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Milidonis, Andreas

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An Empirical Investigation of CDS spreads using a Regime Switching Default Risk Model

Andreas Milidonis

Insurance Risk & Finance Research Centre,
Nanyang Business School,
Nanyang Technological University,
Singapore.

University of Cyprus,
Cyprus

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Abstract

Default risk in equity returns can be measured by structural models of default. In this paper we propose a credit warning signal (*CWS*) based on the Merton default risk (*MDR*) model and a Regime-switching default risk (*RSDR*) model. The *RSDR* model is a generalization of the *MDR* model, comprises regime-switching asset distribution dynamics and thus produces more realistic default probability estimates in cases of deteriorating credit quality. Alternatively, it reduces to the *MDR* model. Using the dataset of US credit default swap (CDS) contracts we construct rating-based indices to investigate the *MDR* and *RSDR* implied probabilities of default in relation to the market-observed CDS spreads. The proposed *CWS* measure indicates an increase in default probabilities several months ahead of notable increases in CDS spreads.

Keywords: Default Risk, Regime Switching, Credit Warning Signal, Credit Default Swaps.

JEL classification: G13, G17.

1. Introduction

Default risk governs daily transactions at a personal, corporate and sovereign level. Actuaries and risk professionals have long managed default risk within large financial institutions while regulators require even more advanced default risk management techniques after the 2008 collapse of the credit markets, e.g. in the European Framework for Insurance Solvency – Solvency II. Default risk can be assessed either through ratings published by Credit Rating Agencies, credit derivative markets, reduced-form (actuarial) models of default (e.g. Jarrow and Turnbull 1995, Lando 1998, Madan and Unal 1998, Duffie and Singleton 1999) or through structural models of default which use option pricing theory to estimate implied default probabilities (Black and Scholes 1973, Merton 1973, 1974). Regardless of model choice, the implementation of default risk models that can accommodate non-normality features, in particular heavy-tailedness and structural shifts in market variables, is becoming more attractive given the ongoing banking and sovereign debt crises in Europe.

In this paper we propose a credit warning signal (*CWS*) based on the ratio of default probability predictions of two structural default risk models. The first model is the Regime Switching Default Risk (*RSDR*) model (Milidonis and Chisholm, 2013), which employs regime switching between two normal distributions to model a firm's unobserved asset log-returns. The second model is the classic Merton's default risk (*MDR*) model which assumes a normal distribution for asset log-returns. The *CWS* is the logarithmic transformation of the ratio of the *RSDR* default probability divided by the *MDR* default probability. In the case of deteriorating default risk, the ability of the *RSDR* model to incorporate extreme asset log-returns in the underlying distribution (vs. the *MDR* model which is constrained by the inflexibility of its underlying distributional assumption), allows the *CWS* measure to indicate significant increases in default probabilities. In the opposite case the *RSDR* model can be reduced to the *MDR* model, thus allowing the *CWS* measure to oscillate around the value of zero.

For many years, default (credit) risk assessment was largely done through bond (credit) ratings. Bond ratings represent the opinions of credit rating agencies (CRA) of the ability and willingness of rated firms to fulfill their financial obligations. Recently, the timeliness and accuracy of bond ratings have been

scrutinized (e.g. SEC, 2003, Johnson, 2004, Beaver et al., 2006, SEC, 2008, Cheng and Neamtiu, 2009). Several academic studies find evidence that markets react in the period before and up to the period of changes in bond ratings' announcements for several reasons. For example, there could be association between the timeliness of changes in ratings and the compensation incentives of credit rating agencies (Beaver, Shakespeare and Soliman, 2006, Milidonis, 2013). Additionally, changes in individual corporate bond ratings have been associated with increased volatility in the underlying stock's returns (Vassalou and Xing, 2005), while positive (negative) abnormal stock returns have been associated with subsequent upgrades (downgrades) in bond ratings (Goh and Ederington, 1993, Goh and Ederington, 1999, Hand, Holthausen and Leftwich, 1992). Furthermore, Milidonis and Wang (2007) show that there are different regimes in stock returns around the announcements of timely downgrades.¹ Information related to forthcoming default risk events (e.g. sovereign debt downgrades) is also important at a global level, as shown by Michalides, Milidonis, Nishiotis and Papakyriakou (2015). They find economically significant declines in international stock markets preceding sovereign debt downgrades, which they attribute to evidence consistent with leakage of information, in the discussion between credit rating agencies and local government officials, before the change in rating is officially announced.

The *MDR* model has been extended in several directions in recent years (e.g. Black and Cox, 1976, Leland 1994, Longstaff and Schwarz, 1995, Leland and Toft, 1996, Zhou 2001). Siu, Erlwein and Mamon (2008) have developed a Markov modulated model to price credit default swaps and produce a default probability estimate via a coupled-PDE (partial differential equations) approach. Siu et al. (2008) demonstrate the properties of their model through simulation.

Empirical applications of *MDR* extensions are less common. One reason could have been the absence of a methodology to implement an advanced option pricing framework on real data. Duan's (1994, 2000) proposed transformed likelihood method has bridged the gap between observed and unobserved variables used in most mathematical models. Milidonis and Chisholm (2013) present a regime-switching

¹ Timely downgrades are defined as the earlier downgrade to the same rating of two rating companies: a NRSRO (nationally recognized statistical rating organization; a title awarded by the Securities and Exchange Commission) and a non-NRSRO.

framework for the evolution of the implied value of assets (*RSDR*), which they implement using previously developed option pricing models under regime switching (Naik, 1993, Hardy, 2001, Boyle and Draviam, 2007) and Duan's (1994, 2000) econometric method of transformed likelihood functions. Their model uses a two-state asset evolution process where assets are allowed to switch between two Geometric Brownian motions governed by an unobserved Markov Chain. Their model uses the celebrated structural model of default (Black and Scholes, 1973 and Merton 1973, 1974) where a company's assets evolve continuously through time based on a continuous diffusion process with constant expected rate of return and volatility. According to the model any debt issued is given seniority over cash and other assets and is repayable at a pre-determined future maturity date with no coupons. In the event that the value of assets falls below the value of debt at maturity, the company defaults. The underlying assumption is that the unobserved value of assets follows a Geometric Brownian motion, which is linked to the observed equity value through the option pricing formula. In a setting of fixed maturity, the value of the option should equal the equity value and the default boundary should be debt outstanding at maturity. From this framework, the value and volatility of assets are estimated and used in the calculation of the default probability as a function of a standardized measure of default called distance-to-default. Distance-to-default is a standardized measure of the number of standard deviations of the log-value of assets, that the expected log-value of assets will be away from the default boundary (liabilities) of the company at a future time period. In section 2 we describe distance-to-default and explain how the shortcomings of standardizations may be overcome by the *RSDR* model.

Given that the combination of increased volatility and positive (negative) abnormal *equity* returns have been documented to precede upgrades (downgrades) in bond ratings, then since equity is viewed as a call option on the value of assets, then the implied distribution of assets is also expected to be affected². Since Hamilton's important paper (1989), regime switching models have been widely documented to accommodate changes in both the mean and volatility of an underlying variable. Furthermore option prices under regime switching models have been developed in the literature (Naik 1993, Hardy 2001,

² For example, for companies with very low debt, the volatility of equity is close to the volatility of assets.

Boyle and Draviam, 2007) and also discussed by Guo (2001) and Elliot et al. (2005). Therefore using the value of equity as the observed European option price, assets are assumed to follow a regime switching process of two Geometric Brownian motions.

Our paper implements both the *RSDR* and the *MDR* models on the aggregate stock indices of heterogeneous credit quality and proposes a risk measure as a credit warning signal to detect changes in model-implied default probabilities for publicly traded firms. The major difference between the two models is the implied distribution governing the asset log-return process. Through the *RSDR* model the implied asset distribution could be characterized by multiple modes, skewness and excess kurtosis thus being able to capture sudden and/or persistent periods of changes in the volatility of asset values. A jump-diffusion extension of the *MDR* model of default could accommodate jumps in asset values (Zhou, 2001). However, it would not be able to capture changes in asset return volatility as typically happens before corporate bond downgrades (Milidonis and Wang, 2007). Furthermore, allowing for both positive and negative jumps in a jump-diffusion model, could easily produce a net, average jump that is close to zero.

The flexible distribution of the *RSDR* model is valuable in measuring default risk, since default happens by definition at an extreme percentile of the implied asset distribution. To demonstrate the differences between the *RSDR* and *MDR* models, we independently run them on the sample of US companies that have traded credit default swap (CDS) contracts for the period of January 2004 to September 2010, thus including the 2008 crisis period. Using goodness of fit tests we find that the *RSDR* model prevails the *MDR* model for the largest part of our sample, but especially around time that extreme negative equity (hence asset) returns are observed and the *MDR* model cannot incorporate such extreme returns in its underlying distribution. Furthermore, the *RSDR* model is more responsive to deteriorating default risk, and produces higher default probabilities, earlier than the *MDR* model. This is reflected in the proposed *CWS* measure, which when compared to the respective, market-perceived credit risk measures (i.e. CDS spreads), we document a notable lead effect of the *CWS* measure over CDS spreads.³

³ The performance of the *CWS* measure in the period before the largest crisis of recent years, serves as a motivation for a full-fledged analysis of the *CWS* as an early warning signal in the context of Abiad (2007).

