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A hybrid dynamic modeling of active chilled beam terminal unit

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Abstract: This paper proposes a hybrid dynamic model of active chilled beam (ACB) terminal unit. The model encapsulates mechanical and thermal aspects of the confined air jet and the cooling coil contained in the terminal unit and could be divided into two sub-models respectively. The models for the primary air, secondary air and mixing of them are together taken as the confined air jet sub-model. Another sub-model is the heat transfer description of the cooling coil. The model is kept simple and practical, avoiding sophisticated jet flow theories as well as heat transfer theories. Thus, in deriving the model using first principles and estimating it experimentally, a reasonable compromise is made between capturing exact underlying physics and suitability for engineering applications. Supported by experimental results from a pilot plant, unknown model parameters are identified by either a linear or nonlinear least-squares method. It is shown that static and dynamic performances of the model are satisfied, which reflect the effectiveness of this hybrid modeling technique as well. The model developed in this work is expected to have wide control and optimization applications.

Keywords: Active chilled beam, Hybrid dynamic model, Parameter identifications, Experiment verification

1. Introduction

Heating, ventilation and air conditioning (HVAC) systems provide comfortable and healthy indoor environment for occupants, but they account for a lot of energy consumption. Compared with conventional variable air volume (VAV) all-air HVAC systems, air-water ones consume substantially less energy. One of the most promising options in air-water HVAC systems is active chilled beam (ACB) which has been used for more than 20 years in Europe and is becoming more and more popular as an outstanding means of managing large sensible cooling loads all over the world [1, 2]. According to the energy saving figures of several energy retrofit projects in America [3], the total power demand of ACB is about 25%-30% less than that of VAV. In terms of enhancing operation efficiency further, many practitioners have paid great efforts on control and optimal operation of ACB during recent years [1, 2, 4, 5], while studies from the researchers and experts in HVAC field are still inadequate. As a consequence, in most of the existing applications, simple and model-free control and optimization strategies lead to much degeneration in energy performance. It is absolutely necessary for the researchers and experts to devote themselves into this field and develop advanced and model-based counterparts. Then, proposing a proper model of ACB terminal unit for such a purpose becomes the first task.

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Preparatory to the review of related studies, schematic diagram of ACB terminal unit is depicted in Fig.1. It comprises of a primary air plenum, dozens of specially designed nozzles, a mixing chamber and a cooling coil. A fixed pressure is maintained in the primary air plenum by a fan, so that a certain amount of cooled and dehumidified primary air is forced through the nozzles, through the mixing chamber and out into conditioned-zone. The nozzles are designed in such a way that a negative pressure kernel is generated in the region beyond the nozzle exit. It induces the flow of zone air known as secondary air through the cooling coil and into the mixing chamber. This induction of the secondary air by virtue of nozzle design is called entrainment effect, the better the design the stronger it is. Since the cooling coil is supplied with chilled water, the secondary air passing it is cooled. Finally, the cooled secondary air mixes with the primary air in the mixing chamber and the resultant supply air exits the unit into the zone.

For the modeling of such kind of terminal unit, there is no dedicated study yet. However, there have been some research works on the entrainment effect and cooling coil which may be able to be incorporated into the modeling. In fact, the turbulent merging confined jet phenomenon which happens in ACB is very complicated. In most cases, it is explored with the assistance of computational fluid dynamics software and massive experiments. Morton et al. [6] presented the entrainment effect as an incorporation of fluid from the surrounding into the jet by turbulent eddies generated by the shear existing between the two regions. Because the turbulent energy dissipation can't be measured, some empirical hypotheses are employed to close the equation system in jet models. Based on entrainment hypothesis and spreading hypothesis, Enjalbert et al. [7] and Wang [8] individually derived the entrainment effect models for single turbulent jet from conservations of momentum and mass. Given Reichardt's hypothesis, the entrainment effect modeling becomes more powerful and flexible. Hodgson et al. [9] and Wang et al. [10] demonstrated some models of complex processes involving complicated boundary conditions and multiple jets by the method of superposition of particular solutions. Unfortunately, all these models require the details of system configurations, such as nozzle dimensions, arrangements and so on. More importantly, the independent variables of these models, spreading constant and virtual point, should be determined empirically with a lot of experiments as well. As a result, they are generally useful in active control of jets. Targeted at control and optimization applications rather than at developing innovative terminal units, such kind of confined air jet models are unsuitable here. Then, this paper captures the entrainment effect via an empirical model.

Considering the modeling of the cooling coil, a wide range of models is currently available. Lebrun [11] and Brandemuehl [12] developed two theoretical cooling coil models for ASHRAE

HVAC Toolkits. The models require dimensions of the fin and the tube thickness, diameter, and spacing as inputs in order to calculate the heat transfer coefficients but present insufficient robustness. Stoecher [13] proposed an empirical model, which can predict the performance of a coil regardless of the type of fluid in the tubes with a given set of constants. Its defect is that for different fluids different sets of constants are needed. Either theoretical or empirical modeling approach has some inherent disadvantages. Conversely, a hybrid modeling approach takes advantages of both theoretical and empirical modeling approaches. It can offer acceptable accuracy and robustness with simple models. The model structures are derived from first principles, while the unknown parameters are identified by catalog or experiment data. A variety of models are achieved in this way. Braum et al. [14] established an effective model for cooling coils via introducing the concept of air saturation specific heat. Rabehl et al. [15] relaxed some of the assumptions and complications in Braum's model and capture the effect of geometry on performance. Furthermore, the model is simplified to several unknown lumped parameters without any geometry descriptions in references [16-20].

