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1 Analysis of geomembrane whale due to liquid flow through composite liner

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12
13 **ABSTRACT**

14 Due to the defects of geomembrane liner (GL), leaking water could infiltrate dry soil below,
15 replace the pore air and thus generate geomembrane whale (GW). An analytical solution is
16 proposed in this paper to analyze the geometry and tensile force of axisymmetric GW.
17 Parametric studies are also conducted to identify the influences from key factors and provide
18 predicting charts for practical usage. It is concluded from the parametric studies that the
19 maximum height and tensile force of GW can be achieved when the GW is just about to be
20 submerged by external water. For a given level of external water, the height, width and
21 tensile force of GW versus volume of leaking water curves are bilinear in a log-log graph,
22 with a higher slope before a turning point but a smaller slope after that, whereas gas pressure
23 in GW has reverse trends. It is also observed in this paper that GL with higher $E\delta$ has a strong
24 capacity to confine the gas pressure and thus generate lower and wider GW with higher
25 tensile force and gas pressure.

26
27 **KEYWORDS:** geosynthetic, geomembrane liner, geomembrane whale

28 1 INTRODUCTION

29 The composite liner provides an ideal impermeable layer for the construction of water
30 reservoirs, city wetlands, wastewater lagoons, solid waste disposals and some other projects
31 that need a waterproof liner. The composite liner is usually constructed using a layer of
32 geomembrane liner (GL) over a compacted clay liner (CCL) or a geosynthetic clay liner
33 (GCL). However, water will leak through GL into the bottom dry soil due to the defect of GL.
34 The defect of GL may arise from manufacturing defects, handling of the geomembrane rolls,
35 on-site placement and seaming, placement of drainage gravel over the liner system, traffic on
36 the liner or the overlying protection layer, placement of the waste in a landfill or cleaning of
37 residue from a leachate lagoon and stress cracking as the geomembrane ages (Row, 2012).
38 The water leaking into the dry soil liner will infiltrate the soil and thus replace the pore gas or
39 generate gas due to a microbiological reaction. The gas will then migrate and aggregate
40 underneath the wrinkle or at a high spot below the GL; see step (1) in Figure 1. When
41 sufficient gas aggregates under the GL, gas pressure larger than the hydraulic pressure acts on
42 its top and the GL is lifted from the ground, thus generating a geomembrane whale (GW), as
43 in step (2) in Figure 1. Although generation of GW will release the gas pressure under GL, it
44 will elongate the GL, making it thinner, increasing the risk of failure and reducing the storage
45 ability of the reservoir. Some in-situ test results have been reported by Cao et al. (2015).

46

47 Theoretical calculation methods for the leakage rate, Q (m^3/yr), due to the defect of GL
48 have been proposed by many researchers, e.g., Giroud et al. (1989), Giroud et al. (1992),
49 Giroud and Bonaparte (1989a, b), Giroud (1997), Rowe and Booker (1998), Touze-Foltz
50 et al. (1999), Foose et al. (2001), Cartaud et al. (2005), Rowe and Abdelatty (2012), Rowe
51 (2012) and El-Zein et al. (2012). Generally, the leakage rate of water through the GL is a
52 function of the number and size of holes, permeability of the clay liner, water head
53 difference, interface between the geomembrane liner and clay liner beneath, wrinkles of the
54 constructed geomembrane and properties of the reserved wastewater (Rowe 2012). As the topic
55 is relatively large and complex, it will not be covered in this paper. The following
56 discussion assumes that the leakage rate of GL, Q (m^3/yr), is known or has been calculated
57 based on the mentioned theoretical calculation methods. Then, the total volume of leaking
58 water, V_l (m^3), could be calculated as

59

$$V_l = Qt \quad (1)$$

60 where t is the time of leakage (yr).

61

62 The volume of replaced gas in the GW will be the same as the total volume of leaking
63 water, V_1 , if the pore gas in the dry soil is assumed to be fully infiltrated by the leaking
64 water (very low ground water level) and neither the gas generated by the microbiological
65 reaction between leaking water and soil nor the water absorbed by the thin clay liner in
66 GCL is ignored. After the replaced gas migrates and aggregates underneath the wrinkle or
67 at a high spot, the volume of the replaced gas will change from V_1 to V_2 because the gas
68 pressure changes from gas pressure P_1 in the ground to gas pressure P_2 underneath the GL.
69 The gas removed from the ground to the bottom of the GL will also have temperature
70 changes that will also influence the volume of gas in the two statuses. The aggregated gas
71 follows the ideal gas law, i.e., $PV = nRT$, where the letters denote pressure (Pa), volume
72 (m^3), amount (in moles), ideal gas constant and temperature of the gas in kelvin (K and 0
73 $^{\circ}C = 273 K$), respectively; thus, we have

$$\frac{P_1 V_1}{P_2 V_2} = \frac{T_1}{T_2} \quad (2)$$

74 where T_1 and T_2 (units in K) are the temperature of gas before and after replacement,
75 respectively.

76

77 In this paper, an analytical solution is proposed to analyze the geometry and tensile force
78 along the GW. Parametric studies are also conducted in this paper to identify the influences
79 from the key factors and provide predicting charts for practical usage. The proposed solution
80 is capable of calculating the axisymmetric GW generated in construction projects on dry soil
81 related to landfills, water reservoirs and lagoons for wastewater treatment.

82

83 2 PROPOSED ANALYTICAL METHOD

84 2.1 Basic Assumptions

85 In deriving the solutions, the following assumptions need to be made: (1) the GW is an
86 axisymmetric problem; (2) the geomembrane shell is thin, elastic and weightless; (3) the
87 frictions between the geomembrane and soil liner or water are not considered; (4) the soil
88 surface is horizontal and its deformation due to infiltration is not considered; (5) ground
89 water level is very low and thus the same volume of gas is replaced by the same volume of
90 leaking water.

