Probabilistic seismic loss estimation by stochastic simulation algorithms

Cheung, Sai Hung; Bansal, Sahil

2013

Cheung, S. H., & Bansal, S. (2014). Probabilistic seismic loss estimation by stochastic simulation algorithms. *Safety, Reliability, Risk and Life-Cycle Performance of Structures and Infrastructures* (pp. 4107-4113): CRC Press.

<https://hdl.handle.net/10356/79595>

<https://doi.org/10.1201/b16387-596>

© 2013 Taylor & Francis Group, London. This is the author created version of a work that has been peer reviewed and accepted for publication by *Safety, Reliability, Risk and Life-Cycle Performance of Structures and Infrastructures*, Taylor and Francis Group. It incorporates referee's comments but changes resulting from the publishing process, such as copyediting, structural formatting, may not be reflected in this document. The published version is available at: [<http://dx.doi.org/10.1201/b16387-596>].

Downloaded on 26 Apr 2023 22:05:20 SGT

Probabilistic Seismic Loss Estimation by Stochastic Simulation Algorithms

S.H. Cheung & S. Bansal

School of Civil & Environmental Engineering, Nanyang Technological University, Singapore

ABSTRACT: Prior estimation of losses in a structure due to future uncertain earthquakes, especially those which can lead to disastrous consequences, is essential to reduce potential losses and assist recovery. In this paper, we are particularly interested in the evaluation of seismic loss probability distribution including its tail by evaluating the seismic loss exceedance probability. Taking into account uncertainties from structural modeling, the modeling of future uncertain excitation and the models for damage and loss analysis, the evaluation of the exceedance probability will unavoidably involve the computation of high-dimensional integrals. A new method based on stochastic simulation algorithms is proposed to compute such integrals more efficiently. The effectiveness and efficiency of the proposed method are shown by an illustrative example involving a multi-storey inelastic building.

1 INTRODUCTION

Prior estimation of losses in a structure due to future uncertain earthquakes, especially those which can lead to disastrous consequences, is essential to reduce potential losses and assist recovery. Taking into account uncertainties from the modeling of the uncertain future excitation, structural modeling, and the models for damage and loss analysis, the seismic loss analysis will unavoidably involve the computation of high-dimensional integrals. The Pacific Earthquake Engineering Center (PEER) has developed a modular approach for PBEE, presented in e.g., Porter (2003) and Moehle and Deierlein (2004), that involves four fundamental steps: hazard analysis, response analysis, damage assessment, and loss evaluation. The mapping between structural response and economic loss can be done using, e.g., the Assembly-based vulnerability framework proposed by Porter *et al.* (2001). In most applications of the above framework, since only a small suite of scaled, ground motion recordings is used to compute the structural response quantities, the more complete probabilistic information of the performance of the system (especially the tail parts of the performance probability distribution function, PDF) is not obtained. The framework falls short of accurately considering small exceedance probability (e.g., <0.1) that characterizes rare events which can lead to significant consequences.

A new method based on stochastic simulation algorithms is proposed for the estimation of seismic

loss probability distribution function with the adoption of stochastic ground motion models coupled with nonlinear stochastic dynamic models, probabilistic damage assessment and loss evaluation models. A stochastic simulation based reliability analysis is performed where the failure is defined as unsatisfactory performance of the system, i.e., the exceedance of decision variables over a threshold.

The proposed approach involves the modification of the simulation algorithms in Subset Simulation proposed by Au and Beck (2001) and the development of new estimators to tackle the estimation problem of seismic loss probability distribution function including its tail. The conditional failure probability in each level can be estimated making use of samples of the decision variables drawn from a properly selected conditional probability distribution based on conditional failure samples. A family of specially designed stochastic simulation procedures is proposed to simulate these samples which can reduce the variance of the exceedance probability estimators without requiring additional dynamic analyses. The output of the proposed method is the exceedance probability of loss at various threshold levels, i.e., the complementary cumulative distribution function (CDF) of loss as a function of threshold. The approach is robust to the number of random variables including both continuous and discrete ones due to stochastic modeling of ground motion and a large number of combinations of discrete damage measures (and corresponding loss functions) that describe the condition of the structure. Perfor-

mance assessment of an existing reinforced-concrete (RC) seven-story hotel located in Van Nuys, California is considered to show the application of the proposed approach.

