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Collective coherence in nearest neighbor coupled metamaterials: A metasurface ruler equation

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The collective coherent interactions in a meta-atom lattice are the key to myriad applications and functionalities offered by metasurfaces. We demonstrate a collective coherent response of the nearest neighbor coupled split-ring resonators whose resonance shift decays exponentially in the strong near-field coupled regime. This occurs due to the dominant magnetic coupling between the nearest neighbors which leads to the decay of the electromagnetic near fields. Based on the size scaling behavior of the different periodicity metasurfaces, we identified a collective coherent metasurface ruler equation. From the coherent behavior, we also show that the near-field coupling in a metasurface lattice exists even when the periodicity exceeds the resonator size. The identification of a universal coherence in metasurfaces and their scaling behavior would enable the design of novel metadevices whose spectral tuning response based on near-field effects could be calibrated across microwave, terahertz, infrared, and the optical parts of the electromagnetic spectrum. © 2015 AIP Publishing LLC. [<http://dx.doi.org/10.1063/1.4934256>]

I. INTRODUCTION

There is a strong thrust on developing application oriented metamaterial devices for radio, microwave, terahertz, infrared, optical, and ultra-violet regimes.^{1–3} The targeted applications are mainly in cloaking, data storage, telecommunications, optics, flexible electronics, active photonics, and nanoscale photolithography. Recently, due to the complexities involved in the fabrication of three-dimensional metamaterials at subwavelength scales, several groups have focused on developing two-dimensional planar metasurfaces that have shown tremendous potential to be used for flat optics.^{4,5} Lasing spaser has been another important innovation in which lasing has been achieved by integrating planar metasurfaces with a gain medium.⁶ In such a system, each resonator oscillates in phase which is synchronized by their nearest neighbor interactions and emits coherent radiation normal to the metasurface. Thus, the nearest neighbor coupling among the metamaterial resonators in a large array can significantly affect the collective coherent behavior of the metasurfaces that are intended for several applications including flat optics, lasing spasers, plasmon rulers, and sensing.^{1,7–12} Several recent works highlighted the intra-unit cell near-field coupling between dipolar resonator pairs in two dimensional and 3D plasmonic arrays that has also resulted in excellent plasmonic rulers.^{13–16} Plasmon rulers can predict the spectral shift of the plasmonic resonance based on the strength of the inter resonator near-field coupling within a unit cell. This unique property allow plasmon rulers to determine extremely small angstrom scale distances in

biomolecules.¹³ However, in most of the previous plasmonic coupling works, only the near-field coupling between the particle pairs in the unit cell of a large array has been studied where the coupling between different unit cells has been negligible.^{13–23} There has been few experiments that aimed at probing the coherent collective behavior of nearest neighbor coupled plasmonic resonators where all the resonators in the entire lattice are coupled to its nearest neighbor through their near fields.^{24–30}

In this contribution, we have investigated the collective coherence of the nearest neighbor coupling in planar split-ring resonator (SRR) arrays that are also known as metasurfaces. The essential finding is the identification of the collective scaling behavior of the coherently coupled metasurfaces in which we observed the decay of strongly overlapping near fields in the SRR lattice due to the mutual inductive coupling. The electromagnetic field decay manifests itself in the form of an exponentially decaying inductive-capacitive (LC) resonance's fractional frequency shift with the decreasing nearest neighbor distance or the lattice period irrespective of the size of the SRRs. We found that the fractional frequency shift in metasurface lattices can be represented by a simple equation that we identify here as a “*coherent metasurface ruler*” (CMR) equation. Thus, we show a CMR equation for inter unit cell coupling in metasurface lattices using rigorous simulations, experimentation, and a simple analytical model. The CMR equation accurately predicts the near-field inductive coupling induced spectral shift in different size metasurfaces with varying lattice constant. This collective coherence behavior is of utmost significance in understanding various functionalities of metasurface based photonic devices such as lasing spaser, polarization control devices, sensors, and flat optics.

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II. EXPERIMENTAL AND NUMERICAL DESIGN

