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Multi-colour microscopic interferometry for optical metrology and imaging applications

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ABSTRACT

Interferometry has been widely used for optical metrology and imaging applications because of their precision, reliability, and versatility. Although single-wavelength interferometry can provide high sensitivity and resolution, it has several drawbacks, namely, it fails to quantify large-discontinuities, large-deformations, and shape of unpolished surfaces. Multiple-wavelength techniques have been successfully used to overcome the drawbacks associated with single wavelength analysis. The use of colour CCD camera allows simultaneous acquisition of multiple interferograms. The advances in colour CCD cameras and image processing techniques have made the multi-colour interferometry a faster, simpler, and cost-effective tool for industrial applications. This article reviews the recent advances in multi-colour interferometric techniques and their demanding applications for characterization of micro-systems, non-destructive testing, and bio-imaging applications.

Keywords: Interferometry, Speckle, Multiple-wavelength, Colour CCD, Microsystems, Bio-imaging, Shape, Deformation, NDT

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1. Introduction

Interferometry has been widely used in research and development (R&D), and industrial applications because of its precision, reliability, and versatility. It is a well-established full-field, non-contact, non-invasive optical tool which offers excellent sensitivity and resolution for metrology applications[1-3]. Interferometer has been used as an important investigating tool in the fields of fiber optics, optical metrology, surface profiling, microfluidics, mechanical stress/strain measurement, velocimetry, biology and medicine etc[3-7]. Interferometric techniques can be used to inspect both reflecting (smooth) and scattering (diffusive) surfaces in wide range of sizes. The inspection can be done under static, quasi-static or dynamic conditions[8-10]. Conventional interferometry can only handle smooth (mirror like) samples. If the test surface is diffusive, the interference between reference and test beam results in no-visible fringe pattern. So, conventional interferometry fails to characterize samples with diffusive (speckled) surface. The interferometry that can study rough or scattering surfaces is known as Electronic / Digital speckle pattern interferometry (ESPI/DSPI) or TV holography (TVH)[8, 11]. TV holography under microscopic configuration was demonstrated for characterization of shape, deformation, NDT on microsystems such as MEMS (Micro-Electro-Mechanical Systems) [2, 4, 12-15]. In conventional or speckle interferometry, the desired information about the object under study is encoded in the fringe pattern, which in fact represents the phase distribution.

The analysis of the interferogram produced by a test surface gives the parameters of interest relating to the surface. Several multiple-frame[1, 16-18] and single-frame[19-24] methods have been reported for quantitative analysis of fringes. Temporal phase shifting, polarization phase shifting techniques etc. are popular multiple-frame methods. These methods can provide phase with high accuracy but cannot be used in dynamic situations and industrial measurement environment. Single-shot measurements where only a single frame or multiple frames in one go are recorded and analyzed for phase by an appropriate procedure have been reported. In recent years, numerous single-frame analysis methods such as Fast Fourier Transform (FFT)[25-27], Huang-Hilbert transformation (HHT)[26, 28], Hilbert Transform (HT)[29-31], Pixelated-polarization phase shifting (PPPS) [20, 32]etc. have been applied for single-

interferogram analysis. Single (visible) wavelength measurements are accurate, but using single-wavelength greatly limits the wide spread applications of interferometry.

The major drawbacks associated with single wavelength fringe analysis are: (i) for surface profiling of discontinuous surfaces, the unambiguous step-height measurement range is limited to half-a-wavelength ($\lambda/2$), (ii) for deformation measurement, it suffers from overcrowding of the fringes under relatively large loading conditions. The high frequency of the fringes sets a limitation due to speckle de-correlation for quantitative analysis, (iii) it cannot reveal the shape of a rough surface, (iv) it cannot resolve the discontinuities between polished and unpolished surfaces. The approaches adopted to overcome the problems associated with single-wavelength are based on scanning white light interferometry[33-35], spectrally resolved white light interferometry[36-39] and multiple-wavelength interferometry[17, 40-46].

Scanning white light interferometry (WLI) is a state-of-the-art technique for measuring discontinuous surface profiles [33-35]. This makes use of the short coherence length of the white light source. High contrast fringe occurs only when the optical path difference (OPD) is close to zero. The 3-D plot of the axial positions of the zero OPD along the optical axis represents the surface profile of the object under test. Compared to single wavelength phase shifting interferometry, the scanning white light interferometry is rather slow, as the number of frames to be recorded and evaluated is large[33-35]. The spectrally resolved white light interferometry (SRWLI) is a variation of WLI in which the white light interferogram output of WLI is spectrally resolved into its constituent colour interferograms [36-39]. All the colour interferograms are analyzed for respective phases, which are then used to determine the profile. The phases at all the wavelengths can be evaluated by phase shifting technique. However this procedure gives only a line profile of the object, although the requirement on number of frames is similar to the single wavelength phase shifting interferometry.

In multiple-wavelength techniques more than one visible (e.g. red (R), green (G), blue (B)) laser wavelengths are used for measurements. Each wavelength will generate its own interference pattern which contains the desired information of the object. The acquisition of the multiple-wavelength interferograms can be done in two different

ways: (i) Sequential illumination mode[17, 47] in which **interferograms** are recorded with different wavelength one after another sequentially. This is a time consuming process, and (ii) Simultaneous illumination mode [48-51] in which all the interferograms are recorded in one go by using a RGB CCD camera. This approach makes the fringe acquisition as simple as in single-wavelength case. Three different approaches have been used for 3-colour recording:(a) **Bayer filter 1-CCD sensor** [52-55], (b) **3-CCD sensor** [24, 49, 52, 56], and (c) **Foveon X3 sensor** [46, 52, 57]. We will discuss these three strategies for simultaneous recording of 3-colours in Section-4.

The **Bayer filter** 1-CCD RGB camera has been demonstrated for surface profiling of large discontinuities[23, 51, 58, 59], and simultaneous measurement of shape and deformation[23, 48], non-destructive testing (NDT) of large defects[60], simultaneous acquisition of blood flow, blood volume, and oxygenation on human fingers using dual-wavelength laser imaging [61]etc. Recently, white light illumination combined with 1-CCD RGB camera was successfully demonstrated for surface profiling of micro-lens array and large-discontinuities[30], biological cells such as human Red-Blood-Cell (RBCs)[21, 50, 62], onion skin/cells[22], fish-eye cornea [55]etc. This approach was further simplified by using single-shot methods instead of multiple-frame methods for phase measurements[21, 23, 24].

The 3-CCD RGB camera has been demonstrated for surface profiling of large discontinuities using RGB laser interferometry[49, 63, 64], **analysis of aerodynamic flow using a three-colour differential interferometry** [44], simultaneous measurement of in-plane and out-of-plane displacement derivatives using RG speckle interferometry[65], 3D displacement measurement by two-wavelength (RB) simultaneous DSPI and DSP (Digital speckle photography)[66]. The 3-CCD camera was also combined with white light scanning interferometry for surface profiling of microstructures[49, 63, 64]. The 3-CCD can provide high-resolution imaging, but they are expensive. The advances in 1-chip colour CCD technology, allowed using this as an alternative for 3-CCD camera.

