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Enhanced-Efficiency Operating Variables Selection for Vapor Compression Refrigeration Cycle System

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- Enhanced-efficiency selection of operating variables for VCC system is presented.
- Operating variables are analyzed and selected based on SOC methods.
- Influence of operating variables on operation stability and system efficiency is evaluated.
- Experimental results confirm the effectiveness of the proposed operating variables.

Abstract

In this paper, a novel enhanced-efficiency selection of operating variables based on self-optimizing control (SOC) method for the vapor compression refrigeration cycle (VCC) system is proposed. An objective function is proposed to maximize the energy efficiency of the VCC system while meeting with the demand of indoor thermal comfort. With the detailed analysis of operating variables, three unconstrained degrees of freedom are selected among all the candidate operating variables. Then two SOC methods are applied to determine the optimal individual controlled variables (CVs) and measurement combinations as CVs. The model predictive control (MPC) method based controllers and PID controllers are designed for different sets of CVs, and the experimental results indicate that the proposed selection of CVs can achieve a good trade-off between optimal (or near optimal) stable operation and enhanced-efficiency of the synthesized control structure.

Keywords: Vapor compression refrigeration cycle; Operating variables; Self-optimizing control; Model predictive control; Energy efficiency.

1. Introduction

The significantly increasing energy consumption of the heating, ventilating, and air-conditioning (HVAC) systems has attracted many scholars' attention during the past decades. Numerous researchers have focused on the optimization and control strategies of HVAC systems for enhancing the energy efficiency (Thompson & Dexter, 2005; Schurt, Hermes, & Neto, 2009; Elliott & Rasmussen, 2013; Kamar, Ahmad, Kamsah, & Mustafa, 2013; Afram & Sharifi, 2014; Fallhsohi, Changenet, Place, et al., 2010; Salazar & Méndez, 2014). The effective control strategies adjust the operating variables of the system to ensure the system reliability and reduce the energy consumption.

The vapor compression refrigeration cycle (VCC) system, which is a complex multivariable system, is a core component in HVAC system. As the essential step in controller design of VCC system, appropriate selection of controlled variables (CVs) and manipulated variables (MVs) not only affects the control system performance but also influences the overall plant operation and costs (Rangaiah & Kariwala, 2012; Van De Wal & De Jager, 2001). However, the selection of operating variables is based on practice experience in most of the previous studies. It seems that less attention has been devoted to the influence of different sets of CVs for the operation stability and system efficiency of the VCC system.

The researches focused on the selection of operating variables can be classified into relative gain array (RGA) based criterion and SOC method. Neera Jain et al. analyzed a reduced set of CVs for VCC system, and then proposed a decentralized feedback structure by using RGA number to choose input-output (I/O) pairs (Jain, Keir, et al., 2010). He and Cai (2009) developed the relative normalized gain array (RNGA) algorithm as variable selection criterion which provided more accurate selection results than RGA did. Based on the RNGA method,

Shen, Cai, and Li (2010) proposed a decoupling control strategy to the temperature control of HVAC systems. The experiment results above proved the effectiveness of I/O pairs, while the choices of effective CVs didn't take the energy efficiency into account.

Recently, SOC method is widely applied in industrial process systems. "Self-optimizing control" is defined as a tradeoff between economic performance and acceptable loss achieved without re-optimization when disturbances occur. Skogestad and Postlethwaite (1996) provided an analysis of minimum singular value within configuration of SOC to choose the optimized CVs. Halvorsen, Skogestad, et al. (2003) proposed a local optimization rule to minimize the loss of worst-case with which to choose the optimized CVs. Kariwala, Cao, and Janardhanan (2008) further improved this optimization rule and presented an approach to minimize the average loss for SOC. Alstad and Skogestad (2007) proposed null space method to select optimal measurement combinations as CVs, the advantages of which were simple in computing and realizing. Cao and Kariwala developed branch and bound methods for SOC and applied to large-scale processes to demonstrate the computational efficiency (Cao & Kariwala, 2008; Kariwala & Cao, 2009; Kariwala & Cao, 2010). Zumoffen and Musulin (2013) proposed a novel CVs selection approach based on spectral graph theory by which the energies were related to the deviations in manipulated and controlled variables. While there are few researches on the application of SOC could be found when it comes to select the CVs of actual VCC systems. Jensen and Skogestad (Jensen & Skogestad, 2007a; Jensen & Skogestad, 2007b) considered the steady-state operation of two different refrigeration cycles. Five degrees of freedom were considered and 2% savings of compressor power had been obtained with the proposed selection of CVs, while only one unconstrained degree of freedom

was used to optimize the operation.

In this paper, the objective is to select the CVs for the VCC system to enhance the energy effectiveness and disturbance rejection for VCC system. According to the dynamic model of VCC system, the relationship among operating variables are analyzed and the constraints for the optimal operation are detailed. Three unconstrained degrees of freedom are selected among all the candidate operating variables. The objective function is proposed to minimize the energy consumption of the VCC system while meeting with the cooling demands. Then the optimal individual CVs and measurement combinations as CVs are selected by applying the SOC methods. The effectiveness of the proposed selection is verified by the control performance of the MPC controllers and PID controllers which are designed for different structures of operating variables.

The remainder of the paper is organized as follows. Section 2 details the dynamic model of the VCC system. Section 3 provides the analysis and optimization based on SOC method for VCC system. Section 4 selects the optimal individual CVs and combined CVs of VCC system, and the experimental results indicate the efficiency of the proposed selection. Section 5 summarizes the main conclusions.

2. Description of VCC System

A typical VCC system fully utilizes a circulating refrigerant as the medium which absorbs and removes heat from the spaces and subsequently achieves the desired objectives. It includes four main components: evaporator, compressor, condenser, and expansion valve (see Fig. 1). The relationship between pressure and enthalpy of VCC system is shown in Fig. 2, which is useful to describe the system from the point of view of energy consumption as below.

- 1) From point 1 to point 2 shown in Fig. 2, the circulating refrigerant as a saturated vapor enters the compressor and is compressed into a superheated vapor which has a higher

pressure and higher temperature.

- 2) The compressed, superheated refrigerant vapor then routes through the condenser, in which the refrigerant is cooled and condensed into liquid phase by flowing through a coil or tubes with cool fluid flowing across the coil or tubes (from point 2 to point 3).
- 3) After being condensed, the refrigerant as a subcooled liquid at a high pressure enters the expansion valve where the pressure is reduced abruptly. At the exit of the expansion valve the refrigerant is generally two-phase with low pressure.
- 4) At point 4, the cold fluid then enters the evaporator, where the fluid evaporates and the heat of ambient air is transferred to the fluid. After the evaporation process, the refrigerant as a saturated vapor is routed back into the compressor to complete the refrigeration cycle.

Figure 1 Vapor compression refrigeration cycle system

Figure 2 Pressure and enthalpy diagram of VCC system

The dynamic model of each component of VCC system is derived based on the mass conservation and energy balance principles, which is briefly listed below.

Compressor: The dynamics of the compressor are considered to be much faster than those of heat exchangers. Therefore, its mass flow rate can be modeled as a static component (Arora, 2000).

$$\dot{m}_{com} = F_{com} \left(D_{com1} - D_{com2} \left(\frac{P_{como}}{P_{comi}} \right)^{D_{com3}} \right) \quad (1)$$

where $\dot{m}_{com}$ is the mass flow rate of refrigerant through compressor; F_{com} is the rotation speed of compressor; P_{comi} and P_{como} are the inlet pressure and outlet pressure across compressor; D_{com1} , D_{com2} and D_{com3} are model coefficients obtained by system identification method.

Expansion valve: The expansion valve is also modeled as a static component; its mass flow rate can be calculated from the orifice equation (Arora, 2000).

$$\dot{m}_v = (C_{v1} + C_{v2}v_o) [\rho(P_{vi} - P_{vo})]^{1/2} \quad (2)$$

where $\dot{m}_v$ is the mass flow rate of refrigerant through the expansion valve; C_{v1} and C_{v2} are the orifice coefficients; v_o is the opening degree of expansion valve; ρ is the refrigerant density; P_{vi} and P_{vo} are the inlet pressure and outlet pressure across the

expansion valve, respectively.

Heat exchangers: The heat exchangers in this typical VCC system are condenser and evaporator. According to the state of refrigerant, the condenser can be divided into three sections: a subcooled liquid section, a two-phase section, and a superheated vapor section. Similar to the condenser, the evaporator could be divided into two regions, i.e., a two-phase region with a mean void fraction and a superheated region. They can be modeled according to the conservation of refrigerant mass and energy:

$$\frac{\partial(\rho_i A_i)}{\partial t} + \frac{\partial(\dot{m}_i)}{\partial z} = 0 \quad i = c, e \quad (3)$$

$$\frac{\partial(\rho_i A_i h_i - A_i P_i)}{\partial t} + \frac{\partial(\dot{m}_i h_i)}{\partial z} = \dot{Q}_i \quad i = c, e \quad (4)$$

where $i = c$ and $i = e$ represent condenser and evaporator, respectively. ρ_i , $\dot{m}_i$, h_i , P_i , A_i , p_i , and $\dot{Q}_i$ are the refrigerant density, the mass flow rate of the refrigerant, the enthalpy, the pressure through the heat exchangers, the cross-sectional area, the inner perimeter of heat exchangers, and the energy discharged or absorbed by refrigerant through the heat exchangers, respectively.