2 RELIABILITY ANALYSIS FOR LOSS ESTIMATION

To evaluate the performance of the system for any possible future event, different threshold levels of performance quantity of interest need to be considered and the corresponding exceedance probabilities need to be estimated. In this study, the aim is to obtain the PDF of the *Loss* including the tail part due to the combinations of scenarios which can lead to extreme consequences. *Loss* may refer to dollar loss (repair and restoration cost), loss in functionality (downtime), or human loss (casualties) etc. Let L denote any *Loss* quantity. To estimate the probability distribution of L including its tail, the following integral as a function of c should be evaluated:

$$P_F(c) = \int P(L > c | \theta_g, \mathbf{Z}, \theta_s) p(\theta_g) p(\mathbf{Z}) p(\theta_s) dL d\theta_g d\mathbf{Z} d\theta_s \quad (1)$$

where θ_g is the stochastic ground motion model parameters vector, $\mathbf{Z}$ is the stochastic excitation vector (white noise), and θ_s is the structural model parameters vector. It should be noted that in addition to accounting for ground motion uncertainty, uncertainty resulting from, the size and location of the earthquake, and the structural parameters are also directly introduced. The PDF of any loss for a scenario event (which will still correspond to many possible ground motions) can be obtained by fixing θ_g . In Equation (1), $p(L | \theta_g, \mathbf{Z}, \theta_s)$ is equivalent to $p(L | \mathbf{EDP})$ where **EDP** denotes the engineering demand parameter vector (e.g., maximum interstory drifts, peak floor accelerations). There is a one-to-one mapping from $[\theta_g, \mathbf{Z}, \theta_s]$ to **EDP**, as for every unique set of input parameters a unique structural model output is obtained during the structural dynamic analysis. The PDF of L conditioned on $[\theta_g, \mathbf{Z}, \theta_s]$ is given by:

$$p(L | \theta_g, \mathbf{Z}, \theta_s) \equiv p(L | \mathbf{EDP}) = \sum_{\forall \mathbf{DM}} p(L | \mathbf{DM} = \mathbf{DM}) p(\mathbf{DM} = \mathbf{DM} | \mathbf{EDP}) \quad (2)$$

$$p(\mathbf{DM} = \mathbf{DM} | \mathbf{EDP}) = p(\bigcap_{j=1}^{N_c} \{DM_j = dm_j\} | \mathbf{EDP}) \quad (3)$$

$$p(L | \mathbf{DM} = \mathbf{DM}) = p(\sum_{j=1}^{N_c} L_j | DM_j = dm_j) \quad (4)$$

$$p(L | \mathbf{DM} = \mathbf{DM}) = p(g(L_1, \dots, L_{N_c}) | \mathbf{DM} = \mathbf{DM}) \quad (5)$$

where $\mathbf{DM} = [DM_1, DM_2, \dots, DM_{N_c}]$ is a damage state random vector and $dm_j \in \{1, 2, \dots, nds_j\}$; N_c is the total number of damageable components and nds_j is the total number of discrete damage states for the j -th component. There are $nds_1 \cdot nds_2 \cdot \dots \cdot nds_{N_c}$ possible

realizations of **DM** and the PDF of total loss is given by the summation over each possible realization **dm**. $p(DM_j = dm_j | \mathbf{EDP})$ can be obtained using the fragility functions for the j -th component (fragility functions are probability distributions that are used to indicate the probability that a component will be damaged to a given or more severe damage state as a function of demand parameters). $p(L_j | DM_j = dm_j)$ is the PDF for j -th component in damage state dm_j and is defined using a loss function (that gives the probability of occurrence of a certain level of loss when a certain damage state has occurred in the component). Depending on the type of loss quantity, $p(L | \mathbf{DM} = \mathbf{DM})$ can take the form in Equation (4) or a general form as in Equation (5). Uncertainty in parameters that define the fragility and loss functions can also be incorporated in the above equations; however it is not discussed in the present study. Loss is not uncorrelated to the seismic parameters or structural parameters that can be modeled as independent variable. For clarity in presentation, all uncertain independent variables will now be referred by $\theta = [\theta_g, \mathbf{Z}, \theta_s]$.

