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2016

Chiba, S., & Leong, K. (2016). Behavioral Economics of Crime Rates and Punishment Levels. *Journal of Institutional and Theoretical Economics*, 172(4), 727-754.

<https://hdl.handle.net/10356/81339>

<https://doi.org/10.1628/093245616X14631368691817>

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Behavioral Economics of Crime Rates and Punishment Levels

by

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Received July 24, 2014; in revised form August 25, 2015;

accepted March 8, 2016

Empirical studies have shown, paradoxically, that increasing the probability of apprehension can correlate with an increase in the total number of criminal actions. To examine this phenomenon, we develop a dynamic model of “personal rules” in which forgetfulness and hyperbolic discounting together can cause a potential criminal to commit more crimes as the probability of apprehension increases. At the time of the future decision, he may commit a crime due to hyperbolic discounting, even if it is not profitable. Hence, he may choose not to commit a crime today as a commitment device to abstain from crime in the future. However, increased prosecution can limit the effectiveness of the commitment device. (JEL: D03, D81, K42)

1 Introduction

A key allure of substance abuse is the promise of instant gratification: feeling “high” or helping one “forget” anxieties (National Institute on Drug Abuse, 2008). In such crimes, the potential criminal chooses between instant gratification through substance abuse and the delayed benefits of abstinence, such as good health. Because of the temptation of instant gratification in such crimes, self-control, an internal commitment device, is key for a person who is trying to prevent himself from committing these crimes. Hence, a potential substance abuser, when considering the future ramifications of substance abuse, might choose to exert self-control to override the present temptation of instant gratification for the sake of future benefit. In view of the importance of self-control in preventing crime, a natural question would be whether increasing the probability of apprehension affects one’s self-control – and consequently the rates of crime.

* Kyoto University, Japan, and (corresponding author) Nanyang Technological University, Singapore. We are particularly grateful to Faruk Gul and Bart Lipman for their guidance and patience during the entire project. We thank Andrew Ellis, Sambuddha Ghosh, Keith N. Hylton, Ching-To Albert Ma, Michael Manove, Dilip Mookherjee, William Neilson, Andrew Newman, Rahel O’More, Chun Wing Tse, and Tak Yuen Wong as well as seminar participants at Boston University, Princeton University, and 2011 Econometric Society North American Summer Meetings for helpful comments. Research assistance by Ng Jian Jie, Duoxi Li, Eugene Lim, Haiyan Long, and Jeff Zhou is gratefully acknowledged. The first author acknowledges use of the facilities of the Department of Management at Università Ca’ Foscari Venezia. All remaining errors are our own.

Becker (1968) argued that an increase in the probability of apprehension led to a monotonic decline in crime if criminals were rational. However, Becker’s view (1968) has not always been consistent with empirical literature, which suggested that increasing apprehension rates or severity of punishment could have a positive relationship with criminal activity (Cerro and Meloni, 2005; Myers, 1983; Tittle and Rowe, 1974). This paper proposes an alternative theory to Becker’s that reconciles these counterintuitive empirical findings. We focus on the effect of apprehension rates on crime rates. Specifically, we consider soft crimes (e.g., illegal drug use) rather than hard crimes (i.e., armed robbery).

We develop a dynamic model of *personal rules* in which imperfect recall (or forgetfulness) and hyperbolic discounting (or temptation) together can cause a potential criminal to commit more crimes as the probability of apprehension increases. Our work builds on Bénabou and Tirole’s (2004) theory of “personal rules” using self-reputation as a mechanism for achieving self-control.

There are two crucial ingredients in our model. The first is hyperbolic discounting – when a potential criminal is presented with an opportunity for crime, the temptation of immediate gains from crime causes him to overweigh these gains relative to future payoffs from honest work. This sometimes causes a criminal to commit a crime even when it is not in his best interests to do so *ex post*. Criminologists have suggested that criminal actions are essentially associated with impulsivity (Gottfredson and Hirschi, 1990; Kahneman and Tversky, 1979).

The second ingredient is imperfect recall – a potential criminal with no past criminal experience is unable to accurately recall past information about his criminal productivity without the aid of hard evidence about his productivity. Piccione and Rubinstein (1997a,b) first identified the importance of imperfect recall for studying decision-making problems. Likewise, we examine how imperfect recall affects decision-making in a multiple-period model. Moreover, like Bénabou and Tirole (2004), our model involves only one player, and the effects of imperfect recall are related to the player’s self-inference.

Our model includes two periods. A potential criminal, DM, has two opportunities for criminal actions, one in each period. Before period 1, he is endowed with criminal productivity (which determines his payoffs from crime) and receives a noisy signal about his criminal productivity. This signal is soft information and will be forgotten in period 2 if DM does not commit a crime in period 1. Trope (1978), Tulving and Thomson (1973), and Tversky and Kahneman (1971) all demonstrated imperfect recall of soft information. If DM commits a crime in period 1, he will have hard evidence (for example, incarceration) concerning his criminal productivity. In every period, DM faces temptation (hyperbolic discounting) that would cause him to commit a crime in the period against his better interests. In period 1, DM anticipates facing temptation (hyperbolic discounting) in period 2.

Cutoff – DM’s criminal productivity cutoff – at each period characterizes an equilibrium. In each period, DM commits a crime if he infers that his criminal productivity is below each period’s cutoff, meaning that his expected gain from committing a crime in the period exceeds the expected loss. If the inferred productivity is below the period’s

cutoff, his expected loss from committing a crime in the period exceeds the expected gain. In our dynamic model, he can abstain from crime in period 1 as an internal control mechanism to prevent himself from committing a crime in period 2. DM's abstinence in period 1 causes him to forget the signal about his criminal productivity in period 2 because of imperfect recall. As a result, DM has to infer his criminal productivity from his first-period cutoff: he attributes his first-period abstinence to his criminal productivity being below his first-period cutoff, and is able to infer what it is. If his inferred criminal productivity is below his second-period cutoff, he does not commit a crime in period 2. If he knows that he will not commit a crime tomorrow when he does not commit a crime today, his choice of abstinence in period 1 acts as a commitment device to avoid a criminal action in period 2. DM may resist today's temptation and choose not to commit a crime in period 1 in order to prevent his future mistake – committing a crime in period 2 due to temptation.

Section 2 considers a model with a constant level of temptation between two periods. In this case, the commitment device works: DM will not commit a crime in period 2 if he does not commit a crime in period 1 and forgets his signal. We claim that the probability that he will commit a crime in period 2 given he commits a crime in period 1 decreases in the probability of apprehension. This claim of conditional monotonicity is supported in Emons (2003) and Polinsky and Shavell (1998). In addition, the cutoff in each period increases with the probability of apprehension. As a result, the total number of criminal actions across periods also decreases in the probability of apprehension.

Section 3 considers models with asymmetric levels of temptation between two periods. We show that the total number of criminal actions across periods can *increase* with the probability of apprehension because a commitment device may not work for every level of probability of apprehension. As the cutoff in each period increases with the probability of apprehension, DM's inferred criminal productivity in period 2 increases with the probability of apprehension. As a result, if DM's level of temptation increases over time, for a certain range of the probability of apprehension, his inferred criminal productivity can exceed his second-period cutoff. As a result, DM commits a crime in period 2 although he did not in period 1. Hence, conditional on DM's abstinence from crime in period 1, his criminal action in period 2 can be nonmonotonic with the probability of apprehension. An increase in the probability of apprehension can undermine the effectiveness of self-control as an internal commitment mechanism.

Section 4 considers extended models and discusses the robustness of the nonmonotonic relationship result in section 3. We can observe the nonmonotonic relationship result if we consider endogenous imperfect recall instead of increasing temptation. Section 5 concludes. Proofs are gathered in the appendix.

1.1 Related Literature

Inclusion of Behavioral Factors in Classical Analysis of Criminal Behavior. Many studies in criminology identified the limitations of the classical approach towards analyzing criminal behavior despite recognizing the merits of such an approach. Jolls, Sunstein, and Thaler (1998) as well as Garoupa (2003) modified the classical model in order to

achieve more realistic results. Similarly, we modify the classical model by allowing for hyperbolic discounting and imperfect recall.

Personal Rules. People are aware of their own tendencies to exhibit present bias and hence rely on cognitive rules to prevent themselves from giving in to such temptation. According to Wertenbroch (1998), people impose personal rules on themselves in the form of precommitment so that a present decision today eliminates the possibility of engaging in myopic behavior in the future.

Bénabou and Tirole (2004) developed a theory of personal rules using imperfect recall and hyperbolic discounting. People have imperfect knowledge of their willpower, and hence see their own choices as indicative of “what kind of a person” they are. Lapses (incidents where one succumbs to temptation) can cause one to lose faith in oneself and hence succumb to temptation in the future. Due to the presence of imperfect recall, an individual chooses his memory to have confidence in his willpower, which helps him exercise self-control effectively and avoid future mistakes. On the other hand, the effects of imperfect recall and hyperbolic discounting do not necessarily offset each other in our model.

Akerlof and Dickens (1982) and Dickens (1986) predicted nonmonotonicity between severity of punishment and a number of criminal actions. They used a non-Bayesian framework that assumes DM directly chooses his belief about the value of the crime to avoid a psychic cost of cognitive dissonance. On the other hand, our study predicts nonmonotonicity between the probability of apprehension and a number of criminal actions. We use a Bayesian framework that assumes DM does not directly choose his future inference.

Self-Signaling. As psychologists suggested, people often make choices to preserve favorable self-images because they often learn about themselves by observing past actions. In Bénabou and Tirole (2006b), people consider monetary payoffs, values of their self-images, and values of social reputations for altruism. Then, the lower probability of apprehension may increase the value of self-image or social reputation as a result of avoiding a crime, and therefore result in a smaller number of criminal actions. On the other hand, only considering monetary payoffs, we show that lower probability of apprehension may decrease individuals’ confidence in the criminal productivity, which may result in a smaller number of criminal actions.