91

92 2.2 *Theoretical Derivations*

93 A 2D analytical model of an axisymmetric GW could be simplified from a 3D model, as
 94 shown in Figure 2(a). The coordinates of the system are set-up with x in the horizontal
 95 direction and y in the vertical direction. The origin of this coordinate is taken as the center of
 96 the contact edge with the ground surface. The unit weight of the external water and height are
 97 written as γ_w and H_w , respectively. The tensile force along the GW is written as T . The height
 98 and width of the cross-section of the GW are denoted as H and B , respectively. An
 99 infinitesimal small curve with a length of ds at an arbitrary point $S(x, y)$ can be treated as an
 100 arc with a radius of r , as shown in Figure 2(b). The angle between the tangential direction at
 101 point $S(x, y)$ and the x -axis is denoted as θ . Then, two geometrical differential equations
 102 relating θ and the x and y coordinates can be written as shown in Eqs. (3) and (4). The minus
 103 sign in Eq. (4) identifies that the integral direction on $S(x, y)$ is along the direction from $y = H$
 104 to $y = 0$. Because the frictions between water/gas and GW or those between GW and the
 105 ground surface are neglected, the tension force along the cross-section is constant and thus
 106 we have the expression shown in Eq. (5). The hydraulic pressure from stored water acting
 107 externally on the GW at a given depth, y , is written as $\beta\gamma_w(H_w - y)$, where $\beta = 1$ when $H_w > y$
 108 and $\beta = 0$ when $H_w < y$. The force equilibrium equations in the normal and tangential
 109 directions along the infinitesimal element yield the expression shown in Eq. (6).

110

$$\frac{dx}{ds} = \cos \theta \tag{3}$$

$$\frac{dy}{ds} = -\sin \theta \tag{4}$$

$$\frac{dT}{ds} = 0 \tag{5}$$

$$\frac{d\theta}{ds} = \frac{1}{T} [P_2 - \beta\gamma_w(H_w - y)] \tag{6}$$

111

112 2.3 *Boundary Conditions*

113 To solve Eqs. (3) to (6), some more boundary conditions related to the tensile forces and
 114 geometries have to be established. It should be noted that the reason for generating the GW is
 115 because of the elongation and wrinkle of the flexible GL. The final perimeter of GW, L , in
 116 the 2D axisymmetric model is elongated from the initial length of the GL (or B in Figure

117 2(a)) and length of the wrinkle. Their relationships in this axisymmetric analytical model
 118 could be expressed as

$$L = (1 + \frac{T}{E\delta})(1 + \varepsilon)B \quad (7)$$

119 where δ and E are the thickness and elastic modulus of GL, respectively; ε is the ratio of the
 120 wrinkle, which could be calculated as the ratio of the initial wrinkle length to the total length
 121 of GL.

122

123 A free body diagram of the 2D axisymmetric cross-section is presented in Figure 3. Because
 124 the tensile force is balanced in the axisymmetric cross-section and along its tangential
 125 directions, the direction of tensile forces on the top point is horizontal, as shown in Figure
 126 3(a). The ground surface is horizontal, making the direction of T on the bottom edge also
 127 horizontal; see Figure 3(a). Then, the forces on the horizontal direction only involve the
 128 external hydraulic pressure and internal gas pressure. The force equilibrium in the horizontal
 129 direction between the two forces yields

$$P_2H = \frac{1}{2}\gamma_w H_w^2 - \frac{1}{2}\eta\gamma_w (H_w - H)^2 \quad (8)$$

130 where η is the calculation constant and $\eta = 0$ when $H_w < H$; $\eta = 1$ when $H_w > H$.

131

132 It can be seen from Figure 2(a) that there is an inflection point on the cross-section at which
 133 the curve changes from being concave to convex. Mathematically, it is the point defined as y
 134 $= h_i$, at which $d\theta/ds = 0$ in Eq. (6); $d\theta/ds > 0$ when $y > h_i$ and $d\theta/ds < 0$ when $y < h_i$.
 135 Substituting $y = h_i$ and $d\theta/ds = 0$ into Eq. (6), we can obtain a balance between internal gas
 136 pressure and external hydraulic pressure at this inflection point, as shown in Eq. (9). Note that
 137 $\beta = 1$ in this derivation because the inflection point will always be lower than the external
 138 water level, or $h_i < H_w$, because the gas pressure in Eq. (6) is always greater than zero, or $P_2 >$
 139 0 , if GM could be generated. Figure 3(b) shows a selected free body from the cross-section
 140 with y -coordinates ranging from $y = H$ to $y = h_i$. It can be seen that the forces acting on this
 141 free body along the horizontal direction involve the internal gas pressure, external hydraulic
 142 pressure and tensile forces. By denoting the angle between the tangential direction at the
 143 inflection point and the x -axis as θ_i , the force equilibrium along the horizontal direction could
 144 yield the expression shown in Eq. (10). Because the problem is axisymmetric, the volume of
 145 the GW could be integrated by using Eq. (11).

146

$$P_2 = \gamma_w (H_w - h_i) \quad (9)$$

$$T(1 - \cos \theta_i) = P_2(H - h_i) - \frac{1}{2} \gamma_w [(H_w - h_i)^2 - \eta(H_w - H)^2] \quad (10)$$

$$V_2 = \int_0^H \pi x^2 dy \quad (11)$$

147

148 2.4 Iteration Method

149 By combining Equations (1) to (11), the geometry and tensile force of the GW could be
 150 solved by using a computer program. The unit weight and height of stored water, γ_w and H_w ,
 151 respectively; leakage volume, V_l ; wrinkle ratio, ε ; elastic modulus, E ; and thickness of GL, δ ,
 152 are taken as inputs. The following two boundary conditions are also adopted for the
 153 calculation of the curved section: 1) when $y = H$ and $\theta = 0$ and 2) when $y = 0$ and $\theta = 0$.

