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Onset of space charge limited current for field emission from a single sharp tip

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A protrusive model of the Child-Langmuir (CL) law is proposed for non-uniform electron field emission from a sharp tip of either Lorentzian or hyperboloid shape. The model is expressed as $J_c = \mu \times J_{1D}$ at large electric field higher than a critical value E_c , and J_{1D} is the classical one-dimensional CL law. It is found that the enhancement μ over the 1D CL law depends only on the calculated field enhancement factor (β) of the emitter and is independent of the work function and gap spacing. In particular, μ increases with β and it converges to a constant at $\beta \gg 1$ for a very sharp tip with small emission area. © 2012 American Institute of Physics.

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I. INTRODUCTION

Space-charge-limited (SCL) electron flow describes the maximum current density allowed for steady-state electron beam transport across a planar diode of spacing D and applied voltage V_g . In the one-dimensional (1D) classical regime, it is known as the Child-Langmuir (CL) law^{1,2} given by

$$J_{1D} = \frac{4\epsilon_0}{9} \sqrt{\frac{2e}{m}} \frac{V_g^{3/2}}{D^2}, \quad (1)$$

where e and m are the charge and mass of the electron, respectively, and ϵ_0 is the permittivity of free space. Extensive studies have been done to revise the 1D classical CL law to multi-dimensional models,^{3–8} quantum regime,^{9–13} ultra-short time scale,^{14–16} and Coulomb blockade regime.¹⁷ Recently, nano-scale field emitter^{18–21} is able to emit very high current at relatively low voltages or electric fields, which corresponds to a very high local electron current density that space charge effects cannot be ignored. Similar high current density can also be obtained by using ultrafast lasers to induce photo-field electron transmission from nanotips.^{22–28} Thus, it is of interests to have a protrusive model to estimate the onset of the space charge limited electron *nonuniform* emission from a single sharp tip and calculate its enhancement factor as compared to the traditional 1D CL law.^{1,2}

It is important to note that the prior 2D CL law^{3–6} is not applicable in our situation as the emission area of the nanotip is much smaller than the gap spacing, and it has a non-uniform emission profile on its emitting surface. Furthermore, prior models that studied the transition from field emission or Fowler Nordheim (FN) law²⁹ to CL law has been restricted to flat emitting surface.^{30–32} It is only recently that a 3D model was developed to study the space charge effects of a field emitter array.^{33,34} Thus, it is interesting to see if one can develop a simple protrusive CL model to

account for the sharpness effect of the cathode, which is absent from the prior 2D CL law developed for a pristine surface.^{3–6} It is generally argued that the space charge effects can be ignored for a very sharp emitter, so it is also necessary to have a protrusive CL law to determine if such assumption is valid.

The basic approach in the development of the prior 2D CL law is to use the electron density profile obtained from the 1D CL law to estimate the amount of electron current density (from a finite emission area larger than the gap spacing) in order to suppress the electric field at the center of the emitting surface to zero due to the space charge effects. For a sharp emitter, it is clear that these assumptions are not valid as the vacuum electric field on the tip is not uniform and the profile of the emitted electron density is very sensitive to the localized electric field controlled by the geometrical shape of the emitter. The characteristic size of the emission area for the sharp tip is also much smaller than the gap spacing.

In our model, we will use the electron density profile obtained from the field emission model (without the space charge effects) to estimate the required emitted current density (or charge density) that is able to produce a sufficient large space charge field to suppress the applied local electric field at the apex of the emitter. At a critical applied field or voltage, we will see that the resultant space charge field will completely suppress the local electric field at the tip's apex to zero, and thus reach the space charge limited condition assumed in the CL law. From our model, this critical applied field E_c will be calculated as a function of work function Φ , gap spacing D , and field enhancement β for two geometrical types of emitters: Lorentzian shape³⁵ and hyperboloid shape.³⁶ The corresponding normalized SCL current density μ (in terms of the 1D CL law) can then be calculated at this critical field E_c .

The detail models for emitters of Lorentzian shape and hyperboloid shape are presented, respectively, in Secs. II and III. Further analysis and discussions are shown in Sec. IV. Finally, we provide concluding remarks in Sec. V.

