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2014

Zhang, D., Cai, W., & Wang, Q.-G. (2014). Energy-efficient H^∞ filtering for networked systems with stochastic signal transmissions. *Signal Processing*, 101, 134-141.

<https://hdl.handle.net/10356/81731>

<https://doi.org/10.1016/j.sigpro.2014.01.032>

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Energy-efficient H_∞ Filtering for Networked Systems with Stochastic Signal Transmissions[☆]

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Abstract

This paper is concerned with the energy-efficient filtering for a class of wireless sensor networks (WSNs). Due to the power limitation of WSNs, the measurement signal is transmitted to the remote filter infrequently and stochastically. A stochastic framework is proposed to formulate the filtering problem for such systems. A sufficient condition is established such that the filtering error system is mean-square stable and achieves a prescribed disturbance attenuation level in the H_∞ sense. The optimal filter design is presented to determine the filter gains. Relations between the transmission parameters, e.g., transmission probability, transmission intervals and the filtering performance are obtained. Finally, a continuous stirred tank reactor (CSTR) system is employed to evaluate the effectiveness of the proposed design.

Keywords: energy-efficient, H_∞ filtering, stochastic transmission, networked systems, sensor networks.

1. Introduction

Wireless Sensor Networks (WSNs) have received much attention in the last decade due to the extensive applications in target surveillance, informa-

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tion collection, environment monitoring and so on [1]. A typical WSN consists of a large number of wireless sensor nodes, and each sensor is equipped with limited computing and communication resources. These constraints have brought new challenges in developing control and estimation algorithms over networks. Specifically, the state estimation of dynamical systems based on the incomplete measurements is known to be challenging [2]-[6]. For example, the authors in [2] studied the H_∞ filtering problem for a class of networked systems with repeated scalar nonlinearities under unreliable communication links, and they showed that the optimal filter gain design can be determined by solving a linear matrix inequality. The authors in [5] investigated the H_∞ filtering problem for a class of discrete-Time Systems with stochastic incomplete measurement and mixed delays, and a unified modelling was presented to capture the random discrete delay and distributed delay. Based on the Lyapunov stability theory and some stochastic system analysis, the filter gains were finally determined by solving an optimization problem.

Sensor nodes in WSN are usually equipped with battery. Hence, power constraint in WSNs becomes a major problem that limits the application of WSNs. Experiments show that the major power consumption in sensor nodes comes from the communication process [7]. In the last decades, many efforts have been devoted to designing the energy efficient communication protocol for WSNs, e.g., multi-rate transmission protocol. Under such a protocol, each sensor transmits its measurement at the pre-given time instant. Though the transmission instants for different sensors are different, each sensor must transmit the measurement with a sequence (which is pre-given) and is not allowed to vary. For example, sensor 1 transmits its measurement with the 1st instant, sensor 2 transmits its measurement with the 2nd instant, and so on. Related state estimation algorithms for such transmission protocol can be found in [8]-[15].

It is noted that the above multi-rate transmission protocol is a deterministic one as the transmission instant is pre-set. This may lead to poor estimation performance. In order to reduce the conservatism of the deterministic transmissions proposed in [8]-[15]. Recently, the authors in [16]-[18] proposed a stochastic transmission one and the optimal estimator designs were also given. However, it should be pointed out that there still exist some limitations in the results [16]-[18], e.g., they only considered the Kalman filtering problem under the stochastic transmission protocol. It is well known that the standard Kalman filter is sensitive to modeling errors, and the dis-

tribution of the noise should be Guassian white noise. In practice, modeling error is inevitable and one may not always know the distribution of the noise. In these scenarios, estimator design methods in [16]-[18] fail to work and one should resort to other estimator design, e.g., the H_∞ filter design [19],[20], which is much robust than the Kalman filter. Further, in [16]-[18], they only used the transmission rate to design the filter, while other statistic information, e.g., the probability of successive non-transmissions has not been used, which leads to much conservatism. Hence, the energy-efficient filtering for networked systems with stochastic transmission protocol deserves research attention, which motivates the present study.

In this paper, the H_∞ filtering problem is considered for a class of networked systems, where the measurement signal is transmitted to the remote filter stochastically. Unlike the design results in [16]-[18], more statistic information is utilized to reduce the design conservatism. A new formulation is proposed to address such a filtering problem. A sufficient condition is obtained such that the filtering error system is asymptotically stable in the mean-square sense and achieves a prescribed H_∞ performance level. A case study on CSTR system is given to show the effectiveness of the proposed design.

The main contributions of this paper are summarized as follows:

- Most authors have considered the Kalman filtering problem with stochastic signal transmissions, see, e.g., [16]-[18], while it is well known that the standard Kalman filter is sensitive to modeling errors, and the distribution of the noise must be Guassian white noise. In practice, modeling error is inevitable and one may not always know the distribution of the noise. In these scenarios, estimator design methods in these works fail to work and one should resort to other estimator design, e.g., the H_∞ filter design, which is much robust than the Kalman filter. So, this motivates us to investigate the H_∞ filtering problem, and there has been no related paper reported on the H_∞ filtering with the stochastic signal transmissions.
- In the existing results, they only used the transmission rate to design the filter, while other statistic information, e.g., the probability of successive non-transmissions has not been used, which leads to much conservatism. In this paper, more information is used to design the filter, and one can see the effectiveness of our design in the Example Section.