The Foveon X3 sensor based camera has been used in white-light interferometric applications [67], aerodynamic flow analysis using digital three-colour holographic interferometry [46, 52], surface profiling using snap-shot fringe projection profilometry [57]. The RGB interferometry that uses a RGB CCD camera to acquire multiple wavelength interferograms simultaneously is a simpler, faster, and dynamic tool for metrology and imaging applications. In this article, we will discuss the recent advances in RGB interferometric techniques and their applications for optical metrology and imaging with examples.

2. Multiple-wavelength metrology :Theoretical background

2.1 Surface profiling of discontinuous surfaces

In interferometry, the desired information about the test object is encoded in the fringe pattern. Analysis of the fringe pattern (interferogram) gives the parameters of interest of the test surface. The interference between the reference and object beams results in a visible fringe pattern which can be written as[1, 68]

$$I = I_o(1 + V \cos \phi) \quad (1)$$

where I_o is the bias intensity, V is the fringe contrast, $\phi = (4\pi/\lambda)z$ is the phase at the wavelength λ , z is the profile height or depth. The intensity distribution of interference patterns obtained at different wavelengths can be written as[1, 68]

$$I_i = I_o(1 + V \cos[\phi_i]) \quad (2)$$

where $\phi_i = (4\pi/\lambda_i)z$ is the phase at λ_i . When two different wavelengths (say Red and Green) simultaneously illuminate the test and reference surfaces, the resultant interference signal is modulated as shown in Fig.1a. Fig.1(b-e) shows typical inference patterns from red (R), red-green (RG), red- green-blue (RGB) wavelengths acquired with a 1- CCD RGB camera. The colour fringe patterns can be separated in to individual components using image processing tools as shown in Fig.1c.

Once the individual components are extracted, the quantitative phase evaluation can be done using multiple frame methodsor single frame methods. Multiple frame methods such as phase shifting techniques can provide high resolution phase information than

single frame methods such as Hilbert transform, Fourier transform etc. Implementing phase shifting technique requires 3 or more frames for phase calculation. The intensity distribution of phase shifted frames at multiple-wavelengths can be written as[1, 16-18, 69-71]

$$I_{iN} = I_o(1 + V \cos[\phi_i + (N-1)\alpha_i]) \quad (3)$$

where N is the number of the phase shifted frames, and α_i is the phase shift at λ_i . If the phase shift is chosen as $\pi/2$ the phase can be evaluated using any of the phase shifting algorithms shown in Table-1. When multiple wavelength interferometry is carried out, the phase shift can be $\pi/2$ only at one wavelength. If a phase shifter (such as PZT) is calibrated for a known phase shift of 90° at wavelength λ_{cal} , and same motion of the PZT is used for other wavelengths (λ_i), then the phase error introduced at all other λ_i wavelengths can be expressed as[1, 16, 53, 70-72]

$$\alpha_i = (\lambda_{cal}/\lambda_i)90^\circ \quad (4)$$

For example, if a PZT is calibrated at $\lambda_2 = 532$ nm for phase shift $\alpha_2 = 90^\circ$. Then, the phase shift values become $\alpha_1 = (\lambda_2/\lambda_1) \pi/2 = 75.7^\circ$ for $\lambda_1 = 632.8$ nm and $\alpha_3 = (\lambda_2/\lambda_3) \pi/2 = 108.4^\circ$ for $\lambda_3 = 441.6$ nm. In such cases, the phase shift algorithm can be chosen based on the amount of phase shift error expected during phase shifting from the nominal value of $\pi/2$. **Phase shift errors as large as 20° can be adequately compensated using 8-step algorithm[18]. A detailed analysis of phase shift errors and their compensation using various phase shifting algorithms can be found in reference [37].**

The calculated phase ϕ contains the information about the test surface. If the test surface is continuous or has discontinuities less than $\lambda/2$, single wavelength phase measurement is unambiguous. For example, using $\lambda = 633$ nm, step-height less than 316 nm can be measured unambiguously. If the test surface has large discontinuities ($\geq \lambda/2$), phases measured at multiple wavelength are required to obtain ambiguity-free phase. Using the phases at multiple wavelengths, the unambiguous step-height measurement of single-wavelength interferometry can be extended in two ways: (a) Phase subtraction method [42, 58, 73], and (b) Fringe order method [41, 48, 60].

2.1(a) Phase subtraction method (PSM)

In phase subtraction method, the phase at one wavelength is subtracted from that of the other to generate phase at an effective (synthetic) wavelength $\Lambda_{12}(= \lambda_1\lambda_2/|\lambda_1-\lambda_2|)$ as shown in Fig.2. If ϕ_1 and ϕ_2 are the phases at two different wavelengths, λ_1 and λ_2 , the phase at effective wavelength is governed by the relation [42, 58, 73, 74]

$$\phi_1 - \phi_2 = \frac{4\pi}{\Lambda_{12}} z \quad (5)$$

For example, using $\lambda_1 = 632.8 \text{ nm}$ (He-Ne laser), $\lambda_2 = 532 \text{ nm}$ (Nd:YAG laser), $\lambda_3 = 441.6 \text{ nm}$ (He-Cd laser), the effective wavelengths that can be generated and corresponding unambiguously step-height measurement ranges are listed in Table-2. With these three wavelengths, we find that $\Lambda_{23} \sim \Lambda_2$, and $\Lambda_{31} \sim \Lambda_3$. Hence these wavelengths are not included in the Table-2. The phase subtraction method could increase the measurement range of single-wavelength measurement, but the drawbacks associated with this method are [16, 73, 75]: (a) Its unambiguous range is again limited to $\Lambda/2$, (b) The signal-to-noise ratio (SNR) of the surface profile measured at Λ is reduced by λ/Λ times compared to that measured at λ .

2.1(b) Fringe order method (FOM)

To overcome the drawbacks with phase subtraction method (PSM) for step-height measurement, fringe order method has been used. To implement FOM, phases measured at three different wavelengths are required. The phase ϕ_i is governed by the following equation [16, 30, 53, 74]

$$\phi_i = \frac{4\pi}{\lambda_i} z = 4\pi\sigma_i z \quad (6)$$

where $\sigma_i = 1/\lambda_i$ is the wavenumber. The variation of ϕ_i with σ_i is linear in Eq.(6). Using the wrapped phases at three wavelengths, it is possible to adjust the wrapped phase data at any pixel such that they lie on a best fit line by addition or subtraction of multiples of 2π . The slope of this line gives the absolute height at the pixel $as z_a = 1/4\pi (\Delta\phi/\Delta\sigma)$. The value of z_a obtained is less precise but it is quite close to the actual value. It can be used to remove the ambiguity in single wavelength phase by determining the fringe order at each pixel. The ambiguity-free phase with precision of phase shifting interferometry can be expressed as [30, 41, 76, 77]

$$\Phi_i = \phi_i - 2\pi \text{int} \left(\frac{\phi_i - 4\pi\sigma_i z_a}{2\pi} \right) \quad (7)$$

The function $\text{int}()$ sets the argument to its nearest integer. An improved value of z with resolution provided by the phase shifting technique can be obtained as $z = (\lambda_i/4\pi)\Phi_i$. Thus we have a long measurement range and high precision of the surface profile of an object under study.

