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Thermal interaction between free convection and forced convection along a vertical conducting wall

J.-J. Shu, I. Pop

Abstract

A theoretical study is presented in this paper to investigate the conjugate heat transfer across a vertical finite wall separating two forced and free convection flows at different temperatures. It is assumed that the heat conduction in the wall is only in the transversal direction. We also assume that countercurrent boundary layers are formed on the both sides of the wall. The governing equations of this problem and their corresponding boundary conditions are all cast into a dimensionless form by using a non-similarity transformation. These resulting equations, which are singular at the points $\xi_c = 0$ and 1, are solved numerically using a very efficient singular perturbation method. The effects of the resistance parameters and of the Prandtl numbers on heat transfer characteristics are investigated and presented in a table and ten figures.

List of symbols

A	constant
b	thickness of the plate
C	constant
f	reduced stream function
g	acceleration due to gravity
h_x	local heat transfer coefficient
k	thermal conductivity
L	length of the plate
Nu_x	local Nusselt number
Nu	average Nusselt number
Pr	Prandtl number
q_x	local heat flux
Q	overall heat flux
Re	Reynolds number
R_t	forced convection thermal resistance parameter
R_t^*	free to forced convection parameter
t	ambient temperature
Δt	characteristic ambient temperature
T	fluid temperature
u, v	velocity components
U_∞	free stream velocity
x, y	Cartesian coordinates

Greek letters

α	thermal diffusivity
β	coefficient of thermal expansion
θ	dimensionless temperature
λ	dummy variable
ν	kinematic viscosity

ξ, η reduced coordinates
 ψ stream function

Subscripts

c cold fluid system
 h hot fluid system
 s solid wall
 w condition at the wall

Introduction

The study of thermal interaction between two semi-infinite fluid reservoirs at different temperatures through a vertical conductive wall is a very important topic in heat transfer because of its numerous engineering applications. This heat transfer process applies to reactor cooling, heat exchangers, thermal insulation, nuclear reactor safety, etc. Additionally, such interaction mechanisms is, for the most part, inherent in the design of heat transfer apparatus. On the other hand, it is worth mentioning that recent demands in heat transfer engineering have requested researchers to develop new types of equipments with superior performances, especially compact and lightweight ones. The need for small-size units, requires detailed studies on the effects of interaction between the thermal field in both fluids and of the wall conduction, which usually degrades the heat exchanger performance.

The problem of heat exchange between two free convection systems separated by a finite vertical conductive wall has not been studied extensively because of difficulties in solving the developments of flow and thermal boundary layers simultaneously. Lock and Ko [1] applied the local similarity method to investigate theoretically the effect of thermal coupling produced by conduction through a finite vertical wall separating two free convection systems. The study was done under the assumption that the wall conduction is only in the transversal direction. The governing boundary layer equations were transformed by introducing semi-similar variables and then solved numerically using a finite-differences method. This problem was also treated by Viskanta and Lankford [2] following a more simple analysis based on a super-position method. The authors have also conducted interferometric experiments which confirm the validity of their approximated theoretical results. Next, Anderson and Bejan [3] have employed the modified Oseen linearized method to solve such a conjugate problem when the Prandtl number is very large. It was shown that the overall heat transfer rate is relatively independent by the Prandtl number. Sakakibara et al. [4] have recently extended the problem of coupled heat transfer between two free convection systems separated by a finite vertical conducting wall assuming the two-dimensional conduction equation in the wall. In other words, the wall conduction takes place in both axial and transversal directions. Numerical

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solutions for both free convection systems and the analytical solution for the wall conduction were combined to obtain final solutions for the flow and heat transfer characteristics which fit the conjugate boundary conditions at both sides of the wall. Experiments were also conducted for air-air systems with the conducting wall made of aluminum or glass. It was found that theoretical results describe well the experimental temperature distributions. More recently, in a very interesting paper, Treviño et al. [5] reported numerical and asymptotic solutions of the free convection boundary layers on both sides of a vertical conducting wall for all possible values of two main parameters.

We notice to this end that there were also published several papers by Poulikakos [6], Méndez and Treviño [7], and Chen and Chang [8, 9] on the problem of thermal interaction between laminar film condensation of a saturated vapor and a forced or free convection system separated by a vertical conducting wall.

In this paper we intend to propose a new theoretical (mathematical) method to predict the heat transfer between free convection on one side of a finite vertical conducting wall and forced convection flow on the other side of the wall with the consideration of the wall thermal resistance. It is assumed that the conduction in the wall is in the transversal direction only. Since both the plate temperature and the heat flux through the plate are unknown a priori in this problem, the boundary layer equations on both sides of the wall and the one-dimensional heat conduction equation for the wall are solved simultaneously. The numerical method used is a new comprehensive and non-iterative scheme based on the singular perturbation method described in a recent paper by Shu and Pop [10]. We notice that this method differs by the iterative guessing technique proposed by Chen and Chang [8, 9]. Heat transfer characteristics have been derived for some main parameters entering this problem.

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Basic equations

The physical model under consideration along with the coordinate systems is shown in Fig. 1, where the vertical plate with length L and thickness b separates two semi-infinite fluid reservoirs at different temperatures. The warmer reservoir contains a stagnant fluid with temperature t_h , while the ambient temperature on the cold side of the plate is t_c . Obviously, t_h is higher than t_c . The upper left corner of the plate coincides with the origin of a Cartesian coordinate system whose y axis points in the direction normal to the plate, while the x axis points downward in the plate's longitudinal direction. Due to gravity, a free convection laminar boundary layer appears on the hot side of the plate and flows downward along the plate. A forced convection flow of the cooling fluid with velocity U_∞ is imposed on the right lateral surface of the plate thus generating a forced convection boundary layer on this surface, which develops with increasing thickness downstream. Accordingly, two fluid streams move in opposite directions. Due to this assumption, the present problem can be formulated in terms of the boundary layer equations for two different heat transfer systems. These governing differential equations need to be considered separately and they are:

Hot fluid

$$\frac{\partial u_h}{\partial x} + \frac{\partial v_h}{\partial y} = 0 \quad (1)$$

$$u_h \frac{\partial u_h}{\partial x} + v_h \frac{\partial u_h}{\partial y} = v_h \frac{\partial^2 u_h}{\partial y^2} - g\beta(T_h - t_h) \quad (2)$$

$$u_h \frac{\partial T_h}{\partial x} + v_h \frac{\partial T_h}{\partial y} = \alpha_h \frac{\partial^2 T_h}{\partial y^2} \quad (3)$$

where u_h and v_h denote the velocity components of the hot fluid in the x and y directions, respectively, T_h is the temperature of the hot fluid, g is the gravitational acceleration, β is the

thermal expansion coefficient of the hot fluid, and v_h and α_h are the kinematic viscosity and thermal diffusivity of the heat fluid, respectively.

