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Simulation and detection of photonic Chern insulators in a one-dimensional circuit-QED lattice

Feng Mei,^{1,2,*} Jia-Bin You,² Wei Nie,² Rosario Fazio,^{2,3,†} Shi-Liang Zhu,^{1,4,‡} and L. C. Kwek^{2,5,6,§}

¹National Laboratory of Solid State Microstructures, Department of Physics, Nanjing University, Nanjing 210093, China

²Centre for Quantum Technologies, National University of Singapore, 3 Science Drive 2, Singapore 117543, Singapore

³NEST, Scuola Normale Superiore and Istituto Nanoscienze, CNR, I-56126 Pisa, Italy

⁴Synergetic Innovation Center of Quantum Information and Quantum Physics, University of Science and Technology of China, Hefei 230026, China

⁵Institute of Advanced Studies, Nanyang Technological University, 60 Nanyang View, Singapore 639673

⁶National Institute of Education, Nanyang Technological University, 1 Nanyang Walk, Singapore 637616

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We introduce a conceptually simple and experimentally feasible method to realize and detect photonic topological Chern insulators with a one-dimensional circuit quantum electrodynamics lattice. By periodically modulating the couplings in this lattice, we show that this one-dimensional model can be mapped into a two-dimensional Chern insulator model. In addition to allowing the study of photonic Chern insulators, this approach also provides a natural platform to realize experimentally Laughlin's pumping argument. Remarkably, based on the scattering theory of topological insulators and input-output formalism, we find that both the photonic edge state and topological invariant can be unambiguously probed with a simple dissipative few-resonator circuit-QED network.

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Introduction. The recent rapid experimental developments in circuit QED have turned this system into one of the leading platforms for studying quantum optics and quantum computation [1]. This system possesses high coherence superconducting qubits [2] and well controllable coupling parameters [3,4]. Putting qubits and microwave resonators into a lattice further allows this system to be used for solid-state quantum simulation. The quantum state can be easily manipulated and detected at single-site level in such a lattice. Combined with on-site nonlinearity or photon blockade, circuit-QED lattices have been widely explored as quantum simulators for investigating photon- or polariton-based many-body physics [5–7]. Experimentally, mimicking quantum spin models with qubit arrays recently has been successfully demonstrated [8].

Topological photonics nowadays has become a very active area of research [9]. The photonic topological insulator was first predicted in a two-dimensional photonic crystal [10] and subsequently has been extensively studied [11–15]. Based on the engineering of an artificial magnetic field and spin-orbit coupling, photonic quantum integers and spin Hall states have also been studied in two-dimensional coupled resonators [16–23] and a linear circuit lattice [24,25]. Experimentally, the propagation of photonic chiral edge states has been observed [12,13,15,16,26]. It would be highly desirable, in these experiments, to measure the photonic topological invariants. Due to the different statistics of the carrier, photonic topological invariants cannot be measured using the same methods devised for electronic systems. Recently, how to detect photonic topological invariants has been studied based on probing the Berry curvature [27] and the dynamics of

edge states [28,29]. However, although both circuit QED and topological photonics have rapidly developed, the connection between them has been less studied.

In this paper we propose a conceptually simple method to simulate a two-dimensional photonic topological Chern insulator with a one-dimensional transmission line resonator photonic lattice. By identifying a periodic parameter introduced in the system as the quasimomentum in a second artificial dimension, we show that this one-dimensional photonic lattice can be mapped onto a two-dimensional lattice exhibiting the Chern insulator state. Compared to previous methods of engineering photonic topological insulators, our method employs a one-dimensional photon lattice and does not require mimicking gauge fields and spin-orbit coupling, which opens a different and simple route to study and probe high-dimensional photonic topological states. It is also interesting to note that such a one-dimensional system can provide a natural platform to realize Laughlin's pumping argument [30] and to test the scattering theory of topological insulators [31]. Based on such a feature, we generalize such a scattering theory to take into account dissipation and demonstrate that both photonic edge states and the topological invariant still can be clearly probed from the final steady state. We believe that our results could simulate more studies on the dissipation effect in topological states. It is also noteworthy to stress that the topological features emerge even in a dissipative few-resonator circuit-QED network. A topological invariant defined on a two-dimensional topological lattice can be unambiguously detected with a small-size one-dimensional simulator.

