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Abstract

The use of bender elements to measure small-strain soil properties in the laboratory has become increasingly accessible in recent years. Coupled with the fact that bender elements can be incorporated into conventional test apparatuses such as the triaxial test brings about great savings in time and resources as the bender element tests can be conducted concurrently with conventional geotechnical test for the same soil specimen. While primary (P) and shear (S) waves in bender element tests can provide soil stiffnesses reliably, damping ratio of the soil was seldom determined. This paper attempts to show numerically and experimentally that it is possible to determine the damping ratio of soils by applying the Hilbert transform method (HTM) to the bender element test results.

Keywords: Bender element; damping ratio; soil; S-wave; Hilbert transform

1 Introduction

Determination of small-strain soil properties using bender element tests is becoming more common. One great advantage of bender elements is that bender elements can be incorporated into existing apparatuses such as the triaxial test apparatus. To date, bender elements test is frequently used to determine small-strain stiffness but not damping ratio.

The objective of this paper is to investigate the feasibility of determining damping ratio from bender element tests, numerically and experimentally using the Hilbert transform method (HTM). First, bender element tests were simulated using finite element modelling and the damping ratio determined using HTM. Second, results of bender element tests on standard Ottawa 20-30 sand were used to obtain its damping ratio under various confining pressures.

The damping ratios were compared with those from the literature obtained using the resonant column apparatus.

2 Determination of Damping Ratio

There are several methods of determining damping ratio from a seismic trace. One of the first used in the field of geophysics is the spectral ratio method (SRM) [1]. The SRM when applied to bender element tests has two configurations and involves the comparisons of the signals obtained from two specimens. The first configuration makes use of a reference specimen of similar dimensions and known damping ratio [2-5]. The second configuration involves using two identical specimens but of different heights [6-8]. The damping ratio can be obtained from either configuration using the equation below:

$$\ln\left(\frac{A_1}{A_2}\right) = \ln\left(\frac{GF_1}{GF_2}\right) + \left(\frac{\pi x_2}{Q_2 V_2} - \frac{\pi x_1}{Q_1 V_1}\right)f \quad (1)$$

where subscripts 1 and 2 represents specimens 1 and 2, respectively, A is the amplitude of the wave, f is the frequency, x is the distance travelled by the wave, GF is a frequency independent geometrical factor which includes spreading, reflections etc., V is the wave velocity and Q is the quality factor which can be correlated to the damping ratio ξ using Equation 2.

$$\frac{1}{Q} = 2\xi \quad (2)$$

However, the spectral ratio method has various shortcomings. The first configuration of the spectral ratio method carries the implicit assumption that the frequency spectra of the reference specimen's signal and the tested specimen's signal have the same frequency range. Difference in specimens' stiffness will lead to mismatch in the frequency spectra leading to a large scatter in the damping ratio when using SRM [9]. The second configuration suffers

from practical limitation as it is almost impossible to obtain identical soil specimens of different heights unless they are reconstituted specimens.

Another widely popular method used to determine damping ratio is the Logarithmic Decrement Method (LDM). The LDM is utilised in resonant column tests to determine the damping ratio of soil specimens [10]. However, the LDM only works well in the resonant column test where the entire soil specimen was subjected to steady state vibration at its resonant frequency and the excitation was cut off to allow the free vibration decay curve to be obtained [10]. In the bender element test where only a small perturbation is introduced into the soil specimen, the transient nature of the propagating wave is easily affected by interference from reflected waves. This interference give rises to irregularities which cannot be removed via digital signal processing and would thus affect the application of the LDM. Moreover, the number of decay cycles is usually insufficient to apply LDM reliably [11]. In view of the limitations of bender element test, the HTM is able to provide more reliable results than LDM as demonstrated in the finite element simulations and experimental results presented.

The Hilbert transform method (HTM) is conceptually similar to the LDM and was first used by Agneni and Balis-Crema [12] to derive damping ratios of composite materials using free vibration decay data. The Hilbert transform is an operator which convolutes a signal by $1/\pi x$. In other words, it is a filter which transforms the signal by shifting their phases by $\pm \pi/2$ while maintaining the magnitudes of their respective spectral components. In most cases, determination of the dynamic response of engineering structures involves measuring vibration responses subjected to random wind and other background vibrations. These engineering structures respond in different modes of vibration. Determination of damping

ratio thus requires the use of the ‘empirical mode decomposition’ method [13] to decompose complicated signals into their respective modal components to yield well-behaved Hilbert transforms [13-19]. However, in bender element test where a single sinusoidal pulse is introduced and the receiver bender element records the free vibration decay in a known mode (flexural for shear wave), such decomposition of the signal is not required.