2.2 Simultaneous measurement of shape and deformation on rough surfaces

If the test surface is rough, the interference between reference and test beams does not produce a visible fringe pattern. So, TV holography is used to study speckled surfaces. In TV holography, the intensity distributions of speckle patterns acquired before and after deformation are subtracted to generate speckle correlation fringes.

The intensity distribution of speckle pattern before (I^b) and after (I^a) deformation can be written as

$$I^b = I_o(1 + V\cos[\phi^b]) \quad (8)$$

$$I^a = I_o(1 + V\cos[\phi^b + \Delta\phi]) \quad (9)$$

where a, and b stand for after and before deformation, ϕ^b the random speckle phase, $\Delta\phi (= 4\pi w/\lambda)$ is the desired (deformation) phase, w is the out-of-plane deformation.

The subtraction of Eq.(8) from Eq.(9), results in deformation fringes [1, 8]

$$\begin{aligned} |I^a - I^b| &= |I_o(1 + V\cos[\phi^b + \Delta\phi]) - I_o(1 + V\cos[\phi^b])| \\ &= I_o V(\cos[\phi^b + \Delta\phi] - \cos[\phi^b]) \end{aligned} \quad (10)$$

Fig.3 shows the flow chart for generating the correlation fringes in multi-colour speckle interferometry.

The intensity distribution of phase shifted frames before and after deformation at different wavelength can be written as[14, 17]

$$I_{iN}^b = I_o(1 + V\cos[\phi_i^b + (N-1)\alpha_i]) \quad (11)$$

$$I_{iN}^a = I_o(1 + V\cos[\phi_i^a + (N-1)\alpha_i]) \quad (12)$$

The phases before ϕ_i^b and after $\phi_i^a (= \phi^b + \Delta\phi_i)$ deformation can be calculated using any one of the phase shifting algorithm given in Table-1. Using these phases, the possible multiple-wavelength measurements with sensitivities are shown in Table-3.

3. Multi-colour microscopic interferometry

In multi-colour interferometry, the light source is tunable or broad band and the detector is a colour CCD camera. Several configurations have been reported with colour CCD cameras : 2λ -interferometry with a 1-CCD[47, 60], 3λ -interferometry with 1-CCD [48]and 3-CCD[49], and white light interferometry with 1-CCD [50, 53, 78]and 3-CCD[49, 63]. A typical system based on white light illumination and a 1-CCD colour camera is depicted in Fig.4[30, 53]. The interference device is a micro-Mirau interference system equipped with PZT for phase shifting. A convex lens (CL) is used to collimate the light. The collimated beam illuminates the micro-specimen via the beam splitter (BS) and the Mirau objective (MO). The micro-specimen is mounted on a 3-axis stage for alignment. The microscopic imaging system **consists in** microscopic objectives with different magnifications. The interference pattern can be acquired using a colour camera. The camera is interfaced to PC with a frame grabber card. A PZT is attached to the Mirau objective for implementing temporal phase shifting technique. The PZT is driven by the PC with a Data acquisition card (DAQ) card. Real-time fringe visualization, storing phase shifting, and quantitative fringe analysis were carried out in LABVIEW and MATLAB software.

4. Colour CCD configurations

To acquire single wavelength interferogram, a monochrome CCD would be sufficient. But in multiple-wavelength interferometry, each wavelength generates its own interferogram. If R, G, B wavelengths simultaneously illuminate the object and reference mirror, the interferograms can be acquired **in single-shot**. **Three different approaches have been used for simultaneous recording of three-colours: (a) Bayer filter 1-CCD sensor, (b) 3-CCD sensor, and (c) Foveon X3 sensor. Schematic diagrams of these sensors are shown in Fig.5. A comparative study of the three sensorshas been done in digital 3λ holographic interferometry [52].** In a 1-CCD colour camera, a mosaic

color filter array (CFA) is used on a single image sensor. Different primary filters are distributed over the pixels of the sensor. Each filter has a specific transmission band. Commonly used CFA in digital cameras is Bayer RGB CFA as shown in Figure 5a. The Bayer filter pattern is 50% green, 25% red and 25% blue. The raw output from CCD is an incomplete RGB image which is then CFA interpolated to obtain a full RGB image at resolution reduced by $\sim 25\%$. In 3-CCD colour camera, a beam splitter is used to split and project the interferogram into the three image sensors. Each sensor has its own primary color filter. Thus interferograms at R, G, B can be collected at high resolution. The schematic of the prism-based 3-CCD is shown in Figure 5b. The advantages with 3-CCD camera are high spatial resolution and higher color precision. But these cameras are very expensive for industrial applications. In recent years, 1-CCD camera has been used for interferometric applications. But, the spatial resolution of Bayer filter based 1-CCD colour camera is $\sim 25\%$ less compared to a monochrome sensor or 3-CCD sensor technology where all the photons are used for imaging. However, this can be overcome by using 1-CCD camera with more pixels which are easily available in the market. The schematic diagram of Foveon X3 sensor is shown in Fig. 5c. It is stack of the layers of colour-specific pixels in the silicon and as a result it can capture light at every pixel unlike Bayer filter array. Thus the three-layers of pixels directly capture the entire colour in Foveon X3 camera.

5. Applications of RGB interferometry

5.1 Surface profiling of continuous surfaces : Micro-lens array

Measurement of the shape, surface quality, and optical performance of micro-lenses, as well as, the uniformity of the parameters across the wafer is an important issue[5, 79]. A single-wavelength laser interferometry can be used for surface profiling of microlens array, but the long coherence length results in speckle noise, which could introduce error in the surface profile[19]. Hence white light is ideal for testing microlens array[80, 81]. 3-D surface profiling of a fused silica micro lens array was demonstrated using the white light system shown in Fig. 4. The micro lens array and the reference mirror were illuminated with a white light source and interferograms were collected using a 1-CCD RGB camera. The three wavelengths corresponding to the three interferograms were determined by the spectral bands of the colour CCD used. The

CCD has three separate spectral bands, R, G, B centered at Red (620 nm), Green (540 nm), and Blue (460 nm) wavelengths respectively.

