE I
MAJOR AND MINOR OCV TEST PROCEDURE

	SOC (%)
Major OCV 1	100-95-90-85-...-0
Major OCV 2	0-5-10-15-...-100
Minor OCV 1	100-50-55-60-...-100
Minor OCV 2	100-60-65-70-...-100
Minor OCV 3	100-70-72-74-76-...-100
Minor OCV 4	100-80-82-84-86-...-100
Minor OCV 5	100-90-92-94-96-...-100
Minor OCV 6	0-70-66-62-58-54-50
Minor OCV 7	0-80-70-60-50
Minor OCV 8	0-90-80-70-60-50

C. Validation Test Procedure

The test procedure is to validate the performance of battery model from the parameterization test procedure. In parameterization test procedure, it only covers OCV test, whereas in validation test procedure, it has dynamic response of battery voltage covering both with current pulse and without current pulse (rest) sections. The test procedure is as shown in Fig. 8.

IV. MODEL PARAMETER IDENTIFICATION

The OCV tests from Parameterization Test Procedure are used to train weights of the discrete preisach model. The weights determine the contribution of each hysteron. The input training is SOC and output training is OCV. SOC is obtained through Coulomb Counting Method. The training process is done offline. It starts with updating the hysteron state by using (1), then weights can be determined by applying least square method on hysteron state and output training (OCV). The obtained weights value will be minimized the error between measured value and modeled value. The training process is depicted in Fig. 9. After the training is finished, the weights are used in the preisach model as model parameters.

Fig. 8: (a) Current profile and (b) voltage profile of validation test procedure

Fig. 9: Flow chart of discrete Preisach OCV model's parameter training.

V. RESULTS

In this section, the simulated OCV in parameterization test procedure as well as simulated battery voltage in validation test procedure are presented.

Comparison between measured OCV and modelled OCV at parameterization test procedure is shown in Fig. 10(a)-(b). Measured OCV value and modelled OCV value have shown good agreement. Hence, model is able to take into account both major OCV as well as minor OCV.

The simulated battery voltages are based on SOEC model as presented in [23] and the proposed improved SOEC model based on the discrete Preisach method for estimation of the OCV. Proposed model has similar structure of electrical element (e.g. series resistor and parallel RC element) as the SOEC model except in modelling of SOC, instead of using look-up table, discrete Preisach model is employed to model OCV of the battery. In validation test procedure, the mean error produced by proposed model and SOEC model are 0.251% and 0.526% respectively. The maximum error produced by proposed model and SOEC model are 3.510% and 4.682% respectively. It can be seen the proposed model with discrete Preisach technique of OCV estimation is able to capture hysteretic behaviour in OCV much better than the simulated SOEC model alone.

Fig. 10: (a) Comparison between measured and simulated Major OCV and (b) Comparison between measured and simulated Minor OCV 3.

Fig. 11: Comparison between measured and simulated voltage in validation test procedure.

VI. CONCLUSION

A mathematical method to model OCV hysteresis in LiFePO_4 battery is presented based on the discrete preisach technique. The validation of the model integrated with SOEC is also presented. The model is able to reconstruct the battery OCV precisely without any need for relaxation. It just depends on SOC and history of the battery. The mean error produced by the proposed battery model is less than the model without OCV hysteresis in consideration.

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