The boundary conditions for the free convection system are

$$\begin{aligned} u_h = v_h = 0, \quad T_h = T_{wh}(x) \quad \text{on } y = 0 \\ u_h \rightarrow 0, \quad T_h \rightarrow t_h \quad \text{as } y \rightarrow \infty \end{aligned} \quad (4)$$

where $T_{wh}(x)$ denotes the wall temperature facing the hot side of the plate.

Cold fluid

$$\frac{\partial u_c}{\partial x_c} + \frac{\partial v_c}{\partial y_c} = 0 \quad (5)$$

$$u_c \frac{\partial u_c}{\partial x_c} + v_c \frac{\partial u_c}{\partial y_c} = v_c \frac{\partial^2 u_c}{\partial y_c^2} \quad (6)$$

$$u_c \frac{\partial T_c}{\partial x_c} + v_c \frac{\partial T_c}{\partial y_c} = \alpha_c \frac{\partial^2 T_c}{\partial y_c^2} \quad (7)$$

where x_c and y_c are Cartesian coordinates on the forced convection side, u_c and v_c denote the velocity components of the cold fluid in the x_c and y_c directions, and T_c , v_c and α_c are the temperature, kinematic viscosity and thermal diffusivity of the cold fluid, respectively.

The boundary conditions for the forced convection system are

$$\begin{aligned} u_c = v_c = 0, \quad T_c = T_{wc}(x_c) \quad \text{on } y_c = 0 \\ u_c \rightarrow U_\infty, \quad T_c \rightarrow t_c \quad \text{as } y_c \rightarrow \infty \end{aligned} \quad (8)$$

where $x_c = L - x$ and $T_{wc}(x_c)$ denotes the wall temperature facing the forced convection side.

It is assumed that heat conduction along the plate is neglected in comparison with transverse heat conduction. Under this condition, the heat flux entering the left face of the plate will be equal to that leaving the right face at any given vertical position x , i.e.

$$k_s \frac{T_{wh} - T_{wc}}{b} = -k_c \left. \frac{\partial T_c}{\partial y_c} \right|_{y_c=0} = k_h \left. \frac{\partial T_h}{\partial y} \right|_{y=0} = q_{xc} \quad (9)$$

where k_s , k_h and k_c denote the thermal conductivities of the solid plate, hot fluid and cold fluid, respectively, and q_{xc} is the local heat flux through the plate. A correlation between T_{wh} and T_{wc} can be obtained from Eq. (9) as

$$T_{wc} = T_{wh} - \frac{bk_h}{k_s} \left. \frac{\partial T_h}{\partial y} \right|_{y=0}. \quad (10)$$

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Solution

To solve Eqs. (1)-(3) and (5)-(7), we introduce the following dimensionless variables

$$\begin{aligned} \xi = x/L, \quad \eta = (C_h y)/(L\xi^{1/4}) \\ \psi_h = 4v_h C_h \xi^{3/4} f_h(\xi, \eta), \quad \theta_h(\xi, \eta) = [T_h - (t_h + t_c)/2]/\Delta t \\ \xi_c = x_c/L = 1 - \xi, \quad \eta_c = (y_c Re^{1/2})/(L\xi_c^{1/2}) \\ \psi_c = (U_\infty v_c L \xi_c)^{1/2} f_c(\xi_c, \eta_c), \quad \theta_c(\xi_c, \eta_c) = [T_c - (t_h + t_c)/2]/\Delta t \end{aligned} \quad (11)$$

where $C_h = [g\beta\Delta t L^3 / (4v_h^2)]^{1/4}$, $\Delta t = t_h - t_c$, $Re = U_\infty L / v_c$ is the Reynolds number for the forced convection flow, and ψ_h and ψ_c are the stream functions of the hot and cold fluids, respectively, which are defined as

$$u_h = \frac{\partial \psi_h}{\partial y}, \quad v_h = \frac{\partial \psi_h}{\partial x}$$

$$u_c = \frac{\partial \psi_c}{\partial y_c}, \quad v_c = \frac{\partial \psi_c}{\partial x_c} \quad (12)$$

Due to the definition of (11) and (12), Eqs. (1)-(3) and (5)-(7) can be transformed into the following form

$$\frac{\partial^3 f_h}{\partial \eta^3} + 3f_h \frac{\partial^2 f_h}{\partial \eta^2} - 2\left(\frac{\partial f_h}{\partial \eta}\right)^2 - \left(\theta_h - \frac{1}{2}\right) = 4\xi \left(\frac{\partial f_h}{\partial \eta} \frac{\partial^2 f_h}{\partial \eta \partial \xi} - \frac{\partial^2 f_h}{\partial \eta^2} \frac{\partial f_h}{\partial \xi}\right) \quad (13)$$

$$\frac{1}{Pr_h} \frac{\partial^2 \theta_h}{\partial \eta^2} + 3f_h \frac{\partial \theta_h}{\partial \eta} = 4\xi \left(\frac{\partial f_h}{\partial \eta} \frac{\partial \theta_h}{\partial \xi} - \frac{\partial \theta_h}{\partial \eta} \frac{\partial f_h}{\partial \xi}\right) \quad (14)$$

$$\frac{\partial^3 f_c}{\partial \eta_c^3} + \frac{1}{2} f_c \frac{\partial^2 f_c}{\partial \eta_c^2} = \xi_c \left(\frac{\partial f_c}{\partial \eta_c} \frac{\partial^2 f_c}{\partial \eta_c \partial \xi_c} - \frac{\partial^2 f_c}{\partial \eta_c^2} \frac{\partial f_c}{\partial \xi_c}\right) \quad (15)$$

$$\frac{1}{Pr_c} \frac{\partial^2 \theta_c}{\partial \eta_c^2} + \frac{1}{2} f_c \frac{\partial \theta_c}{\partial \eta_c} = \xi_c \left(\frac{\partial f_c}{\partial \eta_c} \frac{\partial \theta_c}{\partial \xi_c} - \frac{\partial \theta_c}{\partial \eta_c} \frac{\partial f_c}{\partial \xi_c}\right) \quad (16)$$

where $Pr_h = v_h/\alpha_h$ and $Pr_c = v_c/\alpha_c$ are the Prandtl numbers of hot and cold fluids, respectively.

