Regenerated Phase-shifted Sinusoid-assisted Empirical Mode Decomposition

Wang, Chenxing; Kemao, Qian; Da, Feipeng

2016

Wang, C., Kemao, Q., & Da, F. (2016). Regenerated Phase-Shifted Sinusoid-Assisted Empirical Mode Decomposition. *IEEE Signal Processing Letters*, 23(4), 556-560.

<https://hdl.handle.net/10356/80310>

<https://doi.org/10.1109/LSP.2016.2537376>

© 2016 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collective works, for resale or redistribution to servers or lists, or reuse of any copyrighted component of this work in other works. The published version is available at: [<http://dx.doi.org/10.1109/LSP.2016.2537376>].

Downloaded on 26 Apr 2023 18:58:28 SGT

Regenerated Phase-shifted Sinusoids Assisted Empirical Mode Decomposition

Wang Chenxing, Qian Kemao, and Da Feipeng

C. Wang is with the School of Automation, Southeast University, China, and also with the MAGIC, Nanyang Technological University, Singapore (email: w.chenxing@gmail.com).

Q. Kemao is with the School of Computer Engineering, Nanyang Technological University, Singapore (email: mkmqian@ntu.edu.sg).

F. Da is with the School of Automation, Southeast University, Nanjing, China (e-mail: dafp@seu.edu.cn)

Abstract—The effectiveness of the renowned empirical mode decomposition (EMD) is affected by the mode-mixing problem if a signal contains intermittent modes. The ensemble EMD (EEMD) and several modified and extended algorithms solve this problem by adding random white noises. However, the necessary large size of the ensemble and the inevitable manual intervention limits the application of EEMD. In this paper, a novel regenerated phase-shifted sinusoids assisted EMD (RPSEMD) is proposed. Sinusoids with different scales are iteratively generated and added to cope with all possible mode-mixing problems in different intrinsic modes, where each sinusoid is designed adaptively and automatically. Furthermore, the sinusoids are shifted for better retaining the details of each intrinsic mode and eliminating the added sinusoids. In the comparison experiments, the RPSEMD provides more reasonable results with less computation time.

Index Terms— Signal analysis, ensemble empirical mode decomposition, time-frequency analysis, mode mixing problem.

I. INTRODUCTION

EMPIRICAL mode decomposition (EMD) has been widely used as a powerful time-frequency analysis tool [1], [2], [3]. It decomposes a signal into several intrinsic mode functions (IMFs) by a

sifting process based on the extrema of the signal, thus it is regarded an adaptive method driven by data and needs no priori basis function. Since the EMD is sensitive to the extrema, it often suffers from the mode-mixing problem (MMP) if the extrema of a signal are distributed unevenly [4]. Solving the MMP is important for getting more meaningful results.

The ensemble EMD (EEMD) is one of the most effective techniques for solving the MMP [5]. In the EEMD, the original signal is added by a random noise and the newly formed signal is decomposed by the EMD. This process is repeated many times, typically more than 200 as recommended in [5], and then all the IMFs are averaged for the ensemble to yield the final IMFs. The success of the EEMD is attributed to the following factors: 1) the added white noise has scales populated throughout the time-frequency space [6], which makes the extrema distribute more evenly in all scales to avoid the MMP; 2) even if some added noises fail to solve the MMP, the influence of the failures is suppressed through the ensemble average; 3) the added noise will all be cancelled out if the ensemble size is large enough, because the statistic mean of the white noise is 0. The effectiveness of the EEMD leads to some modifications, such as the complementary EEMD [7] and the complete EEMD with adaptive noise (CEEMDAN) [8], [9] that aim to reduce the residual noise or spurious modes. Further, these methods are extended to process multidimensional signals [10], [11]. However, the EEMD based methods still face some challenges. The amplitude of the noise is selected empirically [12], [13], which is a difficulty in practice, e.g., for a signal with drastically different modes, the added noise with a small amplitude is of no effect, while with a larger one may lead to spurious modes. Moreover, the ensemble size of the EEMD is required to be very large, resulting in heavy computation.