An 8f confocal terahertz time-domain spectrometer (THz-TDS) based on the photoconductive switch is employed to measure the transmitted terahertz signal through the SRR samples and a blank silicon wafer reference. The samples are excited by the 3.5 mm beam waist at normal incidence, and electric field is oriented along the gap arm of the square SRR with a sample area of 100 mm². The frequency-domain spectrum of the amplitude transmission is obtained by $|E_s(\omega)/E_r(\omega)|$ through the Fourier transform of the time domain transmission signal through the sample and the reference, respectively. The square SRR arrays are patterned using conventional photolithography on n-type 640 μm blank silicon wafer, followed by 200 nm of Aluminum film deposition. Three sets of different sized SRR, 18, 22, and 36 μm , in side length were chosen. For the 18 μm SRR size, we fabricated samples with periodicities ranging from 60 to 25 μm ; for the 22 μm SRRs the period varied from 100 to 27 μm ; and for the 36 μm side length of SRRs, the sample period varied from 180 to 40 μm in a decreasing order. The microscopic images of the fabricated SRR samples with sizes of 22 and 36 μm with varying periodicities are showed in Fig. 1. The intrinsic (uncoupled) frequency f_0 of the LC resonance is mainly determined by the absolute dimensions of the square SRRs, and the resonance frequency is given by $f = \omega/2\pi = 1/2\pi (LC)^{1/2}$, where L is the inductance of the SRR loop and C is the capacitance of the SRR gap. The intrinsic frequency f_0 of 18, 22, and 36 μm SRRs is the frequency of LC resonance at 60, 90, and 180 μm periodicities, respectively, when excited at normal incidence with electric field polarized along the gap arm of the SRRs. Numerically simulated transmission spectra for all the three different size SRRs with various periodicities using CST Microwave studio frequency domain solver with tetrahedral mesh are shown in Fig. 2. The boundary conditions are set as unit cell for metasurfaces plane and open (add space) for terahertz wave propagation direction. The lossy aluminum with $\sigma = 3.56 \times 10^7 \text{ S/m}$ and lossy silicon with $\epsilon = 11.90$ are used in the simulation.

III. RESULTS AND DISCUSSION

Figures 2(a)–2(c) show the respective amplitude transmission resonance response of the 18, 22, and 36 μm sized

SRRs with various lattice constants. We observe a common trend in all three different sized SRR sets. The blue-shift of the LC resonances persists with the decrease in the period of each of the different sized SRR metasurfaces. The measured transmission spectra validate the simulated results by showing identical behavior in Figs. 2(d), 2(e), and 2(f) for the 18, 22, and 36 μm sized SRRs, respectively. The blue-shifts of the LC resonances in all of the different sized SRRs with varying periods happen due to enhanced near-field coupling among the nearest neighbors. As the lattice period decreases, the nearest neighbor SRRs give rise to stronger mutual inductance due to overlap of the magnetic flux that is generated from the circulating currents in the SRRs at the LC resonance frequency. As a result of overlapping magnetic flux, the net inductance of the individual SRR is given by $L = L_s - M$ reduces with L_s being the self-inductance and M being the mutual inductance between the nearest neighbors. Thus, the decrease in periodicity leads to the increase in the mutual inductance of the SRRs that causes the blue-shift of the LC resonance in all the different sized SRR sets. The kink at ~ 1.45 THz for 60 μm period in Figs. 2(a) and 2(d) is due to the fundamental order diffractive mode that depends on the index of the substrate and the period of the structure.²⁶

We analyzed the LC resonance shifts in all of SRR sets with varying periodicities. We define the frequency shift as $\Delta f = f - f_0$, where f_0 is the intrinsic uncoupled LC resonance of individual SRRs at the largest periodicity. Figures 3(a) and 3(b) show the simulated and measured frequency shift as a function of periodicity. We notice that the frequency shift in these SRRs decreases exponentially with an increase in the lattice periodicity but with a different decay constant which we address here as exponential decay length. The frequency shifts were fit to exponentially decaying curves with a general behavior of exponential decay lengths $y = Ae^{-x/a}$, as shown in Figs. 3(a) and 3(b). The simulated exponential decay lengths $a1 = 18.52 \pm 1.58$ for the 18 μm SRRs, $a2 = 31.90 \pm 2.41$ for the 22 μm SRRs, and $a3 = 58.29 \pm 4.48$ for the 36 μm SRRs, while the respective decay lengths from the measurements were found to be $a1 = 18.26 \pm 2.87$, $a2 = 25.16 \pm 3.80$, and $a3 = 55.14 \pm 4.69$. The exponential decay lengths are largest for the largest SRRs which signify that the magnetic flux from individual SRR extends over much larger distances in case of larger SRRs and the nearest neighbors begin to interact at comparatively larger lattice periods.

FIG. 1. (a) Schematic unit cell of the SRR with dimensions: $l = 18 \mu\text{m}$ (22 μm , 36 μm), $g = 3 \mu\text{m}$, and $w = 6 \mu\text{m}$. (b) Microscopic images of the 22 μm SRR arrays with varying periodicities P : 27, 45, 60, 75, 90, and 100 μm . (c) Microscopic images of the 36 μm SRR arrays with varying periodicities P : 40, 60, 90, 120, 150, and 180 μm .

FIG. 2. Simulated and measured amplitude transmission. (a), (b), and (c) Simulated LC resonance of the 18, 22, and 36 μm SRR with varying periodicities, respectively. (d), (e), and (f) Measured LC resonances of the 18, 22, and 36 μm SRR with varying periodicities, respectively.

