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Hybrid Selection and Switch-and-Examine Combining Strategy for DF Relaying Systems Over Nakagami- m Fading Channels

Swaminathan R and Rajarshi Roy

Abstract

Distributed switched diversity combining has drawn significant attention in the recent past due to its low complexity nature in terms of channel state information (CSI) requirement at the receiving end to achieve full diversity order. In this paper, we propose a hybrid selection and switch-and-examine combining (HSSEC) scheme, which is a combination of selection combining (SC) and switch-and-examine combining (SEC) schemes, for a multi-relay decode-and-forward (DF) system to improve the performance of distributed switched diversity combining schemes proposed in the literature. Furthermore, the performance of HSSEC scheme is investigated over non-identical Nakagami- m fading channels. Average end-to-end symbol error probability (SEP) expression is derived for M -ary phase-shift keying (MPSK) signaling and in addition, asymptotic SEP expression is also derived to analyse diversity order. From the derived asymptotic expression, it is inferred that the HSSEC scheme attains full diversity order over Nakagami- m fading channels except for the case when threshold signal-to-noise ratio (SNR) is very much lesser than the average SNR. Furthermore, exact outage probability expression is derived for the HSSEC scheme. Finally, in the numerical results, the outage and SEP performances are compared with other schemes proposed in the literature.

Index Terms

Decode-and-forward (DF) relay, End-to-end symbol error probability (SEP), Hybrid selection and switch-and-examine combining (HSSEC), M -ary phase shift keying (MPSK), Nakagami- m fading, Outage probability, Switch-and-examine combining (SEC).

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I. INTRODUCTION

Diversity combining is one of the important techniques to mitigate the adverse effects of fading for a multiple-input-multiple-output (MIMO) system. Conventional diversity combining schemes such as selection combining (SC), maximal-ratio combining (MRC), equal-gain combining (EGC), etc. require channel state information (CSI) of all the links to achieve full diversity order. Since estimating the CSI of all the links is an arduous task, low complexity switched diversity combining schemes, which require less CSI, such as switch-and-stay combining (SSC), switch-and-examine combining (SEC), generalized SEC (GSEC), generalized sort, switch, and examine combining (GSSEC) had been proposed and the performance analysis was carried out in [1]-[5]. In the case of SSC scheme [1], the receiver switches and stays in the next branch only if the instantaneous signal-to-noise ratio (SNR) of the current branch falls below a threshold SNR γ_t . However, in the case of SEC scheme [1]-[3], if the instantaneous SNR of the current branch falls below γ_t , then the receiver examines the next branch. If its instantaneous SNR is above γ_t , then the receiver switches to the next branch, else it examines the subsequent branches. Out of N available branches, if the instantaneous SNR of $N - 1$ branches fail to satisfy the threshold SNR value, eventually the receiver switches to the N^{th} branch without examining. Next, in the case of GSEC scheme [4], out of N available branches, N_c branches (i.e. $N_c \leq N$) with instantaneous SNR value greater than γ_t will be selected unlike the SEC scheme and MRC will be performed at the receiver. In addition, the branch updation algorithm in GSEC scheme for a given switching period will replace the unaccepted branches (i.e. branches with SNR lesser than γ_t) with branches that satisfy the threshold SNR in no particular order. Finally, GSSEC [5] scheme is a modified version of GSEC scheme and the former varies compared to the later in branch updation process. Here, the GSSEC scheme sorts the unaccepted branches in decreasing order and starts replacing the branch which is having the lowest SNR with the one that satisfies the threshold SNR. The branch updation process will continue as mentioned before for the subsequent unaccepted branches. It was shown that all the proposed switched diversity

schemes require less CSI compared to the conventional diversity combining schemes. In this paper, we propose a new scheme which can be considered as a combination of SC and SEC schemes for cooperative diversity systems and the same has been termed as hybrid selection and switch-and-examine combining (HSSEC) scheme. Note that the HSSEC scheme is proposed for cooperative diversity scenario unlike the schemes explained previously for receiver diversity scenario. Moreover, the proposed scheme selects a particular path that satisfies the threshold SNR and neglects the remaining paths unlike GSEC or GSSEC scheme.

Cooperative diversity is also an important technique to mitigate the adverse effects of fading with the help of randomly distributed relay nodes. Moreover, it provides distributed spatial diversity to suppress the multipath fading effects with the help of relaying protocols such as decode-and-forward (DF), amplify-and-forward (AF), etc. [6]. In the case of DF relaying, the relay node decodes, re-encodes and forwards the received message signal from the source node to the destination node. In the case of AF relaying, the relay node amplifies and then forwards the amplified version of the received message signal from the source node to the destination node. If analog processing is affordable at relay nodes, then AF relaying can be preferred. Unfortunately, for mitigating the existing coupling effects in AF relaying systems, expensive RF chains are required [7]. This coupling effect, which distorts the received signal at the relay node, is mainly due to partial reception of the relay transmit antenna signal [8]. Hence, this motivates to choose DF-relaying-based cooperative system, which has garnered special attention due to its signal regeneration capability in contrast to AF relaying.

In [9] and [10], performance analysis of maximum-likelihood (ML) and piecewise linear (PL) decoding based single-DF-relay cooperative diversity system was studied and symbol error probability (SEP) expressions were derived considering coherent and non-coherent modulation schemes. In [11], capacity expression was derived for DF-relaying-based cooperative system over Rician fading channels. Furthermore, relay selection scheme based on the CSI of source-to-relay (S-R) link was proposed in [12] for DF cooperative system and bit error rate (BER) analysis of the

same was carried out over Gamma-Gamma fading free space optical (FSO) links. Conventional SC scheme proposed for receiver diversity scenario had been extended to DF-relaying-based cooperative diversity scenario and the SEP performances of partial and full CSI SC schemes over Rayleigh fading channels were investigated in [13] and [14], respectively. In addition, the capacity analysis of full and partial CSI SC schemes was carried out in [15] for Rayleigh fading scenario. Furthermore, performance analysis of hop-by-hop beamforming-and-combining-based dual-hop AF and DF MIMO relay systems with two antennas at each node over Nakagami- m fading channels was investigated in [16] and [17], respectively. In [18], SC-based path selection technique along with beamforming for DF relaying MIMO system had been proposed and the performance analysis of the same was studied over Nakagami- m fading channels.

Two new antenna selection strategies were proposed in [19] for two-way MIMO relay networks considering a single AF relay. After that the proposed strategies were extended to single-user and multi-user multi-AF-relay networks. The first proposed joint user, antenna, and relay selection criteria was based on maximizing the end-to-end SNR, thereby reducing the outage probability, whereas the second proposed selection criteria was based on maximizing the sum-rate of the AF relay networks. The performance of the proposed strategies in terms of outage probability was analyzed and from the high-SNR approximations, diversity gain was evaluated for all the strategies. Similarly, in [20], the best transmit/receive antenna pair for multi-user multi-DF-relay networks was selected based on end-to-end SNR. In addition, exact and asymptotic SEP and capacity expressions were derived for flat Rayleigh fading environment considering outdated CSI as well as correlation between the antennas. Note that in [19], the relay selection was based on maximizing the end-to-end SNR or sum-rate. Further, in [20], all the relays will forward the message signal to the destination node in orthogonal time slots. But the proposed relay selection scheme in our work is based on switched diversity combining scheme and it is different from the strategies proposed in [19] and [20]. Moreover, the current work is restricted to one-way relaying and all the nodes are employed with single transmit and receive antenna.

Subsequently, the switched diversity combining schemes proposed for the receiver diversity system have also been extended to the cooperative diversity system and are commonly termed as distributed switched diversity schemes. The performance analysis of a distributed SSC (DSSC) scheme for single-relay and two-relay systems was reported in [21]-[23]. In [21], the BER expression assuming binary phase-shift keying (BPSK) modulation and outage probability expression for the DSSC scheme considering DF relaying protocol were derived over Rayleigh fading channels. According to the DSSC scheme for a single-relay system, if the instantaneous SNR value of source-to-destination (S-D) link is greater than γ_t , then the destination will decode the information symbol from direct S-D link without choosing the relay. If the SNR value is lesser than γ_t , then irrespective of the SNR value of S-R and relay-to-destination (R-D) links, the message signal will be received and decoded from the relay node at the destination. An incremental relaying scheme similar to DSSC scheme was proposed in [22] for a single-DF-relay system and performance analysis of the same was carried out over Rayleigh fading channels by deriving BER and spectral efficiency expressions. In [21], it was assumed that the CSI of S-R link is known at the destination and the instantaneous SNR of S-R link was taken into consideration for formulating the switching decision rule. In contrast, instantaneous SNR of S-R link was not taken into consideration at the destination in [22]. But optimal switching threshold SNR γ_t^{opt} was derived incorporating the instantaneous SNR value of S-R link. In [23], the DSSC scheme was extended to a two-relay system and the outage and BER expressions over Rayleigh fading channels were derived for analyzing the performance. Moreover, the performance was studied for both the AF and DF relaying systems with and without the presence of direct S-D link and the DSSC scheme was used for selecting the best relay. With the presence of direct S-D link, the information symbols from S-D and source-to-relay-to-destination (S-R-D) links were combined with the help of simple MRC scheme. Since the combining weights were not evaluated using the CSI of S-R link, full diversity order was not achieved for the DF relaying case. In [24], outage analysis of two-way AF relay networks was investigated over Rayleigh fading channels

considering two relays and outage probability expression was derived in closed-form.

In [25], the DSSC scheme was proposed for the multi-DF-relay system and BER analysis was performed. Considering the DSSC scheme for the selection relay networks proposed in [25], if the instantaneous SNR value of direct S-D link is lesser than γ_t , then the instantaneous SNR values of all S-R and R-D links are evaluated and the destination will receive the information symbols from the relay node which is having the highest γ_i value, where $\gamma_i = \min(\gamma_{sr_i}, \gamma_{r_id})$, $i \in \{1, 2, \dots, N\}$, γ_{sr_i} and γ_{r_id} denote the instantaneous SNR values of source-to- i^{th} relay (S-R_{*i*}) and i^{th} relay-to-destination (R_{*i*}-D) links, respectively. If the SNR value of direct S-D link is greater than γ_t , then all the relays are omitted. Furthermore, the performance of the SEC-based multi-relay cooperative diversity system was studied in [26]-[28] for AF and DF relaying schemes over Rayleigh and Nakagami- m fading channels. In [28], the distributed SEC scheme was proposed in such a way that if direct S-D link fails to satisfy γ_t , then γ_1 was evaluated for the first relay. If $\gamma_1 < \gamma_t$, then end-to-end SNR of second relay (i.e. γ_2) was calculated. In a similar fashion, end-to-end SNR values till $N - 1^{th}$ relay will be evaluated and compared with γ_t . If all $N - 1$ relays fail to satisfy the threshold SNR criteria, then N^{th} relay will be selected by default. Note that if any one of the links (i.e. direct and relay links) satisfy the threshold SNR criteria, then all the other relay links will be omitted without examining the end-to-end SNR value. Hence, by neglecting the relay links in [28], CSI information requirement has been decreased at the destination, thereby reducing the complexity.