Inspired by the idea of getting assistance from random noise, getting assistance from regular signals has also been explored. In [14], a high-frequency sinusoid was added to solve the MMP, while in [15], [16], a pair of complementary sinusoids is added. In [16], the design of the added sinusoids is elaborated. However, these methods can only solve the MMP appearing in high-frequency IMFs. In this paper, a novel regenerated phase-shifted sinusoids assisted empirical mode decomposition (RPSEMD) is proposed. In the RPSEMD, first, different sinusoids are iteratively generated and added to solve the MMP in different scales; second, sinusoids are adaptively designed according to the intrinsic modes of a signal; third, in each stage of decomposition, the sinusoid is shifted by a phase to better retain the

details of each separated intrinsic mode.

The rest of the paper is organized as follows. In Section II, the idea of adding auxiliary sinusoids is explained. In Section III, the proposed RPSEMD and the design of the sinusoids are elaborated. Comparison experiments are given in Section IV. The paper is concluded in Section V.

II. EMPIRICAL MODE DECOMPOSITION WITH AUXILIARY SIGNALS

A. The necessity of an auxiliary signal for EMD

The EMD algorithm is first summarized as follows [1]:

Algorithm 1: EMD

1. Detect the maxima and the minima of $x(t)$;
 2. Interpolate the maxima and the minima to get the upper envelope and the lower envelope respectively;
 3. Compute the mean, $m(t)$, of the two envelopes;
 4. Let $d(t)=x(t)-m(t)$ to sift the signal towards an IMF;
 5. Repeat the steps 1-4 to get the first IMF $c_1(t)=d(t)$ until the stop criterion is met;
 6. Remove $c_1(t)$ and repeat the steps 1-5 on the residue to obtain all other IMFs. The final residue is denoted as $r(t)$.
-
-

The result of EMD can be expressed as

$$x(t) = \sum_{k=1}^K c_k(t) + r(t), \quad (1)$$

where k is the index and K is the total number of IMFs. We always expect that an IMF is a mono component containing only one intrinsic mode (IM) of $x(t)$. If so, an IMF can be represented as $\text{IMF}(t) = a(t)\cos[2\pi f(t)t]$ [1], [17], where $a(t)$ is the instantaneous amplitude and $f(t)$ is the instantaneous phase. Unfortunately, the MMP often appears [5], causing that an IM is split into two adjacent IMFs or different IMs exists in one IMF. Then an IMF containing J IMs will follow the form

$$\text{IMF}(t) = \sum_{j=1}^J a_j(t)\cos[2\pi f_j(t)t]. \quad (2)$$

The study in [18] indicates that a two-mode signal, with a_1, f_1 and a_2, f_2 as their respective amplitudes

and frequencies, can be separated only if the cutoff frequency ratio is no more than 0.67, which can be expressed as

$$f_1/f_2 > 1.5 \text{ (or } f_2/f_1 < 0.67), \quad (f_1 > f_2). \quad (3)$$

Thus, we consider that two points at t_1 and t_2 belong to the same IM if $f(t_1)/f(t_2) \in [0.67, 1.5]$. Further, the extrema detected in the sifting of EMD will all belong to the high-frequency mode if

$$a_1 f_1 > a_2 f_2, \quad (f_1 > f_2). \quad (4)$$

We take (3) and (4) as the criteria for separating two IMs.

Fig. 1 (a) Two IMs and their combination $x(t)$; (b) the IMFs got by EMD.

As an instance, Fig. 1(a) shows a signal $x(t)$ and its two IMs simulated referring to [5], where IM1 and IM2 have amplitudes 0.1 and 1 as well as frequencies 0.06 and 0.0037, respectively. It can be seen that the detected extrema of $x(t)$, marked by blue dots, neither belong to IM1 alone nor belong to IM2 alone. In other words, the extrema of IM1 distributes unevenly and thus a mixed-mode $c_1(t)$ is resulted, which contains the whole IM1 and parts of IM2 near the two borders, as shown in Fig. 1(b).

In view of the cause of MMP, making the extrema of each IM distribute evenly becomes an important factor to solve the MMP, which can be achieved by adding auxiliary signals.

