Effective indexing for approximate constrained shortest path queries on large road networks

Wang, Sibo; Xiao, Xiaokui; Yang, Yin; Lin, Wenqing

2016

Wang, S., Xiao, X., Yang, Y., & Lin, W. (2016). Effective indexing for approximate constrained shortest path queries on large road networks. Proceedings of the VLDB Endowment, 10(2), 61-72.

https://hdl.handle.net/10356/81353

https://doi.org/10.14778/3015274.3015277

Effective Indexing for Approximate Constrained Shortest Path Queries on Large Road Networks

Sibo Wang[1] Xiaokui Xiao[1] Yin Yang[2] Wenqing Lin[3]
[1]Nanyang Technological University [2]Hamad Bin Khalifa University [3]Qatar Computing Research Institute
{wang0759, xkxiao}@ntu.edu.sg {yyang, wlin}@qf.org.qa

ABSTRACT

In a *constrained shortest path (CSP)* query, each edge in the road network is associated with both a length and a cost. Given an origin s, a destination t, and a cost constraint θ, the goal is to find the shortest path from s to t whose total cost does not exceed θ. Because exact CSP is NP-hard, previous work mostly focuses on approximate solutions. Even so, existing methods are still prohibitively expensive for large road networks. Two main reasons are (i) that they fail to utilize the special properties of road networks and (ii) that most of them process queries without indices; the few existing indices consume large amounts of memory and yet have limited effectiveness in reducing query costs.

Motivated by this, we propose *COLA*, the first practical solution for approximate CSP processing on large road networks. *COLA* exploits the facts that a road network can be effectively partitioned, and that there exists a relatively small set of landmark vertices that commonly appear in CSP results. Accordingly, *COLA* indexes the vertices lying on partition boundaries, and applies an on-the-fly algorithm called α-*Dijk* for path computation within a partition, which effectively prunes paths based on landmarks. Extensive experiments demonstrate that on continent-sized road networks, *COLA* answers an approximate CSP query in sub-second time, whereas existing methods take hours. Interestingly, even without an index, the α-*Dijk* algorithm in *COLA* still outperforms previous solutions by more than an order of magnitude.

1. INTRODUCTION

Nowadays, route planning via online mapping/navigation services has become an essential part of driving in many places. Most popular online maps today, such as Google Maps [3], compute routes based on a single criterion, which is usually either the total route length or the total travel time. In practice, however, the user often needs to consider multiple criteria when planning a route. Besides travel distance and time, a common criterion is toll payment. For example, many cities charge the road user a fee to use highways (e.g., in Tokyo), bridges and undersea tunnels (e.g., in New York City); additionally, some densely populated metropolitan areas impose congestion charges (e.g., in London). Another common consideration is safety. For instance, in 2015, members of the New York City Council requested that Google Maps reduces the number of left turns in its suggested routes, since left turns lead to a higher rate of pedestrian crashes[1]. Finally, the shortest path is not necessarily the fastest or the most pleasant, e.g., the user may rather prefer driving through a university campus than on the highway. Currently, most online navigation systems address the problem by returning multiple paths, allowing the user to manually modify a path, and providing multiple options on how the best route is determined. None of these solutions is ideal since they do not take into consideration multiple criteria simultaneously.

The *constrained shortest path (CSP)* [19,24] addresses this problem by finding the best path based on one criterion with a constraint on another criterion. For instance, the user may want to compute a CSP that minimizes total travel time within a budget for toll payment. In an online navigation system, the constraint can be presented to the user in the form of a slider bar, which drastically simplifies user-system interactions. We focus on single-constraint CSP, because (i) tuning multiple parameters burdens the user, e.g., many parameter combinations may lead to no feasible solution and (ii) processing single-constraint CSP efficiently is already very challenging; for existing solutions, a single query may take hours on a continent-sized road network. Hence, we focus on single-constraint CSP and leave multiple-constraint CSP as future work.

Specifically, in CSP, each edge is assigned two attributes, which are used in the optimization objective and constraint respectively. Without loss of generality, we assume that these two attributes are edge length and cost, respectively. Given an origin s, a destination t, and a cost constraint θ, CSP finds the path from s to t that minimizes its total length, while satisfying that its total cost does not exceed θ. Besides online navigation systems, CSP also finds applications in railroad management, military aircraft management systems, telecommunications, etc. [29]. The CSP problem has been proven to be NP-hard [13, 19]. Hence, the majority of existing work (e.g., [19, 24, 31]) focuses on approximate solutions, which guarantee that the resulting path length is no longer than α times of the optimal path length (where α is a user specified approximation ratio), subject to the cost constraint θ. Although there exist polynomial-time algorithms for approximate CSP (e.g. [19,24,31]), as we show in experiments, the current state-of-the-art solutions are still prohibitively expensive for large road networks. There are two main reasons for their inefficiency. First, they aim at answering approximate CSPs on general graphs, rather than specifically on road networks; consequently, they fail to utilize the latter's special properties. Second and more importantly, most of them process queries without an index. The few known indices are all designed for exact

[1]http://www.digitaltrends.com/cars/new-york-city-to-google-reduce-the-number-of-left-turns-in-maps-navigation-directions/

Proceedings of the VLDB Endowment, Vol. 10, No. 2

CSPs, and they consume large amounts of memory; furthermore, none of them succeeds at reducing query cost to a practical level.

We thus propose a novel and practical solution *COLA* for index-based approximate CSP processing on large road networks. *COLA* mainly exploits two important properties of the road network. First, real road networks are often (roughly) planar, and, thus, can be effectively split into partitions, each of which contains only a relatively small number of boundary vertices. Accordingly, *COLA* partitions the network, builds an overlay graph on the partitions, and indexes a set of selected paths between pairs of boundary vertices. Second, in practice there often exist a relatively small number of landmark vertices [16] in the road network that commonly appear in CSP results. Based on this property, we design an index-free algorithm α-*Dijk* as a component of *COLA* for path computation within a partition, which achieves effective pruning using a landmark set. Extensive experiments using real continent-sized road networks containing tens of millions of vertices show that *COLA* answers an approximate CSP query within a second, whereas previous solutions need several hours. Further, even when an index is not available (e.g., when the edge lengths or costs change frequently), the α-*Dijk* module still outperforms existing methods by over an order of magnitude.

2. BACKGROUND

2.1 Formal Definitions

Let $G = (V, E)$ be a directed road network with a vertex set V and an edge set E. Each edge $e \in E$ is associated with a *length* $\ell(e) \geq 0$ and a *cost* $c(e) \geq 0$. For a path $P = \langle e_1, e_2, \cdots, e_k \rangle$ in G, the *length* and *cost* of P are defined as $\ell(P) = \sum_{i=1}^{k} \ell(e_i)$ and $c(P) = \sum_{i=1}^{k} c(e_i)$, respectively. Following previous work [19, 24, 31], we assume that the length of each edge is an integer. In practice this can be done by measuring the edge length in a sufficiently small unit, e.g., foot or meter, if the edge length represents travel distance. Similarly, we assume that the cost of each edge is also an integer. We use ℓ_{max} (resp. c_{max}) to denote the maximum length (resp. cost) of an edge in G. Meanwhile, we assume that ℓ_{max} (resp. c_{max}) is non-zero; otherwise, the problem can be trivially regarded as a conventional shortest path problem.

Given an origin vertex $s \in V$, a destination vertex $t \in V$, and a cost constraint θ, a *constrained shortest path (CSP)* query asks for the shortest path P among all paths from s to t with costs no more than θ. If there exist multiple CSPs with the same length, we break ties by the cost of the paths. The CSP problem has been proven to be NP-hard, if both ℓ_{max} and c_{max} can be arbitrarily large [13, 19]. On the other hand, if either ℓ_{max} or c_{max} is polynomial to the number of vertices, there exist polynomial-time solutions for CSP, e.g., [19, 21]. Nevertheless, as we review in Sections 2.2 and 2.3, these algorithms incur tremendous costs for large graphs, and, thus, are far from practical. As such, recently much effort has been devoted to solving approximate versions of the CSP problem. This paper follows a popular definition called α-CSP, defined as follows.

DEFINITION 1 (α-CSP QUERY). *Given an origin s, a destination t, a cost constraint θ, and an approximation ratio α, an α-CSP query returns a path P, such that $c(P) \leq \theta$, and $\ell(P) \leq \alpha \cdot \ell(P_{opt})$, where P_{opt} is the optimal answer to the exact CSP query with origin s, destination t, and cost constraint θ.* □

EXAMPLE 1. Figure 1 illustrates an example of exact- and α-CSP on a graph with 5 vertices $v_1, v_2, \cdots, v_5$. The length and cost for each edge are also shown in the figure. For example, the edge from v_1 to v_2 has cost $c = 1$ and length $\ell = 2$. Given origin $s = v_1$, destination $t = v_5$, and cost constraint $\theta = 6$, the CSP query returns the path $P_{opt} = \langle (v_1, v_3), (v_3, v_5) \rangle$, since (i) $c(P_{opt}) = 6 \leq \theta$ and (ii) the length $\ell(P_{opt}) = 5$ is the smallest among all paths from v_1 to v_5 with a cost no more than θ. Meanwhile, for $\alpha = 1.2$, a valid result for the α-CSP query with the same parameters $s = v_1, t = v_5$ and $\theta = 6$ is $P_\alpha = \langle (v_1, v_2), (v_2, v_5) \rangle$, since $c(P_\alpha) = 5 \leq \theta$, and $\ell(P_\alpha) = 6 \leq \alpha \cdot \ell(P_{opt}) = 6$. □

Table 1: List of notations.

Symbol	Meaning
$G = (V, E)$	Input graph
n, m	Numbers of vertices and edges in G
$\ell(e), c(e)$	Length and cost of an edge e
ℓ_{max}, c_{max}	Maximum length and cost for any edge in G
α	Approximation ratio in α-CSP
s, t	Query origin and destination vertices
$\mathcal{T}$	A partitioning of graph G
G_s, G_t	Subgraph in $\mathcal{T}$ containing s and t, respectively
$G^\circ = (V^\circ, E^\circ)$	Overlay graph of G (refer to Section 3.1)

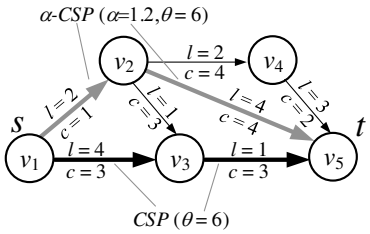


Figure 1: Example of exact CSP and α-CSP

Two important concepts in solving α-CSP are *dominance relationship* and *skyline*. We define dominance for α-CSP as follows.

DEFINITION 2 (α-DOMINANCE). Let P_1 and P_2 be two paths connecting the same origin and destination vertices. P_1 α-*dominates* P_2 iff $c(P_1) \leq c(P_2)$ and $\ell(P_1) \leq \alpha \cdot \ell(P_2)$. □

For instance, consider paths $P_1 = \langle (v_1, v_2), (v_2, v_5) \rangle$ and $P_2 = \langle (v_1, v_3), (v_3, v_5) \rangle$ in the above example. When $\alpha = 1.2$, P_1 α-dominates P_2, since (i) the cost of P_1 is $c(P_1) = 5 \leq c(P_2) = 6$, and (ii) the length of P_1 is $\ell(P_1) = 6 \leq \alpha \cdot \ell(P_2) = 1.2 \times 5 = 6$.