- Though some papers have been reported on the event-based H_∞ filtering for networked systems, e.g., the reference [21]. In [21], the time delay system approach was used to study the event-based filtering problem, it should be pointed out that the main results are only applicable onto the continuous-time systems and the single sensor case. Unlike [21], our attention is focused on the energy-efficient filtering for discrete-time networked systems with multiple distributed sensors. Moreover, our analysis and design method is much simpler than that of [21].
- In the filter design with the stochastic signal transmission, one of the main difficulties is how to quantify the effect of transmission parameters into the filtering performance. In our work, interesting relations are established, specifically, the more frequently the measurement is transmitted, the better filtering performance one obtains. Moreover, the filtering performance level is a monotonic function of the largest transmission span of sensors. Such relations have not been established in literature, e.g., [21].

Notation: The notation is fairly standard. We use W^T to denote the transpose of any square matrix W . We use $W > 0$ to denote a positive-definite matrix W and I_n to denote the $n \times n$ identity matrix. Let $\mathbb{R}^n$ denote the n dimensional Euclidean space. $\mathbb{R}^{m \times n}$ is the set of all $m \times n$ real matrices. The notation $l_2[0, \infty)$ refers to the space of square summable infinite vector sequences with the norm $\|\bullet\|_2$. The symbol “*” will be used in some matrix expressions to represent the symmetric terms. Moreover, $\mathbb{E}\{\bullet\}$ stands for the mathematical expectation.

2. Problem Formulation

Consider the following discrete-time system:

$$x(k+1) = Ax(k) + Bw(k) \quad (1)$$

where $x(k) \in \mathbb{R}^{n_x}$ is the state, $w(k) \in \mathbb{R}^{n_w}$ is the unknown disturbance signal belonging to $l_2[0, +\infty)$. A and B are the constant matrices with appropriate dimensions. Suppose there are m distributed sensors for the system and the p -th sensor produces its measurement as

$$y_p(k) = C_p x(k) + D_p w(k) \quad (2)$$

where $y_p \in \mathbb{R}^1$ is a scale signal, C_p and D_p are the constant matrices with appropriate dimensions. To save energy, the measurement may not be transmitted to the remote filter at each time instant. On the other hand, to estimate the state reasonably well, the filter needs recent measurements. Thus, a reasonable communication protocol is that the measurement signal is transmitted at least once over N_p time steps, where $N_p > 0$ is an integer. No other restrictions are made on the protocol, implying that the transmission can happen at any time in these N_p steps and is random. One possible transmission scenario is depicted in Fig. 1, where $y_p(k)$ is the measurement signal sequence and $\tilde{y}_p(k)$ is the transmitted one with $N_p = 3$.

Figure 1: Transmission process

It follows from the transmission mechanism that there could be no transmission at some time instants. In the case of no transmissions, the filter has to use the last transmitted measurement signal as its input. Then, at each time instant k , the filter input, $\bar{y}_p(k)$, will be the most recent member of the transmitted subset of $\{y_p(k), y_p(k-1), \dots, y_p(k-N_p+1)\}$. To reflect this random selection of one member, a set of stochastic variables, $\alpha_{p,s}(k) \in \{0, 1\}$, $s = 0, 1, \dots, N_p - 1$, is introduced such that $\alpha_{p,s}(k) = 1$ if $y_p(k-s)$ is selected at k as the filter input, which happens when there is transmission at $k-s$ but none at $k-s+1, k-s+2, \dots, k$; $\alpha_{p,s}(k) = 0$,

otherwise. For the scenario considered before, the selected $\bar{y}_p(k)$ is shown in Fig. 1 as the 3rd line. The above filter input selection rule implies that at each k , there is one and only one measurement point (most recent one) is selected as the filter input, and equivalently, there is one and only one $\alpha_{p,s}(k) = 1$ with all other variables being zero, so that $\sum_{s=0}^{N_p-1} \alpha_{p,s}(k) = 1$. In this paper, the probabilities $\mathbb{E}\{\alpha_{p,s}(k) = 1\} = \bar{\alpha}_{p,s}$ are assumed to be known, which is reasonable as the data on actual transmissions could be collected on any given system and the estimates of these probabilities can be obtained with their frequencies. Obviously, there holds $\sum_{s=0}^{N_p-1} \bar{\alpha}_{p,s} = 1$. Moreover, different sensors are assumed to operate independent of each other and their transmission sequences are then also independent.

It follows from the above transmission protocol and selection rule that the filter input can be expressed as

$$\bar{y}_p(k) = \alpha_{p,0}(k)y_p(k) + \alpha_{p,1}(k)y_p(k-1) + \cdots + \alpha_{p,N_p-1}(k)y_p(k-N_p+1) \quad (3)$$

We now verify (3) for the scenario in Fig. 1. With $N_p = 3$, one readily sees that

- when $k = 2$, the measurement signal is transmitted immediately so that $\alpha_{p,0}(k) = 1$ and $\alpha_{p,s}(k) = 0$ for $s = 1, 2$. It follows from (3) that $\bar{y}_p(2) = y_p(2)$, which is true from Fig.1.
- when $k = 3$, the measurement is not transmitted but was done at the last step, so that $\alpha_{p,1}(k) = 1$ and $\alpha_{p,s}(k) = 0$ for $s = 0, 2$. It follows from (3) that $\bar{y}_p(3) = y_p(3-1) = y_p(2)$, which is true from Fig.1.
- when $k = 4$, the measurement is not transmitted again, so that $\alpha_{p,2}(k) = 1$ and $\alpha_{p,s}(k) = 0$ for $s = 0, 1$. It follows from (3) that $\bar{y}_p(4) = y_p(4-2) = y_p(2)$, which is true from Fig.1.

