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Incipient sediment motion with upward seepage

Mouvement de sédiment sous l'effet d'une infiltration ascendante par le fond

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ABSTRACT

This study investigates the effect of upward bed seepage on the critical condition of incipient sediment motion in open channel flow both analytically and experimentally. The critical condition was derived by analyzing the forces acting on a sediment particle lying on a permeable horizontal bed subject to seepage. The ratio of the critical shear velocity with seepage to that without seepage depends on the ratio of the hydraulic gradient of seepage to its critical value under the quick condition. Experimental results concerning incipient motion of cohesionless uniform sediments in open channel flow show that for a particular size of sediment, the critical shear velocity decreases with increasing seepage velocity. All measurements generally support the theoretically derived expression of the critical shear velocity in the presence of an upward seepage.

RÉSUMÉ

Les présents travaux étudient sur les plans analytiques et expérimentaux l'effet de l'infiltration ascendante au travers du fond sur les conditions critiques du mouvement de sédiment dans les canaux à surface libre. Les conditions critiques sont dérivées de l'analyse des forces qui s'exercent sur un lit horizontal soumis à l'infiltration. Le rapport de la vitesse critique de cisaillement en présence d'infiltration à celle obtenue sans infiltration dépend du rapport du gradient hydraulique à sa valeur sous conditions rapides. Les résultats expérimentaux sur le mouvement de sédiment uniforme non cohésif dans des canaux à surface libre montre que, pour une taille de sédiment donnée, la vitesse de cisaillement critique décroît lorsque la vitesse d'infiltration s'accroît. Les mesures confirment l'expression analytique de la vitesse de cisaillement.

1 Introduction

The critical shear stress for the initial motion of cohesionless sediment particles under a unidirectional flow has been investigated experimentally by numerous researchers. The widely cited work to date is that done by Shields (1936). Some forms of the Shields diagram are frequently used by subsequent researchers to describe the incipient condition of sediment transport.

A successful application of the Shields diagram in predicting the initiation of sediment transport is limited to a few conditions, such as sediments with a uniform size distribution, horizontal or near-horizontal bed slopes and unidirectional flows. A deviation from these conditions will render the application of the Shields diagram ineffective in predicting the critical shear velocity. One typical example is the threshold condition for sediment transport over a large longitudinal bed slope. Many investigators such as Dyer (1986), Whitehouse and Hardisty (1988), and Chiew and Parker (1994) have related the ratio of the critical shear stress on a slope to that on a horizontal bed to the angle of the slope. Chiew and Parker conducted a series of experiments using a closed-conduit flow to study the threshold condition for sediment entrainment on a streamwise slope. The critical flow conditions established by both experimental and analytical methods show that the streamwise bed slope has a significant influence on the incipient condition for sediment transport. Another type of deviation from the Shields diagram is caused by the non-uniform size distribution of the bed sediment. Recent developments on the threshold condition of non-uniform sediments have been made by Parker et al. (1982), Wilcock and Southard (1988) and Kuhnle (1993).

When sediment transport takes place on a porous bed subject to seepage, the sediment particles experience an additional hydrodynamic force. This also leads to the invalidation of the Shields diagram. Seepage through a permeable bed can take place with an arbitrary angle relative to the bed surface. Two typical types of seepage can be identified (Cheng, 1997). One is parallel seepage, in which the direction is tangential to the boundary and is thus parallel to the free-surface flow. The other type of seepage occurs normal to the boundary, which is referred to as normal seepage. The studies on parallel seepage have been conducted by Zippe and Graf (1983), Mendoza and Zhou (1992), and Zhou and Mendoza (1993). This paper only investigates normal seepage, and upward seepage specifically. A search in the literature shows that very few studies have been conducted to examine the seepage effect on the critical condition of incipient sediment motion. Martin and Aral (1971) addressed the instability of sediment particles on an inclined plane, on which both inflow and outflow types of seepage were applied, to determine the seepage force on interfacial bed particles. Oldenziel and Brink (1974) suggested that the Shields parameter be modified in view of the fact that the submerged weight force of bed particles was reduced by an upward seepage. As far as the writers are aware, there is as yet no published data on how the threshold condition of sediment particles is affected by seepage. The objective of this study is to perform a force analysis for the threshold condition of sediment transport by including the effect of a force due to upward seepage, and to conduct laboratory experiments to verify the theoretical derivation. Although both upward and downward flows can occur in normal seepage, only upward seepage is examined in this study.

