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Observer-Based Control of Rijke-type Combustion Instability

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Abstract. In this work, observer-based feedback control of combustion instability in a Rijke-type thermoacoustic system is considered. A generalized thermoacoustic model with distributed monopole-like actuators is developed. The model is linearized and formulated in state-space and it is assumed that pressure sensors are the only information available for feedback. It is shown that a system of this form is observable. As a Linear-Quadratic-Gaussian (LQG) controller is implemented to tune the actuators, the system becomes asymptotically stable. The performance of the controller is evaluated with a system involving two modes. The successful demonstration indicates that the observer-based feedback controller can be applied to a real combustion system with multiple modes.

Keywords: Combustion instability, LQG control, non-normality.

INTRODUCTION

Recently, there has been a surge of interest in the control of thermoacoustic oscillations in Rijke-type combustion systems. Both passive and active control methods have been developed to prevent thermoacoustic instability, which is regarded as a major challenge for land-based gas turbine and aero-engine manufacturers ([11], [13]).

Passive control [14] involves either applying acoustic dampers such as Helmholtz resonators ([7], [19]) and acoustic liners ([5], [24]) to increase the damping or redesigning the system, such as changing the flame anchoring position or changing the operating conditions. Passive approach is simple but it is effective only over a limited range of operating conditions. On the other hand, active control approaches can be applied to a wide range of operations and can be implemented to various types of combustion systems.

There are two typical actuation actions that can be applied to such systems. One is to modulate the acoustic field by using a monopole-like sound source such as a loudspeaker ([22]). The other is to modulate the unsteady heat release rate by using a secondary fuel injector ([16]). An experimental study of using loudspeakers to minimize the thermoacoustic instabilities was conducted by [4] on a swirl-stabilized non-premixed spray combustor. Model-based controllers such as linear quadratic Gaussian (LQG)/loop transfer recovery (LTR) and H^∞ loop-shaping were proposed in the literature ([3]). It is found that these controllers resulted in better attenuation of the pressure oscillations than the phase-delay approach. The traditional linear controllers designed for a thermoacoustic system have focused on the most prominent eigenmodes of the system. The objective of these controllers is to make these eigenmodes decay exponentially. However, if the eigenmodes of the thermoacoustic system are non-orthogonal, controlling the dominant eigenmode alone may cause other potential mode being excited due to the coupling between the acoustic modes [1].

The non-orthogonality (i.e., non-normality) of thermoacoustic eigenmodes has recently received attention ([9], [20]). It has been shown that the non-orthogonality is due to the presence of unsteady heat release or the complex impedance boundary conditions. It has also been shown that in a linearly stable system but with non-orthogonal eigenmodes, there can be significant transient energy growth of small perturbations before their eventual decay. If the transient growth rate is large enough, thermoacoustic instability might be triggered by causing small-amplitude disturbances to grow to amplitudes high enough to make nonlinear effects significant.

In this work, a simplified Rijke-type thermoacoustic system with multiple distributed actuators is considered. In Section 2, the dimensionless governing equations are described and the flow perturbations are expanded by using Galerkin series following [21]. The model is linearized and formulated in state-space form. Section 3 introduces the

design of a Linear-Quadratic-Gaussian controller. In Section 4, pressure measurements are introduced and an observer design proposed. A case study for two modes is presented and its performance is evaluated in Section 5. It is shown that combustion instabilities can be asymptotically stabilized.

DESCRIPTION OF ACTUATED THERMOACOUSTIC MODEL

The thermoacoustic system examined in this paper, a horizontal Rijke tube with distributed actuators, is identical to that studied in [23]. However, for completeness, a brief description of the actuated thermoacoustic model is presented here.

