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2012

Tang, L., Yang, Y., & Wu, H. (2012). Modeling and Experiment of a Multiple-DOF Piezoelectric Energy Harvester. Proceedings of SPIE - The International Society for Optical Engineering, 8341.

<https://hdl.handle.net/10356/79901>

<https://doi.org/10.1117/12.914764>

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Modeling and Experiment of a Multiple-DOF Piezoelectric Energy Harvester

Lihua TANG, Yaowen YANG*, Hao WU

School of Civil and Environmental Engineering, Nanyang Technological University
50 Nanyang Avenue, 639798, Singapore

ABSTRACT

Vibration energy harvesters have been usually designed as single-degree-of-freedom (1DOF) systems. The fact that such harvesters are only efficient near sole resonance limits their applicability in frequency-variant and random vibration scenarios. In this paper, a novel multiple-DOF piezoelectric energy harvester model (PEHM) is developed, which comprises a primary mass and n parasitic masses. The parasitic masses are independent of each other but attached to the primary mass. The piezoelectric element is placed between the primary mass and the base for energy generation. First, a 2DOF model is analyzed and characterized. Through parametric analysis, it is found that with a slight increase of the overall weight to the original 1DOF harvester (without parasitic masses), two close and effective peaks or one effective peak with significantly enhanced magnitude can be achieved in the power response. Subsequently, the 2DOF model is generalized to an n -DOF model and its analytical solution is derived. This solution provides a convenient tool for parametric study and design of a multiple-DOF piezoelectric energy harvester (PEH). Useful multimodal energy harvesting can be achieved with a slight increase of the overall weight. Finally, a prototype of the proposed multiple-DOF model is devised for proof of concept.

Keywords: vibration, multiple-DOF, piezoelectric energy harvester

1. INTRODUCTION

Vibration energy harvesting provides a promising solution for self-powered wireless sensing electronics and has attracted immense research interests in recent years. Conventional vibration energy harvesters reported in the literature have been usually designed as 1DOF systems, which are only efficient near sole resonance. Unfortunately, the vast majority of practical vibration sources are present in the frequency-variant or random form. Hence, a critical issue in this research field is how to improve the functionality of energy harvesters in practical wideband vibration scenarios [1].

Resonance tuning is one approach to enable a vibration energy harvester adaptive in frequency-variant scenarios. Mechanical preload [2,3] and magnetic force [4] have been frequently exerted to tune the stiffness and thus the fundamental resonant frequency of a harvester. Besides, broadband energy harvesting can be achieved by introducing nonlinearity into the harvester. Monostable [5,6], bistable [7-10] and mechanical stopper [11,12] have been reported as nonlinear configurations to implement a broadband energy harvester. To date, however, how to enable these nonlinear harvesters to persist in high energy orbits for large-amplitude oscillations remains a challenging issue. On the other hand, a system with multiple modes is also capable of harvesting broadband vibration energy. Roundy *et al.* [13] first proposed the idea of multiple-DOF system incorporating multiple proof masses to achieve wider bandwidth. Based on this idea, Yang *et al.* [14] developed an electromagnetic energy harvesting beam with multiple magnets as proof masses and voltage inducing components. Ou *et al.* [15] theoretically modeled a two-mass cantilever beam for broadband energy harvesting. Tadesse *et al.* [16] presented a cantilever harvester integrated with hybrid energy harvesting schemes, each of which is efficient for a specific mode. However, in these designs, the high-order modes are far away from the fundamental mode.

When designing a multimodal energy harvester, having its multiple modes close to each other is rationally preferable for practical random or frequency-variant vibrations. Aldraihem and Baz [17] and Arafa *et al.* [18] studied a 2DOF piezoelectric energy harvester with one mass served as a dynamic magnifier. Although close resonances could be achieved, the magnifier required a huge weight, which places limitations in practice. Erturk *et al.* [19] introduced an L-shaped cantilever harvester where the second natural frequency approximately doubles the first. Jang *et al.* [20] and Kim

* cywyang@ntu.edu.sg, phone: (+65) 67904057, fax: (+65) 67910676.

et al. [21] developed 2DOF electromagnetic and piezoelectric energy harvesting devices in which translation and rotation vibration modes of a single rigid mass were exploited and the two natural frequencies could be designed to be very close to each other. Another way to achieve multiple close resonant frequencies is to assemble an array of cantilever harvesters on a common rigid base. The geometry and proof mass of harvesters can be carefully designed such that their resonant frequencies can be close to each other [22,23]. However, such configuration of achieving broad bandwidth might significantly sacrifice the power density due to the pronounced increase in the weight and volume of the system. Hence, a fine multimodal energy harvesting system is expected to achieve multiple close and effective resonant peaks in targeted bandwidth with least sacrifice of power density.

