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Parameter Estimation of Block and Helical Scan Interleavers in the Presence of Bit Errors

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Abstract

In data communication and digital storage applications, interleaver, which rearranges the encoded data stream, follows the channel encoder to prevent burst errors. In general, forward error correction (FEC) codes provide protection against randomly distributed errors, whereas interleaver provides protection against error bursts. To de-interleave the interleaved data, accurate information about the interleaver parameters is required at the receiver. However, in non-cooperative context, the parameters are either not known or only partially known at the receiver. Therefore, blind/semi-blind estimation of interleaver parameters is mandatory at the receiver in order to successfully de-interleave the interleaved data. In this context, this paper proposes innovative algorithms for the estimation of interleaver parameters such as interleaver period (or size of the interleaver matrix), number of rows and columns of the interleaver matrix, and helical array step size in the presence of bit errors provided the knowledge of type of interleaver is known at the receiver. In addition, it has been assumed that the block and helical scan interleavers follow a convolutional encoder.

Keywords: Blind/semi-blind estimation, Block interleaver, Helical array step size, Helical scan interleaver, Interleaver period, and Non-cooperative context.

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1. Introduction

Forward error correction (FEC) codes play a vital role in improving the bit error rate (BER) performance of a digital communication system by counter-acting randomly distributed errors. However, for improving the reliability of a system against burst errors, interleaver plays a significant role. FEC encoder together with an interleaver nullify both the random and burst errors, respectively. The concept of bit-interleaving at the convolutional encoder output to improve the performance of coded modulation over Rayleigh fading channels was first proposed in [1] and the same was called as bit interleaved coded modulation (BICM). In [2], theory behind the BICM scheme from information-theoretic perspective was discussed.

Figure 1: Matrix block interleaver operation assuming $N_r = 6$ and $N_c = 4$ considering input values [1 : 24]

Different types of interleavers such as block or matrix interleaver, helical scan interleaver, convolutional interleaver, helical interleaver, random interleaver, etc. were proposed in the literature in protecting the data stream from error bursts. In this paper, we restrict our discussion to block and helical scan interleavers, since they are more prominently used in error control systems compared to other interleavers. The data stream is segregated into multiple blocks each of size equal to size of interleaver matrix or interleaver period β , where $\beta = N_r \times N_c$, N_r and N_c denote the number of rows and columns of the interleaver matrix, respectively. Both interleavers store each block of data row wise and perform permutation of data stream. In the case of block interleaver, it outputs the

stored data column wise. An example for block or matrix interleaver with $N_r=6$ and $N_c=4$ is shown in Fig.1. In the case of helical scan interleaver, it outputs the stored data symbols using matrix diagonals according to helical array step size d . The helical scan interleaver operation assuming $N_r=6$, $N_c=4$, and $d=1$ is shown in Fig.2.

Figure 2: Helical scan interleaver operation assuming $N_r=6$, $N_c=4$, and $d=1$ considering input values $[1 : 24]$

To de-interleave the interleaved data stream, exact information about the interleaver parameters used in the transmitter should be known at the receiver. But for non-cooperative scenarios, where the interleaver parameters are not available at the receiver, there is a need to estimate the parameters from the received data sequence. Furthermore, due to uninterrupted evolution of modern digital communication systems, it is an expensive as well as time consuming process to design communication devices for every standards in use around the world as mentioned in [3] and [4]. To counteract this problem, software-defined radio (SDR) systems are introduced. As a matter of fact, an ideal SDR should be able to blindly estimate the transmission parameters from the received data stream. Since the usage of interleaver block is inevitable in modern digital communication systems, there is a need for the SDR systems to blindly/semi-blindly estimate the interleaver parameters to de-interleave the interleaved data stream successfully.

In prior works, the parameter estimation of block and convolutional codes was extensively studied. In [5], the parameter estimation of single linear non-

systematic block and convolutional codes was considered and the code parameters for block and convolutional codes were estimated. However, the estimation process was carried out considering ideal channel conditions without any bit errors. In [6], the parameters of rate $\frac{1}{n}$, where n denotes the codeword length or number of output symbols, convolutional codes were estimated in a noiseless environment and the same was extended to rate $\frac{k}{n}$ in [7], where k denotes number of input symbols, considering both noiseless and noisy environments. Furthermore, expectation-maximization-based parameter estimation algorithm from noisy observations was presented in [8] for rate $\frac{1}{n}$ convolutional codes. In [3], [4], and [9], the semi-blind identification of convolutional code parameters was carried out in a noisy environment based on dual-code concept, where the estimation process was restricted to Galois field $\text{GF}(2)$. In [10] and [11], the semi-blind estimation of code parameters was extended to non-binary convolutional and block codes (i.e. $\text{GF}(2^m)$), respectively. A theoretical justification behind rank criterion method of estimating convolutional code parameters was investigated in [12].

The parameter estimation of block and helical scan interleavers had been reported in earlier works. In [13] and [14], the parameter estimation was restricted to block interleaver with hamming block code. Algorithms for identification of interleaver period, estimation of code rate of block code along with steps to synchronize the interleaved data stream in the presence of bit errors were described based on evaluating the number of ones in the lower part of column echelon form F . In [15], the interleaver period estimation process was carried out for non-binary Reed Solomon coded data streams transmitted over additive white Gaussian noise (AWGN) channel considering block interleaver. The parameter estimation of block encoded block interleaver was investigated in [16] for non-erroneous or error-free scenarios. In [17], the helical scan interleaver parameters like interleaver period along with N_r and N_c were estimated for hamming block code over AWGN channel considering the synchronized case. The semi-blind parameter estimation of convolutional interleaver for block codes was also investigated in the prior works assuming the type of interleaver is known at the

receiving end. In [18], the proposed approach to estimate the convolutional
75 interleaver parameters was based on recognizing the data bits from the same
codewords by recording their row and column indices from the data matrix. In
[19], a rank-deficiency-based methodology was proposed to estimate the inter-
leaver period. Further, by considering two different rank deficiency matrices,
rest of the interleaver parameters over robust environment were estimated based
80 on normalized rank values. Finally, in [20], the proposed algorithm to estimate
the interleaver parameters was based on finding the positions of the dependent
columns of the data matrix.

1.1. Motivations

The main motivations behind the proposed work are as follows:

- 85 • In most of the practical applications, accurate information about coding
and interleaver parameters is known at the receiving end. However, in
a non-cooperative context and applications such as cognitive radio, spec-
trum surveillance, etc., the coding and interleaver parameters need not
be known at the receiver. Therefore, blind/semi-blind recognition of the
90 parameters is essential and it will be helpful for further analysis.
- In earlier works [13]-[16], only interleaver period for block interleaver was
estimated, which is not sufficient to de-interleave the data. The exact
values of N_r and N_c should also be estimated. Even though interleaver
period along with N_r and N_c had been estimated in [17] for helical scan
95 interleaver, helical array step size d was not estimated.
- Apart from recognizing the convolutional interleaver parameters in [18],
the proposed algorithm cannot recover the delay of the received data to
perform synchronization. Though the proposed algorithm in [19] success-
fully estimated the convolutional interleaver parameters, it works with
100 100% accuracy only for the scenario when bit error rate (BER) is less
than or equal to 6×10^{-3} .

- In all the prior works, the interleaver matrix size or interleaver period was assumed as a multiple of codeword length n . Moreover, the interleaver period estimation process had been carried out assuming block encoded data stream.
- Finally, in all the prior works, the interleaver parameter estimation had been investigated without puncturing and also only in the presence of random errors. To the best of our knowledge, the estimation of interleaver parameters had not been carried out considering punctured convolutional codes and also in the presence of burst errors.

