Energy and spectral efficiency of leakage-based precoding for large-scale MU-MIMO systems

Tran, Tuong Xuan; Teh, Kah Chan

2015

Tran, T. X., & Teh, K. C. (2015). Energy and Spectral Efficiency of Leakage-Based Precoding for Large-Scale MU-MIMO Systems. IEEE Communications Letters, 19(11), 2041-2044.

<https://hdl.handle.net/10356/82923>

<https://doi.org/10.1109/LCOMM.2015.2475757>

© 2015 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collective works, for resale or redistribution to servers or lists, or reuse of any copyrighted component of this work in other works. The published version is available at: [<http://dx.doi.org/10.1109/LCOMM.2015.2475757>].

Downloaded on 26 Apr 2023 20:25:44 SGT

Energy and Spectral Efficiency of Leakage-Based Precoding for Large-Scale MU-MIMO Systems

Tuong Xuan Tran, *Student Member, IEEE*, and Kah Chan Teh, *Senior Member, IEEE*

Abstract—We study the capacity performance of signal-to-leakage-and-noise ratio precoding scheme (SLNR-PS) for large-scale multiuser multiple-input multiple-output systems. We derive capacity bounds of SLNR-PS when the number of base station antennas N is large. A trade-off between energy efficiency (EE) and spectral efficiency (SE) is defined for the case of equal power allocation. Based on this trade-off, the closed-form expressions of optimal SE and optimal number of BS antennas for maximizing EE are derived, respectively. In addition, we consider the EE optimization problem under both maximum transmit power and quality of service constraints. The energy-efficient power allocation scheme is obtained to solve this optimization problem. Numerical simulations verify the benefits of the proposed scheme.

Index Terms—Massive MIMO system, SLNR-based scheme.

I. INTRODUCTION

LINEAR precoding techniques including zero-forcing (ZF), minimum mean square error, block diagonalization and maximizing signal-to-leakage-and-noise ratio (SLNR) have been considered as practical solutions for multiuser multiple-input multiple-output (MU-MIMO) communication systems [1]. The SLNR-based scheme does not require any dimension condition on the base station (BS) antennas. Moreover, it also takes into account the influence of noise when determining the beamforming vectors for all users. Recently, it has been shown that the large-scale MIMO (LS-MIMO) systems can greatly improve system performance and reliability [2]. Motivated by these advantages, we analyze the energy efficiency (EE) and spectral efficiency (SE) of the SLNR precoding scheme (SLNR-PS) for LS-MIMO systems.

In [3]–[5], the energy efficiencies of the uplink and downlink LS-MIMO systems have been studied based on simulated results only. Thus, the trade-off between SE and EE has not been analyzed. In [3] and [4], the authors adopted a power consumption model (PCM) that only considers the emitted power consumption. However, for LS-MIMO systems, the emitted power consumption, the radio frequency circuit power consumption, as well as the power consumption for performing digital signal processing (DSP) should be considered [5], [6]. In this paper, we first derive capacity bounds of the SLNR-PS when the number of BS antennas is large. Applying the realistic PCM, we analyze the trade-off between SE and EE based on the upper bound (UB) on the system capacity at high signal-to-noise ratio (SNR) region for the case of equal power allocation (EPA). Based on this trade-off, the optimal value of SE and the

optimal number of BS antennas that can maximize the overall system EE are also presented, respectively. We obtain energy-efficient power allocation scheme to maximize the EE under the constraints of the transmit power (TP) and quality-of-service (QoS). In addition, the optimal rates of individual users are defined by solving the rate profile optimization problem under the constraints of the TP, QoS, and EE target.

II. SYSTEM MODEL

We consider a downlink MU-MIMO system which has one BS equipped with N transmit antennas and K single-antenna users. The received signal at user k is given by [7]

$$y_k = \sqrt{p_k} \mathbf{g}_k^H \mathbf{w}_k s_k + \sum_{i=1, i \neq k}^K \sqrt{p_i} \mathbf{g}_k^H \mathbf{w}_i s_i + n_k, \quad (1)$$

where s_k is the data symbol, p_k is the transmit power for user k with $\sum_k p_k = P$, and n_k is the noise term with $n_k \sim \mathcal{CN}(0, \sigma^2)$. The channel vector of user k is given by $\mathbf{g}_k = \mathbf{h}_k \sqrt{\beta_k}$, where $\mathbf{h}_k$ is the fast fading vector of user k with $\mathbf{h}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$; $\sqrt{\beta_k}$ is the large-scale fading coefficient. The beamforming vector $\mathbf{w}_k$ is defined as [1]