The exponential rise in the frequency blue-shift of smaller sized SRRs with decreasing periods indicate stronger coupling in smaller SRRs where the mutual inductance tends to increase very sharply at close nearest neighbor distances. The decay lengths are shorter in case of smaller SRRs, since the near-field coupling in the lattices is absent in case of larger periodicities because the magnetic flux of each SRR does not extend up to the magnetic flux of the nearest neighbors, and thus they tend to behave as uncoupled nearest neighbors at larger periods. On the other hand, the nearest neighbor coupling in larger SRRs extends over larger periodicities, and the blue-shifting of the LC resonance is not as sharp as it is for the smaller SRRs.

A. Coherent metasurface ruler

In order to probe the size scaling behavior, we plotted the ratio of the resonance blue-shifts and its intrinsic LC resonance as $\Delta f/f_0$ (fractional LC resonance shift) versus the ratio of period to the SRR size as P/l (l = side length of square SRRs) in Figs. 4(a) and 4(b). To our surprise, we notice that the overall fractional LC resonance shift versus P/l for all the different sized SRRs with different periodicities show a collective behavior obeying a single exponential decay equation

$y = Ae^{-x/a}$, where $a = 1.38$ is the decay length extracted from simulated data plotted in Fig. 4(a). The corresponding value of the decay length from the measured data shown in Fig. 4(b) is found to be $a = 1.33$. Figures 4(c) and 4(d) show the scaling behavior of fractional frequency shift with periodicity/size ratio on a log-log plot. The linear fit suggests that collective coherent behavior of SRRs obeys the exponent law and their collective coherent response is a universal behavior. The simulated and measured collective exponential decay lengths in all different sized and different period coherent metasurfaces were found to be in good agreement. Thus, the collective coherent metasurface ruler (CMR) equation is formulated as

$$\Delta f/f_0 = 0.28 \exp[-(P/l)/1.38]. \quad (1)$$

The fractional LC resonance shift exponentially decays with increase in the lattice period to the SRR size ratio indicating that the near-field coupling among the nearest neighbor SRRs declines exponentially as the lattice size (P) increases with respect to the SRR size (l). This eventually becomes uncoupled when the P/l ratio exceeds five times (Figs. 4(a) and 4(b)), since the fractional LC resonance blue-shifting almost becomes negligible for $P/l > 5$. Largest

FIG. 3. (a) and (b) The simulated and measured frequency shifts of the 18, 22, and 36 μm SRR arrays as a function of periodicity and their exponential decay fitting curves $y = Ae^{-x/a}$.

fractional shift of the LC resonance is observed when the lattice size tends to approach the size of the SRRs. Intuitively, this behavior is obvious from the fact that the SRRs have the strongest near-field coupling when they are closest to their nearest neighbors with strongest overlap of their magnetic flux created by the circular currents in the

SRRs at the LC resonance frequency. The disagreement between the exponential fit and the experimental data at large P/l in the fractional frequency shift is due to the reduced mutual inductance effect between the neighboring SRRs resulting from a weaker interaction. As a result, the blue-shift of the LC resonance frequency does not occur anymore, and the resonance frequency tends to be close to the intrinsic resonance frequency. Therefore, the fractional shift of the LC resonance of different SRRs deviates from the CMR equation at large P/l values. At intrinsic LC resonance frequency where the nearest neighbor coupling is absent, P/l values correspond to 3.33 and 5 for 18 and 36 μm SRRs, respectively. Typically, electric coupling causes red-shift of the LC resonance and the magnetic coupling causes the blue-shift. The dominant electric coupling phenomena are observed only in rectangular lattices.^{11,21,22} However, in this work, we do not observe any red-shifting of the LC resonance which is a signature of minimal electric coupling, and the blue-shift of the resonance is due to dominant magnetic coupling.

We also considered the effect of disorder on the collective coherent behavior of the metasurface arrays. If we introduce disorder in the metamaterial lattice, we observe broadening and decrease in the amplitude of the transmission resonance due to the loss of coherence in the system. The disordered arrangement of the unit cells reduces the cooperative interactions between the SRRs, thus degrading the collective coherent behavior of the resonators. We observed an extremely weak spectral blue-shift of the LC resonance in the disordered lattice when compared to the periodic arrangement. We attribute this to the loss of coherence and the suppression of the mutual inductance effect. Thus, the randomly arranged structures do not show a coherent behavior due to extremely weak nearest neighbor interactions which leads to weak average mutual inductance. As a result, the coherent metasurface ruler equation does not hold in disordered metamaterial systems. Our observations agree well

FIG. 4. (a) and (b) Fractional frequency shift extracted from the simulated and measured transmission amplitude spectra versus period/size ratio of SRRs. The simulated and measured data fits with coherent metasurface ruler (CMR) equation $\Delta f/f_0 = 0.28\exp[-(P/l)/1.38]$ and $\Delta f/f_0 = 0.35\exp[-(P/l)/1.33]$, respectively. (c) and (d) The simulated and measured period/size ratio of SRRs on a log-log plot and the corresponding linear fit.