In this paper, a novel multiple-DOF piezoelectric energy harvester model (PEHM) is presented, which comprises a primary mass and n parasitic masses. First, the 2DOF model is analyzed and characterized. Different from the previous literature [17,18], the piezoelectric element is placed between the primary mass and the base. The advantage of this configuration is that with a slight increase of overall weight to the original 1DOF harvester (without parasitic mass), one can achieve two close and effective peaks in power response (though unable to significantly outperform the 1DOF harvester in magnitude), or one peak with significantly enhanced magnitude. Thus, this configuration overcomes the limitations in the previous 2DOF models. Subsequently, this 2DOF configuration is generalized to an n -DOF PEHM and its analytical solution is provided. This solution provides a convenient tool for parametric study and design of a multiple-DOF piezoelectric energy harvester (PEH). Useful multimodal energy harvesting can be achieved with a slight increase of the overall weight. Finally, a prototype of the proposed multiple-DOF PEHM is devised for proof of concept.

2. ANALYTICAL MODELING

2.1 2DOF Model

Though a distributed parameter system such as a cantilever beam with proof mass carries multiple modes, the contribution to energy harvesting from the high-order modes is usually neglected as they are far away from the fundamental one. Hence, a conventional distributed parameter system can be reduced to a single-mode harvester, which is actually an improved 1DOF model with some correction factor introduced to forcing amplitude [24]. Obviously, it is preferable not only that an energy harvester has multiple modes or multiple DOFs but also that the corresponding peaks in the response are close to each other and with effective magnitudes to contribute to energy harvesting. To design such a multiple-DOF PEHM is the motivation of this paper.

We begin the modeling and analysis of multiple-DOF PEHM from the 2DOF lumped parameter model. The schematic of the 2DOF model is illustrated in Figure 1. The piezoelectric element is placed between the base and mass m_1 . To our best knowledge, such configuration has not been studied in the literature. It comprises two subsystems. The primary subsystem, composed of the primary mass m_1 , spring k_1 , damper η_1 and piezoelectric element, is actually the same as the 1DOF model. The parasitic subsystem is composed of the parasitic mass m_2 , spring k_2 and damper η_2 . Thus, we can regard the 2DOF model as the combination of original 1DOF model with an attached parasitic subsystem.

Figure 1. 2DOF PEHM

By setting $y=u_2-u_1$, $x=u_1-u_0$, the governing equations of the system can be written as,

$$\begin{cases} m_2 \ddot{y} + \eta_2 \dot{y} + k_2 y = -m_2 \ddot{x} - m_2 \ddot{u}_0 \\ (m_1 + m_2) \ddot{x} + \eta_1 \dot{x} + k_1 x + \theta V + m_2 \ddot{y} + (m_1 + m_2) \ddot{u}_0 = 0 \\ -\theta \dot{x} + C^S \dot{V} + V/R_l = 0 \end{cases} \quad (1)$$

where C^S is the clamped capacitance of the piezoelectric element; θ is the electromechanical coupling coefficient of the system; R_l is the electric load; V is the voltage across R_l ; $-\theta V$ is the force induced by backward electromechanical coupling in the dynamic equation; u_0 , u_1 and u_2 are the displacements of base, mass m_1 and m_2 , respectively. Letting

$$\omega_1 = \sqrt{\frac{k_1}{m_1}}, \omega_2 = \sqrt{\frac{k_2}{m_2}}, \zeta_1 = \frac{\eta_1}{2\sqrt{k_1 m_1}}, \zeta_2 = \frac{\eta_2}{2\sqrt{k_2 m_2}}, \mu = \frac{m_2}{m_1} \quad (2)$$

and applying the Laplace transform for (1), we obtain

$$\begin{cases} s^2 \hat{Y} + 2\zeta_2 \omega_2 s \hat{Y} + \omega_2^2 \hat{Y} = -s^2 \hat{X} - s^2 \hat{U}_0 \\ (1 + \mu) s^2 \hat{X} + 2\zeta_1 \omega_1 s \hat{X} + \omega_1^2 \hat{X} + (\theta/m_1) \hat{V} + \mu s^2 \hat{Y} + (1 + \mu) s^2 \hat{U}_0 = 0 \\ -\theta s \hat{X} + C^S s \hat{V} + \hat{V}/R_l = 0 \end{cases} \quad (3)$$