Dimensionless governing equations

For convenience, the variables are non-dimensionalized as follows:

$$u = \frac{\tilde{u}}{u_0}, \quad p = \frac{\tilde{p}}{\gamma M_0 p_0}, \quad \dot{Q}_s = \frac{\dot{\tilde{Q}}_s}{\gamma p_0 u_0}, \quad x = \frac{\tilde{x}}{L_0}, \quad t = \frac{\tilde{t} c_0}{L_0}, \quad \frac{\delta(x - x_f)}{L_0} = \tilde{\delta}(\tilde{x} - \tilde{x}_f),$$

where the variables with tilde represent dimensional quantities; the quantity with a subscript 0 denotes the mean value; x denotes the location along the duct (actuators and heat source are located at x_{ak} and x_f , respectively, as shown in Fig. 1), t denotes time, M_0 is the Mach number, c_0 is the sound speed, L_0 is the length of the duct, and γ is the ratio of specific heats.

Figure 1. Schematic of diagram of the combustion system with distributed actuators modelled as monopole-like moving piston.

The non-dimensional acoustic equations of such a thermoacoustic system in the presence of K actuators can then be written as

$$\frac{\partial u}{\partial t} + \frac{\partial p}{\partial x} = 0, \quad (1)$$

$$\frac{\partial p}{\partial t} + \zeta p + \frac{\partial u}{\partial x} = (\gamma - 1) \dot{Q}_s \delta(x - x_f) + \gamma \sum_{k=1}^K \alpha_{ak} v_{ak} \delta(x - x_{ak}), \quad (2)$$

where ζp describes the thermo-viscous and friction damping across the heating element, and α_{ak} describes the ratio of the cross-sectional area S_{ak} of the k^{th} actuator to the cross-sectional area S of the duct, i.e., $\alpha_{ak} = S_{ak}/S$. The non-dimensional heat release rate $\dot{Q}_s$ can be written as

$$\dot{Q}_s = \mathcal{K} \left[\sqrt{\left| \frac{1}{3} + u_f(t - \tau) \right|} - \sqrt{\frac{1}{3}} \right], \quad (3)$$

with

$$\mathcal{K} = \frac{2L_w(T_w - \bar{T}_0)}{\sqrt{3}u_0 S \gamma p_0} \sqrt{\pi \lambda c_v \rho_0 \frac{d_w}{2}}, \quad (4)$$

where d_w , L_w , and T_w denote the diameter, length and temperature of the heated wire, respectively; ρ denotes the density, T is the temperature, λ and c_v are the thermal conductivity and specific heat capacity at constant volume, respectively; S corresponds to the cross-sectional area of the duct and τ denotes the non-dimensional time delay describing the difference between the time when the oncoming velocity perturbation acts and the time when the corresponding heat release is felt. A subscript ak or f refers to the variable taken at the actuator or heat source location, respectively. The gas is assumed to be perfect, inviscid and non-heat conducting. The heating element is assumed to be acoustically compact. It is also assumed that there is no mean temperature gradient.

The acoustic pressure $p(x, t)$ and velocity $u(x, t)$ inside the duct can be written as a superposition of the duct natural modes as:

$$p(x, t) = - \sum_{j=1}^N \frac{\sin(j\pi x)}{j\pi} \dot{\eta}_j(t), \quad (5)$$

$$u(x, t) = \sum_{j=1}^N \cos(j\pi x) \eta_j(t), \quad (6)$$

where N represents the number of modes.

The actuation signal v_{ak} of the k^{th} monopole-like source such as loudspeaker [6] can now be expressed as

$$v_{ak} = \mathcal{R}_k u(x_{ak}) + \mathcal{S}_k p(x_{ak}), \quad (7)$$

where $\mathcal{R}_k$ and $\mathcal{S}_k$ are dimensionless control parameters of the actuators. Discretizing the governing equations by substituting Eqs. (5) and (6) into Eq. (2) and simplifying yields

$$\frac{\ddot{\eta}_j}{j\pi} + j\pi \eta_j + \zeta_j \frac{\dot{\eta}_j}{j\pi} = -2(\gamma - 1) \left[\dot{Q}_s(x_f, t - \tau) \sin(j\pi x_f) + \frac{\gamma}{\gamma - 1} \sum_{k=1}^K \alpha_{ak} v_{ak}(x_{ak}, t) \sin(j\pi x_{ak}) \right]. \quad (8)$$

