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TOWARDS COMPRESSIVE SENSING FOR GROUND-TO-AIR MONOSTATIC RADAR

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ABSTRACT

Recently, it is shown that the fundamental problem of range-Doppler estimation can be solved efficiently by compressed sensing (CS) from single-pulse radar return. The performance of CS radar particularly degrade significantly with noise and hence the primary concern is to determine the regime where the operation of CS radar is satisfactory. Most of the studies on CS radar are often conducted under unrealistic conditions where the SNR is much higher than in typical radar applications (e.g., SNR $>$ 5dB). In this paper, we investigate how to improve the CS reconstruction by using coherent integration over N pulses. We consider two scenarios: i) coherent integration is performed *before* CS reconstruction; ii) coherent integration is performed *after* CS reconstruction. We provide numerical results for both scenarios, and demonstrate that a proportional reduction in reconstruction error is obtained if coherent integration is carried out *before* CS reconstruction, corresponding to an effective gain of $\text{SNR}_g = 10\log_{10} N$. On the other hand, when coherent integration is performed after applying CS reconstruction to single pulse radar returns, there is negligible gain. Both observations can be explained by the fact that CS reconstruction exhibits a threshold phenomenon with regard to SNR. By boosting the effective SNR through coherent integration, one can obtain more reliable CS reconstructions.

1. INTRODUCTION

One of the basic use of radar is the estimation of *range* and *velocity* of moving targets [1]. A time-varying linear system is often used to model the radar target space, referred to as $\mathbf{H}$. The target space $\mathbf{H}$ is usually estimated from the radar return $\mathbf{y} = \mathbf{H}\mathbf{f}$, where $\mathbf{f}$ denotes the probing signal. The reflected signal is typically searched for all possible time-delays and Doppler shifts within the radar search range to estimate the range and velocity of the targets. The fundamental problem is how efficiently one can obtain an accurate and reliable estimate for the time delay, Doppler shift, and the reflection coefficient from the reflected signal $\mathbf{y}$. For successful target detection, one needs to mitigate clutter and interference.

Usually the number of targets in the imaging plane of the radar is unknown. Here, we consider the targets to be airborne, hence there will be very few targets against a wide sky. Traditional Radar use matched filter for constructing the range-Doppler space, whose resolution is principally governed by the time-frequency limitation of the probing signal [2]. Compressed Sensing (CS), on the other hand, looks at the problem from a different mathematical perspective, being built entirely upon the *sparsity* of the target scene. CS Radar can provide a better resolution compared to matched filtering under *favorable* conditions [3], and hence should be operated in a regime where it is deemed to be successful. The presence of noise and clutter can severely degrade the performance of CS algorithms and often produce undesirable output. Most of the works on CS Radar [3–5] are tested under unrealistic conditions with high SNRs. In typical radar scenarios, the SNR of the observed signal can be very low; hence reliably solving the fundamental problem of effective and accurate estimation of the range-velocity information using CS is still open. In this paper, we discuss how to tackle this problem using coherent integration and present numerical experiments that supports the claim in a favorable direction.

In Section 2, we explain the classical and the compressed sensing approach of estimating the range-Doppler plane. Some key shortcomings of CS radar are given in Section 3, which is followed by how coherent integration can be combined with CS to overcome them. We present the results and discussions in Section 4, which is followed by concluding remarks in Section 5.

2. RADAR – CLASSIC AND COMPRESSED SENSING APPROACH

Consider a 1-dimensional, far-field, narrowband radar where the transmitter and the receiver are collocated (*monostatic*). Assume a target radially located at a distance x and moving with a *constant velocity* v. The radar emits a probing signal f and observes a reflected signal r, given by the following equation.

$$r(t) = s_{xv} \cdot f(t - \tau_x) \cdot e^{2\pi i \omega_v t}, \qquad (1)$$

where τ_x and ω_v represents the time shift and Doppler shift (frequency shift), respectively; s_{xv} refers to the reflectivity coefficient of the target. The range of the target x can be estimated from the round-trip time $\tau_x = 2x/c$, where c is the velocity of the light. The velocity of the target holds the following relationship with the observed Doppler shift $\omega_v \approx -2v/\lambda$. Thus, the range-velocity information (x, v) can be obtained from the time delay-Doppler shift (τ_x, ω_v) estimates.