The combined cycle model of whole VCC can be obtained by appropriately combining the component models according to the relations between the variables of systems as shown in Eq. (5). The complete representations of the VCC system are included in Appendix A, and the operating variables are shown in Table 1. The mathematic model of VCC system is complicated and multivariable, which is a great challenge for the controller design.

$$\dot{m}_{cri} = \dot{m}_{com}, \dot{m}_{cro} = \dot{m}_v, \dot{m}_{eri} = \dot{m}_v, \dot{m}_{cri} = \dot{m}_{com} \quad (5)$$

Table 1 The operating variables of VCC system

Var.	Description	Var.	Description
F_{com}	Compressor speed	v_o	Opening of Expansion valve
$\dot{m}_v$	Mass flow rate of the refrigerant through the expansion valve	$\dot{m}_{com}$	Mass flow rate of the refrigerant through the compressor

$\dot{m}_{ca}$	Air flow rate of condenser	$\dot{m}_{ea}$	Air flow rate of evaporator
P_c	Condensing pressure	P_e	Evaporating pressure
h_{cri}	Enthalpy at inlet of condenser	h_{eri}	Enthalpy at inlet of evaporator
h_{cro}	Enthalpy at outlet of condenser	h_{ero}	Enthalpy at outlet of evaporator
T_{cai}	Air temperature at inlet of condenser	T_{eai}	Air temperature at inlet of evaporator
T_{cri}	Refrigerant temperature at inlet of condenser	T_{eri}	Refrigerant temperature at inlet of evaporator
T_{cro}	Refrigerant temperature at outlet of condenser	T_{ero}	Refrigerant temperature at outlet of evaporator
T_{crsat}	Condensing temperature	T_{ersat}	Evaporating temperature
T_{csh}	Superheat degree of condenser	T_{esh}	Superheat degree of evaporator
T_{csc}	Subcool degree of condenser	T_{amb}	Environment temperature

3. Analysis on SOC Method for VCC System

The selection of effective CVs is not only important in decentralized control strategies such as PID controller design, but also plays a key role in centralized control scheme, for example, MPC controller design which is a predicted process tracking the set points of the selected CVs. In this section, we apply the SOC method to select the best individual and combined self-optimizing CVs for the VCC system. With offline optimization, the SOC method selects reasonably optimal CVs of systems for the feedback controller to track when the disturbances occur, the brief steps of which are as follows:

Step 1: Define the objective function of the system and detail the constraints for operation;

Step 2: Determine the operational degrees of freedom and main disturbances of the system;

Step 3: Solve the optimization problem under disturbances;

Step 4: Select the optimal CVs under the candidate CVs.

Fig. 3 shows the structure of the proposed enhanced-efficiency operating variables selection for VCC system. The operating variables and the energy variables of VCC system are

analyzed by the optimizer with the SOC theory, and the optimization results obtained by the optimizer provide the optimal set points and CVs to the feedback controller, which regulates the VCC system to maintain an optimal or near-optimal operation when the disturbances occur.

The experimental platform used for experimental analysis and validation in this research is located on process instrumentation laboratory of Nanyang Technological University of Singapore (see Fig. 4). It is composed of a semi-hermetic reciprocating compressor, an electronic expansion valve, an air-cooled evaporator, and an air-cooled finned-tube condenser followed by a liquid receiver. The temperature and pressure sensors are installed on each side of the components in the refrigeration cycle system. Equipped with the variable speed drives (VSDs), the operations of the compressor, the evaporator fan and the condenser fan are continuously adjustable. The working fluid used in this system is R134a.

Figure 3 Enhanced-efficiency operating variables selection structure for VCC system

Figure 4 The VCC experimental platform: (a) Photograph of system (b) Schematic of system

3.1 Define the objective function and constraints

The economic objective of this research is to minimize the energy consumption of the compressor subject to cooling load requirements, which can be solved by maximize the coefficient of performance (COP) of system.

$$J = \frac{\dot{W}_{com}}{\dot{Q}_e} \quad (6)$$

where $\dot{W}_{com}$ is the power consumption of compressor, $\dot{Q}_e$ is the cooling load which can be expressed:

$$\dot{Q}_e = C_p \dot{m}_{ea} (T_{eai} - T_{eao}) \quad (7)$$

$$\dot{W}_{com} = \frac{\dot{Q}_{com}}{\eta_{com}} = \frac{1}{\eta_{com}} C_{com1} F_{com} \dot{m}_{com} P_e \left(\left(\frac{P_c}{P_e} \right)^{C_{com2}} - 1 \right) \quad (8)$$

Substituting Eq. (7) and Eq. (8) into Eq. (6), yields

$$J = \frac{C_{com1} F_{com} \dot{m}_{com} P_e \left(\left(\frac{P_c}{P_e} \right)^{C_{com2}} - 1 \right)}{\eta_{com} C_p \dot{m}_{ea} (T_{eai} - T_{eao})} \quad (9)$$

where the coefficients C_{com1} and C_{com2} are constants for the given compressor, $\dot{m}_{com}$ can

be calculated by Eq. (1), η_{com} is the delivery coefficient of compressor, C_p refers to the special heat of the air flow. The remaining variables are listed in Table 1.

Furthermore, to ensure the operational reliability and system stability, physical constraints should be imposed to restrict the variables. The inequality constraints are as follows:

- 1) The subcool degree of condenser $T_{csc} \geq 0$. Subcool slightly may give savings in energy usage. While high subcooling degree indicates the heat transfer for condensation is used for subcooling.
- 2) The opening of expansion valve $0 < v_o \leq 1$. Considering the inherent characteristic of real industrial systems, a valve cannot open past 100% open or close past 0% open.
- 3) The compressor speed $F_{commin} \leq F_{com} \leq F_{commax}$. It is determined by working range of given compressor.
- 4) The evaporating pressure $P_{emin} \leq P_e \leq P_{emax}$. It is equal to the inlet pressure of compressor, which should not be too low to startup.
- 5) The condensing pressure $P_{cmin} \leq P_c \leq P_{cmax}$. The condensing pressure should be bounded for protecting compressor from being damaged.
- 6) The evaporating temperature $T_{eri} \leq T_{ersat} \leq T_{ero}$. The evaporating temperature should be higher than the refrigerant temperature at evaporator inlet while lower than refrigerant temperature at evaporator outlet.
- 7) The condensing temperature $T_{cro} \leq T_{crsat} \leq T_{cri}$. The refrigerant is condensed during condensation process which results the decrease of condensing temperature.
- 8) The air mass flow rate of evaporator $\dot{m}_{eamin} \leq \dot{m}_{ea} \leq \dot{m}_{eamax}$. The range of air mass flow rate is determined by the given evaporator fan.
- 9) The air mass flow rate of condenser $\dot{m}_{camin} \leq \dot{m}_{ca} \leq \dot{m}_{camax}$. Similar to evaporator fan, it is closely related to the characteristic of given condenser fan.
- 10) The mass flow rate of refrigerant $\dot{m}_{rmin} \leq \dot{m}_r \leq \dot{m}_{rmax}$. Both of the minimal and maximal refrigerant mass flow rates of the system are determined by system characteristics.

Moreover, an important equality constraint is proposed and need to be followed:

- 11) The superheat degree of evaporator T_{esh} . Keeping superheat at low values is an important and necessary factor to achieve energy efficiency and safety operation. Minimal stable superheat (MSS) is defined as a critical minimal degree of stable refrigerant superheat under the condition of corresponding evaporating pressure and cooling load (Chen, Deng, Xu, & Chan, 2008). Therefore, the superheat degree is active when it equals to the MSS value under the corresponding demand of cooling load, which can be expressed as $T_{esh} = MSS$. The calculation are as follows:

$$MSS = T_{ero} - T_{ersat} \quad (10)$$

The saturated temperature of refrigerant T_{ersat} in Eq. (10) is completely determined by the corresponding evaporating pressure, thus it can be obtained by evaporating pressure P_e . Then, T_{ero} can be expressed as a function of MSS and P_e . Enthalpy is closely related to corresponding pressure and temperature. Thus the enthalpy h_{ero} can be transformed into the function of MSS and P_e (Zhao, Cai, Ding, & Chang, 2013):

$$h_{ero} = f_{ero}(P_e, MSS) = a_{ero1}P_e^2 + a_{ero2}P_e + b_{ero}MSS + c_{ero} \quad (11)$$

where a_{ero1} , a_{ero2} , b_{ero} , and c_{ero} are the coefficients, which can be calculated by curve fitting according to the given refrigerant.

The energy absorbed by the refrigerant through the whole evaporator, denoted by Q_e , can be obtained:

$$\dot{Q}_e = \dot{m}_{ero}h_{ero} - \dot{m}_{eri}h_{eri} \quad (12)$$

Substituting Eq. (11) into Eq. (12), the equality constraint is formulated as:

$$\dot{Q}_e = \dot{m}_{ero}(a_{ero1}P_e^2 + a_{ero2}P_e + b_{ero}MSS + c_{ero}) - \dot{m}_{eri}h_{eri} \quad (13)$$

By taking consideration of the effect on the objective function, there are four constraints active at the optimal nominal operation: the superheat degree of evaporator; the refrigerant temperature at inlet of evaporator; the evaporating temperature; the subcool degree of condenser.