2 Force analysis

Consider a plane bed consisting of uniform, cohesionless sediment particles, where the bed slope is at an angle ϕ_b from the horizontal plane, and the bed sediment is subject to both seepage and open-channel flow (see Fig. 1).

At the onset of motion, a cohesionless sediment particle on the porous bed experiences the drag force F_D , the lift force F_L , the submerged weight force W , and the seepage force F_S . All the forces acting on the particle can be divided into two components: normal and tangential to the bed surface. The drag force and one component of the submerged weight force $W \sin \phi_b$ act on the particle in the direction tangential to the bed plane. If one assumes that the direction of the seepage flow is normal to the bed, the seepage force will act in the same direction as the lift force, i.e., normal to the bed slope. With these considerations, the summation of the tangential forces and that of the normal forces can be related to the angle of repose of the bed sediment at the incipience of sediment motion. Assuming that the flow approaches the particle with an incident angle α , as shown in Fig. 1, the relationship can be expressed as

$$\tan \theta = \frac{\sqrt{(F_D \sin \alpha + W \sin \phi_b)^2 + F_D^2 \cos^2 \alpha}}{W \cos \phi_b - F_L - F_S} \quad (1)$$

where θ = angle of repose of the sediment particles. A similar equation to Eq. (1) for zero seepage can be found in literature, for example, Chien and Wan (1983). For a streamwise slope of $\alpha = 90^\circ$, Eq. (1) becomes

$$\tan \theta = \frac{F_D + W \sin \phi_b}{W \cos \phi_b - F_L - F_S} \quad (2)$$

Fig. 1. Forces exerting on a bed particle.

Approximating the sediment particles as spheres for simplicity, the effective weight force, the drag force and the lift force can then be expressed as follows:

$$W = (\rho_s - \rho)g \frac{\pi d^3}{6} \quad (3)$$

$$F_D = C_D \frac{\pi d^2}{8} \rho u_b^2 \quad (4)$$

$$F_L = C_L \frac{\pi d^2}{8} \rho u_b^2 \quad (5)$$

where C_D , C_L = drag and lift coefficients, respectively; d = diameter of sediment particles; ρ = density of fluid; ρ_s = density of particles; g = gravitational acceleration; and u_b = approaching velocity of flow over the bed at the particle. Furthermore, the approaching velocity can be related to the shear velocity (Chiew and Parker, 1994):

$$u_b = \frac{u_{*c}}{\sqrt{f^*}} \quad (6)$$

where f_* = a form of friction factor; and u_c = critical shear velocity in the case of the bed seepage. The coefficients C_D , C_L and f_* generally depend on the boundary Reynolds number, $Re_* = \rho u_{*c} d / \mu$, and they can be considered to be constant when the effect of fluid viscosity is neglected.

The seepage force exerting on a porous medium per unit volume is expressed as (Bear, 1988):

$$S = i \rho g \quad (7)$$

where i = hydraulic gradient for seepage. The number of particles per unit volume is

$$N = \frac{6(1 - \varepsilon)}{\pi d^3} \quad (8)$$

where ε = porosity. Thus, the seepage force exerting on a particle is equal to

$$F_s = \frac{S}{N} = \frac{i \rho g \pi d^3}{6(1 - \varepsilon)} \quad (9)$$

Substituting Eq. (3) through Eq. (5) and Eq. (9) into Eq. (2) leads to

$$\frac{u_{*c}^2}{\Delta g d} = \frac{\frac{4}{3} f_* \left[\cos \phi_b - \frac{\sin \phi_b}{\tan \theta} - \frac{i}{\Delta(1 - \varepsilon)} \right]}{C_L + \frac{C_D}{\tan \theta}} \quad (10)$$

where $\Delta = (\rho_s - \rho) / \rho$. In the case of zero seepage, $i = 0$ and Eq. (10) reduces to