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II. LORENTZIAN SHAPE

Lorentzian model (Fig. 1 insert)³⁵ is a mathematic model frequently used to describe the shape of a field emitter. The apex of the emitter is located at $(x=0, y=h)$, and the planar cathode and anode is, respectively, at $y=0$ and $y=D$, where $D \gg h$ is the gap spacing, h and w is the height and the full-width-at-half-maximum (FWHM) of the emitter, respectively. Unless it is mentioned specifically in the paper, $D = 10 \mu\text{m}$ and $h = 0.1 \mu\text{m}$ or $h/D = 0.01$ is assumed in the calculation. The profile of the field enhancement β along the surface of the emitter is characterized by the aspect ratio h/w , which can be expressed as

$$\beta = \frac{\sqrt{X_0^2 + (Y_0 + \epsilon)^2}}{\{[1 - (Y_0 + \epsilon)^2 + X_0^2]^2 + [2X_0(Y_0 + \epsilon)]^2\}^{1/4}}, \quad (2)$$

where $\epsilon \approx w/3.464h$, X_0 and Y_0 are the normalized emission points on the emitter surface as shown by Eq. (5) in Ref. 35. Figure 1 shows the dependence of β for a Lorentzian type emitter for different values of h/w , as a function of the vertical displacement (Δy) measured from the apex. It is clear that the maximum enhancement factor $\beta_{\max}$ is always at the tip of the emitter, which decreases very rapidly away from the apex, e.g., at $\Delta y > 1 \text{ nm}$. Note the enhancement values at the apex are $\beta_{\max} = 34.64, 346.4$, and 3464 for $h/w = 10, 100$, and 1000 , respectively.

For a given applied electric field $E_0 = V_g/D$, where V_g is the applied gap voltage, we can always express the exact local field strength at any points (x_0, y_0) on the emitter surface as $E_s = E_0 \times \beta$. For a given E_s , we can then determine its local emission current density J (without space charge effect) by using the FN law²⁹ given by

$$J = AE_s^2 e^{-B/E_s}, \quad (3)$$

where $A = 1.54 \times 10^{-6} \exp(10.4/\sqrt{\Phi})/\Phi$, $B = 6.44 \times 10^9 \Phi^{3/2}$, and Φ [eV] is the material work function. Equation (3)

FIG. 1. The local enhancement factor profile for Lorentzian shape at different h/w ratio. The Δy represents the vertical displacement from the apex position.

clearly indicates that the local emission current density J is very sensitive to the local enhancement factor β . Since β decreases very fast from the apex, the local emission current density J will be much larger near the tip region as compared to the region at the tail. For a work function of $\Phi = 4 \text{ eV}$, $h/w = 10$, and fixed $E_0 = 10^8 \text{ V/m}$, we have $\beta_{\max} = 34.64$ and $J = 2.9087 \times 10^8 \text{ A/m}^2$ at the apex. In comparison, we have $J \approx 0$ at the tail region with $\beta = 1$. This again implies that most emitted electrons will concentrate near to the tip, and it is not necessary to record for the entire emission over a large surface. For Lorentzian shape, we thus ignore all emission points in the region with $\beta < 1$.

Consider a macro electron is emitted from a given point at the surface of the emitter, carrying a current density J determined by Eq. (3), it will be accelerated towards the anode, and its trajectory can be determined by the vacuum electric field $\mathbf{E} = E_x + iE_y$ given by

$$E_x + iE_y = \frac{-iE_0[x - i(y+b)]}{\{[a^2 - (y+b)^2 + x^2] - 2ix(y+b)\}^{1/2}}, \quad (4)$$

where $a \approx h + \frac{w}{3.464}$ and $b \approx \frac{w}{3.464} (1 + \frac{1}{3.464h/w})$. In Fig. 2, we have depicted several emitting electron trajectories emitted from different surface locations (x_0, y_0) such as from the apex (red line) and the tail with $\beta = 1$ (cyan line).