In this experiment, the PZT was calibrated at 540 nm for phase shift 90° . From Eq.(4), the maximum phase shift error involved is $\sim 16^\circ$, which can be compensated by using 8-step algorithm (Table-1). Eight phase shifted frames were acquired and decomposed each frame in to its individual components and the corresponding phases at R, G, B were calculated using 8-step algorithm. Any one of them can be used to reconstruct the surface profile of microlens array. The white light colour interferogram obtained on a micro-lens array is shown in Fig.6a. Fig. 6(b-d) shows the individual interferograms decomposed from interferogram in Fig.6a. The wrapped phase map at 620 nm and unwrapped 3-D surface profile of the array are shown in Fig.6(e) and 6(f), respectively. The array has well-defined $220\mu\text{m}$ diameter hemi-spherical micro lenses with pitch $220\mu\text{m}$.

5.2 Surface profiling of discontinuous surfaces

5.2.1 RGB interferometry with 3-chip colour CCD

A 3-CCD camera based RGB (3λ) interferometry was demonstrated for large step-height measurements[49, 64]. The measurement system consisted of a Linnik interference microscope with as 3-CCD camera and three lasers with R, G, B wavelengths 632.8 nm (He-Ne laser), 532 nm (Nd:YAG laser) , and 473 nm (Nd:YAG laser), respectively. An etched silica sample with large discontinuities was simultaneously illuminated with these three wavelengths. The colour interferograms were simultaneously captured using 3 R, G, B channels (Fig.5a) of the camera. The phases at R, G, B were calculated using an error compensating 5-step algorithm (Table-1). The three wrapped phases at R, G, B were then used to reconstruct the ambiguity-free phase using fringe order approach (Section.2.1b). Fig.7a shows the total 3-D profile of an etched silica sample with large steps. Same sample surface was measured with a scanning white light interferometry (SWLI). Figure 7b shows the difference between a RGB-measurement and a white-light measurement. The RMS of the differences is $\sim 3.5\text{ nm}$ [49].

5.2.2 RGB interferometry with white light illumination and 1-chip colour CCD

Fig.8 shows the surface profiling of a silica sample with large discontinuities using the system shown in Fig.4[30, 53]. The white light interferogram, the interferogram at 620 nm, and corresponding phase map calculated using 8-step algorithm (Table-1) are shown in Fig.8 (a-c), respectively. The 3-D profile obtained using fringe order method (Section.2.1b) and phase subtraction method (Section.2.1a) are shown in Fig.8 (d) and 8(e), respectively. The step-height (z) values measured from fringe order method, and phase subtraction method are 645 nm, and 651 nm, respectively. The phase data at 620 nm and 540 nm were subtracted to generate an effective wavelength phase at 4185 nm. So, step-height less than 2092 nm can be unambiguously measured using this synthetic phase. The phase subtraction methods could resolve the large discontinuity, but the vertical resolution is affected at effective wavelength as expected. The fringe order method could resolve the large discontinuity, and also retain the single-wavelength resolution. The RMS noise (~ 9.5 nm) in the profile measured at effective wavelength is (Λ/λ) times the RMS noise (~ 1.4 nm) in the profile measured at a single wavelength as shown in Fig.8f. The maximum step-height that can be measured using this approach is determined by the spectral band widths of the R, G, B channels. The bandwidth of R, G, B channel of the 1-CCD JAI BB-500GE 2/300 GigE vision camera used for this experiment are ~ 80 nm, ~ 100 nm, ~ 90 nm, and hence the coherence lengths of these channels become ~ 3600 nm, ~ 2916 nm, 2351 nm ($L = \lambda^2/\Delta\lambda$), [78, 82] respectively. So, the maximum step-height measurement is limited by the coherence length of the channel. To increase the measurement range the channel spectral width needs to be reduced.

Both RGB interferometry with 3-CCD and white light interferometry with 1-CCD allow simultaneous acquisition of multi-colour interferograms. In RGB interferometry the object profile is not scanned like in white light scanning interferometry and hence the maximum depth is limited by the depth of focus of the imaging optics. The system with three R, G, B lasers and 3-CCD camera is expensive. On the other hand, in the white light phase shifting interferometer (WLPSI) with 1-CCD RGB camera, the spectral band width of R, G, B channels limits the maximum step height that can be measured. Thus WLPSI with 1-CCD RGB camera could be an inexpensive and simple alternative for 3-CCD RGB interferometer with lasers within a limited measurement range. To make this measurement further simpler, and a single-shot approach using

Hilbert transformation single frame analysis was demonstrated for profiling of large discontinuities[22, 30].

5.2.3 Crosstalk compensation in multi-colour interferometry

Using colour CCD allows simultaneous acquisition of multi-colour fringes. But in multi-colour fringe analysis, many issues must be resolved as discussed in reference [83]. One of the factors affecting the phase retrieval accuracy is the colour crosstalk. When R, G, B wavelengths are used, the crosstalk is not negligible due to the spectral sensitivity of the colour CCD camera. To compensate for the crosstalk, a linear model is discussed in References [51, 83]. A typical crosstalk coefficient matrix obtained using linear model is given below [83]:

$$\begin{pmatrix} 1 & 0.05 & 0.00 \\ 0.28 & 1 & 0.04 \\ 0.01 & 0.04 & 1 \end{pmatrix}$$

The coefficients were obtained by tuning only one wavelength (say blue) and capture the blue image. Using regression analysis, the regression coefficients between B and G, B and R were obtained as 0.28 and 0.01, respectively. From similar experiments, the coefficient matrix was obtained. Using these coefficients, the crosstalk compensation can be achieved so that the phase accuracy will not be affected.

5.2.4 Temporal unwrapping for large step-height measurement

The approaches discussed in Section-5.2.1 and 5.2.2, use spatial unwrapping for step-height measurement. It involves searching for 2π discontinuities in the spatial domain. Only one phase map is required, but phase errors can propagate out from regions of high noise, affect rest of the image. If spatial unwrapping is used to quantify large discontinuities, single-wavelength analysis results in ambiguous measurements and hence requires multiple wavelength phases to extend the unambiguous measurement range of the system as discussed in Section-2.1.

Single wavelength interferometry can also quantify large discontinuities by using temporal phase unwrapping method described in Reference [84]. Temporal unwrapping treats each pixel independently of the others, with unwrapping being carried out in the time domain. But many intermediate phase maps are required, the approach is inherently simple and robust, and phase errors are constrained to remain within regions of low signal-to-noise ratio. This approach cannot be used for dynamic measurements.

Fig.9a shows wrapped phase maps of a test object consisting of three 4-mm-wide vertical grooves, on 6-mm centers, machined in a flat piece of plastic. The depths of the three grooves were approximately 1, 2, and 4 mm. It is impossible to unwrap this phasemap correctly by conventional spatial methods because the phase jumps at the groove edges fall outside the range $(-\pi, \pi)$. The incremental phase maps were calculated using 4-step method. Fig.9b shows the result of unwrapping the phasemap of Fig. 9a through 22 intermediate images. The differences in height between the three grooves have been successfully detected. This has been achieved by direct implementation of the temporal phase-unwrapping algorithm [85] without data smoothing, windowing, or manual intervention; this would not have been possible with any of the existing spatial phase-unwrapping algorithms.