It should be noted that most of the existing cooling coil models, including the referenced ones [11-20], are usually static models. They are not sufficient in many cases. For example, compared to controlling the fluid flow rate, controlling the exit temperature of the cold fluid with dynamic models is better to maximize the coil capacity. In other word, a dynamic model is more convenient to make full use of the coil cooling capacity in ACB application. Meanwhile, the condensation can be strictly avoided if the transient behavior can be predicted. The study on dynamic description of the fin-tube heat exchanger was first attempted by Shah [21]. General formulations of different problems, novel approaches and various techniques and so on were addressed step by step [22-24]. However, these comprehensive dynamic models are developed for computer simulation, which are too complex to be applied. Additionally, Wang et al. [25] presented a nonlinear dynamic model and designed a nonlinear controller, but the model performance was compromised because of ignoring several important heat transfer properties, such as variations of the heat transfer coefficients, heat storage of the coil and so on. Jin et al. [26] showed another dynamic model and the model is able to offer acceptable accuracy and robustness with five or six unknown parameters. In references [27-29], similar dynamic models were also developed for automotive waste heat recovery systems, borehole and ground applications. Constrained by the ACB terminal unit, the secondary air outlet temperatures of and the coil temperature are very difficult to be measured. As a consequence, these available modeling

techniques [25-29] become unfeasible here. This paper, however, tries to maintain the important characteristics and achieve a new dynamic model with less information.

In this paper, a hybrid dynamic model is presented, which is the first reported ACB terminal unit model. It is divided with two sub-models. The models for the primary air, induced secondary air and thermal and mechanical mixing of them are lumped together as one sub-model. Another sub-model is a thermal model of the cooling coil. Due to the necessary simplification of the entrainment effect, it is described by an empirical function. In contrast, tremendous effort has been invested on the detailed modeling of the cooling coil based on incomplete information. Using heat transfer mechanism and energy balance principle, a dynamic cooling coil model with no more than five parameters that represent the lumped geometric terms is developed. The unknown model parameters are identified by either a linear or nonlinear least-squares method with the experimental results from a pilot plant. Meanwhile, the experiment data and simulation results are compared to validate the model and illustrate its effectiveness.

2. Model development

Without loss of generality, some assumptions are adopted for ease of mathematical modeling:

1. The mixing of the primary and secondary air is considered instantaneous and homogenous.
2. The tubes are adiabatic in the part of return bends and branch joints.
3. The cooling coil is always under dry condition.
4. The heat storage associated with the moisture in the secondary air is ignored.
5. The flow and thermal properties of the fluids are supposed to be independent of temperature.
6. The temperature field of the cooling coil is assumed to be evenly distributed.

Assumption 1 is quite general for air mixing and assumptions 2-5 are good approximations to the practical situation. For example, the tube bends and joints are well thermal insulated in the experiments; the dry condition is strictly kept during ACB operation; the maximum moisture content of the secondary air is about 3% by mass so that its heat storage is small; the flow and thermal properties variances of the fluids are also very small because the fluids' temperature variances in ACB application are limited. The last assumption is adopted simply because the cooling coil is considered as a whole in the present study.

2.1 Confined air jet

Generally, air ducts or diffusers in HVAC systems are modeled by their flow resistances. For the flow of a particular fluid through a flow passage at a particular volume flow rate, the energy expended to sustain the flow appears as the drop in pressure along the flow passage, the higher the flow resistance of the passage, the higher the pressure drop for the same volume flow rate. Here, the resistance of the primary air offered by its total path is denoted as R . This flow resistance depends on many factors, including outlet area of the nozzles and outlet shape, nozzle length, nozzle arrangement, air properties and so on, while it is treated as a constant during the normal operating conditions of the terminal unit. Then the primary air volume flow rate $\dot{V}_{pri}$ can be calculated with the following equation.

$$\dot{V}_{pri} = \sqrt{\frac{P}{R}} \quad (1)$$

where P is the gauge pressure in the primary air plenum.

The entrainment effect is measured by entrainment ratio (ER), which is the most important assessment criterion of ACB terminal unit performance. ER is defined as the ratio of the secondary air volume flow rate $\dot{V}_{sec}$ to the primary air volume flow rate.

$$ER = \frac{\dot{V}_{sec}}{\dot{V}_{pri}} \quad (2)$$

As mentioned in the introduction, an empirical relationship is adopted to reflect the entrainment effect avoiding sophisticated jet flow theories. Although the entrainment ratio is defined by the air volume flow rates, the primary air plenum pressure is actually the only manipulated variable in this confined air jet sub-model. Therefore, the entrainment effect is described by:

$$ER = aP^b \quad (3)$$

where a and b are unknown constant coefficients.

Since the instantaneous and homogenous mixing of the primary and secondary air forms the supply air to the zone with assumption 2, the volume flow rate $\dot{V}_{supply}$ and temperature T_{supply} are calculated as:

$$\dot{V}_{supply} = (1 + ER)\dot{V}_{pri} \quad (4)$$

$$T_{supply} = \frac{T_{pri} + ER * T_{sec}}{(1 + ER)} \quad (5)$$

Where T_{pri} and T_{sec} are the primary air temperature and the outlet temperature of the secondary air.

It is known that the cooling capacity supplied to the zone is evaluated by the supply air volume flow rate and temperature. Based on Eqs. (4-5), it can be seen that the gauge pressure P is the only variable determines the supply air volume flow rate $\dot{V}_{supply}$ except from the primary air temperature T_{pri} and the supply air temperature T_{supply} depends on the outlet temperature of the secondary air T_{sec} as well. That is described by the following cooling coil sub-model.