In HTM, if $x(t)$ is the time domain signal (Equation 3a) and $x^H(t)$ is the Hilbert transform of the time domain signal (Equation 3b), the combination will give the analytic signal $x_a(t)$ (Equation 3c) [14, 19].

$$x(t) = Ae^{-\xi\omega_n t} \sin(\omega_n t \sqrt{1-\xi^2}) \quad (3a)$$

$$x^H(t) = Ae^{-\xi\omega_n t} \cos(\omega_n t \sqrt{1-\xi^2}) \quad (3b)$$

$$x_a(t) = x(t) - ix^H(t) \quad (3c)$$

where A is the amplitude of the signal, ξ is the damping ratio, ω_n is the natural frequency in radians, t is the time in seconds and i is the imaginary number. The magnitude of the analytic signal gives the time signal $x_a(t)$ and is shown in Equation 4.

$$|x_a(t)| = Ae^{-\xi\omega_n t} \quad (4)$$

From Equation (4), the damping ratio ξ can be separated as shown in Equation (5).

$$\ln(|x_a(t)|) = \ln(A) - (\xi\omega_n)t \quad (5)$$

Equation 5 shows that the gradient from the plot of $\ln(|x_a(t)|)$ with t used to derive the damping ratio has to be negative. The damping ratio can be derived from the gradient m ($=\xi\omega_n$) as shown in Equation 6. When considering the time window to obtain the gradient, it

is important to avoid both ends of the analytic signal which can be distorted by the Hilbert transform [20, 21].

$$\xi = \frac{m}{\omega_n} = \frac{m}{2\pi f_n} \quad (6)$$

where m is the gradient and f_n is the natural frequency of the signal.

Equation 6 shows that in order to obtain the damping ratio, the natural frequency of the signal will be necessary. The situation is not straight-forward as the signal detected in bender element test is a broad frequency band signal and the natural frequency is affected by the soil properties [22]. Lee and Santamarina [23] showed that the first resonant frequency (f_r) of an equivalent bender element–soil system can be obtained as:

$$f_r = \frac{1}{2\pi} \sqrt{\frac{1.875^4 \frac{E_b I}{(\alpha L_b)} + 2\eta V_s \rho_s (1+\nu) L_b}{\rho_b b h (\alpha L_b) + (\rho_s b^2 L_b) \chi}} \quad (7)$$

where L_b is length of the bender element, E_b is the Young's modulus of the bender element, α is an effective length factor ($\alpha = 1$ for perfectly fixed base condition and $\alpha > 1$ for flexible base condition), ρ_b is the mass density of the bender element, I is the area moment of inertia of the bender element ($I = bh^3/12$), b is the width of the bender element, h is the thickness of the bender element, V_s is the S-wave velocity of the soil, ρ_s is the soil density, ν is the Poisson's ratio of the soil, χ is an experimental factor related to the volume of soil mass affecting the vibration of the bender element, and $\eta \approx 2$ is the mean displacement influence factor at the soil–element interface. To use Equation 7, α needs to be estimated by measuring the resonant frequency of the bender elements in air and χ needs to be obtained iteratively by comparing the estimated values from Equation 7 and the measured resonant frequencies for different confining pressures (i.e. different shear wave velocities). However, this can be

avoided by calculating a representative natural frequency f_n from the second moment of the power spectrum following Barnes [24] after filtering as shown in Equation 8.

$$f_n = \sqrt{\frac{\int_0^{f_e} f^2 P(f) df}{\int_0^{f_e} P(f) df}} \quad (8)$$

where f_e is the frequency corresponding to the upper limit of the frequency band and $P(f)$ is the ordinate of the power spectrum.

Hence unlike LDM, HTM uses the gradient from the plot of $\ln(|x_a(t)|)$ with t to determine damping ratio from the free vibration decay and thus overcomes the limitations of insufficient decay cycles and interference from noise and reflected waves.

3 Finite Element Simulations

The finite element (FE) program LS-DYNA [25] was used to determine if the input damping ratio can be recovered from the bender element test using HTM. The soil is modelled as a viscoelastic material (MAT_006). The soil specimen was modelled as a 50 mm diameter and 50 mm height three-dimensional cylindrical model with 35000 elements (FIG. 1) with the bottom face fixed. Excitation of the bender element was simulated by applying a sinusoidal horizontal displacement to a row of five nodes, corresponding to 5.3 mm, at the middle of the top surface of the FE model. The responses at a node in the middle and 4 mm from the bottom end of the FE model were analysed. Excitation frequencies of 3, 5 and 10 kHz were used. The input parameters for the viscoelastic material (MAT_006) are tabulated in Table 1.