Type of Information. Our model is closely related to Bénabou and Tirole (2009), in which the memorability of information depends on the type of information. “Hard information,” which leaves a verifiable record, is perfectly recalled. “Soft information,” which does not leave a verifiable record, may not be recalled perfectly.

Imperfect Recall. The idea of imperfect recall is widely supported by empirical studies in biology and psychology. In this model, as in Bénabou and Tirole (2009), when DM chooses a payoff-relevant action, he considers not only his immediate payoffs, but also

his future payoffs determined by his memory in the future, which can be changed by his prior action.

Optimal Punishment. The optimal level of punishment for criminals, especially for repeat offenders and first-time offenders, has always been a controversial subject of interest in criminological studies. Polinsky and Shavell (1998) proposed an alternative model to the standard economic model of deterrence. Similar to our model, they adopted a two-period approach and showed that punishing repeat offenders more heavily than first-time offenders can enhance the effect of deterrence. On the other hand, Emons (2003) showed that increasing punishment levels for first-time offenders is generally more effective in deterring crime than increasing punishment levels for the repeat offenders, because the first-time offenders have more wealth to lose than repeat offenders.

1.2 Evidence in Criminology about the Mechanism

We briefly review empirical evidence in criminology.

Type of Crime. Our model applies more plausibly to soft crime (e.g., illegal drug use) than to hard crime (e.g., armed robbery). Fagan, Kupchik, and Liberman (2007) pointed out that adolescents prosecuted and punished in juvenile court are more likely to commit drug offenses repeatedly.

Using experimental data, Corman and Mocan (1996) found that the law-enforcement elasticity of occurrences of drug abuse is quite small in magnitude and smaller than that for hard crime. They also found evidence that arrests have short-duration impacts on drug abuse, which aligns with our model, where the probability of apprehension may not bring down the crime rate in the case of soft crime.

According to the survey conducted in 89 large U.S. metropolitan areas, Friedman et al. (2006) found that legal repressiveness measures, including hard-drug arrests per capita, police employees per capita, and corrections expenditure per capita, are not associated with drug injectors per capita. This finding suggests that the probability of apprehension has little deterrent effect on the rate of drug injection.

2 Model

2.1 Setup

This game considers a decision maker (DM) with a horizon of two periods, $t \in \{1, 2\}$. At each period t , DM can either commit a crime ($a_t = 1$) or not ($a_t = 0$). Before the start of the game, DM is endowed with criminal productivity v , which represents how much he earns by committing a crime. At $t = 1$, DM receives a noisy but informative signal σ . The signal is correlated with v . After observing a signal, DM commits a crime ($a_1 = 1$) or not ($a_1 = 0$). If DM commits a crime ($a_1 = 1$), DM will be apprehended with probability $p \in (0, 1)$. DM knows the probability of apprehension, p . If he is not apprehended, DM will have the payoff, which is the criminal productivity $v > 0$. If he is

apprehended, he will be fined $F > 0$. If DM does not commit a crime ($a_1 = 0$), he will receive wage income $W > 0$ in the end of the period. At $t = 2$, DM commits a crime ($a_1 = 1$) or not ($a_1 = 0$) again. The costs and benefits associated with his action are the same as at $t = 1$.

2.2 Assumptions

We explain the assumptions in this model and discuss each of them.

ASSUMPTION 1 (HYPERBOLIC DISCOUNTING) *At each period $t \in \{1, 2\}$, when DM chooses $a_t = 1$ or $a_t = 0$, he values the payoffs for $a_t = 1$ realized at period t as*

$$(1 - p) \cdot \frac{v}{\beta} - p \cdot F,$$

and he values the payoffs for $a_t = 0$ realized at period t as W , where $\beta \in (0, 1]$. Moreover, β is predetermined and known to DM in the beginning of the game.¹

ASSUMPTION 2 (IMPERFECT RECALL) *Given $a_1 = 1$, DM learns v and perfectly recalls it at $t = 2$. Given $a_1 = 0$, DM will not learn v at all and completely forget σ at $t = 2$. Let $\hat{\sigma}$ denote DM's memory related to his criminal productivity at $t = 2$. Then, we define $\hat{\sigma}$ as follows: $\hat{\sigma} = v$ if $a_1 = 1$ and DM observes v for any v ; and $\hat{\sigma} = \emptyset$ if $a_1 = 0$.*

ASSUMPTION 3 *Let $f_v(\cdot)$ denote the probability density function of v . Then, $f_v(v) = e^{-v}$ for $v \geq 0$, and $f_v(v) = 0$ for $v < 0$ (i.e., v is drawn from an exponential distribution with parameter 1). DM knows the functional form of $f_v(\cdot)$.*

ASSUMPTION 4 $\sigma = v + \varepsilon$. *Let $f_\varepsilon(\cdot)$ denote the probability density function of the noise ε . Let $f_{v,\varepsilon}(\cdot, \cdot)$ denote the joint probability density function of v and ε . Then, $f_\varepsilon(\varepsilon) = e^{-\varepsilon}$ for $\varepsilon \geq 0$, and $f_\varepsilon(\varepsilon) = 0$ for $\varepsilon < 0$ (i.e., ε is drawn from an exponential distribution with parameter 1). Furthermore, $f_{v,\varepsilon}(v, \varepsilon) = f_v(v) \cdot f_\varepsilon(\varepsilon)$ (i.e., ε and v are independently distributed). DM does not directly observe v or ε , but he knows the structure of σ . He also knows the functional forms of $f_\varepsilon(\cdot)$ and $f_{v,\varepsilon}(\cdot, \cdot)$.*

Discussion of Assumption 1. DM exhibits time inconsistency in his preferences. At each period, when DM makes a decision, he is tempted to obtain immediate gratification through criminal activities instead of working hard. A criminal action yields immediate gains, v , to DM. In contrast, the benefit from honest work W , and the cost from apprehension, F , are only realized at the end of the period. Hence, when DM makes a decision, he discounts the delayed benefits and costs; equivalently, he values the immediate gratification from a criminal action at v/β instead of v but values W and F as they are in each period. Thus, β can be interpreted as the rate of hyperbolic discounting, or the strength of DM's temptation (which decreases in β). This form of

¹ β represents hyperbolic discounting, which we will explain in the discussion of Assumption 1.

hyperbolic discounting only applies to immediate payoffs from a criminal action, i.e., at each period $t \in \{1, 2\}$, he values the payoffs for today's action, $a_t = 1$ at v/β , while at period 1 he values the payoffs for future action, $a_2 = 1$ at v .

This form of hyperbolic discounting is closely related to that of Bénabou and Tirole (2004, 2006a). Because of the present-period temptation, DM will commit a crime even though it is unprofitable from an ex post standpoint.

Discussion of Assumption 2. We make a distinction between *hard information*, which leaves verifiable hard evidence, and *soft information*, which does not leave such evidence. In this model, hard information includes his action a_t and the payoff v he earns from committing a crime. The knowledge of his criminal productivity that he obtains by committing a crime is hard information discovered through tangible gains, which is verifiable evidence. In addition, his past action a_1 is also hard evidence. If he commits a crime, he remembers the action committed.

Soft information includes his signal σ at $t = 1$, because it is his impression about his criminal productivity obtained through transient interactions with other people. This impression fades from memory over time, because it is not verifiable.

Furthermore, we assume that DM perfectly recalls hard information, and imperfectly recalls soft information under some conditions. Hence, if DM commits a crime at $t = 1$, he discovers v and perfectly recalls it. For simplicity, apprehension does not affect his memory of v . This is a reasonable assumption because the severity of punishment from apprehension of soft crime is milder (e.g., a shorter prison sentence) than that from felonies. On the other hand, if DM does not commit a crime at $t = 1$ ($a_1 = 0$), he will not discover v and will completely forget σ .

Figure 1 illustrates this path-dependent information.

Discussion of Assumption 3. Kanazawa (2003) stated that the age distribution of criminals is similar to the distribution of research productivity. Simonton (1988) pointed out that research productivity is drawn from an exponential distribution. The similarity of

mechanism behind criminal productivity and research productivity allows us to assume that criminal productivity follows an exponential distribution.

Discussion of Assumption 4. This assumption implies $\sigma \geq v$. This assumption does not lead to the overestimation of v , because DM knows the structure of σ .

2.3 Timeline

The sequence of events is shown on the timeline in Figure 2.

Figure 2
Timeline

2.4 Equilibrium Concept

We consider a mixed strategy. At each period $t \in \{1, 2\}$, DM chooses the probability of committing a crime ($a_t = 1$) at period t , denoted $\mu_t \in [0, 1]$.²

Let U_t denote DM's payoffs perceived by him when he chooses μ_t . Then, at $t = 1$,

$$(1) \quad U_1 := \underbrace{E \left[\mu_1 \cdot \left((1-p) \cdot \frac{v}{\beta} - p \cdot F \right) + (1 - \mu_1) \cdot W \mid \sigma = \sigma' \right]}_{\text{perception of payoffs that will be realized at } t=1} + \underbrace{E \left[\mu_2 \cdot \left((1-p) \cdot v - p \cdot F \right) + (1 - \mu_2) \cdot W \mid \sigma = \sigma' \right]}_{\text{perception of payoffs that will be realized at } t=2},$$

where $\sigma' \in \mathbb{R}_+$. At $t = 2$,

$$(2) \quad U_2 := \underbrace{E \left[\mu_2 \cdot \left((1-p) \cdot \frac{v}{\beta} - p \cdot F \right) + (1 - \mu_2) \cdot W \mid \hat{\sigma} = i, a_1 = j \right]}_{\text{perception of payoffs that will be realized at } t=2},$$

where $i \in \mathbb{R}_+ \cup \{\emptyset\}$ and $j \in \{1, 0\}$.

Following Bénabou and Tirole (2002, 2004), we define a perfect Bayesian equilibrium (PBE) of this dynamic game.