It should be mentioned that ω_1 and ω_2 are the natural frequencies when the primary and parasitic subsystems work separately. Solving (3), we have

$$\hat{V} = \frac{\left(\frac{\mu s^2}{s^2 + 2\zeta_2 \omega_2 s + \omega_2^2} - (1 + \mu) \right) s^2 \hat{U}_0}{\left((1 + \mu) s^2 + 2\zeta_1 \omega_1 s + \omega_1^2 - \frac{\mu s^4}{s^2 + 2\zeta_2 \omega_2 s + \omega_2^2} \right) \frac{R_l C^S s + 1}{R_l \theta s} + \frac{\theta}{m_1}} \quad (4)$$

Setting $s=j\omega$, the dimensionless voltage and power on the resistor R_l are written as

$$|\tilde{V}| = \left| \frac{\hat{V}}{\frac{m_1 \omega^2 \hat{U}_0}{\theta}} \right| = \left| \frac{(1 + \mu) + \frac{\mu \Omega^2}{(\alpha^2 - \Omega^2 + j2\zeta_2 \alpha \Omega)}}{\left(1 - (1 + \mu) \Omega^2 + j2\zeta_1 \Omega - \frac{\mu \Omega^4}{(\alpha^2 - \Omega^2 + j2\zeta_2 \alpha \Omega)} \right) \frac{j r \Omega + 1}{j r k_e^2 \Omega} + 1} \right| \quad (5)$$

$$|\tilde{P}| = \left| \frac{\hat{P}}{\frac{m_1 (\omega^2 \hat{U}_0)^2}{\omega_1}} \right| = \frac{r k_e^2 \Omega^2}{(r \Omega)^2 + 1} \left| \frac{(1 + \mu) + \frac{\mu \Omega^2}{(\alpha^2 - \Omega^2 + j2\zeta_2 \alpha \Omega)}}{\left(1 - (1 + \mu) \Omega^2 + j2\zeta_1 \Omega - \frac{\mu \Omega^4}{(\alpha^2 - \Omega^2 + j2\zeta_2 \alpha \Omega)} \right) + \frac{j r k_e^2 \Omega}{j r \Omega + 1}} \right|^2 \quad (6)$$

where the dimensionless parameters are

$$\alpha = \frac{\omega_2}{\omega_1}, \Omega = \frac{\omega}{\omega_1}, r = \omega_1 C^S R_l, k_e^2 = \frac{\theta^2}{C^S k_1} \quad (7)$$

Given $\mu \rightarrow 0$, the 2DOF model can degrade to 1DOF model, which can be found in [25].

Two Resonant Frequencies

To be versatile in frequency-variant or random vibrations with known dominant bandwidth, the two resonant frequencies of the 2DOF model are preferred to be close to each other. Thus, we need to understand how to tune the system parameters to achieve this. For open circuit condition (i.e., $r \rightarrow \infty$), the dimensionless open circuit voltage of the 2DOF PEHM can be derived from (5) as

$$|\tilde{V}_{oc}| = \frac{\left| k_e^2 \left((1+\mu) + \frac{\mu \Omega^2}{(\alpha^2 - \Omega^2 + j2\zeta_2 \alpha \Omega)} \right) \right|}{\left| \left(1 - (1+\mu)\Omega^2 + j2\zeta_1 \Omega - \frac{\mu \Omega^4}{(\alpha^2 - \Omega^2 + j2\zeta_2 \alpha \Omega)} \right) + k_e^2 \right|} \quad (8)$$

For undamped condition, the following equation from (8)

$$(1 - \Omega^2)(\alpha^2 - \Omega^2) - \mu \alpha^2 \Omega^2 + k_e^2 (\alpha^2 - \Omega^2) = 0 \quad (9)$$

yields the two undamped open circuit resonant frequencies as

$$\Omega_{1,2} = \sqrt{\frac{(1+\mu)\alpha^2 + (1+k_e^2)}{2} \pm \frac{\sqrt{((\mu+1)\alpha^2 + 1 + k_e^2)^2 - 4\alpha^2(1+k_e^2)}}{2}} \quad (10)$$

Figure 2 shows the difference of the two resonant frequencies $\Delta\Omega_{1,2}$ versus α and μ for different coupling coefficient k_e . Though $\Delta\Omega_{1,2}$ tends to increase with μ , we can tune α to an optimal value to minimize $\Delta\Omega_{1,2}$ for each specific μ . We note that the optimal α (for minimal $\Delta\Omega_{1,2}$) decreases with the increase of μ and shifts to the right of α axis with the increase of k_e . According to these results, when designing a 2DOF PEHM, we should carefully select α according to μ and k_e to reduce $\Delta\Omega_{1,2}$. Moreover, with a relatively larger μ , the region near optimal α is much flatter. Thus, the selection of α can be relatively flexible.

