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Indoor Tracking with the Generalized t -Distribution Noise Model

Le Yin, Shuo Liu, Weng Khuen Ho, and Keck Voon Ling

Abstract—An indoor tracking system with forgetting factor and Generalized t -distribution (GT) noise model is proposed in this paper. It consists of first using the weighted centroid formulas to give an estimate of the position and then a filter with GT noise model to improve on the estimate. A common problem with indoor tracking is the noisy disturbances and this paper uses the GT distribution to model them. By being a superset encompassing Gaussian, uniform, t , Cauchy, and double exponential distributions, GT distribution has the flexibility to characterize noise with Gaussian or non-Gaussian statistical properties. Because of the more accurate noise model, the filter with GT noise model can produce a better estimate than that of the Kalman filter which makes the usual assumption of Gaussian noise. An equation to compute the variance of the estimation error is also derived in the paper. For verification, 200 tracking experiments were conducted. The variance obtained from the experiments matched the variance calculated from the equation. The variance of the estimation error from the filter with GT noise model is smaller than that of the Kalman filter. Another experiment at the lift landing showed that the proposed filter with GT noise model is also less affected by outliers.

Index Terms—Indoor positioning and tracking, robust estimation, GT noise model.

I. INTRODUCTION

INDOOR positioning systems have proven to be useful in applications such as indoor navigation, equipment tracking and inventory management [1]. For instance, indoor tracking systems have been used in hospitals to keep track of expensive equipment and elderly patients [2], [3]. Furthermore, position information can also be used in smart buildings for occupancy detection, energy conservation and demand-driven operations [4], [5].

Fingerprinting is a popular indoor localization technique [6], [7]. This technique utilizes fingerprint matching as the basis for position determination. Fingerprint here referred to the unique characteristic of the received signal at different indoor positions. Although these fingerprint based systems have shown encouraging results, the process of site survey during the training phase to build up the database is time-consuming and labor-intensive. SLAM (Simultaneous Localization and Mapping) techniques [8] have made it possible to automatically build a radio fingerprint map, but are still subject to poor initial accuracy and slow convergence [9].

Instead of forming databases of fingerprints and then searching them, the target position can also be estimated using simple geometry. For example, path loss models [10] were used to establish a relationship between the Received Signal Strength (RSS) and the target-receiver distance, and using simple geometrical process of lateration [11] or weighted centroid [12] the position of the target can be estimated. Besides RSS, other measurements such as time of arrival or angle of arrival can also be used to infer the target position [13], [14]. Hybrid solutions such as multi-sensing modalities [15], [16], opportunistic discovery [9], [17] and dead reckoning enhancement [18] are also effective but require additional hardware and sensor fusion.

A common problem with indoor tracking is the disturbances encountered in dynamic and complex indoor environments. Wireless signal variations due to obstacles and multipath [19], geomagnetic field anomalies caused by ferromagnetic materials and electronic devices [20], and even changes in user's posture [9] can all impair the performance of localization techniques. Therefore, the estimate from a localization algorithm is usually highly corrupted by noise and outliers [21], [22].

Conventionally, the zero-mean Gaussian distribution is used to model the unknown noise. However, the occurrence of outliers, transient data in steady-state measurements, instrument failure, human error, model nonlinearity, etc. can all induce non-Gaussian noise [23], [24]. Moreover, even high-quality data may not fit the Gaussian distribution and the presence of a single outlier can spoil the statistical analysis completely for the case of least squares estimation and the Kalman filter [23], [25].

The Generalized t -distribution (GT) has been employed in many applications, from industrial manufacturing to statistical finance [26], [27]. GT was also explored in [28] to help understand the nature of genetic association signal. By being a superset encompassing Gaussian, uniform, t , Cauchy, and double exponential distributions, GT distribution has the flexibility of characterizing noise with Gaussian or non-Gaussian statistical properties [29]. In practice, noise modelling in the GT setting can proceed by likelihood methods analogous to those for the Gaussian distribution. For instance, Fig. 1 shows the histogram for the error (estimated position – true position) distribution of 3000 x-position estimates of a target obtained from the weighted centroid localization algorithm. The maximum likelihood criterion was used to fit a GT distribution and a Gaussian distribution. It can be seen that the GT distribution (solid-line) fits the histogram better than the Gaussian distribution (dashed line). Hence we can expect

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a more precise estimation result if the GT distribution is used.

In this paper, we illustrate the benefit of using filter with GT noise model by filtering indoor positions obtained from weighted centroid method. However, the approach is by no means restricted to the weighted centroid method. Other methods such as triangulation, lateration or hybrid localization schemes can also be used to obtain the initial estimate of indoor positions and then use a filter with GT noise model to obtain a more accurate estimate.

A filter with GT noise model was used in [30] to solve a simple one-dimensional tracking problem where movement along a one-dimensional pathway is tracked by placing a row of sensors along the pathway. A target must also be on the pathway in order to be discovered. In this paper, we extend the problem to a more realistic two-dimensional case where the target can move in any direction instead of being restricted to a pathway. Furthermore, a new equation to compute the variance of the estimation error for the filter with GT noise model and forgetting factor is derived.

A two-dimensional tracking system based on RSS and weighted centroid was implemented. Two dimensional indoor tracking experiments were conducted. We choose RSS for its ubiquitous availability, large user base and hence representativeness [31]–[33]. The weighted centroid method is chosen for its simplicity and robustness to changes in wireless propagation properties compared to other RSS-based methods such as lateration [12]. The variance of the experimental results was also calculated and it matched the variance calculated theoretically from equation (51). This is useful as the variance equation can be used for the design and assessment of the indoor tracking system before implementation. The variance of the estimation error from the filter with GT noise model is less than that from the Kalman filter. The experiments also showed that the filter with GT noise model could handle outlier better than the Kalman filter. This is the first time that GT distribution is used to model the noise in a realistic two-dimensional indoor tracking problem. In summary, the contributions are

- Robust and accurate indoor tracking: We proposed a novel filtering approach based on Generalized t -distribution noise model which improves the accuracy of the tracking system compared with the conventional Kalman filter. We demonstrated good performance even in the presence of outliers.
- Theoretical verification: We derived the equation to compute the variance of the estimation error. The variance equation can be used as an analytical tool for designing and assessing the tracking system. We verified the equation through experimental results.
- Real-environment validation: The experiments were conducted in two different indoor environments: office and lift landing. In both cases, the proposed filter with GT noise model gives a smaller variance for the estimation error than the Kalman filter.