Remark 1. In the traditional estimator designs for single sensor systems, it may not be necessary to update the estimator state when no new measurement arrives at the estimator side. However, in the large scale wireless sensor networks, one can not know how many sensor measurements may arrive at the filter side at certain time instant, hence, in this paper we require the estimator to update the state at each time instant.

Remark 2. The signal may not be transmitted to the remote estimator, e.g., the measurement signal varies only a little compared with the ones transmitted in the last time instant. Hence, the measurement is transmitted intermittently. We formulate such a transmission as a stochastic one as in [16]-[18]. However, compared with the filtering results in [16]-[18], our follow-up filtering results would be much robust and more applicable as we do not assume the Guassian white noise here.

Remark 3. It is seen that (3) reduces to measurement models in [16]-[18] when $\alpha_{p,s}(k) = 0$ for all $s = 1, \dots, N_p - 1$. Incorporating these statistics information to our new filter design can achieve a better estimation performance level than that of [16]-[18], as will be shown in simulation later.

Remark 4. In this paper, we pay our attention on the stochastic transmission issue, however the packet may be lost under the networked environment. It should be pointed out that the proposed modelling (3) has also captured the successive packet dropout phenomenon. When one further considers the packet dropout problem, one may replace N_p in (3) by $M_p + \bar{N}_p$, where M_p is the upper bound of successive packet dropout and $\bar{N}_p$ is the largest transmission span.

Let $N = \max_{p=1,2,\dots,m} \{N_p\}$, and define

$$\begin{aligned} X(k) &= \begin{bmatrix} x^T(k) & x^T(k-1) & \cdots & x^T(k-N+1) \end{bmatrix}^T, \\ W(k) &= \begin{bmatrix} w^T(k) & w^T(k-1) & \cdots & w^T(k-N+1) \end{bmatrix}^T. \end{aligned}$$

Substituting (2) to (3) yields

$$\bar{y}_p(k) = \sum_{s=0}^{N_p-1} \alpha_{p,s}(k) \{C_p E_{ps} X(k) + D_p H_{ps} W(k)\} \quad (4)$$

where $E_{ps} = \begin{bmatrix} 0 & \cdots & 0 & I_{n_x} & 0 & \cdots & 0 \end{bmatrix}$, and $H_{ps} = \begin{bmatrix} 0 & \cdots & 0 & I_{n_w} & 0 & \cdots & 0 \end{bmatrix}$, in which all elements are zeros except for the $(s+1)$ -th block is I_{n_x} and I_{n_w} , respectively.

Let $\alpha_p(k) = [\alpha_{p,0}(k), \alpha_{p,1}(k), \dots, \alpha_{p,N_p-1}(k)]$, then the stochastic vector $\alpha_p(k)$ has N_p possible realizations as

$$\alpha_p(k) \in \{[1, 0, 0, \dots, 0], \dots, [0, 0, 0, \dots, 1]\} \quad (5)$$

Define $\alpha(k) = [\alpha_1(k), \alpha_2(k), \dots, \alpha_m(k)]$. The total number of possible realizations of $\alpha(k)$ is $N = N_1 \times N_2 \times \cdots \times N_m$. Let

$$\begin{aligned}\bar{E}_{\alpha(k)} &= [E_{1,\alpha_1(k)}^T \quad E_{2,\alpha_2(k)}^T \quad \cdots \quad E_{m,\alpha_m(k)}^T]^T, \\ \bar{H}_{\alpha(k)} &= [H_{1,\alpha_1(k)}^T \quad H_{2,\alpha_2(k)}^T \quad \cdots \quad H_{m,\alpha_m(k)}^T]^T.\end{aligned}$$

One particular realization of $\alpha(k)$ means one sequence of $\alpha(k)$, which specifies one particular case of $(\bar{E}_{\alpha(k)}, \bar{H}_{\alpha(k)})$. For ease of exposition, define an integer set $\Gamma = \{1, 2, \dots, N\}$, and introduce a new set of stochastic variables, $\beta_i(k) \in \{1, 0\}, i \in \Gamma$, one variable for one possible realization of $\alpha(k)$, such that $\beta_1(k) = 1$ if and only if $\alpha_p(k) = [1, 0, \dots, 0], p = 1, 2, \dots, m$; $\beta_2(k) = 1$ if and only if $\alpha_p(k) = [1, 0, \dots, 0], p = 1, 2, \dots, m-1, \alpha_m(k) = [0, 1, \dots, 0]$; and so on. By our construction, there is one and only one realization of $\alpha(k)$ at any time instant, so that $\sum_{i=1}^N \beta_i(k) = 1$.

The probability, $\mathbb{E}\{\beta_i(k) = 1\} = \bar{\beta}_i$, can be computed by the probabilities of sensor transmissions, $\bar{\alpha}_{p,s}$. For example, we have two sensors and each sensor transmits the measurement within two time steps stochastically with their probabilities being $\bar{\alpha}_{1,0}, \bar{\alpha}_{1,1}$ and $\bar{\alpha}_{2,0}, \bar{\alpha}_{2,1}$, respectively. Then, it follows from simple probability rules that $\bar{\beta}_1 = \bar{\alpha}_{1,0}\bar{\alpha}_{2,0}$, $\bar{\beta}_2 = \bar{\alpha}_{1,0}\bar{\alpha}_{2,1}$, $\bar{\beta}_3 = \bar{\alpha}_{1,1}\bar{\alpha}_{2,0}$ and $\bar{\beta}_4 = \bar{\alpha}_{1,1}\bar{\alpha}_{2,1}$.