Based on the above definition, a set S of paths is called a *skyline set*, iff no path in S is α-dominated by another in the same set S. We say that a path P is a *skyline path* if P is in a skyline set. Note that if two paths P_1 and P_2 have the same cost, it is possible that they α-dominate each other, in which case we put the path with smaller length in a skyline set. For exact CSP, we define dominance relationship and skyline in the same way, by simply fixing $\alpha = 1$. Table 1 summarizes common symbols throughout the paper.

2.2 State of the Art

We present the current state of the art for CSP processing. Additional literature review appears in Section 2.3.

Exact CSP without index. The state of the art index-free solution for exact CSP problem is the one proposed in [18], which we call *Sky-Dijk* because it follows the general idea of Dijkstra's algorithm [10]. The main difference between *Sky-Dijk* and Dijkstra's algorithm is that the former incrementally maintains a set of paths at each vertex, rather than a single shortest path. Specifically, *Sky-Dijk* maintains a label set $L(v)$ for each vertex v, which contains the current set of skyline paths from the origin s to v, i.e., those not dominated by another path in $L(v)$. Similar to Dijkstra's algorithm, each $L(v)$ is initialized to empty and updated iteratively.

Meanwhile, akin to Dijkstra's algorithm, *Sky-Dijk* maintains a heap H of paths originating from s, in ascending order of their

costs[2]. Initially, H contains only one trivial path with no edge, which both starts and ends at the origin vertex s. Then, in each iteration, *Sky-Dijk* pops the top path P from H. Let v be the last vertex in P. If $v \neq t$, i.e., P has not reached the query destination, the algorithm enumerates each path P' that can be obtained by appending an edge (v, v') at the end of P, and checks whether P' exceeds the cost limit θ or is dominated by any path in $L(v')$. If so, P' is simply discarded; otherwise, *Sky-Dijk* adds P' to both H and $L(v')$, and updates $L(v')$ to eliminate paths dominated by P'. The algorithm terminates when H is empty, and returns the path in $L(t)$ with the minimum length, where t is the destination vertex.

EXAMPLE 2. Consider again the example in Figure 1 with the same exact CSP query with origin $s = v_1$, destination $t = v_5$, and cost constraint $\theta = 6$. *Sky-Dijk* initializes the heap H with a trivial path P_0 from v_1 to v_1 with no edge, and zero cost / length. Then, it pops P_0 from H, and extends it to obtain paths $P_1 = \langle (v_1, v_2) \rangle$ with cost $c_1 = 1$ and length $\ell_1 = 2$, and $P_2 = \langle (v_1, v_3) \rangle$ with $c_2 = 3$ and $\ell_2 = 4$. The algorithm adds P_1 and P_2 to the label set $L(v_2)$ and $L(v_3)$ respectively, and both of them to H.

Next, P_1 is popped from H, and *Sky-Dijk* extends it to obtain paths $P_4 = \langle (v_1, v_2), (v_2, v_3) \rangle$, $P_5 = \langle (v_1, v_2), (v_2, v_4) \rangle$ and $P_6 = \langle (v_1, v_2), (v_2, v_5) \rangle$, which are added to $L(v_3)$, $L(v_4)$ and $L(v_5)$ respectively. Note that now $L(v_3)$ contains two paths P_2 ($c_2 = 3, \ell_2 = 4$) and P_4 ($c_2 = 4$, $\ell_2 = 3$); neither dominates the other. Meanwhile, note that P_6 does not need to be inserted to H, as it has reached the destination $t = v_5$, and, thus, cannot be extended further. After that, P_2 is popped from H; extending P_2 generates $P_7 = \langle (v_1, v_3), (v_3, v_5) \rangle$, which is eventually returned as the CSP result. The algorithm terminates until H becomes empty, at which time it inspects $L(v_5)$, and returns P_7. □

The time complexity of *Sky-Dijk* is $\mathcal{O}(\ell_{max} mn \cdot \log(\ell_{max} n))$, where m (resp. n) is the number of edges (resp. vertices) in the graph, and ℓ_{max} is the maximum edge length [18]. Clearly, *Sky-Dijk* is a polynomial-time algorithm when ℓ_{max} is polynomial to n. However, on real road networks, the performance of *Sky-Dijk* is very poor, since it does not optimize for such datasets at all. As shown in [30] and also in our experiments, *Sky-Dijk* incurs enormous costs for large graphs, and is clearly impractical.

α-CSP without index. The current state-of-the-art solution for α-CSP is developed by Tsaggouris and Zaroliagis [31], dubbed as *CP-Dijk* in the following. Specifically, given an α-CSP with origin s, destination t, cost constraint θ, and approximation ratio α, *CP-Dijk* applies the same data structures and follows the same steps as *Sky-Dijk*, with a single modification: that each label set $L(v)$ maintains the set of paths that are not $\sqrt[n]{\alpha}$-dominated (Definition 2) by another path in $L(v)$, where n is the total number of vertices in the input graph G. Because $\sqrt[n]{\alpha}$-dominance is a relaxed condition of exact (i.e., 1-) dominance, this modification leads to faster query processing. However, for a large graph, $\sqrt[n]{\alpha}$ is very close to 1 even for a large α. Consequently, the performance improvement of *CP-Dijk* over *Sky-Dijk* is often negligible, as shown in our experiments.

The time complexity of *CP-Dijk* is $\mathcal{O}(\kappa mn \cdot \log(\kappa n))$, where $\kappa = \log(n \cdot \ell_{max}/\ell_{min})/(\alpha - 1)$ [31], and ℓ_{max}, ℓ_{min} are the maximum and minimum non-zero values of an edge length, respectively [31]. As explained before, in terms of practical performance, *CP-Dijk* obtains only marginal improvement over *Sky-Dijk*; nevertheless, *CP-Dijk* is at least no worse than *Sky-Dijk*. As discussed in Section 2.3, other polynomial-time solutions for α-CSP can be far more costly. The fact that *CP-Dijk* is the state-of-the-art for α-CSP processing reveals that previous research focuses mostly on asymptotic complexity, not practical performance.

Exact CSP with index. The state-of-the-art for indexed CSP processing is *CSP-CH* [30], which accelerates *Sky-Dijk* with contraction hierarchies [15], an indexing technique that has been shown to be effective for accelerating conventional shortest path processing on road networks [33]. Similar to [15], in each iteration *CSP-CH* removes a vertex from the graph, and substitutes it with new shortcut edges for the remaining vertices. Each shortcut (u, w) created during the removal of vertex v represents a path $\langle (u, v), (v, w) \rangle$ that is not dominated by any other path from u to w. After that, *CSP-CH* answers query with a bidirectional *Sky-Dijk* search from both the origin s and the destination t simultaneously, utilizing the shortcuts to reduce the number of nodes to be traversed.

The problem of *CSP-CH* is that unlike conventional shortest path search, in CSP there can be numerous shortcuts (i.e., multiple skyline sets) for each removed vertex, leading to a prohibitively large index size. *CSP-CH* uses heuristics to alleviate this problem, e.g., by adding only a set of selected shortcuts, and by keeping the vertex in the graph instead of removing it. Such compromises, however, dramatically decrease the effectiveness of the index. Consequently, its query processing cost is still impractically high.

α-CSP with index. To our knowledge, we are not aware of any indexed solution for α-CSP processing. In sum, none of the state-of-the-art methods optimizes for road networks, applies indexing effectively, or obtains acceptable query time for large networks.

2.3 Other Related Work

Joksch [21] first studies the CSP problem, and proposes a dynamic programming algorithm for exact CSP. Subsequently, Handler and Zang [17] propose two methods for exact CSP processing: one method formulates CSP as an integer linear programming (ILP) problem, and solves it with a standard ILP solver. This same methodology is used by Mehlhorn and Ziegelmann [25]. However, as shown in [25], these ILP-based solutions scale poorly, and incur tremendous processing costs on large road networks. The other solution in [17] reduces CSP to a *k-shortest path* problem, and repeatedly computes the next shortest path (in terms of total length) until reaching one that satisfies the cost constraint. Afterwards, Hansen [18] proposes an augmented Dijkstra's algorithm [10] for exact CSP queries and is shown in [28] to outperform the *k-shortest path* solution. The state-of-the-art methods for exact CSP are described in Section 2.2, for index-free and indexed processing, respectively. Meanwhile, most recently, Sedeno et al. [27] propose several pruning strategies to improve the efficiency of *k-shortest path* search, and is shown to outperform the existing *k-shortest path* solutions. However, it does not compare with the *Sky-Dijk* solution. In our experiment, we include Sedeno et al.'s solution [27] as one of our competitors.

Regarding α-CSP, Hansen [18] proposes the first solution, which runs in polynomial time but has a high complexity: $\mathcal{O}(m^2 \frac{n^2}{\alpha - 1} \log \frac{n}{\alpha - 1})$. Lorenz and Raz. [24] improve the complexity to $\mathcal{O}(nm \cdot (\log \log \frac{\ell_{max}}{\ell_{min}} + \frac{1}{\alpha - 1}))$. However, this solution is orders of magnitude slower than an exact CSP algorithm based on *k-shortest path*, as shown in [23]. Later on, Tsaggouris et al. [31] propose *CP-Dijk* based on the conservative pruning technique, i.e., the current state-of-the-art for α-CSP as described in Section 2.2.

Delling et al. [9] study a related query, which returns the entire set of skyline paths between two given vertices. Their solution creates shortcuts similarly as *CSP-CH* [30], and can be adapted to answer CSP queries. However, this method is not scalable to

[2]Note that, similar to the Dijkstra's algorithm, it suffices to store in H only the length, cost and the last two vertices for each path. For ease of presentation we assume H contains full paths.

large graphs, as shown in [12, 30]. In order to reach an acceptable processing time, [9] proposes to modify the problem setting by relaxing the definition of dominance. However, with this relaxation, the method can no longer be used to answer CSP or α-CSP queries. Another related query type is to find the shortest path in terms of a weighted sum of edge costs [11, 14]. These methods, however, cannot be used to answer CSP or α-CSP queries.

Finally, we briefly review classic shortest path and distance queries. One notable class of solutions [8, 20, 22] employ graph partitioning, as in the proposed method *COLA*. The representative is MLD [8], which combines partitioning and contraction hierarchy to improve query efficiency. Yu et al. [34] propose *CI-Rank*, which first identifies a number of star vertices, and then builds an overlay graph on these star vertices. The proposed method *COLA* differs from *CI-Rank* in two major aspects. First, in terms of data structure, the overlay graph in *COLA* correspond to skyline paths, rather than simple shortest paths as in *CI-Rank*. Second, in terms of algorithm, *COLA* builds an additional index structure on top of overlay graph, whereas *CI-Rank* processes the query directly using the overlay graph. Another important indexing technique is *2-hop labelling (2HL)* [6]. The state-of-the-art *2HL* algorithms precompute an order of vertices in the graph, and construct a *2HL* index based on this order, e.g., [4, 7, 32, 35]. None of these methods applies to CSP or α-CSP. Hence, we omit further discussions on classic shortest path processing for brevity, and we refer the reader to a recent survey [5].