$$\mathbf{w}_k \propto \max \text{.eigenvector}(\mathbf{A}_k), \quad (2)$$

where $\|\mathbf{w}_k\|^2 = 1$, $\mathbf{A}_k = (\bar{\mathbf{G}}_k \bar{\mathbf{G}}_k^H + \frac{\sigma^2}{p_k} \mathbf{I})^{-1} \mathbf{g}_k \mathbf{g}_k^H$, and $\bar{\mathbf{G}}_k = [\mathbf{g}_1, \dots, \mathbf{g}_{k-1}, \mathbf{g}_{k+1}, \dots, \mathbf{g}_K]$, respectively [1]. The resulting maximum SLNR value φ_k is the largest eigenvalue λ_k of $\mathbf{A}_k$, which is obtained by solving the following characteristic equation [1], [7]

$$\det \left[\left(\mathbf{G}_k \mathbf{G}_k^H + \frac{\sigma^2}{p_k} \mathbf{I} \right) - \left(\frac{\lambda + 1}{\lambda} \right) \mathbf{g}_k \mathbf{g}_k^H \right] = 0, \quad (3)$$

where $\mathbf{G}_k = [\mathbf{g}_1, \dots, \mathbf{g}_K]$. Let us define $\Phi^{-1} = (\mathbf{G}_k \mathbf{G}_k^H + \frac{\sigma^2}{p_k} \mathbf{I})$, $\mathbf{c} = -\frac{\lambda+1}{\lambda} \mathbf{g}_k$ and $\mathbf{d}^H = \mathbf{g}_k^H$. Applying the matrix determinant lemma, i.e., $\det(\Phi^{-1} + \mathbf{c} \mathbf{d}^H) = (1 + \mathbf{d}^H \Phi \mathbf{c}) \det(\Phi^{-1})$, φ_k is defined as

$$\varphi_k = \lambda_k = \frac{1}{1 - \mathbf{g}_k^H (\mathbf{G}_k \mathbf{G}_k^H + \frac{\sigma^2}{p_k} \mathbf{I})^{-1} \mathbf{g}_k} - 1. \quad (4)$$

According to [7], the power of the interference plus noise can be estimated from the power of the leakage plus noise, i.e., $\sum_{i \neq k} p_i \|\mathbf{g}_k^H \mathbf{w}_i\|^2 + \sigma^2 \approx \sum_{i \neq k} p_i \|\mathbf{g}_i^H \mathbf{w}_k\|^2 + \sigma^2$. This estimation is generally tight when the leakage power is small compared with the noise power. Moreover, as $K < N$, the leakage power converges to zero at low and high SNR regions [7, Eq. (19)]. Hence, the signal-to-interference-plus-noise ratio (SINR) of the k th user can be approximated by the corresponding SLNR.

Manuscript received April 1, 2015; revised August 26, 2015; accepted August 27, 2015. Date of publication September 2, 2015; date of current version November 9, 2015. The associate editor coordinating the review of this paper and approving it for publication was Q. Du.

The authors are with the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798 (e-mail: xuantuon001@ntu.edu.sg; ekcteh@ntu.edu.sg).

Digital Object Identifier 10.1109/LCOMM.2015.2475757

III. ERGODIC ACHIEVABLE CAPACITY

From (4) the ergodic capacity of the k th user is defined as

$$R_k = \mathbb{E} \left\{ \log_2 \left(\frac{1}{\left[\left(\frac{p_k}{\sigma^2} \mathbf{G}_k^H \mathbf{G}_k + \mathbf{I} \right)^{-1} \right]_{kk}} \right) \right\}. \quad (5)$$