Note that in Eq. (8) the damping ζ is taken into account by assigning a damping parameter ζ_j to each mode. For the thermoacoustic system examined in this paper, the pressure p and the velocity gradient $\frac{\partial u}{\partial x}$ are both set to zero at the ends of the duct (i.e., there is no acoustical energy being dissipated at the end boundaries). Furthermore, the acoustic waves are assumed to be planar such that there is also no acoustic energy being dissipated in the viscous and thermal boundary layers at the duct walls. Both types of dissipations are modelled by the damping parameter for each mode:

$$\zeta_j = \frac{1}{2\pi} \left[c_1 j + c_2 j^{-1/2} \right], \quad (9)$$

where c_1 and c_2 are to be the same for each mode. This model, which is based on the correlations developed by [12], has been widely used ([9]).

The actuated nonlinear thermoacoustic model as described above contains the most influential characteristics as discussed in more sophisticated flame-involved control models.

Linearization of the governing equations

The non-dimensional heat release rate $\dot{Q}_s$ can be linearized as

$$\dot{Q}_s(x_f, t - \tau) \approx \frac{\sqrt{3}\mathcal{K}}{2} u_f(t - \tau), \quad (10)$$

where it has been assumed that $|u_f(t - \tau)| < 1/3$ and keeping terms that are linear in τ only, $u_f(t - \tau)$ can be approximated as

$$u_f(t - \tau) \approx \sum_{j=1}^N \cos(j\pi x_f) [\eta_j(t) - \tau \dot{\eta}_j(t)]. \quad (11)$$

Define

$$\begin{aligned}\mathbf{x} &= (\eta_1, \eta_2, \dots, \eta_N)^T, \quad \dot{\mathbf{x}} = (\dot{\eta}_1, \dot{\eta}_2, \dots, \dot{\eta}_N)^T, \\ \Psi &= (\cos(\pi x), \cos(2\pi x), \dots, \cos(N\pi x))^T, \quad \Phi = \left(\frac{\sin(\pi x)}{\pi}, \frac{\sin(2\pi x)}{2\pi}, \dots, \frac{\sin(N\pi x)}{N\pi} \right)^T, \\ \mathbf{M} &= \text{diag} \left(\frac{1}{\pi}, \dots, \frac{1}{N\pi} \right), \quad \mathbf{D}_d = \text{diag} \left(\frac{\zeta_1}{\pi}, \dots, \frac{\zeta_N}{N\pi} \right),\end{aligned}$$

so that the heat release rate can be expressed as

$$\dot{Q}_s(x_f, t - \tau) \approx \frac{\sqrt{3}\mathcal{K}}{2} \Psi_f^T (\mathbf{x} - \tau \dot{\mathbf{x}}). \quad (12)$$

and, hence, system (8) can be rewritten as

$$\begin{aligned}\mathbf{M}\ddot{\mathbf{x}} + \mathbf{D}_d\dot{\mathbf{x}} + \mathbf{M}^{-1}\mathbf{x} &= (1 - \gamma)\sqrt{3}\mathcal{K}\Psi_f^T (\mathbf{x} - \tau \dot{\mathbf{x}}) \mathbf{M}^{-1}\Phi_f \\ &\quad - 2\gamma \left[\sum_{k=1}^K \alpha_{ak} \mathcal{R}_k \mathbf{M}^{-1} \Phi_{ak} \Psi_{ak}^T \right] \mathbf{x} + 2\gamma \left[\sum_{k=1}^K \alpha_{ak} \mathcal{S}_k \mathbf{M}^{-1} \Phi_{ak} \Phi_{ak}^T \right] \dot{\mathbf{x}},\end{aligned} \quad (13)$$