One-dimensional circuit-QED lattice. We start with a transmission line resonator lattice, as shown in Fig. 1(a). The photon hopping between nearest-neighbor resonators is mediated through the coupling capacitors and the connected flux qubits. Note that this coupling method has recently been demonstrated in two-resonator circuit-QED experiments [3,4]. In both cases, the coupling has been designed to make sure that the resonator lattice has the alternating hopping configuration.

*tianfengmei@gmail.com

†rosario.fazio@sns.it

‡slzhu@nju.edu.cn

§cqtklc@nus.edu.sg

FIG. 1. (Color online) (a) Setup for the one-dimensional circuit QED lattice, where the cavity input-output process has been explored to probe the edge state and topological invariant. (b) Artificial torus constructed by combining the parameter θ with the quasimomentum k_x of the photonic lattice.

Each unit cell of this lattice has two resonators, labeled a and b . The capacitively coupled resonator lattice is described by the Hamiltonian

$$H_0 = \sum_n J_1 a_n^\dagger b_n + J_2 a_n^\dagger b_{n-1} + \text{H.c.}, \quad (1)$$

where J_1 and J_2 are the intracell and intercell hopping rates, respectively.

For the qubit-assisted hopping, we assume that the two resonators within the same unit cell are both coupled to the flux qubit Q_1 , while the two resonators belonging to the two nearest-neighbor unit cells are both coupled with the flux qubit Q_2 . The purpose of this coupling is to provide an alternating parametric modulation on the hopping rates and the on-site energies. In the dispersive regime, when all the qubits are in the ground state, the coupling between the resonator and the qubit can be removed, leading to an effective transmission resonator lattice with photon hopping assisted by the connected qubits. Combined with the previous capacitively coupled resonator lattice, the total Hamiltonian of this circuit-QED lattice (in a rotating frame with respect to the external driving frequency ω_d and also in the interaction picture with respect to the qubit energy $\omega_{1,2}$) takes the form

$$H = \sum_n \left(J_1 - \frac{g_1 g_2}{\Delta} \right) a_n^\dagger b_n + \left(J_2 + \frac{g_1 g_2}{\Delta} \right) a_n^\dagger b_{n-1} + \text{H.c.} \\ + \frac{g_2^2 - g_1^2}{\Delta} (a_n^\dagger a_n - b_n^\dagger b_n) + \Delta_c (a_n^\dagger a_n + b_n^\dagger b_n), \quad (2)$$

where g_1 and g_2 describe the coupling strengths between the qubit Q_1 (Q_2) and the resonators a_n and b_n (b_n and a_{n+1}), $\Delta = \omega_1 - \omega_d = \omega_d - \omega_2$ is the detuning of the qubit energies, and $\Delta_c = \omega_c - \omega_d$ is the detuning of the resonator frequency. The qubit-assisted hopping and on-site modulation terms are introduced in order to map into the second dimension for the construction of the photonic Chern insulator.

Two-dimensional lattice mapping. To simulate the two-dimensional Chern insulator Hamiltonian [32], we write the qubit-resonator coupling strengths in the above lattice Hamiltonian in parameter space as

$$g_1 = g_0 \sin(\theta/2), \quad g_2 = g_0 \cos(\theta/2), \quad (3)$$

where the mixing angle $\theta = 2 \arctan(g_1/g_2)$ and $g_0 = \sqrt{g_1^2 + g_2^2}$. The parameter θ is determined by the ratio between the coupling strengths g_1 and g_2 . Note that the coupling strengths between the flux qubit and the resonators can be individually controlled through using superconducting quantum interference (SQUIDs) devices and changing the external magnetic fluxes applied on the SQUID loops [33]. Then θ can be engineered from 0 to 2π for subsequent two-dimensional mapping. Moreover, the topological feature demonstrated below in this model endows this system with topological protection, which allows our methods to be robust to practical deformations in the parameters engineering. By substituting the above equation into the total lattice Hamiltonian and further writing it in momentum space, one can get $H = \sum_k C_k^\dagger h(k) C_k$, where $C_k = (a_k, b_k)^T$. The momentum density has the form

