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Weyl points and topological nodal superfluids in a face-centered-cubic optical latticeLi-Jun Lang,^{1,2} Shao-Liang Zhang,^{1,3} K. T. Law,⁴ and Qi Zhou^{1,5,*}¹*Department of Physics, The Chinese University of Hong Kong, Shatin, New Territories, Hong Kong, China*²*Division of Physics and Applied Physics, School of Physical and Mathematical Sciences, Nanyang Technological University, Singapore 637371, Singapore*³*School of Physics, Huazhong University of Science and Technology, Wuhan, China*⁴*Department of Physics, Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong, China*⁵*Department of Physics and Astronomy, Purdue University, West Lafayette, Indiana 47906, USA*

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We point out that a face-centered-cubic (fcc) optical lattice, which can be realized by a simple scheme using three lasers, provides one a highly controllable platform for creating Weyl points and topological nodal superfluids in ultracold atoms. In noninteracting systems, Weyl points automatically arise in the Floquet band structure when shaking such fcc lattices, and sophisticated design of the tunneling is not required. More interestingly, in the presence of attractive interaction between two hyperfine spin states, which experience the same shaken fcc lattice, a three-dimensional topological nodal superfluid emerges, and Weyl points show up as the gapless points in the quasiparticle spectrum. One could either create a double Weyl point of charge 2, or split it into two Weyl points of charge 1, which can be moved in the momentum space by tuning the interactions. Correspondingly, the Fermi arcs at the surface may be linked with each other or separated as individual ones.

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Fascinating progresses have been made in studying topological matters of ultracold atoms in the past few years. A number of topological models and topological phenomena difficult to access in solid materials have been realized [1–5]. For instance, the Harper-Hofstadter model [6] and the topological Haldane model [7] have been delivered in optical lattices [1–4]. In the continuum, a two-dimensional (2D) synthetic spin-orbit coupling has created a single stable Dirac point that can be moved anywhere in the momentum space [5]. So far, most of these studies have been focusing on one or two dimensions. A large class of three-dimensional (3D) topological phenomena remain unexplored at the moment.

A Weyl point is a characteristic 3D topological band structure [8], which provides an analog of Weyl fermions, a building block in quantum field theory. It also serves as an ideal platform to explore a wide range of topological phenomena in gapless quantum systems, such as Fermi arc [9] and chiral anomalies [10]. Weyl points and Weyl semimetals have recently been discovered in certain solid materials [11–13]. Whereas this development represents a major advancement in the current frontier of condensed matter physics, challenges remain on manipulating Weyl points since microscopic parameters are essentially fixed in a given solid material. Furthermore, a fundamentally important question regarding the interplay between Weyl semimetal and interaction remains unsolved. Although theoretical studies have predicted a variety of interesting results, including novel superconductivity in doped Weyl semimetals [14–20] and the emergent supersymmetry [19], there has been no experimental observation of such phenomena. It is therefore desirable to have a highly controllable platform to investigate Weyl points and the resultant quantum phenomena in interacting systems.

In this paper, we show that a face-centered-cubic (fcc) optical lattice provides physicists a unique means to create and

manipulate Weyl points in both noninteracting and interacting systems. An intrinsic property of a fcc lattice is that, the lowest two bands, which are labeled as *A* and *B*, respectively, have “inverted” band structures, i.e., tunnelings with opposite signs. This comes from a simple fact that the Brillouin zone (BZ) of a fcc lattice is the one folded from a simple cubic (sc) lattice, as shown in Figs. 1(a) and 1(b). As a result, Weyl points naturally arise in the Floquet band structure, if one simply uses a periodic shaking to overcome the band gap and couple the *A* and *B* bands. This is distinct from the majority of previous proposals [21–25], which require sophisticated designs to engineer tunnelings along all three directions. Moreover, uploading two hyperfine spin states onto such optical lattice, the highly tunable attractive interaction between fermionic atoms allows one to create a 3D nodal superfluid [26,27], which is composed of layered 2D chiral superfluids in the momentum space. Strikingly, Weyl points show up in quasiparticle spectrum of such superfluid. One could either glue two Weyl points with the same chirality to a monopole of charge 2, or further split such multiple-charge monopoles into multiple charge-1 ones, and move them around in the BZ by tuning interactions. Correspondingly, two Fermi arcs emerge at the surfaces, and can be either linked with or separated from each other, depending on locations of the Weyl points in the bulk spectrum.