5.3 Biological imaging applications

5.3.1 Quantitative phase imaging of Red Blood Cells (RBCs) and Onion cells (OCs)

Quantitative measurement of the refractive index of biological cells is important for extracting vital information, such as, concentration of hemoglobin, thickness, average mass of cell, etc [6, 7, 82, 86, 87]. The refractive index $[n(\lambda)]$ of a biological cell is wavelength dependent, hence requires multiple wavelength techniques. Several microscopy techniques [86, 88] such as Spectroscopic phase microscopy [89], Diffraction phase microscopy, Dispersion microscopy [90] etc., have been used to measure the refractive index of human red blood cells (RBCs) at multiple wavelengths.

Recently, quantitative measurement of phase and wavelength-dependent refractive index of RBCs was reported using a white light phase shifting interferometry and 1-CCD colour camera [50]. The measurement system used is similar to the system shown in Fig.4. The RGB camera used for this experiment has three spectral bands centered at red (620 nm), green (560 nm), blue (450 nm). The RBCs in a saline water medium were placed on a cover glass. The refractive indices of saline water at red, green, blue wavelengths were measured to be 1.3342, 1.3347, and 1.3349, respectively. Phase shifted frames were stored and corresponding phase at R, G, B were calculated using 5-step algorithm [72]. The unwrapped phases were used to measure the cell refractive indices at R, G, B using the relation [50, 55]:

$$n_i^{cell} = \frac{\lambda_i}{2\pi t} \Delta\phi_i + n_i^{med} \quad (13)$$

where n_i^{cell} , and n_i^{med} are refractive indices of cell and medium respectively, $\Delta\phi$ is the unwrapped phase, t is the thickness of the cell. Fig.10 shows the quantitative analysis for red (620 nm) wavelength data. The refractive index of RBCs was found to be different for different wavelengths. The indices determined at R, G, and B were 1.380, 1.392, and 1.401, respectively. These indices values are in agreement with the values reported by spectroscopic phase microscopy[89].

For the phase imaging of RBCs discussed above, phase shifting (multiple-frame) method was used. To make the measurements simpler and single-shot, a Hilbert transformation single-frame method was used for quantitative phase analysis of polystyrene spheres, biological (RBC) cells, onion skin etc[21, 22]. The quantitative phase map of onion skin obtained using single-shot Hilbert transformation method is shown in Fig.11. The experiment was conducted for the quantitative phase imaging of onion cell for three color wavelengths (R, G, B). The onion sample on a glass slide was placed on a translation stage. The white light interferogram of onion was recorded as shown in Fig. 11 (a) and was decomposed into its red, green and blue colourinterferogram. The fringe analysis was done by Hilbert transform method. The unwrapped phase maps are shown in Fig. 11(b) for three colours red, green and blue respectively. Fig.11(c) shows the refractive indices of the onion cell for three wavelengths red, green and blue and was measured to be 1.59, 1.55 and 1.51, respectively [22].

5.3.2 Surface profiling of Fish cornea

Cornea is a smooth membrane that covers the front of the eye and plays an important role in focusing images on the retina. Any minor changes in the surface shape of the cornea will affect clarity of the image. The cornea is a major refractive surface of the eye which contributes ~ 70% to the total dioptic power[91]. Corneal topography information is helpful for finding the health of cornea, and for treatment or surgery. White light source which has short coherence length can provide high axial resolution

for precise cross-sectional imaging of Cornea. White light was already used for biological tissue imaging applications[55, 92].

Using the same WLPSI with 1-CCD RGB camera system (Fig.4), topography and tomography of fish cornea were obtained[55]. The sample (fish) was mounted on a 3-axis translation stage for precise alignment of fish-eye under the Mirau-objective. White light colourinterferograms at different depths were recorded by moving the sample (fish) axially. For each depth position, 5 phase shifted frames were acquired to calculate the wrapped phase map using error compensating 5-step algorithm (Table.1). From the unwrapped phase maps, the corneal topography and refractive index were obtained. And from the amplitude/intensity images the cross-sectional image of fish cornea was obtained. The results obtained on corneal topography of fish-eye are shown in Fig.12 [55]. Thus the white light interferometry with colour 1- CCD camera is a promising tool for biological imaging applications.

5.4 Simultaneous measurement of deformation and shape on MEMS pressure sensor

Shape analysis of the test object is essential for the complete deformation studies where a careful analysis of sensitivity vector orientation across the field-of-view is necessary for reliable quantitative measurements[12, 93-96]. 3-D shape evaluation has also found useful applications in non-destructive testing, quality control, reverse engineering, robot vision and tribology. In addition, the surface measurement can be applied to determine the dimensional accuracy of the manufactured micro-components. Shape measurements have been achieved by changing the refractive index of the surrounding medium or the illumination angle or the wavelength of the laser source[42, 93, 94]. In addition to shape measurement, use of multiple wavelengths also enables the characterization of deformation with reduced sensitivity. For large deformation, the single wavelength configuration suffers from overcrowding of fringes that sets it a limit due to speckle de-correlation for quantitative fringe analysis. Multiple-wavelength method yields speckle correlation fringes with reduced and variable measuring sensitivity to extend the range of measurements. The effective wavelength plays an important role determining the measuring sensitivity. The difference between the two phases obtained at two different wavelengths provides the macroscopic deformation / shape information of the test object at effective wavelength.

Simultaneous shape and deformation measurements were successfully performed on a $1500 \mu\text{m}^2$ MEMS pressure sensor with sensor diaphragm thickness $\sim 25 \mu\text{m}$ [17, 48]. The sensor was loaded by applying pressure externally in a controlled manner. Multiple-wavelengths $\lambda_1 = 632.8 \text{ nm}$ (He-Ne laser), $\lambda_2 = 532 \text{ nm}$ (Nd:YAG laser), and $\lambda_3 = 441.6 \text{ nm}$ (He-Cd laser) were used for measurements. In this experiment, the PZT was calibrated at $\lambda_2 = 532 \text{ nm}$ for phase shift $\alpha_2 = 90^\circ$. So, from Eq.(4) the maximum phase shift error involved at other wavelengths is $\sim 18^\circ$. From the Table-1, it can be compensated by using 8-step method. Eight phase shifted frames were stored at each wavelength before and after loading the sensor. Using the calculated phases ϕ_i^b ($i=1,2,3$) and ϕ_i^a ($i=1,2,3$), it is possible to generate shape and deformation of the sensor using the Table-3. Fig.13(a-c) show the deformation wrapped phase maps obtained under external pressure 250 kPa at individual wavelengths λ_1 , λ_2 , and λ_3 . The large deflection resulted in over crowding of fringes, and hence proper unwrapping was not feasible as shown in Fig. 13(d) which represents a line scan profiles along the central axis. The phase maps evaluated at effective wavelengths also contains the combined information pertaining to shape and deformation as discussed in Table-3.