$$h(k) = h_0 + h_x \sigma_x + h_y \sigma_y + h_z \sigma_z, \quad (4)$$

where $h_0 = \Delta_c$; $\mathbf{h} = \{h_x, h_y, h_z\} = \{2J \cos(k_x), 2\delta \sin(k_x) - J_e \sin(\theta) \sin(k_x), J_e \cos(\theta)\}$ with $J = (J_1 + J_2)/2$, $\delta = (J_1 - J_2)/2$, and $J_e = g_0^2/\Delta$; and $\sigma_{x,y,z}$ are the Pauli matrices spanned by a_k and b_k . Interestingly, by associating the mixing angle θ with the quasimomentum k_y in the second spatial direction, one can find that the above one-dimensional circuit QED lattice can be exactly mapped into a two-dimensional Chern insulator Hamiltonian. As plotted in Fig. 1(b), the x direction quasimomentum k_x and the mixing angle θ can form a two-dimensional Brillouin zone $k_x \in [0, \pi]$ and $\theta \in [0, 2\pi]$, which can be rolled into a torus [26,34–36] for analyzing the underlying topology in the artificial two-dimensional lattice.

Photonic Chern insulator. The topological properties of the model introduced above are captured by the Chern number of the Bloch band and the edge state spectrum. By mapping the two-dimensional torus to a spherical surface, the Chern number of the occupied ground band can be expressed as $C = \frac{1}{4\pi} \int \int dk_x d\theta (\partial_{k_x} \hat{\mathbf{h}} \times \partial_\theta \hat{\mathbf{h}}) \cdot \hat{\mathbf{h}}$, where the unit vector $\hat{\mathbf{h}} = (h_x, h_y, h_z)/|h|$ with $|h| = \sqrt{h_x^2 + h_y^2 + h_z^2}$. By substituting $h_{x,y,z}$ into the above formula one can get the Chern number of the ground band as

$$C = \begin{cases} 1 & \text{for } -J_e < 2\delta < J_e \\ 0 & \text{otherwise.} \end{cases} \quad (5)$$

One can change the hopping difference δ to engineer the photonic topological phase transition. It is also worth pointing out that, when the coupling strength $g_2 = -g_0 \cos(\theta/2)$, the ground state can be prepared as a Chern insulator with $C = -1$. According to the bulk-edge correspondence, the appearance of an edge state is a hallmark of a topological insulator. In Fig. 2(a) we have plotted the edge state spectrum for the topological insulator. There is one pair of edge states at the in-gap energy. The density distribution for the left and right edge states has been plotted in the inset in Fig. 2(a). One finds that there is one edge state localized at each edge. For the topological trivial insulator shown in Fig. 2(b), there is no edge state in the gap.

Scattering formulation of topological invariant. Based on Laughlin's [30] pumping argument, recent scattering theory of topological insulators shows that the topological invariant can be described by the reflection matrices at the Fermi level [31].

FIG. 2. (Color online) Edge state spectrum for the photonic Chern insulator with (a) Chern number $C = 1$ for $\delta = 0$ and (b) Chern number $C = 0$ for $\delta = 0.6J_e$. For the Chern insulator, there are two edge states at the in-gap energy denoted by the red dashed line. The inset shows the density distribution of the two edge states. The other parameter are chosen as $J = J_e$ and the lattice size $L = 10$.

The basic experimental setup is achieved by rolling a two-dimensional topological system into a cylinder and threading it with a magnetic flux. For our one-dimensional photonic simulator, if we regard the left and right edges of the photonic lattice as the two ends of the cylinder, the periodic parameter

$$m_1 = 4\delta J + [E_p + J_e \cos(\theta)][\Delta_c + J_e \cos(\theta)] - 2JJ_e \sin(\theta) - \sqrt{[E_p^2 - 4J^2 - J_e^2 \cos^2(\theta)][E_p^2 - 4\delta^2 - J_e^2 + 4J_e \delta \sin(\theta)]},$$

$m_2 = [E_p + J_e \cos(\theta)]\sqrt{J^2 - (E_p + \Delta_c)^2}$, and E_p is the in-gap energy. By substituting Eq. (7) into Eq. (6), we get

$$\begin{aligned} Q &= \frac{1}{2\pi} \int_0^{2\pi} d\theta \left(\arctan \frac{m_2^2 - m_1^2}{2m_1 m_2} \right) \\ &= \frac{1}{2} [\text{sgn}(2\delta + J_e) - \text{sgn}(2\delta - J_e)] \\ &= C. \end{aligned} \quad (8)$$

One finds that the winding number of the phase of the reflection coefficients is exactly equal to the topological invariant of this system. In the following section we show that the information regarding the photonic reflection coefficient can be probed spectroscopically using the cavity input-output process. The photonic Chern insulator is then detected by counting the winding number of reflection coefficient phase.