The rest of the paper is organized as follows. Section II introduces the weighted centroid formulas and the path loss model. Section III describes the filter with GT noise model. Experimental results are given in Section IV and conclusions in Section V.

Fig. 1. The histogram of the error distribution of 3000 x-position estimates, z_1 , of a target obtained from the weighted centroid formula of Eq. (1). The maximum likelihood fit of GT ($p_1 = 2$, $v_1 = 1.5$, $\sigma_1 = 0.42$) and Gaussian (standard deviation = 0.44) distributions are given by the solid and dashed line respectively.

II. THE WEIGHTED CENTROID METHOD

Weighted centroid localization requires only simple calculations. Receivers are placed at known positions. The target broadcasts a unique identifier and the RSS values at the receivers are transformed into respective target-receiver distances through a path loss model. The distances are then employed as weights to estimate the target position as the weighted centroid [12], [34].

Consider m RSS values. The x-position and y-position of the target denoted by z_1 and z_2 respectively are estimated as the weighted centroid:

$$z_1 = \frac{\sum_{i=1}^m w(i) x_r(i)}{\sum_{i=1}^m w(i)} \quad (1)$$

$$z_2 = \frac{\sum_{i=1}^m w(i) y_r(i)}{\sum_{i=1}^m w(i)} \quad (2)$$

$$w(i) = D(i)^{-g} \quad (3)$$

where $w(i)$, $x_r(i)$ and $y_r(i)$ are the weight, x-position and y-position coordinates of the receiver with the i^{th} RSS and $g > 0$ is an empirical tuning parameter that controls the relative weight of the receivers.

The target-receiver distance $D(i)$ can be obtained from the path loss model. One commonly used path loss model for indoor localization is the International Telecommunication Union (ITU) indoor propagation model [35]–[37]. For 2.4 GHz single floor applications, it is given as

$$RSS = RSS_0 + 10\eta \log_{10}(D) \quad (4)$$

where RSS is in dBm and D in meters. The reference coefficient, RSS_0 , and the path loss exponent, η , can be determined through calibration.

Note that the procedure of calibration is different from the concept of site survey in fingerprinting. What we need to know is the relationship between distance and signal strength as shown in Fig. 2, instead of the signal characteristic over all the places of interest. Additionally, all the receivers can be calibrated simultaneously, saving labor cost and time.

Fig. 2. The path loss model (solid-line) is obtained by fitting the RSS in the least-squares sense. The RSS is given by the $\square$.

III. FILTERING WITH GT NOISE MODEL

The positions z_1 and z_2 estimated from the weighted centroid formulas of Eqs. (1) and (2) are highly corrupted by noise and it is necessary to pass them through a filter.

A. The Auto-Regressive-Integrated-Moving-Average and Constant-Velocity Model

Consider

$$A(q^{-1})z(k) = C(q^{-1})\varepsilon(k) \quad (5)$$

where $k = 1, 2, \dots, N$ is the sampling instance, q^{-1} is the backward shift operator and

$$\begin{aligned} A(q^{-1}) &= A_0 + A_1 q^{-1} + A_2 q^{-2} \\ C(q^{-1}) &= C_0 + C_1 q^{-1} + C_2 q^{-2} \\ A_0 &= A_2 = C_0 = I \\ A_1 &= -2I \end{aligned}$$

This is the well-known second-order multivariate Auto-Regressive-Integrate-Moving-Average (ARIMA) model which, in state-space form, is the so-called constant-velocity model commonly used in tracking problems [38]–[40]. Equation (5) in the state-space form is

$$x(k+1) = \Phi_A x(k) + \Omega \varepsilon(k) \quad (6)$$

$$z(k) = H x(k) + \varepsilon(k) \quad (7)$$

where

$$\begin{aligned} x(k) &= [x_1(k) \ x_2(k) \ x_3(k) \ x_4(k)]^T \\ \varepsilon(k) &= [\varepsilon_1(k) \ \varepsilon_2(k)]^T, z(k) = [z_1(k) \ z_2(k)]^T \\ \Phi_A &= \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \\ \Omega &= \begin{bmatrix} C_1(1,1)+2 & C_1(1,2) \\ C_1(2,1) & C_1(2,2)+2 \\ \frac{C_1(1,1)+C_2(1,1)+1}{T} & \frac{C_1(1,2)+C_2(1,2)}{T} \\ \frac{C_1(2,1)+C_2(2,1)}{T} & \frac{C_1(2,2)+C_2(2,2)+1}{T} \end{bmatrix} \end{aligned} \quad (8)$$

and T is the sampling interval. The states $x_1(k)$ and $x_2(k)$ are the positions in x and y directions; $x_3(k)$ and $x_4(k)$ are the speeds in x and y directions. The noise input, $\varepsilon(k)$, in the constant-velocity model of Eq. (6) allows for motion uncertainty and hence speed variation [40]. The measurements $z_1(k)$ and $z_2(k)$ are the estimated positions from the weighted centroid formulas of Eqs. (1) and (2).