1.2. Contributions

Therefore, the main contributions of our work are as follows

- The parameter estimation process is carried out for block and helical scan interleavers considering unsynchronized convolutional encoded data in the presence of bit errors based on evaluating the zero mean ratio from column echelon form F .
- The interleaver parameter identification has been carried out for a generic case without restricting the interleaver period as a multiple of n .
- Along with the estimation of interleaver period, algorithms for joint estimation of N_r , N_c , d , and number of bits to be shifted for achieving proper synchronization of interleaved data have also been proposed.
- The parameter estimation is also investigated over punctured convolutional codes and also over burst error scenario.
- The performance of both the interleavers is investigated for different values of constraint length K .

- It has been observed that the proposed algorithms can estimate the interleaver parameters with 100% accuracy for the case when $\text{BER} \leq 3 \times 10^{-2}$.

130 The rest of the paper is organized as follows. In Section 2, a brief explanation for interleaver parameter estimation process is given using a generic block diagram. Section 3 describes the algorithms for estimating the interleaver period of both block and helical scan interleavers. In Section 4, a rank-based methodology for estimating the codeword length and code rate over erroneous
135 and error-free scenarios is discussed. Section 5 explains the algorithms for estimating the interleaver parameters N_r , N_c , and d along with number of bits to be shifted for achieving synchronization. In Section 6, simulation results over various scenarios assuming different interleaver and code parameter values are presented with detailed discussions. Finally, concluding remarks are given in
140 Section 7.

2. Interleaver parameter estimation process

A generic block diagram showing the necessary blocks required to estimate the interleaver parameters is given in Fig. 3. The interleaver block always follows the convolutional encoder block to counteract the burst errors. After interleaving, the convolutional encoded data stream is transmitted. To estimate
145 all the interleaver parameters at the receiver side, interleaver period should be estimated along with the codeword length from the received non-synchronized data stream with bit errors. After estimating the interleaver parameters, convolutional code parameters such as constraint length and generator polynomial
150 must also be estimated. In this paper, estimation of convolutional code parameters has not been discussed, as this issue has been already investigated comprehensively in [3] and [4]. It is to be noted that the type of interleaver is assumed to be known at the receiver in order to estimate all the interleaver parameters.

Figure 3: Generic block diagram for parameter estimation process considering block and helical scan interleavers

155 3. Interleaver Period Estimation

The interleaver period estimation process for error-free scenario considering block code is already described in earlier works and the estimation process is restricted for the case when $\beta = \gamma \times n$ or β is a multiple of n , where γ denotes positive integer. Here, the procedure for estimating the interleaver period for
160 a convolutional code in an error-free scenario has been explained without any restriction and it is given as follows:

- Step 1: Assume that the received data stream is encoded using convolutional encoder and interleaved using block or helical scan interleaver considering specific interleaver period β .
- 165 • Step 2:¹ Reshape the column wise interleaved data stream into a matrix S , referred as data matrix, of size $a \times b$, where b is the number of columns and a is the number of rows of data matrix S , which is at least greater than or equal to $2 \times b$.

¹ N_r and N_c denote the number of rows and columns of interleaver matrix (refer Fig. 1 and Fig. 2), respectively, whereas a and b denote the number of rows and columns of data matrix S , respectively. It should be noted that data matrix S contains interleaved data streams.

- Step 3: Convert the data matrix S into column echelon form F through Gauss elimination process [21]. The rank of the data matrix S in $GF(2)$ by varying b is computed by calculating the number of non-zero columns in F . Further, the rank ratio p is given by $p = \text{rank}(S)/b$.
- Step 4: The difference between successive number of columns with rank deficiency is observed and the same gives β or $\text{lcm}(n, \beta)$, where lcm represents least common multiple.

Proposition 1: While varying the number of columns, when $b = \alpha \times \beta$ (i.e. b is a multiple of β), where α is a positive integer and $\beta = \gamma \times n$ (i.e. β is a multiple of n), then rank deficiency will be obtained and the deficient rank ($\rho(S)$) of the data matrix S is given by

$$\rho(S) = \alpha \times \gamma \times k + m. \quad (1)$$

However, if $b \neq \alpha \times \beta$ (i.e. b is not a multiple of β) and $\beta = \gamma \times n$, then full rank will be obtained i.e. $\rho(S) = b$.

Proof: It is to be noted that in convolutional codes, the output n data bits depend on k present and m previous input bits (i.e. $k + m$ bits), where m is the memory of the encoder which is equal to $K - 1$. Therefore, $b = \alpha \times \beta$ output bits, where $\beta = \gamma \times n$, depend on $\alpha \times \gamma \times k + m$ input bits. Otherwise, if $b = \alpha \times \beta$ and if β is a multiple of n , then one or more interleaved data block of size equal to $\alpha \times \gamma \times n$ output bits in a particular row will be a linear combination of $\alpha \times \gamma \times k + m$ input bits. This is applicable to all the other rows. Since $\alpha \times \gamma$ codewords in all the rows will be aligned properly in the same column, rank deficiency will be observed for the mentioned case after converting S into column echelon form F through Gauss elimination process. However, if $b \neq \alpha \times \beta$, then $\alpha \times \gamma$ codewords will be segregated in different rows and will not be aligned properly in the same column resulting in full rank.

Proposition 2: Consider the case when $\beta \neq \gamma \times n$ (i.e. β is not a multiple of n). While varying the number of columns, when $b = \alpha \times \text{lcm}(n, \beta)$ (i.e. b is a multiple of $\text{lcm}(n, \beta)$), then only rank deficiency will be obtained and the deficient rank

($\rho(S)$) is given by (1). Moreover, for the case when $b \neq \alpha \times lcm(n, \beta)$ (i.e. b is not a multiple of $lcm(n, \beta)$), full rank will be obtained.

195 **Proof:** Note that if $\beta \neq \gamma \times n$ and $b = \beta$, then one interleaved data block will not contain γ codewords aligned in a single row. Moreover, γ codewords will be segregated in different rows and hence, it will result in full rank. Furthermore, only when $b = \alpha \times lcm(n, \beta)$, $\alpha \times \gamma$ codewords within one or more interleaved data block in different rows will be aligned properly in the same column resulting in
200 rank deficiency, since $lcm(n, \beta)$ is a multiple of n or $lcm(n, \beta) = \gamma \times n$.

From (1), the interleaver period β or $lcm(n, \beta)$ can be calculated by observing the difference between two successive columns of the data matrix with rank deficiency. Let $b = \alpha \times \gamma \times n$ and $b' = (\alpha + 1) \times \gamma \times n$ denote two successive columns with rank deficiency and their difference is given by

$$b' - b = \gamma \times n \quad (2)$$

Note that from proposition 1, $\beta = \gamma \times n$ and from proposition 2, $lcm(n, \beta) = \gamma \times n$.