The boundary conditions (4) and (8) become

$$f_h = \frac{\partial f_h}{\partial \eta} = 0, \quad \theta_h = \theta_{wh}(\xi) \quad \text{at } \eta = 0$$

$$\frac{\partial f_h}{\partial \eta} \rightarrow 0, \theta_h \rightarrow \frac{1}{2} \quad \text{as } \eta \rightarrow \infty \quad (17)$$

$$f_c = \frac{\partial f_c}{\partial \eta_c} = 0, \quad \theta_c = \theta_{wh}(\xi) - R_t R_t^* \xi^{-1/4} \frac{\partial \theta_h}{\partial \eta} \Big|_{\eta=0} \quad \text{at } \eta_c = 0$$

$$\frac{\partial f_c}{\partial \eta_c} \rightarrow 1, \theta_c \rightarrow -\frac{1}{2} \quad \text{as } \eta_c \rightarrow \infty \quad (18)$$

where $R_t = bk_c Re^{1/2}/(k_s L)$ denotes the thermal resistance ratio of the forced convection flow to the wall, $R_t^* = (k_h C_h)/(k_c Re^{1/2})$ can be regarded as the thermal resistance of the hot fluid to the cold fluid, $\theta_{wh} = [T_{wh} - (t_h + t_c)/2]/\Delta t$ and $\theta_{wc} = [T_{wc} - (t_h + t_c)/2]/\Delta t$. Substituting variables (11) into Eq. (9), we get

$$R_t^* \left(\xi_c^{1/2}/\xi^{1/4}\right) \frac{\partial \theta_h}{\partial \eta} \Big|_{\eta=0} + \frac{\partial \theta_c}{\partial \eta_c} \Big|_{\eta_c=0} = 0 \quad \text{at any given position } x_c. \quad (19)$$

Based on Eq. (9), the local heat transfer coefficient h_{xc} for the forced convection system can be expressed as

$$h_{xc} = [-k_c \frac{\partial T_c}{\partial y_c} \Big|_{y_c=0}] / [T_c(x_c, 0) - t_c] = q_{xc} / [T_c(x_c, 0) - t_c]. \quad (20)$$

The local Nusselt number for the forced convection system can be expressed as

$$Nu_{xc} = \frac{x_c h_{xc}}{k_c} = -Re^{1/2} \xi_c^{1/2} \frac{\partial \theta_c}{\partial \eta_c} \Big|_{\eta_c=0} / [\theta_{wc}(\xi_c) + \frac{1}{2}]. \quad (21)$$

The total heat flux Q through the surface facing the cold fluid is obtained by integrating the local heat flux over the entire height of the plate and can be expressed as

$$Q = k_c \int_0^L \left[-\frac{\partial T_c}{\partial y_c} \right]_{y_c=0} dx_c . \quad (22)$$

Substituting (11) into (22) yields the average Nusselt number for the forced convection system as

$$Nu_c = \frac{Q}{k_c \Delta t} = Re^{1/2} \int_0^1 \left[-\frac{\partial \theta_c}{\partial \eta_c} \right]_{\eta_c=0} / \xi_c^{1/2} d \xi_c = A Re^{1/2} \quad (23)$$

where

$$A = \int_0^1 \left[-\frac{\partial \theta_c}{\partial \eta_c} \right]_{\eta_c=0} / \xi_c^{1/2} d \xi_c . \quad (24)$$

To solve Eq. (13) to (16), a comprehensive and non-iterative numerical scheme is proposed, which is in contrast with the iterative process purposed by Chen and Chang [8,9] using a guessing strategy. Based on Eq. (19), the boundary conditions can be rewritten as

$$f_h = \frac{\partial f_h}{\partial \eta} = 0, \quad \frac{\partial \theta_h}{\partial \eta} = \xi^{1/4} \lambda^*(\xi) \quad \text{at } \eta = 0$$

$$\frac{\partial f_h}{\partial \eta} \rightarrow 0, \theta_h \rightarrow \frac{1}{2} \quad \text{as } \eta \rightarrow \infty \quad (25)$$

$$f_c = \frac{\partial f_c}{\partial \eta_c} = 0, \quad \frac{\partial \theta_c}{\partial \eta_c} = -R_t^* \xi_c^{1/2} \lambda^*(\xi) \quad \text{at } \eta_c = 0$$

$$\frac{\partial f_c}{\partial \eta_c} \rightarrow 1, \theta_c \rightarrow -\frac{1}{2} \quad \text{as } \eta_c \rightarrow \infty \quad (26)$$

and

$$\lambda = 0 \quad \text{at } \eta = 0, \quad \eta_c = 0 \quad (27)$$

where the dummy variable λ is defined as

$$\lambda(\xi, \eta) = R_t R_t^* \lambda^*(\xi) - \theta_h + \theta_c. \quad (28)$$

The system (13)-(16) and (28), together with the boundary conditions (25), (26) and (27), are then solved using the singular perturbation method and the difficulties associated with the guessed interfacial conditions have been obviated. Since this procedure was described in a recent paper by Shu and pop [10], we will not repeat it here. Note that the points $\xi_c = 0$ and 1 are singular which make the problem more difficult.

4

Results and discussion

In this section we discuss the effects of the Prandtl numbers Pr_h and Pr_c , and resistance parameters R_t and R_t^* on the interface temperatures, heat transfer rates and Nusselt numbers. Figures 2 and 3 show variation of $\theta_{wc}(\xi_c)$ with ξ_c for some values of R_t and R_t^* when the two working fluids have $Pr_h = Pr_c = 1$. The results of these figures show that for increasing values of R_t the temperature of the cold side of the wall decreases. It happens because when R_t is increased the wall becomes more effective insulation between the two forced and free convection flows. In contrast, $\theta_{wc}(\xi_c)$ increases as the free convection becomes dominant, i.e.

when the parameter R_t^* increases. It turns out that an increase of R_t leads to a reduction of the heat transfer rates at the cold side of the wall and to an increase of these rates with increasing R_t^* , as can be seen from Fig. 4 and 5. It is evident from fig. 6 and 7 that the effects of R_t and R_t^* on the local Nusselt number are insignificant. Further, fig. 8 to 11 illustrate the variation of $\theta_{wc}(\xi_c)$ and $Nu_{xc}(\xi_c)$ with the distance ξ_c along the wall. As can be seen from Fig. 9 and 11 the temperature and local Nusselt number of the cold side of the wall are less affected by Pr_h . To this end it should be mentioned that the same trends persist for the heat transfer characteristics of the free convection system. But, they are not presented here for the sake of space conservation.

Finally, values of the average Nusselt number are given in Table 1 for $R_t = R_t^* = 1$ and some values of Pr_h and Pr_c . We notice from this table that $Nu_c/Re^{1/2}$ increases with decreasing Pr_h or increasing Pr_c .

5 Conclusions

A conjugate problem of heat transfer between a laminar forced convection flow and a laminar free convection separated by a vertical finite wall was studied theoretically. The axial thermal conduction in the wall was neglected. The governing boundary layer equations subject to conjugate boundary conditions were solved numerically using a very efficient method which differs from the one used by other authors. As a result the temperature distributions and heat transfer rates at both sides of the wall have been determined. The results show that the resistance parameters influence substantially the interactive heat transfer characteristics. We have given particular attention to the case when the resistance parameters $R_t = R_t^* = 1$ which include the situation in which the fluids at both sides of the wall are the same.