The economic significance of the *RSDR* model is reflected in the cross section of estimated default probabilities, mainly because *MDR*'s proxy for default (distance-to-default (*DD*)) fails to produce any meaningful estimates of default probabilities, for firms close to and higher than the investment-grade category (Kealhofer, 2003).⁴ Therefore, if investors' models cannot differentiate between investment and non-investment grade bonds, they are likely to be accompanied by economically significant excess losses when investing in such assets. The *RSDR* model helps remedy the underestimation of default probabilities by the Merton model of default. The more accommodating regime switching distribution allows the derivation of a company specific asset distribution which allows continuous monitoring of default risk in investment targets, without having to rely entirely on analysts' reports and changes in credit ratings.

There is also empirical value in the time series characteristics of the *RSDR* model. Structural changes in equity and assets are smoothed out in the *MDR* model based on year-long observations of equity data. This means that periods of high (low) volatility are underestimated (overestimated) by the assumption of constant volatility and their impact on default probabilities appears with a lag or disappears. In the *RSDR* model such changes are incorporated into the estimation on a periodic basis and their impact is weighted into default probabilities using maximum likelihood estimation. Therefore short-term changes in default risk can be captured more effectively through periodic (e.g. daily) update of regime switching parameters.

In the next section (section 2) we describe the two default risk models and the two methods to implement them. Emphasis is given on Duan's (1994, 2000) econometric transformation of observed equity values for publicly traded companies, and the implementation of the *RSDR* model. In section 3 we describe the data and empirically estimate default probabilities from the *RSDR* and *MDR* models and the *CWS* measure for several US Credit Default Swap (CDS) constructed indices. Section 4 concludes.

⁴ *DD* comprises several financial elements: asset value; debt value; asset drift and volatility; time to maturity of debt (section 2.2, equation (7)).

2. Model and Methodology

In this section we first describe the *MDR* model and its estimation using the popular Ronn and Verma (1986) approach. Then we explain Duan's (1994, 2000) econometric transformation and describe the implementation of the *MDR* model. The *RSDR* model is then introduced and positioned within Duan's framework to compute comparable implied default probabilities to those of the *MDR* model.

2.1. Merton Default Risk (*MDR*) model

The *MDR* model has been widely employed by risk professionals to estimate default probabilities. The model assumes that a continuous diffusion process governs the evolution of the value of assets of a firm, A_t , which have constant expected rate of return μ_A and volatility σ_A :

$$dA_t/A_t = \mu_A dt + \sigma_A dZ_t \quad (1)$$

where Z_t denotes a Wiener process, with respect to a real world probability. The European option pricing formula maps the observed value of equity at time t , E_t , onto the implied value of assets, since equity can be viewed as a call option on the residual assets of the firm (Black and Scholes, 1973):

$$E_t = A_t N(d_1) - \Omega N(d_2) \exp(-\xi T) \quad (2)$$

where Ω is debt at maturity, T is the time to maturity, ξ is the continuously compounded risk-free rate,

$$d_1 = [\ln(A_t/\Omega) + (\xi + \sigma_A^2/2) * T] / \sigma_A \sqrt{T}, \quad (3)$$

$$d_2 = d_1 - \sigma_A \sqrt{T} \quad (4)$$

and $N(\cdot)$ is the Normal Cumulative Density Function. The partial derivative of the option value with respect to the price of the underlying asset, Δ , can be computed as:

$$\Delta = \partial E_t / \partial A_t = N(d_1). \quad (5)$$

2.2. Estimation of *MDR* model: Ronn and Verma (1986)

The first widely used attempt to estimate A_t and σ_A was proposed by Ronn and Verma (1986, 1989). Li and Wong (2008) call this approach the “volatility-restriction” method, since the volatility of equity (σ_E) is assumed equal a weighted value (A_t/E_t) of the asset volatility (σ_A) and the European option's delta (Δ):

$$\sigma_E = (A_t/E_t) \Delta \sigma_A \quad (6)$$

The volatility-restriction method is used widely by both academics (e.g. Lyden and Sarniti, 2001, Huang and Huang, 2002, Vassalou and Xing, 2004) and industry professionals (Crosbie and Bohn, 2003). Below we outline the steps to implement the volatility-restriction method to estimate the probability of default using the *MDR* model, and then provide the relevant equations in sub-sections 2.2.1 and 2.2.2.

The first step is to gather historical equity returns for the previous year (252 trading days) to compute an estimate of equity's volatility (σ_E) using the sample standard deviation. This estimate is used as a proxy for assets' volatility (σ_A). The market value of equity (E_t) equals the value of the call option on the firm's market value of assets with exercise price equal to the default boundary (Ω). Time to maturity is assumed to be one year and the risk-free rate (ξ) is approximated using the one-year government zero rate. Then, eq. 2 is solved for each of the previous 252 days to produce daily estimates of the unobserved market value of assets (A_t).

Asset returns are the estimated from the implied value of assets over the previous year, to produce a new estimate for σ_A using the sample's standard deviation. Values of assets are re-estimated using the new value for σ_A in eq. 2. The same procedure is repeated until the estimate of σ_A converges. At this point μ_A is estimated as the sample mean of assets' log-returns. The estimates of μ_A , σ_A and the last value of A_t are then used to compute distance-to-default (*DD*) for the following year:

$$DD = [\ln(A_t/\Omega) + (\mu_A - \sigma_A^2) * T] / \sigma_A \sqrt{T}. \quad (7)$$

The negative value of value of *DD* (left tail of the asset distribution) is substituted in the standard normal cumulative density function to give the Merton Probability of Default (*MPD*):

$$MPD = N(-DD). \quad (8)$$

MPD measures the probability that the log-value of assets will fall below the default boundary at the one-year horizon. It must be acknowledged that the estimated probability only reflects the estimated asset price dynamics and does not accurately reflect empirical default probabilities (Vassalou and Xing, 2004).

2.2.1. Equity Mean Return and Volatility

The main observed variable needed, is the reference entity's market capitalization⁵ for a period of g days ($g = 1, 2, 3 \dots G$) prior to the estimation day. Vassalou and Xing (2004) suggest using $G = 252$. The daily number of shares outstanding, N_g , is multiplied by the closing equity price, S_g to give the entity's market capitalization, E_g ; $E_g = N_g S_g$. For each day g , equity log-returns, r_{Eg} , are calculated as:

$$r_{Eg} = \log \frac{E_g}{E_{g-1}}. \quad (9)$$

The mean and variance of equity log-returns are approximated by the sample mean, $E[r_E] = \frac{1}{G-1} \sum_{g=2}^{g=G} r_{Eg}$, and variance, respectively, $E[(r_E - E[r_E])^2] = \frac{1}{G-2} \sum_{g=2}^{g=G} (r_{Eg}^2 - (G-1)E[r_E]^2)$. Then,

the volatility estimate is annualized to give $\sigma_E = \sqrt{\frac{E[(r_E - E[r_E])^2]}{1/252}}$.

2.2.2. Asset Values, A_g , volatility, σ_A , and drift, μ_A .

To estimate the value of assets, (A_g) , the values of equity, (E_g) , equity volatility, (σ_E) , default boundary, (Ω) , risk free rate, (ξ) , and time to maturity, (T) , are substituted in eq. 2 for each day g . Then, the solution to the equation: $c(u) = E_g(u) - E_g = 0$, is computed using the Newton Raphson method, where u is the value of A_g that will make this equation approximately zero. To solve this equation, elements of the Taylor's expansion series are used, $c(u + \delta u) \approx c(u) + c'(u)\delta u$, where $\delta u = -c(u)/\Delta(u)$, to make the value of $c(u + \delta u)$ equal to zero ($\Delta(u)$ is the option delta). The initial values for u and δu are set equal to $E_g + \Omega$, as assets can be approximated by the sum of debt and equity, and $(E_g + \Omega)/10$, respectively. For each g , a new value of A_g is computed until δu approaches zero.

To estimate the volatility of asset returns, (σ_A) , this procedure is repeated for all g observations of A_g . The values of assets are then used to compute estimates of σ_A following similar equations to those in section 2.2.1. This process continues until σ_A converges. The values of assets, (A_g) over the previous G

⁵ A reference entity can be one firm, a portfolio of firms or even an entire industry index where total market capitalization will be the sum of individual capitalizations.

days are also used to estimate the drift of assets, (μ_A) . Assets are assumed to follow a geometric Brownian motion (eq. 1) over small time periods Δt , which implies a lognormal asset distribution:

$$\log \frac{A_t}{A_{t-\Delta t}} = \left(\mu_A - \frac{1}{2} \sigma_A^2 \right) \Delta t + \sigma_A \sqrt{\Delta t} \varepsilon, \quad (10)$$

where ε is standard normal random variable and expected value and variance of the asset log-return, r_A , are: $E[r_A] = \left(\mu_A - \frac{\sigma_A^2}{2} \right) \Delta t$, and $E[(r_A - E[r_A])^2] = \sigma_A^2 \Delta t$, respectively. Hence:

$$\mu_A = \frac{E[r_A]}{\Delta t} + \frac{\sigma_A^2}{2}. \quad (11)$$

2.3. Duan's Methodology (1994, 2000)

In reference to the Ronn and Verma (1986) approach to implement the *MDR* model, Duan (1994) states that “...the volatility relationship used in Ronn and Verma is a redundant condition which provides a restriction only because the equity volatility is inappropriately treated as a constant.” The assumption of constant volatility is not consistent with the assumptions of the *MDR* model and also, it is not obvious how asymptotic standard errors for the parameter estimates can be produced. Duan's transformed likelihood approach introduces two improvements: (a) it estimates both the parameters and their asymptotic standard errors and (b) it estimates μ_A , under the physical measure.

