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Triple-well potential with a uniform depth: Advantageous aspects in designing a multi-stable energy harvester

Pilkee Kim,¹ Dowung Son,² and Jongwon Seok^{2,a)}

¹School of Mechanical and Aerospace Engineering, Nanyang Technological University, Singapore 639798, Singapore

²School of Mechanical Engineering, College of Engineering, Chung-Ang University, Seoul 156-756, South Korea

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Analytical expressions for the bi- and tri-stable conditions of a multi-stable energy harvester (MEH) are derived on the basis of bifurcation analyses, and the associated multi-stable regions are characterized in a 2-D parametric space. It is found that a special boundary condition exists for a triple-well with a uniform depth (T_U boundary condition), originating from a degenerate pitchfork bifurcation (DPF) point. Interestingly, the outermost well-to-well distance of the triple-well potential, when subjected to the condition that the maximum well depth is kept constant, becomes widest when the well depth is uniform. Accordingly, instead of investigating all possible parametric conditions, the design parameters for the optimal well configuration of the MEH can be sought most efficiently by simply tracing them on the T_U boundary. A detailed examination of the potential well configurations along the T_U boundary reveals that the most efficient energy harvesting from low-intensity ambient vibrations can be achieved on a T_U boundary point, near the DPF point but inevitably a certain distance apart, by inducing an enlarged interwell motion. This investigation is experimentally validated. *Published by AIP Publishing.* [<http://dx.doi.org/10.1063/1.4954169>]

During the last decade, nonlinear oscillators such as softening (or hardening) mono-stable oscillators^{1–6} and bi-stable (or multi-stable) oscillators^{7–26} have been intensively investigated for vibration energy harvesting applications. In particular, bi-stable energy harvesters (BEHs)^{7–20} have been considered to be able to extract high electrical power from ambient vibration sources over a broad frequency band, particularly when a potential escape phenomenon (inducing a large-amplitude interwell motion) occurs. Because the BEHs' potential energy barrier and resonance hysteresis can inhibit the occurrence of interwell motions under an actual environmental vibration source, the identification of their characteristics must be a top priority. Many studies have been conducted for performance optimization or system improvement in order to find solutions to characterize the BEHs.²⁰ In addition, many different types of vibration sources (such as swept sine⁹ and Gaussian white noise¹⁴) have been used to estimate the realistic performance of the BEHs.

Recently, multi-stable energy harvesters (MEHs), including not only the bi-stable but also tri-stable and even quad-stable^{21,22} devices have been proposed, and several theoretical and experimental studies^{21–25} show that a shallow MEH potential well configuration (with a low potential barrier) can be beneficial to vibration energy harvesting from a low-intensity vibration source. Extensive numerical bifurcation analyses of MEHs^{21,23} have been useful in thoroughly uncovering the formation mechanisms of the multi-stability and provide a mathematical means to draw stability maps in parametric spaces. Impact-driven interwell motion²⁴ was used to promote high-energy orbits of the MEH, regardless of the hysteresis at resonance, and the effect of the potential

well depth²⁵ was also examined. MEHs commonly require at least two parameters to describe multiple-well configurations (generally one more than the BEHs),^{21–25} which makes their design and optimization process complicated. The present study proposes an efficient analytical approach for designing the potential well configuration of an MEH, as shown in Fig. 1. The MEH under consideration acts as a bi- or tri-stable oscillator, strongly depending on the relative locations of three permanent magnets.²² In this study, rigorous stability criteria are obtained by analytical bifurcation analyses, and the potential well configurations in the resulting multi-stable regions are thoroughly investigated. The advantages of the triple-well potential, possessing a uniform depth, are found and discussed. It should be noted that in both dynamic simulations and experiments of this study, it is postulated that a certain initial perturbation initiates high-energy orbit motion (if it exists), regardless of the resonant hysteresis, such as that used in the impact-driven energy harvester.²⁴

FIG. 1. Schematic diagram of a multi-stable energy harvester composed of a bimorph cantilever and three neodymium magnets ($2 \times 10 \times 6 \text{ mm}^3$). The stainless steel beam ($70 \times 12 \times 0.3 \text{ mm}^3$) partially covered with two piezoelectric polyvinylidene fluoride laminates ($52 \times 12 \times 0.052 \text{ mm}^3$) is used for the bimorph beam. Two distance parameters, d_x and d_z , are used to completely determine the relative locations of these magnets.