It is worth mentioning that the results reported in this paper are in general in agreement with those from the open literature. However, the accuracy of these results can be further evidenced through experiments.

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	$\text{Pr}_h = 0.1$	$\text{Pr}_h = 1$	$\text{Pr}_h = 10$
$\text{Pr}_c = 0.1$	0.4945	0.4941	0.4899
$\text{Pr}_c = 1$	0.4974	0.4970	0.4927
$\text{Pr}_c = 10$	0.5168	0.5164	0.5116

Table 1.

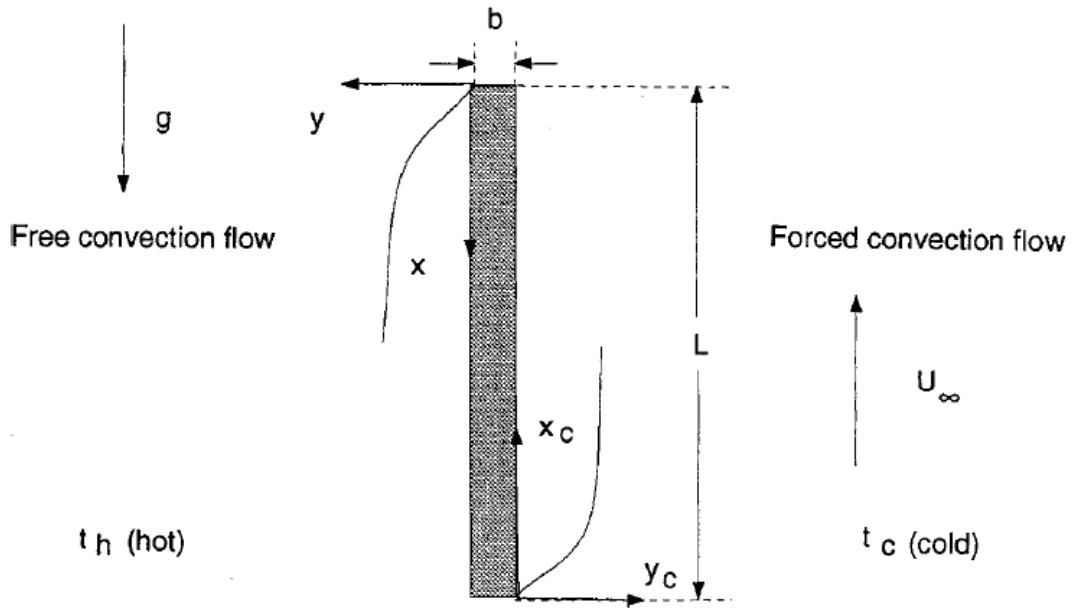


Fig. 1.

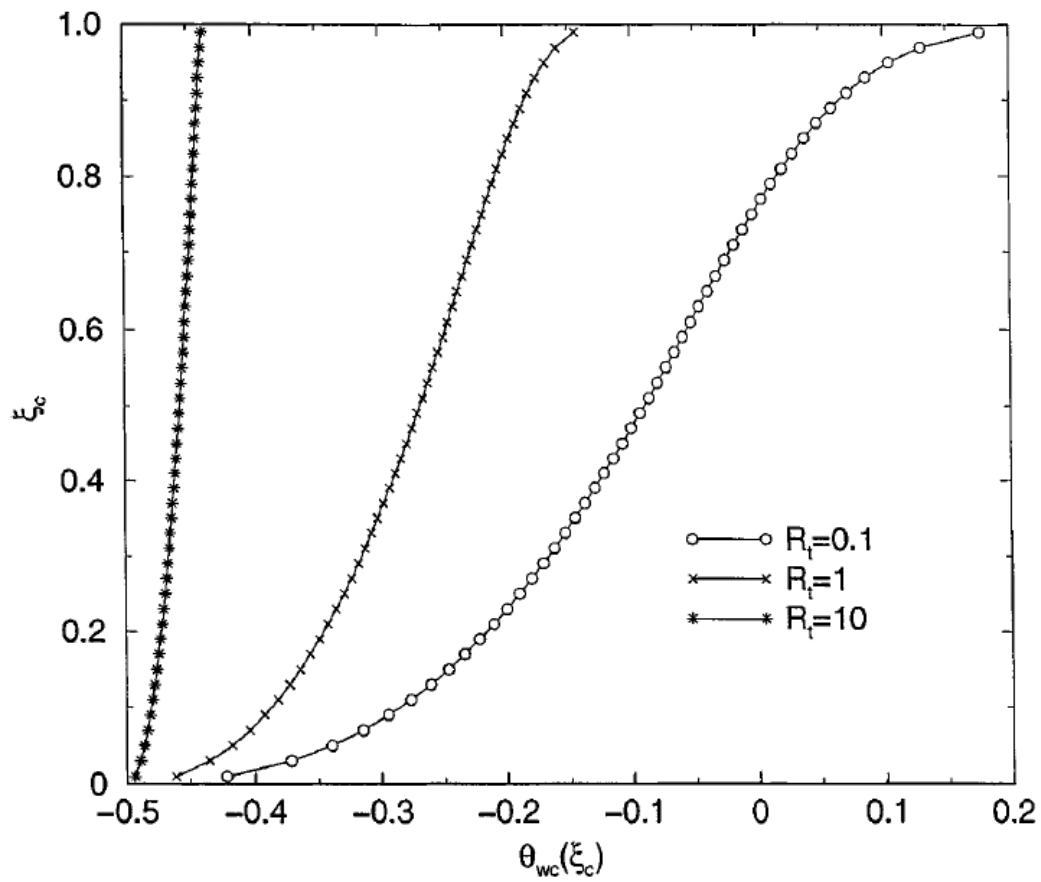


Fig. 2.

