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Reliability Assessment of Ultimate Limit State of Twin Caverns

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ABSTRACT

Construction of a cavern in close proximity to an existing cavern modifies the state of stresses and movements in a zone around the existing cavern, as some degree of interaction between these two caverns generally takes place. This study investigates the interaction of two parallel caverns and the influence of such interaction on stress-induced global stability in terms of a global factor of safety. A series of finite difference analyses were performed to derive the global factor of safety of a system of two parallel and adjacent caverns. A mathematical response surface model was then built using the multivariate adaptive regression splines (MARS) approach and a series of charts based on this surrogate model were developed to relate the global factor of safety to the critical parameters. The built MARS model is of high accuracy and is simple to interpret and can be used to perform probabilistic assessment of ultimate limit state of twin caverns.

Keywords: reliability assessment; ultimate limit state; factor of safety; twin caverns; interaction; multivariate adaptive regression splines.

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1. Introduction

The construction of a new cavern close to an existing cavern modifies the state of stresses and movements in a zone around the existing cavern. For multiple caverns, the size of this influence zone depends on the ground type, the in situ stress, the cavern span, the width of the pillar separating the caverns, and the excavation sequences. If two adjacent excavations are constructed far apart such that their influence zones do not overlap, then the individual cavern can be considered separately as single isolated caverns and analyzed as such. However, if the influence zones of the two caverns do overlap, some degree of interaction between the two caverns will take place. Interaction of the two caverns will affect the global stability, the state of stress and the deformations around the caverns.

The subject of interaction between parallel caverns/tunnels has been studied by several authors who have reported the results of field measurements or analytical studies of the problem. [Barla and Ottoviani \(1974\)](#) adopted the finite element (FE) method to study the case of two parallel unlined tunnels excavated at the same time in rock. Their results indicate that the interaction between two tunnels become negligible for $S_c/D = 1$ or greater (S_c is the width of the pillar separating the tunnels and D is the diameter of the tunnels). A series of FE analyses on two parallel circular tunnels performed by [Ghaboussi and Ranken \(1977\)](#) suggested that interaction effects are small at $S_c/D=1$, and that at a S_c/D value of approximately two the two tunnels should behave independently of each other. [Gercek \(2005\)](#) numerically analyzed the interaction between closely spaced and parallel underground openings with complicated geometry and concluded that no interaction exists between the parallel openings when $S_c/B > 1.5$ (B is the span of the opening). [Zhao and Ma \(2009\)](#) set up FEM models to study the influence of cavern spacing on the stability of adjacent caverns. Their research indicates that regardless of the strength of the rocks, once the cavern spacing is less than one cavern characteristic length (taken to the biggest width of a cavern in the group), the stability of adjacent caverns is dramatically affected by cavern spacing. [Mortazavi et al. \(2009\)](#) carried out an extensive numerical analysis to study the pillar deformation and failure process under natural loading conditions. They found that as S_c/h (h is the pillar height) ratio reduces, pillar load-bearing capacity drops and at low S_c/h ratios, it is the inherent material strength that governs the failure. At high S_c/h ratios, the pillar behaves in a very stiff manner in the elastic range demonstrating a high load-bearing capacity. [Karademir](#)

(2010) performed 3D FE analyses on excavations in weak rock and concluded that the interaction effects may be eliminated by increasing S_c/D ratio equal to or larger than an approximate value of 2.5-3.0. Based on observations of stable and failed cases, Esterhuizen et al. (2011) supplemented numerical models and suggested that, for room-and-pillar mining in stone mines, pillars having a S_c/h ratio of less than 0.8 should be avoided.

Conventional deterministic evaluation of stability of geotechnical structures and underground openings involves the calculation of global factor of safety FS . Even after obtaining the limit state surface, this single FS value cannot quantify the margin of safety with absolute certainty since the stochastic nature of the design parameters is usually not considered. The alternative is to use probabilistic design approaches which take uncertainty and complexity into account to assess the probability of failure P_f . The calculation of P_f involves the determination of the joint probability distribution of the resistance R and the load (stress) S and the integration of the probability density function (PDF) over the failure domain. For a problem with multiple n random variables, the calculation of P_f involves the determination of a multi-dimensional joint PDF of the random variables and the integration of the PDF over the failure domain. A Monte Carlo simulation (MCS) is a procedure, which seeks to simulate stochastic processes by random selection of input values to an analytical model in proportion to their joint PDF. The basis of MCS is the use of random numbers which were originally either manually or mechanically generated by using such techniques as spinning wheels, dice rolling or card shuffling. It is a powerful technique that is applicable to both linear and non-linear problems.

