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COMPARISONS OF SIDEWALL CORRECTION OF BED SHEAR STRESS IN OPEN-CHANNEL FLOWS

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Abstract: When investigating sediment transport in laboratory open-channel flows, it is often necessary to remove sidewall effects for computing effective bed shear stress. Previous sidewall correction methods include some assumptions that have not been completely verified, and therefore different values of the bed shear stress may be obtained, depending on the approach used in making sidewall corrections. This study provides a quantitative assessment of the existing correction procedures by comparing them to a new sidewall correction model proposed in this study. The latter was derived based on the shear stress function and equivalent roughness size for both rigid and mobile bed conditions obtained directly from experimental measurements. The comparisons show that the bed shear stress is generally overestimated by the Vanoni and Brooks (1957) correction approach but underestimated by the Einstein (1941) formula. In addition, low precision was obtained from the prediction by the Williams' (1970) empirical equation. This study also demonstrates that the Vanoni and Brooks approach can be well

approximated by a simple formula derived based on the Blasius resistance function. The sidewall effects, when removed in the different ways, would consequently affect the presentation of the bedload function. Experimental results of bedload transport, when plotted as the dimensionless transport rate against the dimensionless shear stress with the latter being corrected using the present model, exhibit less scatter than those associated with the previous procedures.

Introduction

For laboratory open-channel flows over rigid or mobile sediment beds, evaluation of the bed shear stress using bulk flow parameters, such as, flow depth, average velocity and energy slope, are often subject to sidewall friction effects. The procedure used for removing the sidewall effects is referred to as sidewall correction, which is usually required in studies of sediment transport conducted in open-channel flows. Such an approach described by Vanoni and Brooks (1957) has been employed in many other research works. This approach was derived based on several assumptions including the average velocity associated with the wall-affected flow region being taken to be the same as that associated with the bed-affected flow region. This assumption of velocity equality was also used by Einstein (1941, 1950) in deriving his bedload function. However, Einstein developed an independent approach for correcting the bed shear stress. Meyer-Peter and Mueller (1948) adopted a sidewall correction procedure which was exactly the same as that used by Einstein (1941). It should be mentioned that in these approaches, bed information such as roughness size were not directly taken into account. It is understood that the roughness of the bed, even if it comprises the same sediment, may vary with transport stage. In addition, it is also not clear if a sidewall correction procedure, which is derived for rigid beds, is also applicable for mobile beds. Therefore, it is obvious

that the shear stresses corrected in the different ways may vary significantly, as is demonstrated later in this study.

Sediment transport is dependent on the bed shear stress. If the sidewall effects on the bed shear stress are not properly corrected, the transport rate would not be predicted with good precision. We note that formulations related to sediment transport, for example, those for computing bedload transport rates, often differ significantly from one to another. This difference may partly be associated with the method that is used to perform sidewall correction of the bed shear stress. At present, however, it is still unknown how the bedload function varies if the sidewall effects are to be removed following different procedures given in the literature.

This study aims to provide a quantitative assessment of various sidewall correction processes. First, typical correction formulas available in the literature are reviewed. They are then assessed using a new sidewall correction model, which is proposed in this study based on direct laboratory measurements of boundary shear stresses in open channel flows and experimental data of equivalent roughness sizes for mobile sediment bed conditions. Relevant computations are conducted using the classical database of sediment transport. To avoid complications associated with the presence of bedform-induced drag, only flat bed conditions are considered.

Review of existing sidewall correction approaches

Einstein (1941) formula

Einstein used the Manning roughness coefficient to differentiate flow resistance components associated with bed and wall, respectively. He defined the average bed shear stress, τ_b , as

$$\tau_b = \rho g R S \left(\frac{n_b}{n} \right)^{1.5} \quad (1)$$

where ρ = fluid density, g = gravitational acceleration, $R = bh/(b + 2h)$ = hydraulic radius, b = channel width, h = flow depth, S = energy slope, n = total Manning roughness coefficient, and n_b = bed-related Manning roughness coefficient. The different roughness components were then related in the form,

$$n_b = \left[\frac{(2h + b)n^{1.5} - 2hn_w^{1.5}}{b} \right]^{1/1.5} \quad (2)$$

where n_w = wall-related roughness coefficient. Since $n = R^{2/3} S^{1/2} / V$, where V = cross-sectional average velocity, substituting Eq. (2) into Eq. (1) and manipulating yields

$$\frac{\tau_b}{\rho g h S} = 1 - \frac{2n_w^{1.5} V^{1.5}}{b S^{0.75}} \quad (3)$$

It is noted that the same correction method as given by Eq. (3) was also used by Meyer-Peter and Mueller (1948) in deriving their bedload formula. For smooth glass sidewalls, the roughness coefficient, n_w , may be taken as approximately 0.009 (Daugherty et al. 1989), and therefore Eq. (3) can be simplified as

$$\frac{\tau_b}{\rho g h S} = 1 - 0.0017 \frac{V^{1.5}}{b S^{0.75}} \quad (4)$$

Eq. (4) implies that the sidewall correction of the bed shear stress could be simply made based on measurements of the flow depth, average flow velocity and energy slope.