For evaluation of the deflection and the shape, the effective wavelength phase maps were generated by subtracting the single wavelength phase maps obtained at $\lambda_1 = 632.8 \text{ nm}$, and $\lambda_2 = 532 \text{ nm}$ respectively before and after loading. Shape of the sensor before deformation was obtained by subtracting the phase maps generated before deflection at λ_1 and λ_2 as shown in Fig. 13(e). Similarly, the shape of the sensor after deformation was obtained by subtracting the phase maps generated after deflection at λ_1 and λ_2 as shown in Fig.13(f). The wrapped phase map in Fig.13(g) contains the information about the deflection along with the shape of the sensor. The subtraction of the shape before from shape after deformation resulted in deformation phase at $\Lambda_{12} = 3340 \text{ nm}$ as shown in Fig. 13(g). The evaluated unwrapped line scans along the central axis are given in Fig. 13(h). The line scan profile-A in Fig.13(h) represents the shape of the sensor before deflection. It indicates that the silicon membrane is projecting out about $1.6 \mu\text{m}$ with reference to the silicon wafer surface. The line scan profile-B represents the deflection along with the shape, while profile-C gives the deflection profile of the sensor[17].

5.5 Surface profiling of combined (polished and unpolished) surfaces

Many engineered structures such as micro systems (or MEMS), consists of both polished and unpolished surfaces on a single platform [12, 35, 79, 96, 97]. For quality testing of such samples, 1λ -interferometry cannot be used. The 2λ -interferometry was successfully used to study polished-unpolished surfaces [42, 47]. Interference between reference and object beams from polished surfaces results in visible fringes. But in case of unpolished surfaces the interference is between a scattered object beam and a smooth reference beam. No fringes were visible due to random speckle background. A sample shown in Fig.14(a) had a smooth region (S) along the boundary while central region is rough (R). Fig. 14(b) and (c) represent the wrapped phase maps at $\lambda_1 = 632.8$ nm and $\lambda_2 = 532$ nm respectively. It can be seen from the interferograms that in the rough surface region, no fringes are observed, while interference fringes are visible in the smooth surface region. To reveal surface contour phase map fringes in the rough surface region, phases in Fig. 14(b) and (c) were subtracted. Fig. 14(d) shows the effective wavelength wrapped phase map measured at $\Lambda_{12} = 3340$ nm, where one can now observe the contour fringes in the rough surface region. The evaluated 3-D surface profile and the corresponding line profile along the central x-axis are shown in Fig. 14(e) and (f) respectively [47].

5.6 Two-wavelength Non-destructive testing (NDT)

Non-destructive testing (NDT) of mechanical elements is an important requirement in many industrial applications. TV holography (TVH) and TV shearography are two independent optical techniques widely used by the industry as a prominent tool for NDT [11, 12, 15, 98-100]. TVH is sensitive to out-of-plane deformation whereas TVS is sensitive to the gradient of deformation. Microscopic TV shearography and a dual function microscopic TV holo-shearography systems have been demonstrated for NDT of microsystems [15]. Any defect on the object surface will induce anomaly in the fringe pattern in and around the defect location. For quantitative measurement of the defect, generation of phase map using a phase evaluation approach is necessary. Single wavelength TVH with phase shifting facility is widely used for deformation measurements and NDT of engineering structures [8, 98, 101]. But, if the deformation

in the defect location results in overcrowding of fringes, it is difficult to quantify such defects using the single wavelength data. Usually, the speckle de-correlation sets a limit for quantifying the large deformation [17]. This problem can be eliminated using a two-wavelength phase subtraction method (Section.2.1a). Shape of the test object can also be obtained using this method. This approach desensitizes the measurement by synthetically increasing the wavelength. Thus it can convert the high frequency crowded fringes at single wavelength to low frequency less number of fringes at effective wavelength, which then can be quantified easily using a conventional phase unwrapping algorithm.

Two-wavelength speckle fringe analysis for quantifying large defects was demonstrated [60, 102]. Experiments were conducted on a flat 4 X 3 mm² specimen with a simulated defect in 1 mm² area. The defect was a blind hole in 1mm² area. The sample was loaded by applying pressure externally. The specimen surface as well as the reference mirror was simultaneously illuminated with the collimated red (λ_1) and green (λ_2) beams. The object and reference waves are recombined at the colour CCD plane. In this experiment, the PZT was calibrated for phase shift $\alpha_2=90^\circ$ at $\lambda_2=532$ nm, so the phase shift error at $\lambda_1= 632.8$ nm is 14.3° , so to compensate this error 8-step algorithm was used (Table-1). Eight frames before and eight frames after loading the sample were acquired. The real-time colour fringe pattern is shown in Fig.15 (a). The colour phase shifted frames then decomposed into its monochromatic components. The correlation fringes, and phase maps at single wavelength are shown in Figs. 15(b,c) and 15(d,e) respectively. These phase maps show overcrowded fringes in the defect location. The 3-D deformation plots at λ_1 and λ_2 are shown in Figs. 15(f), and (g). Due to the high frequency of the wrapped phase in the defect location, quantification of the defect is not accurate. The wrapped phases at λ_1 (Fig.15d) and λ_2 (Fig.15e) were subtracted to generate an effective wavelength phase at $\Lambda_{12} = 3340$ nm (Fig.15f) which clearly shows the contour fringes in the defect location. The phase map at effective wavelength was then unwrapped. The 3-D view of the defect is in Fig.15(i) which clearly shows the enhanced defect. Thus the two-wavelength analysis allows quantification of large defects and also it can provide the shape of the test object at effective wavelength [60, 102].

5.7. Zero-order fringe analysis in digital 3 λ holographic interferometry

In interferometry zero-order fringe (ZOF) means zero optical path difference (OPD). In white light interferometry, ZOF is identifiable but can be used to only measure small path differences correctly. In laser interferometry, the ZOF cannot be identified but by counting the fringe number, large path differences can be measured. In RGB-laser interferometry, the ZOF generated by the R, G, B lasers is always recognizable and the colors always remain identifiable for small and large path differences [103].

The zero-fringe analysis was used to visualize the variations in refractive index induced by candle flame using digital 3 λ (RGB) holographic interferometry [46]. The optical set-up used and the colour fringes obtained in the candle flame are presented in Fig.16a and 16b, respectively. The color holograms are generated and recorded with a Foveon X3 sensor allowing a simultaneous recording with a high spatial resolution. From the colour fringes, the R,G,B components were separated and corresponding wrapped phases were calculated using FFT. At zero order fringe, all the three (R, G, B) wrapped phase values are identically null. From Fig.16b, it is clear that ZOF is shifted and located inside the candle flame. This indicates that the background has moved slightly between the reference and object recording. The colors in the flame are compared with simulation data as shown in Fig.16c. The MIDI software, developed by ONERA, calculates the color fringes versus the OPD. By comparing the reference color table with experimental colours, a variation of $\delta = 3.55\mu\text{m}$ in the OPD was measured between the center of the flame and the right side of the field of view. The developed digital RGB holographic interferometry will find applications in aerodynamics, visualizing gas in motion, characterization of complex flows etc.