Figure 2. $\Delta\Omega_{1,2}$ versus α and μ for different k_e : (a) $k_e=0.02$ (b) $k_e=0.3$ (c) $k_e=0.6$

Effects of α and μ on Peak Magnitude

For a 2DOF model, tuning the two resonant frequencies to be close to each other is only one aspect in system design. To ensure advantageous performance, another aspect is to ensure that both peaks in the power response have significant magnitudes for energy harvesting. Thus, it is equally important to understand how the system parameters affect the magnitudes of the two peaks.

In the following case studies, we select $k_e=0.02$, $\zeta_1=0.02$ and $\zeta_2=0.004$. The dimensionless optimal power (impedance matching is achieved at each frequency) versus Ω and μ for different α is illustrated in Figure 3. For the optimal $\alpha=1$ when $\mu \rightarrow 0$, we note in Figure 3(b) that the two peaks gradually approach each other and merge into one peak (1DOF response). With the increase of μ , the magnitude of the first peak initially decreases then constantly increases, while the magnitude of the second peak monotonically decreases. For $\alpha > 1$ (Figure 3(a)), the first peak of the 2DOF model degrades to the 1DOF peak when $\mu \rightarrow 0$. With the increase of μ , the magnitude of the first peak initially decreases then constantly increases, while the second peak appears, increases and finally decreases. For $\alpha < 1$ (Figure 3(c) and 3(d)), the second peak of the 2DOF model degrades to the 1DOF peak when $\mu \rightarrow 0$. With the increase of μ , the first peak of power appears and monotonically increases, while the second peak monotonically decreases. In general, we can conclude that except for the initial limited range, the magnitude of the first peak increases and the magnitude of the second peak decreases with the increase of μ , regardless of α . Besides, based on these observations, we can conclude that given $\alpha < 1$ and certain small μ , it is possible to achieve both peaks with equally significant magnitudes. This is one important aspect for the design of a 2DOF PEHM.

Given specific μ , Figure 4 further depicts how the two peaks in power response are affected by α . For a very small μ (e.g. 0.04), it is observed that the magnitudes of both peaks are sensitive to α . Besides, the two peaks can be close and have significant contribution to energy harvesting if α is carefully tuned (e.g., $\alpha=0.85$ can give two peaks with nearly equal magnitudes). While, for a relative larger μ (e.g. 0.2), the magnitude of the first peak barely varies with α . Though the first peak has much larger magnitude as compared to that for $\mu=0.04$, the second peak is drastically lower than the first peak and thus its contribution to energy harvesting is negligible. In a word, if one's intention is to have one larger peak response, a relatively larger μ can be selected (i.e., use bigger parasitic mass) and α can be flexibly chosen. While, if one's intention is to have two close peaks with significant magnitudes to contribute to broadband energy harvesting, a relatively small μ should be selected and α should be carefully tuned.

Figure 3. Dimensionless optimal power output versus Ω and μ for different α . (a) $\alpha=1.05$ (b) $\alpha=1$ (c) $\alpha=0.95$ (d) $\alpha=0.9$

Figure 4. Dimensionless optimal power output versus Ω for different α . (a) $\mu=0.04$ and (b) $\mu=0.2$

It should be mentioned that the conditions for small $\Delta\Omega_{1,2}$ and those for two effective peaks or one enhanced peak may conflict. For relatively large μ , fortunately, the selection of α can be flexible to achieve both small $\Delta\Omega_{1,2}$ and one enhanced peak. For relatively small μ , the values of α to achieve small $\Delta\Omega_{1,2}$ and two effective peaks may be inconsistent. For example, with $\mu=0.04$ and $k_e=0.02$, the minimal $\Delta\Omega_{1,2}$ requires $\alpha=0.96$ while two peaks with equal magnitudes requires $\alpha=0.85$. In such case, certain tradeoff should be made.

