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Path-independent digital image correlation with high accuracy, speed and robustness

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Abstract

The initial guess transferring mechanism is widely used in iterative DIC algorithms and leads to path-dependence. Using the known deformation at a processed point to estimate the initial guess at its neighbouring points could save considerable computation time, and a cogitatively-selected processing path contributes to the improved robustness. In this work, our experimental study demonstrates that a path-independent DIC method is capable to achieve high accuracy, efficiency and robustness in full-field measurement of deformation, by combining an inverse compositional Gauss-Newton (IC-GN) algorithm for sub-pixel registration with a fast Fourier transform-based cross correlation (FFT-CC) algorithm to estimate the initial guess. In the proposed DIC method, the determination of initial guess accelerated by well developed software library can be a negligible burden of computation. The path-independence also endows the DIC method with the ability to handle the images containing large discontinuity of deformation without manual intervention. Furthermore, the possible performance of the proposed path-independent DIC method on parallel computing device is estimated, which shows the feasibility of the development of real-time DIC with high-accuracy.

Keywords: Digital image correlation; Fast Fourier transform; Inverse compositional algorithm; Gauss-Newton method; Path-independence

1. Introduction

Since Peters and Ranson introduced digital image correlation (DIC) to measure the displacement on solid surfaces in 1982 [1], this technique has been undergoing accelerating growth in the past three decades, and has become one of the most important and popular technologies in experimental mechanics for non-contact full-field measurements of shape, deformation and motion[2, 3].

In the methodology study of DIC, three key issues have been investigated persistently and still remain as the major challenges nowadays, i.e. measurement accuracy, computation efficiency and robustness. To achieve high accuracy, various sub-pixel registration algorithms have been developed to reach accuracies ranging from 0.01 to 0.5 pixel, including the coarse-fine search method [1, 4], the correlation coefficient curve-fitting method [5, 6], the gradient-based method [7, 8], and the iterative method represented by the Newton-Raphson (NR) algorithm [9, 10]. However, the high accuracy of sub-pixel registration algorithms is at the price of high computation cost since most of these algorithms consist of an interpolation operation. In particular, the NR algorithm, though gives the best performance in accuracy and stability [11], is computationally expensive due to its nature of non-linear optimization. During each iteration step of the NR algorithm, the intensity and intensity gradient at sub-pixel locations in the target (deformed) image have to be reconstructed based on interpolation, accompanied with the re-evaluation of the inverse of the Hessian matrix. This problem becomes increasingly critical with the explosion of the number of the points of interest (POIs) to be processed by DIC, due

to the rapid development and application of higher resolution digital cameras for image acquisition. To overcome it, an inverse compositional Gauss-Newton (IC-GN) algorithm [12, 13] was introduced into DIC recently [2, 14]. In contrast with the traditional forward additive NR algorithm which matches the affinely warped target image with the reference (un-deformed) image iteratively, the IC-GN algorithm warps the reference image in a small range and then matches it with the warped target image, whereby the algorithm avoids the repeated calculation of intensity gradients and the inverse of the Hessian matrix during the iterations. The IC-GN algorithm has been proven to be mathematically equivalent to the classic forward additive algorithm [12, 13], and shows similar accuracy and convergence criteria to the NR algorithm in experimental study [11, 14, 15].

The robustness of DIC refers to its adaptability to handle complex deformation fields and its resistance to intensity variation and noise. In recent years, the strategies to enhance the robustness of DIC, e.g. the choice of the subset size [16, 17], correlation criterion [18, 19], and the processing path [20, 21], have been studied extensively. Pan et al [19] compared commonly used correlation criteria, and found the zero-mean normalized cross-correlation (ZNCC) criterion, zero-mean normalized sum of squared difference (ZNSSD) criterion and the parametric sum of squared difference (PSSD_{ab}) criterion (with two additional unknown parameters a and b) are insensitive to the variation of intensity before and after deformation.

The importance of processing path stems from a widely adopted approach to get the initial guess for iterative DIC algorithm. Based on the assumption of continuous deformation, the displacement and its gradient at a POI can be used to estimate the displacement at the neighbouring POIs, supposing that they are close enough. Therefore, correct deformation information at a POI helps to provide reliable initial guess for the DIC calculation at its neighbours and avoid the time-consuming computation of initial guess at each POIs. On the other hand, unfortunately, error of calculation at a POI can be spread out through the same way. Inspired by the strategy used in phase unwrapping, Pan proposed a reliability-guided processing path scheme to prevent the spread of error [14, 20, 22]. This scheme starts from a pre-set seed POI, then processes the four POIs adjacent to the seed and weighs the reliability of these POIs according to their ZNCC values, afterwards inserts these POIs into a queue in a descending order of ZNCC values. The POI on the top of the queue is popped out as the new seed. Its adjacent POIs, if not processed, are correlated, weighed and inserted into the queue, followed by a re-ordering of the queue. By repeating this processing-weighing-queuing procedure, the processing path is guided to follow the direction towards the area with a good chance to get accurate DIC results. The cogitatively selected path can reduce the disturbance of area with poor quality of speckle pattern or subject to discontinuous deformation, as compared to a simple scanning path along rows or columns. Due to the aforementioned advantages, a DIC method combining ZNCC-based reliability-guided tracking with a ZNSSD-based IC-GN algorithm was expected to be a new standard approach [14].