3. COLA FRAMEWORK

This section presents the general framework of our proposed solution *constrained labeling (COLA)*. The implementation of several important components in *COLA* is described in Section 4. Basically, *COLA* partitions the road network and constructs an *overlay graph* on top of the partitions. The index structure in *COLA* is then built on the overlay graph, whose size is much smaller than the original graph, leading to much less query processing costs. In the following, Section 3.1 describes the overlay graph; Section 3.2 explains the index structure of *COLA*; Section 3.3 elaborates on query processing based on the *COLA* index; Section 3.4 presents several optimizations that significantly reduce query costs.

3.1 Overlay Graph

Given an input graph G, a partitioning of G consists of a set $\mathcal{T} = \{G_1, G_2, \cdots, G_{|\mathcal{T}|}\}$ of edge-disjoint subgraphs of G, such that the union of all G_i ($1 \leq i \leq |\mathcal{T}|$) equals G. Given $\mathcal{T}$, we say that a vertex v is a *boundary vertex*, if v appears in more than one subgraphs in $\mathcal{T}$. Graph partitioning is a well-studied problem, and *COLA* could use any of the existing solutions, e.g., [8, 20, 22]. In our implementation, we use a state-of-the-art approach for road networks by Delling et al. [8].

We formally define an overlay graph as follows.

DEFINITION 3 (OVERLAY GRAPH). *Given an input graph G, a partitioning $\mathcal{T}$ of G, and the query approximation ratio α[3], a graph $G^\circ = (V^\circ, E^\circ)$ is an overlay graph of G with respect to $\mathcal{T}$, if it satisfies the following three conditions:*

1. *V° consists of all boundary vertices with respect to $\mathcal{T}$;*
2. *For each edge $e^\circ \in E^\circ$ that starts at vertex v and ends at vertex v', there exists a path P in G that goes from v to v', such that $c(P) = c(e^\circ)$ and $\ell(P) = \ell(e^\circ)$;*
3. *For any pair of origin and destination vertices $s, t \in G^\circ$, and any path P in G from s to t, there exists a path P° in G° that goes from s to t that α-dominates P, i.e., $c(P^\circ) \leq c(P)$ and $\ell(P^\circ) \leq \alpha \cdot \ell(P)$.* □

Intuitively, an overlay graph compresses the input graph by (i) including only the boundary vertices of the partitions and removing all other vertices, (ii) using edges to represent paths in G, and (iii) reducing the number of edges by removing paths that are α-dominated by others. Note that the above definition does not require the overlay graph to be minimal, i.e., for the same graph G and partitioning $\mathcal{T}$, there may be another possible overlay graph with fewer edges. Hence, there can be different ways to build an overlay graph, and we explain one such algorithm later in Section 4.2. Besides, for any two vertices v and v' in the overlay graph G°, there can be multiple edges from v to v', when there are multiple paths from v to v' in G.

EXAMPLE 3. Consider the input graph G in Figure 1. Figures 2(a), (b), and (c) show three subgraphs G_1, G_2 and G_3 of G respectively. Clearly, $\mathcal{T} = \{G_1, G_2, G_3\}$ is a partitioning of G, since (i) the set of edges of each subgraph is disjoint with the other subgraphs, and (ii) the union of edges in G_1, G_2 and G_3 constitutes the set of edges of G. Meanwhile, v_2, v_3, and v_5 are the boundary vertices w.r.t. $\mathcal{T}$, since they appear in more than one subgraphs.

Assume that $\alpha = 1.1$, Figure 2(d) shows an overlay graph G° w.r.t. $\mathcal{T}$. The set of vertices of G° is $V^\circ = \{v_2, v_3, v_5\}$, which consists of the boundary vertices of $\mathcal{T}$. Observe that there is exactly one path from v_2 to v_3, i.e. $\langle (v_2, v_3) \rangle$; hence, G° contains an edge $e_1^\circ = (v_2, v_3)$ with length $\ell(e_1^\circ) = 1$ and cost $c(e_1^\circ) = 3$. Similarly, there is exactly one path from v_3 to v_5; thus, G° also includes an edge $e_2^\circ = (v_3, v_5)$ with $\ell(e_2^\circ) = 1$ and $c(e_2^\circ) = 3$. From v_2 to v_5 there are three paths: $P_1 = \langle (v_2, v_3), (v_3, v_5) \rangle$, $P_2 = \langle (v_2, v_5) \rangle$, and $P_3 = \langle (v_2, v_4), (v_4, v_5) \rangle$. Note that P_2 α-dominates P_3, and the two edges in path P_1 already exist in G°. Hence, G° only includes one edge e_3° from v_2 to v_5 with length $\ell(e_3^\circ) = \ell(P_2) = 4$ and cost $c(e_3^\circ) = c(P_2) = 4$. □

Since the overlay graph can be pre-computed and has a smaller size than the original graph, it can be used as a low-cost index to accelerate α-CSP processing, as follows. Given an α-CSP query q on G with an origin s, a destination t, and a cost threshold θ, we first identify the subgraphs G_s and G_t in $\mathcal{T}$ that contain s and t, respectively. Then, we construct graph G^q (which we call an *extended graph*) by merging G_s, G_t and G°, i.e., $G^q = (V_s \cup V_t \cup V^\circ, E_s \cup E_t \cup E^\circ)$, where V_s (resp. E_s) and V_t (resp. E_t) are the vertex (resp. edge) sets of G_s and G_t, respectively. After that, we run an α-CSP algorithm on G^q; its result corresponds to a result for the original α-CSP query, according to the following lemma[4].

LEMMA 1. Let P_{opt} be a result of a CSP query on graph G with origin s, destination t, and cost constraint θ. Meanwhile, let $G_s = (V_s, E_s), G_t = (V_t, E_t)$ be the subgraphs in a partitioning $\mathcal{T}$ that contains s and t, respectively. Then, any result P of an α-CSP with parameters s, t, α on the extended graph $G^q = (V_s \cup V_t \cup V^\circ, E_s \cup E_t \cup E^\circ)$ satisfies $c(P) \leq \theta$ and $\ell(P) \leq \alpha \cdot \ell(P_{opt})$. □

To translate an α-CSP on G^q to an α-CSP on G, we "unfold" each edge in E° into a path in G, which is done according to Condition 2 in Definition 3. Because G^q can be viewed as a compressed version of G with significantly fewer vertices and edges, searching for an α-CSP on G^q is expected to be faster than doing so on G. On the other hand, the speedup using an overlay graph is limited, since the query processing algorithm is the same, albeit on a smaller graph. Next we introduce a much more powerful index structure.

[3]Note that the overlay graph and the *COLA* index both require the knowledge of α, which we consider as a system parameter. The choice of α is discussed further in Section 6.4.

[4]We include all proofs in a technical report [1].

3.2 Constrained Labeling Index

The main *COLA* index is constructed on the overlay graph $G^\circ = (V^\circ, E^\circ)$ described in the previous subsection. For each vertex $v^\circ \in G^\circ$, the index contains two label sets for v°: an *in-label set* $B_{in}(v^\circ)$ and an *out-label set* $B_{out}(v^\circ)$. Each entry in $B_{out}(v^\circ)$ corresponds to a path[5] from v° to another vertex in G°. Symmetrically, each label in $B_{in}(v)$ corresponds to a path from another vertex in G° to v°. The paths in the label sets are carefully chosen such that given any pair of origin and destination vertices $s^\circ, t^\circ \in G^\circ$, and a cost constraint θ, we can construct the α-CSP from s° to t° subject to θ using only the paths in $B_{out}(s^\circ)$ and $B_{in}(t^\circ)$. In other words, with the *COLA* index we do not need to search for the α-CSP result; instead, we simply combine pre-computed paths in the label sets to form a result.

Formally, we define the *COLA* index as follows.

DEFINITION 4 (COLA INDEX). *Given an overlay graph G°, a* COLA *index contains label sets $B_{in}(v^\circ)$ and $B_{out}(v^\circ)$ for each vertex $v^\circ \in G^\circ$ satisfying the following conditions:*

1. *Each entry in $B_{in}(v^\circ)$ corresponds to a path from another vertex in G° to v°;*
2. *Each entry in $B_{out}(v^\circ)$ corresponds to a path from v° to another vertex in G°;*
3. *For any path P between any two vertices $s^\circ, t^\circ \in V^\circ$ (P may contain vertices in $V \setminus V^\circ$), the* COLA *index contains both an out-label in $B_{out}(s^\circ)$ with cost c_o and length ℓ_o and an in-label in $B_{in}(t^\circ)$ with cost c_i and length ℓ_i such that $c_o + c_i \leq c(P)$ and $\ell_o + \ell_i \leq \alpha \cdot \ell(P)$.* □

Condition 3 in the above definition indicates that for any path P connecting two vertices s° and t° in the overlay graph, we can derive another path P' by concatenating two paths P_o and P_i from the out-label set of s° and in-label set of t° respectively, such that P' α-dominates P. Therefore, according to the definition of α-CSP (Definition 1) and α-dominance (Definition 2), we can obtain an α-CSP result between s° and t° by joining the paths from their label sets, without searching for the result from scratch.

EXAMPLE 4. Consider the overlay graph G° in Example 3 with $\alpha = 1.1$. Let $P_1^\circ = \langle (v_2, v_3) \rangle$, $P_2^\circ = \langle (v_3, v_5) \rangle$, and $P_3^\circ = \langle (v_2, v_5) \rangle$. Let P_1', P_2', and P_3' be three trivial paths that go from v_2 to v_2, v_3 to v_3, and v_5 to v_5, with zero cost / length, respectively.

Then $B_{out}(v_2) = \{P_1^\circ, P_1'\}$, $B_{in}(v_2) = \{P_1'\}$, $B_{out}(v_3) = \{P_2'\}$, $B_{in}(v_3) = \{P_2'\}$, $B_{out}(v_5) = \{P_3'\}$, and $B_{in}(v_5) = \{P_2^\circ, P_3^\circ, P_3'\}$ constitute an instance $\mathcal{L}$ of *COLA* index. To explain, clearly, $\mathcal{L}$ satisfies Conditions 1 and 2 in Definition 4. It remains to verify that $\mathcal{L}$ satisfies Condition 3. Note that in the input graph G, there are five paths concerning nodes in G°, i.e., $P_1 = \langle (v_2, v_4), (v_4, v_5) \rangle$, $P_2 = \langle (v_2, v_3) \rangle$, $P_3 = \langle (v_3, v_5) \rangle$, $P_4 = \langle (v_2, v_5) \rangle$, and $P_5 = \langle (v_2, v_3), (v_3, v_5) \rangle$. Consider the first path $P_1 = \langle (v_2, v_4), (v_4, v_5) \rangle$ with cost $c_1 = 6$ and $\ell_1 = 5$. P_1° in $B_{out}(v_2)$ and P_2° in $B_{in}(v_5)$ satisfy that $c(P_1^\circ) + c(P_2^\circ) \leq c(P_1)$ and $\ell(P_1^\circ) + \ell(P_2^\circ) \leq \alpha \cdot \ell(P_1)$. Similarly, we can verify that $\mathcal{L}$ also fulfills Condition 3 for the other four paths. □

One may wonder why we need both an in-label set and an out-label set, instead of just one of them. For example, given a pair of origin and destination vertices s° and t°, if the out-label set $B_{out}(s^\circ)$ of s° contains a path that ends at t° and satisfies the cost constraint, we could simply return this path as the α-CSP result, without checking the in-labels of t°. The problem with having only out- (or in-) labels is that we must store for each vertex the complete set of labels containing skyline paths to (or from) every other vertex in G°, leading to a prohibitively large index size. In contrast, by using both in-labels and out-labels, each label set only contains path to (or from) a selected subset of vertices, leading to a significantly reduced index size. This is akin to database normalization, where storing two separate base tables consumes less space than their join results.