Matched filtering of radar return (r) over the search range will yield a 2-dimensional (2-D) time-frequency (range-Doppler) plane, which is given by the following equation.

$$\begin{aligned} \mathcal{A}(\tau,\omega) &= \int r(t) f(t-\tau) e^{-2\pi i \omega t} dt, \\ \mathcal{A}(\tau,\omega) &= |s_{xv}| \mathcal{A}_f(\tau - \tau_x, \omega - \omega_v). \end{aligned} \tag{2}$$

In case of multiple targets, the time-frequency plane $\mathcal{A}$ is the superposition of scaled self-ambiguity function $\mathcal{A}_f$ of the probing function at the location of targets. Hence, the time-frequency operator basis forms a natural way of representing the radar systems and the time-frequency uncertainty principle translates to the radar uncertainty principle. The footprint of the self-ambiguity function on the time-frequency plane determines the resolution of the range-Doppler plane. A spread in the self-ambiguity function results in blurring the location of the targets and adds uncertainty to the number of targets located in a particular region of the range-Doppler plane.

In compressed sensing, the range-Doppler plane $\mathbf{H}$ is discretized and assumed sparse, i.e., there are only very few significant elements. An orthonormal basis (ONB) $(\mathbf{H}_i)_{i=0}^{N^2-1}$ for $\mathbb{C}^{N\times N}$ is used to represent $\mathbf{H} = \sum_{i=0}^{N^2-1} s_i \mathbf{H}_i$ and finding coefficients $(s_i)_{i=0}^{N^2-1}$ is equivalent to finding $\mathbf{H}$. The received signal is given by,

$$\begin{aligned} \mathbf{y} &= \mathbf{H}\mathbf{f}, \\ \mathbf{y} &= \sum_{i=0}^{N^2-1} s_i \mathbf{H}_i \mathbf{f} = \sum_{i=0}^{N^2-1} s_i \varphi_i = \mathbf{\Phi}\mathbf{s}, \end{aligned} \tag{3}$$

where $\varphi_i = \mathbf{H}_i \mathbf{f}$ is the ith atom, $\mathbf{\Phi} = (\varphi_0 | \cdots | \varphi_{N^2-1}) \in \mathbb{C}^{N\times N^2}$, and $\mathbf{s} \in \mathbb{C}^{N^2\times 1}$. As $\mathbf{\Phi}$ being a rectangular matrix, the system of equations mentioned in eq. (3) is clearly underdetermined.

Perfect recovery of $\mathbf{s}$ from $\mathbf{y}$ (from underdetermined system – eq. (3)) is guaranteed by CS atop two important conditions: (1) sufficient sparsity of $\mathbf{s}$ (sparsity level $K \ll N^2$), and (2) incoherence of $\mathbf{\Phi}$. Sparsity is determined by the number of targets in the imaging plane which is not under control; however, the algorithm must be designed to support the maximum possible sparsity. Secondly, the incoherence of $\mathbf{\Phi}$ is determined by the waveform chosen for probing and the basis used for representation. Assuming both the above mentioned conditions are satisfied, then CS recovers $\mathbf{s}$ with with *overwhelming* probability. This recovery problem can be stated as a convex optimization problem as mentioned below.

$$\min_{||\mathbf{s}'||_1} \quad \text{s.t.} \quad \mathbf{\Phi}\mathbf{s}' = \mathbf{y}. \tag{4}$$

If the signal $\mathbf{y}$ is corrupted with additive noise $\mathbf{e}$, the measurements are of the form $\mathbf{y} = \mathbf{\Phi}\mathbf{s} + \mathbf{e}$. If noise in each element obeys $|e_n| \leq \varepsilon$, then the eq. (4) can be reformulated as

$$\min_{||\mathbf{s}'||_1} \quad \text{s.t.} \quad |(\mathbf{\Phi}\mathbf{s}' - \mathbf{y})_n| \leq \varepsilon. \tag{5}$$

Thus, CS recovery can be used to solve for $\mathbf{s}$ mentioned in eq. (3) and in turn the range-Doppler plane $\mathbf{H}$.