Effects of Damping on Peak Magnitude

Other than α and μ , the damping in the system is another factor affecting the magnitudes of the two peaks in power response. We assume $k_e=0.02$, $\alpha=1$. Two cases are studied, that is, (1) $\zeta_2=0.004$ and ζ_1 varies and (2) $\zeta_1=0.02$ and ζ_2 varies. For Case (1), it is noted in Figure 5 that ζ_1 significantly affects the performance of both the 2DOF and the original 1DOF harvester (with parasitic subsystem removed). Fortunately, the influence of ζ_1 is almost equivalent to both harvesters. Hence, with various ζ_1 , for $\mu=0.04$, the 2DOF harvester always exhibits two peaks that have the same level of magnitude as the 1DOF harvester; and for $\mu=0.2$, the first peak of the 2DOF harvester always has significantly larger magnitude than the 1DOF harvester. For Case (2), Figure 6 shows that ζ_2 has very minor influence on the performance of the 2DOF harvester. The 1DOF harvester does not include damper η_2 thus is not affected by ζ_2 . Based on these results, when designing a 2DOF PEHM, one should devote efforts to reducing ζ_1 for performance improvement rather than ζ_2 .

Figure 5. Dimensionless optimal power output versus Ω for different ζ_1 . (a) $\mu=0.04$ and (b) $\mu=0.2$

Figure 6. Dimensionless optimal power output versus Ω for different ζ_2 . (a) $\mu=0.04$ and (b) $\mu=0.2$

Summary of 2DOF Model

We have discussed the characteristics of the proposed 2DOF model in terms of $\Delta\Omega_{1,2}$ and magnitudes of two peaks in power responses, as summarized in Table 1. These characteristics provide important guidelines for designing a 2DOF

PEHM. Previously reported 2DOF PEHM [17,18] can achieve two close and magnify the power output as well but huge increase of the overall weight is required, which may not help improve the performance in terms of power density. In general, the advantage of the 2DOF model proposed in this paper is that with a slight increase of overall weight (parasitic mass added to original 1DOF model), the system can be tuned to provide: **(a)** two close and effective peaks (though their magnitudes may not be increased), or **(b)** one peak with significantly enhanced magnitude than the 1DOF model.

Table 1. Characteristics of proposed 2DOF PEHM

$\Delta\Omega_{1,2}$	Magnitudes of two peaks
<ul style="list-style-type: none"> ◆ A small $\Delta\Omega_{1,2}$ requires a small μ (slight increase of weight to original 1DOF model provides two close resonances) ◆ Slowly increases with μ at optimal α ◆ Optimal α for minimum $\Delta\Omega_{1,2}$ increases with k_e and decreases with μ ◆ For larger μ, $\Delta\Omega_{1,2}$ versus α is flat around optimal α (minor mistuning of α still ensures $\Delta\Omega_{1,2}$ close to minimum) 	<ul style="list-style-type: none"> ◆ Except for the limited range close to $\mu \rightarrow 0$, the magnitude of first peak increases while the second peak decreases with μ. ◆ For a very small μ, the performance is sensitive to α and both peaks can contribute to energy harvesting (though the magnitudes may not be larger than that of the 1DOF model). While for a relatively large μ, the performance is insensitive to α and only the first peak has significant contribution to energy harvesting and can outperform the 1DOF model. ◆ The performance is sensitive to ζ_1 but insensitive to ζ_2. Besides, ζ_1 has nearly equivalent influence on both 2DOF and 1DOF models.

2.2 Generalized n -DOF Model

With more small parasitic masses attached, we can generalize the 2DOF PEHM to an n -DOF PEHM, as shown in Figure 7. It is expected that more effective peaks can be obtained with minor increase of overall weight.

Figure 7. n -DOF PEHM

Setting $x=u_1-u_0$, $y_1=u_2-u_1$, $y_2=u_3-u_1$, ..., $y_{n-1}=u_n-u_1$, the governing equations of the n -DOF PEHM can be written as

$$\begin{cases} \sum_{p=1}^n m_p \ddot{x} + \eta_1 \dot{x} + k_1 x + \theta V + \sum_{p=2}^n m_p \ddot{y}_{p-1} + \sum_{p=1}^n m_p \ddot{u}_0 = 0 \\ m_2 \ddot{y}_1 + \eta_2 \dot{y}_1 + k_2 y_1 = -m_2 \ddot{x} - m_2 \ddot{u}_0 \\ m_3 \ddot{y}_2 + \eta_3 \dot{y}_2 + k_3 y_2 = -m_3 \ddot{x} - m_3 \ddot{u}_0 \\ \vdots \\ m_n \ddot{y}_{n-1} + \eta_n \dot{y}_{n-1} + k_n y_{n-1} = -m_n \ddot{x} - m_n \ddot{u}_0 \\ -\theta \dot{x} + C^S \dot{V} + V/R_l = 0 \end{cases} \quad (11)$$