[5]Similar to *Sky-Dijk* and *CP-Dijk*, it suffices to store the important path parameters such as its length, cost and last two vertices. These details are clarified in Section 5.2 and the full version [1]; for now, we assume that each entry in a label set is a path for simplicity.

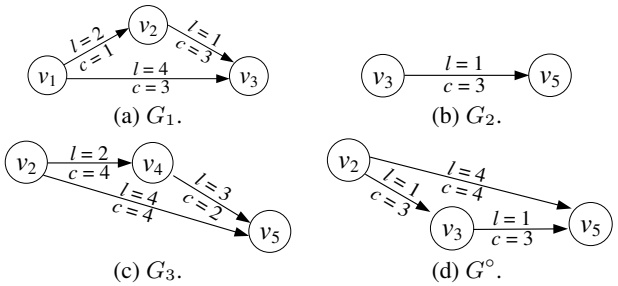


Figure 2: A partition $\mathcal{T} = \{G_1, G_2, G_3\}$ of G in Figure 1 and the corresponding overlay graph G°.

3.3 Query Processing

This subsection clarifies the processing of an α-CSP query with a pair of origin and destination vertices $s, t \in G$ and a cost constraint θ, using the overlay graph and the *COLA* index described in previous subsections. Note that if both s and t belong to the overlay graph G°, we can simply join $B_{out}(s)$ and $B_{in}(t)$, and select the α-CSP result by concatenating a path from $B_{out}(s)$ and another from $B_{in}(t)$, according to Condition 3 in Definition 4. However, either s or t may not appear in the overlay graph, where the α-CSP queries with s and t cannot be answered purely by the *COLA* index which is built on the overlay graph. Note that we could build the *COLA* index on the original graph G instead of the overly graph G°. Nevertheless, doing so may lead to a prohibitively large index size, since G is far larger than G°.

The main idea of *COLA* query processing is to build $B_{out}(s)$ and $B_{in}(t)$ during query time, using the *COLA* index as well as subgraphs $G_s, G_t \in \mathcal{T}$ containing s and t, respectively. In particular, $B_{out}(s)$ and $B_{in}(t)$ must satisfy that the α-CSP result can be obtained by concatenating a path from $B_{out}(s)$ and another from $B_{in}(t)$. Formally, for any path P between s and t, there must exist paths P_o and P_i in $B_{out}(s)$ and $B_{in}(t)$ respectively, such that the concatenation of P_o and P_i α-dominates P. Given this property, the α-CSP result can be obtained by joining $B_{out}(s)$ and $B_{in}(t)$ similarly as the case when s and t are boundary vertices.

The main challenge thus lies in the computation of $B_{out}(s)$ and $B_{in}(t)$. We first focus on the former, initialized to empty. Let $G_s \in \mathcal{T}$ be the sub-graph containing s. We perform a Dijkstra-like search from vertex s to every boundary vertex of G_s. This can be done, for example, using a slightly modified version (i.e., with multiple destinations) of the *CP-Dijk* algorithm described in Section 2.2. In our implementation, we use a novel algorithm *α-Dijk*, detailed in Section 4, which is significantly more efficient than *CP-Dijk*. After we finish the Dijkstra search, we extract the set of skyline paths (c.f. Section 2.1) $L(v^\circ)$ for each boundary vertex $v^\circ \in G_s \cap G^\circ$. Then, we join $L(v^\circ)$ with $B_{out}(v^\circ)$ and add the results to $B_{out}(s)$. Specifically, for each path P_1 in $L(v^\circ)$ from s to v°, and each path P_2 in $B_{out}(v^\circ)$ from v° to another boundary vertex (say, $w^\circ \in G^\circ$), we concatenate P_1 and P_2 into a path P from s to w°, and insert P to $B_{out}(s)$ if the latter does not contain a path that α-dominates P. After that, we purge from $B_{out}(s)$ all paths from s to w° that are α-dominated by P. The computation for $B_{in}(t)$ is

symmetric and omitted for brevity. Algorithm 1 summarizes the *COLA* query processing algorithm.

EXAMPLE 5. Consider the overlay graph in Example 3, and the *COLA* index $\mathcal{L}$ in Example 4 with $\alpha = 1.1$. Given an α-CSP query from v_1 to v_5 with cost threshold $\theta = 7$, *COLA* first checks whether v_1 and v_5 are boundary vertices. Since v_5 is a boundary vertex, the method directly obtains $B_{in}(v_5) = \{P_2^\circ, P_3^\circ, P_3'\}$. On the other hand, since v_1 is not a boundary vertex, its out-label set $B_{out}(v_1)$ needs to be computed on the fly. To do this, *COLA* initiates a Dijkstra-like search from v_1, and computes the set of skyline paths from v_1 to the boundary nodes of v_1's subgraph. In particular, it retrieves the skyline set from v_1 to v_2, which contains only $P_1 = \langle (v_1, v_2) \rangle$. Then, it joins the skyline set with $B_{out}(v_2)$, and adds the joined paths into $B_{out}(v_1)$ if they are not α-dominated by any path in $B_{out}(v_1)$. After that, we have $B_{out}(v_1) = \{P_1, P_1 \cdot P_1^\circ\}$, where $P_1 \cdot P_1^\circ$ denotes the concatenation of P_1 and P_1°.

Similarly, *COLA* retrieves the skyline set from v_1 to v_3, and joins paths in the skyline set with $B_{out}(v_3)$. These paths are $P_2 = \langle (v_1, v_3) \rangle$ and $P_3 = \langle (v_1, v_2), (v_2, v_3) \rangle$. Note that P_3 is identical to $P_1 \cdot P_1^\circ$. Hence, P_3 is not added to $B_{out}(v_1)$, which ends up with $B_{out}(v_1) = \{P_1, P_1 \cdot P_1^\circ, P_2\}$. By joining $B_{out}(v_1)$ and $B_{in}(v_5)$, *COLA* retrieves a skyline set for all paths from v_1 to v_5. Finally, it inspects the results, unfolds edges in G° whenever necessary, and returns path $P = \langle (v_1, v_2), (v_2, v_3), (v_3, v_5) \rangle$. □

The query processing algorithm described so far contains several nested-loop join operations, which can be rather expensive for large label / skyline sets. Next we present effective optimizations that reduce the cost of such joins.

3.4 Optimizations

First we optimize the join between $B_{out}(s)$ and $B_{in}(t)$, which produces the α-CSP result based on the following observation.

OBSERVATION 1. Let P_o and P_i be two arbitrary paths in $B_{out}(s)$ and $B_{in}(t)$ respectively that can be joined, i.e., P_o ends at the starting vertex of P_i. We have the following:

1. If $c(P_o) + c(P_i) > \theta$, then joining P_o (resp. P_i) with any path in $B_{in}(t)$ (resp. $B_{out}(s)$) with cost higher than P_i (resp. P_o) cannot lead to an α-CSP result;
2. If $c(P_o) + c(P_i) \leq \theta$, then joining P_o (resp. P_i) with any path in $B_{in}(t)$ (resp. $B_{out}(s)$) with length longer than P_i (resp. P_o) can be discarded. □

Based on the above observation, we accelerate the join between $B_{out}(s)$ and $B_{in}(t)$ through a careful ordering of the labels. Specifically, *COLA* sorts paths in $B_{out}(s)$ by the IDs of their end vertices, breaking ties by total cost (in ascending order). Note that $B_{out}(s)$ is a skyline set, meaning that paths with the same end vertex are also automatically sorted in descending order of their lengths (otherwise one path would dominate another). Similarly, *COLA* sorts paths in $B_{in}(t)$ firstly by the IDs of their origin vertices, and secondly in ascending of their lengths / descending order of their costs. With such ordering, we propose a novel algorithm *LabelJoin* (as shown in Algorithm 2), which joins $B_{out}(s)$ and $B_{in}(t)$ with a *linear scan of each set*.

LEMMA 2. *Algorithm 2 correctly computes the α-CSP result from $B_{out}(s)$ and $B_{in}(t)$ in linear time.* □

EXAMPLE 6. Consider an α-CSP query from s to t with $\alpha = 1.1$ and cost threshold $\theta = 13$. Assume that $B_{out}(s) = \{P_1, P_2\}$ where both P_1 and P_2 end at w, with $c(P_1) = 4$, $\ell(P_1) = 7$, $c(P_2) = 7$, and $\ell(P_2) = 4$; $B_{in}(t) = \{P_3, P_4, P_5\}$ where all three paths start at w, with $c(P_3) = 7$, $\ell(P_3) = 5$, $c(P_4) = 6$, $\ell(P_4) = 6$, $c(P_5) = 5$, and $\ell(P_5) = 7$. Clearly, $B_{out}(s)$ (resp. $B_{in}(t)$) is sorted in ascending (resp. descending) order of cost.

By Algorithm 2, we first check P_1 and P_3, to see if the concatenated path $P' = P_1 \cdot P_3$ satisfies the cost constraint. As $c(P') = 11 \leq \theta = 13$ and P^* is empty, the found shortest path under cost constraint is hence updated to $P^* = P'$. Afterwards, the *LabelJoin* algorithm proceeds to the next path P_2 in $B_{out}(s)$, and concatenate it with P_3. Here, P_1 is not further concatenated with P_4 and P_5 due to Observation 1.2, i.e., concatenating P_1 with P_4 or P_5 produces a path with a larger length than P'.

Consider the concatenated path $P_1' = P_2 \cdot P_3$. Note that the cost of P_1' exceeds the cost threshold, and Algorithm 2 proceeds to the next path, i.e. P_4, in $B_{in}(t)$. Consider $P_2' = P_2 \cdot P_4$. The cost of P_2' is 13, which satisfies the cost constraint, and the length of P_2' is 10, smaller than P^*. Hence P^* is updated to P_2'. Since there is no more path in $B_{out}(s)$, the *LabelJoin* algorithm stops checking the labels and returns P^* as the result. □

Algorithm 1: *COLA*

input : $s, t, \theta, \alpha, G, G^\circ, B_{out}(s)$, and $B_{in}(t)$
output: A path for the α-CSP query with the origin s, the destination t, and cost threshold θ on G

```
1  Initialize both B_out(s) and B_in(t) to empty;
2  Perform a Dijkstra search from s within its partition G_s to obtain the
   set of skyline paths L(v^o) from s to each boundary vertex v^o in G_s;
3  for each boundary vertex v^o ∈ G_s ∩ G° do
4      Join L(v^o) and B_out(v^o);        // optimized in Section 3.4
5      for each joined path P from s to w^o ∈ G° do
6          if c(P) > θ or P is α-dominated by a path in B_out(s) to
           w^o then
7              Discard P;
8          else
9              Add P to B_out(s);
10             Delete all paths in B_out(s) from s to w^o that are
               α-dominated by P;
11 Compute B_in(t) similarly as Lines 2-9 (see Section 3.3);
12 Join B_out(s) and B_in(t);             // optimized in Section 3.4
13 return the α-CSPs from the above join result;
```

Next, we focus on the join between a skyline set $L(v^\circ)$ and a label set $B_{out}(v^\circ)$ in the *COLA* index (Line 4 of Algorithm 1). The case for joining $L(v^\circ)$ with $B_{in}(v^\circ)$ is symmetric and omitted for brevity. The basic idea for the optimization is not to compute the complete join results, but only those results that can possibly lead to an α-CSP result. Specifically, we avoid generating certain join results based on the following observation.