Duan's (1994, 2000) method can be applied in the following way: after choosing a structural default risk model, the option pricing equation that will map the firm's (unobservable) value of assets onto the value of (observable) equity is derived. Duan's method requires a unique transformation from the value of assets to equity (and vice-versa), and that the function mapping assets onto equity is continuous and twice differentiable. Since, the chosen model dictates a probability density function for the unobserved value of assets (i.e. *MDR* model's lognormal distribution), then the equity likelihood function can be derived as a function of the asset distribution. This means that assets' parameters (i.e. *MDR* model's μ_A and σ_A) can be estimated directly from equity observations, by maximizing the log-likelihood function that should also satisfy the option pricing equation for each equity observation. The value of assets from the option price equation (i.e. eq. 2 for the *MDR* model), since parameters of the asset value distribution are now known.

In mathematical terms, assume that a continuous random variable Y has a probability density function $h_Y(y)$. Then a new random variable X , can be defined as $x = b(y)$, where transformation $b(y)$ is monotonic and twice differentiable. Then, following Duan (1994, 2000), the probability density function (*p.d.f.*) of $X = b(Y)$, is:

$$f_X(x) = h_Y(b^{-1}(x))/|b'(b^{-1}(x))|. \quad (12)$$

2.3.1. Implementation of *MDR* : Duan (1994, 2000)

The evolution of assets (eq. 1), implies that changes in $\ln(A_g)$, are normally distributed between time periods g_{i-1} and g_i , under the real-world probability:

$$h(\ln(A_g)|\ln(A_{g-1})) = \frac{1}{\sigma_A \sqrt{2\pi(g_i - g_{i-1})}} \cdot \exp\left(-\frac{(\ln(A_g) - \ln(A_{g-1}) - (\mu_A - \sigma_A^2/2)(g_i - g_{i-1}))^2}{2\sigma_A^2(g_i - g_{i-1})}\right). \quad (13)$$

where $h(\cdot)$ is the normal density function. The European Black Scholes Call option (eq. 2) relates the value of equity E_g to the value of assets, A_g . The derivative of this equation with respect to the value of A_g is given by eq. 5. Using eq. 2 and 5, we can then formulate the log-likelihood function of E_g as a function of the unobserved value of A_g :

$$L(\mu_A, \sigma_A) = \sum_{g=2}^{g=G} \ln(f(E_g|E_{g-1}, \mu_A, \sigma_A)), \quad (14)$$

where $f(\cdot)$ is the density function of E_g . The one-to-one mapping between assets and equity ensures that $\widehat{A}_g$ is the only solution to eq. 2, hence the transformed density function of equity becomes:

$$f(E_g|E_{g-1}, \mu_A, \sigma_A) = h(\ln(\widehat{A}_g)|\ln(\widehat{A}_{g-1}), \mu_A, \sigma_A) / (\widehat{A}_g \cdot N(d_1)|_{A=\widehat{A}_g}). \quad (15)$$

The log-likelihood function then becomes:

$$L(\mu_A, \sigma_A) = \sum_{g=2}^{g=G} \left[\ln\left(h(\ln(\widehat{A}_g)|\ln(\widehat{A}_{g-1}))\right) - \ln\left(\widehat{A}_g \cdot N(d_1)|_{A=\widehat{A}_g}\right) \right], \quad (16)$$

which must satisfy eq. 2 for all observations G in the sample period:

$$\max_{\mu_A, \sigma_A} L(\mu_A, \sigma_A),$$

$$s.t. \ E_g = A_g \cdot N(d_1) - \Omega \cdot \exp(\xi T) \cdot N(d_2), \ \forall g = 1, 2, \dots G. \quad (17)$$

2.3.2. Computation of Merton's Probability of Default (MPD)

The maximum likelihood estimation of the parameters μ_A and σ_A over the 252-day period prior to current time, t , allows the computation of the value of assets at time t , A_t . To compute Merton's probability of default (MPD) n days ahead of current time t , then the value of assets at that time, A_{t+n} , is estimated from the values of A_t , μ_A and σ_A . A_{t+n} can then be used to estimate the distance-to-default (DD); the standardized difference between the estimated asset value and the face value of liabilities at time $t + n$. At first the default boundary is subtracted from the estimated value of assets at the n -day horizon to obtain the random variable:

$$\log A_{t+n} - \log \Omega = \log A_t + \left(\mu_A - \frac{\sigma_A^2}{2} \right) n + \sigma_A \sqrt{n} \varepsilon - \log \Omega, \quad (18)$$

which is distributed normally with variance $\sigma_A^2(n)$. Normalizing the mean of this distribution gives DD:

$$DD_{t+n} = \frac{\log \frac{A_t}{\Omega} + \left(\mu_A - \frac{\sigma_A^2}{2} \right) n}{\sigma_A \sqrt{n}}. \quad (19)$$

The probability that assets will fall short of the face value of liabilities at the n -day horizon is:

$$MPD = N(-DD_{t+n}). \quad (20)$$

2.4. The Regime Switching Default Risk (RSDR) model using Duan (1994, 2000)

Milidonis and Chisholm (2013) developed a regime switching default risk (RSDR) model to assess individual company default probabilities. Their rationale for developing the RSDR model was founded on the publicly available information preceding changes in bond ratings and default risk in general, that is priced in equity returns and hence can be reflected in short-term default probabilities. The RSDR model assumes that assets evolve through time according to a combination of several geometric Brownian motions, whose state at any point in time is governed by a hidden Markov Chain that captures idiosyncratic firm but not systematic risk.

2.4.1. *RSDR* framework

We let, r_{A_g} represent entity's asset log-returns, where $g = 1, 2 \dots G$. r_{A_g} follows a normal distribution whose mean and variance are state-dependent, i.e.:

$$r_{A_g} \sim N(\mu_{A, \rho_g}, \sigma_{A, \rho_g}^2), \quad (20)$$

where ρ_g represents the regime where the process is classified on day g . The properties of the unobservable Markov chain dictate memory-less states, i.e. the probability that the process lies in a regime in day g , only depends on where the process was in the previous day ($g - 1$). Hamilton (1989) improved the model by Quandt (1958) and Goldfeld and Quandt (1973) and illustrated the estimation of the transition probability matrix. The *RSDR* model assumes a two-state process, ($\rho_g = 1$ or 2) which implies the following transition probabilities:

$$p_{1,2} = Pr[\rho_g = 2 | \rho_{g-1} = 1], \text{ and } p_{2,1} = Pr[\rho_g = 1 | \rho_{g-1} = 2]. \quad (21)$$

Hence the following probability transition matrix with constant transition probabilities through time governs the hidden Markov chain:

$$\begin{bmatrix} p_{1,1} & 1 - p_{1,1} = p_{1,2} \\ 1 - p_{2,2} = p_{2,1} & p_{2,2} \end{bmatrix} \quad (22)$$

The *RSDR* model, is therefore fully characterized by six parameters, $\theta: \{\mu_1, \mu_2, \sigma_1, \sigma_2, p_{1,2}, p_{2,1}\}$, which will be estimated following the steps required by Duan (1994, 2000).⁶

To estimate these parameters the following steps are needed: (a) the derivation of the European call option price under regime-switching, such that a unique link exists between the value of equity and assets; (b) the derivation of the this option's delta to apply Duan's transformation (eq. 12); (c) the derivation of the transformed equity log-likelihood function as a function of the six parameters; (d) the maximization of the log-likelihood function over all observations g subject to the constraint that the option price equation (in (a) above) is satisfied for all observations g .⁷

⁶ We omit subscript A from θ ; as an example we use μ_1 instead of $\mu_{A,1}$.

⁷ The *RSDR* model can be extended to more than two regimes following Hardy (2001) or Siu, Erlwein, and Mamon (2008). The empirical implementation could be done following Duan (1994, 2000). Since our paper focuses in

2.4.2. Two State Sojourn Probability Function

A necessary step before defining the European call option price under regime-switching is to define the sojourn probability function for a two state process. Following Hardy (2001), we let S_g , be the random variable of the total number of days spent in regime 1 over the period $[g, m)$, $m \leq G$. Also, let the probability of having s days in regime 1 be $Pr[S_0 = s]$, where $s \in [0, m]$. $Pr[S_0 = s]$ can be constructed from the regime transition probabilities $Pr[\rho_{g+1} = 1 | \rho_g = 1]$ and $Pr[\rho_{g+1} = 2 | \rho_g = 2]$ and the periodic conditional probability of being in each regime $Pr[S_g = s | \rho_{g-1}]$ (Hardy, 2001). More specifically to compute $Pr[S_0 = s | \rho_{-1}]$, the process is initialized as follows:

$$\begin{aligned}
 Pr[S_{m-1} = 0 | \rho_{g-1} = 1] &= Pr[\rho_{g+1} = 2 | \rho_g = 1] \\
 Pr[S_{m-1} = 1 | \rho_{g-1} = 1] &= Pr[\rho_{g+1} = 1 | \rho_g = 1] \\
 Pr[S_{m-1} = 0 | \rho_{g-1} = 2] &= Pr[\rho_{g+1} = 2 | \rho_g = 2] \\
 Pr[S_{m-1} = 1 | \rho_{g-1} = 2] &= Pr[\rho_{g+1} = 1 | \rho_g = 2]
 \end{aligned} \tag{23}$$