I. HAMILTONIAN

We consider a circularly shaken 3D fcc optical lattice, whose Hamiltonian in the Floquet framework is written as $H_0 - i\hbar\partial_\tau$, where τ is time,

$$H_0 = \frac{\mathbf{p}^2}{2M} + V(x + f \cos \omega\tau, y + f \sin \omega\tau, z), \quad (1)$$

and f and ω are the shaken amplitude and frequency, respectively. M is the atom mass. Formally, the above equation is a direct generalization of the scheme of shaking a 2D

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FIG. 1. (a) Schematic of the fcc lattice with A and B sublattices. (b) The first BZ (red truncated octahedron) of the fcc lattice folded from that (black cubic) of the sc lattice. (c) The lowest two bands (blue solid), $\epsilon_{A\mathbf{k}}^0$ and $\epsilon_{B\mathbf{k}}^0$, of the fcc lattice. Red dashed line represents the subband of $\epsilon_{A\mathbf{k}}^0$ after absorbing one photon via shaking. The parameters are $V = 8E_R$, $\hbar\omega = 0.7E_R$, $\alpha = 0.06$, $f = 0.16d_0$. (d) One laser setup to realize the fcc lattice. Red (purple) arrows represent the lasers with wavelength λ (λ') and frequency ω (ω'). Details can be referred to in the Supplemental Material [29].

checkerboard lattice [28] to a 3D fcc lattice, whose potential is written as

$$V(x, y, z) = -V \left(\cos^2 \frac{\pi}{a} x + \cos^2 \frac{\pi}{b} y + \cos^2 \frac{\pi}{c} z \right) + \alpha \cos \frac{\pi}{a} x \cos \frac{\pi}{b} y \cos \frac{\pi}{c} z, \quad (2)$$

where a , b , and c are constants. Interestingly, such a 3D optical lattice of fundamental importance in solids has never been produced in ultracold atoms. We point out that it can be produced by three lasers with directions and polarizations arranged in a few ways, one of which is shown in Fig. 1(d). Three pairs of lasers are used, where lasers 1 and 2 interfere with each other and form a z -dependent checkerboard lattice in the x - y plane, while laser 3 with a different frequency forms a standing wave along the z direction (see Supplemental Material [29]). This setup has already been realized by Esslinger's group [3].

To concretize the discussion, we focus on a symmetric case $a = b = c$. All results here can be easily generalized to an arbitrary choice of a, b, c . The exact band structure of the static lattice $V(x, y, z)$, where $f = 0$, can be solved exactly using plane-wave expansions. The results for the lowest two bands, denoted as $\epsilon_{A\mathbf{k}}^0$ and $\epsilon_{B\mathbf{k}}^0$, are shown in Fig. 1(c). Both of them can be well approximated by the tight-binding results $\epsilon_{A\mathbf{k}}^0 = 4t(\cos k_x d_0 \cos k_y d_0 + \cos k_y d_0 \cos k_z d_0 + \cos k_z d_0 \cos k_x d_0) + 2t'(\cos 2k_x d_0 + \cos 2k_y d_0 + \cos 2k_z d_0)$, and $\epsilon_{B\mathbf{k}}^0 = -\epsilon_{A\mathbf{k}}^0 + \Delta$, where t and t' characterize the nearest- and next-nearest-neighbor tunnelings in A sublattice, and Δ is the band separation of them. The inverted structure between A and B bands could be understood qualitatively from the folding of BZ, i.e., $\epsilon_{A\mathbf{k}}^0 \approx \epsilon_{\mathbf{ck}}$, $\epsilon_{B\mathbf{k}}^0 \approx \epsilon_{\mathbf{ck}+\mathbf{G}} + \Delta$, where $\epsilon_{\mathbf{ck}}$ is the ground band dispersion of the SC

lattice, and $\mathbf{G} = (1, 1, 1)\pi/d_0$. $\tilde{d}_0 = \sqrt{2}d_0$ and d_0 are the lattice spacings of the fcc and the sc lattices, respectively. Applying the shaking, the A band by absorbing a photon $\hbar\omega$ couples with the B band in the Floquet Hamiltonian, which can be written as $H_0 = (\epsilon_{A\mathbf{k}} + \epsilon_{B\mathbf{k}})I/2 + K$ with

$$K = \begin{pmatrix} (\epsilon_{A\mathbf{k}} - \epsilon_{B\mathbf{k}})/2 & \Omega_{\mathbf{k}} e^{i\varphi_{\mathbf{k}}} \\ \Omega_{\mathbf{k}}^* e^{-i\varphi_{\mathbf{k}}} & (\epsilon_{B\mathbf{k}} - \epsilon_{A\mathbf{k}})/2 \end{pmatrix}, \quad (3)$$