The relationship between β and ξ for the viscoelastic model obtained through numerical simulations of a resonant column test [26] is

$$\xi = 0.4249 \frac{\beta}{2\pi f_n} \quad (9)$$

The soil cylinder model was first subjected to an implicit eigenvalue analysis to obtain the natural frequencies of the model followed by an explicit analysis to obtain the time domain signal for the HTM analysis. The soil cylinder has natural frequencies of 0.983 kHz and 1.413 kHz for the first and second modes of vibration.

The signal obtained from the bender elements test is different from that of the resonant column test which shows free vibration decay after excitation of the soil specimen is cut off upon reaching the resonant frequency. In the bender element test, the receiver signal will contain a transient part which is caused by the initial excitation pulse and a free vibration decay part [27]. A typical receiver signal from the experiments is shown in FIG. 2. Oscillations in the free vibration decay correspond to the natural frequency of the specimen. The natural frequency obtained from the implicit eigenvalue analysis would indicate the plausible natural frequency to be used for HTM.

Typical receiver signals from the FE explicit analyses are shown in Figs. 3a and b. The receiver signals in the frequency domain are shown in Fig. 3c. The first peak frequency in Fig. 3c is between the first and second mode natural frequencies of the soil cylinder model (0.983 kHz and 1.413 kHz, respectively) obtained from implicit eigenvalue analyses. Hence the signals were filtered using a bandpass to retain the peak at the natural frequency of 1 kHz. The filtered signals are shown in Figs. 3a and b. A Hilbert transform function was then used to convert the filtered signal into the analytic signal.

The natural logarithm of the analytic signal with time is shown in Fig. 3d. Magalas and Majewski [21] show that the true decay envelope coincides with the Hilbert transform envelope at the central portion of the plot of natural logarithm of the analytic signal with time. The damping ratio is calculated using the gradient in Fig. 3d and Equation 5. Comparison of the input damping ratio and the damping ratio derived from HTM is summarized in Table 2.

Differences between the input damping ratio and those derived from HTM are within $\pm 10\%$. Fig. 2 and Table 2 further show that using excitation frequency of 3 kHz gave a clearer receiver signal and closer agreement between the computed and input damping ratio compared to excitation frequencies of 5 and 10 kHz. It is surmised that the better agreement in damping ratios is due to the excitation frequency being slightly above the natural frequency of the soil cylinder model (≈ 1 kHz).

The finite element simulations described above modelled the soil as a continuum, similar to Arroyo et al. [28], were able to capture the important phenomenon of wave reflections at the model boundaries but not wave reflections at the particle contacts which can be capture if the simulations was performed using the discrete element method [29]. Nonetheless, the general observation is that numerical simulations results in signals that are less noisy then in the physical experiments [29].

4 Verification

4.1 Material used and specimen preparation

Bender element tests were conducted on Ottawa 20-30 sand specimens to verify that the HTM can be applied reliably in experiments. Ottawa 20-30 sand was chosen as there is abundant damping ratio data from resonant column tests in the literature (Table 3). The bender element tests were conducted in a triaxial apparatus and the set-up has been described in Leong et al. [37] and Cheng and Leong [38]. The Ottawa 20-30 sand conforming to ASTM C778-13 [39] was prepared as 50 mm diameter and 100 mm high dry soil specimens at void ratio of 0.55. Void ratio of 0.55 was chosen as it is the average void ratio in Table 3. Confining pressures of 50, 100, 200, 400 and 800 kPa were used which are within the range of confining pressures used in the tests listed in Table 3. The bender element was excited using a sinusoidal pulse at frequencies of 3, 5, and 10 kHz. Signals were recorded at a sampling frequency of 2 MHz for duration of 5 ms.

4.2 Signal processing for HTM

The receiver signal was first filtered using a fourth order Butterworth bandpass filter to retain the portion containing the natural frequency and then processed as previously described to obtain the analytic signal with time. Typical raw and filtered signals from different excitation frequencies are plotted in time and frequency domains in Fig. 4. The natural frequency of the Ottawa 20-30 sand specimen lies between 1 and 4 kHz indicated by the twin peaks in Fig. 4b which shows the frequency domain of the receiver signal at excitation frequency of 3 kHz. Peaks at higher frequencies (>15 kHz) are attributed to interference from waves reflected

from the specimen's boundary. Figs 4e and 4h show that at higher excitation frequencies of 5 and 10 kHz, respectively, the twin peaks present between 1 to 4 kHz have substantially lower magnitude. The experimental observation agrees with those from the finite element simulations.