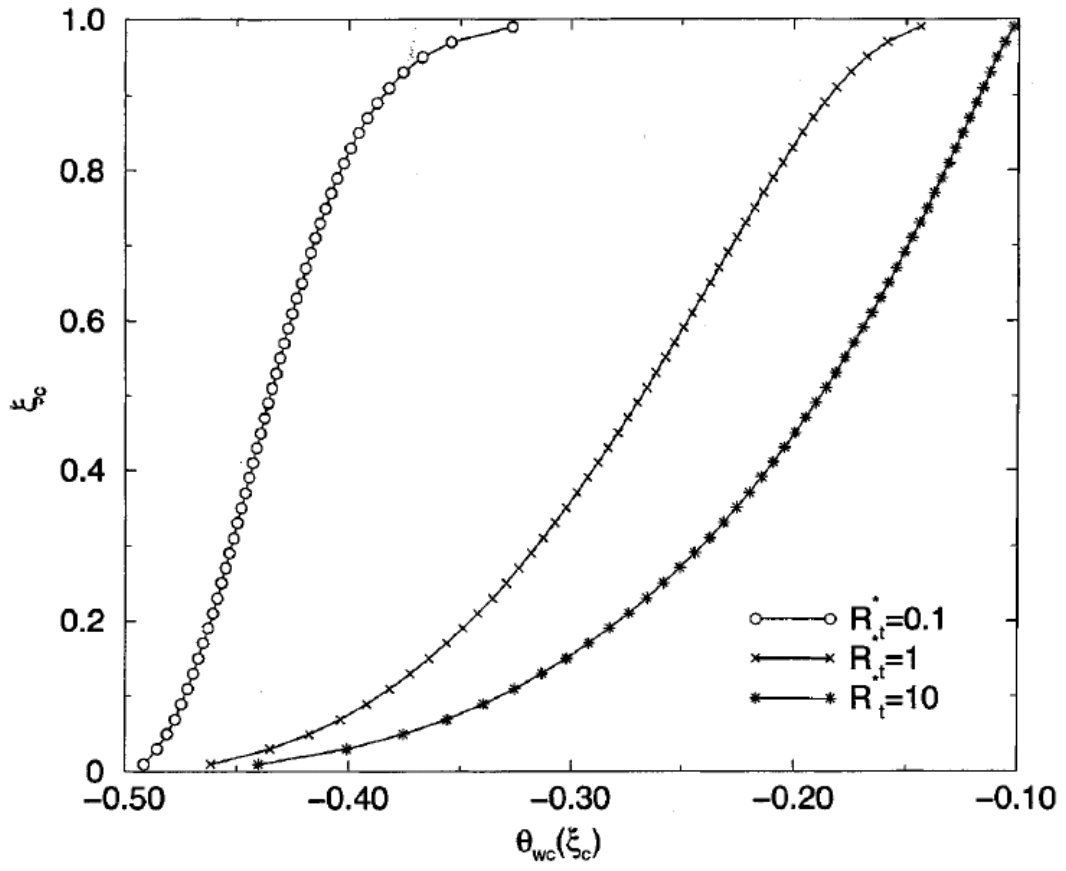


Fig. 3.

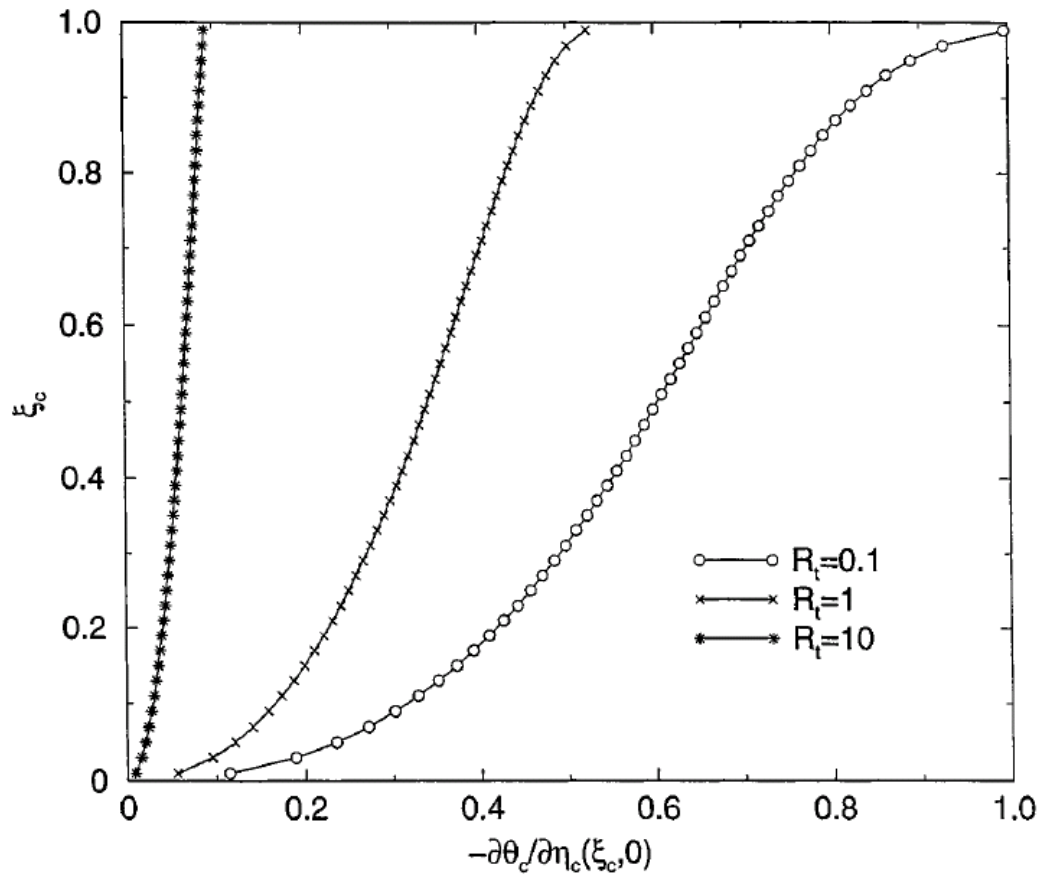


Fig. 4.