In view of the above development, we can express the filter input vector as

$$\bar{y}(k) = \sum_{i=1}^N \beta_i(k) \{ \bar{C} \bar{E}_i X(k) + \bar{D} \bar{H}_i W(k) \} \quad (6)$$

where $\bar{C} = \text{diag}\{C_1, C_2, \dots, C_m\}$, $\bar{D} = \text{diag}\{D_1, D_2, \dots, D_m\}$. In this paper, we aim to estimate the following signal:

$$z(k) = Lx(k) \quad (7)$$

where $z(k) \in \mathbb{R}^{n_z}$ and L is a constant matrix with appropriate dimension. In order to estimate the signal in (7), we propose the following filter:

$$\begin{cases} x_f(k+1) = A_f x_f(k) + B_f \bar{y}(k) \\ z_f(k) = C_f x_f(k) \end{cases} \quad (8)$$

where $x_f(k) \in \mathbb{R}^{n_x}$ is the state of the filter and $z_f(k) \in \mathbb{R}^{n_z}$ is the estimate of $z(k)$. A_f, B_f , and C_f are the filter parameters to be designed.

In order to derive the filtering error system, we rewrite state equation in (1) and the estimation equation in (7) as

$$\begin{cases} X(k+1) = \bar{A}X(k) + \bar{B}W(k) \\ z(k) = \bar{L}X(k) \end{cases} \quad (9)$$

where

$$\bar{A} = \begin{bmatrix} A & 0 \\ I_{nN} & 0 \end{bmatrix}, \bar{B} = \begin{bmatrix} B & 0 \\ 0 & 0 \end{bmatrix}, \bar{L} = [L \quad 0].$$

Define the augmented state as $\eta(k) = [X^T(k) \quad x_f^T(k)]^T$ and the estimation error as $e(k) = z(k) - z_f(k)$. Then the filtering error system is given by

$$\begin{cases} \eta(k+1) = \tilde{A}\eta(k) + \tilde{B}W(k) + \sum_{i=1}^N (\beta_i(k) - \bar{\beta}_i) [\hat{A}_i\eta(k) + \hat{B}_iW(k)] \\ e(k) = \tilde{L}\eta(k) \end{cases} \quad (10)$$

where

$$\begin{aligned} \tilde{A} &= \begin{bmatrix} \bar{A} & 0 \\ B_f \bar{C} \bar{E} & A_f \end{bmatrix}, \tilde{B} = \begin{bmatrix} \bar{B} \\ B_f \bar{D} \bar{H} \end{bmatrix}, \tilde{L} = [\bar{L} \quad -C_f], \\ \hat{A}_i &= \begin{bmatrix} 0 & 0 \\ B_f \bar{C} \bar{E}_i & 0 \end{bmatrix}, \hat{B}_i = \begin{bmatrix} 0 \\ B_f \bar{D} \bar{H}_i \end{bmatrix}, \bar{E} = \sum_{i=1}^N \bar{\beta}_i \bar{E}_i, \bar{H} = \sum_{i=1}^N \bar{\beta}_i \bar{H}_i. \end{aligned}$$

Our later development makes use of the following definitions.

Definition 1. The system (10) with $w(k) = 0$ is said to be asymptotically stable in the mean-square sense, if the solution $\eta(k)$ of system (10) satisfies $\lim_{k \rightarrow \infty} \mathbb{E}\{\|\eta(k)\|\} = \eta(0)$ for any initial condition.

Definition 2. For given scalars $\gamma > 0$, system (10) is said to be asymptotically stable in the mean-square sense and achieves a prescribed H_∞ performance $\gamma > 0$, if it is asymptotically stable and under zero initial condition, $\sum_{s=0}^{+\infty} \mathbb{E}\{e^T(s)e(s)\} \leq \sum_{s=0}^{+\infty} \gamma^2 w^T(s)w(s)$ holds for all nonzero $w(k) \in l_2[0, \infty)$.

The filtering problem is summarized as follows:

Filtering Problem: Design a filter in form of (8) such that the filtering error system (10) is asymptotically stable in the mean-square sense and achieves a prescribed H_∞ performance level in the presence of stochastic transmissions.

3. The proposed solution

Based on the stochastic analysis method, a sufficient condition is presented for the solvability of considered filtering problem in the following theorem.