This study presents a study of the interaction of two parallel caverns and the influence of such interaction on global stability in terms of global factor of safety (FS_{g-t}) with regard to stress-induced instability. Since FS_{g-t} is not known explicitly, it is determined through repeated point-by-point numerical analyses which take into account design variables including the cavern span B , the width of the pillar separating the two caverns S_c , and the rock mass classification Q . Based on the numerical results, closed-form limit state surfaces are then constructed artificially using the multivariate adaptive regression splines (MARS) approach. Subsequently, probabilistic assessment on ultimate limit state of twin caverns was performed using MCS. Charts potentially useful for preliminary design and checking of twin cavern stability are proposed. The developed MARS model can be implemented into MCS to calculate P_f for twin cavern stability.

2. Details of MARS

[Friedman \(1991\)](#) introduced MARS as a statistical method for fitting the relationship between a set of input variables and dependent variables. It is a nonlinear and nonparametric regression method based on a divide and conquer strategy in which the training data sets are partitioned into separate piecewise linear segments (splines) of differing gradients (slope). No specific assumption about the underlying functional relationship between the input variables and the output is required. The end points of the segments are called knots. A knot marks the end of one region of data and the beginning of another. The resulting piecewise curves (known as basis functions), give greater flexibility to the model, allowing for bends, thresholds, and other departures from linear functions.

MARS generates basis functions by searching in a stepwise manner. An adaptive regression algorithm is used for selecting the knot locations. MARS models are constructed in a two-phase procedure. The forward phase adds functions and finds potential knots to improve the performance, resulting in an overfit model. The backward phase involves pruning the least effective terms. An open source code on MARS from [Jekabsons \(2010\)](#) is used in carrying out the analyses presented in this paper.

Let y be the target output and $\mathbf{X} = (X_1, \dots, X_P)$ be a matrix of P input variables. Then it is assumed that the data are generated from an unknown “true” model. In case of a continuous response this would be

$$y = f(X_1, \dots, X_P) + e = f(\mathbf{X}) + e \quad (1)$$

in which e is the distribution of the error. MARS approximates the function f by applying basis functions (BFs). BFs are splines (smooth polynomials), including piecewise linear and piecewise cubic functions. For simplicity, only the piecewise linear function is expressed. Piecewise linear functions are of the form $\max(0, x-t)$ with a knot occurring at value t . The equation $\max(.)$ means that only the positive part of $(.)$ is used otherwise it is given a zero value. Formally,

$$\max(0, x-t) = \begin{cases} x-t, & \text{if } x \geq t \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

The MARS model $f(X)$, is constructed as a linear combination of BFs and their interactions, and is expressed as

$$f(X) = \beta_0 + \sum_{m=1}^M \beta_m \lambda_m(X) \quad (3)$$

where each $\lambda_m(X)$ is a basis function. It can be a spline function, or the product of two or more spline functions already contained in the model (higher orders can be used when the data warrants it; for simplicity, at most second-order is assumed in this paper). The coefficients β are constants, estimated using the least-squares method.

Figure 1 shows a simple example of how MARS would use piecewise linear spline functions to attempt to fit data. The MARS mathematical equation is expressed as

$$y = -44.08 + 4.24 \times BF1 - 3.67 \times BF2 + 6.31 \times BF3 - 2.50 \times BF4 \quad (4)$$

where $BF1 = \max(0, 16 - x)$, $BF2 = \max(0, x - 10)$, $BF3 = \max(0, x - 5.5)$ and $BF4 = \max(0, 5.5 - x)$. The knots are located at $x = 5.5, 10$ and 16 . They delimit four intervals where different linear relationships are identified.