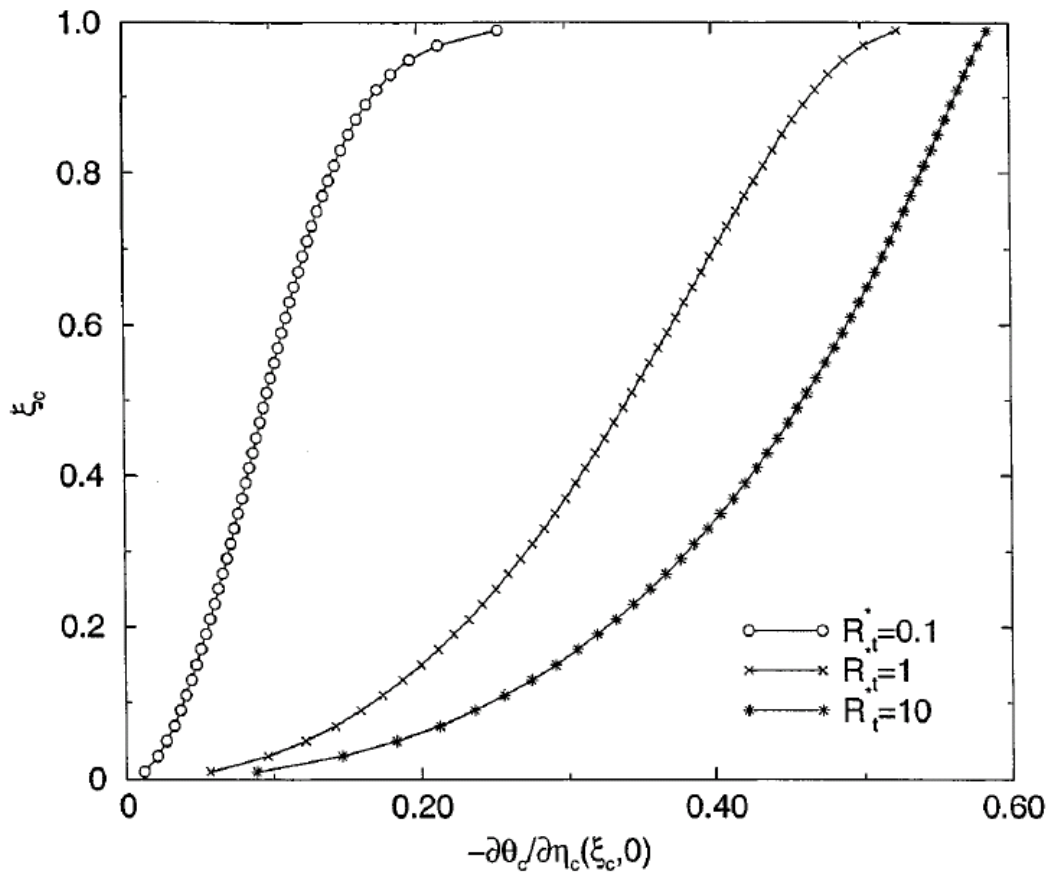


Fig. 5.

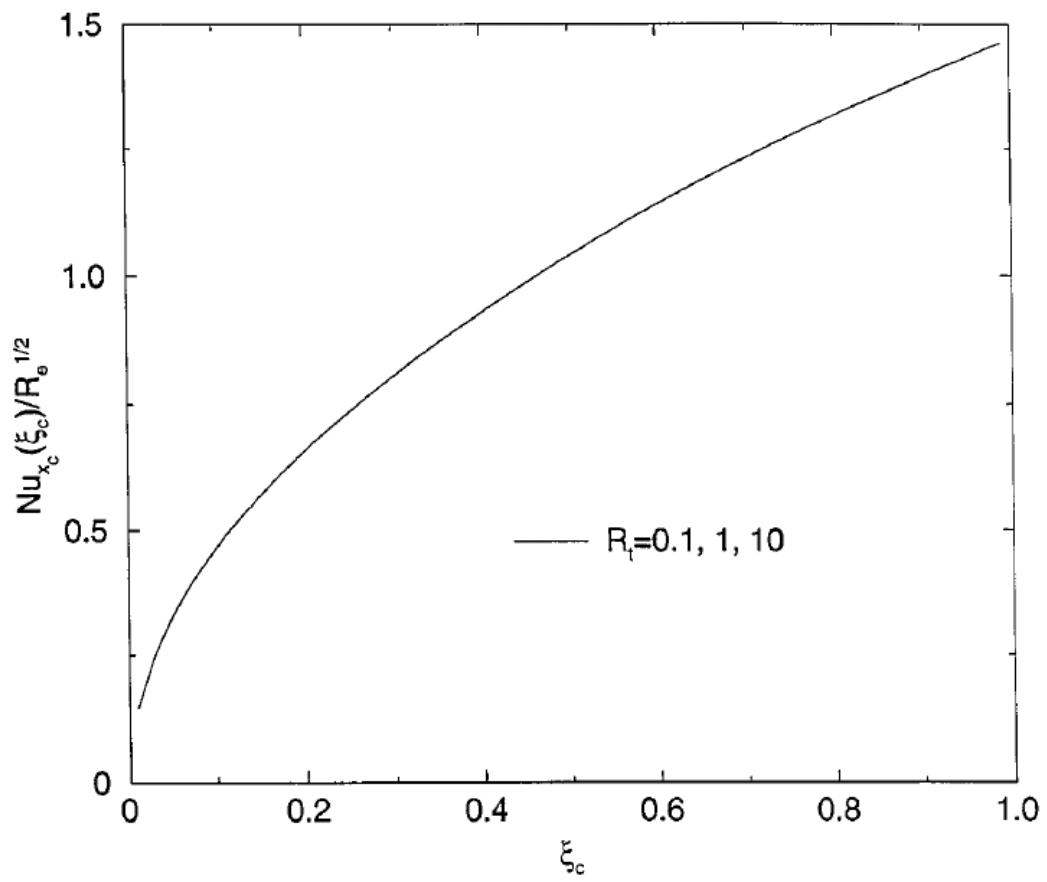


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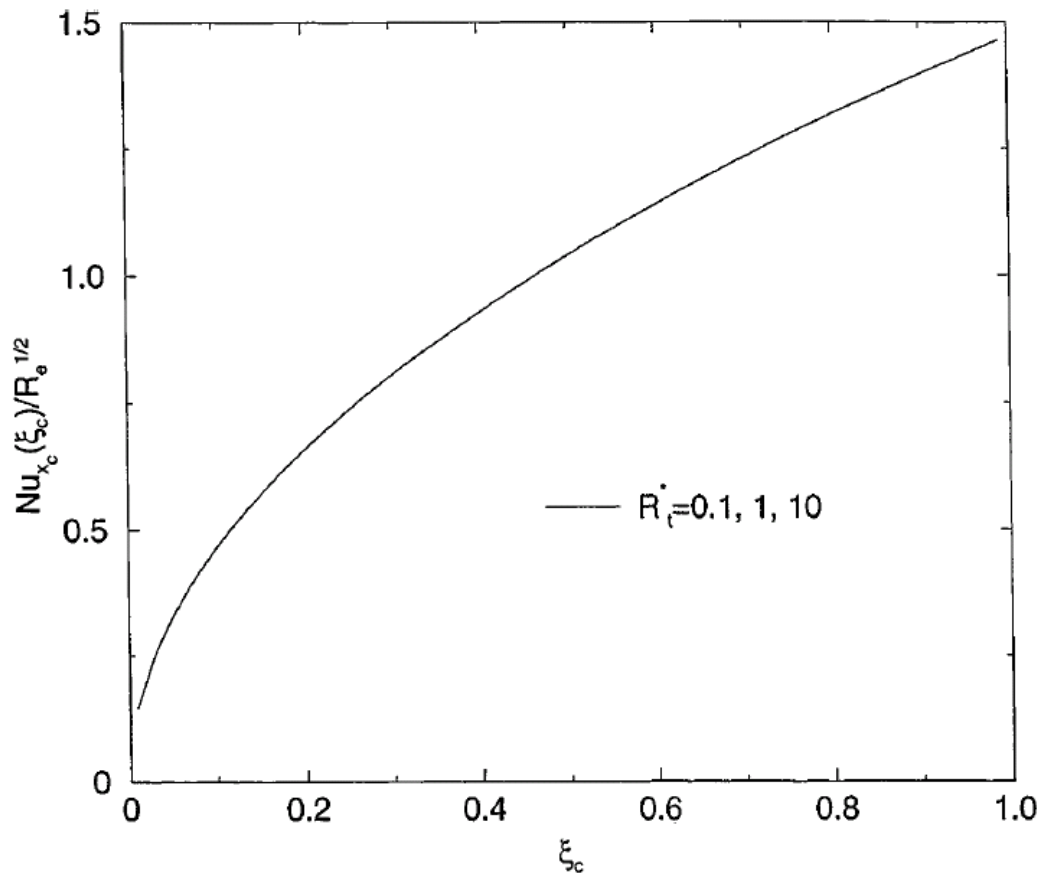