Then the conditional probabilities can be calculated by working recursively from $g = m - 2$ to $= 0$:

$$\begin{aligned}
 Pr[S_g = s | \rho_{g-1} = 1] \\
 &= Pr[\rho_{g+1} = 1 | \rho_g = 1] Pr[S_{g+1} = s - 1 | \rho_g = 1] \\
 &\quad + Pr[\rho_{g+1} = 2 | \rho_g = 1] Pr[S_{g+1} = s | \rho_g = 2]
 \end{aligned}$$

and

$$\begin{aligned}
 Pr[S_g = s | \rho_{g-1} = 2] \\
 &= Pr[\rho_{g+1} = 1 | \rho_g = 2] Pr[S_{g+1} = s - 1 | \rho_g = 1]
 \end{aligned}$$

detecting short-term changes in default risk, and given the standard approach in the finance literature to use one year of data to estimate it (about 250 trading days), then extending to more than 2 regimes will come at the cost of increased parameter uncertainty. If the estimation period is extended, then older stock prices will be included which will decrease primarily the responsiveness of the *MDR* model.

$$+Pr[\rho_{g+1} = 2|\rho_g = 1]Pr[S_{g+1} = s|\rho_g = 2]. \quad (24)$$

The conditional probabilities at each recursion are computed for the entire set of $s \in [0, m]$. At equilibrium the unconditional sojourn probability function $Pr[S_0 = s]$ equals:

$$Pr[S_0 = s] = \pi_1 Pr[S_0 = s|\rho_{-1} = 1] + \pi_2 Pr[S_0 = s|\rho_{-1} = 2] \quad (25)$$

where π_1 and π_2 are the stationary probabilities of regime 1 and 2 respectively, and they exist since the hidden Markov chain is homogeneous. Hamilton (1989) and, later, Kim and Nelson (1999) show that a recursive filter can be used to estimate parameter set θ (details in Appendix A). In our context, we modify Hamilton's filter with the transformed equity log-likelihood to estimate θ (section 2.4.4).

2.4.3. Mapping Asset Values onto Equity Likelihood

The first ingredient is a European call option price under regime-switching. In a regime-switching environment, the market is incomplete. However, the *RSDR* model assumes that only idiosyncratic risk is priced, and also that $Pr[S_0 = s]$ remains unchanged when changing from the physical to the risk-neutral measure. Hence, following Guo (2001), Hardy (2001) and Elliott et al. (2005), we can assume a unique transformation from assets to equity:

$$E_g = \sum_{s=0}^m \left(A_g N(d_1(s)) - \Omega * \exp(-m * \xi) N(d_2(s)) \right) Pr[S_0 = s], \quad (26)$$

where

$$d_1(s) = \frac{\left(\log \frac{A_g}{\Omega} + m * \xi + \frac{s}{2} \sigma_1^2 + (m-s) \frac{1}{2} \sigma_2^2 \right)}{\sqrt{s \sigma_1^2 + (m-s) \sigma_2^2}}, \quad (27)$$

and

$$d_2(s) = d_1(s) - \sqrt{s \sigma_1^2 + (m-s) \sigma_2^2}, \quad (28)$$

A necessary step in the application of Duan's (1994, 2000) method is the calculation of the derivative of the option delta, where the option can be thought of as a weighted average of individual option values weighted by the time spent in each regime:

$$E_g = \sum_{s=0}^m E_g(A_g, s) Pr[S_0 = s] \quad (29)$$

thus the option delta is equal to:

$$\Delta_g = \frac{\partial E_g}{\partial A_g} = \sum_{s=0}^{s=m} N(d_1(s)) \Pr[S_0 = s]. \quad (30)$$

2.4.4. Equity Likelihood function

Given the equation for the European option price under regime-switching, its unique solution $\widehat{A}_g$, and its option delta, the next step is to derive the transformed equity likelihood as a function of the parameters of the unobserved value of assets. The likelihood of each equity observation depends on the regime in which the process is found (regime 1 or 2, respectively):

$$f(E_g | E_{g-1}, \rho_g = 1) = h(\ln(\widehat{A}_g) | \ln(\widehat{A}_{g-1}), \mu_1, \sigma_1) / (\widehat{A}_g \cdot \Delta_g |_{A=\widehat{A}_g}), \quad (31)$$

$$f(E_g | E_{g-1}, \rho_g = 2) = h(\ln(\widehat{A}_g) | \ln(\widehat{A}_{g-1}), \mu_2, \sigma_2) / (\widehat{A}_g \cdot \Delta_g |_{A=\widehat{A}_g}). \quad (32)$$

In the case of observed variables, Hamilton's (1989) recursive filter is employed to estimate parameters of Markov Switching processes (Hamilton, 1990, Hamilton and Susmel 1994), where the transition probabilities are estimated recursively as explained in section 2.4.2 and appendix A. In the context of default risk, where the underlying variable is unobserved, we modify Hamilton's filter using the transformed likelihood function for each regime as shown above (eq. 31 and 32). The log-likelihood function is then maximized to estimate the six parameters, subject to the option pricing constraint:

$$\max_{\theta} L(\theta), \quad (33)$$

$$s.t. E_g = \sum_{s=0}^{s=m} \left(A_g N(d_1(s)) - \Omega * \exp(-m * \xi) N(d_2(s)) \right) \Pr[S_0 = s], \quad \forall g = 1, 2, \dots G.$$

2.4.5. Regime Switching Probability of Default (RSPD)

Using the asset value parameter set θ , and the sojourn probability function, $\Pr[S_0 = s]$, we can estimate the Regime Switching Probability of Default (RSPD), as the probability that the value of assets n -days ahead of the current time t , A_{t+n} , will be less than the default boundary, Ω .⁸ RSPD can also be defined as the probability that the net asset return from t to $t + n$, Q_{A_n} , will be less than Ω/A_t , where $A_{t+n} = A_t * Q_{A_n}$. To estimate RSPD, we define the mean and standard deviation for the asset log-value (Hardy 2001):

$$\mu^*(s) = s\mu_1 + (n - s)\mu_2 \quad (34)$$

⁸ In this section we assume that for the sojourn probability function, $s \in [0, n]$.

$$\sigma^*(s) = \sqrt{s\sigma_2^1 + (n-s)\sigma_2^2} \quad (35)$$

and the unconditional probability density function as follows:

$$f_{Q_{A_n}}(k) = \sum_{s=0}^n \frac{\exp\left(-\frac{(\log(k) - \mu^*(s))^2}{2(\sigma^*(s))^2}\right)}{\sigma^*(s)\sqrt{2\pi}} \Pr[S_0 = s]. \quad (36)$$

2.5. Goodness of Fit between *MDR* and *RSDR*

Since both models are estimated using maximum likelihood, then typical goodness-of-fit models can be applied to evaluate which model fits the data better. Examples are the Akaike Information Criterion (AIC) and the Schwartz Bayes Criterion (SBC), which we apply in our empirical investigation in section 3 and show in appendix B. Since the *RSDR* model is a general case of the *MDR* model, then, in cases where the distributional assumptions of the *MDR* model are satisfied by the data, the *RSDR* model is expected to be reduced to one regime and the goodness-of-fit test to support the *MDR* model. Alternatively, we expect goodness-of-fit support for the *RSDR* model due to the more flexible regime-switching distribution.

3. Empirical Results

3.1. Credit Warning Signal (*CWS*)

To identify whether the *RSDR* model provides an advantage over the *MDR* model in detecting deteriorating credit quality of publicly traded firms in a short-term horizon, we define a credit warning signal (*CWS*) mechanism that uses the estimated default probabilities from the two models:

$$CWS = \log(RSPD/MPD) \quad (37)$$

In deteriorating credit conditions, if the *RSDR* is more responsive than the *MDR* model to identify increases in default risk through equity prices, then *CWS* is expected to increase above the value of zero. Alternatively, *CWS* is expected to oscillate around the value of zero.

3.2. Data and Descriptive Statistics

To evaluate the potential empirical value of the *CWS*, we compare it to US corporate Credit Default Swap (CDS) spreads before and after the 2008 crisis. To save space and time, we aggregate data into indices with the same S&P Domestic Short-Term Issuer Credit Ratings (ICRs) downloaded from

Compustat North America database. This aggregation works in favour of the *MDR* model as firm idiosyncratic equity patterns are aggregated in one index with less jumps and changes in volatility. ICRs represent S&P's opinion of the firm's ability to meet its financial obligations maturing in no more than one-year. S&P ICRs range from A-1 (S&P states: "strong capacity to meet financial obligations") to C (S&P states: "currently vulnerable"). Letter grades have been transformed to numerical categories, 10 being the highest rating, 1 being the lowest (Table 1).

The universe of CDS spreads as reported by Datastream for all US publicly traded companies has been collected. CDS contracts trade at several maturities usually from one to ten years ahead. Since we focus on short-term changes in default risk, we use CDS contracts of one-year maturity.⁹ The variable of interest to our analysis is chosen to be "Spread-mid" which represents the mid-market quote between the spread bid and offer prices. Spreads are reported in basis points hence no other transformations are needed to make it comparable across companies or time. CDS spread data are reported on a daily basis. Estimation of default probabilities however needs to be done at a lower frequency than daily since liability values (hence default boundaries) are reported on a quarterly basis. We interpolate default boundaries on a monthly basis and also take monthly averages of reported daily CDS data.