Let the two independent noises $\varepsilon_1(k)$ and $\varepsilon_2(k)$ be modeled by the zero-mean GT probability density functions.

$$f_i(\varepsilon_i) = \frac{p_i}{2\sigma_i v_i^{1/p_i} \beta(1/p_i, v_i) \left(1 + \frac{|\varepsilon_i|^{p_i}}{v_i \sigma_i^{p_i}}\right)^{v_i+1/p_i}} \quad (9)$$

where σ_i is the scale parameter, p_i and v_i are the shape parameters, $i = 1, 2$. The beta function is given by $\beta(a, b) = \int_0^1 \theta^{a-1} (1-\theta)^{b-1} d\theta$. By different choices of p_i and v_i , the GT distribution can represent a wide range of distributions that one commonly meets in practice as shown in Fig. 3.

Fig. 3. Different well-known distributions can be obtained by different choices of the GT distribution shape parameters p and v [24].

Substituting Eq. (7) into Eq. (6) gives

$$x(k+1) = \Phi x(k) + \Omega z(k) \quad (10)$$

where

$$\begin{aligned} \Phi &= \Phi_A - \Omega H \\ &= \begin{bmatrix} -(C_1(1,1)+1) & -C_1(1,2) & T & 0 \\ -C_1(2,1) & -(C_1(2,2)+1) & 0 & T \\ -\frac{C_1(1,1)+C_2(1,1)+1}{T} & -\frac{C_1(1,2)+C_2(1,2)}{T} & 1 & 0 \\ -\frac{C_1(2,1)+C_2(2,1)}{T} & -\frac{C_1(2,2)+C_2(2,2)+1}{T} & 0 & 1 \end{bmatrix} \end{aligned} \quad (11)$$

Iterating from the initial value $x(1)$, Eqs. (7) and (10) give

$$\begin{aligned} x(2) &= \Phi x(1) + \Omega z(1) \\ x(3) &= \Phi^2 x(1) + \Phi \Omega z(1) + \Omega z(2) \\ &\vdots \\ x(N) &= \Phi^{N-1} x(1) + \bar{x}(N) \end{aligned} \quad (12)$$

$$z(N) = H \Phi^{N-1} x(1) + H \bar{x}(N) + \varepsilon(N) \quad (13)$$

where

$$\bar{x}(N) = \sum_{k=1}^{N-1} \Phi^{k-1} \Omega z(N-k) \quad (14)$$

B. Maximum Likelihood Estimation

Given N measurements $z(k)$, $k = 1, \dots, N$, and using Eq. (13), the noise vectors $\varepsilon(k)$ can be expressed as

$$\varepsilon(k) = z(k) - H\Phi^{k-1}x(1) - H\bar{x}(k) \quad (15)$$

The initial condition, $x(1)$, can be estimated by minimizing the following maximum likelihood objective function

$$J = - \sum_{i=1}^2 \sum_{k=1}^N \ln f_i(\varepsilon_i(k)) \quad (16)$$

where $f_i(\cdot)$ is the GT probability density function given by Eq. (9) and $\varepsilon_i(k)$ is the i^{th} element of $\varepsilon(k)$ given by Eq. (15). Let $\psi(\varepsilon)$ be the partial derivative of J with respect to $x(1)$. It follows from Eq. (16) that

$$\begin{aligned} \psi(\varepsilon) &= \frac{\partial J}{\partial x(1)} \\ &= - \sum_{i=1}^2 \sum_{k=1}^N \frac{\partial \varepsilon(k)}{\partial x(1)} \frac{\partial f_i(\varepsilon_i(k))}{\partial \varepsilon(k)} \frac{\partial \ln f_i(\varepsilon_i(k))}{\partial f_i(\varepsilon_i(k))} \end{aligned} \quad (17)$$

$$= - \sum_{k=1}^N (H\Phi^{k-1})^T \begin{bmatrix} \frac{(p_1 v_1 + 1) \varepsilon_1(k) |\varepsilon_1(k)|^{p_1-2}}{v_1 \sigma_1^{p_1} + |\varepsilon_1(k)|^{p_1}} \\ \frac{(p_2 v_2 + 1) \varepsilon_2(k) |\varepsilon_2(k)|^{p_2-2}}{v_2 \sigma_2^{p_2} + |\varepsilon_2(k)|^{p_2}} \end{bmatrix} \quad (18)$$

and the maximum likelihood estimate of $x(1)$ is obtained as the solution of the equation

$$\psi(\varepsilon) = 0 \quad (19)$$

Denoting the estimate of $x(1)$ at the N^{th} sample as $\hat{x}(1|N)$, Eq. (19) can be solved for $\hat{x}(1|N)$ numerically for example using the Newton-Raphson algorithm. Once $\hat{x}(1|N)$ is solved, $\hat{x}(N|N)$ the estimate of $x(N)$ at the N^{th} sample can be obtained from Eq. (12). However, unlike recursive algorithms such as the recursive least-squares estimator, solving Eq. (19) numerically using iterative methods is not suitable for real-time applications such as indoor tracking.

C. Influence Function Approximation

The influence function $IF(\varepsilon)$ [23], [41] can be used to solve Eq. (19) approximately. The solution is given in [42] as

$$\begin{aligned} \hat{x}(1|N) &= IF(\varepsilon) \\ &= - \left(\int_{-\infty}^{+\infty} \frac{\partial \psi(\varepsilon)}{\partial x(1)} dF(\varepsilon) \right)^{-1} \psi(\varepsilon) \end{aligned} \quad (20)$$

where $dF(\varepsilon) = f_1(\varepsilon_1) f_2(\varepsilon_2) d\varepsilon_1 d\varepsilon_2$.

Remark: We can interpret Eq. (20) intuitively. Consider the first-order Taylor series approximation of the nonlinear function $\psi(\varepsilon)$. If the approximation is taken about an operating point that is defined as zero then it is given by $\psi(\varepsilon) = \frac{\partial \psi(\varepsilon)}{\partial x(1)} x(1)$. If we do not consider the sign then taking expectation of $\frac{\partial \psi(\varepsilon)}{\partial x(1)}$ gives Eq. (20). A formal derivation is given in [23], [41], [42].