2.2 Cooling coil

Different from the widely used finite element analysis method, the cooling coil is treated as the only control volume here. Otherwise, the model has to be described by partial differential equations. Without spatial variation considerations, the model can be represented in the form of differential equations, which is better for the subsequent control and optimization applications. Fig. 2 shows schematic diagram of a simplified cooling coil. The secondary air is forced by the entrainment effect with inlet temperature T_{zone} and volume flow rate $\dot{V}_{sec}$ and its outlet temperature decreases to T_{sec} . Similarly, the chilled water flows is forced by the pump with inlet temperature $T_{w.in}$ and volume flow rate $\dot{V}_w$ and the outlet temperature increases to $T_{w.out}$.

Both the secondary air and chilled water are driven mechanically by external forces, so the heat transfer mechanisms are forced convections. By dimensional analysis, forced convection heat transfer coefficient for a single phase flow can be calculated by Reynolds number Re_D and Prandtl number Pr as:

$$h = \frac{k}{D} C Re_D^e Pr^f = \frac{k}{D} C \left(\frac{\rho u D}{\mu} \right)^e \left(\frac{C_p \mu}{k} \right)^f \quad (6)$$

where k is the fluid thermal conductivity, D is the fluid characteristic length, C , e and f are unknown constant coefficients, ρ is the fluid density, u is the fluid velocity, μ is the fluid absolute viscosity, C_p is the fluid specific heat at a constant pressure. In reality, these flow and

thermal properties depend on the fluid temperature, while they are always evaluated at the fluid bulk temperature thus avoiding iteration. Moreover, assumption (6) ensures that they are constant in the proposed model. In the present study, the fluid is measured by its volume flow rate rather than velocity. Then, Eq. (6) is rewritten.

$$h = c \dot{V}^e \quad (7)$$

where $c = \frac{k}{D} C \left(\frac{\rho D}{\mu} \right)^e \left(\frac{C_p \mu}{k} \right)^f \left(\frac{1}{A} \right)^e$, $\dot{V}$ is the fluid volume flow rate and A is the effective fluid section area.

In the secondary air loop, the rate of convective heat transferred from the secondary air to the cooling coil Q_{ac} can be expressed in form of Newton's law of cooling.

$$Q_{ac} = h_{ac} A_a \eta (T_a - T_c) \quad (8)$$

where h_{ac} , A_a , η , T_a and T_c are the corresponding convective heat transfer coefficient, effective heat transfer area, finned surface efficiency, bulk temperature of the secondary air and cooling coil.

Refer to Eq. (7), the convective heat transfer coefficient h_{ac} is obtained.

$$h_{ac} = c_a \dot{V}_{sec}^{e_a} \quad (9)$$

where c_a and e_a are the counterparts of constants c and e in Eq. (7) for the secondary air loop.

Rather than inlet temperature difference or log mean temperature difference, the bulk temperature difference is adopted in Eq. (8). The bulk temperature of the secondary air is its average temperature.

$$T_a = \frac{T_{zone} + T_{sec}}{2} \quad (10)$$

The resultant heat increment rate Q_a of the secondary air is calculated by:

$$Q_a = C_a \rho_a \dot{V}_{sec} (T_{sec} - T_{zone}) \quad (11)$$

where C_a and ρ_a are the specific heat and density of dry air.

In the same manner, the following formulas can be used to denote the chilled water loop.

$$Q_{cw} = h_{cw} A_w (T_c - T_w) \quad (12)$$

$$h_{cw} = c_w \dot{V}_w^{e_w} \quad (13)$$

$$T_w = \frac{T_{w.in} + T_{w.out}}{2} \quad (14)$$

$$Q_w = C_w \rho_w \dot{V}_w (T_{w.out} - T_{w.in}) \quad (15)$$

where h_{cw} , A_w , and T_w are the convective heat transfer coefficient, effective heat transfer area and bulk temperature of the chilled water, c_w and e_w are the counterparts of constants c and e in Eq. (7) for the chilled water loop, Q_w is the resultant heat increment of the chilled water, C_w and ρ_w are the specific heat and density of water.

Actually, the operation of such a cooling coil can be considered as three interacting dynamic processes – describing the energy storage of the secondary air, coil, and chilled water respectively. However, for the sake of developing a model that is simple and practicable, two of the processes are neglected and only the energy storage of the coil temperature is kept as an intermediate. Hence, the simplified model will be of the form.

$$M_c C_c \frac{dT_c}{dt} = Q_{ac} - Q_{cw} \quad (16)$$

$$0 = Q_{ac} + Q_a \quad (17)$$

$$0 = Q_{cw} + Q_w \quad (18)$$

Refer to [30], the coil temperature dynamic is the only source of the dynamics of the secondary air outlet temperature and the chilled water outlet temperature. The dynamic relationship between these variables can be obtained by taking derivative to both sides of Eq. (17) and Eq. (18),

$$\frac{dT_{sec}}{dt} = \frac{h_{ac} A_a \eta}{C_a \rho_a \dot{V}_{sec} + \frac{1}{2} h_{ac} A_a \eta} \frac{dT_c}{dt} \quad (19)$$

$$\frac{dT_{w.out}}{dt} = \frac{h_{cw} A_w}{C_w \rho_w \dot{V}_w + \frac{1}{2} h_{cw} A_w} \frac{dT_c}{dt} \quad (20)$$