3.2 Operational degree of freedom

The typical VCC system has eight operational degrees of freedom which have a significant effect on the objective function value. Furthermore, they interconnect and interplay with other system variables through the interaction equations. They are selected as follows:

- 1) The compressor speed
- 2) The opening of expansion valve
- 3) The rotating speed of evaporator fan
- 4) The rotating speed of condenser fan
- 5) The evaporating pressure
- 6) The condensing pressure
- 7) The refrigerant temperature at outlet of evaporator
- 8) The refrigerant temperature at outlet of condenser

3.3 Identification of disturbances

The main disturbances for VCC system are the variations of air mass flow rates of condenser ($\dot{m}_{ca}$), the ambient temperature (T_{amb}), and the indoor temperature (T_{eai}). According to the actual situation in Singapore, the disturbances are restricted within the following limits:

$$d_1: \dot{m}_{ca} = 0.555 \text{ kg/s} \sim 0.405 \text{ kg/s}$$

$$d_2: T_{amb} = 22^\circ\text{C} \sim 35^\circ\text{C}$$

$$d_3: T_{eai} = 20^\circ\text{C} \sim 27^\circ\text{C}$$

Consequently, we have 8 steady state degrees of freedom, 4 active constraints, and take consideration of given cooling load requirement. Therefore, we have 3 remaining unconstrained degrees of freedom.

3.4 Optimization

In this section, two local optimization methods are adopted to find the optimal CVs of VCC system, which are exact local method based on worst-case loss minimization and local optimization method based on average loss minimization (Halvorsen, Skogestad, Morud, et al., 2003; Kariwala, V., Cao, Y., Janardhanan, S., 2008).

3.4.1 Exact local method based on worst-case loss minimization

As an effective tool for selecting CVs, the exact local method based on worst-case loss minimization analyzes candidate sets of CVs and finds the set with the smallest worst-case loss (Halvorsen, Skogestad, Morud, et al., 2003). The brief procedure is as follows:

First, we assume that the set points are nominally optimal, and Table 2 lists the optimal values of variables at a steady state operating point for VCC system. Under the optimal operating point, the model of system can be linearized as Eq. (13).

Table 2 The optimal values at a steady state operating point

Variables	F_{com}	v_0	$\dot{m}_{ea}$	P_c	T_{cri}	T_{cro}	T_{csh}
	(Hz)		(kg/s)	(bar)	(°C)	(°C)	(°C)
Opt. Values	35	0.5360	0.2332	10.27	58.04	27.69	17.65
Variables	T_{csc}	T_{cao}	P_e	T_{eri}	T_{ero}	T_{esh}	T_{eao}
	(°C)	(°C)	(bar)	(°C)	(°C)	(°C)	(°C)
Opt. Values	12.36	34.93	3.69	27.58	12.84	6.32	12.38

$$y = G^y u + G_d^y W_d d + W_n^y n \quad (13)$$

where y , u , d , and n are the system outputs, inputs, varying disturbances, and the measurement and implementation errors, respectively. G^y and G_d^y are the gains of the selected measurements from inputs and disturbances. W_d and W_n^y are the expected magnitudes of the disturbances and implementation errors.

Let c_{opt} represent the optimal points, $J_{opt}(d)$ is the optimal value of cost function at the optimal operating point and the corresponding disturbances:

$$J_{opt}(d) = J(c_{opt}, d) \quad (14)$$

The loss L is defined as the difference between the actual value of the cost function and $J_{opt}(d)$. Different values of the cost function can be obtained by different candidate sets of CVs. Then the optimization problem is the minimization of the loss

L over any set of disturbances and implementation errors:

$$L = J_c(d, n) - J_{opt}(d) \quad (15)$$

The calculation of Eq. (15) is given as the following (Halvorsen, Skogestad, Morud, et al., 2003):

$$L_{worst-case} = \frac{\| [M_d \ M_n] \|_2^2}{2} \quad (16)$$

where

$$M_d = J_{uu}^{1/2} \left(J_{uu}^{-1} J_{ud} - (HG^y)^{-1} (HG_d^y) \right) W_d \quad (17)$$

$$M_n = J_{uu}^{1/2} (HG^y)^{-1} H W_n^y \quad (18)$$

Here, J_{uu} and J_{ud} represent $\partial^2 J / \partial u^2$ and $\partial^2 J / \partial u \partial d$. The complete representations of them are included in Appendix B.

H is a selection or combination matrix which gives a set of CVs (denoted by Δc). The set of CVs can be composed by linear combination of measurements as Eq. (19). If individual measurements are selected as CVs, then H is a unit matrix with $n_u \times n_y$ dimension. While if combination of measurements are chose as CVs, the H is a full matrix and its dimension is related with the number of measurements.

$$\Delta c = H \Delta y \quad (19)$$

The optimization problem of Eq. (16) is then equivalent to minimization of the worst-case loss which can be expressed as:

$$L_{worst-case} = \frac{\bar{\sigma}([M_d \ M_n])^2}{2} \quad (20)$$

s.t. $\text{rank}(H) = n_u$

$\bar{\sigma}([M_d \ M_n])$ represents the maximum singular value of matrix $[M_d \ M_n]$. The set c with the smallest value of $\bar{\sigma}([M_d \ M_n])$ minimizes the loss L , meanwhile has the optimal disturbance rejection performance. The optimal choice of CVs is then depend on the value of matrix H with the rank n_u .

3.4.2 Local optimization method based on average loss minimization

The local optimization method based on average loss minimization is an extension of the exact local method based on worst-case loss minimization. By minimizing the average loss, it also minimizes the worst-case loss over all disturbances and implementation errors.

The procedure of average loss minimization is similar to that of worst-case loss minimization. The average loss over the allowable combination of disturbances and implementation errors is defined as:

$$L_{average} = \frac{\| [M_d \ M_n] \|_F^2}{6(n_y + n_d)} \quad (21)$$

where $\| \cdot \|_F$ denotes the Frobenius norm, $y, n \in \mathbb{R}^{n_y}$, $u \in \mathbb{R}^{n_u}$, and $d \in \mathbb{R}^{n_d}$.

This average loss can be effectively minimized with the following optimization problem, which can be solved by select X as Eq. (23):

$$\begin{aligned} & \min_{H, X} \frac{tr(X)}{6(n_y + n_d)} \\ & s.t. \ H \left(G^y J_{uu}^{-\frac{1}{2}} X J_{uu}^{-\frac{1}{2}} (G^y)^T - YY^T \right) H^T \geq 0 \\ & \quad X \geq 0 \\ & \quad rank(H) = n_u \end{aligned} \quad (22)$$

$$X = \frac{1}{J_{uu}^{-\frac{1}{2}} (G^y)^T (YY^T)^{-1} G^y J_{uu}^{-\frac{1}{2}}} \quad (23)$$

4 Selection of Candidate Controlled Variables

Based on the analysis and optimization methods above, the optimal individual CVs and combined CVs of VCC system are selected in this section. We consider the manipulated variables composed of the compressor speed (u_1), the opening of expansion valve (u_2), and the air mass flow rate of evaporator (u_3). As the enthalpy can be expressed as the function of corresponding pressure and temperature, 11 measurements are considered as candidate controlled variables:

y_1 The condensing pressure P_c

y_2 The refrigerant temperature at inlet of condenser T_{cri}

y_3 The refrigerant temperature at outlet of condenser T_{cro}

y_4 The superheat degree of condenser T_{csh}

y_5 The subcool degree of condenser T_{csc}

y_6 The air temperature at outlet of condenser T_{cao}

y_7 The evaporating pressure P_e

y_8 The refrigerant temperature at inlet of evaporator T_{eri}

y_9 The refrigerant temperature at outlet of evaporator T_{ero}

y_{10} The superheat degree of evaporator T_{esh}

y_{11} The air temperature at outlet of evaporator T_{eao}

The implementation errors for the candidate measurements are assumed to be the following: temperature 0.3 , pressure 0.3%, and mass flow 1.6%. The J_{uu} , J_{ud} , G^y , and G_d^y using in the SOC methods are calculated at the nominally optimal operating point, shown in Eq. (24).

$$J_{uu} = \begin{bmatrix} 2.7215 & 15983 & -11.6702 \\ 15983 & 0 & -2398800 \\ -11.6702 & -2398800 & 1751.5 \end{bmatrix}, \quad J_{ud} = \begin{bmatrix} 0.1095 & -0.0074 & -0.1890 \\ 87.7727 & -5.9066 & -38855 \\ -47.9244 & 3.2250 & 14.1855 \end{bmatrix}$$

$$G^y = \begin{bmatrix} -0.0157 & 0.2749 & 0.0154 \\ 0.0209 & -0.9737 & 0.0999 \\ 0.0232 & -0.8293 & 0.2766 \\ 0.0265 & -0.5 & -1.6306 \\ -0.0497 & 0.8920 & 1.0212 \\ 0.0066 & -0.6201 & 0.0501 \\ -0.3035 & 2.0947 & 2.5608 \\ 0.0021 & -0.0916 & 0.0575 \\ 0.0043 & -1.6707 & 0.2255 \\ 6.6044 & -38.6374 & 6.7033 \\ -0.0001875 & -0.0088 & 0.1197 \end{bmatrix}, \quad G_d^y = \begin{bmatrix} -0.0046 & 0.0108 & 0.0035 \\ -0.0283 & -0.0298 & -0.0902 \\ -0.1151 & 0.0029 & -0.1002 \\ 0.6890 & -0.1108 & -0.0348 \\ -0.4364 & 0.1701 & 0.0779 \\ -0.0191 & -0.0188 & -0.0659 \\ -1.1335 & 0.1163 & 0.0119 \\ -0.0233 & -0.0103 & -0.0115 \\ -0.0963 & 0.0867 & -0.2364 \\ -1.0147 & 2.2527 & -3.5311 \\ -0.0518 & 0.0049 & -0.0138 \end{bmatrix} \quad (24)$$

4.1 Individual measurements as controlled variables

Under the restriction of 3 remaining unconstrained degrees of freedom, 3 individual CVs are selected with the minimum loss L . Consequently, there are 165 possible single CV candidate sets. The calculation is as follows:

$$C(11,3) = \frac{11!}{3!(11-3)!} = 165 \quad (25)$$

Here the selection of individual CVs is considered, therefore, the H in both Eq. (19) and Eq. (22) is a unit matrix with 3×3 dimension. By applying the SOC analyses based on worst-case loss minimization and average loss minimization, five best sets of single CVs are listed in Table 3. The optimization results obtained by worst-case loss minimization are consistent with that solved by average loss minimization. The losses for all the candidate sets can be computed and the smallest value is given by controlling the condensing pressure P_c , evaporating pressure P_e and superheat of evaporator T_{esh} . The structure of the best individual self-optimizing CVs is shown in Fig. 5 where the P_c can be computed by the average values of P_2 and P_3 . From a physical point of view, the selection results from SOC analysis not only guarantee the safe and stable operation under the temperature, humidity and other exterior environment influence, but also improve the energy efficiency of VCC system.