It is noteworthy to point out that any spatial path-dependent strategy has its inherent deficiency which limits the potential of DIC in further improvement of computation efficiency and robustness. Current DIC method is basically a block-by-block, or windowed processing technique, which is suitable for parallel computing. Enlightened by the recent progress in the general-purpose computing on graphics processing units (GPU), the computation speed of integer-pixel DIC algorithms are found to speed up for a couple of orders of magnitude when running the DIC program on commercially available GPU platforms [23-25]. However, the spatial path-dependent strategy based on sequential processing mechanism apparently does not meet the essential requirement of parallel computing, i.e. the operation performed in parallel processing unit should be independent.

In the aspect of robustness, even for an intelligent strategy like reliability-guided displacement tracing, there is high probability to fail in the full-field analysis of deformation without manual intervention if the images contain large areas of discontinuous deformation. For example, on the lateral surface of a composite laminate subjected to delamination, the boundary of spatial discontinuities can transverse the whole view field. In this case it is difficult to compute the full-field deformation using the path-dependent strategy unless multiple seed POIs are preliminarily set.

To replace the path-dependent DIC method with a path-independent one, a question emerges naturally: is the estimation of initial guess without the information

transferred from neighbouring POI unbearably expensive in computation? In this paper, we demonstrate that the time spent for the independent calculation of initial guess at each POI (with integer-pixel accuracy) can be reduced to a negligible level by using a fast Fourier transform-based cross-correlation (FFT-CC) algorithm with ZNCC criterion, which is accelerated by one of the fastest FFT software libraries (FFTW [26]). The initial guess obtained using FFT-CC algorithm is able to make the ZNSSD-based IC-GN algorithm converge fast and reach sub-pixel accuracy. A systematic experimental study shows how path-independent DIC can achieve improved efficiency and robustness without sacrifice of accuracy.

The rest of the paper is structured as follows. Sec. 2 and Sec. 3 provide brief introductions of the FFT-CC algorithm for integer-pixel registration and the IC-GN algorithm for sub-pixel registration, respectively. A path-independent DIC method that combines these two algorithms is proposed in Sec. 4. In Sec. 5, the proposed method is verified using simulated speckle images and real experimental results. The paper is concluded in Sec. 6.

2. Fast Fourier transform-based cross correlation algorithm

2.1. Principle and implementation

Cross correlation method, first introduced by Yamaguchi to tracking the in-plane speckle displacement on solid surface [27, 28], is a powerful technique in particle image velocimetry (PIV) to measure velocity field of flows [29-31]. By tracking the translation of the spherical tracer particles in fluids, which are like artificial speckles

in the recorded images, the velocity distribution of flows can be determined. Cross correlation of function R and function T is defined mathematically as the integral of the product of $R^*(x)$ with $T(x)$

$$C_{CC}(\Delta x) = R \otimes T = \int_{-\infty}^{+\infty} R^*(x)T(x + \Delta x)dx, \quad (1)$$

where the symbol $\otimes$ denotes the operation of cross correlation, and the superscript $*$ indicates the complex conjugate. Δx is a shift of distance between the two functions, which can be determined by searching the peak of $C_{CC}(\Delta x)$. Suppose that a square subset includes N discrete pixels, and let R_i and T_i represent the grayscale values of the i th pixel in the reference subset (before deformation) and the target subset (after deformation), respectively. The discrete form of Eq. (1) combining with the ZNCC criterion can be written as

$$C_{ZNCC} = \frac{\sum_i \bar{R}_i \bar{T}_i}{\sqrt{\sum_i (\bar{R}_i)^2 \sum_i (\bar{T}_i)^2}}, \quad (2)$$

where $\bar{R}_i = R_i - R_m$ and $\bar{T}_i = T_i - T_m$, with R_m and T_m as the mean values of the intensity in the two subsets, i.e. $R_m = \frac{1}{N} \sum_i R_i$ and $T_m = \frac{1}{N} \sum_i T_i$. Notice that the superscript $*$ has been omitted here since R contains only real numbers and thus $R^*(x) = R(x)$.

The calculation of Eq. (2) can be simplified by introducing the operation in frequency domain. According to Fourier theory, the operation of cross correlation in space domain is equivalent to a simple product in frequency domain, i.e.

$$R \otimes T = FT^{-1} \left\{ [FT(R)]^* \cdot FT(T) \right\}, \quad (3)$$

where $FT(\cdot)$ and $FT^{-1}(\cdot)$ denote the Fourier transform and inverse Fourier transform, respectively.

Figure 1 illustrates the flow chart of the ZNCC-based FFT-CC algorithm. At first, two matrices containing zero-mean normalized values, i.e. $[\bar{R}]$ and $[\bar{T}]$, are constructed as the reference subset and the target subset respectively. Fourier transform is then performed on the two matrices, yielding $FT([\bar{R}])$ and $FT([\bar{T}])$. By performing inverse Fourier transform on the product of $FT^*([\bar{R}])$ and $FT([\bar{T}])$, a matrix of $C_{ZNCC}(u, v)$ in space domain can be obtained. The location of the positive peak of C_{ZNCC} is the desired integer-pixel displacement (u, v) .