It has been observed that the DSSC scheme proposed in [23] combines the information symbols from S-D and S-R-D links using simple MRC. Since the combining weights did not contain the knowledge of S-R link at destination, full diversity order was not obtained as mentioned in [7]. Hence, to collect full diversity, recently, a hybrid selection and switch-and-stay combining (HSSSC) scheme was proposed in [29] and the performance analysis of the same for DF relaying was investigated over Rayleigh fading channels. The relay and link selection strategies for HSSSC scheme were based on SSC and SC schemes, respectively, considering

the knowledge of S-R link at destination. Findings from the numerical results illustrated that improvement in the SEP performance of the HSSSC scheme was obtained compared to the DSSC scheme [23], partial-CSI-based SC, and scaled selection combining schemes proposed in [13] and [30], respectively. Moreover, it was also shown that the HSSSC scheme requires less CSI at destination compared to the full CSI SC scheme [28]. However, the performance analysis of HSSSC scheme in [29] was restricted to two-relay system and Rayleigh fading scenario only.

In this paper, we propose a HSSEC scheme to improve the SEP performance of distributed SEC scheme proposed in our prior work [28] for a multi-relay scenario. The SEP performance of HSSEC scheme is carried out over Nakagami- m fading channels assuming M -ary phase shift keying (MPSK) modulation scheme. Furthermore, the relay selection methodology is based on the SEC scheme in contrast to the SSC scheme for two-relay system proposed in [29]. Therefore, the proposed scheme is called HSSEC scheme, which is a combination of SC and SEC schemes for link and relay selection, respectively. Note that the HSSSC scheme of [29] is a special and restricted case of HSSEC scheme for $N = 2$, where N is the number of relays, and Rayleigh fading scenario. Therefore, the scenario we do deal with is more general, practical, complicated, and mathematically rigorous compared to the scenario in [29]. In the current work, the performance analysis has been carried out over Nakagami- m fading channels in contrast to other fading channels and the reason is given as follows:

- Compared to Rayleigh, log-normal, or Rice distributions, Nakagami- m distribution is a better fit for some experimental data with greater accuracy and flexibility. To justify the statement, it has been shown in [31] that Nakagami- m distribution fits urban radio multipath channel data with better accuracy in comparison to Rayleigh or Rician distribution.
- It has been mentioned in [32] that a generalized distribution known as Nakagami- m distribution matches experimental data far better than Rayleigh or Rician distribution.
- It is a known fact that Nakagami- m distribution includes Rayleigh fading ($m=1$) and one-sided Gaussian distribution ($m=0.5$) as special cases. Furthermore, it has been mentioned

in [32] that Nakagami- m fading can be closely approximated to Rician fading for the case when $m > 1$.

- Finally, experimental observations in [33] has confirmed that both ionospheric and tropospheric modes of propagation can be modeled using Nakagami- m distribution with better accuracy.

The main contributions are listed as follows:

- Average end-to-end SEP expression is derived for the HSSEC scheme over slow, flat, independent, and non-identical Nakagami- m fading channels assuming MPSK signaling.
- Since it is difficult to obtain the closed-form expression for exact average SEP, upper bound expression for the same is derived in closed-form.
- From the upper bound expression, closed-form asymptotic SEP expression is derived and diversity order analysis is also performed.
- Monte-Carlo simulations are also carried out to validate the derived SEP expression.
- Exact outage probability expression is derived over non-identical Nakagami- m fading channels.
- The SEP and outage performances of the HSSEC scheme are also compared with DSSC scheme for two-relay networks [23], DSSC for selection relay networks (SSCSR), incremental relaying for selection relay networks (IRSR) proposed in [25], distributed SEC scheme, full CSI SC scheme proposed in [28], and PL-detection-based single-relay cooperative diversity system proposed in [9].

We present the system model of HSSEC scheme in section II. Furthermore, exact and asymptotic SEP expressions are derived in section III. In Section IV, exact outage probability analysis is investigated. Section V presents the numerical results and related discussions. Finally, concluding remarks are given in section VI.

II. SYSTEM MODEL

We consider N DF relay nodes (R_i , where $i = 1, 2, \dots, N$) cooperating with a single source (S) and destination (D) nodes, which are employed with single antenna, in transmitting the baseband information-bearing MPSK symbol s as shown in Fig. 1. In the first phase of symbol transmission, S transmits s to R_i and D. The received complex baseband symbol at R_i and D are, respectively, given by

$$r_{sr_i} = h_{sr_i}s + n_{sr_i}, r_{sd} = h_{sd}s + n_{sd}, \quad (1)$$

where h_{sr_i} and h_{sd} indicate the fading channel gains of S- R_i and S-D links, respectively. The additive white Gaussian noise (AWGN) samples at R_i and D are denoted by n_{sr_i} and n_{sd} , respectively. From the received complex baseband symbol at i^{th} DF relay, the transmitted MPSK symbol can be detected according to the following decision rule given by

$$\hat{s}_i = \left\{ \operatorname{argmax}_{s \in S'} \operatorname{Re}(s^* h_{sr_i}^* r_{sr_i}) \right\}, \quad (2)$$

where S' belongs to MPSK signal space and $i = 1, 2, \dots, N$. We assume R_k is chosen according to the HSSEC scheme. Therefore, in the orthogonal phase of symbol transmission, R_k will forward the detected MPSK symbol $\hat{s}_k$ to D and the received complex baseband symbol at D is given by

$$r_{r_kd} = h_{r_kd}\hat{s}_k + n_{r_kd}, \quad (3)$$

where h_{r_kd} and n_{r_kd} denote the fading channel gain of R_k -D link and AWGN noise sample at D, respectively.

The fading channel coefficient h_j , where $j \in \{sd, sr_i, r_id\}$, is modeled as $2m$ dimensional column vector following zero-mean Gaussian distribution and the norm of h_j follows Nakagami- m distribution. Furthermore, the AWGN noise sample n_j follows complex Gaussian distribution with zero-mean and variance $2N_0$. Note that all the fading channel coefficients and AWGN noise samples are mutually independent and are independent of each other. The instantaneous

and average SNR values are, respectively, given by

$$\gamma_j = \frac{E_s |h_j|^2}{N_0}, E[\gamma_j] = \Gamma_j = \frac{E_s \Omega_j m_a}{N_0}, \quad (4)$$

where $E[\cdot]$ denotes the expectation operator, $|\cdot|$ denotes the norm operator, $a \in \{1, 2i, 3i\}$, and m_1, m_{2i} , and m_{3i} indicate the fading severity parameters of S-D, R_i-D, and S-R_i links, respectively. Since the norm of h_j follows Nakagami- m distribution, γ_j follows gamma distribution.

A. HSSEC relay and link selection

In this subsection, steps involved to implement the HSSEC scheme in practice are discussed followed by discussions related to other existing schemes.

- *Step 1:* Firstly, S broadcasts an ready-to-send (RTS) packet to all the relay nodes in the guard period before each transmission phase.
- *Step 2:* The relay nodes after receiving the packet starts a timer.
- *Step 3:* Assume that the timer of relay node R_l gets expired first. Then the same node updates the value of its counter $l = l + 1$. Moreover, if $l \neq N$, then CSI of S-R link (i.e. h_{sr_l}) is estimated by R_l and the same information is transmitted along with RTS packet to D.
- *Step 4:* Meanwhile, all the other relay nodes by overhearing the RTS packet from another relay node update the value of their counters $l = l + 1$ and will enter into listening mode.
- *Step 5:* After receiving the RTS packet, CSI of R-D link (i.e. $h_{r_l d}$) is estimated by D and $\gamma_l = \min(\gamma_{sr_l}, \gamma_{r_l d})$ is evaluated.
- *Step 6:* If $\gamma_l > \gamma_t$, then a positive acknowledgement (ACK) is sent to R_l by D indicating that the same node has been selected for second phase transmission. Moreover, a positive flag message is sent by R_l to other nodes. All the other relay nodes, which are in the listening mode, will enter into idle mode after receiving the flag message.
- *Step 7:* If $\gamma_l < \gamma_t$, then a negative ACK is sent to R_l by D. Further, a negative flag message

is sent by R_l to rest of the relay nodes to start their timer. The relay node whose timer gets expired next will execute from Step 3 to Step 7.

- *Step 8:* If $l = N$ in Step 3, then it indicates all the other relay nodes except N^{th} relay node has failed to meet the threshold criteria. Therefore, N^{th} relay node will be selected without estimating the CSI and the same sends a positive flag message to rest of the nodes.
- *Step 9:* After selecting the cooperative path (assuming S- R_k -D link is chosen), it is compared with the direct S-D link according to the decision rule $\max(\gamma_{sd}, \min(\gamma_{sr_k}, \gamma_{r_kd}))$.
- *Step 10:* Finally, after selecting S- R_k -D or S-D link, D asks S to start the transmission phase.

In the case of SEC scheme proposed in [28], S-D link will not be compared with S- R_k -D link similar to the HSSEC scheme according to the decision rule given in Step 9. If $\gamma_{sd} > \gamma_t$, then the SEC scheme in [28] neglects S-R-D link. But in the case of HSSEC scheme, after selecting the best cooperative path, SC is performed according to the selection rule given in Step 9 to improve the SEP performance compared to the SEC scheme. Furthermore, in the case of full CSI SC scheme, the CSI of all S-R links will be estimated at the relay nodes after receiving the RTS packets. Moreover, the estimated CSI of S-R link along with another RTS packet will be sent to D by the relay nodes in orthogonal time slots to avoid interference. From the received RTS packets, D estimates the CSI of all R-D links and evaluate the decision rule $D' = \max(\gamma_{sd}, \max_{i \in 1, \dots, N} (\min(\gamma_{sr_i}, \gamma_{r_id})))$ to select either S-D or S-R-D link. Here, we can observe that the full CSI SC scheme is not spectrally efficient, since it requires CSI of all the links to be estimated in the guard period and estimating the same involves high complexity. However, in the case of proposed HSSEC scheme, estimating the CSI of all the links is not mandatory at every instant of time in the guard period. Either S-D link or a particular S-R-D link satisfying the threshold SNR criteria will be selected in the guard period and the transmission phase will begin. Hence, the proposed scheme is spectrally efficient compared to the full CSI SC scheme.