² This model needs to consider mixed strategies at $t = 2$ on the path $a_1 = 0$, as discussed later.

DEFINITION A *perfect Bayesian equilibrium (PBE)* of this dynamic game is a pair $(\mu_1^*, \mu_2^*) \in [0, 1] \times [0, 1]$ that maximizes U_1 and U_2 defined in (1) and (2). Furthermore, at $t = 1$, on receiving σ , DM updates his belief on v using Bayes's rule. At $t = 2$, given his memory $\hat{\sigma}$ and recalled action a_1 , DM updates his belief on v using Bayes's rule that takes into account his action at $t = 1$ and his strategy at $t = 1$.

This definition means that when DM decides at $t = 1$, he takes into account how his decision at $t = 1$ will affect his belief on v at $t = 2$ and hence his decision at $t = 2$. Moreover, the Bayesian rationality of DM at $t = 2$ means that he is aware of himself at $t = 1$ choosing μ_1 to maximize (1). Then, he uses this optimal μ_1 in updating his belief on v at $t = 2$.

We study a mixed PBE. In equilibrium, DM decides (μ_1^*, μ_2^*) at the beginning of the game.

DM's inference off the equilibrium path is assumed as follows: If the outcome at $t = 1$ is $a_1 = 0$ after the deviation from his equilibrium strategy, he will infer his criminal productivity (v) based on his past action ($a_1 = 0$) and his strategy at $t = 1$ (μ_1^*). If the outcome at $t = 1$ is $a_1 = 1$ after the deviation from his equilibrium strategy, he will discover v perfectly.

2.5 Results

Period 2. We first analyze DM's equilibrium strategy at $t = 2$ fixing his strategy at $t = 1$ and his action at $t = 1$.

LEMMA 1 Define a cutoff $Y(p, \beta)$:

$$(3) \quad Y(p, \beta) := \left(\frac{p \cdot F + W}{1 - p} \right) \cdot \beta.$$

Then, DM's equilibrium strategy at $t = 2$ is $\mu_2^*(\hat{\sigma}, a_1) = 1$ if $E[v|\hat{\sigma} = i, a_1 = j; \mu_1^*] > Y(p, \beta)$, $\mu_2^*(\hat{\sigma}, a_1) \in [0, 1]$ if $E[v|\hat{\sigma} = i, a_1 = j; \mu_1^*] = Y(p, \beta)$, and $\mu_2^*(\hat{\sigma}, a_1) = 0$ if $E[v|\hat{\sigma} = i, a_1 = j; \mu_1^*] < Y(p, \beta)$, where $i \in \mathbb{R}_+ \cup \{\emptyset\}$ and $j \in \{1, 0\}$.

PROOF Consider DM's problem at $t = 2$ by fixing his information, $\hat{\sigma} = i$, and action at $t = 1$, $a_1 = j$, where $i \in \mathbb{R}_+ \cup \{\emptyset\}$ and $j \in \{1, 0\}$. It follows from (2) that when he chooses μ_2 , DM simply considers payoffs at $t = 2$ given $a_2 = 1$ minus payoffs at $t = 2$ given $a_2 = 0$:

$$(4) \quad \frac{(1 - p) \cdot E[v|\hat{\sigma} = i, a_1 = j; \mu_1^*]}{\beta} - p \cdot F - W.$$

Hence, the claim holds.

$Y(p, \beta)$ increases with p and β respectively. We explain the intuition behind (4). The first term denotes the perceived benefits of a criminal action, while the other two terms denote the cost of a criminal action. When p increases, the first and second terms

decrease, i.e., the benefit decreases and the cost increases. When β increases (the level of his temptation decreases), the first term decreases, i.e., DM tends not to magnify immediate gain through criminal activity. Hence, as p (or β) increases, DM should expect larger criminal productivity in order to commit a crime.

His temptation causes him to make a wrong decision – DM will commit a crime even though it is not profitable for him ex post (and at $t = 1$) – if

$$E[v|\hat{\sigma} = i, a_1 = j; \mu_1^*] \in (Y(p, \beta), Y(p, 1)),$$

where $i \in \mathbb{R}_+ \cup \{\emptyset\}$ and $j \in \{1, 0\}$. DM's decision also depends on the path – whether DM committed a crime at $t = 1$ (i.e., $a_1 = 1$) or not (i.e., $a_1 = 0$). Given $a_1 = 1$, he discovers his criminal productivity v , and simply compares v and $Y(p, \beta)$. Given $a_1 = 0$, he forgets his signal, and hence infers v from his strategy at $t = 1$ (i.e., μ_1^*) and his past action (i.e., $a_1 = 0$). As a result, he compares inference $E[v|\hat{\sigma} = \emptyset, a_1 = 0; \mu_1^*]$ and $Y(p, \beta)$.

Period 1. Let $f_{v|\sigma}$ denote the probability density function of v conditional on σ , which is derived from Assumptions 3 and 4 (see appendix A.1 for the formal definitions). Define a value function at $t = 1$ denoted by $V(\mu_1, \sigma; \mu_2^*, p, \beta)$:

$$\begin{aligned} (5) \quad & V(\mu_1, \sigma; \mu_2^*, p, \beta) \\ &:= \underbrace{\mu_1 \cdot \left((1-p) \cdot \frac{E[v|\sigma = \sigma']}{\beta} - p \cdot F \right) + (1-\mu_1) \cdot W}_{\text{payoffs at } t=1} \\ &+ \underbrace{\mu_1 \cdot \left(\int_{Y(p, \beta)}^{\infty} \{(1-p) \cdot v - p \cdot F\} \cdot f_{v|\sigma}(v|\sigma = \sigma') \cdot dv + \int_0^{Y(p, \beta)} W \cdot f_{v|\sigma}(v|\sigma = \sigma') \cdot dv \right)}_{\text{payoffs at } t=2 \text{ given } a_1=1} \\ &+ (1-\mu_1) \cdot \underbrace{(\mu_2^*(\emptyset, 0) \cdot ((1-p) \cdot E[v|\sigma = \sigma'] - p \cdot F) + (1-\mu_2^*(\emptyset, 0)) \cdot W)}_{\text{payoffs at } t=2 \text{ given } a_1=0}. \end{aligned}$$

Recall that $\mu_2^*(\emptyset, 0)$ denotes DM's strategy at $t = 2$ on the path $a_1 = 0$. Hence, $\partial V(\mu_1, \sigma; \mu_2^*, p, \beta) / \partial \mu_1$ – the marginal benefit of selecting $a_1 = 1$ instead of $a_0 = 1$ – monotonically increases with σ .

LEMMA 2 *For any p, β , there is a unique cutoff $X(p, \beta)$ such that*

$$X(p, \beta) \in \{\sigma \in \mathbb{R}_+ : \partial V(\mu_1, \sigma; \mu_2^*, p, \beta) / \partial \mu_1 = 0\}.$$

Then, DM's equilibrium strategy at $t = 1$ is $\mu_1^(\sigma) = 1$ if $\sigma > X(p, \beta)$, $\mu_1^*(\sigma) \in [0, 1]$ if $\sigma = X(p, \beta)$, and $\mu_1^*(\sigma) = 0$ if $\sigma < X(p, \beta)$.*

$X(p, \beta)$ is a solution to a fixed-point problem that involves DM's problems at both periods. At $t = 2$, DM knows that he compared σ with $X(p, \beta)$ and chose μ_1 at $t = 1$. Even if he has forgotten his signal (i.e., $\hat{\sigma} = \emptyset$), he knows that he did not commit a crime ($a_1 = 0$), and he knows that $a_1 = 0$ implies $\sigma < X(p, \beta)$. As p increases, the cost of committing a crime increases. Hence, DM should expect larger criminal productivity to commit a crime, i.e., the cutoff $X(p, \beta)$ increases with p .

DM's inference about his criminal productivity v is given by

$$E[v|\hat{\sigma} = \emptyset, a_1 = 0; \mu_1^*] = E[v|\sigma < X(p, \beta)] = I(X(p, \beta)),$$

where

$$I(t) := 1 - \left(\frac{t^2}{2} + t + 1 \right) \cdot \exp(-t).$$

$I(\cdot)$ is strictly increasing in its element and bounded above.³

On this path ($a_1 = 0$), DM's strategy at $t = 2$ is $\mu_2^*(\emptyset, 0) = 1$ (i.e., he commits a crime at $t = 2$ with probability 1) if

$$\underbrace{I(X(p, \beta))}_{\substack{\text{inference at } t=2 \\ \text{(perceived benefit)}}} > \underbrace{(p, \beta)}_{\substack{\text{cutoff value at } t=2 \\ \text{(perceived cost)}}},$$

$\mu_2^*(\emptyset, 0) \in [0, 1]$ if $I(X(p, \beta)) = Y(p, \beta)$, and $\mu_2^*(\emptyset, 0) = 0$ if $I(X(p, \beta)) < Y(p, \beta)$.

DM's inference about his criminal productivity, $I(X(p, \beta))$, is interpreted as his benefits from a criminal action perceived by DM at $t = 2$, given $a_1 = 0$. The cutoff at $t = 2$, $Y(p, \beta)$, is interpreted as the costs of a criminal action perceived by DM at $t = 2$. As Figure 3 shows, $Y(p, \beta)$ increases with p . Moreover, $Y(p, \beta)$ approaches infinity as p approaches 1 – the cost is infinite if you know that you will be surely punished. On the other hand, his inference $I(X(p, \beta))$ increases with p but is bounded.⁴ As p increases, DM abstains from crime even given a larger signal (the cutoff at $t = 1$, $X(p, \beta)$, increases), and eventually he abstains given almost any signal. However, the inference is at most the unconditional expectation.

LEMMA 3 *Fix any β and p . Then, there exists a unique equilibrium almost everywhere.*

PROOF This directly follows from Lemmas 1 and 2.