^{a)}E-mail: seokj@cau.ac.kr

The MEH system under consideration (Fig. 1) can be modeled as a piezoelectric composite beam, subjected to a magnetic load exerted on its free end. The governing equations and boundary conditions are derived on the basis of the Euler–Bernoulli beam theory and linear piezoelectricity.¹⁷ The geometric nonlinearity²¹ caused by the possible large deflection of the beam is also considered, and the associated nonlinear terms are maintained up to the third order in the model. The nonlinear magnetic force is described using the magnetic dipole model.²³ In this study, the magnetic force is approximated by a fifth-order polynomial that is the simplest form for expressing the multi-stable features of the MEH. Within the abovementioned modeling framework, the electromechanical oscillator model of the MEH can be obtained in dimensionless form

$$\ddot{x} + \eta \dot{x} + (1 - \gamma_1)x + \gamma_3 x^3 + \gamma_5 x^5 - \kappa v - N_b = f \sin \Omega t, \quad (1a)$$

$$\dot{v} + \rho v + 2\dot{x} = -\beta_4 \dot{x} x^2, \quad (1b)$$

with

$$N_b = -\beta_1 \dot{x} x^2 + \beta_2 x^2 v - \beta_3 (\dot{x}^2 x + \ddot{x} x^2), \quad (1c)$$

where x and v are the tip deflection and voltage, respectively; a dot over a symbol indicates a derivative with respect to time t ; η is the damping coefficient; κ and ρ are the electro-mechanical coupling coefficient and cut-off frequency, respectively; γ_1 , γ_3 , and γ_5 are the nonlinear spring constants varying with d_x and d_z ; and N_b includes the inertial effect and electromechanical coupling effect owing to the geometric nonlinearity with the coefficients β_1 , β_2 , β_3 , and β_4 . Refer to supplementary material²⁷ for the system parameters and the detailed expressions of γ_1 , γ_3 , and γ_5 .

The dimensionless restoring force F_R and potential energy function Φ can be expressed as

$$F_R = -x(1 - \gamma_1 + \gamma_3 x^2 + \gamma_5 x^4), \quad (2a)$$

$$\Phi = \frac{1}{2} x^2 \left(1 - \gamma_1 + \frac{1}{2} \gamma_3 x^2 + \frac{1}{3} \gamma_5 x^4 \right). \quad (2b)$$

Clearly, the static equilibrium of the MEH always satisfies $F_R = 0$.

To investigate the multi-stability characteristics of the MEH, a bifurcation analysis for the static equilibrium state was analytically performed with two bifurcation parameters, $\alpha = 1 - \gamma_1$ and $\mu = \gamma_3 / \sqrt{4\gamma_5}$. Three different types of static bifurcations were observed with the following conditions: $\alpha = 0$ ($\mu > 0$) for supercritical pitchfork (PF_{sup}) bifurcation, $\alpha = 0$ ($\mu < 0$) for subcritical pitchfork (PF_{sub}) bifurcation, and $\alpha = \mu^2$ ($\mu < 0$) for saddle-node (SN) bifurcation. The MEH system becomes structurally unstable on these bifurcation conditions [illustrated in Fig. 2(a)] and undergoes an immediate change in its equilibrium state. As depicted by three representative phase portraits in Fig. 2(a), a single trivial center exists in the region of R1 in Fig. 2(a) (i.e., the MEH is mono-stable), a trivial saddle and two non-trivial centers exist in the region of R2 (i.e., bi-stable), and three trivial or non-trivial centers and two non-trivial saddles exist in the region of R3 (i.e., tri-stable). In particular, the pitchfork bifurcation is degenerate at the point of degenerate pitchfork bifurcation (DPF) (0, 0), a demarcation point of regions R1–R3, at which the criticality of the pitchfork bifurcation changes. Three bifurcation boundaries, PF_{sup}, PF_{sub}, and SN, associated with the multi-stability formation (examined analytically in this study) are related to the stability map reported in the previous study (obtained numerically).²³ The main contribution of this study is the advantageous aspects of triple-well potential with a uniform depth along the T_U boundary, which remarkably reduces the design complexity of the MEH. It should be noted that the T_U boundary is an additional concept that is different from the three boundaries (PF_{sup}, PF_{sub}, and SN) and was not introduced in the previous stability map.²³ A detailed investigation of the T_U boundary follows.