III. PERFORMANCE ANALYSIS

In this section, we derive the average end-to-end SEP of HSSEC scheme by individually adding the SEPs of S-D link (given in subsection A) and S-R-D link (given in subsection B), since both the error events are mutually exclusive. As the instantaneous SNR of all the links follows gamma distribution, the probability density function (PDF) of gamma distribution is given by [32, eq.(2.21)]

$$f_{\gamma_j}(t) = \frac{m_a^{m_a} e^{-\frac{m_a t}{\Gamma_j}} t^{m_a-1}}{(\Gamma_j)^{m_a} \Gamma(m_a)}, \quad (5)$$

where $\Gamma(\cdot)$ represents the gamma integral function. From (5), the cumulative distribution function (CDF) of gamma distribution can be written as

$$F_{\gamma_j}(t) = \frac{\gamma(m_a, \frac{m_a t}{\Gamma_j})}{\Gamma(m_a)}, \quad (6)$$

where $\gamma(\cdot, \cdot)$ represents the lower incomplete gamma function [34, eq.(8.350-1)]. Now let us define a random variable $V_i = \min(\gamma_{sr_i}, \gamma_{r_id})$ and its corresponding CDF can be written as

$$F_{V_i}(t) = 1 - \int_t^\infty \int_t^\infty f_{\gamma_{sr_i}}(t_3) f_{\gamma_{r_id}}(t_2) dt_2 dt_3 = 1 - \frac{\Gamma(m_{2i}, \frac{m_{2i} t}{\Gamma_{r_id}}) \Gamma(m_{3i}, \frac{m_{3i} t}{\Gamma_{sr_i}})}{\Gamma(m_{2i}) \Gamma(m_{3i})}, \quad (7)$$

where $\Gamma(\cdot, \cdot)$ represents the upper incomplete gamma function [34, eq.(8.350-2)]. After expanding $\Gamma(\cdot, \cdot)$ using [34, eq.(8.352-4)] and differentiating (7), the PDF of V_i is given by

$$\begin{aligned} f_{V_i}(t) = \exp \left\{ - \left(\frac{m_{2i}}{\Gamma_{r_id}} + \frac{m_{3i}}{\Gamma_{sr_i}} \right) t \right\} & \left\{ \frac{\left(\frac{m_{2i}}{\Gamma_{r_id}} \right)^{m_{2i}}}{\Gamma(m_{2i})} \sum_{u=0}^{m_{3i}-1} \left(\frac{m_{3i}}{\Gamma_{sr_i}} \right)^u \frac{t^{m_{2i}-1+u}}{u!} \right. \\ & \left. + \frac{\left(\frac{m_{3i}}{\Gamma_{sr_i}} \right)^{m_{3i}}}{\Gamma(m_{3i})} \sum_{v=0}^{m_{2i}-1} \left(\frac{m_{2i}}{\Gamma_{r_id}} \right)^v \frac{t^{m_{3i}-1+v}}{v!} \right\} \end{aligned} \quad (8)$$

A. SEP analysis of S-D link

The conditional error probability of MPSK signaling [32, eq.(8.22)] conditioned on γ , which is the instantaneous SNR of any given link, is given by

$$P_e(\gamma) = \frac{1}{\pi} \int_0^{\phi_0} \exp \left(\frac{-\gamma \sin^2(\frac{\pi}{M})}{\sin^2(\phi)} \right) d\phi, \quad (9)$$

where $\phi_0 = \frac{\pi(M-1)}{M}$ and M denotes the modulation order. An upper bound on conditional SEP of MPSK signaling conditioned on γ is given by [32, eq.(8.24)]

$$P_e(\gamma) \leq \frac{M-1}{M} \exp \left(-\gamma \sin^2 \left(\frac{\pi}{M} \right) \right). \quad (10)$$

Here, we first assume S-D link is chosen among S-D and S- R_k -D links, where $k \neq N$ (i.e. excluding the last N^{th} relay), under the condition $\min(\gamma_{sr_l}, \gamma_{r_ld}) < \gamma_t$, where $l = 1, 2, \dots, k-1$, $\min(\gamma_{sr_k}, \gamma_{r_kd}) > \gamma_t$, and $\gamma_{sd} > \min(\gamma_{sr_k}, \gamma_{r_kd})$. Now the average SEP of S-D link can be expressed as

$$P'_{eSD_k} = \prod_{l=1}^{k-1} F_{V_l}(\gamma_t) \left\{ \int_{\gamma_t}^{\infty} \int_{t_6}^{\infty} P_e(t_1) f_{\gamma_{sd}}(t_1) f_{V_k}(t_6) dt_1 dt_6 \right\}. \quad (11)$$

For convenience, we define some functions as given by (12). Substituting the conditional SEP of MPSK signaling conditioned on γ_{sd} , which is given by (9), PDF of gamma distribution, which is given by (5), and the PDF of V_i , which is given by (8), in (11), the inner integral is simplified using [34, eq.(8.350-2)]. After expanding the resultant expression using [34, eq.(8.352-4)], the outer integral is simplified again by using [34, eq.(8.350-2)]. Now the simplified SEP expression for S-D link is given by

$$P'_{eSD_k} = \prod_{l=1}^{k-1} F_{V_l}(\gamma_t) \left\{ \frac{1}{\pi} \int_0^{\phi_0} \left(\frac{m_1}{\Gamma_{sd}} \right)^{m_1} \sum_{i=0}^{m_1-1} \rho_i(m_1, \Gamma_{sd}, \phi) \left\{ \frac{\left(\frac{m_{2k}}{\Gamma_{r_kd}} \right)^{m_{2k}}}{\Gamma(m_{2k})} \sum_{u=0}^{m_{3k}-1} \left(\frac{m_{3k}}{\Gamma_{sr_k}} \right)^u \right. \right. \\ \left. \left. \times \chi(i, u, m_{2k}, \phi, \gamma_t) + \frac{\left(\frac{m_{3k}}{\Gamma_{sr_k}} \right)^{m_{3k}}}{\Gamma(m_{3k})} \sum_{v=0}^{m_{2k}-1} \left(\frac{m_{2k}}{\Gamma_{r_kd}} \right)^v \chi(i, v, m_{3k}, \phi, \gamma_t) \right\} d\phi \right\}. \quad (13)$$

The upper bound for P'_{eSD_k} is obtained by substituting (10) instead of (9) in (11). After simplification using [34, eq.(8.350-2)], the closed-form upper bound expression for average SEP of the S-D link is given by

$$P'^{UB}_{eSD_k} = \prod_{l=1}^{k-1} F_{V_l}(\gamma_t) \left(\frac{M-1}{M} \right) \left(\frac{m_1}{\Gamma_{sd}} \right)^{m_1} \sum_{i=0}^{m_1-1} \rho_i(m_1, \Gamma_{sd}, \pi/2) \left\{ \frac{\left(\frac{m_{2k}}{\Gamma_{r_kd}} \right)^{m_{2k}}}{\Gamma(m_{2k})} \sum_{u=0}^{m_{3k}-1} \left(\frac{m_{3k}}{\Gamma_{sr_k}} \right)^u \right. \\ \left. \times \chi(i, u, m_{2k}, \pi/2, \gamma_t) + \frac{\left(\frac{m_{3k}}{\Gamma_{sr_k}} \right)^{m_{3k}}}{\Gamma(m_{3k})} \sum_{v=0}^{m_{2k}-1} \left(\frac{m_{2k}}{\Gamma_{r_kd}} \right)^v \chi(i, v, m_{3k}, \pi/2, \gamma_t) \right\}. \quad (14)$$

$$\begin{aligned}
dd(i) &= \frac{m_{2i}}{\Gamma_{r_i d}} + \frac{m_{3i}}{\Gamma_{sr_i}}, \quad cc(i) = \frac{m_1}{\Gamma_{sd}} + dd(i), \\
\rho_i(m_a, \Gamma_c, \phi) &= \frac{1}{i!} \left(\frac{m_a}{\Gamma_c} + \frac{\sin^2(\frac{\pi}{M})}{\sin^2 \phi} \right)^{i-m_a}, \quad L(i, \phi) = \frac{\sin^2 \frac{\pi}{M}}{\sin^2 \phi} + cc(i), \\
L'(i, \phi) &= \frac{\sin^2(\frac{\pi}{M})}{\sin^2 \phi} + dd(i), \quad \chi(i, j, m_a, \phi, \gamma_t) = \frac{\Gamma(m_a + j + i, \gamma_t(L(l, \phi)))}{i! (L(l, \phi))^{m_a+j+i}}, \\
\chi'(j, m_a, \phi, \gamma_t) &= \frac{\Gamma(m_a + j, \gamma_t(L'(l, \phi)))}{(L'(l, \phi))^{m_a+j}}, \quad B(i, \phi'_1, \phi'_2) = \frac{\sin^2(\frac{\pi}{M})}{\sin^2 \phi'_1} + \frac{\sin^2(\frac{\pi}{M})}{\sin^2 \phi'_2} + cc(i) \\
B'(i, \phi'_1, \phi'_2) &= \frac{\sin^2(\frac{\pi}{M})}{\sin^2 \phi'_1} + \frac{\sin^2(\frac{\pi}{M})}{\sin^2 \phi'_2} + dd(i), \\
\zeta'(j, m_a, \phi'_1, \phi'_2, \gamma_t) &= \frac{\Gamma(m_a + j, \gamma_t(B'(l, \phi'_1, \phi'_2)))}{(B'(l, \phi'_1, \phi'_2))^{m_a+j}}, \text{ and} \\
\zeta(i, j, m_a, \phi'_1, \phi'_2, \gamma_t) &= \frac{\Gamma(m_a + j + i, \gamma_t(B(l, \phi'_1, \phi'_2)))}{i! (B(l, \phi'_1, \phi'_2))^{m_a+j+i}} \tag{12}
\end{aligned}$$

In (11), we did not choose S-D link comparing with S-R_N-D link. Now we assume S-D link is chosen between S-D and S-R_N-D links. The average SEP of S-D link under the condition $\min(\gamma_{sr_l}, \gamma_{r_l d}) < \gamma_t$, where $l = 1, 2, \dots, N-1$, and $\gamma_{sd} > \min(\gamma_{sr_N}, \gamma_{r_N d})$ is given by

$$P'_{eSD_N} = \prod_{l=1}^{N-1} F_{V_l}(\gamma_t) \left\{ \int_0^\infty P_e(t_1) f_{\gamma_{sd}}(t_1) F_{V_N}(t_1) dt_1 \right\}. \tag{15}$$

After substituting (9), (5), and (7) in (15) and further simplifying the integral using the definition of gamma integral function, the average SEP of S-D link is given by

$$\begin{aligned}
P'_{eSD_N} &= \prod_{l=1}^{N-1} F_{V_l}(\gamma_t) \left\{ \frac{m_1^{m_1}}{\pi(\Gamma_{sd})^{m_1} \Gamma(m_1)} \int_0^{\phi_0} \left\{ \frac{\Gamma(m_1)}{\left(\frac{\sin^2(\frac{\pi}{M})}{\sin^2 \phi} + \frac{m_1}{\Gamma_{sd}} \right)^{m_1}} - \sum_{i=0}^{m_{2N}-1} \sum_{j=0}^{m_{3N}-1} \right. \right. \\
&\quad \left. \left. \frac{\left(\frac{m_{2N}}{\Gamma_{r_N d}} \right)^i \left(\frac{m_{3N}}{\Gamma_{sr_N}} \right)^j}{j!} \chi(i, j, m_1, \phi, 0) \right\} d\phi \right\}. \tag{16}
\end{aligned}$$

Now (16) is represented in the form of simple integrals, which can be evaluated easily. But similar to (14), less complicated upper bound expression for P'_{eSD_N} in closed-form after

substituting (10) is given by

$$P'_{eSD_N}{}^{UB} = \prod_{l=1}^{N-1} F_{V_l}(\gamma_t) \frac{(M-1) m_1^{m_1}}{M (\Gamma_{sd})^{m_1} \Gamma(m_1)} \left\{ \frac{\Gamma(m_1)}{\left(\sin^2\left(\frac{\pi}{M}\right) + \frac{m_1}{\Gamma_{sd}}\right)^{m_1}} - \sum_{i=0}^{m_{2N}-1} \sum_{j=0}^{m_{3N}-1} \frac{\left(\frac{m_{2N}}{\Gamma_{rNd}}\right)^i \left(\frac{m_{3N}}{\Gamma_{srN}}\right)^j}{j!} \chi(i, j, m_1, \pi/2, 0) \right\}. \quad (17)$$

B. SEP analysis of S-R-D link

Firstly, the conditional error probability of S-R_k-D link is given by [13, eq.(21)]¹

$$P_{eSR_kD}(\gamma_{sr_k}, \gamma_{r_kd}) = P_e(\gamma_{sr_k}) + P_e(\gamma_{r_kd}) - P_e(\gamma_{sr_k}) P_e(\gamma_{r_kd}). \quad (18)$$

Here, we first assume S-R_k-D link, where $k \neq N$, is chosen based on the following conditions: 1) $\min(\gamma_{sr_l}, \gamma_{r_ld}) < \gamma_t$, where $l=1, 2, \dots, k-1$, 2) $\min(\gamma_{sr_k}, \gamma_{r_kd}) > \gamma_t$, and 3) $\min(\gamma_{sr_k}, \gamma_{r_kd}) > \gamma_{sd}$.