Differentiating $\psi(\varepsilon)$ in Eq. (18) with respect to $x(1)$ gives

$$\frac{\partial \psi(\varepsilon)}{\partial x(1)} = \sum_{k=1}^N (H\Phi^{k-1})^T L H \Phi^{k-1} \quad (21)$$

where

$$L = \begin{bmatrix} l_1 & 0 \\ 0 & l_2 \end{bmatrix}$$

and for $i = 1, 2$

$$l_i = \frac{(p_i v_i + 1) [(p_i - 1) v_i \sigma_i^{p_i} - |\varepsilon_i|^{p_i}] |\varepsilon_i|^{p_i-2}}{(v_i \sigma_i^{p_i} + |\varepsilon_i|^{p_i})^2}$$

Substituting Eq. (21) into Eq. (20) gives

$$\begin{aligned} \hat{x}(1|N) &= \left(\sum_{k=1}^N (H\Phi^{k-1})^T S H \Phi^{k-1} \right)^{-1} \\ &\times \left(\sum_{k=1}^N (H\Phi^{k-1})^T S W(k) \right) \end{aligned} \quad (22)$$

where

$$\begin{aligned} W(k) &= \begin{bmatrix} w_1(k) \\ w_2(k) \end{bmatrix} \\ S &= \begin{bmatrix} s_1 & 0 \\ 0 & s_2 \end{bmatrix} \end{aligned}$$

and for $i = 1, 2$

$$w_i(k) = \frac{(p_i v_i + 1) \varepsilon_i(k) |\varepsilon_i(k)|^{p_i-2}}{v_i \sigma_i^{p_i} + |\varepsilon_i(k)|^{p_i}} \times \frac{1}{s_i}$$

$$s_i = \int_{-\infty}^{+\infty} \frac{(p_i v_i + 1) [(p_i - 1) v_i \sigma_i^{p_i} - |\varepsilon_i|^{p_i}] |\varepsilon_i|^{p_i-2}}{(v_i \sigma_i^{p_i} + |\varepsilon_i|^{p_i})^2} f_i(\varepsilon_i) d\varepsilon_i$$

Once $\hat{x}(1|N)$ is found, $\hat{x}(N|N)$ can be obtained from Eq. (12) as

$$\hat{x}(N|N) = \Phi^{N-1} \hat{x}(1|N) + \bar{x}(N) \quad (23)$$

D. The Recursive Algorithm with Forgetting-Factor

Notice that Eq. (22) is the weighted least-squares solution $\hat{x}(1|N)$ from the minimization of the weighted least-squares loss function

$$\begin{aligned} V &= \frac{1}{2} \sum_{k=1}^N (W(k) - H\Phi^{k-1} \hat{x}(1|N))^T \\ &\times S (W(k) - H\Phi^{k-1} \hat{x}(1|N)) \end{aligned} \quad (24)$$

In other words, through influence function approximation, the maximum likelihood objective function in Eq. (16) can be approximated by the weighted least-squares objective function in Eq. (24).

In the loss function of Eq. (24), it is common to reduce the influence of old data by introducing a forgetting-factor as follows

$$\begin{aligned} V &= \frac{1}{2} \sum_{k=1}^N \lambda^{N-k} (W(k) - H\Phi^{k-1} \hat{x}(1|N))^T \\ &\times S (W(k) - H\Phi^{k-1} \hat{x}(1|N)) \end{aligned} \quad (25)$$

The forgetting-factor, $\lambda \leq 1$, is a measure of how fast old data are forgotten. When the loss function of Eq. (25) is used to obtain the least squares estimate, Eq. (22) is changed to

$$\hat{x}(1|N) = P(1|N) \left(\sum_{k=1}^N (H\Phi^{k-1})^T \lambda^{N-k} S W(k) \right) \quad (26)$$

TABLE I
THE FILTER WITH GT NOISE MODEL. THE OUTPUT IS $\hat{x}(1|N)$

$$\begin{aligned}
 P(1|N) &= [P(1|N-1) - K(N)H\Phi^{N-1}P(1|N-1)]\frac{1}{\lambda} \quad (28) \\
 K(N) &= P(1|N-1)(H\Phi^{N-1})^T \\
 &\quad \times [\lambda S^{-1} + H\Phi^{N-1}P(1|N-1)(H\Phi^{N-1})^T]^{-1} \quad (29) \\
 \hat{x}(1|N) &= \hat{x}(1|N-1) \\
 &\quad + K(N)[W(N) - H\Phi^{N-1}\hat{x}(1|N-1)] \quad (30)
 \end{aligned}$$

where

$$P(1|N) = \left(\sum_{k=1}^N (H\Phi^{k-1})^T \lambda^{N-k} S H \Phi^{k-1} \right)^{-1} \quad (27)$$

and the recursive solution in Table I is given in many textbooks that discuss least-squares [43].

Table I gives $\hat{x}(1|N)$, the estimate of $x(1)$ at sample N . The filtering problem is to produce $\hat{x}(N|N)$, the estimate of $x(N)$ at sample N . This can be easily done by iterating from $\hat{x}(1|N)$ to $\hat{x}(N|N)$ using Eq. (12). The results are given in Table II. For easy reference, the derivation is given in Appendix A. For initialization, $P(1|0)$ can be set as an identity matrix multiplied by some large number.

E. The Kalman Filter Connection

Fig. 3 shows that the GT distribution reduces to the Gaussian distribution when $p = 2$, $v = \infty$. Furthermore, it is well-known that Gaussian noise is assumed in the Kalman filter. The connection between the proposed filter with GT noise model and the Kalman filter will be shown below.