The lower bound (LB) on the ergodic capacity for the k th user can be obtained by applying Jensen's inequality

$$R_k \geq \log_2 \left(1 + (\mathbb{E}\{1/\alpha_k\})^{-1} \right), \quad (6)$$

where $\alpha_k = \frac{1}{\left[\left(\frac{p_k}{\sigma^2} \mathbf{G}_k^H \mathbf{G}_k + \mathbf{I} \right)^{-1} \right]_{kk}} - 1$. Following [8], the probability density function (pdf) of α_k is approximated by a Gamma function, i.e., $\alpha_k \sim \Gamma(\psi_k, \vartheta_k)$ with shape parameter $\psi_k = \frac{(N-K+1+(K-1)\mu)^2}{N-K+1+(K-1)\theta}$ and scale parameter $\vartheta_k = \frac{N-K+1+(K-1)\theta}{N-K+1+(K-1)\mu} p_k \beta_k$, respectively. Therefore, the LB on the ergodic capacity can be approximated as

$$\hat{R}_k \approx \log_2 (1 + (\psi_k - 1)\vartheta_k). \quad (7)$$

The values of parameters μ and θ are defined by solving the following equations

$$\begin{cases} \mu &= \frac{1}{K-1} \sum_{i=1, i \neq k}^K \frac{1}{\phi_k} \\ \theta \left(1 + \sum_{i=1, i \neq k}^K \frac{p_k \beta_i}{\phi_k^2} \right) &= \frac{1}{K-1} \sum_{i=1, i \neq k}^K \frac{m_k}{\phi_k^2}, \end{cases} \quad (8)$$

where $\phi_k = N p_k \beta_i \left(1 - \frac{K-1}{N} + \frac{K-1}{N} \mu \right) + 1$ and $m_k = p_k \beta_i \mu (K-1) + 1$, respectively.

In addition, (4) can also be rewritten as [7], [9]

$$\varphi_k = \frac{p_k}{\sigma^2} \mathbf{g}_k^H \mathbf{\Upsilon}_k \mathbf{g}_k + \mathbf{g}_k^H \mathbf{\Xi}_k \mathbf{g}_k, \quad (9)$$

where $\mathbf{\Upsilon}_k = \mathbf{I} - \bar{\mathbf{G}}_k (\bar{\mathbf{G}}_k^H \bar{\mathbf{G}}_k)^{-1} \bar{\mathbf{G}}_k^H$ and $\mathbf{\Xi}_k = \bar{\mathbf{G}}_k (\bar{\mathbf{G}}_k^H \bar{\mathbf{G}}_k + \frac{\sigma^2}{p_k} \mathbf{I})^{-1} (\bar{\mathbf{G}}_k^H \bar{\mathbf{G}}_k)^{-1} \bar{\mathbf{G}}_k^H$. Following [9], we have $\rho_{zf,k} = \frac{p_k}{\sigma^2} \mathbf{g}_k^H \mathbf{\Upsilon}_k \mathbf{g}_k = \frac{p_k/\sigma^2}{[(\mathbf{G}_k^H \mathbf{G}_k)^{-1}]_{kk}}$, hence $\mathbb{E}\{\rho_{zf,k}\} = \frac{p_k \beta_k}{\sigma^2} (N-K+1)$ due to $1/[(\mathbf{H}_k^H \mathbf{H}_k)^{-1}]_{kk} \sim \chi_{2(N-K+1)}^2$, where $\mathbf{H}_k = \{\mathbf{h}_1, \dots, \mathbf{h}_K\}$. We define $\rho_{ad,k} = \mathbf{g}_k^H \mathbf{\Xi}_k \mathbf{g}_k$; $\rho_{ad,k}$ and $\rho_{zf,k}$ are statistically independent, and $\rho_{ad,k}$ is a nondecreasing function of p_k/σ^2 [9]. We denote $\mathbf{q}_k = \mathbf{U}_k^H \mathbf{g}_k \in \mathbb{C}^{(K-1) \times 1}$, where $\mathbf{U}_k$ is obtained from the singular value decomposition of $\bar{\mathbf{G}}_k = \mathbf{U}_k \mathbf{\Lambda}_k \mathbf{V}_k^H$. We consider a matrix $\mathbf{Q}_k \in \mathbb{C}^{N \times (K-1)}$ that has the same distribution of $\bar{\mathbf{G}}_k$, and it is independent of $\mathbf{q}_k$. According to [9, Eq. (32)], at high SNR region, $\rho_{ad,k}$ has an identical distribution of $\|\mathbf{q}_k\|^2 [(\mathbf{Q}_k^H \mathbf{Q}_k)^{-1}]_{ii}$, hence $\mathbb{E}\{\rho_{ad,k}\} \simeq \mathbb{E}\{\|\mathbf{q}_k\|^2 [(\mathbf{Q}_k^H \mathbf{Q}_k)^{-1}]_{ii}\} = \frac{K-1}{N-K+1}$. Therefore, the UB on the capacity at high SNR region is obtained as