DM uses a pure strategy almost everywhere. At $t = 2$ given $a_1 = 0$, there is a unique $\mu_2^*(\emptyset, 0) \in [0, 1]$ because this is a solution to the fixed point problem. On the other hand, at $t = 1$, he compares continuous σ and the unique $X(p, \beta)$, and he decides μ_1 according to Lemma 2. Hence, given $\sigma = X(p, \beta)$, he is indifferent among any $\mu_1 \in [0, 1]$, and the equilibrium strategy at $t = 2$ remains unchanged for any $\mu_1 \in [0, 1]$. Similarly, at $t = 2$ given $a_1 = 1$, DM compares continuous v and $Y(p, \beta)$, and decides $\mu_2(v, 1)$ according to Lemma 1. Hence, given $v = Y(p, \beta)$, he is indifferent among any $\mu_2 \in [0, 1]$, and the

³ See appendix for a mathematical derivation of I .

⁴ Precisely $I(X(p))$ is convex in p for small p and concave in p for large p .

Figure 3
 $I(X(p, \beta))$ and $Y(p, \beta)$ Given $\beta = 0.3$, $F = 1.5$, and $W = 0.4$

equilibrium strategy at $t = 1$ or $t = 2$ on the path $a_1 = 0$ remains unchanged for any $\mu_2 \in [0, 1]$.

Without loss of generality, we assume $\mu_1^*(\sigma) = 0$ if $\sigma = X(p, \beta)$ and $\mu_2^*(v, 1) = 0$ if $v = Y(p, \beta)$ (i.e., DM uses a pure strategy at $t = 1$ as well as $t = 2$ given $a_1 = 1$). However, it is not obvious whether DM should use a mixed strategy as the unique strategy at $t = 2$ given $a_1 = 0$.

PROPOSITION 1 *Fix any β and p . Then, in equilibrium, $\mu_2^*(\emptyset, 0) = 0$. Moreover,*

$$X(p, \beta) = \left(\frac{2 + \sqrt{\beta^2 - \beta + 2}}{\beta + 1} \right) \cdot Y(p, \beta).$$

DM stops himself from committing a crime at $t = 2$ if he does not commit a crime in period 1 (i.e., $\mu_2^*(\emptyset, 0) = 0$). As Figure 3 shows, $Y(p, \beta)$ and $I(X(p, \beta))$ do not cross, and $I(X(p, \beta)) < Y(p, \beta)$ holds for any p and β .

Next, we define the ex ante expected total number of criminal actions for two periods, denoted $E[a_1 + a_2]$:

$$\begin{aligned} E[a_1 + a_2] := & \int_0^{X(p, \beta)} (\mu_1^*(\sigma) + \mu_2^*(\emptyset, 0)) \cdot f_\sigma(\sigma) \cdot d\sigma \\ & + \int_{Y(p, \beta)}^\infty \int_{X(p, \beta)}^\infty (\mu_1^*(\sigma) + \mu_2^*(v, 1)) \cdot f_{\sigma, v}(\sigma, v) \cdot d\sigma \cdot dv. \end{aligned}$$

COROLLARY 1 $E[a_1 + a_2]$ monotonically decreases in p .

PROOF According to Lemma 1 and Proposition 1, $\partial X(p, \beta)/\partial p > 0$ and $\partial Y(p, \beta)/\partial p > 0$. Moreover, $\mu_2^*(\emptyset, 0) = 0$ for any β and p . Hence, the claim holds.

The higher probability of apprehension p leads to fewer criminal actions at every period and hence leads to smaller $E[a_1 + a_2]$. An increase in probability of apprehension increases the expected costs of criminal action in both periods. This implies that a higher

criminal productivity is needed before criminal action becomes viable in both periods, reducing the number of DMs who find it viable to commit crimes in both periods.

Commitment Device and Self-Control. In equilibrium, DM at $t = 1$ knows that his choice $a_1 = 0$ at $t = 1$ acts as a commitment device to $a_2 = 0$ at $t = 2$. DM has incentives to exert self-control on himself through the application of his personal rule. Bénabou and Tirole (2004) described personal rules as a kind of cognitive measure used by individuals to regulate their future behavior. They said that an individual is aware of his own proclivity towards instant gratification and therefore willing to put himself through a commitment device that serves to protect himself from giving in to such temptation.

If DM ignores the effect of his action at $t = 1$ on his action at $t = 2$, he should commit a crime at $t = 1$ if $\sigma > 2Y(p, \beta)$.⁵ Hence, we interpret $X(p, \beta) > 2Y(p, \beta)$ as DM exerting self-control on himself at $t = 1$. In particular, given $\sigma \in (2Y(p, \beta), X(p, \beta))$, DM abstains from crime today in order to abstain from tomorrow even if he perceives the positive payoffs he will earn at $t = 1$ by committing a crime today. According to the definition, $X(p, \beta)/2Y(p, \beta)$ decreases in β so that $X(p, \beta) > 2Y(p, \beta)$ if and only if $\beta < 2/3$. DM exerts more self-control as his temptation increases.

3 Asymmetric Temptation

We relax the assumption about temptation (Assumption 1) and consider asymmetric temptation between two periods. Now DM's temptation β at period t is denoted β_t , where smaller β_t means stronger temptation. We consider two cases, $\beta_1 < \beta_2$ (decreasing temptation over time) and $\beta_1 > \beta_2$ (increasing temptation over time). In particular, given $\beta_1 > \beta_2$, we can observe a nonmonotonic relationship between the total number of criminal actions and the apprehension rate p .

3.1 Decreasing Temptation

For the case of decreasing temptation, $\beta_1 < \beta_2$, we simply assume $\beta_2 = 1$.

The equilibrium cutoff values are $X^{DT}(p, \beta_1)$ for period $t = 1$ and $Y(p, 1)$ for period $t = 2$, and both of $X^{DT}(p, \beta_1)$ and $Y(p, 1)$ increase with p .⁶

LEMMA 4 *Fix any p and β_1 . Then, in equilibrium, $\mu_2^*(\emptyset, 0) = 0$ and*

$$X^{DT}(p, \beta_1) = \begin{cases} \left(\frac{2 + \sqrt{3 - 1/\beta_1}}{1 + \beta_1} \right) \cdot Y(p, \beta_1) & \text{if } \beta_1 \geq \frac{1}{2}, \\ 2Y(p, \beta_1) & \text{if } \beta_1 < \frac{1}{2}. \end{cases}$$

⁵ $E[v|\sigma = \sigma'] > Y(p, \beta) \Leftrightarrow \sigma' > 2Y(p, \beta)$ from Assumptions 3 and 4. See appendix for details.

⁶ $Y(\cdot, \cdot)$ is defined in (3).

COROLLARY 2 *Fix any β_1 . $E[a_1 + a_2]$ monotonically decreases in p .*

PROOF This directly follows from Lemma 4.

Both cutoff values, $X^{DT}(p, \beta_1)$ and $Y(p, 1)$, increase with p . Moreover, when his temptation decreases over time, he resists the temptation at $t = 2$ if he does at $t = 1$ (i.e., $\mu_2^*(\emptyset, 0) = 0$). Hence, the higher probability of apprehension p results in fewer criminal actions.

As in the model in section 3, DM's action $a_1 = 0$ at $t = 1$ acts as a commitment device to his action $a_2 = 0$ at $t = 2$. $X^{DT}(p, \beta_1) > 2Y(p, \beta_1)$ is interpreted as DM exerting self-control at $t = 1$. However, in this case of decreasing temptation, the result is $X^{DT}(p, \beta_1) < 2Y(p, \beta_1)$, for any p and β_1 . He has small incentives to use the commitment device at $t = 1$ because his temptation is weak at $t = 2$ (in our simplified case, $\beta_2 = 1$, there is no temptation problem at $t = 2$).

3.2 Increasing Temptation

In the case of increasing temptation, $\beta_1 > \beta_2$, we simply assume $\beta_1 = 1$.

The equilibrium cutoff values are $X^{IT}(p, \beta_2)$ for period $t = 1$ and $Y(p, \beta_2)$ for period $t = 2$, and both of $X^{IT}(p, \beta_2)$ and $Y(p, \beta_2)$ increase with p .⁷ However, because of the increasing temptation, DM can fail to resist the temptation to commit a crime at $t = 2$ even if he had succeeded in resisting it at $t = 1$. Moreover, the probability of apprehension p can sometimes result in more criminal actions.

PROPOSITION 2 *There exist cutoff values B_1 and B_2 , where $0 < B_1 < B_2 < 1$, such that:*

- (1) *For $\beta_2 \in (B_1, B_2)$, there exist P_1 and P_2 , where $0 < P_1 < P_2 < 1$, such that in equilibrium, $\mu_2^*(\emptyset, 0) > 0$ for $p \in (P_1, P_2)$, and $\mu_2^*(\emptyset, 0) = 0$ for the remaining p .*
- (2) *For $\beta_2 < B_1$, $\mu_2^*(\emptyset, 0) = 1$ for any p .*
- (3) *For $\beta_2 > B_2$, $\mu_2^*(\emptyset, 0) = 0$ for any p .⁸*

Moreover, for any p and β_2 :

$$X^{IT}(p, \beta_2) = \frac{1}{\beta_2} \cdot \left(1 + \sqrt{1 - \frac{(2 - \beta_2) \cdot \beta_2}{2 - \mu_2^*(\emptyset, 0)}} \right) \cdot Y(p, \beta_2).$$

Proposition 2 means that if period 2's temptation is small (i.e., $\beta_2 > B_2$), the commitment device works well (i.e., for any p , DM will not commit a crime at $t = 2$ if he does not commit a crime at $t = 1$). However, if period 2's temptation is large (i.e., $\beta_2 < B_1$), the commitment device ceases to work (i.e., for any p , DM will commit a crime at $t = 2$

⁷ $Y(\cdot, \cdot)$ is defined in (3). See appendix for the definition of $X^{IT}(\cdot, \cdot)$.