154

155 To solve the differential equations, a computer program has been developed using the
 156 adaptive Runge-Kutta-Merson method (RKM4), which has already been coded by Press et al.
 157 (2007), Xu (1995), Christiansen (1970) and Lukehart (1963). A mathematical programming
 158 scheme, the Complex Method proposed by Box (1965), was used to search for the unknown
 159 parameters in their specific ranges. For more details of the Complex Method, one can refer to
 160 Lipson (1977) and Xu (1995).

161

162 3 PARAMETRIC STUDIES

163 3.1 Influence of external water level

164 The effects of the external water level are studied to investigate the influence from water
 165 level changes that may arise from rainfall, drought, water filling or discharge. The parameter
 166 studies are conducted by assuming that the gas pressure P_l in soil is the standard atmospheric
 167 pressure and $P_l = 101.235 \text{ kPa}$. The influence from the length of the wrinkle is not
 168 considered ($\varepsilon = 0$). The unit weight of stored water is $\gamma_w = 10 \text{ kN/m}^3$. The product of elastic
 169 modulus, E , and thickness of GL, δ , is taken as one parameter: $E\delta = 300 \text{ kN/m}$. The solutions
 170 are given for different external water heights, H_w , and different volumes of leaking water, V_l .

171

172 For a certain volume of leaking water, $V_l = 300 \text{ m}^3$, three cross-sections of GM with respect to
 173 different external water levels were plotted, as shown in Figure 4. To have a better angle of
 174 view, the 2D axisymmetric cross-sections are reflected along the y -axis. It can be seen that

175 when the external water level increases from 2 m to 4.9 m, the cross-section is protruded,
176 with its width decreased from 8.01 m to 7.31 m, whereas its height increased from 4.03 m to
177 4.93 m. This is because the external water level is lower than the height of GW and thus the
178 hydraulic pressure only acts around the GW. If water levels continue to increase, the external
179 water submerges the GW and compresses the width and height of the cross-section. For
180 example, when H_w increased from 4.9 m to 10 m (Figure 4), the width of the cross-section
181 decreased from 7.31 m to 6.72 m and height from 4.93 m to 3.71 m.

182

183 This phenomenon can be further illustrated through the heights of GW versus external water
184 heights curves, as shown in Figure 5. Generally, for each certain volume of leaking water, V_l ,
185 the cross-section of the GWs has one maximum height at which the GW is just about to be
186 submerged by external water. Before this maximum height, the external water protrudes the
187 cross-section and thus makes the height of GW increase nonlinearly with increasing external
188 water level. Take $V_l = 200 \text{ m}^3$ for example; the height of GW increased by a factor of 2.8
189 from 1.44 to 4.02 when the external water level increased from $H_w = 0.2 \text{ m}$ to $H_w = 4 \text{ m}$. After
190 the peak, the external water submerged and compressed the GW and thus the height of GW
191 decreased with increasing external water level. Take $V_l = 200 \text{ m}^3$ for example; the height of
192 GW decreased by a factor of 0.62 from 4.02 to 2.51 when the external water level increased
193 from $H_w = 4 \text{ m}$ to $H_w = 20 \text{ m}$. One more very interesting phenomenon that can be found in
194 Figure 4 is a linear relationship between the maximum heights of GW and height of the
195 external water, with a slope of 1.0 and intercept of 0.0. Figure 6 shows the tensile force along
196 the GW versus height of external water curves. The same trends as those for the height of
197 GW versus external water level curves are observed. The peak values of the tensile forces for
198 different volumes of leaking water are also in a linear relationship with respect to the height
199 of the external water, with a slope of 15.3.

200

201 The gas pressure in GW increases nonlinearly with increasing external water height before
202 reaching the peak height of GW but linearly after the peak height; see Figure 7. This is
203 because part of the gas pressure was released and thus protruded GM before it was
204 submerged by the external water. However, after GM was submerged by the external water,
205 the gas was compressed and confined in GM such that its pressure depends on the
206 compression from the hydraulic pressure. The higher the hydraulic pressure, the higher the
207 gas pressure generated. One more phenomenon that could be observed in Figure 7 is that the
208 lower the volume of leaking water, the higher the gas pressure in the GW when all other

209 conditions are constant. This is because GW is a natural phenomenon that occurs to release
210 the gas pressure. A large volume of GW will release the gas pressure but, at the same time,
211 will increase the tensile force along the GW. Figure 8 shows the width of GW versus height
212 of the external water level curves. It can be seen that the width of GW decreases nonlinearly
213 with increasing external water level. The widths of GW decrease quickly with respect to the
214 external water level before the peak height of GW but slowly after that. This is also
215 consistent with the observations of the external water effects of protruding and compressing.

216

217 3.2 *Influence of leakage volume*

218 The effect of water leakage volume, V_l , is studied to investigate the influence from leaking
219 time, t , and leakage rate, Q . The parameter study is conducted by using the same parameters
220 as those used in section 3.1, except that the water leakage volumes range from 1 m^3 to 1000
221 m^3 . The heights of GW versus leakage volume curves in the log-log graph are shown in
222 Figure 9. It can be seen that the curves are bilinear, with high slopes before a turning point
223 and small slopes afterwards. The turning points in these curves indicate the volumes of
224 leakage water when GW is just about to emerge from the external water. It is easy to
225 understand that the deeper the external water, the higher the volume of leakage water
226 required for the GW to emerge from the external water. Take the case of $H_w = 2 \text{ m}$, $E \delta = 300$
227 kN/m for example; the turning point is at $V_l = 50 \text{ m}^3$. For the case of $H_w = 5 \text{ m}$, $E \delta = 300$
228 kN/m , the turning point increases to $V_l = 300 \text{ m}^3$. Another phenomenon that can be observed
229 from the two lines is that the GWs are higher for the case of $H_w = 2 \text{ m}$ than for the case of H_w
230 $= 5 \text{ m}$ before $V_l = 50 \text{ m}^3$. This agrees with our expectation that lower hydraulic pressure has
231 lower compression effect on the GW. However, this trend reversed after the volume of
232 leaking water became larger than 50 m^3 . The tensile forces along GW versus volumes of
233 leakage water curves in the log-log graph are shown in Figure 10. Generally, the same
234 phenomenon has been observed as those from the height of GW versus volumes of leaking
235 water curves. The widths of GW versus volumes of leaking water curves in the log-log graph
236 are shown in Figure 11. It can be seen that the curves are bilinear, with lower slopes before
237 the turning point and larger slopes afterwards. However, the differences between their slopes
238 are not very clear. The lower external water level also induces a lower width of GW, which is
239 consistent with our expectations. Figure 12 shows the gas pressures versus volumes of
240 leaking water curves in the log-log graph. It can be seen that the gas pressures decrease
241 slowly with increasing volume of leaking water before a turning point and increase heavily
242 afterwards. Take the case of $H_w = 5 \text{ m}$, $E \delta = 300 \text{ kN/m}$ for example; the decreasing rate is