It is to be observed that after converting data matrix S into column echelon form F through Gauss elimination process, all the dependent columns in S will be transformed into all-zero columns in F for error-free scenario and the rank value can be calculated from the number of non-zero or independent columns. But for erroneous case, all the dependent columns in S will not be transformed into all-zero columns in F due to erroneous bits. Hence, full rank will be obtained regardless of the number of columns and interleaver period β or $lcm(n, \beta)$ cannot be estimated from the rank deficiency property of S . Therefore, instead of evaluating β or $lcm(n, \beta)$ from rank deficiency, they can be evaluated from zero mean ratio $\mu(b)$ for erroneous case. This is because, the rank deficient data matrix with erroneous bits will contain more number of zeros compared to a full rank data matrix. Firstly, to calculate $\mu(b)$, the mean value of number of zeros $\sigma(c)$, where c denotes the column index and $c \in \{1, 2, \dots, b\}$, in each column is obtained from F for a particular value of b and it is given by

$$\sigma(c) = \frac{\phi(c)}{a}, \quad c \in \{1, 2, \dots, b\} \quad (3)$$

where $\phi(c)$ indicates the number of zeros in c^{th} column, and a denotes the number of rows in data matrix S . After that zero mean ratio $\mu(b)$ is evaluated from $\sigma(c)$ as follows

$$\mu(b) = \frac{\sum_{c=1}^b \sigma(c)}{b}. \quad (4)$$

The following are the steps given for estimating β or $lcm(n, \beta)$ based on $\mu(b)$ over erroneous channel conditions:

- Step 1: Convert the given interleaved data matrix S into column echelon form F using Gauss elimination process.
 - Step 2: Fix the number of rows a greater than or equal to twice the number of columns b (i.e. $a \geq 2 \times b$) for S .
 - Step 3: Calculate $\mu(b)$ by varying b
 - Step 4: The zero mean ratio $\mu(b)$ for a particular value of b is normalized with respect to the peak or maximum value of zero mean ratio obtained for different values of b . Let us denote the normalized zero mean ratio as $\mu'(b)$ and the peak or maximum value of zero mean ratio as μ_{max} . Now $\mu'(b)$ can be written as
- $$\mu'(b) = \frac{\mu(b)}{\mu_{max}} \quad (5)$$
- Step 5: The difference between successive number of columns with $\mu'(b)$ greater than Γ_2^{th} is observed, where Γ_2^{th} denotes a normalized threshold value, and this gives β or $lcm(n, \beta)$.

It is to be noted that the procedure for estimating β or $lcm(n, \beta)$ is the same for both interleavers and it is suitable for both erroneous and error-free channel conditions.

4. Codeword length and Code Rate Estimation

While estimating the rest of interleaver parameters, it has been assumed that codeword length n is known. The codeword length and code rate r estimation process follows interleaver period estimation process. For error-free case, r

can be estimated from the slope of rank deficiency curves. The difference between two successive columns of the data matrix with rank deficiency is already given by (2). Now the difference between rank values (i.e. $\rho'(S) - \rho(S)$) for two successive columns b' and b can be calculated as follows

$$\begin{aligned}\rho'(S) - \rho(S) &= \{(\alpha + 1) \times \gamma \times k + m\} \\ &\quad - \{(\alpha) \times \gamma \times k + m\} \\ &= \gamma \times k,\end{aligned}\tag{6}$$

where $\rho(S)$ and $\rho'(S)$ denote the ranks associated with columns b and b' , respectively. Dividing (6) by (2) gives the code rate r and the same is given by

$$r = \frac{\rho'(S) - \rho(S)}{b' - b} = \frac{k}{n}.\tag{7}$$

Hence, from (7) and its denominator, the parameters r and n can be estimated, respectively, for error-free scenario.

For erroneous scenario, the rank deficiency calculations will not be suitable, since full rank will be obtained for all values of b due to erroneous bits. However, deficient rank or rank ratio for a particular value of b , which is a multiple of β or $lcm(n, \beta)$, can be evaluated by calculating the number of independent columns based on the mean value of number of zeros $\sigma(c)$ in each column, where $c \in \{1, 2, \dots, b\}$. It is to be noted that the mean value of number of zeros $\sigma(c)$ for dependent columns will be higher compared to independent columns considering the erroneous scenario. Let us denote b_1 and b_2 as two different columns with rank deficiency, where $b_2 > b_1$, and both are fixed as a multiple of β or $lcm(n, \beta)$. Now the number of independent columns considering b_1 (i.e. $N_{ind}(b_1)$) can be evaluated by counting the number of columns with $\sigma(c)$ lesser than a threshold value Γ_3^{th} , which is given by

$$N_{ind}(b_1) = \mathbf{card} \left(c \in \{1, 2, \dots, b_1\} \mid \sigma(c) < \Gamma_3^{th} \right).\tag{8}$$

In (8), $\mathbf{card}(\cdot)$ indicates the cardinality operation, which counts the number of columns with $\sigma(c)$ lesser than Γ_3^{th} and c denotes the column index of data

matrix S , which varies from 1 to b_1 . We know that $\sigma(c) = \frac{\phi(c)}{a}$ and a indicates the number of rows of S , which is fixed according to b_1 . Since we know that the number of independent columns denote the rank value, after calculating $N_{ind}(b_2)$ similar to (8), r can be estimated as

$$r = \frac{N_{ind}(b_2) - N_{ind}(b_1)}{b_2 - b_1} \quad (9)$$

225 5. Estimation of rest of the interleaver parameters

After estimating n and β or $lcm(n, \beta)$, block interleaver parameters such as N_r and N_c and helical scan interleaver parameters such as N_r , N_c , and d should be estimated to complete the interleaver parameter estimation process. The following algorithm explains the steps for estimating the interleaver parameters N_r , N_c , and d .

5.1. Algorithm 1: Estimation of interleaver parameters

Input : codeword length n , $\zeta = \beta$ or $\zeta = lcm(n, \beta)$, where β denotes interleaver period,

Output : N_r^{est} , N_c^{est} , d^{est} , and ϕ^{est}

- 235 • Step:1 Assume $\beta = N_r \times N_c$.
- Step:2 **For** $i = 1 : n$
 Get all possible combinations of two factors N'_r and N'_c
 such that $N'_r \times N'_c = \frac{\zeta}{i}$, where N'_r and N'_c are integers.
end
- 240 • Step:3 **Block interleaver**: De-interleave and evaluate the rank (for error-free case) or normalized zero mean ratio $\mu'(b)$ (for erroneous case) considering all values of N'_r and N'_c by simultaneously varying b and also by varying the starting bit position ϕ from 0 to $\zeta - 1$.
Helical scan interleaver: De-interleave and evaluate the rank or normalized zero mean ratio $\mu'(b)$ considering all values of N'_r and N'_c by

simultaneously varying b , helical array step d' from 1 to $N'_r - 1$, and also by varying the starting bit position ϕ from 0 to $\zeta - 1$.

- Step:4 Obtain the difference between two successive columns δ with rank deficiency or normalized zero mean ratio $\mu'(b)$ greater than Γ_2^{th} for all possible checks.

- Step:5 **Block interleaver:**

Find $[N_r^{est}, N_c^{est}, \phi^{est}] = \underset{N'_r, N'_c, \phi}{\operatorname{argmin}}(\delta)$.

Helical scan interleaver:

Find $[N_r^{est}, N_c^{est}, d^{est}, \phi^{est}] = \underset{N'_r, N'_c, d', \phi}{\operatorname{argmin}}(\delta)$.

It is to be observed that for $N'_r = N_r$, $N'_c = N_c$, $d' = d$, and synchronized bit position, the minimum value of δ will be equal to n (i.e. $\delta' = n$) and for rest of the cases $\delta > n$. Algorithm 1 is a time consuming process, since while de-interleaving the number of columns are varied for all possible checks. In addition, for error-free case we have to calculate the rank and for erroneous case $\mu'(b)$ should be computed. Therefore, there are two different methodologies which will again increase the computational complexity. In this context, this paper proposes a generic algorithm with reduction in computational complexity suitable for both erroneous and error-free cases for helical scan and block interleavers. The necessary steps for the modified algorithm are explained below.