2.2. Discussion

The FFT-CC algorithm, according to its principle, is limited to determine the translation of speckle pattern. However, it benefits to a large extent from the mature development of software and hardware optimized for FFT, and is still used for the measurement of simple deformation. In recent years, integer-pixel DIC programs based on the FFT-CC algorithm were developed on NVIDIA multi-processor graphic card [23-25]. The reported work well demonstrates the power of GPU for DIC calculation.

It should be noticed that a sub-pixel registration can also be achieved by applying the FFT-CC algorithm in the interpolated images. Although an initial guess with sub-pixel accuracy may be helpful to reduce the number of iteration in IC-GN algorithm, this attempt can be skipped since a reliable initial guess at integer-pixel level is enough to ensure rapid convergence of the IC-GN algorithm, as will be shown in our experimental study in Sec. 5.

3 Inverse compositional Gauss-Newton algorithm

3.1. Principle and implementation

The IC-GN algorithm was originally proposed by Baker and Matthews in 2001 as an efficient image alignment method [12], which was adopted by Sutton et al in DIC later [2]. Very recently, Pan et al used this algorithm, combining with the ZNSSD criterion, as a sub-pixel DIC registration method [14]. The principle of the IC-GN algorithm is briefly introduced in the section. More details can be found in Refs. [12-14].

In DIC, a point (x, y) in the reference subset is supposed to move to a point (x', y') in the target subset. The relationship between the locations of the two points in their respective subsets can be expressed by

$$\begin{aligned} x' &= x + u + u_x \Delta x + u_y \Delta y \\ y' &= y + v + v_x \Delta x + v_y \Delta y \end{aligned} \quad (4)$$

where u and v are the displacement components of the subset center in x -axis and y -axis respectively, u_x , u_y , v_x and v_y represent their gradient components, the terms Δx and Δy are the distances from the subset center to point (x, y) respectively. Define a vector of deformation parameters $p = [u, u_x, u_y, v, v_x, v_y]^T$ and its increment $\Delta p = [\Delta u, \Delta u_x, \Delta u_y, \Delta v, \Delta v_x, \Delta v_y]^T$, the ZNSSD criterion with an affine warp function can be expressed as

$$C_{ZNSSD}(\Delta p) = \sum_{\xi} \left\{ \frac{\bar{R}[\psi + W(\xi, \Delta p)]}{\bar{R}_n} - \frac{\bar{T}[\psi + W(\xi, p)]}{\bar{T}_n} \right\}^2, \quad (5)$$

where $\psi = [x, y, 1]^T$ denotes the global coordinates of a subset center (i.e. POI) in the reference or target image, while $\xi = [\Delta x, \Delta y, 1]^T$ is the local coordinates of a pixel in the reference or target subset. $\bar{R}_n = \sqrt{\sum_{\xi} \{\bar{R}[\psi + W(\xi, \Delta p)]\}^2}$ and $\bar{T}_n = \sqrt{\sum_{\xi} \{\bar{T}[\psi + W(\xi, p)]\}^2}$ are normalization factors in the reference subset and the target subset, respectively. The warp function $W(\xi, p)$ describes the change of the position and shape of the target subset against the reference subset. An affine warp function often used in DIC can be written as

$$W(\xi, p) = \begin{bmatrix} 1+u_x & u_y & u \\ v_x & 1+v_y & v \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ 1 \end{bmatrix}. \quad (6)$$

Correspondingly, the incremental warp function $W(\xi, \Delta p)$ used to adjust the reference subset can be written as

$$W(\xi, \Delta p) = \begin{bmatrix} 1 + \Delta u_x & \Delta u_y & \Delta u \\ \Delta v_x & 1 + \Delta v_y & \Delta v \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ 1 \end{bmatrix}. \quad (7)$$

The goal of the IC-GN algorithm is to minimize the ZNSSD criterion $C_{ZNSSD}(\Delta p)$ in Eq. (5). Considering a small Δp and knowing $W(\xi, 0) = \xi$, the non-linear expression in Eq. (5) can be linearized by performing a first-order Taylor expansion on $\bar{R}[\psi + W(\xi, \Delta p)]$, yielding

$$C_{ZNSSD}(\Delta p) = \sum_{\xi} \left\{ \frac{\bar{R}(\psi + \xi) + \nabla R(\psi + \xi) \frac{\partial W}{\partial p} \Delta p}{\bar{R}_n} - \frac{\bar{T}[\psi + W(\xi, p)]}{\bar{T}_n} \right\}^2, \quad (8)$$

where $\nabla R(\psi + \xi) = [\frac{\partial R(\psi + \xi)}{\partial x}, \frac{\partial R(\psi + \xi)}{\partial y}]$ is the gradient in the reference subset,

and $\frac{\partial W}{\partial p} = \begin{bmatrix} 1 & \Delta x & \Delta y & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \Delta x & \Delta y \end{bmatrix}$ is the Jacobian matrix of the warp function.