OBSERVATION 2. Let P be a path from s to w° from the join result of $L(v^\circ)$ and $B_{out}(v^\circ)$. P cannot possibly lead to an α-CSP result, if any of the following is true.

1. w° cannot possibly reach t on the input graph G;
2. The minimum cost for any path from w° to t exceeds $\theta - c(P)$;
3. There exists a path P' from s to t satisfying the cost constraint, such that the minimum length of any path from w° to t exceeds $\ell(P')/\alpha - \ell(P)$. □

According to the above observation, before performing any join operation, we first select a set of end vertices W for the join results that can possibly lead to an α-CSP result, which is incrementally pruned using Observation 2. Then, we filter the *COLA* index, and use only the labels that reach a vertex in W. Algorithm 3 shows the algorithm for computing W. Filtering the *COLA* index before joining it with the skyline sets can be understood as using a semi-join to improve join performance in database systems. Note that the

Algorithm 2: LabelJoin

input : θ, α, $B_{out}(s)$, and $B_{in}(t)$
output: An α-CSP P^* with a cost not larger than θ from s to t

```
1  Sort the paths in B_out(s) first in ascending order of end vertex ID, and
   then in ascending order of cost (i.e., in descending order of length);
2  Sort the paths in B_in(t) first in ascending order of end vertex ID, and
   then in ascending order of length (i.e., in descending order of cost);
3  Initialize α-CSP result P* to empty;
4  repeat
5      Scan both B_out(s) and B_in(t) simultaneously, until reaching a
       matching pair P_o ∈ B_out(s) and P_i ∈ B_out(t);
6      while P_o matches P_i, i.e., P_o ends at the origin vertex of P_i do
7          if c(P_o) + c(P_i) > θ then
8              Set P_i to the next entry in B_out(s);
9          else
10             Update P* to the concatenation of P_o and P_i if the
               combination of P_o and P_i has length smaller than P*;
11             Set P_o to next entry in B_in(t);
12 until reaching the end of either B_out(s) or B_in(t);
13 return P*;
```

Algorithm 3: PruneLabel

input : s, t, θ, α, L, B_{out}, and B_{in}
output: B_{out} and B_{in} with labels filtered

```
1  Initialize vertex set W with all boundary vertices reachable from both
   s and t, according to B_out(s) and B_in(t);    // Pruning condition 1
2  for each vertex w in W do
3      for each vertex v^o in G_s do
4          Compute the minimum cost c_1 from s to v^o in L(v^o);
5          Compute the minimum cost c_2 from v^o to w in B_out(v^o);
6          Set c_s(w) = c_1 + c_2;
7      Compute c_t(w) similarly as Lines 3-6;
8      if c_s(w) + c_t(w) > θ then
9          Remove w from W;                        // Pruning condition 2
10 Initialize ℓ_max to −∞;
11 for each vertex w in W do
12     for each boundary vertex v^o in G_s do
13         Compute the max length ℓ^1_max from s to v^o in L(v^o), and
           the max length ℓ^2_max from v^o from w in B_out(v^o);
14         Set ℓ^s_max(w) = ℓ^1_max + ℓ^2_max;
15     Compute ℓ^t_max(w) similarly as Lines 12-14;
16     Update ℓ_max if ℓ^s_max(w) + ℓ^t_max(w) > ℓ_max;
17 for each vertex w in W do
18     Compute minimum lengths ℓ^s_min(w) and ℓ^t_min(w) similarly as
       Lines 12-14;
19     if ℓ^s_min(w) + ℓ^t_min(w) > ℓ_max then
20         Remove w from W;                        // Pruning condition 3
21 Filter B_out (resp. B_in) by removing paths that do not end (resp.
   originate) at a vertex in W;
22 return B_out and B_in;
```

COLA index is constructed before we know the query parameters, hence, it usually contains a large number of labels not needed for answering the query at hand.

Algorithm 3 shows the pseudo-code for computing W. W is initialized with all boundary vertices reachable from both s and t. Then, we prune those vertices that cannot lead to a path from s to t within cost threshold θ. After that, the algorithm computes an upper bound for the length of a path from s to t, and uses it to prune more vertices in W based on the third condition in Observation 2.

4. α-DIJK

In this section, we present α-*Dijk*, which is used in our query processing to compute skyline paths. Apart from this, α-*Dijk* has three other main uses: (i) for intra-partition search during query processing in *COLA*, (ii) for building the *COLA* index, and (iii) as a standalone index-free solution for α-CSP.

Similar to *CP-Dijk* (refer to Section 2.2), α-*Dijk* is based on *Sky-Dijk* with enhanced pruning with the relaxed α-dominance definition. On the other hand, the pruning strategy of α-*Dijk* is radically different from that in *CP-Dijk*. The intuition is as follows. Imagine that we have a total "budget" for pruning along a path; the larger the budget allocated to a vertex, the stronger pruning power it is allowed to apply to reduce the size of its set of associated paths. *CP-Dijk* simply distributes this budget equally to each vertex on the path. Since the path may have a large number of vertices, each of them only receives little pruning power, leading to ineffective pruning everywhere. In contrast, α-*Dijk* concentrates the pruning power to vertices associated with a large number of paths, and does not allocate pruning power at all to vertices with relatively few paths. In a real road network, there are usually a small number of landmark vertices that appear frequently in CSPs, which tend to accumulate a large number of paths. As a result, concentrating the pruning power to such vertices leads to effective reduction of the total number of paths to be examined, and thus, accelerates query processing.

Algorithm 4 shows the pseudo-code of α-*Dijk*. Given an α-CSP query with an origin vertex s, a destination vertex t, and a cost threshold θ, α-*Dijk* first invokes the vanilla Dijkstra's algorithm to compute, for each vertex v, the minimum-length path $P_\ell(v)$ and the minimum-cost path, i.e., the path with the minimum cost (ties broken by smaller length), $P_c(v)$ from s to v (Line 1). Then it records the length of $P_\ell(v)$ and $P_c(v)$ as $\ell^\perp(v)$ and $\ell^\top(v)$ (Line 2), respectively. Note that $\ell^\perp(v)$ and $\ell^\top(v)$ are lower- and upper- bounds on the total path length, respectively. To explain why $\ell^\top(v)$ is an upper-bound, consider a path P' that has length larger than $\ell^\top(v)$. Clearly, P' can be α-dominated by $P_c(v)$. Thus, we can keep only $P_c(v)$ and discard all paths with length larger than $P_c(v)$, meaning that its length $\ell^\top(v)$ is the upper bound for total path length.

Next, α-*Dijk* creates a min-heap H of skyline paths sorted on path costs similarly as in *Sky-Dijk* and *CP-Dijk*, except that in α-*Dijk* each heap entry $\rho = \langle P, \tau \rangle$ contains both a path P and an additional *pruning surrogate* $\tau \in [\ell/\alpha, \ell]$ that facilitates adaptive allocation of the "pruning budget" as mentioned earlier [6]. The heap H is initialized with a single entry that contains a trivial path that contains only one vertex s, and a pruning surrogate with value 0 (Line 5). In addition, α-*Dijk* initializes an list $L(v)$ of heap entries for each vertex v (Line 4).

After that, α-*Dijk* iteratively pops the top entry $\rho = \langle P, \tau \rangle$ from H and processes it as follows. Let v be the last vertex in P. α-*Dijk* first examines $L(v)$, and retrieves the entry $\rho' = \langle P', \tau' \rangle$ in $L(v)$ with the largest path cost (Line 8). The algorithm guarantees that the cost of P' is smaller than that of P, since H is sorted in ascending order of path costs. Then, α-*Dijk* compares the pruning surrogates τ and τ' of the two entries, and prunes ρ iff. $\tau' \leq \tau$ (Lines 9-10). Let $l(P)$ be the total length of P and $l(P')$ be the length of P', we have:

$$\ell(P') \leq \alpha \cdot \tau' \leq \alpha \cdot \tau \leq \alpha \cdot \ell(P),$$

which means that P' α-dominates P (note that the former also has a lower cost explained above). One may wonder why we prune

[6] To conserve memory, in our implementation instead of a full path P each entry only stores its length ℓ, cost c, last vertex v and the vertex v_{pred} before v. The full path can be reconstructed by using v and v_{pred}. Details can be found in the full version [1].

Algorithm 4: α-*Dijk*

input : An α-CSP query on G with an origin s, a destination t, and a cost threshold θ
output: An answer P to the α-CSP query

1 Calculate the minimum-length path $P_\ell(v)$ and minimum-cost path $P_c(v)$ from s to each vertex v;
2 Let $\ell^\perp(v)$ and $\ell^\top(v)$ be the length of $P_\ell(v)$ $P_c(v)$, respectively;
3 Creates a min-heap H with entries in the form $\langle P, \tau \rangle$, sorted in ascending order of path costs, breaking ties with path lengths;
4 Create an entry list $L(v)$ for each vertex v in G;
5 Insert an entry $\langle P = \langle s \rangle, \tau = 0 \rangle$ into H;
6 **while** H *is not empty* **do**
7 Pop the top entry $\rho = \langle P, \tau \rangle$ in H;
8 Let $\langle P', \tau' \rangle$ be the entry in $L(v)$ with the largest cost;
9 **if** $\tau' \leq \tau$ **then**
10 **continue**; // ρ is pruned
11 Insert ρ into $L(v)$;
12 **if** $v \neq s$ *and* $|L(v)| > \log_\alpha \frac{\ell^\top(v)}{\ell^\perp(v)}$ **then**
13 Modify ρ to set $\tau = \max\{\ell/\alpha, \ell^\perp(v)\}$;
14 **for each** *outgoing edge* $e = (v, v')$ *of* v **do**
15 Construct path P_{new} by extending P with e;
16 **if** $c + c(e) \leq \theta$ **then**
17 Push $\langle P_{new}, \tau + \tau(e) \rangle$ into H;
18 **return** the length-shortest path in $L(t)$;

Algorithm 5: Overlay Graph Construction

input : A partition $\mathcal{T} = \{G_1, G_2, \ldots, G_k\}$ of G
output: An overlay graph $G^\circ = (V^\circ, E^\circ)$ of G

1 Let $E^\circ = \emptyset$, and V° be the set of boundary vertices defined by $\mathcal{T}$;
2 **for each** *subgraph* $G_i \in \mathcal{T}$ **do**
3 **for each** *boundary vertex* v° *in* G_i **do**
4 Feed v° and G_i as input to the single-source α-*Dijk*, which outputs an entry list $L(v)$ for each vertex v in G_i;
5 **for each** *boundary vertex* v *in* G_i **do**
6 Let $S = \emptyset$;
7 **for each** *entry* ρ *in* $L(v)$ *in ascending order of* $c(\rho)$ **do**
8 Let ρ' be the last entry inserted into S;
9 **if** S *is empty or* $\ell(\rho') > \alpha \cdot \tau(\rho)$ **then**
10 Insert ρ into S;
11 **else**
12 $\tau(\rho') = \min\{\tau(\rho'), \tau(\rho)\}$; // ρ is pruned
13 **for each** *entry* ρ *in* S **do**
14 Insert an edge $e^\circ = (v^\circ, v))$ into E°, with $c(e^\circ) = c(\rho)$, $\ell(e^\circ) = \ell(\rho)$, and $\tau(e^\circ) = \tau(\rho)$;
15 **return** $G^\circ = (V^\circ, E^\circ)$;

based on surrogates rather than the path lengths. The reason is that the surrogates τ and τ' control the amount of pruning at vertex v. To see this, if we set $\tau' = \ell'/\alpha$ and $\tau = \ell$, then P' can prune P whenever the former α-dominates the latter, which indicates that we use the maximum pruning power at v. Conversely, when $\tau' > \ell'/\alpha$ or $\tau < \ell$, pruning at v is not performed with full power, i.e., it is possible that P is not pruned even if P' α-dominates P. Note that *pruning with full power everywhere leads to incorrect results*, and a counter-example can be found in the full version [1]. Meanwhile, *pruning with the same power everywhere is inefficient*, as explained at the beginning of this subsection. The use of surrogates enables *adaptive pruning*, the key idea of α-*Dijk*.