The model in Eqs. (16-18) then can be reduced to two dynamic processes on the secondary air outlet temperature and chilled water outlet temperature. Substitute Eqs. (8-11) and Eq. (19) into Eq. (16) and rearrange the equation properly,

$$\begin{aligned} M_c C_c \left(\frac{C_a \rho_a \dot{V}_{sec}}{c_a \dot{V}_{sec}^e A_a \eta} + \frac{1}{2} \right) \frac{dT_{sec}}{dt} = & - \left[C_a \rho_a \dot{V}_{sec} + \frac{C_w \rho_w \dot{V}_w c_w \dot{V}_w^e A_w}{C_w \rho_w \dot{V}_w + \frac{c_w \dot{V}_w^e A_w}{2}} \left(\frac{1}{2} + \frac{C_a \rho_a \dot{V}_{sec}}{c_a \dot{V}_{sec}^e A_a \eta} \right) \right] T_{sec} \\ & + \left[C_a \rho_a \dot{V}_{sec} - \frac{C_w \rho_w \dot{V}_w c_w \dot{V}_w^e A_w}{C_w \rho_w \dot{V}_w + \frac{c_w \dot{V}_w^e A_w}{2}} \left(\frac{1}{2} - \frac{C_a \rho_a \dot{V}_{sec}}{c_a \dot{V}_{sec}^e A_a \eta} \right) \right] T_{zone} + \frac{C_w \rho_w \dot{V}_w c_w \dot{V}_w^e A_w}{C_w \rho_w \dot{V}_w + \frac{c_w \dot{V}_w^e A_w}{2}} T_{w.in} \end{aligned} \quad (21)$$

Similarly, corresponding equation for the chilled water loop can be obtained.

$$\begin{aligned} M_c C_c \left(\frac{C_w \rho_w \dot{V}_w}{c_w \dot{V}_w^e A_w} + \frac{1}{2} \right) \frac{dT_{w.out}}{dt} = & - \left[C_w \rho_w \dot{V}_w + \frac{C_a \rho_a \dot{V}_{sec} c_a \dot{V}_{sec}^e A_a \eta}{C_a \rho_a \dot{V}_{sec} + \frac{c_a \dot{V}_{sec}^e A_a \eta}{2}} \left(\frac{1}{2} + \frac{C_w \rho_w \dot{V}_w}{c_w \dot{V}_w^e A_w} \right) \right] T_{w.out} \\ & + \left[C_w \rho_w \dot{V}_w - \frac{C_a \rho_a \dot{V}_{sec} c_a \dot{V}_{sec}^e A_a \eta}{C_a \rho_a \dot{V}_{sec} + \frac{c_a \dot{V}_{sec}^e A_a \eta}{2}} \left(\frac{1}{2} - \frac{C_w \rho_w \dot{V}_w}{c_w \dot{V}_w^e A_w} \right) \right] T_{w.in} + \frac{C_a \rho_a \dot{V}_{sec} c_a \dot{V}_{sec}^e A_a \eta}{C_a \rho_a \dot{V}_{sec} + \frac{c_a \dot{V}_{sec}^e A_a \eta}{2}} T_{zone} \end{aligned} \quad (22)$$

Reviewing above cooling coil sub-model, it can be observed that its air side operation depends on the confined air jet in terms of the secondary air volume flow rate $\dot{V}_{sec}$, while the chilled water inlet temperature $T_{w.in}$ and volume flow rate $\dot{V}_w$ on the water side could be independent variables to regulate the secondary air outlet temperature T_{sec} and finally adjust the output cooling capacity.

3. Model identification

Least-squares method (LSM) is widely used to estimate unknown parameters of linear models, which can be represented by:

$$\Psi X = \Gamma + \Delta \quad (23)$$

where Ψ , Γ are measurements, X is unknown model parameters to be estimated and Δ is measurement error. The best estimated X^* of X can be found using standard LSM as

$$X^* = (\Psi^T \Psi)^{-1} \Psi^T \Gamma \quad (24)$$

For the confined air jet sub-model, the primary air resistance and the description function of the entrainment effect are estimated with LSM. Experiments are conducted N times for different primary air plenum pressures. Eq. (1) is rearranged to be linear form.

$$\dot{V}_{pri}^2 * R = P \quad (25)$$

and Eq. (25) is considered in the form of Eq. (23).

$$X = R, \Psi = \begin{bmatrix} \dot{V}_{pri_1}^2 \\ \vdots \\ \dot{V}_{pri_N}^2 \end{bmatrix}, \Gamma = \begin{bmatrix} P_1 \\ \vdots \\ P_N \end{bmatrix}$$

Similarly, Eq. (3) is rewritten to be:

$$\ln(a) + b \ln(P) = \ln(ER) \quad (26)$$

and then,

$$X = \begin{bmatrix} \ln(a) & b \end{bmatrix}, \Psi = \begin{bmatrix} 1 & \ln(P_1) \\ \vdots & \vdots \\ 1 & \ln(P_N) \end{bmatrix}, \Gamma = \begin{bmatrix} \ln(ER_1) \\ \vdots \\ \ln(ER_N) \end{bmatrix}$$

The estimation procedure of the cooling coil sub-model is separated into two steps: steady state estimation and transient state estimation. In steady state, the heat transfer processes are balance. Based on Eqs. (8-13) and (15), the following equation can be established.