The average SEP of S-R_k-D link incorporating the aforementioned conditions is given by

$$P'_{eSR_kD} = \prod_{l=1}^{k-1} F_{V_l}(\gamma_t) \left\{ \underbrace{\int_{\gamma_t}^{\infty} \int_{t_{2k}}^{\infty} P_{eSR_kD}(t_{3k}, t_{2k}) f_{\gamma_{sr_k}}(t_{3k}) f_{\gamma_{r_kd}}(t_{2k}) F_{\gamma_{sd}}(t_{2k}) dt_{3k} dt_{2k}}_{\text{Part-A } \mathbf{I_1(m_{2k}, \Gamma_{r_kd}, m_{3k}, \Gamma_{sr_k}, \gamma_t) + I_2(m_{2k}, \Gamma_{r_kd}, m_{3k}, \Gamma_{sr_k}, \gamma_t) - I_3(m_{2k}, \Gamma_{r_kd}, m_{3k}, \Gamma_{sr_k}, \gamma_t)}} + \underbrace{\int_{\gamma_t}^{\infty} \int_{t_{3k}}^{\infty} P_{eSR_kD}(t_{3k}, t_{2k}) f_{\gamma_{sr_k}}(t_{3k}) f_{\gamma_{r_kd}}(t_{2k}) F_{\gamma_{sd}}(t_{3k}) dt_{2k} dt_{3k}}_{\text{Part-B } \mathbf{J_1+J_2-J_3}} \right\}. \quad (19)$$

Now we divide (19) into six terms and the final expressions for all the six terms along with upper bound expressions are given in Appendix A.

Now we assume S-R_N-D link is chosen based on the following conditions: 1) $\min(\gamma_{sr_l}, \gamma_{r_ld}) < \gamma_t$, where $l=1, 2, \dots, N-1$ and 2) $\min(\gamma_{sr_N}, \gamma_{r_Nd}) > \gamma_{sd}$. Now the average SEP of S-R_N-D link

¹Note that the final term in [13, eq.(21)] is omitted, since it does not add any significant change in the end-to-end average SEP as shown in [35] and also our simulation results shown in Fig.1 exactly agree with the computed SEP values.

is given by

$$P'_{eSRND} = \prod_{l=1}^{N-1} F_{V_l}(\gamma_t) \left\{ \underbrace{\int_0^\infty \int_{t_{2N}}^\infty P_{eSRND}(t_{3N}, t_{2N}) f_{\gamma_{srN}}(t_{3N}) f_{\gamma_{rNd}}(t_{2N}) F_{\gamma_{sd}}(t_{2N}) dt_{3N} dt_{2N}}_{\text{Part-A } \mathbf{I}'_1 + \mathbf{I}'_2 - \mathbf{I}'_3} + \underbrace{\int_0^\infty \int_{t_{3N}}^\infty P_{eSRND}(t_{3N}, t_{2N}) f_{\gamma_{srN}}(t_{3N}) f_{\gamma_{rNd}}(t_{2N}) F_{\gamma_{sd}}(t_{3N}) dt_{2N} dt_{3N}}_{\text{Part-B } \mathbf{J}'_1 + \mathbf{J}'_2 - \mathbf{J}'_3} \right\} \quad (20)$$

Again we divide (20) into six terms and the final expressions remain same as that of (19) except equating $\gamma_t = 0$. Therefore, all the six terms are given by (48a) in Appendix B. In addition, the upper bound expressions are also given.

C. End-to-end SEP and diversity order analysis

The average end-to-end SEP expression for HSSEC scheme over non-identical Nakagami- m fading channels considering MPSK signaling is obtained by adding the individual error probabilities of S-D link (refer (13) and (16)) and S-R-D link (refer (19) and (20)) and is given by

$$P_e = \sum_{k=1}^{N-1} (P'_{eSD_k} + P'_{eSR_kD}) + P'_{eSD_N} + P'_{eSR_ND} \quad (21)$$

By adding (14), (17), (47a), and (50a), the closed-form upper bound expression for the average SEP of HSSEC scheme is given by

$$P_e \leq \sum_{k=1}^{N-1} (P'^{UB}_{eSD_k} + P'^{UB}_{eSR_kD}) + P'^{UB}_{eSD_N} + P'^{UB}_{eSR_ND} \quad (22)$$

For deriving the asymptotic SEP expression, we assume $\Gamma = \Gamma_{sd} = \Gamma_{sr_i} = \Gamma_{r_id}$, where $i = 1, 2, \dots, N$. Now we consider the upper bound for the conditional SEP of MPSK signaling, which is given by (10). Substituting the conditional SEP in (15) and (20) results in the upper bound for the average SEPs of P'_{eSD_N} and P'_{eSR_ND} and are expressed as (17) and (50a), respectively. These two equations play a significant role in deriving the asymptotic expression along with high-SNR approximations [36, eq.(30)] which are given by

$$\gamma\left(m, \frac{m}{\Gamma}\right) \approx \frac{m^{m-1}t^m}{\Gamma^m}, \quad \Gamma\left(m, \frac{m}{\Gamma}\right) \approx 1 - \frac{m^{m-1}t^m}{\Gamma^m},$$

$$1 - \left(\frac{\Gamma(m_2, \frac{m_2 t}{\Gamma})}{\Gamma(m_2)} \frac{\Gamma(m_3, \frac{m_3 t}{\Gamma})}{\Gamma(m_3)} \right) \approx \frac{m_3^{m_3-1} t^{m_3}}{\Gamma(m_3) \Gamma^{m_3}} + \frac{m_2^{m_2-1} t^{m_2}}{\Gamma(m_2) \Gamma^{m_2}}, \text{ and } \sin^2(\pi/M) + \frac{m}{\Gamma} \approx \sin^2(\pi/M).$$

After simplification using high-SNR approximations, the closed-form asymptotic SEP of the HSSEC scheme for MPSK signaling considering S-D link is derived and the same is given by

$$P_{eSD}^{asy} = \prod_{l=1}^{N-1} \left[\frac{m_{3l}^{m_{3l}-1} \gamma_t^{m_{3l}}}{\Gamma(m_{3l}) \Gamma^{m_{3l}}} + \frac{m_{2l}^{m_{2l}-1} \gamma_t^{m_{2l}}}{\Gamma(m_{2l}) \Gamma^{m_{2l}}} \right] \left\{ \frac{(M-1)}{M \Gamma(m_1)} \left(\frac{m_1}{\Gamma} \right)^{m_1} \right. \\ \left. \times \left[\frac{m_{3N}^{m_{3N}-1} \Gamma(m_1 + m_{3N})}{\Gamma^{m_{3N}} \Gamma(m_{3N}) (\sin^2(\frac{\pi}{M}))^{m_1+m_{3N}}} + \frac{m_{2N}^{m_{2N}-1} \Gamma(m_1 + m_{2N})}{\Gamma^{m_{2N}} \Gamma(m_{2N}) (\sin^2(\frac{\pi}{M}))^{m_1+m_{2N}}} \right] \right\}. \quad (23)$$

Similarly, using high-SNR approximations, the closed-form asymptotic SEP of the HSSEC scheme for MPSK signaling considering S-R-D link is derived and is given by

$$P_{eSRD}^{asy} = \prod_{l=1}^{N-1} \left[\frac{m_{3l}^{m_{3l}-1} \gamma_t^{m_{3l}}}{\Gamma(m_{3l}) \Gamma^{m_{3l}}} + \frac{m_{2l}^{m_{2l}-1} \gamma_t^{m_{2l}}}{\Gamma(m_{2l}) \Gamma^{m_{2l}}} \right] \left\{ \frac{m_1^{m_1-1}}{\Gamma^{m_1} \Gamma(m_{2N}) \Gamma(m_{3N}) \Gamma(m_1)} \right. \\ \left. \times \left[\left(\frac{m_{2N}}{\Gamma} \right)^{m_{2N}} \frac{\Gamma(m_1 + m_{2N})}{(\sin^2(\frac{\pi}{M}))^{m_1+m_{2N}}} + \left(\frac{m_{3N}}{\Gamma} \right)^{m_{3N}} \frac{\Gamma(m_1 + m_{3N})}{(\sin^2(\frac{\pi}{M}))^{m_1+m_{3N}}} \right] \right\}. \quad (24)$$

Adding (23) and (24), the asymptotic SEP of the HSSEC scheme considering MPSK signaling is given by

$$P_e^{asy} = \frac{(M-1)}{M} \prod_{l=1}^{N-1} \left[\frac{m_{3l}^{m_{3l}-1} \gamma_t^{m_{3l}}}{\Gamma(m_{3l}) \Gamma^{m_{3l}}} + \frac{m_{2l}^{m_{2l}-1} \gamma_t^{m_{2l}}}{\Gamma(m_{2l}) \Gamma^{m_{2l}}} \right] \left(\left(\frac{m_1}{\Gamma} \right)^{m_1} \frac{1}{\Gamma(m_1)} \right. \\ \times \left[\frac{m_{3N}^{m_{3N}-1} \Gamma(m_1 + m_{3N})}{\Gamma^{m_{3N}} \Gamma(m_{3N}) (\sin^2(\frac{\pi}{M}))^{m_1+m_{3N}}} + \frac{m_{2N}^{m_{2N}-1} \Gamma(m_1 + m_{2N})}{\Gamma^{m_{2N}} \Gamma(m_{2N}) (\sin^2(\frac{\pi}{M}))^{m_1+m_{2N}}} \right] \\ + \left\{ \frac{m_1^{m_1-1}}{\Gamma^{m_1} \Gamma(m_{2N}) \Gamma(m_{3N}) \Gamma(m_1)} \left[\left(\frac{m_{2N}}{\Gamma} \right)^{m_{2N}} \frac{\Gamma(m_1 + m_{2N})}{(\sin^2(\frac{\pi}{M}))^{m_1+m_{2N}}} \right. \right. \\ \left. \left. + \left(\frac{m_{3N}}{\Gamma} \right)^{m_{3N}} \frac{\Gamma(m_1 + m_{3N})}{(\sin^2(\frac{\pi}{M}))^{m_1+m_{3N}}} \right] \right\} \right). \quad (25)$$

Now (25) can be written as

$$P_e^{asy} \propto \frac{G_1}{\Gamma^{m_1 + \sum_{l=1}^{N-1} m_{2l} + m_{2N}}} + \frac{G_2}{\Gamma^{m_1 + \sum_{l=1}^{N-1} m_{3l} + m_{3N}}}, \quad (26)$$

where G_1 and G_2 are constants which are independent of Γ . From (25) and (26), it has been inferred that the HSSEC scheme achieves full diversity order of $m_1 + \sum_{l=1}^N \min(m_{2l}, m_{3l})$ over non-identical fading channels. Considering identical fading links, the diversity order will be equal

to $m(N + 1)$, where $m = m_1 = m_{2l} = m_{3l}$. It is to be noted that the expression given by (25) is valid for all the cases excluding for the case when $\gamma_t \ll \Gamma$. For the case when $\gamma_t \ll \Gamma$, most likely $\min(\gamma_{SR_1}, \gamma_{R_1D}) > \gamma_t$ and SC combining will be employed between the S-D and S-R₁-D links. Therefore, $P_e^{asy} \propto G_3/(\Gamma)^{m_1+m_{21}} + G_4/(\Gamma)^{m_1+m_{31}}$, where G_3 and G_4 are constants which are independent of Γ . Hence, the diversity order for the case when $\gamma_t \ll \Gamma$ will be reduced to $m_1 + \min(m_{21}, m_{31})$ considering non-identical fading scenario. In addition, the diversity order for identical case is equal to $2m$.