⁸ See proof of Proposition 2 in appendix for the mathematical definitions of B_1 , B_2 , P_1 , and P_2 .

even if he does not commit a crime at $t = 1$). If period 2's temptation is medium (i.e., $\beta_2 \in (B_1, B_2)$), the commitment device works only for small or large apprehension rates (i.e., $p \leq P_1$ or $p \geq P_2$). We explain the intuition behind this result.

Recall that $\mu_2^*(\emptyset, 0)$ denotes the probability of realizing $a_2 = 1$ given $a_1 = 0$. $Y(p, \beta_2)$ is interpreted as the marginal cost of a criminal action at $t = 2$, and $I(X^{IT}(p, \beta_2))$ (DM's inference about his criminal productivity on the path $a_1 = 0$) is interpreted as the marginal benefit of a criminal action at $t = 2$ on this path. $\mu_2^*(\emptyset, 0) = 0$ holds only if $I(X^{IT}(p, \beta_2)) \leq Y(p, \beta_2)$.

In the case of increasing temptation, $a_2 = 1$ given $a_1 = 0$ (i.e., $I(X^{IT}(p, \beta_2)) > Y(p, \beta_2)$) can occur only for medium p . We consider three cases: p is high, medium, or low.

When p is low, the outcome is $a_2 = 0$ given $a_1 = 0$ (i.e., $I(X^{IT}(p, \beta_2)) < Y(p, \beta_2)$). Because $X^{IT}(p, \beta_2)$ is low, DM does not commit a crime at $t = 1$ when he receives a low signal $\sigma < X^{IT}(p, \beta_2)$. At $t = 2$, he infers that he did not commit a crime because he has very low criminal productivity, which prevents him from committing a crime at $t = 2$.

When p is high, $a_2 = 0$ given $a_1 = 0$ (i.e., $I(X^{IT}(p, \beta_2)) < Y(p, \beta_2)$). Because $X^{IT}(p, \beta_2)$ is high, he infers that he is of high criminal productivity at $t = 2$. However, the cost of committing a crime is larger than the benefit at $t = 2$.

On the other hand, when p is medium, $a_2 = 1$ given $a_1 = 0$ (i.e., $I(X^{IT}(p, \beta_2)) > Y(p, \beta_2)$) occurs. For DM at $t = 2$, the fact that he did not commit a crime does not necessarily mean he has low criminal productivity. And p is only medium. Besides, DM is more tempted to commit a crime at $t = 2$ than at $t = 1$. Thus, DM fails to resist temptation at $t = 2$ even if he had succeeded at $t = 1$. Figure 4 offers a graphical representation.

Figure 4
 $I(X(p, \beta_2))$ and $Y(p, \beta_2)$ Given $\beta_2 = 0.3$, $F = 1.5$, and $W = 0.4$

This nonmonotonic relationship between $E[a_2|a_1 = 0]$ and p is observed only when DM's temptation is medium (i.e., $\beta_2 \in (B_1, B_2)$). If his temptation is strong (i.e., $\beta_2 < B_1$), he will fail in resisting temptation at $t = 2$ on the path $a_1 = 0$ for any level of

p . The result is monotonic. If his temptation is weak (i.e., $\beta_2 > B_2$), he will succeed in resisting temptation at $t = 2$ on the path $a_1 = 0$ for any level of p . The result is again monotonic.

COROLLARY 3 For $\beta_2 \in (B_1, B_2)$, $E[a_1 + a_2]$ is not monotonically decreasing in p .

For $\beta_2 \in (B_1, B_2)$, $E[a_2|a_1 = 0]$ can decrease in p while both $E[a_1]$ and $E[a_2|a_1 = 1]$ decrease in p . The first effect dominates the second effect. $E[a_1 + a_2]$ is nonmonotonic in p , as shown in Figure 5.

Figure 5

Nonmonotonic Relationship between $E[a_1 + a_2]$ and p Given $\beta_2 = 0.3$, $F = 1.5$, and $W = 0.4$

Empirical Support for Increasing Temptation. There are empirical findings in criminological studies supporting increasing temptation ($\beta_1 > \beta_2$).

We first consider the case $a_1 = 1$, meaning that DM committed a crime at $t = 1$. We introduce empirical findings pertaining to drug usage because a vast majority of the general criminal population abuse drugs.⁹ Volkow (2014, p. 1) stated, “repeated drug use disrupts well-balanced systems in the human brain in ways that persist, eventually replacing a person’s normal needs and desires with a one-track mission to seek and use drugs.” This statement implies that for an individual with prior usage of drugs (i.e., $a_1 = 1$), his desire to use drugs grows with time.

Polinsky and Shavell (1999) claimed that some criminals discount the disutility of the later years of imprisonment more than they do for the earlier years. This claim is also consistent with the assumption $\beta_1 > \beta_2$, since a more discounted net benefit necessarily leads to a higher level of crime temptation. Fajnzylber, Lederman, and Loayza (2002, p. 1328) also pointed out the presence of “criminal hysteresis or inertia” in criminals, implying that an individual’s past criminal activities increase his own propensity towards committing a crime during subsequent time periods.

⁹ According to Bureau of Justice Statistics (2004), more than 2/3 of the prisoners in the United states have drug problems.

Next, we consider the case $a_1 = 0$, meaning that DM did not commit a crime at $t = 1$. According to studies on drug abuse, curiosity is one of the major reasons why individuals start using drugs, and curiosity-induced temptation towards the first drug use increases with time. When an individual is provided with sufficient information regarding the thrills of drug usage, his desire to use drugs lingers and increases until he eventually does.¹⁰ These findings regarding the first drug use are consistent with our assumption $\beta_1 > \beta_2$ given $a_1 = 0$.

On the other hand, some empirical findings in criminological studies may appear to contradict the assumption $\beta_1 > \beta_2$. However, upon closer scrutiny, we find out that our assumption does not contradict these findings. For example, Levitt and Miles (2007, p. 460) stated, “criminal propensities decline with age, as widely observed.” This decline in crime propensity is caused by a decline in the individual criminal productivity due to the age-imposed physical limitations. Polinsky and Shavell (2007, p. 443) also stated, “evidence exists suggesting that the harm caused by individuals declines with their age.” Again, this statement is associated with increasing physical restrictions with age, rather than a decline in criminal intent. Thus, Levitt and Miles (2007) as well as Polinsky and Shavell (2007) indicated that criminal productivity v declines with age. Moreover, they considered a longer period than we do. Our time period, which is required for temptation towards crime to increase, is significantly shorter than what is required for aging to affect individuals’ criminal productivity.

4 Discussions

We now discuss two extensions to our model to gain insight into the nonmonotonicity between apprehension rates and crimes rates. The first extension examines the declining effectiveness of deterrence on repeat offenders; the second extension examines endogenous recall.

4.1 Declining Effectiveness of Deterrence

In this extension, we show that the nonmonotonicity between apprehension rates and crime rates can be attributed to the declining effectiveness of deterrence on repeat offenders: the cost of apprehension (F) in period 2 decreases if DM was apprehended in period 1. We give two reasons why this might be so. First, after incarceration, DM may find it challenging to reintegrate into society and might prefer prison life. Second, after incarceration, DM may associate less disutility with apprehension because he is familiar with the punishment.

Like the model of increasing temptation, this model of deterrence can also result in nonmonotonicity between p and $E[a_1 + a_2]$. We fix $\beta = 1$ and modify Assumption 1 as follows: When DM chooses μ_2 , he values the benefits and costs at $t = 2$ as follows:

$$\mu_2 \cdot ((1 - p) \cdot v - p \cdot \gamma \cdot F) + (1 - \mu_2) \cdot W,$$

¹⁰ According to Lord (2003), curiosity is a feeling that does not (just) fall into our lap fortuitously and rest there.

where $\gamma \in (0, 1)$. γ is predetermined and known to DM at the beginning of the game. This model quantitatively resembles the model of increasing temptation in section 3.2. In particular, when W is sufficiently small, e.g., $W = 0$, the equilibrium outcome is equivalent to the outcome in the model of increasing temptation in section 3.2. The lack of deterrence can lead DM to commit a crime even if he does not at $t = 1$ for medium p . As a result, $E[a_1 + a_2]$ does not monotonically decrease with decreasing p , and in some cases it will rise with decreasing p . (See appendix for further details.)

Past incarceration carries social stigma that makes it difficult for ex-convicts to reintegrate into society, particularly in the areas of rebuilding social ties and finding secure employment. These difficulties could thus cause ex-convicts to prefer prison life to freedom, because his basic needs are taken care of in prison. This preference could then cause the ex-convict to discount the cost of apprehension, thus undermining the effectiveness of apprehension as a deterrence to crime. Visser and Travis (2003) found that ex-convicts who were unable to rebuild relationships with family members and former friends were more likely to become repeat offenders. Similarly, Kubrin and Stewart (2006) found that gaining secure employment was a significant challenge that made it difficult for released ex-convicts to reintegrate into society.

Past incarceration might also cause an ex-convict to associate less disutility with future incarceration. As Polinsky and Shavell (1999) found, the ex-convict's disutility from incarceration may decline because he is more familiar with prison life, or because he has grown distant from his friends and family and no longer desires to return to them. This causes him to discount the costs of apprehension and induces recidivism.

4.2 Endogenous Recall

Lastly, we show that nonmonotonicity in apprehension rates and crime rates can be observed without increasing temptation if we endogenize imperfect recall of soft information.

Endogenous imperfect recall was first introduced by Bénabou and Tirole (2002). They built a model of a dynamic game between the individual's temporal selves – with the different degrees of “willpower” or “salience of the present,” the individual can select different degrees of distortion in memory. He may choose either to forget all his information, to recall some of his information, or to recall it completely. There may be multiple perfect Bayesian equilibria due to the individual's Bayesian rationality or “metacognition”: when he recalls previous events, he is aware of the fact that his past self has deliberately managed his memory previously so as to maximize his total utility.