$$\frac{u_{*oc}^2}{\Delta g d} = \frac{\frac{4}{3} f_* \left(\cos \phi_b - \frac{\sin \phi_b}{\tan \theta} \right)}{C_L + \frac{C_D}{\tan \theta}} \quad (11)$$

where u_{*oc} = critical shear velocity caused solely by the channel flow. Eq. (11) is applicable to the threshold condition of the particle on the streamwise bed without seepage and was previously presented by Chiew and Parker (1994). Dividing Eq. (10) by Eq. (11), one gets

$$\frac{u_{*c}^2}{u_{*oc}^2} = 1 - \frac{i}{\Delta(1 - \varepsilon)} \frac{\tan \theta}{\cos \phi_b \tan \theta - \sin \phi_b} \quad (12)$$

Because the streamwise bed slope ϕ_b is generally smaller than the angle of repose,

$$\frac{\tan \theta}{\cos \phi_b \tan \theta - \sin \phi_b} > 0 \quad (13)$$

It can therefore be inferred from Eq. (12) that the critical shear stress required for a particle to move over the porous bed will decrease in the presence of an upward seepage. Note that this is also confirmed by the data presented in this paper.

Furthermore, if the velocity of the surface flow is very small, which leads to $F_L = F_D = 0$, Eq. (2) simply reduces to

$$\tan \theta (W \cos \phi_b - F_s) = W \sin \phi_b \quad (14)$$

It means that, without any surface flow over the bed, the seepage force is the sole driving force acting on sediment particles except for a streamwise gravitational component. Under the threshold condition, the corresponding hydraulic gradient for seepage is referred to as critical hydraulic gradient, i_c . By substituting Eq. (3) and Eq. (9) into Eq. (14), i_c can be solved to be

$$i_c = \left(\cos \phi_b - \frac{\sin \phi_b}{\tan \theta} \right) \Delta (1 - \varepsilon) \quad (15)$$

For a horizontal bed, $\phi_b = 0$ and Eq. (15) reduces to

$$i_c = \Delta (1 - \varepsilon) \quad (16)$$

Eq. (16) is the equation that describes the quick condition, at which the seepage force acting on a particle just balances its submerged weight force at the inception of motion. Substituting Eq. (15) into Eq. (12) yields

$$\left(\frac{u_{*c}}{u_{*oc}} \right)^2 = 1 - \frac{i}{i_c} \quad (17)$$

Eq. (17) shows the relationship between the critical shear velocities, with and without seepage, under the initial condition for sediment transport on a streamwise bed slope. It can be seen that increasing the hydraulic gradient results in a reduction of the critical shear velocity; and the critical shear velocity decreases to zero when the quick condition occurs.

The hydraulic gradient represents the driving force that causes seepage and can be related to seepage velocity. If the seepage flow through the porous medium is linear, its equation of motion can be described by Darcy's law (see Bear, 1988), which states that the seepage velocity is directly proportional to the hydraulic gradient, and the constant of proportionality is known as the coefficient of permeability, K :

$$v_s = Ki \quad (18)$$

where v_s = seepage velocity. However, it must be noted that Darcy's law specifically neglects the kinetic energy of the flow through the porous medium. For a porous medium consisting of fine particles, the kinetic energy of the flow is small and the flow is thus within the laminar region and the assumption of Darcy's law is valid. However, the flow velocity in coarse granular materials can be quite large and the corresponding kinetic energy becomes significant. The loss in kinetic energy due

to turbulence can cause deviations from Darcy's law. Extensive attempts have been made to develop nonlinear relationships between the seepage velocity and the hydraulic gradient for transition and turbulence regions. Comprehensive reviews have been presented, among others, by Kovacs (1981) and Bear (1988). Kovacs (1981) summarized various nonlinear relationships between the seepage velocity and the hydraulic gradient and stated that they fall into two main categories:

(a) binomial function

$$i = av_s + bv_s^2 \quad (19)$$

(b) exponential function

$$i = cv_s^m \quad (20)$$

where a, b, c = empirical coefficients and m = exponent = 1-2.