Figure 1

The MARS modeling is a data-driven process. To fit the model in Eq. (3), first a forward selection procedure is performed on the training data. A model is constructed with only the intercept, β_0 , and the basis pair that produces the largest decrease in the training error is added. Considering a current model with M basis functions, the next pair is added to the model in the form

$$\hat{\beta}_{M+1} \lambda_m(X) \max(0, X_j - t) + \hat{\beta}_{M+2} \lambda_m(X) \max(0, t - X_j) \quad (5)$$

with each β being estimated by the method of least squares. As a basis function is added to the model space, interactions between BFs that are already in the model are also considered. BFs are added until the model reaches some maximum specified number of terms leading to a purposely overfit model.

To reduce the number of terms, a backward deletion sequence follows. The aim of the backward deletion procedure is to find a close to optimal model by removing extraneous variables. The backward pass prunes the model by removing the BFs with the lowest contribution to the model until it finds the best sub-model. Thus, the BFs maintained in the final optimal model are selected from the set of all candidate BFs, used in the forward selection step. Model subsets are compared using the less computationally expensive method of Generalized Cross-Validation (GCV). The GCV equation is a goodness of fit test that penalizes large numbers of BFs and serves to reduce the chance of overfitting. For the training data with N observations, GCV for a model is calculated as follows (Hastie et al. 2009)

$$GCV = \frac{\frac{1}{N} \sum_{i=1}^N [y_i - f(x_i)]^2}{\left[1 - \frac{M + d \times (M - 1) / 2}{N}\right]^2} \quad (6)$$

in which M is the number of BFs, d is the penalizing parameter, N is the number of observations, and $f(x_i)$ denotes the predicted values of the MARS model. The numerator is the mean squared error of the evaluated model in the training data, penalized by the denominator. The denominator accounts for the increasing variance in the case of increasing model complexity. Note that $(M - 1) / 2$ is the number of hinge function knots. The GCV penalizes not only the number of the model's basis functions but also the number of knots. A default value of 3 is assigned to penalizing parameter d (Friedman 1991). At each deletion step a basis function is removed to minimize Eq. (3), until an adequately fitted model is found. MARS is an adaptive procedure because the selection of BFs and the variable knot locations are data-based and specific to the problem at hand.

After the optimal MARS model is determined, by grouping together all the BFs that involve one variable and another grouping of BFs that involve pairwise interactions (and even higher level interactions when applicable), the procedure known as analysis of variance (ANOVA) decomposition (Friedman 1991) can be used to assess the contributions from the input variables and the BFs through comparing (testing) variables for statistical significance. Previous applications of MARS algorithm in civil engineering can be found in Attoh-Okine et al. (2009), Lashkari (2012), Mirzahosseinia et al. (2011), Zarnani et al. (2011), Samui (2011), Samui and

Karup (2011), Zhang and Goh (2013) and Goh et al. (2013). Zhang and Goh (2013) systematically explored the generalization capacities and accuracy of MARS approach in comparison to the back-propagation neural network algorithm and found that the main advantages of MARS were its capacity to produce simple, easy-to-interpret models, its ability to estimate the contributions of the input variables, and its computational efficiency. However, use of MARS in underground excavations is limited. In this study, the MARS model was found to be more accurate than the commonly used logarithmic regression (LR) method. For brevity, the LR results have been omitted from this paper.

3. Numerical Modeling

The FLAC^{3D} code (Itasca, 2005) was utilized for the numerical experiments even though only plane strain conditions are considered in this paper as the project will subsequently consider 3D effects and installation of supports. FS_{g-t} is solved through the shear strength reduction (SSR) technique (also called c - ϕ -reduction method), in which the shear strengths are systematically reduced until failure occurs. This procedure was proposed by Zienkiewicz et al. (1975), improved by Brinkgreve and Bakker (1991) and is now available in many commercial finite element (FEM) and finite difference (FDM) programs. This procedure essentially involves repeated analyses by progressively reducing the shear strength properties until collapse occurs. For a Mohr-Coulomb material, by reducing the shear strength by a factor F the shear strength equation becomes:

$$\frac{\tau_f}{F} = \frac{c}{F} + \sigma_n \frac{\tan \phi}{F} \quad (7)$$

$$F = \frac{\tau_f}{c^* + \sigma_n \tan \phi^*} \quad (8)$$

where τ_f is the shear strength, σ_n is the normal stress, and $c^* = c/F$ and $\phi^* = \arctan(\tan \phi / F)$ are the new Mohr-Coulomb shear strength parameters. Systematic increments of F are performed until the finite element or finite difference model does not converge to a solution (i.e. failure occurs). The critical strength reduction value which corresponds to non-convergence is taken to be the global factor of safety FS . It has been applied to caverns (Hammah et al., 2007; Zhang and