Minimizing $C_{ZNSSD}(\Delta p)$ in Eq. (8) is a least-squares problem with respect to Δp . Thus Δp can be solved as

$$\Delta p = H^{-1} \sum_{\xi} \left\{ \left[\nabla R(\psi + \xi) \frac{\partial W}{\partial p} \right]^T \left[\frac{\bar{R}_n}{\bar{T}_n} \bar{T}[\psi + W(\xi, p)] - \bar{R}(\psi + \xi) \right] \right\}, \quad (9)$$

where H^{-1} is the inverse of the Hessian matrix H

$$H = \sum_{\xi} \left\{ \left[\nabla R(\psi + \xi) \frac{\partial W}{\partial p} \right]^T \left[\nabla R(\psi + \xi) \frac{\partial W}{\partial p} \right] \right\}. \quad (10)$$

Thus the incremental warp function $W(\xi, \Delta p)$ can be determined. Afterwards, the deformation of the reference subset is inversely imposed on the target subset by updating the warp function $W(\xi, p)$ via

$$W(\xi, p) \leftarrow W(\xi, p) \circ W^{-1}(\xi, \Delta p), \quad (11)$$

where $W^{-1}(\xi, \Delta p)$ denotes the inverted incremental warp function of $W(\xi, \Delta p)$, and the operator $\circ$ means the composition of two warp functions. Finally, the vector of deformation p is updated using the elements in the new warp matrix according to Eq. (6).

Figure 2 illustrate procedure of the IC-GN algorithm. The iterative procedure can be summarized as four steps:

a) Construct a warped target subset $[\bar{T}[\psi + W(\xi, p)]]$ using an interpolation algorithm;

b) Compute the matrix

$$\sum_{\xi} \left\{ \left[\nabla R(\psi + \xi) \frac{\partial W}{\partial p} \right]^T \left[\frac{\bar{R}(\psi + \xi) + \nabla R(\psi + \xi) \frac{\partial W}{\partial p} \Delta p}{\bar{R}_n} - \frac{\bar{T}[\psi + \xi W(\xi, p)]}{\bar{T}_n} \right] \right\};$$

c) Compute the increment vector of deformation parameters Δp using Eq. (9);

d) Update the warp function $W(\xi, p)$ according to Eq. (11), then calculate p according to Eq. (6).

This four-step procedure is repeated until one of the following two pre-set convergence conditions is satisfied:

$$\text{i)} \quad \|\Delta p\| = \sqrt{(\Delta u)^2 + [\max(\Delta x) \times \Delta u_x]^2 + [\max(\Delta y) \times \Delta u_y]^2 + (\Delta v)^2 + [\max(\Delta x) \times \Delta v_x]^2 + [\max(\Delta y) \times \Delta v_y]^2} < 0.001;$$

ii) The maximum iteration number, e.g. 20, is reached.

3.2. Discussion

The gradient $\nabla R(\psi + \xi)$, the Jacobian matrix $\frac{\partial W}{\partial p}$, the steepest descent images $\nabla R(\psi + \xi) \frac{\partial W}{\partial p}$ and the inverse of the Hessian matrix H^{-1} can be pre-computed at the beginning of the IC-GN algorithm since these parameters keep invariable during the iteration. This pre-computation enables the IC-GN algorithm to surpass the classic forward additive NR algorithm in computation efficiency.

To accelerate the IC-GN algorithm further, Pan and Li proposed a strategy of pre-computed interpolation coefficients [14, 22]. During every iteration step, the intensity map need to be reconstructed in the warped target subset, and a bicubic interpolation is recommended for high accuracy [32]. However, to get the intensity at a sub-pixel location, 16 interpolation coefficients have to be determined using the intensity information of a small grid of 4×4 -integer-pixels surrounding this location [33], which is time consuming. Noticing that these 16 interpolation coefficients associated with every 4×4 -pixel grid in the target image are invariable and can be reused, a global interpolation coefficient look-up table can be constructed in the

pre-computation stage to avoid redundant calculations. By this way, the time spent for interpolation can be reduced for nearly two orders of magnitude [22].

4. A path-independent DIC method

The basic idea of DIC is the tracking of a geometric point before and after deformation, which can be done without knowing the deformation of the other POIs. DIC from its origin is a path-independent measurement technique. However, path-dependent strategies, either by simple line-scanning or by more sophisticated reliability guidance, are predominant nowadays. The main reason is to supply a reliable initial guess for an iterative matching algorithm with less computation cost. In other words, the path-dependence is not indispensable if quick and reliable initial guess can be attained for every POI independently. A path-independent DIC method will have two obvious advantages:

- i) It is suitable for parallel computing mechanism. By processing all the POIs simultaneously, the speed of DIC on a pair of high-resolution images can be increased dramatically.
- ii) The calculation at each POI is not disturbed by the other POI. This immunity make it applicable to the images containing large spatial discontinuities (e.g. long and wide crack or steep change of deformation), which may be problematic to any guided processing path.