When attributing the deviation from Darcy's law to inertial force due to turbulence, the binomial form of the relationship between the seepage velocity and the hydraulic gradient is acceptable. This is because the inertial force is proportional to the square of the velocity. Similarly, the exponent, m , in the exponential function ranges from 1 to 2. For convenience, Eq. (20) is adopted in the following analysis to delineate the nonlinear relationship between the seepage velocity and the hydraulic gradient. Furthermore, by assuming that Eq. (20) is applicable up to the quick condition and that c remains constant for a particular sediment, one gets

$$\frac{i}{i_c} = \left(\frac{v_s}{v_{sc}} \right)^m \quad (21)$$

where v_{sc} = critical seepage velocity in the case of quick sand. Depending on the characteristics of fluid and sediment beds, the m -value is expected to be equal to 1 when seepage flow is within the laminar region and increases to 2 in the turbulent region. Using Eq. (21), Eq. (17) changes to

$$\left(\frac{u_{*c}}{u_{*oc}} \right)^2 = 1 - \left(\frac{v_s}{v_{sc}} \right)^m \quad (22)$$

Eq. (22) can be used for the determination of the critical shear velocity for various cases of upward seepage.

3 Experimental setup and procedure

3.1 Experimental apparatus

The experimental apparatus comprised of two seepage conduits and an open-channel flume (see Fig. 2). The two seepage conduits were made of perspex and they were used to conduct seepage tests through granular materials. One was a cylinder 50 cm in height and 11 cm in diameter. The other conduit was a square, 20 cm by 20 cm in cross section and 100 cm in height. Water was pumped through the bottom of the conduit. Special filter was inserted at the lower end of the conduit to ensure uniform flow into the granular materials. Four piezometric holes were arranged on the wall of the conduit and they were connected to a manometer for piezometric head measure-

ments. A glass-sided horizontal flume 7.6 m long, 0.21 m wide and 0.4 m deep was employed to conduct experiments of incipient sediment motion. Located at 5.5 m from the inlet of the flume was a seepage zone that was 0.5 m long, 0.21 m wide and 0.3 m deep. The structure of the seepage zone was similar to that of the seepage conduit. A sand bed was prepared in the upper part of the seepage zone and the seepage flow was introduced from the bottom of the seepage zone. The same sand particles used in the seepage zone were also glued to the impermeable bed to furnish a consistent roughness throughout the channel for each test.

The velocity measurements were performed using an 8 mm minipropeller current meter, which was designed to measure velocity in the range of 4 to 300 cm/s. Two types of flowmeters were utilized to measure the flow rate. The flow rate through the head tank of the flume was monitored using an electromagnetic flowmeter. A turbine flowmeter was used to determine the seepage discharge through the seepage conduits or the seepage zone in the flume. The depth at the test sections in the flume was measured using a depth gauge. The hydraulic gradient of the seepage through porous beds was determined using the measured piezometric heads.

Altogether six sets of uniform sediments were tested in this study. The properties of the sediment particles used are summarized in Table 1.

Fig. 2. Sketch of experimental apparatus: (a) Seepage conduit; (b) Seepage zone in the flume.

Table 1. Properties of sediments used in study.

No.	Median Grain Diameter (mm)	Geometric Standard Deviation	Experimental Series
1	0.63	1.23	I, II
2	0.93	1.28	I
3	1.02	1.26	I, II
4	1.64	1.23	I
5	1.95	1.24	I, II
6	5.83	1.12	I

3.2 Procedure

The experimental program consists of two series of tests to study (I) seepage through porous beds; and (II) the seepage effect on the threshold condition for sediment transport.

Experimental series I was designed specially to investigate seepage through a packed bed of sediment particles using the two seepage conduits. As Darcy's law has been found to be inadequate for porous media consisting of coarse particles, this series aimed to modify Darcy's law and to develop an alternative equation of motion to describe the flow through such porous media.

All the six sediments outlined in Table 1 were tested in this series. The following procedure was applied to each test. The seepage conduit was first filled with water and the sand to be tested. Second, the sand was stirred to release the air bubbles before the sand surface was leveled. After this, the pump was turned on and the valve was slowly opened to allow water to flow into the conduit. Upon stabilization of the flow, the piezometric heads were recorded from the manometer for calculating the hydraulic gradient. Next, by gradually increasing the seepage velocity, a series of hydraulic gradients were thus obtained for the sand. This step continued until either the quick condition took place or the seepage velocity achieved its maximum.