If the Gaussian instead of GT distribution noise is assumed in the algorithm of Table II then it will be reduced to the Kalman filter in [43] which is given in Table III for easy reference. Let $p_i = 2$, $v_i = \infty$ and $\lambda = 1$. Firstly, notice that Eq. (39) gives $s_i = \frac{2}{\sigma_i^2}$, hence Eqs. (37) and (46) give $S^{-1} = \sigma^2$ and Eq. (34) is reduced to Eq. (45). Secondly, substituting Eqs. (11) and (34) into Eq. (33) gives Eq. (44). Thirdly, Eq. (38) gives $w_i(N) = \varepsilon_i(N)$ and Eqs. (35), (36) and (40) gives $\xi(N) = z(N)$. Substituting $\xi(N) = z(N)$ into Eq. (31) gives Eq. (42). Finally, substituting Eqs. (11) and (31) into Eq. (32) gives Eq. (43). Hence the algorithm in Table II is reduced to the Kalman filter in Table III.

IV. STATISTICAL ANALYSIS

The variance is an indication of precision and in this section we derive the equations for the expectation and variance of the estimation error.

A. Expectation

Consider the estimation error

$$\tilde{x}(N) = x(N) - \hat{x}(N|N) \quad (47)$$

Substituting Eqs. (12) and (23) into Eq. (47) gives

$$\tilde{x}(N) = \Phi^{N-1}x(1) - \Phi^{N-1}\hat{x}(1|N) \quad (48)$$

TABLE II
THE FILTER WITH GT NOISE MODEL. THE OUTPUT IS $\hat{x}(N|N)$

Filtered Estimate:

$$\hat{x}(N|N) = \hat{x}(N|N-1) + K_f(N)[\xi(N) - H\hat{x}(N|N-1)] \quad (31)$$

Predicted Estimate:

$$\hat{x}(N+1|N) = \Phi\hat{x}(N|N) + \Omega z(N) \quad (32)$$

Covariance Update:

$$P(N+1|N) = \Phi[I - K_f(N)H]P(N|N-1)\Phi^T \frac{1}{\lambda} \quad (33)$$

where

Kalman Gain:

$$K_f(N) = P(N|N-1)H^T[\lambda S^{-1} + HP(N|N-1)H^T]^{-1} \quad (34)$$

Transformed Output:

$$\xi(N) = W(N) + H\bar{x}(N) \quad (35)$$

$$W(N) = [w_1(N) \ w_2(N)]^T \quad (36)$$

$$S = \text{diag}(s_1, s_2) \quad (37)$$

$$w_i(N) = \frac{(p_i v_i + 1)\varepsilon_i(N)|\varepsilon_i(N)|^{p_i-2}}{v_i \sigma_i^{p_i} + |\varepsilon_i(N)|^{p_i}} \times \frac{1}{s_i} \quad (i = 1, 2) \quad (38)$$

$$s_i = \int_{-\infty}^{+\infty} \frac{(p_i v_i + 1)[(p_i - 1)v_i \sigma_i^{p_i} - |\varepsilon_i|^{p_i}]\varepsilon_i |^{p_i-2}}{(v_i \sigma_i^{p_i} + |\varepsilon_i|^{p_i})^2} f_i(\varepsilon_i) d\varepsilon_i \quad (39)$$

$$\varepsilon(N) = z(N) - H\bar{x}(N) \quad (40)$$

$$\bar{x}(N+1) = \Phi\bar{x}(N) + \Omega z(N) \quad (41)$$

TABLE III
THE KALMAN FILTER

Filtered Estimate:

$$\hat{x}(N|N) = \hat{x}(N|N-1) + K_f(N)[z(N) - H\hat{x}(N|N-1)] \quad (42)$$

Predicted Estimate:

$$\hat{x}(N+1|N) = \Phi_A \hat{x}(N|N-1) + K(N)[z(N) - H\hat{x}(N|N-1)] \quad (43)$$

Covariance Update:

$$P(N+1|N) = \Phi_A P(N|N-1)\Phi_A^T + R_1 - K(N)[HP(N|N-1)H^T + R_2]K^T(N) \quad (44)$$

Kalman Gain:

$$K_f(N) = P(N|N-1)H^T[R_2 + HP(N|N-1)H^T]^{-1} \quad (45)$$

where

$$R_1 = \Omega\sigma^2\Omega^T, \quad R_{12} = \Omega\sigma^2, \quad R_2 = \sigma^2, \quad \sigma = \text{diag}\left(\frac{\sigma_1}{\sqrt{2}}, \frac{\sigma_2}{\sqrt{2}}\right) \quad (46)$$

$$K(N) = [\Phi_A P(N|N-1)H^T + R_{12}] \times [HP(N|N-1)H^T + R_2]^{-1}$$

Substituting Eqs. (26) and (27) into Eq. (48) and then taking expectation yields

$$\begin{aligned}
 E \tilde{x}(N) &= E(\Phi^{N-1}x(1) - \Phi^{N-1}\hat{x}(1|N)) \\
 &= \Phi^{N-1}E x(1) - \Phi^{N-1}E \hat{x}(1|N) \\
 &= \Phi^{N-1}E x(1) \\
 &\quad - \Phi^{N-1} \left(\sum_{k=1}^N (H\Phi^{k-1})^T \lambda^{N-k} S H \Phi^{k-1} \right)^{-1} \\
 &\quad \times \left(\sum_{k=1}^N (H\Phi^{k-1})^T \lambda^{N-k} S E W(k) \right) \\
 &= 0 \quad (49)
 \end{aligned}$$

given the assumption that $x(1) = 0$ and according to Eq. (38) $W(k)$ depends on $\varepsilon_1(k)$ and $\varepsilon_2(k)$ which are zero-mean independent random variables.