For the final sample of CDS data, we isolate unique companies and match them with COMPUSTAT database to retrieve balance sheet information (liability values) and CRSP database to retrieve daily stock prices. For each company we construct a default boundary following the academic and professional literature (e.g. Vassalou and Xing, 2004; Crosbie and Bohn, 2003) at the one year horizon. The default boundary is the sum of short-term liabilities and one half of long-term liabilities of the firm, so that, both the short- and long-term refinancing concerns of the firm, are incorporated. Our sample is then matched with the monthly S&P ICR data and any missing observations are deleted. Furthermore, all banks and financial institutions are removed from the sample by deleting all firms with a standard industry classification code between 6000 and 7000, following similar empirical studies in finance (e.g. Brockman

⁹ In unreported results (available from the authors) we plot CDS spreads for both one-year and five-year maturities and observe similar patterns. Hence, the main conclusions of the paper are unchanged regardless of maturity.

et al., 2010). Also, companies with stale CDS spreads are removed to ensure a sample with stock prices that efficiently reflect any valuable information about the firm. Then monthly averages are computed to allow matching our dataset with monthly S&P ICR and default boundaries.

The third and final step in our data collection is constructing the market capitalization variable for each firm and also downloading the market risk-free rate.¹⁰ Market capitalization is calculated as the product of number of shares outstanding and the closing share price on a daily basis. Our final sample covers the period from January 2004 to September 2010.¹¹

For each rating category we construct a CDS index as the weighted average (by market-capitalization) of estimated monthly spreads. In Table 1 we provide descriptive statistics of the number of firms present in each of the ten indices. Figure 1 shows the time-series evolution of the number of firms per rating index on a monthly basis. We observe that lower category indices are either absent from the data (for example *SPR1* and *SPR2* respectively), or have discontinued coverage (*SPR5* and *SPR6*), or include only a few company-month observations which change a lot through time (*SPR4*). Since categories *SPR 1-7* have numerous months with less than 10 individual companies per index, to avoid any artificial distortions in the input variables of the *MDR* and *RSDR* models, we focus on indices with the largest and time-consistent volume of traded contracts, which are *SPR8*, *SPR9* and *SPR10* rating indices.¹²

The weighted-average (by market capitalization) CDS spreads in basis points that describe each of the three rating indices are plotted together in Figure 2. We observe a consistent increase in CDS spreads across the three rating categories starting towards the third quarter of 2007, which declines close to the second quarter of 2008 and then a steep incline that leads to the highest observed values for CDS spreads. Even though there seems to be some changes in CDS trends at the beginning of the sample period (before

¹⁰ One-month T-bill rate from Kenneth R. French's website: <http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/>.

¹¹ Since we used Datastream for our CDS data, the natural ending point for our analysis was September 2010, because the firms covered after this date from Datastream change significantly from the companies covered before this date. This ending point does not affect our results since this paper shows that the *RSDR* model can show an earlier deterioration in default risk than the *MDR*, with reference to the 2008 crisis (peak point at around December 2008), which is included in our sample period.

¹² As shown in Figure 1, these three categories represent a very high percentage of all rated firms with available data. Specifically, the percentage of firms in these top three rating categories relative to all firms in the sample, has an average of 86% with a standard deviation of 5%.

2005), these may be associated with noisy signals at the beginning of the sample period, and also the small number of firms used to create each index (Figure 1).

The inputs for each of the *MDR* and *RSDR* models for the three rating indices (*SPR8*, *SPR9* and *SPR10*) are shown in Figures 3(a), 3(b) and 3(c). For each index, the market value of equity (MVE) and the respective default boundary are plotted in millions of \$US (right-vertical axis) against the weighted average of observed monthly CDS spreads, shown in basis points on the left-vertical axis. Starting from panel A (lowest credit quality), we observe CDS spreads reaching above 200 basis points towards the end of 2008, and the gap between the market value of equity and debt to be increasing until about the end of 2007 and then decreasing. Starting shortly after the beginning of 2009 when the gap reaches one of the lowest points, it again starts to widen. Similar trends obtain for panels B (*SPR9*) and C (*SPR10*).

Descriptive statistics of the basic input variables (MVE and default boundary) for each CDS index are shown in the first two lines of Tables 2(a), 2(b) and 2(c). In each table we report descriptive statistics of 81 monthly observations (01/2004 to 09/2010). We observe that the ratio between the average market value of equity and average default boundary increases, as credit quality increases. Specifically as we move from *SPR8* to *SPR9* to *SPR10* the ratio increases from about 2.2 (1022/472), to 2.6 (1511/571) to 4.3 (1696/396) respectively. A similar relation obtains for median values.

We also, report descriptive statistics of the CDS spreads for each rating index, in lines 3-6 of Tables 2(a), 2(b) and 2(c). Specifically we show statistics for CDS contracts with one-year (five-year) maturity where CDS spreads are estimated on an equal-weighted or value-weighted basis and they are denoted by *CDSI_m* and *CDSI_wm*, (*CDS5_m* and *CDS5_wm*,) respectively. As expected, the average and median value of CDS spreads decreases as credit quality increases (from Table 2 Panel A to B to C).

3.3. Results

3.3.1. Parameter Estimates

We report parameter estimates of the *MDR* and *RSDR* models under “Model Parameters” in Table 2, for the *SPR8*, *SPR9* and *SPR10* in panels A, B and C respectively. It is not easy to compare parameters across rating indices, except perhaps their direction and possibly the relative size of parameters within the

same rating index. For the *MDR* model, we observe an overall positive drift, μ , is obtained for the implied value of assets averaging a bit over 4% for *SPR8* and *SPR9* and slightly above 2% for *SPR10*. The average annual volatility estimate, σ , (around 12-13%) also seems consistent across ratings indices.

Focusing now on the *RSDR* parameters, it is important to note that no restrictions have been imposed on the parameter set $\theta: \{\mu_1, \mu_2, \sigma_1, \sigma_2, p_{1,2}, p_{2,1}\}$ during the estimation. However, since the initial values of σ_1 (σ_2) were set to be higher (lower) than the sample's volatility estimate, then in most cases σ_1 (σ_2) ended up being the high-volatility (low-volatility) regime. For the purposes of reporting the parameter estimates of the model, we sorted and renamed all parameters estimates in a way that σ_1 is defined as the high-volatility regime. Interestingly we observe that on average, the average and median values of σ_1 are about twice the value of σ_2 across all rating indices. Moreover, we observe that the high volatility regime (σ_1) is associated with a lower mean (μ_1) when compared to the low-volatility regime which has a larger mean ($\mu_1 < \mu_2; \sigma_1 > \sigma_2$).¹³

In terms of the Markovian transition probabilities we observe a balanced allocation between the high-volatility and low-volatility regimes (regime 1 and 2 respectively) across all rating indices. Transition probabilities can be used to estimate the stationary probability of each regime, that is, unconditional probability of being in each regime (i.e. π_1, π_2). π_1 is calculated as $(1 - p_{2,2}) / ((1 - p_{2,2}) + (1 - p_{1,1}))$. Hence, for each of the three indices, the unconditional probability of being in regime 1 is between 47% to 50% which leaves the remaining unconditional probability to regime 2 (since $\pi_1 + \pi_2 = 1$). Finally, when comparing the parameters of the *MDR* and *RSDR* models for the three rating indices we observe that average and median values of the parameters of the *MDR* model (μ, σ) fall in-between the respective values of the *RSDR* model ($\mu_1 < \mu < \mu_2; \sigma_1 > \sigma > \sigma_2$).

3.3.2. Model Outputs

The implied Merton Probability of Default (*MPD*) and the Regime-switching Probability of Default (*RSPD*) are estimated according to sections 2.3.4 and 2.4.5 respectively. As explained in section 2,

¹³ Standard Errors can be computed as explained in Appendix C of Milidonis and Chisholm (2013).

implied asset values from the two models, MDR_VA and $RSDR_VA$, are the same (rounding errors account for the small differences). Similar to the ratio between MVE and DB , the average ratio of asset value and DB is also increasing, as credit quality increases (3.2 to 3.7 to 5.3 for $SPR8$ to $SPR9$ to $SPR10$ respectively). Given the relatively high credit quality of these rating indices most default probabilities are very small; hence the descriptive statistics for the actual probabilities are not very informative. Instead we take the logarithmic transformation of MPD and $RSPD$, which are also used to calculate CWS . As we observe from Table 2, both the average and median CWS per rating index are positive thus giving early indications that the $RSDR$ model produces higher default probabilities in our sample period which includes deteriorating credit quality.

Figure 4 plots the credit warning signal measure (CWS) and the logarithmic transformation of MPD (Log_MPD) and $RSPD$ (Log_RSPD), on the right vertical axis, in panels A-C for the three rating indices $SPR8-10$, respectively. Also, the respective value-weighted CDS spreads corresponding to one-year maturity ($CDSI_wm$) is plotted on the left vertical axis in basis points.

Results are striking across all panels, as the CWS measure provides a clear lead signal over the observed CDS spreads, several months in advance. Starting from $SPR8$ (panel A), and specifically from the model-implied default probabilities, we observe almost overlapping trends until about the end of 2005. From that point onwards and until mid-2007, the MPD increases significantly, however the $RSPD$ increases even more, which is the reason why the CWS measure enters a period of highly positive values. Goodness of fit tests provide support for the $RSDR$ over the MDR model, especially where it is needed the most, i.e. around the time that the MDR model is constrained by its own distributional assumption, as explained in detail below.¹⁴ The gap between MPD and $RSPD$ can be explained by the rigidity of the MDR asset distribution and, on the opposite end, the flexibility of the $RSDR$'s asset distribution, which allows even a few extreme, negative observations to be taken into account in the $RSDR$ model.