From the figure, it is obvious that the electron trajectories will not intersect with each other. This property implies that we may assume the magnitude of the current density $|\vec{J}(x, y)|$ along a given trajectory is always equal to the local emission current density $J(x_0, y_0)$ (calculated by Eq. (3)), where (x_0, y_0) is the initial emitting position on the emitter surface. Thus for a given trajectory, we could obtain the electron charge density $n(x, y)$ in the vacuum space by solving $en(x, y)|\vec{v}(x, y)| = J(x_0, y_0)$, where the velocity $\vec{v}(x, y)$ is obtained by solving the equation of motion based on the vacuum electric field distribution (Eq. (4)). It is important to note that in our calculations so far, we have completely ignored the space charge effects in the calculation to obtain the electron density $n(x, y)$, which is to be used later in

FIG. 2. The emitted electron trajectories (x, y) from various emitting positions (x_0, y_0) (from apex to $\beta = 1$) for Lorentzian shape emitter at $h/w = 10$ and $E_0 = 10^8 \text{ V/m}$.

solving the Gauss law [see Eqs. (5) and (8)] to estimate the space charge field.

The remaining task is to find out the critical applied electric field E_c at which the SCL condition is reached. For a protrusive shape emitter (Lorentzian or hyperboloid), we define that the SCL condition is reached when the resulting space charge field E_{sp} (in y -direction due to the vacuum charge density $n(x, y)$) calculated at the apex of the emitter is equal to the applied local electric field $E_s = E_0 \times \beta_{max}$ at the same location ($x_0 = 0, y_0 = h$). By solving the Gauss law, we obtain

$$E_{sp} = \int_0^\infty dx \int_{y(x)}^D dy \frac{(y-h)\rho(x, y)}{\pi\epsilon_0[x^2 + (y-h)^2]}, \quad (5)$$

where $\rho(x, y) = -en(x, y)$, $y(x)$ is the corresponding y -coordinate of the emitting location on surface and $y(0) = h$ represents the emitter apex.

A sample of the calculated values of E_{sp} (solid line) is illustrated in Fig. 3(a) for $\beta_{max} = 34.64$ as a function of applied electric field E_0 . The critical value of $E_0 = E_c$ reaching the SCL condition is obtained by having $E_{sp} = E_s = E_c \times \beta_{max}$ (dashed line). For example, in this case, E_c is

FIG. 3. For Lorentzian shape at $h/w = 10$ or $\beta_{max} = 34.64$. (a) The overall space charge field E_{sp} (solid line) at the apex as a function of applied electric field E_0 . E_s (dashed line) is the surface electric field at the apex and E_c (dotted line) is the critical electric field at which $E_{sp} = E_s$. (b) The emission current density at the apex as a function of applied electric field E_0 . J_{FN} (solid line) is calculated using FN law, E_c is the same critical electric field in (a), and the intersection is marked as SCL current J_{CL} .

about $E_c = 0.21$ V/nm (dotted line). Initially when the applied electric field E_0 is small, the surface electric field E_s is higher than the space charge field E_{sp} , and thus the space charge effect is not important. When E_0 is approaching E_c , the space charge effect becomes more and more important, and the total electric field is zero at $E_0 = E_c$ which fulfills the SCL condition. Subsequently by plotting the FN law J_{FN} as a function of E_0 as shown in Fig. 3(b) (solid line), we can easily find the corresponding SCL current density at $E_0 = E_c$ denoted as J_{CL} . The comparison between the calculated J_{CL} and the 1D CL law will be shown later for various values of h/w or β_{max} .

III. HYPERBOLOID SHAPE

Another well-known mathematical model used for field emitter is the hyperboloid shape (Fig. 4 inset).³⁶ In this model, the distance between the emitter apex and the anode is given by $D = a \cos \theta$, where a is the half-foci distance and θ is the tip half-angle. To maintain consistency, we fix the gap spacing $D = 10 \mu\text{m}$ and vary the half-foci distance a to control the sharpness of the emitter. For the ease of formulation, we have used the prolate spheroidal coordinate system: ξ, η , and ϕ , where lines of constant ξ and η correspond to the prolate-spheroidal surface and the hyperboloidal surface, respectively, and ϕ is the azimuthal angle of rotation about the major axis. The values of $\xi = 1$ and $\xi \gg 1$ denote the location of the apex and the tail of the emitter, respectively. The corresponding local enhancement factor profile is given by

$$\beta^{-1} = Q_0(\cos \theta) \sqrt{\xi^2 - \cos^2 \theta} \tan \theta, \quad (6)$$

where $Q_0(\cos \theta) = (1/2) \ln[(1 + \cos \theta)/(1 - \cos \theta)]$ is the Legendre polynomial of second kind. Figure 4 shows the calculated β profile as a function of the vertical displacement Δz measured from the apex at different sharpness conditions. The maximum enhancement β_{max} is again at the apex of the