B. Variance

Using Eqs. (48) and (49), the variance of the estimation error can be obtained as follows.

$$\begin{aligned} \text{Var } \hat{x}(N) &= E \hat{x}(N) \hat{x}^T(N) \\ &= E (\Phi^{N-1} \hat{x}(1|N)) (\Phi^{N-1} \hat{x}(1|N))^T \end{aligned} \quad (50)$$

Substituting Eq. (26) into Eq. (50) gives

$$\text{Var } \hat{x}(N) = \sum_{k=1}^N \lambda^{2(N-k)} M_N (H \Phi^{k-1})^T \Gamma (H \Phi^{k-1}) M_N^T \quad (51)$$

where

$$M_N = \Phi^{N-1} \left(\sum_{k=1}^N (H \Phi^{k-1})^T \lambda^{N-k} S H \Phi^{k-1} \right)^{-1} \quad (52)$$

$$\Gamma = \begin{bmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{bmatrix} \quad (53)$$

and for $i = 1, 2$

$$\gamma_i = \int_{-\infty}^{+\infty} \left(\frac{(p_i v_i + 1) \varepsilon_i |\varepsilon_i|^{p_i-2}}{v_i \sigma_i^{p_i} + |\varepsilon_i|^{p_i}} \right)^2 f_i(\varepsilon_i) d\varepsilon_i \quad (54)$$

The experimental verification of Eq. (51) is described in the next section.

V. EXPERIMENT

A. Hardware Implementation

The tracking system was implemented in a typical office environment. The photograph and the floor plan are shown in Figs. 4 and 5 respectively. The Texas Instrument CC2530 ZigBee Development Kit with 2.4 GHz omni-directional antenna was programmed to work as transducers. Nine receivers were placed in the office to form a rectangular grid covering the monitored area as shown in Figs. 4 and 5. The target broadcasts packages with a unique identifier to the receivers. The receivers then send the measured RSS together with their coordinates to a computer equipped with a CC2531 USB Dongle which then calculates the position of the target at $T = 0.3$ second interval. The block diagram of the proposed tracking system is shown in Fig. 6. The inputs to the system are the RSS. They are first converted into target-receiver distances, D , through the log-distance path loss model of Eq. (4). The distances are used as weights in the weighted centroid formula of Eqs. (1) and (2) to give $z(N)$ and subsequently through the proposed filter with GT noise model (Table II), the estimated state $\hat{x}(N|N)$.

Fig. 4. The receivers are numbered in the photograph.

Fig. 5. The receivers are numbered and represented by the ● in the floor plan. The estimates from the weighted centroid, Kalman filter and filter with GT noise model are given by the ×, Δ and □ respectively. The true path is given by the dotted-line.

Fig. 6. The tracking system.

B. Parameter Selection

Before the experiment, all the receivers were calibrated simultaneously with 1000 RSS measured at 7 different distances. The parameters in the path loss model of Eq. (4) were obtained from the experimental data using least-squares estimation giving $RSS_0 = -49.87$ dBm and $\eta = -2.11$. The achieved result is shown in Fig. 2, where the squares represent RSS measurements and the solid line refers to the path loss model obtained by least-squares fitting.

To obtain the GT parameters, 3000 initial x-position estimates, z_1 , of a target obtained from the weighted centroid

formula of Eq. (1) were collected and compared with true positions. The error (estimated position – true position) distribution is represented by the histograms in Fig. 1. The GT distribution of Eq. (9) was fitted to the noise. For simplicity, we fixed $p_1 = 2$, $v_1 = 1.5$ and then used maximum likelihood estimation to obtain $\sigma_1 = 0.42$. For the Gaussian distribution, a standard deviation of 0.44 was also obtained.

The quantile-quantile plot was used as an assessment of goodness of fit [44] and the results for both GT and Gaussian distributions are shown in Fig. 7. It is clear that the plot for the GT distribution follows the 45-degree reference line more closely, especially at the tails. This indicates that the GT distribution provides a more accurate representation of the actual noise distribution and better describes the heavy-tailed samples and outliers.

Fig. 7. The quantile-quantile plots for the Gaussian and GT distributions are given by the top and bottom graphs respectively. The quantile values of the 3000 samples are plotted against the theoretical values for the fitted distributions and the results are given by +. A 45-degree solid reference line is also plotted in each graph.

The fitted curves for both GT (solid-line) and Gaussian (dashed-line) distributions are also shown in Fig. 1. It can be seen that the GT distribution (solid-line) fits the histograms better than the Gaussian distribution (dashed line). Hence we can expect a more precise estimation result if the GT

distribution is used. For y-position, the same values were obtained for the GT and Gaussian distribution parameters.

The polynomial C was chosen as to give

$$\Omega = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \\ 0.08 & 0 \\ 0 & 0.08 \end{bmatrix}$$

in Eq. (8) to model the process noise. A detail discussion on the choice of the process noise model for tracking with the constant velocity model is given in [40]. The value of g in Eq. (3) was empirically determined as $g = 2.5$. The forgetting factor λ is problem dependent. For our application, we tested different values of λ in the range of 0.4 to 1 and 0.5 gave the smallest average distance error as defined in [35]. The filter with GT noise model was implemented according to Table II with $p_1 = p_2 = 2$, $v_1 = v_2 = 1.5$, $\sigma_1 = \sigma_2 = 0.42$. The Kalman filter was implemented according to Table II with $p_1 = p_2 = 2$, $v_1 = v_2 = \infty$, $\sigma_1 = \sigma_2 = 0.44\sqrt{2}$. Note that if $\lambda = 1$ was chosen then Tables II and III give the same estimation results for the Kalman filter.

C. Statistical Results

The variance equation (51) can be verified experimentally. Thirteen samples of x-position and y-position estimates were determined in an experiment as a target moved from point “a” to point “b” as shown in Fig. 5 and to obtain statistical results, the experiment was repeated 200 times. The computational time needed by Matlab 2016a on a standard laptop (Intel i5-3230M, 8GB RAM, 3.2 GHz) for both the filter with GT noise model and Kalman filter to process the data are 0.1402 s and 0.1264 s, respectively.