B. Assisted by noise or by a sinusoid?

The EEMD is the mainstream technique to solve the MMP, with the white noise as the auxiliary signal [5]. As described in Section I, in the procedure of EEMD, there are many sets of temporary IMFs resulted from numbers of noise-assisted signals to be averaged to yield final IMFs. For $x(t)$ in Fig. 1(a), one of the successful temporary results is shown as Fig. 2, where the added noise $n_1(t)$ has amplitude 0.1. Unfortunately, the MMP is not always resolved successfully as the auxiliary noise is randomly generated, e.g., Fig. 3 shows an example of the failure temporary results. These occasional failures will be suppressed if the ensemble size is very large, but unavoidably the computation of the EEMD will become very heavy.

Fig. 2 A successful temporary result of the signal $x(t)+n_1(t)$ in EEMD.

Fig. 3 A failure temporary result of the signal $x(t)+n_2(t)$ in EEMD.

The above observation stimulates us to test the feasibility of using a regular auxiliary signal instead. A successful attempt is given in Fig. 4, by adding to $x(t)$ a sinusoid $s(t)$. It is clear that $IM1+s(t)$ are separated from $IM2$ satisfactorily. If doing the similar operation by subtracting $s(t)$ from $x(t)$, we can eliminate $s(t)$ from $c_1(t)$ by averaging the IMFs and leave $IM1$ alone. This example shows that the EMD assisted by actively designed sinusoids can be more effective than using the random noise. Such proposals have appeared in [14], [15], [16], but these approaches are far too simple to solve the MMP in a complicated signal with multiscale IMs. Therefore, in this paper, we propose the RPSEMD for general signals.

Fig. 4 The signal $x(t)$ added with a sinusoid $s(t)$ and the resulted two IMFs.

III. REGENERATED PHASE-SHIFTED SINUSOID ASSISTED EMPIRICAL MODE DECOMPOSITION (RPSEMD)

A. The main idea

The proposed RPSEMD consists of three novel points.

First, a general-form sinusoid is used as the auxiliary signal,

$$s_k(t|a_k, f_k, \theta_k) = a_k \cos(2\pi f_k t + \theta_k), \quad (5)$$

where k denotes the k -th stage of decomposition to extract the IMF $c_k(t)$; a_k, f_k , and θ_k are the amplitude, frequency, and phase, respectively. This form enhances the flexibility to cope with the diverse MMP even if a signal has high complexity.

Second, $s_k(t)$ is designed based on the IM selected after IM clustering analysis. This active way ensures that the RPSEMD is deterministic and is able to solve the MMP more efficiently.

Third, $s_k(t)$ is shifted by θ_k to change the positions of extrema, not only helping to retain more details of the separated IMs but also ensuring $s_k(t)$ be completely canceled out in final results.

The algorithm of RPSEMD is summarized below.

Algorithm 2: RPSEMD

0. Initialize $k=1$;

1. Apply EMD to $x(t)$; determine a_k and f_k with the resulted IMFs; θ_{ki} is uniformly sampled in $[0, 2\pi]$ with phase shifting number I ($1 \leq i \leq I$); $s(t|a_k, f_k, \theta_{ki})$ is then generated;

2. Apply EMD to $x(t)+s(t|a_k, f_k, \theta_{ki})$ to get the first IMF; average all I such first IMF to get the final IMF $c_k(t)$;

3. Remove $c_k(t)$ from $x(t)$: $x(t) \leftarrow x(t) - c_k(t)$, and let $k=k+1$;

4. Repeat steps 1-3 until no more IMF can be obtained; the final $x(t)$ is treated as a residue $r(t)$.

The results of RPSEMD can be written in the same form to (1), but they are more reasonable as the MMP is largely solved.

B. Determining the parameters of $s_k(t)$

The MMP arises because the extrema of the IM in a signal distribute unevenly. Therefore, the target of adding $s_k(t)$ is to imitate a homogenous IM k .

1) Determining a_k and f_k of $s_k(t)$

As described above, the first task is to find the target IM to be imitated. We have stated in (2) that an IMF is generally a composition of J IMs and $J>1$ if the MMP exists. In this sense, the initial IMFs by EMD, denoted as $ci_{k'}(t)$ ($1 \leq k' \leq K'$), can be used as a reference to obtain the target IM.