Goh 2012; Goh and Zhang 2012), and circular tunnels (Vermeer et al. 2002). The assumptions of the numerical analysis, the cross-section layout of the twin caverns, and the basic design parameters used are described in this section.

3.1 Assumptions of numerical analysis

The basic assumptions of numerical analyses in this case are:

- a) the study was a two-dimensional plane strain problem;
- b) the discontinuous nature of the rock is incorporated implicitly in the Mohr-Coulomb constitutive relationship used to represent the mass as an equivalent continuum;
- c) the rock material obeyed Mohr-Coulomb failure criterion that follows the elastic perfectly-plastic stress-strain relationship;
- d) the caverns are unsupported and only hydrostatic in situ stress fields considered;
- e) the twin caverns are of equal size, both horse-shoe shaped, with semi-circular roof, the span-to-side wall height ratio (B/H) is 2, and horizontally aligned;
- f) the excavation involves six stages: heading, first benching, second benching of the right cavern, followed by heading, first benching, second benching of the left cavern;
- g) the effect of creep was not considered in the analysis.

3.2 Cross-section layout

One significant parameter influencing the interaction is the cavern span B . In this study, B values of 10, 20 and 30 m are considered. In the numerical models, the cavern crown is 65 m below the ground surface. The plane strain conditions are enforced by including a thin 1 m slice of material in the longitudinal direction and imposing boundary conditions on the two off-plane surfaces that allow movement vertically but are restrained against displacements normal to these planes. Outer boundaries are located far from the cavern wall. No surface loading above-ground surface is considered. The initial vertical in situ stress σ_v is induced by self-weight of the rock. The physical and geometrical model including the twin caverns and the design variables considered are shown in Figure2.

Figure 2

3.3 Ranges of design parameters

The main factors affecting interaction between the twin caverns are found to be the cavern geometrical characteristics, the rock strength properties, the in situ stress field, and the excavation sequence. While the effects of some of the factors such as the strength properties and the in situ stresses are measurable, the effects of the others such as excavation sequence cannot be quantified. For simplicity, excavation in six stages as described in Figure 2 is regarded as deterministic. The other important factors are shown in Table 1.

Table 1

For the numerical analyses, the rock mechanical properties are indirectly (through *RMR*) determined from the *Q* system by means of the commonly used empirical equations and correlations.

3.4 Determination of rock mass strength parameters

In the preliminary stage of an engineering design, the need for an approximate estimate of the rock mass parameters frequently arises (Barton et. al. 1980; Hoek and Brown, 1997; Basarir, 2006; Aksoy et al., 2010). For the numerical analyses carried out in this study, the following Equations (Equations 9-15) are adopted for determining the rock mass properties.

$$RMR = 7 \ln Q + 36 \quad (\text{Tugrul 1998}) \quad (9)$$

$$E_m (GPa) = 10^{(RMR-10)/40} \quad (RMR \leq 50) \quad (\text{Serafim and Pereira, 1983}) \quad (10)$$

$$E_m (GPa) = 2RMR - 100 \quad (RMR > 50) \quad (\text{Bieniawski 1978}) \quad (11)$$