It remains to clarify how α-*Dijk* computes the surrogate values. This is done in Lines 12-13. In particular, if ρ passes pruning, it is inserted into $L(v)$. At this point, the algorithm adjusts its surrogate τ based on the following heuristic: if the number of entries $|L(v)| > \log_\alpha \frac{\ell^\top(v)}{\ell^\perp(v)}$, then α-*Dijk* sets $\tau = \max\{\ell/\alpha, \ell^\perp(v)\}$, which grants ρ the maximum pruning power (Line 13). Otherwise, α-*Dijk* sets $\tau = \ell$, minimizing the pruning capability of ρ. To explain, observe that $\log_\alpha \frac{\ell^\top(v)}{\ell^\perp(v)}$ is an upper bound of $|L(v)|$ when we apply α-dominance in the construction of $L(v)$. Intuitively, if the size of the current $L(v)$ exceeds this upper bound, then applying aggressive pruning in $L(v)$ is likely to reduce $|L(v)|$ and help improve query efficiency. On the other hand, if $|L(v)| \leq \log_\alpha \frac{\ell^\top(v)}{\ell^\perp(v)}$, then pruning entries in $L(v)$ tends to be ineffective, in which case it is more preferable to omit pruning in $L(v)$ in order to enable aggressive pruning at other vertices.

After that, for an entry that is not pruned, the algorithm continues to extend the corresponding path P, by adding one more edge $e = (v, v')$ (Lines 14-15). If the resulting path P_{new} satisfies the cost constraint, α-*Dijk* creates a new entry and inserts it into H. The surrogate value for P_{new} is computed by adding τ and the *edge surrogate value* $\tau(e)$ (Line 17), obtained as follows. If edge e is an original edge in the input graph, then $\tau(e)$ is simply the length of e. Otherwise, i.e., e is added during the construction of the overlay graph which corresponds to a path in the original graph, $\tau(e)$ is the surrogate value corresponding to that path, as we clarify in the next subsection. Finally, after H depletes, α-*Dijk* retrieves the entry in $L(t)$ with the smallest length, and returns the corresponding path as the the answer to the α-CSP query.

Theoretical Analysis. The following theorem establishes the correctness of α-*Dijk*.

THEOREM 1. *For each vertex v, let $L(v)$ be the entry list of v when α-Dijk terminates. Then, for any path P from s to v with a cost smaller than θ, there exists an entry ρ in $L(v)$ with a cost at most $c(P')$ and a length at most $\alpha \cdot \ell(P')$.* □

Next, we discuss the time complexity of our α-*Dijk* algorithm. For each vertex v, the number of stored entries in $L(v)$ is bounded by $\ell_{max} \cdot n$. When adding labels for outgoing edges of each vertex v, it incurs at most $d_v \cdot \ell_{max} \cdot n$ labels, where d_v is the out-degree of v. To sum up with, there are at most $\ell_{max} \cdot n \cdot m$ labels added in the whole procedure. To insert / pop $\ell_{max} \cdot n \cdot m$ entries into the heap, it requires $\mathcal{O}(\log(\ell_{max} n))$ time for each operation, ending up with $\mathcal{O}(\ell_{max} mn \cdot \log(\ell_{max} n))$ time complexity.

5. COLA IMPLEMENTATION

5.1 Overlay Graph Construction

Given a partitioning $\mathcal{T} = \{G_1, G_2, \ldots, G_k\}$ of G, we construct an overlay graph $G^\circ = (V^\circ, E^\circ)$ defined in Definition 3 using α-*Dijk* with two minor modifications: (i) there is no cost constraint, i.e., $\theta = +\infty$ and (ii) instead of a path, the algorithm returns the entry list $L(v)$ for every vertex v. We refer to this modified algorithm as *single-source* α-*Dijk*. To simplify our notations, in the following we use $c(\rho)$, $l(\rho)$ and $\tau(\rho)$ to denote the path cost, path length and pruning surrogate of an entry $\rho \in L(v)$, respectively.

Algorithm 5 shows the pseudo-code of our overlay graph construction algorithm. Initially, the algorithm sets V° to the set of all boundary vertices defined by $\mathcal{T}$, and E° to empty (Line 1). After that, it processes each subgraph G_i (Lines 2-14). In particular, for each boundary vertex v° in G_i, it invokes the single-source α-*Dijk* to compute an entry list $L(v)$ for each vertex v in G_i (Line 4). If v is also a boundary vertex in G_i, then some of the entries in $L(v)$

may be converted into edges between v° and v in E° (Lines 5-14), as follows. First, the algorithm creates a set $S = \emptyset$ for storing entries (Line 6). Then, it inspects the entries in $L(v)$ in ascending order of their costs, and compares each entry ρ with the last entry ρ' inserted into S. If ρ' does not exist (i.e., S is empty) or $c(\rho') \leq c(\rho)$ or $\ell(\rho') \leq \alpha \cdot \tau(\rho)$, then the algorithm inserts ρ into S as an entry to be converted into an edge in E° (Lines 9-10). Otherwise, the path represented by ρ must be α-dominated by ρ', and hence, the algorithm omits ρ, and modifies ρ' to set $\tau(\rho') = \min\{\tau(\rho'), \tau(\rho)\}$ (Lines 11-12). The change of $\tau(\rho')$ is important to ensure that the resulting graph G° satisfies Definition 3. After all entries in $L(v)$ are examined, the algorithm retrieves each entry ρ that has been inserted in S, and converts it into an edge $e^\circ \in E^\circ$ with $c(e^\circ) = c(\rho)$ and $\ell(e^\circ) = \ell(\rho)$. In addition, we define the surrogate value of e° as $\tau(e^\circ) = \tau(\rho)$, which is used in our *COLA* index construction, clarified in the next subsection. Once all subgraphs in $\mathcal{T}$ are processed, the algorithm terminates and returns $G^\circ = (V^\circ, E^\circ)$. We have the following theorem.

THEOREM 2. *Algorithm 5 correctly constructs an overlay graph that satisfies Definition 3.* □

5.2 Labeling Index Construction

This subsection details the construction of the *COLA* index. Note that the index structure is not unique, and there are various ways to build it. In our implementation, we apply a standard technique in the literature of *2-hop labeling* (e.g., [4, 7, 32, 35]) for conventional shortest paths, which introduces a ranking function r of all vertices in G°, whose values reflect the relative importance of the vertices. Then, for each $v^\circ \in G^\circ$, we require that (i) each entry in $B_{in}(v^\circ)$ is a path P_i originating at a vertex w that has the highest rank among all vertices in P_i, and symmetrically, (ii) each entry in $B_{out}(v^\circ)$ is a path P_o ending at a vertex w that has the highest rank among all vertices in P_o. To reduce memory consumption, in each entry we can substitute a full path with a tuple $\langle v, c, l, v_{pred}\rangle$, where c and l are the cost and length of the path and v and v_{pred} are the last and second-to-last vertices, respectively. Similar to conventional 2-hop labeling, the choice of the ordering plays an important role for the effectiveness of the index. We follow a similar approach as in previous work [32], and refer the reader to a full version [1] for details. We further define the rank of a path as follows.

DEFINITION 5 (RANK OF A PATH). *Let P be a path in G°. The rank $r(P)$ of P is the highest rank among all vertices in P.* □

Our index construction algorithm runs in iterations. After finishing i iterations, the labels constructed in B_{out} and B_{in} guarantee that for any path P from u to v ($u, v \in G^\circ$) whose rank is no more than i, there exists a label entry $\rho_u \in B_{out}(u)$ corresponding to a path P_1, and a label entry $\rho_v \in B_{in}(v)$ concerning a path P_2 in G° such that $c(\rho_u) + c(\rho_v) \leq c(P)$ and $\ell(\rho_u) + \ell(\rho_v) \leq \alpha \cdot \ell(P)$. In other words, by concatenating P_1 and P_2, we find a path that α-dominates P.

Algorithm 6 shows the procedure for the index construction. We explain how labels in B_{in} are computed, and omit the case for B_{out} for brevity. In the i-th iteration, the vertex v_i° with $r(v_i^\circ) = i$ is selected. A modified version of single-source α-*Dijk* is invoked to produce a set of label lists $L(v^\circ)$ for $v^\circ \in G^\circ$. The modification is that when deriving the lower bound of the length $\ell_\perp(v^\circ)$ from v_i° to v°, it uses the τ value of a path instead of its length (Line 4). The reason is that since G° is a simplified version of G , the minimum length path P in G might not be preserved in G°; meanwhile, the τ value of a path denotes the lower bound of the minimum length path that it has pruned, indicating that the τ value of the minimum-τ path is a lower bound of the length from v_i° to v°.

Algorithm 6: *Index Construction*

input : An overlay graph G°, and a rank for all vertices in G°
output: a COLA index $\mathcal{L}$

```
1  Let G°_1 = G°;
2  for i = 1, 2, ..., n do
3      Let v°_i be the vertex whose rank is i;
4      Invoke Algorithm 4 on G°_i except that (i) to derive ℓ⊥(v°), it
       computes the path P with minimum τ value instead of
       minimum-length; and (ii) it outputs the label list L;
5      for w° ∈ V° whose L(w°) is not empty do
6          for each entry ρ ∈ L(w°) do
7              if ∄ρ_o ∈ B_out(v°_i), ρ_i ∈ B_in(w°) such that
                 c(ρ_o) + c(ρ_i) ≤ c ∧ ℓ(ρ_o) + ℓ(ρ_i) ≤ α · ρ(τ) then
8                  if ∄ρ' ∈ B_in(w°) that can dominate ρ then
9                      Add the path corresponding to ρ into B_in(w°);
10     Repeat the above procedure with a backward modified α-Dijk and
       add entries into B_out;
11     Let G°_{i+1} be the graph by removing u_i and its incident edges in
       G°_i;
12 return B_in, B_out;
```

When the α-*Dijk* algorithm finishes, it outputs the label list into L. For each vertex w°, it then iteratively selects entries from $L(w^\circ)$ to add into $B_{in}(w^\circ)$ (Lines 5-9). Note that our algorithm removes some redundant labels. For a label entry ρ, we check whether there exists a label $\rho^+ \in B_{out}(u_i)$ and a label $\rho^- \in B_{in}(v)$ such that $c(\rho^+) + c(\rho^-) \leq c(\rho)$ and $\ell(\rho^+) + \ell(\rho^-) \leq \tau(\rho)$. If so, ρ is pruned. Meanwhile, if ρ can be α-dominated by an existing label entry in $B_{in}(w^\circ)$, then it is also pruned. A label that passes pruning is added into $B_{in}(w^\circ)$.