Obtain the target IM to be imitated:

Since EMD sifts a signal from high frequency to low frequency, we analyze the IMs of $ci_1(t)$ and pick out the one with the highest frequency as the target IM. However, $x(t)$ is unknown and the IMs of $ci_1(t)$ are quite arbitrary, while the only certainty is that the frequencies of different IMs are constrained by (3). In view of these issues, clustering analysis is introduced to cluster the instantaneous frequencies of the extrema in $ci_1(t)$, in order to classify the IMs of $ci_1(t)$. We select hierarchical clustering since it is a flexible and non-parametric approach [19], [20]. Specifically, based on the instantaneous frequencies of extrema, the hierarchical clustering is executed as follows: first the Euclidean distances between instantaneous frequencies are calculated, then a tree of hierarchical clusters with these distances is

created, and finally the clusters are constructed.

As the number of the clusters P is unknown, we start with $P=K'$ to ensure $P \geq J$, and iteratively reduce P by P^m (the number of the pairs of two adjacent clusters not satisfying (3)) until P^m is 0. Finally, we exclude the clusters whose mean frequencies do not satisfy (3) when comparing with that of $s_{k-1}(t)$ obtained in the $(k-1)$ -th stage of decomposition.

Determining a_k and f_k :

The final P clusters are treated as P IMs with amplitudes ac_p ($1 \leq p \leq P$) and mean frequencies fc_p , and the one with the highest frequency, say, the p^0 -th cluster, is determined as the target IM. We then set $f_k = fc_{p^0}$ and $a_k = ac_{p^0}$.

Two adjustments of a_k are executed based on (3)-(4) to ensure the added $s_k(t)$ can be separated from the IMs that are much easier to be mixed with $s_k(t)$, which are described as:

First, to separate from the other IMs in $ci_1(t)$, the current a_k is increased to $\max(a_k, \max(ac_p \times fc_p / f_k))$ according to (4);

Second, to separate from all IMs in $ci_2(t)$, a_k is further adjusted as $a_k + a_k^*$ if a_k, f_k and a_k^*, f_k^* do not satisfy (3)-(4), where a_k^* and f_k^* are obtained by using the above procedure of obtaining a_k and f_k to $ci_2(t)$. This adjustment means to merge the target IM because it is judged to be split into $ci_1(t)$ and $ci_2(t)$.

The algorithm of determining a_k and f_k is summarized below.

Algorithm 3: Determining a_k and f_k of $s_k(t)$

1. For the extrema of $ci_1(t)$, get their instantaneous amplitudes $ai_1(e)$ and instantaneous frequencies $fi_1(e)$, where e indicates the index of an extremum;
 2. Repeatedly classify $fi_1(e)$ into P clusters by adjusting $P \leftarrow P - P^m$ until any two clusters satisfy (3);
 3. Find the p^0 -th cluster and set $f_k = fc_{p^0}$ and $a_k = ac_{p^0}$;
 4. Adjust a_k to ensure $s_k(t)$ can be separated from the IMs clustered in $ci_1(t)$ and $ci_2(t)$.
-
-

2) Determining θ_{ki} of $s_k(t)$

Including the phase θ_{ki} in $s_k(t)$ has two intensions: first, make the auxiliary sinusoid is added complementarily in pairs to make sure they can be eliminated finally; second, change the relative positions of the extrema of the added sinusoid and the signal in order to largely retain the details of the

separated IMs.

The second intension is the response to the fact that, occasionally some extrema describing the details of an IM may be hidden and cannot be detected after adding a sinusoid; while shifting the sinusoid by a phase can make those extrema appear. For example, in Fig. 5(a), some extrema of IM1 in Fig. 1(a), marked by the circle, fail to be detected after adding a sinusoid, whereas all extrema are detected successfully in Fig. 5(b) by shifting the sinusoid.

Fig. 5. The extrema distribution of: (a) $x(t)$ in Fig. 1 added with a sinusoid; (b) $x(t)$ added with the similar sinusoid that includes a phase.

The shifted phase is designed as

$$\theta_{ki} = \frac{2\pi}{N}i, \quad (0 \leq i \leq N-1) \quad (6)$$

where N denotes the number of shifts. To achieve the first intention, N must be an even number so that $\theta_{ki+N/2} = \theta_{ki} + \pi$ ($0 \leq i \leq N/2$) and thus $s_{ki+N/2}(t) = -s_{ki}(t)$. For the second intention, we set $N = 2\mathbf{R}(1/2f_1)$ where f_1 comes from $s_1(t)$ in the first stage of decomposition and $\mathbf{R}(\cdot)$ is a rounding operation. We select f_1 to determine N in order to balance the calculation and the ability of retaining details of IMs, because usually the details of a high-frequency IM is much easier to be lost.