Assuming that all the parasitic masses have the same weight, i.e., $m_2=m_3=\dots=m_n$ and setting,

$$\mu = \frac{m_2}{m_1}, \omega_1 = \sqrt{\frac{k_1}{m_1}}, \omega_2 = \sqrt{\frac{k_2}{m_2}}, \dots, \omega_n = \sqrt{\frac{k_n}{m_n}}, \zeta_1 = \frac{\eta_1}{2\sqrt{k_1 m_1}}, \zeta_2 = \frac{\eta_2}{2\sqrt{k_2 m_2}}, \dots, \zeta_n = \frac{\eta_n}{2\sqrt{k_n m_n}} \quad (12)$$

we can write

$$\begin{cases} (1 + (n-1)\mu)\ddot{x} + 2\zeta_1\omega_1\dot{x} + \omega_1^2 x + (\theta/m_1)V + \mu \sum_{p=2}^n \ddot{y}_{p-1} + (1 + (n-1)\mu)\ddot{u}_0 = 0 \\ \ddot{y}_1 + 2\zeta_2\omega_2\dot{y}_1 + \omega_2^2 y_1 = -\ddot{x} - \ddot{u}_0 \\ \ddot{y}_2 + 2\zeta_3\omega_3\dot{y}_2 + \omega_3^2 y_2 = -\ddot{x} - \ddot{u}_0 \\ \vdots \\ \ddot{y}_{n-1} + 2\zeta_n\omega_n\dot{y}_{n-1} + \omega_n^2 y_{n-1} = -\ddot{x} - \ddot{u}_0 \\ -\theta\dot{x} + C^S\dot{V} + V/R_l = 0 \end{cases} \quad (13)$$

Applying the Laplace transform for (13), it is not difficult to find

$$\hat{V} = \frac{\left(-(1 + (n-1)\mu) + \mu s^2 \sum_{p=2}^n \frac{1}{s^2 + 2\zeta_p\omega_p s + \omega_p^2} \right) s^2 \hat{U}_0}{\left((1 + (n-1)\mu)s^2 + 2\zeta_1\omega_1 s + \omega_1^2 - \mu s^4 \sum_{p=2}^n \frac{1}{s^2 + 2\zeta_p\omega_p s + \omega_p^2} \right) \frac{R_l C^S s + 1}{R_l \theta} + \frac{\theta}{m_1}} \quad (14)$$

Setting $s = j\omega$, the dimensionless voltage and power on the resistor R_l are obtained as

$$|\tilde{V}| = \left| \frac{\hat{V}}{\frac{m_1 \omega^2 \hat{U}_0}{\theta}} \right| = \left| \frac{(1 + (n-1)\mu) + \mu \Omega^2 \sum_{p=2}^n \frac{1}{\alpha_{p-1}^2 - \Omega^2 + j2\zeta_p \alpha_{p-1} \Omega}}{\left(1 - (1 + (n-1)\mu)\Omega^2 + j2\zeta_1 \Omega - \mu \Omega^4 \sum_{p=2}^n \frac{1}{\alpha_{p-1}^2 - \Omega^2 + j2\zeta_p \alpha_{p-1} \Omega} \right) \frac{j r \Omega + 1}{j r k_e^2 \Omega} + 1} \right| \quad (15)$$

$$|\tilde{P}| = \left| \frac{\hat{P}}{\frac{m_1 (\omega^2 \hat{U}_0)^2}{\omega_1}} \right| = \frac{r k_e^2 \Omega^2}{(r \Omega)^2 + 1} \left| \frac{(1 + (n-1)\mu) + \mu \Omega^2 \sum_{p=2}^n \frac{1}{\alpha_{p-1}^2 - \Omega^2 + j2\zeta_p \alpha_{p-1} \Omega}}{\left(1 - (1 + (n-1)\mu)\Omega^2 + j2\zeta_1 \Omega - \mu \Omega^4 \sum_{p=2}^n \frac{1}{\alpha_{p-1}^2 - \Omega^2 + j2\zeta_p \alpha_{p-1} \Omega} \right) + \frac{j r k_e^2 \Omega}{j r \Omega + 1}} \right|^2 \quad (16)$$

where the dimensionless parameters are

$$\alpha_1 = \frac{\omega_2}{\omega_1}, \alpha_2 = \frac{\omega_3}{\omega_1}, \dots, \alpha_{n-1} = \frac{\omega_n}{\omega_1}, \Omega = \frac{\omega}{\omega_1}, r = \omega_1 C^S R_l, k_e^2 = \frac{\theta^2}{C^S k_1} \quad (17)$$