$$\frac{1}{c_a A_a \eta \dot{V}_{sec}^{e_a}} + \frac{1}{c_w A_w \dot{V}_w^{e_w}} = \frac{T_{zone} - \frac{T_{w.in} + T_{w.out}}{2}}{C_w \rho_w \dot{V}_w (T_{w.out} - T_{w.in})} - \frac{1}{2C_a \rho_a \dot{V}_{sec}} \quad (27)$$

In normal engineering applications, the rule of thumb value $e_a = e_w = 0.5$ is often adopted. Then, Eq. (27) has only two aggregated unknown parameters. Repeat the experiment N times and rewrite them in the form of Eq. (23).

$$X = \begin{bmatrix} \frac{1}{c_a A_a \eta} \\ \frac{1}{c_w A_w} \end{bmatrix}, \Psi = \begin{bmatrix} \left(\frac{1}{\dot{V}_{sec}^{e_a}} \right)_1 & \left(\frac{1}{\dot{V}_w^{e_w}} \right)_1 \\ \vdots & \vdots \\ \left(\frac{1}{\dot{V}_{sec}^{e_a}} \right)_N & \left(\frac{1}{\dot{V}_w^{e_w}} \right)_N \end{bmatrix}, \Gamma = \begin{bmatrix} \left(\frac{T_{zone} - \frac{T_{w.in} + T_{w.out}}{2}}{C_w \rho_w \dot{V}_w (T_{w.out} - T_{w.in})} - \frac{1}{2C_a \rho_a \dot{V}_{sec}} \right)_1 \\ \vdots \\ \left(\frac{T_{zone} - \frac{T_{w.in} + T_{w.out}}{2}}{C_w \rho_w \dot{V}_w (T_{w.out} - T_{w.in})} - \frac{1}{2C_a \rho_a \dot{V}_{sec}} \right)_N \end{bmatrix}$$

If the parameters e_a and e_w are regarded as unknown parameters, Eq. (27) has total four unknown parameters. Since this equation becomes nonlinear, nonlinear least-squares method (NLSM), widely known as Levenberg-Marquardt method, is used by defining an objective function as

$$f(x) = \sum_{i=1}^N r_i^2(x) = \sum_{i=1}^N \left(\frac{T_{zone} - \frac{T_{w.in} + T_{w.out}}{2}}{C_w \rho_w \dot{V}_w (T_{w.out} - T_{w.in})} - \frac{1}{2C_a \rho_a \dot{V}_{sec}} - \frac{1}{c_a A_a \eta \dot{V}_{sec}^{e_a}} - \frac{1}{c_w A_w \dot{V}_w^{e_w}} \right)^2 \quad (28)$$

where $r(x)$ are residuals $f(x)$ is sum of squares of the residuals, $x = [c_a A_a \eta, c_w A_w, e_a, e_w]$ is unknown parameter vector, N is number of fitting points.

After the steady state estimation, the model time constant is the major unknown parameter to be estimated. Consequently, Eq. (22) is rewritten as

$$\theta \frac{dT_{w.out}(t)}{dt} = \varnothing(t) \quad (29)$$

where

$$\theta = M_c C_c \left(\frac{C_w \rho_w \dot{V}_w(t)}{c_w \dot{V}_w^{e_w}(t) A_w} + \frac{1}{2} \right)$$

$$\begin{aligned} \varnothing(t) = & - \left[C_w \rho_w \dot{V}_w(t) + \frac{C_a \rho_a \dot{V}_{\text{sec}}(t) c_a \dot{V}_{\text{sec}}^{e_a}(t) A_a \eta}{C_a \rho_a \dot{V}_{\text{sec}}(t) + \frac{c_a \dot{V}_{\text{sec}}^{e_a}(t) A_a \eta}{2}} \left(\frac{1}{2} + \frac{C_w \rho_w \dot{V}_w(t)}{c_w \dot{V}_w^{e_w}(t) A_w} \right) \right] T_{w.out}(t) \\ & + \left[C_w \rho_w \dot{V}_w(t) - \frac{C_a \rho_a \dot{V}_{\text{sec}}(t) c_a \dot{V}_{\text{sec}}^{e_a}(t) A_a \eta}{C_a \rho_a \dot{V}_{\text{sec}}(t) + \frac{c_a \dot{V}_{\text{sec}}^{e_a}(t) A_a \eta}{2}} \left(\frac{1}{2} - \frac{C_w \rho_w \dot{V}_w(t)}{c_w \dot{V}_w^{e_w}(t) A_w} \right) \right] T_{w.in}(t) + \frac{C_a \rho_a \dot{V}_{\text{sec}}(t) c_a \dot{V}_{\text{sec}}^{e_a}(t) A_a \eta}{C_a \rho_a \dot{V}_{\text{sec}}(t) + \frac{c_a \dot{V}_{\text{sec}}^{e_a}(t) A_a \eta}{2}} T_{zone} \end{aligned}$$

Starting from this initial steady state at $t = 0$, step inputs of the secondary air volume flow rate, chilled water volume flow rate or the inlet their temperatures are given for testing. Assuming that the intercept of the tangent to the step response that has the largest slope with respect to the horizontal axis gives H and integrating the differential terms in Eq. (29) from $t = 0$ to $t = \tau$ ($\tau \leq H$) results in the following equation

$$\theta \int_0^\tau \frac{dT_{w.out}(t)}{dt} dt = \int_0^\tau \varnothing(t) dt \quad (30)$$

With the sampling interval of T_s , and $\tau = nT_s$. When the perturbation of the input variable is very small, the following linear form equation is achieved.