It can be expected that the dimension of the integral in Equation (1) is high due to a large number of random variables involved and the failure region has complicated geometry. To evaluate the integral in Equation (1), a new stochastic simulation based approach for the estimation of seismic loss PDF (including its tail) is presented in the following section.

3 PROPOSED APPROACH

Au and Beck (2001, 2003) proposed a very robust algorithm, Subset Simulation (SS), for computing small failure probabilities of dynamic systems. The application of SS for efficiently computing small failure probabilities encountered in seismic risk problems involving structural dynamic analysis can be found in Au and Beck (2003). The basic idea of subset simulation is to consider a sequence of failure events $F_1 \supset F_2 \supset \dots \supset F_m = F$ (one being the subset of another) converting a rare-event problem into a problem with a sequence of more frequent events. This enables the computation of the small failure probability as a product of conditional probabilities $P(F_{i+1} | F_i)$, $i=0, 1, \dots, m$ where $P(F_1 | F_0) = P(F_1)$ and the target failure probability is given by the last element in the sequence:

$$P_F = P(F_m) = \left[\prod_{i=1}^{m-1} P(F_{i+1} | F_i) \right] P(F_1) \quad (6)$$

The intermediate thresholds c_i , where $c_1 < c_2 < \dots < c_m$ are chosen ‘‘adaptively’’ so that the conditional PDFs $P(F_{i+1} | F_i)$ are approximately equal to some specified value, p_0 . In SS, it is assumed that (a) $P(F_{i+1} | F_i)$ and $P(F_1)$ are evaluated by considering the

integrals as shown in Equation (8)-(9) where $I_{F_i} = 1$ if $\theta \in F_i$ and otherwise $I_{F_i} = 0$. The conditional failure probability $P(F_{i+1}|F_i)$ is estimated by the sample average of $I_{F_{i+1}}(\theta)$ which is just the fraction of number of samples following the distribution $p(\theta|F_i)$ that lead to failure F_{i+1} ; (b) $p(\theta)$ can be written as a product of marginal PDFs of its component groups of random variables, i.e., $p(\theta) = p(\theta^{(1)}) p(\theta^{(2)}) \dots p(\theta^{(G)})$ where $\theta = [\theta^{(1)}, \theta^{(2)}, \dots, \theta^{(G)}]$. According to this, the transition of individual groups are independent, so the transition PDF of the Markov chain between two states in F can be expressed as a product of the group transition PDFs as shown in Equation (7).

$$P(F_1) = \int I_{F_1}(\theta) p(\theta) d\theta \quad (8)$$

$$P(F_{i+1} | F_i) = \int I_{F_{i+1}}(\theta) p(F_i | \theta) d\theta \quad (9)$$

$$p(\theta_{k+1} | \theta_k) = \prod_{j=1}^G p_j(\theta_{k+1}^{(j)} | \theta_k^{(j)}) \quad (10)$$

The modified subset simulation based method proposed in this paper for loss probability distribution estimation considers the same “sequence of failure events” as in the original SS to compute the target failure probability, with several important differences as described as follows. The following integrals are first considered:

$$P(F_1) = \int P(F_1 | \theta) p(\theta) d\theta = \int P(L > c_1 | \theta) p(\theta) d\theta \quad (11)$$