Experimental series II was conducted in the open-channel flume to explore the effect of an upward seepage on the threshold condition for sediment transport. Three uniform sediments with median grain diameter of 0.63 mm, 1.02 mm and 1.95 mm, respectively were used as the bed materials (see Table 1). The bed surface in the seepage zone was first leveled to the elevation of the neighboring roughened bed. The pump for seepage was then turned on and the seepage discharge was gradually adjusted to the predetermined value. Following this, the water for the main flow was slowly delivered to the flume at a constant rate. The tailgate was then adjusted to obtain the required water depth in the channel.

Given the water depth and the flow rate in the flume, the tail gate and the valve controlling the seepage were slowly and alternately adjusted to achieve the threshold condition for sediment transport. The so-called "weak movement" as described in Vanoni (1975) was used to define the threshold condition at the incipient motion of bed particles. With this method, some subjectivity is expected and a certain amount of scatter in the experimental data is unavoidable. When the condition for incipient sediment motion is reached, the velocity measurement at the middle section of the seepage zone was finally performed.

4 Evaluations of basic parameters

4.1 Critical shear velocity without seepage

In the case of zero seepage, the Shields diagram can be used to determine the critical shear velocity for the incipient motion of cohesionless sediment particles on a horizontal bed. As additional experimental data on the threshold condition for sediment entrainment become available, the original Shields diagram has been updated. Recent examples were provided by Yalin and Karahan (1979) and Chien and Wan (1983). Based on the available experimental data in the literature, Yalin and Karahan (1979) defined an averaged curve in place of the Shields curve while Chien and Wan (1983) furnished a strip to amend the original Shields diagram presented by Vanoni (1975).

To avoid an iterative procedure encountered in determining the critical shear velocity using the Shields diagram, the relation of the Shields parameter and the boundary Reynolds number Re_* can be re-plotted as Re_* against the dimensionless diameter d_* (see Fig. 3), where

$$d_* = \left(\frac{\Delta \rho^2 g}{\mu^2} \right)^{\frac{1}{3}} d \quad (23)$$

Given the value of the dimensionless diameter, Fig. 3 can be used, without a trial-and-error procedure, for the evaluation of the critical shear velocity in the case of zero-seepage.

Fig. 3. Threshold condition in the case of zero seepage.

4.2 Critical shear velocity in the presence of upward seepage

For a two-dimensional open-channel flow without seepage, there are numerous methods available for the evaluation of the bed shear stress. One typical method is to compute the bed shear stress by fitting the logarithmic law of the wall to the measured velocity profiles.

Another way is to measure the water surface slope, S_w and to compute the bed shear stress according to $\tau_b = ghS_w$. Yet another method is to directly measure the boundary shear stress using the Preston tube. When the profile of the Reynolds shear stress is available, the bed shear stress can also be estimated by extrapolating it to the boundary. However, in the presence of seepage, the boundary conditions of the flow are changed. This change leads to the invalidation of all the above-mentioned methods except for the last one in evaluating the bed shear stress.

Cheng and Chiew (1998a) derived a momentum integral equation in the case of boundary seepage, which shows that the bed shear stress can be computed using the water surface slope, seepage velocity and the other flow parameters. The bed shear stresses computed using the momentum integral equation are in good agreement with those obtained by extending the Reynolds shear stress distributions to the boundary. Cheng and Chiew (1998b) also proposed a modified logarithmic law by including the seepage effect on velocity profiles in open-channel flow. They showed that the shear velocity can be related to the water depth h , the depth-averaged velocity U , the seepage velocity v_s and the roughness function B as follows:

$$\frac{U}{u_*} = \frac{1}{\kappa} \left(\ln \frac{h}{y_o} - 1 \right) + \frac{v_s}{4\kappa^2 u_*} \left[\ln^2 \frac{h}{y_o} - 2 \left(\ln \frac{h}{y_o} - 1 \right) \right] \quad (24)$$

where

$$y_o = k_s \exp(-\kappa B) \quad (25)$$

k_s = equivalent sand roughness = $2d$ and κ = von Karman's constant = 0.4 for water. With the negligible effect of fluid viscosity, the roughness function B is only dependent on the seepage velocity in the form:

$$B = \frac{8.5}{1 + v_s/u_*} \quad (26)$$

The shear velocities calculated using Eq. (24) show a good agreement with those computed using the momentum integral equation (Cheng and Chiew, 1998b). Eq. (24) is used to evaluate the bed shear stress in this study. The critical shear velocity in the presence of seepage is computed using the velocity profile measured at the middle section of the seepage zone, which was measured under the condition of incipient sediment motion.