In order to elucidate the superiority of the path-independent DIC, a method based on path-independent strategy is proposed, combining the FFT-CC algorithm with the IC-GN algorithm. The ZNCC-based FFT-CC algorithm is performed to get the integer-pixel displacement (u, v) of the interrogated POI (see Fig. 1). The obtained vector of deformation parameters $p_0 = [u, 0, 0, v, 0, 0]^T$ is then transferred to the ZNSSD-based IC-GN algorithm as the initial guess for sub-pixel registration (see Fig. 2). Besides, two small but useful techniques are applied to improve the performance of the proposed method.

First, in the FFT-CC algorithm, the Fourier transform and inverse Fourier transform are accelerated using a software library called Fastest Fourier Transform in the West (FFTW, 3rd version) developed by Frigo and Johnson [34]. According to the published benchmarking results, FFTW, even compared with commercial software libraries, shows a top-class performance in both speed and accuracy [26].

Second, to reduce the calculation in the IC-GN algorithm, an alternative method is used to update the warp function. The composition of two warp functions in Eq. (11) is directly calculated using Eq. (12), whereby the numerical calculation of the inverse of incremental warp matrix $W(\xi, \Delta p)$ is avoided.

$$\begin{aligned}
& \begin{bmatrix} 1+u_x & u_y & u \\ v_x & 1+v_y & v \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1+\Delta u_x & \Delta u_y & \Delta u \\ \Delta v_x & 1+\Delta v_y & \Delta v \\ 0 & 0 & 1 \end{bmatrix}^{-1} \\
&= \gamma^{-1} \begin{bmatrix} (1+u_x)(1+\Delta v_y) - u_y \Delta v_x & u_y(1+\Delta u_x) - (1+u_x)\Delta u_y & \gamma u + \alpha u_y - \beta(1+u_x) \\ v_x(1+\Delta v_y) - (1+v_y)\Delta v_x & (1+v_y)(1+\Delta u_x) - v_x \Delta u_y & \gamma v + \alpha(1+v_y) - \beta v_x \\ 0 & 0 & \gamma \end{bmatrix} \\
&\gamma = (1+\Delta u_x)(1+\Delta v_y) - \Delta u_y \Delta v_x \neq 0 \\
&\alpha = \Delta u \Delta v_x - \Delta u_x \Delta v - \Delta v \\
&\beta = \Delta u \Delta v_y + \Delta u - \Delta u_y \Delta v
\end{aligned}$$

(12)

Theoretically speaking, the proposed DIC method, incorporating the ZNSSD-based IC-GN algorithm, is able to achieve the registration accuracy as high as other IC-GN algorithm-based DIC methods. As regards the computation efficiency, it should be mentioned that we have not run the proposed method on a parallel processing device yet. In this work the POIs are processed following a natural point-wise scanning path along columns and then rows. Thus the additional time consumed by the proposed method, compared to a reliability-guided IC-GN algorithm, is due to the FFT-CC calculation at each POI. Nevertheless, this marginal cost in computation may be offset by the evaluation of reliability and the adjustment of path in the reliability-guided algorithm.

In order to evaluate the possible computation efficiency that the proposed method could reach on a GPU platform, the time spent by the FFT-CC algorithm and the IC-GN algorithm during DIC calculation is precisely counted. Moreover, the time for the pre-computation described in Sec. 3.2 is counted separately because the

pre-computation is carried out on the full image sequentially, prior to the iteration in the IC-GN algorithm. From a theoretical point of view, this pre-computation can also be speed up on a GPU platform, by dividing the image into a number of 4×4 -pixel grids and assigning the determination of interpolation coefficients in every grid to a parallel processing unit. The implementation of our path-independent DIC method on a GPU platform is in progress.

5. Experimental verification

The proposed DIC method was coded using C++ language and run on a laptop computer (Intel (R) Core (TM) i5-2557M CPU with main frequency of 1.70 GHz, 4.0 GB RAM). Computer-simulated speckle images and real experimental photos are utilized to study the performance of the proposed method, in the aspects of registration accuracy, computational efficiency and robustness.

5.1. Verification using computer-simulated examples

Figure 3 displays a 512×512 -pixel speckle image, which is numerically generated by simulating the grayscale intensity map with a number of randomly distributed Gaussian speckles [7]. The image contains approximately 4000 speckles with radius of 2 pixels. A rectangular region of interest (ROI) is set to cover the whole image.

To evaluate the registration accuracy and precision of the proposed DIC method, a series of speckle images were generated with pre-set sub-pixel displacements along y-axis varying from 0 to 1 pixel. The displacement between two successive images is

set to be 0.05 pixels. Two kinds of subsets with sizes of 33×33 pixels and 65×65 pixels were employed for comparison purpose. The accuracy and precision are quantitatively evaluated using two indicators: mean error and standard deviation. The mean error of the v -component of the measured displacement is defined by

$$e_v = \frac{1}{M} \sum_{i=1}^M (v_i - v_d), \quad (13)$$

where M denotes the number of POIs. In this example, M was set to be 2209 and 1849 for the two subsets, respectively. v_i is the calculated displacement at i th POI, and v_d is the pre-set displacement in the simulated speckle image. The standard deviation is defined as:

$$\sigma_v = \sqrt{\frac{1}{M-1} \sum_{i=1}^M (v_i - \bar{v})^2}, \quad (14)$$

where $\bar{v} = \frac{1}{M} \sum_{i=1}^M v_i$. These two indicators, if needed, can be similarly defined for the other deformation components.