3. CS RADAR AND COHERENT INTEGRATION

Although CS seems to be very promising in radar applications, we need to clearly define its satisfactory working regime for its correct usage. We put forth some of the unanswered questions below and try to answer them in this paper.

- In most of the CS radar literature, the results are often only simulated, without any field tests or using the real-time data [4, 5]. Usually, the single-pulse radar returns have very low SNRs, e.g., around -50 dB, and in such cases CS fails miserably [6]. Clearly, the SNR needs to be boosted before applying CS.

- In the CS radar literature, so far the prime motive is how to utilize CS in radar as a replacement for the existing approaches, but not on how it can coexist with the current techniques, supplement them, and improve the accuracy of the overall radar system. On the other hand, maintaining Nyquist rate allows us realize the conventional processing when the target scene does not obey the conditions imposed by CS for satisfactory reconstruction. Merging standard and CS radar, operating CS at Nyquist rate may provide a higher resolution in some specific cases.

- As CS reconstruction is built atop sparsity constraints, its violation will cause a complete failure. The presence of clutter can damage the sparsity of the signal and hence CS is prone to fail if the clutter is not removed sufficiently before applying CS. A similar argument can be made for a noisy signal returns too. Principally, the noise level should be sufficiently low, below the signal floor, for the CS to be effective.

The single-pulse radar returns often have very low SNR, which degrades the performance of target detection. Here, we use coherent integration along with CS reconstruction in two different settings. In first case, the N single pulse radar returns are integrated before applying CS reconstruction; this case is referred as *Integration before CS*. Here, the CS reconstruction is applied after a coherent processing interval (CPI)

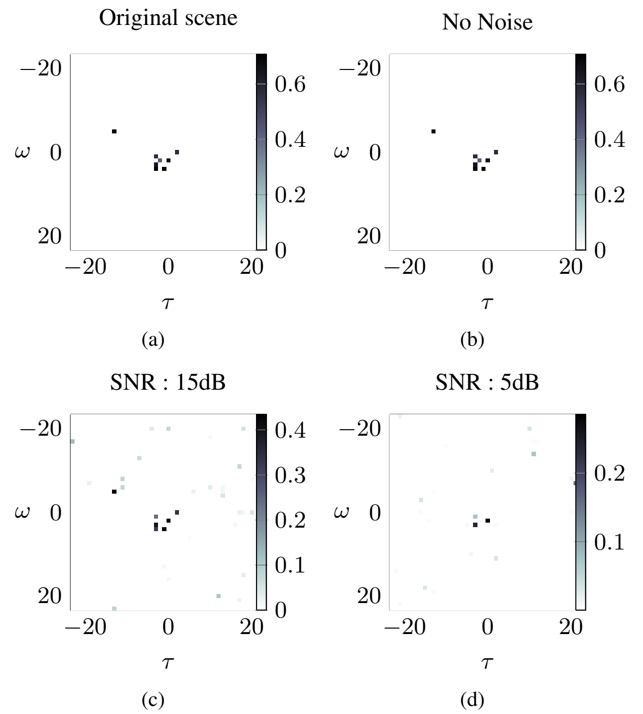


Fig. 1. CS radar simulations with $K = 8$ targets on a 47×47 time-frequency plane at various noise levels. (a) Original scene; CS reconstructions with (b) No noise, (c) SNR$=$ 15 dB, and (d) SNR$=$ 5 dB.