FIG. 2. (a) Bifurcation sets: PF_{sup}, PF_{sub}, and SN on the parametric space (μ , α). In the phase portrait insets, the closed and open circle points indicate the stable equilibrium (called center) and unstable equilibrium (called saddle), respectively. (b) Contour plot of the potential well depth in the multi-stable region. Note that for triple-well potentials composed of non-uniform depths, the maximum well depth was used for the contour plot. Variations in potential energy functions obtained (c) at the points P1–P4 along the PF_{sup} boundary and (d) at the points A–F along a contour line of a representative energy (33 mJ).

The bifurcation boundaries enable us to instantly recognize that the MEH acts as either a bi- or tri-stable energy harvester (called MEH-B and MEH-T, respectively, in this study). For an MEH-B, two potential wells always appear mirror-symmetric, whereas for an MEH-T, not all three wells are in the same configuration. In fact, the triple-well potential of the MEH-T can be classified into three different types:²³ Type 1, where the depth of the inner well is the deepest; type 2, where the depths of all three wells are the same; and type 3, where the depths of two outer wells are the deepest. This study analytically derived the parametric condition for each type of triple-well potential. Notice that the potential energy function $\Phi(x)$ for type 2 is always zero at non-trivial centers $x = x_c$; i.e., $\Phi(x_c) = 0$, which gives the following analytical condition:

$$\alpha = \frac{3}{4}\mu^2 \quad (\mu < 0). \quad (3)$$

Remarkably, the type 2 condition comprises another boundary curve [denoted as T_U in Fig. 2(a)] dividing the tri-stable region into type 1 and type 3 regions. Accordingly, the present stability map can completely characterize all the potential well configurations of the MEH in the space of (μ, α) . It is worth noting here that the T_U boundary is important in designing the potential energy function of the MEH. To address this point, the variation in the potential energy function is globally investigated in the parametric space. Fig. 2(b) shows the contour plot of the (maximum) potential well depth in the multi-stable regions. The potential well depth grows from the PF_{sup} bifurcation boundary, and the well depth increases with distance from the PF_{sup} line. Interestingly, the widest potential flat in the potential energy function [thick line in Fig. 2(c)] appears at the point of DPF, so that the MEH becomes an ultra-low frequency resonator. For the potential functions of the MEH-B with an identical well depth [A-C in Fig. 2(d)], the well-to-well distance increases as μ decreases, and it is maximized at the PF_{sub} boundary, on which a flat-top saddle is observed. For the MEH-T, the outermost well-to-well distance further continues to increase, passing through the type 3 region until it meets the T_U condition [E in Fig. 2(d)]; it then starts to decrease, moving into the type 1 region. Such variations in the potential energy functions of the MEH [discussed in Fig. 2(d)] can be consistently observed along each contour line of potential well depth [presented in Fig. 2(b)]. Figure 3 shows the frequency responses of (a) the tip deflection and (b) the generated mean power for the fully developed periodic interwell motion, obtained with four different types of potential well configurations when the base acceleration is fixed at 0.21 g. In this figure, the interwell motion of the type-2 tri-stable case is accompanied by the largest amplitude over its frequency band [Fig. 3(a)], and the resulting output power is maximized [Fig. 3(b)] (refer to the supplementary material²⁷ for more details). For the above observations, it can be concluded that the MEH-T on the T_U boundary (MEH-TU) is an MEH with optimally designed multiple potential wells, under the condition that the maximum well depth is unchanged.

The variations in the potential wells of the MEH-TU from the point DPF along the T_U boundary are shown in Fig. 4. Both the overall well width and depth tend to increase as

FIG. 3. Frequency responses of (a) tip deflections and (b) generated mean powers for interwell motions obtained with four different potential well configurations: one bi-stable case and three types (types 1–3) of tri-stable cases, all of which have an identical (maximum) well depth of 5.3 mJ. The base acceleration is fixed at 0.21 g.

μ decreases; thereby, interwell motions are enlarged and the potential barriers are raised. These tendencies are mutually contradictory in terms of energy harvesting performance; a larger interwell oscillation produces higher power, whereas a higher barrier further hinders the occurrence of that motion. However, the difference in their increasing rates may provide a clue to resolve this conflict. The enlarging effect of the interwell motion is strong near the DPF point but it is rapidly weakened as μ decreases, whereas the raising effect of the potential barriers is negligibly weak near the DPF point but is rapidly strengthened. Thus, in designing the MEH-TU, a

FIG. 4. (a) Variations in the well-to-well distance and depth of equi-depth potential wells along the T_U boundary ($\alpha = 3\mu^2/4$). At the TU1, TU2, and TU3 points, the well-to-well distance and depth are (9.33 mm, 1.42 mJ), (11.13 mm, 5.15 mJ), and (12.04 mm, 9.58 mJ), respectively. (b) Potential energy functions obtained with four selected parameter sets corresponding to the four points in (a).