where $\epsilon_{A\mathbf{k}} = \epsilon_{A\mathbf{k}}^0 + \hbar\omega$ and $\epsilon_{B\mathbf{k}} = \epsilon_{B\mathbf{k}}^0$. One-photon detuning, $\delta \equiv \Delta - \hbar\omega$, can be defined to characterize the separation of $\epsilon_{A\mathbf{k}}$ and $\epsilon_{B\mathbf{k}}$. The interband coupling is written as $\Omega_{\mathbf{k}} e^{i\varphi_{\mathbf{k}}} = \Omega(i \sin k_x d_0 - \sin k_y d_0) e^{-ik_x d_0}$ with $\varphi_{\mathbf{k}} = \arg[(i \sin k_x d_0 - \sin k_y d_0) e^{-ik_x d_0}]$. This coupling is the same as that in a shaken checkerboard lattice [28] since $V(x, y, z)$ in Eq. (2) reduces to a 2D checkerboard lattice for each plane with a given value of z .

II. WEYL POINTS FROM SHAKING

A Weyl point requires that all matrix elements in Eq. (3) become zero at some $\mathbf{k}_0$ in BZ. In previous proposals [21–23], this is realized by engineering the tunneling along all three dimensions, which often requires sophisticated designs of microscopic models. Here, a shaken fcc lattice automatically provides Weyl points. The off-diagonal term could vanish at $(k_x, k_y) = (0, 0)$, $(0, \pi)$, or $(\pi, 0)$. Meanwhile, since $\epsilon_{B\mathbf{k}}^0 = -\epsilon_{A\mathbf{k}}^0 + \Delta$ is readily satisfied in the static lattice, $\epsilon_{A\mathbf{k}} - \epsilon_{B\mathbf{k}} = 0$ can be easily satisfied if δ is small enough, i.e., in the strong interband hybridization regime with the shaken frequency tuned near resonance. Whereas the total number of pairs of Weyl points depends on the microscopic parameters, here we focus on the simplest case with only one pair of Weyl points, i.e., $\mathbf{k}_0 = (0, 0, k_{0z})$, so that $|K| = 0$, where k_{0z} satisfies $4t(1 + 2 \cos k_{0z} d_0) + 2t'(2 + \cos 2k_{0z} d_0) = \delta/2$. Near $\mathbf{k}_0$, the matrix can be linearized

$$K = \begin{pmatrix} v_z(k_z - k_{0z}) & i v_x k_x - v_y k_y \\ -i v_x k_x - v_y k_y & -v_z(k_z - k_{0z}) \end{pmatrix}, \quad (4)$$

where $v_x = v_y = \Omega d_0 \equiv v_{\parallel}$, $v_z = -8d_0 \sin(k_{0z} d_0)(t + t' \cos k_{0z} d_0)$. Equation (4) indeed describes a Weyl point in the BZ. When $\mathbf{k} = \mathbf{k}_0$, $|K| = 0$, and the Weyl point represents a monopole in the momentum space with a topological charge ± 1 . The sign of the charge is determined by the sign of $v_x v_y v_z$. Since for each k_{0z} , there is always another solution $-k_{0z}$ to satisfy $|K| = 0$ with an opposite chirality, one sees that the total chirality in BZ is zero. The positions of the Weyl points are determined by the microscopic parameters in the system, such as the detuning δ . Changing the value of $|\delta|$, Weyl points move in BZ, and once a pair of Weyl points with opposite chiralities meet in BZ, they annihilate each other. For large enough $|\delta|$, the disappearance of Weyl points indicates that the 3D band structure becomes topologically trivial in such cases with weak interband hybridization.