For instance, each new month, about 21 new trading days (the most recent ones) enter the estimation period, while the oldest 21 trading days leave the sample. This means that the $RSDR$ model can

¹⁴ In appendix B we plot goodness of fit tests of the $RSDR$ and MDR models based on the AIC and SBC.

potentially classify the new and (say) extreme negative observations into a new regime, while the *MDR* model will have to stretch the existing model parameters to fit the new observations under the distribution fitting the majority of observations. Furthermore, as the number of the new extreme observations enters the estimation period (i.e. the estimation period now includes a significant part of the crisis period), the *MPD* would slowly adjust to match the *RSPD*.

This delay of the *MDR* model to catch up with the *RSPD* model is the main message of this paper. This is what happens in Figure 4 (A) starting towards the end of 2007 when *MPD* is slowly increasing to match a seemingly stable *RSPD*. From the beginning of 2008 onwards, the two default probabilities seem to be overlapping, hence the *CWS* measure is now back to about the value of zero.

Results are similar for Figure 4 panel B and C. *CWS* increases significantly and peaks in the period before the crisis and before CDS spreads begin to rise. Once the *MDR* model has enough observations in the estimation period to shift away from the non-crisis asset distribution, the gap between *MPD* and *RSPD* decreases until they converge completely. By that time however, the *CWS* measure has given consistent signals of upcoming deteriorating quality.

3.4. Future Research

In this paper the *CWS* measure is used around the 2008 crisis (which is used as an exogenous, systematic shock to the economy) to show that it can signal large changes in individual portfolio default risk. To be able to address the strengths and weakness of the *CWS* measure a potential investment tool, it would be beneficial to conduct a comprehensive analysis about potential false positive or false negative signals that the *CWS* measure would give. Future research could focus on analysing individual firm risk, where a firm specific “risk threshold” can be defined as a function of individual firm characteristics. Then, using, for instance, the methods described by Abiad (2007) the effectiveness of a potential trading strategy similar to the framework of Barber, Lehavy, and Trueman (2010), could be examined.¹⁵

¹⁵ We would like to thank an anonymous referee for this suggestion.

4. Conclusion

In this paper we propose a credit warning signal (*CWS*) using implied default probabilities from the Merton default risk (*MDR*) model and a regime-switching default risk (*RSDR*) model (Milidonis and Chisholm, 2013). We use the *CWS* to empirically investigate changes in default probabilities in relation to CDS spreads around the 2008 US crisis. The *CWS* measure highlights the flexibility of the *RSDR* model in incorporating extreme equity (and hence asset) returns, relative to the less flexible asset distributional assumption of the *MDR* model, which results in a lead effect of the *RSDR* over the *MDR* model in responding to deteriorating credit quality.

The flexibility of the *RSDR* lies in the unobservable two-state Markov process, which governs the switching process of the assets of a publicly traded firm between two Geometric Brownian motions. The model replaces the assumption of log-normality for the implied value of assets with a more flexible regime-switching distribution that can accommodate bi-modality, skewness and excess kurtosis. In this fashion the implied probability of default of a firm can make use of (either) sudden changes in the value of assets and (/or) even extended periods of significantly different mean and volatility in the log-value of assets, by classifying them into a separate regime. Estimating probabilities of default requires integration to a very “distant” value from the expected value of the distribution, hence extra attention should be paid in correctly capturing the tail of a distribution, a feature that characterises the *RSDR* model better than the *MDR* model.

Our empirical analysis uses all US traded credit default swap contracts for the period January 2004 until September 2010. Indices are constructed by aggregating market capitalization and liability values of individual firms which share a common Standard and Poor’s ratings. Each index is treated as a publicly traded firm whose equity and default boundary along with the market risk-free interest rate are used as inputs to the *MDR* and the *RSDR* models. Our results show the existence of at least two regimes, one of which having on average about double the volatility of the lower volatility regime. The *CWS* measure reveals a gap in the implied default probabilities of the *MDR* and *RSDR* model preceding the beginning of

the credit crunch crisis. Future research could investigate the early warning abilities of the regime-switching default risk model, perhaps in a similar manner to Abiad (2007).

The *RSDR* model can provide useful insights for the academic and practising actuary. For instance, the upcoming implementation of Solvency II in Europe and its impact on the US market (Cummins and Phillips, 2009) provides not only an opportunity but rather a necessity for insurance companies to employ an advanced credit risk model. Investing in fixed income securities has become risky in recent times and the role of credit rating agencies in reflecting credit risk has been debated extensively in the accounting and finance literature (Johnson, 2004, Beaver et al., 2006, SEC, 2003, SEC, 2008, Cheng and Neamtiu, 2009). With few exceptions, credit ratings have been described as un-timely, thus investment decisions based on changes in ratings hide losses. In contrast, using an advanced model of default risk that has the ability to produce heavy tails on the asset value distribution could at least produce some indication of changes in the default risk of investment targets.

Appendix A: Hamilton's (1989) Filter

Hamilton (1989) introduced a filter that was used on the two-state switching normal model. We implement his filter here using the following initial values for the parameter set θ :

- the means of both regimes were set to zero
- transition probabilities were set at 50%, and
- initial values for the respective deviations were assigned as a function of sample equity observations: the initial value of the standard deviation for one regime was set at twice the sample standard deviation and the initial value for the other regime was set at one-half the value of the equity's sample standard deviation.

The stationary probabilities of the asset process at time g conditional on the information φ_g generated by this process up to time g are given by

$$Pr[\rho_0 = 1|\varphi_0] = \pi_1$$

and

$$Pr[\rho_0 = 2|\varphi_0] = \pi_2.$$

π_1 and π_2 are computed as a function of the transition probabilities by making use of the memory-less property of Markov Chains. For example the unconditional probability that regime 1 at time $g + 1$ governs the asset process, is then:

$$\begin{aligned} Pr[\rho_{g+1} = 1] = \\ Pr[\rho_{g+1} = 1|\rho_g = 1]Pr[\rho_g = 1] + Pr[\rho_{g+1} = 1|\rho_g = 2]Pr[\rho_g = 2] \end{aligned}$$

and:

$$\begin{aligned} Pr[\rho_{g+1} = 2] = \\ Pr[\rho_{g+1} = 2|\rho_g = 1]Pr[\rho_g = 1] + Pr[\rho_{g+1} = 2|\rho_g = 2]Pr[\rho_g = 2]. \end{aligned}$$

By definition, the stationary probability implies that the process will have the same probability to be in each state, hence:

$$Pr[\rho_{g+1} = 1] = Pr[\rho_g = 1] = \pi_1$$

and

$$Pr[\rho_{g+1} = 2] = Pr[\rho_g = 2] = \pi_2.$$

Given that the sum of transition probabilities referring to the same state is one, i.e.:

$$Pr[\rho_{g+1} = 1|\rho_g = 2] + Pr[\rho_{g+1} = 2|\rho_g = 2] = 1$$

and

$$Pr[\rho_{g+1} = 1|\rho_g = 1] + Pr[\rho_{g+1} = 2|\rho_g = 1] = 1,$$

then the stationary probabilities can be written as a function of transition probabilities:

$$\pi_1 = \frac{Pr[\rho_{g+1} = 1|\rho_g = 2]}{Pr[\rho_{g+1} = 1|\rho_g = 2] + Pr[\rho_{g+1} = 2|\rho_g = 2]}$$

and

$$\pi_2 = \frac{Pr[\rho_{g+1} = 2|\rho_g = 1]}{Pr[\rho_{g+1} = 1|\rho_g = 2] + Pr[\rho_{g+1} = 2|\rho_g = 2]}.$$

In each state the log-return process follows a normal distribution:

$$f[r_{A_g}|\rho_g = j] = \frac{\exp - \left(\frac{(r_{A_g} - \mu_j)^2}{2\sigma_j^2} \right)}{\sqrt{2\pi} \sigma_j}$$

(j and i taking values of 1 or 2) and the probabilities of being in either state in the next time period

($Pr[\rho_g = j|\varphi_{g-1}]$) can be computed using:

$$Pr[\rho_g = j|\varphi_{g-1}] = \sum_{i=1}^{i=2} Pr[\rho_{g-1} = i|\varphi_{g-1}] Pr[\rho_g = j|\rho_{g-1} = i]. \quad (A.1)$$

Therefore, the joint conditional density functions, $f[r_{A_g}, \rho_g = j|\varphi_{g-1}]$, can be given by:

$$f[r_{A_g}, \rho_g = j|\varphi_{g-1}] = \frac{\exp - \left(\frac{(r_{A_g} - \mu_j)^2}{2\sigma_j^2} \right)}{\sqrt{2\pi} \sigma_j} Pr[\rho_g = j|\varphi_{g-1}].$$

with marginal density $f[r_{A_g}|\varphi_{g-1}]$ taken to be the integral over the regimes:

$$f[r_{A_g}|\varphi_{g-1}] = \sum_{j=1}^{j=2} f[r_{A_g}, \rho_g = j|\varphi_{g-1}].$$

Also, given that:

$$f[r_{A_g}, \rho_g = j|\varphi_{g-1}] = f[r_{A_g}|\varphi_{g-1}] Pr[\rho_g = j|\varphi_{g-1}, r_{A_g}]$$

then given r_{A_g} the probability of being in each state can be computed as:

$$Pr[\rho_g = j|\varphi_g] = Pr[\rho_g = j|\varphi_{g-1}, r_{A_g}] = \frac{f[r_{A_g}, \rho_g = j|\varphi_{g-1}]}{f[r_{A_g}|\varphi_{g-1}]}.$$

Finally we are ready to compute the log-likelihood function for all observations r_{A_g} by running the same process starting from eq. A1:

$$\ell(\theta) = \sum_{g=1}^{g=G} \log f[r_{A_g}|\varphi_{g-1}]$$

Appendix B: Goodness of Fit Tests between *RSDR* and *MDR* models

This figure shows the goodness-of-fit tests for the *MDR* and *RSDR* models across three rating categories (S&P rating “A-2”, “A-1” and “A-1+”). Specifically:

- Average credit warning signal (*CWS*) across the three rating categories.
- Average of the AIC test (details below) across the three rating categories.
- Average of the SBC test (details below) across the three rating categories.