$$\theta [T_{w.out}(nT_s) - T_{w.out}((n-1)T_s)] = \int_{(n-1)T_s}^{nT_s} \varnothing(t) dt \approx \frac{T_s}{2} [\varnothing(nT_s) - \varnothing((n-1)T_s)] \quad (31)$$

and

$$X = [\theta], \Psi = \begin{bmatrix} T_{w.out}(T_s) - T_{w.out}(0) \\ \vdots \\ T_{w.out}(nT_s) - T_{w.out}((n-1)T_s) \end{bmatrix}, \Gamma = \begin{bmatrix} \frac{T_s}{2} [\varnothing(T_s) - \varnothing(0)] \\ \vdots \\ \frac{T_s}{2} [\varnothing(nT_s) - \varnothing((n-1)T_s)] \end{bmatrix}$$

4. Experimental setup

The pilot plant of ACB performance test is shown in Fig.3. The plant consists of two physical loops. The air loop is designed to blow primary air into the ACB primary plenum and force it through the nozzles, while the water loop is used to circulate chilled water between the cooling coil and a dedicated chiller system. It mainly contains following components:

- an ACB terminal unit;
- two centrifugal fans;
- a water circulating pump;
- a self-contained chiller system;
- a monitoring and control system.

The test is conducted on a specially designed two-way discharge ACB terminal unit. Its face dimensions are of 0.6m×1.2m and 29 circular rubber nozzles are distributed equally on each side. The plain fin and tube cooling coil has face dimensions of 0.3m×1.1m and the aluminum fin thickness and pitch are 0.5mm and 4.35mm respectively. The coil is roughly comprised of 0.69kg copper pipe, 1.92kg aluminum fin, 0.71kg aluminum casing and 0.10 kg steel inlet and outlet pipes. To keep a certain gauge pressure in the primary air plenum, two centrifugal fans are installed in series to overcome the unexpected pressure drop across the vortex precession flowmeters. A water circulating pump is placed in the cycle to circulate chilled water between ACB terminal unit and the chiller system. A self-contained chiller system provides chilled water by a vapor compressor cycle and stores it in a water bath. Meanwhile, the water bath is heated by a solid-state circuit breaker controlled heater with power output of 4kW. The temperature is controlled by a SHIMADEN PID controller. The function of the monitoring and control system is to measure the primary air plume pressure, the primary and secondary air and chilled water volume flow rates, the environment temperature and the inlet and outlet chilled water temperatures. The transmitters include:

- A Dwyer series MS Magnesense® differential pressure transmitter for the primary air plume pressure with error of $\pm 1\%$.
- Two intelligent vortex precession flowmeters for the different measurement spans of the primary air volume flow rates with error of $\pm 1.5\%$.
- A TSI model 8710 DP-CALCTM micromanometer for the secondary air volume flow rate with error of $\pm 3\%$.
- A float flowmeter for the chilled water volume flow rate with error of $\pm 1.6\%$.
- Two PT1000 platinum resistance temperature sensors with error of $\pm 0.3^{\circ}\text{C}$.

All the motors, including the fans and pump, are equipped with VSDs and all the water pipes, fittings and temperature transmitters are thermally insulated properly to minimize the heat loss and measurement inaccuracy.

5. Experiments

As the main concern of the present study is to model the ACB terminal unit, it is desired to be accurate in various working conditions. Except from the primary air temperature, all the other independent variables, including the gauge pressure in the primary air plenum, chilled water inlet temperature and volume flow rate are tried to be varied during the experiments. Although the environment temperature can't be controlled without a dedicated thermal insulated lab, the experiments are conducted in different weather conditions so that the environment temperature is adjusted in a passive way. The most of typical working conditions are covered. Experimental parameters setting are summarized in Tab. 1.

All the experimental data are recorded under steady-state besides the model dynamic property considerations. The attention is focused on the stabilization of the secondary air volume flow rate and the thermal equilibrium of the cooling coil. The micromanometer reading for the secondary air is valid only if its variation is within $\pm 10\text{L/h}$ for 5 minutes. Since the outlet secondary air temperature cannot be available, the thermal equilibrium of the cooling coil is only determined via the chilled water loop. Once the outlet chilled water temperature keeps within $\pm 0.1^\circ\text{C}$ limits for 20 minutes, the steady state is confirmed.

As same as the model development, the proposed model is identified and validated in terms of two sub-models. The cooling coil is the only dynamic element in the whole model and therefore it is more reasonable as well as convenient to evaluate it as a whole.

5.1 Static performance

For the confined air jet sub-model, totally 40 data sets are collected consisting of the primary air plenum gauge pressure, primary air volume flow rate and secondary air volume flow rate. Randomly select 28 data sets for model fitting while the rest 12 ones are used for model validation.

The primary air resistance and the description function of the entrainment effect are estimated.

$$\dot{V}_{pri}^2 * 0.003997 = P$$

$$ER = 1.563 * P^{0.107}$$

The model fitting and validation results are given in Figs. (4-7). Given the confidence level of 95%, it can be observed that the maximum model errors are about 5%. The inconsistency of the experiment data shown by the red circles in Fig. (4) and Fig. (6) is due to the different flowmeters.

For the cooling coil sub-model, totally 41 data sets are obtained, including the environment temperature, pressure air plenum gauge pressure, chilled water inlet temperature and chilled water volume flow rate. Since the cooling coil model is relatively complex, more data are adopted for model validation. About 50% of the data, 21 sets, are used for model fitting and the rest 20 ones for model validation.