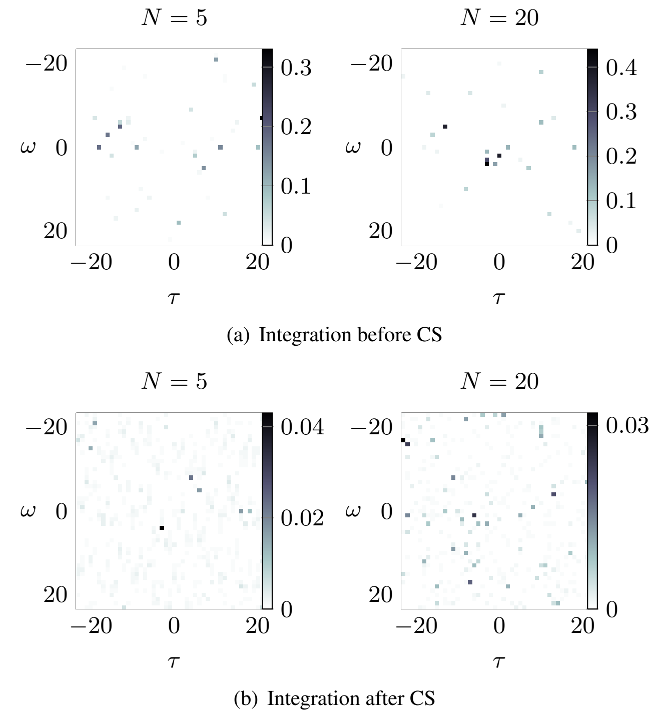


Fig. 2. CS radar simulations along with coherent integration of the original scene shown in Fig. 1(a). The noise of single radar return is SNR $= 0$ dB. (a) Integration is performed before CS, and (b) Integration is performed after CS.

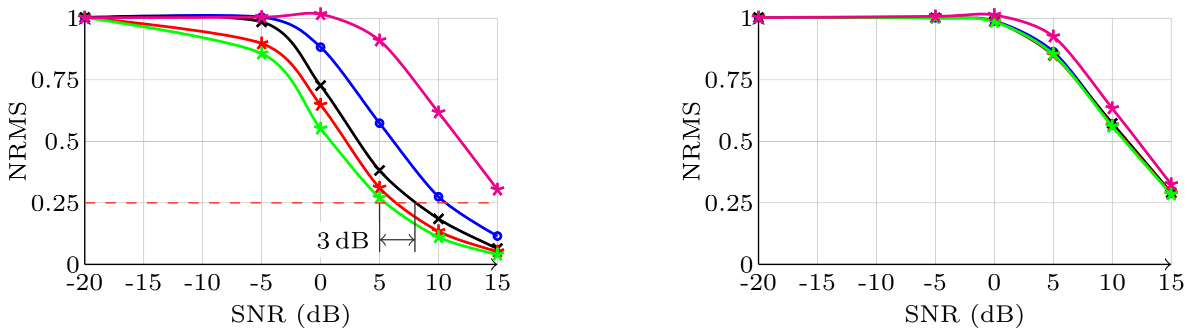
and hence it takes one CPI to produce the first output. In this case, CS reconstruction can be thought of as a slow-time operation. In the second case, we apply the CS reconstruction to single pulse radar returns and integrate the range-Doppler plane for N successive pulses; this case is referred as *Integration after CS*. This operation is very similar to conventional processing where CS reconstruction can be thought of as an equivalent operation (in time scale) to the matched filtering.

4. RESULTS AND DISCUSSION

All simulations are performed on a random time-frequency scene–47×47 grid with $K = 8$ targets. For better evaluation of the reconstruction capacity of the algorithms, the target space contains both individual and clustered targets. CS reconstruction of the time-frequency plane is given in Fig. 1 for various noise levels. As discussed in Section 3, we performed the integration before and after CS reconstruction and summarized the results in Fig. 2 & Fig. 3. In all cases, the results are averaged over 50 different noise realizations for consistency.