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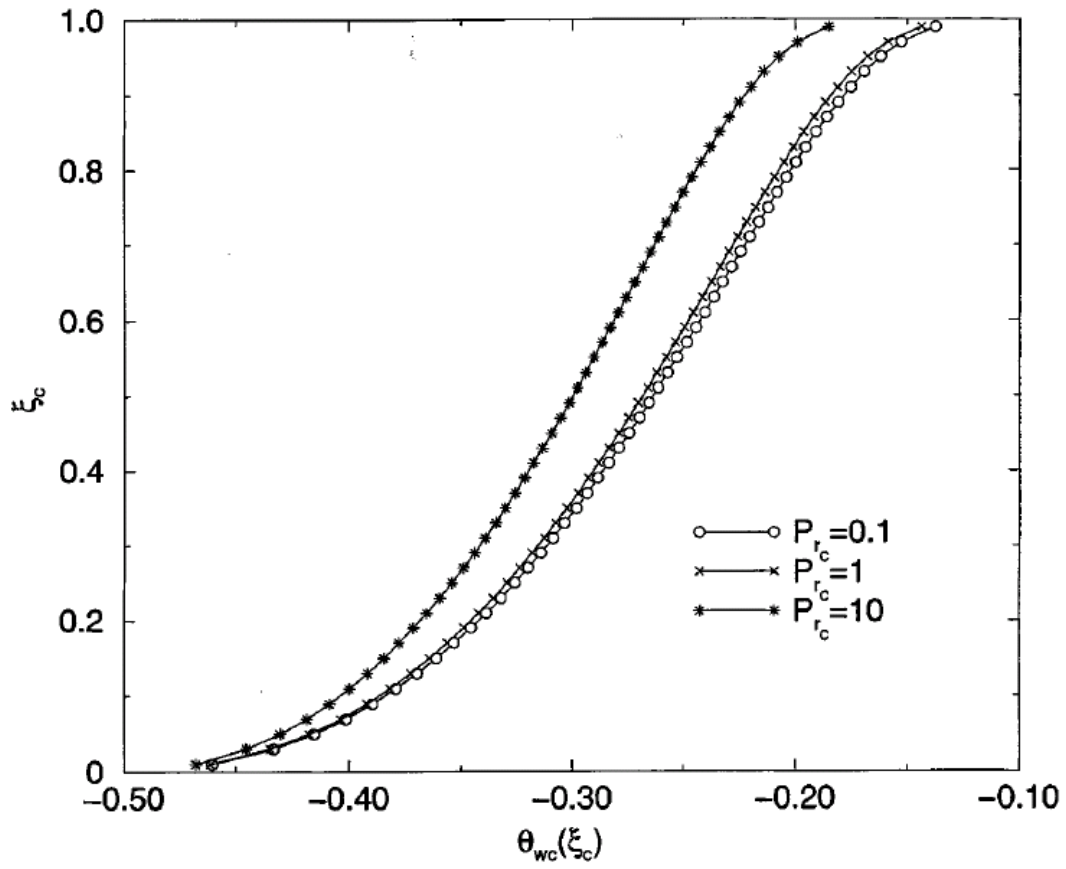


Fig. 8.