5.2. Algorithm 2: Estimation of interleaver parameters with reduced complexity

Input : $n, \zeta = \beta$ or $\zeta = \operatorname{lcm}(n, \beta)$, **Output :** N_r^{est} , N_c^{est} , d^{est} , and ϕ^{est}

- Step:1 Assume $\beta = N_r \times N_c$.

- Step:2 **For** $i = 1 : n$

Get all possible combinations of two factors N'_r and N'_c

such that $N'_r \times N'_c = \frac{\zeta}{i}$.

end

- Step:3 Fix the number of columns as a multiple of ζ (i.e. $b = \lambda \times \zeta$, where λ is a positive integer).

- Step:4 **Block interleaver:** De-interleave and evaluate $\mu(b)$ considering
 all possible values of N'_r and N'_c by varying the starting bit position ϕ
 from 0 to $\zeta - 1$.
- Step:4 **Helical scan interleaver:** De-interleave and evaluate $\mu(b)$ con-
 sidering all possible values of N'_r and N'_c by varying helical array step
 $d' = 1 : N'_r - 1$ and starting bit position $\phi = 0 : \zeta - 1$.
- Step:5 **Block interleaver:**
 Find $[N_r^{est}, N_c^{est}, \phi^{est}] = \underset{N'_r, N'_c, \phi}{argmax}(\mu(b))$.
Helical scan interleaver:
 Find $[N_r^{est}, N_c^{est}, d^{est}, \phi^{est}] = \underset{N'_r, N'_c, d', \phi}{argmax}(\mu(b))$.

Note that for $N'_r = N_r$, $N'_c = N_c$, $d' = d$, and synchronized bit position, the value
of zero mean ratio $\mu(b)$ will be maximum due to rank deficiency.

It is to be noted that in algorithm 1, $\mu(b)$ is evaluated by varying b and also
by de-interleaving with all possible combinations of interleaver parameters as
mentioned in step 3. Further, the estimation process for algorithm 1 is different
for error-free and erroneous cases. For error-free scenario, the estimation is based
on detecting the minimum difference between two successive columns with rank
deficiency. For erroneous scenario, the estimation is based on detecting the
minimum difference between two successive columns with $\mu'(b)$ greater than
a particular threshold value Γ_2^{th} . It has been mentioned in step 5 that for
correct interleaver and synchronization parameters, the difference between two
successive columns with rank deficiency or with $\mu'(b)$ greater than Γ_2^{th} will
be minimum and is equal to the codeword length n . This is because, if the
convolutional coded and block/helical scan interleaved data symbols are de-
interleaved with correct parameters, after de-interleaving, the data symbols will
be convolutional coded. For convolutional coded symbols, the rank deficiency
will be obtained only for the case when b is a multiple of n as stated in [3] and
[4]. Let $b = \alpha \times n$ and $b' = (\alpha + 1) \times n$ denote two successive columns with rank
deficiency or with $\mu'(b)$ greater than Γ_2^{th} after de-interleaving. The difference

(i.e. $b' - b$) will give the estimate of n as stated in algorithm 1. However, if the convolutional coded data symbols are de-interleaved with incorrect interleaver and synchronization parameters, the data symbols will not get de-interleaved correctly. Therefore, the difference between two successive columns with rank deficiency or with $\mu'(b)$ greater than Γ_2^{th} will give the estimate of $N'_r \times N'_c$ (refer Table 1 given in Section 6), which will be greater than the codeword length n of convolutional codes. Hence, the correct combination of $[N'_r, N'_c, \phi]$ (for block interleaver) or $[N'_r, N'_c, d', \phi]$ (for helical scan interleaver), which gives minimum value equal to codeword length n , has been chosen.

Instead of estimating the interleaver parameters by varying the number of columns b , fixed value for b is assumed in algorithm 2. The main reason behind this simplification is given as follows: While varying b in algorithm 1, it is observed that the zero mean ratio values are higher for correct combination of interleaver and synchronization parameters for rank deficient columns. Hence, in algorithm 2, b is fixed as one of the rank deficient columns (i.e. b is fixed as a multiple of β or $lcm(n, \beta)$) to reduce the computational complexity.

In algorithm 2, by fixing b as a multiple of β or $lcm(n, \beta)$, correct interleaver parameters are estimated successfully by searching the correct combination of $[N'_r, N'_c, \phi]$ or $[N'_r, N'_c, d', \phi]$, which gives maximum zero mean ratio value $\mu(b)$. This is because, setting b as a multiple of β (i.e. for the case when β is a multiple of n) or $lcm(n, \beta)$ (i.e. for the case when β is not a multiple of n) as stated in algorithm 2 is also equivalent to setting b as a multiple of n . Therefore, after de-interleaving with correct interleaver and synchronization parameters, the data symbols will be convolutional encoded. Since b is fixed as a multiple of n , rank deficiency will be obtained as mentioned in [3] and [4] after converting the data matrix S into column-echelon form F . It is to be noted that $\mu(b)$ will be higher for rank deficient matrix in erroneous scenario. Moreover, if the convolutional coded data symbols are de-interleaved with incorrect interleaver and synchronization parameters, it is highly impossible to obtain lesser deficient rank value compared to the rank value obtained from correct combination and hence, $\mu(b)$ will be lower for such cases. Thus, the correct combination of

335 $[N'_r, N'_c, \phi]$ or $[N'_r, N'_c, d', \phi]$, which gives maximum $\mu(b)$, has been chosen as the estimated interleaver parameters.

5.3. Computational Complexity

Here, the computational complexity of both the algorithms mentioned in the previous subsections are compared in terms of total number of search operations required to estimate the interleaver parameters. Let us assume the number of factors of $\frac{\zeta}{i}$ that satisfy $N'_r \times N'_c = \frac{\zeta}{i}$ is equal to M . Similarly, let us assume b is varied in the range $[b_{min}, b_{max}]$. In addition, as already mentioned in the algorithm, the starting bit position ϕ is also simultaneously varied in the range $[0, \zeta - 1]$. Hence, the number of search operations T_1 required for successful estimation of block interleaver parameters according to algorithm 1 is given by

$$T_1 = M \times \zeta \times (b_{max} - b_{min}) \quad (10)$$

Similarly, for helical scan interleaver, along with ϕ , the helical array step size d is also varied. Since $d < N_r$, it should be varied in accordance with N'_r . Hence, let us assume the number of variation is equal to $Q(N'_r)$. The total number of search operations T'_1 required for correct recognition of helical scan interleaver parameters based on algorithm 1 is given by

$$T'_1 = M \times \zeta \times Q(N'_r) \times (b_{max} - b_{min}) \quad (11)$$

In algorithm 2, b has been fixed as a multiple of ζ . Therefore, the total number of search operations for both block and helical scan interleavers (i.e. T_2 and T'_2) based on algorithm 2 are, respectively, given by

$$\begin{aligned} T_2 &= M \times \zeta \\ T'_2 &= M \times \zeta \times Q(N'_r) \end{aligned} \quad (12)$$

340 Clearly, from (10), (11), and (12), it has been inferred that T_2 and T'_2 are much smaller compared to T_1 and T'_1 . Hence algorithm 2 is computationally more efficient compared to algorithm 1. However, the main advantage of algorithm 1 is the estimation of n along with other interleaver parameters. Hence,

Figure 4: Variation of rank with number of columns (b) for block interleaver ($C(2, 1, 7)[133, 171]$, $N_r=3$, $N_c=3$, $BER=0$)

Table 1: Block interleaver after de-interleaving for $C(2, 1, 7)[133, 171]$ with $BER=0$

N'_r	N'_c	δ
2	9	18
3	3	2
3	6	18
6	3	18
9	2	18

the estimation of n from algorithm 1 is useful for validation. Note that n has
345 been already estimated from the procedure mentioned in Section 4 and it cannot be estimated using algorithm 2, since fixed value for number of columns is assumed. Therefore, if algorithm 2 is taken into consideration, then the convolutional code parameters including n can be estimated using code parameters estimation block shown in Fig. 3 after the estimation of interleaver parameters.