The variance of the estimation error (estimated position – true position) in the experimental results is shown in Fig. 8. It can be seen that variance calculated from Eq. (51) matched the variance from the experiment. A sample calculation for the variances at $N = 2$ using Eq. (51) is given in Appendix B.

As shown in Fig. 1, the GT distribution can model the noise more accurately than the Gaussian distribution used in the Kalman filter. Hence the variance of the estimation error of the filter with GT noise model is less than that of the Kalman filter. For large N , Fig. 8 shows that the variance of the estimation error from the filter with GT noise model is about half of the Kalman filter.

D. Outliers

A snapshot of the estimation errors for the 200 experiments at $N = 7$ is shown in Fig. 9. Notice that the quantile-quantile plot of Fig. 7 shows the GT distribution can model the outliers more accurately than the Gaussian distribution used in the Kalman filter. Hence in Fig. 9, for Experiments 2, 18, 86, 150, 160 and 162, the estimates (triangles) from the Kalman filter are affected by the outliers (crosses) but the estimates (squares) from the filter with the GT noise model are not.

The target now moved along the path “abcd” as shown in Fig. 5. The estimates from the weighted centroid formulas of

Fig. 8. The variances of the estimation errors in the 200 experiments are given by $\square$ and $\triangle$ for the filter with GT noise model and Kalman filter respectively. Variances calculated from Eq. (51) for the filter with GT noise model and Kalman filter are given by $\times$ and $*$ respectively.

Eqs. (1) and (2), the Kalman filter and the filter with GT noise model are shown in Figs. 5 and 10 by the crosses, triangles and squares respectively.

In Fig. 5, large deviations of the weighted centroid estimates (crosses) from the true path (dotted-line) are observed. This is probably due to the wireless-unfriendly office environment where magnetic whiteboards are installed at every cubicles. Signal blockage, attenuation and multipath effect easily spoil the RSS measurements and hence proper filtering is needed.

Consider the x-position estimates in Fig. 10 (top). For the outliers (crosses) at $N = 10, 19, 22, 26, 33$ and 62 , the estimates (squares) from the filter with GT noise model are close to the true positions (circles) while the estimates (triangles) from the Kalman filter are faraway. Note that for $30 \leq N \leq 46$ the true x-position is constantly at 8 meters because the target was traveling along the path “bc” during that time.

E. Lift Landing Experiment

Another experiment was conducted at the lift landing and the floor plan is shown in Fig. 11. In Fig. 12 the parameters obtained by fitting the distribution to the noise using the maximum likelihood objective function are for the GT probability density function $p_1 = 2$, $q_1 = 1.5$, $\sigma_1 = 0.22$ and for the Gaussian probability density function, standard deviation of 0.28. The results of the estimates when the target moved along the path “ab” and path “bc” in Fig. 11 are shown in Fig. 13. It can be seen that the deviations of the weighted centroid estimates (crosses) from the true positions (circles) are larger along the path “bc” where there are six lifts with metal doors as shown in Fig. 14 which exacerbated the effect of multipath. Compared to the Kalman filter which assumed Gaussian noise, the filter with the GT noise model fitted to the actual noise distribution in Fig. 12 is less affected by the outliers at $N = 9$ and 32 in Fig. 13 (right).

Fig. 11. The floor plan for the lift landing experiment. The path “ab” and path “bc” are given by the solid and dashed line respectively. The receivers are represented by the $\bullet$.

Fig. 12. The histogram of the error distribution of the weighted centroid estimates collected in the lift landing experiment. The maximum likelihood fit of GT ($p_1 = 2$, $v_1 = 1.5$, $\sigma_1 = 0.22$) and Gaussian (standard deviation = 0.28) distributions are given by the solid and dashed line respectively.

Fig. 14. Photograph showing the path “bc” where six metal lift doors exacerbate the effect of multipath.

Fig. 9. A snapshot of the estimation error for the 200 experiments at $N = 7$. The estimates from the weighted centroid, filter with GT noise model and Kalman filter are given by $\times$, $\square$ and $\triangle$ respectively.

Fig. 10. Estimates for x-position and y-position as the target moved along the path “abcd” are given by the top and bottom graphs respectively. The true position, estimates from the weighted centroid, Kalman filter and filter with GT noise model are given by $\circ$, $\times$, $\triangle$ and $\square$ respectively. The filter with GT noise model was implemented according to Table II with $p_1 = p_2 = 2$, $v_1 = v_2 = 1.5$, $\sigma_1 = \sigma_2 = 0.42$. The Kalman filter was implemented according to Table II with $p_1 = p_2 = 2$, $v_1 = v_2 = \infty$, $\sigma_1 = \sigma_2 = 0.44\sqrt{2}$.

Fig. 13. Estimates for x-position as the target moved along the path “ab” and y-position as the target moved along the path “bc” are given by the left and right graphs respectively. The true position, estimates from the weighted centroid, Kalman filter and filter with GT noise model are given by $\circ$, $\times$, $\triangle$ and $\square$ respectively. The filter with GT noise model was implemented according to Table II with $p_1 = p_2 = 2$, $v_1 = v_2 = 1.5$, $\sigma_1 = \sigma_2 = 0.22$. The Kalman filter was implemented according to Table II with $p_1 = p_2 = 2$, $v_1 = v_2 = \infty$, $\sigma_1 = \sigma_2 = 0.28\sqrt{2}$.

VI. CONCLUSION

This paper makes use of the GT noise model to model the actual noise distribution instead of making the usual Gaussian noise assumption which may not be applicable to the indoor environment. Because of better noise modeling, more precise estimation results can be expected. An equation to compute the variance of the estimation error from the filter with GT noise model and the Kalman filter is also derived. The variance of the estimation error from the tracking experiments matched the variance calculated from the equation. This is useful as the equation can be used for the design and assessment of the indoor tracking system before implementation. The variance equation and experimental results also showed that the variance of the estimation error from the proposed filter with GT noise model is smaller than that of the Kalman filter. Hence the proposed filter with GT noise model produces a more precise estimate.