$$R_k \leq \hat{R}_k = \log_2 \left(1 + \frac{p_k \beta_k}{\sigma^2} (N-K+1) + v \right), \quad (10)$$

where $v = \frac{K-1}{N-K+1}$. In the case of EPA, since K and N grow large without bound, i.e., $N, K \rightarrow \infty$, but $\frac{N}{K} = x < \infty$ and $x \gg 1$, then the UB expression in (10) is converged to

$$\hat{R}_k \rightarrow \log_2 (1 + P \beta_k (x-1)). \quad (11)$$

This result is similar to the one which is obtained by using ZF-PS in [2]. Thus, the capacity of the SLNR-PS converges to that of the ZF-PS when SNR is high. Moreover, it is more convenient to obtain (10) than (7), hence the closed-form UB (10) is used for further analysis due to its low complexity.

IV. ENERGY EFFICIENCY OPTIMIZATION

We now study the optimal SE, optimal N , as well as optimal power allocation (OPA) for maximizing EE performance.

A. Optimum SE and Optimum N for Maximizing EE

We assume that the system has unit bandwidth, $p_k = P/K$ and $\sigma^2 = 1$. Thus, the system EE is given by [5], [6]

$$\eta = \frac{R_s}{\frac{1}{\omega} \left(\frac{P}{\gamma(1-\sigma_{feed})} + P_c + P_{sp} \right)}, \quad (12)$$

where $\omega = (1 - \sigma_{DC})(1 - \sigma_{MS})(1 - \sigma_{cool})$, γ is the power amplifier, and $R_s = \sum_k R_k$ is the SE. The parameters σ_{feed} , σ_{DC} , σ_{MS} , σ_{cool} are the loss factors of antenna feeder, direct current - direct current power supply, main power supply and active cooling system, respectively. P_{sp} is the power consumption for performing DSP. P_c is the circuit power consumption, i.e., $P_c = N(p_{dac} + p_{mix} + p_{filt}) + p_{syn}$, where p_{dac} , p_{mix} , p_{filt} , p_{syn} denote the power consumptions from a digital-to-analog converter, a mixer, a filter, and a frequency synthesizer, respectively. Note that P can be defined by obtaining the inverse function of R_s . Based on (10), the solution of $f^{-1}(R_s)$ can be obtained as

$$P = f^{-1}(R_s) = \frac{K 2^{R_s/K} - K(1+v)}{\beta_k(N-K+1)}. \quad (13)$$

Substituting (13) into (12), the trade-off function between the SE and EE can be rewritten as

$$\eta = \frac{R_s b}{K 2^{R_s/K} - K(1+v) + a}, \quad (14)$$

where $a = (P_c + P_{sp})\gamma\beta_k(1 - \sigma_{feed})(N-K+1)$ and $b = \omega\gamma\beta_k(1 - \sigma_{feed})(N-K+1)$. Since $\frac{\partial^2 \eta(R_s)}{\partial R_s^2} < 0$, the system EE in (14) is a concave function. By letting $\frac{\partial \eta(R_s)}{\partial R_s} = 0$, the optimum value of SE for maximizing the EE is obtained as

$$R_s^{opt} = \frac{K}{\ln 2} \left(1 + \Psi \left(\frac{a/K - v - 1}{e} \right) \right), \quad (15)$$

where $\Psi(\cdot)$ is the Lambert function, i.e., $\bar{x} = \Psi(\bar{x})e^{\Psi(\bar{x})}$, and e is Euler's number.

For a MIMO system with a specific R_s requirement, we can obtain the optimal number of BS antennas for maximizing EE. Let us define $t = N - K + 1$, (14) can be rewritten as

$$\eta = \frac{R_s c_1 t^2}{t^3 c_2 c_4 + t^2 c_4 c_6 + c_5 t - K(K-1)}, \quad (16)$$

where $c_1 = \omega\gamma(1 - \sigma_{feed})\beta_k$, $c_2 = p_{dac} + p_{mix} + p_{filt}$, $c_3 = p_{syn} + P_{sp}$, $c_4 = \gamma(1 - \sigma_{feed})\beta_k$, $c_5 = K(2^{R_s/K} - 1)$, and $c_6 = (K - 1)c_2 + c_3$. By letting $\frac{\partial \eta}{\partial t} = 0$, we have