Similarly, we consider a model where DM may invest in his ability to recall. We modify the model's setup as follows: At $t = 1$ after observing σ , DM selects a probability $\lambda \in [0, 1]$ that he will recall this signal at $t = 2$. With probability $1 - \lambda$, he will forget σ . His information at $t = 2$ is $\hat{\sigma} = \sigma$ if he recalls his signal, and $\hat{\sigma} = \emptyset$ if he forgets his signal. At $t = 1$, DM incurs the cost $m \cdot \lambda$, where $m > 0$ is sufficiently small. DM's problem at $t = 2$ is similar to the one in section 2: given his information $\hat{\sigma}$, he chooses the probability μ_2 of committing a crime. The equilibrium strategy is a pair of $\lambda(\sigma)$ and $\mu_2^{\text{recall}}(\hat{\sigma})$ such that at $t = 1$, given his current information σ and his strategy in period

2, $\mu_2^{\text{recall}}(\cdot)$, DM selects a recall rate λ that maximizes

$$\begin{aligned} & -m \cdot \lambda + \underbrace{\lambda \cdot (\mu_2^{\text{recall}}(\sigma) \cdot ((1-p) \cdot v - p \cdot F) + (1 - \mu_2^{\text{recall}}(\sigma)) \cdot W)}_{\text{perception of payoffs when he recalls } t=2} \\ & + \underbrace{(1-\lambda) \cdot (\mu_2^{\text{recall}}(\emptyset) \cdot ((1-p) \cdot v - p \cdot F) + (1 - \mu_2^{\text{recall}}(\emptyset)) \cdot W)}_{\text{perception of payoffs when he recalls } t=2}. \end{aligned}$$

At $t = 2$, given his current information $\hat{\sigma}$ and his strategy in period 1, $\lambda(\cdot)$, he infers v and chooses μ_2 that maximizes

$$\mu_2 \cdot \left((1-p) \cdot \frac{E[v|\hat{\sigma} = i]}{\beta} - p \cdot F \right) + (1 - \mu_2) \cdot W,$$

where $i \in \mathbb{R}_+ \cup \{\emptyset\}$.

There can be multiple equilibria. In one equilibrium, there is a cutoff $Z(p) > Y(p, \beta)$ such that DM chooses $\lambda = 1$ (perfect recall) if $\sigma > Z(p)$ and $\lambda = 0$ otherwise at $t = 1$, and he will choose not to commit a crime if he forgets his signal, $\mu_2^{\text{recall}}(\hat{\sigma}) = 0$. In the other equilibrium, there is a cutoff $z(p, \beta) \leq Y(p, \beta)$ such that DM chooses $\lambda = 1$ if $\sigma < z(p, \beta)$ and $\lambda = 0$ otherwise at $t = 1$, and then $\mu_2^{\text{recall}}(\hat{\sigma}) = 1$ at $t = 2$.¹¹ In any equilibrium, if he recalls σ , he will commit a crime if and only if $\sigma > Y(p, \beta)$. For $\sigma \in (Y(p, \beta), Y(p, 1))$, DM will make the mistake of committing a crime if he recalls σ . However, in the former equilibrium, DM can avoid this mistake. Hence, we focus on the former equilibrium if it exists. We have found that the former equilibrium exists only for large p and small p for some medium level of β . For medium p , only the latter equilibrium exists. Hence, the expected number of criminal actions at $t = 2$ does not monotonically decrease in p .

Meanwhile, the existence of multiple equilibria for both monotonic and nonmonotonic relationships is consistent with many empirical studies. For example, using data from Argentina during 1990–1999, Cerro and Meloni (2005) found that an increase in the probability of apprehension decreased crime rates if criminals preferred risk. Polinsky and Shavell (1998) as well as Emons (2003) also demonstrated the existence of (conditional) monotonicity. In contrast, Tittle and Rowe (1974), using data from Florida, found a nonmonotonic relationship between apprehension rates and crime rates: when apprehension rates were at least 30 percent, they had a negative correlation with crime rates; when apprehension rates were under 30 percent, they had a positive correlation with crime rates.

5 Conclusion

This paper has shown how a model incorporating hyperbolic discounting and imperfect recall can theoretically explain the nonmonotonic relationship between apprehension rates and crime rates in existing empirical literature. If a government desires to minimize the crime rate, but is unable to substantially increase the probability of apprehension

¹¹ $Y(\cdot, \cdot)$ is defined in (3). See appendix for the definition of $Z(\cdot)$ and $z(\cdot, \cdot)$.

due to resource constraints, our model suggests that a zero increase in the probability of apprehension can be better than a small increase. Moreover, this study offers a testable mechanism that explains the observed effect of apprehension levels on criminal activity.

Appendix

A.1 Definitions

Let $f_\sigma(\sigma)$ denote the unconditional probability density function of the signal σ . From Assumptions 3 and 4 in section 2.2, $f_\sigma(\sigma)$ and $f_{v|\sigma}(v|\sigma)$ are given by

$$f_\sigma(\sigma) = \begin{cases} \sigma \cdot \exp(-\sigma) & \forall \sigma > 0, \\ 0 & \text{otherwise,} \end{cases}$$

and

$$(A1) \quad f_{v|\sigma}(v|\sigma) = \begin{cases} \frac{1}{\sigma} & \text{for } v \in (0, \sigma), \\ 0 & \text{otherwise,} \end{cases}$$

where $f_\sigma(\sigma)$ is derived from $F_\sigma(\sigma)$ such that

$$F_\sigma(s) = \Pr(v + \varepsilon < s) = \int_0^s f_v(v) \left(\int_0^{s-v} f_\varepsilon(\varepsilon) d\varepsilon \right) dv.$$

$f_{v|\sigma}(v|\sigma)$ is derived from $f_{v|\sigma}(v|\sigma) = f_{v,\sigma}(v, \sigma) / f_\sigma(\sigma)$, where $f_{v,\sigma}(v, \sigma) = f_v(v) f_\varepsilon(\sigma - v)$.

A.2 Proof of Lemma 2 and Proposition 1

Fixing period 2's strategy, then, we define a cutoff (s) related to period 1's strategy.

CLAIM A1 Let $\mu_2^*(\emptyset, 0) = \mu$, and fix any $\mu \in [0, 1]$. Define a set $x(\mu, p, \beta)$:

$$(A2) \quad x(\mu, p, \beta) := \{\sigma \in (0, \infty) : \partial V(\mu_1, \sigma; \mu_2^*, p, \beta) / \partial \mu_1 = 0\}.$$

Define $M(\beta)$:

$$M(\beta) := \frac{1}{2}(\beta^2 - 2\beta + 4) - \frac{1}{2}\sqrt{(2 - \beta)(4 - 4\beta + 2\beta^2 - \beta^3)},$$

where $M(\beta) \in (1/(2 - \beta), \infty)$. Then, $x(\mu, p, \beta)$ is a singleton if $\mu \notin (1/(2 - \beta), M(\beta))$, and it includes three elements if $\mu \in (1/(2 - \beta), M(\beta))$.

PROOF Consider DM's problem at $t = 1$. Let $\mu_2^*(0, \emptyset) = \mu$, and fix any $\mu \in [0, 1]$. Then, it follows from (5) and (A1) that

$$(A3) \quad \frac{\partial V(\mu_1, \sigma; \mu_2^*, p, \beta)}{\partial \mu_1} = \left(\left(\frac{E[v|\sigma]}{\beta} - Y(p, 1) \right) + \int_{Y(p, \beta)}^{\infty} (v - Y(p, 1)) \cdot f_{v|\sigma}(v|\sigma) \cdot dv \right. \\ \left. - \mu \cdot (E[v|\sigma] - Y(p, 1)) \right) \times (1 - p),$$

where $E[v|\sigma] = \sigma/2$ and

$$\begin{aligned} & \int_{Y(p,\beta)}^{\infty} (v - Y(p, 1)) \cdot f_{v|\sigma}(v|\sigma) \cdot dv \\ &= \begin{cases} E[v|\sigma] - Y(p, 1) + \frac{\beta(2-\beta)}{2\sigma} \cdot Y(p, 1)^2 & \text{for } \sigma \geq Y(p, \beta), \\ 0 & \text{for } \sigma < Y(p, \beta). \end{cases} \end{aligned}$$

$\partial V(\mu_1, \sigma; \mu_2^*, p, \beta)/\partial \mu_1$ is continuous at every $\sigma > 0$.

From (A3), if $\mu \notin (1/(2-\beta), M(\beta))$, then $x(\mu, p, \beta)$ is a singleton:

$$\begin{aligned} x(\mu, p, \beta) = & \begin{cases} \frac{2-\mu}{1+1/\beta-\mu} \cdot \frac{p \cdot F + W}{1-p} \cdot \left(1 + \sqrt{1 - \frac{(1+1/\beta-\mu)\beta(2-\beta)}{(2-\mu)(2-\mu)}} \right) & \text{if } \mu \in \left[0, \frac{1}{2-\beta} \right], \\ \frac{2(1-\mu)}{1/\beta-\mu} \cdot \frac{pF + W}{1-p} & \text{if } \mu \in [M(\beta), 1]. \end{cases} \end{aligned}$$

$x' \in x(\mu, p, \beta)$ is above $Y(p, \beta)$ if $\mu \in [0, 1/(2-\beta)]$ and below $Y(p, \beta)$ if $\mu \in [M(\beta), 1]$.

On the other hand, if $\mu \in (1/(2-\beta), M(\beta))$, then $x(\mu, p, \beta)$ includes three elements:

$$\begin{aligned} & x(\mu, p, \beta) \\ &= \frac{2(1-\mu)}{1/\beta-\mu} \cdot \frac{pF + W}{1-p}, \frac{2-\mu}{1+1/\beta-\mu} \cdot \frac{pF + W}{1-p} \cdot \left(1 \pm \sqrt{1 - \frac{(1+1/\beta-\mu)\beta(2-\beta)}{(2-\mu)(2-\mu)}} \right). \end{aligned}$$

In the bracket, the first element is below $Y(p, \beta)$ and the last two elements are above $Y(p, \beta)$.