$$c (MPa) = 0.005(RMR - 1.0) \quad (\text{Bieniawski, 1989}) \quad (12)$$

$$\phi (^\circ) = 0.5RMR + 4.5 \quad (\text{Bieniawski, 1989}) \quad (13)$$

$$\sigma_{cm} (MPa) = RMR \quad (\text{Palmstrom 2000}) \quad (14)$$

$$\sigma_t(MPa) = \sigma_{cm}/10 \quad (15)$$

where E_m is the deformation modulus of rock mass, c is the cohesive strength, ϕ is the friction angle, σ_{cm} is the Uniaxial Compressive Strength (*UCS*) and σ_t is the tensile strength. Adopting the above empirical equations, the Q value of each category and its corresponding Mohr-Coulomb rock properties to be used in the numerical calculations are shown in Table 2. In Table 2, the Poisson's ratio μ values are assumed. For simplicity, density of 2670 kg / m^3 is assumed for rock mass of all the ranges of Q . Similar correlations were used for the analysis of single caverns (Goh and Zhang 2012).

Table 2

4. Modeling Results and Analyses

4.1 Modeling results of FS_{g_t}

Based on the parameter combinations, a total of 180 cases were analyzed to determine the global factor of safety for twin caverns FS_{g_t} . The FS_{g_t} values are shown in Table 3. Also shown in Table 3 for comparison are the global factors of safety FS_{g_s} for a single cavern.

Table 3

It is obvious from Table 3 that both Q and B significantly affect FS_{g_t} . Figure 3 shows that for the same Q and B , FS_{g_t} increases with the increase of the pillar width. Figure 3 also plots the single cavern FS_{g_s} values (dots with hollow symbol). It is obvious that as the pillar width S_c gradually increases, FS_{g_t} converges to the FS_{g_s} . The difference between FS_{g_t} and FS_{g_s} is generally less than 7% for $S_c/B \geq 2.0$.

Figure 3

4.2 Determination of FS_{g_t} using MARS

Based on the above results, this problem is analyzed by means of MARS. Of the 60 observations, 48 patterns were randomly selected as the training data and the remaining 12 data were used for

testing. The testing data sets are listed in Table 4. The MARS training results together with the relative error e ($e(\%) = \frac{abs(u_{max_s_MARS} - u_{max_s_FDM})}{u_{max_s_FDM}} \times 100$, in which $abs()$ is absolute value function) are shown in Table 5.

Table 4

Table 5

The MARS model adopted 10 BFs of linear spline functions with second-order interaction. A plot of MARS predicted global factor of safety $FS_{g_t_MARS}$ versus the $FS_{g_t_FDM}$ determined using FLAC^{3D} as shown in Figure 4 indicates that the MARS model is reasonably accurate.

Figure 4

Table 6 displays the ANOVA decomposition of the developed MARS model. The first column in Table 6 lists the ANOVA function number. The second column gives an indication of the importance of the corresponding ANOVA function, by listing the GCV score for a model with all BFs corresponding to that particular ANOVA function removed. This GCV score can be used to evaluate whether the ANOVA function is making an important contribution to the model, or whether it just slightly helps to improve the global GCV score. The third column provides the standard deviation of this function. The fourth column gives the number of BFs comprising the ANOVA function. The last column gives the particular input variables associated with the ANOVA function. Table 7 gives the parameter relative importance, which is evaluated by the increase in the GCV value caused by removing the considered variables from the developed MARS model. It can be observed that B is the most important parameter in determining FS_{g_t} , followed by Q .

Table 6

Table 7

Table 8 lists the BFs of the MARS model for FS_{g_t} and their corresponding equations. The interpretable MARS model to predict FS_{g_t} is given by

$$FS_{g_t_MARS} = 1.322 - 0.0224 \times BF1 - 0.0113 \times BF2 + 0.0484 \times BF3 + 0.1082 \times BF4 - 0.17 \times BF5 - 0.00057 \times BF6 + 0.0034 \times BF7 - 0.0496 \times BF8 + 0.0173 \times BF9 - 0.0028 \times BF10 \quad (16)$$

Table 8

The response surfaces relating FS_{g_t} to Q and B , B and S_c/B , Q and S_c/B are illustrated in Figure 5 (a), (b) and (c), respectively. These response surfaces provide a practical design tool, especially during early stages of design and construction. By specifying the pillar geometries S_c/B for a given Q as well as the cavern span B , the expected global factor of safety FS_{g_t} can be estimated quickly. Or by assuming the threshold FS_{g_t} for a given Q and B , the approximate pillar width can be estimated. Figure 6 a-c plots the knot locations for Q , B , and S_c/B respectively. Since MARS explicitly defined these intervals (boundaries) for the inputs Q , B , and S_c/B , the model enables engineers to have an insight and understanding of where significant changes of FS_{g_t} may occur.