243 0.054 before $V_l = 200 \text{ m}^3$ and changes to 0.4 after $V_l = 200 \text{ m}^3$. For the case of $H_w = 10 \text{ m}$, E
244 $\delta = 300 \text{ kN/m}$, the curve nearly reaches the turning point, which is not plotted in the range of
245 the log-log graph.

246

247 3.3 Influence of elastic modulus

248 The effect from the elastic modulus, E , and thickness of GL, δ , are investigated by taking
249 their product, $E\delta$, as one parameter. Cases of $E\delta$ of 300 kN/m and 1000 kN/m are studied in
250 the parametric studies. The solutions are given for different external water heights, H_w , and
251 different volumes of leaking water, V_l . As shown in Figure 9, the higher the $E\delta$, the lower the
252 GW generated. This is because a higher $E\delta$ has a strong ability to confine the gas pressure
253 and thus generates lower GW. It can also be seen that a higher $E\delta$ could increase the turning
254 point, or a strong GL needs more gas to emerge from the external water. Take the case of H_w
255 $= 2 \text{ m}$ for example; the turning point for $E\delta = 300 \text{ kN/m}$ is $V_l = 50 \text{ m}^3$, but for $E\delta = 1000$
256 kN/m , it increases to $V_l = 100 \text{ m}^3$. A higher $E\delta$ will increase the tensile force along GW, as
257 shown in Figure 10. Their differences are almost constant in the log-log graph before and
258 after the turning points. It is easy to understand that a strong GL could hold the GW at a
259 lower volume, making the gas pressure hard to release (Figure 12) and generating a larger
260 tensile force. A strong GL will also induce wider GW, as shown in Figure 11. With respect to
261 different $E\delta$, the differences between the widths of GW versus leaking water volume curves
262 in the log-log graph are almost constant.

263

264 4 CONCLUSIONS

265 Leaking water due to the defects of a geomembrane liner (GL) could infiltrate the dry soil
266 below, replace the pore air and thus generate a geomembrane whale (GW). A new analytical
267 solution was proposed in this paper to analyze the geometry of GW and the tensile force
268 along it. Parametric studies were also conducted in this paper to identify the influences from
269 the key factors and provide designing charts for practical usage

270

271 When the water level changes due to rain, drought, water filling or discharge, the maximum
272 height and tensile force of GW can be achieved, at which the GW is just about to be
273 submerged by external water. There is a linear relationship between the maximum heights
274 and tensile force of GW and height of the external water, with slopes of 1.0 and 15.3,
275 respectively. However, the gas pressure in GW continues to increase, whereas the width of
276 GW decreases, with increasing external water level.

277

278 The effect of the leakage volume was studied to investigate the influence from leaking time
279 and rate. For a certain external water, the height of GW and tensile force along the GW
280 versus volume of leaking water curves are bilinear in the log-log graph, with higher slopes
281 before the turning point and smaller slopes afterwards, whereas gas pressure in the GW has
282 different trends.

283

284 The product of the elastic modulus and thickness of GL, $E\delta$, also influences the tensile force
285 and geometry of GW. A GL with higher $E\delta$ has a strong ability to confine the gas pressure
286 and thus generate lower GW with higher influence on width. However, strong GL will induce
287 higher tensile force along the GW and gas pressure in the GW under same other conditions.

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290 gratefully acknowledged.

291

292 5 NOTATIONS

293	B	Influence width of cross-section
294	E	Elastic modulus of GL
295	H	Height of cross-section
296	h_i	Height of inflection point
297	H_w	Height of external water
298	L	Perimeter of cross-section
299	P_1	Gas pressure in ground
300	P_2	Gas pressure in GW
301	Q	Rate of water leakage through GL
302	r	Radius of the infinitesimal element
303	T	Tensile force along GW
304	t	Time of leakage
305	T_1	Temperature of gas in soil
306	V_1	Volume of leaking water
307	T_2	Temperature of gas in GW
308	V_2	Volume of gas in GW
309	γ_w	Unit weight of water
310	δ	Thickness of GL

311	ε	Ratio of wrinkle
312	θ	Angle between x axis and the tangential direction along the curve
313	θ_i	Value of θ at inflection point