$$c_2 c_4 t^3 - c_5 t + 2K(K - 1) = 0. \quad (17)$$

The solutions of (17) are defined as $t_j = -\frac{1}{3c_2 c_4} \left(u_j \bar{Z} + \frac{\delta_0}{u_j \bar{Z}} \right)$ with $j \in \{1, 2, 3\}$, where $u_1 = 1$, $u_2 = \frac{-1 + i\sqrt{3}}{2}$, $u_3 = \frac{-1 - i\sqrt{3}}{2}$, and $\bar{Z} = \sqrt[3]{(\delta_1 + \sqrt{\delta_1^2 - 4\delta_0^3})/2}$ with $\delta_0 = 3c_2 c_4 c_5$ and $\delta_1 = 54(c_2 c_4)^2 K(K - 1)$. If t_j is real and $t_j > K - 1$, the optimal number of BS antennas is obtained as $N_{opt} = \min\{(\lceil t_j \rceil + K - 1)\}$, where $\lceil \cdot \rceil$ denotes the round number.

B. Optimal Power Allocation for Maximizing EE

The optimization problem for maximizing EE under both transmit power and QoS constraints is formulated as

$$(Q_1) \begin{cases} \max_{p_k} & \frac{\sum_k^K R_k(p_k)}{\xi \sum_k^K p_k + u} \\ \text{s.t. } (C_1) : R_k(p_k) \geq C_k^{min}, (C_2) : \sum_k^K p_k \leq P, \end{cases} \quad (18)$$

where $\xi = \frac{1}{\omega\gamma(1 - \sigma_{feed})}$ and $u = (P_c + P_{sp})/\omega$. The objective function Q_1 is a ratio of two functions of p_k . The result in (18) is a fractional programming (FP) problem which is nonconvex. By applying the properties of FP, the objective function is equivalent to $\sum_k^K R_k(p_k) - \eta^* (\xi \sum_k^K p_k + u)$, where η^* is the EE when p_k is equal to the optimal value p_k^* , i.e., $\eta^* = \frac{\sum_k^K R_k(p_k^*)}{\xi \sum_k^K p_k^* + u}$ [10]. Thus, (18) can be rewritten as

$$\max_{p_k} \sum_k^K R_k(p_k) - \eta^* \left(\xi \sum_k^K p_k + u \right), \text{ s.t. } C_1 \text{ and } C_2. \quad (19)$$

The optimization in (19) has been transformed into a convex optimization problem. The Lagrange function is obtained as

$$\mathcal{L}(p_k, \gamma, \tau) = \sum_k^K R_k(p_k) - \eta^* \left(\xi \sum_k^K p_k + u \right) + \sum_k^K \varrho_k (R_k(p_k) - C_k^{min}) - \tau \left(\sum_k^K p_k - P \right). \quad (20)$$

Based on the Karush-Kuhn-Tucker (KKT) conditions, the OPA for the k th user is obtained as $p_k^* = \max \left\{ 0, \frac{1 + \varrho_k}{(\tau + \xi \eta^*) \ln 2} - \frac{1 + v}{\beta_k (N - K + 1)} \right\}$, where ϱ and τ are Lagrange multipliers, respectively. They are updated by the gradient method which can be defined as $\varrho_k(i + 1) = \max\{0, \varrho_k(i) - \Delta \varrho_k(R_k - C_k^{min})\}$ and $\tau(i + 1) = \max\{0, \tau(i) - \Delta \tau(P - \sum_k^K p_k)\}$, where $\Delta \varrho_k$ and $\Delta \tau$ are the positive iteration steps.

We consider the rate optimization problem for individual users based on C_1 and C_2 , and the EE target η_t , i.e., $(C_3) : \sum_k^K R_k \geq \eta_t (\xi \sum_k^K p_k + u)$. Let us define the achievable rate region $\mathcal{R}$ as a set which composes of rate-tuples for K users, i.e., $\mathcal{R} = \{\mathbf{r} \in \mathbb{R}_+^K | r_k = R_k(p_k) \text{ with } C_1, C_2, \text{ and } C_3\}$ [11]. We need to obtain the Pareto boundary (PB) of $\mathcal{R}$, at which it is impossible to improve a user's rate without degrading others' rate. Any rate-tuple on the PB of $\mathcal{R}$ can be achieved as a solution of the rate profile optimization problem, i.e.,

$$\max_{\{p_k\} \in \mathcal{R}_\Sigma} R_\Sigma, \text{ s.t. } R_k(p_k) \geq z_k R_\Sigma, C_1, C_2, \text{ and } C_3, \forall k, \quad (21)$$

Fig. 1. Capacity bounds versus SNR for both cases of EPA and OPA.