With the LSM and NLSM produces defined in model identification section, the unknown parameters of the cooling coil model are summarized in Tab. 2. The model fitness is included as well, which is used to compare the two-parameters model and the four-parameters model. The fitness of a model is evaluated through the root mean square error (RMS), which is defined by:

$$RMS = \sqrt{\frac{\sum_{i=1}^N (Q_c - Q_m)^2}{N}}$$

Where N is the number of fitted points, Q_c is the heat transfer of point i calculated by the model, Q_m is the heat transfer of point i measured from experiments.

The model fitting and validation results of the cooling coil sub-models are successively illustrated in Figs. (8-11). The maximum model errors are also about 5% with 95% confidence levels.

5.2 Dynamic performance

According to the estimation procedure defined above, the time constant is directly identified from the experimental step responses without any normalization processes. The effects of steady state errors would be introduced into this process. In order to minimize them, particular attention is paid on the data selection. The steady states of the experimental setup and the model are tried to be as close as possible before and after the step input. 2 sets of step responses for the primary air chamber gauge pressure change, from 106Pa to 40 Pa and from 20 Pa to 50 Pa, are selected. This selection is also due to the fact that ACB is normally configured in the form of modulating air loop and on-off water loop. The primary air volume flow rate, pressure air plenum gauge pressure,

environment temperature, chilled water inlet temperature as well as chilled water volume flow rate are recorded. Using the time constant estimation procedure, the unknown parameter $M_c C_c$ is identified to be 5946 J/°C. The model fitting results are summarized in Figs. (12-13), where red lines mean model simulation outputs and blue lines represent experiment outputs.

The model validation is conducted under the real-time working conditions. The time varying chilled water inlet temperature and primary air plenum gauge pressure are given in Fig. 14. The environment temperatures before and after the experiment are 24.8 °C and 25.1 °C, while it is assumed to be a constant 25 °C in the model simulation. The practical system outputs in terms of the chilled water outlet temperature and heat transfer rate are compared to the model prediction results. For comparison, the number of the unknown parameters is considered and the time constant is also estimated by the compositions and the specific heat of them, 2674 J/°C. The results are given in Figs. (15-18).

From the comparison of blue circles in Figs (15-18), the dynamic performance of the model with estimated $M_c C_c$ value should be better. That means something else ignored in the $M_c C_c$ value calculation can also store heat, such the coil structure, unit casing and so on. Meanwhile, it should be pointed out that the smaller calculated $M_c C_c$ value seems to get a slower dynamic response from the figures, which is quite strange. The reason may be that the final dynamic responses contain the effect of time-varying chilled water inlet temperature and the faster dynamic response with a smaller $M_c C_c$ value may be cancelled with it.

6. Conclusion

A dynamic model of ACB terminal unit is presented in this paper. It's integrated by two sub-models for the confined air jet and cooling coil respectively. Blending the first principles and experimental results in a hybrid manner, an accurate but robust model is established based on limited information. From the experiment results, we can summarize the following conclusions:

1. The confined air jet sub-model is very simple, avoiding the sophisticated jet flow theories, but practical and accurate in a wide operation range, confined within errors of $\pm 5\%$.
2. For the cooling coil, the steady state performances of both two-parameters model and four-parameters model are both satisfied, within $\pm 5\%$ error ranges.

3. $M_c C_c$ value estimated by LMS can precisely reflect the transient performance of the model. It provides better performance than the value estimated by the compositions of the cooling coil.

4. Totally, the two-parameters model for the cooling coil seems to be better than the four-parameters one. Although its steady state accuracy is a little worse than that of four-parameter model, it is much better in the dynamic performance test. In the future, more experiments will be used to prove this conclusion.

The significance of the proposed model is that it has broad potential to apply to real-time control and optimization applications. For example, to avoid condensation, a dynamic tracking control scheme to control the off coil temperature of the secondary air above the dew point temperature can be well done. As the first reported model of ACB terminal unit, there is a lot more work to do, such as to conduct more comprehensive identifications and validations, to simplify further the complexity of the model and so on.

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References

- [1] Alexander D, O'Rourke M. Design considerations for active chilled beams. ASHRAE J 2008;50:50-4+6+8.
- [2] Loudermilk K. Designing Chilled Beams for Thermal Comfort. ASHRAE J 2009;51:58-60.
- [3] Sachs HM, Lin W, Lowenberger AK. Emerging Energy Saving-HVAC Technologies and Practices for the Buildings Sector (2009). American Council for an Energy-Efficient Economy; 2009.
- [4] Livchak A, Lowell C. Don't Turn Active Beams Into Expensive Diffusers. ASHRAE J 2012;54:52-60.
- [5] Loudermilk KJ, Alexander DS. Efficient space humidity control with active chilled beam systems. ASHRAE J 2012;54:28-38.
- [6] Morton B, Taylor G, Turner J. Turbulent gravitational convection from maintained and instantaneous sources. In: Proceedings of the Royal Society of London Series A Mathematical and Physical Sciences. 1956;234:1-23.
- [7] Enjalbert N, Galley D, Pierrot L. An entrainment model for the turbulent jet in a coflow. C R Mec 2009;337:639-44.
- [8] Wang H. Jet interaction in a still or co-flowing environment. Hong Kong: Hong Kong Univ. of Science and Technology; 2000.
- [9] Hodgson JE, Moawad AK, Rajaratnam N. Concentration field of multiple circular turbulent jets. J Hydraul Res 1999;37:249-56.
- [10] Wang H, Davidson M. Jet interaction in a still ambient fluid. J Hydraul Eng 2003;129:349-57.