Figure 4(a) shows the mean error of the v -component, in which a quasi-sinusoidal relation with maximum amplitude of 3.4×10^{-3} pixels can be seen in a period of 1 pixel. Such a regular distribution is attributed to the error of intensity interpolation during the construction of the speckle images with sub-pixel displacement [35]. The standard deviation of the v -component stays at a stable level of 1.8×10^{-3} pixels (33×33 -pixel subset) or 8.0×10^{-4} pixels (65×65 -pixel subset), as shown in Fig. 4(b). The results are indicative of good accuracy and precision of the proposed DIC

method, as compared with the path-dependent DIC method based on the IC-GN algorithm [14] or the NR algorithm [11]. In particular, the standard deviations are very close to zero in the cases of pre-set integer-pixel displacements, which means the calculated value at every POI is almost identical to each other.

Table 1 shows the statistics of the time consumed in the DIC calculation. At a POI, when using the 33×33 -pixel subset, the FFT-CC algorithm-based integer-pixel registration takes only 0.05 milliseconds (ms). Using the initial guess obtained by the FFT-CC algorithm, the IC-GN algorithm converges quickly to the correct results after 3-4 iterations. The iteration number decreases to 1 or 2 for pre-set integer-pixel displacements (i.e. 0 and 1 pixel). The average time spent at one POI by the IC-GN algorithm is 0.59 ms. On a 512×512 -pixel image, the average pre-computation time is about 69.66 ms. Therefore, with the current sequential implementation, the computation speed of the proposed DIC method is about 1489 point/sec. This speed, taking into account the relatively inferior CPU used in this work, is comparable to that of the reliability-guided IC-GN algorithm reported in Ref. [14]. However, if the proposed DIC method is running on a GPU platform, the most optimistic time cost for a pair of high-resolution images could be the same as that for one POI, which is less than 1 ms. To be more practical, considering the limited number of processing units in current GPU devices, the relatively slow individual processing unit compared with CPU, and the time spent for communication, real-time DIC at video frame rate (i.e. 25 frame/sec or 40 ms/frame) is possible.

The effect of the subset size can be observed in Fig. 4 and Table 1. It is found that the size of a subset exerts trivial influence on the registration accuracy but discernable influence on the precision. As regards the calculation speed, a large subset size, on one hand, reduces the number of iteration if the linear shape function is a good approximation in the subset, but on the other hand, slows down the computation due to the involvement of more pixels. According to Table 1, the 33×33 -pixel subset is regarded as a good option and will be employed in the following examples.

In order to evaluate the performance of the proposed DIC method in measurement of strain, another series of speckle images containing uniform pre-set strain u_y from 500 to 5000 $\mu\epsilon$ were generated with a step of 500 $\mu\epsilon$. The results show that the difference between the mean values of calculated u_y and pre-set u_y is less than 12 $\mu\epsilon$, i.e. the mean error of calculated u_y falls in a range from $-5.3 \mu\epsilon$ to $11.4 \mu\epsilon$, as shown in Fig. 5(a). However, the standard deviation of calculated u_y is at a level around 220 $\mu\epsilon$ (Fig. 5(b)), which indicates a discernible scattering of calculated u_y among the POIs. To obtain precise strain field from DIC results, further processing is required [36-39].

The robustness of the proposed DIC method is examined using an example containing the disturbance of intensity variation and deformation discontinuity. This example is designed to imitate a typical shear on the two sides of an rigid layer in composite materials. Two target speckle images with different displacements (in both direction and magnitude) are generated based on a reference speckle image (Fig. 6(a)).

To simulate the change of illumination condition after the deformation, the intensity of one target image is adjusted with 5% decrease in brightness and 15% increase in contrast, and the other one with 15% increase in brightness and 5% decrease in contrast. Afterwards, an upper triangular part is extracted from the first target image ($[u, v] = [3.8, 2.1]$ pixels), and joined with a low triangular part extracted from the second one ($[u, v] = [-2.3, -4.7]$ pixels), as shown in Fig. 6(b). A boundary line of discontinuity with grayscale intensity of 128 along the diagonal is added into the image.

Figure 7 gives the full-field distribution of displacement vector calculated using the proposed DIC method without any manual intervention. The proposed DIC method gives the correct results at most of POIs, except for those near discontinuity boundary. Around the boundary, the subsets include the pixels from both of the two triangles, which sometimes make the calculated value a compromise between the displacements of the two target images. It is found that decrease in intensity affects the DIC calculation very slightly, leading to small mean error of u -component and v -component (-0.0013 and 0.0026 pixels respectively) within the upper triangular area in Fig. 7. Nevertheless, increase in intensity bothers the calculation remarkably and results in higher mean error of u -component and v -component (-0.0051 and 0.0051 pixels respectively) within the lower triangular area in Fig. 7, as compared to the level displayed in Fig. 4(a). This effect may be attributed to the truncation of grayscale values at the pixels when their brightness is increased to a certain extent.