Probing edge states and reflection coefficients. In contrast to the Fermi system, one can directly probe the edge state and its scattering feature in our photonic simulator. The reason is that bosonic photons can occupy one particular eigenstate at the same time. This could be done by externally driving the resonators with the driven frequency tuned as the eigenenergy of the lattice and then the corresponding eigenmode would be occupied by some weights. In the rotating frame with respect to the driving frequency, the driven Hamiltonian is

θ as the external magnetic flux, and the in-gap energy as the Fermi level, our system can be naturally used to simulate the experimental setup in Laughlin's pumping argument and to test the scattering theory of topological insulators. When the frequency of the incident photon towards one edge is tuned into the in-gap energy and the external periodic parameter θ is tuned over one period, the pumping particle number per cycle can be expressed as

$$Q = \frac{1}{2\pi i} \int_0^{2\pi} d\theta \frac{d}{d\theta} \ln r(\theta), \quad (6)$$

where $r(\theta)$ is the reflection coefficient of the incident photon from one edge. In this way, based on the scattering theory of topological insulators [31], the topological invariant can be characterized by the winding number of the reflection coefficient phase [29].

To further demonstrate this point, we use the Green's function to analytically derive the reflection coefficient from the left edge of the above one-dimensional lattice (see Ref. [33]), giving

$$r(\theta) = -\frac{m_1 + im_2}{m_1 - im_2}, \quad (7)$$

where

$H_d = \sum_n (\Omega_{na} a_n^\dagger + \Omega_{nb} b_n^\dagger) + \text{H.c.}$, where $\Omega_{na, nb}$ are the driven amplitudes in the n th unit cell. In the presence of dissipation, the expectation value of the cavity field a_j in a steady state can be derived from the solution of the Lindblad master equation $\langle \dot{a}_j \rangle = -i \langle [a_j, H + H_d] \rangle + \kappa \sum_n \langle L[a_n] a_j \rangle$, where the Lindblad term $L[a_n] a_j = a_n a_j a_n^\dagger - \{a_n^\dagger a_n, a_j\}/2$ and κ is the cavity decay rate. In the new bases $\vec{a} = (\langle a_1 \rangle, \langle b_1 \rangle, \dots, \langle a_n \rangle, \langle b_n \rangle)^\top$ and $\vec{\Omega} = (\Omega_{1a}, \Omega_{1b}, \dots, \Omega_{na}, \Omega_{nb})^\top$, with $\top$ representing the transpose of the matrix, based on the condition of the steady-state solution $\langle \dot{a}_j \rangle = 0$, we can write the expectation value of the cavity fields in the steady state as

$$\vec{a} = -\left(\Delta_c + T - i\frac{\kappa}{2} \right)^{-1} \vec{\Omega}, \quad (9)$$

where the elements of matrix T are defined by $T_{na, nb} = T_{nb, na} = J_1 - J_e \sin(\theta)/2$, $T_{na, (n-1)b} = T_{(n-1)b, na} = J_2 + J_e \sin(\theta)/2$, and $T_{na(b), na(b)} = \pm J_e \cos(\theta)$.

To probe the edge states, we need to occupy this edge states first. As shown in Fig. 2(a), there is one pair of edge states at the in-gap energy for the photonic topological insulator. In particular, we choose to excite the left edge state through externally driving the leftmost resonator [see Fig. 1(a)], with the driven microwave pulses chosen as $\vec{\Omega} = (\Omega_{1a}, 0, \dots, 0, 0)^\top$

FIG. 3. (Color online) Average photon number in the steady state when the left edge state (circles) and one bulk state (squares) are driven to be occupied by tuning (a) $\Delta_c = 0.5J_e$ and (b) $\Delta_c = 1.2J_e$. For the edge state case, the average photon numbers are also plotted when the driven pulse is applied on the middle (diamonds) and rightmost (stars) resonators. Also shown are the reflection coefficients from the left edge for topological (c) nontrivial and (d) trivial insulators. The lattice size is (a), (c), and (d) $L = 10$ and (b) $L = 4$, the driving amplitude $\Omega_{1a} = 0.1J_e$, and the cavity decay rates $\kappa = 0.1J_e$ (solid line), $\kappa = 0.7J_e$ (dashed line), and $\kappa = 1.5J_e$ (dash-dotted line).