III. 3D NODAL SUPERFLUID

We now turn to the interaction effects. Whereas the interplay between Weyl fermions and interaction has been studied in the literature [14–20], our shaken lattices provide

FIG. 2. (a)–(c) Fermi arcs with (100) surface of (a) noninteracting single particles for either spin up or spin down, and (b), (c) Bogoliubov quasiparticles for half-filling with $(U_A, U_B) = (0.15, 0)E_R$ and $(0.15, 0.08)E_R$, respectively. $\tilde{k}_{y,z} = (k_z \pm k_y)/\sqrt{2}$. (d)–(f) Schematics of nodal points in the bulk with no interaction, only one interaction, and both interactions, respectively. Pink (green) and red (blue) dots represent the nodal points with $+(-)1$ and $+(-)2$ charges, respectively. Spheres in (e) and (f) enclosing the nodal points have the same total Chern number, -2 , of the lowest two bands. Parameters for single particles are the same as those in Fig. 1(c).

one a unique system to explore new physics that has not been explored before. We introduce two hyperfine spin states into the fcc optical lattice, each of which has the same single-particle Hamiltonian specified by Eq. (3). This corresponds to a spin-independent shaken lattice, where we have two Weyl points with the same chirality at the same positions in the momentum space. Without interaction, the many-body ground state is simply composed of two identical copies of Weyl semimetals, if the chemical potential is tuned right at the Weyl point. Unlike the electronic spins, the hyperfine spin is conserved in ultracold atoms. This gives rise to two copies of topological charge of 1 in the single-particle level, as shown in Fig. 2(d). However, introducing attractive interaction inevitably leads to particle-hole mixing between the two hyperfine spin states. A natural question is then as follows: What is the fate of such Weyl semimetals? Alternatively, one could consider tuning the chemical potential away from Weyl points so that the Fermi surface becomes finite. Such finite Fermi surface indicates that the low-lying excitations in the interacting system are actually located at momenta away from the Weyl points of the noninteracting system. It is thus desired to explore whether the Weyl point, which is now embedded inside the Fermi sea, is relevant to the emergent superfluidity when the attractive interaction is turned on.

Using $\sigma = \uparrow, \downarrow$ to denote these two hyperfine spin states, the onsite interaction can be written as $\hat{V} = -U_A \sum_{i \in A} \hat{n}_{i\uparrow} \hat{n}_{i\downarrow} - U_B \sum_{i \in B} \hat{n}_{i\uparrow} \hat{n}_{i\downarrow}$, where $\hat{n}_{i\sigma}$ is the density operator for spin- σ particles at site i , and $U_{A(B)} > 0$ is the onsite interaction strength for $A(B)$ sites. Our Hamiltonian is different from the ones previously studied in the literature, where the degree of freedom that participates in the interaction is the same as that provides the band crossing [14–20]. Here, we consider the attractive interaction between

two Weyl semimetals or two normal metals with small Fermi surfaces. Define the pairing order parameters $\Delta_{A(B)} = -U_{A(B)} \langle \hat{\Psi}_{A(B), -\mathbf{k}\downarrow} \hat{\Psi}_{A(B), \mathbf{k}\uparrow} \rangle / N$, where N is the number of unit cells, the BCS Hamiltonian can be written as

$$\hat{H}_{\text{BCS}} = \begin{pmatrix} \epsilon_{A\mathbf{k}} - \mu & \Omega_{\mathbf{k}} e^{i\varphi_{\mathbf{k}}} & \Delta_A & 0 \\ \Omega_{\mathbf{k}} e^{-i\varphi_{\mathbf{k}}} & \epsilon_{B\mathbf{k}} - \mu & 0 & \Delta_B \\ \Delta_A^* & 0 & \mu - \epsilon_{A\mathbf{k}} & -\Omega_{\mathbf{k}} e^{-i\varphi_{-\mathbf{k}}} \\ 0 & \Delta_B^* & -\Omega_{\mathbf{k}} e^{i\varphi_{-\mathbf{k}}} & \mu - \epsilon_{B\mathbf{k}} \end{pmatrix}, \quad (5)$$

where Δ_A and Δ_B are solved self-consistently for a fixed density $n = n_{\uparrow} + n_{\downarrow}$. Since U_A and U_B could be controlled independently, very rich physics emerges.

We first consider $n_{\uparrow} = n_{\downarrow} > \frac{1}{2}$, i.e., a finite Fermi surface surrounding each Weyl point in the BZ. Turning on one of the interactions, say U_A or U_B , it turns out that there exists one point on each Fermi surface remaining gapless, whereas the superfluid gap opens anywhere else, as shown in Fig. 2(e). We note that the BCS Hamiltonian (5) is block-diagonalized if one sets $k_x = k_y = 0$. When $U_A = 0$, which naturally leads to $\Delta_A = 0$, all matrix elements of one block vanish at the momenta $\mathbf{k}^* = (0, 0, \pm k_{0z} \pm k_F)$, i.e., the Bogoliubov quasiparticle spectrum remains gapless. The other branch, which comes from the other block, opens a gap due to a finite Δ_B . Alternatively, when $U_B = 0$ the spectrum is gapless at $\mathbf{k}^* = (0, 0, \pm k_{0z} \mp k_F)$.