IV. OUTAGE PROBABILITY ANALYSIS

When the instantaneous SNR of the selected link is lesser than a threshold SNR γ_{out} , then the HSSEC system is said to be in a state called outage, where the target BER cannot be supported. Similar to the SC scheme [37], the outage probability of HSSEC scheme can be evaluated from the CDF of S-D and S-R-D links. It is to be noted that the CDF of single hop and dual-hop links are already given by (6) and (7), respectively, and using both the equations, outage probability of HSSEC scheme can be derived. If S-R_k-D link, where $k \neq N$, is selected based on the HSSEC scheme, then the system reaches the state of outage when $\max(\gamma_{sd}, \min(\gamma_{sr_k}, \gamma_{r_kd})) < \gamma_{out}$. Assuming $\gamma_t < \gamma_{out}$, the outage probability is obtained based on the following conditions: (1) $\min(\gamma_{sr_l}, \gamma_{r_ld}) < \gamma_t$, where $l = 1, 2, \dots, k-1$, (2) $\min(\gamma_{sr_k}, \gamma_{r_kd}) > \gamma_t$, and 3) $\gamma_{sd} < \gamma_{out}$, $\min(\gamma_{sr_k}, \gamma_{r_kd}) < \gamma_{out}$ and the same is given by

$$P_{out}^k = \prod_{l=1}^{k-1} F_{V_l}(\gamma_t) \left\{ \int_0^{\gamma_{out}} \int_{\gamma_t}^{\gamma_{out}} f_{\gamma_{sd}}(t_1) f_{V_k}(t_6) dt_1 dt_6 \right\}. \quad (27)$$

After simplification, (27) can be written as

$$P_{out}^k = \prod_{l=1}^{k-1} F_{V_l}(\gamma_t) \{ F_{\gamma_{sd}}(\gamma_{out}) [F_{V_k}(\gamma_{out}) - F_{V_k}(\gamma_t)] \}. \quad (28)$$

If S-R_N-D link is selected based on the HSSEC scheme, then the system reaches the state of outage when $\max(\gamma_{sd}, \min(\gamma_{sr_N}, \gamma_{r_Nd})) < \gamma_{out}$. Assuming $\gamma_t < \gamma_{out}$, the outage probability is

obtained based on the following conditions: (1) $\min(\gamma_{sr_l}, \gamma_{r_ld}) < \gamma_t$, where $l = 1, 2, \dots, N - 1$,
(2) $\gamma_{sd} < \gamma_{out}$, $\min(\gamma_{sr_N}, \gamma_{r_Nd}) < \gamma_{out}$ and the same is given by

$$P_{out}^N = \prod_{l=1}^{N-1} F_{V_l}(\gamma_t) \left\{ \int_0^{\gamma_{out}} \int_0^{\gamma_{out}} f_{\gamma_{sd}}(t_1) f_{V_N}(t_6) dt_1 dt_6 \right\}. \quad (29)$$

After simplification, (29) can be written as

$$P_{out}^N = \prod_{l=1}^{N-1} F_{V_l}(\gamma_t) \{F_{\gamma_{sd}}(\gamma_{out}) F_{V_N}(\gamma_{out})\}. \quad (30)$$

Summing (28) and (30), the final outage probability expression is given by

$$P_{out} = \sum_{k=1}^{N-1} P_{out}^k + P_{out}^N. \quad (31)$$

If $\gamma_t \geq \gamma_{out}$, then the only possibility with which the HSSEC system reaches the state of outage is when S-R_N-D link is selected. Therefore, considering $\gamma_t \geq \gamma_{out}$, the outage probability is given by $P_{out} = P_{out}^N$. Note that for a given switching threshold γ_t , the outage probability will be minimum if $\gamma_t = \gamma_{out}$ [23].

The average outage probability expression derived for single antenna case can also be extended to multiple antenna case. Let us assume that S and D are employed with N_t and N_r antennas, respectively. Moreover, we also assume that the best transmit and receive antennas at S and D are selected based on the instantaneous SNR of best S-D link alone and the selection rule is given as follows:

$$\{x, y\} = \underset{1 \leq i \leq N_t, 1 \leq b \leq N_r}{\operatorname{argmax}} (\gamma_{s_i d_b}), \quad (32)$$

where x and y represent the best transmit and receive antenna indices of S and D, respectively and $\gamma_{S_i D_b}$ denotes the instantaneous SNR between i^{th} source antenna and b^{th} destination antenna. It is to be noted that the antenna selection rule is sub-optimal, since it is based on the instantaneous SNR of S-D link alone. Let us assume that S_x-R_k-D_y link, where $k \neq N$, is selected based on the HSSEC scheme. The system reaches the state of outage when $\max(\gamma_{s_x d_y}, \min(\gamma_{s_x r_k}, \gamma_{r_k d_y})) < \gamma_{out}$. Assuming $\gamma_t < \gamma_{out}$, the outage probability is obtained based on the following conditions:

(1) $\min(\gamma_{s_x r_l}, \gamma_{r_l d_y}) < \gamma_t$, where $l = 1, 2, \dots, k-1$, (2) $\min(\gamma_{s_x r_k}, \gamma_{r_k d_y}) > \gamma_t$, and (3) $\gamma_{s_x d_y} < \gamma_{out}$, $\min(\gamma_{s_x r_k}, \gamma_{r_k d_y}) < \gamma_{out}$ and it is given by

$$P_{out}^{k/ant} = N_t N_r \prod_{l=1}^{k-1} F_{V_l}(\gamma_t) \left\{ \int_0^{\gamma_{out}} \int_{\gamma_t}^{\gamma_{out}} f_{\gamma_{sd}}(t_1) (F_{\gamma_{sd}}(t_1))^{N_t N_r - 1} f_{V_k}(t_6) dt_1 dt_6 \right\}. \quad (33)$$

Note that we assume $\Gamma_{sr_k} = \Gamma_{s_x r_k}$, $\Gamma_{r_k d} = \Gamma_{r_k d_y}$, and $\Gamma_{sd} = \Gamma_{s_x d_y}$. After substituting (5) and (6) in (33), the final closed-form expression for $P_{out}^{k/ant}$ can be obtained after simplification using [34, eq.(8.352-6)] and [38, eq.(9)] and the same is given by

$$P_{out}^{k/ant} = N_t N_r \prod_{l=1}^{k-1} F_{V_l}(\gamma_t) [F_{V_k}(\gamma_{out}) - F_{V_k}(\gamma_t)] \left(\frac{m_1}{\Gamma_{sd}} \right)^{m_1} \frac{1}{\Gamma(m_1)} \sum_{a=0}^{N_t N_r - 1} \binom{N_t N_r - 1}{a} (-1)^a \\ \times \prod_{i=1}^{m_1 - 1} \left[\sum_{n_i=0}^{n_{i-1}} \binom{n_{i-1}}{n_i} \left(\frac{1}{i!} \right)^{n_i - n_{i+1}} \left(\frac{m_1}{\Gamma_{sd}} \right)^{n_i} \right] \frac{\gamma \left(A + m_1, \gamma_{out} \left[\frac{m_1(1+a)}{\Gamma_{sd}} \right] \right)}{\left[\frac{m_1(1+a)}{\Gamma_{sd}} \right]^{A+m_1}}, \quad (34)$$

where $A = n_1 + n_2 + \dots + n_{m_1 - 1}$. It is to be noted that for $m_1 = 1$,

$$\prod_{i=1}^{m_1 - 1} \left[\sum_{n_i=0}^{n_{i-1}} \binom{n_{i-1}}{n_i} \left(\frac{1}{i!} \right)^{n_i - n_{i+1}} \left(\frac{m_1}{\Gamma_{sd}} \right)^{n_i} \right] = 1.$$

If S_x - R_N - D_y link is selected based on the HSSEC scheme, then the system reaches the state of outage based on the following conditions: (1) $\min(\gamma_{s_x r_l}, \gamma_{r_l d_y}) < \gamma_t$, where $l = 1, 2, \dots, N-1$, and (2) $\gamma_{s_x d_y} < \gamma_{out}$, $\min(\gamma_{s_x r_N}, \gamma_{r_N d_y}) < \gamma_{out}$. The outage probability expression based on the mentioned conditions can be written as

$$P_{out}^{N/ant} = N_t N_r \prod_{l=1}^{N-1} F_{V_l}(\gamma_t) \left\{ \int_0^{\gamma_{out}} \int_0^{\gamma_{out}} f_{\gamma_{sd}}(t_1) (F_{\gamma_{sd}}(t_1))^{N_t N_r - 1} f_{V_N}(t_6) dt_1 dt_6 \right\}. \quad (35)$$

It is to be noted that we assume $\Gamma_{sr_N} = \Gamma_{s_x r_N}$, $\Gamma_{r_N d} = \Gamma_{r_N d_y}$, and $\Gamma_{sd} = \Gamma_{s_x d_y}$. After substituting (5) and (6) in (35), the final closed-form expression can be obtained after simplification using [34, eq.(8.352-6)] and [38, eq.(9)]. Now $P_{out}^{N/ant}$ is given by

$$P_{out}^{N/ant} = N_t N_r \prod_{l=1}^{N-1} F_{V_l}(\gamma_t) F_{V_N}(\gamma_{out}) \left(\frac{m_1}{\Gamma_{sd}} \right)^{m_1} \frac{1}{\Gamma(m_1)} \sum_{a=0}^{N_t N_r - 1} \binom{N_t N_r - 1}{a} (-1)^a \\ \times \prod_{i=1}^{m_1 - 1} \left[\sum_{n_i=0}^{n_{i-1}} \binom{n_{i-1}}{n_i} \left(\frac{1}{i!} \right)^{n_i - n_{i+1}} \left(\frac{m_1}{\Gamma_{sd}} \right)^{n_i} \right] \frac{\gamma \left(A + m_1, \gamma_{out} \left[\frac{m_1(1+a)}{\Gamma_{sd}} \right] \right)}{\left[\frac{m_1(1+a)}{\Gamma_{sd}} \right]^{A+m_1}}, \quad (36)$$

By adding (34) and (36), the end-to-end average outage probability expression for multiple antenna case assuming $\gamma_t < \gamma_{out}$ is given by

$$P_{out}^{ant} = \sum_{k=1}^{N-1} P_{out}^{k/ant} + P_{out}^{N/ant}. \quad (37)$$

Similar to single antenna case, the outage probability for the case when $\gamma_t \geq \gamma_{out}$ is given by $P_{out}^{ant} = P_{out}^{N/ant}$. It is to be observed that by substituting $N_t = N_r = 1$ in (33) and (35), the outage probability expression for single antenna case is obtained (refer to (27) and (29)).