Table 3 The best 5 sets of individual self-optimizing controlled variables

Number	Set			Worst-case loss (10^{-3})	Average loss (10^{-3})
1	P_c	P_e	T_{esh}	17.884	0.43047
2	T_{cri}	P_e	T_{esh}	56.147	13.590
3	T_{cri}	T_{csh}	T_{esh}	137.89	34.560
4	P_c	T_{csh}	T_{esh}	144.59	42.794
5	T_{cri}	T_{csh}	P_e	301.49	72.161

Figure 5 Structure of the proposed self-optimizing CVs of VCC system

The disturbance rejection performance for implementation of the proposed set 1 is analyzed as shown in Table 4. The allowance disturbance set corresponds to $\pm 10\%$ variation in the air mass flow rate through the condenser $\dot{m}_{ca}$, and $\pm 10\%$ variation in indoor temperature T_{eai} and environment temperature T_{amb} around their normal values. The values of loss are all equal or smaller than the maximum losses given by the analysis, which illustrate the effectiveness of proposed self-optimizing CVs (P_c , P_e , T_{esh}).

Table 4 Disturbances rejection performances for implementation of the proposed CVs

Disturbance	Worst-case loss	Average loss
	(10^{-3})	(10^{-3})
10% decrease in the air mass flow rate through condenser $\dot{m}_{ca}$	17.884	0.43047

10% decrease in indoor temperature T_{ei}	17.783	0.42791
10% increase in environment temperature T_{amb}	17.679	0.42531

In order to further demonstrate the system performance with the proposed individual self-optimizing CVs, energy efficient model predictive control strategies are proposed for the implementation of No.1 and No.2 sets of CVs in Table 3, respectively. The performance index of the controllers ($J_{MPC}(k)$) is as follows:

$$\min_{\Delta u(k)} J_{MPC}(k) = \sum_{j=1}^{n_y} \sum_{i=1}^{P_y} \left\| \tilde{y}_{sp,j}(k+i|k) - y_j(k+i|k) \right\|_{q_j} + \sum_{j=1}^{n_{mv}} \sum_{i=1}^M \left\| \Delta u_j(k+i|k) \right\|_{r_j} + \sum_{i=1}^M \left\| \frac{1}{COP(k+i|k)} \right\| \quad (26)$$

where k , $k+i$, P_y , n_y , q_j , r_j , M , n_{mv} , and $\Delta u(k)$ are the current sampling interval, the future sampling interval, the prediction horizon, the number of plant outputs, the weighting matrix of outputs and input increments, the control horizon, the number of the inputs, and the increment of manipulated variables, respectively. $\tilde{y}_{sp}(k+i|k)$ is the desired output at instant $k+i$; $y_j(k)$ is the actual output at instant k . The system coefficient of performance (COP) is an index of system efficiency, which can be defined as the ratio of cooling provided to electrical energy consumed. Eq. (26) computes the weighted sum of squared deviations for the deviation of the outputs from the set points, the incremental manipulated variables, and the system coefficient COP. At each sample time, an optimal control signal can be obtained by searching for values of Δu over the control horizon which minimizes the objective function subject to the constraints specified.

To validate the effectiveness of the proposed MPC strategies, a traditional PID control strategies are also developed. By applying RGA-NI-RNGA analysis, the loop pairings of No.1 and No.2 sets of CVs in Table 3 with manipulated variables (F_{com} , v_o , $\dot{m}_{ea}$) are determined. The specific pairings and the fine-tuned gain, integral time, and

differential time values of PID controllers are all included in Table 5. The parameters of corresponding MPC controllers are given in Table 6. Three disturbance variations are carried out in the experiments at time $t_1=70\text{min}$, $t_2=117\text{min}$, and $t_3=177\text{min}$, respectively. Fig. 6 and Fig. 7 display the closed-loop VCC performance for the No.1 and No.2 sets of CVs with same manipulated variables (F_{com} , v_o , $\dot{m}_{ea}$). The results clearly indicate that the MPC controllers exhibit less oscillations and faster tracing in reference tracking and better disturbance rejection performance compared to PID controllers. Furthermore, the results of Fig. 6 demonstrate that the proposed self-optimizing CVs (P_c , P_e , T_{esh}) are controlled effectively by MPC controller. At time t_1 , a drop of 2 percent in the $\dot{m}_{ca}$ is occurred, which leads to small increase of P_c while shows little influence on the P_e and T_{esh} . With the increases of environment temperature at time t_2 and indoor temperature at time t_3 , the MPC controllers regulate rapidly to maintain the CVs at their setting values shown in Fig. 6 and Fig. 7. While compared with the No.2 set of CVs shown in Fig. 7, the P_e and T_{esh} in No.1 set of CVs shows better rising time and lower overshoot performance, which demonstrate the good disturbance rejection performance of the proposed CVs. In order to evaluate the energy efficiency of the VCC system, the average value of system coefficient of performance (COP) is evaluated over the operation time. According to Eq. (27), the average value of COP under No.1 set of CVs is 3.3, which increases 6% compared with No.2 set of CVs.

$$\overline{COP} = \frac{\sum_{t=t_0}^{t_{end}} \left| \frac{Q_e(t)}{W_{com}(t)} \right|}{|t_{end} - t_o|} \quad (27)$$

where the t_0 and t_{end} are the start and end time in the experiments, COP is the system coefficient of performance which equals to the ratio of cooling load (Q_e) to compressor power (W_{com}).

Table 5 The parameters of PID controllers for different sets of CVs

Loop pairing		Parameters of PID controllers	
No.1 set of CVs	$P_c - \dot{m}_{ea}$	K_C	4.365
		τ_I	0.6816
		τ_d	6.9107
	$P_e - v_o$	K_C	0.0882
		τ_I	0.0931
		τ_d	-0.0293
	$T_{esh} - F_{com}$	K_C	15.988
		τ_I	0.4275
		τ_d	-3.151
	$T_{cri} - \dot{m}_{ea}$	K_C	2.619
		τ_I	0.4089
		τ_d	4.1464
No.2 set of CVs	$P_e - v_o$	K_C	0.097
		τ_I	0.1024
		τ_d	-0.0323
	$T_{esh} - F_{com}$	K_C	15.99
		τ_I	0.4275
		τ_d	-3.151

Table 6 The parameters of MPC controllers for different sets of CVs

Parameters of MPC	Value in No.1 set of CVs	Value in No.2 set of CVs
Number of outputs	3	3

Number of inputs	3	3
Prediction horizon	30	30
Control horizon	15	15
Weight of outputs	$diag\{5,5,10\}$	$diag\{1,10,10\}$
Weight of inputs	$diag\{0.1,0.1,0.1\}$	$diag\{0.1,0.1,0.1\}$

Figure 6 Experimental results of the No.1 set of CVs

Figure 7 Experimental results of the No.2 set of CVs

4.2 Combinations of operating variables as controlled variables

To further reduce the loss L , combinations of operating variables as CVs are analyzed in this part. Three CVs are required which is the same with the selection of individual CVs. Different numbers of measurements could be combined as CVs. The brief steps based on worst-case loss minimization method are as follows:

Step 1: Let num denote the numbers of variables used for combination, thus the H is a full matrix with $3 \times num$ dimension.

Step 2: Let $y_{N1}, y_{N2}, \dots, y_{Nnum}$ denote the num operating variables chosen for combination. Compute the corresponding G^y , G_d^y , and W_n^y , denoting as G_{num}^y , $G_{d,num}^y$, and $W_{n,num}^y$, respectively:

$$G_{num}^y = \begin{bmatrix} G^y(N1,1) & G^y(N1,2) & G^y(N1,3) \\ G^y(N2,1) & G^y(N2,2) & G^y(N2,3) \\ \vdots & \vdots & \vdots \\ G^y(Nnum,1) & G^y(Nnum,2) & G^y(Nnum,3) \end{bmatrix}$$

$$G_{d,num}^y = \begin{bmatrix} G_d^y(N1,1) & G_d^y(N1,2) & G_d^y(N1,3) \\ G_d^y(N2,1) & G_d^y(N2,2) & G_d^y(N2,3) \\ \vdots & \vdots & \vdots \\ G_d^y(Nnum,1) & G_d^y(Nnum,2) & G_d^y(Nnum,3) \end{bmatrix}$$

$$W_{n,num}^y = \begin{bmatrix} W_n^y(N1) & 0 & \dots & 0 \\ 0 & W_n^y(N2) & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & W_n^y(Nnum) \end{bmatrix}$$

Step 3: Solve the optimization problem by finding the optimal H:

$$H = \arg \min_H \bar{\sigma}([M_d \quad M_n]) = \arg \min_H \bar{\sigma}(HF)$$

$$s.t. \quad HG_{num}^y = J_{uu}^{1/2}$$

$$where \quad F = [F_d \quad F_n] \quad (28)$$

$$F_d = (G_{num}^y J_{uu}^{-1} J_{ud} - G_{d,num}^y) W_d$$

$$F_n = W_{n,num}^y$$

Step 4: Traverse all possible combinations of the operating variables, and solve the optimization problem in Step 3. The set of CVs with the smallest value of $\bar{\sigma}([M_d \quad M_n])$ are the optimal CVs for combinations of num operating variables, and matrix H is the according combinational coefficients.