5.8. Sensors influence in digital RGB holographic interferometry

In digital holographic interferometry, the resolution of the reconstructed image depends on the pixel size and pixel number of the sensor used for recording. When different wavelengths are simultaneously used for the interferometer, the shape and the overlapping of three filters of a color sensor strongly influence the three reconstructed images. To better understand these problems 3 different sensors, Bayer filter CCD, 3CCD sensor, and Foveon sensor, have been tested. The influence of sensor can be directly visualized in 2D Fourier planes on R, G and B channels.

The fringes generated by the superimposition of 3 reference waves and 3 object waves are simultaneously recorded with the 3 different sensors using the digital 3λ holographic interferometry (Fig.16a). The colour fringes were separated into its individual components. The B-channel data was processed using 2D FFT, the details of which can be found in the References [46, 104]. From Fig.17a, where Foveon sensor is used, it is clear that in both reference and measurement spectra the parasitic images of the G and R light can be seen. It is easy to filter -1 and 0 orders but difficult to filter the parasitic images of the G and R components. Although this camera is not very expensive, it can only be used to record stationary events. From Fig.17b, where Bayer mosaic sensor is used, the parasitic signals are present in the 4 quarters of Fourier planes. The waste light and gaps in the color data, affect the imaging resolution. It is difficult to distinguish good data of the 1^{st} order from the parasitic signals. Fig.17c obtained with the 3CCD sensor, no parasitic lines coming from R and G are visible in the B-channel data. So the filtering of the spectrum will be very easy. The study shows that quantities results obtained with 3CCD sensor are better than those from the other two sensors.

7. Conclusions

In this article, we reviewed multi-colour interferometry and their applications for optical metrology and bio-imaging applications. Applications such as shape and deformation characterization of micro systems (MEMS), refractive index imaging of RBCs, and NDT of large defects were discussed. The multiple-wavelength RGB techniques have several advantages over single wavelength techniques: (i) shape of rough surface can be measured at the effective wavelength, (ii) simultaneous measurement of shape and deformation with variable sensitivities, (iii) relatively large deformation measurements possible, (iv) large step-height measurements possible, (iv) quantification of large defects for NDT applications, (v) spectroscopic imaging of biological cells, (vi) the use of colour CCD allows the image acquisition as simple as single wavelength case. Thus the multi-colour interferometry could be a promising tool that can provide simpler, cheaper, and faster quantitative measurements for optical inspection and imaging applications.

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List of Figure Captions

Fig.1.(a) Modulated interference signal generated in 2λ -interferometry, Typical colour fringes acquired with a 1-CCD RGB camera : (b) Single wavelength (R) interference pattern, (c) RG wavelength interference pattern. The corresponding decomposed R and G components are also shown, (d) RGB- interference pattern, and (e) white light interference pattern.

Fig.2. Generating the effective wavelength phase map by subtracting the single-wavelength phase maps.

Fig.3. Generating speckle correlation fringes in multiple-wavelength speckle interferometry: (a) speckle patterns acquired using 1-CCD RGB camera, (b) decomposed speckle patterns, and (c) subtraction correlation fringes at individual wavelengths.

Fig.4. Schematic of a white light interferometer with a colour CCD and a provision for PZT phase shifting : CL- Collimating lens, IML-Imaging lens, CBS- Cubic beam splitter, MO- Microscopic objective, RM- Reference mirror, PBS- Partial reflecting beam splitter, PZT-Piezoelectric transducer; DAQ-Data acquisition card. Figure reproduced from[53]with permission.

Figure 5. Demonstration of simultaneous recording of R, G, B colours using different sensors : (a) Bayer Mosaic filter 1-CCD colour sensor, (b) Prism-based 3-CCD colour sensor, and (c) Foveon X3 sensor.

Fig.6. 3-D Surface profiling of the fused silica microlens-array: (a) white light colour interferogram, interferogram at (b) 620 nm, (c) 540 nm, (d) 460 nm, and (e) wrapped phase map at 620 nm, (f) 3-D surface profile of the array. Figure reproduced from[105]with permission.

Figure 7. (a) 3-D surface profile of an etched fused silica sample measured with the RGB-interferometer, (b) Difference between a RGB-measurement and a SWLI-measurement. The RMS value is ~ 3.5 nm. Figure reproduced from [64] with permission.

Fig.8. 3-D Surface profile analysis of an etched silicon sample with large discontinuity: (a) white light interferogram, (b) the extracted interferogram at 620 nm, (c) wrapped phase at 620 nm, (d) 2π ambiguity corrected 3-D surface profile using fringe order method, (e) 3-D surface profile measured using phase subtraction method at effective wavelength 4185 nm, and (f) The surface profiles obtained using fringe order method (Profile-A) and phase subtraction method (Profile B). The RMS noise values are indicated on the profiles. A shift is given to the profiles for clarity.

Fig.9. Temporal unwrapping for large step-height measurement: (a) wrapped phase map, (b) unwrapped 3D profile. Figure reproduced from [85] with permission.

Fig.10. Quantitative phase imaging of red blood cells (RBCs) : (a) white light interferogram acquired using 1-CCD RGB camera, (b) decomposed interferogram at 620 nm and (c) unwrapped phase map at 620 nm, (d) 3-D refractive index profile and (e) line profile of a single RBC (identified in Fig.9c) for 620 nm. Figure reproduced from [50] with permission.

Fig. 11. (a) White light interferogram of onion skin, (b) unwrapped phase map of onion for three colors red, green and blue, respectively, and (c) Refractive index of onion for three color wavelengths red, green and blue, respectively. Figure reproduced from [22] with permission.

Fig.12.(a) Schematic of Mirau interferometer and fish-eye, (b) white light colour interferograms at 10 μm , 20 μm and 38 μm , respectively, along axial direction, and (c) 3D unwrapped phase map at 38 μm . Figure reproduced from [55] with permission.

Fig.13. Multiple-wavelength deflection fringe analysis on a MEMS pressure sensor under large pressure load, $P=250$ kPa: Wrapped phase at (a) $\lambda_1 = 632.8$ nm, (b) $\lambda_2 = 532$

nm, (c) $\lambda_3 = 441.6$ nm, (d) deflection line scan profiles at individual wavelengths, Effective wavelength ($\Lambda_{12}=3340$ nm) analysis : (e) shape before loading, (f) shape after loading, (g) deflection phase obtained by subtracting the phase maps in Fig.(e) and (f), and (h) lines scan profiles (A- shape before loading, B-shape after loading, C-deflection profile). Figure reproduced from[17]with permission.