Multiple Resonant Frequencies

For open circuit condition (i.e., $r \rightarrow \infty$), the voltage is obtained as,

$$|\tilde{V}_{oc}| = \left| \frac{\hat{V}_1}{\frac{m_1 \omega^2 \hat{U}_0}{\theta}} \right| = \left| \frac{k_e^2 \left((1 + (n-1)\mu) + \mu \Omega^2 \sum_{p=2}^n \frac{1}{\alpha_{p-1}^2 - \Omega^2 + j2\zeta_p \alpha_{p-1} \Omega} \right)}{\left(1 - (1 + (n-1)\mu)\Omega^2 + j2\zeta_1 \Omega - \mu \Omega^4 \sum_{p=2}^n \frac{1}{\alpha_{p-1}^2 - \Omega^2 + j2\zeta_p \alpha_{p-1} \Omega} \right) + k_e^2} \right| \quad (18)$$

The undamped open circuit resonant frequencies can be determined with the following equation from (18),

$$\left(1 - (1 + (n-1)\mu)\Omega^2 - \mu \Omega^4 \sum_{p=2}^n \frac{1}{\alpha_{p-1}^2 - \Omega^2} \right) + k_e^2 = 0 \quad (19)$$

It is difficult to express the multiple resonant frequencies explicitly from the above equation, but they can be determined with numerical method.

Case Study

In the following case studies, a 3DOF model is analyzed. We assume $k_e=0.02$, $\zeta_1=0.02$ and $\zeta_2=\zeta_3=0.004$. The dimensionless optimal power versus Ω and μ with various combinations of α_1 and α_2 is shown in Figure 8. Obviously, the effects of μ , α_1 and α_2 on the three peaks in the power response are more complicated than the 2DOF model. First, for $\alpha_1=1$ or $\alpha_2=1$, two of the three peaks will merge when $\mu \rightarrow 0$, as shown in Figures 8(a) and 8(b). If μ is not extremely small, for all cases the magnitude of first peak increases while the magnitudes of other two decreases with the increase of μ . Moreover, among the four combinations of α_1 and α_2 , only for $\alpha_1=1$ and $\alpha_2 < 1$ (Figure 8(b)), all three peaks have significant magnitudes at certain μ (swap between α_1 and α_2 , i.e., $\alpha_2=1$ and $\alpha_1 < 1$ does not affect this conclusion).

With fixed $\alpha_1=1$, Figures 9 and 10 illustrates the dimensionless open circuit voltage and optimal power for various α_2 at specific mass ratios $\mu=0.04$ and $\mu=0.12$. The damping is still assumed as $\zeta_1=0.02$ and $\zeta_2=\zeta_3=0.004$. When α_2 decreases from 1.1 to 1 then further to 0.8, the second peak declines, disappears (for $\alpha_1=\alpha_2$), reappears and increases. For $\mu=0.04$, though the magnitudes of the three peaks may not be larger than that of the 1DOF model, all of them can be tuned to have significant values that contribute to energy harvesting (for example, $\alpha_2=0.8$), as shown in Figure 9(a) and 10(a). For a relatively large $\mu=0.12$, the first peak contributes most to energy harvesting. Its magnitude can greatly surpass that of the 1DOF model and almost insensitive to the change of α_2 . Thus, we can design a multiple-DOF PEHM to have either multiple effective peaks with reasonable magnitudes, or one peak with significantly enhanced magnitude.

It should be mentioned that with more DOFs introduced, the effects of parameters on system behavior become more complicated. Fortunately, the analytical solution given in (15) and (16) provides a way to investigate the system performance through parametric study, which would greatly assist in designing a multiple-DOF PEHM.