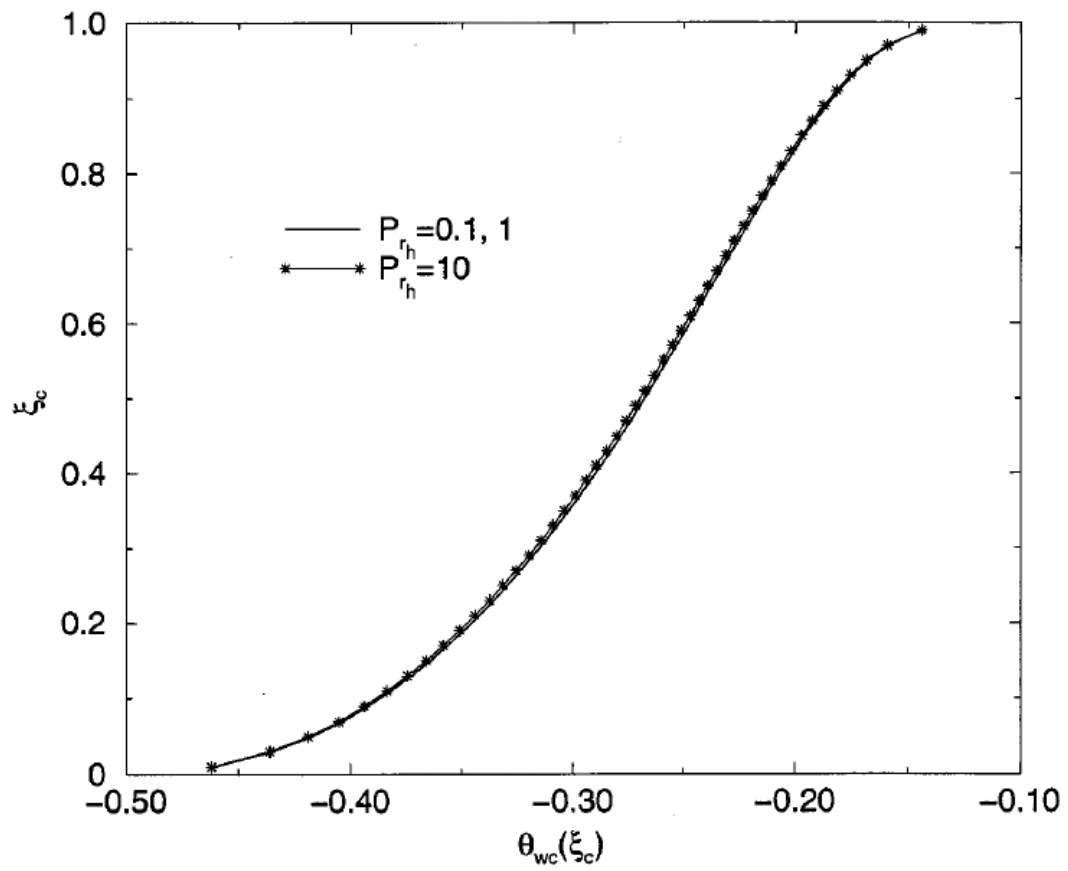


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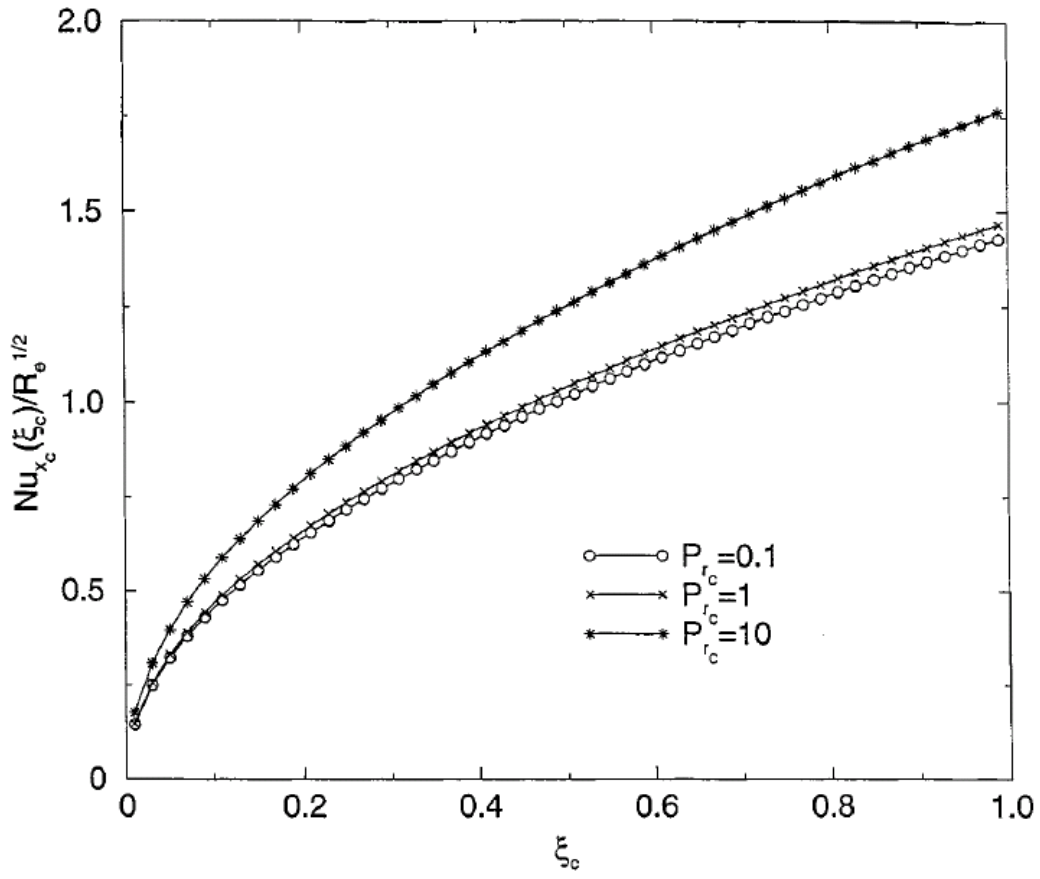


Fig. 10.

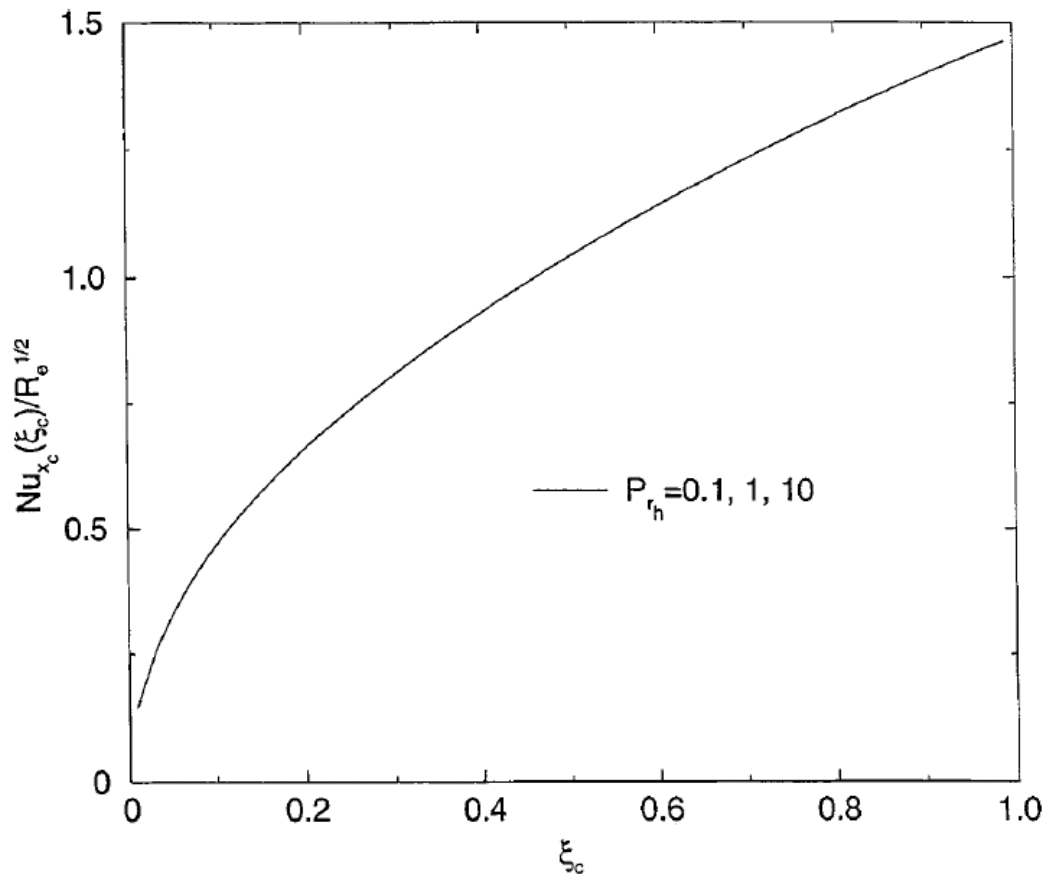


Fig. 11.