V. NUMERICAL RESULTS AND DISCUSSIONS

Note that all the curves are plotted assuming $\Gamma = \Gamma_{sd} = \Gamma_{sr_i} = \Gamma_{r_id}$, where $i = 1, 2, \dots, N$, except Fig. 7(b) and Fig. 8. The numerical optimization of γ_t for different average SNR values is shown in Fig. 2 for $N=3$ and $M=4$ case over Rayleigh fading channels. From the curves, we obtain γ_t^{opt} value, which gives minimum SEP, for different average SNR values. The obtained γ_t^{opt} values are substituted in (21) and the SEP performance curves are plotted for optimum case. From Fig. 2, it is observed that as the average SNR value increases, γ_t^{opt} value also increases. It is to be noted that the final exact average SEP expression given by (21) is not in closed-form. Moreover, the derived upper-bound SEP expression given by (22) is also very complicated. Therefore, it is difficult to obtain an analytical expression for γ_t^{opt} by differentiating the exact and upper-bound SEP expressions. Hence, numerical optimization is conducted and γ_t^{opt} values minimizing P_e are obtained for different average SNR values considering various Nakagami- m channel conditions.

Fig. 3 demonstrates the SEP performance of HSSEC scheme for different values of M and fading severity parameter. The performance curves are plotted assuming $N=3$, $m = m_{2i} = m_{3i}$, and $\gamma_t = \gamma_t^{opt}$. From the figure, it is observed that the SEP performance of BPSK signaling performs better than quadrature phase-shift keying (QPSK) signaling as expected. Furthermore, the SEP performance of $m_1=1, m=4$ case is better than $m_1=m=2$ case. Because the diversity order for $m_1=1, m=4$ case is 13 and it is greater than the diversity order for $m_1=m=2$ case,

which is equal to 8, as observed from the asymptotic SEP expressions. In addition, improvement in the SEP performance is observed for $m_1 = m = 2$ compared to $m_1 = 1, m = 2$ and $m_1 = m = 1$ cases as expected. From Fig. 3, it is also evident that the Monte-Carlo simulation results well agree with the theoretical results.

In Fig. 4(a), the average SEP of HSSEC scheme considering QPSK modulation is plotted against switching threshold SNR γ_t for three different values of N (i.e. $N=2, 4$, and 6) assuming average SNR value $\Gamma = 20$ dB and $m_1 = m = 2$ case, where $m = m_{2i} = m_{3i}$. From the plots, it is observed that if γ_t is very much lesser or greater than Γ , then SEP performance improvement is not achieved with increase in the number of relays. This is because if $\gamma_t \ll \Gamma$, then all the S-R-D paths will be acceptable. However, according to HSSEC relay selection decision rule, S-R₁-D link will be chosen without examining other relay nodes and the same will be compared with S-D link. Moreover, if $\gamma_t \gg \Gamma$, then $(N - 1)$ S-R-D links will be unacceptable and always N^{th} S-R-D link will be chosen and the same will be compared with S-D link. Hence, in both the cases increasing the number of relays will not improve the SEP performance. From the plots, it is also inferred that if the value of switching threshold SNR γ_t is comparable to the value of average SNR Γ , then SEP performance gain is obtained by increasing the number of relays due to diversity order benefit.

The SEP performance of the HSSEC scheme is shown in Fig. 4(b) considering QPSK modulation and $m_1 = m = 2$ for two different values of γ_t and three different values of N . Considering $\gamma_t = 10$ dB, it is observed that as the value of N increases, improvement in the SEP performance and diversity order is not obtained in the high-SNR region. Since $\gamma_t \ll \Gamma$ in the high-SNR region, all the links will be acceptable and based on the HSSEC decision rule always S-R₁-D path will be chosen and compared with direct S-D link irrespective of increasing the number of relays. Therefore, increasing the value of N will not play a major role in improving the SEP performance in the high-SNR region as already mentioned for the case when $\gamma_t \ll \Gamma$ in Fig. 4(a). However, it is also observed that in the SNR region comparable to $\gamma_t = 10$ dB

performance gain is obtained. This is because, if γ_t is comparable with Γ , always the same path will not be selected unlike $\gamma_t \ll \Gamma$ case and hence, additional paths will improve the SEP performance due to increase in diversity order. Therefore, by considering $\gamma_t = 18$ dB, it is observed that for average SNR value greater than 15 dB, increasing the number of relays tend to increase the SEP performance gain as well as diversity gain. In addition, degradation in the SEP performance is noticed for the case when $\gamma_t = 18$ dB and $N = 6$ compared to $\gamma_t = 10$ dB in the low-SNR region. But in the high-SNR region (i.e. more than 18 dB) improvement in the SEP performance as well as diversity gain is observed for the same. It is to be noted that above the average SNR value of 40 dB, degradation in the diversity order could be observed for $\gamma_t = 18$ dB. But receiver operating at high-SNR value close to 40 dB and SEP value less than 10^{-10} is not very practical.

In Fig. 5(a), the exact, asymptotic, and upper bound SEP performances of the HSSEC scheme are compared considering BPSK and QPSK signaling schemes over Nakagami- m fading channels assuming $m = m_{2i} = m_{3i}$ and $N = 3$. Firstly, the upper bound SEP performance converges well with the asymptotic SEP curves. From the asymptotic SEP performance plotted using (25), it is inferred that HSSEC scheme with $\gamma_t = 20$ dB assuming $N = 3$ and Rayleigh fading scenario offers full diversity order of 4 (i.e. $m(N + 1)$) in the region of operation of interest. This is because, as the switching threshold is fixed, degradation in the diversity order will be obtained at very high-SNR region when $\gamma_t \ll \Gamma$ similar to the performance trends shown in Fig. 4(b). Since high-SNR more than 40 dB is not very practical, the inferences related to the diversity order and the SEP performance at very high-SNR and very low SEP value less than 10^{-10} can be neglected. The asymptotic SEP performance curve for $\gamma_t = 8$ dB case is plotted assuming $m = 1$. Since $\gamma_t \ll \Gamma$, the diversity order converges to 2 (i.e. $m + m_1$). Finally, considering $\gamma_t = \gamma_t^{opt}$ case, it is observed that the HSSEC scheme attains full diversity order as the SEP performance curve converges well with the asymptotic SEP curve plotted using (25). This is because, if optimum values are chosen for each average SNR value, then the scenario will be

similar to that of the case when γ_t is comparable with Γ . Since the same path will not be selected every time, options are available to select the best path according to HSSEC decision rule from N available paths, which will eventually give full diversity order benefit.

In a nutshell, full diversity order is obtained for the case when $\gamma_t = \gamma_t^{opt}$. Moreover, if γ_t value is fixed and if the same satisfies $\gamma_t \geq \gamma_{30}^{opt}$, where γ_{30}^{opt} is the optimum γ_t value for average SNR value of 30 dB, full diversity order is obtained in the region of operation of interest. Further, if γ_t is fixed and if $\gamma_t < \gamma_{30}^{opt}$, then diversity order of $m + m_1$ is obtained for the HSSEC scheme. For example, the optimum γ_t value which minimizes the average SEP for $\Gamma = 30$ dB assuming $N = 3, M = 2, m_1 = 1, m = 2$ is equal to 18 dB. Hence, by fixing $\gamma_t = 20$ dB, full diversity order is obtained for HSSEC scheme as shown in Fig. 5(a). However, if $\gamma_t < \gamma_{30}^{opt}$, diversity order degradation is noticed for the case when $\gamma_t = 8$ dB.

It is complicated to perform numerical optimization of γ_t for all average SNR values at D. Hence, instead of optimizing γ_t for all SNR values, there are other ways to set the threshold value appropriately and the same is discussed as follows. Firstly, if the main concern of the receiver is to get full diversity order benefit along with improved SEP performance in high-SNR region, then we can find the optimum value for one particular high-SNR value, say 30 dB, as suggested and fix $\gamma_t \geq \gamma_{30}^{opt}$ for all other average SNR values to get the diversity order benefit. Similar methodology has also been used in [39] for fixing the threshold SNR value. Therefore, by fixing $\gamma_t \geq \gamma_{30}^{opt}$, diversity order benefit along with SEP performance improvement in high-SNR region are obtained compared to $\gamma_t < \gamma_{30}^{opt}$ case as shown in Fig. 5(a). Furthermore, if the main concern of the receiver is to get better SEP performance in the low-SNR region, then by fixing $\gamma_t < \gamma_{30}^{opt}$, performance improvement in low-SNR region can be achieved along with degradation in the diversity order as shown in Fig. 5(a). However, diversity order degradation leads to less number of CSI requirement at D compared to $\gamma_t \geq \gamma_{30}^{opt}$ case in the high-SNR region. This is because in the high-SNR region, most likely S-R₁-D path will be selected all the time and hence, along with S-D link, CSI of only three links will be required at D. Therefore, depending upon

the receiver requirements, γ_t can be fixed appropriately. Finally, if the receiver is capable of calculating optimum γ_t for every average SNR value, then γ_t^{opt} can be fixed and it has been shown in the previous results that both full diversity order and SEP performance improvement can be achieved for the case when $\gamma_t = \gamma_t^{opt}$.

In Fig. 5(b), the upper bound and asymptotic SEP performance curves of the HSSEC scheme are compared over non-identical Nakagami- m fading channels assuming $m_2 = m_{2i}$, $m_3 = m_{3i}$, $N = 3$, and $\gamma_t = 20$ dB. Firstly, it is observed that the upper bound SEP performance curves converge well with the asymptotic SEP curves. Since $\gamma_t \geq \gamma_{30}^{opt}$, it is inferred from the figure that full diversity order of $m_1 + N\min(m_2, m_3)$ is obtained.

In Fig. 6(a), the SEP performance of the HSSEC scheme considering $N=2$, $M=2$, $\gamma_t = \gamma_t^{opt}$, and Rayleigh fading scenario is compared with SEC [28], HSSSC [29], SSCSR, IRSR [25], full CSI SC [28], and DSSC [23] schemes proposed in the literature. Firstly, from the SEP performance curves, SNR gain values of 1.5 dB and 0.3 dB at 10^{-4} are observed for the HSSEC scheme compared to the SSCSR and IRSR schemes, respectively. Moreover, the performance of the HSSEC scheme well agree with the performance of the HSSSC scheme proposed in [29] for the case when $N=2$ and $M=2$ and it also proves that the HSSSC scheme is a special case of the HSSEC scheme with $N=2$ and $m = m_1 = 1$. Considerable performance improvement (i.e. more than 3 dB at 10^{-4}) is observed in the case of the HSSEC scheme compared to the DSSC scheme proposed in [23] for the two-relay system. Since the combining weights employed by MRC in the DSSC scheme does not consider the CSI of S-R link at D, considerable degradation in the SEP performance is observed. Furthermore, for MRC scheme, both amplitude and phase information of the channel are required at the receiving end. However, in the case of HSSEC scheme, only amplitude information of the channel is sufficient, since selection is based on the instantaneous SNR values. Moreover, considerable SEP performance improvement of 2 dB at 10^{-4} is observed for the HSSEC scheme compared to the SEC scheme [28]. This is because of employing SC between S-D and S-R-D links in the HSSEC scheme unlike SEC scheme. From

the performance comparison of HSSEC and full CSI SC schemes [28] in Fig. 6(a), it can be observed that for the case when $N=2$, full CSI SC scheme outperforms HSSEC scheme. This is because, for selecting the best path, the decision rule of the full CSI SC scheme is given by $\max(\gamma_{sd}, \max_{i \in 1, \dots, N}(\min(\gamma_{sr_i}, \gamma_{r_i d})))$. Here, with the help of CSI of all the links, either direct link or the best S-R-D link is selected. However, in the case of the HSSEC scheme, if the instantaneous SNR value of a particular link or path under consideration is greater than γ_t , then the same will be selected without examining the further links.