In the above equation (or Equation (1)) failure is still uncertain given θ , and is certain if L is known. The PDF of L as a function of θ is given by Equations (2)-(5). For this problem, to estimate $P(F_1)$ in Equation (6), the failure probability estimator similar to that in standard Monte Carlo Simulation (MCS) can be used:

$$P(F_1) \approx \tilde{P}_1 = \frac{1}{N} \sum_{k=1}^N \frac{1}{M} \sum_{j=1}^M I_{F_1}(L_{kj}) \quad (12)$$

where L_{kj} : $j=1, \dots, M$ are independently and identically distributed (i.i.d.) samples according to PDF $p(L|\theta_k)$ and θ_k , $k=1, \dots, N$, are i.i.d. samples according to the PDF $p(\theta)$. Similarly, $P(F_{i+1}|F_i)$ in Equation (6) can be estimated as:

$$P(F_{i+1} | F_i) \approx \tilde{P}_{i+1} = \frac{1}{N} \sum_{k=1}^N \frac{1}{M} \sum_{j=1}^M I_{F_{i+1}}(L_{kj}^{(i)}) \quad (13)$$

where samples $L_{kj}^{(i)}$: $k=1, \dots, N$; $j=1, \dots, M$ distributed according to the conditional PDF $p(L|\theta_k, F_i)$ and θ_k , $k=1, \dots, N$, distributed according to $p(\theta|F_i)$ are simulated by the stochastic sampling procedures proposed in sections 4 and 5. The proposed stochastic sampling technique does not assume that the transition of individual groups is independent. The output

of the proposed method is the exceedance probability of loss at various threshold levels.

It should be noted that the case with $M=1$ may look more natural to most of the readers as each θ sample will generate one L sample. However, the advantage of having $M>1$ is that the estimates of the exceedance probabilities has smaller variance than for the case with $M=1$. The increase in the computation cost due to $M>1$ is negligible because this does not need any additional dynamic analysis. Special case with $M=1$ is presented in Cheung and Bansal (2012). For more details about the proposed algorithm including its correctness, the readers are encouraged to refer to the journal version of this paper.

4 CONDITIONAL SAMPLES

Samples distributed according to PDF $p(L|F_i)$ cannot be simulated directly, therefore a Markov Chain Monte Carlo (MCMC) simulation based technique is proposed to generate $L_{kj}^{(i)}$: $k=1, \dots, N$; $j=1, \dots, M$ samples using those that lie in F_i available from the previous simulation level, F_{i-1} . The stationary distribution of the Markov chain is equal to:

$$p(L, \mathbf{DM}, \theta | F_i) = \frac{P(F_i | L, \mathbf{DM}, \theta) p(L, \mathbf{DM}, \theta)}{P(F_i)} \quad (14)$$

$$\propto I_{F_i}(L) p(L | \mathbf{DM} = \mathbf{dm}) p(\mathbf{DM} = \mathbf{dm} | \theta) p(\theta)$$

According the above set of equations, sample distributed according to the conditional PDF $p(L, \mathbf{DM}, \theta | F_i)$ can be generated by starting Markov chains from the samples distributed according to the conditional PDF $p(\theta | F_i)$. Seed samples for the Markov chains distributed according to the conditional PDF $p(\theta | F_i)$ are obtained by propagating $p_0 N \theta$ samples from F_{i-1} to F_i according to the following weights:

$$\theta_u^{(i)} = \theta_k^{(i-1)} \text{ w.p. } \frac{1}{p_0 N M} \sum_{j=1}^M I_{F_i}(L_{kj}^{(i-1)}) \quad \begin{matrix} u = 1, \dots, N \times p_0 \\ k = 1, \dots, N \end{matrix} \quad (15)$$

It is possible that some θ samples are selected multiple times. In that case the starting point of Markov chain for the repeated sample will be the last sample of the previous chain generated using the same sample or one can have the number of Markov chains generated, each with the starting point being the same θ sample as many as the number of times it is being selected.

Given a sample θ_k distributed according to the conditional PDF $p(\theta | F_i)$, a new sample θ_{k+1} also distributed according to the conditional PDF $p(\theta | F_i)$ is generated by first simulating a candidate $\tilde{\theta}_{k+1}$ sample using Metropolis-Hastings algorithm as described later. Using $\tilde{\theta}_{k+1}$, a candidate damage state random vector $\tilde{\mathbf{DM}}_{k+1}$ is simulated according to Equation (3), and conditioned on this, a candidate loss sample $\tilde{L}_{k+1}$

is simulated according to Equation (4) or (16). At last, if the simulated $\tilde{L}_{k+1}$ falls in the failure domain F_i (i.e., $L > c_i$), the candidate sample $\tilde{\theta}_{k+1}$ is accepted as the next sample. If not, the current sample θ_k is taken as the next sample.