For comparison purpose, the example is processed using the IC-GN algorithm with path-dependent strategy. Fig. 8(a) shows a simple path design for the IC-GN algorithm, where the red dot is a seed POI manually selected in the upper triangular area, far from the discontinuity boundary. The deformation at the seed is calculated using FFT-CC algorithm and then IC-GN algorithm. Afterwards, the DIC computation follows the two paths. At the i th POI, the initial guess $(p_0)_i = \left[(u)_i, (u_x)_i, (u_y)_i, (v)_i, (v_x)_i, (v_y)_i \right]^T$ for the IC-GN algorithm is obtained from the deformation vector of the previously processed POI

$(p_0)_{i-1} = \left[(u)_{i-1}, (u_x)_{i-1}, (u_y)_{i-1}, (v)_{i-1}, (v_x)_{i-1}, (v_y)_{i-1} \right]^T$, according to

$$(p_0)_i = \begin{bmatrix} 1 & x_i - x_{i-1} & y_i - y_{i-1} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x_i - x_{i-1} & y_i - y_{i-1} \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} (p)_{i-1}, \quad (15)$$

where x_i and y_i denote the global coordinates of the i th POI.

Figure 8(b) shows that the path-dependent DIC method is severely disturbed by the discontinuity boundary. After the processing path crosses the boundary a couple of times, the error of initial guess becomes too large to mislead the IC-GN algorithm and induces the failure to converge after maximum iteration steps at some POIs (marked as diamonds in Fig8 (b)). Finally the accumulated error leads to an initial guess corresponding to a warped target subset located out of ROI, which terminates the IC-GN algorithm at the very beginning at these POIs marked as asterisks. It is noteworthy that a more intelligent path-dependent strategy, such as reliability-guided

tracking [20], might avoid this trouble without much manual intervention by finding a path crossing the discontinuous boundary as few as once. But it is not guaranteed, especially in more complicated cases.

5.2. Verification using real experimental results

In this section, the proposed DIC method is employed to process the photos from a real test. The grayscale images are appreciatively obtained from the website of the Society for Experimental Mechanics (Digital Image Correlation Challenge) [40]. Figure 9 shows a plate specimen (with a round hole in the middle), which is subjected to unidirectional tension along x -axis (for the convenience of display in the paper, the original TIF images with a size of 400×1040 pixels are rotated by 90° anticlockwise). A rectangular ROI with a size of 900×280 pixels, containing 1955 (85×23) POIs, is set within the specimen.

Figure 10 shows the computed distributions of the u -component and v -component of displacement. In Fig. 10(a) a uniformly increasing displacement along x -axis can be seen in the areas far from the hole. However, the distribution of the u -component is distorted near the hole, which demonstrates how the hole affects the deformation of the specimen. It is interesting that the hole is well outlined according to the DIC results. At the POIs that fall in this range, the subsets include too many black pixels and lose their characteristic features. Consequently, the FFT-CC algorithm does not get correct initial guesses, and the IC-GN algorithm fails to converge after maximum iteration steps, and thus gives the wrong values that are relatively large in magnitude.

Figure 10(b) shows the v -component field. In the most area of the ROI, the calculated displacements along y -axis are very small (less than 0.5 pixels in magnitude). The displacement at such sub-pixel level may be ascribed to the slight shake when the specimen was stretched by the testing machine. A distorted distribution of v -component field near the hole is also well visualized.

Table 2 lists the calculation time consumed by the DIC method. The pre-computation still takes the largest share of the computation cost, i.e. 64.23 ms. The FFT-CC algorithm keeps its speed (0.05 ms). The IC-GN algorithm spends much more time (1.29 ms) for processing the real experimental images, which means the iteration step increases considerably due to the relatively poor quality of speckle patterns and the existence of invalid region (i.e. the hole).

6. Concluding remarks

A path-independent DIC method combining the FFT-CC algorithm with the IC-GN algorithm was proposed and examined through numerical and real experiments. Some valuable conclusions can be drawn from the study:

- a) The independent estimation of integer-pixel initial guess at a POI is not as computationally expensive as it was thought to be. Actually, the computation cost of integer-pixel registration, using the FFT-CC algorithm accelerated with FFTW, is negligible compared to the more time-consuming iterative DIC algorithm. With this trivial burden, the path-independent DIC, even

implemented in a sequential way that processes the POIs one by one, can work as fast as the path-guided DIC.

- b) In the proposed DIC method, the IC-GN algorithm converges quickly using the initial guess transferred from the FFT-CC algorithm, and achieves as high accuracy and precision as its path-dependent counterpart.
- c) Path-independent strategy entirely avoids the issue of error spreading, which is an inherent drawback of the path-dependent DIC method. The proposed DIC method demonstrates outstanding capability to analyze the images containing large discontinuity of deformation, without any manual intervention.