- [11] LeBrun J, Bourdouxhe J-P, Grodent M. HVAC1 toolkit: A toolkit for primary HVAC System energy calculation. Atlanta: ASHRAE Inc. 1999.
- [12] Bourdouxhe J-PH, Saavedra C, Grodent M, Silva KL, Lebrun JJ. Toolkit for primary HVAC system energy calculation- part 2: reciprocating chiller models. ASHRAE Trans 1994;100:774-86.
- [13] Stoecker WF. Procedures for simulating the performance of components and systems for energy calculations. New York: ASHRAE Inc; 1975.
- [14] Braun J, Klein S, Mitchell J. Effectiveness models for cooling towers and cooling coils. ASHRAE Trans 1989;95.
- [15] Rabehl RJ, Mitchell JW, Beckman WA. Parameter estimation and the use of catalog data in modeling heat exchangers and coils. HVAC&R Res 1999;5:3-17.
- [16] Ding X, Cai W, Jia L, Wen C. Evaporator modeling - A hybrid approach. Appl Energy 2009;86:81-8.
- [17] Wang X, Cai W, Lu J, Sun Y, Ding X. A hybrid dehumidifier model for real-time performance monitoring, control and optimization in liquid desiccant dehumidification system. Appl Energy 2013;111:449-55.
- [18] Ding X, Cai W, Jia L, Wen C, Zhang G. A hybrid condenser model for real-time applications in performance monitoring, control and optimization. Energy Convers Manage 2009;50:1513-21.
- [19] Jin GY, Cai WJ, Lu L, Lee EL, Chiang A. A simplified modeling of mechanical cooling tower for control and optimization of HVAC systems. Energy Convers Manage 2007;48:355-65.
- [20] Wang YW, Cai WJ, Soh YC, Li SJ, Lu L, Xie L. A simplified modeling of cooling coils for control and optimization of HVAC systems. Energy Convers Manage 2004;45:2915-30.
- [21] Shah R. Transient response of heat exchangers. Conf NATO Adv Study Inst Heat Exchangers: Thermal-Hydraulic Fund & Design; 1980. p. 915-54.
- [22] Ataer ÖE. An approximate method for transient behavior of finned-tube cross-Flow heat exchangers. Int J Refrig 2004;27:529-39.
- [23] Ataer ÖE, Ileri A, Göğüş Y. Transient behaviour of finned-tube cross-flow heat exchangers. Int J Refrig 1995;18:153-60.
- [24] Roetzel W, Xuan Y. Transient response of parallel and counterflow heat exchangers. J Heat Transf 1992;114.
- [25] Wang QG, Zhang Y, Cai WJ, Bi Q. Model-based nonlinear controller for cooling coil unit. In: Proceedings of international conference on systems, signals, controls, computers, 1998; South Africa.
- [26] Jin GY, Cai WJ, Wang YW, Yao Y. A simple dynamic model of cooling coil unit. Energy Convers Manage 2006;47:2659-72.
- [27] Horst TA, Rottengruber H-S, Seifert M, Ringler J. Dynamic heat exchanger model for performance prediction and control system design of automotive waste heat recovery systems. Appl Energy 2013;105:293-303.
- [28] Florides GA, Christodoulides P, Pouloupatis P. An analysis of heat flow through a borehole heat exchanger validated model. Appl Energy 2012;92:523-33.
- [29] Li M, Lai ACK. Analytical model for short-time responses of ground heat exchangers with U-shaped tubes: Model development and validation. Appl Energy 2013;104:510-6.
- [30] Jun W, Yan W. Performance improvement of VAV air conditioning system through feedforward compensation decoupling and genetic algorithm. Appl Therm Eng 2008;28:566-74.

Fig.1. Schematic diagram of an ACB terminal unit (Lateral Cross Section View)

Fig.2. Schematic diagram of a simplified cooling coil

Fig.3. Pilot plant of ACB test

Fig.4. Experiment fitting for the primary air resistance

Fig.5. Model validations for the primary air resistance

Fig.6. Experiment fitting for the entrainment effect

Fig.7. Model validation for the entrainment effect

Fig.8. Experiment fitting by two-parameter model for the cooling coil

Fig.9. Experiment fitting by four-parameter model for the cooling coil

Fig.10. Model validation by two-parameter model for the cooling coil

Fig.11. Model validation by four-parameter model for the cooling coil

Fig.12. Experiment fitting for the time constant with a primary air chamber pressure drop

Fig.13. Experiment fitting for the time constant with a primary air chamber pressure increase

Fig.14. Time varying the chilled water inlet temperature and the primary air plenum pressure

Fig.15. Dynamic performance of two-parameter model with $M_c C_c$ estimated by cooling coil compositions

Fig.16. Dynamic performance of four-parameter model with $M_c C_c$ estimated by cooling coil compositions

Fig.17. Dynamic performance of two-parameter model with $M_c C_c$ estimated by LMS

Fig.18. Dynamic performance of four-parameter model with $M_c C_c$ estimated by LMS

Table 1 Summary of experimental parameters setting

Parameter	Range	Unit
Environment temperature	25.0-29.0	°C
Primary air plenum pressure	20-260	Pa
Chilled water volume flow rate	50-400	L/h
Chilled water inlet temperature	13.6-18.5	°C

Table 2 Summary of heat transfer model parameters and their performances

Model	e_a	e_w	$c_a A_a \eta$	$c_w A_w$	RMS
Two-parameters	0.5	0.5	11.6279	13.1406	14.5786
Four-parameters	0.4465	0.3426	25.7086	22.4594	17.5950