Taking the combinations of four operating variables as example, then the dimension of matrix H is 3×4 . Let $y_{N1}, y_{N2}, y_{N3}, y_{N4}$ denote the four operating variables chosen for combination. According to step 2, the G_4^y , $G_{d,4}^y$, and $W_{n,4}^y$ under the selected set of variables can be obtained. By searching all the combinations out of the operating variables, the optimization problem in Eq. (28) is solved and the optimal combination of four operating variables are found as $P_c, T_{cri}, P_e, T_{esh}$, and the matrix H is shown in Eq. (29). The optimal CVs after combined are shown in Eq. (30), which reduce the loss to 1.3781×10^{-3} :

$$H = \begin{bmatrix} -0.0067 & 0.1356 & 0.0708 & 0.0377 \\ -0.1671 & 1.1795 & 0.0637 & -0.1788 \\ 0.1618 & -0.1470 & -0.0665 & 0.0501 \end{bmatrix} \quad (29)$$

$$\begin{bmatrix} CV1 \\ CV2 \\ CV3 \end{bmatrix} = H \begin{bmatrix} P_c \\ T_{cri} \\ P_e \\ T_{esh} \end{bmatrix} = \begin{bmatrix} -0.0067P_c + 0.1356T_{cri} + 0.0708P_e + 0.0377T_{esh} \\ -0.1671P_c + 1.1795T_{cri} + 0.0637P_e - 0.1788T_{esh} \\ 0.1618P_c - 0.1470T_{cri} - 0.0665P_e + 0.0501T_{esh} \end{bmatrix} \quad (30)$$

As the principles of combined CVs using average loss minimization are similar to that of worst-case loss minimization, they will not be mentioned in this part. We calculate all the losses when different numbers of operating variables are used to compose CVs. Fig. 8 shows the results of all the minimum worst-case losses and average losses under different number of measurements using for combined CVs. The results show that the combination of all the available operating variables gives the smallest worst-case loss 1.1×10^{-3} and average loss 0.03×10^{-3} . The use of combinations of four or five operating variables as CVs offers a reasonable trade-off between simplicity of the control system and the operational loss. The best candidates for $num = 4, 5, 6, 7$ are obtained using the two SOC methods and the results are summarized in Table 7 and Table 8.

Figure 8 The losses with different numbers of operating variables

Table 7 The optimal combinations of operating variables as self-optimizing CVs using worst-case loss method

Number	of Self-optimizing CVs	Worst-case
--------	------------------------	------------

measurements		loss (10^{-3})
4	$CV1 = -0.0067P_c + 0.1356T_{cri} + 0.0708P_e + 0.0377T_{esh}$ $CV2 = -0.1671P_c + 1.1795T_{cri} + 0.0637P_e - 0.1788T_{esh}$ $CV3 = 0.1618P_c - 0.1470T_{cri} - 0.0665P_e + 0.0501T_{esh}$	1.3781
5	$CV1 = 1.2695P_c + 0.0020T_{cri} - 0.0273T_{cao} + 0.7234P_e + 0.2956T_{esh}$ $CV2 = -0.2698P_c + 0.1009T_{cri} + 0.1230T_{cao} + 1.2799P_e + 0.6986T_{esh}$ $CV3 = 0.1331P_c + 0.4036T_{cri} + 1.6486T_{cao} - 0.0596P_e - 0.0150T_{esh}$	1.3176
6	$CV1 = 0.0247P_c - 0.1582T_{cri} + 6.7733P_e + 0.0018T_{eri} + 0.0612T_{ero} + 0.0453T_{esh}$ $CV2 = 0.0433P_c + 0.4523T_{cri} + 0.3731P_e + 0.0811T_{eri} + 0.0306T_{ero} - 0.0180T_{esh}$ $CV3 = 0.0784P_c + 0.3439T_{cri} - 5.5135P_e + 0.0624T_{eri} - 0.0377T_{ero} - 0.0512T_{esh}$	1.2860
7	$CV1 = 1.7687P_c + 0.7536T_{cri} + 6.6865P_e + 0.1765T_{eri} - 1.3481T_{ero} + 15.3071T_{esh} - 0.9538T_{eao}$ $CV2 = 0.3185P_c + 1.2586T_{cri} + 0.3919P_e + 0.1437T_{eri} + 0.1021T_{ero} + 3.6944T_{esh} - 0.2197T_{eao}$ $CV3 = 0.8057P_c + 0.8697T_{cri} - 5.5630P_e + 0.1475T_{eri} - 0.3417T_{ero} + 8.5370T_{esh} - 0.4767T_{eao}$	1.2736

Table 8 The optimal combinations of operating variables as self-optimizing CVs using Average Loss

Number of Self-optimizing CVs	Average loss
measurements	(10^{-3})
4	$CV1 = 1.0099P_c - 0.0247T_{cri} + 0.9945P_e + 0.0565T_{esh}$

	$CV2 = 0.9163P_c + 0.4123T_{cri} + 0.1601P_e + 0.3517T_{esh}$	
	$CV3 = 0.7862P_c - 0.0508T_{cri} + 0.0938P_e + 0.3800T_{esh}$	
5	$CV1 = -62.618P_c - 5.307T_{cri} - 83.613T_{cao} + 32.176P_e$ $+ 8.649T_{esh}$	0.0355
	$CV2 = 51.393P_c + 4.578T_{cri} + 67.448T_{cao} - 23.965P_e$ $- 0.6986T_{esh}$	
	$CV3 = 484.09P_c + 38.24T_{cri} + 617.48T_{cao} - 231.9P_e$ $- 62.428T_{esh}$	
6	$CV1 = 0.8043P_c - 0.4875T_{cri} + 6.7606P_e + 0.6218T_{eri}$ $+ 2.8301T_{ero} + 0.0986T_{esh}$	0.0354
	$CV2 = -0.3532P_c + 0.7066T_{cri} + 0.4024P_e - 0.1516T_{eri}$ $+ 0.1675T_{ero} - 0.0133T_{esh}$	
	$CV3 = 0.9107P_c + 0.1793T_{cri} - 5.5121P_e + 0.5413T_{eri}$ $- 2.3396T_{ero} + 0.0105T_{esh}$	
7	$CV1 = 1.5565P_c - 67.8303T_{cri} - 9.5512P_e + 13.9774T_{eri}$ $- 80.0186T_{ero} - 37.5297T_{esh} - 1.6148T_{eao}$	0.0354
	$CV2 = 5.7377P_c - 42.661T_{cri} + 13.355P_e - 16.223T_{eri}$ $- 50.856T_{ero} + 17.699T_{esh} - 0.5658T_{eao}$	
	$CV3 = -2.6726P_c + 49.217T_{cri} + 30.360P_e + 9.6702T_{eri}$ $+ 56.308T_{ero} - 8.3247T_{esh} + 1.8440T_{eao}$	

In order to validate the closed-loop performance of the combinations of operating variables as CVs, the energy efficient model predictive control strategy and traditional PID control strategy for the optimal combination of four operating variables as self-optimizing CVs are carried out. However, one drawback of measurement combinations as CVs is that the combined CVs lack physical meanings. Therefore, the dynamic simulations are provided to demonstrate the closed-loop performance. The parameters of the MPC controller and PID controller are well-tuned based on a good control performance and summarized as shown in Table 9. The simulation results for

the tracking performance of the VCC system under the two control strategies are illustrated in Fig. 9. In this research, a drop of 2 percent in $\dot{m}_{ca}$ is occurred at time $t=200s$, and then further increases in environment temperature and indoor temperature are occurred at time $t=400s$ and time $t=600s$. Simulation results show that both of the two control strategies are able to immediately restrain the disturbances and follow the setting values, while the proposed MPC strategy has better tracking performance and disturbances rejection performance than PID strategy. The proposed combined self-optimizing CVs can efficiently guarantee stability and performance of the closed-loop system.