Given θ samples with PDF $p(\theta|F_i)$, a large number of loss samples distributed according to PDF $p(L|\theta_k, F_i)$ can be obtained by first simulating $\{L, \mathbf{DM}\}$ 4.1 samples according to $p(L, \mathbf{DM}|\theta_k)$ (using Equations (3)-(5)) and then accepting only those that fall in F_i . The resulting L samples will be distributed according to $p(L|\theta_k, F_i)$. For more details about the procedures including the proof for their correctness, the readers are encouraged to refer to the journal version of this paper.

4.1 Summary of Steps for Simulating Samples

The steps for the proposed stochastic simulation procedures for generating samples with limiting stationary PDF equal to $p(L|F_i)$ are as follows:

1. Given a current sample θ_k with distribution PDF $p(\theta|F_i)$, generate a new candidate sample $\tilde{\theta}_{k+1}$ using the Metropolis Hastings algorithm (described below).
2. Given $\tilde{\theta}_{k+1}$ generate a loss sample $\tilde{L}_{k+1}$ from the conditional PDF $p(\tilde{L}|\tilde{\theta}_{k+1})$ (Equations (2)-(5)).
3. Set $\theta_{k+1} = \begin{cases} \tilde{\theta}_{k+1} & \text{if } \tilde{L}_{k+1} > c_i \\ \theta_k & \text{otherwise} \end{cases}$
4. Given θ_{k+1} , generate $L_{k+1,j}^{(i)}$ samples by repeating the following for $j=1, \dots, M$
 - i) Generate $\tilde{L}_{k+1,j}$ from PDF $p(\tilde{L}|\theta_{k+1})$.
 - ii) Accept $L_{k+1,j} = \tilde{L}_{k+1,j}$ if $\tilde{L}_{k+1,j} > c_i$, else return to step (i).

Assuming θ is divided into G independent groups, i.e., $\theta = [\theta^{(1)}, \theta^{(2)}, \dots, \theta^{(G)}]$. Given a current sample θ_k , a candidate $\tilde{\theta}_{k+1}$ sample from the PDF $p(\theta)$ is generated using the Metropolis Hastings algorithm by repeating the following steps from $j=1$ till $j=G$:

1. For the j -th group, draw a pre-candidate component ξ_{k+1}^j from the proposal PDF q^j and compute the acceptance ratio r_{k+1}^j

$$r_{k+1}^j = \min \left\{ 1, \frac{p(\xi_{k+1}^j) q^j(\theta_k^j | \xi_{k+1}^j)}{p(\theta_k^j) q^j(\xi_{k+1}^j | \theta_k^j)} \right\}$$

2. Set $\tilde{\theta}_{k+1}^j = \begin{cases} \xi_{k+1}^j & \text{with probability } \min(1, r_{k+1}^j) \\ \theta_k^j & \text{with probability } 1 - \min(1, r_{k+1}^j) \end{cases}$

5 ILLUSTRATIVE EXAMPLE

The example used in this study is the hotel structure located in Van Nuys. This building is a seven-story reinforced concrete (RC) structure that was severely damaged during the 1994 Northridge Earthquake. In this study, it is assumed that the building is in its original condition prior to the occurrence of the Northridge earthquake. The stochastic ground motion records are generated by adopting a point-source model characterized by the moment magnitude Mag and the epicentral distance r . For illustration and convenience, uncertainty in structural parameters is not considered at this point and a simplified nonlinear degradation structural model (Ching et al., 2006) is used for structural dynamic analysis. Economic loss (building repair cost) because of earthquake induced damage to drift sensitive structural and non structural components is considered as a risk related decision variables. Seismic hazard with fixed Mag and r is considered ($Mag=6.5$ and $r=20$).