Another way to deal with non-Gaussian noise is to use the particle filter. However, this may come at the expense of heavy computational load. One approach that has been proposed for improving particle filtering is to combine it with another filter such as the extended kalman filter or the unscented kalman filter [45], [46]. For future work, we can combine the filter with GT noise model with particle filter just like the combination of Kalman filter and particle filter [47].

APPENDIX A

DERIVATION OF THE FILTER WITH GT NOISE MODEL AND FORGETTING-FACTOR IN TABLE II

Instead of estimating $\hat{x}(1|N)$ in Eq. (30) we can estimate $\hat{x}(N|N)$ directly as shown in Eq. (31). The recursive filter algorithm in Table II can be derived as follows.

Firstly, notice that Eq. (10) gives Eq. (32).

Secondly, multiplying Eq. (30) by Φ^{N-1} and then adding $\bar{x}(N)$ to both sides of the equation gives

$$\begin{aligned} & \Phi^{N-1}\hat{x}(1|N) + \bar{x}(N) \\ &= \Phi^{N-1}\hat{x}(1|N-1) + \bar{x}(N) + \Phi^{N-1}K(N) \\ & \quad \times [W(N) + H\bar{x}(N) - H\Phi^{N-1}\hat{x}(1|N-1) - H\bar{x}(N)] \end{aligned} \quad (55)$$

Using Eq. (12), Eq. (55) can be written as

$$\begin{aligned} \hat{x}(N|N) &= \hat{x}(N|N-1) + \Phi^{N-1}K(N) \\ & \quad \times [W(N) + H\bar{x}(N) - H\hat{x}(N|N-1)] \end{aligned} \quad (56)$$

Defining

$$K_f(N) = \Phi^{N-1}K(N) \quad (57)$$

in Eq. (56) gives Eqs. (31) and (35).

Thirdly, substituting Eq. (29) into Eq. (57) gives

$$\begin{aligned} K_f(N) &= \Phi^{N-1}P(1|N-1)(H\Phi^{N-1})^T \\ & \quad \times [\lambda S^{-1} + H\Phi^{N-1}P(1|N-1)(H\Phi^{N-1})^T]^{-1} \\ &= P(N|N-1)H^T \\ & \quad \times [\lambda S^{-1} + HP(N|N-1)H^T]^{-1} \end{aligned} \quad (58)$$

where

$$P(N|N-1) = \Phi^{N-1}P(1|N-1)(\Phi^{N-1})^T \quad (59)$$

Notice that Eq. (58) is also Eq. (34).

Finally, replacing N by $N+1$ in Eq. (59) gives

$$P(N+1|N) = \Phi^N P(1|N)(\Phi^N)^T \quad (60)$$

and substituting Eq. (28) into Eq. (60) gives

$$\begin{aligned} & P(N+1|N) \\ &= [\Phi P(N|N-1)\Phi^T - \Phi K_f(N)HP(N|N-1)\Phi^T] \frac{1}{\lambda} \\ &= \Phi[I - K_f(N)H]P(N|N-1)\Phi^T \frac{1}{\lambda} \end{aligned}$$

which is Eq. (33).

APPENDIX B SAMPLE CALCULATION

The integral in Eq. (54) can be viewed as an expectation and calculated as a sample average. For $p_1 = p_2 = 2$, $v_1 = v_2 = 1.5$, $\sigma_1 = \sigma_2 = 0.42$, $\lambda = 0.5$, Eq. (54) gives

$$\begin{aligned}\gamma_1 &= \int_{-\infty}^{\infty} \left(\frac{4\varepsilon_1}{0.265 + \varepsilon_1^2} \right)^2 f_1(\varepsilon_1) d\varepsilon_1 \\ &= \frac{1}{2600} \sum_{k=1}^{2600} \left(\frac{4\varepsilon_1(k)}{0.265 + \varepsilon_1(k)^2} \right)^2 \\ &= 8.706\end{aligned}$$

where $f_1(\varepsilon_1)$ is given by the empirical discrete distribution obtained from the 200 experiments $\times$ 13 samples = 2,600 data samples i.e. $\varepsilon_1(k)$ and $k = 1, 2, \dots, 2600$. Similar calculations for γ_2 gives

$$\Gamma = \begin{bmatrix} 8.706 & 0 \\ 0 & 8.692 \end{bmatrix}$$

for Eq. (53). For $N = 2$, Eq. (52) gives

$$\begin{aligned}M_2 &= \Phi (H^T \lambda S H + (H\Phi)^T S H \Phi)^{-1} \\ &= \begin{bmatrix} 0 & 0 & 0.441 & 0 \\ 0 & 0 & 0 & 0.441 \\ -0.637 & 0 & 2.957 & 0 \\ 0 & -0.637 & 0 & 2.957 \end{bmatrix}\end{aligned}$$

where from Eq. (39)

$$S = \begin{bmatrix} 7.559 & 0 \\ 0 & 7.559 \end{bmatrix}$$

Finally, Eq. (51) gives

$$\begin{aligned}\text{Var } \tilde{x}(2) &= \lambda^2 M_2 H^T \Gamma H M_2^T + M_2 (H\Phi)^T \Gamma (H\Phi) M_2^T \\ &= \begin{bmatrix} 0.152 & 0 & 0.508 & 0 \\ 0 & 0.152 & 0 & 0.507 \\ 0.508 & 0 & 2.577 & 0 \\ 0 & 0.507 & 0 & 2.573 \end{bmatrix}\end{aligned}$$

where the first two diagonal elements of 0.152 are the variances of the x-position and y-position estimation errors shown in Fig. 8 by the crosses at $N = 2$.

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