314

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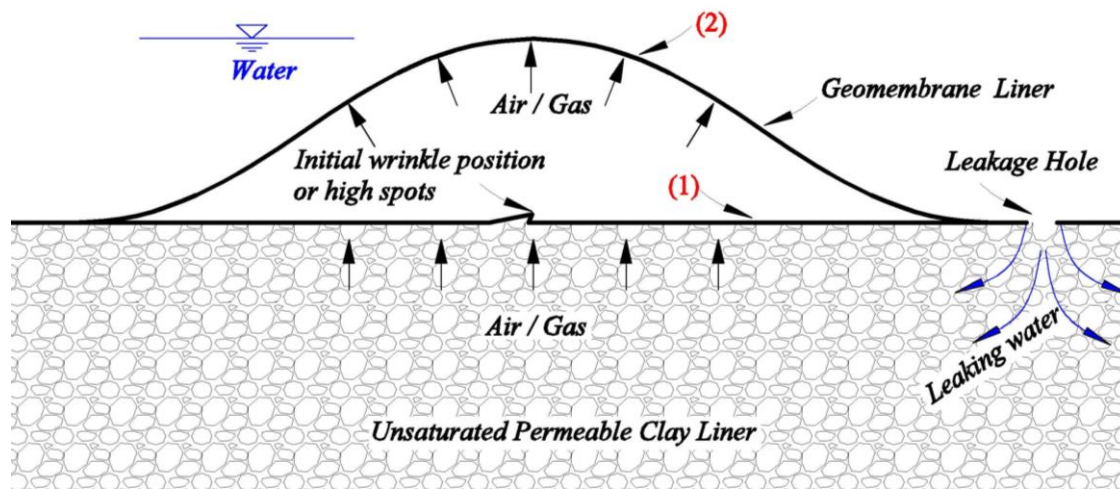
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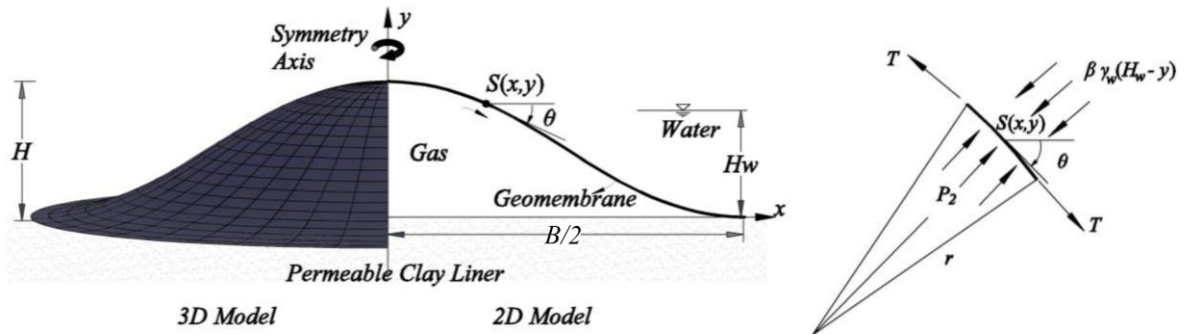
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364 Figure 1 Generation process of GW due to water leakage through GL for (1) Initial
365 position of GL and (2) Final position of GL (GW)

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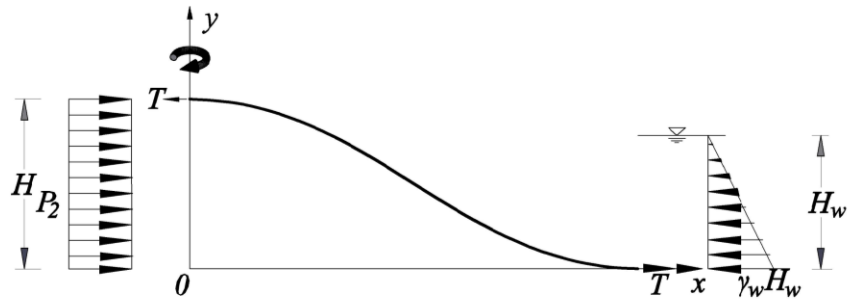
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371 (a) method of simplifying from 3D to 2D model (b) free body diagram of an infinitesimal
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373 Figure 2 Simplified analytical model of GW

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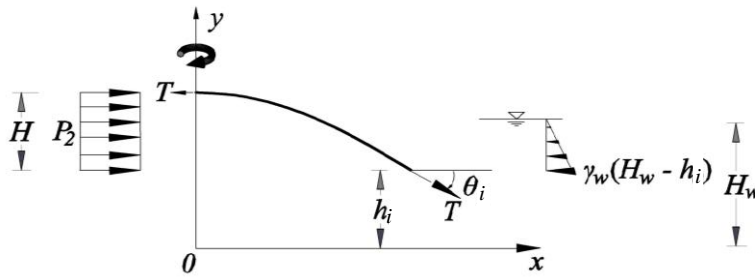
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(a) Free body diagram of axisymmetric model

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(b) Free body diagram of the selected curve

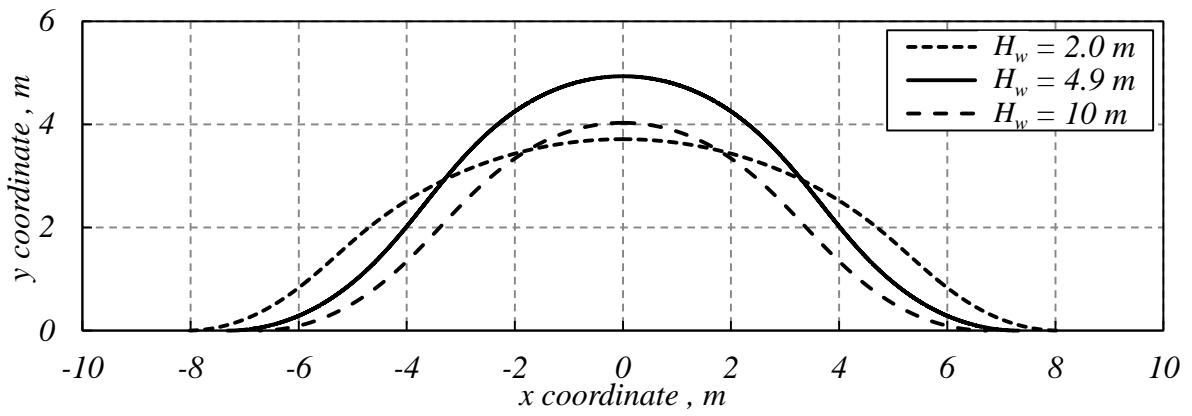
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Figure 3 Free body diagrams for the calculation of boundary conditions