FIG. 4. The local enhancement factor profile for hyperboloid shape at different $\cos \theta$. The Δz represents the vertical displacement from the apex position.

emitter, but it decreases less rapidly compared to the Lorentzian shape. To make the maximum field enhancement comparable to those obtained from the Lorentzian shape, we vary the values of a accordingly in the hyperboloid model. For example, we shall have $\beta_{\max} = 34.786, 341.51, \text{ and } 3449.4$ at their corresponding $a[\mu\text{m}] = 10.04818, 10.00337, \text{ and } 10.00026$ (or $\cos\theta = 0.995205, 0.999663, \text{ and } 0.999974$).

With the known β profile, we could calculate the emission current density from the emitter surface using the FN law [see Eq. (3)] at any applied electric field E_0 ($E_s = E_0 \times \beta$). Since the emitted electrons also concentrate near the tip region, the effective emission area is limited within tip half angle $\lesssim 20^\circ$, which is enough to include majority of the emitted electrons. For example, at $E_0 = 0.1 \text{ V/nm}$ and $\Phi = 4 \text{ eV}$, we have $\beta_{\max} = 34.786$ and $J = 2.7122 \times 10^8 \text{ A/m}^2$ at 0° (the apex), while they are $\beta = 2.6895$ and $J \approx 0$ at tip half angle $= 20^\circ$.

Once the electrons are emitted from the surface, they will be accelerated towards the anode under the influence of the vacuum electric field, which can be determined by taking the derivation of the electric potential field (without space charge field), given by³⁶

$$V(\eta) = V_0 \frac{\ln[(1+\eta)/(1-\eta)(1-\eta_1)/(1+\eta_1)]}{\ln[(1+\eta_2)/(1-\eta_2)(1-\eta_1)/(1+\eta_1)]}, \quad (7)$$

where $\eta_1 = \cos\theta$ is the cathode surface and $\eta_2 = 0$ is the planar anode. By solving the equation of motion, we obtain the trajectories of the emitted electrons, which are plotted in polar coordinate system (r, z) as shown in Fig. 5 for different emitting locations (r_0, z_0) ranging from the apex (red line) to tip half angle 20° (cyan line). Here, the apex of the emitter and the planar anode are located, respectively, at $(r=0, z=D)$ and $z=0$. Similar to the approach mentioned in Sec. II, since the trajectories are not across each other, the magnitude of the current density $J(r, z)$ along the same trajectory is equal to the initial value $J(r_0, z_0)$ calculated at the surface position (r_0, z_0) , and subsequently, we could obtain the velocity profile $\vec{v}(r, z)$ and also the electron charge density $n(r, z)$.

FIG. 5. The emitted electron trajectories from various emitting positions (r_0, z_0) (from apex to half angle 20°) for hyperboloid shape emitter at $\cos\theta = 0.995205$ and $E_0 = 10^8 \text{ V/m}$.

z). To find out the SCL condition, we use the exact same approach, and the space charge field (in z -direction due to $n(r, z)$) determined by the Gauss law at the apex is

$$E_{sp} = \int_0^\infty dr \int_0^D dz \frac{(D-z)r\rho(r, z)}{2\epsilon_0[r^2 + (z-D)^2]^{3/2}}. \quad (8)$$

Figure 6(a) shows the calculated E_{sp} (solid line) at $\beta_{\max} = 34.786$ as a function of applied electric field E_0 . Its intersection with the surface electric field at the emitter apex $E_s = E_0 \times \beta_{\max}$ (dashed line) defines the critical point $E_c \approx 0.23 \text{ V/nm}$ (dotted line), where the SCL condition is fulfilled. Similarly, Fig. 6(b) illustrates the SCL current J_{cL} at this critical E_c .

IV. RESULTS AND DISCUSSION

Figure 7 shows the dependence of the critical applied electric field E_c (solid lines) on the maximum field enhancement factor $\beta_{\max}$ at different work functions $\Phi = 4, 3, \text{ and } 2 \text{ eV}$ (top to the bottom) for (a) Lorentzian and (b) hyperboloid shapes. The corresponding h/w ratio and $\cos\theta$ values are

FIG. 6. For hyperboloid shape at $\cos\theta = 0.995205$ or $\beta_{\max} = 34.786$. (a) The overall space charge field E_{sp} (solid line) at the apex as a function of applied electric field E_0 . E_s (dashed line) is the surface electric field at the apex and E_c (dotted line) is the critical electric field at which $E_{sp} = E_s$. (b) The emission current density at the apex as a function of applied electric field E_0 . J_{FN} (solid line) is calculated using FN law, E_c is the same critical electric field in (a), and the intersection is marked as SCL current J_{cL} .