In the present study, the aspect ratio of flows is less than 5, the velocity profiles along the centerline of the flume would deviate from those of two dimensional flows. It follows that both the depth-averaged velocity U and the water depth h in Eq. (24) should be replaced with the values associated with the bed denoted by U_b and h_b , respectively. In this paper, h_b is taken to be the distance from the bed surface to the maximum velocity point in the centerline profile and U_b is obtained by integrating the measured velocity distribution from $y = 0$ to $y = h_b$.

4.3 Determination of m -values

As discussed earlier, the exponent m in Eq. (22) depends on the properties of the fluid and the porous medium. Thus it can generally be expressed as

$$m = f(\rho, \mu, l, g) \quad (27)$$

where l = characteristic dimension of voids within the sediment bed. Using dimensional analysis, Eq. (27) can be rewritten in the dimensionless form as follows:

$$m = f \left[\left(\frac{\rho^2 g}{\mu^2} \right)^{\frac{1}{3}} l_{\epsilon} \right] \quad (28)$$

The characteristic dimension of the voids can be related to both the particle diameter and the porosity for uniform sediments. By analogy with the hydraulic radius in pipe flows, the characteristic dimension of the voids can be shown to be proportional to the ratio of the value of the voids to the surface area of particles per unit volume of a porous medium (Churchill, 1988). Since the volume of the voids per unit volume is equal to the porosity, ϵ , the surface area of particles per unit volume, α_a , is equal to

$$\alpha_a = \frac{(1 - \epsilon)\pi d^2}{\pi d^3/6} = \frac{6(1 - \epsilon)}{d} \quad (29)$$

Therefore, the characteristic dimension of voids can be defined as

$$l_{\epsilon} = \frac{\epsilon}{\alpha_a} = \frac{\epsilon d}{6(1 - \epsilon)} \quad (30)$$

Substituting Eq. (30) into Eq. (28) yields

$$m = f(l_{\epsilon*}) \quad (31)$$

where the dimensionless characteristic dimension of voids l is defined as

$$l_{\epsilon*} = \left(\frac{\rho^2 g}{\mu^2} \right)^{\frac{1}{3}} \frac{\epsilon d}{6(1 - \epsilon)} \quad (32)$$

To determine the functional relationship of Eq. (31), the value of m must first be evaluated using experimental data of seepage through different porous media. This can be achieved by plotting the hydraulic gradient against the seepage velocity. Fig. 4 shows such a plot in which the experimental data were collected from experimental series I in this study, and the data were fitted to the exponential function Eq. (20) to determine m . Next, the derived m -values are plotted in Fig. 5 against the dimensionless parameter, $l_{\epsilon*}$. Besides the data collected in this study, the results from Mints and Shubert (1957) and Oldenzil and Brink (1974) are also included in the figure. All the experimental data in Fig. 5 appear to collapse onto one curve which can approximately be expressed, with the consideration of m varying from 1 to 2, in the form:

$$m = \frac{1 + 2\delta l_{\epsilon*}^2}{1 + \delta l_{\epsilon*}^2} \quad (33)$$

where $\delta = 0.0027$. It is unfortunate that there are no data for very large values of $l_{\epsilon*}$, and the largest in the present experimental data set is 22.44. The empirical equation (33) is fitted by assuming that m is equal to the limiting value of 2 when l approaches 1000.

Fig. 4. Hydraulic gradient versus seepage velocity (The solid lines represent the exponential functions fitted to the data with the least square error).

Fig. 5. Relationship of m and l .