CS reconstruction of target space is given in Fig. 1 for various noise levels. All the simulations use Alltop sequence [5] for the CS reconstruction. In the no noise case (SNR $= \infty$), it is clear from Fig. 1 that CS can accurately reconstruct the target scene; the closer targets can also be resolved perfectly. In other words, CS is able to accurately resolve the *smallest* possible cell in the time-frequency plane under favorable conditions. For SNR of 15 dB, the performance slightly degrades but all the targets are still detectable. Some false positives start to appear and the amplitude of the targets is reduced slightly. When SNR is 5 dB, a significant reduction in target amplitude is noticed with very few targets detected correctly and many false positives starting to appear. As mentioned earlier, single pulse radar returns have very low SNRs, at least less than -20 dB, and hence CS definitely fails under those conditions.

Fig. 2 shows the reconstructed target plane for the single radar return having SNR $= 0$ dB integrated over 5 and 20 pulse returns. Fig. 2(a) shows that when integration is performed before CS, it leads to a significant increase in detected target amplitudes. For a particular threshold, the probability of detecting an actual target increases with increasing N. For example, for the case of $N = 20$, most of the targets appeared along with only a few false positives. In contrast, Fig. 2(b) shows that for integration after CS, increase in amplitude is negligible; not even a single actual target is detected correctly. Strong noise often violates the sparsity constraint and render the output of CS unusable.



(a) Integration before CS　　(b) Integration after CS

Fig. 3. Reconstruction error performance for integration (a) before, and (b) after CS reconstruction for the different coherent integration durations, N=1 (magenta), N=5 (blue), N=10 (black), N=15 (red), and N=20 (green). Sparsity level is $K = 8$ with cell size of 47×47. NRMS stands for normalized root-mean-square between original and reconstructed target scene.

Figure 3 gives the variation of reconstruction error in normalized root-mean-square (NRMS) with SNR of single-pulse radar returns. Let us first discuss the results for "Integration after CS" shown in Fig. 3(b). For easier interpretation, we divide the plot in Fig. 3(b) into two parts: SNR < 0 dB and SNR > 0 dB. Consider the observed signal $\mathbf{y} = \mathbf{Hf} + \mathbf{e}$, where noise in each element must satisfy $|e_n| \leq \epsilon$ for a stable K-sparse solution [3]. The value of ϵ is estimated from the noise, and hence it determines the *sparsity* and *stability* of the solution. When SNR < 0 dB, the signal level is below the noise level and the estimate of ϵ is large enough to mask the sparsity thereby affecting the stability of solution. Hence, this leads to a non-sparse and near-zero solution, which can be verified from the Fig. 2(b). When SNR > 0 dB, the signal is above the noise level, which in turn reduces the estimate of ϵ, leading to improved reconstruction. The plot clearly shows that the error depends only on the value of SNR and not on the value of N. This implies that integration after CS has very little or no effect in improving the error and detection of the targets.

Next, we discuss "Integration before CS" case, where visible difference is noticed with increasing N. Integration over N pulses will result in improvement of SNR by $10 \log_{10} N$. For example, the improvement between $N = 10$ and $N = 20$ is nearly 3 dB which is shown in Fig. 3(a) for NRMS $= 0.25$. Hence, integration before CS leads to higher effective SNR, a lower estimate of ϵ (due to reduced noise), which in turn unveils the sparsity of the observed signal and hence results in a more stable solution compared to its counterpart. Note, however, that CS obtains an unstable solution for SNR$= -20$ dB where the integration is not sufficient to pull the signal over the noise level. On the other hand, a stable solution is obtained for all other SNR values once the signal crosses the noise level. As a final note, all targets are assumed stationary over the period of integration; a change in the target state leads to a reduction in the achievable SNR.

5. CONCLUSION

In this paper, we discussed the problem of estimation of delay-Doppler plane using compressed sensing, particularly the algorithmic failure at high noise scenarios. We propose to use coherent integration as a possible solution to this problem, and apply it in two different settings with a view to improve the delay-Doppler estimation. If coherent integration is carried out before CS, the effective SNR is improved by $10 \log_{10} N$, which in turn leads to improved target detection. In near future, we are planning to investigate frameworks that involves both CS and conventional method to improve the accuracy and robustness of the overall radar system.

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