which can be further simplified as

$$\mathbf{M}\ddot{\mathbf{x}} + \left(\mathbf{D}_d - (\gamma - 1)\sqrt{3}\mathcal{K}\tau \mathbf{M}^{-1}\Phi_f \Psi_f^T \right) \dot{\mathbf{x}} + \left(\mathbf{M}^{-1} + (\gamma - 1)\sqrt{3}\mathcal{K}\mathbf{M}^{-1}\Phi_f \Psi_f^T \right) \mathbf{x} = \mathbf{u}, \quad (14)$$

where $\mathbf{u}$ is the control input defined by

$$\mathbf{u} = -2\gamma \left[\sum_{k=1}^K \alpha_{ak} \mathcal{R}_k \mathbf{M}^{-1} \Phi_{ak} \Psi_{ak}^T \right] \mathbf{x} + 2\gamma \left[\sum_{k=1}^K \alpha_{ak} \mathcal{S}_k \mathbf{M}^{-1} \Phi_{ak} \Phi_{ak}^T \right] \dot{\mathbf{x}}. \quad (15)$$

Let $\chi_1 = \mathbf{x}$ and $\chi_2 = \mathbf{M}\dot{\mathbf{x}}$ so that the state-space equations of the actuated thermoacoustic system is given as

$$\begin{aligned}\dot{\chi}_1 &= \mathbf{M}^{-1}\chi_2, \\ \dot{\chi}_2 &= - \left(\mathbf{M}^{-1} + (\gamma - 1)\sqrt{3}\mathcal{K}\mathbf{M}^{-1}\Phi_f \Psi_f^T \right) \chi_1 - \left(\mathbf{D}_d - (\gamma - 1)\sqrt{3}\mathcal{K}\tau \mathbf{M}^{-1}\Phi_f \Psi_f^T \right) \mathbf{M}^{-1}\chi_2 + \mathbf{u}.\end{aligned} \quad (16)$$

Denote by $\chi = (\chi_1^T \chi_2^T)^T$ the state vector. Then the linearized state space equations can be written as

$$\dot{\chi} = \mathbf{A}\chi + \mathbf{B}\mathbf{u}, \quad (17)$$

where

$$\begin{aligned}\mathbf{A} &= \begin{bmatrix} \mathbf{0} & \mathbf{M}^{-1} \\ \mathbf{A}_1 & \mathbf{A}_2 \end{bmatrix}, \quad \mathbf{B} = [\mathbf{0} \quad \mathbf{I}]^T, \\ \mathbf{A}_1 &= - \left(\mathbf{M}^{-1} + (\gamma - 1)\sqrt{3}\mathcal{K}\mathbf{M}^{-1}\Phi_f \Psi_f^T \right), \quad \mathbf{A}_2 = - \left(\mathbf{D}_d - (\gamma - 1)\sqrt{3}\mathcal{K}\tau \mathbf{M}^{-1}\Phi_f \Psi_f^T \right) \mathbf{M}^{-1}.\end{aligned}$$

DESIGN OF A LQR CONTROLLER

Consider a LQR (Linear-Quadratic-Regulator) controller of the form

$$\mathbf{u} = -\mathbf{K}\chi, \quad (18)$$

that minimizes the quadratic cost function

$$\mathcal{J} = \int_0^\infty (\chi^T \mathbf{Q}\chi + \mathbf{u}^T \mathbf{R}\mathbf{u}) dt, \quad (19)$$

where $\mathbf{Q}$ is a symmetric positive-semidefinite weighting matrix and $\mathbf{R}$ is a positive-definite weighting matrix. The optimal control gain matrix $\mathbf{K}$ is found by solving the corresponding matrix Riccati equation (or using MATLAB's *lqr* function). This controller is then applied to the actual nonlinear system.