4.4 Critical seepage velocity

The critical seepage velocity under the quick condition for a particular size of sediment can be directly derived from experimental series I. With increasing seepage velocity, the hydraulic gradient first increases steadily. This trend is maintained until the hydraulic gradient reaches a peak. Following that, the hydraulic gradient decreases for higher seepage velocities. The seepage velocity corresponding to the maximum hydraulic gradient is defined as the critical seepage velocity, at which the inception of the quick condition takes place.

5 Upward seepage influence on initial motion of sediment particles

Three sizes of sediment particles were used in experimental series II, which was designed to examine the seepage effect on the incipient motion. Table 2 shows the properties of the sediments used. The critical shear velocity for the incipient sediment motion in the case of no-seepage can be evaluated using the Shields diagram, the modified Shields diagram (Fig. 3) and through experimental observa-

tion. As shown in Table 2, the experimentally determined values of the critical shear velocity, which were obtained by fitting the measured velocity profiles at the incipient condition to the logarithmic law, are found to be closest to the lower limits of those evaluated using Fig. 3. The critical shear velocities derived from the Shields diagram are slightly higher than the experimental results. The experimentally determined values of the critical shear velocity are used in the data analysis.

Table 3 summarizes all the results obtained in Experimental Series II. Fig. 6 shows the results of the critical shear velocity plotted against the seepage velocity for the three sediments used. The data show that the presence of seepage causes a significant reduction of the critical shear velocity. Fig. 6 is also superimposed with the curves computed using Eq. (22). It shows that the computed curves agree reasonably well with the experimental data.

Table 2. Critical shear velocity with zero seepage and critical seepage velocity for tested sediments.

d (mm)	u_{*oc} (cm/s)			v_{sc} (cm/s)
(1)	(2) ^a	(3) ^b	(4) ^c	(5)
0.63	1.8	1.7	1.7	0.17
1.02	2.4	2.1	2.0	0.68
1.95	3.5	3.2	3.1	1.45

^acalculated using the Shields diagram.

^bcalculated using the modified Shields diagram (lower limit) by Chien and Wan (1983).

^cexperimental results.

Fig. 6. Reduction of critical shear velocity for sediment transport caused by upward seepage.

Table 3. Critical shear velocity subject to seepage.

No	d (mm)	v_s (cm/s)	U_b (cm/s)	h_b (cm)	ku_{*oc} (cm/s)	u_{*c} (cm/s)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	1.95	0.70	34.0	2.9	2.31	2.17
2	1.95	0.00	38.6	5.3	3.10	3.09
3	1.95	0.73	36.6	4.8	2.27	1.81
4	1.95	0.00	39.9	4.8	3.10	3.25
5	1.95	0.68	39.8	3.3	2.34	2.49
6	1.95	0.70	31.9	2.7	2.31	2.07
7	1.95	0.98	27.1	2.8	1.84	1.52
8	1.95	1.23	21.9	4.7	1.27	0.76
9	1.95	1.30	23.4	4.3	1.05	0.87
10	1.95	1.38	27.9	4.3	0.72	1.09
11	1.95	1.33	32.2	4.3	0.94	1.32
12	1.95	0.99	35.4	4.3	1.82	1.68
13	1.95	0.37	38.7	4.3	2.74	2.50
14	1.95	1.30	30.8	6.0	1.05	0.99
15	1.95	1.18	33.3	6.0	1.40	1.16
16	1.02	0.62	24.9	6.5	0.61	0.65
17	1.02	0.65	16.4	6.5	0.40	0.30
18	1.02	0.62	19.2	6.0	0.62	0.44
19	1.02	0.57	23.4	7.6	0.81	0.56
20	1.02	0.58	26.1	6.7	0.78	0.72
21	1.02	0.00	29.6	7.1	2.00	1.99
22	1.02	0.36	29.2	6.2	1.40	1.17
23	1.02	0.00	31.3	6.2	2.00	2.15
24	1.02	0.25	31.5	6.2	1.60	1.49
25	1.02	0.33	29.6	5.0	1.45	1.34
26	1.02	0.31	29.3	3.6	1.50	1.55
27	1.02	0.57	28.8	5.3	0.83	0.98
28	1.02	0.54	26.0	4.8	0.92	0.91
29	1.02	0.55	28.3	5.9	0.89	0.91
30	1.02	0.41	29.1	4.0	1.28	1.33
31	1.02	0.45	28.2	3.1	1.17	1.37
32	0.63	0.11	13.9	3.0	0.99	0.71
33	0.63	0.12	20.2	3.0	0.90	1.12
34	0.63	0.10	18.5	3.0	1.08	1.06
35	0.63	0.11	9.4	3.3	1.02	0.41
36	0.63	0.12	17.4	3.3	0.95	0.92
37	0.63	0.00	23.7	3.1	1.70	1.69
38	0.63	0.13	18.3	4.0	0.87	0.90
39	0.63	0.14	15.3	4.1	0.74	0.68
40	0.63	0.13	12.3	4.1	0.78	0.50
41	0.63	0.12	19.7	4.0	0.89	1.00
42	0.63	0.14	16.8	5.3	0.73	0.70
43	0.63	0.14	14.3	4.3	0.69	0.60
44	0.63	0.16	11.3	5.0	0.47	0.38
45	0.63	0.17	9.0	5.0	0.23	0.25
46	0.63	0.11	21.2	5.0	0.98	1.05
47	0.63	0.16	19.1	5.5	0.49	0.79
48	0.63	0.16	15.9	4.5	0.43	0.66
49	0.63	0.16	13.8	5.5	0.43	0.48
50	0.63	0.12	23.2	6.0	0.96	1.12