5.1 Stochastic ground motion model

The total radiation Fourier spectrum developed for California seismicity, and modified for soil site is used here for illustration.

$$A(f; Mag, r) = A_0(f) V(f) G(f) \exp(-\gamma(f) R) \exp(-\pi f \kappa) \quad (17)$$

where A_0 is the ‘equivalent point-source spectrum’ based on two corner frequencies (Atkinson and Silva, 2000) defined at a unit distance described by

$$A(f; Mag, r) = C(2\pi f)^2 M_0 \left\{ \frac{1 - \varepsilon}{1 + (f/f_a)^2} + \frac{\varepsilon}{1 + (f/f_b)^2} \right\} \quad (18)$$

The constant C is given by $C = RVF_s / (4\pi R_0 \rho \beta^3)$, where $R=0.55$ is the radiation pattern, $V=1/\sqrt{2}$ represent the partition of the total shear wave velocity onto two horizontal components, $F_s=2$ is the free surface amplification $R_0=1\text{km}$ is a reference distance, and ρ and β are the density and shear-wave velocity, in the vicinity of the earthquake source assumed equal to 2.8 g/cm^3 and 3.5 km/s , respectively; M_0 is the seismic moment; f_a and f_b , are the lower and higher corner frequencies are related to the size of the finite fault and subfault size, and ε is a relative weighing parameter.

$$\begin{aligned} M_0 &= 10^{1.5(Mag+10.7)} \\ f_a &= 10^{2.181-0.496M} \\ f_b &= 10^{2.410-0.408M} \\ \varepsilon &= 10^{0.605-0.255M} \end{aligned} \quad (19)$$

In Equation (17), V describes the amplification factor for the crustal velocity gradient, G is the geometric spreading factor, γ is the anelastic attenuation factor, κ is the near-surface attenuation factor, and $R = \sqrt{(r^2 + h^2)}$, $h = 10^{-0.05 + 0.15Mag}$ is the ‘equivalent point-source depth’.

Non-stationarity in the ground motion amplitude is achieved by using an empirical window function $e(t; Mag, r)$, which also depends on Mag and r (Boore, 2003).

$$e(t; Mag, r) = a(t/t_\eta)^b \exp(-ct/t_\eta) \quad (20)$$

where parameters a , b , and c are determined such that $e(t) = 1$ when $t = \varepsilon \times t_\eta$ and $e(t) = \eta$ when $t = t_\eta$. The time t_η is given by $t_\eta = 2T_w$ where T_w is the duration of the ground motion. The parameters ε and η are taken to be $\varepsilon = 0.2$ and $\eta = 0.05$.

$$\begin{aligned} b &= -\varepsilon \ln \eta / (1 + \varepsilon (\ln \varepsilon - 1)) \\ c &= b / \varepsilon \\ a &= (\exp(1) / \varepsilon)^b \\ T_w &= 1 / 2 f_a + 0.05R \end{aligned} \quad (21)$$

To obtain a sample ground motion record $u_g(t)$ for a given scenario event, first a discrete-time white noise sequence $\mathbf{Z}$ with unit spectral intensity for the sampling interval Δt is generated. The noise is then windowed by multiplying it by an envelope function $e(t; Mag, r)$. The windowed noise is transformed into frequency domain and the spectrum is normalized by the square-root of the mean square amplitude spectrum. The resulting spectrum is multiplied by the point-source spectrum $A(f; Mag, r)$ which is then transformed back to the time domain to yield a sample of the ground acceleration time history. The synthetic ground motion $u_g(t)$ generated from the model is thus a function of the additive excitation parameters $\mathbf{Z}$, and model parameters Mag and r (Boore, 2003, Rezaeian and Der Kiureghian, 2008).

5.2 Simplified Structural Model

Ching et al. (2006), described in detail the procedure for developing a simplified nonlinear degradation model that was able to capture the E-W responses of the Van Nuys hotel building. It was a seven-DOF lumped-mass shear building model and was able to produce relationships for inter-story restoring force versus inter-story drift similar to those produced by the FE model (Beck, 2002). With the lumped mass assumption, the stiffnesses and damping parameters are given by:

$$k_{i,t} = k_{i,0} e^{-\eta d_{i,t}^2} \quad (22)$$

$$c_{i,t} = \delta + \lambda \mu_{i,t} \quad (23)$$

where $\mu_{i,t}$ is the interstory drift ratio at time t (for i -th story, with story height H_i), and is given by:

$$\mu_{i,t} = \max_{0 \leq k \leq t} (|x_{i,k}| / H_i) \quad (24)$$

$$\mu_{i,t} = \max_{0 \leq k \leq t} (|x_{i,k} - x_{i-1,k}| / H_i) \quad (25)$$

5.3 Fragility and Loss Functions

Loss due to earthquake induced damage to drift sensitive structural and nonstructural components is considered here as the performance function. The mapping between **EDP** and economic loss adopted here follows the ABV framework (Porter et al., 2001). Peak interstory drift ratio (IDR) at each story is used as the **EDP**. For illustration and convenience, it is assumed in this example that (a) once an individual structural component has experienced extensive damage (loss of vertical carrying capacity), a local or global collapse mode has occurred in the structure that triggers the need to replace the building, which is highly conservative (b) the damage experienced by different components for a given **EDP** vector are statically independent. Component specific fragility functions are defined using the lognormal distribution and their statistical parameters, logarithmic mean θ and logarithmic standard deviation β , are taken from (Aslani and Miranda, 2005). Given the fragility functions, the probability of j -th component being in a damage state k (i.e., $DM_j = k$) is given by

$$\begin{aligned} P(DM_j = k | \mathbf{EDP}) \\ = \Phi\left(\frac{\ln(\mathbf{EDP}) - \theta_{j,k}}{\beta_{j,k}}\right) - \Phi\left(\frac{\ln(\mathbf{EDP}) - \theta_{j,k+1}}{\beta_{j,k+1}}\right) \end{aligned} \quad (26)$$

where Φ is standard normal CDF. The direct economic loss, L is calculated by summing the economic loss for all components conditioned on no local/global collapse and is equal to the replacement cost of the building in case of local/global collapse. Economic loss functions follow a lognormal distribution with logarithmic mean μ and logarithmic standard deviation σ (Aslani and Miranda, 2005) and the CDF when the j -th component is in damage state k is given by

$$P(L_j \leq l | DM_j = k) = \Phi\left(\frac{\ln(l) - \mu_{j,k}}{\sigma_{j,k}}\right) \quad (27)$$

Samples of L distributed according to $p(L|Mag, R, \mathbf{Z})$ can be generated by MCS by first simulating a random damage state for each component given the **EDP** using Equation (3), and then given the damage state, random economic loss sample for each component is generated using Equation (4). Finally the sum of economic losses in individual components

gives a sample of L distributed according to $p(L|Mag,R,\mathbf{Z})$, if the sum is less than the replacement cost, else the sample is set equal to the building replacement cost.

5.4 Simulation Parameters

A discrete-time white noise sequence $\mathbf{Z}$ with corresponding to duration of input ground motion $T=100\text{sec}$ and sampling interval $\Delta t=0.01\text{sec}$ is considered. To obtain the estimate of exceedance probability the proposed method is applied with a conditional failure probability at each level approximately equal to $p_0=0.1$ and with the number of samples set to $N=500$ at each conditional level. One-dimensional chain adaptive symmetric uniform distribution is adopted as proposal for each additive excitation random variable. For each analysis (or set of **EDPs**) 1000 damage state vectors are simulated and for each damage state vector a single loss sample is simulated. Therefore, in total there are 500,000 loss samples at each conditional level. Results are also obtained for $M=1$ for comparison.

5.5 Results

Fig. 1. Mean estimates of the exceedance probability estimator

Figure 1 shows the mean estimates of the exceedance probability estimator for different threshold levels of economic loss normalized by the replacement cost of the building. The mean of the loss exceedance probability estimator is estimated using 50 independent simulation runs. Results computed using 25,000 MCS computed samples are shown for comparison. As expected, it is observed that the proposed exceedance probability estimator with $M=1$ (S1) and $M=1000$ (S2) is practically unbiased.

The sample c.o.v. for the exceedance probability estimator estimated using 50 independent simulation runs is shown in Figure 2. The sample c.o.v. of the proposed loss exceedance probability estimator using $M=1000$ is much smaller than the case using $M=1$, which is much smaller than the MCS estimator, showing a substantial improvement in the quality of the estimator when M increases and a significant improvement in computational efficiency.