Theorem 1. For given scalars τ and $\bar{\beta}_i, i = 1, 2, \dots, N$, if there exists a positive-definite matrix P such that the following inequality

$$\begin{bmatrix} \Phi_1 & \Phi_2 & \sqrt{\bar{\beta}_1}\Xi_1 & \cdots & \sqrt{\bar{\beta}_N}\Xi_N & \Phi_3 \\ * & -P^{-1} & 0 & 0 & 0 & 0 \\ * & * & -P^{-1} & 0 & 0 & 0 \\ * & * & * & \ddots & 0 & 0 \\ * & * & * & * & -P^{-1} & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0 \quad (11)$$

hold, then the filtering error system (10) is asymptotically stable in the mean-square sense and achieves a prescribed H_∞ performance level $\gamma = \tau\sqrt{N}$, where

$$\Phi_1 = \begin{bmatrix} -P & 0 \\ 0 & -\tau^2 I \end{bmatrix}, \Phi_2 = [\tilde{A} \quad \tilde{B}]^T, \Xi_i = [\hat{A}_i \quad \hat{B}_i]^T, \Phi_3 = [\tilde{L} \quad 0]^T.$$

Proof : We first prove the stability of the filtering error system (10) with $w(k) = 0$. Suppose the following Lyapunov function for system (10):

$$V(k) = \eta^T(k)P\eta(k) \quad (12)$$

Then, one sees that

$$\begin{aligned} & \mathbb{E} \{V(k+1) - V(k)\} \\ &= \begin{bmatrix} \tilde{A}\eta(k) \end{bmatrix}^T P \begin{bmatrix} \tilde{A}\eta(k) \end{bmatrix} - \eta^T(k)P\eta(k) \\ &+ \mathbb{E} \left\{ \left[\sum_{i=1}^N \phi_i(k)\hat{A}_i\eta(k) \right]^T P \left[\sum_{i=1}^N \phi_i(k)\hat{A}_i\eta(k) \right] \right\} \end{aligned} \quad (13)$$

where, $\phi_i(k) = \beta_i(k) - \bar{\beta}_i$. On the other hand, we have

$$\begin{aligned}
& \mathbb{E} \left\{ \left[\sum_{i=1}^N \phi_i(k) \hat{A}_i \eta(k) \right]^T P \left[\sum_{i=1}^N \phi_i(k) \hat{A}_i \eta(k) \right] \right\} \\
&= \mathbb{E} \left\{ \begin{aligned} & \sum_{i=1}^N (\phi_i(k))^2 \left[\hat{A}_i \eta(k) \right]^T P \left[\hat{A}_i \eta(k) \right] \\ & + \sum_{i=1}^N \sum_{j=1, j \neq i}^N \phi_i(k) \phi_j(k) \left[\hat{A}_i \eta(k) \right]^T P \left[\hat{A}_j \eta(k) \right] \end{aligned} \right\} \\
&= \sum_{i=1}^N \bar{\beta}_i \left[\hat{A}_i \eta(k) \right]^T P \left[\hat{A}_i \eta(k) \right] \\
&\quad - \left[\sum_{i=1}^N \bar{\beta}_i \hat{A}_i \eta(k) \right]^T P \left[\sum_{i=1}^N \bar{\beta}_i \hat{A}_i \eta(k) \right] \\
&\leq \sum_{i=1}^N \bar{\beta}_i \left[\hat{A}_i \eta(k) \right]^T P \left[\hat{A}_i \eta(k) \right]
\end{aligned} \tag{14}$$

Hence,

$$\begin{aligned}
& \mathbb{E} \{ V(k+1) - V(k) \} \\
&\leq \left[\tilde{A} \eta(k) \right]^T P \left[\tilde{A} \eta(k) \right] \\
&\quad - \eta^T(k) P \eta(k) + \sum_{i=1}^N \bar{\beta}_i \left[\hat{A}_i \eta(k) \right]^T P \left[\hat{A}_i \eta(k) \right]
\end{aligned} \tag{15}$$

It is seen that the right hand side of (15) is negative under (11), and then system (10) is asymptotically stable in the mean-square sense.

Now we consider the H_∞ performance of the filtering error system (10). It follows the above analysis method, we have

$$\begin{aligned}
& \mathbb{E} \{ V(k+1) - V(k) + e^T(k) e(k) - \tau^2 W^T(k) W(k) \} \\
&= \left[\tilde{A} \eta(k) + \tilde{B} W(k) \right]^T P \left[\tilde{A} \eta(k) + \tilde{B} W(k) \right] - \eta^T(k) P \eta(k) \\
&\quad + \mathbb{E} \left\{ \begin{aligned} & \left[\sum_{i=1}^N (\beta_i(k) - \bar{\beta}_i) \left(\hat{A}_i \eta(k) + \hat{B}_i W(k) \right) \right]^T \\ & \times P \left[\sum_{i=1}^N (\beta_i(k) - \bar{\beta}_i) \left(\hat{A}_i \eta(k) + \hat{B}_i W(k) \right) \right] \end{aligned} \right\} \\
&\quad + \left[\tilde{L} \eta(k) \right]^T \left[\tilde{L} \eta(k) \right] - \tau^2 W^T(k) W(k) \\
&\leq \bar{\eta}^T(k) \left[\Phi_1 + \Phi_2 P \Phi_2^T + \sum_{i=1}^N \bar{\beta}_i \Xi_i P \Xi_i^T + \Phi_3 \Phi_3^T \right] \bar{\eta}(k)
\end{aligned} \tag{16}$$

where, $\bar{\eta}(k) = \begin{bmatrix} \bar{\eta}^T(k) & W^T(k) \end{bmatrix}^T$. By using the Schur Complement [22], it is easy to see

$$\mathbb{E} \{V(k+1) - V(k) + e^T(k)e(k) - \tau^2 W^T(k)W(k)\} < 0 \quad (17)$$

Summing both side of (17) from $k = 0$ to $k = T$, then

$$\mathbb{E} \left\{ V(k+1) - V(0) + \sum_{k=0}^T \{e^T(k)e(k) - \tau^2 W^T(k)W(k)\} \right\} < 0 \quad (18)$$

which implies, by the zero initial condition and positiveness of $V(k+1)$, that

$$\mathbb{E} \left\{ \sum_{k=0}^T \{e^T(k)e(k) - \tau^2 W^T(k)W(k)\} \right\} < 0 \quad (19)$$

Let $T \rightarrow +\infty$, then

$$\sum_{k=0}^{+\infty} \mathbb{E} \{e^T(k)e(k)\} \leq \tau^2 W^T(k)W(k) = \gamma^2 w^T(k)w(k) \quad (20)$$

Hence, system (10) achieves a prescribed H_∞ performance level as well. This completes the proof.