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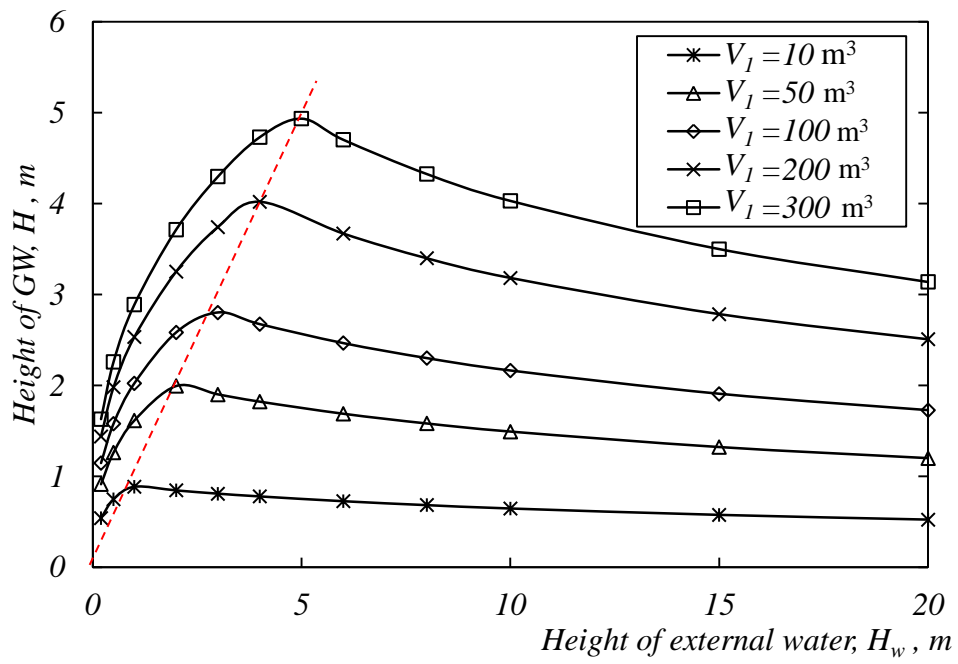
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Figure 4 Cross-section of GW with respect to different external water levels

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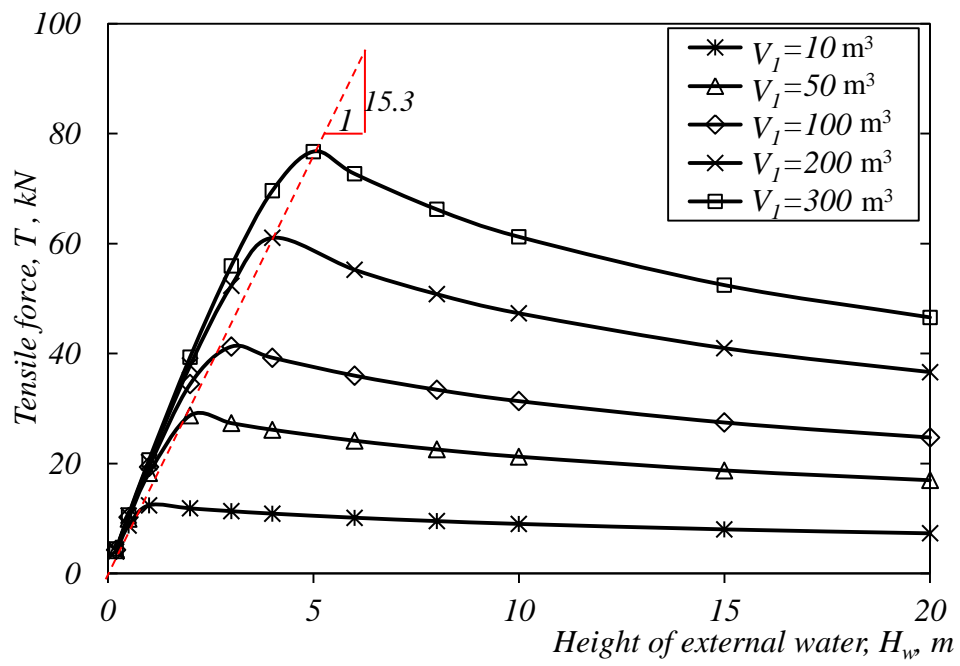
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Figure 5 Height of GW vs external water height curves

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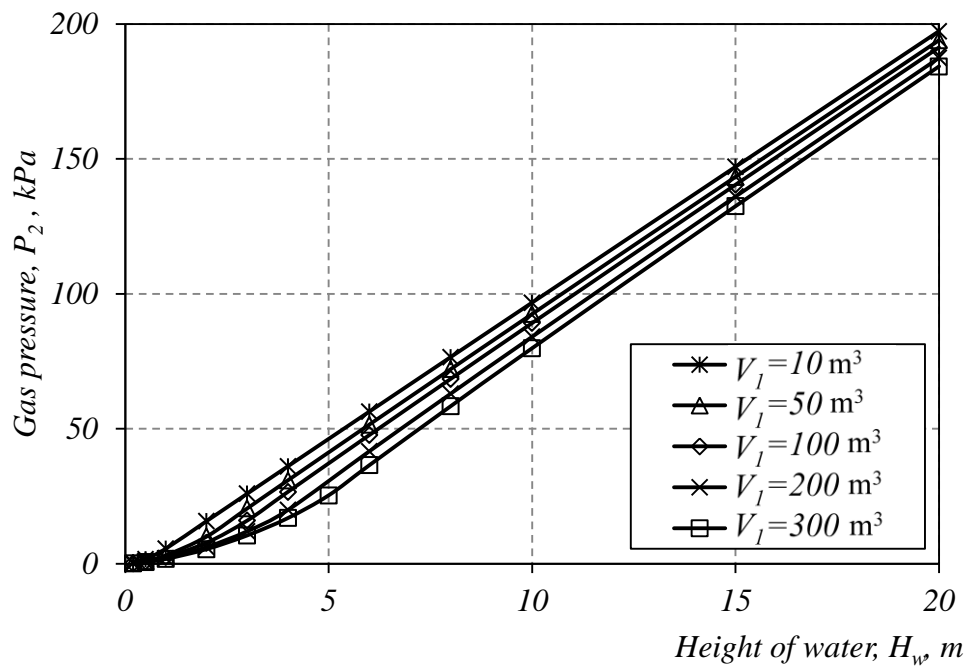
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Figure 6 Tensile force along GW versus height of external water curves