350 6. Simulation Results and Discussions

Consider a transmission system with an interleaver together with a convolutional encoder, which is denoted as $C(n, k, K)[g_1^1, \dots, g_i^j, \dots, g_k^n]$, where n, k , and K represent the number of outputs, number of inputs, and constraint length, respectively, and g_i^j denotes the octal representation of generator polynomial between input i and output j . Furthermore, the threshold values Γ_2^{th} and Γ_3^{th} are associated with $\mu'(b)$ and $\sigma(c)$, respectively. It is to be noted that for erroneous scenario, the number of zeros or ones in a particular column of S decide whether the column is dependent or not. Moreover, if the column is dependent, then the number of zeros is relatively higher compared to number of ones; and if the column is independent, then the probability that the particular column contains equal number of zeros and ones is high. So the number of zeros in a rank deficient data matrix is higher compared to a full rank data matrix and approximately equal number of ones and zeros can be observed from a full rank data matrix. Therefore, to distinguish between dependent and independent columns or between rank deficient and full rank data matrices, the threshold values associated with $\mu'(b)$ and $\sigma(c)$ should be more than 0.5. For our simulations we have chosen $\Gamma_3^{th} = 0.55$ to distinguish between dependent and independent columns, and $\Gamma_2^{th} = 0.7$ to distinguish between rank deficient and full rank data matrices. In addition, the justification for choosing Γ_3^{th} and Γ_2^{th} is explained using Fig. 5, Fig. 8, Fig. 12, and Fig. 17.

Firstly, in Fig. 4, rank deficiency curve before de-interleaving was plotted for convolutional encoded block interleaver assuming $C(2, 1, 7)[133, 171]$, $N_r = 3$ and $N_c = 3$ considering synchronized and error-free cases to estimate β or $lcm(n, \beta)$. Since $\beta \neq \gamma \times n$, from the figure it is inferred that the difference between successive number of columns with rank deficiency is equal to 18 (i.e. $lcm(n, \beta)$) according to proposition 2. In addition, the rank values plotted against b do match with (1). Moreover, from the slope of the rank deficiency curves, n and r are estimated according to (7). After de-interleaving the interleaved data with all possible values of N_r' and N_c' according to algorithm 1, it is observed from

380 Table 1 that the difference between successive number of columns with rank deficiency (i.e. δ) for $N'_r=3$ and $N'_c=3$ is minimum and is equal to n . Hence, the interleaver parameters are estimated using algorithm 1.

In Fig. 5(a) and Fig. 5(b), the histogram plots for mean value of number of zeros $\sigma(c)$ in each column for two different values of b (i.e. $b_1 = 36$ and
385 $b_2 = 54$) considering block interleaver are shown for $C(2, 1, 5)[23, 35]$ assuming $N_r = 3$ and $N_c = 3$ with $BER = 10^{-2}$ to calculate r and n . These two values of b are fixed as multiples of $lcm(n, \beta)$ as mentioned in Section 4. From Fig. 5(a), it can be observed that the value of $\sigma(c)$ for 22 columns out of 36 columns is in between 0.5 to 0.52. Furthermore, it has also been observed that the
390 mean value of number of zeros $\sigma(c)$ is greater than 0.65 or 65% for the rest of 14 columns. Since $\sigma(c)$ for dependent columns will be higher compared to independent columns for erroneous scenario, 14 columns with $\sigma(c)$ greater than 0.65 can be categorized as dependent columns. It is to be noted that these 26 columns should have been all-zero columns over error-free scenario. In addition,
395 if a particular column is independent, then it is logical to assume that the probability of mean value of number of zeros or ones in that particular column will be around 0.5. Therefore, from the histogram plots, it can be inferred that by setting a safe threshold value Γ_3^{th} between 0.53 to 0.65, dependent and independent columns can be segregated. Similarly, from Fig. 5(b), it can be
400 inferred that $\sigma(c)$ for 31 columns out of 54 columns is in between 0.5 to 0.52 and for the rest of the 23 columns, $\sigma(c)$ is greater than 62%. Therefore, 31 columns out of 54 columns can be classified as independent columns by fixing a safe threshold value Γ_3^{th} between 0.53 to 0.6. It has also been observed that for the rest of rank deficient columns, the value of $\sigma(c)$ considering all the independent
405 columns is observed to be around 0.5 and for the dependent columns it is greater than 0.6. Now the number of independent columns ($N_{ind}(b_1)$ and $N_{ind}(b_2)$) or rank values have been calculated from (8), Fig. 5(a), and Fig. 5(b) and are shown in Table 2. From the tabulated values, the code rate according to (9) is given by $r = \frac{31-22}{54-36} = \frac{1}{2}$ and from the denominator, codeword length is given
410 by $n = 2$. Note that for estimating rest of the interleaver parameters using

(a)

(b)

Figure 5: (a) Histogram plot for $\sigma(c)$ considering block interleaver, $N_r = 3$, $N_c = 3$, $b_1 = 36$, $C(2, 1, 5)[23, 35]$, and $\text{BER} = 10^{-2}$. (b) Histogram plot for $\sigma(c)$ considering block interleaver, $N_r = 3$, $N_c = 3$, $b_2 = 54$, $C(2, 1, 5)[23, 35]$, and $\text{BER} = 10^{-2}$

Table 2: Rank calculation for erroneous scenario

No. of Columns	No. of Dependent Columns	Rank
36	14	$N_{ind}(36) = 22$
54	23	$N_{ind}(54) = 31$

algorithm 2, estimation of n is essential as mentioned before in Section 4 and as shown in Fig. 3.

In Fig. 6(a), the normalized zero mean ratio $\mu'(b)$ versus number of columns b before de-interleaving is shown for $C(2, 1, 7)[133, 171]$ assuming $N_r = 3$ and $N_c = 3$ with $BER = 10^{-2}$ and delay $\chi = 7$, where χ represents the bit or symbol position with which the receiver starts receiving the data. From the plot, it has been inferred that the difference between successive number of columns with $\mu'(b)$ greater than Γ_2^{th} (i.e. $\delta > \Gamma_2^{th}$) is equal to 18, where $\Gamma_2^{th} = 0.7$. Since β is not a multiple of n , $\delta > \Gamma_2^{th}$ is equal to $lcm(n, \beta)$ according to proposition 2. In Fig. 6(b), the zero mean ratio $\mu(b)$ plot is shown for the same case after de-interleaving. According to algorithm 2, the interleaved data is de-interleaved with $[N'_r, N'_c] = \{[6, 3], [3, 6], [9, 2], [2, 9], [3, 3]\}$ by fixing b as a multiple of $lcm(n, \beta)$ and also by simultaneously varying the starting bit position ϕ from 0 to 17. Among all possible combinations, it has been observed from the figure that for $N'_r = 3$, $N'_c = 3$, and $\phi = 12$, $\mu(b)$ is maximum and hence, the block interleaver parameters are successfully estimated using the proposed algorithm.