Remark 5. In the filtering of stochastic transmitted systems, one of the main difficulties is how to quantify the transmission parameters into the filtering performance. In Theorem 1, it is interesting to see that the filtering performance level is a monotonic function of the largest transmission span N .

It should be pointed out that theorem 1 can not be used to determine the filter gain directly due to the co-existence of P and P^{-1} . In the following theorem, we present the filter gain design algorithm.

Theorem 2. For given scalars τ and $\bar{\beta}_i, i = 1, 2, \dots, N$, if there exist a positive-definite matrix P and a matrix G with appropriate dimensions, such that the following inequality

$$\begin{bmatrix} \Phi_1 & \bar{\Phi}_2 & \sqrt{\bar{\beta}_1} \bar{\Xi}_1 & \cdots & \sqrt{\bar{\beta}_N} \bar{\Xi}_N & \bar{\Phi}_3 \\ * & \bar{P} & 0 & 0 & 0 & 0 \\ * & * & \bar{P} & 0 & 0 & 0 \\ * & * & * & \ddots & 0 & 0 \\ * & * & * & * & \bar{P} & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0 \quad (21)$$

hold, then the considered filtering performance is solvable. Moreover, the filter gains are determined by $A_f = G_3^{-T} A_F$, $B_f = G_3^{-T} B_F$, $C_f = C_F$. where

$$\bar{\Phi}_2 = \begin{bmatrix} \bar{\Phi}_2^1 \\ \bar{\Phi}_2^2 \end{bmatrix}, \bar{\Xi}_i = \begin{bmatrix} \bar{\Xi}_i^1 \\ \bar{\Xi}_i^2 \end{bmatrix}, \bar{\Phi}_3 = \begin{bmatrix} \bar{\Phi}_3^1 \\ 0 \end{bmatrix}, \bar{P} = P - G - G^T,$$

with

$$\begin{aligned} \bar{\Phi}_2^1 &= \begin{bmatrix} \bar{A}^T G_1 + E^T B_F^T M & \bar{A}^T G_2 + E^T B_F^T \\ A_F^T M & A_F^T \end{bmatrix}, \\ \bar{\Phi}_2^2 &= \begin{bmatrix} \bar{B}^T G_1 + \bar{H}^T B_F^T M & \bar{B}^T G_2 + \bar{H}^T B_F^T \end{bmatrix}, \\ \bar{\Xi}_i^1 &= \begin{bmatrix} \bar{E}_i^T \bar{C}^T B_F^T M & \bar{E}_i^T \bar{C}^T B_F^T \\ 0 & 0 \end{bmatrix}, \\ \bar{\Xi}_i^2 &= \begin{bmatrix} \bar{H}_i^T \bar{D}^T B_F^T M & \bar{H}_i^T \bar{D}^T B_F^T \end{bmatrix}, \\ \bar{\Phi}_3^1 &= \begin{bmatrix} \bar{L}^T \\ -C_F^T \end{bmatrix}, \quad P = \begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix}, \\ G &= \begin{bmatrix} G_1 & G_2 \\ G_3 M & G_3 \end{bmatrix}, \quad M = \begin{bmatrix} I & \cdots & I \end{bmatrix}. \end{aligned}$$

Proof : By pre and post multiplying (11) with $\text{diag} \left\{ I, G^T, \underbrace{G^T, \dots, G^T}_N, I \right\}$

and its transpose respectively, (11) is equivalent to

$$\begin{bmatrix} \Phi_1 & \Phi_2 G & \sqrt{\beta_1} \Xi_1 G & \cdots & \sqrt{\beta_N} \Xi_N G & \Phi_3 \\ * & \tilde{P} & 0 & 0 & 0 & 0 \\ * & * & \tilde{P} & 0 & 0 & 0 \\ * & * & * & \ddots & 0 & 0 \\ * & * & * & * & \tilde{P} & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0 \quad (22)$$

where $\tilde{P} = -G^T P^{-1} G$. On other hand, it is easy to see that $-G^T P^{-1} G^T \leq P - G - G^T$ always hold for any matrix G . Then, (22) holds if

$$\begin{bmatrix} \Phi_1 & \Phi_2 G & \sqrt{\beta_1} \Xi_1 G & \cdots & \sqrt{\beta_N} \Xi_N G & \Phi_3 \\ * & \bar{P} & 0 & 0 & 0 & 0 \\ * & * & \bar{P} & 0 & 0 & 0 \\ * & * & * & \ddots & 0 & 0 \\ * & * & * & * & \bar{P} & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0 \quad (23)$$

which is the same as (21) with $P = \begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix}$, $G = \begin{bmatrix} G_1 & G_2 \\ G_3 M & G_3 \end{bmatrix}$, $A_F = G_3^T A_f$, $B_F = G_3^T B_f$, and $C_F = C_f$. This completes the proof.