Furthermore, if the wall friction is described using the Darcy-Weisbach friction factor instead of the Manning coefficient, Eq. (3) can be presented in an alternative form. Herein, we assume that the wall-related friction can be estimated using Blasius expression,

$$f_w = \frac{0.316}{(4VR_w/\nu)^{0.25}} \quad (5)$$

where R_w = wall-related hydraulic radius, and ν = kinematic viscosity of fluid. Noting that $f_w = 8gR_wS/V^2$, and making this substitution in Eq. (5), we obtain

$$R_w = 0.057 \left(\frac{V^7 \nu}{S^4 g^4} \right)^{0.2} \quad (6)$$

This finally yields

$$\tau_b = \rho g h S \left(1 - \frac{0.114}{b} \left(\frac{V^7 \nu}{S^4 g^4} \right)^{0.2} \right) \quad (7)$$

If taking $\nu = 10^{-6} \text{ m}^2/\text{s}$, and $g = 9.81 \text{ m/s}^2$, Eq. (7) can be re-written as

$$\frac{\tau_b}{\rho g h S} = 1 - 0.0012 \frac{V^{1.4}}{b S^{0.8}} \quad (8)$$

Obviously, Eq. (8) closely resembles Eq. (4). In addition, as is demonstrated subsequently in this study, Eq. (7) can be used as a good alternative to the Vanoni and Brooks (1957) correction procedure.

Vanoni and Brooks (1957) formula

Vanoni and Brooks argued that the Darcy-Weisbach friction factor has a sounder theoretical basis than the Manning roughness coefficient, the latter being used in the Einstein's consideration. Using the friction factor, the bed shear stress could be expressed as

$$\frac{\tau_b}{\rho g h S} = \frac{b}{b + 2h} \frac{f_b}{f} \quad (9)$$

where $f = \frac{8gRS}{V^2}$, $f_b = f + \frac{2h}{b}(f - f_w)$ = bed friction factor, and f_w = wall friction factor.

The wall friction factor, f_w , was further related to the ratio of Re/f , where $Re = 4VR/\nu$. This relationship, which was originally given as a graph of f_w against Re/f by Vanoni and Brooks, can also be described herein by the following function

$$f_w = \frac{1}{20(Re/f)^{0.1} - 39} \quad (10)$$

Eq. (10) was obtained by curve-fitting.

Williams (1970) formula

The approach provided by Williams was experimentally achieved, which suggested that the bed shear stress could be adjusted simply according to

$$\frac{\tau_b}{\rho ghS} = \frac{b^2}{b^2 + 0.055h} \quad (11)$$

where b and h are measured in meters. Being different from the approaches given by Einstein (1941) and Vanoni and Brooks (1957), Eq. (11) does not include the average velocity for the correction. Wiberg and Smith (1988) used Eq. (11) in their bedload study and obtained a good fit to the data. However, it should be mentioned that in their study, data points for which the calculated shear stresses required more than 5% correction were discarded.

Model proposed in this study

As discussed above, the assumption of the average velocity associated with the bed flow region being equal to that associated with the wall region plays an important role in the derivation of the equations given by Einstein (1941) and Vanoni and Brooks (1957). However, this assumption has not been theoretically proven although it is generally supported by Knight and MacDonald's (1979) estimates based on limited experimental results. In the following, an alternative correction is proposed using a boundary shear

function, which was derived from direct stress measurements for open channel flows (Knight 1981).

Knight (1981) obtained a satisfactory method for computing the average wall and bed shear stresses for rectangular open-channel flows over a wide range of roughness and width to depth ratios, based on a series of laboratory shear stress data. The experiments were conducted in a 15-m flume with rigid beds, and the boundary shear stresses were measured by a Preston tube or deduced from semi-logarithmic plotting of velocity profiles. By fitting the experimental data, Knight related the average bed shear stress to the aspect ratio of the channel flow and the Nikuradse equivalent sand roughness, yielding

$$\frac{\tau_b}{\rho ghS} = 1 - 0.01\alpha \left[\tanh \pi\beta - 0.5(\tanh \pi\beta - \beta)^2 \right] \quad (12)$$

where $\alpha = \exp[6.211 - 3.264 \log(b/h + 3)]$, $\beta = 1 - 0.2 \log(k_{sb}/k_{sw})$, k_{sb} = bed roughness size, and k_{sw} = smooth-wall roughness size taken as 0.0015 mm. Eq. (12) implies that the average bed shear stress can be computed provided that the aspect ratio and equivalent bed roughness size are known. It is also noted that the average velocity is not included in Eq. (12).

In this study, we assume that Eq. (12) can be used for evaluating the bed shear stress for both rigid and mobile bed conditions. For rigid sediment beds, the roughness size can be simply characterized by the sediment size. It is usually taken as approximately $2d$, where d = sediment particle diameter, i.e.

$$\frac{k_{sb}}{d} \approx 2 \quad (13)$$

This result may also be applicable for the mobile bed with weak sediment transport or subject to low shear stresses.

If a sediment bed experiences intense motion, however, its equivalent bed roughness may not be directly related to the sediment size. For example, in the presence of bed

undulations, it may be scaled by the thickness of bedform, which is much greater than the sediment particle diameter. For the case of sheet flows, where bedforms disappear and the bed shear stress is high, the equivalent roughness size has been reported (Wilson 1989) to be proportional to the dimensionless bed shear stress or Shields number:

$$\frac{k_{sb}}{d} \approx 5\theta \quad (14)$$

where $\theta = \tau_b/[(\rho_s - \rho)gd]$ = dimensionless bed shear stress, ρ = fluid density, and ρ_s = particle density.