Table 9 The parameters of MPC controller and PID controller for optimal combination of four operating variables as CVs

Controller	Parameters of controllers		
MPC	Prediction horizon		30
	Control horizon		15
	Weight of outputs		$diag\{5,5,8\}$
	Weight of inputs		$diag\{0.1,0.1,0.1\}$
PID	CV1 – v_o	K_C	2.1825
		τ_I	0.3407
		τ_d	3.455
	CV2 – F_{com}	K_C	0.0882
		τ_I	0.0931
		τ_d	-0.0293
	CV3 – $\dot{m}_{ea}$	K_C	20.78
		τ_I	0.5557
		τ_d	-4.096

Figure 9 Simulation results of optimal combination of four operating variables as self-optimizing CVs

5. Conclusions

The influence of different operating variables sets on the operation stability and system efficiency of the VCC system is evaluated in this paper. A performance index is proposed to minimize the energy consumption of the VCC system while meeting with the cooling demands. Two SOC methods are applied in this research, and both of them can achieve satisfactory optimization results. Moreover, MPC controllers and PID controllers are designed for optimal control structure and the comparison experiments are carried out between the different control structures and controllers. The experimental results confirm that the proposed selection of self-optimizing CVs can not only provide the energy efficient control configuration for the VCC system, but also ensure the near-optimal stable operation for all disturbance scenarios. Future research will focus on the cascade control strategy of the VCC system. Moreover this control strategy will be extended to the HVAC system with Active Chilled Beams for solving the multi-objective optimization problem. These researches are currently in progress and will be reported later.

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Appendix A

The state space representation of the VCC system can be simply characterized with certain accuracy by 7 system states, 6 system inputs, and 16 system outputs as:

$$\begin{bmatrix} Z_e(x_e) & 0 \\ 0 & Z_c(x_c) \end{bmatrix} \begin{bmatrix} \dot{L}_{e1} \\ \dot{P}_e \\ \dot{h}_{ero} \\ \dot{L}_{c1} \\ \dot{L}_{c2} \\ \dot{P}_c \\ \dot{h}_{cro} \end{bmatrix} = \begin{bmatrix} (h_{eri} - h_{eg})(C_{v1} + C_{v2}v_o)[\rho(P_c - P_e)]^{1/2} + Q_{e2} \\ (h_{eg} - h_{ero})F_{com} \left(D_{com1} - D_{com2} \left(\frac{P_c}{P_e} \right)^{D_{com3}} \right) + Q_{e1} \\ (C_{v1} + C_{v2}v_o)[\rho(P_c - P_e)]^{1/2} - F_{com} \left(D_{com1} - D_{com2} \left(\frac{P_c}{P_e} \right)^{D_{com3}} \right) \\ (h_{cri} - h_{cg})F_{com} \left(D_{com1} - D_{com2} \left(\frac{P_c}{P_e} \right)^{D_{com3}} \right) + Q_{c1} - 0.5A_c \rho_{cg} L_{c1} \dot{h}_{cri} \\ F_{com} \left(D_{com1} - D_{com2} \left(\frac{P_c}{P_e} \right)^{D_{com3}} \right) h_{cg} - (C_{v1} + C_{v2}v_o)[\rho(P_c - P_e)]^{1/2} h_{cf} + Q_{c2} \\ (h_{cf} - h_{cro})(C_{v1} + C_{v2}v_o)[\rho(P_c - P_e)]^{1/2} + Q_{c3} \\ F_{com} \left(D_{com1} - D_{com2} \left(\frac{P_c}{P_e} \right)^{D_{com3}} \right) - (C_{v1} + C_{v2}v_o)[\rho(P_c - P_e)]^{1/2} \end{bmatrix}$$

$$\begin{bmatrix} L_{e1} \\ P_e \\ h_{ero} \\ T_{eao} \\ T_{ero} \\ T_{esh} \\ \dot{m}_{er} \\ L_{c1} \\ L_{c2} \\ P_c \\ h_{cro} \\ T_{cao} \\ T_{cro} \\ T_{csh} \\ T_{csc} \\ \dot{m}_{cr} \end{bmatrix} = \begin{bmatrix} L_{e1} \\ P_e \\ h_{ero} \\ T_{eai} - Q_e / (C_p \dot{m}_{ea}) \\ \Gamma(P_e, h_{ero}) \\ \Gamma(P_e, h_{ero}) - \Gamma_{sat}(P_e) \\ \rho_{eg} A_e L_{e1} + (\rho_{eg} \gamma_e + \rho_{ef} (1 - \gamma_e)) A_e (L_e - L_{e1}) \\ L_{c1} \\ L_{c2} \\ P_c \\ h_{cro} \\ T_{cai} + Q_c / (C_p \dot{m}_{ca}) \\ \Gamma(P_c, h_{cro}) \\ \Gamma(P_c, h_{cro}) - \Gamma_{sat}(P_c) \\ \Gamma_{sat}(P_c) - \Gamma(P_c, h_{cro}) \\ \rho_{cg} A_c L_{c1} + (\rho_{cg} \gamma_c + \rho_{cf} (1 - \gamma_c)) A_c L_{c2} + \rho_{cf} A_c (L_c - L_{c1} - L_{c2}) \end{bmatrix}$$

where

$$x = [L_{e1} \quad P_e \quad h_{ero} \quad L_{c1} \quad L_{c2} \quad P_c \quad h_{cro}]^T$$

$$u = [F_{com} \quad v_o \quad \dot{m}_{ea} \quad \dot{m}_{ca} \quad T_{eai} \quad T_{cai}]^T$$

$$y = [L_{e1} \quad P_e \quad h_{ero} \quad T_{eao} \quad T_{ero} \quad T_{esh} \quad \dot{m}_{ea} \quad L_{c1} \quad L_{c2} \quad P_c \quad h_{cro} \quad T_{cao} \quad T_{cro} \quad T_{csh} \quad T_{csc} \quad \dot{m}_{ca}]^T$$

$$Z_e(x_e) = \begin{bmatrix} z_{e11} & z_{e12} & 0 \\ z_{e21} & z_{e22} & z_{e23} \\ z_{e31} & z_{e32} & 0 \end{bmatrix}$$

$$Z_c(x_c) = \begin{bmatrix} z_{c11} & 0 & z_{c13} & 0 \\ z_{c21} & z_{c22} & z_{c23} & 0 \\ z_{c31} & z_{c32} & z_{c33} & z_{c34} \\ z_{c41} & z_{c42} & z_{c43} & 0 \end{bmatrix}$$

Table A1 Elements of matrix $Z_e(x_e)$

z_{e11}	$(\rho_{ef} h_{eg} - \rho_{ef} h_{ef})(1 - \gamma_e) A_e$
z_{e12}	$\left[\frac{d\rho_{ef}}{dP_e} (1 - \gamma_e) (h_{ef} - h_{eg}) + \frac{dh_{ef}}{dP_e} (1 - \gamma_e) \rho_{ef} + \frac{dh_{eg}}{dP_e} \gamma_e \rho_{eg} - 1 \right] A_e (L_e - L_{e1})$
z_{e21}	$0.5 A_e \rho_{eg} (h_{eg} - h_{ero})$

z_{e22}	$0.5A_e \left[(h_{ero} - h_{eg})L_{e1} \frac{d\rho_{eg}}{dP_e} + \rho_{eg} L_{e1} \frac{dh_{eg}}{dP_e} - 2L_{e1} \right]$
z_{e23}	$0.5\rho_{eg} L_{e1}$
z_{e31}	$A_e \left((\rho_{eg} - \rho_{ef})(1 - \gamma_e) \right)$
z_{e32}	$A_e \left(\left(\frac{d\rho_{eg}}{dP_e} \gamma_e + \frac{d\rho_{ef}}{dP_e} (1 - \gamma_e) \right) L_e + \left(\frac{d\rho_{eg}}{dP_e} - \frac{d\rho_{ef}}{dP_e} \right) (1 - \gamma_e) L_{e1} \right)$

Table A2 Elements of matrix $Z_c(x_c)$

c_{11}	$0.5A_c \left(\rho_{cg} (h_{cri} - h_{cg}) + 2P_c \right)$
c_{13}	$A_c \left(0.5(h_{cri} - h_{cg}) \frac{d\rho_{cg}}{dP_c} + 0.5\rho_{cg} \frac{dh_{cg}}{dP_c} \right) L_{c1}$
c_{21}	$A_c \left(\rho_{cg} h_{cg} - \rho_{cf} h_{cf} \right)$
c_{22}	$A_c \left(\rho_{cg} h_{cg} - \rho_{cf} h_{cf} \right) \gamma_c$
c_{23}	$\left(\left(\frac{d\rho_{cg}}{dP_c} h_{cg} + \frac{dh_{cg}}{dP_c} \rho_{cg} \right) \gamma_c + \left(\frac{d\rho_{cf}}{dP_c} h_{cf} + \frac{dh_{cf}}{dP_c} \rho_{cf} \right) (1 - \gamma_c) - 1 \right) A_c L_{c2}$ $+ \frac{d\rho_{cg}}{dP_c} h_{cg} A_c L_{c1} + \frac{d\rho_{cf}}{dP_c} h_{cf} A_c L_{c3}$
c_{31}	$0.5A_c \left(h_{cro} - h_{cf} \right) \rho_{cf}$
c_{32}	$0.5A_c \left(h_{cro} - h_{cf} \right) \rho_{cf}$
c_{33}	$\left(0.5(h_{cro} - h_{cf}) \frac{\partial \rho_{cf}}{\partial P_c} + 0.5\rho_{cf} \frac{dh_{cf}}{dP_c} - 1 \right) A_c L_{c3}$
c_{34}	$0.5A_c \rho_{cf} L_{c3}$
c_{41}	$A_c \left(\rho_{cg} - \rho_{cf} \right)$
c_{42}	$A_c \left(\rho_{cg} - \rho_{cf} \right) \gamma_c$
c_{43}	$A_c \left[\frac{d\rho_{cg}}{dP_c} L_{c1} + \left(\frac{d\rho_{cg}}{dP_c} \gamma_c + \frac{d\rho_{cf}}{dP_c} (1 - \gamma_c) \right) L_{c2} + \frac{d\rho_{cf}}{dP_c} L_{c3} \right]$