Remark 6. In order to obtain the minimum H_∞ performance γ^* , one can solve the following optimization problem:

$$\begin{aligned} \min \quad & \nu \\ \text{subject to} \quad & (21) \text{ with } \nu = \tau^2 \end{aligned} \quad (24)$$

and find the minimum H_∞ performance γ^* by $\gamma^* = \sqrt{\nu^* N}$.

4. Illustrative Examples

In this section, a modified CSTR system [23] is given to show the usefulness of the proposed filter design. The CSTR is depicted in Fig. 2, and there is the following series-parallel reactions:

where $\mathbb{A}$ = cyclopentadiene, $\mathbb{B}$ = cyclopentenol, $\mathbb{C}$ = cyclopentanediol and $\mathbb{D}$ = dicyclopentadiene. The reactor inflow contains only the educt $\mathbb{A}$ in low concentration $c_{\mathbb{A}0}$. The desired product is the component $\mathbb{B}$, the intermediate component in the series reaction. Assuming constant density and an ideal residence time distribution within the reactor, the balance equations are given as follows [23]:

$$\begin{aligned} \frac{dc_{\mathbb{A}}}{dt} &= \frac{\dot{V}}{V_R}(c_{\mathbb{A}0} - c_{\mathbb{A}}) - k_1 c_{\mathbb{A}} - k_3 c_{\mathbb{A}}^2 \\ \frac{dc_{\mathbb{B}}}{dt} &= -\frac{\dot{V}}{V_R} c_{\mathbb{B}} + k_1 c_{\mathbb{A}} - k_2 c_{\mathbb{B}} \\ \frac{d\vartheta}{dt} &= \frac{\dot{V}}{V_R}(\vartheta_0 - \vartheta) + \frac{k_w A_R}{\zeta C_p V_R}(\vartheta_K - \vartheta) \\ &\quad - \frac{k_1 c_{\mathbb{A}} \Delta H_R^{\mathbb{A}\mathbb{B}} + k_2 c_{\mathbb{B}} \Delta H_R^{\mathbb{B}\mathbb{C}} + k_3 c_{\mathbb{A}}^2 \Delta H_R^{\mathbb{A}\mathbb{D}}}{\zeta C_p} \end{aligned} \quad (26)$$

where $c_{\mathbb{A}}$ and $c_{\mathbb{B}}$ are the concentrations of educt $\mathbb{A}$ and the desired product $\mathbb{B}$ within the reactor, ϑ denotes the reactor temperature. The rate factors $k_1 - k_3$ depend exponentially on the reactor temperature ϑ via Arrhenius law

$$k_i(\vartheta) = k_{0i} \exp\left(\frac{-E_{Ai}}{R\vartheta}\right) \quad (27)$$

For the reaction system at hand, we let $k_1 = k_2$. The values of model parameters and the steady-state values of the main operating point of the

Figure 2: Continuous stirred tank reactor

Table 1. Model parameters and main operating point

Model parameters	$A_R = 0.215\text{m}^2$
$k_{01,2} = 1.287 \times 10^{12}\text{h}^{-1}$	$V = 10.01$
$k_{03} = 9.043 \times 10^9\text{mol h}^{-1}$	$\vartheta_0 = 403.15\text{K}$
$E_{A1,2}/R = 9758.3\text{K}$	$k_w = 4032\text{kJ/h m}^2 \text{K}$
$E_{A3}/R = 8560.0\text{K}$	Main operating point
$\Delta H_R^{\text{AB}} = 4.2\text{kJ/mol}$	$C_{\text{As}} = 1.235\text{mol/l}$
$\Delta H_R^{\text{BC}} = -11\text{kJ/mol}$	$C_{\text{Bs}} = 0.9\text{mol/l}$
$\Delta H_R^{\text{AD}} = -41.85\text{kJ/mol}$	$\vartheta_s = 407.29\text{K}$
$\rho = 0.9342\text{kg/l}$	$\dot{V}/V_R = 18.83\text{h}^{-1}$
$C_P = 3.01\text{kJ/kg K}$	$C_{\text{A}0s} = 5.1\text{mol/l}$

reactor are listed in Table 1. Detailed background of CSTR can be found in [23].

Linearizing (26) at the operating point, we have the following state space model

$$\dot{x}(t) = A_p x(t) + B_p u(t) \quad (28)$$

where

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} c_{\mathbb{A}} - c_{\mathbb{A}s} \\ c_{\mathbb{B}} - c_{\mathbb{B}s} \\ \vartheta - \vartheta_s \end{bmatrix}, \quad u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \dot{V} - \dot{V}_s \\ c_{\mathbb{A}0} - c_{\mathbb{A}0s} \end{bmatrix},$$

A_p and B_p are Jacobian matrices with

$$A_p = \begin{bmatrix} -86.0962 & 0 & 4.2077 \\ 50.6146 & -69.4446 & -0.9974 \\ 172.2263 & 197.9985 & -65.5149 \end{bmatrix},$$

$$B_p = \begin{bmatrix} 0.3861 & 18.83 \\ -0.0899 & 0 \\ -0.4136 & 0 \end{bmatrix}.$$