The troughs observed in Figs. 4c, 4f and 4i are due to irregularities from destructive interference of reflected waves. The peaks in the steady-state part of the signals in Figs. 3c, 3f and 3i were used to construct the best fit line to determine the gradient as shown. The representative natural frequency was calculated using Equation 8. Using the gradient and the natural frequency, the damping ratio was obtained using Equation 6.

4.3 Comparison of damping ratios

The damping ratios (ξ_s) of dry Ottawa sand determined using the resonant column tests from the literature were compared with those obtained using the bender element tests by HTM in this study and shown in Fig. 5. Fig. 5 shows that the damping ratio generally decreases as the effective confining pressure increases. The range of values of the damping ratio obtained from bender element tests using HTM agrees well with those from the resonant column tests.

6 Conclusion

In this paper, it was demonstrated that damping ratio can be obtained from bender element tests numerically and experimentally using the Hilbert transform method. The Hilbert transform method is suggested as a more reliable and robust method compared to the logarithmic decrement method to determine damping ratio from bender element tests. The

procedures undertaken to process the raw signals for determination of damping ratio were elaborated. Good agreement was obtained between damping ratios of Ottawa 20-30 sand obtained from bender element tests using HTM and those reported in the literature from resonant column tests.

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References

- [1] Bath M. Spectral analysis in geophysics - Developments in solid earth geophysics. Elsevier Scientific Publishing Company, Amsterdam, The Netherlands; 1974.
- [2] Toksöz MN, Johnston DH, Timur A. Attenuation of seismic waves in dry and saturated rocks: I. Laboratory measurements. *Geophysics* 1979; 44(4): 681–690.
- [3] Johnston DH, Toksöz MN. Ultrasonic P and S wave attenuation in dry and saturated rocks under pressure. *Journal of Geophysical Research* 1980; 85(B2): 925–936.
- [4] Frempong P, Butt SD, Donald A. Frequency dependent spectral ratio technique for Q estimate. In: *Proceedings of Rainbow in the Earth – 2nd International Workshop*, Lawrence Berkeley National Laboratory, Berkeley, California; 2005. pp. 1–4.
- [5] Frempong P, Donald A, Butt, SD. The effect of pore pressure depletion and injection cycles on ultrasonic velocity and quality factor in a quartz sandstone. *Geophysics* 2007; 72(2): E43–E51.
- [6] Wang YH, Yan WM, Lo KF. Damping-ratio measurements by the spectral ratio method. *Canadian Geotechnical Journal* 2006; 43(11): 1180-1194.
- [7] Karl L, Haegeman W, Degrande G, Dooms D. Determination of the material damping ratio with the bender element test. *Journal of Geotechnical and Geoenvironmental Engineering* 2008; 134(12): 1743-1756.

- [8] Khan Z, Cascante G, and Naggar MHE. Measurement of dynamic properties of stiff specimens using ultrasonic waves. *Canadian Geotechnical Journal* 2011; 48(1): 1-15
- [9] Sams M, Goldberg D. The validity of Q estimates from borehole data using spectral ratios. *Geophysics* 1990; 55(1): 97-101.
- [10] ASTM Standard D4015-07. Standard test methods for modulus and damping of soils by resonant-column method, American Society for Testing and Materials, West Conshohocken, United States; 2007.
- [11] Cheng ZY, Leong EC. Effect of confining pressure and degree of saturation on damping ratios of sand, *International Conference on Unsaturated Soils: Research & Applications* 1, Sydney, Australia, pp. 227-232; 2014.
- [12] Agneni A, Balis-Crema L. Damping measurements by Hilbert transform on composite materials. In: *Damping Proceedings Volume 2*, Las Vegas, Nevada, United States; 1986: FI1–FI13.
- [13] Huang NE, Shen Z, Long SR, Wu MC, Shih HH, Zheng Q, Yen NC, Tung CC, Liu HH. The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis. *Proceedings of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences*; 1998, 454(1971): 903-995.
- [14] Salvino LW. Empirical mode analysis of structural response and damping. In: *Proceedings of the 18th International Modal Analysis Conference*, San Antonio, Texas, United States; 2000. pp. 503-509.
- [15] Zhang L, Tamura Y. Damping estimation of engineering structures with ambient response measurements. In: *Proceeding of the 21st International Modal Analysis Conference*, Kissimmee, Florida, United States; 2003.