IV. EXPERIMENTS

Fig.6 shows the result by using the RPSEMD on $x(t)$ in Fig. 1, where the resulted IMFs corresponds to the ideal IMs very well.

Fig. 6. The result of RPSEMD applied to $x(t)$ in Fig. 1.

We further add a new mode IM3 to $x(t)$ to increase the complexity, as shown in Fig. 7. The signal is processed by the EEMD [5], [21], the improved CEEMDAN (ICEEMDAN) [9], [22] and the proposed RPSEMD [23] successively. The results of the EEMD and the ICEEMDAN are the best ones after many attempts with different amplitudes of noises and ensemble sizes. The optimal amplitude of the noise is set as 0.4 and the ensemble size is 200. Fig. 8 shows that the RPSEMD results with better correspondence with the ideal IMs. Further, the orthogonality of successive IMFs (OSI) referring to [1] and the time cost of these methods are given in Table 1 as quantitative and objective measures, showing that the RPSEMD outperforms the other two. In addition, the reconstruction error, computed by the standard deviation of the difference of $x(t)$ and $\sum c_k(t) + r(t)$, is noticeable for EEMD (0.0334) but almost 0 for the ICEEMDAN (5.3×10^{-17}) and the RPSEMD (5.6×10^{-16}).

Fig. 7. The IM3 and the new formed signal $x(t)$.

Fig. 8. The results of (a) the EEMD, (b) the ICEEMDAN, and (c) the RPSEMD.

TABLE 1

THE COMPARISON OF PERFORMANCE FOR DIFFERENT METHODS

	Simulation data		Real data	
	OSI (10^{-4})	Time (s)	OSI (10^3)	Time (s)
EEMD	765	38.5	12	48.8
ICEEMDAN	405	22.4	1.9	50.6
RPSEMD	7	1.6	0.12	5

A portion of a real ECG signal, obtained from [22], is shown in Fig. 9(a). The signal is similar to discrete Dirac δ functions and has severe noise, so severe MMP is easily produced. The signal is also processed with the three methods used earlier and the quantitative evaluation is given in Table 1. Both Table 1 and Fig. 9 show that the RPSEMD is superior due to the high efficiency and the fewer but reasonable IMFs [8].

Fig. 9. (a) The ECG signal; the results by (b) ICEEMDAN; (c) RPSEMD.

Finally, it is worth mentioning that both EEMD and ICEEMDAN as noise-assisted methods, give different results in different attempts due to the randomness of noise, while the RPSEMD, being deterministic, always gives the same result.

V. CONCLUSIONS

In this paper, the RPSEMD method is proposed to solve the MMP in EMD. The RPSEMD involves: (i) an active means of iteratively solving the MMP in each IM with regenerated sinusoids; (ii) an effective way for designing the sinusoids; (iii) a phase-shifting scheme for retaining the details of the separated IMs. Due to these novel points, the RPSEMD is able to solve the MMP effectively and efficiently even when the MMP is complicated, which are experimentally validated.

ACKNOWLEDGEMENT

This work was supported by the Multi-plATform Game Innovation Centre (MAGIC) funded by the Singapore National Research Foundation under its IDM Futures Funding Initiative and administered by the Interactive & Digital Media Programme Office, Media Development Authority, the National Natural Science Foundation of P. R. China under Grant 61405034, China Postdoctoral Science Foundation under Grant 2014M551486.

REFERENCES

- [1] N. E. Huang, Z. Shen, S. Long, M. Wu, H. Shin, Q. Zheng, N. Yen, C. Tung, and H. Liu, "The empirical mode decomposition and Hilbert spectrum for non-linear and non-stationary time series analysis," in *Proc. Roy. Soc. A*, vol. 454, pp. 903-995, 1998.
- [2] D. P. Mandic, N. U. Rehman, Z. Wu, and N. E. Huang, "Empirical mode decomposition- based time-frequency analysis of Multivariate signals," *IEEE Signal Process. Mag.*, vol. 36, no. 6, pp. 741786, 2013.
- [3] N. E. Huang and S. S. P. Shen, "Hilbert-Huang transform and its applications," World Scientific, Singapore, 2005.
- [4] N. E. Huang, M. Wu, S. Long, S. Shen, W. Qu, P. Gloerson, and K. Fan, "A confidence limit for the empirical mode decomposition and Hilbert spectral analysis," in *Proc. Roy. Soc. A*, vol. 459, no. 2037, pp. 2317-2345, 2003.