We point out that the emergent 3D nodal superfluid is a topological one, which can be seen from the topological charges carried by the nodal points in the quasiparticle spectrum. Expanding the quasiparticle spectrum near such gapless point, an effective Hamiltonian is obtained,

$$H_{\text{eff}}^{\pm} = \begin{pmatrix} \pm v_z^* q_z & 0 \\ 0 & \mp v_z^* q_z \end{pmatrix} + \frac{v_{\parallel}^2 (q_x^2 + q_y^2)}{|\Delta_{\xi}|^2 + 4\mu^2} \begin{pmatrix} -2\eta\mu & \Delta_{\xi} e^{-2i\theta_q} \\ \Delta_{\xi} e^{2i\theta_q} & 2\eta\mu \end{pmatrix}, \quad (6)$$

where $\mathbf{q}$ is the momentum measured from the gapless point $\mathbf{k}^*$ or $\mathbf{k}'^*$, $\theta_q = \arg(iq_x - q_y)$, and $v_z^* = 8 \sin(k_{0z} + \eta k_F)[t + t' \cos(k_{0z} + \eta k_F)]$. $\eta = +1$ and $\xi = A$ for $U_B = 0$ while $\eta = -1$ and $\xi = B$ for $U_A = 0$. The superscript $\pm$ is the valley index, representing the two Fermi surfaces surrounding the two Weyl points in noninteracting systems. $H_{\text{eff}}^{\pm}$ describes a Bogoliubov quasiparticle spectrum, which is linear along the q_z direction and quadratic along the q_x and q_y directions, since the off-diagonal term $\sim (iq_x \pm q_y)^2$. It thus describes a monopole of charge 2 in the momentum space. We thus see that the topological charge when introducing attraction interaction shows up in Bogoliubov quasiparticle spectrum.

Turning on the other interaction, the monopole of charge 2 splits to two charge-1 ones, i.e., two Weyl points, in the k_x - k_y plane, the positions of which are denoted as $\mathbf{k}_1^*$ and $\mathbf{k}_2^*$, respectively. This can be understood from the conservation of the total charge of the monopoles enclosed in the spheres, as shown in Figs. 2(e) and 2(f). Near these two Weyl points, the dispersion becomes linear along all three directions. Since the noninteracting system has a fourfold rotation symmetry about the k_z axis, and such a splitting reduces the symmetry to a

twofold one, the choice of the direction for the splitting is a consequence of spontaneous symmetry breaking, along either the k_x or k_y direction. By changing the ratio U_A/U_B , these two Weyl points move in BZ before meeting their counterparts with opposite chiralities emerged from the other valley. For noninteracting systems, it is known that tuning the parameters leads to the movement of Weyl points without opening the gap. In our system, tuning interaction also offers such opportunity to control the positions of Weyl points in the Bogoliubov quasiparticle spectrum in BZ.

There is an alternative way to understand why the 3D superfluid remains nodal with turning on a small U_A . As shown in Eq. (5) and Figs. 2(d)–2(f), in 3D BZ each 2D plane with fixed k_z defines a 2D $s + (d + id)$ superfluid, similar to the one emerged from a 2D shaken lattice [28]. Whereas tuning k_z effectively changes the chemical potential of such 2D superfluid, its Chern number can be computed straightforwardly. When $|k_z| < k_z^*$, where k_z^* is the z component of $\mathbf{k}_1^*$ (and $\mathbf{k}_2^*$), this 2D chiral superfluid is topologically nontrivial with a Chern number of 2, as the chiral component $(d + id)$ is dominant for small U_A . When $|k_z| > k_z^*$, $C = 0$. The topological transition of the 2D superfluid just corresponds to the nodal points. Thus, the nodal points cannot be suddenly gapped, and the 3D superfluid remains nodal when U_A is small enough.