Appendix B

The detailed representation of J_{uu} and J_{ud} are provided as follows:

At a steady-state, the mass flow rate is a constant during the cycle:

$$\dot{m}_{com} = \dot{m}_v = (C_{v1} + C_{v2}v_o) [\rho(P_c - P_e)]^{1/2} \quad (B1)$$

Substituting Eq. (B1) into Eq. (9), yields

$$J = \frac{C_{com1} F_{com} (C_{v1} + C_{v2}v_o) [\rho(P_c - P_e)]^{1/2} P_e \left(\left(\frac{P_c}{P_e} \right)^{C_{com2}} - 1 \right)}{\eta_{com} C_p \dot{m}_{ea} (T_{eai} - T_{eao})} \quad (B2)$$

Take the first derivative with respect to each input:

$$\frac{\partial J}{\partial F_{com}} = \frac{2C_{com1} F_{com} \left(D_{com1} - D_{com2} \left(\frac{P_c}{P_e} \right)^{D_{com3}} \right) P_e \left(\left(\frac{P_c}{P_e} \right)^{C_{com2}} - 1 \right)}{C_p \dot{m}_{ea} (T_{eai} - T_{eao}) \eta_{com}} \quad (B3)$$

$$\frac{\partial J}{\partial v_o} = \frac{C_{v2} C_{com1} F_{com} [\rho(P_c - P_e)]^{1/2} P_e \left(\left(\frac{P_c}{P_e} \right)^{C_{com2}} - 1 \right)}{\eta_{com} C_p \dot{m}_{ea} (T_{eai} - T_{eao})} \quad (B4)$$

$$\frac{\partial J}{\partial \dot{m}_{ea}} = - \frac{C_{com1} F_{com}^2 \left(D_{com1} - D_{com2} \left(\frac{P_c}{P_e} \right)^{D_{com3}} \right) P_e \left(\left(\frac{P_c}{P_e} \right)^{C_{com2}} - 1 \right)}{\eta_{com} C_p \dot{m}_{ea}^2 (T_{eai} - T_{eao})} \quad (B5)$$

It is a symmetric matrix, thus $J_{uu} = J_{uu}^T$. Take the second derivative with respect to all of the

inputs, then the J_{uu} can be obtained:

$$J_{uu} = \begin{bmatrix} \frac{\partial^2 J}{\partial F_{com}^2} & \frac{\partial^2 J}{\partial F_{com} \partial v_o} & \frac{\partial^2 J}{\partial F_{com} \partial \dot{m}_{ea}} \\ \frac{\partial^2 J}{\partial v_o \partial F_{com}} & \frac{\partial^2 J}{\partial v_o^2} & \frac{\partial^2 J}{\partial v_o \partial \dot{m}_{ea}} \\ \frac{\partial^2 J}{\partial \dot{m}_{ea} \partial F_{com}} & \frac{\partial^2 J}{\partial \dot{m}_{ea} \partial v_o} & \frac{\partial^2 J}{\partial \dot{m}_{ea}^2} \end{bmatrix} \quad (B6)$$

where

$$\begin{aligned}
 \frac{\partial^2 J}{\partial F_{com}^2} &= \frac{2C_{com1} \left(D_{com1} - D_{com2} \left(\frac{P_c}{P_e} \right)^{D_{com3}} \right) P_e \left(\left(\frac{P_c}{P_e} \right)^{C_{com2}} - 1 \right)}{C_p \dot{m}_{ea} (T_{eai} - T_{eao}) \eta_{com}} \\
 \frac{\partial^2 J}{\partial v_o^2} &= 0 \\
 \frac{\partial^2 J}{\partial \dot{m}_{ea}^2} &= \frac{2C_{com1} F_{com}^2 \left(D_{com1} - D_{com2} \left(\frac{P_c}{P_e} \right)^{D_{com3}} \right) P_e \left(\left(\frac{P_c}{P_e} \right)^{C_{com2}} - 1 \right)}{\eta_{com} C_p \dot{m}_{ea}^3 (T_{eai} - T_{eao})} \\
 \frac{\partial^2 J}{\partial v_o \partial F_{com}} &= \frac{\partial^2 J}{\partial F_{com} \partial v_o} = \frac{C_{v2} C_{com1} [\rho(P_c - P_e)]^{1/2} P_e \left(\left(\frac{P_c}{P_e} \right)^{C_{com2}} - 1 \right)}{\eta_{com} C_p \dot{m}_{ea} (T_{eai} - T_{eao})} \\
 \frac{\partial^2 J}{\partial \dot{m}_{ea} \partial F_{com}} &= \frac{\partial^2 J}{\partial F_{com} \partial \dot{m}_{ea}} = - \frac{2C_{com1} F_{com} \left(D_{com1} - D_{com2} \left(\frac{P_c}{P_e} \right)^{D_{com3}} \right) P_e \left(\left(\frac{P_c}{P_e} \right)^{C_{com2}} - 1 \right)}{C_p \dot{m}_{ea}^2 (T_{eai} - T_{eao}) \eta_{com}} \\
 \frac{\partial^2 J}{\partial \dot{m}_{ea} \partial v_o} &= \frac{\partial^2 J}{\partial v_o \partial \dot{m}_{ea}} = - \frac{C_{v2} C_{com1} F_{com} [\rho(P_c - P_e)]^{1/2} P_e \left(\left(\frac{P_c}{P_e} \right)^{C_{com2}} - 1 \right)}{\eta_{com} C_p \dot{m}_{ea}^2 (T_{eai} - T_{eao})}
 \end{aligned}$$

Based on Eq. (B3)-Eq. (B5), we take the second derivative of them with respect to all the disturbances ($\dot{m}_{ca}$, T_{cai} , and T_{eai}). According to the energy balance equation, the $\dot{Q}_e$ and $\dot{W}_{com}$ can be represented by

$$\dot{Q}_e = \dot{Q}_c - \eta_{com} \dot{W}_{com} \quad (B7)$$

$$\dot{W}_{com} = \frac{1}{\eta_{com}} (\dot{Q}_c - \dot{Q}_e) \quad (B8)$$

Substituting Eq. (B7) and Eq. (B8) into Eq. (9), we get:

$$J_{ud} = \begin{bmatrix} \frac{\partial^2 J}{\partial F_{com} \partial \dot{m}_{ca}} & \frac{\partial^2 J}{\partial F_{com} \partial T_{cai}} & \frac{\partial^2 J}{\partial F_{com} \partial T_{eai}} \\ \frac{\partial^2 J}{\partial v_o \partial \dot{m}_{ca}} & \frac{\partial^2 J}{\partial v_o \partial T_{cai}} & \frac{\partial^2 J}{\partial v_o \partial T_{eai}} \\ \frac{\partial^2 J}{\partial \dot{m}_{ea} \partial \dot{m}_{ca}} & \frac{\partial^2 J}{\partial \dot{m}_{ea} \partial T_{cai}} & \frac{\partial^2 J}{\partial \dot{m}_{ea} \partial T_{eai}} \end{bmatrix} \quad (B10)$$