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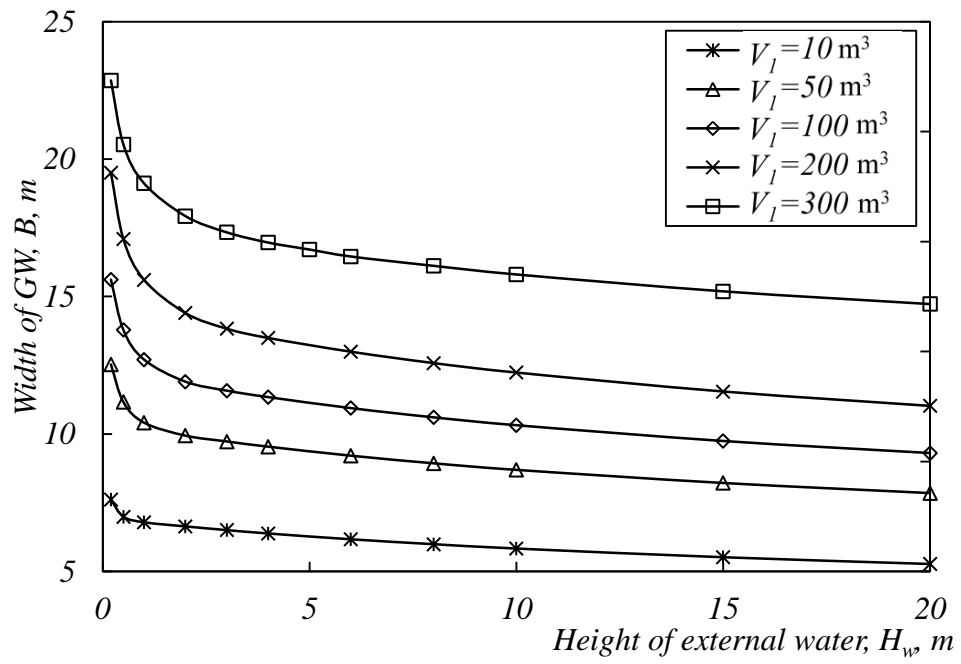
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Figure 7 Gas pressure in GW vs height of external water curves

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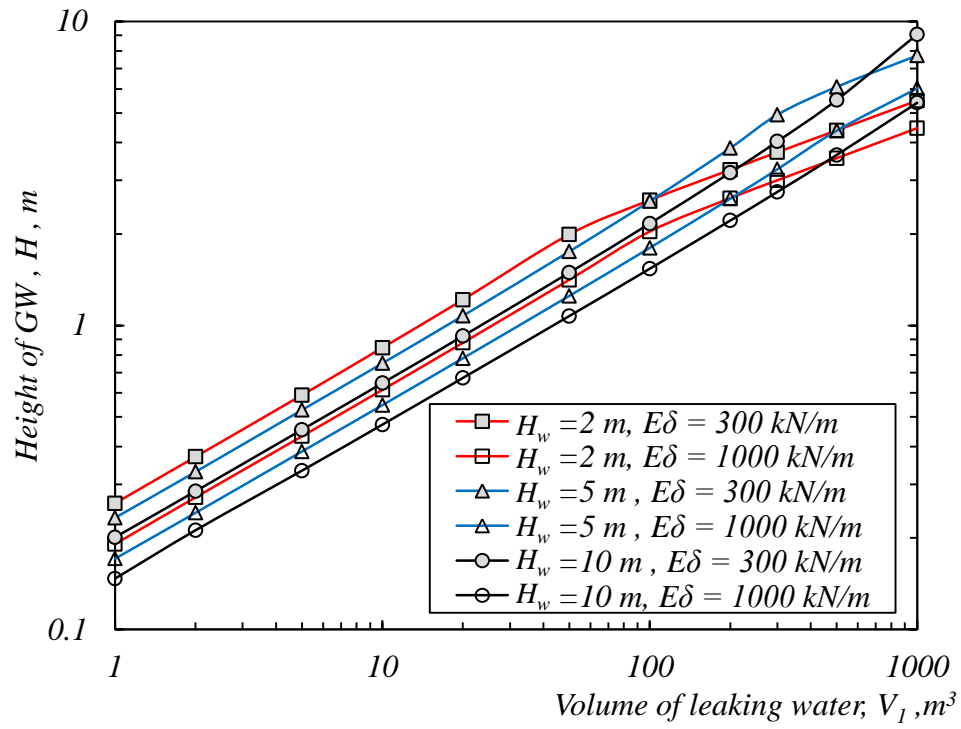
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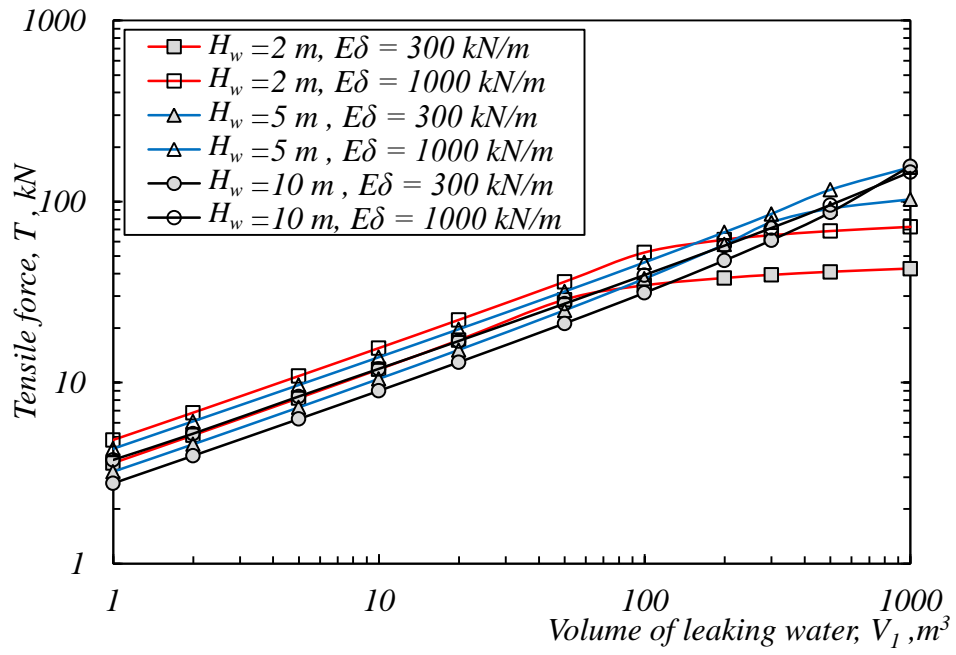
Figure 8 Width of GW vs height of external water curves