Between the two extreme cases given by Eqs. (13) and (14), a transition exists but it is hard to formulate the variation of k_{sb}/d theoretically. Alternatively, Yalin (1992) proposed an empirical interpolation based on experimental data for flat beds, which takes the following form

$$\frac{k_{sb}}{d} = 2 \quad \text{for } \theta \leq 1 \quad (15)$$

$$\frac{k_{sb}}{d} = 5\theta + (\theta - 4)^2(0.043\theta^3 - 0.289\theta^2 - 0.203\theta + 0.125) \quad \text{for } 1 < \theta < 4 \quad (16)$$

$$\frac{k_{sb}}{d} = 5\theta \quad \text{for } \theta \geq 4 \quad (17)$$

In fact, the functional relationship, Eqs. (15) to (17), can be represented by a continuous function given by

$$\frac{k_{sb}}{d} = \frac{2 + 5\theta \exp(2\theta - 4.2)}{1 + \exp(2\theta - 4.2)} \quad (18)$$

This function is plotted in Fig. 1, which shows good agreement with the results of Yalin (1992).

Now, we can use Eqs. (12) and (18) to compute the average bed shear stress for the condition of mobile sediment beds (flat bed only) if the parameters, b , h , S , d , ρ and ρ_s , are known. It should be mentioned that an iterative computation is required because k_{sb} is also dependent on τ_b as given in Eq. (18). First, k_{sb} was computed using Eq. (18) with an initial guess of τ_b . Then, a new value of τ_b was computed using Eq. (12). The computation was

terminated when the difference between the two consecutive values of τ_b was not considerable. It was found that the computation converged quickly when the initial value of τ_b was taken as ρghS .

In the following, the results so computed will be used to assess the bed shear stresses corrected with the equations by Einstein (1941), Vanoni and Brooks (1957) and Williams (1970).

Comparisons

To single out sidewall friction effects, only bed conditions without bedforms are considered in this study. Therefore, comparisons are made by performing sample computations with classical experimental data, which were collected under flat bed conditions, i.e., plane beds at upper or lower stages. The data used comprise 250 runs by Gilbert (1914), 27 runs by Guy et al. (1966), and 26 runs by Williams (1970), which are all available in the compilation by Brownlie (1981). The relevant characteristics of flow and sediment for these data are summarized in Table 1. The flat bed measurements from the same sources were also selected by Bridge and Dominic (1984) for verifying their saltation model for bedload transport.

Figs. 2 - 5 show the relative deviations of the bed shear stress computed using the formulas given by Vanoni and Brooks, Einstein, and Williams, respectively, and the present model. First, the two well-known approaches developed by Einstein (1941) and Vanoni and Brooks (1957) are compared. Fig. 2 plots the relative difference against the dimensionless shear stress calculated using the uncorrected hydraulic radius, $\theta_R = \rho gRS / [(\rho_s - \rho)gd]$. The relative difference is expressed as $100(\theta_{VB} - \theta_E) / \theta_E$,

where $\theta_{VB} = \tau_{VB} / [(\rho_s - \rho)gd]$, $\theta_E = \tau_E / [(\rho_s - \rho)gd]$, τ_{VB} = bed shear stress corrected with the Vanoni and Brooks procedure, and τ_E = bed shear stress corrected using the Einstein formula. From Fig. 2, it can be observed that the shear stress corrected by the Vanoni and Brooks formula is generally larger than that predicted by the Einstein formula. The difference for most of the data points can be up to 10%, while the average difference is 3.7%.

By comparing to the present model, the correction based on the Vanoni and Brooks formula gives relatively higher shear stresses for most cases (the average difference is 2.2%, see Fig. 3), while the Einstein procedure generates slightly smaller stresses largely for $\theta_R > 0.3$ (the average difference is -0.7%, see Fig. 4). In Figs. 3 and 4, $\theta_P = \tau_P / [(\rho_s - \rho)gd]$, where τ_P = bed shear stress corrected using the present model. Noting that the average differences associated with the formulas given by Einstein (1941) and Vanoni and Brooks (1957) are not so large, it seems that for the mobile bed condition, the sidewall correction could be still done reasonably through the measured flow depth, average velocity and energy slope (such as those included in the two formulas) instead of using direct bed characteristics such as the bed roughness size, the latter varying with sediment transport intensity.

The worst case obtained in the comparisons is associated with the correction approach provided by Williams; the predicted magnitudes of the shear stress have a deviation up to 44% (the average is 14%) compared to those given by the present model. This is shown in Fig. 5, where $\theta_W = \tau_W / [(\rho_s - \rho)gd]$ and τ_W = bed shear stress corrected using the Williams formula.

Additional computation is also conducted for comparing Eq. (7) and the Vononi and Brooks formula, which both include the friction factor rather than the Manning coefficient in the derivations. The computed results indicate that the shear stress corrected using Eq.

(7) is almost the same to that given by the Vanoni and Brooks formula (see Fig. 6), the average absolute difference being less than 0.3%. This suggests that the well-known Vanoni and Brooks correction method, which includes the use of the inconvenient empirical graph of f_w against Re/f , could be simply replaced with Eq. (7).