- [16] Yang JN, Lei Y, Lin S, Huang N. Identification of natural frequencies and dampings of in situ tall buildings using ambient wind vibration data. *Journal of Engineering Mechanics* 2004; 130(5): 570-577.
- [18] Le TH, Tamura Y, Yoshida A. Modal parameter estimation from ambient data using time-frequency analysis. In: *Proceeding of the 24th International Conference of Noise and Vibration*, Leuven, Belgium; 2010. pp. 1-15.
- [19] Iglesias AM. Investigating various modal analysis extraction techniques to estimate damping ratio [Masters Thesis]. Virginia Polytechnic Institute and State University, United States; 2000.
- [20] Luo H, Fang X, Ertas B. Hilbert transform and its engineering applications. *American Institute of Aeronautics and Astronautics Journal* 2009; 47(4): 923-932.
- [21] Magalas LB, Majewski M. Hilbert-Twin—A novel Hilbert transform-based method to compute envelope of free decaying oscillations embedded in noise, and the logarithmic decrement in high-resolution mechanical spectroscopy HRMS. *Archives of Metallurgy and Materials* 2015; 60(2): 1091-1098.
- [22] Camacho-Tauta J, Cascante G, Viana da Fonseca A, Santos JA. Time and frequency domain evaluation of bender element systems. *Géotechnique* 2015; 65(7): 548–562.
- [23] Lee J-S, Santamarina JC. Bender elements: performance and signal interpretation. *Journal of Geotechnical and Geoenvironmental Engineering* 2005; 131(9): 1063–1070.
- [24] Barnes AE. Instantaneous spectral bandwidth and dominant frequency with applications to seismic reflection data. *Geophysics* 1993; 58(3): 419-428.
- [25] LSTC LS-DYNA User Manual R7.0. Livermore Software Technology Corporation, California, United States; 2013.
- [26] Cheng Z, Leong EC. Finite element simulations of wave propagation in soils using a viscoelastic model. *Soil Dynamics and Earthquake Engineering* 2016; 88: 207-214

- [27] Rossing TD, Fletcher NH. Principles of Vibration and Sound 2nd Edition, Springer Science+Business Media, New York, United States; 2004.
- [28] Arroyo M, Muir Wood D, Greening PD, Medina L, Rio J. Effects of sample on bender-based axial G_0 measurements. *Géotechnique* 2006; 56(1): 39-52.
- [29] O'Donovan J, Ibraim E, O'Sullivan C., Hamlin S, Muir Wood D, Marketos G. Micromechanics of seismic wave propagation in granular materials. *Granular Matter* 2016; 18:56. DOI 10.1007/s10035-015-0599-4
- [30] Hall JR Jr. Effect of amplitude on damping and wave propagation in granular materials [PhD thesis]. University of Florida, Gainesville, Florida; 1962.
- [31] Hardin BO. The nature of damping in sands. *Journal of Soil Mechanics & Foundations Division* 1965; 91(SM1): 63-97.
- [32] Ray RP, Woods RD. Modulus and damping due to uniform and variable cyclic loading. *Journal of Geotechnical Engineering* 1988; 114(8): 861-876.
- [33] Shen CK, Li XS, Gu YZ. Microcomputer based free torsional vibration test. *Journal of Geotechnical Engineering* 1985; 111(8): 971-986.
- [34] Maher MH, Ro KS, Welsh JP. High strain dynamic modulus and damping of chemically grouted sand. *Soil Dynamics and Earthquake Engineering* 1994; 13(2): 131-138.
- [35] Hardin KO, Drnevich VP, Wang J, Sams CE. Resonant column testing at pressures up to 3.5 MPa (500 psi). *Dynamic Geotechnical Testing II* 1994; STP 1213: 222-233.
- [36] Zhang XJ, Aggour MS. Damping determination of sands under different loadings. In: *Proceedings of the 11th World Conference on Earthquake Engineering, Acapulco, Mexico*; 1996.
- [37] Leong EC, Cahyadi J, Rahardjo H. Measuring shear and compression wave velocities of soil using bender-extender elements. *Canadian Geotechnical Journal* 2009; 46(7): 792-812.

- [38] Cheng Z, Leong EC. A hybrid bender element–ultrasonic system for measurement of wave velocity in soils. *Geotechnical Testing Journal* 2014; 37(3): 1-12
- [39] ASTM Standard C778-13 Standard Specification for Standard Sand, American Society for Testing and Materials, West Conshohocken, United States; 2013.