and the driven frequency ω_d tuned to the in-gap energy. The reason is that the left edge state has maximal probability occupying the leftmost resonator. In Fig. 3(a) we have plotted the average photon number in the steady state for this case and find that the photons are most localized in the left edge resonator. In contrast, if the middle and rightmost resonators are driven with same laser, the occupied probability of the left edge mode is very small and then there will almost be no resonant eigenmode and all the photons will finally decay into a vacuum in the steady state. In contrast, when the driven frequency is tuned as the bulk energy, the photons are extensively populated in the lattice, which satisfies the feature of the Bloch bulk state. The results are the same even if we choose two unit cells [see Fig. 3(b)]. Therefore, the photonic edge state can be directly observed by measuring the corresponding average photon number in the steady state.

The detection of the photonic reflection coefficient is naturally related to the cavity input-output process [37]. Using the input-output formalism, the reflected output photon a_1^{out} from the left edge resonator is related to the input photon through $a_1^{\text{out}} = a_1^{\text{in}} + \sqrt{\kappa}a_1$, where the input field a_1^{in}

is related to the external driving by $\sqrt{\kappa}a_1^{\text{in}} = i\Omega_{1a}$ [38]. Using Eq. (9), the photonic reflection coefficient from the left edge is obtained as

$$r_L(\theta) = \frac{\langle a_1^{\text{out}} \rangle}{\langle a_1^{\text{in}} \rangle} = 1 + i\kappa \left[\left(\Delta_c + T - i\frac{\kappa}{2} \right)^{-1} \right]_{11}. \quad (10)$$

In Figs. 3(c) and 3(d) we plot the numerical results of reflection coefficients for the photonic topological nontrivial insulator (Chern number $C = 1$) and the trivial insulator (Chern number $C = 0$). The results show that the winding numbers of the reflection coefficient phase of r_L are 1 and 0, respectively, which yield the photonic topological invariants. This method also applies for the right edge case and the conclusion is the same.

Moreover, we also calculate the corresponding winding numbers even when the lattice size is $L = 4$ (two unit cells) and find that the corresponding trajectories are the same and all the results are the same, which means that topological states could be implemented with only a few-resonator lattice. Such a remarkable feature is quite attractive to circuit-QED experimenters. The current circuit-QED experiment has already realized the capacitive and qubit-assisted coupling between two resonators [4] (one unit cell). Compared with previous works, our scheme is very promising and it is expected to be experimentally demonstrated. In all cases, we also take into account the influence of the cavity decay. The results show that, if the cavity decay rate is not larger than the energy gap $2J_e$, the in-gap energy will remain in the energy gap and the winding number will remain the same; then our measurement is very robust to fluctuations of the frequency of the input photon.

Experimental discussion. Before concluding, a detailed estimate of the experimental parameters involved is in order. For circuit-QED experiment [4], with a typical choice of $\omega_d = 5\Delta$ and $g_0 = 0.1\Delta$, the qubit-assisted hopping rate J_e can approach the order of 10 MHz. For the current coupled transmission line resonator experiment [39], the hopping rate $J_{1,2}$ can be tuned within the range 1–100 MHz. One can easily check that the experimental parameters required in our work are within the experimentally accessible regimes. For the experimental detection of the reflection coefficient phase, we assume an initial input of driven lasers on the leftmost (rightmost) resonator prepared in a coherent state $|\alpha\rangle$ and then the reflected output photon pulse will be in the coherent state $||r_L(r_L)(\theta)|e^{i\phi}\alpha\rangle$, where ϕ is the reflection coefficient phase. Based on homodyne detection by interfering the output photons with a local oscillator, the phase ϕ in the output coherent state $||r_L(r_L)|e^{i\phi}\alpha\rangle$ can be extracted. In this way, the winding number of the photonic reflection coefficient phase is measured, yielding the photonic topological invariant.

Summary. We have introduced a conceptually simple method to realize a photonic Chern insulator in a one-dimensional circuit-QED lattice. Based on Laughlin's pumping argument and input-output formalism, we have further demonstrated that the photonic edge states and topological invariant can be unambiguously measured even in a dissipative few-resonator network, which may take a significant step towards observing a topological invariant with circuit QED. Our method also provides an alternative route and simple means to

study and probe high-dimensional photonic topological states. By introducing an effective photon interaction, the exotic fractional photonic Chern insulator could be further studied in such a framework.

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