The next claim suffices to show $\mu_2^*(\emptyset, 0) = 0$ for any p and β .

CLAIM A2 Let $\mu_2^*(\emptyset, 0) = \mu$, and fix any $\mu \in [0, 1]$. Let $x(\mu, p, \beta)$ be as defined in Claim A1. Then, $I(x') - Y(p, \beta) < 0$ holds for any $x' \in x(\mu, p, \beta)$.

PROOF First consider $\mu \notin (1/(2-\beta), M(\beta))$, i.e., $x(\mu, p, \beta)$ is a singleton. From the definitions of $I(\cdot)$ and $f_{\sigma}(\cdot)$, DM's conditional inference at $t = 2$ is

$$\begin{aligned} E[v|\hat{\sigma} = \emptyset, a_1 = 0; \mu_1^*] &= \int_{\sigma=0}^{x'} E[v|\sigma] \cdot f(\sigma) \cdot d\sigma \\ &\equiv I(x'), \end{aligned}$$

where $x' \in x(\mu, p, \beta)$. Then, $\mu = 0$ is dominant if $I(x') - Y(p, \beta) < 0$.

Hence, we will show $I(x') - Y(p, \beta) < 0$ for $x' \in x(\mu, p, \beta) \forall \mu \notin (1/(2-\beta), M(\beta))$ by using the next result. Fix any $x' > 0$, and take $t = Y(p, \beta)/x'$. Then

$$I(x') - Y(p, \beta) \equiv I(x') - tx' < 0 \quad \text{if } 2 \exp(-2) < t$$

because $\partial(I(x') - tx')/\partial x' < 0 \forall x' > 0$ if $2\exp(-2) < t$ and because $I(0) = 0$.

Letting $x' \in x(\mu, p, \beta)$ and $t = Y(p, \beta)/x'$, then, $I(x') - Y(p, \beta) < 0$ holds $\forall \mu \notin (1/(2 - \beta), M(\beta))$ because $2\exp(-2) < t$:

$$t = \begin{cases} \frac{1 - \mu \cdot \beta}{2(1 - \mu)} & \text{if } \mu \in [M(\beta), 1], \\ \frac{1 + \beta - \mu \cdot \beta}{2 - \mu + \sqrt{(2 - \mu)^2 - (1 + \beta - \mu \cdot \beta) \cdot (2 - \beta)}} & \text{if } \mu \in \left[0, \frac{1}{2 - \beta}\right]. \end{cases}$$

Last consider $\mu \in (1/(2 - \beta), M(\beta))$. Similarly, fix any $x' \in x(\mu, p, \beta)$, and take $t = Y(p, \beta)/x'$. Then, $I(x') - Y(p, \beta) < 0$ holds because $2\exp(-2) < t$:

$$t \in \left[\frac{2(1 - \mu)}{1/\beta - \mu} \cdot \frac{p \cdot F + W}{1 - p}, \frac{2 - \mu}{1 + 1/\beta - \mu} \cdot \frac{p \cdot F + W}{1 - p} \cdot \left(1 + \sqrt{1 - \frac{(1 + 1/\beta - \mu) \cdot \beta \cdot (2 - \beta)}{(2 - \mu) \cdot (2 - \mu)}} \right) \right].$$

From Claim A1 and Claim A2, the only outcome is $\mu_2^*(\emptyset, 0) = 0$, and hence $X(p, \beta)$ is uniquely determined as defined in Lemma 2.

A.3 Proof of Lemma 4

For this case of decreasing temptation ($\beta_1 \in (0, 1)$ and $\beta_2 = 1$), we take the same approach as for Lemma 2 and Proposition 1.

A.4 Proof of Proposition 2

For this case of increasing temptation ($\beta_1 = 1$ and $\beta_2 \in (0, 1)$), we take the same approach as for Lemma 2 and Proposition 1.

In this proof, let $\mu_2^{IT}(\emptyset, 0) = \mu$, and fix any $\mu \in [0, 1]$. The value function is denoted by V^{IT} , and

$$\begin{aligned} \frac{\partial V^{IT}(\mu_1, \sigma; \mu_2^{IT}, p, \beta_2)}{\partial \mu_1} &= \left((1 - \mu) \cdot (E[v|\sigma] - Y(p, 1)) \right. \\ &\quad \left. + \int_{Y(p, \beta_2)}^{\infty} (v - Y(p, 1)) \cdot f_{v|\sigma}(v|\sigma) \cdot dv \right) \times (1 - p). \end{aligned}$$

Define a set $x^{IT}(\mu, p, \beta_2)$:

$$x^{IT}(\mu, p, \beta_2) := \{ \sigma \in (0, \infty) : \partial V^{IT}(\mu_1, \sigma; \mu_2^{IT}, p, \beta_2) / \partial \mu_1 = 0 \}.$$

$x^{IT}(\mu, p, \beta_2)$ is a singleton for any $\mu \in [0, 1]$. Hence, from now on, we let $x^{IT}(\mu, p, \beta_2)$ denote the unique element in the set $x^{IT}(\mu, p, \beta_2)$:

$$(A4) \quad x^{IT}(\mu, p, \beta_2) = \frac{pF + W}{1 - p} \cdot \left(1 + \sqrt{1 - \frac{(2 - \beta_2)\beta_2}{2 - \mu}} \right).$$

Then, a pair $X^{IT}(p, \beta_2)$ and $\mu_2^{IT}(\emptyset, 0)$ is a solution to DM's problem at $t = 1$ and his problem at $t = 2$ on the path $a_1 = 0$:

$$(X^{IT}(p, \beta_2), \mu_2^{IT}(\emptyset, 0))$$

$$\in (x, \mu) \in [0, 1]^2 : x = x^{IT}(\mu, p, \beta_2), \begin{cases} \mu = 1 & \text{if } I(x) - Y(p, \beta_2) > 0, \\ \mu \in [0, 1] & \text{if } I(x) - Y(p, \beta_2) = 0, \\ \mu = 0 & \text{if } I(x) - Y(p, \beta_2) < 0. \end{cases}$$

Next, define a function $h^{IT}(\mu, p, \beta_2)$:

$$(A5) \quad h^{IT}(\mu, p, \beta_2) := I(x^{IT}(\mu, p, \beta_2)) - Y(p, \beta_2),$$

where $\mu_2^{IT}(\emptyset, 0) = 1$ if $h^{IT}(\mu, p, \beta_2) > 0$, $\mu_2^{IT}(\emptyset, 0) \in [0, 1]$ if $h^{IT}(\mu, p, \beta_2) = 0$, and $\mu_2^{IT}(\emptyset, 0) = 0$ if $h^{IT}(\mu, p, \beta_2) < 0$. Moreover, $h^{IT}(\mu, p, \beta_2)$ and $x^{IT}(\mu, p, \beta_2)$ are differentiable and strictly monotone with respect to μ such that

$$\frac{\partial x^{IT}(\mu, p, \beta_2)}{\partial \mu} < 0$$

and

$$\frac{\partial h^{IT}(\mu, p, \beta_2)}{\partial \mu} = \frac{\partial (I(x^{IT}(\mu, p, \beta_2)) - y(p, \beta_2))}{\partial \mu} \propto \frac{\partial x^{IT}(\mu, p, \beta_2)}{\partial \mu} < 0.$$

Hence, $\mu_2^{IT}(\emptyset, 0)$ is well defined. As a result, $X^{IT}(p, \beta_2)$ and $\mu_2^*(\emptyset, 0)$ are well defined for any p and β_2 .

For the existence of a nonmonotonic relationship between $\mu_2^*(\emptyset, 0)$ and p for some β_2 , it suffices to show the next claim.

CLAIM A3 *For any β_2 ,*

$$(A6) \quad \lim_{p \nearrow 1} h^{IT}(0, p, \beta_2) < 0.$$

There is a cutoff $B_1 \in (0, 1)$ such that

$$(A7) \quad \lim_{p \searrow 0} h^{IT}(0, p, \beta_2) < 0 \quad \text{iff } \beta > B_1.$$

There is also a cutoff $B_2 \in (B_1, 1)$ such that

$$(A8) \quad \max_{p \in (0, 1)} h^{IT}(1, p, \beta_2) > 0 \quad \text{iff } \beta_2 < B_2.$$

The inequality (A6) means that for any β_2 , at the limit $p = 1$, DM does not commit a crime at $t = 2$ on the path $a_1 = 0$. The inequality (A7) means that there is the lower bound B_1 such that for $\beta_2 > B_1$, at the limit $p = 0$, DM does not commit a crime at $t = 2$ on the path $a_1 = 0$. The inequality (A8) means that there is the upper bound B_2 such that for $\beta_2 < B_2$, for some $p \in (0, 1)$, DM does not commit a crime at $t = 2$ on the path $a_1 = 0$.

PROOF DM's inference is bounded $I(x) < 1$, but $\lim_{p \nearrow 1} Y(p, \beta_2) = \infty$. Hence, $\lim_{p \nearrow 1} h^{IT}(\mu, p, \beta_2) < 0$ for any $\mu \in [0, 1]$ for any $\beta_2 \in [0, 1]$. Hence, (A6) holds.