Fig. 2. Capacity bounds versus N for both cases of EPA and OPA when SNR = 5 dB.

where z_k is the target ratio between the achievable rate of user k and the users' sum rate [11]. For a given rate-profile vector $\mathbf{z}$ with $\sum_k^K z_k = 1$, the optimal solution of (21) R_Σ^* , can be defined by applying bisection method, hence the corresponding Pareto optimal rate-tuple for user k is obtained as $z_k R_\Sigma^*$.

V. NUMERICAL RESULTS AND DISCUSSIONS

The active users are assumed to be uniformly distributed in the cell with radius $R = 800$ m, the large-scale fading is $\beta_k = \frac{u_k}{(r_k/r_h)^{3.8}}$, where u_k is a log-normal random variable with standard deviation $\sigma_{sh} = 8$ dB, and $r_h = 100$ m [2]. We set $K = 10$ users, $p_{dac} = 15.6$ mW, $p_{mix} = 30.3$ mW, $p_{filt} = 20$ mW, $p_{syn} = 50$ mW, $P_{sp} = 25$ mW, $\sigma_{DC} = 7.5\%$, $\sigma_{MS} = 9\%$, $\sigma_{cool} = 10\%$, and $\sigma_{feed} = -3$ dB, respectively [5], [6].

Firstly, we examine the accuracy of the derived capacity bounds of SLNR-PS for both cases of EPA and OPA. For the case of OPA, we use the well-known water-filling algorithm (WFA) to allocate power for each user while the QoS requirement is satisfied. We set $N = 30$ and $\sigma^2 = 1$. In Fig. 1, the SLNR-PS outperforms the ZF-PS. The LB on the capacity is very close to simulated results and the UB on the capacity is very tight to the simulation at high SNR region. This fact can be shown more evidently by observing Fig. 2 where N is increased. Clearly, all bounds are very tight when $N > 50$. These results validate our theoretical analysis. The WFA provides an improvement in capacity at low SNR region, but there is no difference in system performance between two cases of EPA and OPA at high SNR region or N is large.

In Fig. 3, we investigate the trade-offs between EE and SE, and between EE and N based on both the UB and the LB on the capacity via numerical results. The y-axis on the left and

Fig. 3. For the case of EPA and $\beta_k = 1$: a) EE versus SE with different N ; b) EE versus N with different R_s requirements.

Fig. 4. For both cases of EPA and OPA, $\beta_k \neq 1$: a) EE versus SE with different N ; b) EE versus N with different SNR.

x-axis on the bottom of Fig. 3, i.e., named Fig. 3(a), shows the EE versus the SE with different N , whereas the y-axis on the right and x-axis on the top of Fig. 3, i.e., named Fig. 3(b), shows the EE versus N with different R_s requirements. In this example, we set $\beta_k = 1$, then we have optimum values of SE, which is obtained from (15), equal to $R_s^{opt} = 19.56, 31.39$, and 44.77 bits/s/Hz corresponding to $N = 50, 100$, and 200 , respectively. We observe that the R_s^{opt} is increased as N increases from 50 to 200, but the maximum EE at R_s^{opt} is decreased because P_c is increased. On a different note, there are two operating points of SE for a given EE as shown in Fig. 3(a). However, the operating point corresponding to lower SE should be avoided. In Fig. 3(b), the optimal numbers of N for maximizing EE are equal to $N_{opt} = 83, 166$, and 233 corresponding to the given $R_s = 30, 50$, and 60 bits/s/Hz, respectively. The accuracy of the optimal values has been demonstrated in Fig. 3, respectively, where the maximum EEs achieved at the optimum values of SE and N are marked by a circle. Moreover, the maximum EEs at R_s^{opt} and N_{opt} obtained from the UB on the capacity are close to the corresponding ones which are obtained from the LB on the capacity.