Figure 5

Figure 6

4.3 Design curves of FS_{g_t}

Based on Equation (16), charts relating FS_{g_t} to Q , B and S_c have been developed as shown in Figure 7. The proposed charts are potentially useful for preliminary design and checking of twin cavern stability.

Figure 7

5. Probabilistic Assessment on Ultimate Limit State

Assuming the critical FS_{g_t} value is $FS_{g_t_cr}$, the ultimate limit state function of the MARS model is:

$$g(x) = FS_{g_t_MARS} - FS_{g_t_cr} = 1.322 - 0.0224 \times BF1 - 0.0113 \times BF2 + 0.0484 \times BF3 + 0.1082 \times BF4 - 0.17 \times BF5 - 0.00057 \times BF6 + 0.0034 \times BF7 - 0.0496 \times BF8 + 0.0173 \times BF9 - 0.0028 \times BF10 - FS_{g_t_cr} \quad (17)$$

The expressions of BFs in this equation can be found in Table 8. With the determination of the limit state function - Equation (17), reliability assessment of the FS_{g_t} can be carried out using MCS. In this study, the MCS was carried out using the software package @RISK (<http://www.palisade.com>). For each MCS simulation, the number of iterations is 1 000 000 and Latin Hypercube sampling is adopted. The MCS procedures and the calculation of P_f are illustrated in Figure 8 (the critical $FS_{g_t_cr}$ is assumed as 1). Failure in terms of global stability occurs if the predicted FS_{g_t} is smaller than unity. For illustration, the assumed statistical information of the input variables for the MCS is listed in Table 9.

Figure 8

Table 9

Figure 9a plots the influences of the choice of $FS_{g_t_cr}$ on the calculated P_f . It is obvious that $FS_{g_t_cr}$ substantially influences P_f . Figure 9b and 9c plot the influences of design parameter statistics on P_f , assuming $FS_{g_t_cr}$ value of 1. The results indicate that both the *COV* and the average value of Q significantly influence P_f . It is also clear that P_f of twin caverns with large spans is much higher than that with small spans. In addition, P_f decreases with the increase of S_c/B .

Figure 9

6. Conclusions

This study investigates the interaction between two adjacent horse-shoe shaped caverns in terms of the global FS_{g_t} . MARS model was developed to predict FS_{g_t} in terms of Q , B and S_c/B . A series of charts relating FS_{g_t} to Q , B and S_c have been developed, which are potentially useful for preliminary design and checking of twin cavern stability. For the MARS model, the modeling process, the mathematical expression, the parametric relative importance, the response surfaces, and the knot locations of the input variables are elaborated. Probabilistic assessments on the ultimate limit state can be performed once the limit state functions are determined and statistical information of the design parameters are known. Probabilistic assessments are carried out based on MCS and the limit state function derived by MARS model is used. This paper explores the

use of MARS and illustrates a feasible approach to perform probabilistic assessment of ultimate limit state of twin caverns.

It should be emphasized that the rock mass properties in this study is assumed to obey the Mohr-Coulomb failure criterion. Other constitutive models such as the Hoek-Brown model will also be considered in future research, for which [Hoek and Diederichs \(2006\)](#) and [Hoek et al. \(2002\)](#) have proposed relationships for estimating rock mass properties from Geological Strength Index (GSI). **It should also be noted that the proposed model of Eq. (16) is only applicable to the stress controlled instability.**

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Table captions

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Table 1 Cavern design parameters

Parameter	Description	Values
B	Cavern span (m)	10, 20, 30
S_p/B	Ratio of pillar width to cavern span	1, 1.5, 2, 2.5
Q	Tunneling quality index	1, 4, 10, 40, 100