The complexity of HSSEC, IRSR, full CSI SC, and SEC schemes is compared in the Table I for Rayleigh fading scenario in terms of the average number of CSI required at D for different average SNR values assuming $\gamma_t = \gamma_t^{opt}$, $N = 3$, and $M = 4$. It is to be noted that for the full CSI SC scheme proposed in [28], number of CSI required at D is equal to seven (i.e. six channels or three relays + CSI of direct link). From the tabulated values, it is inferred that on an average, SEC scheme requires CSI of maximum two links out of seven links and IRSR scheme requires CSI of three links, whereas HSSEC scheme requires more CSI compared to the IRSR and SEC schemes but lesser than the full CSI SC scheme. Since additional SC is performed between S-D and S-R-D links, HSSEC scheme requires more CSI in contrast to the SEC and IRSR schemes. Therefore, HSSEC scheme outperforms SEC and IRSR schemes in terms of SNR gain (refer Fig. 6(a)) with more CSI requirement for the case when $\gamma_t = \gamma_t^{opt}$. It is also to be noted that full CSI SC scheme outperforms HSSEC scheme, however, at the expense of more CSI requirement at D as shown in Table I.

In Fig. 6(a), the SEP performances of the HSSEC and IRSR schemes are also compared considering $N=2$, $M=2$, $\gamma_t=25$ dB, and Rayleigh fading scenario. From the SEP performance curves, it has been observed that the proposed HSSEC scheme achieves an SNR gain value of more than 1 dB at the SEP value of 10^{-3} compared to IRSR scheme proposed in [25]. Note that by fixing $\gamma_t = \gamma_t^{opt}$, the SNR gain value of 0.3 dB is obtained. However, by fixing $\gamma_t = 25$ dB, improvement in the SNR gain is observed. From Table II, it has been observed that both

the schemes require CSI of nearly 5 links for the mentioned scenario. But HSSEC scheme outperforms IRSR scheme especially in the low-SNR region.

In Fig. 6(b), the SEP performance of HSSEC scheme is compared with PL-detection-based DF cooperative diversity system proposed in [9] considering three node cooperative setup for $M=4$ and $M=8$. From the figure, it can be inferred that the proposed HSSEC scheme slightly losses the SEP performance in the high-SNR region compared to the PL-detection-based cooperative system. This is because, in [9], the average SNR of S-R link is utilized at D for decoding the transmitted message signal. However, in the proposed HSSEC scheme, only the amplitude of the CSI of S-R link is assumed to be known at D and hence, slight degradation in the SEP performance especially in the high-SNR region is observed for both $M=4$ and $M=8$ cases considering $N=1$.

Firstly, in Fig. 7(a), the outage performance of the HSSEC scheme is compared for different values of γ_{out} and γ_t assuming $N=3$, $m_1=1$, and $m_2=m_3=3$. From the figure, it can be observed that the outage performance of $\gamma_{out}=10$ dB case is better than the outage performance of $\gamma_{out}=12$ dB case as expected. This is because for higher value of γ_{out} , the HSSEC system reaches the state of outage more frequently compared to the lower value of γ_{out} . Furthermore, it can be also observed that the outage performance of the case when $\gamma_t > \gamma_{out}$ is better compared to the case when $\gamma_t < \gamma_{out}$. This is due to the fact that for the case when $\gamma_t > \gamma_{out}$, the only possibility with which the HSSEC system reaches the state of outage is when S- R_N -D link is selected and instantaneous SNR of the same is lesser than γ_{out} along with S-D link. This means when rest of the $N-1$ relay links are selected, the HSSEC system will not reach the state of outage. However, for the case when $\gamma_t < \gamma_{out}$, the probability of any selected link reaching the state of outage is higher and hence, the outage performance degradation is observed for the same compared to the case when $\gamma_t > \gamma_{out}$. In addition, the best outage performance is observed for $\gamma_t = \gamma_{out}$ scenario. Finally, it can be also noticed that the Monte-Carlo simulation results well agree with the theoretical results.

In Fig. 7(b), the outage performance of the HSSEC scheme is compared with the SEC [28], full CSI SC [28], DSSC [23], and IRSR [25] schemes proposed in the literature over Rayleigh fading channels assuming $N = 2$, $\gamma_t = 8$ dB, $\gamma_{out} = 12$ dB, and $\Gamma_{sd} = \frac{\Gamma}{4}$, where $\Gamma = \Gamma_{sr_i} = \Gamma_{r_id}$. Firstly, as expected full CSI SC scheme has a better outage performance compared to all the other schemes including the proposed HSSEC scheme. Since CSI of all the links is available at D, full CSI SC scheme has an improved outage performance. For the case when $\gamma_t < \gamma_{out}$, the proposed HSSEC scheme outperforms IRSR and SEC schemes in terms of outage performance. In addition, the outage performances of the DSSC and HSSEC schemes are observed to be equal.

In Fig. 8, the outage performance plot for single antenna and multiple antenna systems considering HSSEC-based relay selection scheme is compared assuming $N = 2$, $\Gamma_{sd} = \frac{\Gamma}{4}$, where $\Gamma = \Gamma_{sr_i} = \Gamma_{r_id}$, and $m = 2$, where $m = m_1 = m_{2i} = m_{3i}$. Firstly, from the plot, it can be inferred that improvement in the outage performance is obtained for multiple-antenna-based HSSEC system compared to single antenna system due to improvement in the diversity gain. In addition, it is also observed that the outage performance for the case when $\gamma_t > \gamma_{out}$ is better compared to the case when $\gamma_t < \gamma_{out}$ considering multiple-antenna-based HSSEC system. Finally, for $N_t = N_r = 1$, the outage performance curves, which are plotted based on (37), well agree with the performance curves plotted based on (31) and thus validates the derived outage probability expression for multiple antenna case.

VI. CONCLUSION

To sum up, HSSEC scheme has been proposed for a multi-relay DF system to improve the performance of distributed switched diversity combining schemes proposed in the literature. The performance analysis of the HSSEC scheme is investigated by deriving average end-to-end SEP expression for MPSK signaling and outage probability expression over non-identical Nakagami- m fading channels. Additionally, upper bound and asymptotic SEP expressions are derived in closed-form and diversity order is evaluated from the asymptotic expression. It is inferred from the asymptotic SEP expressions that full diversity order, which is equal to $m_1 +$

$\sum_{l=1}^N \min(m_{2l}, m_{3l})$, is obtained considering non-identical Nakagami- m fading scenario except for the case when $\gamma_t \ll \Gamma$. In addition, the diversity order for the case when $\gamma_t \ll \Gamma$ is equal to $m_1 + \min(m_{21}, m_{31})$. Moreover, a less complicated method to fix the threshold SNR value in order to attain full diversity order benefit at D has also been suggested. Findings from the numerical results illustrate that the HSSEC scheme performs better in terms of SEP and outage probability compared to the switched diversity schemes proposed in the literature with more CSI requirement.

APPENDIX A

AVERAGE SEP OF S-R_k-D LINK

The first term of Part-A in (19) is given by

$$I_1(m_{2k}, \Gamma_{r_k d}, m_{3k}, \Gamma_{sr_k}, \gamma_t) = \int_{\gamma_t}^{\infty} \int_{t_{2k}}^{\infty} P_e(t_{3k}) f_{\gamma_{sr_k}}(t_{3k}) f_{\gamma_{r_k d}}(t_{2k}) F_{\gamma_{sd}}(t_{2k}) dt_{3k} dt_{2k}. \quad (38a)$$

After substituting the conditional SEP of MPSK signaling conditioned on γ_{sr_k} and (5) in (38a), the inner integral is simplified using [34, eq.(8.350-2)]. Further, expanding $F_{\gamma_{sd}}(t_{2k})$ and simplifying the outer integral using [34, eq.(8.352-6)] and [34, eq.(8.350-2)], respectively, the final simplified expression is given by

$$I_1(m_{2k}, \Gamma_{r_k d}, m_{3k}, \Gamma_{sr_k}, \gamma_t) = \frac{1}{\pi} \left(\frac{m_{2k}}{\Gamma_{r_k d}} \right)^{m_{2k}} \left(\frac{m_{3k}}{\Gamma_{sr_k}} \right)^{m_{3k}} \frac{1}{\Gamma(m_{2k})} \int_0^{\phi_0} \sum_{j=0}^{m_{3k}-1} \rho_j(m_{3k}, \Gamma_{sr_k}, \phi) \\ \times \left\{ \chi'(j, m_{2k}, \phi, \gamma_t) - \sum_{i=0}^{m_1-1} \left(\frac{m_1}{\Gamma_{sd}} \right)^i \chi(i, j, m_{2k}, \phi, \gamma_t) \right\} d\phi. \quad (39a)$$

where $\chi'(\cdot, \cdot, \cdot, \cdot)$ and $\rho_j(\cdot, \cdot, \cdot)$ are given by (12). However, (39a) is expressed in the form of simple integral. Hence, the closed-form upper bound for (39a) is obtained after substituting (10) in (38a) and is given by

$$P_1(m_{2k}, \Gamma_{r_k d}, m_{3k}, \Gamma_{sr_k}, \gamma_t) = \frac{M-1}{M} \left(\frac{m_{2k}}{\Gamma_{r_k d}} \right)^{m_{2k}} \left(\frac{m_{3k}}{\Gamma_{sr_k}} \right)^{m_{3k}} \frac{1}{\Gamma(m_{2k})} \sum_{j=0}^{m_{3k}-1} \rho_j(m_{3k}, \Gamma_{sr_k}, \pi/2) \\ \times \left\{ \chi'(j, m_{2k}, \pi/2, \gamma_t) - \sum_{i=0}^{m_1-1} \left(\frac{m_1}{\Gamma_{sd}} \right)^i \chi(i, j, m_{2k}, \pi/2, \gamma_t) \right\}, \quad (40a)$$