Fig.14. 3-D surface profiling of a polished-unpolished sample using 2λ -interferometry: (a) photograph of the sample, phase map at (b) 632.8 nm, (c) 532 nm,(d) 3340 nm, (e) 3-D surface shape, and (f) corresponding central line-scan profile showing the quantified depth of the rough region with respect to smooth boundary. Figure reproduced from[47]with permission.

Fig.15. Two-wavelength NDT of large-defects: (a) real-time color fringes, fringes at (b) $\lambda_1 = 632.8$ nm, (c) $\lambda_2 = 532$ nm, phase maps at (d) λ_1 , (e) λ_2 , (f) Λ_{12} , 3D-view of the defect at (g) λ_1 , (h) λ_2 , (i) Λ_{12} . The large defect is clearly visible in effective wavelength phase. Figure reproduced from[60]with permission.

Fig.16. (a) Digital three-color holographic interferometer : DM-Dichroic mirror, AOC-Acousto-optical cell, CBS- Cubic beam splitter, AL-Achromatic lens, FM-Flat mirror, (b) Experimental color fringes in a candle flame, (c) Numerical colour fringes. Figure reproduced from[46] with permission.

Fig.17. 2D Fourier spectra for different sensors : (a) Foveon sensor, (b) Bayer mosaic sensor, (c) 3CCD sensor. Figure reproduced from [52] with permission.

Table 1. Various phase shifting algorithms (PSA) and their percentage tolerance for phase shift error from the nominal value of $\pi/2$ [18, 37, 69, 71, 72]

PSA	Equation for Phase calculation	Tolerance (%)
3-step	$\phi^3 = \tan^{-1} \left(\frac{I_1 - 2I_2 + I_3}{I_1 - I_3} \right)$	0
4A-step	$\phi^{4A} = \tan^{-1} \left(\frac{I_2 - I_4}{-I_1 + I_3} \right)$	0
4B-step	$\phi^{4B} = \tan^{-1} \left(\frac{I_1 - 3I_2 + I_3 + I_4}{I_1 + I_2 - 3I_3 + I_4} \right)$	± 3
5-step	$\phi^5 = \tan^{-1} \left(\frac{2(I_2 - I_4)}{-I_1 + 2I_3 - I_5} \right)$	± 3
6-step	$\phi^6 = \tan^{-1} \left(\frac{I_1 - I_2 - 6I_3 + 6I_4 + I_5 - I_6}{4(I_2 - I_3 - I_4 + I_5)} \right)$	± 10
7-step	$\phi^7 = \tan^{-1} \left(\frac{4I_2 - 8I_4 + 4I_6}{-I_1 + 7I_3 - 7I_5 + I_7} \right)$	± 10
8-step	$\phi^8 = \tan^{-1} \left(\frac{-I_1 - 5I_2 + 11I_3 + 15I_4 - 15I_5 - 11I_6 + I_7 + I_8}{I_1 - 5I_2 - 11I_3 + 15I_4 + 15I_5 - 11I_6 - I_7 + I_8} \right)$	± 20

Table 2. Effective wavelengths and their unambiguous step-height measurement range.

Wavelength λ (nm)	Effective wavelength Λ (nm)	Phase equation	Unambiguous range $\Lambda/2$ (nm)
$\lambda_1 = 632.8\text{nm}$ $\lambda_2 = 532\text{nm}$	$\Lambda_1 = 3330$	$\phi_1 - \phi_2 = \frac{4\pi}{\Lambda_1} z$	1670
$\lambda_2 = 532\text{ nm}$ $\lambda_3 = 441.6\text{ nm}$	$\Lambda_2 = 2599$	$\phi_2 - \phi_3 = \frac{4\pi}{\Lambda_2} z$	1300
$\lambda_3 = 441.6\text{ nm}$ $\lambda_1 = 632.8\text{ nm}$	$\Lambda_3 = 1462$	$\phi_3 - \phi_1 = \frac{4\pi}{\Lambda_3} z$	731
$\Lambda_1 = 3330\text{ nm}$ $\Lambda_2 = 2599\text{ nm}$	$\Lambda_{12} = 11839$	$(\phi_1 - \phi_2) - (\phi_2 - \phi_3) = \frac{4\pi}{\Lambda_{12}} z$	5920

Table 3. Multiple-wavelength analysis of deformation and shape measurements at variable sensitivities. Table reproduced from [17] with permission.

Wavelength (nm)	Fringe equation	Measurement	Sensitivity (nm)
$\lambda_1 = 632.8\text{nm}$	$\phi_1^a - \phi_1^b = \frac{4\pi}{\lambda_1} w$	Deformation	316
$\lambda_2 = 532\text{ nm}$	$\phi_2^a - \phi_2^b = \frac{4\pi}{\lambda_2} w$	Deformation	266
$\lambda_3 = 441.6\text{ nm}$	$\phi_3^a - \phi_3^b = \frac{4\pi}{\lambda_3} w$	Deformation	221
$\Lambda_1 = \frac{\lambda_1 \lambda_2}{ \lambda_1 - \lambda_2 }$	$\phi_1^b - \phi_2^b = \frac{4\pi}{\Lambda_1} z$	Shape	1665
$\Lambda_1 = 3330\text{ nm}$	$\phi_1^a - \phi_2^a = \frac{4\pi}{\Lambda_1} (w + z)$	Deformation + Shape	
$\Lambda_2 = \frac{\lambda_2 \lambda_3}{ \lambda_2 - \lambda_3 }$	$\phi_2^b - \phi_3^b = \frac{4\pi}{\Lambda_2} z$	Shape	1300
$\Lambda_2 = 2599\text{ nm}$	$\phi_2^a - \phi_3^a = \frac{4\pi}{\Lambda_2} (w + z)$	Deformation + Shape	
$\Lambda_3 = \frac{\lambda_1 \lambda_3}{ \lambda_1 - \lambda_3 }$	$\phi_1^b - \phi_3^b = \frac{4\pi}{\Lambda_3} z$	Shape	731
$\Lambda_3 = 1462\text{ nm}$	$\phi_1^a - \phi_3^a = \frac{4\pi}{\Lambda_3} (w + z)$	Deformation + Shape	
$\Lambda_{12} = \frac{\Lambda_1 \Lambda_2}{ \Lambda_1 - \Lambda_2 }$	$(\phi_1^b - \phi_2^b) - (\phi_2^b - \phi_3^b) = \frac{4\pi}{\Lambda_{12}} z$	Shape	5920
$\Lambda_{12} = 11839\text{ nm}$	$(\phi_1^a - \phi_2^a) - (\phi_2^a - \phi_3^a) = \frac{4\pi}{\Lambda_{12}} (w + z)$	Deformation + Shape

Fig.1.

Fig.2.

Fig.3.

Fig.4.

Fig. 5.

Fig.6.

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Fig.8.

Fig.9

Fig.10.

Fig. 11.

Fig.12.

Fig.13.

Fig.14

Fig.15.

Fig.16

Fig.17