In the state estimation problem, one may also treat the control input as the unknown input signal. Since CSTR is open loop stable based on our parameters, we choose $u = 0$ for simplicity. By further considering the unknown disturbance in CSTR, we have the following discrete-time state space model by sampling period $T_0 = 1$ min:

$$x(k+1) = Ax(k) + Bw(k) \quad (29)$$

where

$$A = \begin{bmatrix} 0.2747 & 0.0345 & 0.0206 \\ 0.2323 & 0.3152 & 0.0033 \\ 1.2566 & 1.1042 & 0.3671 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

In practice, it may be necessary for one to know the product concentration $c_{\mathbb{B}}$ for other use, but the direct measure of concentration $c_{\mathbb{B}}$ by using traditional chemical approaches is usually expensive. An alternative yet non-expensive approach is to use signal processing approaches to estimate the concentration. In this example, we deploy two sensors to measure the educt concentration $c_{\mathbb{A}}$ and the reactor temperature ϑ , and the purpose is to estimate

the product concentration $c_{\mathbb{B}}$. Hence, $C_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$, $C_2 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$, $D_1 = 0.3$, $D_2 = 0.2$ and $L = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$. Suppose the noise varies in $w(k) \in [-1, 1]$. Note that this assumption is only for simulation purpose. One may find the up and lower bound of the noise in real industrial fields. The estimation task is carried out for 100 minutes.

Suppose that the transmission occurs if one of the following events happens:

$$\|y_p(k) - y_{last,p}\| \geq \delta_{y,p} \quad (30)$$

$$k - k_{last,p} > \theta_{k,p} \quad (31)$$

where $y_{last,p}$ is the last transmitted signal of sensor p , $k_{last,p}$ is the last transmitted time instant of sensor p . $\delta_{y,p}$ and $\theta_{k,p}$ are the measurement threshold and time threshold, respectively. In this example, we assume no packet dropout occurs only for simulation simplicity, and we set $\delta_{y,1} = 0.1$, $\delta_{y,2} = 0.2$; $\theta_{k,1} = \theta_{k,2} = 1$. It implies that $N_1 = N_2 = 2$. By doing 200 random samples for the transmissions, we have $\bar{\alpha}_{1,0} = 0.69$, $\bar{\alpha}_{1,1} = 0.31$; $\bar{\alpha}_{2,0} = 0.44$, $\bar{\alpha}_{2,1} = 0.56$.

With all information available, we solve the optimization problem (24) to get $\gamma^* = 1.9069$. The corresponding filter gains are

$$A_f = \begin{bmatrix} 0.2171 & 0.0420 & 0.0167 \\ 0.4788 & -0.0179 & 0.0159 \\ 6.0850 & -4.3532 & 0.6279 \end{bmatrix}, \quad B_f = \begin{bmatrix} -1.2479 & 0.1254 \\ -1.4373 & 0.1444 \\ -4.2860 & 0.4305 \end{bmatrix},$$

$$C_f = \begin{bmatrix} -1.5297 & 0.7602 & 0.0040 \end{bmatrix}.$$

One sample trajectories of $z(k)$ and $z_f(k)$ are shown in Fig. 3. By simple calculation, we have $\sqrt{\sum_{k=0}^{100} e^T(k)e(k)} / \sqrt{\sum_{k=0}^{100} w^T(k)w(k)} = 0.9342 < \gamma^* = 1.9069$. In order to see how robust our method is, we run 500 simulations. In each simulation, the transmission process is randomly generated and the transmission probabilities of each simulation are different (also different from the statistical one). It is shown in Fig. 4 that the true performance levels are smaller than the optimal one γ^* . Hence, the estimator design is robust to the probability uncertainties from the simulation point of view. Please note that if the transmission protocol in [16]-[18] is used, the optimal H_∞ performance is $\gamma^* = 2.0362$, which is much larger than that of our ones.

We are now on the position to see how the transmission parameters affect the filtering performance. Here, we increase the measurement threshold $\delta_{y,1}$.

Figure 3: Trajectories of $z(k)$ and $z_f(k)$

Figure 4: True performance level of 500 samples

The relation between transmission probability and the filtering performance is listed in the following table. It is seen that the more frequently the measurement is transmitted, the better filtering performance one obtains. In this scenario, more energy would be consumed by the sensors. The trade-off between the energy consumption and the filtering performance should be considered in design of the WSNs, for example, if one requires that the filtering performance level is below 2.1, then, one may set $\delta_{y,1} = 0.15$ but not necessary to choose $\delta_{y,1} = 0.1$.

Table 2. Relation between $\bar{\alpha}_{1,0}$ and γ^*

$\delta_{y,1}$	0.1	0.15	0.2	0.25	0.3
$\bar{\alpha}_{1,0}$	0.69	0.58	0.49	0.43	0.39
γ^*	1.9069	2.0926	2.1378	2.1670	2.1859

5. Conclusions

In this paper, the energy-efficient filtering for a class of networked systems has been investigated. A new stochastic transmission protocol is proposed and formulated. A sufficient condition is given such that the filtering error system is asymptotically stable in the mean-square sense and achieves a prescribed H_∞ performance level. A filter design is presented and illustrated with a case study of CSTR system. In our work, relations between the transmission parameters, e.g., the transmission interval, the transmission probability and the filtering performance have been established.

Acknowledgement

The authors would like to thank the Editor-in-Chief, the Associate Editor, and anonymous reviewers for their constructive comments that have helped to improve this paper significantly. The work was jointly funded by A*STAR-Ministry of National Development Green Building Joint Grant (1121760027) and Energy Research Institute @ NTU.

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