Taking the long view, the return to path-independent strategy may open up a new vista of DIC methodology research. By breaking the dependence of DIC calculation at a POI on the deformation information at the other processed POIs, the proposed method meets the essential requirement of parallel computing. The development of path-independent DIC on parallel processing device makes it possible to achieve sub-pixel accuracy at a computation cost much lower than any of the present high-accuracy DIC algorithms. One can expect that the proposed DIC method, running on a GPU platform, performs real-time DIC calculation with high-accuracy, which is an ultimate goal aspired to for years.

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Figures and tables

Figure 1. Schematic flow chart of integer-pixel registration using the FFT-CC algorithm

Figure 2. Schematic flow chart of sub-pixel registration using the IC-GN algorithm

Figure 3. One of the computer-simulated speckle images used in numerical experiments (with the coordinate system).

Figure 4. (a) Mean error and (b) standard deviation of the v -component of the measured displacement as a function of the pre-set sub-pixel displacement.

Figure 5. (a) Mean error and (b) standard deviation of the measured strain as a function of the pre-set strain.

Figure 6. (a) Reference image and (b) target image joined using two images with different deformation. Before the two deformed images are joined together, one image (upper triangle) is adjusted with 5% decrease in brightness and 15% increase in contrast, and the other one (lower triangle) with 15% increase in brightness and 5% decrease in contrast. The boundary of discontinuity is along the diagonal.

Figure 7. Distribution of displacement vectors for an example containing intensity variation and discontinuous deformation calculated using the path-independent DIC method.

Figure 8. (a) A simple path design for DIC computation. (b) DIC results of the example in Figure 6 calculated using the path-guided DIC method (via the paths in (a)). Red dot represents a manually selected seed POI. Diamonds represent the POIs where the IC-GN algorithm fails to converge after maximum iteration steps. Asterisks represent the POIs where the warped target subset is located out of ROI at the beginning of IC-GN algorithm.

Figure 9. A real photo of plate specimen with a hole subjected to unidirectional tension (obtained from DIC challenge [40]). The solid-line rectangle outlines the ROI for DIC calculation.

Figure 10. Displacements of the stretched specimen calculated using the proposed DIC method: (a) u -component field, and (b) v -component field.

Table 1. Computation time consumed by the proposed DIC method using 33×33 -pixel subset and 65×65 -pixel subset

Pre-set displacement [pixel]	Pre-computation time (s1 / s2)* [millisecond]	Mean time of the calculation at one POI		Average iteration number in the IC-GN algorithm (s1 / s2)
		FFT-CC algorithm	IC-GN algorithm	
		(s1 / s2) [millisecond]	(s1 / s2) [millisecond]	
0.0	67.20 / 67.28	0.05 / 0.18	0.25 / 1.14	1.00 / 1.00
0.05	65.50 / 69.09	0.05 / 0.18	0.52 / 1.81	2.73 / 2.04
0.1	70.17 / 67.03	0.05 / 0.18	0.55 / 1.98	3.11 / 2.31
0.15	68.60 / 70.87	0.05 / 0.18	0.58 / 2.39	3.24 / 3.00
0.2	67.70 / 65.71	0.05 / 0.18	0.58 / 2.34	3.31 / 3.00
0.25	70.54 / 65.79	0.05 / 0.18	0.58 / 2.34	3.36 / 3.00
0.3	68.05 / 65.87	0.05 / 0.18	0.60 / 2.39	3.42 / 3.00
0.35	66.47 / 69.41	0.05 / 0.18	0.59 / 2.34	3.47 / 3.00
0.4	68.16 / 70.06	0.05 / 0.18	0.61 / 2.34	3.53 / 3.00
0.45	69.92 / 72.16	0.05 / 0.18	0.61 / 2.37	3.61 / 3.00
0.5	65.49 / 65.43	0.05 / 0.18	0.62 / 2.38	3.71 / 3.00
0.55	80.54 / 67.60	0.05 / 0.18	0.70 / 2.39	3.78 / 3.01
0.6	66.10 / 71.13	0.05 / 0.18	0.65 / 2.38	3.81 / 3.02
0.65	67.47 / 65.14	0.05 / 0.18	0.63 / 2.38	3.82 / 3.02
0.7	68.94 / 65.30	0.05 / 0.18	0.65 / 2.41	3.80 / 3.01
0.75	70.98 / 68.47	0.05 / 0.18	0.64 / 2.36	3.77 / 3.01
0.8	69.42 / 66.96	0.05 / 0.18	0.63 / 2.37	3.73 / 3.01
0.85	66.76 / 66.89	0.05 / 0.18	0.62 / 2.38	3.69 / 3.01
0.9	70.76 / 70.68	0.05 / 0.18	0.62 / 2.03	3.61 / 2.39
0.95	74.35 / 71.14	0.05 / 0.18	0.66 / 1.86	3.36 / 2.14
1.0	79.65 / 68.01	0.05 / 0.17	0.52 / 1.23	2.40 / 1.14

*s1 and s2 denote the 33×33 -pixel subset and the 65×65 -pixel subset, respectively.

Table 2. Computation time consumed by the proposed DIC method in a real experimental case.

Number of processed POI	Pre-computation time [millisecond]	Mean computation time at one POI	
		FFT-CC algorithm [millisecond]	IC-GN algorithm [millisecond]
1955	64.23	0.05	1.29