Fig. 2. Sample c.o.v. for the exceedance probability estimator

6 CONCLUSION

A new stochastic simulation based approach for the estimation of seismic loss probability function is proposed with the adoption of stochastic ground motion model. Exceedance probabilities at various loss thresholds are obtained using the proposed method that efficiently computes small failure probabilities. The proposed approach involves the modification of the simulation algorithms in Subset Simulation and the development of new estimators to tackle the estimation problem of seismic loss probability distribution function including its tail. A family of specially designed stochastic simulation procedures is proposed to simulate the required samples which can reduce the variance of the exceedance probability estimators without requiring additional dynamic analyses. The approach is robust to the number of random variables and is directly applicable even when considering both seismic uncertainty and other modeling uncertainty including structural modeling uncertainty, damage and loss modeling uncertainty. It is observed that significant reduction in the c.o.v. of the estimates can be obtained using the proposed method if more samples in part of the problem can be simulated without necessarily increasing the computation cost. An example was presented to show the applicability, effectiveness and efficiency of the proposed approach.

The quality of the prediction of course depends on (a) the stochastic ground motion model to provide realistic description of the characteristic of the ground motions expected to happen at the specific site; (b) modeling of the physical system and collapse; (c) the quality of fragility functions and loss functions for structural and nonstructural components.

REFERENCES

- Aslani, H. and Miranda, E., 2005. Probabilistic earthquake loss estimation and loss disaggregation in buildings. Stanford, John A. Blume Earthquake Engineering Center.
- Atkinson, G. M. and Silva, W., 2000. Stochastic modeling of California ground motions. *Bulletin of the Seismological Society of America*, 90, 255.
- Au, S. and Beck, J. L., 2001. Estimation of small failure probabilities in high dimensions by subset simulation. *Probabilistic Engineering Mechanics*, 16, 263-277.
- Au, S. and Beck, J. L., 2003. Subset simulation and its application to seismic risk based on dynamic analysis. *Journal of Engineering Mechanics*, 129, 901.
- Beck, J. L., Et Al., 2002. Impact of seismic risk on lifetime property values. *Final Rep., CUREe-Kajima Joint Research Program Phase IV, Consortium of Universities for Research in Earthquake Engineering, Richmond, Calif.* <http://resolver.caltech.edu/CaltechEERL:2002.EERL-2002-04>.
- Boore, D. M., 2003. Simulation of ground motion using the stochastic method. *Pure and Applied Geophysics*, 160, 635-676.
- Cheung, S. H. and Bansal, S., 2012. A Stochastic Simulation Based Approach for Seismic Loss Analysis and Probability Function Computation. *15th World Conference on Earthquake Engineering*, Lisbon, Portugal,
- Chib, S. and Greenberg, E., 1995. Understanding the Metropolis-Hastings Algorithm. *The American Statistician*, 49, 327-335.
- Ching, J., Beck, J., Porter, K. and Shaikhutdinov, R., 2006. Bayesian State Estimation Method for Nonlinear Systems and Its Application to Recorded Seismic Response. *Journal of Engineering Mechanics*, 132, 396-410.
- Moehle, J. and Deierlein, G. G., 2004. A framework methodology for performance-based earthquake engineering. *13th World Conference on Earthquake Engineering*, Vancouver, Canada, 3812-3814.
- Porter, K. A., 2003. An overview of PEER's performance-based earthquake engineering methodology. *Proc. Ninth International Conference on Applications of Statistics and Probability in Civil Engineering (ICASP9)*, 973-980.
- Porter, K. A., Kiremidjian, A. S. and Legerue, J. S., 2001. Assembly-Based Vulnerability of Buildings and Its Use in Performance Evaluation. *Earthquake spectra*, 17, 291-312.
- Rezaeian, S. and Der Kiureghian, A., 2008. A stochastic ground motion model with separable temporal and spectral nonstationarities. *Earthquake Engineering & Structural Dynamics*, 37, 1565-1584.