The pairing here is an intervalley BCS pairing between $-\mathbf{k}$ and $\mathbf{k}$. There have been studies of the competition between the intervalley and intravalley pairings near Weyl points [14–19]. Here, it is the hyperfine spin states that incorporate the interaction effect. We find out that, at least in the mean field level, the intervalley pairing wins. This can be qualitatively understood from the phase-space argument. In the intervalley pairing, a paired state $(\mathbf{k}_0 + \mathbf{q}\uparrow, -\mathbf{k}_0 - \mathbf{q}\downarrow)$ can be scattered to both $(\mathbf{k}_0 + \mathbf{q}'\uparrow, -\mathbf{k}_0 - \mathbf{q}'\downarrow)$ and $(-\mathbf{k}_0 + \mathbf{q}'\uparrow, \mathbf{k}_0 - \mathbf{q}'\downarrow)$. For the intravalley pairing, a paired state $(\mathbf{k}_0 + \mathbf{q}\uparrow, \mathbf{k}_0 - \mathbf{q}\downarrow)$ could only be scattered to $(\mathbf{k}_0 + \mathbf{q}'\uparrow, \mathbf{k}_0 - \mathbf{q}'\downarrow)$. Thus, the intervalley pairing gains more energy than the intravalley pairing (see Supplemental Material [29]). Another mechanism favoring the intervalley pairing is that $\epsilon_{\mathbf{k}\sigma} = \epsilon_{-\mathbf{k}\sigma}$ is satisfied. Since the shaking is spin independent, $\epsilon_{\mathbf{k}_0+\mathbf{q}\uparrow} = \epsilon_{-\mathbf{k}_0-\mathbf{q}\downarrow}$ is thus valid. However, near the same Weyl point, $\epsilon_{\mathbf{k}_0+\mathbf{q}}$ is not exactly the same as $\epsilon_{\mathbf{k}_0-\mathbf{q}}$, when $\mathbf{q}$ is large enough and the linear approximation for the single-particle energy is no longer accurate. This energy mismatch disfavors the intravalley pairing.

We now consider half-filling $n = \frac{1}{2}$. In such Weyl semimetal, due to the vanishing Fermi surfaces, a weak attractive interaction is no longer relevant [19]. Our calculations indeed show that the pairing remains vanishing before either or both interactions reach a critical value. The critical interaction strength by itself is not universal, in the sense that it depends on the details of the single-particle spectrum at both low and high energies. For a purely linear dispersion, the critical value is written as $U_A^c = 6\pi^2 v_z \Omega^2 / \Lambda^2$ for $U_B = 0$, where Λ is the high-energy cutoff. In realistic systems, the high-energy part

of the spectrum is no longer linear, and is also important to the critical interaction. Thus, there is no simple expression of U_A^c . Nevertheless, if $U_A > U_A^c$, the results become similar to those with a finite Fermi surface. A double Weyl point of charge 2 emerges in the quasiparticle spectrum. Turning on U_B , such multiple-charge monopole splits to two Weyl points of 1. Using typical values in Fig. 1(c) for single particles and in Fig. 2(c) for interactions, we can estimate the order parameters as $\Delta_A \sim 12.02$ nK and $\Delta_B \sim 1.71$ nK.

IV. FERMI ARCS

A characteristic feature of Weyl points in noninteracting systems is the existence of Fermi arc [9] at the surface, an unclosed line as the zero energy state. Whereas in the absence of interaction, such Fermi arc is indeed observed in our system as shown in Fig. 2(a), it is more interesting to explore the interacting case when the nodal superfluids have emerged. We solve the Bogoliubov–de Gennes equation for half-filling self-consistently with (100) surface such that k_y and k_z are still good quantum numbers, and the zero-energy surface state is shown in Figs. 2(b) and 2(c).

Without interactions, two identical Fermi arcs, each of which comes from one hyperfine spin state, are on top of each other. Turning on one of the interactions, two Fermi arcs connect with each other at two points in the 2D BZ, which are just the projection of the two charge-2 monopoles in the bulk spectrum onto the surface. Turning on the other interaction, accompanied with the splitting of each charge-2 monopole into two Weyl points, the two Fermi arcs split, and the four ending points are simply the projections of the four Weyl points in the bulk. One could understand these Fermi arcs from the layered 2D superfluids. When the 2D superfluid is topologically nontrivial with Chern number 2, its two zero-energy edge states have momenta $k_{y1}(k_z)$ and $k_{y2}(k_z)$, respectively. Changing k_z leads to different values of k_{y1} and k_{y2} , and gives rise to Fermi arcs as the trajectories of zero-energy states on the surface. Such zero-energy states merge into the bulk spectrum when $|k_z| > k_z^*$ and the 2D superfluid becomes topologically trivial.

We have shown that Weyl points are readily achievable in current ultracold atom experiments. The interplay between the interaction and the topological band structure leads to a 3D topological nodal superfluid with Weyl points and Fermi arcs, which are highly controllable via tuning the interaction. We hope that our work will stimulate more studies on 3D topological matters in ultracold atoms.

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