Next, let $\mu_2^*(\emptyset, 0) = \mu$. Then, the hyperplane h^{IT} is expressed as follows:

$$\begin{aligned} h^{IT}(\mu, p, \beta_2) &= I(x^{IT}(\mu, p, \beta_2)) - Y(p, \beta_2) \\ &\equiv I(x^{IT}(\mu, p, \beta_2)) - \frac{\beta_2}{1 + \sqrt{1 - [(2 - \beta_2)\beta_2]/(2 - \mu)}} \cdot x^{IT}(\mu, p, \beta_2). \end{aligned}$$

Hence, we define $l^{IT}(x, \mu, \beta_2)$ such that

$$l^{IT}(x, \mu, \beta_2) := I(x) - \frac{\beta_2}{1 + \sqrt{1 - [(2 - \beta_2)\beta_2]/(2 - \mu)}} \cdot x.$$

For (A7), the lower bound B_1 is well defined because $\partial h^{IT}(\mu, p, \beta_2)/\partial \beta_2 < 0$ over $\beta_2 \in (0, 1)$, $h^{IT}(\mu, 0, 0) > 0$ and $h^{IT}(\mu, 0, 1) < 0$.

Last, we will consider the upper bound B_2 . Define a set $T(\cdot)$:

$$T(\beta_2) \in \left\{ x > 2 : x = \arg \max_{x'} l(x', 1, \beta_2), \text{ and } l(x, 1, \beta_2) = 0 \right\}.$$

$l^{IT}(x, 1, \beta_2)$ is convex for $x < 2$ and concave for $x > 2$. Hence, $T(\beta_2)$ is a singleton such that $(x^2/2) \exp(-x) = \beta_2/(2 - \beta_2)$ for $x \in T(\beta_2)$. Then

$$\begin{aligned} l(x, \mu = 1, \beta_2)|_{x \in T(\beta_2)} &= 1 - \left(\frac{x^2}{2} + x + 1 \right) \cdot \exp(-x) - \frac{\beta_2}{2 - \beta_2} \cdot x \\ &= 1 - \left(\frac{x^3}{2} + \frac{x^2}{2} + x + 1 \right) \cdot \exp(-x) \\ &=: k(x). \end{aligned}$$

In the third line, $\beta_2/(2 - \beta_2) = (x^2/2) \exp(-x)$ is substituted.

From the definition of $k(\cdot)$, we have $k(0) = 0$, and $\partial l^{IT}(x, \mu = 1, \beta_2)/\partial x|_{x \in T(\beta_2)} < [>] 0$ for $x < [>] 2$. Hence, there should be a unique $x > 2$ such that $x \in T(\beta_2)$ and $k(x) = 0$.

Consequently, if $\beta_2 \in (B_1, B_2)$, there exist cutoffs $0 < P_1 < P_3 < P_4 < P_2 < 1$ such that

$$(A9) \quad h^{IT}(0, P_1, \beta_2) = h^{IT}(0, P_2, \beta_2) = 0 \quad \text{and} \quad h^{IT}(1, P_3, \beta_2) = h^{IT}(1, P_4, \beta_2) = 0.$$

and hence

$$\mu_2^{IT}(\emptyset, 0) \in \begin{cases} \{0\} & \text{if } p \in (0, P_1), \\ (0, 1) & \text{if } p \in (P_1, P_3), \\ \{1\} & \text{if } p \in (P_3, P_4), \\ (0, 1) & \text{if } p \in (P_4, P_2), \\ \{0\} & \text{if } p \in (P_2, 1). \end{cases}$$

$\mu_2^{IT}(\emptyset, 0)$ is increasing in p for $p \in (P_1, P_3)$, while it is decreasing in p otherwise.

A.5 Proof of Corollary 3

In this proof, we consider $\beta_2 \in [B_1, B_2]$ where $\mu_2^{IT}(\emptyset, 0)$ is not monotonic with p . Let $\mu_2^{IT}(\emptyset, 0) = \mu$ and $x' = x^{IT}(\mu, p, \beta_2)$, where $x^{IT}(\mu, p, \beta_2)$ is defined by (A4).

Then, given $\mu = 1$,

$$E[a_1 + a_2] = \begin{cases} 1 + (x' + 1) \cdot \exp(-x') & \text{if } \mu = 1, \\ 2(x' + 1) \cdot \exp(-x) & \text{if } \mu = 0. \end{cases}$$

In either case, $E[a_1 + a_2]$ decreases in p . Hence, it suffices to show the following inequality:

$$(A10) \quad 2(x + 1) \cdot \exp(-x) < 1 + (x' + 1) \cdot \exp(-x'),$$

where $x = x(0, P_1, \beta_2)$ and $x' = x(1, P_3, \beta_2)$. P_1 and P_3 are determined according to (A9). We can show (A10) holds if $x = x'$. Moreover, (A10) is equivalent to $1 < [1 + (x' + 1) \cdot \exp(-x')]/[2(x + 1) \cdot \exp(-x)]$, where $[1 + (x' + 1) \cdot \exp(-x')]/[2(x + 1) \cdot \exp(-x)]$ increases with x'/x . And x'/x increases as $h^{IT}(0, p, \beta_2)/h^{IT}(1, p, \beta_2)$ increases; $h^{IT}(0, p, \beta_2)/h^{IT}(1, p, \beta_2)$ increases with β_2 . Hence, it suffices to find the upper bound of β_2 over $[B_1, B_2]$ such that (A10) holds.

We can show (A10) holds given $\beta_2 = B_2$ as follows: According to (A4), (A5), and (A9), if $x = x(0, P_1, \beta_2)$ and $x' = x(1, P_3, \beta_2)$, then (A10) is equivalent to

$$\begin{aligned} 2 - \frac{2\beta_2}{2 - \beta_2} - \frac{2x \cdot \beta_2}{1 + \sqrt{[1 + (1 - \beta_2)^2]/2}} &< 2 - \frac{\beta_2}{2 - \beta_2} - \frac{x' \cdot \beta_2}{2 - \beta_2} \\ \Leftrightarrow \frac{x' - 1}{x} &< \frac{2(2 - \beta_2)}{1 + \sqrt{[1 + (1 - \beta_2)^2]/2}}. \end{aligned}$$

The last inequality holds because

$$\frac{x' - 1}{x} \equiv \frac{2 - \beta_2 - (1 - p)/(pF + W)}{1 + \sqrt{[1 + (1 - \beta_2)^2]/2}} < \frac{2(2 - \beta_2)}{1 + \sqrt{[1 + (1 - \beta_2)^2]/2}}.$$

A.6 Declining Effectiveness of Deterrence

We show that there is an equilibrium characterized by a cutoff for $t = 1$, denoted $X^d(p, \gamma)$, a cutoff for $t = 2$, denoted $Y^d(p, \gamma)$, and DM's choice at $t = 2$ conditional on $a_1 = 0$, denoted $\mu_2^d(\emptyset, 0)$. Here $Y^d(p, \gamma)$ is

$$Y^d(p, \gamma) := \frac{p \cdot \gamma \cdot F + W}{1 - p}.$$

Let $\mu_2^d(\emptyset, 0) = \mu$, and fix any $\mu \in [0, 1]$. The value function is denoted V^d , and

$$\begin{aligned} \frac{\partial V^d(\mu_1, \sigma; \mu_2^d, p, \gamma)}{\partial \mu_1} &= \left((1 - \mu) \cdot (E[v|\sigma] - Y^d(p, 1)) \right. \\ &\quad \left. + \int_{Y^d(p, \gamma)}^{\infty} (v - Y^d(p, 1)) \cdot f_{v|\sigma}(v|\sigma) \cdot dv \right) \times (1 - p). \end{aligned}$$

Hence, a pair $X^d(p, \gamma)$ and $\mu_2^d(\emptyset, 0)$ is well defined. (See section A.4 in the appendix for the remaining steps.) If $W = 0$, the equilibrium outcome is equivalent to that in the model of increasing temptation.

A.7 Endogenous Recall

Define cutoffs $Z(p)$ and $z(p, \beta)$ introduced in section 4.2 as follows:

$$Z(p) := \frac{2m}{1-p} + 2Y(p, 1),$$

$$z(p, \beta) := \min \left\{ 2Y(p, 1) - \frac{2m}{1-p}, 2Y(p, \beta) \right\}.$$

The value function at $t = 1$ is denoted V^{recall} , and

$$\begin{aligned} & \partial V^{\text{recall}}(\lambda, \sigma'; \mu_2^{\text{recall}}, p, \beta) / \partial \lambda \\ &= -m + (1-p) \cdot (\mu_2^{\text{recall}}(\sigma') - \mu^{\text{recall}}(\emptyset)) \cdot (E[v|\sigma = \sigma'] - Y(p, 1)). \end{aligned}$$

where $\mu_2^{\text{recall}}(\sigma') = 1$ if $E[v|\sigma = \sigma'] > Y(p, \beta)$, and $\mu_2^{\text{recall}}(\sigma') = 0$ if $E[v|\sigma = \sigma'] \leq Y(p, \beta)$, where $\sigma' \in \mathbb{R}_+$.

In the former equilibrium, $\lambda(\sigma) = 1$ if $\sigma > Z(p)$ and $\lambda(\sigma) = 0$ otherwise at $t = 1$. Then, $\mu_2^{\text{recall}}(\sigma) = 1$ when DM recalls σ , while $\mu^{\text{recall}}(\emptyset) = 0$. In the latter equilibrium, $\lambda(\sigma) = 1$ if $\sigma < z(p, \beta)$ and $\lambda(\sigma) = 0$ otherwise at $t = 1$. Then, $\mu_2^{\text{recall}}(\sigma) = 0$ when DM recalls σ , while $\mu^{\text{recall}}(\emptyset) = 1$. The former equilibrium results in smaller $E[a_2]$ than the latter equilibrium.

Last, we find conditions for the existence of the former equilibrium. It suffices to check whether $I(Z(p)) - Y(p, \beta) \leq 0$ holds, because DM's inference when he forgets is $E[v|\sigma < Z(p)] = I(Z(p))$ in the former equilibrium. (See section A.4 in the appendix for the remaining steps.) For example, given $W = 1/2$, $F = 1/2$, $m = 1/10$, and $\beta = 1/4$, the former equilibrium exists for small p (e.g., $p = 1/5$) and large p (e.g., $p = 4/5$), but it does not exist for medium p (e.g., $p = 1/2$).

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