Implications for bedload function

Given the possible deviations of the bed shear stress after sidewall correction, the bedload function may vary considerably. For example, depending on which sidewall correction procedure is applied, the coefficient included in the Meyer-Peter and Mueller's bedload formula [$\phi = 8(\theta - 0.047)^{1.5}$, where ϕ = dimensionless transport rate and θ = dimensionless bed shear stress], which was originally taken as 8, may increase or decrease. If the corrected shear stress is increased by 15% when compared to Meyer-Peter and Mueller's results, then the coefficient would be reduced to approximately 6.5. A reduced coefficient was also presented recently by Wong (2003), who reported that the best coefficient was 4.93, with the exponent taken as 1.6 for the Meyer-Peter and Mueller bedload formula, when the originally used datasets were reprocessed in a different way for removing sidewall and bedform effects.

In Fig. 7, the three datasets outlined in Table 1 are used to demonstrate possible variations in the resulted bedload function if sidewall effects are removed following different procedures. The experimental data of bedload transport are plotted in terms of the dimensionless transport rate and the dimensionless shear stress. It is interesting to notice that the data points generally exhibit the similar trend, but those associated with the present model are less scattered and therefore are able to represent the trend better. This is because the evaluation of shear stresses and equivalent roughness size made for the

present model, though empirical, are all based on direct measurements, while the previous formulas involve some assumptions, which may not hold for some situations. As a consistent reference for each comparison, also plotted is a semi-empirical bedload formula (denoted by the solid line) given by

$$\phi = 13\theta^{1.5} \exp\left(\frac{-0.05}{\theta^{1.5}}\right) \quad (19)$$

which is applicable for a variety of transport stages (Cheng 2002).

Conclusions

When investigating sediment transport in laboratory open-channel flows, removal of sidewall effects is often considered to be an essential procedure in computing effective bed shear stress. Various sidewall correction methods have been used in previous studies of sediment transport. However, these methods are subject to some assumptions that have not been completely verified, and therefore different values of the bed shear stress may be obtained, depending on the approach used in making sidewall corrections.

This study provides a quantitative assessment of the previous correction procedures by comparing them to a new sidewall correction model proposed in this study. The latter was derived based on the shear stress function and equivalent roughness size for both rigid and mobile bed conditions, which were obtained directly from experimental measurements. The comparisons show that the bed shear stress is generally overestimated by the Vanoni and Brooks correction approach but underestimated by the Einstein formula. In addition, low precision was obtained from the prediction by the Williams' empirical equation. This study also demonstrates that the Vanoni and Brooks approach can

be well approximated by a simple formula [Eq. (7)], derived based on the Blasius resistance function.

The sidewall effects, when removed in the different ways, would consequently have an effect on the presentation of the bedload function. For example, the Meyer-Peter and Mueller bedload formula may have a smaller coefficient if fitted to the shear stress which is corrected by other approaches. The bedload function in terms of the dimensionless transport rate and dimensionless shear stress, with the latter being corrected using the present model, seems to show a better data trend, and is subject to less scatter than those associated with the other procedures considered in this study.

Notation

The following symbols are used in this paper:

b = channel width;

d = sediment particle diameter;

f_b = bed-related friction factor;

$f_w = 8gR_w S/V^2$ = wall-related friction factor;

g = gravitational acceleration;

h = flow depth;

k_{sb} = bed roughness size;

k_{sw} = smooth-wall roughness size taken as 0.0015 mm;

n = total Manning roughness coefficient;

n_b = bed-related Manning roughness coefficient;

n_w = wall-related roughness coefficient;

$R = bh/(b + 2h)$ = hydraulic radius;

R_w = wall-related hydraulic radius;

$$Re = 4VR/\nu;$$

S = energy slope;

V = cross-sectional average velocity;

α = coefficient;

β = coefficient;

$\theta = \tau_b/[(\rho_s - \rho)gd]$ = dimensionless bed shear stress;

$\theta_E = \tau_E /[(\rho_s - \rho)gd]$;

$\theta_P = \tau_P /[(\rho_s - \rho)gd]$;

$\theta_R = \rho gRS /[(\rho_s - \rho)gd]$;

$\theta_{VB} = \tau_{VB} /[(\rho_s - \rho)gd]$;

$\theta_W = \tau_W /[(\rho_s - \rho)gd]$;

ν = kinematic viscosity of fluid;

ρ = fluid density;

ρ_s = particle density;

τ_b = average bed shear stress;

τ_E = bed shear stress corrected using the Einstein formula;

τ_P = bed shear stress corrected using the present model;

τ_{VB} = bed shear stress corrected with the Vanoni and Brooks formula;

τ_W = bed shear stress corrected using the Williams formula; and

ϕ = dimensionless transport rate.

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Table 1. Experimental data used for computations.