AIC is estimated as the value of: $[\log\text{-likelihood value} - \text{number of parameters}]$. For the *MDR* model there are 2 parameters and for the *RSDR* model there are 6 parameters. Hence:

- AIC is estimated for each of the *MDR* and *RSDR* on a daily basis.
- *MDR* and *RSDR* log-likelihood values are obtained using equations 16 and 33 respectively.
- The plotted value for AIC is: “AIC test” = (AIC for *RSDR*) – (AIC for *MDR*).
- If the value of “AIC test” is positive then the *RSDR* model outperforms the *MDR* model.

SBC is estimated as the value of:

$[\text{Log-likelihood value} - 0.5 * (\text{number of parameters}) * \log(\text{number of observations})]$. For the *MDR* model there are 2 parameters and for the *RSDR* model there are 6 parameters, while both models are estimated on 252 observations. Hence:

- SBC is estimated for each of the *MDR* and *RSDR* on a daily basis.
- *MDR* and *RSDR* log-likelihood values are obtained using equations 16 and 33 respectively.
- The plotted value for SBC is: “SBC test” = (SBC for *RSDR*) – (SBC for *MDR*).
- If the value of “SBC test” is positive then the *RSDR* model outperforms the *MDR* model.

In summary, the *RSDR* model outperforms the *MDR* model. This happens mostly when the *MDR* is constrained by its own distributional assumption. For example, in periods of abnormal equity activity the *MDR* model cannot accommodate extreme variations in equity log-returns but the *RSDR* model has the potential to use jumps or extended periods of changing volatility in the underlying equity prices.

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This chart shows the number of unique firms (at the end of each month from January 2004 to September 2010), included in each of the ten rating categories used by Standard and Poor's (S&P) in assigning Domestic Short-Term Issuer Credit Ratings (ICR). Variables of companies with the same rating at the end of every month (market value of equity and default boundary) are aggregated to form the input dataset for the Merton Default Risk (*MDR*) and Regime Switching Default Risk (*RSDR*) models.

Figure 2: Credit Default Swap (CDS) spreads from 01/2004 to 09/2010, classified by S&P Rating.

This chart shows the weighted average (by market value of equity) *CDS* spread, with one year maturity, at the end of each month from January 2004 to September 2010. The top three indices (by credit quality) used by Standard and Poor's (S&P) in assigning Domestic Short-Term Issuer Credit Ratings (ICR) are shown. We focus on these three because there is a limited number of companies in the remaining indices (e.g. SPR 1-7 have numerous months with less than 10 companies in each index).

Figure 3: CDS Spreads, Market Value of Equity (MVE) and Default Boundary (DB)

The three panels in this chart shows the weighted average *CDS* spread for the top three indices (by credit quality) used by Standard and Poor's (S&P) in assigning Domestic Short-Term Issuer Credit Ratings (ICR). *CDSI_wm* is the Credit Default Swap (*CDS*) spread weighted by each company's market value of equity plotted in basis points on the left vertical axis. It also shows the respective market value of equity (*MVE*) and the default boundary (*DB*) for each index for the same period (January 2004 to September 2010) plotted in \$ million on the right vertical axis. Panel A shows these variables for the index rated "A-2" (SPR 8), panel B shows results for index rated "A-1" (SPR 9), and panel C shows the same results for the index rated "A-1+" (SPR 10).

Figure 4: Credit warning signal (CWS) and Credit Default Swap (CDS) spread.

The three panels in this chart show the same market variables for each of the three ratings-based indices. Panel A refers to the index rated “A-2” (SPR 8), panel B to the index rated “A-1” (SPR 9), and panel C to the index rated “A-1+” (SPR 10). On the left vertical axis, we plot the $CDSI_{wm}$, which is the Credit Default Swap (CDS) spread weighted by each company’s market value of equity, plotted in basis points. The remaining variables are plotted on the right vertical axis (intercept at 0 also shown for convenience): Log_RSPD (Log_MPD) is the logarithmic transformation of the 1-year default probability estimated using the $RSDR$ (MDR) model. CWS is the credit warning signal calculated as $(Log_RSPD - Log_MPD)$.

Table 1: Structure of the Rating Aggregate Indices

This table shows the available rating categories used by Standard and Poor's (S&P) in assigning Domestic Short-Term Issuer Credit Ratings (ICR). Companies with the same rating (also denoted as categories 1-10) are aggregated (their market value of equity and default boundary) to be used in the Merton Default Risk (*MDR*) and Regime Switching Default Risk (*RSDR*) models. Since estimates are produced at the end of each month (January 2004 to September 2010), the number of companies rated at the end of each month could change, and also companies could change rating. Descriptive statistics of the number of companies are provided in the last four columns.

S&P ICR Rating	Rating Index Code	Number of Companies included in Indices			
		Mean	Median	Min	Max
A-1+	SPR 10	15	15	13	18
A-1	SPR 9	37	39	18	42
A-2	SPR 8	51	56	12	65
A-3	SPR 7	12	11	3	21
B	SPR 6	1	1	1	2
B-1	SPR 5	5	5	1	7
B-2	SPR 4	4	2	1	11
B-3	SPR 3	3	3	2	4
C	SPR 2	0	0	0	0
D	SPR 1	0	0	0	0

Table 2: Descriptive Statistics and Model Estimates of Index with S&P Rating "A-2" (SPR 8)

This table shows the descriptive statistics of the rating index constructed from individual companies with an S&P ICR rating of "A-2" (SPR 8). *Mean* is the average value over the 81 monthly observations from January 2004 to September 2010. *Std Dev* is the standard deviation, *Skew* is the skewness and *Kurt* is the kurtosis. Under observable variables: *MVE* is the sum of the market value of equity; *DB* is the sum of the default boundaries; *CDS1_m* (*CDS5_m*) is the average 1-year (5-year) maturity, credit default swap (CDS) spread; *CDS1_wm* (*CDS5_wm*) is the weighted average by *MVE* of the 1-year (5-year) maturity, credit default swap (CDS) spread. Under model parameters, μ and σ is the Merton Default Risk (*MDR*) mean and volatility of the estimated asset log-returns. μ_1 and μ_2 are the Regime Switching Default Risk (*RSDR*) model's estimated means of the two regimes characterizing the estimated asset log-returns. σ_1 and σ_2 are the *RSDR* model's estimated volatilities. The *RSDR* model's estimated transition probability from regime 1 (2) to regime 1 (2) is p_{11} (p_{22}). Under model estimates, *MDR_VA* (*RSDR_VA*) is the market Value of assets estimated from the *MDR* (*RSDR*) model; *MPD* (*Log_MPD*) is the 1 year (logarithmic transformation of) the default probability estimate from the *MDR* model; *RSPD* (*Log_RSPD*) is the 1 year (logarithmic transformation of) the default probability estimate from the *RSDR* model. *CWS* is the Credit Warning Signal, estimated as (*Log_RSPD* - *Log_MPD*).

Details	N	Mean	Median	Std Dev	Min	Max	Skew	Kurt
Observable Variables								
<i>MVE</i>	81	1,021,879	1,002,400	362,921	142,786	1,615,054	(0.19)	(0.75)
<i>DB</i>	81	471,888	476,644	120,930	83,057	625,916	(1.21)	1.71
<i>CDS1_m</i>	81	43.306	25.184	54.630	6.099	250.614	2.306	4.953
<i>CDS5_m</i>	81	73.231	47.729	56.886	27.664	278.511	2.139	4.346
<i>CDS1_wm</i>	81	40.605	29.424	45.072	4.961	216.594	2.178	4.643
<i>CDS5_wm</i>	81	69.046	51.416	46.793	21.645	243.102	2.018	4.245
Model Parameters								
μ	81	4.17%	8.67%	14.22%	-39.98%	26.54%	-1.276362	1.106062
σ	81	12.58%	9.47%	6.01%	7.69%	26.87%	1.431134	0.699091
μ_1	81	-30.00%	-15.52%	68.96%	-446.33%	71.01%	-3.154481	16.577658
μ_2	81	21.18%	15.51%	24.59%	-35.35%	83.52%	0.655393	0.249199
σ_1	81	17.15%	14.49%	10.36%	3.54%	43.83%	1.501728	1.252165
σ_2	81	8.45%	7.35%	3.72%	1.74%	19.87%	1.589223	2.237210
$p_{1,1}$	81	55.54%	63.60%	40.76%	0.00%	99.60%	-0.270791	-1.675641
$p_{2,2}$	81	60.73%	88.48%	42.31%	0.00%	99.56%	-0.467358	-1.638501
Model Outputs								
<i>MDR_VA</i>	81	1,497,146	1,509,659	447,883	225,501	2,189,910	(0.39)	(0.24)
<i>MPD</i>	81	0.20%	0.00%	0.010	0.00%	8.28%	7.518	61.628
<i>Log_MPD</i>	81	-80.28	-96.62	48.97	-153.89	-2.49	0.29	-1.41
<i>RSDR_VA</i>	81	1,497,141	1,509,659	447,887	225,501	2,189,910	(0.39)	(0.24)
<i>RSPD</i>	81	0.41%	0.00%	0.013	0.00%	6.62%	3.562	12.506
<i>Log_RSPD</i>	81	-62.82	-46.23	46.91	-160.98	-2.72	-0.51	-1.03
<i>CWS</i>	81	17.46	2.13	28.00	-11.56	83.00	1.23	-0.03