Notes: The values in Column 6 were computed using (22) and $k = [1 - (v_s/v_{sc})^m]^{0.5}$.

The values of the critical shear velocity listed in Column 7 were determined using (24).

6 Conclusions

This paper examines the effect of upward seepage on the critical condition of incipient sediment motion. The critical condition was first established by analyzing the forces acting on a sediment particle lying on a permeable bed subject to normal seepage. It has been shown that the critical shear velocity is reduced in the presence of upward seepage. The ratio of the critical shear velocity with seepage to that without seepage is dependent on the ratio of the hydraulic gradient of seepage to its value at the quick condition.

To relate the hydraulic gradient to the seepage velocity, Darcy's law was generalized with an exponential function so that it can be applied to the porous medium consisting of coarse grains. The general relation between the hydraulic gradient and the seepage velocity was substantiated using experimental results for uniform sediments. The ratio of the critical shear velocity with seepage to that without seepage can thus be expressed as a function of the ratio of the seepage velocity to its critical value at the quick condition.

The experimental data were analyzed to confirm that the upward seepage through a permeable bed affects the critical condition of sediment entrainment. For a particular size of sediment, the critical shear velocity was found to decrease with increasing seepage velocity. All measured data generally support the theoretically derived expression of the critical shear velocity in the presence of seepage.

Notations

a	coefficient
b	coefficient
B	roughness function
c	coefficient
C_D	drag coefficients
C_L	lift coefficients
d	diameter of sediment particle
d_*	dimensionless diameter
f_*	a measure of friction
F_D	drag force
F_L	lift force
F_S	seepage force
g	gravitational acceleration
h	water depth
h_b	distance from the bed surface to the maximum velocity point
i	hydraulic gradient for seepage
i_c	critical hydraulic gradient
k_s	equivalent sand roughness
K	coefficient of permeability
l	characteristic dimension of voids
l	dimensionless characteristic dimension of voids
m	exponent
N	number of particles per unit volume
S	seepage force exerting on the porous media per unit volume
u_b	approaching velocity of flow

u_*	shear velocity
u_{oc}	critical shear velocity without seepage
u_c	critical shear velocity in the presence of seepage
U	depth-averaged velocity
U_b	averaged velocity associated with the bed
v_s	seepage velocity
v_{sc}	critical seepage velocity under quick condition
Re_*	boundary Reynolds number = $\rho u_* d / \mu$
W	submerged weight force
y	normal distance from bed
y_o	$k_s \exp(-0.4B)$
α	incident angle of flow approaching particles
α_a	surface area of particles per unit volume of porous media
δ	constant
Δ	$(\rho_s - \rho)/\rho$
ε	porosity
θ	angle of repose of submerged sediment particle
κ	constant
μ	dynamic viscosity of fluid
ρ	density of fluid
ρ_s	density of particles
τ_*	Shields parameter
ϕ_b	bed slope angle.

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