Source	Channel width $b(\text{m})$	Flow depth $h(\text{m})$	Energy slope S	Average velocity $V(\text{m/s})$	Re $=VR/\nu$	Fr $=V/(gh)^{0.5}$	Sediment size $d(\text{mm})$
Gilbert (1914)	0.07-0.60	0.02-0.12	0.0039-0.0275	0.49-1.42	$(1.1-6.9)\times 10^4$	0.8-2.1	0.31-4.94
Guy et al. (1966)	0.61-2.44	0.09-0.31	0.0003-0.0079	0.40-1.62	$(5.1-23.0)\times 10^4$	0.2-1.6	0.19-0.93
Williams (1970)	0.08-0.61	0.03-0.16	0.0011-0.0367	0.33-2.35	$(0.7-11.0)\times 10^4$	0.4-3.5	1.35

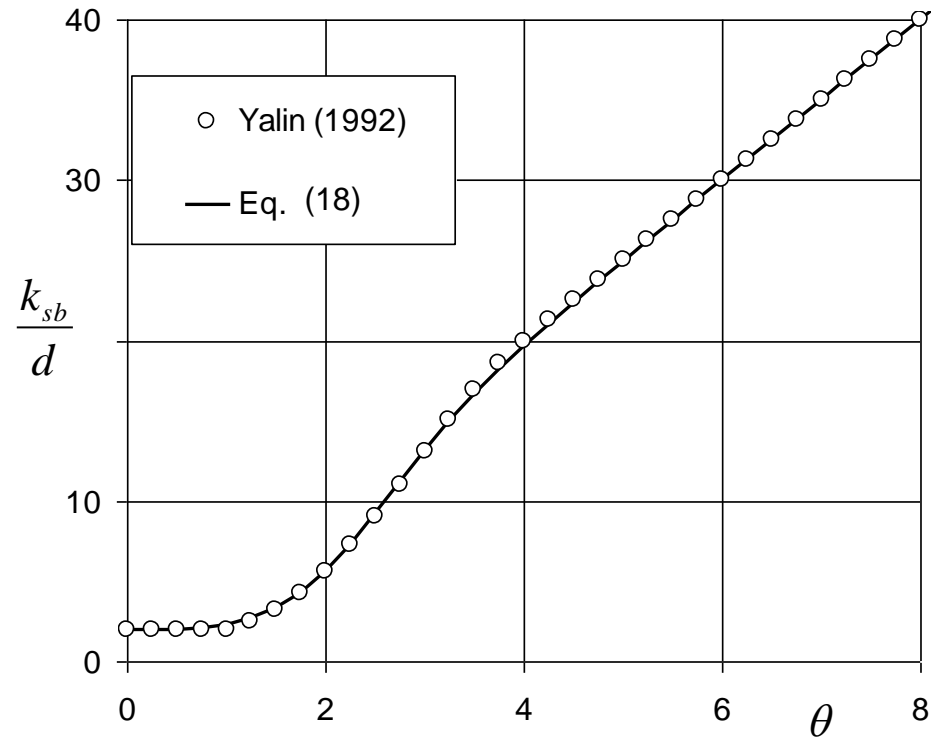


Fig. 1. Variation of equivalent roughness size with dimensionless bed shear stress.

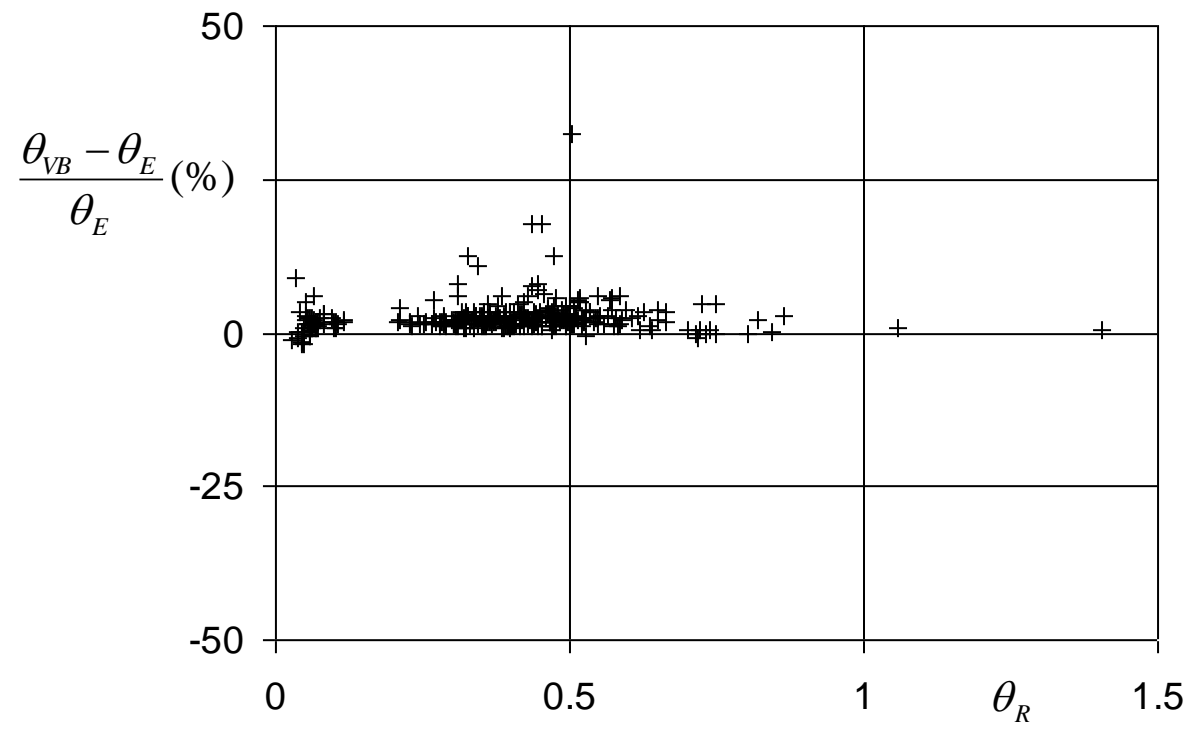


Fig. 2. Comparison of shear stresses corrected using formulas presented by Einstein (1941) and Vanoni and Brooks (1957).