- [5] Z. Wu and N. E. Huang, "Ensemble empirical mode decomposition: A noise-assisted data analysis method," *Adv. Adapt. Data. Anal.*, vol. 1, no. 1, pp. 1-41, 2009.
- [6] Z. Wu and N. E. Huang, "A study of the characteristics of white noise using the empirical mode decomposition method," in *Proc. Roy. Soc. A*, vol. 460, pp. 1597-1611, 2004.
- [7] J. R. Yeh and J. S. Shieh, "Complementary ensemble empirical mode decomposition: a novel noise enhanced data analysis method," *Adv. Adapt. Data. Anal.*, vol. 2, no. 2, pp. 135-156, 2010.
- [8] M. E. Torres, M. A. Colominas, G. Schlotthauer, and P. Flandrin, "A complete ensemble empirical mode decomposition with adaptive noise," in *Proc. IEEE ICASSP*, pp. 4144-4147, 2011.
- [9] M. A. Colominas, G. Schlotthauer, M. E. Torres, "Improved complete ensemble EMD: A suitable tool for biomedical signal processing," *Biomed. Signal Process. Control.*, vol. 14, pp. 19-29, 2014.
- [10] Z. Wu, "The multi-dimensional ensemble empirical mode decomposition method," *Adv. Adapt. Data. Anal.*, vol. 1, no. 3, pp. 339-372, 2009.
- [11] A. Humeau-Heurtier, G. Mahe, and P. Abraham, "Multi-dimensional complete ensemble empirical mode decomposition with adaptive noise applied to laser speckle contrast images," *IEEE Trans. Medical Imaging.*, to be published.
- [12] R. K. Niazy, C. F. Beckmann, J. M. Brady, and S. M. Smith, "Performance evaluation of ensemble empirical mode decomposition," *Adv. Adapt. Data. Anal.*, vol. 1, no. 2, pp. 231-242, 2009.
- [13] M. A. Colominas, G. Schlotthauer, M. E. Torres, and P. Flandrin, "Noise-assisted EMD methods in action," *Adv. Adapt. Data. Anal.*, vol. 4, no. 4, pp. 1250025, 2012.
- [14] C. Wang and F. Da, "Phase retrieval for noisy fringe pattern by using empirical mode decomposition and Hilbert Huang transform," *Opt. Eng.*, vol. 51, no. 6, pp. 061306, 2012.
- [15] R. Deering and J. F. Kaiser, "The use of a masking signal to improve empirical mode decomposition," in *Proc. IEEE ICASSP*, pp. 485-488, 2005.
- [16] C. Wang and F. Da, "Differential signal-assisted method for adaptive analysis of fringe pattern," *Appl. Opt.*, vol. 53, no. 27, pp. 6222-6229, 2014.
- [17] R. C. Sharpley and V. Vatchev, "Analysis of intrinsic mode functions," *Constr. Approx.*, vol. 21, no. 1, pp. 17-47, 2006.
- [18] G. Rilling and P. Flandrin, "One or two frequencies? The empirical mode decomposition answers," *IEEE trans. Signal Process.*, vol. 56, no. 1, pp. 85-95, 2008.
- [19] S. C. Johnson, "Hierarchical clustering scheme," *Psychometrika*, vol. 32, no. 3, pp. 241-254, 1967.

- [20] F. Murtagh, "A survey of recent advances in Hierarchical clustering algorithms," *The computer journal*, vol. 26, no. 4, pp. 354-359, 1983.
- [21] <http://rcada.ncu.edu.tw/research1.htm>.
- [22] <http://bioingenieria.edu.ar/grupos/ldnlys>.
- [23] <http://www3.ntu.edu.sg/home/mkmaqian/RPSEMD.htm>