After the label entries are updated, a backward version of α-*Dijk* is performed, and entries are added in to B_{out}. Then, we update the input graph of the next iteration by removing vertex u_i and its incident edges. After n iterations, the algorithm terminates and returns B_{out} and B_{in}. We have the following theorem.

THEOREM 3. *Algorithm 6 correctly produces a COLA index that satisfies Definition 4.* □

6. EXPERIMENTS

This section experimentally evaluates *COLA* against the current state-of-the-art methods. Section 6.1 explains the experimental settings. Sections 6.2 and 6.3 present evaluation results in terms of query efficiency and indexing overhead, respectively. Section 6.4 provides insights for choosing an appropriate value for α.

6.1 Experimental Settings

All methods are implemented in C++, compiled with full optimizations, and tested on a Linux machine with an Intel Xeon 2.6GHz CPU and 64GB RAM. We repeat each experiment 5 times and report the average results.

Datasets. We use 8 real road networks from the 9th DIMACS Implementation Challenge [2] as shown in Table 2. In the datasets, each vertex represents a road junction, and each edge represents a road segment. Among the eight road networks, EU is directed and the others are undirected. Each edge in the dataset contains two attributes: the travel time and the travel distance. Following previous work [27], we use the travel time as the cost, and the travel distance as the length. The dataset sizes vary from small cities to the full USA road networks. Table 2 summarizes the properties of the datasets, where $|V|$, $|E|$, $|V^\circ|$ and $|E^\circ|$ are the cardinalities of

Table 2: Road networks (K=10^3, M=10^6).

dataset	$\|V\|$	$\|E\|$	$\|V^\circ\|$	$\|E^\circ\|$
New York City (NY)	0.3M	0.7M	4.8K	0.2M
Florida (FLA)	1.1M	2.7M	4.2K	0.2M
Northwest USA (NW)	1.2M	2.8M	4.2K	0.2M
Northeast USA (NE)	1.5M	3.9M	7.0K	0.7M
Great Lakes (LKS)	2.8M	6.9M	10.9K	1.8M
Western USA (W)	6.3M	15.2M	7.8K	1.2M
Europe (EU)	18.0M	42.2M	16.9K	9.3M
Full USA (USA)	23.9M	58.3M	17.7K	9.5M

vertices in the road network, edges in the road network, vertices in the overlay graph and edges in the overlay graph, respectively.

Query sets. For each dataset, we generate 5 query sets Q1-Q5, each containing 100 queries. Q1-Q5 are generated as follows. First, we randomly choose the query origin s and destination t among the vertices of the road networks. Then, we classify the query into one of the 5 query sets, based on the length from s to t. Specifically, we first compute a lower bound for the graph diameter (the longest length of all shortest paths) using an existing approximate algorithm [26], which is at most 2 times the value of diameter. Let d_{min} be this lower bound. We then generate queries as follows: if the length of these two nodes is in range $[d_{min}/2, +\infty)$, we add it into Q5; if the distance is in range $[d_{min}/4, d_{min}/2)$, we add it into Q4; if the distance is in the range $[d_{min}/8, d_{min}/4)$, we add it into Q3, etc. For each query set, we report the average time for the 100 queries. The differences among results for different query sets reflect the impact of the travel distance.

Next, we clarify the generation of the cost constraint θ. For each query, we first compute c_{min}, the minimum cost of any path, and c_{max}, the cost of the minimum-length path from s to t; then, we select θ uniformly at random from $[c_{min}, c_{max}]$. Note that if $\theta < c_{min}$, the query cannot return any result; if $\theta > c_{max}$, the minimum-length path is always a valid result to the CSP query.

Another important parameter is α, which determines the approximation guarantee of α-CSP queries. We view α as a system parameter rather than part of the query, since α controls the trade-off between query accuracy and system efficiency, e.g., space consumption, which is more relevant to the system environment than individual queries. In order to find a suitable value for α, we performed a set of experiments with varying α. The results shown in Section 6.4 demonstrate that the value $\alpha = 1.1$ leads to a good balance among the space consumption, preprocessing time, query efficiency and query accuracy. Hence, in the rest of our experiments, we fix $\alpha = 1.1$.

Methods. We compare *COLA* against three state-of-the-art methods: *Sky-Dijk* [18], *CP-Dijk* [31] and *CSP-CH* [30], described in Section 2.2. Meanwhile, we also include a most recent *k-shortest path* based solution [27] as our competitor, dubbed as *KSP*. Besides, we also include *α-Dijk* (see Section 4) in our comparisons, which answers α-CSP queries without an index.

6.2 Query Efficiency

Figure 3 plots the average query execution time (in seconds) for all methods. For brevity, we only show the results on 4 representative datasets: NY, LKS, EU, and USA. On these 4 datasets, we inspect the performance on all 5 query sets Q1-Q5. Note that the y-axis is in logarithmic scale, and we use error bars to present the variations of the query performance for *COLA* and *α-Dijk*.

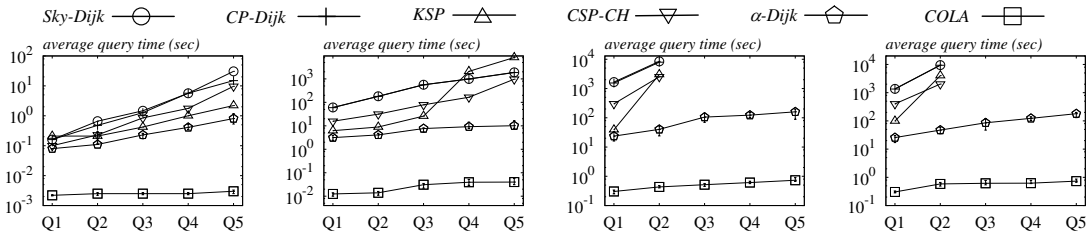
The most apparent observation is that regardless of the dataset or query set, our main proposal *COLA* consistently outperforms all other methods by several orders of magnitude. Further, the query time of *COLA* is always within one second, even on the largest network covering the entire USA. Besides, on large directed road networks, e.g., EU, *COLA* still outperforms the existing methods by two orders of magnitude, which demonstrates the effectiveness of our *COLA* index on both directed and undirected road networks. For state-sized networks, *COLA* always finishes within milliseconds. In contrast, the rest of the methods require at least 1 second to finish, except for a few settings that involve both a small dataset and a query set with very close query origin and destination. For queries with relatively far apart origin / destination pairs, these methods (except for *α-Dijk*) can take several hours to process one query. From these observations, we conclude that *COLA* not only outperforms its competitors, but brings down the query processing cost from prohibitively high to a practically low. In other words, *COLA* makes α-CSP feasible on current hardware.

Besides *COLA*, the next fastest method is our index-free algorithm *α-Dijk*, again in all settings. The performance gap between *α-Dijk* and the competitors is also notable, often more than an order of magnitude. Moreover, both *COLA* and *α-Dijk* demonstrate small variation for the query processing time, which is close to the average. In contrast, as shown in [27, 30],existing CSP algorithms vary significantly in terms of query time, and may incur much longer query time than the average. This demonstrates the robustness of our methods to the input query. Interestingly, our index-free *α-Dijk* outperforms the indexed method *CSP-CH*, which is given the advantage of large amounts of pre-computations. This suggests that exact CSP as a query type may not be practical for large networks; in such situations, it is necessary to relax the query definition.

Comparing the four existing methods, *CSP-CH* outperforms *CP-Dijk* and *Sky-Dijk* in all settings, and outperforms *KSP* for queries with far apart origin / destination pairs. Meanwhile, *KSP* turns out to be very efficient when the origin and destination are close. However, as the distance between the origin and destination increases, the query efficiency degrades rapidly, due to the exponential growth in the number of paths from the origin to the destination. In addition, we observe that the performance improvement of *CP-Dijk* (for α-CSP) over *Sky-Dijk* (for exact CSP) is negligible, in all settings. This confirms that the *CP-Dijk* (and also other previous work on α-CSP, as *CP-Dijk* is the current state of the art) focuses on theoretical improvement in terms of asymptotic complexity rather than practical performance.

The impact of the distance between the origin s and destination t becomes apparent, once we compare results for different query sets. When they are close to each other (e.g., Q1 and Q2), the query cost for all methods are small, especially on small networks. As s and t become farther apart, query costs increase rapidly. This is expected, as the longer the trip is, the more vertices are involved during CSP processing. An important observation is that *COLA*'s performance is robust against varying query sets. This is mainly due to its use of the overlay network. Specifically, *COLA* utilizes the pre-computed distances between boundary vertices of different partitions, and the number of boundary vertices involved slowly increases with the distance between s and t, compared to the number of vertices in the network. Besides, the query cost of our *α-Dijk* algorithm increases slowly, which demonstrates the effectiveness of its adaptive pruning. On the contrary, the other four existing methods scale poorly; for large datasets, they usually require hours to process a single query. Furthermore, on the USA dataset, none of these four methods can answer queries in Q3-Q5 due to prohibitive memory consumption (>64GB).

Finally, we compare results on different datasets. As expected, the larger the network is, the more expensive it is to process a CSP query. Once again, unlike its competitors, the query cost of *COLA* grows slowly with the network size, thanks to its use of the overlay



(a) NY (b) LKS (c) EU (d) USA

Figure 3: Query efficiency vs. Query sets.

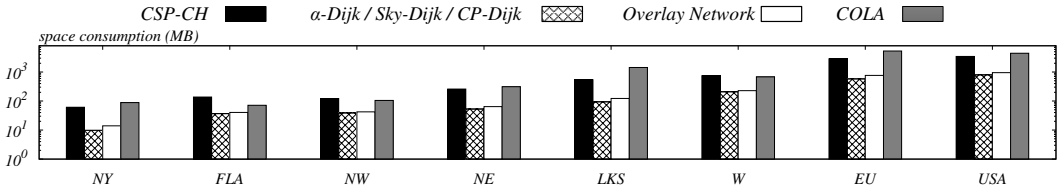


Figure 4: Space consumption.

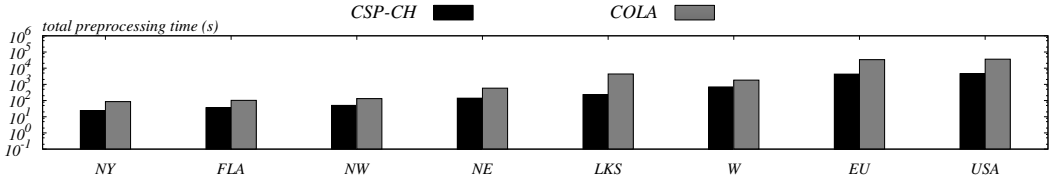


Figure 5: Total preprocessing time.

network and the constrained labeling index, whose effects become more pronounced as the dataset becomes larger.