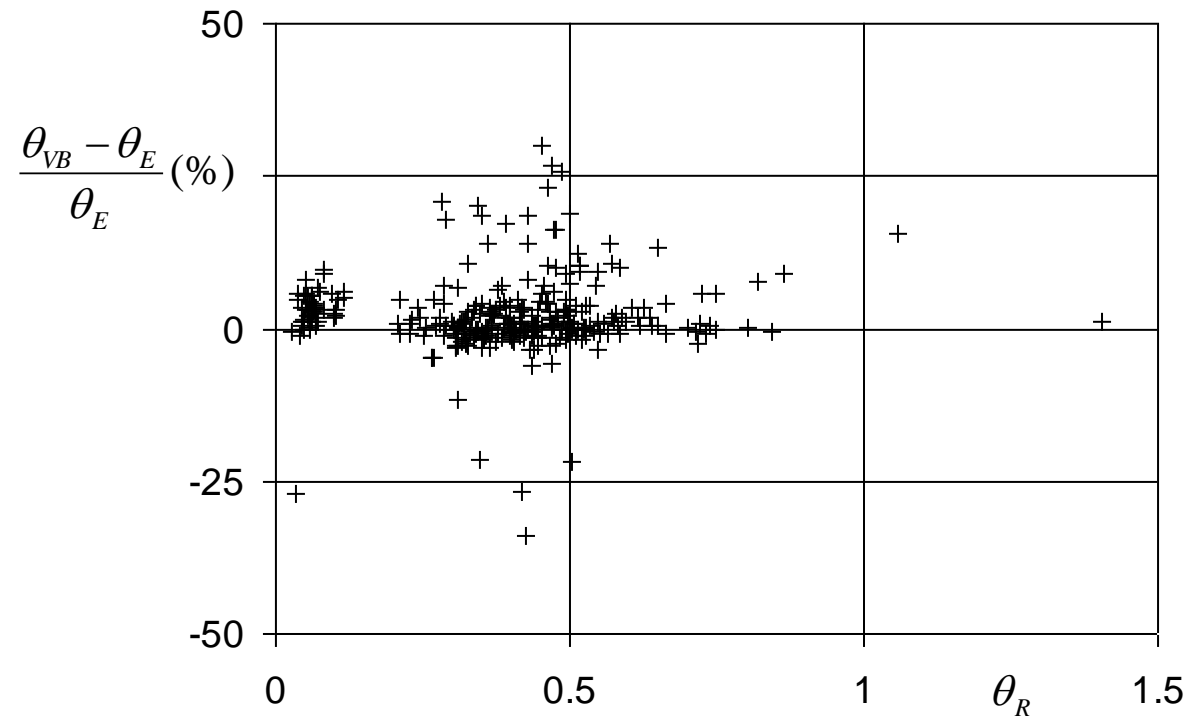


Fig. 3. Comparison of shear stresses corrected using Vanoni and Brooks formula and present model.