FIG. 5. Comparison of the exponential decay fitting curve $y = Ae^{-x/a}$, where $a = 1.17$ is calculated from the mutual inductance model and the CMR equation. The inset shows the surface currents and mutual inductance of the four nearest neighbor SRRs surrounding the central SRR.

with some of the recently reported works on disordered metamaterials.^{31–35}

B. Mutual inductance model

To understand the mechanism further, we chose a simple mutual inductance model due to high magnetic polariz-

ability¹⁰ at terahertz frequencies that describes the collective exponential decay of the fractional LC resonance shift with respect to the lattice period and the SRR size. The LC resonance frequency of a particular sized SRR with varying periodicities depends on the change of the net inductance and the net capacitance of the LC circuit formed by the individual SRR. Based on the mutual inductance model, the LC resonance frequency is estimated and fitted with an exponential function in order to compare with the CMR equation. For each SRR, there are four nearest neighbor SRRs that surround the individual SRR from its four side arms, as shown in the inset of Fig. 5. However, only the nearest parallel SRR arms with the opposite currents have dominant mutual inductance effect on the frequency shift based on the near-field magnetic coupling effect. For two parallel wires with the opposite current flows have the net inductance of $L = L_s - M$, with L_s being the self-inductance of each SRR and M being the mutual inductance. The mutual inductance M is given in Eq. (2) with l as the length of the conductors and d is the center-to-center distance between two wires

$$M = \frac{\mu_0}{2\pi} l \left[\ln \left(\frac{l}{d} + \sqrt{1 + \frac{l^2}{d^2}} \right) - \sqrt{1 + \frac{d^2}{l^2}} + \frac{d}{l} \right]. \quad (2)$$

For each specific size of SRRs with varying periodicities, the net capacitance and the self-inductance are constant, while the mutual inductance varies with the change in the separation between the four neighboring SRRs. Then, the modified LC resonance frequency becomes

$$f = 1/2\pi \sqrt{\left(L_s - 4 \frac{\mu_0}{2\pi} l \left[\ln \left(\frac{l}{P-l} + \sqrt{1 + \frac{l^2}{(P-l)^2}} \right) - \sqrt{1 + \frac{(P-l)^2}{l^2}} + \frac{(P-l)}{l} \right] \right) C}. \quad (3)$$

Here, $d = P - l$, the frequency f can be plotted as a function of P/l . For the $36 \mu\text{m}$ SRR, $L_s = 144 \text{ pH}$ is calculated from Eq. (4) where l , w , and t are length, arm-width, and thickness of the SRRs, respectively³⁶

$$L_s = \frac{4\mu_0}{2\pi} \times 2l \left[\ln \left(\frac{l}{w+t} + 0.2235 \times \frac{w+t}{l} \right) + 0.726 \right], \quad (4)$$

$$\Delta f = (f - f_0)/f_0 = \alpha(\sqrt{L_s - M}/\sqrt{L_s - M_0} - 1). \quad (5)$$

The fractional frequency shift from the mutual inductance model can be calculated by Eq. (5), where f_0 is the intrinsic frequency when periodicity is $180 \mu\text{m}$ for the $36 \mu\text{m}$ SRRs with mutual inductance M_0 . A scaling fitting coefficient $\alpha = 3.15$ is added to denote the geometrical correction of the LC featured resonator. Then, the fractional frequency shift using the mutual inductance model is plotted in Fig. 5. Compared to the CMR equation with exponential decay length of $a = 1.38$, the fractional LC resonance frequency

shift calculated from a simplistic mutual inductance model is reasonably close to the experimental and simulated decay lengths. The difference in the value comes from the limitation of the model which is based only on individual SRR surrounded by four nearest neighbor SRRs, whereas in the measurements and simulations, the response comes from the entire metasurface lattice.

IV. CONCLUSION

We demonstrate that the collective coherent metasurface ruler equation can predict the lattice size or the distance between the two nearest meta-atoms by observing the fractional LC resonance shift in the transmission spectra of the split-ring resonator based metasurface array. The dominant coupling mechanism in our square lattice is magnetic in nature. The near-field coupling decreases exponentially with the increasing lattice period to the SRR size ratio and

eventually disappears when this ratio exceeds five times. The identification of the CMR equation opens up avenues to design metasurface resonance based rulers that could be used to measure distances in dynamic material/biological systems. Identification of the collective coherence in metasurfaces across the broad electromagnetic domain would allow efficient design of flat metasurface based lasers and optics, such as lasing spasers, flat lenses, and ultrasensitive planar sensors.

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