Figure 8. Dimensionless optimal power output versus Ω and μ . (a) $\alpha_1=1.1$, $\alpha_2=1$ (b) $\alpha_1=1$, $\alpha_2=0.8$ (c) $\alpha_1=0.9$, $\alpha_2=0.8$ (d) $\alpha_1=1.2$, $\alpha_2=0.8$

Figure 9. Dimensionless open circuit voltage versus Ω for different α_2 . (a) $\mu=0.04$ and (b) $\mu=0.12$

Figure 10. Dimensionless optimal power output versus Ω for different α_2 . (a) $\mu=0.04$ and (b) $\mu=0.12$

3. EXPERIMENT STUDY

To prove the concept of multiple-DOF PEH, a 3DOF PEH is fabricated for the experimental test. Figure 11 shows the schematic and dimensions of the prototype.

Figure 11. Schematic and dimensions of 3DOF PEH prototype

The 3DOF PEH comprises a primary beam and two parasitic beams. A piezoelectric transducer (MFC, model: M2814P2) is bonded at the root of the primary beam. A steel proof mass of 25g, including the weight of the screws, is fixed at the free end of the primary beam. First, without the parasitic beams, i.e., a conventional 1DOF harvester, the resonant frequency of the primary beam is measured as 53.5Hz. Two parasitic beams are then attached with the same small proof mass (0.86g or 2.58g). One of the parasitic beams has a fixed length of 51mm or 37.6mm for the proof mass of 0.86g or 2.58g, respectively. This ensures its resonant frequency to be 53.5Hz and thus $\alpha_1=1$. For another parasitic beam, the length is tuned to achieve different α_2 and the corresponding position is marked. The equivalent mass of a cantilever beam with proof mass can be estimated by $m_{eq}=33/140\rho L+M_p$, where ρ is mass per length of the cantilever and M_p is the mass of the proof mass. Thus, the equivalent mass of the primary beam is 32.78g. For the parasitic beam with fixed length of 51mm (for proof mass of 0.86g) or 37.6mm (for proof mass of 2.58g), the equivalent mass is 1.11g or 2.76g, respectively. For the other parasitic beam with tunable length, since the distributed mass of the aluminum cantilever is much smaller than the steel proof mass, its equivalent mass is assumed to be unchanged when its length is tuned. Hence, the mass ratio μ_{eq} of the parasitic beam and the primary beam is 0.034 and 0.084 for the parasitic proof mass of 0.86g and 2.58g, respectively.

The parasitic beams and the primary beam compose the 3DOF harvester for the experimental test. The overall experimental setup is shown in Figure 12. The harmonic excitation signal is generated by the function generator, adjusted by the power amplifier and finally fed to the seismic shaker. In the experiment, the excitation frequency is manually swept from 20Hz to 90Hz. During this sweeping procedure, the acceleration is monitored by an acceleration data logger as feedback loop and controlled at 1m/s^2 . The open circuit voltage output from the MFC is logged by the digital multimeter.

Figure 12. Experiment setup

Figure 13. Frequency response of open circuit voltage for different α_2 . (a) $\mu_{eq}=0.034$ and (b) $\mu_{eq}=0.084$

Similar to the analytical study, with fixed $\alpha_1=1$, we measured the frequency response of open circuit voltage for different α_2 at specific mass ratios $\mu_{eq}=0.034$ and $\mu_{eq}=0.084$, as shown in Figure 13. For $\mu_{eq}=0.034$, the three peaks are sensitive to α_2 and it is possible to tune their magnitudes to have nearly equal contribution for energy harvesting. For a relatively large mass ratio $\mu_{eq}=0.084$, the first peak in the frequency response is almost insensitive to α_2 and its magnitude always surpasses that of the 1DOF harvester. In general, the trends shown in Figure 13 are consistent with the analytical prediction, as compared to Figure 9. However, it is found that the third peak in experiment also has significant output even for $\mu_{eq}=0.084$, which is some kind different from the analytical estimation.

4. CONCLUSION

This paper presents a novel multiple-DOF PEH. First, a 2DOF PEHM is investigated and characterized. The advantage of the proposed 2DOF PEHM is that with a slight increase of the overall weight to the original 1DOF harvester, one can achieve two close and effective peaks in power response, or one peak with significantly enhanced magnitude. Subsequently, the 2DOF configuration is generalized to an n -DOF PEHM and its analytical solution is provided. This solution provides a convenient tool for parametric study and design of a multiple-DOF PEH. It is found that useful multimodal energy harvesting is achievable with a slight increase of the overall weight. Finally, a prototype of the proposed multiple-DOF PEHM is devised and the experimental test proves the fidelity and feasibility of the proposed multiple-DOF harvester.

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