Table 2 Rock mass properties with different Q values

Q	c (MPa)	ϕ ($^{\circ}$)	E_m (GPa)	μ	σ_i (MPa)
1	0.18	22.5	4.47	0.35	2.40
4	0.22	27.4	7.81	0.20	3.05
10	0.26	30.6	11.30	0.20	3.47
40	0.30	35.4	19.75	0.16	4.12
100	0.34	38.6	28.57	0.16	4.55

Table 3 Results from numerical experiments for FS_{g_t}

B (m)	Q	FS_{g_t}				FS_{g_s}
		$S_c/B=1$	$S_c/B=1.5$	$S_c/B=2$	$S_c/B=2.5$	
30	100	1.38	1.47	1.59	1.66	1.67
	40	1.24	1.31	1.42	1.48	1.48
	10	1.04	1.10	1.19	1.25	1.25
	4	0.90	0.96	1.03	1.08	1.09
	1	0.72	0.77	0.83	0.87	0.88
20	100	1.55	1.64	1.72	1.79	1.82
	40	1.38	1.46	1.53	1.61	1.63
	10	1.17	1.23	1.29	1.35	1.36
	4	1.01	1.06	1.12	1.17	1.19
	1	0.81	0.85	0.90	0.94	0.96
10	100	2.13	2.15	2.20	2.24	2.25
	40	1.90	1.92	1.96	2.00	2.02
	10	1.61	1.62	1.65	1.68	1.69
	4	1.39	1.40	1.43	1.46	1.47
	1	1.11	1.12	1.15	1.17	1.19

Table 4 Testing data sets for FS_{g-t}

Q	B (m)	S_c/B	FS_{g-t_FDM}	FS_{g-t_MARS}	Q	B (m)	S_c/B	FS_{g-t_FDM}	FS_{g-t_MARS}
4	30	1	0.90	0.888	4	20	2	1.12	1.140
10	30	1.5	1.10	1.144	40	20	2.5	1.61	1.551
40	30	2	1.42	1.384	4	10	1	1.39	1.365
4	30	2.5	1.08	1.081	10	10	1.5	1.62	1.569
40	20	1	1.38	1.358	40	10	2	1.96	1.980
10	20	1.5	1.23	1.257	4	10	2.5	1.46	1.471

Table 5 Training data sets for FS_{g_t} and relative error

Q	B (m)	S_c/B	$FS_{g_t_FDM}$	$FS_{g_t_MARS}$	e (%)	Q	B (m)	S_c/B	$FS_{g_t_FDM}$	$FS_{g_t_MARS}$	e (%)
1	30	1	0.72	0.663	7.9	1	20	2	0.9	0.915	1.7
10	30	1	1.04	1.059	1.9	10	20	2	1.29	1.311	1.7
40	30	1	1.24	1.245	0.4	40	20	2	1.53	1.497	2.2
100	30	1	1.38	1.445	4.7	100	20	2	1.72	1.697	1.3
1	30	1.5	0.77	0.748	2.8	1	20	2.5	0.94	0.969	3.1
4	30	1.5	0.96	0.973	1.3	4	20	2.5	1.17	1.194	2.0
40	30	1.5	1.31	1.330	1.5	10	20	2.5	1.35	1.365	1.1
100	30	1.5	1.47	1.530	4.1	100	20	2.5	1.79	1.751	2.2
1	30	2	0.83	0.802	3.3	1	10	1	1.11	1.123	1.2
4	30	2	1.03	1.027	0.3	10	10	1	1.61	1.571	2.4
10	30	2	1.19	1.198	0.7	40	10	1	1.9	1.928	1.5
100	30	2	1.59	1.584	0.4	100	10	1	2.13	2.129	0.1
1	30	2.5	0.87	0.856	1.6	1	10	1.5	1.12	1.121	0.1
10	30	2.5	1.25	1.252	0.2	4	10	1.5	1.4	1.363	2.6
40	30	2.5	1.48	1.438	2.9	40	10	1.5	1.92	1.926	0.3
100	30	2.5	1.66	1.639	1.3	100	10	1.5	2.15	2.127	1.1
1	20	1	0.81	0.776	4.2	1	10	2	1.15	1.175	2.2
4	20	1	1.01	1.001	0.9	4	10	2	1.43	1.417	0.9
10	20	1	1.17	1.172	0.2	10	10	2	1.65	1.623	1.6
100	20	1	1.55	1.558	0.5	100	10	2	2.2	2.181	0.9
1	20	1.5	0.85	0.861	1.3	1	10	2.5	1.17	1.229	5.1
4	20	1.5	1.06	1.086	2.4	10	10	2.5	1.68	1.677	0.2
40	20	1.5	1.46	1.443	1.2	40	10	2.5	2	2.034	1.7
100	20	1.5	1.64	1.643	0.2	100	10	2.5	2.24	2.235	0.2