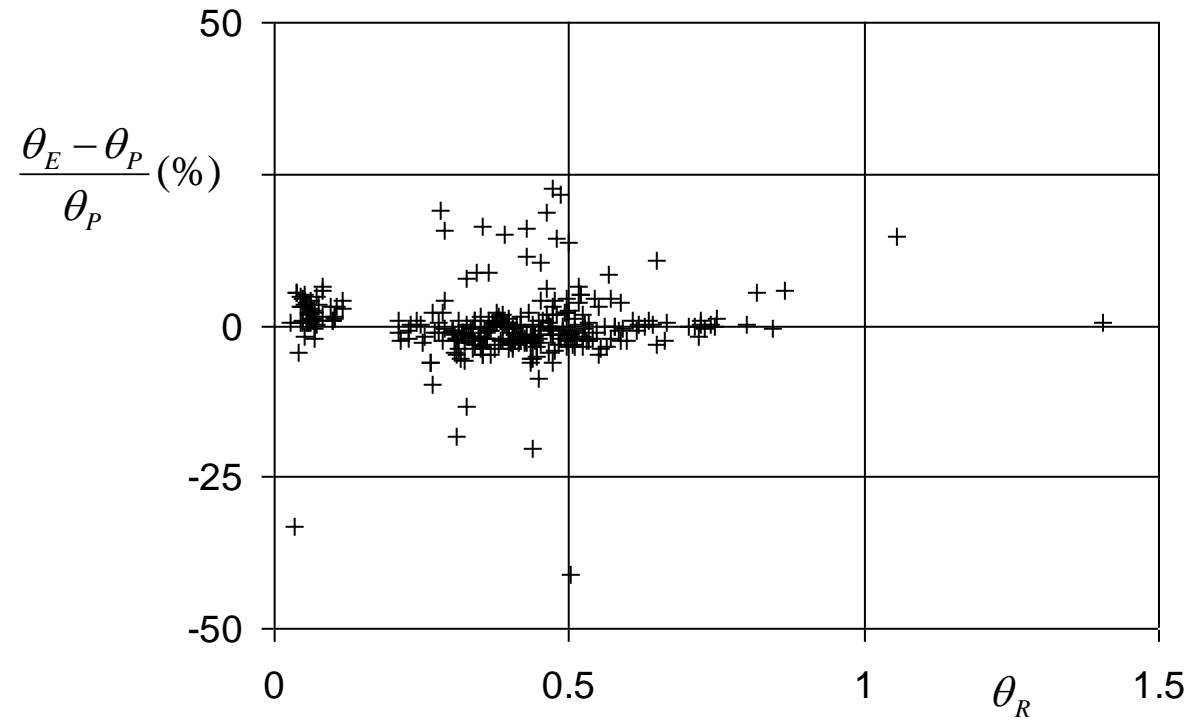


Fig. 4. Comparison of shear stresses corrected using Einstein formula and present model.

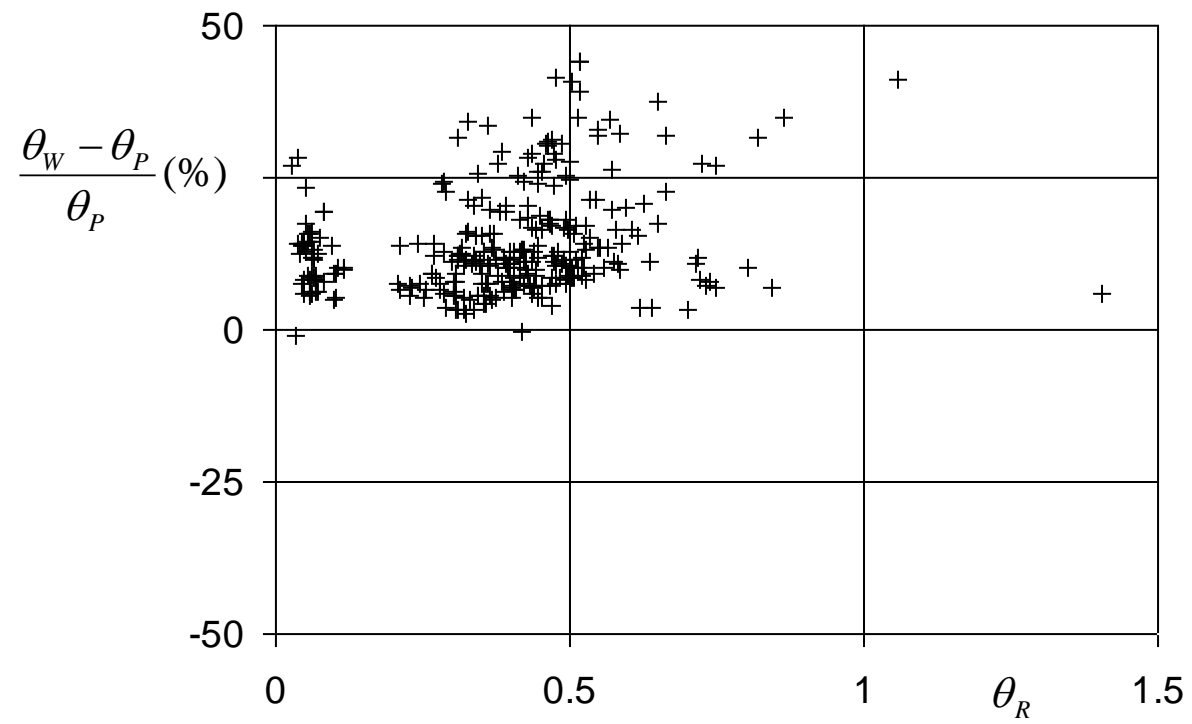


Fig. 5. Comparison of shear stresses corrected using Williams formula and present model.