Fig. 7(a) and Fig. 7(b) show the zero mean ratio plots before and after de-interleaving, respectively, for helical scan interleaver with $N_r = 7$, $N_c = 4$, $d = 4$, $BER = 10^{-2}$, and $\chi = 8$ considering $C(4, 1, 7)[133, 171, 117, 165]$. From Fig. 7(a), it is inferred that $\delta > 0.7$ is equal to 28. Since $\beta = \gamma \times n$, $\delta > 0.7$ is equal to β according to proposition 1. By de-interleaving with all possible factors of $\frac{28}{i}$, where $i = 1 : 4$, together with varying d' from 1 to $N'_r - 1$ and ϕ from 0 to 27, it is noticed from Fig. 7(b) that for $N'_r = 7$, $N'_c = 4$, $d' = 4$, and $\phi = 21$, $\mu(b)$ reaches maximum. Therefore, the helical scan interleaver parameters are recognized successfully using algorithm 2.

(a)

(b)

Figure 6: (a) Variation of normalized zero mean ratio with b for block interleaver $(C(2, 1, 7)[133, 171], BER = 10^{-2}, N_r = 3, N_c = 3, \chi = 7)$. (b) Variation of zero mean ratio with all possible values of $[N'_r, N'_c, \phi]$ for block interleaver $(C(2, 1, 7)[133, 171], BER = 10^{-2}, N_r = 3, N_c = 3, \chi = 7)$

(a)

(b)

Figure 7: (a) Variation of normalized zero mean ratio with b for helical scan interleaver ($C(4, 1, 7)[133, 171, 117, 165]$, $BER = 10^{-2}$, $N_r = 7$, $N_c = 4$, $d = 4$, $\chi = 8$). (b) Variation of zero mean ratio with all possible values of $[N'_r, N'_c, \phi, d']$ for helical scan interleaver ($C(4, 1, 7)[133, 171, 117, 165]$, $BER = 10^{-2}$, $N_r = 7$, $N_c = 4$, $d = 4$, $\chi = 8$)

Table 3: Rank calculation for erroneous scenario

No. of Columns	No. of Dependent Columns	Rank
28	12	$N_{ind}(28) = 16$
56	33	$N_{ind}(56) = 23$

In Fig. 8(a) and Fig. 8(b), the histogram plots for $\sigma(c)$ are shown for two different values of b (i.e. $b_1 = 28$ and $b_2 = 56$) considering helical scan interleaver assuming $C(4, 1, 7)[133, 171, 117, 165]$, $N_r = 7$, $N_c = 4$, and $d = 4$ with $BER = 10^{-2}$. As mentioned in Section 4, b_1 and b_2 are fixed as multiples of β . From Fig. 8(a), it can be observed that the value of $\sigma(c)$ for 16 columns out of 28 is in between 0.5 to 0.53 and hence, they can be categorized as independent columns. Furthermore, for the rest of 12 columns, $\sigma(c)$ is greater than 0.70. As more than 70% of the elements are zeros, they can be segregated as dependent columns. Note that these 12 columns should have been all-zero columns, if no errors were introduced. Hence, from the histogram plots, it can be inferred that by setting a safe threshold value Γ_3^{th} between 0.54 to 0.68, dependent and independent columns can be classified. Similarly, from Fig. 8(b), it can be inferred that $\sigma(c)$ for 23 columns out of 56 columns is in between 0.5 to 0.53 and for the rest of the 33 columns, $\sigma(c)$ is greater than 66%. Therefore, 33 columns out of 56 can be classified as dependent columns by fixing Γ_3^{th} between 0.54 to 0.65. Now the number of independent columns ($N_{ind}(b_1)$ and $N_{ind}(b_2)$) or rank values have been calculated from (8), Fig. 8(a), and Fig. 8(b) and are shown in Table 3. From the tabulated values, the code rate according to (9) is given by $r = \frac{23-16}{56-28} = \frac{1}{4}$ and from the denominator, codeword length is given by $n = 4$.

From the histogram plots of Fig. 5, Fig. 8, and from the simulation studies of other test cases, it has been identified that Γ_3^{th} can be safely fixed within 0.55 to 0.6 for categorizing the dependent and independent columns to evaluate rank and codeword length n . Hence, we fixed $\Gamma_3^{th} = 0.55$ for all the test cases to evaluate n .

In Fig. 9(a) and Fig. 9(b), the plots are shown for $C(3, 1, 4)[13, 15, 17]$

(a)

(b)

Figure 8: (a) Histogram plot for $\sigma(c)$ considering helical scan interleaver assuming $b_1 = 28$, $C(4, 1, 7)[133, 171, 117, 165]$, $N_r = 7$, $N_c = 4$, $d = 4$, and $\text{BER} = 10^{-2}$. (b) Histogram plot for $\sigma(c)$ considering helical scan interleaver assuming $b_2 = 56$, $C(4, 1, 7)[133, 171, 117, 165]$, $N_r = 7$, $N_c = 4$, $d = 4$, and $\text{BER} = 10^{-2}$

assuming helical scan interleaver with $N_r = 5$, $N_c = 4$, $d = 3$, $BER = 10^{-2}$, and $\chi = 26$. From Fig. 9(a), it is observed that $\delta > \Gamma_2^{th}$ is equal to 60, where
465 $\Gamma_2^{th} = 0.7$. According to proposition 2, since $\beta \neq \gamma \times n$, $\delta > \Gamma_2^{th}$ is equal to $lcm(n, \beta)$. Furthermore, from Fig. 9(b), it is observed that the zero mean ratio curve reaches maximum for $N'_r = 5$, $N'_c = 4$, and $d' = 3$, which exactly matches the assumed helical scan interleaver parameters at the transmitter.

Fig. 10(a) and Fig. 10(b) show the variation of zero mean ratio before and
470 after interleaver de-interleaving, respectively, assuming $C(3, 1, 7)[133, 165, 171]$, $BER = 10^{-2}$, $N_r = 10$, $N_c = 9$, $\chi = 5$. It is to be observed from Fig. 10(a) that the difference between two successive columns with $\mu'(b)$ greater than Γ_2^{th} gives the correct estimate of much longer interleaver period $\beta = 90$. In addition, by de-interleaving with all possible combinations of N'_r and N'_c as given in
475 algorithm 2, it has been inferred from Fig. 10(b) that for $[N'_r, N'_c, \phi] = [10, 9, 86]$, $\mu(b)$ reaches maximum and hence, the correct estimate of block interleaver and synchronization parameters are obtained.

In Fig. 11(a) and Fig. 11(b), the zero mean ratio curves are plotted before and after de-interleaving, respectively, considering $C(2, 1, 3)[5, 7]$ with puncturing
480 pattern P , which is given by $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$, assuming helical scan interleaver, $N_r = 4$, $N_c = 3$, $d = 3$, $BER = 10^{-2}$, and $\chi = 9$. After puncturing, the equivalent punctured codeword length (n_p) and number of input bits (k_p) are given by 3 and 2, respectively. Usually, the number of ones in P indicates n_p , number of columns in P indicates k_p , and $r_p = \frac{k_p}{n_p}$ denotes the equivalent code rate after
485 puncturing [22]. From the normalized zero mean ratio plot, it is inferred that $\delta > \Gamma_2^{th}$ is equal to 12 (i.e. β). Furthermore, findings from Fig. 11(b) illustrate that after de-interleaving with all possible combinations of N'_r , N'_c , and d' , the interleaver parameters $N_r = 4$, $N_c = 3$, and $d = 3$ along with synchronization parameter $\phi = 4$ have been detected successfully using the proposed algorithm.