Next, we consider the EE optimization problem (19) with the effect of β_k in Fig. 4. Similar to Fig. 3, we have Fig. 4(a) and 4(b) which compare the EE of the proposed algorithm with that of EPA. At low SNR region or when N is not large (such as $N = 50$ in Fig. 4(a)), the EE is substantially improved by applying the OPA algorithm. For example, as we set $C_k^{min} = 1$ bit/s/Hz and SNR = 0 dB, in Fig. 4(b), the system with OPA has an 13.7% improvement of the EE at N_{opt} as compared to that of the EPA. Moreover, the optimum values of R_s^{opt} and N_{opt} for the case of OPA are closed to the corresponding values for the case of EPA as N is large or at high SNR region. Observing

Fig. 5. Plot of three-user rate region at SNR = 10 dB.

Figs. 3 and 4, the effect of β_k significantly decreases the system EE and SE performance.

Finally, by solving optimization problem in (21) with different rate-profile vector $\mathbf{z}$, the complete Pareto boundary of $\mathcal{R}$ is illustrated in Fig. 5 for $K = 3$ users when $N = 200$, $C_k^{min} = 1$ bit/s/Hz, SNR = 10 dB, and $\eta_t = 1$ bit/Joule. The boundary looks like a box with rounded edges, and the colour bar show the individual user rate. For example, for a given $\mathbf{z} = \{0.263; 0.146; 0.591\}$, we have $R_\Sigma^* = 11.18$ bits/s/Hz, then Pareto optimal per-user rates are $R_1 = 2.94$, $R_2 = 1.63$ and $R_3 = 6.61$ bits/s/Hz, respectively.

VI. CONCLUSION

The capacity bounds of the SLNR-PS and the trade-off between SE and EE have been derived and validated by numerical results when N is large. The closed-form expressions of the optimal SE and optimal N corresponding to maximum EEs have also been derived based on this trade-off for the case of EPA. By applying the energy-efficient power allocation, our results have shown that the SLNR-PS with OPA can improve the system EE as compared to that of EPA at low SNR region.

REFERENCES

- [1] M. Sadek, A. Tarighat, and A. H. Sayed, "A leakage-based precoding scheme for downlink multi-user MIMO channels," *IEEE Trans. Wireless Commun.*, vol. 6, no. 5, pp. 1711–1721, May 2007.
- [2] F. Rusek *et al.*, "Scaling up MIMO: Opportunities and challenges with very large arrays," *IEEE Signal Process. Mag.*, vol. 30, no. 1, pp. 40–60, Jan. 2013.
- [3] H. Q. Ngo, E. G. Larsson, and T. L. Marzetta, "Energy and spectral efficiency of very large multiuser MIMO systems," *IEEE Trans. Commun.*, vol. 61, no. 4, pp. 1436–1449, Apr. 2013.
- [4] H. Yang and T. L. Marzetta, "Performance of conjugate and zero forcing beamforming in large-scale antenna system," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 2, pp. 172–179, Feb. 2013.
- [5] D. Ha, K. Lee, and J. Kang, "Energy efficiency analysis with circuit power consumption in massive MIMO systems," in *Proc. IEEE PIMRC*, London, U.K., Sep. 2013, pp. 938–942.
- [6] G. Auer, *et al.*, "How much energy is needed to run a wireless network?" *IEEE Trans. Wireless Commun.*, vol. 18, no. 5, pp. 40–49, Oct. 2011.
- [7] P. Patcharamaneepakorn, S. Armour, and A. Doufexi, "On the equivalence between SLNR and MMSE precoding schemes with single-antenna receivers," *IEEE Commun. Lett.*, vol. 16, no. 7, pp. 1034–1037, Jul. 2012.
- [8] P. Li, D. Paul, R. Narasimhan, and J. Cioffi, "On the distribution of SINR for the MMSE MIMO receiver and performance analysis," *IEEE Trans. Inf. Theory*, vol. 52, no. 1, pp. 271–286, Jan. 2006.
- [9] Y. Jiang, M. K. Varanasi, and J. Li, "Performance analysis of ZF and MMSE equalizers for MIMO systems: An in-depth study of the high-SNR regime," *IEEE Trans. Inf. Theory*, vol. 57, no. 4, pp. 2008–2026, Apr. 2011.
- [10] D. W. K. Ng, E. S. Lo, and R. Schober, "Energy-efficient resource allocation for secure OFDMA systems," *IEEE Trans. Veh. Technol.*, vol. 61, no. 6, pp. 2572–2585, Jul. 2012.
- [11] R. Mochaourab, P. Cao, and E. Jorswieck, "Alternating rate profile optimization in single stream MIMO interference channels," *IEEE Signal Process. Lett.*, vol. 21, no. 2, pp. 221–224, Feb. 2014.