Now the second term of Part-A in (19) can be written similar to (38a) by replacing $P_e(t_{3k})$ with $P_e(t_{2k})$. After substituting $P_e(\cdot)$ and (5) and simplifying the resultant expression using [34, eq.(8.350-2)] and [34, eq.(8.352-6)], the final expression for second term is given by

$$I_2(m_{2k}, \Gamma_{r_k d}, m_{3k}, \Gamma_{sr_k}, \gamma_t) = \frac{1}{\pi} \left(\frac{m_{2k}}{\Gamma_{r_k d}} \right)^{m_{2k}} \frac{1}{\Gamma(m_{2k})} \sum_{j=0}^{m_{3k}-1} \frac{\left(\frac{m_{3k}}{\Gamma_{sr_k}} \right)^j}{j!} \left\{ \int_0^{\phi_0} \chi'(j, m_{2k}, \phi, \gamma_t) \right. \\ \left. - \sum_{i=0}^{m_1-1} \left(\frac{m_1}{\Gamma_{sd}} \right)^i \chi(i, j, m_{2k}, \phi, \gamma_t) \right\} d\phi. \quad (41a)$$

Using (10), the upper bound expression for (41a) is obtained after simplification using [34, eq.(8.350-2)] and [34, eq.(8.352-6)]. The final closed-form expression for upper bound of $I_2(\cdot, \cdot, \cdot, \cdot, \cdot)$ is given by

$$P_2(m_{2k}, \Gamma_{r_k d}, m_{3k}, \Gamma_{sr_k}, \gamma_t) = \frac{M-1}{M} \left(\frac{m_{2k}}{\Gamma_{r_k d}} \right)^{m_{2k}} \frac{1}{\Gamma(m_{2k})} \sum_{j=0}^{m_{3k}-1} \frac{\left(\frac{m_{3k}}{\Gamma_{sr_k}} \right)^j}{j!} \left\{ \chi'(j, m_{2k}, \pi/2, \gamma_t) \right. \\ \left. - \sum_{i=0}^{m_1-1} \left(\frac{m_1}{\Gamma_{sd}} \right)^i \chi(i, j, m_{2k}, \pi/2, \gamma_t) \right\}. \quad (42a)$$

Similar to I_1 and I_2 , the third term of Part-A can be obtained by replacing $P_e(t_{3k})$ with $P_e(t_{2k})P_e(t_{3k})$ in (38a). After substituting the conditional SEP of MPSK signaling, PDF and CDF of gamma distribution, the final expression after simplification using [34, eq.(8.350-2)] and [34, eq.(8.352-6)] is given by

$$I_3(m_{2k}, \Gamma_{r_k d}, m_{3k}, \Gamma_{sr_k}, \gamma_t) = \frac{1}{\pi^2} \left(\frac{m_{2k}}{\Gamma_{r_k d}} \right)^{m_{2k}} \left(\frac{m_{3k}}{\Gamma_{sr_k}} \right)^{m_{3k}} \frac{1}{\Gamma(m_{2k})} \int_0^{\phi_0} \int_0^{\phi_0} \sum_{j=0}^{m_{3k}-1} \rho_j(m_{3k}, \Gamma_{sr_k}, \phi_2) \\ \times \left\{ \zeta'(j, m_{2k}, \phi_1, \phi_2, \gamma_t) - \sum_{i=0}^{m_1-1} \left(\frac{m_1}{\Gamma_{sd}} \right)^i \zeta(i, j, m_{2k}, \phi_1, \phi_2, \gamma_t) \right\} d\phi_1 d\phi_2. \quad (43a)$$

The upper bound expression for (43a) in closed-form is given by

$$P_3(m_{2k}, \Gamma_{r_k d}, m_{3k}, \Gamma_{sr_k}, \gamma_t) = \left(\frac{M-1}{M} \right)^2 \left(\frac{m_{2k}}{\Gamma_{r_k d}} \right)^{m_{2k}} \left(\frac{m_{3k}}{\Gamma_{sr_k}} \right)^{m_{3k}} \frac{1}{\Gamma(m_{2k})} \sum_{j=0}^{m_{3k}-1} \\ \rho_j(m_{3k}, \Gamma_{sr_k}, \pi/2) \left\{ \zeta'(j, m_{2k}, \pi/2, \pi/2, \gamma_t) - \sum_{i=0}^{m_1-1} \left(\frac{m_1}{\Gamma_{sd}} \right)^i \zeta(i, j, m_{2k}, \pi/2, \pi/2, \gamma_t) \right\}, \quad (44a)$$

where $\zeta'(\cdot, \cdot, \cdot, \cdot, \cdot)$ and $\zeta(\cdot, \cdot, \cdot, \cdot, \cdot)$ are given by (12). Similar to Part-A, three terms in Part-B is obtained as follows

$$\begin{aligned} J_1 &= I_2(m_{3k}, \Gamma_{sr_k}, m_{2k}, \Gamma_{r_k d}, \gamma_t), J_2 = I_1(m_{3k}, \Gamma_{sr_k}, m_{2k}, \Gamma_{r_k d}, \gamma_t), \\ J_3 &= I_3(m_{3k}, \Gamma_{sr_k}, m_{2k}, \Gamma_{r_k d}, \gamma_t). \end{aligned} \quad (45a)$$

The equivalent upper bound expression in closed-form for Part-B is given by

$$\begin{aligned} Q_1 &= P_2(m_{3k}, \Gamma_{sr_k}, m_{2k}, \Gamma_{r_k d}, \gamma_t), Q_2 = P_1(m_{3k}, \Gamma_{sr_k}, m_{2k}, \Gamma_{r_k d}, \gamma_t), \\ Q_3 &= P_3(m_{3k}, \Gamma_{sr_k}, m_{2k}, \Gamma_{r_k d}, \gamma_t). \end{aligned} \quad (46a)$$

The derivation procedure is similar to that of Part-A and hence, we omit the detailed derivation steps. From (40a), (42a), (44a), and (46a), the upper bound expression for $P'_{eSR_k D}$ in (19) is obtained in closed-form and is expressed as

$$\begin{aligned} P'_{eSR_k D}{}^{UB} &= P_1(m_{2k}, \Gamma_{r_k d}, m_{3k}, \Gamma_{sr_k}, \gamma_t) + P_2(m_{2k}, \Gamma_{r_k d}, m_{3k}, \Gamma_{sr_k}, \gamma_t) \\ &\quad - P_3(m_{2k}, \Gamma_{r_k d}, m_{3k}, \Gamma_{sr_k}, \gamma_t) + Q_1 + Q_2 - Q_3. \end{aligned} \quad (47a)$$

APPENDIX B

AVERAGE SEP OF S-R_N-D LINK

The final expressions for Part-A and Part-B in (20) is given by

$$\begin{aligned} I'_1 &= I_1(m_{2N}, \Gamma_{r_N d}, m_{3N}, \Gamma_{sr_N}, 0), I'_2 = I_2(m_{2N}, \Gamma_{r_N d}, m_{3N}, \Gamma_{sr_N}, 0), \\ I'_3 &= I_3(m_{2N}, \Gamma_{r_N d}, m_{3N}, \Gamma_{sr_N}, 0) J'_1 = I_2(m_{3N}, \Gamma_{sr_N}, m_{2N}, \Gamma_{r_N d}, 0), \\ J'_2 &= I_1(m_{3N}, \Gamma_{sr_N}, m_{2N}, \Gamma_{r_N d}, 0), J'_3 = I_3(m_{3N}, \Gamma_{r_N d}, m_{2N}, \Gamma_{r_N d}, 0). \end{aligned} \quad (48a)$$

The closed-form upper bound expressions for (48a) is expressed as follows

$$\begin{aligned} P'_1 &= P_1(m_{2N}, \Gamma_{r_N d}, m_{3N}, \Gamma_{sr_N}, 0), P'_2 = P_2(m_{2N}, \Gamma_{r_N d}, m_{3N}, \Gamma_{sr_N}, 0), \\ P'_3 &= P_3(m_{2N}, \Gamma_{r_N d}, m_{3N}, \Gamma_{sr_N}, 0), Q'_1 = P_2(m_{3N}, \Gamma_{sr_N}, m_{2N}, \Gamma_{r_N d}, 0), \\ Q'_2 &= P_1(m_{3N}, \Gamma_{sr_N}, m_{2N}, \Gamma_{r_N d}, 0), Q'_3 = P_3(m_{3N}, \Gamma_{sr_N}, m_{2N}, \Gamma_{r_N d}, 0). \end{aligned} \quad (49a)$$

From (49a), the closed-form upper bound for $P'_{eSR_N D}$, which is given by (20), is expressed as

$$P'^{UB}_{eSR_N D} = P'_1 + P'_2 - P'_3 + Q'_1 + Q'_2 - Q'_3. \quad (50a)$$

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Fig. 1. Multi-DF-relay system with single antenna source, relay and destination nodes

TABLE I
COMPLEXITY COMPARISON OF HSSEC, IRSR, FULL CSI SC, AND SEC SCHEMES IN TERMS OF AVERAGE NUMBER OF CSI REQUIRED AT D OVER RAYLEIGH FADING SCENARIO ASSUMING $\gamma_t = \gamma_t^{opt}$, $N = 3$, AND $M = 4$

SNR (dB)	Average No. of CSI required at D			
	SEC [28]	IRSR [25]	Full CSI SC [28]	HSSEC
0	2.0	3.1	7	5.9
5	2.2	3.3	7	5.3
10	1.8	2.9	7	4.5
15	1.5	2.2	7	3.9

TABLE II
COMPLEXITY COMPARISON OF IRSR AND HSSEC SCHEMES IN TERMS OF AVERAGE NUMBER OF CSI REQUIRED AT D OVER RAYLEIGH FADING SCENARIO ASSUMING $\gamma_t = 25dB$, $N = 2$, AND $M = 2$

SNR (dB)	Average No. of CSI required at D	
	HSSEC	IRSR [25]
0	5.0	5
5	5.2	5
10	5.0	4.9
15	4.9	4.8

Fig. 2. Numerical optimization of switching threshold SNR for different average SNR values over Rayleigh fading channels

Fig. 3. SEP performance of HSSEC scheme for different modulation schemes and fading severity parameter over Nakagami- m fading channels

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Fig. 4. (a) Average SEP of the HSSEC scheme considering $M=4$ and $m_1=m=2$ versus switching threshold SNR value γ_t for three different values of N (i.e. Number of relays) assuming average SNR $\Gamma=20$ dB. (b) Performance of the HSSEC scheme considering QPSK modulation and $m_1=m=2$ for two different values of γ_t and three different values of N

Fig. 5. (a) Comparison of the asymptotic, upper bound, and exact SEPs of the HSSEC scheme considering MPSK signaling over Nakagami- m fading channels assuming $N=3$. (b) Comparison of the upper bound and asymptotic SEPs of the HSSEC scheme considering QPSK signaling over non-identical Nakagami- m fading channels assuming $N=3$ and $\gamma_t=20$ dB

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Fig. 6. (a) Performance comparison of HSSEC scheme with other schemes proposed in the literature over Rayleigh fading channels. (b) Performance comparison of HSSEC scheme with PL detector proposed in [9] over Rayleigh fading channels

Fig. 7. (a) Outage performance of HSSEC scheme for different outage and switching threshold SNR values assuming $N=3$, $m_1=1$, and $m_2=m_3=3$. (b) Outage probability comparison of HSSEC scheme with other schemes proposed in the literature assuming $N=3$ and $m=2$, where $m = m_1 = m_2 = m_3$.

Fig. 8. Outage probability comparison of single antenna and multiple-antenna-based HSSEC systems assuming $N = 2$ and $m = 2$, where $m = m_1 = m_{2i} = m_{3i}$