FIG. 7. The critical applied electric field E_c as a function of emitter sharpness β_{max} and work function Φ [eV] (top to bottom) = 4 (black), 3 (red), and 2 (blue) for (a) Lorentzian and (b) hyperboloid shapes.

also shown in the upper x-axis. Note that in order to hold the assumption in Ref. 35, Lorentzian shape is only valid for $h/w \geq 1$, or $\beta_{max} \geq 3.6055$, and hence the calculations at smaller $h/w < 1$ are not shown. From the figure, we can summarize the following points:

1. For both shapes, as the sharpness (β_{max}) increases, the critical applied field E_c will decrease. It is quite straight forward since for larger β , smaller applied electric field E_0 is needed to produce a large surface electric field $E_s = E_0 \times \beta$, which gives larger emission current density J (Eq. (3)). Thus for a shaper emitter (higher β), smaller critical electric field E_c is needed to reach SCL condition.
2. For a very sharp emitter ($\beta_{max} \geq 30$), E_c is inversely proportional to β_{max} , which implies that $E_c \times \beta_{max} = \text{constant}$. This is a rather useful scaling because once this *constant* is determined for one specific β_{max} , it will allow us to predict other E_c values at other sharpness conditions. As an example, for Lorentzian shape, at $\beta_{max} = 34.64$, $E_c = 0.20195$ V/nm and $E_c \times \beta_{max} = 6.9955$ V/nm, while at $\beta_{max} = 346.4$, $E_c = 0.019965$ V/nm and $E_c \times \beta_{max} = 6.9159$ V/nm, where the difference is only 1.13%. Similarly for hyperboloid shape, we have $E_c = 0.2337$ V/nm for $\beta_{max} = 34.786$ and $E_c = 2.267$ V/ μm for $\beta_{max} = 3449.4$ and the difference is only 3.8%.

3. For both shapes, the numerical fitting of E_c will approach to the 1D limit value at small $\beta_{max} \approx 1$, e.g., for hyperboloid case at $\Phi = 4$ eV (black line) as shown in Fig. 7(b), the 1D limit is around $E_c = 5.65$ V/nm. This 1D limit value can also be obtained by equating the 1D CL law (Eq. (1)) and the FN law (Eq. (3)) at $\beta_{max} = 1$ (1D flat surface), which gives $E_c = 5.78$ V/nm, with a difference of only 2.21%.

Due to the non-uniform emission of electrons from a protrusive surface, it is necessary to define two parameters to characterize the space charge limit current occurred at the critical condition E_c . The first one is J_{max} , which is simply the maximum current density emitted from the apex of the emitter. The second one is J_{ave} , which is the average current density (over the effective emission area) calculated by $J_{ave} = \int \vec{J} \cdot d\vec{A} / \int dA$, where dA is the unit area element on the emitting surface. Recalled that the effective emission area is, respectively, determined by $\beta \gtrsim 1$ for Lorentzian shape and within half angle $\lesssim 20^\circ$ for hyperboloid shape.

In terms of the 1D limit current density J_{1D} (= 1D CL law at the critical condition E_c), the normalized values of J_{max} [$\mu_{max} = J_{max}/J_{1D}$ (dashed lines)], and J_{ave} [$\mu_{ave} = J_{ave}/J_{1D}$ (solid lines)], are plotted in Fig. 8(a) for Lorentzian shape (circle) and hyperboloid shape (inverted triangle). It is

FIG. 8. (a) The enhancement μ of the protrusive CL model over 1D CL law for J_{ave} (solid lines) and J_{max} (dashed lines) as a function of β_{max} for Lorentzian (circle) and hyperboloid (inverted triangle) shapes. (b) The corresponding deviation of the calculated μ_{ave} shown in (a) for 2 (solid lines) and 5 (dashed lines) times more emitting macroparticles.

clear that both μ increase with the sharpness characterized by $\beta_{\max}$, which will converge to a saturated value at large $\beta_{\max}$. The saturated values are about 20 and 70, respectively, for Lorentzian and hyperboloid shapes. The convergence is due to the corresponding decreasing of the effective emission area when the tip becomes sharper. At the limit case of $\beta_{\max} = \infty$, all electrons will emit from the apex position, where the effective emission area is restricted to the single apex point, we shall have $\mu_{\text{ave}} = \mu_{\max}$. Note the hyperboloid shape has a higher value of μ than the Lorentzian shape as the former has a more uniform surface enhancement profile near to the apex (see Figs. 1 and 4).