To recover the dimensionless control parameters $\mathcal{R}_k$ and $\mathcal{S}_k$, the gain matrix is partitioned as $\mathbf{K} = [\mathbf{K}_1 \ \mathbf{K}_2]$ so that the control input $\mathbf{u}$ can be rewritten as

$$\mathbf{u} = -\mathbf{K}_1 \chi_1 - \mathbf{K}_2 \chi_2, \quad (20)$$

where $\mathbf{K}_i \in \mathbb{R}^{N \times N}$, $i = 1, 2$. Hence, it can be clearly seen that the following relations hold

$$\mathbf{K}_1 = 2\gamma \sum_{k=1}^K \alpha_{ak} \mathcal{R}_k \mathbf{M}^{-1} \Phi_{ak} \Psi_{ak}^T, \quad (21)$$

$$\mathbf{K}_2 = -2\gamma \sum_{k=1}^K \alpha_{ak} \mathcal{S}_k \mathbf{M}^{-1} \Phi_{ak} \Phi_{ak}^T \mathbf{M}^{-1}. \quad (22)$$

Given an optimal gain matrix $\mathbf{K}$, generally $2N^2$ linear equations are required to solve Eqs. (21)-(22) for the dimensionless control parameters $\mathcal{R}_k$ and $\mathcal{S}_k$. This defines the relationship between the number of actuators and the number of eigenmodes to be controlled such that in general N^2 actuators are needed to realize an LQR controller.

OBSERVER-CONTROLLER DESIGN

To implement the LQR controller explained in the previous section, knowledge of the full state vector is required. It can be easily seen that, by locating a certain number of velocity and pressure sensors along the tube, one can use Eqs. (5)-(6) at those locations to calculate the state vector. Nevertheless, only one of this type of sensors is typically used. In this paper, the subsequent development is restricted to the assumption that only pressure sensors are available. Its extension to velocity sensors is straight forward and thus omitted here. It is shown that even with a partial knowledge of the state vector, the system can be still controlled using an observer design. This observer also accounts for process and measurement noises normally present in this kind of systems.

Introducing sensors which can only measure the pressure values at M different locations x_{sk} , $k = 1, \dots, M$, and adding white Gaussian noises to the state and observer equations, the linearized system (17) can be rewritten as

$$\dot{\chi} = \mathbf{A}\chi + \mathbf{B}\mathbf{u} + \mathbf{w}, \quad (23)$$

$$\mathbf{y} = \mathbf{C}\chi + \mathbf{v}, \quad (24)$$

where $\mathbf{y} = [p(x_{s1}, t) \ p(x_{s2}, t) \ \dots \ p(x_{sM}, t)]^T$ is the measured output,

$$\mathbf{C} = [\mathbf{0} \quad -\Phi_S^T \mathbf{M}^{-1}], \quad (25)$$

where

$$\Phi_S = [\Phi_{s1} \ \Phi_{s2} \ \dots \ \Phi_{sM}],$$

and $\mathbf{w}$, $\mathbf{v}$ are assumed to be zero-mean white Gaussian noises, uncorrelated and with covariance matrices $\mathbf{W}$ and $\mathbf{V}$, respectively. It can be easily shown that this system is observable since

$$\text{rank} [\mathbf{C}^T \ \mathbf{A}^T \mathbf{C}^T \ \dots \ (\mathbf{A}^T)^{2N-1} \mathbf{C}^T] = 2N.$$

According to the stationary Kalman-Bucy filter solution, define the mathematical model of the observer to be

$$\dot{\hat{\chi}} = \mathbf{A}\hat{\chi} + \mathbf{B}\mathbf{u} + \mathbf{L}(\mathbf{y} - \mathbf{C}\hat{\chi}), \quad (26)$$

where $\hat{\chi}$ denotes the estimated state based on the measurements of $\mathbf{y}$, and use the observed state feedback as

$$\mathbf{u} = -\mathbf{K}\hat{\chi}. \quad (27)$$

Here the observer gain matrix $\mathbf{L}$ is chosen as

$$\mathbf{L} = \mathbf{P}\mathbf{C}^T \mathbf{V}^{-1}, \quad (28)$$

where $\mathbf{P}$ is the solution of the following Riccati equation

$$\mathbf{0} = \mathbf{A}\mathbf{P} + \mathbf{P}\mathbf{A}^T - \mathbf{P}\mathbf{C}^T \mathbf{V}^{-1} \mathbf{C}\mathbf{P} + \mathbf{W}.$$