where

$$\begin{aligned}
 \frac{\partial^2 J}{\partial F_{com} \partial \dot{m}_{ca}} &= \frac{2C_q (T_{cao} - T_{cai}) C_{com1} \left(D_{com1} - D_{com2} \left(\frac{P_c}{P_e} \right)^{D_{com3}} \right) P_e \left(\left(\frac{P_c}{P_e} \right)^{C_{com2}} - 1 \right) F_{com}}{\eta_{com} \left(C_q \dot{m}_{ca} (T_{cao} - T_{cai}) - C_{com1} F_{com}^2 \left(D_{com1} - D_{com2} \left(\frac{P_c}{P_e} \right)^{D_{com3}} \right) P_e \left(\left(\frac{P_c}{P_e} \right)^{C_{com2}} - 1 \right) \right)^2} \\
 &\quad - \frac{4C_q^2 \dot{m}_{ca} (T_{cao} - T_{cai})^2 C_{com1} \left(D_{com1} - D_{com2} \left(\frac{P_c}{P_e} \right)^{D_{com3}} \right) P_e \left(\left(\frac{P_c}{P_e} \right)^{C_{com2}} - 1 \right) F_{com}}{\eta_{com} \left(C_q \dot{m}_{ca} (T_{cao} - T_{cai}) - C_{com1} F_{com}^2 \left(D_{com1} - D_{com2} \left(\frac{P_c}{P_e} \right)^{D_{com3}} \right) P_e \left(\left(\frac{P_c}{P_e} \right)^{C_{com2}} - 1 \right) \right)^3} \\
 \frac{\partial^2 J}{\partial F_{com} \partial T_{cai}} &= \frac{-2C_q \dot{m}_{ca} C_{com1} \left(D_{com1} - D_{com2} \left(\frac{P_c}{P_e} \right)^{D_{com3}} \right) P_e \left(\left(\frac{P_c}{P_e} \right)^{C_{com2}} - 1 \right) F_{com}}{\eta_{com} \left(C_q \dot{m}_{ca} (T_{cao} - T_{cai}) - C_{com1} F_{com}^2 \left(D_{com1} - D_{com2} \left(\frac{P_c}{P_e} \right)^{D_{com3}} \right) P_e \left(\left(\frac{P_c}{P_e} \right)^{C_{com2}} - 1 \right) \right)^2} \\
 &\quad + \frac{4C_q^2 \dot{m}_{ca}^2 (T_{cao} - T_{cai}) C_{com1} \left(D_{com1} - D_{com2} \left(\frac{P_c}{P_e} \right)^{D_{com3}} \right) P_e \left(\left(\frac{P_c}{P_e} \right)^{C_{com2}} - 1 \right) F_{com}}{\eta_{com} \left(C_q \dot{m}_{ca} (T_{cao} - T_{cai}) - C_{com1} F_{com}^2 \left(D_{com1} - D_{com2} \left(\frac{P_c}{P_e} \right)^{D_{com3}} \right) P_e \left(\left(\frac{P_c}{P_e} \right)^{C_{com2}} - 1 \right) \right)^3} \\
 \frac{\partial^2 J}{\partial F_{com} \partial T_{eai}} &= - \frac{2C_{com1} F_{com} \left(D_{com1} - D_{com2} \left(\frac{P_c}{P_e} \right)^{D_{com3}} \right) P_e \left(\left(\frac{P_c}{P_e} \right)^{C_{com2}} - 1 \right)}{C_p \dot{m}_{ea} (T_{eai} - T_{eao})^2 \eta_{com}} \\
 \frac{\partial^2 J}{\partial v_o \partial \dot{m}_{ca}} &= \frac{C_q (T_{cao} - T_{cai}) C_{v2} C_{com1} F_{com} [\rho(P_c - P_e)]^{1/2} P_e \left(\left(\frac{P_c}{P_e} \right)^{C_{com2}} - 1 \right)}{\eta_{com} \left(C_q \dot{m}_{ca} (T_{cao} - T_{cai}) - C_{com1} F_{com} (C_{v1} + C_{v2} v_o) [\rho(P_c - P_e)]^{1/2} P_e \left(\left(\frac{P_c}{P_e} \right)^{C_{com2}} - 1 \right) \right)^2} \\
 &\quad - \frac{2(C_q (T_{cao} - T_{cai}))^2 \dot{m}_{ca} C_{v2} C_{com1} F_{com} [\rho(P_c - P_e)]^{1/2} P_e \left(\left(\frac{P_c}{P_e} \right)^{C_{com2}} - 1 \right)}{\eta_{com} \left(C_q \dot{m}_{ca} (T_{cao} - T_{cai}) - C_{com1} F_{com} (C_{v1} + C_{v2} v_o) [\rho(P_c - P_e)]^{1/2} P_e \left(\left(\frac{P_c}{P_e} \right)^{C_{com2}} - 1 \right) \right)^3}
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial^2 J}{\partial v_o \partial T_{cai}} &= - \frac{C_q \dot{m}_{ca} C_{v2} C_{com1} F_{com} [\rho(P_c - P_e)]^{1/2} P_e \left(\left(\frac{P_c}{P_e} \right)^{C_{com2}} - 1 \right)}{\eta_{com} \left(C_q \dot{m}_{ca} (T_{cao} - T_{cai}) - C_{com1} F_{com} (C_{v1} + C_{v2} v_o) [\rho(P_c - P_e)]^{1/2} P_e \left(\left(\frac{P_c}{P_e} \right)^{C_{com2}} - 1 \right) \right)^2} \\
 &+ \frac{2(C_q \dot{m}_{ca})^2 (T_{cao} - T_{cai}) C_{v2} C_{com1} F_{com} [\rho(P_c - P_e)]^{1/2} P_e \left(\left(\frac{P_c}{P_e} \right)^{C_{com2}} - 1 \right)}{\eta_{com} \left(C_q \dot{m}_{ca} (T_{cao} - T_{cai}) - C_{com1} F_{com} (C_{v1} + C_{v2} v_o) [\rho(P_c - P_e)]^{1/2} P_e \left(\left(\frac{P_c}{P_e} \right)^{C_{com2}} - 1 \right) \right)^3} \\
 \frac{\partial^2 J}{\partial v_o \partial T_{eai}} &= - \frac{C_{v2} C_{com1} F_{com} [\rho(P_c - P_e)]^{1/2} P_e \left(\left(\frac{P_c}{P_e} \right)^{C_{com2}} - 1 \right)}{\eta_{com} C_p \dot{m}_{ea} (T_{eai} - T_{eao})^2} \\
 \frac{\partial^2 J}{\partial \dot{m}_{ea} \partial \dot{m}_{ca}} &= - \frac{C_q (T_{cao} - T_{cai})}{\eta_{com} C_p \dot{m}_{ea}^2 (T_{eai} - T_{eao})} \\
 \frac{\partial^2 J}{\partial v_o \partial T_{cai}} &= - \frac{C_q \dot{m}_{ca} C_{v2} C_{com1} F_{com} [\rho(P_c - P_e)]^{1/2} P_e \left(\left(\frac{P_c}{P_e} \right)^{C_{com2}} - 1 \right)}{\eta_{com} \left(C_q \dot{m}_{ca} (T_{cao} - T_{cai}) - C_{com1} F_{com} (C_{v1} + C_{v2} v_o) [\rho(P_c - P_e)]^{1/2} P_e \left(\left(\frac{P_c}{P_e} \right)^{C_{com2}} - 1 \right) \right)^2} \\
 &+ \frac{2(C_q \dot{m}_{ca})^2 (T_{cao} - T_{cai}) C_{v2} C_{com1} F_{com} [\rho(P_c - P_e)]^{1/2} P_e \left(\left(\frac{P_c}{P_e} \right)^{C_{com2}} - 1 \right)}{\eta_{com} \left(C_q \dot{m}_{ca} (T_{cao} - T_{cai}) - C_{com1} F_{com} (C_{v1} + C_{v2} v_o) [\rho(P_c - P_e)]^{1/2} P_e \left(\left(\frac{P_c}{P_e} \right)^{C_{com2}} - 1 \right) \right)^3} \\
 \frac{\partial^2 J}{\partial \dot{m}_{ea} \partial T_{cai}} &= \frac{C_q \dot{m}_{ca}}{\eta_{com} C_p \dot{m}_{ea}^2 (T_{eai} - T_{eao})} \\
 \frac{\partial^2 J}{\partial \dot{m}_{ea} \partial T_{eai}} &= \frac{C_{com1} F_{com}^2 \left(D_{com1} - D_{com2} \left(\frac{P_c}{P_e} \right)^{D_{com3}} \right) P_e \left(\left(\frac{P_c}{P_e} \right)^{C_{com2}} - 1 \right)}{\eta_{com} C_p \dot{m}_{ea}^2 (T_{eai} - T_{eao})^2}
 \end{aligned}$$

Nomenclature

a, b, c Coefficient

d Disturbance

A	Cross-sectional area (m^2)
C, D	Coefficient
C_p	Specific heat capacity at constant pressure ($\text{kJ/kg}^\circ\text{C}$)
F_{com}	Frequency of compressor (Hz)
G	Gain of the selected measurement
H	Selection or combination matrix
h_{cri}	Enthalpy at inlet of condenser (kJ/kg)
h_{cro}	Enthalpy at outlet of condenser (kJ/kg)
h_{eri}	Enthalpy at inlet of evaporator (kJ/kg)
h_{ero}	Enthalpy at outlet of evaporator (kJ/kg)
J	Cost function
L	Loss in the self-optimizing method
$\dot{m}_{ca}$	Air flow rate of condenser (kg/s)
$\dot{m}_{com}$	Mass flow rate of refrigerant through compressor (kg/s)
$\dot{m}_{ea}$	Air flow rate of evaporator (kg/s)
$\dot{m}_{eri}$	Mass flow rate of refrigerant at inlet of evaporator (kg/s)
$\dot{m}_{ero}$	Mass flow rate of refrigerant at outlet of evaporator (kg/s)
$\dot{m}_r$	Mass flow rate of refrigerant (kg/s)
$\dot{m}_v$	Mass flow rate of refrigerant through expansion valve (kg/s)
MSS	Minimal stable superheat of evaporator ($^\circ\text{C}$)
n	Measurement and implementation error
P_e	Evaporating pressure (bar)

P_c	Condensing pressure (bar)
$\dot{Q}_e$	Evaporator energy transfer rate (kJ/s)
T_{amb}	Environment temperature ($^{\circ}C$)
T_{cai}	Air temperature at inlet of condenser ($^{\circ}C$)
T_{cri}	Refrigerant temperature at inlet of condenser ($^{\circ}C$)
T_{cro}	Refrigerant temperature at outlet of condenser ($^{\circ}C$)
T_{crsat}	Condensing temperature ($^{\circ}C$)
T_{csc}	Subcool degree of condenser ($^{\circ}C$)
T_{csh}	Superheat degree of condenser ($^{\circ}C$)
T_{eai}	Air temperature at inlet of evaporator ($^{\circ}C$)
T_{eri}	Refrigerant temperature at inlet of evaporator ($^{\circ}C$)
T_{ero}	Refrigerant temperature at outlet of evaporator ($^{\circ}C$)
T_{ersat}	Evaporating temperature ($^{\circ}C$)
T_{esh}	Superheat degree of evaporator ($^{\circ}C$)
u	System input
$\dot{W}_{com}$	Power consumption rate of compressor (kJ/s)
y	System output
v_o	Opening of expansion valve
η_{com}	Delivery coefficient of compressor
ρ	Density (kg / m^3)

Subscripts

a	Air
-----	-----

<i>c</i>	Condenser
<i>com</i>	Compressor
<i>e</i>	Evaporator
<i>i</i>	Inlet
<i>o</i>	Outlet
<i>r</i>	Refrigerant
max	Maximal value allowed
min	Minimal value allowed

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