Similar as Fig. 7, both μ_{ave} and $\mu_{\max}$ will approach to unity as $\beta_{\max}$ approaching the 1D limit, especially for the hyperboloid shape, we have $\mu_{\text{ave}} \approx \mu_{\max} \approx 1$ at $\beta_{\max} \approx 1$. Besides the dependence on the emitter sharpness $\beta_{\max}$, we also investigate the effects of the gap spacing D ($1\mu\text{m} - 100\mu\text{m}$) and the work function Φ (2–4 eV). Although the absolute magnitude of SCL current density J depends on the specific values of D and Φ , the enhancement factor μ shown in Fig. 8(a) is not sensitive to D and Φ . The maximum variance on μ is within 8% for $D = 1$ to $100\mu\text{m}$ and 5% for $\Phi = 2$ to 4 eV. In Fig. 8(b), we show the deviation of our calculated μ_{ave} by increasing the number of emitting macroparticles or trajectories [solid line: 2 times more and dashed lines: 5 times more] for Lorentzian shape (circle) and hyperboloid shape (inverted triangle), respectively. The comparison shows good convergence in terms of numerical accuracy which is within 2% to 5% over a wide range of $\beta_{\max}$.

It is important to note that the hyperboloid type already offers an excellent physical approximation for the experimental geometrical features of a field emission microtip (see Fig. 2 in Ref. 36). Here, we would like to comment on the two types of geometries that were not studied in this paper, namely, a perfectly sharp conical shape³⁷ and knife-edge shape.³⁸ For a perfectly sharp conical shape,³⁷ we believe the results should be quite similar to the hyperboloid shape since they are both cylindrical symmetry about the central axis. The only difference is that the field enhancement is less uniform for the conical shape (enhancement decreases faster as moving away from the apex), and the electron beam will be more concentrated around the emitter tip. Thus, the onset critical field E_c and SCL current enhancement μ for a conical shape will be less than the hyperboloid shape. For the knife-edge emitter,³⁸ it is infinitely long in the z -direction similar to the Lorentzian shape. Thus, we may speculate that the results will be closer to the Lorentzian shape. However, the knife-edge geometry has two sharp tips (see points B and C in Fig. 1(a) of Ref. 38); it is possible that the onset of SCL condition may not occur at the center of the emitter (point h) but at the two sharp edges. Thus to apply the model proposed in this paper for a knife-edge emitter will require comprehensive studies, which is beyond the scope of this paper. Furthermore, the two sharp edge points have very large maximum enhancement value approaching infinity [see Eqs. (1) and (4) in Ref. 38], and careful numerical treatment must be applied. We shall explore in details in our future work and may even try to extend it to a composite geometry of two ascending metallic protrusions to investigate the effect of multi-sharp tips.

V. SUMMARY

In summary, we have proposed a simple method to determine the onset of SCL current from a protrusive field emitter of either Lorentzian or hyperboloid shape. The critical applied electric field E_c on the onset of SCL current is calculated and we establish an useful scaling $E_c \times \beta_{\max} = \text{constant}$ for a very sharp emitter ($\beta_{\max} > 30$). The corresponding SCL current density at the critical applied field is also calculated as a function of the emitter's sharpness characterized by the maximum field enhancement factor $\beta_{\max}$ at the apex of the emitter. In terms of the 1D CL law, the enhancement factor of this calculated SCL current density $\mu = J_c/J_{1D}$ only depends on $\beta_{\max}$ and is independent of gap spacing and work function of the emitter. This model should be useful in the quick assessment of SCL electron emission from a sharp tip in applications such as using sharp emitters to produce high current electron beam required for high power microsources.^{39,40} After further verification with well-resolved PIC simulation, it may be implemented into gun-code or PIC code to avoid fine meshing (at the cathode surface) for device-scale simulation.

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