SIMULATIONS

The feedback control law developed in the previous section is implemented here for a system with two modes. The actuator locations are chosen at $x_{a1} = 0.2$, $x_{a2} = 0.3$, $x_{a3} = 0.4$, and $x_{a4} = 0.5$. The sensor is located at $x_s = 0.9$. The physical parameters used in the simulations are given in Table I. Initial conditions at the heat source location are $p(x_f, 0) = 0$ and $u(x_f, 0) = -0.1675$ for $x_f = 0.7$.

Table 1. Parameters.

Parameter	Value	Parameter	Value
ρ	1.025 kg/m ³	λ	0.0328 W/m · K
c_v	719 J/kg · K	γ	1.4
L_0	1 m	L_w	2.5 m
c	344 m/s	u_0	0.3 m/s
T_0	295 K	T_w	1680 K
d_w	0.5×10^{-3} m	S	1.56×10^{-3} m
P_0	8.69×10^4 Pa	τ	0.25 ms
ζ_1	0.0440	ζ_2	0.1657
α_{a1}	0.01	α_{a2}	0.01

The LQR gain matrix $\mathbf{K}$ found by choosing the weighting matrices as

$$\mathbf{Q} = \frac{1}{2} \begin{bmatrix} \mathbf{I}_{2 \times 2} & \mathbf{0}_{2 \times 2} \\ \mathbf{0}_{2 \times 2} & \mathbf{I}_{2 \times 2} / \pi \end{bmatrix}, \quad \mathbf{R} = \mathbf{I}_{2 \times 2},$$

yields the following eigenvalues for the closed-loop system matrix $\mathbf{A} - \mathbf{BK}$:

$$(-0.4198 \pm 6.7538i, -0.5782 \pm 2.3303i).$$

From relations (21) and (22), the dimensionless control parameters are recovered as

$$\mathcal{R}_1 = 87.81, \mathcal{R}_2 = -172.27, \mathcal{R}_3 = 157.64, \mathcal{R}_4 = -64.46,$$

$$\mathcal{S}_1 = -76.80, \mathcal{S}_2 = \mathcal{S}_3 = 24.88, \mathcal{S}_4 = -41.39.$$

A computer implementation of the LQR control law given by (18) with the above feedback gain matrix was performed to achieve the stabilization of the thermoacoustic system.

This controller is applied to the actual nonlinear system. It has been assumed that the output vector is affected by an independent zero-mean white Gaussian noise with $\sigma_y^2 = 10^{-3}$. The covariance matrices have been chosen as

$$\mathbf{W} = 10^{-4} \text{diag}\{1, 1, 1, 1\}, \quad \mathbf{V} = 10^{-3}.$$

The results of the simulation are shown in Figs. 2-3.

As can be seen in Fig. 2, the nondimensionalized velocity and pressure perturbations at the heat source location converge for the given control input. Figure 3 shows that, in the absence of any control input, the system is linearly unstable.

Figure 2. Time responses of velocity u and pressure p at the heat source location (LQG) and of control input u .

Figure 3. Time responses of velocity u and pressure p at the heat source location (zero input case).

CONCLUSION

In this work, feedback control of combustion instability in a Rijke-type thermoacoustic system has been considered. A generalized thermoacoustic model with distributed monopole-like actuators has been proposed and a Linear-Quadratic Regulator has been developed to stabilize the combustion system. It has been shown that in the presence of pressure sensors only, the system is observable and, subsequently, an observer design has been introduced. The performance of the observer-based controller has been illustrated through a thermoacoustic system involving two modes. Our future work includes extensions to cope with the nonlinear dynamics of the combustion system.

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