FIG. 5. (a) Experimental setup for frequency response measurement and (b) an enlarged picture of the multi-stable energy harvester. Refer to supplementary material²⁷ for more detailed pictures of experimental setup.

parametric condition should be chosen to be close to the DPF point, but keeping a certain distance apart [such as with the TU1–TU3 points in Fig. 4(a)], to induce large-amplitude interwell motions under a given ambient vibration. The numerical results of the generated voltage responses were obtained with the TU1–TU3 conditions, when base acceleration was given at 0.12 g, 0.17 g, and 0.27 g, and then compared with experimental results for validation. The detailed experimental setup is shown in Fig. 5. The base of the MEH system was excited by a vibration exciter (Type 4808, B&K) connected with a function generator (AFG3021b, Tektronics). An accelerometer (352C34, PCB Piezotronics) was used to measure the harmonic base acceleration. The base acceleration and the generated voltage were simultaneously measured by a signal acquisition system (Pulse 3560B, B&K). As shown in Figs. 6(a) and 6(b), when the potential

barrier is relatively small (1.42 mJ for the TU1 condition), a high-energy orbit motion appears for all excitation intensities under consideration, and the associated frequency bandwidth tends to be broadened with the excitation intensity. On the other hand, a higher potential barrier tends to suppress the occurrence of a high-energy orbit motion, as seen from the results for the TU2 [Figs. 6(c) and 6(d)] and TU3 [Figs. 6(e) and 6(f)], where the barrier height increases to 5.15 mJ and 9.58 mJ, respectively. Such an undesirable effect of the potential barrier is obviously more significant than the enlarging effect of the interwell motion, along the path from TU1 to TU3. Although the voltage output generated from the interwell motion is somewhat enhanced, it seems to be quite small even after the overall well width has been grown to a certain level. The numerical frequency responses for the voltage output (left column in Fig. 6) are in good agreement with the experimental ones (right column in Fig. 6). Moreover, both results correspond with the predictions based on potential well configurations.

In existing reports,^{23,25} the performance of the MEH was evaluated using an arbitrarily chosen well depth value for each type of potential energy function, which is not suitable for making a comparison. On the other hand, this study proposes that the most reasonable way to compare different types of potential energy functions in the MEH is to follow the contour line of the potential well depth [presented in Fig. 2(b)], because the potential energy functions of the MEH are then mutually comparable in following each contour line. The broadening of the potential well configuration causes a relatively small change in the frequency band of the interwell motion. From these observations, it can be found that the largest amplitude of interwell motion and the highest generated power could be obtained under the T_U boundary condition. All of the theoretical and experimental observations firmly support the assertion that the satisfaction of the T_U boundary condition, originating from the DPF, could be one of the most efficient ways to optimally design the MEH.

FIG. 6. Theoretical results (left column) and experimental results (right column) for the generated voltages obtained with three selected conditions, TU1 (first row), TU2 (second row), and TU3 (third row), when the base accelerations of 0.12 g, 0.17 g, and 0.27 g were given.

In particular, two contradictory effects (enlarging the interwell motion and raising the potential barrier) along the T_U boundary should be properly balanced or compromised by considering the characteristics of the ambient vibration source. For a given specific vibration source, the design parameter μ should be assigned its minimum value for activating an interwell motion, at which the oscillation amplitude (thus, the generated power) is also maximized. In addition, all analytical expressions of the bifurcation boundaries and T_U boundary derived in the present study could provide substantial stability criteria accompanied with several underlying rules for multi-stable energy harvesters. Note that the aforementioned conclusion has been drawn with the assumption that an initial external perturbation causes the MEH to take an interwell motion (if it exists) rather than an intrawell motion. We firmly believed that further theoretical and experimental research studies are needed to derive a universal optimal condition of the MEH through the investigation of all possible types of operational conditions including the cases out of the present input condition.

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²⁷See supplementary material at <http://dx.doi.org/10.1063/1.4954169> for (I) system parameters and the detailed expressions of coefficients γ_1 , γ_3 , and γ_5 in Eqs. (1a)–(1c), (II) dynamic behaviors of the interwell motions for four types of potential well configurations of the MEH, and (III) detailed experiment set-up.