490 In Fig. 12(a) and Fig. 12(b), the histogram plots for $\sigma(c)$ are given considering $b_1 = 24$ and $b_2 = 36$ assuming helical scan interleaver, $C(2, 1, 3)[5, 7]$, $P = [1; 1; 0; 1]$, $N_r = 4$, $N_c = 3$, and $d = 3$ with $BER = 10^{-2}$. As mentioned before,

(a)

(b)

Figure 9: (a) Variation of normalized zero mean ratio with b for helical scan interleaver $(C(3, 1, 4)[13, 15, 17])$, $BER=10^{-2}$, $N_r=5$, $N_c=4$, $d=3$, $\chi=26$. (b) Variation of zero mean ratio with all possible values of $[N_r', N_c', \phi, d']$ for helical scan interleaver $(C(3, 1, 4)[13, 15, 17])$, $BER=10^{-2}$, $N_r=5$, $N_c=4$, $d=3$, $\chi=26$

(a)

(b)

Figure 10: (a) Variation of normalized zero mean ratio with b for block interleaver $(C(3, 1, 7)[133, 165, 171])$, $BER = 10^{-2}$, $N_r = 10$, $N_c = 9$, $\chi = 5$. (b) Variation of zero mean ratio with all possible values of $[N'_r, N'_c, \phi]$ for block interleaver $(C(3, 1, 7)[133, 165, 171])$, $BER = 10^{-2}$, $N_r = 10$, $N_c = 9$, $\chi = 5$

(a)

(b)

Figure 11: (a) Variation of normalized zero mean ratio with b for helical scan interleaver ($C(2, 1, 3)[5, 7]$, $BER=10^{-2}$, $N_r=4$, $N_c=3$, $d=3$, $\chi=9$, $P = [1;1;0;1]$). (b) Variation of zero mean ratio with all possible values of $[N'_r, N'_c, \phi, d']$ for helical scan interleaver ($C(2, 1, 3)[5, 7]$, $BER=10^{-2}$, $N_r=4$, $N_c=3$, $d=3$, $\chi=9$, $P = [1;1;0;1]$)

Table 4: Rank calculation for erroneous scenario

No. of Columns	No. of Dependent Columns	Rank
24	6	$N_{ind}(28) = 18$
36	10	$N_{ind}(56) = 26$

the dependent and independent columns can be classified by fixing $\Gamma_3^{th} = 0.55$ and the same are shown in Table 4. From the tabulated values, the punctured
495 code rate according to (9) is given by $r_p = \frac{26-18}{36-24} = \frac{2}{3}$ and from the denominator, punctured codeword length is given by $n_p = 3$. Note that since $C(2, 1, 3)[5, 7]$ is punctured using $P = [1; 1; 0; 1]$, instead of estimating parent code rate $r = 0.5$ and codeword length $n = 2$, the proposed algorithm in Section 4 recognizes punctured code parameters, which are given by $r_p = \frac{2}{3}$ and $n_p = 3$. It is also
500 to be noted that the proposed algorithms for estimating interleaver period and rest of interleaver parameters cannot be used to estimate puncturing pattern P and dedicated algorithms for estimating puncturing pattern and parent convolutional codes are given in [22].

In Fig. 13(a) and Fig. 13(b), the zero mean ratio curves are plotted before
505 and after de-interleaving, respectively, considering $C(2, 1, 3)[5, 7]$ assuming block interleaver with puncturing pattern P , which is given by $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$, $N_r = 5$, $N_c = 4$, $BER = 10^{-2}$, and $\chi = 10$. After puncturing, the equivalent punctured code parameters are given by $n_p = 5$ and $k_p = 4$. From the normalized zero mean ratio plot, it is inferred that the difference between successive columns with
510 $\mu'(b)$ greater than Γ_2^{th} is equal to 20. Here, the interleaver period β is a multiple of n_p and hence, the difference between successive number of columns with $\mu'(b)$ greater than Γ_2^{th} is equal to β according to proposition 1. Furthermore, findings from Fig. 13(b) illustrate that after de-interleaving with all possible combinations of N'_r and N'_c , the zero mean ratio curve reaches maximum for
515 the case when $N_r = 5$ and $N_c = 4$ along with synchronization parameter $\phi = 11$ and thus, the interleaver and synchronization parameters have been recognized successfully using the proposed algorithm.

(a)

(b)

Figure 12: (a) Histogram plot for $\sigma(c)$ considering helical scan interleaver assuming $N_r = 4$, $N_c = 3$, $d = 3$, $b_1 = 24$, $C(2, 1, 3)[5, 7]$, $P = [1; 1; 0; 1]$, and $\text{BER} = 10^{-2}$. (b) Histogram plot for $\sigma(c)$ considering helical scan interleaver assuming $N_r = 4$, $N_c = 3$, $d = 3$, $b_2 = 36$, $C(2, 1, 3)[5, 7]$, $P = [1; 1; 0; 1]$, and $\text{BER} = 10^{-2}$

(a)

(b)

Figure 13: (a) Variation of normalized zero mean ratio with b for block interleaver $(C(2, 1, 3)[5, 7], BER=10^{-2}, N_r=5, N_c=4, \chi=10, P = [1; 1; 0; 1; 0; 1; 0; 1])$. (b) Variation of zero mean ratio with all possible values of $[N'_r, N'_c, \phi]$ for block interleaver $(C(2, 1, 3)[5, 7], BER=10^{-2}, N_r=5, N_c=4, \chi=10, P = [1; 1; 0; 1; 0; 1; 0; 1])$

Table 5: Test code set

No.	Code rate (r)	n	k	K	g_i^j
1	1/2	2	1	3	[5,7]
2				4	[15,17]
3				5	[23,35]
4				6	[65,57]
5				6	[75,53]
6				7	[133,171]
7				8	[345,237]
8				9	[561,753]
9				10	[1167,1545]
10				11	[2335,3661]
11	1/3	3	1	4	[13,15,17]
12				7	[133,165,171]
13				11	[2353,2671,3175]
14	1/4	4	1	7	[133,171,117,165]
15				10	[1117,1365,1633,1653]
16	2/3	3	2	[5,4]	[23 35 0; 0 5 13]

In Fig. 14(a), the interleaver period estimation for block interleaver is shown in the presence of burst error of length equal to 500. Similarly, in Fig. 14(b), the interleaver period estimation is shown for helical scan interleaver in the presence of random burst error of maximum burst length equal to 500. The simulations are carried out with $\text{BER}=10^{-2}$ considering $C(3, 1, 11)[2353, 2671, 3175]$ and $C(3, 2, [5, 4])[23\ 35\ 0; 0\ 5\ 13]$ in Fig. 14(a) and Fig. 14(b), respectively. From Fig. 14(a) and Fig. 14(b), observe the difference between successive columns with normalized zero mean ratio greater than $\Gamma_2^{th} = 0.7$ similar to Fig. 6(a), 7(a), 9(a), 10 (a), 11(a), and 13(a) and the same gives the estimate of interleaver period β , which is equal to 18 and 15, respectively.

Apart from the given simulation results, the interleaver parameter estimation algorithms are also tested successfully for different code rates and constraint lengths listed in Table 5.