Table 6 ANOVA decomposition of the developed MARS model for FS_{g_t}

Functions	GCV	STD	#basis	variable(s)
1	0.205	0.297	4	Q
2	0.262	0.261	2	B
3	0.02	0.072	2	S_c/B
4	0.033	0.08	1	Q, B
5	0.005	0.024	1	$B, S_c/B$

Table 7 Relative importance of the parameters

Parameters	Relative importance (%)
Q	78.0
B	100
S_c/B	6.6

Table 8 BFs and corresponding equations of MARS model for FS_{g-t}

BFs	Equation	BFs	Equation
BF1	$\max(0, 10 - Q)$	BF6	$BF3 \times \max(0, 40 - Q)$
BF2	$\max(0, B - 20)$	BF7	$\max(0, Q - 4)$
BF3	$\max(0, 20 - B)$	BF8	$\max(0, 4 - Q)$
BF4	$\max(0, S_c/B - 1.5)$	BF9	$BF3 \times \max(0, 1.5 - S_c/B)$
BF5	$\max(0, 1.5 - S_c/B)$	BF10	$\max(0, 40 - Q)$

Table 9 Input variables for reliability assessment of ultimate limit state

Variable	Distribution type	Mean	coefficient of variation (<i>COV</i>)
Q	Normal	4, 10	0.1, 0.2, 0.3, 0.4
B	Normal	10, 20, 30	0.1
S_c/B	Normal	1, 1.5, 2, 2.5	0.1

Figure captions

Figure 1 Knots and linear splines for a simple MARS example

Figure 2 Geometrical model and basic design parameters

Figure 3 Influence of S_c on the FS_{g-t} : (a) $B=10$ m, (b) $B=20$ m, and (c) $B=30$ m

Figure 4 Comparison between FS_{g-t_FDM} and FS_{g-t_MARS}

Figure 5 Response surfaces relating FS_{g-t} to: (a) Q and B , (b) B and S_c/B , and (c) Q and S_c/B

Figure 6 Knot locations for: (a) Q , (b) B , and (c) S_c/B

Figure 7 Design curves for FS_{g-t} : (a) $FS_{g-t}=1.0$, (b) $FS_{g-t}=1.2$, and (c) $FS_{g-t}=1.5$

Figure 8 Implementation of MARS model into MCS

Figure 9 Influence of design parameters on P_f of ultimate limit state: (a) the choice of FS_{g-t_cr} , (b) COV of Q , mean values of B and Q and (c) S_c/B and mean values of B and Q

Figure 1 Knots and linear splines for a simple MARS example

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Figure 3 Influence of S_c on the $FS_{g,t}$: (a) $B=10$ m, (b) $B=20$ m, and (c) $B=30$ m

Figure 4 Comparison between $FS_{g_t_FDM}$ and $FS_{g_t_MARS}$

(a)

(b)

(c)

Figure 5 Response surfaces relating FS_{gt} to: (a) Q and B , (b) B and Sc/B , and (c) Q and Sc/B

(a)

(b)

(c)

Figure 6 Knot locations for: (a) Q , (b) B , and (c) Sc/B

(a)

(b)

(c)

Figure 7 Design curves for $FS_{g,t}$: (a) $FS_{g,t}=1.0$, (b) $FS_{g,t}=1.2$, and (c) $FS_{g,t}=1.5$

(b)

Figure 8 Implementation of MARS model into MCS

(a)

(b)

(c)

Figure 9 Influence of design parameters on P_f of ultimate limit state: (a) the choice of $FS_{g_t_cr}$, (b) COV of Q , mean values of B and Q and (c) S_c/B and mean values of B and Q