Table 3: Descriptive Statistics and Model Estimates of Index with S&P Rating "A-1" (SPR 9)

This table shows the descriptive statistics of the rating index constructed from individual companies with an S&P ICR rating of "A-1" (SPR 9). *Mean* is the average value over the 81 monthly observations from January 2004 to September 2010. *Std Dev* is the standard deviation, *Skew* is the skewness and *Kurt* is the kurtosis. Under observable variables: *MVE* is the sum of the market value of equity; *DB* is the sum of the default boundaries; *CDS1_m* (*CDS5_m*) is the average 1-year (5-year) maturity, credit default swap (CDS) spread; *CDS1_wm* (*CDS5_wm*) is the weighted average by *MVE* of the 1-year (5-year) maturity, credit default swap (CDS) spread. Under model parameters, μ and σ is the Merton Default Risk (*MDR*) mean and volatility of the estimated asset log-returns. μ_1 and μ_2 are the Regime Switching Default Risk (*RSDR*) model's estimated means of the two regimes characterizing the estimated asset log-returns. σ_1 and σ_2 are the *RSDR* model's estimated volatilities. The *RSDR* model's estimated transition probability from regime 1 (2) to regime 1 (2) is p_{11} (p_{22}). Under model estimates, *MDR_VA* (*RSRD_VA*) is the market Value of assets estimated from the *MDR* (*RSDR*) model; *MPD* (*Log_MPD*) is the 1 year (logarithmic transformation of) the default probability estimate from the *MDR* model; *RSPD* (*Log_RSPD*) is the 1 year (logarithmic transformation of) the default probability estimate from the *RSDR* model. *CWS* is the Credit Warning Signal, estimated as (*Log_RSPD* - *Log_MPD*).

Details	N	Mean	Median	Std Dev	Min	Max	Skew	Kurt
Observable Variables								
<i>MVE</i>	81	1,510,718	1,554,940	425,905	449,693	2,249,101	(0.60)	(0.02)
<i>DB</i>	81	570,763	599,736	182,004	152,812	776,186	(0.60)	(0.77)
<i>CDS1_m</i>	81	19.096	11.046	19.623	3.441	85.416	1.985	3.609
<i>CDS5_m</i>	81	39.929	29.442	24.997	13.715	132.050	1.565	2.447
<i>CDS1_wm</i>	81	18.281	12.514	18.459	2.876	82.536	1.994	3.764
<i>CDS5_wm</i>	81	38.944	29.275	23.517	11.236	120.874	1.305	1.679
Model Parameters								
μ	81	4.58%	6.47%	12.82%	-30.76%	27.20%	-1.053419	0.848398
σ	81	12.20%	10.07%	6.02%	7.02%	26.49%	1.452430	0.832352
μ_1	81	-5.07%	1.54%	79.26%	-297.43%	245.76%	-0.329358	2.831165
μ_2	81	12.73%	5.92%	35.97%	-60.83%	105.95%	0.904171	0.758138
σ_1	81	17.54%	12.46%	12.87%	5.17%	50.48%	1.405055	0.532912
σ_2	81	7.56%	6.48%	3.86%	0.27%	17.60%	1.014842	1.037020
$p_{1,1}$	81	44.35%	37.44%	39.08%	0.00%	99.53%	0.243588	-1.602309
$p_{2,2}$	81	47.60%	42.88%	39.25%	0.00%	99.67%	0.157865	-1.543718
Model Outputs								
<i>MDR_VA</i>	81	2,083,506	2,178,253	568,189	602,316	2,924,380	(0.72)	(0.08)
<i>MPD</i>	81	0.01%	0.00%	0.001	0.00%	0.46%	6.388	45.258
<i>Log_MPD</i>	81	-109.91	-119.40	66.21	-223.32	-5.39	0.19	-1.35
<i>RSDR_VA</i>	81	2,083,503	2,178,233	568,188	602,316	2,924,380	(0.72)	(0.08)
<i>RSPD</i>	81	0.10%	0.00%	0.003	0.00%	1.45%	3.028	9.499
<i>Log_RSPD</i>	81	-102.46	-76.73	70.10	-254.50	-4.23	-0.21	-1.17
<i>CWS</i>	81	7.44	1.11	30.88	-31.30	125.96	2.53	6.57

Table 4: Descriptive Statistics and Model Estimates of Index with S&P Rating "A-1+" (SPR 10)

This table shows the descriptive statistics of the rating index constructed from individual companies with an S&P ICR rating of "A-1+" (SPR 10). *Mean* is the average value over the 81 monthly observations from January 2004 to September 2010. *Std Dev* is the standard deviation, *Skew* is the skewness and *Kurt* is the kurtosis. Under observable variables: *MVE* is the sum of the market value of equity; *DB* is the sum of the default boundaries; *CDS1_m* (*CDS5_m*) is the average 1-year (5-year) maturity, credit default swap (CDS) spread; *CDS1_wm* (*CDS5_wm*) is the weighted average by *MVE* of the 1-year (5-year) maturity, credit default swap (CDS) spread. Under model parameters, μ and σ is the Merton Default Risk (*MDR*) mean and volatility of the estimated asset log-returns. μ_1 and μ_2 are the Regime Switching Default Risk (*RSDR*) model's estimated means of the two regimes characterizing the estimated asset log-returns. σ_1 and σ_2 are the *RSDR* model's estimated volatilities. The *RSDR* model's estimated transition probability from regime 1 (2) to regime 1 (2) is p_{11} (p_{22}). Under model estimates, *MDR_VA* (*RSDR_VA*) is the market Value of assets estimated from the *MDR* (*RSDR*) model; *MPD* (*Log_MPD*) is the 1 year (logarithmic transformation of) the default probability estimate from the *MDR* model; *RSPD* (*Log_RSPD*) is the 1 year (logarithmic transformation of) the default probability estimate from the *RSDR* model. *CWS* is the Credit Warning Signal, estimated as (*Log_RSPD* - *Log_MPD*).

Details	N	Mean	Median	Std Dev	Min	Max	Skew	Kurt
Observable Variables								
<i>MVE</i>	81	1,695,813	1,717,990	215,175	1,294,395	2,064,214	(0.03)	(1.02)
<i>DB</i>	81	395,733	401,616	72,587	245,377	517,834	(0.46)	(0.72)
<i>CDS1_m</i>	81	14.278	4.737	17.192	1.864	71.465	1.915	3.163
<i>CDS5_m</i>	81	28.440	14.217	23.733	7.113	108.066	1.574	2.148
<i>CDS1_wm</i>	81	13.477	3.911	16.885	1.589	69.716	1.927	3.198
<i>CDS5_wm</i>	81	26.542	12.918	23.348	5.714	106.018	1.567	2.166
Model Parameters								
μ	81	2.21%	2.88%	9.09%	-21.50%	22.31%	-0.717226	0.768366
σ	81	13.07%	10.06%	6.60%	7.57%	29.35%	1.696299	1.401140
μ_1	81	-18.99%	-25.23%	55.72%	-146.88%	198.48%	1.232881	3.518522
μ_2	81	26.10%	17.32%	39.35%	-82.26%	136.61%	0.411591	0.509855
σ_1	81	19.39%	14.13%	14.12%	7.34%	59.24%	1.572373	1.089136
σ_2	81	8.26%	7.72%	3.47%	1.86%	16.21%	0.661201	-0.075137
$p_{1,1}$	81	51.55%	57.70%	37.57%	0.00%	99.44%	-0.144657	-1.533706
$p_{2,2}$	81	52.17%	62.87%	41.60%	0.00%	99.66%	-0.165157	-1.714740
Model Outputs								
<i>MDR_VA</i>	81	2,094,047	2,124,496	251,477	1,548,742	2,463,495	(0.42)	(0.86)
<i>MPD</i>	81	0.00%	0.00%	0.000	0.00%	0.00%	4.809	24.400
<i>Log_MPD</i>	81	-141.60	-154.67	81.63	-292.49	-10.57	0.05	-0.86
<i>RSDR_VA</i>	81	2,094,046	2,124,496	251,477	1,548,742	2,463,495	(0.42)	(0.86)
<i>RSPD</i>	81	0.03%	0.00%	0.001	0.00%	1.04%	6.533	43.583
<i>Log_RSPD</i>	81	-126.49	-127.51	80.24	-288.28	-4.56	-0.03	-0.99
<i>CWS</i>	81	15.11	6.84	27.35	-28.35	103.14	0.99	0.84