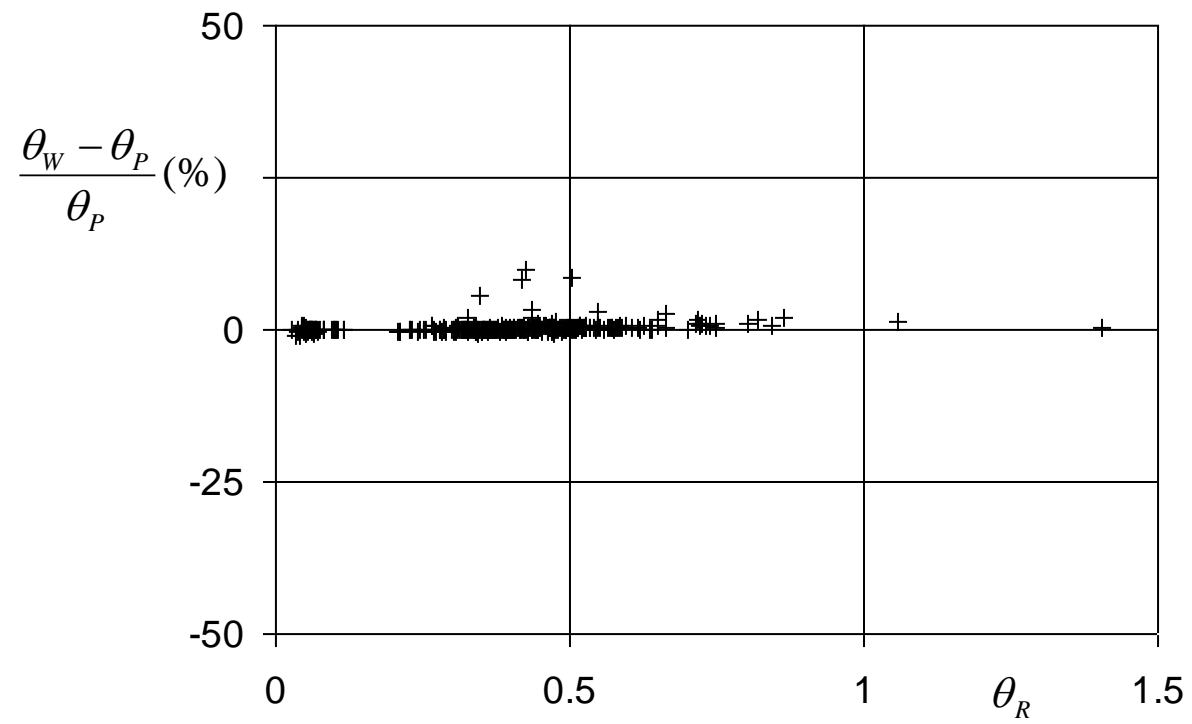


Fig. 6. Comparison of shear stresses corrected using Vanoni and Brooks formula and simplified approach [Eq. (7)].

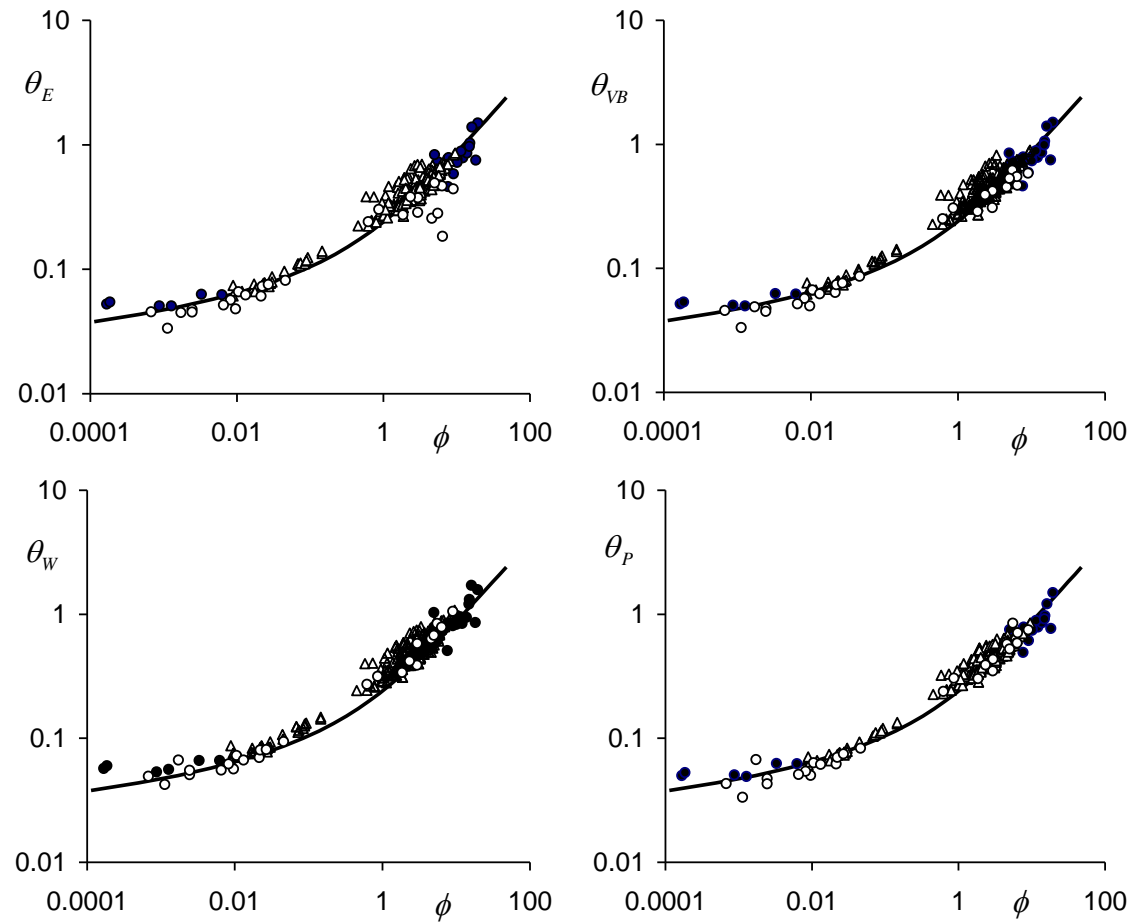


Fig. 7. Bedload functions based on shear stresses corrected using previous formulas and present model. The data used are from Gilbert (1914) (denoted by Δ), Guy et al. (1966) (denoted by \bullet), and Williams (1970) (denoted by \circ). The solid line represents the bedload formula given by Cheng (2002).