In Fig. 15(a) and Fig. 15(b), the accuracy of estimation of interleaver period is shown for both the interleavers considering two different values of constraint length K assuming $N_r = 4$ and $N_c = 3$ with helical array step size $d = 3$. Note that the results are shown assuming $a = 2 \times b$ and $a = 10 \times b$. From the plots

(a)

(b)

Figure 14: (a) Variation of normalized zero mean ratio with b for block interleaver ($C(3, 1, 11)[2353, 2671, 3175]$, $N_r = 6$, $N_c = 3$, $BER = 10^{-2}$, delay $\chi = 4$, constant burst length=500). (b) Variation of normalized zero mean ratio with b for helical scan interleaver ($C(3, 2, [5, 4]) [23 \ 35 \ 0; 0 \ 5 \ 13]$, $N_r = 5$, $N_c = 3$, $d = 4$, $BER = 10^{-2}$, delay $\chi = 8$, maximum burst length=500)

535 given in Fig. 15(a) and Fig. 15(b), it is inferred that the accuracy of estimation
 increases with decrease in the value of K for both the interleavers. This is
 because as K increases, the number of memory elements also increases, which
 in turn increases the minimum number of columns resulting in rank deficiency.
 Hence, the rank deficiency for a particular column is not as low as similar to that
 540 of the case considering lower K values. Therefore, when the BER is assumed to
 be equal to 10^{-2} , the difference between successive columns with rank deficiency
 could not be estimated properly for higher constraint length case and there is a
 need to increase the data size or number of row values accordingly. Hence, by
 increasing $a = 10 \times b$, it is observed from Fig. 15(a) and Fig. 15(b) that the
 545 accuracy of estimation could be improved.

The performance estimation shown in Fig. 15(a) and Fig. 15(b) is given
 with respect to the estimation of interleaver period β or $lcm(n, \beta)$ alone. It
 has been identified that once β or $lcm(n, \beta)$ estimation is correct, rest of the
 interleaver parameters can be estimated with 100% accuracy using algorithm
 1 and 2. However, if β or $lcm(n, \beta)$ estimation is not correct, then rest of the
 550 interleaver parameters cannot be estimated correctly, since β or $lcm(n, \beta)$ is an
 input parameter for the proposed algorithms. Further, this is also because of
 the fact that the proposed algorithms (algorithm 1 and 2) for estimating rest of
 the interleaver parameters such as N_r , N_c , and d are also based on zero mean
 555 ratio $\mu(b)$ computation similar to the methodology proposed for estimating β
 or $lcm(n, \beta)$.

In the case of algorithm 1, while de-interleaving with all possible combina-
 tions of interleaver parameters, $\mu(b)$ is computed by varying b . However, $\mu(b)$ is
 computed by fixing b as a multiple of β or $lcm(n, \beta)$ in algorithm 2. It has been
 560 observed from the simulation studies that the estimation performance of both
 the algorithms are equal, as there is not much difference between the proposed
 methodologies except computational complexity, which is given in Section 5.3.

In Fig. 16(a) and Fig. 16(b), the plots are shown comparing the performance
 of the proposed block and helical scan interleaver period estimation algorithm
 565 with the estimation methodologies discussed in [13] and [17], respectively. Note

(a)

(b)

Figure 15: (a) Performance of block interleaver ($C(3, 1, 4)[13, 15, 17]$, $C(3, 1, 7)[133, 165, 171]$, $N_r=4$, $N_c=3$). (b) Performance of helical scan interleaver ($C(2, 1, 3)[5, 7]$, $C(2, 1, 7)[133, 171]$, $N_r=4$, $N_c=3$, $d=2$)

(a)

(b)

Figure 16: (a) Performance comparison of the proposed algorithm with the algorithm proposed in [13] for block interleaver ($C(3, 1, 4)$ [13, 15, 17], $N_r = 4$, $N_c = 3$). (b) Performance comparison of the proposed algorithm with the algorithm proposed in [17] for helical scan interleaver ($C(2, 1, 3)$ [5, 7], $N_r = 4$, $N_c = 3$, $d = 2$)

that the proposed algorithm in [13] is similar to the one proposed in [14] and the estimation is based on evaluating the number of ones in the lower part of column echelon form F . Further, the estimation methodology in [17] is based on evaluating normalized ratio of number of ones in each row of row echelon form.

Both the interleaver period estimation algorithms proposed in [13] and [17] are simulated for rate 1/2 and 1/3 convolutional codes. From the figures, it can be observed that the proposed algorithm substantially outperforms the algorithms proposed in the literature. For example, considering the block interleaver period estimation performance in Fig. 16(a), it can be observed that for $\text{BER} = 5 \times 10^{-2}$, the proposed algorithm performs with 80% accuracy. However, the algorithm proposed in [13] fails to estimate the correct interleaver period for the same BER value. Similarly, from Fig. 16(b) it can be observed that the proposed algorithm for the case when $\text{BER} = 4 \times 10^{-2}$ performs with 60% accuracy. However, the algorithm proposed in [17] fails to recognize the helical scan interleaver period for the same case.

It is also observed that the algorithm proposed in [19] for the estimation of convolutional interleaver period works with 100% accuracy for the scenario when $\text{BER} \leq 6 \times 10^{-3}$. However, our algorithm can estimate the interleaver period with 100% accuracy for all the cases when $\text{BER} \leq 3 \times 10^{-2}$. Thus, the proposed algorithm performs much better in the presence of higher bit error rate than the methods suggested in the prior works.

In Fig. 17, the accuracy of estimation versus Γ_2^{th} is shown for three different BER values considering block interleaver. From the plots, it can be observed that the interleaver period can be estimated with 100% accuracy for BER values ranging from 2×10^{-2} to 4×10^{-2} provided $\Gamma_2^{th} \in [0.5, 0.7]$. Moreover, for the case when $\text{BER} = 10^{-2}$, it can be observed from Fig. 6(a), 7(a), 9(a), 10 (a), 11(a), 13(a), 14(a), and 14(b) that Γ_2^{th} can be chosen between 0.3 to 0.7 to estimate β or $lcm(n, \beta)$. Hence, a safe threshold value of $\Gamma_2^{th} = 0.7$, which satisfies four different BER values (i.e. from 10^{-2} to 4×10^{-2}) is chosen to estimate interleaver period. Further, for all the test cases given in Table 5, it has been observed from the simulation studies that for the case when $\text{BER} \leq 3 \times 10^{-2}$, the block

Figure 17: Accuracy of estimation versus Γ_2^{th} for three different BER values assuming block interleaver ($C(3, 1, 4)$ [13, 15, 17], $N_r = 4$, and $N_c = 3$)

and helical scan interleaver parameters can be successfully recognized provided $\Gamma_2^{th} \in [0.5, 0.7]$.

7. Conclusion

In this paper, the parameter estimation of block and helical scan interleavers are carried out for convolutional encoded erroneous and non-synchronized data. Firstly, a generic block diagram explaining the parameter estimation process is given. Based on the generic block diagram, interleaver period, codeword length, and code rate parameters are estimated for a generalized case without any restriction being imposed on codeword length and interleaver period. Furthermore, computationally efficient algorithms are proposed for recognizing rest of the parameters like number of rows and columns of interleaver matrix and helical array step size. From the simulation results, different values of interleaver parameters along with number of bits to be shifted for achieving proper synchronization have been estimated successfully in a robust environment in

the presence of significant bit errors. Moreover, interleaver and synchronization parameters have also been identified correctly over burst error scenario and also from the punctured convolutional codes. From the accuracy of estimation plots, it is observed that the probability of correct detection of interleaver parameter improves with decrease in the constraint length as well as increase in the number of rows of data matrix taken into consideration. Finally, from the performance comparison plots, it has been identified that the proposed algorithm based on zero mean ratio computation outperforms other methodologies proposed in the literature.

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