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Figure 9 Height of GW vs volume of leaking water curves in log-log graph

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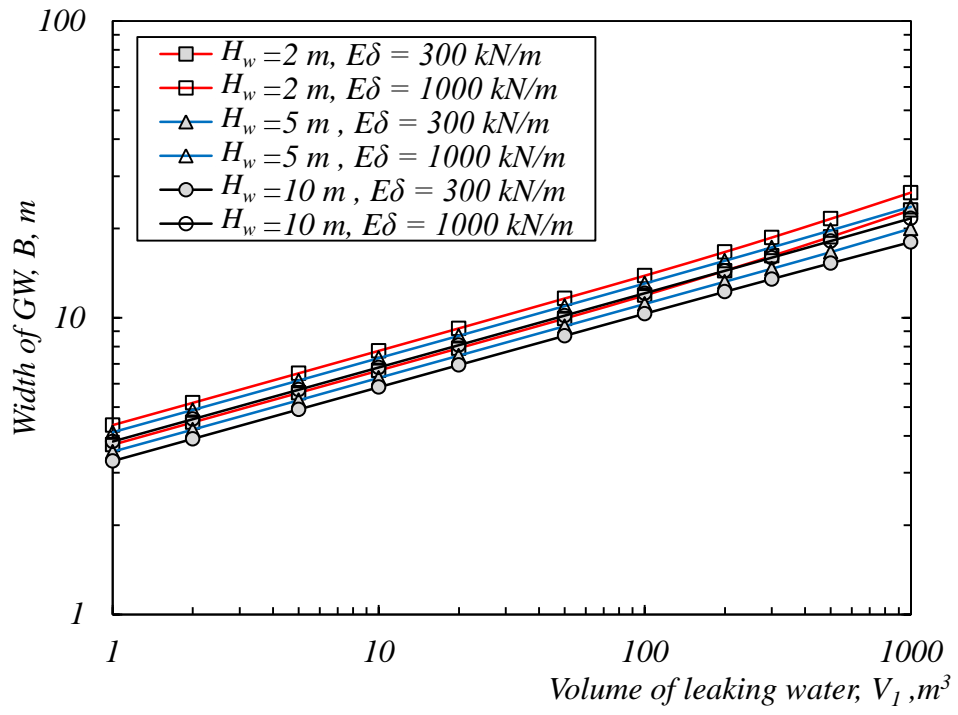


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Figure 10 Tensile force of GW vs volume of leaking water curves in log-log graph

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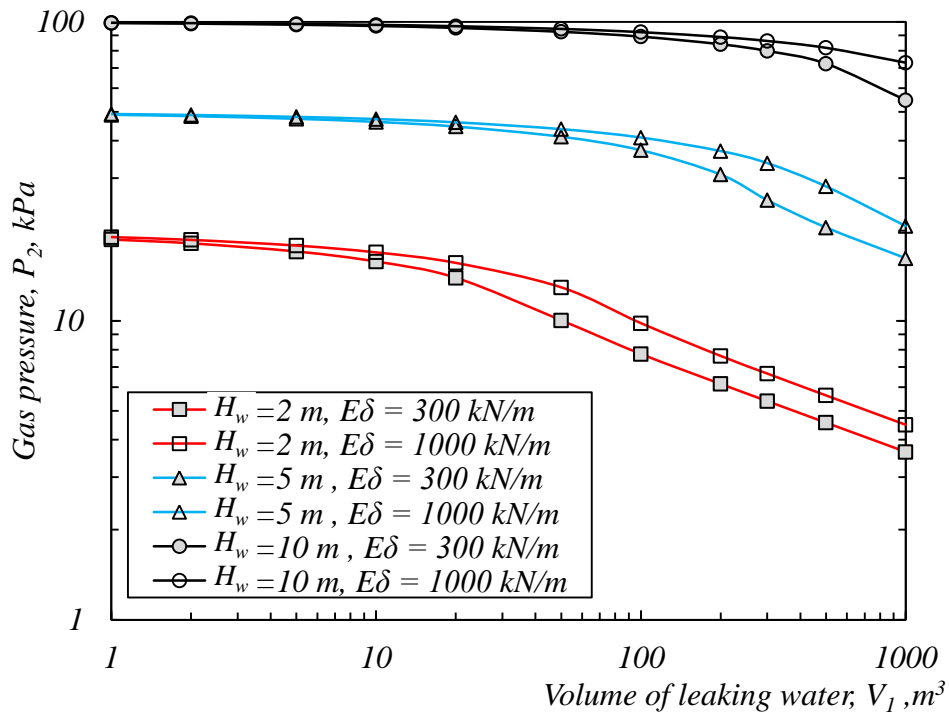
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Figure 11 Width of GW vs volume of leaking water curves in log-log graph

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Figure 12 Gas pressure in GW vs volume of leaking water curves in log-log graph

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