6.3 Index Size and Construction Time

Figure 4 illustrates the memory consumption for all methods, before running any query. The results for the non-indexed approaches *Sky-Dijk, CP-Dijk, KSP* and α-*Dijk* are simply the size of the road network. On the other hand, the results for *CSP-CH* and *COLA* indicate their respective index sizes. In addition, we also show the size of the overlay graph in *COLA* in the same figure.

The most important observation is that the index size of *COLA* is no more than 5GB on the largest dataset USA and such memory requirement can be easily satisfied on a modern server. Comparing *CSP-CH* and *COLA*, the former uses a smaller amount of memory than the latter, and yet the index size of *CSP-CH* is considerable compared to that of non-indexed methods, i.e., the size of the input graph. Although *COLA* requires a larger index, its memory overhead is affordable, which is more than compensated by its high query performance as shown in Figure 3. The results also show that the size of the overlay graphs is negligible, indicating that the space consumption of *COLA* is mainly attributed to its constrained labeling index.

Figure 5 presents the total pre-processing time of *CSP-CH* and *COLA*. Note that the non-indexed methods are not shown as they do not need pre-processing. Compared to *CSP-CH*, *COLA* takes on average 3x to 4x processing time. Nevertheless, the cost of pre-processing in *COLA* is still modest, i.e., within 12 hours on the largest dataset USA, using a single server. Considering that existing methods require hours to process even one query, the pre-processing cost of *COLA* is worth paying for.

Summarizing the experiments, *COLA* effectively reduces the processing time of α-CSP queries from hours to sub-second, with moderate index size (no more than 5GB). Hence, it is clearly the method of choice for α-CSP processing. When an index is not available, we recommend the α-*Dijk* algorithm, which might be suitable for applications that do not require fast response. Since all previous methods are prohibitively expensive and α-CSP has promising use cases, the proposed methods might become key enablers for new online navigation services based on α-CSP.

6.4 Tuning α

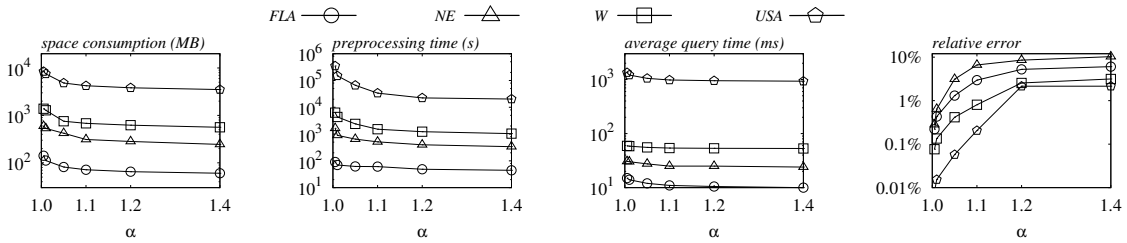
In this set of experiments, we evaluate the impact of α on *COLA*. In particular, we measure the query accuracy and space consumption of *COLA* by varying α from 1.005 to 1.4. Due to space limitations, we show the results for 4 representative datasets.

Figures 6(a)-(b) show the memory consumption, i.e., the index size, and preprocessing costs of our *COLA*, on FLA, NE, W, and USA datasets. As we can observe, when α changes from 1.005 to 1.4, both the space consumption and pre-processing time decrease, since a larger α tends to help the *COLA* labels prune more paths, resulting in a smaller index size.

Note that the impact of α is more pronounced on pre-processing time than on memory consumption. To explain, the index construction algorithm requires examining a large number of paths (even if the paths are added into label sets), and a larger α can help prune more paths, and hence can help save more preprocessing time. However, only a subset of the paths traversed are stored in the index, and hence the pruning effect is less significant in terms of space consumption than pre-processing time.

Besides, the query time of *COLA* is relatively insensitive to α. In particular, when α decreases from 1.4 to 1.005, the query time of *COLA* only increases by around 1.5x. To explain, the size of the *COLA* index shows a moderate increase when α decreases, and the label scanning of the query algorithm can exploit cache locality, making the query time relatively insensitive to α.

In terms of the query accuracy, we use *relative error* of 1000 random queries as the evaluation. Specifically, given a query q, let P^* be the solution of the exact CSP query and P' be an α-CSP, the relative error of the latter is computed as $\ell(P')/\ell(P^*) - 1$. Figure 6(d) shows the relative error of *COLA* on the same datasets. A higher value of α leads to a larger relative error. Nevertheless, the relative error is generally smaller than the worst case bound. For



(a) space consumption (b) preprocessing time (c) query performance (d) relative error

Figure 6: Tuning α for *COLA*.

example, the relative error of α-*Dijk* for $\alpha = 1.1$, is around 3% on FLA dataset, which is less than a third of the worst case bound, i.e., 10%. We set α to 1.1 for *COLA* since it strikes a good balance among query accuracy, query efficiency, space consumption and preprocessing time.

7. CONCLUSIONS

We present *COLA*, a novel and practical solution for approximate constrained shortest path processing. *COLA* utilizes important properties for real road networks, and applies effective indexing which leads to orders of magnitude reduction in query execution time. Meanwhile, *COLA* also includes an algorithm, α-*Dijk*, which significantly outperforms existing techniques for α-CSP processing without an index. As future work, we plan to investigate (i) how to avoid reconstruction of indices for different values of α and (ii) α-CSP processing in denser graphs compared to road networks.

8. ACKNOWLEDGMENTS

This research is supported by grants MOE2015-T2-2-069 from MOE, Singapore and NPRP9-466-1-103 from QNRF, Qatar.

9. REFERENCES

[1] https://sites.google.com/site/colatr2016/.
[2] 9th DIMACS Implementation Challenge. http://www.dis.uniroma1.it/challenge9/download.shtml.
[3] Google Maps. www.google.com/maps.
[4] T. Akiba, Y. Iwata, and Y. Yoshida. Fast exact shortest-path distance queries on large networks by pruned landmark labeling. In *SIGMOD*, pages 349–360, 2013.
[5] H. Bast, D. Delling, A. V. Goldberg, M. Müller-Hannemann, T. Pajor, P. Sanders, D. Wagner, and R. F. Werneck. Route planning in transportation networks. *CoRR*, abs/1504.05140, 2015.
[6] E. Cohen, E. Halperin, H. Kaplan, and U. Zwick. Reachability and distance queries via 2-hop labels. *SIAM J. Comput.*, 32(5):1338–1355, 2003.
[7] D. Delling, A. V. Goldberg, T. Pajor, and R. F. Werneck. Robust distance queries on massive networks. In *ESA*, pages 321–333, 2014.
[8] D. Delling, A. V. Goldberg, I. Razenshteyn, and R. F. F. Werneck. Graph partitioning with natural cuts. In *IPDPS*, pages 1135–1146, 2011.
[9] D. Delling and D. Wagner. Pareto paths with sharc. In *SEA*, pages 125–136, 2009.
[10] E. W. Dijkstra. A note on two problems in connexion with graphs. *Numerische mathematik*, 1(1):269–271, 1959.
[11] S. Funke, A. Nusser, and S. Storandt. On k-path covers and their applications. *PVLDB*, 7(10):893–902, 2014.
[12] S. Funke and S. Storandt. Polynomial-time construction of contraction hierarchies for multi-criteria objectives. In *SOCS*, pages 41–54, 2013.
[13] M. R. Garey and D. S. Johnson. *Computers and Intractability: A Guide to the Theory of NP-Completeness*. 1979.
[14] R. Geisberger, M. Kobitzsch, and P. Sanders. Route planning with flexible objective functions. In *ALENEX*, pages 124–137, 2010.
[15] R. Geisberger, P. Sanders, D. Schultes, and D. Delling. Contraction hierarchies: Faster and simpler hierarchical routing in road networks. In *WEA*, pages 319–333, 2008.
[16] A. V. Goldberg and C. Harrelson. Computing the shortest path: *A* search meets graph theory. In *SODA*, pages 156–165, 2005.
[17] G. Y. Handler and I. Zang. A dual algorithm for the constrained shortest path problem. *Networks*, 10(4):293–309, 1980.
[18] P. Hansen. Bicriterion path problems. In *Multiple criteria decision making theory and application*, pages 109–127, 1980.
[19] R. Hassin. Approximation schemes for the restricted shortest path problem. *Mathematics of Operations research*, 17(1):36–42, 1992.
[20] N. Jing, Y. Huang, and E. A. Rundensteiner. Hierarchical encoded path views for path query processing: An optimal model and its performance evaluation. *IEEE Trans. Knowl. Data Eng.*, 10(3):409–432, 1998.
[21] H. C. Joksch. The shortest route problem with constraints. *Journal of Mathematical analysis and applications*, 14(2):191–197, 1966.
[22] S. Jung and S. Pramanik. An efficient path computation model for hierarchically structured topographical road maps. *IEEE Trans. Knowl. Data Eng.*, 14(5):1029–1046, 2002.
[23] F. A. Kuipers, A. Orda, D. Raz, and P. V. Mieghem. A comparison of exact and *epsilon*-approximation algorithms for constrained routing. In *NETWORKING*, pages 197–208, 2006.
[24] D. H. Lorenz and D. Raz. A simple efficient approximation scheme for the restricted shortest path problem. *Oper. Res. Lett.*, 28(5):213–219, 2001.
[25] K. Mehlhorn and M. Ziegelmann. Resource constrained shortest paths. In *ESA*, pages 326–337, 2000.
[26] U. Meyer and P. Sanders. [delta]-stepping: a parallelizable shortest path algorithm. *J. Algorithms*.
[27] A. Sedeño-Noda and S. Alonso-Rodríguez. An enhanced K-SP algorithm with pruning strategies to solve the constrained shortest path problem. *Applied Mathematics and Computation*, 265:602–618, 2015.
[28] A. J. V. Skriver and K. A. Andersen. A label correcting approach for solving bicriterion shortest-path problems. *Computers & OR*, 27(6):507–524, 2000.
[29] O. J. Smith, N. Boland, and H. Waterer. Solving shortest path problems with a weight constraint and replenishment arcs. *Computers & OR*, 39(5):964–984, 2012.
[30] S. Storandt. Route planning for bicycles-exact constrained shortest paths made practical via contraction hierarchy. In *ICAPS*, volume 4, page 46, 2012.
[31] G. Tsaggouris and C. D. Zaroliagis. Multiobjective optimization: Improved FPTAS for shortest paths and non-linear objectives with applications. *Theory Comput. Syst.*, 45(1):162–186, 2009.
[32] S. Wang, W. Lin, Y. Yang, X. Xiao, and S. Zhou. Efficient route planning on public transportation networks: A labelling approach. In *SIGMOD*, pages 967–982, 2015.
[33] L. Wu, X. Xiao, D. Deng, G. Cong, A. D. Zhu, and S. Zhou. Shortest path and distance queries on road networks: An experimental evaluation. *PVLDB*, 5(5):406–417, 2012.
[34] X. Yu and H. Shi. Ci-rank: Ranking keyword search results based on collective importance. In *ICDE*, pages 78–89, 2012.
[35] A. D. Zhu, W. Lin, S. Wang, and X. Xiao. Reachability queries on large dynamic graphs: a total order approach. In *SIGMOD*, pages 1323–1334, 2014.